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COMMUNICATIONS RECEIVER PERFORMANCE DEGRADATION HANDBOOK

Prepared for

National Telecommunications and Information Administration (NTIA)
1401 Constitution Ave, NW
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JSC Project Engineer

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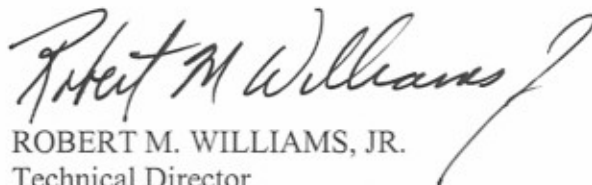Under Contract to
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14. ABSTRACT
This handbook provides the radio frequency (RF) analyst with the capability to calculate the effects of noise and interference on RF communications receivers. A receiver is modeled as a sequence of modules. Each module has a *transfer function* that relates the module outputs to the module inputs. By consecutively analyzing each module in the sequence, the analyst can then relate the receiver outputs (performance) to the receiver inputs (signal characteristics).

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EXECUTIVE SUMMARY

In May 2003, President Bush established the Spectrum Policy Initiative to promote the development and implementation of a United States spectrum policy for the 21st century. In response to the Spectrum Policy Initiative, the Secretary of Commerce established a Federal Government Spectrum Task Force and initiated a series of public meetings to address policies affecting spectrum use by the federal government, state, and local governments, and the private sector. The recommendations resulting from these activities were included in two reports released by the Secretary of Commerce in June 2004. In November 2004, the President directed the federal agencies to develop a plan to implement the 24 recommendations contained in the reports. One of the recommendations directed the National Telecommunications and Information Administration (NTIA) to develop a handbook documenting best practices in spectrum engineering that will be recognized by all regulatory authorities in the United States. The Best Practices Handbook will address the electromagnetic compatibility (EMC) analysis process. Establishing receiver performance degradation levels is required to perform an EMC analysis. The NTIA requested that the Joint Spectrum Center develop a communications receiver performance degradation handbook that could be included as part of the Best Practices Handbook.

The purpose of this handbook is to provide the radio frequency (RF) analyst with the capability to calculate the effects of noise and interference on RF communications receivers. A receiver is modeled as a sequence of modules. Each module has a *transfer function* that relates the module outputs to the module inputs. By consecutively analyzing each module in the sequence, the analyst can then relate the receiver outputs (performance) to the receiver inputs (signal characteristics).

Section 2 describes the general procedures for performing a communications receiver performance analysis. It introduces the fundamental concepts and describes the receiver model. Sections 3-8 present the transfer functions for the receiver modules. Section 9 presents two examples that demonstrate how this handbook can be used to perform a receiver analysis.

The first module in the receiver is designed to amplify the desired signal, convert it to an intermediate frequency (IF), and filter out some interference and noise. Section 3 provides a detailed description of the RF/IF section. It also specifies how interfering signals may be changed by the filters.

Section 4 describes the despreader module in spread spectrum receivers. It specifies how the spread spectrum processing gain can be calculated. It also provides information that can be used to analyze spread spectrum multiple-access systems.

Sections 5 and 6 describe the demodulator module. They provide plots that characterize performance as a function of the input power ratios. Analog voice and broadcast television receivers are considered in Section 5 and digital receivers are considered in Section 6.

Section 7 describes forward error correction (FEC) decoders, which use redundancy bits to reduce the bit-error rate (BER) by correcting some of the bit errors introduced by interference and noise. It provides plots of output BER vs. input BER for several types of FEC decoders.

Section 8 describes source decoders, which convert the information bit sequence to the final format at the receiver output. For a digital voice system, this format is an analog voice waveform. Section 8 provides plots that relate output signal quality to the input BER.

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GLOSSARY

ADC	Analog-to-Digital Converter
ADM	Adaptive Delta Modulation
AI	Articulation Index
AM	Amplitude Modulation
AS	Articulation Score
ASK	Amplitude-Shift Keying
BCH	Bose-Chaudhurie-Hocquenghem
BER	Bit Error Rate
BPSK	Binary Phase-Shift Keying
CFSK	Coherent Frequency-Shift Keying
CPM	Continuous-Phase Modulation
CVSD	Continuously Variable Slope Delta
CW	Continuous Wave
DAC	Digital-to-Analog Converter
DM	Delta Modulation
DPCM	Differential Pulse Code Modulation
DPSK	Differential Phase-Shift Keying
DSB	Double Sideband
E_b/N_o	Ratio of bit energy to noise power density
EIRP	Effective Isotropic Radiated Power
EMC	Electromagnetic Compatibility
FDR	Frequency-Dependent Rejection
FEC	Forward Error Correction
FM	Frequency Modulation
FSK	Frequency-Shift Keying
IF	Intermediate Frequency
MSK	Minimum-Shift Keying
NCFSK	Non-coherent Frequency-Shift Keying
NTIA	National Telecommunications and Information Administration
OQPSK	Offset Quadrature Phase-Shift Keying
PCM	Pulse Code Modulation
PN	Pseudo-noise

PRI	Pulse Repetition Interval
PSD	Power Spectral Density
PSK	Phase-Shift Keying
QAM	Quadrature Amplitude-Modulation
QBL-MSK	Quasi-Bandlimited Minimum-Shift Keying
QPSK	Quadrature Phase-Shift Keying
RF	Radio Frequency
RMS	Root-mean-square
RS	Reed-Solomon
SER	Symbol Error Rate
S/I	Signal-to-Interference Ratio
S/N	Signal-to-Noise Ratio
SSB	Single Sideband
TASO	Television Allocation Study Organization
WER	Word Error Rate

SECTION 1 – INTRODUCTION

1.1 BACKGROUND

In May 2003, President Bush established the Spectrum Policy Initiative to promote the development and implementation of a United States spectrum policy for the 21st century. In response to the Spectrum Policy Initiative, the Secretary of Commerce established a Federal Government Spectrum Task Force and initiated a series of public meetings to address policies affecting spectrum use by the federal government, state, and local governments, and the private sector. The recommendations resulting from these activities were included in two reports released by the Secretary of Commerce in June 2004.[1-1] In November 2004, the President directed the federal agencies to develop a plan to implement the 24 recommendations contained in the reports.[1-2] One of the recommendations directed the National Telecommunications and Information Administration (NTIA) to develop a handbook documenting best practices in spectrum engineering that will be recognized by all regulatory authorities in the United States. The Best Practices Handbook will address the electromagnetic compatibility (EMC) analysis process. Establishing receiver performance degradation levels is required to perform an EMC analysis. The NTIA requested that the Joint Spectrum Center develop a communications receiver performance degradation handbook that could be included as part of the Best Practices Handbook.

1.2 PURPOSE OF THE HANDBOOK

The purpose of the handbook is to provide the radio frequency (RF) analyst with the capability to calculate the effects of noise and interference on RF communications receivers. A receiver is modeled as a sequence of modules. Each module has a *transfer function* that relates the module outputs to the module inputs. By consecutively analyzing each module in the sequence, the analyst can then relate the receiver outputs (performance) to the receiver inputs (signal characteristics).

The first version of this document includes only transfer functions that were immediately available at the time of development. Future versions of the handbook could include an expanded set of functions.

[1-1] *Spectrum Policy for the 21st Century – The President's Spectrum Policy Initiative: Report 1*, Department of Commerce (June 2004), at http://www.ntia.doc.gov/reports/specpolini/presspecpolini_report1_06242004.htm; *Spectrum Policy for the 21st Century – The President's Spectrum Policy Initiative: Report 2*, Department of Commerce (June 2004), at http://www.ntia.doc.gov/reports.html.

[1-2] White House Executive Memorandum, Subject: *Improving Spectrum Management for the 21st Century* (23 November 2004). The latest released document for this subject is *Spectrum Management for the 21st Century – Plan to Implement Recommendations of the President's Spectrum Policy Initiative,* Department of Commerce (March 2006), at http://www.ntia.doc.gov/reports.html.

1.3 OVERVIEW OF HANDBOOK CONTENTS

Section 2 describes the general procedures for performing a communications receiver performance analysis. It introduces the fundamental concepts and describes the receiver model. It does not include detailed descriptions for the individual modules within the receiver.

The first module in the receiver is designed to amplify the desired signal, convert it to an intermediate frequency (IF), and filter out some interference and noise. Section 3 provides a detailed description of the RF/IF section. It also specifies how interfering signals may be changed by the filters.

Section 4 describes the despreader module in spread spectrum receivers. It specifies how the spread spectrum processing gain can be calculated. It also provides information that can be used to analyze spread spectrum multiple-access systems.

Sections 5 and 6 describe the demodulator module. They provide plots that characterize performance as a function of the input power ratios. Analog voice and broadcast television receivers are considered in Section 5 and digital receivers are considered in Section 6.

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Section 8 describes source decoders, which convert the information bit sequence to the final format at the receiver output. For a digital voice system, this format is an analog voice waveform. Section 8 provides plots that relate output signal quality to the input BER.

Section 9 presents two examples that demonstrate how this handbook can be used to perform a receiver analysis.

1.4 HOW TO USE THE HANDBOOK

The first step in using this handbook is to learn about the communications receiver performance analysis model discussed in Section 2. That section introduces some basic concepts and terminology and sets the context in which a receiver analysis may be performed. It also discusses the limitations and scope of the model. It displays the general structure of the receiver model as a sequence of distinct modules, each of which performs a specific signal-processing function.

The second step in using the handbook is to actually perform a receiver analysis. The analysis objective is either to determine a receiver performance measure value (such as BER) for a given set of input signal

conditions or, conversely, to determine the input signal conditions that would yield a given performance measure value.

The receiver modules are represented in the handbook by transfer functions, which are organized by module in Sections 3 through 8. Section 2.4.5 lists the specific steps to follow in using these transfer functions to perform an analysis. Section 9 gives two receiver analysis examples.

SECTION 2 – ANALYSIS PROCEDURES

This section describes the procedures for performing a communications receiver performance analysis. To set the context, it specifies types of RF systems and types of RF signals. Then it introduces some basic analysis concepts with a simple link budget analysis model. Finally, it describes the detailed model that is the subject of this handbook: the receiver performance analysis model.

2.1 SYSTEM TYPES

The focus of this handbook is on RF communications systems. The RF spectrum can be used for many purposes other than what is usually thought of as communications. Although many of the analysis results presented in this handbook may be applied to RF systems in general, all discussions and data will be presented in a communications context.

2.1.1 Communications

RF communications systems are designed to send a message from one point to another by converting the message to an RF signal and transmitting that signal.

2.1.1.1 Point-to-point

The most basic RF communications system is composed of a transmitter at a given location, a receiver at a given location, and the channel through which the RF signal propagates. Such a system is called a point-to-point system because the signal travels between two fixed points. This permits directional antennas to be used for both the transmitter and receiver. These antennas can reduce the output power required from the transmitter by several orders of magnitude. A point-to-point system is also called a communications link.

2.1.1.2 Point-to-multipoint

Another common type of RF communications system has one transmitter and many receivers, which are located at different points within the transmission range of the transmitter. In this case, the transmitter antenna is often non-directional, and must therefore transmit enough power to cover the appropriate area. Such a system is called a point-to-multipoint system. Each transmitter-receiver pair constitutes an independent communications link, and a link analysis may be performed for each one.

2.1.1.3 Mobile

Mobile systems require that the antennas not be constrained to favor one fixed direction over another. This can be accomplished very simply by utilizing omnidirectional antennas. Alternatively, tracking antennas can be used to adapt to changes in the direction of propagation. This adaptation can be accomplished physically, by moving the antennas, or electrically, by means of steerable-beam arrays.

2.1.2 Non-communications

There are many RF non-communications systems that are not designed to send a message from one point to another. RF technology may be used to determine geographic location. Examples include RADAR, global positioning system, and navigation systems. A special type of RADAR can be used for spatial data collection, particularly terrain data. Radio frequency identification tags are becoming commonplace, as a means of tracking the location of various objects. Remote control can be implemented with RF technology, enabling operators to perform tasks at inaccessible or inconvenient locations.

2.2 SIGNAL TYPES

RF signals may be classified in several ways. From the perspective of a receiver, an incoming signal may be either desired or undesired. An RF signal may also be classified according to its waveform properties. These properties are responsible for the ways in which the signal interacts with the receiver components.

2.2.1 Noise

Noise is present in every practical RF system component, and in every propagation channel. Noise imposes practical limits on RF communications, because it is always necessary for the receiver to distinguish the desired signal from the noise. Consequently, RF analysis results are often expressed as the ratio of the desired signal to the noise. Noise is modeled in the time domain as a signal whose amplitude varies according to the Gaussian probability distribution. Noise is modeled in the frequency domain as a signal that has a constant magnitude at all frequencies within the receiver band.

