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Methods of Analysis and Failure Predictions for Adhesively Bonded Joints of Uniform and Variable Bondline Thickness

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16. Abstract Adhesively bonded joints under tensile lap shear loading were analyzed using the finite element method and closed-form solutions. Predictions of the stress distribution and failure prediction are compared with experimental failure load data. Case studies were performed that addressed the finite element meshing strategies of adhesively bonded joints such as h- and p-methods, mesh density around the overlap regions, element types. Comparisons were made with available closed-form solutions. Titanium single lap joints were analyzed using a linear analysis and the effects of bondline thickness and fillet were investigated. Parametric studies showed that the maxima strength of the adhesively bonded single lap joint increased with decreasing adhesive thickness. The proposed use of varying adhesive layer thickness to reduce the stress singularity was investigated by profiling the adherends' thickness quadratically or linearly to reduce or eliminate the shear stress concentration at the ends. Preliminary analysis on the effect of variable thickness along the overlap direction showed that maximum stress occurred at the end of least thickness. Further analysis on profiling the adherend thickness to reduce the stresses at the ends is recommended. To support and validate the analysis, single lap joints were tested with uniform and variable bondline thickness. The specimen used titanium adherends and 3M's DP460 adhesive. Finally, nonlinear analyses of titanium single lap joints, taking into consideration the ductility of the adhesive were performed, and the predicted failure load was shown to be 10% less than the test failure load. The cause of this lower prediction is discussed, and future work is suggested.			
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EXECUTIVE SUMMARY

Adhesively bonded joints under tensile lap shear loading were analyzed using the finite element method and closed-form solutions. Predictions of the stress distribution and failure prediction are compared with experimental failure load data.

Case studies were performed that addressed the finite element meshing strategies of adhesively bonded joints such as h- and p- methods, mesh density around the overlap regions, element types. Comparisons were made with available closed-form solutions.

Titanium single lap joints were analyzed using a linear analysis and the effects of bondline thickness and fillet were investigated. Parametric studies showed that the maxima strength of the adhesively bonded single lap joint increased with decreasing adhesive thickness. The proposed use of varying adhesive layer thickness to reduce the stress singularity was investigated by profiling the adherends' thickness quadratically or linearly to reduce or eliminate the shear stress concentration at the ends. Preliminary analysis on the effect of variable thickness along the overlap direction showed that maximum stress occurred at the end of least thickness. Further analysis on profiling the adherend thickness to reduce the stresses at the ends is recommended.

To support and validate the analysis, single lap joints were tested with uniform and variable bondline thickness. The specimen used titanium adherends and 3M's DP460 adhesive.

Finally, nonlinear analyses of titanium single lap joints, taking into consideration the ductility of the adhesive were performed, and the predicted failure load was shown to be 10% less than the test failure load. The cause of this lower prediction is discussed, and future work is suggested.

1. INTRODUCTION AND FOCUS OF RESEARCH.

Adhesively bonded joints are widely used in the aerospace and automotive industries; evidence of interface failures have been observed in many cases. However, the failure mechanism is not well understood and considerable effort has been devoted to testing, theoretical prediction, and numerical analysis to effectively address this issue.

Adhesively bonded joints can provide an efficient method of joining that would be more extensively used if reliable methods of analysis and failure prediction were available. Improvements in the technology that controls bonding fabrication, such as reliable nondestructive inspection, would also advance the field. In this report, the focus is placed on methods of analysis and failure prediction and the effects of variations in the thickness of the adhesive bondline.

Well-established closed-form expressions, e.g., Volkersen [1] and Goland and Reissner [2], and adaptations thereof, were first used to obtain stress distributions in the adhesive bondlines of conventional single- and double lap configurations. The assumptions upon which such methods are based were reviewed, and predictions were evaluated by comparisons with linear finite element analyses that were developed for the identical configuration.

One recognized difficulty associated with strength prediction for joints involving variations in bondline thickness is addressed. Subsequently, both nonlinear adhesive characteristic and fracture mechanics methods were considered in an attempt to resolve this difficulty for a relatively ductile adhesive system.

2. LITERATURE REVIEW.

2.1 STRESS ANALYSIS.

Adhesively bonded joints were intended to transfer load from one adherend to another simply by a shearing mechanism alone. However, due to load eccentricities, transversely normal stress (also called peel stress) exists in the adherend and adhesive, which could be the major contributor that causes failure in composite adherends with typically low interlaminar strength. The analytical prediction of the stress distribution in lap joints has been studied by many researchers.

In 1938, Volkersen [1] first proposed a simple shear lag model for mechanical joints with many fasteners, and later on, this model was adopted for adhesively bonded lap joints with the assumption that the adherends are in tension and the adhesive is in shear only and both stresses are constant across the thickness. However, the Volkersen solution does not reflect the effects of the adherend bending and shear deformations, which are potentially significant for composite adherends with a low shear and transverse moduli and strength. In 1944, Goland and Reissner [2] (hereafter referred to as the GR solution) took into consideration the effects of the adherend bending and the peel stress, as well as the shear stress, in the adhesive layer in a single lap joint. Subsequent efforts by Oplinger [3] suggested the corrections to the GR solution by using a layered beam theory instead of the classical homogeneous beam model for single lap joints.

The corrections to the shear lag model, or Volkersen solution, include works by Hart-Smith [4 and 5] Tsai, Oplinger, and Morton [6]. Hart-Smith [4 and 5] modified the shear lag model to include the adhesive plasticity. Tsai, Oplinger, and Morton [6] (hereafter referred to as the TOM solution) provided a correction to the shear lag model with the assumption that the shear stress is linear through the adherends. All the models predict that maximum shear stress occurs at the free ends, which is not strictly possible for a free surface condition.

In many cases, an exact solution is not available due to the complex nature of the stress state, complicated geometry, and dissimilar bonded materials. Assumptions can be made to simplify the problem, such as neglecting the stress variation across the adhesive thickness and linear variation in shear stress through adherends.

With the development of modern computational tools, some numerical methods, e.g., the finite difference method used by H. Kim and K. Kedward [7] and the finite element method used by many researchers, e.g., Penado and Dropek [8] and Tessler, Dambach, and Oplinger [9], have been used to analyze adhesively bonded joints. Compared with finite difference methods, finite element methods can be more conveniently applied to almost any geometric shape under loading and is more versatile for the numerical simulation of adhesively bonded joints in general. Due to the relatively thin adhesive layer, finite element modeling, considering the element type and the mesh density used, has to be carefully designed to obtain a reasonable solution. Penado and Dropek [8] indicated several important issues to consider when applying finite element analysis (FEA). Tessler, Dambach, and Oplinger [9] presented an adaptive mesh refinement method in bonded joints and showed close approximation of the traction free conditions. Another significant advantage of FEA is the ability to account for the stress variation through the

thickness. In addition, the geometrical and material nonlinearities can be included, providing that the discretization, boundary conditions, and the loading are applied properly.

2.2 FAILURE PREDICTIONS.

Stress analysis is an important part of the design process. It is also important to predict fracture load and thereby enable safe design concepts. Inevitably, analysis and prediction in conjunction with carefully selected experimental work is necessary. An appropriate failure mechanism should be suggested to guide and better understand the design of adhesively bonded joints. From the literature, there are basically two approaches for predicting the strength of joints: strength of materials and fracture mechanics.

Strength of materials approaches typically suggest that when certain stress and strain components, or an equivalent stress component (e.g., Von Mises equivalent stress), surpass its allowable counterpart, dependent on the material property, failure will occur.

Using the strength of materials approach, closed-form solutions predict that adhesive joints show improved strength with increasing bondline thickness, this is observed by using average stresses along the centerline from linear FEA. However, this is contrary to experimental results that strength decreases with increasing bondline thickness for thick bondline (Tomblin, et al. [10]). Gleich, Tooren and Beukers [11] first pointed out that this apparent contradiction emanates from the wrong interpretation of the stresses from the FEA. The peak shear and peel stress occurring at the interface of the bonded region increase with increasing adhesive thickness, and average adhesive stresses are shown to decrease with increasing bondline thickness. Harris and Adams [12] used a nonlinear finite element technique to predict the mode of failure and failure load for several single lap joints with aluminum alloy adherends and found that different criteria are needed for different adhesive systems.

As for the fracture mechanics approach, researchers have investigated the use of a generalized stress-intensity factor, analogous to the stress-intensity factor in classical fracture mechanics, to predict fracture initiation for bonded joints. Sometimes this stress singularity approach is referred as the fracture mechanics approach with no initial crack. The intent of this approach is to successfully implement a predictive design tool.

Stress singularity order (some authors call this strength of the singularity) at bimaterial interfaces has been studied by many authors [13-19]. Based on numerical and elasticity solutions, Penado [15] studied the stress singularity orders at bimaterial interfaces and concluded that the direction of crack propagation in lap joints with fillets was different from the cases without fillets based on a maximum stress failure criterion. Groth [17] suggested a fracture initiation criterion at the interface corners for bonded structure; it was assumed that initiation of fracture occurs when the generalized stress-intensity factor reaches its critical value. Akisanya and Meng [19] investigated the validity and limitations of the fracture initiation criterion in a butt joint with a thin elastic-plastic adhesive layer between two elastic adherends. The plastic zone size was compared with the extent of the stress singularity to determine the condition for the fracture initiation criterion to be valid.

2.3 THE EFFECT OF VARIATIONS IN BONDLINE THICKNESS.

Using the fracture mechanics approach, it is implied that reducing the singularity factor would improve the strength of the joints. For example, Adams, et al. [20] proposed the use of varying adhesive layer thickness to reduce the stress singularity. This could be realized by profiling the adherends thickness quadratically or linearly to reduce or eliminate the shear stress concentration at the ends. Another way to improve the strength of the joints could be done by forming an adhesive fillet region around the free end of the adhesive-adherend interfaces, this effect was studied by Apalak and Davies [21], Adam and Harris [22], and Tsai and Morton [23]. It was found that by rounding the corner of the adhesive and the adherends, the strength of the joint was improved. Tsai and Morton [23] also stressed that apart from the fillet effect, nonlinear deformation also plays a part in the adhesive stress concentration.

In this report, the finite element modeling strategy is studied in detail in section 3. This strategy is applied to the linear analyses of aluminum and titanium lap joints with some parametric studies including the effects of bondline thickness and the fillet effect. Preliminary work on the effects of variable bondline thickness was also studied. Nonlinear analyses were performed for two different adhesive thicknesses (0.01 and 0.03 inch). The ANSYS general purpose finite element software was used to analyze adhesively bonded titanium joints with adhesive thickness varying from 0.0085 to 0.0225 inch. The results are compared with the experimental observations, and future work on the prediction of joint failure is suggested.

3. ANALYSIS APPROACHES AND CASE STUDIES.

3.1 STRESS DISTRIBUTION IN DOUBLE LAP JOINT CONFIGURATION.

3.1.1 Volkersen.

Volkersen [1] first proposed a simple shear lag model for load transfer from one adherend to another by a simple shearing mechanism alone. In the model, the adherends are assumed in tension and the adhesive is in shear only, and both are constant across the thickness. The important relationships are given by

$$\tau(x) = \frac{\lambda P}{4} \left[\frac{\cosh(\lambda x)}{\sinh(\lambda c)} - \frac{E_i t_i - 2 E_o t_o}{E_i t_i + 2 E_o t_o} \frac{\sinh(\lambda x)}{\cosh(\lambda c)} \right] \quad (3-1)$$

where

$$\lambda^2 = \frac{G_a}{t_a} \left(\frac{1}{E_o t_o} + \frac{2}{E_i t_i} \right)$$

In these expressions, the subscripts o , i , and a denote the respective components relative to the outer adherend, inner adherend, and adhesive. The parameter P denotes the loading applied at the end of the inner adherend, and the parameter c is half the length of the adhesive. The origin of the x coordinate is in the middle of the adhesive, see figure 3-1.

However, the Volkersen solution does not reflect the effect of the adherend bending and shear deformations, which are potentially significant for composite adherends with a low shear and transverse moduli and strength. The TOM solution [6] provided a correction to the shear lag model with the assumption that the shear stress is linear through the adherends. As a result, the λ^2 is replaced by

$$\beta^2 = \lambda^2 \left[1 + \frac{G_a}{t_a} \left(\frac{t_i}{6 G_i} + \frac{t_o}{3 G_o} \right) \right]^{-1} \quad (3-2)$$

3.1.2 Linear Finite Element Analysis.

In this section, a typical balanced double lap joint will be used for the case study. The overlap length is 1 inch, the thickness of the outer adherend, inner adherend, and adhesive are 0.05, 0.05 and 0.005 inch, respectively. Geometrical details are indicated in figure 3-1. The linear-elastic material properties are as follows:

Epoxy Adhesive: Young's modulus $E_a = 0.6$ msi, Poisson's ratio $\nu_a = 0.429$

Aluminum Adherend: $E = 10.4$ msi, $\nu = 0.33$

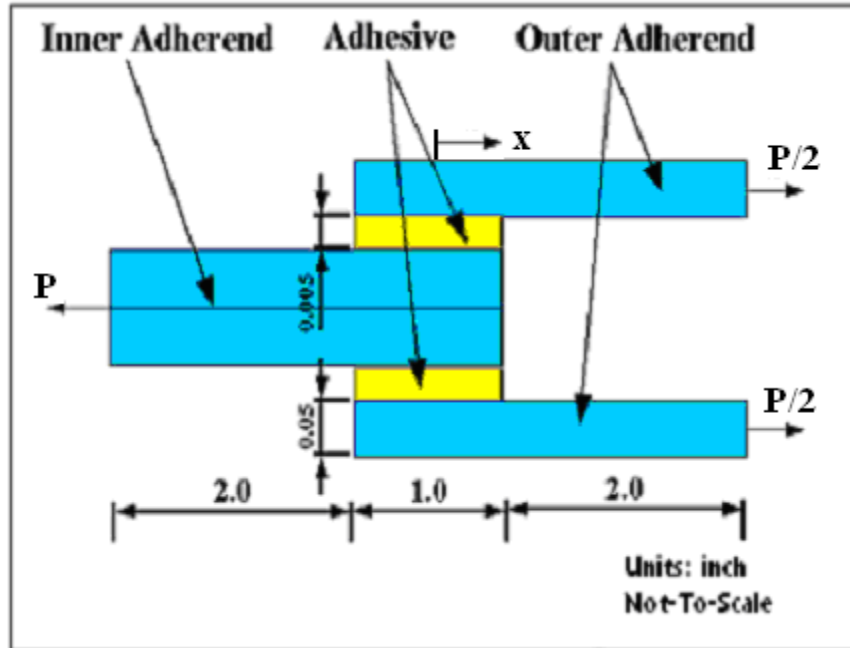


FIGURE 3-1. A TYPICAL DOUBLE LAP JOINT

The theory of stress analysis using the finite element method is well described in many finite element texts, e.g., Zienkiewicz and Taylor [24], and it will not be repeated here.

For this double lap joint, several different modeling schemes with regard to the element type and mesh techniques, assuming plane strain conditions, are discussed below.

3.1.2.1 h-Method.

Due to the symmetry of the loads and structure, only one-half of the joint was considered. Eight-node isoparametric elements were used to discretize the whole joint; figure 3-2 shows the finite element model where the eight-node isoparametric element (ANSYS plane82) is used. The mesh density is biased with a ratio of 12 near the ends of the adhesive due to the high stress concentration in that region. Details of this local region are shown in figure 3-3. There are eight elements and five elements through the thickness of adherends and adhesive, respectively. There are a total of 3,406 elements and 10,625 nodes, the number of degrees of freedom is 20,900. The mesh used here is considered to be suitably refined for the present evaluation.

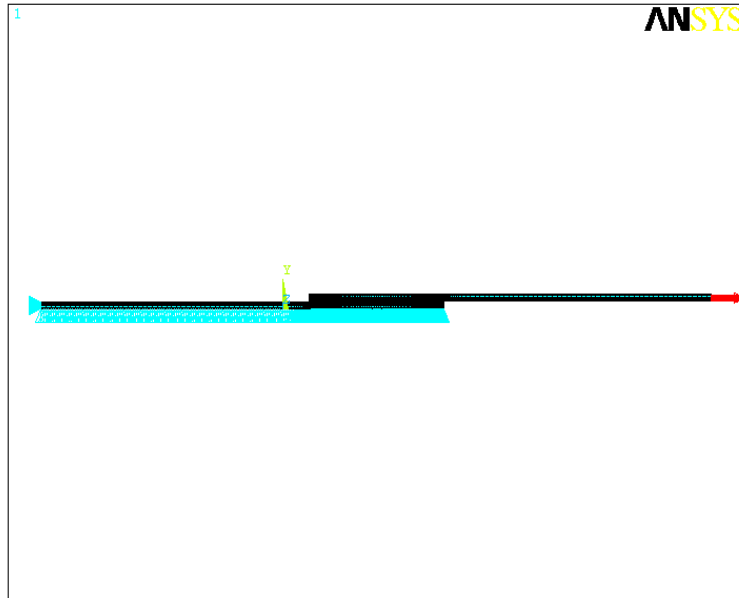


FIGURE 3-2. FINITE ELEMENT MODEL USING PLANE82

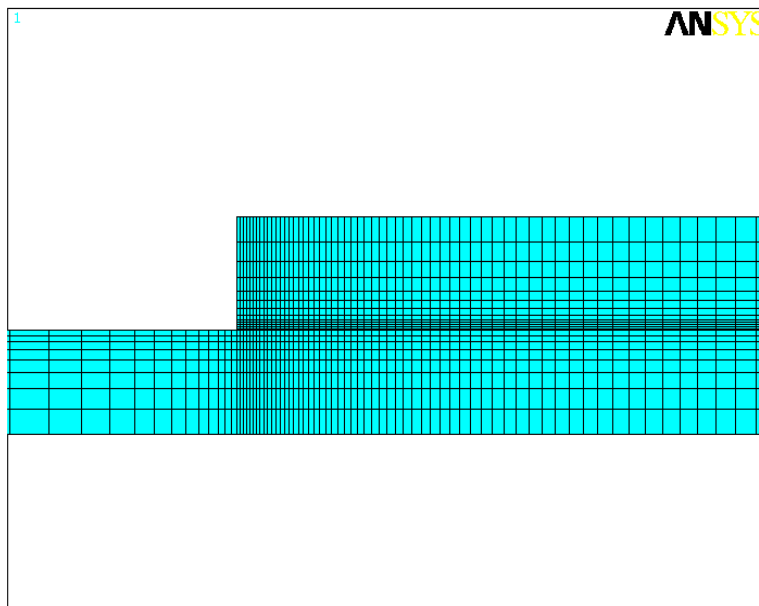


FIGURE 3-3. DETAIL MESHES AROUND THE TERMINATION REGION OF THE ADHESIVE

In ANSYS, the following loads and constraints are applied.