2.2.2 Continuous-Wave

A continuous-wave (CW) signal is a sinusoid. Its properties are, in a sense, the opposite of the properties of a noise signal. A CW signal has a constant-amplitude envelope, and only one frequency.

CW signals are used as RF carriers, beacons, and reference signals. Interfering (undesired) signals are sometimes modeled as CW signals.

2.2.3 Modulated

One signal can be used to control the amplitude, frequency, phase, or other property of another signal through a process called *modulation*. Communications signals are typically modulated signals, since the modulation process can be used to convey information. There are many specific modulation schemes. Each has certain advantages and disadvantages with respect to system performance. This handbook includes analysis results for many common modulation schemes. Modulated signals occupy more RF spectrum than CW signals; the emission bandwidth depends on the particular modulation technique (and on other characteristics such as data rate).

2.2.4 Intermittent

A signal may periodically or randomly exhibit large amplitude fluctuations for various reasons. This intermittent behavior may, in fact, constitute the most important property of a signal. Pulsed and frequency-hopping systems are examples of intermittent signal sources. For intermittent signals, it is not generally sufficient to consider properties (such as power) that have been averaged over a long period of time. Instead, it is necessary to consider the temporal fluctuation of signal properties and to include the effects of those fluctuations in the analysis.

2.3 LINK BUDGET ANALYSIS

A basic *link budget analysis* technique assumes that a communications link can be modeled as a series of independent modules originating at the transmitter and terminating at the receiver input. Each module is represented by its output power, gain, or loss. Link budget analysis determines the effect of each module on the desired or undesired signal power at the receiver input, without tracing a signal through the various components within the receiver. If the analysis shows the signal power to be too low for a given link, then more power must be budgeted or allocated for that receiver (e.g., by moving the antennas closer together).

2.3.1 Link Budget Model

The most basic model is composed of a transmitter and receiver linked by a propagation channel, as shown in Figure 2-1. The transmitter signal is represented by the transmitted power, which is delivered to the propagation channel. The channel subjects the signal to propagation loss, which is a reduction in

power of the transmitted signal. The received signal is simply the transmitted signal, reduced in magnitude by the propagation loss. The model in Figure 2-1 is surprisingly useful, given its simplicity. If there are multiple transmitters (a desired transmitter and one or more interfering transmitters), then the model in Figure 2-1 is used for each transmitter. For the desired transmitter, it yields the desired signal power at the receiver input. For an interfering transmitter, it yields the interfering signal power at the receiver input.
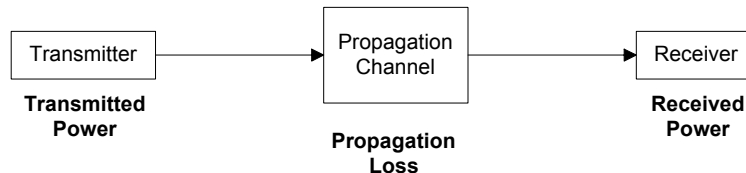
Figure 2-1. Basic Link Budget Model

The model in Figure 2-1 makes no mention of antennas, which are assumed to be lumped together with other modules. This may not be acceptable in some cases because it may be important to consider the antenna effects separately. Figure 2-2 shows a more detailed model. The transmitter and receiver antennas are represented by separate modules, permitting their contribution to the overall performance to be considered separately.
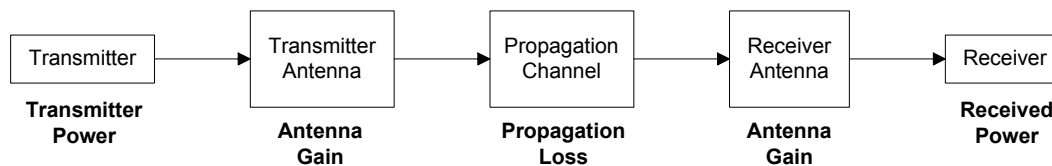
Figure 2-2. More Detailed Link Budget Model

Transmitter. The only transmitter characteristic considered is the transmitter output power P_T, which is typically expressed in decibels referenced to power units, such as decibels relative to one milliwatt (dBm) or decibels relative to one watt (dBW).

Transmitter Antenna. The effect of the transmitter antenna is expressed as a power gain G_T with respect to an isotropic antenna, expressed in units of dBi. Antennas radiate in all directions, but many antennas are designed to favor one or more particular directions. For these directional antennas, the gain will depend on the orientation of the antenna with respect to the propagation path. An antenna pattern, which shows the gain in all directions, may be used to determine the gain for a particular link. Antenna patterns usually describe the far-field radiation pattern.

Effective Isotropic Radiated Power (EIRP). This quantity, which combines the transmitter power with the transmitter antenna gain, is often used to simplify the analysis. If the antenna is directional, the EIRP will be valid only in the direction corresponding to the antenna gain that was used in determining the EIRP. The unit of EIRP is the same as for the transmitter power: dBm, dBW, etc.

Propagation Loss. The propagation loss is expressed as the ratio of the transmitted power to the received power, not including the antenna gains. Since this ratio is always greater than 1, the propagation loss in dB is always a positive quantity.

Receiver Antenna. The gain G_R of the receiver antenna with respect to an isotropic antenna is expressed in dBi. The same considerations of directionality that applied to the transmitter antenna also apply to the receiver antenna.

2.3.2 Link Budget Equation

A typical application of the link budget concept is the calculation of the received-signal power for a line-of-sight communications link:

$$\begin{aligned} P_R &= P_T + G_T + G_R - L \\ &= P_T + G_T + G_R - 20\,\log(d) - 20\,\log(f) + 27.6 \end{aligned}$$

(2-1)

where

$$\begin{aligned} P_R &= \text{signal power at receiver input, in dBm} \\ P_T &= \text{signal power at transmitter output, in dBm} \\ G_T &= \text{transmitter antenna gain, in dBi} \\ G_R &= \text{receiver antenna gain, in dBi} \\ L &= \text{free-space propagation loss, in dB} \\ d &= \text{distance between antennas, in meters} \\ f &= \text{transmitter frequency, in MHz} \end{aligned}$$

Equation 2-1 assumes free-space propagation. If the EIRP is specified instead of the transmitter power, then the following relationship is used:

$$P_R = EIRP + G_R - L$$

(2-2)

2.3.3 Link Budget Performance Measures

The received-signal power (of the desired signal) may be used to predict the overall quality of the communications link. To make such a prediction, it is necessary to have a specification of required performance to which the calculated quantity can be compared. These specifications depend on many factors, will vary from system to system, and should include external noise and interference.

A very basic specification is the minimum signal that the receiver can detect. This may be called the minimum discernable signal or the minimum detectable signal. However, the fact that a signal is strong enough to be discerned or detected does not imply that it is strong enough for the message to be accurately recovered.

A more useful specification is the receiver sensitivity. This is the minimum received signal level that the receiver can use to produce a certain required output. The required output will usually be a particular value of BER or signal-to-noise ratio (S/N).

2.3.4 Limitations of Link Budget Analysis

The simple link-budget analysis technique presented here is often used because of its simplicity. However, that simplicity is based on several assumptions which limit the usefulness of the technique.

It is assumed that average power provides a realistic and meaningful characterization of the input signal. This is valid if each of the independent variables in the analysis (e.g., the independent variables in Equation 2-1) is constant in time. Many systems, however, do exhibit time variation of one or more of these variables.

Equation 2-1 assumes free-space propagation. The effects of the propagation environment, except for distance, are ignored. This equation can be modified to accommodate more complex types of propagation loss. The propagation loss calculation for many practical systems requires the availability of sophisticated propagation models.

A link-budget analysis does not directly address the performance of the receiver. Even when the receiver sensitivity is used as a performance measure, the corresponding output BER or S/N may be unknown or may be inappropriate for the given context. For example, in the presence of an interfering signal the desired signal level must generally exceed the receiver sensitivity. The processing that takes place once the signals have entered the receiver determines the fidelity and usefulness of the final output. It is this final set of signal-processing operations within the receiver that this handbook addresses in detail.

2.4 RECEIVER PERFORMANCE ANALYSIS

In contrast to the link-budget analysis, a system performance analysis directly addresses the performance of the receiver. The system performance model is divided into three parts: the transmitter model, the RF channel model, and the receiver performance model. This handbook presents the methodology for performing an analysis with the receiver performance model.

2.4.1 System Performance Model

The system performance model is shown in Figure 2-3. The transmitter and receiver are modeled as systems which in turn are composed of a number of modules. These modules – or functional blocks – represent distinct signal-processing operations. The transmitter modules systematically prepare the information signal for propagation through the RF channel. The receiver essentially reverses the operations performed by the transmitter to obtain the information. The antennas have been included in the RF channel which, in addition, contains an interference source. The interference propagates through its own RF channel. The interference RF channel is not shown in the illustration.
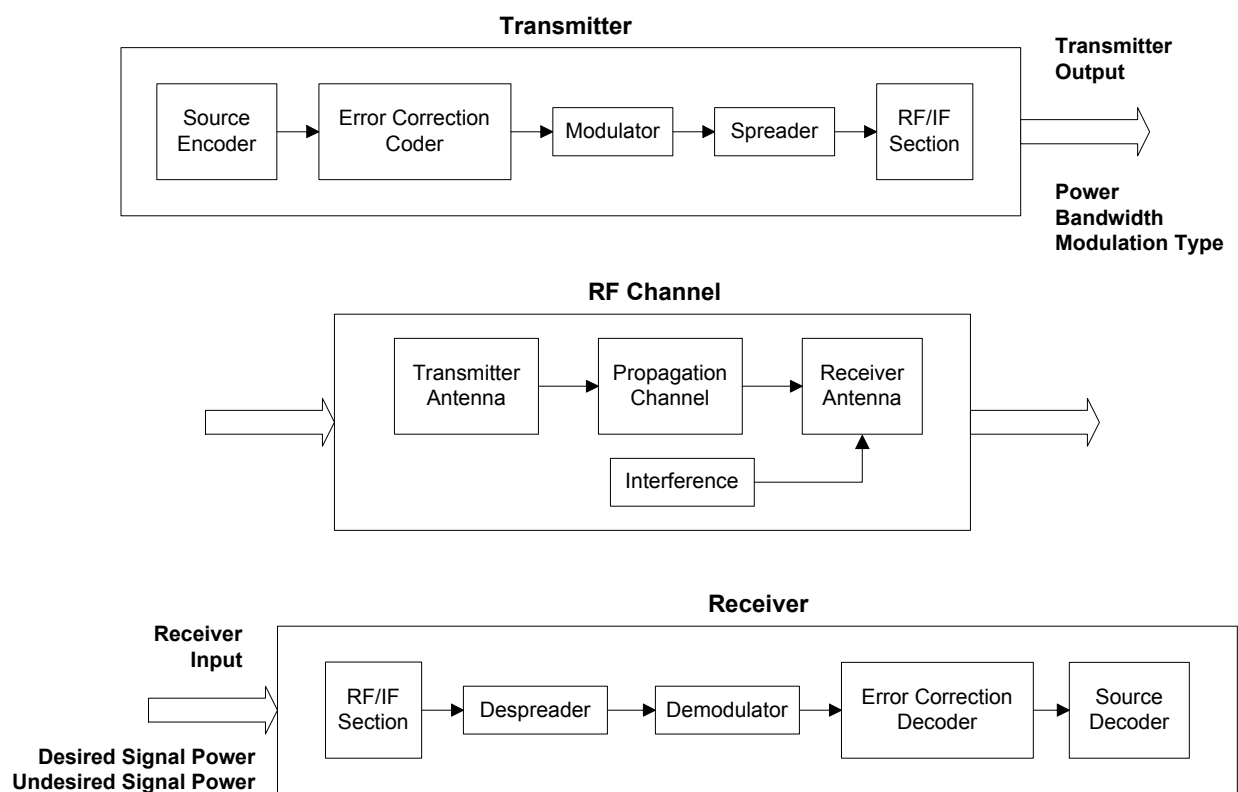
Figure 2-3. System Performance Model

One benefit of placing the antenna models in the RF channel model is that the effects of site location are grouped together. These effects include antenna orientation, separation distances, and environmental phenomena such as atmospheric absorption and ground conductivity. This organization permits analysts who specialize in RF propagation to focus on that part of the communication link. The communications system model in Figure 2-3 has proven to be a very practical and useful representation.

2.4.1.1 Transmitter

As modeled, the transmitter has five modules. Not all transmitters will have all five modules. For example, the transmitter of a purely analog system has only two modules: the RF/IF section and the modulator.

Source Encoder. The source encoder converts the source into a sequence of bits at a certain data rate. If the source is analog, an analog-to-digital converter (ADC) samples and encodes the analog waveform. If the source is digital, the source encoder reformats and retimes the input data if necessary. The output of the source encoder is a sequence of information bits, at the information data rate.

FEC Encoder. The FEC encoder encodes the data from the source encoder to meet requirements dictated by the communication channel. The FEC encoder adds code bits to the information bit sequence. In many cases, this requires that the data rate be increased to accommodate the extra bits. The output of the FEC encoder is a sequence of code bits, at the coded data rate.

Modulator. The modulator creates an analog waveform (carrier) whose properties vary in accordance with the input waveform or bit sequence. The amplitude, frequency, and phase of the carrier may be modified by the modulation process. There are a great many modulation schemes. The output of the modulator is a modulated carrier waveform. The frequency of this waveform is usually not suitable for RF transmission, and will have to be changed (usually increased) in subsequent processing.

Spreader. The digital modulator output waveform will have a bandwidth that is a function of the coded data rate and the modulation type. The spreader creates a so-called "spread-spectrum" signal, which increases this bandwidth. There are two common approaches to spectrum spreading. Frequency-hopping utilizes a number of different carrier frequencies rather than just one. These carriers span a frequency range that is much larger than if the single carrier were used. Direct sequence converts the input data rate to a much higher output data rate. The resulting waveform has a much larger bandwidth than if the lower data rate were used.

RF/IF Section. The RF/IF section provides frequency translation, filtering, and amplification to prepare the signal for transmission. The output of this section is delivered to the transmitter antenna by a cable, waveguide, or some other conductor.

2.4.1.2 RF Channel

Transmitter Antenna. The transmitter antenna may be designed to focus RF energy in a particular direction for a point-to-point system, or it may be omnidirectional, to support point-to-multipoint and mobile communications. The transmitter antenna adds gain (or loss) to the transmitter output.

Propagation Channel. The most basic propagation channel is free space – the transmitted signal is attenuated only by distance. A more realistic propagation channel could include the effects of natural and man-made obstructions, atmospheric effects, ground effects, and other effects. Propagation modeling can be very complex. Many models have been developed to simulate the propagation of RF energy in various environments. The use of these models is somewhat specialized, and is not covered in this handbook. The ultimate purpose of a propagation model is to predict the magnitude (and possibly other characteristics such as phase) of the transmitted signal at the receiver location.

Interference. The interference source is typically another transmitter in the environment. To simplify Figure 2-3, it is shown as a single block that feeds the receiver antenna block. The actual transmitter, of course, creates a signal in its own modules and sends that signal through its antenna and through a separate propagation channel. In addition, there may be multiple interfering transmitters.

Receiver Antenna. The purpose of the receiver antenna is to collect sufficient energy from the transmitted signal so that the transmitted information can be extracted by the receiver system. Depending on its design and orientation, the receiver antenna may add gain or loss to the signal.

2.4.1.3 Receiver

The receiver takes its input from the receiver antenna, and performs processing complementary to that done by the transmitter.

RF/IF Section. The RF/IF section converts the frequency of the received signal to an IF that is more suitable for signal processing. It also filters the composite signal to reduce the undesired signal power.

Despreader. The despreader, in the case of a frequency-hopping system, resolves the multiple-carrier input so that a single IF is obtained. In the case of a direct-sequence system, the despreader removes the spreading code from the desired signal.

Demodulator. A digital demodulator converts the input waveform to a bit sequence. If error correction coding has been applied, this is the coded bit sequence, at the coded data rate. An analog demodulator produces a replica of the original baseband analog waveform (but with some noise, interference, and distortion).

FEC Decoder. The FEC decoder performs the digital processing necessary to reconstruct the original information bit sequence. The output is the information bit sequence, at the information bit rate. This bit sequence may contain residual errors.

Source Decoder. The source decoder converts the information bit sequence to the format required by the user of the receiver. If this format is analog, a digital-to-analog converter (DAC) will be employed at this stage. The output of the source decoder is the output of the receiver. In general, it will differ somewhat from what was transmitted due to noise and interference. This difference can be quantified, and is a measure of the overall system performance.

2.4.2 System Performance Measures

The goal of an RF analysis is to predict the performance of a system quantitatively. The performance prediction can then be compared to certain performance goals to obtain an assessment of the system operation. There are several useful measures of performance.

2.4.2.1 Signal-to-Noise Power Ratio

The S/N is calculated by determining the desired-signal power S and the noise power N at some point in the system. The S/N is always expressed in dB. Note that if S and N are in decibel units, such as dBm, the signal-to-noise "ratio" is actually S - N. Depending on the system and the application, target performance goals for S/N might range from 10 to 30 dB. If noise-like interference is present, then the variable N represents the sum of the interference power and the noise power (added in non-logarithmic units such as mW).

2.4.2.2 Signal-to-Interference Power Ratio

The signal-to-interference power ratio (S/I) is calculated when there is an undesired signal which cannot be adequately modeled as additional noise. For example, in addition to noise there may also be a CW interfering signal.

2.4.2.3 Bit Error Rate

The BER, which obviously applies to digital systems, can be interpreted two ways. It is the probability of a particular bit being wrong, and it is also the ratio of bit errors to total bits received in the long term. The latter interpretation is useful when the error-causing process is constant over the analysis interval. The target BER after error correction is often between 10^{-3} and 10^{-6}.

2.4.2.4 Subjective Performance Measures

Measures of signal power or BER do not directly address the quality of communication as perceived by users of the system. Some subjective performance measures have been defined that do address that quality of communication. For voice communications, the articulation score (AS) is defined as the percentage of words, phrases, sentences, or other message elements correctly identified by a listener panel. The Television Allocation Study Organization (TASO) score is a similar evaluation of television picture quality by a viewer panel. The articulation index (AI) is a calculated quantity that is designed to be a predictor of voice intelligibility.

It is possible to establish a correspondence between objective performance and subjective performance by extensive, careful testing and polling. It is very difficult, however, to extrapolate subjective test results beyond the parameter set used for the tests.

2.4.2.5 Time-varying Performance

When reporting results in terms of S/N or BER, it may be assumed that these quantities are relatively constant. This implies, of course, that the phenomena responsible for these measures of performance are relatively constant. If this is not the case, then the temporal variations should be taken into account. These variations are often expressed statistically. For example, a quality-of-service target might be a BER of less than 10^{-5} at least 90% of the time, and a BER of greater than 10^{-3} no more than 1% of the time.

2.4.3 System Performance Analysis Limitations

In Section 2.3.4, the limitations of a link budget analysis (exclusion of systems with temporal variations, unsophisticated propagation model, no detailed receiver model) were presented. The first two of those limitations are still concerns in a system performance analysis. A system performance analysis does have detailed receiver modeling, but some effects occur within a receiver that are not included; namely, the nonlinear effects that occur when an interfering signal is very strong.

2.4.3.1 Temporal Variations

The analysis techniques and transfer functions presented in the handbook are useful for systems whose performance does not vary significantly with time. These are systems to which non-time-varying quantities such as average power, bit-error rate, S/N, and S/I may be assigned. If a system does exhibit significant temporal variation, then the nature of the variation must be considered. Performance measures must be revised accordingly. Two types of temporal variations are fading and intermittent interference.

Fading. The term *fading* applies to unexpectedly large variations in the desired signal power at the receiver. The cause of the variation may be understood, but may be impractical to model. For example, fading may be caused by multipath interference in mobile communications. Small changes in position can result in large changes in the way the signal replicas combine at the receiver, leading to occasional periods of weak reception known as *fades*.

Fading may be accounted for very simply by specifying a *fade margin*, which is an increase in the magnitude of received-signal power that is required for proper system operation. The size of the fade margin may be determined experimentally by field measurement, or may be based on experiences with similar systems in similar environments.

Outages. An *outage* is a period of time during which system operation is so compromised that for all practical purposes it does not work at all. It is therefore not helpful to attempt to quantify receiver performance during an outage. Instead, the process responsible for the outage should be analyzed, and the effects on the system modeled statistically. An outage may be a single, catastrophic event, such as a lightning strike. In this case, a typical performance measure is simply the probability of occurrence.

Intermittent interference tends to produce repetitive outages. For example, each pulse from a pulsed radar or each co-channel hop from a frequency hopping transmitter may produce an outage. For repetitive outages, the performance measure must be tailored to the system that is affected. Such performance measures typically specify the required performance during the "quiescent" periods between outages, as well as the maximum frequency or duration of the outages. For example, a system may be required to operate below a certain BER a certain percentage of the time. The rest of the time, the system may experience outages during which communications will be interrupted.

Intermittent interference may also result in periods of partially compromised performance, rather than full outages. A statistical analysis is generally required to evaluate such systems.

2.4.3.2 Complex RF Propagation Environments

In any analysis one can assume a free-space propagation path loss. In many cases, this is not a valid assumption. The propagation environment includes natural and manmade obstructions such as terrain, foliage, and buildings. These obstructions affect propagation by blocking, diffracting, absorbing, scattering, and reflecting RF energy. The atmosphere also affects propagation through refraction, absorption, ducting, and scattering. These environmental effects are frequency dependent and, in some cases, intermittent in nature. Nevertheless, loss from propagation is usually the largest factor in the link budget – by many orders of magnitude.

There are many analytical and simulation-based RF propagation models available. These should be employed when needed to predict the RF propagation loss and, in turn, the received-signal power.

2.4.3.3 Collocated RF Equipment

A cosite environment is one in which interference power levels are so high that nonlinear effects such as desensitization and intermodulation must be considered. The interfering signal power level may be high because the transmitter and receiver antennas are close together (collocated). A typical cosite environment is a platform such as a ship, airplane, or tower on which multiple antennas are located.

As an example, consider the case in which a strong interfering signal is far off-tune from the receiver frequency. Because it is far off-tune, it is attenuated so much by the receiver filters that the residual signal has no effect on the receiver. However, before it enters those filters it passes through the RF amplifier. If its power is outside of the intended (linear) operating range of the amplifier, it will generally desensitize the amplifier; that is, reduce the amplifier gain. This, in turn, reduces amplification of the desired signal power so that the desired signal gets lost in the noise. Thus, the positive effects of filtering have been defeated by the nonlinear effect.

Cosite effects are not included in this handbook. For cases in which filtering plays a significant role in determining an interference threshold, the analyst should ensure that the signal power is sufficiently low that nonlinear effects will not occur. If nonlinear effects are possible, cosite analysis techniques should be applied to adjust the frequency and/or power level of the interfering signal that is input to the RF/IF section (see Figure 2-3).

2.4.4 Receiver Performance Analysis Types

There are generally three techniques – analytical, measurement, and simulation – that can be used to assess the performance of a receiver.

In the *analytical* technique, the operations of receiver modules are represented by equations that can be solved in closed form or by numerical methods. Many of the transfer functions in the handbook were obtained analytically.

In the *measurement* technique, receiver equipment is acquired and subjected to laboratory measurement. This technique is useful for investigating the performance of specific hardware, but it is lacking in generality. Measurement was not used directly in developing the receiver transfer functions, but many of the results have been confirmed by measurement.

In the *simulation* technique, software generates samples of the signal waveforms and processes these samples through the simulated receiver components. Simulated measurement devices then determine performance measures such as BER. Software simulation of RF systems is useful for systems that are too difficult to characterize analytically. Simulation permits a fine degree of control over system parameters. Many of the results in the handbook were obtained by simulation.

These techniques can be combined. For example, measurement data for a functional block can be entered into a simulation. A hardware module can even be used directly as a functional block in a simulation system. This technique may be used to test specific modules in the controlled, repeatable environment that a simulation system provides.

2.4.5 Receiver Performance Analysis Steps

A receiver performance analysis is that part of the system performance analysis that focuses on processing within the receiver. The communications receiver model is shown in Figure 2-4, which has been extracted from Figure 2-3. Each processing block is represented by a transfer function that expresses the output of the block in terms of the input.
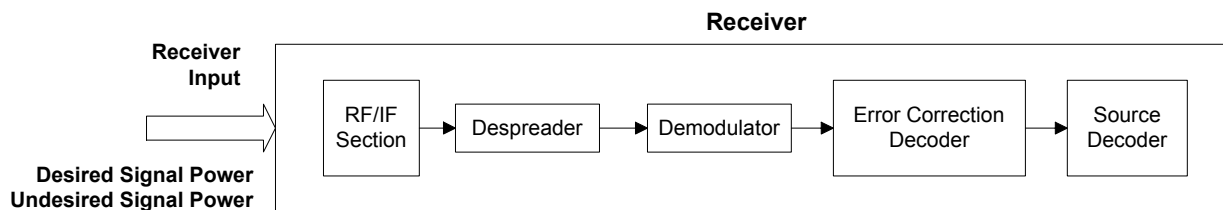
Figure 2-4. Receiver Performance Model

In one common situation, the required performance measure value for the receiver is known and the objective is to determine the corresponding input power ratios (e.g., S/N and S/I). In this case, the

analyst steps through the modules in Figure 2-4 in right-to-left order, although part of the RF/IF calculation must be done first. The analysis steps for this case are specified below:

1. Determine which modules in Figure 2-4 apply to the receiver.

2. Determine the nature of the interference at the output of the RF/IF section. Section 3 describes how this is determined.