Displacement: $v = 0$ (along the bottom edge)

$u = 0$ (along the left edge)

Pressure: $p = -1000$ psi (at the right edge)

Analysis results show that adhesive peel stresses are the largest among all the stress components, and the shear and peel stresses of the adhesive are the most important components. Figures 3-4 and 3-5 portray the total peel and shear stress distribution, whereas figure 3-6 illustrates the stresses along the adhesive centerline. Shear stress contours for the local adhesive termination region using plane82 are shown in figure 3-7. These graphs show that maximum peel and shear stress along the centerline occur at a very small distance from the free edge. Due to the stress singularity, peak stresses occur at the joint corners, shown in figures 3-4 and 3-5. For the current meshing density, maximum peel stress and shear stress are 1.350 and 0.594 times the applied axial stress, respectively.

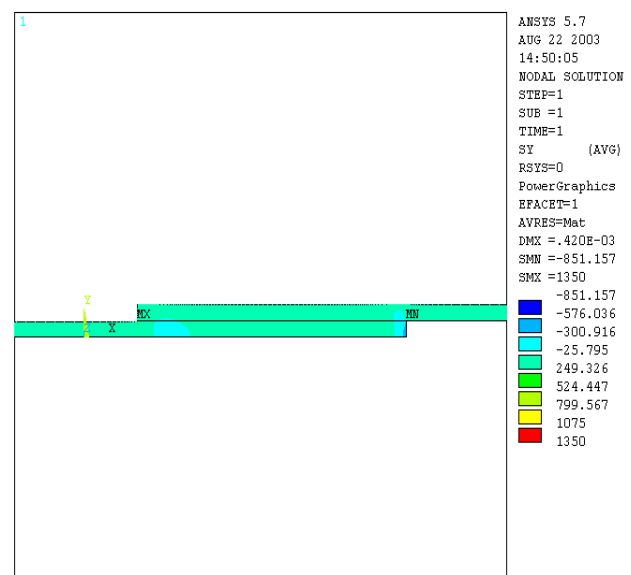


FIGURE 3-4. PEEL STRESS CONTOUR USING PLANE82

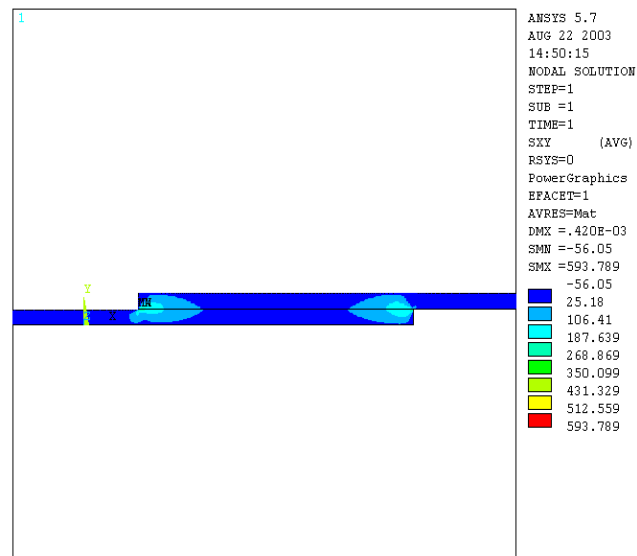


FIGURE 3-5. SHEAR STRESS CONTOUR USING PLANE82

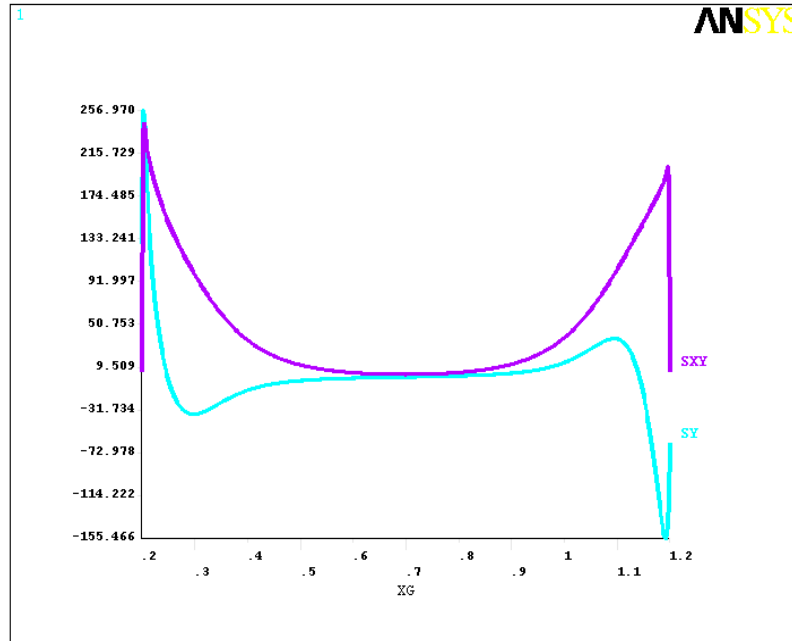


FIGURE 3-6. SHEAR AND PEEL STRESS DISTRIBUTION ALONG THE ADHESIVE CENTERLINE USING PLANE82

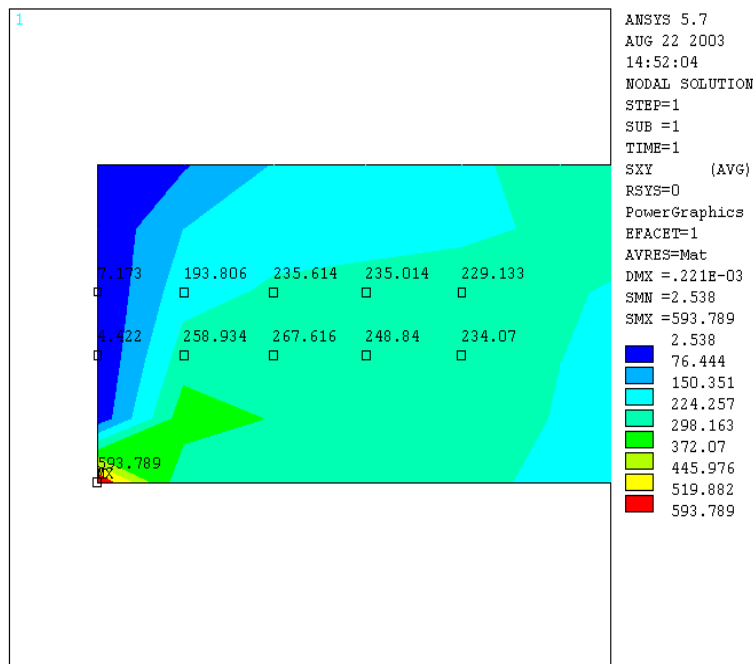
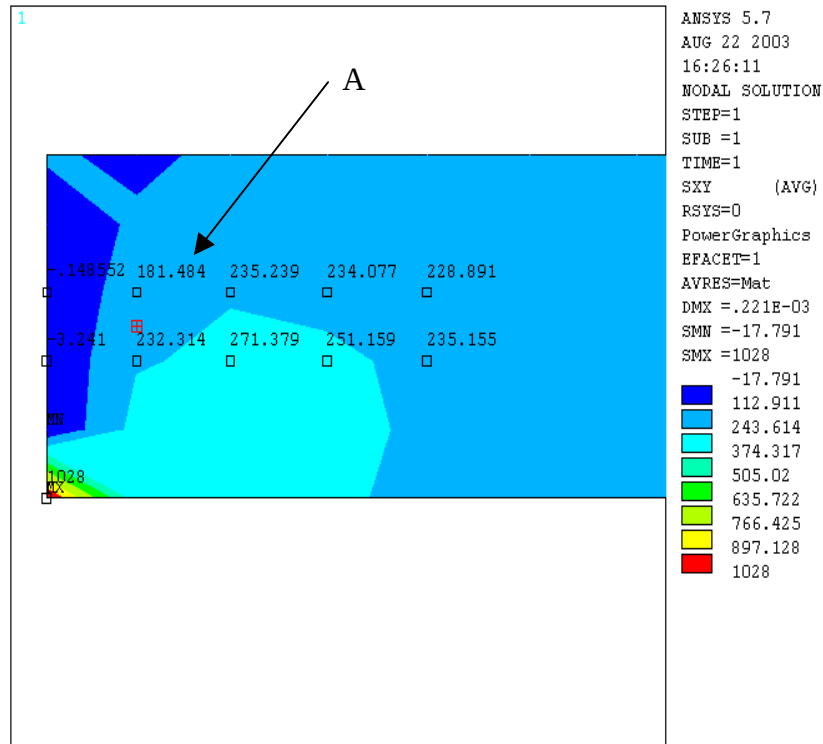


FIGURE 3-7. SHEAR STRESS CONTOURS (LOCAL ADHESIVE TERMINATION REGION) USING PLANE82

3.1.2.2 p-Method.

The p-method, in conjunction with the eight-node elements (plane145), is used in this section. The total number of the elements and nodes are the same as in the h-method. Local convergence criteria at specified locations in the model are used, and it is defined as the tolerance for convergence specifications as 1%, based on shear stress τ_{xy} at a distance of 0.0015 inch from the free edge along the adhesive centerline (point A in figure 3-8).

Peak stresses occurs at the joint corners, and peak peel stress and shear stress are 2.592 and 1.028 of the applied axial stress. The stress distribution along the adhesive centerline is shown in figure 3-9. Detail shear stress contours around the adhesive termination are shown in figure 3-8, where peak shear stress occurs at the left corner of the adhesive. Compared with conventional h-method, the advantage of using p-method includes the ability of adaptive refinement to obtain good results to a required accuracy, and the error estimate can be made locally and globally. The p-level, which refers to the polynomial level used at the local point A, is shown in figure 3-10.



Only the adhesive layer is shown here.

FIGURE 3-8. SHEAR STRESS CONTOURS (LOCAL ADHESIVE TERMINATION REGION) USING THE p-METHOD

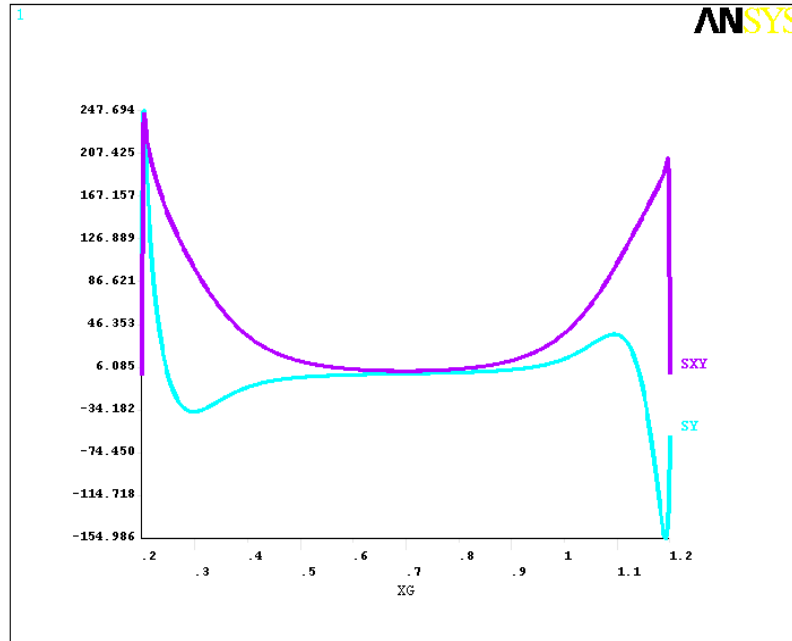


FIGURE 3-9. PEEL AND SHEAR STRESS DISTRIBUTION ALONG THE ADHESIVE CENTERLINE USING THE p-METHOD

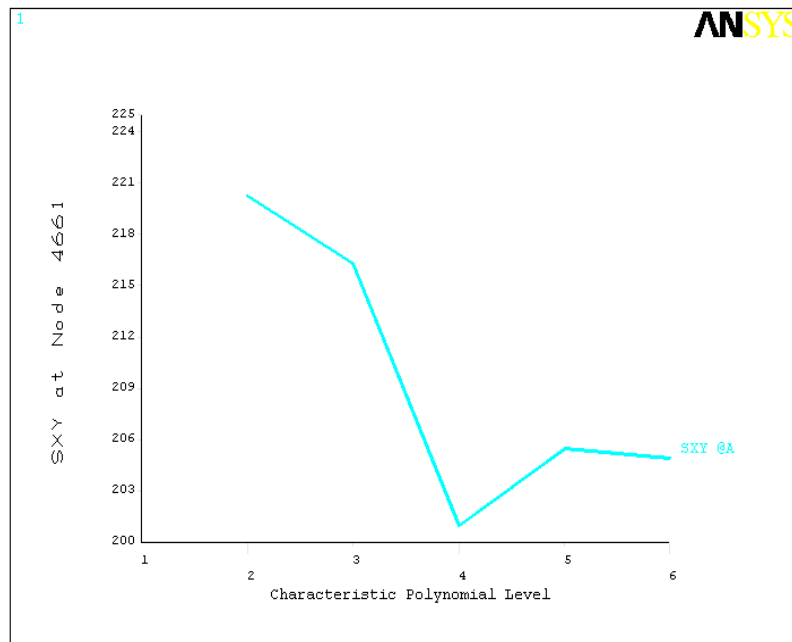


FIGURE 3-10. p-LEVEL USED AT POINT A (SHEAR STRESS VS POLYNOMIAL LEVEL)

3.1.2.3 Mixed Element Modeling Using Spring Elements for Adhesive and Eight-Node Elements for Adherends.

According to Loss and Kedward [25], the adhesive can be modeled by a pair of springs. In ANSYS, this can be realized by using eight-node elements plane82 for the adherend and a couple of spring elements for modeling the shearing and peeling behavior of the adhesive. The stiffness coefficients of the peel and shear springs can be calculated by equations 3-3 and 3-4:

$$k_{peel} = \frac{A_{el} (E_a)_{eff}}{t_a} \quad (3-3)$$

$$k_{shear} = \frac{A_{el} G_a}{t_a} \quad (3-4)$$

where

$$A_{el} = l \times b$$

E_a and ν_a are adhesive Young's modulus and Poisson's ratio

$l, b, t_a, G_a, (E_a)_{eff}$ are in-plane distance between nodes, width of the joint being modeled, adhesive thickness, elastic adhesive shear modulus, and effective elastic modulus of adhesive, respectively. For a triaxial stress state, $(E_a)_{eff}$ is defined below.

$$(E_a)_{eff} = \frac{E_a(1 - \nu_a)}{(1 - \nu_a - 2\nu_a^2)}$$

For uniform meshing along the overlap direction of the adhesive, stiffness coefficients for the peel and shear springs at the very ends of the overlap region is one-half the stiffness of other springs due to representation of one-half the element area.

3.1.3 Correlation Studies of Predicted Stress Distributions.

3.1.3.1 Comparison With Available Closed-Form Solution.

Figure 3-11 (a)-(f) shows the stress comparison of the spring element and plane eight-node element results along the centerline of the adhesive with available closed-form solutions.

The classical Volkersen solution overestimates the maximum shear stress, and the FEA results are closer to the TOM solution (shown in figure 3-11 (d)-(f)). This is because Volkersen assumed a one-dimensional model with only shear deformation in the adhesive layer, the effect of the adherend shear being ignored. The TOM solution corrected for adherend shear deformation by approximating a linear shear stress through the adherends and predicts that maximum shear stress of 272.6 psi occurs at the ends of the lap joint overlap. However, the maximum shear stress by using eight-node elements occurs at a small distance from the free edge. Near the left edge, the maximum shear stress is 245.8 psi, and near the right edge, it is

202.9 psi. Figure 3-11 (d)-(f) shows the comparison of peel stress in the adhesive by using the spring elements and plane82 elements. The peel stress prediction using spring elements is higher than that obtained by using plane82 elements, and maximum stress occurs at the free edge instead of at a small distance away from the free edge. Near the left end, the peel stress is in tension and near the right end, the peel stress is in compression.

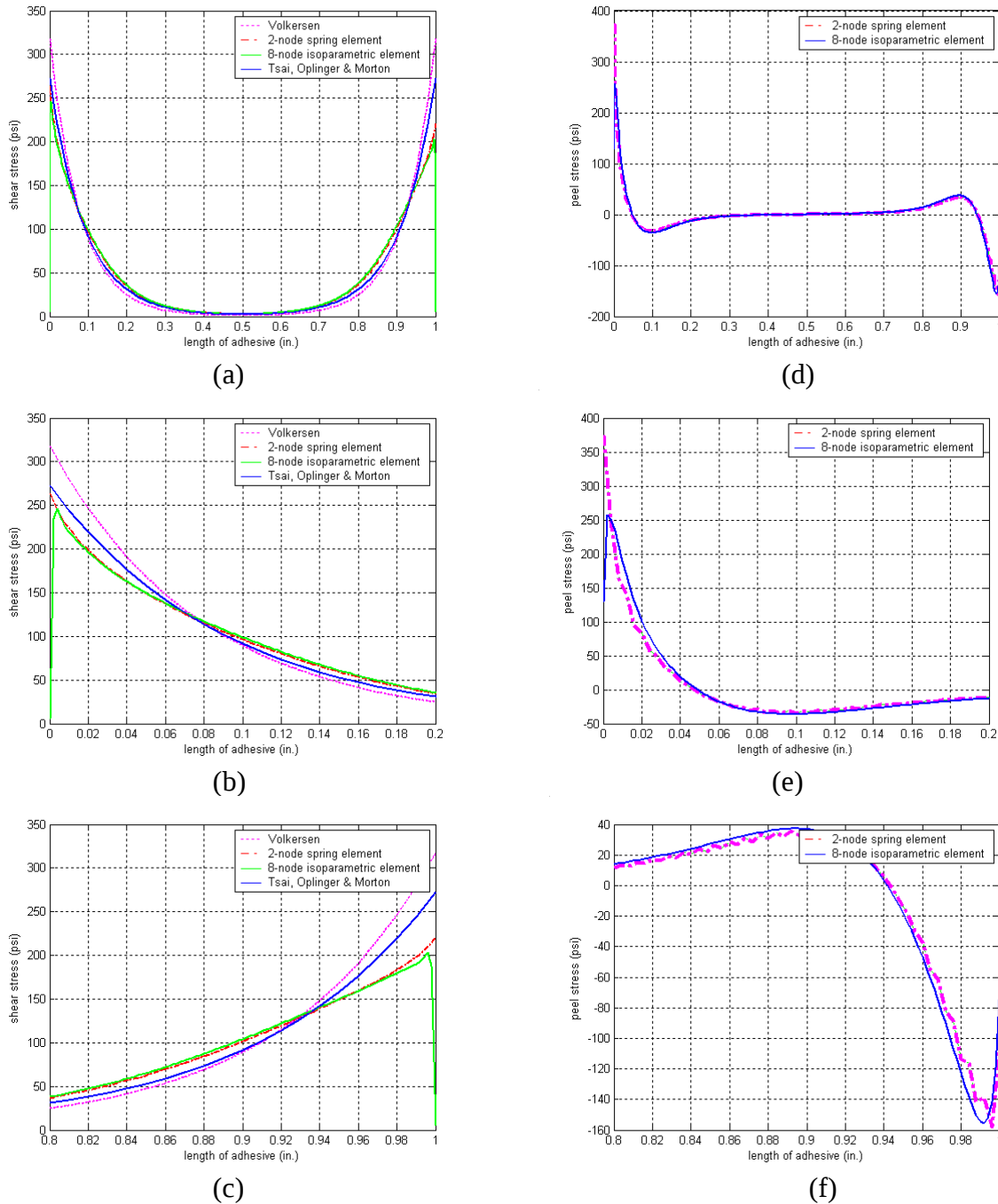


FIGURE 3-11. COMPARISONS OF SHEAR AND PEEL STRESSES ALONG THE CENTERLINE OF THE ADHESIVE

As far as the element type used, for the same meshing density, a high-order element is more accurate than lower-order element. For the eight-node element analysis, the stress varies within the element and the free surface condition is satisfied. For the mixed element modeling, the results are reasonable approximations noting that this is only a relatively coarse mesh (502 spring elements). A finer mesh will involve more manual work in element generation, and it is not convenient to implement. But this mixed element modeling could be very useful for evaluating closed-form predictions when the lap joint is simplified as a beam on an elastic foundation considering axial and transverse effects, and it will help understand the mechanism of the adhesive deformation. Furthermore, the idea of using spring elements to model the adhesive behavior can also be used in the cohesive zone models for investigating interface fracture [26-28].

3.1.3.2 Comparison Along Three Different Paths in the Adhesive Layer.

Figure 3-12 shows three different paths in the adhesive layer, where AB, CD, and EF are interface between the outer adherend and the adhesive, centerline of the adhesive, and interface between the inner adherend and the adhesive, respectively. Figure 3-13 (a)-(f) shows the stress comparison along these paths. Considerable stress variation exists near bondline terminations, where points A, B, C, and D are stress singularity points due to material and geometrical discontinuity. Apart from these regions, stresses can be considered to be similar along these three different paths.

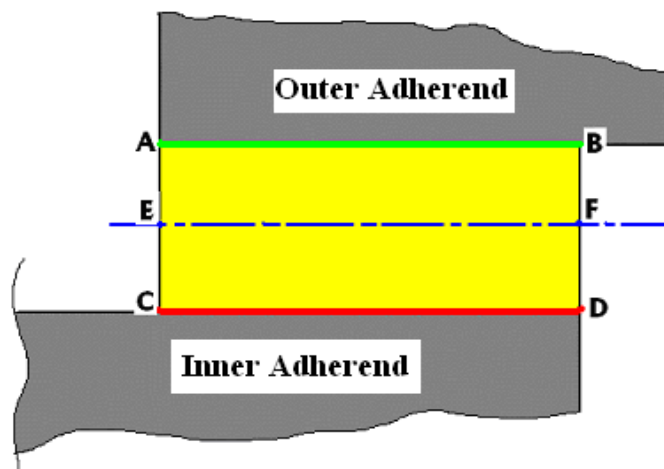
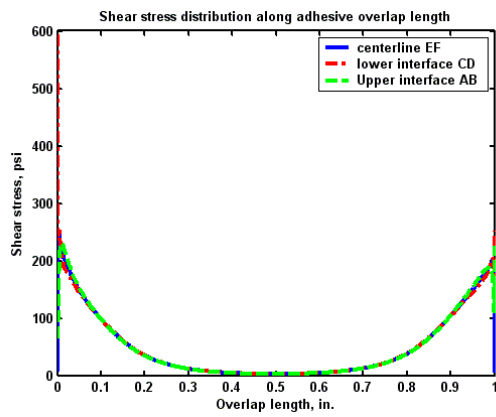
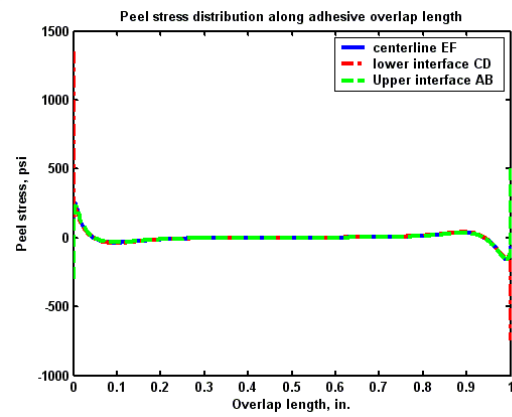


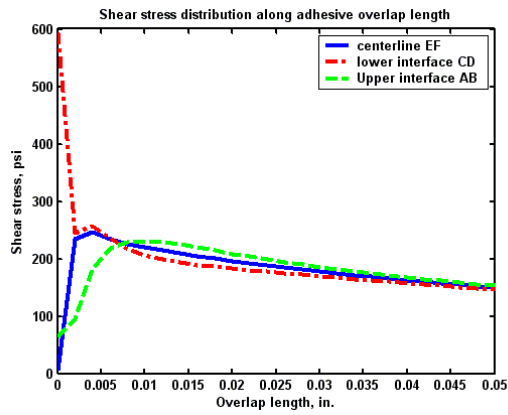
FIGURE 3-12. THREE DIFFERENT PATHS IN THE ADHESIVE LAYER
(AB, CD, AND EF)



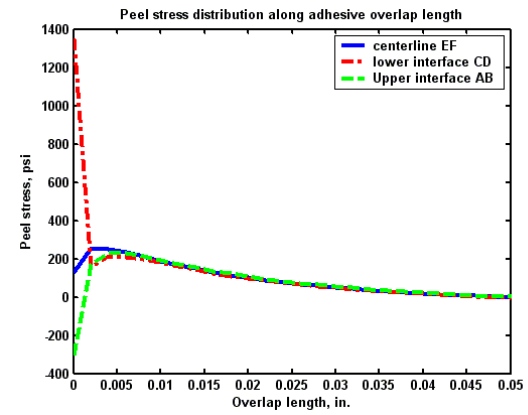
(a)



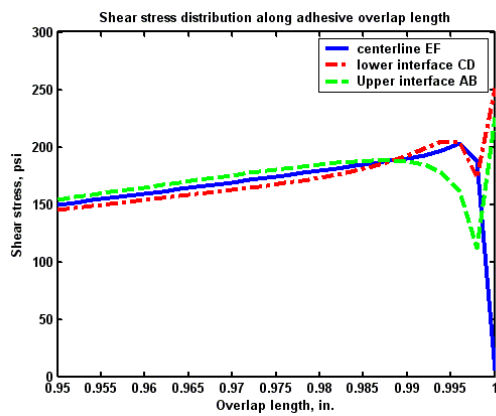
(d)



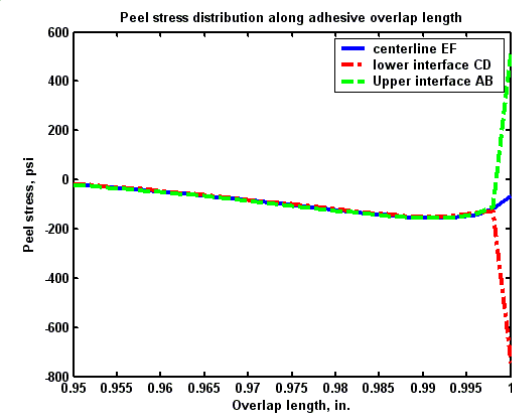
(b)



(e)



(c)



(f)

FIGURE 3-13. COMPARISONS OF SHEAR AND PEEL STRESSES ALONG THREE DIFFERENT PATHS (AB, CD, AND EF SHOWN IN FIGURE 3-12)

3.2 STRESS DISTRIBUTION IN SINGLE LAP JOINT CONFIGURATION.

3.2.1 Goland and Reissner.

The GR [2] solution is the most well known classical treatment of single lap joints. This model assumes that the transverse normal strain and shear strain in the adherends are negligibly small compared with those strains in the adhesive layer. The deformation of the adherends is due to the adherend bending effect, whereas the adhesive layer is analogous to a system of shear and peel springs positioned between the two adherends.

The following equations are predictions for shear and peel stress in single lap joints with relatively flexible adhesive layers:

$$\tau(x) = \frac{-pt}{4l} \left[\frac{\beta l}{2t} (1+3k) \frac{\cosh \frac{\beta(2x-l)}{2t}}{\sinh \frac{\beta l}{2t}} + 3(1-k) \right] \quad (3-5)$$

$$\sigma(x) = \frac{-8pt^2}{l^2 \Delta} \left[(R_2 \lambda^2 \frac{k}{2} + \lambda k' \cosh \lambda \cos \lambda) \cosh \lambda' \cos \lambda' + (R_1 \lambda^2 \frac{k}{2} + \lambda k' \sinh \lambda \sin \lambda) \sinh \lambda' \sin \lambda' \right]$$

where

$$\alpha^2 = \frac{E_a}{(1+\nu_a)t_a E t}, \beta^2 = \frac{4E_a t}{(1+\nu_a)E t_a}$$

$$\lambda = \frac{l}{2t} \sqrt[4]{\frac{6E_a t}{E t_a}}, \lambda' = \lambda \frac{2x-l}{l}$$

$$\Delta = \sinh 2\lambda + \sin 2\lambda$$

$$k = \frac{1}{1 + 2 \sqrt{2 \tanh \frac{l}{2t} \sqrt{\frac{3p(1-\nu^2)}{2E}}}}, k' = \frac{kl}{2t} \sqrt{\frac{3p(1-\nu^2)}{E}}$$

$$R_1 = \sinh \lambda \cos \lambda + \cosh \lambda \sin \lambda, R_2 = \sinh \lambda \cos \lambda - \cosh \lambda \sin \lambda$$

3.2.2 Linear Finite Element Analysis With Aluminum Adherends (ASTM D 1002-99).

A single lap joint (ASTM D 1002-99) is considered (shown in figure 3-14). The lap length is 0.5 inch, the thickness of the outer adherend, inner adherend, and adhesive are 0.064, 0.064, and 0.01 inch, respectively. The grip region is 1 inch at each end, and the total length of the joint is 7.5 inches.

The material for the adherend is aluminum, with Young's modulus $E = 10.4$ msi, and Poisson's ratio $\nu = 0.33$. The material properties of the adhesive are $E_a = 0.6$ msi and $\nu_a = 0.4$.

Testing of single lap joints shows that the joints could be loaded by in-line axial tensile grip loading or by a supported single lap configuration, as shown in figure 3-15. In FEA analysis, for in-line axial tensile grip loading, there are two loading steps. The first loading case is called

alignment which consists of a uniform displacement 0.074 inch along the transverse direction in one grip region. The second loading case is called tensile loading, which is loading of a uniform stress or displacement; for this linear elastic case, a uniform tensile stress of 1000 psi is applied. Since this is a linear elastic analysis, these two load cases are applied simultaneously. For supported single lap configuration or specimen with alignment tabs (figure 3-16), only the second load step is needed.

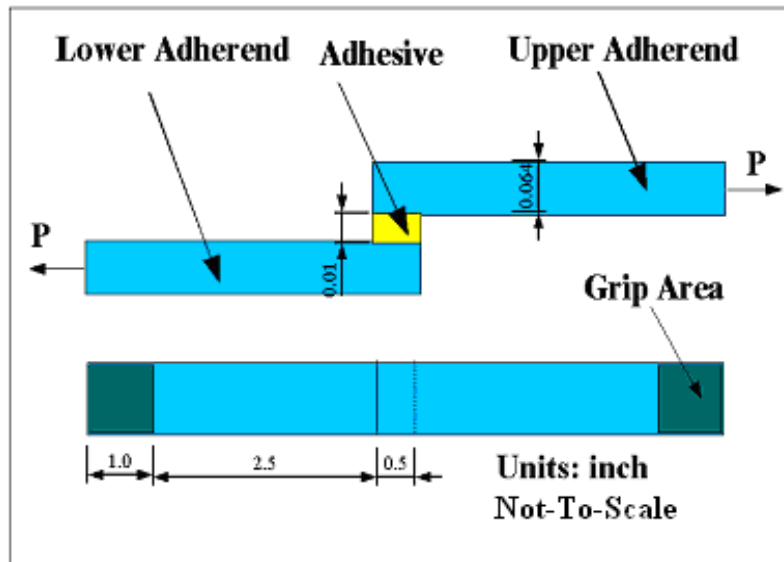


FIGURE 3-14. A SINGLE LAP JOINT (ASTM D 1002-99)

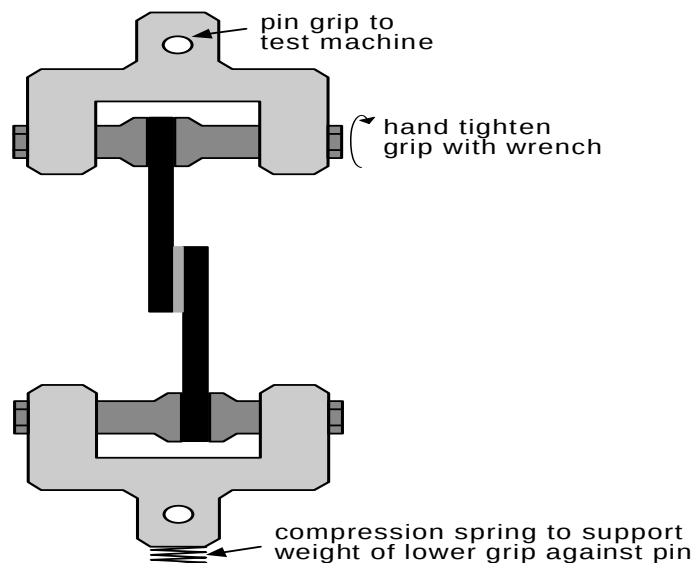


FIGURE 3-15. A SUPPORTED SINGLE LAP CONFIGURATION



FIGURE 3-16. A SINGLE LAP JOINT WITH ALIGNMENT TABS (FROM ADAM, COMYN, AND WAKE [29])

Stress analyses of aluminum and composite single lap joints show that stresses are a few percent larger for in-line axial tensile grip loading than for the supported single lap configuration loading. In the interest of conservatism, all the following analyses of single lap joints are loaded by in-line axial tensile grip.

Using the strategy in section 3.1 of the FEA modeling and analysis, the following finite element model for the single lap joint (figure 3-17) is used, and the results show center symmetric characteristic. Among all the stress components, peel stress has the highest value (shown in figures 3-18 and figure 3-19). Figure 3-20 shows the shear and peel stress distribution along the centerline of the adhesive. It can be seen that with a sufficiently fine mesh, the free surface condition is satisfied.