3. If the interference is intermittent and a simple manual analysis cannot be performed, stop. Section 3 suggests alternate analysis techniques for such cases.

4. Determine the performance criterion at the receiver output. Depending on the receiver, this might be a minimum AI or maximum BER value at the demodulator output, a maximum BER value at the FEC decoder output, or a minimum S/N value at the source decoder output.

5. If there is a source decoder, determine the maximum BER at the source decoder input. Use the Input BER vs. Output S/N curves in Section 8. (The maximum BER corresponds to the minimum S/N value.)

6. If there is a hard-decision FEC decoder, determine the maximum BER at the FEC decoder input. Use the Input BER vs. Output BER curves in Section 7. (The maximum input BER corresponds to the maximum output BER.) Then go to Step 8.

7. If there is a soft-decision FEC decoder, perform the operations described in Steps 8 and 9, except that you will use the soft-decision curves in Section 7 rather than the digital demodulator curves in Section 6. (The soft-decision curves incorporate both demodulator and FEC decoder effects.)

8. If the interference at the output of the RF/IF section is noise-like (as determined in Step 2), determine the minimum S/N value at the demodulator input. In this case, N represents the total noise-like signal power, including receiver noise power and the interference power. Use the analog demodulator curves in Section 5 or the digital demodulator curves in Section 6.

9. If the interference at the output of the RF/IF section is not noise-like (as determined in Step 2), select a particular value of S/I and determine the corresponding minimum S/N value at the demodulator input. In this case, N represents the receiver noise power. Repeat this for several values of S/I. Use the analog demodulator curves in Section 5 or the digital demodulator curves in Section 6.

10. If there is a despreader, calculate the processing gain in dB. Section 4 describes the calculation.

11. Calculate the frequency-dependent rejection (FDR) in dB of the interference in the RF/IF section. Section 3 describes the calculation.

12. Add the processing gain from Step 9 and the FDR from Step 10 to get the total interference power loss in dB.

13. If the interference at the output of the RF/IF section is noise-like (as determined in Step 2), add the total interference power loss in dB from Step 11 to the minimum S/N value in dB from Step 7.

14. If the interference at the output of the RF/IF section is not noise-like (as determined in Step 2), add the total interference power loss in dB from Step 11 to each S/I value in dB from Step 8.

In another common situation, all signal power levels are known and the objective is to determine the performance measure value for the receiver. In this case, the analyst steps through the modules in Figure 2-4 in left-to-right order. The same procedure is followed, except that Steps 4 through 14 are reversed.

SECTION 3 – RF/IF SECTION

This section describes the RF/IF section of a receiver and the model that represents it. It presents the FDR concept and shows how the FDR can be calculated. Then it specifies changes in the interfering signal waveform that may be caused by the RF/IF section. Finally, for cases involving intermittent interference that are beyond the scope of this handbook, it briefly describes the types of analysis that might be required.

3.1 DESCRIPTION

The RF/IF section of a receiver is the first part of the receiver through which a signal from the antenna passes. It generally has amplifiers, mixers, filters, and perhaps other components such as automatic gain control circuits. The numbers of each component type and their specific characteristics vary from one receiver to another. A receiver performance analysis usually focuses on the amplifiers, mixers, and filters.

Typically, there are several amplifiers in the RF/IF section. Each amplifies the total composite signal at its input – including the desired, interfering, and noise components. Because the desired and interfering signals experience the same gain in an amplifier, the S/I does not change. However, the RF/IF amplifiers are the primary source of receiver noise. The noise from these amplifiers passes through all subsequent amplifiers and filters in the RF/IF section. The power of the resultant noise signal is referenced to the receiver input, which is equivalent to pretending that the noise signal enters the receiver from the antenna and experiences the full gain of all the RF/IF amplifiers. Thus, the S/N is also unchanged when the composite signal (including receiver noise) passes through the RF/IF amplifiers. A typical noise power calculation is presented in Section 9.2.7.

If a very strong interfering signal enters a receiver amplifier, it may cause nonlinear effects such as desensitization to occur (Section 2.4.3.3). This may happen, for example, when the interfering transmitter antenna is close to the receiving antenna and there is very little propagation loss to reduce the interfering signal strength. Nonlinear effects occur primarily in the RF/IF section, but they are beyond the scope of this handbook. It is assumed that the interfering signal power is sufficiently low that nonlinear effects are negligible. With this assumption, and given the fact that the amplifiers do not change the S/I or S/N, it is unnecessary to include amplifier effects in the RF/IF section model.

At the receiver input, the desired signal carrier frequency is tunable. The mixers convert that tunable frequency to a fixed frequency. This conversion typically occurs in two or three stages, with a different frequency at each stage. The mixer also converts the carrier frequency of the interfering signal in the

same way. Therefore, the frequency separation between the desired and interfering signals is not changed by the mixing process, and the filter attenuation of the interfering signal is unaffected by mixing. Assuming again that nonlinear effects are negligible, it is also unnecessary to include mixer effects in the RF/IF section model.

Typically, each RF/IF stage contains bandpass filtering to reduce or eliminate unwanted signals. A given bandpass filter is centered on a specific frequency. An interfering signal at a given frequency separation from this center frequency will generally experience some attenuation. The attenuation vs. frequency separation function is the *selectivity* of the filter. Although different filters in the RF/IF section have different center frequencies, the filter selectivities can be aligned and added (in dB) to form one composite function. Thus the RF/IF section model is simply a model of a single composite filter. The selectivity of that filter is known as the *receiver selectivity*.

The RF/IF filter produces two effects that significantly impact performance. The first effect is that interfering signals may be attenuated by the filter. The FDR is the total attenuation integrated across the interfering signal spectrum (Section 3.2). Viewed as a transfer function, the FDR relates the interference power at the RF/IF input to the interference power at the RF/IF output.

The second effect is that the shape of the interfering signal waveform may be changed by the RF/IF filter (Section 3.3). For example, the pulses in a pulsed radar signal may be smeared together so that the resulting signal is no longer pulsed. These waveform changes may result in corresponding changes in the receiver performance measure. A specification of the output waveform characteristics as a function of the input waveform characteristics can be viewed as a (non-mathematical) transfer function for the RF/IF section, because it relates the module outputs to the module inputs.

3.2 FDR

The FDR is the total filter loss experienced by an interfering signal in all the RF/IF stages in a receiver. The FDR, expressed as a pure ratio rather than in dB, is the ratio of the input interference power to the output (filtered) interference power. It is given by:

$$FDR = \frac{\int_{0}^{\infty} s_{max} s(f - f_T)\,df}{\int_{0}^{\infty} s_{max} s(f - f_T)\,r(f - f_R)\,df}$$

(3-1)

where f =   absolute frequency, in MHz

f_T =   transmitter tuned frequency, in MHz

f_R =   receiver tuned frequency, in MHz

s_{max} =   maximum power spectral density of the interfering signal at the receiver input, in W/MHz

$s(f - f_T)$ =   power spectral density of the interfering signal at the receiver input relative to the maximum value (unitless ratio)

$r(f - f_R)$ =   receiver selectivity (unitless ratio)

The integral in the numerator of Equation 3-1 is the power at the receiver input. The integral in the denominator of Equation 3-1 is the power at the output of the receiver RF/IF filters. These integrals can be evaluated numerically, but Equation 3-1 is often approximated by simpler equations, as shown below.

The interfering signal is assumed to be on-tune with the receiver if:

$$\left| \Delta f \right| \leq \mathrm{Max}\left(\frac{B_T}{2}, \frac{B_R}{2} \right)$$

(3-2)

where Δf =   frequency separation between the interfering signal and the desired signal, in MHz

$\mathrm{Max}(a,b)$ =   function that takes the maximum of a and b

B_T =   transmitter 3-dB bandwidth, in MHz

B_R =   receiver 3-dB bandwidth, in MHz

In a common approximate model for the transmitter spectrum, it is assumed that $s(f - f_T) = 1$ within $B_T/2$ of the transmitter tuned frequency and falls quickly to zero outside that band. Thus the numerator of Equation 3-1 is approximately equal to $s_{max} B_T$. Similarly for the receiver, it is assumed that $r(f - f_R) = 1$ within $B_R/2$ of the receiver tuned frequency and falls quickly to zero outside that band.

For an on-tune signal, the FDR is approximately:

$$
\begin{aligned}
FDR &= \frac{s_{max} B_T}{s_{max} B_T} = 1 \qquad \text{if } B_T \leq B_R \\[2ex]
&= \frac{s_{max} B_T}{s_{max} B_R} = \frac{B_T}{B_R} \qquad \text{if } B_T > B_R
\end{aligned}
$$

(3-3)

For an off-tune signal, the FDR is approximately:

$$FDR = \frac{s_{max} B_T}{s_{max} B_R \, s(f_R - f_T) + s_{max} B_T \, r(f_T - f_R)}$$

$$= \frac{1}{\left(\dfrac{B_R}{B_T}\right) s(f_R - f_T) + r(f_T - f_R)}$$

(3-4)

Equation 3-4 is obtained by expressing the integral in the denominator of Equation 3-1 as the sum of two integrals, one over the receiver region ($f \approx f_R$) and one over the transmitter region ($f \approx f_T$). It is then assumed that $s(f - f_T)$ is approximately constant in the receiver region and $r(f - f_R)$ is approximately constant in the transmitter region.

Converting to dB, the FDR is given by:

$$FDR_{dB} = 0 \quad \text{if on-tune and } B_T \leq B_R$$

$$FDR_{dB} = 10 \log\left(\frac{B_T}{B_R}\right) \quad \text{if on-tune and } B_T > B_R$$

$$FDR_{dB} = -10 \log\left[\left(\frac{B_R}{B_T}\right) s(f_R - f_T) + r(f_T - f_R)\right] \quad \text{if off-tune}$$

(3-5)

As an example, suppose that an off-tune interfering signal has a 10-kHz bandwidth and that the receiver has a 4-kHz bandwidth. Also suppose that the transmitter spectrum is 20 dB down at the receiver frequency and the receiver selectivity is 30 dB down at the transmitter frequency. Then the FDR is approximately:

$$FDR_{dB} = -10 \log\left[\left(\frac{B_R}{B_T}\right) s(f_R - f_T) + r(f_T - f_R)\right]$$

$$= -10 \log\left[\left(\frac{0.004}{0.010}\right) 10^{-20/10} + 10^{-30/10}\right] = 23 \text{ dB}$$

Expressing the FDR as a transfer function, the input S/I and output S/I are related as follows:

$$\left(\frac{S}{I}\right)_{\text{out}} = \left(\frac{S}{I}\right)_{\text{in}} + FDR_{\text{dB}}$$

(3-6)

3.3 CHANGES TO INTERFERENCE WAVEFORM

The RF/IF composite filter may change the shape of the interfering signal waveform. A specification of the output waveform characteristics as a function of the input waveform characteristics is therefore an additional transfer function for the RF/IF section. There are three cases of interest: (1) when either the interfering transmitter or the receiver is a frequency hopper, (2) when the interfering signal is pulsed (either unmodulated or swept-frequency), and (3) when the interfering signal is digital.

3.3.1 Frequency Hoppers

3.3.1.1 Fixed-Frequency Receiver

Consider the case in which a frequency-hopping transmitter interferes with a fixed-frequency receiver. The interfering signal typically hops from one carrier frequency to another at some fixed hop rate (e.g., 100 hops/s). The *dwell time* is the duration of transmission at any particular frequency. For a small time interval when the transmitter is switching frequencies, the signal is turned off at the transmitter. This off-time is usually sufficiently small that the dwell time is approximately the reciprocal of the hop rate.

When the interfering signal frequency changes, the frequency separation between the interfering signal and the receiver changes. As a result, the FDR generally changes. Therefore, the interfering signal power at the RF/IF output may vary significantly from one hop to the next. In this case, an input signal that is essentially continuous (except for a small off-time) is converted into an intermittent signal that has large amplitude variations.

3.3.1.2 Frequency-Hopping Receiver

Consider the case in which a narrowband fixed-frequency transmitter interferes with a frequency-hopping receiver. When the desired signal frequency changes, the frequency separation between the interfering signal and the receiver changes, and the FDR changes. Therefore, the interfering signal power at the RF/IF output may vary significantly from one hop to the next. Just as in the previous case, the RF/IF section converts a continuous input signal into an intermittent signal.

If both the interfering transmitter and the receiver are frequency hoppers, essentially the same phenomenon occurs. The only difference is that the interfering signal power at the RF/IF output will

change more frequently, because the FDR generally changes when either the interfering signal or the receiver changes frequencies.

Because the intermittent nature of the output interference is caused by fluctuating FDR values, there are cases involving frequency hoppers that don't result in intermittent output signals. In particular, a direct sequence interfering signal (Section 4) may have a bandwidth that covers the entire range of frequencies in the receiver hopset. In that case, the FDR does not change when the receiver hops because every receiver frequency is on-tune with some portion of the interfering signal band. Therefore, the output signal is not intermittent.

3.3.2 Pulsed Interfering Signals

The RF/IF filters in a receiver may distort a pulsed interfering signal in such a way that the pulsewidth of the signal changes. The pulses may even be smeared together so that the resulting signal is no longer pulsed. In general, the type of distortion depends on whether the pulse is on-tune or off-tune. A pulse is on-tune if Equation 3-2 is satisfied. Although the spectrum of a pulsed signal consists of discrete lines, the signal bandwidth (which appears in Equation 3-2) is normally based on the envelope of the spectrum.

An important characteristic of the RF/IF composite filter is its *impulse response*, which is the reciprocal of the filter bandwidth ($1/B_R$). If B_R is in MHz, then $1/B_R$ is in μs. It characterizes the ability of the filter to resolve waveform events of short duration. For example, if $1/B_R$ is much less than the pulsewidth τ of a single on-tune input pulse, then the filter is able to produce an undistorted output pulse of the same duration τ. However, if $1/B_R$ is greater than τ, then the output pulsewidth is $1/B_R$ (because $1/B_R$ is the smallest resolvable pulsewidth). In this case, the energy of the input pulse is spread out to form a longer pulse of lower amplitude.

Table 3-1 specifies the signal waveform at the RF/IF output when the input interfering signal is pulsed. There are nine different cases shown. In all cases, the input signal is pulsed with a pulsewidth τ and a *pulse repetition interval (PRI)*, which is the time between the leading edges of two successive pulses. In some cases, the frequency of each pulse is linearly swept through a range of frequencies Δf_c. The resulting signals are called *linear frequency modulated* or *chirp* signals.

In Table 3-1, "NA" means that the item is not applicable. In Cases 2 through 8, the output waveform is a pulse train with the same PRI as the input waveform. In Case 9, the output waveform is a train of pulse pairs; the two pulses within a pair are separated by τ and successive pairs are separated by the same PRI as the input waveform.

Table 3-1. RF/IF Output Waveform for Various Interfering Input Signals

Case	Δf	Modulation	Bandwidth Condition	Resolution Condition	Output Waveform
1	On-tune or off-tune	Unmodulated or chirp	NA	$\dfrac{1}{B_R} \geq PRI > \tau$	Continuous CW-like signal
2	On-tune	Unmodulated	NA	$PRI > \dfrac{1}{B_R} > \tau$	One pulse per PRI with pulsewidth $1/B_R$
3	On-tune	Unmodulated	NA	$PRI > \tau \geq \dfrac{1}{B_R}$	One pulse per PRI with pulsewidth τ
4	On-tune	Chirp	$B_T > B_R$	$PRI > \dfrac{1}{B_R} > \dfrac{B_R}{B_T}\tau$	One pulse per PRI with pulsewidth $1/B_R$
5	On-tune	Chirp	$B_T > B_R$	$PRI > \dfrac{B_R}{B_T}\tau \geq \dfrac{1}{B_R}$	One pulse per PRI with pulsewidth $(B_R/B_T)\tau$
6	On-tune	Chirp	$B_R \geq B_T$	$PRI > \dfrac{1}{B_R} > \tau$	One pulse per PRI with pulsewidth $1/B_R$
7	On-tune	Chirp	$B_R \geq B_T$	$PRI > \tau \geq \dfrac{1}{B_R}$	One pulse per PRI with pulsewidth τ
8	Off-tune	Unmodulated or chirp	NA	$PRI > \dfrac{1}{B_R} > \tau$	One pulse per PRI with pulsewidth $1/B_R$
9	Off-tune	Unmodulated or chirp	NA	$PRI > \tau \geq \dfrac{1}{B_R}$	Two pulses per PRI, separated by τ, each with pulsewidth $1/B_R$

3.3.2.1 Unresolvable Pulses

In Case 1, the impulse response $1/B_R$ is greater than the time between pulses (PRI). This case includes on-tune or off-tune signals that may be chirp or unmodulated. A single pulse by itself would be stretched to the duration $1/B_R$. However, because this duration is greater than the time between pulses, the stretched pulses in a pulse train overlap and form a continuous waveform. The exact nature of this waveform depends on where the spectral lines lie relative to the receiver passband. In the worst case, there is one spectral line within the receiver passband. In this case, the output waveform is typically CW-like.

3.3.2.2 On-tune and Unmodulated Pulses

In Cases 2 and 3, the input interfering signal is an on-tune and unmodulated pulsed signal. In Case 2, the impulse response is less than the PRI but greater than the input pulsewidth. Each pulse is stretched to the duration $1/B_R$. Thus, the output waveform has one pulse of pulsewidth $1/B_R$ for each input pulse. In Case 3, the impulse response is less than the input pulsewidth, so the filter is able to produce an undistorted output pulse of the same duration. Thus, the output waveform has one pulse of pulsewidth τ for each input pulse.

3.3.2.3 On-tune Shortened Chirp Pulses

In Cases 4 and 5, the input interfering signal is an on-tune chirp signal. For this type of signal, there is one additional factor to consider. If the frequency of the pulse is swept through a range of frequencies Δf_c that exceeds the IF bandwidth, then the pulse will be attenuated at any frequencies outside the IF bandwidth. This shortens the pulsewidth by the factor $B_R/\Delta f_c$. Because the transmitter bandwidth B_T is nominally equal to Δf_c, the shortening factor is B_R/B_T. The bandwidth condition $B_T > B_R$ specifies that Δf_c does exceed the IF bandwidth. Cases 4 and 5 are the same as Cases 2 and 3, except that the shortened pulsewidth $[(B_R/B_T)\tau]$ replaces the pulsewidth τ.

3.3.2.4 On-tune Unshortened Chirp Pulses

Cases 6 and 7 are the same as Cases 4 and 5, except that the bandwidth condition $B_R \geq B_T$ specifies that Δf_c does not exceed the IF bandwidth. Since the pulse does not sweep to frequencies outside the IF bandwidth, the pulse shortening effect does not occur. Therefore, Cases 6 and 7 are equivalent to Cases 2 and 3, respectively.

3.3.2.5 Off-tune Pulses

In Cases 8 and 9, the input interfering signal is an off-tune pulsed signal. For an off-tune pulse, whether it is an ordinary or a chirp pulse, the filter produces an impulse response at each edge of the pulse that experiences less attenuation (FDR) than the center of the pulse. If $1/B_R$ is less than τ (Case 9), then the resulting output waveform is a pair of pulses of width $1/B_R$, separated by the original pulsewidth. If $1/B_R$ is greater than τ (Case 8), then the two edge responses overlap to form one pulse of width $1/B_R$.

3.3.2.6 Analysis Considerations

Case 1 in Table 3-1 is the only case in which the RF/IF section converts a pulsed signal into a signal that is not intermittent. In this case, the methods of this handbook can be applied. Because the RF/IF output

signal is not pulsed, the *average power* of the signal (averaged over a complete pulse repetition interval), rather than the *peak power* (power during the pulse), should be used in the analysis. These are related as follows:

$$P_{\text{avg}} = P_{\text{peak}} + 10 \log\left(\frac{\tau}{PRI}\right)$$

(3-7)

where P_{avg} =   average power of the signal, in dBm
  P_{peak} =   peak power of the signal, in dBm
  τ =   pulsewidth, in µs
  PRI =   pulse repetition interval, in µs

The output signal in Cases 2 through 8 is pulsed, and so the methods of this handbook cannot be applied to those cases. They have been included in Table 3-1 for those who may wish to apply other methods. Section 3.4 gives a brief introduction to the types of analysis that may be appropriate. In such an analysis, it is necessary to include the fact that the peak power of the output pulse may differ from the peak power of the input pulse, not only because of FDR but also because the energy is spread across a different pulsewidth. These are related as follows:

$$P_{\text{out}} = P_{\text{in}} - FDR_{\text{dB}} + 10 \log\left(\frac{\tau}{\tau_{\text{out}}}\right) - a$$

(3-8)

where P_{out} =   peak power of the output pulse, in dBm
  P_{in} =   peak power of the input pulse, in dBm
  FDR_{dB} =   frequency dependent rejection, in dB (Section 3.2)
  τ =   input pulsewidth, in µs
  τ_{out} =   output pulsewidth, in µs
  a =   3 dB   for Case 9 (each output pulse has half the power)
    =   0 dB   for Cases 2 through 8

3.3.3 Digital Interfering Signals

A digital interfering signal is similar in some ways to an unmodulated pulsed signal. Each bit (or chip for a spread spectrum transmitter) can be thought of as a pulse, but with no "dead time" between pulses. Many of the pulsed signal concepts from Section 3.3.2 are also applicable to digital signals, but most

types of distortion have little effect on receiver performance. However, one type of distortion occurs that is significant: when the interfering signal bit duration is less than the impulse response ($1/B_R$) of the RF/IF composite filter, then the output interfering signal is a noise-like signal.

3.4 ANALYSIS WITH INTERMITTENT SIGNALS

When interfering signals are intermittent, the straightforward analysis method of this handbook cannot be used. A complete description of an appropriate analysis method for that case is beyond the scope of this handbook, but the types of analysis that might be required will be briefly described.

As an initial example, consider a digital receiver with no FEC and an interfering signal that has a power level *I* for 2% of the time and negligible power for the other 98%. It could be a pulsed radar signal for which the pulsewidth is 2% of the pulse repetition interval. Or it could be a narrowband fixed-frequency interfering signal that is in the same channel as one of the 50 hopset frequencies of a frequency-hopping receiver. If the receiver is affected by the interference only when it falls in the same channel, then that co-channel event is like an interference pulse whose duration is 2% (1/50) of the total time spent hopping through the entire hopset. For either case, the BER is given by:

$$BER = 0.98 \, BER_N + 0.2 \, BER_{I+N}$$

(3-9)

where BER =   average BER
   BER_N =   BER in the presence of noise
   BER_{I+N} =   BER in the presence of both interference and noise

Alternatively, rather than calculate a BER, the pulse may be called an outage and the statistics of the outage (e.g., a 10 ms outage that occurs every 500 ms) may simply be reported.

Although this simple example can be analyzed with a simple manual technique, many cases are too complex to analyze manually. For example, there may be multiple radars with different pulsewidths and power levels simultaneously interfering with the receiver. Or the narrowband fixed-frequency signal interfering with the frequency-hopping receiver could have many amplitude levels at the RF/IF output (rather than just two levels) because of changes in the FDR as the receiver hops. Equation 3-9 can be generalized as follows:

$$BER = \sum_j p_j BER_j$$

(3-10)

where *BER* = average BER

    p_j = probability that the jth interference level occurs

    BER_j = BER when the interference is at the jth level

It may be difficult to determine the statistics (e.g., values of p_j) required to use Equation 3-10. In addition, if the receiver has FEC then Equation 3-10 may not be valid. In that case, the relative duration of an interference event and the FEC codeword may be important. (A brief discussion of FEC with intermittent signals is given in Section 7.4.) The best approach for analyzing these complex cases is the simulation approach (Section 2.4.4).

Note that the desired signal may also be intermittent. For example, if a frequency-hopping receiver is subject to frequency-selective multipath fading, then the desired signal power will fluctuate (fade) as the receiver hops.

3.5 SUMMARY

There are two transfer functions associated with the RF/IF section of a receiver. The first is the FDR, which relates the S/I at the RF/IF input to the S/I at the RF/IF output according to Equation 3-6. The FDR is defined by Equation 3-1 and can be approximated in dB by Equation 3-5. The second is a non-mathematical transfer function that relates the interference waveform characteristics at the RF/IF input to the interference waveform characteristics at the RF/IF output. It is specified for frequency hopping, pulsed, and digital signals in Sections 3.3.1, 3.3.2, and 3.3.3, respectively.

SECTION 4 – DESPREADER

This section describes spread spectrum processing and characterizes the processing gain associated with the despreader in a spread spectrum system. Because spread spectrum is frequently associated with multiple access, some simple multiple-access equations are also presented.

4.1 DESCRIPTION

In a transmitter, *spreading* is the process of multiplying the narrowband information signal by a wideband code waveform to dramatically increase the bandwidth of the signal. In the receiver, *despreading* multiplies the wideband desired signal by the same code waveform to recover the original narrowband signal. A *spread spectrum* system is a system that employs spreading and despreading. An RF system may employ spread-spectrum technology for several reasons:

- To enable multiple-access communications with many users utilizing the same portion of the RF spectrum at the same time without unacceptable mutual interference
- To suppress narrowband interference
- To resist multipath fading

The operation of a spread-spectrum system is based on a pseudo-noise (PN) spreading code set, which is known by both the transmitter and receiver. A PN code set is a collection of binary sequences (called keys, spreading codes, or simply codes) with the following properties:

- The autocorrelation of each code is very small.
- The cross-correlation of multiple distinct codes is very small.
- The RF waveform representing each code appears to be noise-like.