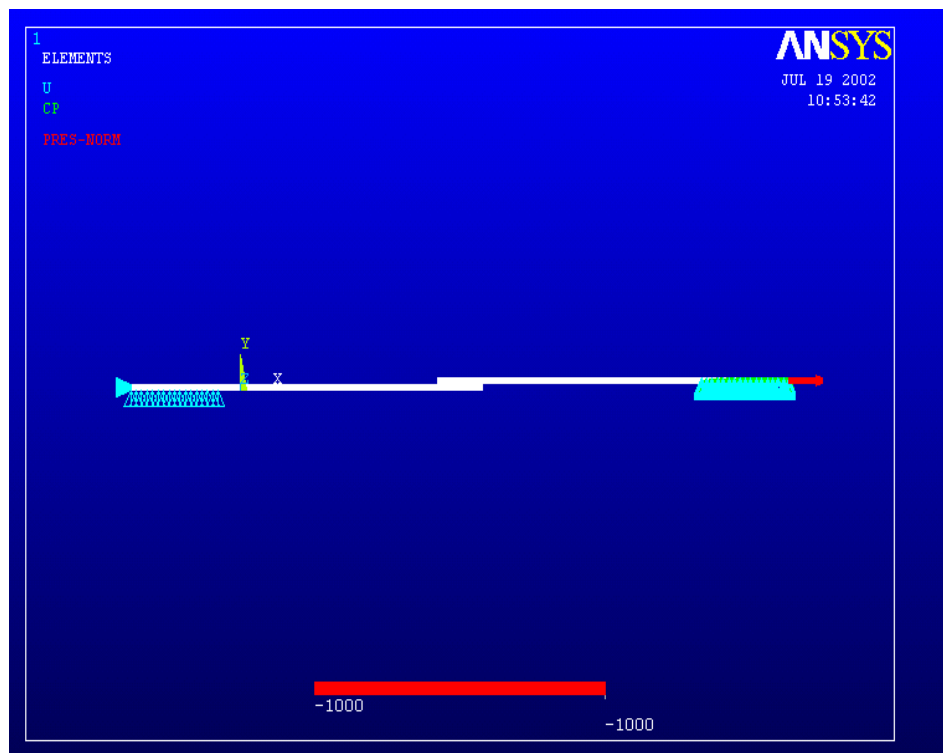


FIGURE 3-17. FINITE ELEMENT MODELS FOR ASTM D 1002-99 SINGLE LAP JOINT

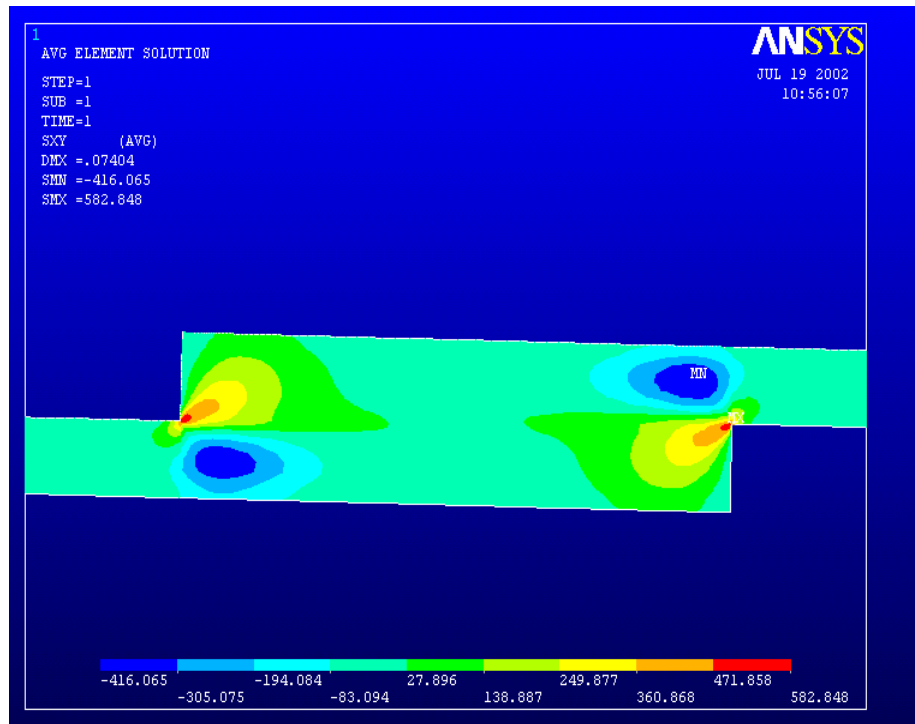


FIGURE 3-18. SHEAR STRESS CONTOURS OF ASTM D 1002-99 SINGLE LAP JOINT

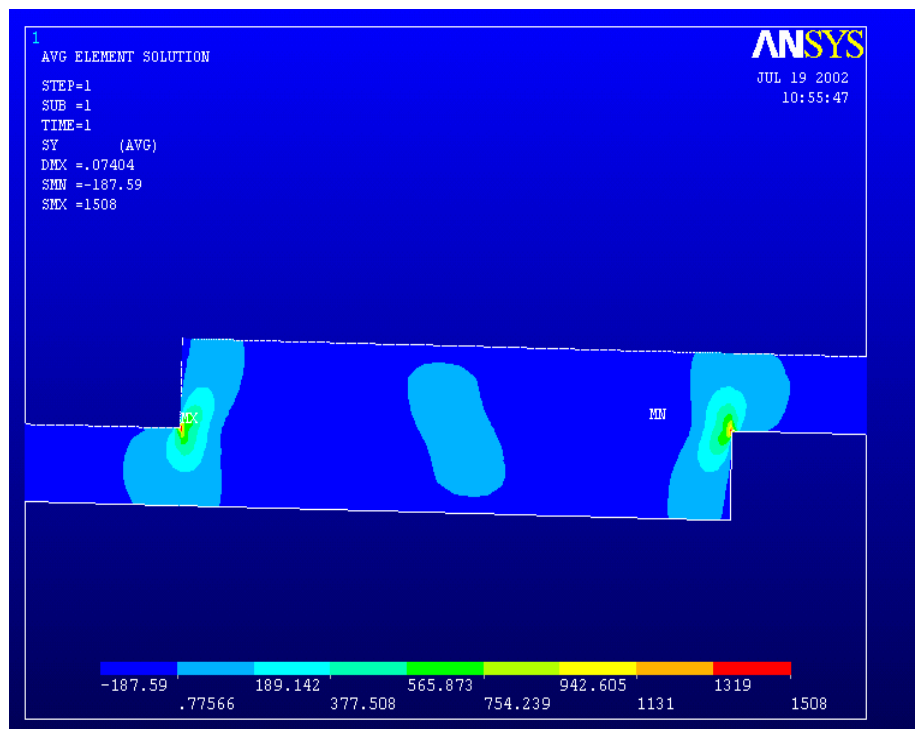


FIGURE 3-19. PEEL STRESS CONTOURS OF ASTM D 1002-99 SINGLE LAP JOINT

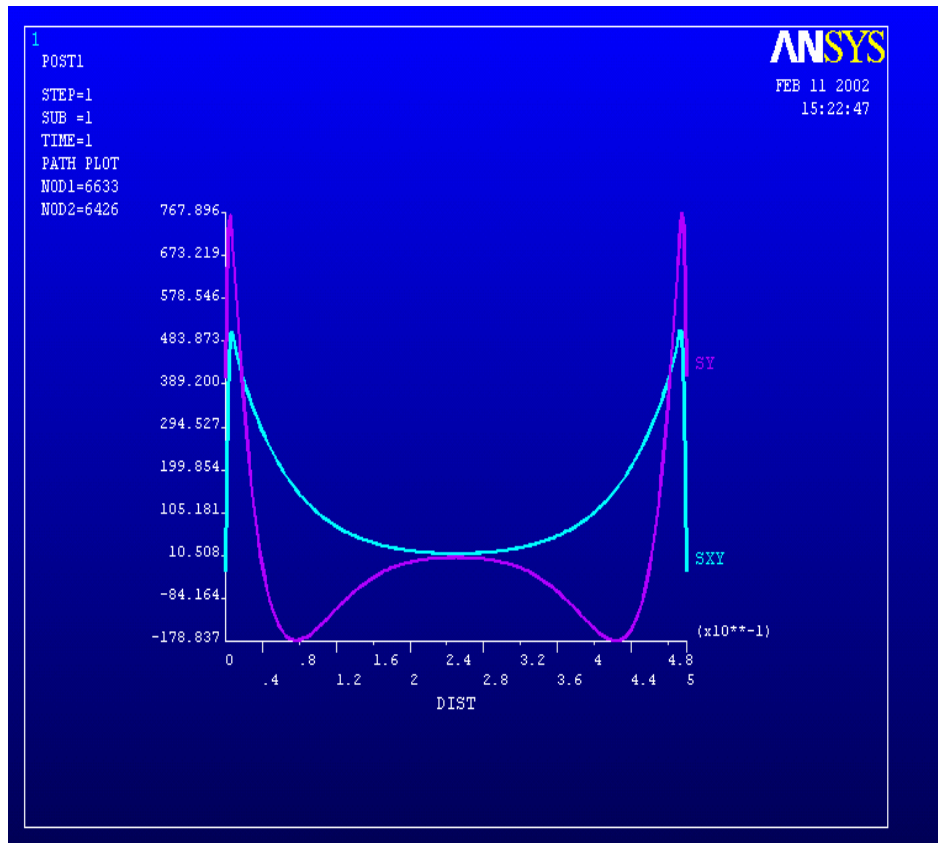


FIGURE 3-20. SHEAR AND PEEL STRESS DISTRIBUTION ALONG CENTERLINE OF THE ADHESIVE FOR SINGLE LAP JOINT

3.2.3 Comparisons Between Single Lap and Double Lap Predictions.

For comparison, a double lap joint with the same dimensions and materials, as specified in ASTM D 1002-99, was also analyzed. For this double lap joint, there was no grip alignment required, and only a tensile load of 1000 psi was applied at the end. There was also a change in boundary condition because of the symmetric constraints that are characteristic of a double lap joint (figure 3-21). Again, shear stress contours, peel stress contours and shear and peel stress distribution along centerline of the adhesive are shown in figures 3-22 to 3-24, respectively. The shear and peel stress along the centerline of the adhesive for double lap joint are less than half of those for single lap joint under the same uniform tensile loading. Also, note that there is no symmetry in the stress distribution along the adhesive centerline for a double lap joint, as shown in the case for a single lap joint. Comparison of FEA and available closed-form solutions are listed in tables 3-1 and 3-2 for two different sets of adhesive properties.

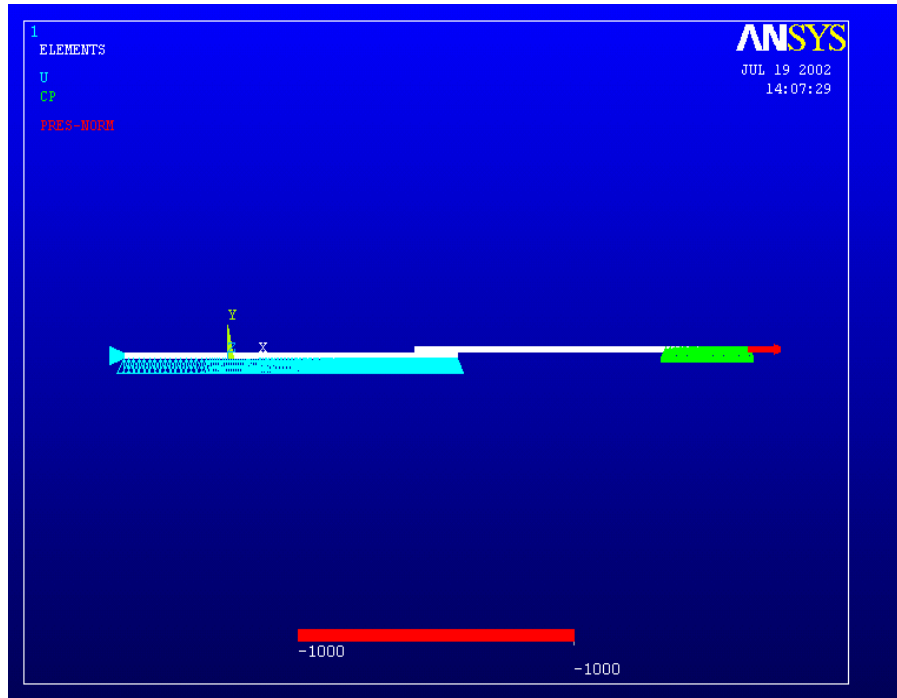


FIGURE 3-21. FINITE ELEMENT MODELS FOR THE ALUMINUM DOUBLE LAP JOINT
(Same dimensions and material as specified in ASTM D 1002-99 single lap joint.)

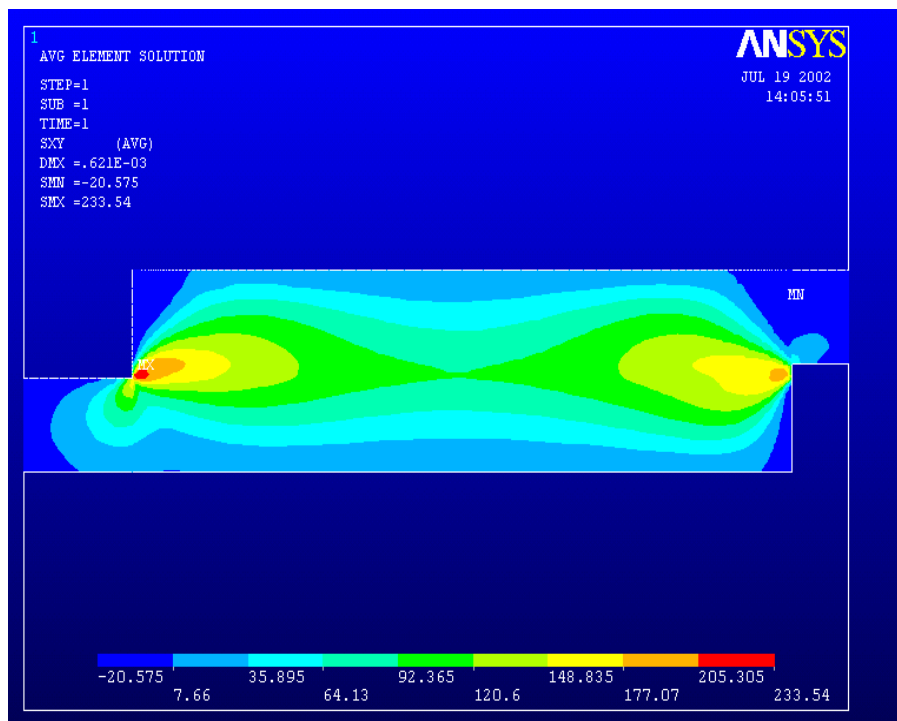


FIGURE 3-22. SHEAR STRESS CONTOURS OF THE ALUMINUM DOUBLE LAP JOINT

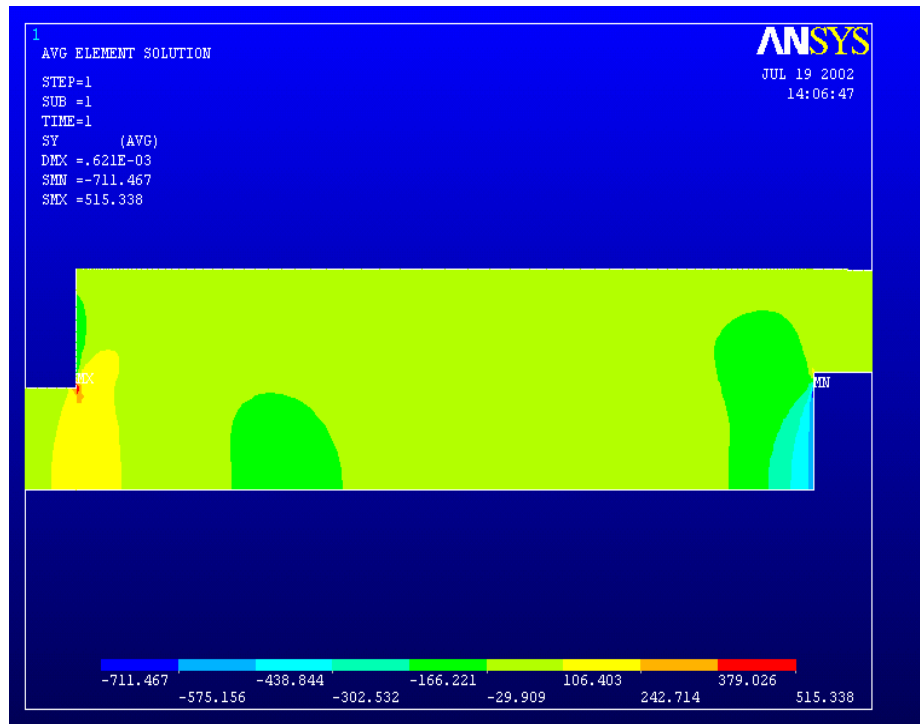


FIGURE 3-23. PEEL STRESS CONTOURS OF THE ALUMINUM DOUBLE LAP JOINT

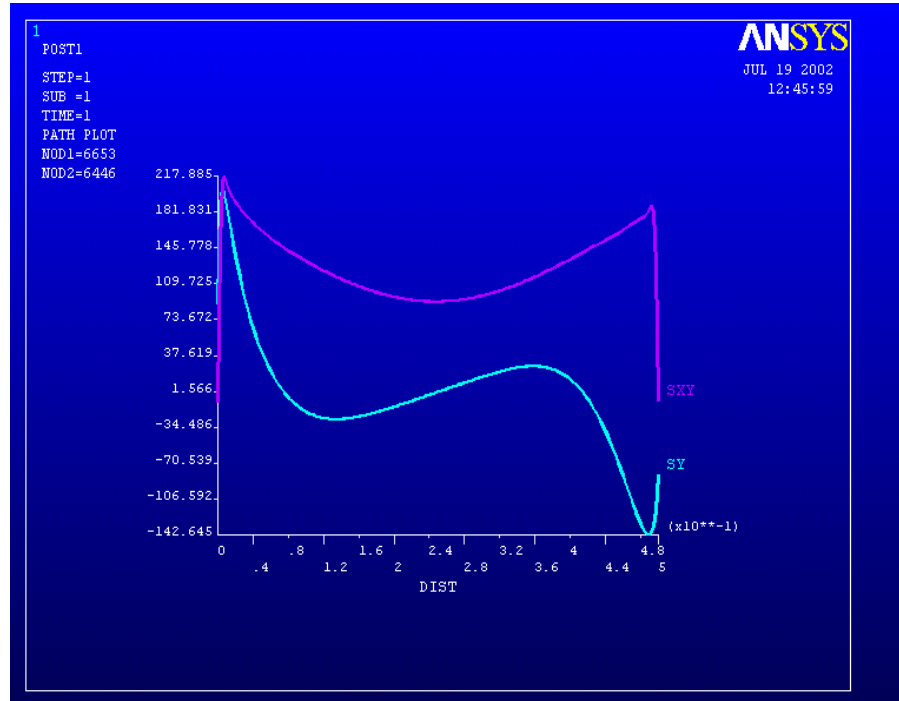


FIGURE 3-24. SHEAR AND PEEL STRESS DISTRIBUTION ALONG CENTERLINE OF THE ADHESIVE FOR THE ALUMINUM DOUBLE LAP JOINT

TABLE 3-1. PEAK PEEL AND SHEAR STRESS COMPARISON
(FOR ADHESIVE $E_a = 0.6$ MSI, $\nu_a = 0.4$)

	FEA		GR Solution		Volkersen	TOM Solution
	Peel	Shear	Peel	Shear	Shear	Shear
Single lap joint	749	498.8	678.8	481.7	NA	NA
Double lap joint	206.6	217.9	NA*	NA	266	244

* Notation: NA means not applicable or not available
Unit of measure = psi

TABLE 3-2. PEAK PEEL AND SHEAR STRESS COMPARISON
(FOR ADHESIVE $E_a = 0.312$ MSI, $\nu_a = 0.4$)

	FEA		GR Solution		Volkersen	TOM Solution
	Peel	Shear	Peel	Shear	Shear	Shear
Single lap joint	576.7	378.3	491	352.2	NA	NA
Double lap joint	154.6	184.2	NA	NA	206.9	199

* Notation: NA means not applicable or not available
Unit of measure = psi

The average shear stress $\bar{\tau}$ (defined as the applied load over the bonded area) is the same for the double lap and single lap configurations. In other words, the load supported by single lap joint is one-half that supported by the double lap joint. However, the peak stresses are higher for a single lap joint than that for a double lap joint, as shown in tables 3-1 and 3-2.

3.3 LINEAR AND NONLINEAR ANALYSIS OF TITANIUM SINGLE LAP JOINT.

A single lap joint with Titanium adherends with both linear and nonlinear adhesive properties is now considered. The adherend material is changed to titanium and the thickness is 0.035 inch (refer to figure 3-14).

The properties of the titanium adherends are: Young's modulus $E = 16.8$ msi, and Poisson's ratio $\nu = 0.34$. The material for the adhesive is DP460, with $E_a = 0.312$ msi and $\nu_a = 0.4$. The engineering stress-strain curve was supplied by A. Pocius from the 3M company [30].

3.3.1 Linear Analysis.

Stress singularity occurs at joint corners, and the peel and shear stresses in the adhesive around the joint corners can be expressed as

$$\sigma_{ij} = Q_{ij} r^{-\lambda} \quad (3-6)$$

where σ_{ij} are the stresses, Q_{ij} are the generalized stress-intensity factors, r is the distance from the point of singularity, and λ is the order of the stress singularity.

According to Bogy [13], the stress singularity orders (termed by some authors as strength of the singularity) for the titanium single lap joint corners can be obtained by solving for a complex equation. It turns out to be $\lambda_1=0.356$ and $\lambda_2=0.329$ for points 1 and 2 (shown in figure 3-25), respectively. The stronger singularity order occurs at the two points (the left lower corner and the right upper corner of the interface between the adherends and the adhesive, shown in figure 3-25), and these two points are believed to be the place where fracture is most likely to initiate.

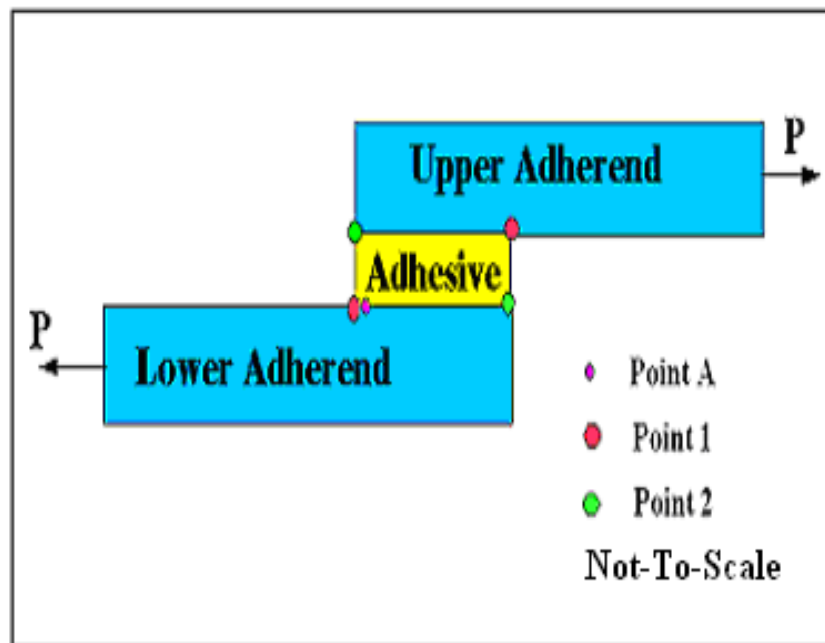


FIGURE 3-25. STRESS SINGULARITY POINTS IN A SINGLE LAP JOINT

3.3.1.1 Bondline Thickness Effect.

For linear analysis, p-method meshing strategy was used, and the convergence criterion was defined as 2% based on shear stress τ_{xy} at point A (0.0006 inch to the right of point 1 in figure 3-25). Some key results on the adhesive thickness effects are shown in tables 3-3 to 3-5.