The desired signal is spread with a particular PN code waveform. When the despreader multiplies the desired signal by that same PN code waveform, it removes the code. As a result, the original narrowband desired signal is recovered. This despreading works because the receiver-generated PN code waveform is synchronized with the PN code waveform on the desired signal and is, therefore, perfectly correlated. After the despreader, the narrowband desired signal passes through a narrow bandpass filter.

If the receiver is part of a multiple-access system and the received signal includes an interfering signal from a different user in the system, then that signal is also multiplied by the receiver's PN code waveform. However, the PN code on the interfering signal does not match the receiver's PN code.

Because the two codes are uncorrelated, the interfering signal is not despread. When the wideband interfering signal at the despreader output then passes through the narrow bandpass filter, much of it is attenuated. This attenuation is what allows a spread spectrum multiple-access receiver to distinguish and extract the desired signal from the composite signal that includes signals from multiple users.

The same mechanism in a spread spectrum receiver suppresses narrowband interference. When the despreader multiplies the narrowband interfering signal by the receiver-generated PN code waveform, the interfering signal is spread (becoming a wideband signal). When that wideband signal then passes through the narrow bandpass filter, much of it is attenuated.

In both of these cases, despreading (followed by narrowband filtering) can therefore produce a dramatic improvement in the S/I. This improvement is called the processing gain. The types of spreading are direct sequence, frequency hopping, and time hopping.

Spread-spectrum systems resist fading. In a frequency-selective multipath fading environment, typically only a small portion of the large spread-spectrum bandwidth experiences fading. The degradation normally caused by propagation delays is minimized by the despreader.

4.2 DIRECT SEQUENCE

In a *direct sequence* system, the code waveform is usually a long pseudorandom sequence of short-duration bits known as *chips*. The transmitter spreads a digital signal by multiplying the signal by the code waveform. The coded waveform is typically transmitted and received using a binary digital modulation scheme, such as phase-shift keying (PSK) or differential PSK (DPSK). The spreading can occur before, during, or after the modulation process.

As an example of one common implementation, consider a case with 100 chips per bit. The information bit sequence is combined with FEC bits to produce the coded bit waveform $b(t)$, which is a series of rectangular pulses. Each pulse has an amplitude ± 1. The FEC-coded bit rate is R_b in bits/s. The spreading sequence is represented by the PN code waveform $c(t)$, which is also a series of rectangular pulses of amplitude ± 1. The chip rate of the PN code is $R_c = 100\ R_b$. In the transmitter, the spreader multiplies the two waveforms together to get the spread baseband waveform $b(t)\ c(t)$. That waveform then modulates a carrier to get a PSK signal. In the receiver, the received PSK signal is demodulated to recover the baseband waveform $b(t)\ c(t)$. The despreader then synchronizes its local PN code waveform $c(t)$ with the input signal and multiplies the two waveforms to get $[b(t)\ c(t)][c(t)] = b(t)\ c^2(t) = b(t)$. Thus, in the absence of interference and noise, the original FEC-coded bit waveform $b(t)$ is recovered. If interference and noise are not negligible, then the received bit sequence will contain bit errors and the FEC decoder will attempt to correct those errors.

4.2.1 Processing Gain

The RF bandwidth B_c associated with the desired signal waveform before despreading is much larger than the bandwidth B_b associated with the desired signal waveform after despreading. The bandpass filter following the despreader has a nominal bandwidth B_b.

When the spread-spectrum desired signal is accompanied by narrowband interference, the despreader expands the interfering signal bandwidth from B_I to approximately B_c. The bandpass filter then attenuates the interference by the factor B_c/B_b. This factor, expressed in dB, is the spread-spectrum processing gain:

$$G_{SS} = 10 \log\left(\frac{B_c}{B_b}\right) \approx 10 \log\left(\frac{R_c}{R_b}\right) = 10 \log(M)$$

(4-1)

where G_{SS} =   spread-spectrum processing gain, in dB
        B_c =   3-dB bandwidth of the desired signal before despreading, in MHz
        B_b =   3-dB bandwidth of the desired signal after despreading, in MHz
        R_c =   chip rate of despreader, in chips/s
        R_b =   bit rate at despreader output, in bits/s
        M =   number of chips per bit

If the interference is wideband with $B_I > B_c$, then the RF/IF section of the receiver (Section 3) reduces the interfering signal bandwidth from B_I to approximately B_c. That bandwidth reduction is accompanied by an attenuation, which is included in the FDR (Section 3.2). When the despreader multiplies the interfering signal by the PN code waveform, the interfering signal bandwidth remains approximately equal to B_c. The bandpass filter then attenuates the interference by the factor B_c/B_b. Therefore, Equation 4-1 also applies to the wideband interference case. Note that the FDR accounts for filter attenuation in the RF/IF section, whereas the processing gain accounts for filter attenuation in the bandpass filter that follows the despreader.

Multipath fading, which is a form of self-interference, can be modeled as narrowband interference to a direct-sequence system. Typically, a fade margin is planned for a communications link in which fading is anticipated. This fade margin can be reduced (by an amount less than or equal to the coding gain) when the system uses direct-sequence spread-spectrum communication.

4.2.2 Multiple-Access Interference

Since direct-sequence spread-spectrum systems are typically designed to support multiple-access communications within a single frequency band, it is frequently necessary to consider the effects of the multiple-access interference as well as noise. If it is assumed that there are K simultaneous users, then there will be one desired signal and K-1 undesired signals. If it is further assumed that the K signals are of equal power at the PSK receiver input, then the BER can be approximated as:[4-1]

$$BER = Q\left(\frac{1}{\sqrt{\frac{K-1}{3M} + \frac{N_o}{2E_b}}} \right)$$

$$(4\text{-}2)$$

where K = total number of simultaneous users
M = number of chips per bit
N_o = noise power density, in W/Hz
E_b = energy per bit, in J (or W/Hz)

The variables E_b and N_o are discussed in Section 6. The function $Q(X)$ is defined as follows:[4-2]

$$Q(X) = \frac{1}{\sqrt{2\pi}} \int_X^\infty e^{-y^2/2} dy$$

$$(4\text{-}3)$$

For a single user the system is limited by noise:

$$BER = Q\left(\sqrt{\frac{2E_b}{N_o}} \right)$$

For a large number of users, the system is limited by the multiple-access interference:

[4-1]   Theodore S. Rappaport, *Wireless Communications Principles and Practice*, 2nd ed., Upper Saddle River, NJ: Prentice Hall PTR, 2002.

[4-2]   John G. Proakis, *Digital Communications*, 3rd ed., McGraw-Hill Series in Electrical Engineering, 1995.

$$BER = Q\left(\sqrt{\frac{3M}{K-1}}\right)$$

These equations assume that signals are received from all users with equal power. If this assumption is not valid, then other more complex equations may be used. To use these complex equations, the analyst must specify statistics describing the distribution of received signal power from all of the users. The only way to obtain such statistics is by field measurements. If such measurements are made, and it is determined that the variation of signal power is limiting the system capacity, the typical response is to reduce the variation. This can be done, for example, by reducing the coverage area or by inserting repeaters into the coverage area.

4.3 FREQUENCY HOPPING

In a *frequency-hopping* system, the desired signal is typically a digital signal that hops from one carrier frequency to another at some fixed hop rate. A *slow hopper* is one that transmits several bits during each hop. A *fast hopper* is one that hops several times during each bit. The system *hopset* is the set of possible frequencies. The transmitter uses a PN code to determine the sequence of frequencies from the hopset that will be used in a transmission. Typically, the PN code waveform is a long pseudorandom sequence of discrete voltages that control the output frequency of an oscillator. The receiver uses the same PN code to convert the hopping desired signal to a fixed-frequency signal.

4.3.1 Processing Gain

Each of the frequencies in a hopset is the center frequency of what is called an instantaneous channel. Each instantaneous channel has an instantaneous bandwidth B_b. Typically, binary frequency-shift keying (FSK) modulation is used, so there are two signalling frequencies within each channel. Assuming that there are M contiguous carrier frequencies and channels, the total system bandwidth is $B_S = MB_b$. (However, a given channel is sometimes removed from a hopset to avoid interference in the environment.)

In a frequency-hopping receiver, a hopping local oscillator signal mixes with the desired signal. This process eliminates the desired signal hopping and converts a non-hopping interfering signal into a hopping signal. As in the case of a direct sequence receiver, this shrinks the desired signal bandwidth and spreads the interfering signal spectrum. A non-hopping narrowband filter then filters out most of the interfering signal. This mixing and narrowband filtering normally occurs in the RF/IF section, so the resulting attenuation is already included in the FDR (Section 3.2). Therefore, although the concept of "processing gain" can be applied to a frequency-hopping receiver, Equation 4-1 is not used.

As discussed in Section 3.3.1, the interfering signal at the RF/IF output in a frequency-hopping receiver is usually intermittent, whether the hopper is slow or fast. Thus, the straightforward analysis method of this handbook cannot be used. Section 3.4 discusses analysis options for such cases.

4.3.2 Multiple-Access Interference

In a frequency-hopping multiple-access system, mutual interference is minimized by designing the hopping sequences so that the probability of two users hopping to the same channel at the same time is small. If two users hop to the same channel at the same time, the event is called a *hit*. For a slow hopper, it may be assumed that during a hit the probability of bit error is 0.5. Adapting Equation 3-10 to this case, the BER is given by:

$$
\begin{aligned}
BER &= \sum_j p_j BER_j \\
&= p_{\text{hit}} BER_{\text{hit}} + p_{\text{miss}} BER_{\text{miss}} \\
&= (1 - p_{\text{miss}})(1/2) + p_{\text{miss}} BER_{\text{miss}}
\end{aligned}
$$

$$(4\text{-}4)$$

where p_{hit} = probability that a bit is in a hit

p_{miss} = probability that a bit is in a miss (not a hit)

BER_{hit} = BER during a hit

BER_{miss} = BER during a miss

The probability of a hit is related to the number of simultaneous users, and the number of instantaneous carrier frequencies in the hopset. From the point of view of a single receiver, it cannot be assumed that all the signals hop synchronously. Even if there were a master system clock, variations in propagation delay would result in varying degrees of offset. In this case, the probability that a bit is not in a hit is (Reference 4-1):

$$
p_{\text{miss}} = \left[1 - \frac{1}{M} \left(1 + \frac{1}{L} \right) \right]^{K-1}
$$

$$(4\text{-}5)$$

where M =   number of frequencies in the hopset

       L =   number of bits per hop

      K =   number of simultaneous users

4.4 TIME HOPPING

In a conventional digital system, the bit duration is the reciprocal of the bit rate. For example, if the bit rate is 1000 bps then the bit duration is 1 ms. In a *time-hopping* system, the conventional bit interval (reciprocal of the bit rate) is conceptually subdivided into small subintervals. The bit is transmitted in one of these subintervals. To point out the similarity to a direct sequence system, the small bit can be referred to as a *chip*. The position of the chip within the conventional bit interval is dictated by the PN code waveform. The frequency does not hop. Spreading occurs because the chip duration is much less than the conventional bit duration.

In a time-hopping receiver, the PN code waveform dictates where in the hopping interval the pulse is located. No energy is collected outside of that pulse window, so most of the interfering signal power is not collected. Equation 4-1 can be used to calculate the processing gain.

For example, consider a system with no FEC for which the conventional bit duration is 100 times the chip duration. If the bit rate is 1000 bps, then the conventional bit duration is 1 ms and the chip duration is 0.01 ms. In the transmitter, the information chip is placed in the nth chip location, where n is a pseudorandom integer between 1 and 100 specified by the PN code. In the receiver, the despreader uses the same PN code to determine the value of n. It then converts the chip waveform back into the conventional bit waveform. The number of chips per bit (ratio of conventional bit duration to chip duration) is 100, so the processing gain is 10 log(100) = 20 dB.

4.5 SUMMARY

The transfer function associated with the despreader is the spread-spectrum processing gain, which relates the S/I at the despreader input to the S/I at the despreader output. The spread-spectrum processing gain is given by Equation 4-1. The S/I is increased by that gain in a direct sequence or time hopping receiver.

SECTION 5 – ANALOG DEMODULATOR

This section describes the demodulator model for amplitude modulation (AM) voice, frequency modulation (FM) voice, and broadcast television receivers. Subjective performance measures of intelligibility for voice systems are discussed. Transfer functions that relate output S/N to input S/N are presented. Curves that specify intelligibility measures as a function of S/N and S/I are also displayed.

5.1 INTRODUCTION

The purpose of the analog demodulator is to remove the RF carrier and restore the baseband signal that was used to modulate the carrier. There are two common types of analog modulation – AM and FM – each with advantages and disadvantages pertinent to RF communications.