- Along the centerline of the adhesive, maximum S_y , S_{xy} , S_1 (maximum principal stress) and $SEQV$ (Von Mises equivalent stress) are shown to decrease with increasing adhesive thickness (table 3-3).
- At the singularity point 1, S_y , S_{xy} , S_1 , and Von Mises equivalent stress increase with increasing adhesive thickness (table 3-4).
- At point A (figure 3-25), S_y , S_{xy} , S_1 , and Von Mises equivalent stress increase with increasing adhesive thickness (table 3-5).

TABLE 3-3. STRESSES ALONG CENTERLINE OF THE ADHESIVE FOR TITANIUM SINGLE LAP JOINT, psi

Thickness	S _x	S _y	S _z	S _{xy}	S ₁	S ₂	S ₃	SEQV
0.01	158.8	348.5	182.33	231.87	483.32	182.33	-2.735	451.3
0.02	102.27	266.20	127.09	187.35	365.4	127.09	-0.317	359.33
0.03	81.76	240.74	107.88	172.13	322.13	107.88	-0.1805	326.17
0.04	71.129	227.62	97.897	163.23	296.49	97.897	0.001	305.88

TABLE 3-4. STRESSES AT THE SINGULARITY POINT 1 FOR TITANIUM SINGLE LAP JOINT, psi

Thickness	S _x	S _y	S _z	S _{xy}	S ₁	S ₂	S ₃	SEQV
0.01	2562.8	3698.7	2504.6	1278.5	4529.7	2504.6	1731.8	2502.7
0.02	2830.1	4076.2	2762.5	1417.0	5001.1	2762.5	1905.2	2768.6
0.03	3173.5	4564.8	3095.3	1590.8	5605.4	3095.3	2132.9	3105.3
0.04	3492.6	5021.5	3405.7	1751.4	6168.1	3405.7	2346.1	3417.7

TABLE 3-5. STRESSES AT POINT A FOR TITANIUM SINGLE LAP JOINT, psi

Thickness	S _x	S _y	S _z	S _{xy}	S ₁	S ₂	S ₃	SEQV
0.01	705.63	914.2	647.93	331.57	1157.5	647.93	462.33	623.45
0.02	796.27	1027.3	729.43	355.34	1285.4	729.43	538.15	672.37
0.03	899.86	1157	822.61	398.26	1447	822.61	609.82	753.38
0.04	994.08	1276.1	908.08	439.37	1596.5	908.08	673.65	830.86

The adhesive used here is an isotropic material. If its ductility is neglected and the maximum equivalent Von Mises stress along the centerline of the adhesive is used, it yields the predicted strength of 506 and 700 lb for 0.01 and 0.03 inch thickness, respectively. The conclusion could be drawn that the thicker the adhesive, the stronger the joint will be.

If the equivalent Von Mises stress at point A is used, the predicted strength will be 366 lb for 0.01 inch thickness and 303 lb for 0.03 inch thickness. This suggests that the thicker the adhesive, the weaker the joint will be. It is noteworthy that the stress predicted at the centerline indicates an increasing stress with decreasing bondline thickness.

Based on the application of fracture mechanics, e.g., the fracture initiation criterion proposed by Groth [17], the strength of the adhesively bonded joint can be evaluated quantitatively. Note that the stress singularity order is independent of the global geometry or load, and it only depends on the local geometry and material properties. Therefore, the stress singularity order for the joint with a 0.01-inch adhesive thickness is the same as that for a 0.03 inch thickness, that is $\lambda(0.01) = \lambda(0.03)$.

From table 3-4, $\sigma_{ij}(0.01) < \sigma_{ij}(0.03)$

Using equation 3-6, the following can be obtained

$$Q_{ij}(0.01) < Q_{ij}(0.03) \quad (3-7)$$

where the 0.01 and 0.03 inside the parentheses denote the adhesive thickness of the corresponding components.

Using equation 3-7, for the same critical generalized stress-intensity factor Q_{crit} , the thicker adhesive layer of 0.03 inch thickness would be weaker. When considering ductile adhesive systems, the stress singularity extent as well as the plastic zone size needs to be investigated.

3.3.1.2 Influence of the Adhesive Fillet.

In an actual joint, there is always some form of fillet existing. To study the effect on the strength of the joints, a titanium single lap joint with a 0.01-inch adhesive thickness and 45° fillet (0.01 by 0.01 inch) is considered (shown in figure 3-26), and the same mesh density along the overlap is used as the case without any fillets. The peak peel and shear stresses occur at point 2, which are 671.9 and 352.5 psi, respectively (figures 3-27 and 3-28, where only the adhesive is shown). While for the case without fillet, the peak stresses are 3698.7 and 1278.5 psi.

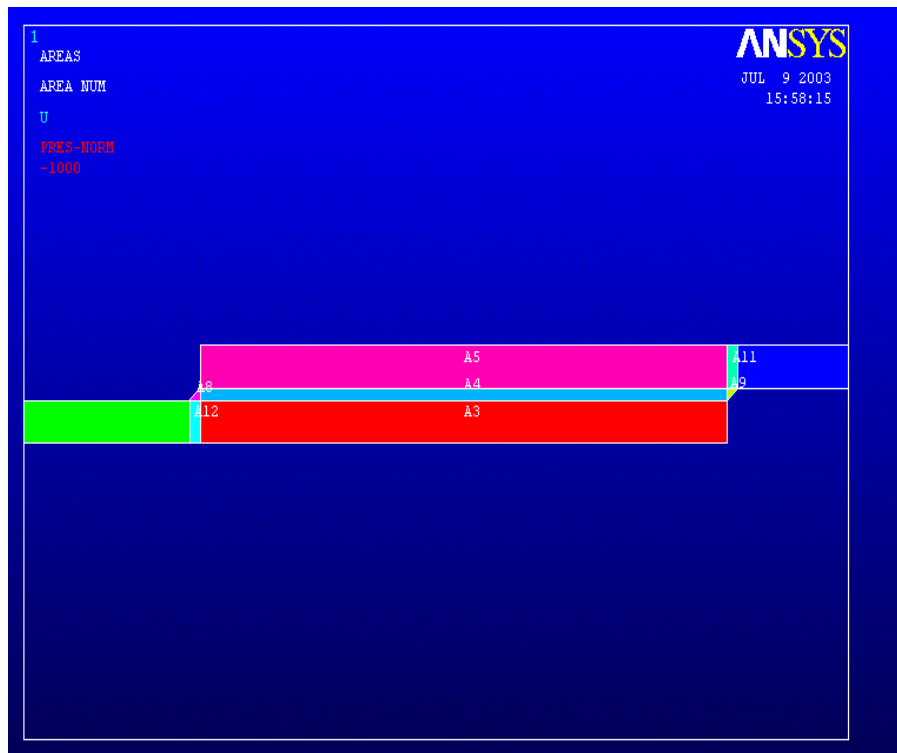


FIGURE 3-26. A 0.01-inch TITANIUM SINGLE LAP JOINT WITH A 45° FILLET

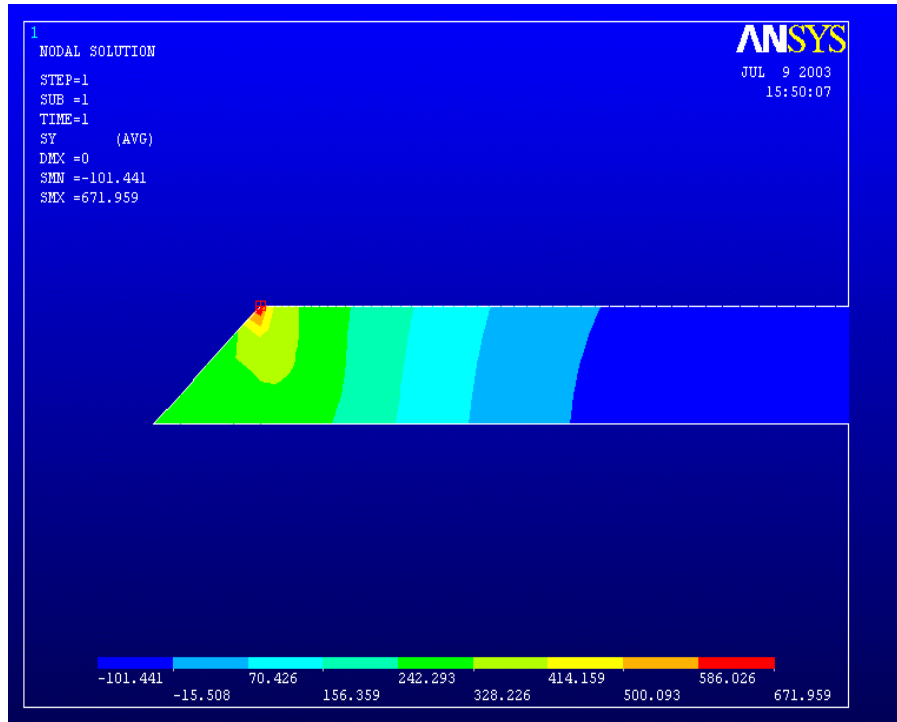


FIGURE 3-27. PEEL STRESS CONTOUR OF 0.01-inch TITANIUM SINGLE LAP JOINT WITH A 45° FILLET

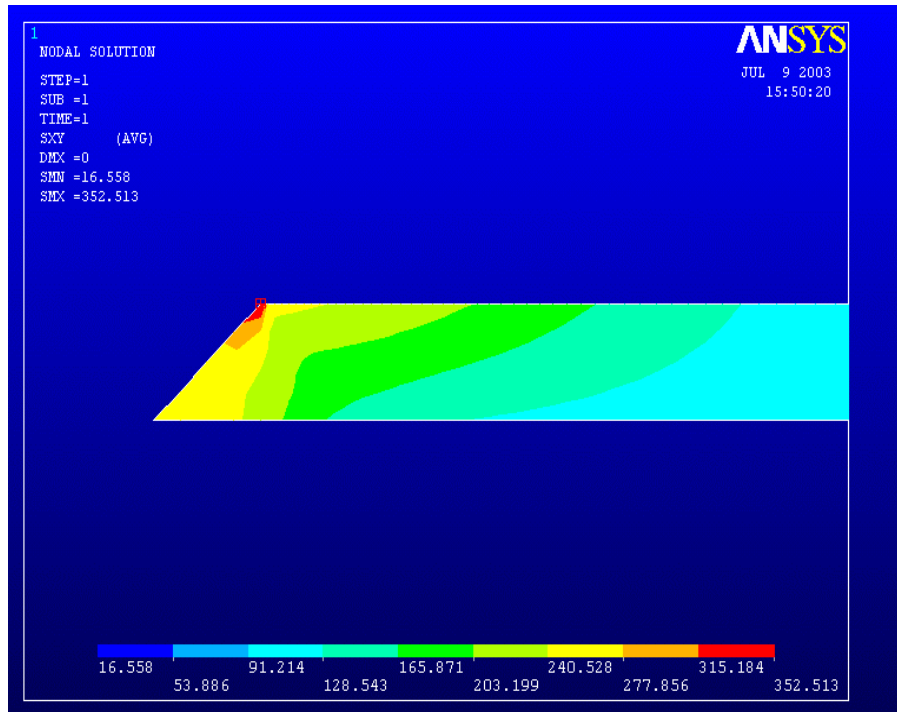


FIGURE 3-28. SHEAR STRESS CONTOUR OF 0.01-inch TITANIUM SINGLE LAP JOINT WITH A 45° FILLET

3.3.1.3 Variable Bondline Thickness Along Overlap Length and Width.

In practice, bondline thickness varies due to imperfections in assembly and material tolerances. A lap shear joint with constant shear stress due to parabolic adhesive thickness (shown in figure 3-29) is derived [31]. The main assumptions are shear stress in the adhesive is constant over the thickness, shear deformation in the adherends is neglected, and tensile stress in the adherends is uniform over the thickness.

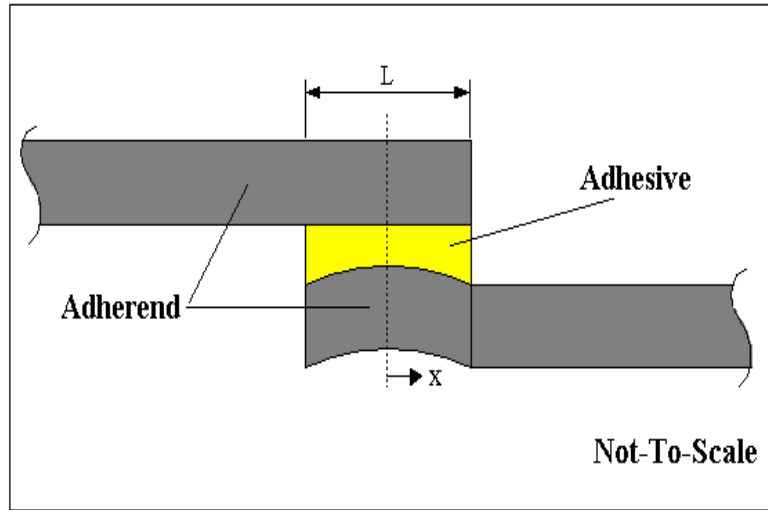


FIGURE 3-29. A LAP JOINT CONFIGURATION WITH CONSTANT SHEAR STRESS IN THE ADHESIVE LAYER

(Based on constant shear stress through adhesive layer and neglecting singularities)

The variation in thickness of the adhesive layer along the overlap length is given by:

$$t_a = \frac{G_a}{Et} x^2 \quad (3-8)$$

where G_a is shear modulus of the adhesive, E is Young's modulus of the adherends, and t is the thickness of the adherends.

It is impracticable to profile the adhesive or the adherends quadratically, this can be achieved approximately by profiling the adherend linearly suggested by Adams, et al. [20]. Some possible configurations include scarf joints and linearly tapered adherends.

To analyze the linear variance effect, a preliminary analysis on the variable adhesive thickness from 0.0085 to 0.0255 inch along overlap length is considered shown in figure 3-30. A plane-strain condition is assumed in the finite element model, and the peel and shear stress distribution along centerline is shown in figure 3-31. It can be seen that maximum stress occurs at the end of smallest thickness. Further analysis on profiling the adherend thickness to reduce the stresses at the ends could be pursued if this is deemed practicable and controllable.

As for the variable thickness along the width direction, a plane-strain model with a uniform adhesive thickness would be unacceptable. Therefore, a three-dimensional finite element model would be needed.

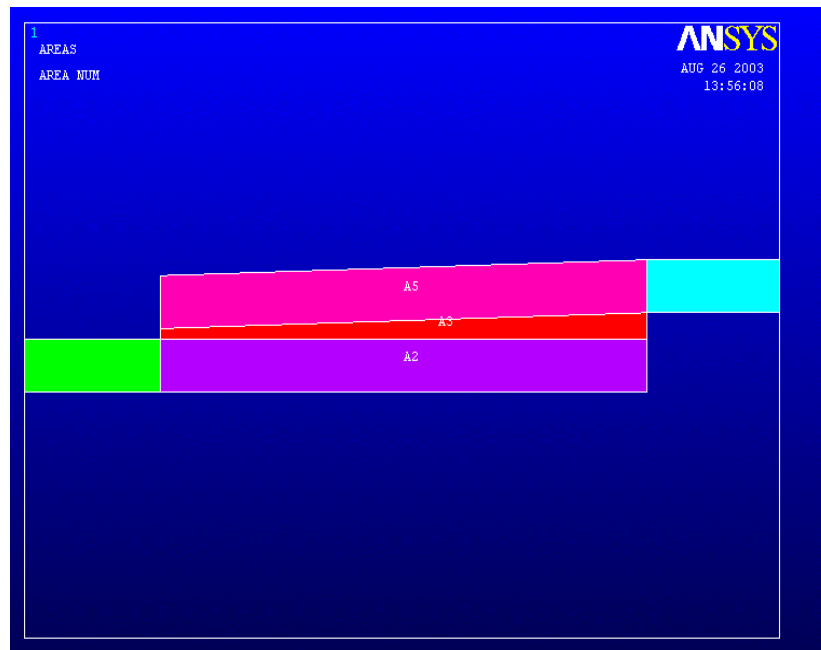


FIGURE 3-30. A SINGLE LAP JOINT WITH VARIANCE IN THE DIRECTION OF OVERLAP LENGTH

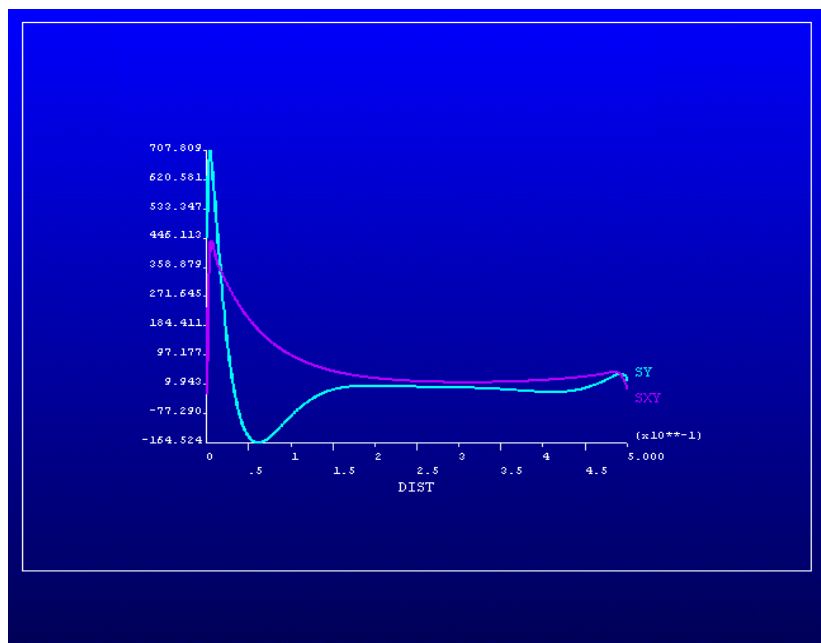


FIGURE 3-31. SHEAR STRESS DISTRIBUTION ALONG THE CENTERLINE OF THE ADHESIVE

3.3.2 Nonlinear Analysis.

For the nonlinear analysis, the tensile true stress-true strain curve in figure 3-32 is used. The load is applied gradually until the joint fails unstably due to large area plastic deformation. Studies have been made to decide whether load control or displacement control should be used for this analysis. It was found that there is only 0.5% difference in the maximum simulation load in load control and in displacement control.

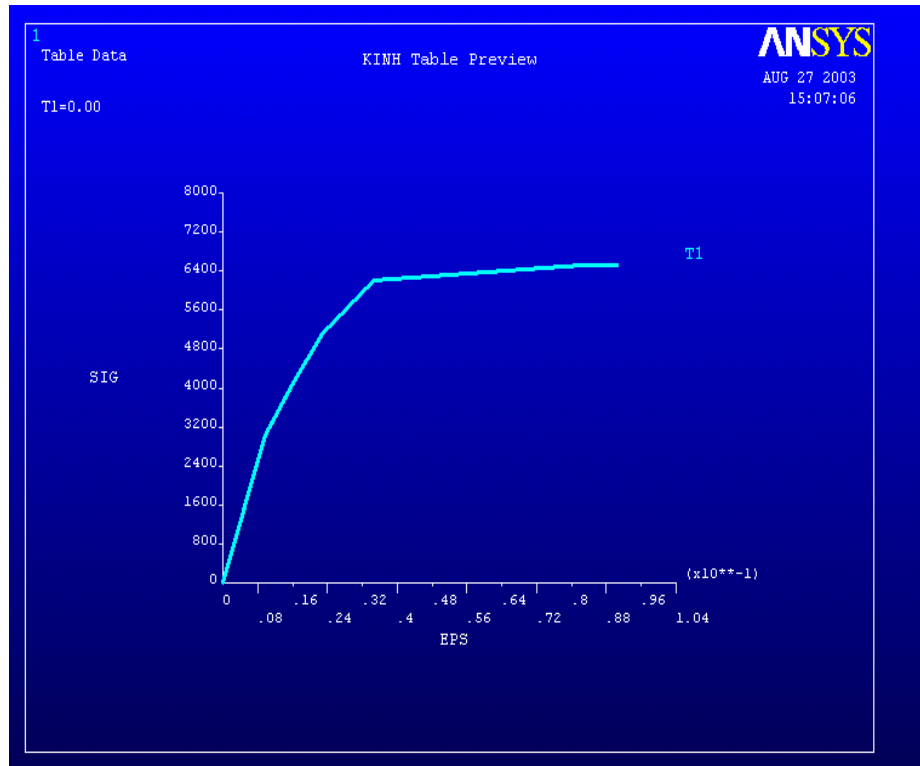


FIGURE 3-32. TRUE STRESS-TRUE STRAIN CURVE FOR ADHESIVE DP460

Analyses for 0.01- and 0.03-inch-thick adhesives yield maximum failure loads of 1870 and 1856 lb. Both are less than the test failure load of the titanium single lap joint, which varies between 2000 to 2200 lb for a range of bondline thickness from 0.01 to 0.035 inch (see table 3-6). Obviously, these predictions are much higher than the predictions from linear elastic analysis. The postprocessing results were the deformed shape of the single lap joint (figure 3-33, which is center symmetric) and the Von Mises stress contours, at the termination region of the adhesive under different load levels (figure 3-34 to 3-37). Von Mises stress contours show the initial yielding starts at the joint corners (stronger singular point), and develops around the joint ends in a certain angle through the thickness of the adhesive until the whole adhesive yields, except a very small region around the weaker singularity point and the free edge.

TABLE 3-6. FAILURE LOAD PREDICTION FOR TITANIUM SINGLE LAP JOINT, lb

Thickness (inch)	Linear Analysis		Nonlinear Analysis	Experiments
	Centerline	Point A (Figure 3-25)		
0.01	506	366	1870	2000-2200
0.03	700	303	1856	
0.017 (variable thickness from 0.0085 to 0.0255 inch)			1865	2150

Unit of measure = lb

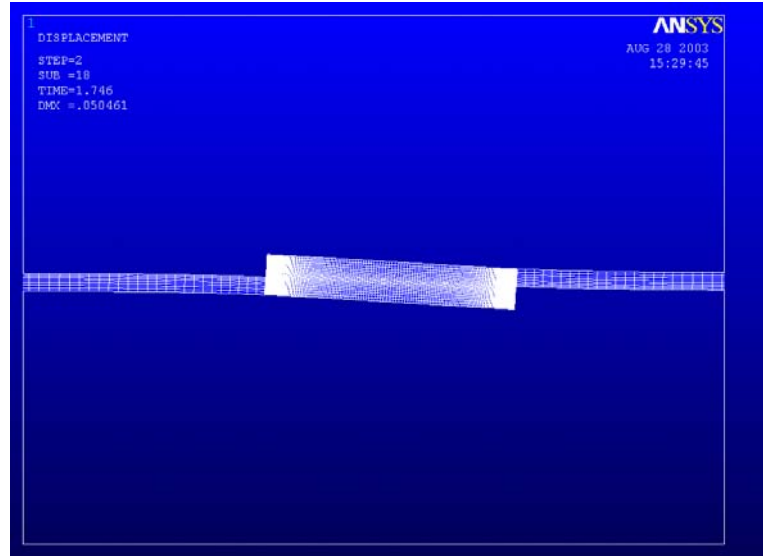


FIGURE 3-33. DEFORMED SHAPE OF THE SINGLE LAP JOINT OF 0.01 inch THICKNESS

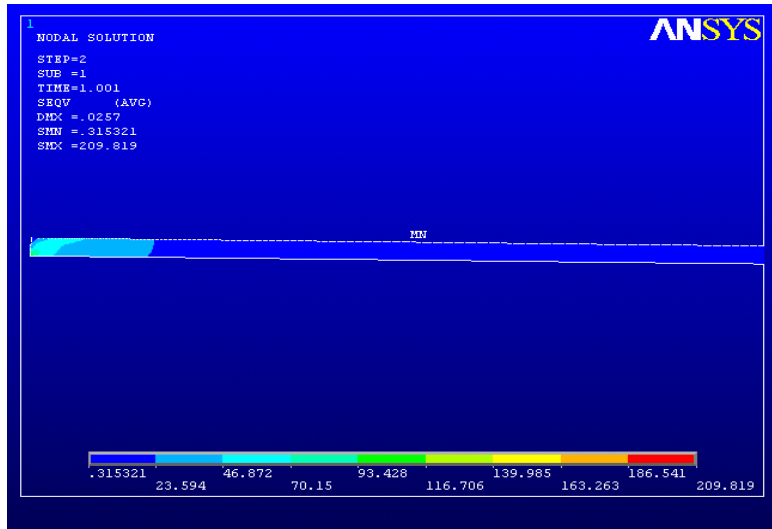


FIGURE 3-34. VON MISES STRESS CONTOUR OF A 0.01-inch-THICK JOINT AT $P = 2.5 \text{ lb}$ ($\bar{\tau} = 5 \text{ psi}$)

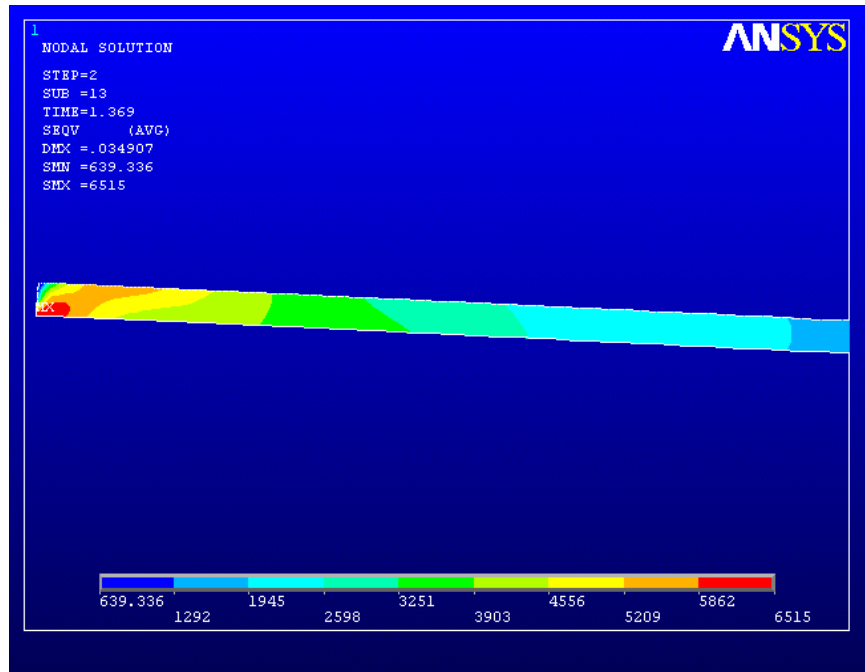


FIGURE 3-35. VON MISES STRESS CONTOURS OF A 0.01-inch-THICK JOINT AT
 $P = 908 \text{ lb}$ ($\bar{\tau} = 1816 \text{ psi}$)

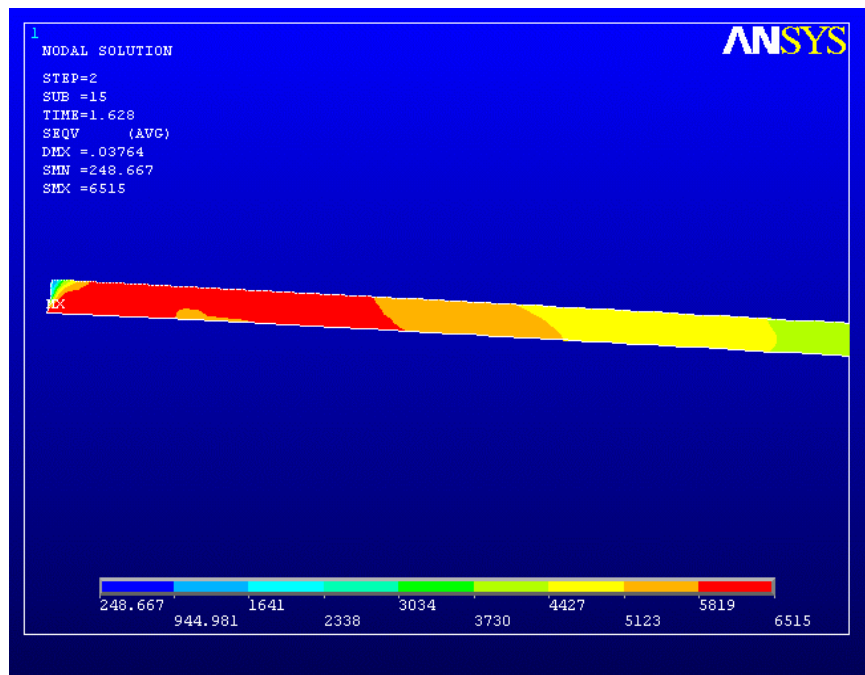


FIGURE 3-36. VON MISES STRESS CONTOURS OF 0.01-inch-THICK JOINT AT
 $P = 1545 \text{ lb}$ ($\bar{\tau} = 3090 \text{ psi}$)

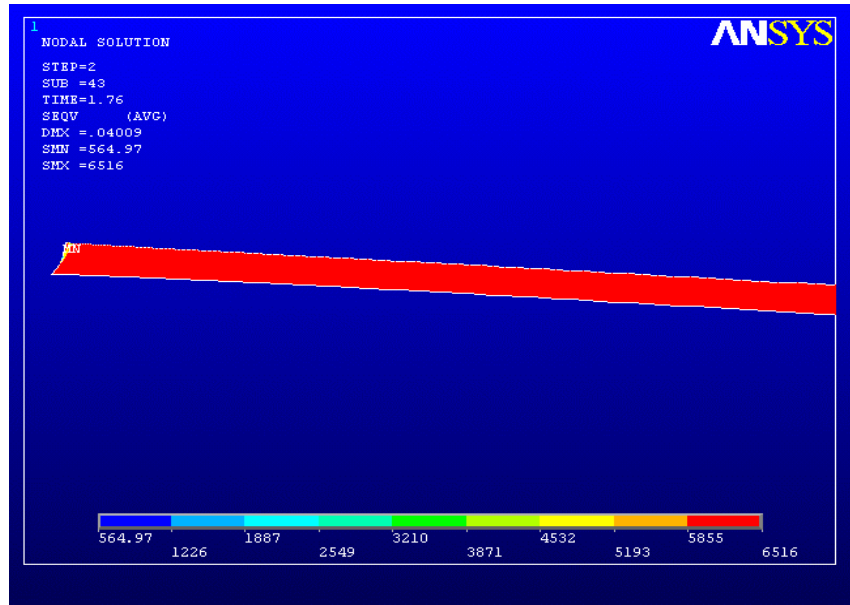


FIGURE 3-37. VON MISES STRESS CONTOURS OF A 0.01-inch-THICK JOINT AT $P = 1870 \text{ lb}$ ($\bar{\tau} = 3740 \text{ psi}$)

Nonlinear analysis of variable bondline thickness from 0.0085 to 0.0255 inch ($\bar{t}_a = 0.017 \text{ inch}$) shows in table 3-6 that the failure load is 1865 lb., while the test failure load is around 2150 lb. Figures 3-38 to 3-40 show the Von Mises stress contours at different load, where a center symmetric characteristic no longer exists and the yield starts from the left corner. With increasing load, the plastic zone increases until most of the adhesive region has yielded.

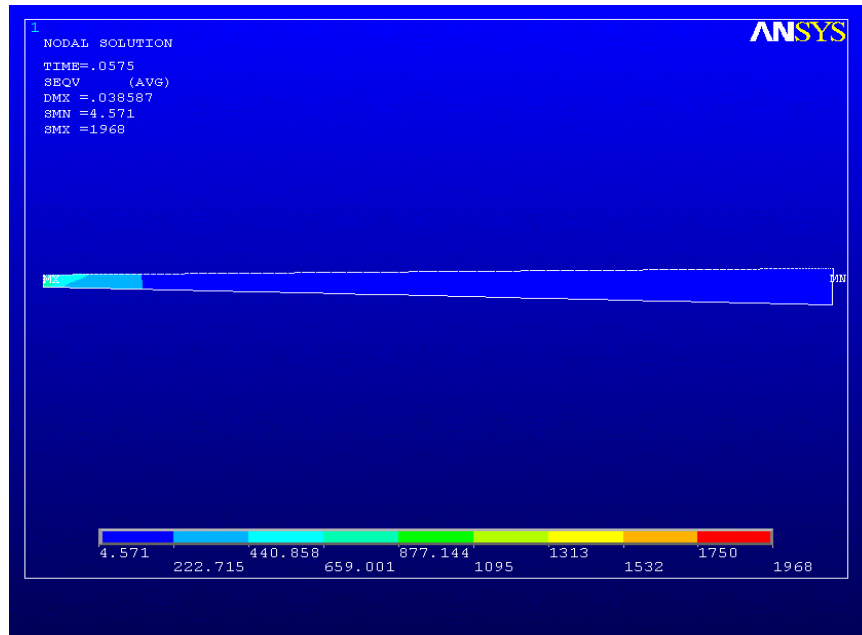


FIGURE 3-38. VON MISES STRESS CONTOURS OF VARIABLE BONDLINE THICKNESS JOINT AT $P = 22 \text{ lb}$ ($\bar{\tau} = 44 \text{ psi}$)

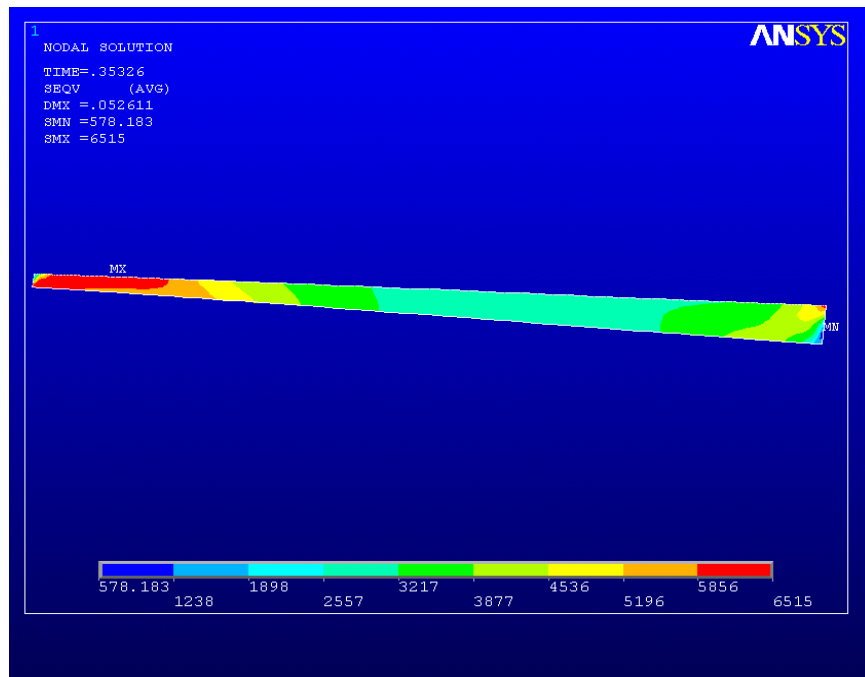


FIGURE 3-39. VON MISES STRESS CONTOURS OF VARIABLE BONDLINE THICKNESS JOINT AT $P = 1122 \text{ lb}$ ($\bar{\tau} = 2244 \text{ psi}$)

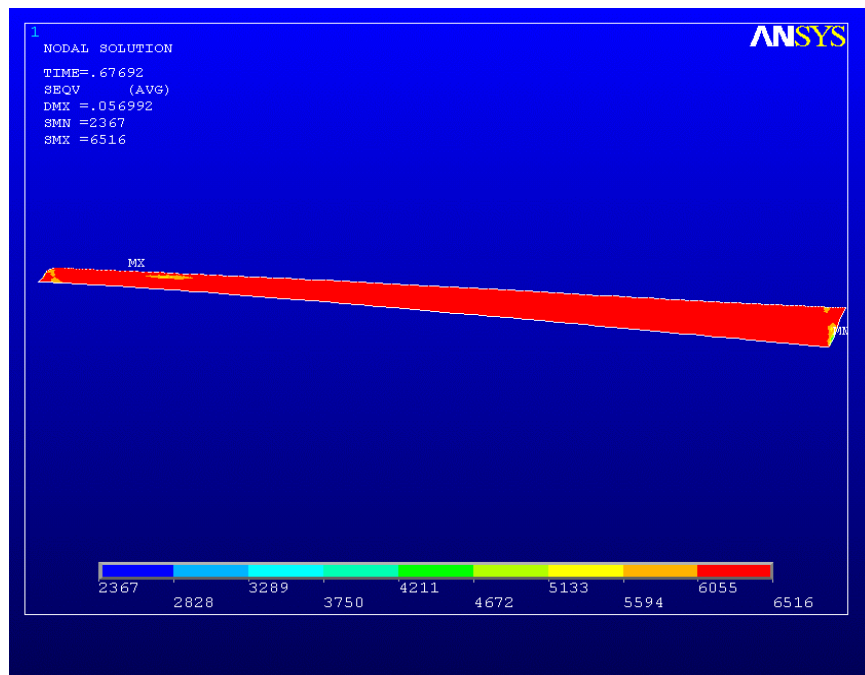


FIGURE 3-40. VON MISES STRESS CONTOURS OF VARIABLE BONDLINE THICKNESS JOINT AT $P = 1865 \text{ lb}$ ($\bar{\tau} = 3730 \text{ psi}$)

Both analysis and test show that the particular variable thickness chosen for this study has no effect on the original uniform configuration.