There are several properties of analog RF systems that affect the system performance:

Bandwidth. AM systems require less bandwidth than FM systems. A typical AM system requires an RF channel that is approximately twice the width of the baseband signal spectrum. Since an analog speech signal has a baseband width of less than 4 kHz, an AM system can accommodate many channels. Single-sideband AM systems require even less bandwidth – approximately the width of the baseband spectrum. FM systems require several times the bandwidth of the baseband spectrum.

Susceptibility to Noise and Interference. FM systems are generally less susceptible to noise, and are particularly less susceptible to intermittent interference and fading than AM systems. FM systems have a property that allows this immunity to be increased by increasing the RF bandwidth. AM systems do not have this property, leaving an increase in transmitter power as the principle means for overcoming noise and interference.

Threshold. Analog systems have a certain threshold, which is a received-signal power below which no usable output is obtained. AM systems can often operate at lower sensitivities than FM systems. FM systems, on the other hand, exhibit the so-called capture effect, which controls the way in which the output performance relates to increasing received-signal power. The capture effect allows the performance of the system, once the sensitivity threshold has been reached, to increase.

5.2 VOICE PERFORMANCE MEASURES

The performance of RF communications receivers is typically quantified in terms of S/N, or some similar ratio of desired-signal power to undesired-signal power. This is an objective measure of performance, in the sense that it can be measured with RF test equipment.

Objective measures of performance do not directly address properties such as fidelity or intelligibility, since these properties depend on human perception. Nevertheless, it is these properties that are ultimately of interest to users of RF communications systems. Consequently, there have been attempts to develop measures of performance that are more closely aligned with human perception.

5.2.1 Articulation Index

One of the most important applications of RF communications technology is the transmission of speech. The spectrum of the speech signal extends from roughly 200 Hz to 2500 Hz. The most basic and fundamental property that can be applied to a speech signal is intelligibility – that is, the ability of the listener to understand the speaker.

Certain parts of the speech spectrum are more sensitive to noise and interference than others. For this reason, a measure of performance was developed that assigns varying weights to undesired signal power, based on the part of the speech spectrum that the signal occupies. This measure of performance, called the AI, is based on empirical data that shows the correlation between the undesired signal spectrum and the intelligibility of the speech. Although the basis of the AI is empirical, the definition of AI is not. The band from 200 Hz to 6100 Hz is divided into 14 contiguous sub-bands of unequal width. The S/I in each sub-band is calculated and normalized to give a value between 0 and 1. The average of these 14 normalized values is the AI. It is effectively a frequency-weighted average because the sub-bands are of unequal width.

In practice, AI is used to characterize the effect of undesired signals on transmitted speech by assigning a qualitative descriptor to a range of AI values. A typical assignment might be:

$1.0 > AI > 0.9$ Good intelligibility

$0.9 > AI > 0.7$ Marginal intelligibility

$0.7 > AI$ Unacceptably poor intelligibility

Because the AI model is based on the long-term average speech and interfering signal spectrum, it cannot be used when either signal is intermittent.

5.2.2 Articulation Score

The ultimate test of system performance is to actually measure the intelligibility of transmitted speech by empirical methods. The experiments are designed to facilitate the comparison of a spoken message with the same message after it has been transmitted and received. The results are quantified by counting

the number of correct vs. incorrect words in the received message as reported by test listeners. The percentage of correct words is called the AS. The AS is thus the most direct measure of system intelligibility.

When designing AS experiments, it is important – but extremely difficult – to minimize the contribution of other variables that affect intelligibility. These variables include:

Age, education, and regional accent of speakers and listeners. AS experiments have been performed primarily on college campuses, using college students as listeners. This introduces the possibility that the uniform level of education (typically first or second year undergraduate) will bias the results. Furthermore, unless a number of colleges in different locations are involved (historically this is not the case), the local accents and speech patterns will bias the results.

Type of message content. The message may be designed to communicate information in small, discrete, uncorrelated units (for example numbers or lists of words). The units may be larger, yet remain discrete and more or less uncorrelated, such as a news broadcast. The message may be highly correlated, where there is one central idea, and individual words are not identically important. The AS experiments favor the small, uncorrelated message units, but many real-world communications are very different.

Style of delivery. The message may be spoken in casual, conversational tones, or it may be delivered with urgency. The focus, concentration, and ultimately the AS of listeners can be affected by whether the message is delivered with a declarative or imperative style.

Vocabulary. If speakers employ a limited vocabulary which is known to and anticipated by the listeners, the AS will be much higher than otherwise. Limited vocabularies and very disciplined delivery styles are used for military, emergency response, scientific, and other types of technical communications. These systems can often be operated in very noisy RF environments.

The impact of these variables is such that AS results have very narrow applicability. The degree to which a particular system resembles a test system with respect to these variables is the degree to which the AS tests will prove useful. There have been attempts to relate AS to AI, since AI is supposed to factor in frequency-dependent intelligibility. Even so, the relationship between AS and AI varies widely, exhibiting strong dependence on the aforementioned variables.

5.3 AM VOICE

5.3.1 Description

The waveform of an AM signal is given by:

$$v(t) = A_c [1 + m(t)] \cos(2\pi f_c t)$$

(5-1)

where $v(t)$ = waveform magnitude at time t, in V
  t = time, in s
  A_c = constant signal amplitude, in V
  $m(t)$ = baseband (modulating) signal, unitless
  f_c = carrier frequency, in Hz

For AM voice, the modulating signal $m(t)$ is the voice signal to be transmitted. The original voice signal is peak-limited and scaled so that the peak amplitude of $|m(t)|$ is not greater than 1. That peak amplitude is called the *modulation index μ* of the AM signal.

Amplitude modulation results in frequency translation of the baseband spectrum. The translated baseband spectrum is referred to as a *sideband*. Equation (5-1) results in a pair of sidebands – each a replica of the baseband spectrum – centered about the carrier frequency f_c. There are several variations of amplitude modulated systems in which parts of the signal are suppressed. This section considers the full AM signal with carrier and two sidebands. The variations are specified in Section 5.4.

5.3.2 Performance

Important considerations are the demodulator threshold, sensitivity, and the output S/N. These are functions of the input S/N. It should be noted that the AM demodulator can never improve upon the input S/N. Ideal AM demodulation would result in an output S/N equal to the input S/N.

The demodulator needs a certain minimum input S/N to function properly. The *AM threshold* is a point on the curve of output S/N vs. input S/N at which the curve changes dramatically. Above threshold, the input vs. output S/N relationship is linear. Below threshold, the output S/N falls off faster than the input S/N. The threshold varies somewhat, depending on the type of demodulator circuit, but a reasonable approximation is an input S/N of approximately 5 dB. This is so low that, even if the demodulator were operating in the linear range, the output would not be usable.

Sensitivity is the minimum received-signal power required for proper receiver operation in an interference-free environment. For analog receivers, "proper operation" is usually specified in terms of the output S/N. Therefore, a design goal for the RF engineer is to ensure that the required received-signal power is available. Sensitivity is usually determined experimentally.

In some analyses not involving interference, it may be sufficient to simply compare the received-signal power with the sensitivity. In other cases, it may be necessary to estimate the actual output S/N. For example, there may be a range of 10 to 15 dB in input S/N between the demodulator threshold and the sensitivity. If the system is operating in this range, the output will be noticably degraded, but possibly still useful. At the other extreme, for critical messages it may be necessary to operate with a higher output S/N than that specified in connection with sensitivity.

5.3.3 Transfer Function

For an AM demodulator operating above threshold with noise (or noise-like interference), the output S/N is related to the input S/N as follows:

$$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{in} + 10 \log\left(\frac{<m^2(t)>}{1 + <m^2(t)>}\right)$$

(5-2)

where $(S/N)_{out}$ = output signal-to-noise power ratio, in dB

$(S/N)_{in}$ = input signal-to-noise power ratio, in dB

$<m^2(t)>$ = time average of the square of $m(t)$, unitless

The numerator in the rightmost term of Equation 5-2 is the power of the modulating signal (in 1 ohm) relative to the carrier power. In a typical application, the root-mean-square (RMS) voltage of the voice signal $m(t)$ is 0.3 (relative to the carrier amplitude). Therefore, $<m^2(t)> = 0.09$ and the rightmost term has the value -11 dB. The output S/N is 11 dB lower than the input S/N because the demodulator eliminates the carrier, which contains most of the power of the input signal.

If the modulating signal is a sinusoidal tone with modulation index μ (rather than a voice signal), then the transfer function is:

$$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{in} + 10 \log\left(\frac{\mu^2/2}{1 + \mu^2/2}\right)$$

(5-3)

For an AM demodulator operating above threshold with AM voice interference, simulations show the transfer function to be:

$$\left(\frac{S}{I}\right)_{out} = \left(\frac{S}{I}\right)_{in} - 8$$

(5-4)

In these simulations, the RMS voltage of each voice signal was set to 0.3 (relative to the carrier amplitude).

5.3.4 AI Curves

Figures 5-1 and 5-2 show AI curves for an AM voice receiver. In these graphs, the term "interference" and the variable I refer to non-noise-like interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: an amplitude-shift keying (ASK) signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the ASK interfering signal power, and the x-axis variable S/N is the ratio of the desired signal power to the total noise-like power (including the receiver noise and the noise-like interfering signal) at the input of the receiver.

The curves were generated by time-domain simulation. The IF bandwidth was 8 kHz. The audio (baseband) bandwidth was from 300 Hz to 3.5 kHz. The modulation index of the desired signal was 0.3. In the last part of the simulation, the signals were transformed to the frequency domain for calculating the AI, as described in Section 5.2.1.

Figure 5-1 shows AI curves for an AM voice receiver with ASK (on-off keying) interference. There are six curves displayed. Each curve is a plot of AI vs. S/N. The top curve (labeled "No interference") applies to the case in which there is no ASK interference. The other five curves are for cases with ASK interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the intelligibility (as indicated by the AI) improves as the S/N increases. As also expected, for a given S/N the intelligibility improves as the S/I increases.