It was assumed that the joint was well bonded and the adhesive was representative of the physical condition, the failure load predicted by nonlinear FEA was lower by $\approx 10\%$ than the test failure load. A possible rationale may be one or more of the following:

- Lower yield strength of the adhesive properties in FEA simulation. Adhesive material properties come from the test results of the adhesive bulk material, in which case, the possibility of more extensive microdefects increases. From the simple Weibull statistics viewpoint, the strength of a small volume of adhesive will be higher.
- Spew fillet effects. There is no fillet included in the FEA analysis, whereas in the actual specimen some form of fillet geometry is typically present. It is expected that with such a spew fillet, the maximum predicted load may be slightly higher as noted for the previous linear analyses, section 3.3.1.2.
- A different yield criterion may apply. Typically, a polymer material has a higher yield stress in compression than in tension, and unlike the case of the metals, its yielding behavior is pressure dependent [12, 29, 32, and 33]. The following modified Von Mises yield criterion is considered to be appropriate for modeling adhesive yielding:

$$\sigma_{vm}^2 - 3(\sigma_{yc} - \sigma_{yt})p = \sigma_{yc}\sigma_{yt} \quad (3-9)$$

Where Von Mises stress: $\sigma_{vm} = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 2$

Hydrostatic stress: $p = -(\sigma_1 + \sigma_2 + \sigma_3) / 3$

and σ_{yc} , and σ_{yt} are yield stress in compression and in tension, respectively. For the adhesive used, the ratio σ_{yc}/σ_{yt} is assumed to be 1.3 [12, 29, and 32].

Using the Von Mises yield criterion for the single lap joint studied, the left corner of the adhesive (point 1 in figure 3-25) yields when the equivalent stress reaches the yield stress of the adhesive. However, a modified Von Mises yield criterion (3-9) will not predict yielding until the hydrostatic stress reaches a certain level. As a result, the yielding process is delayed. Currently, ABAQUS is being used to advance this investigation.

3.4 APPROACHES FOR FAILURE PREDICTION.

3.4.1 Maximum Stress/Strain.

Various methods for failure prediction of bonded joints have been used for many years and are typically based on a maximum shear stress or maximum shear strain criteria. The most common and simplistic of such methods adopt a linear shear lag idealization, e.g., Volkersen [1]. One approach was to use test data based on a given representative single lap or double lap joint

configuration for the same adherend and adhesive materials and the same adhesive thickness as those intended for the design of a given structural joint. It was also recommended that the same surface preparation and processing procedures are adopted for the coupon specimens. From this database, the average shear stress at failure was obtained and the implied peak shear stress at the extremities of the adhesive bondline was computed from the shear lag model. This value of peak shear stress was then used as a design allowable value that was then applied to the actual joint design. Hence, a prediction for the average shear stress at failure can be estimated for the hardware design.

Clearly the above simplified methodology represents a first level analysis and is limited primarily by the assumed linear shear stress-strain behavior of the adhesive system and the absence of any consideration of the peel stress developed at the joint extremities. It was also assumed that the shear stress distribution is constant at all stations through the thickness and that the peak shear stress occurs at the free ends of the adhesive lap.

For a balanced double lap joint, the following useful relationships are assumed to apply:

$$\frac{\tau(x)}{\bar{\tau}} = \frac{\lambda l}{2 \sinh(\lambda l)} \{(\cosh(\lambda(l-x)) + \cosh(\lambda x))\} \quad (3-10)$$

where $\tau(x)$ is the local shear stress developed at distance x from one end of the lap.

$$\lambda^2 = \frac{G_a}{t_a} \frac{2}{Et}$$

$$\bar{\tau} = \frac{P}{wl}$$

This equation gives the same results as equation (3-1), and the maximum shear stress along the centerline of the 0.01-inch-thick adhesive for the aluminum double lap joint is 206.9 psi, while TOM solution gives 199 psi and FEA shows the maximum shear stress of 184.2 psi (table 3-2).

3.4.2 Maximum Stress/Strain Including Nonlinearity of Adhesive.

A common approach introduced by Hart-Smith [5] comprises an elastic-plastic model for the adhesive layer. The three-parameter elastic-plastic model, G_a , τ_p and $\gamma_{\max}$, requires experimental shear stress-strain data for the adhesive for which the basis is the shear strain energy at failure. Hence, the area under the shear stress-strain curve is equated for the idealized elastic perfectly plastic model and the experimentally obtained curve. This enables the determination of the following three parameters: (1) Elastic shear modulus, G_a ; (2) shear stress at yield, τ_p , sometimes termed cutoff value; and (3) ultimate shear strain $\gamma_{\max}$ (elastic plus plastic shear strain $\gamma_e + \gamma_p$).

The key relationship representing the design guidance is given by:

$$P_{\max} = 2\tau_a l \quad (3-11)$$

where

$P_{\max}$ is the failure load per unit width
 τ_a is the average shear stress at failure
 l is the overlap length

The parameter λ for the elastic-plastic case can be expressed:

$$\lambda = \sqrt{\frac{2\tau_p}{tEt_a\gamma}} \quad (3-12)$$

And for practical design guidance, the following lap length is recommended [5]

$$l_R = \left[\frac{P}{2\tau_p} + \frac{2}{\lambda} \right] \times S_F \quad (3-13)$$

where S_F is the factor of safety

Finally, the peel stress developed at the ends of the adhesive layer is approximated by a relationship obtained from a simple beam-on-elastic foundation model, assuming that the shear stress induced yield level had been reached for a finite yielded zone to be established. This relationship is given by:

$$\sigma_{peel} = \tau_p \left[\frac{3E'_z(1-\nu_a^2)t}{Et_a} \right]^{1/4} \quad (3-14)$$

where

E'_z is the effective normal elastic modulus of the adhesive in the peel mode assuming in-plane constraints are imposed by the stiffer and thicker adherends.

ν_a is the Poisson's ratio of the adhesive material

t, t_a are the thicknesses of the adherends and adhesive layer, respectively

E is the in-plane elastic modulus of the adherends

More detailed models, notably finite element-based in most cases, have been used to a significant degree in more recent years. However, the general approach for maximum stress/strain criteria is often adopted by introducing the same fidelity of discretization in the model of the basic test coupon configuration as for the model of the structural hardware that is being designed.

3.4.3 Yielding/Nonlinearity (Development and Yield Zones).

As mentioned in reference 34, lap shear tests give only two pieces of information, the shear strength (actually the average shear stress at failure) and the failure mode. Only a quantitative

assessment of the level of adhesion can be obtained. In FEA modeling, perfect bonding is typically assumed for each analysis.

For adhesively bonded joints, there are three failure modes, depending on the adherend material and the bonding quality: (1) cohesive failure in the adherend, (2) cohesive failure in the adhesive, and (3) interfacial failure at the interface between one of the adherends and the adhesive. A perfectly bonded joint with strong metallic adherends will fail by cohesive failure in the adhesive, while with composite adherends, it usually fails by cohesive failure in the adherends due to its low transverse strength. Interfacial failure occurs at the interface between one of the adherends and the adhesive, and usually reveals a poor surface preparation. In such instances, the joints may fail partly by cohesive failure and partly by interfacial failure.

In section 4, a series of experiments on titanium single lap joint test results are described, the adhesive system for these tests is DP460, a relatively ductile system assumed to be an isotropic material. For such a ductile adhesive system, the Von Mises failure criterion is generally applicable. When considering a joint with composite adherends, a perfect bonded joint will fail by cohesive failure in the adherends, the Von Mises stress criterion is not generally applicable to such, nonisotropic, materials.

From the literature, stresses at the termination of the interfaces between the adherend and adhesive of single lap joints are singular in a linear elastic analysis due to the dissimilar bonded materials; consequently maximum stress/strain criteria cannot be interpreted for the purpose of failure prediction. In FEA, the stresses at the singularity point get larger as the element meshing is refined.

Linear analyses neglecting the ductility of the adhesive predict the strength of 506 and 700 lb for 0.01- and 0.03-inch-thick joints, respectively, if equivalent Von Mises stress along the centerline of the adhesive is employed (table 3-6). While the equivalent Von Mises stress at the point next to the singular point is used, failure is predicted at 366 lb for 0.01-inch-thick joint and 303 lb for 0.03-inch-thick joint. In both cases, the prediction was much lower than that of nonlinear analysis and test results. It was noted that when stress along the centerline of the adhesive is used, it predicts increasing strength with increasing adhesive thickness. This is consistent with the closed form solutions, but this is contrary to the experimental observations and nonlinear analysis. However, if the stresses along the interface around the singular point are used, the prediction would be that the strength slightly decreases with increasing adhesive thickness.

In this report, as the adhesive used in the test and analysis is relatively ductile, with extensive yielding, strength prediction by linear elastic analysis is not applicable. Nonlinear analyses of 0.01-inch-thick, 0.03-inch-thick, and variable thicknesses from 0.0085- to 0.0255-inch-thick adhesive titanium single lap joints with the ductile adhesive DP460 show that failure loads are 1870 lb, 1856 lb, and 1865 lb, respectively. All are less than the test failure load 2000-2200 lb, and possible reasons are listed, see section 3.3.3. As suggested there, a different modeling of the adhesive (i.e., with fillet, modified Von Mises) might be selected for predicting the strength of a joint.

In searching for a workable failure criterion for bonded joints, more extensive studies on testing, postfracture study, and modeling will be necessary in the future.

4. CORRELATION WITH EXPERIMENTS.

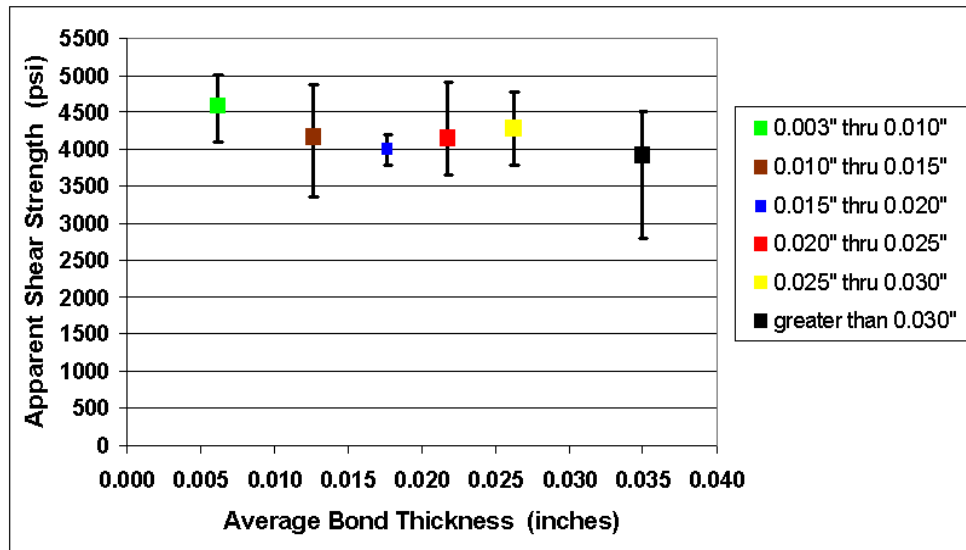
The following section reports the apparent shear strength results of the single lap joint specimens prepared and tested in accordance with the established procedures. Variations of these procedures were studied in the interest of drawing comparisons. Once data for a control group was established, tests were conducted for coupons with altered bond geometry, environmental exposure, testing environment, and surface preparation methods.

This single lap test provided data for the coupon's apparent shear strength that does not generally reflect the true shear strength of the adhesive. Analytical and theoretical predictions were obtained for similar materials with linear-elastic properties as described in section 3. From the analyses, it was evident that the normal (peel) stress along the adhesive-adherend interface, at the edges of the overlap, can dominate coupon failure. Consequently, the results obtained from this series of tests are unique to this configuration and should be used only for comparative studies.

4.1 SINGLE LAP JOINT.

4.1.1 Aluminum/Titanium Adherends.

A basis for comparison was established by testing coupons with constant bond thickness. Figure 4-1 presents data for nonbrushed coupons, bonded with DP460. The apparent shear strength of the coupons is plotted against the average bond thickness. Thirty-one coupons were tested to obtain the results displayed in this chart. Coupons were gathered (see legend) into groups according to their average bond thickness, at approximately 0.005-inch intervals, beginning at 0.005 inch. The data points represent average values of bond thickness and the corresponding apparent shear strength for each group.

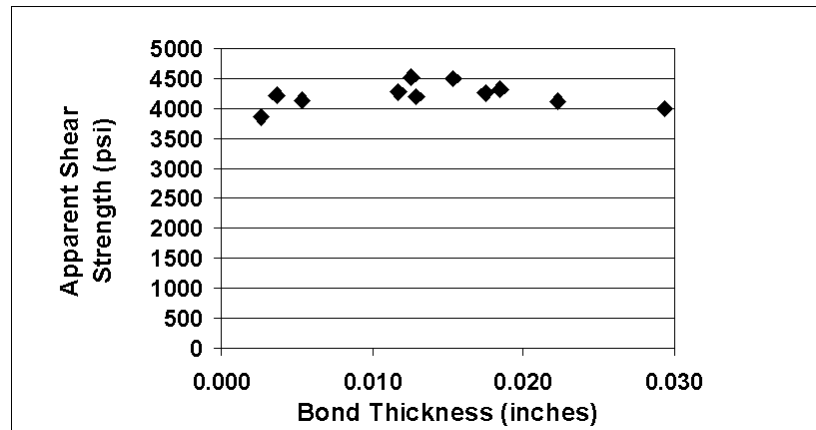


Adhesive: DP460 (unbrushed)
Adherends: Titanium

Cured at 180°F for 1 hour
Tested at room temperature

FIGURE 4-1. EXPERIMENTAL RESULTS FOR SINGLE LAP JOINT TESTS

The next group of coupons was composed of the same adhesive and adherends (DP460 and titanium). However, these coupons differ from those presented in figure 4-1, in that the adhesive was brushed into the bond area. Eleven coupons were tested and are displayed as individual data points in figure 4-2.

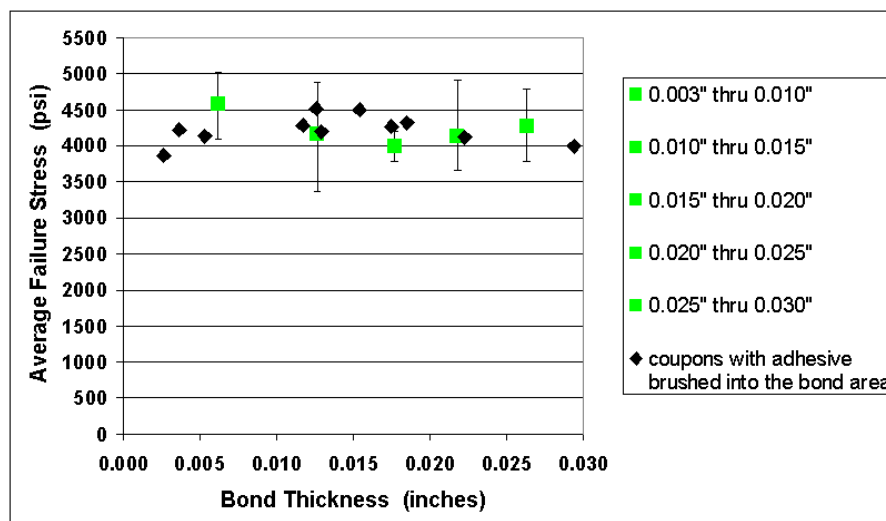


Adhesive: DP460 (brushed)
Adherends: Titanium

Cured at 180°F for 1 hour
Tested at room temperature

FIGURE 4-2. EXPERIMENTAL RESULTS FOR COUPONS WITH ADHESIVE BRUSHED INTO BOND AREA

To emphasize the effect of brushing adhesive into a bond surface, figure 4-3 presents a combination of the preceding two figures. The only effect of brushing appears to be lower scatter strengths for the thin bondline specimens.

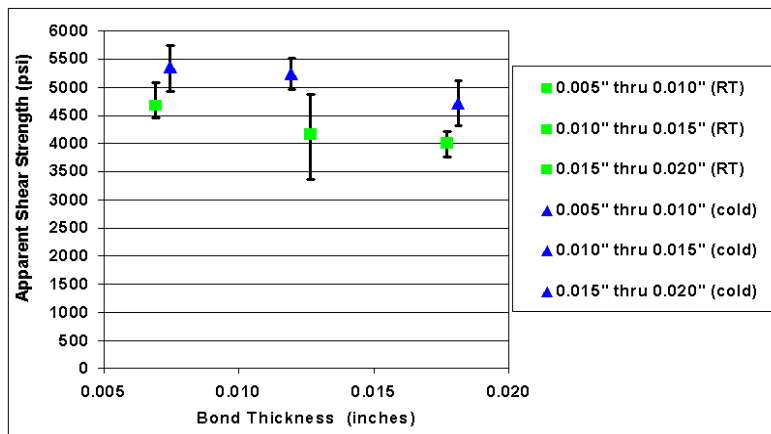


Adhesive: DP460
Adherends: Titanium

Cured at 180° F for 1 hour
Tested at Room Temperature

FIGURE 4-3. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS OF BRUSHED AND NONBRUSHED COUPONS

Data comparing the cooled coupons with coupons tested at room temperature is displayed in figure 4-4. Coupons were gathered into three groups according to their average bond thickness (see legend). The data points represent average values of bond thickness and apparent shear strength for each group.

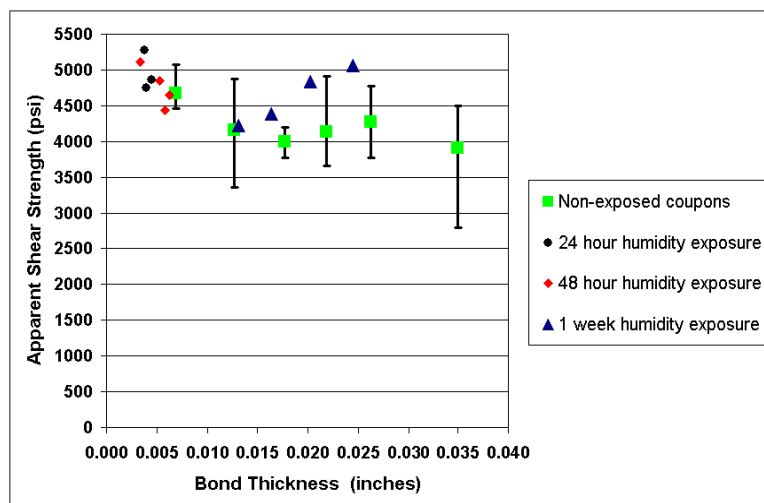


Adhesive: DP460
Adherends: Titanium

(nonbrushed)
Cured at 180°F for 1 hour

FIGURE 4-4. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, COMPARING AMBIENT TEMPERATURE VS LOW TEMPERATURE EXPOSURE

Several tests were conducted to assess the environmental durability of DP460. Figure 4-5 displays data for coupons that were enclosed in a humidity chamber after curing. Humidity ranged from 80% to 100%. Adhesive was not brushed into the bond area. Data for nonexposed coupons is presented for comparison.

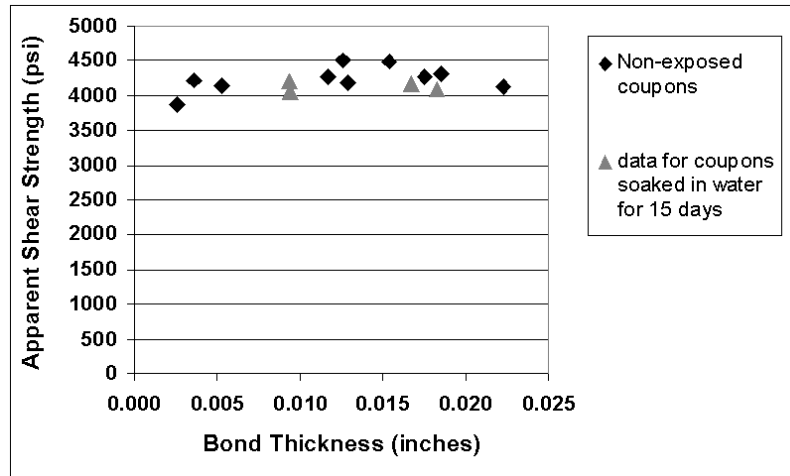


Adhesive: DP460 (nonbrushed)
Adherends: Titanium

Cured at 180°F for 1 hour
Tested at Room Temperature

FIGURE 4-5. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, COMPARING AMBIENT CONDITION VS HIGH HUMIDITY EXPOSURE

Figure 4-6 contains data for several coupons that were submersed in water after curing. Data points represent individual coupons.

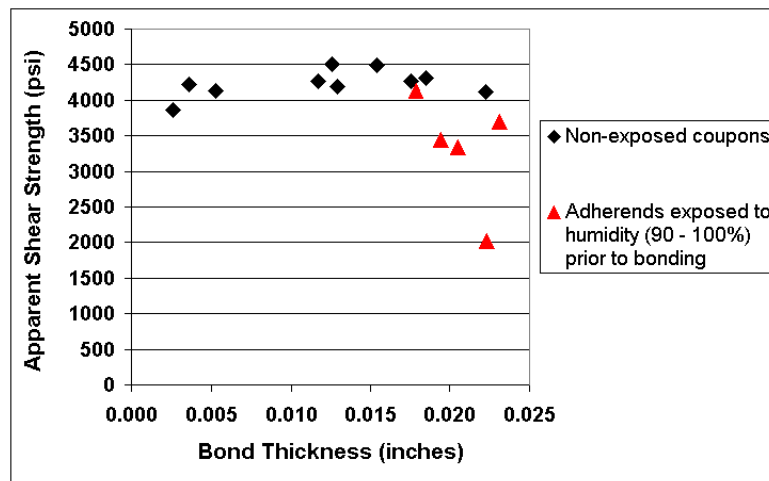


Adhesive: DP460 (brushed)
Adherends: Titanium

Cured at 180° F for 1 hour
Tested at Room Temperature

FIGURE 4-6. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, COMPARING AMBIENT CONDITION VS WATER SOAKED EXPOSURE

Tests were conducted to investigate the effect of humidity exposure during the bonding process. After being gritblasted, coupons were stored in a humidity chamber for 24 hours. Humidity ranged from 90% to 100%. Coupons were bonded within 5 minutes of the time at which they were removed from humidity chamber. Data is presented in figure 4-7, where the points represent individual coupon data.



Adhesive: DP460 (brushed)
Adherends: Titanium

Cured at 180°F for 1 hour
Tested at Room Temperature

FIGURE 4-7. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, COMPARING AMBIENT CONDITION VS HUMIDITY EXPOSURE PRIOR TO BONDING

Typically, a bond surface is rinsed after being gritblasted. Several methods exist to remove the remaining grit media. Figure 4-8 compares coupons blotted with acetone to ones that were rinsed with dry nitrogen air at 100 psi. Data points represent individual coupons.

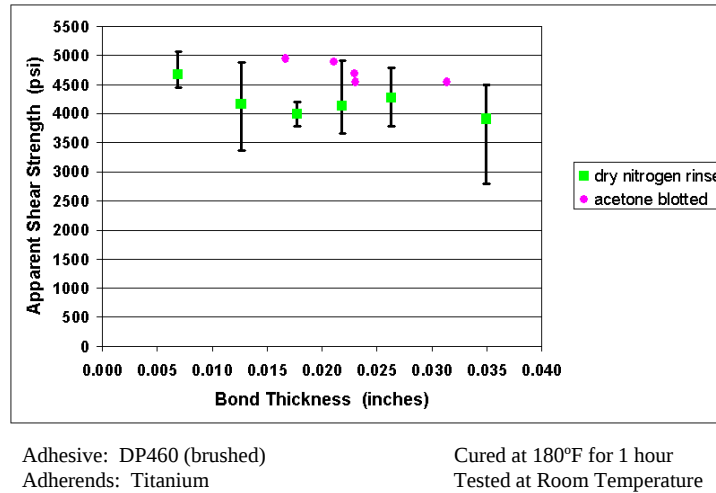


FIGURE 4-8. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, COMPARING ACETONE BLOTING VS DRY NITROGEN RINSING AS PRETREATMENT

4.1.2 Composite Adherends.

Tests were conducted on single lap coupons with composite adherends. The limited database reflects concern regarding the relevance of observed failure modes. Failures tended to initiate in an adherend rather than the adhesive. Figure 4-9 displays data for coupons with composite adherends. The data points represent average values of bond thickness and apparent shear strength for each group.

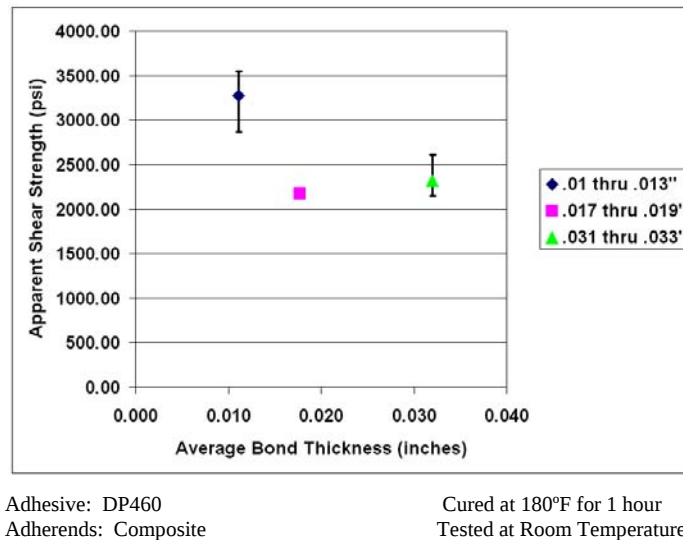
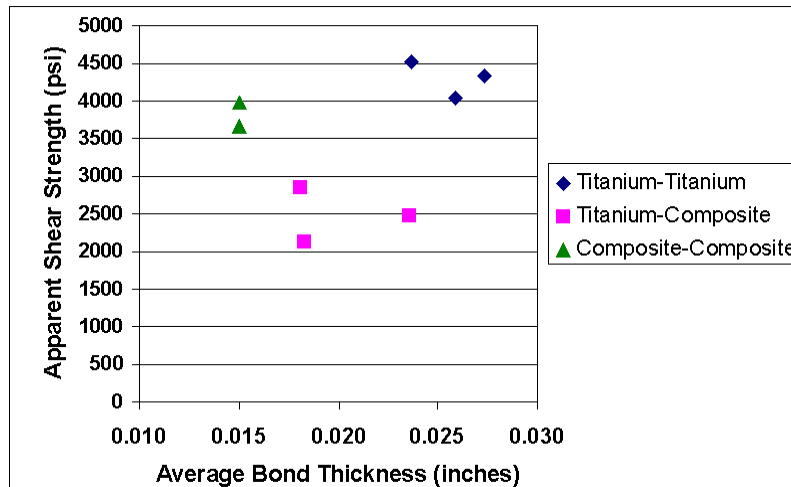


FIGURE 4-9. EXPERIMENTAL RESULTS FOR SINGLE LAP JOINTS WITH COMPOSITE ADHERENDS

4.2 DOUBLE LAP JOINT.

Double lap coupons were constructed by bonding together the adherends of similar single lap coupons. Data for double lap coupons with composite and titanium adherends is presented in figure 4-10. Data points represent individual coupons. The titanium-composite lap joints were lower in strength because of additional peel stresses and fabrication difficulties.



Adhesive: DP460 (brushed)
Adherends: see legend

Cured at 180°F for 1 hour
Tested at Room Temperature

FIGURE 4-10. EXPERIMENTAL DATA FOR DOUBLE LAP COUPONS

4.3 BONDLINE THICKNESS VARIABILITY.

Tests were conducted to assess the effects of bonds that were variable along the lap length and the width (see figure 4-11). Data for brushed coupons with variable and nonvariable bond thickness is presented in table 4-1.

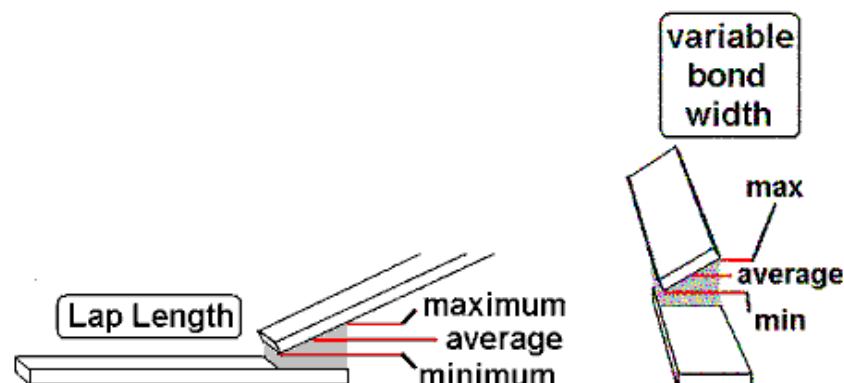


FIGURE 4-11. BOND THICKNESS VARIATION ALONG THE LAP LENGTH AND ACROSS THE LAP WIDTH

If bondline thickness is linearly changing from t_1 (defined as minimum thickness, the least thickness of the bondline thickness) to t_2 (defined as maximum thickness, the maximum thickness of the bondline thickness), then the average thickness t_a will be

$$t_a = \frac{t_1 + t_2}{2} \quad (4-1)$$

TABLE 4-1. GEOMETRY DATA OF ALL THE BONDLINE CASES SHOWN IN FIGURE 4-12

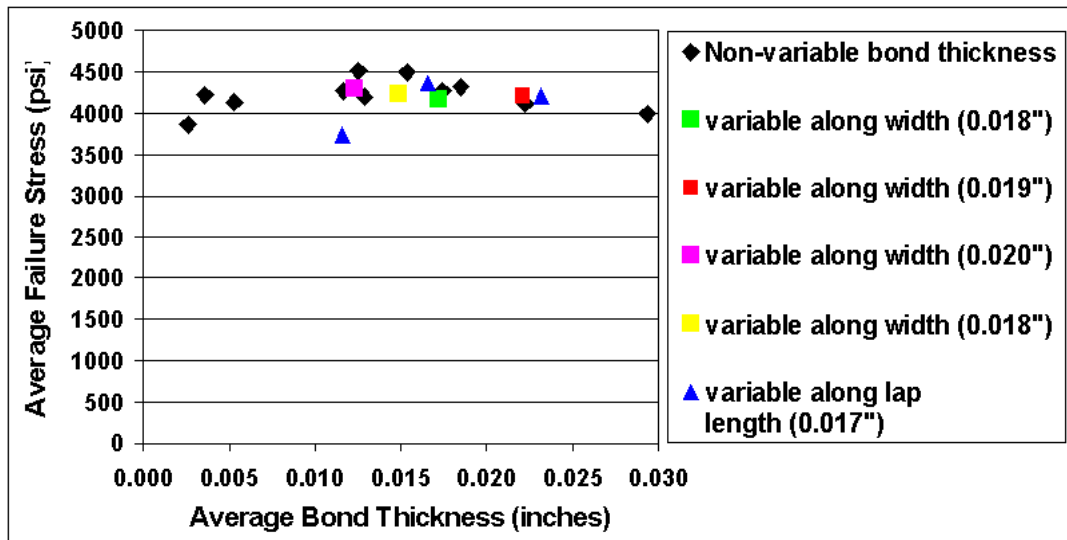
Case No.	Average thickness t_a (inch)	Change of thickness Δt (inch)	Minimum thickness t_1 (inch)	Maximum thickness t_2 (inch)	Average Failure stress τ_a (psi)
1	0.003	0	0.003	0.003	3864
2	0.004	0	0.004	0.004	4219
3	0.005	0	0.005	0.005	4138
4	0.012	0	0.012	0.012	4273
5	0.013	0	0.013	0.013	4510
6	0.013	0	0.013	0.013	4188
7	0.015	0	0.015	0.015	4494
8	0.017	0	0.017	0.017	4265
9	0.019	0	0.019	0.019	4318
10	0.022	0	0.022	0.022	4119
11	0.029	0	0.029	0.029	3992
12	0.012	0.017	0.0035	0.0205	3734
13	0.017	0.017	0.0085	0.0255	4300
14	0.023	0.017	0.0145	0.0315	4209
15	0.012	0.020	0.002	0.022	4305
16	0.015	0.018	0.006	0.024	4246
17	0.017	0.018	0.008	0.026	4160
18	0.022	0.019	0.0134	0.0324	4200

Since this variance is linearly along the lap length, the average thickness occurs at the midpoint along the length, as shown in the first graph of figure 4-11. This is also true for thickness variance across the lap width, where average thickness occurs at the midpoint along the width direction, shown in the second graph of figure 4-11.

If the change of thickness from one end to the other is denoted as Δt (equal to $t_2 - t_1$) and the average bondline thickness t_a is shown (as x axis of figure 4-12), then the minimum thickness t_1 and maximum t_2 can be calculated by

$$t_1 = t_a - \frac{\Delta t}{2} \quad (4-2)$$

$$t_2 = t_a + \frac{\Delta t}{2} \quad (4-3)$$



Adhesive: DP460 (brushed)
Adherends: Titanium

Cured at 180°F for 1 hour
Tested at Room Temperature

FIGURE 4-12. EXPERIMENTAL RESULTS FOR TITANIUM SINGLE LAP JOINT TESTS, EFFECT OF LINEAR VARIATIONS IN BOND THICKNESS

In figure 4-12, the label nonvariable bond thickness represents that the bondline thickness is uniform in both lap length and width directions. Variable along width (or length) means that the bondline is uniform in lap length (or width) direction, but not along width (or length) direction, and the change of thickness Δt from the minimum thickness t_1 to maximum thickness t_2 is indicated inside the parentheses. The x axis is average bondline thickness t_a , and y axis is the average failure stress τ_a defined as the failure load P divided by the bonding area A (in all the cases, $A = 0.5 \text{ in}^2$). Therefore, the failure load P is one half of the average failure stress τ_a (shown in table 4-1).

Table 4-1 shows the geometry data of all the bondline cases shown in figure 4-12. Cases 1 to 11 are cases of uniform bondline thickness brushed, cases 12 to 14 are cases of variable bondline thickness along length, and change of thickness is 0.017 inch for these three cases. Cases 15 through 18 are cases of variable bondline thickness along width, and change of thickness for each case is indicated in the third column of table 4-1.

5. CONCLUSIONS.

Numerical and analytical methods for adhesively bonded joints were reviewed, applied, developed, and evaluated for one specific paste adhesive system, 3M's DP460, which exhibits ductile characteristics. Commencing with simple shear lag analysis, e.g., Volkersen theory, progressively more complex formulations were considered for both stress analysis and failure predictions. Finite element methods, singularities, and plasticity treatments were applied for prediction of failure loads that were compared to experimentally determined values for both single and double lap configurations. The following items summarize the outcome of the research.

1. Predictions based on linear analysis, numerical or analytical, tend to dramatically overestimate shear and peel stress levels and, consequently, greatly underestimate the failure load levels. Of course this assumes that appropriate surface preparation and assembly procedures have been used, resulting in mainly cohesive failure modes. Also, linear elastic analyses of single and double lap joints show that, for the same applied tensile load, the peak stresses at the critical end of a double lap joint are much lower than that in a single lap joint.
2. For similar reasons, to those stated in item 1, the predicted trends for the effect of bondline thickness are significant and misleading, especially for an adhesive with the ductility exhibited by the DP460 system.
3. Nonlinear material simulations approximating adhesive shear and peel stress-strain characteristics, introduced via finite element analyses, produce a more realistic and much closer predictions of failure load when compared with experiments. Since polymer yielding is sensitive to hydrostatic pressure, a modified Von Mises yield criterion is recommended for future studies.
4. Approaches based on the nonlinear characteristics identified in item 3 provide a rationale for the essentially constant failure loads with adhesive bondline thickness over the range of 0.005 through 0.035 inch that was indicated by experiment. It is highly possible that the experimentally indicated tolerance to linear variations of bondline thickness (within a given bondline) can also be based on the same rationale.
5. In single lap shear tests, the indication of high peel stresses appear to drive an initial adhesive failure near the ends of the lap. However, the fracture mechanics approaches attempted herein have not demonstrated that mode I fracture data enables an adequate failure prediction method.
6. For bonded joint stress analysis purposes, finite element analysis (FEA) has been found to be a good choice as long as the mesh, boundary condition, and the loading are applied properly. For FEA modeling using ANSYS, an adequately refined mesh and eight-node element (plane82) are recommended.
7. A limited evaluation of spew fillet effects was studied, although no fillet was included in the nonlinear FEA analyses. In most practical scenarios, there is some evidence of a fillet

at the joint ends, and it is expected that with a spew fillet, the strength of adhesively bonded joints will be somewhat enhanced in accordance with other researcher's results. Furthermore, the yielding of adhesive is pressure dependent, and modified Von Mises yield criterion is more appropriate for modeling adhesive yield behavior. ABAQUS has been adopted for some ongoing studies.

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