In the simulation, the ASK interfering signal had a bit rate of 800 bps, but other simulations showed the results to be insensitive to the bit rate. (However, at very high bit rates the interfering signal is noise-like, as specified in Section 3.3.3.) The ASK signal was tuned 500 Hz away from the receiver, but other simulations showed the results to be insensitive to variations of off-tuning within the same frequency channel.
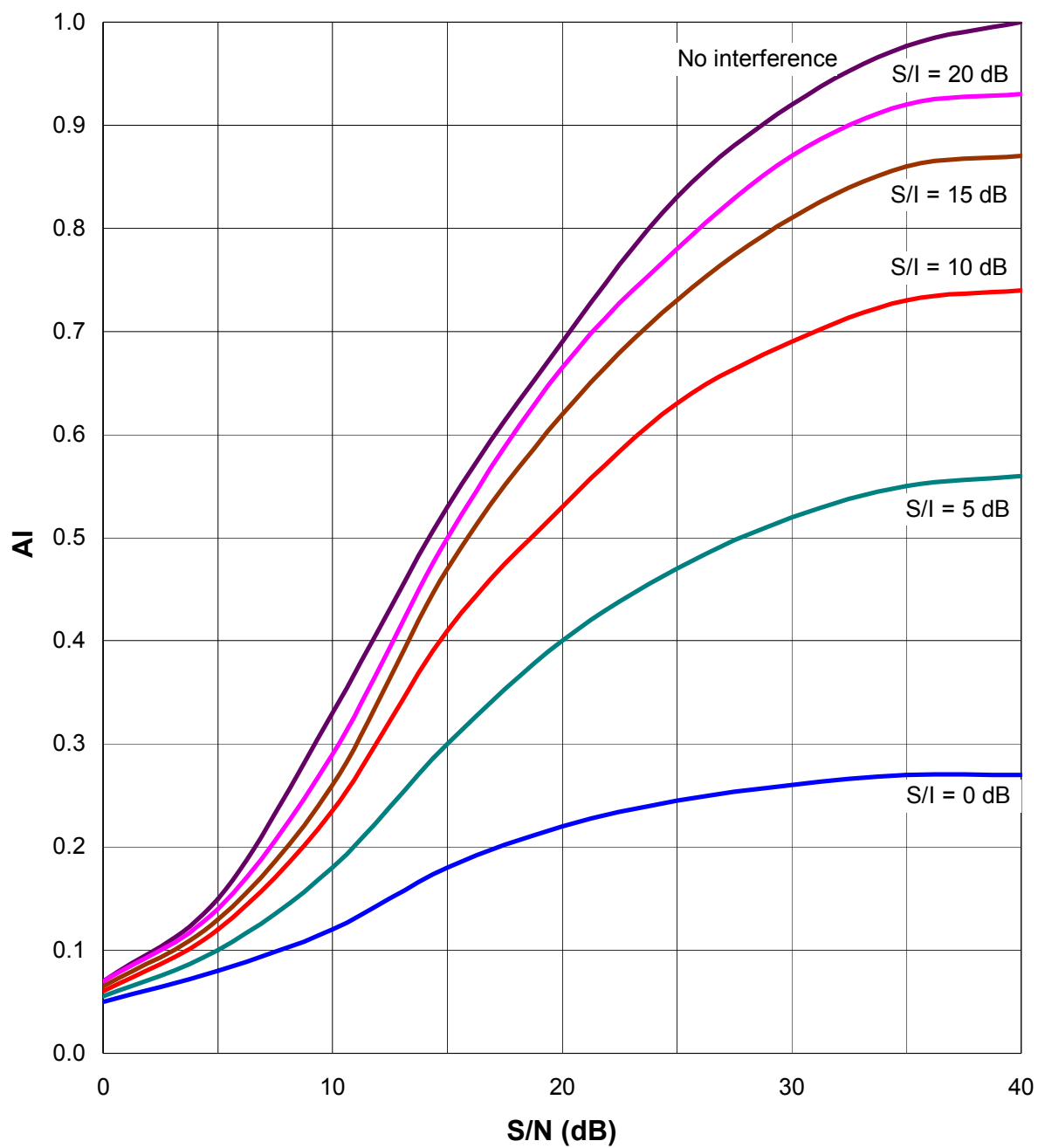
Figure 5-1. AI vs. (S/N)$_{in}$ Curves for AM Voice Receiver with ASK Interference
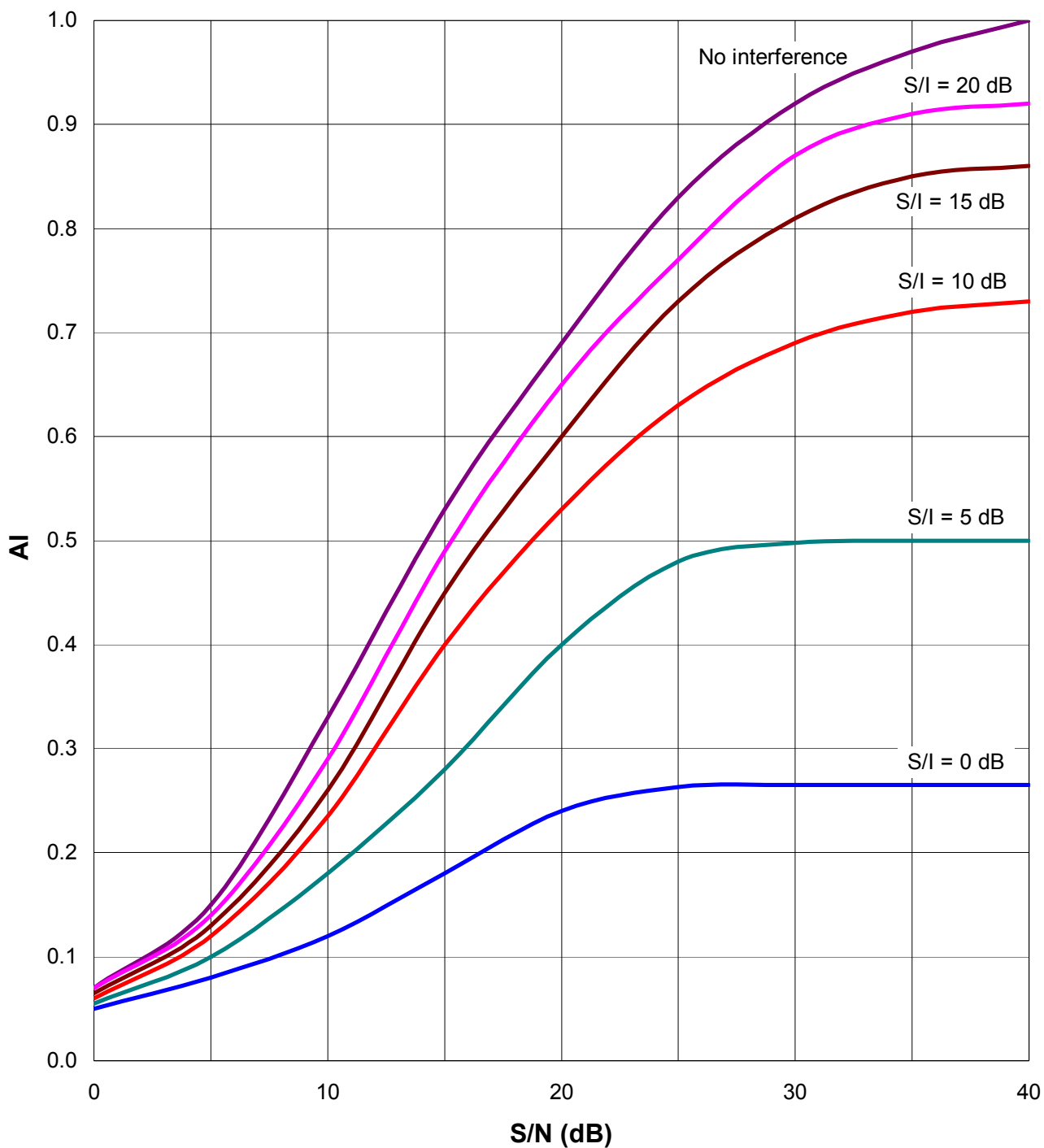
Figure 5-2. AI vs. $(S/N)_{in}$ Curves for AM Voice Receiver with FSK Interference

Figure 5-2 shows AI curves for an AM voice receiver with FSK interference. In the simulation, the FSK interfering signal had a bit rate of 50 bps and an off-tuning of 500 Hz, but other simulations showed the results to be insensitive to the bit rate and to variations of off-tuning within the same frequency channel. The frequency deviation of the FSK signal was ± 200 Hz.

5.4 SINGLE SIDEBAND VOICE

5.4.1 Description

As described in Section 5.3, amplitude modulation results in a pair of sidebands – each a replica of the baseband spectrum – centered about the carrier frequency. There are several variations of amplitude modulated systems:

- **Double sideband (DSB).** This refers to the entire AM signal – the carrier and both sidebands. It is addressed in Section 5.3.

- **DSB with suppressed carrier.** The two sidebands are transmitted, but not the carrier.

- **Single sideband (SSB).** Normally this means that only one of the sidebands is transmitted without the carrier.

- **SSB with pilot tone.** A single sideband is transmitted along with a pilot tone. The pilot tone is transmitted to aid in the demodulation process. This technique is especially useful in mobile systems, since it makes it possible to eliminate the effects of doppler shifts. The pilot tone may be the actual carrier frequency, but it does not have to be.

- **Vestigial sideband.** The carrier, one complete sideband, and a small portion of the other sideband are transmitted. This technique is used by analog broadcast television. It is addressed in Section 5.6.

5.4.2 Transfer Function

For systems that do not transmit the RF carrier, the output S/N is the same as the input S/N, assuming the system is operating above threshold:

$$\left(\frac{S}{N}\right)_{out} = \left(\frac{S}{N}\right)_{in}$$

(5-5)

Equation 5-5 applies to SSB receivers and to DSB with suppressed carrier. For SSB systems with a pilot tone, the pilot tone may be assumed to be small enough that Equation 5-5 applies.

5.4.3 AI Curves

Figures 5-3 and 5-4 show AI curves for a SSB voice receiver. In these graphs, the term "interference" and the variable I refer to non-noise-like interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: an ASK signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the ASK interfering signal power, and the x-axis variable S/N is the ratio of the desired signal power to the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated by time-domain simulation. The IF bandwidth was 2.7 kHz. The audio (baseband) bandwidth was from 300 Hz to 3 kHz. The carrier and lower sideband were suppressed. The center frequency of the IF filter was assumed to be 1650 Hz above the suppressed carrier frequency. In the last part of the simulation, the signals were transformed to the frequency domain for calculating the AI, as described in Section 5.2.1.

Figure 5-3 shows AI curves for a SSB voice receiver with ASK interference. There are six curves displayed. Each curve is a plot of AI vs. S/N. The top curve (labeled "No interference") applies to the case in which there is no ASK interference. The other five curves are for cases with ASK interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the intelligibility (as indicated by the AI) generally improves as the S/N increases. As also expected, for a given S/N the intelligibility generally improves as the S/I increases.

In the simulation, the ASK interfering signal had a bit rate of 800 bps, but other simulations showed the results to be insensitive to the bit rate. (However, at very high bit rates the interfering signal is noise-like, as specified in Section 3.3.3.) The ASK signal was tuned 500 Hz away from the suppressed carrier, but other simulations showed the results to be insensitive to variations of off-tuning within the same frequency channel.

Figure 5-4 shows AI curves for a SSB voice receiver with FSK interference. In the simulation, the FSK interfering signal had a bit rate of 50 bps and an off-tuning of 500 Hz, but other simulations showed the results to be insensitive to the bit rate and to variations of off-tuning within the same frequency channel. The frequency deviation of the FSK signal was ± 200 Hz.
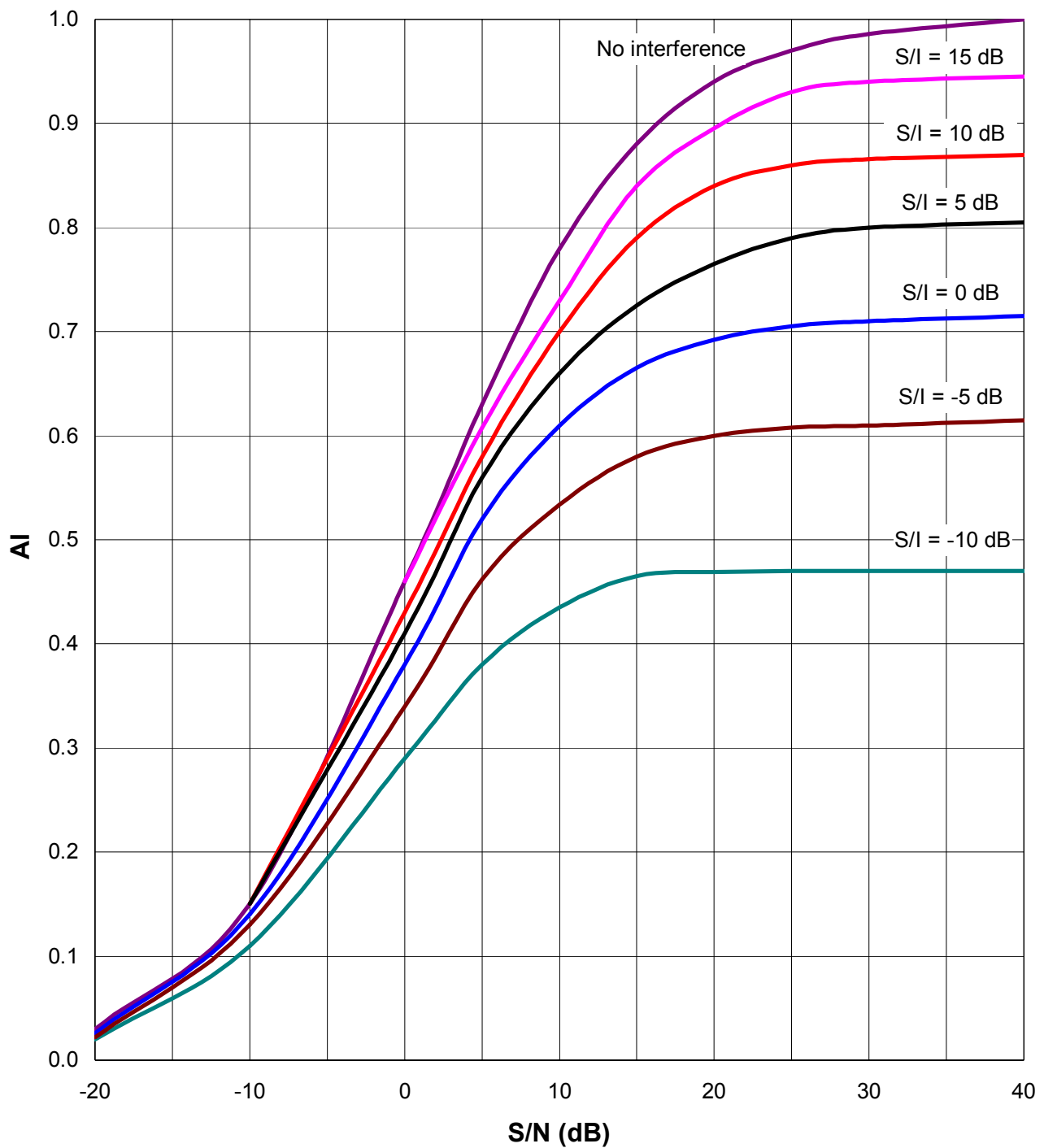
Figure 5-3. AI vs. (S/N)in Curves for SSB Voice Receiver with ASK Interference
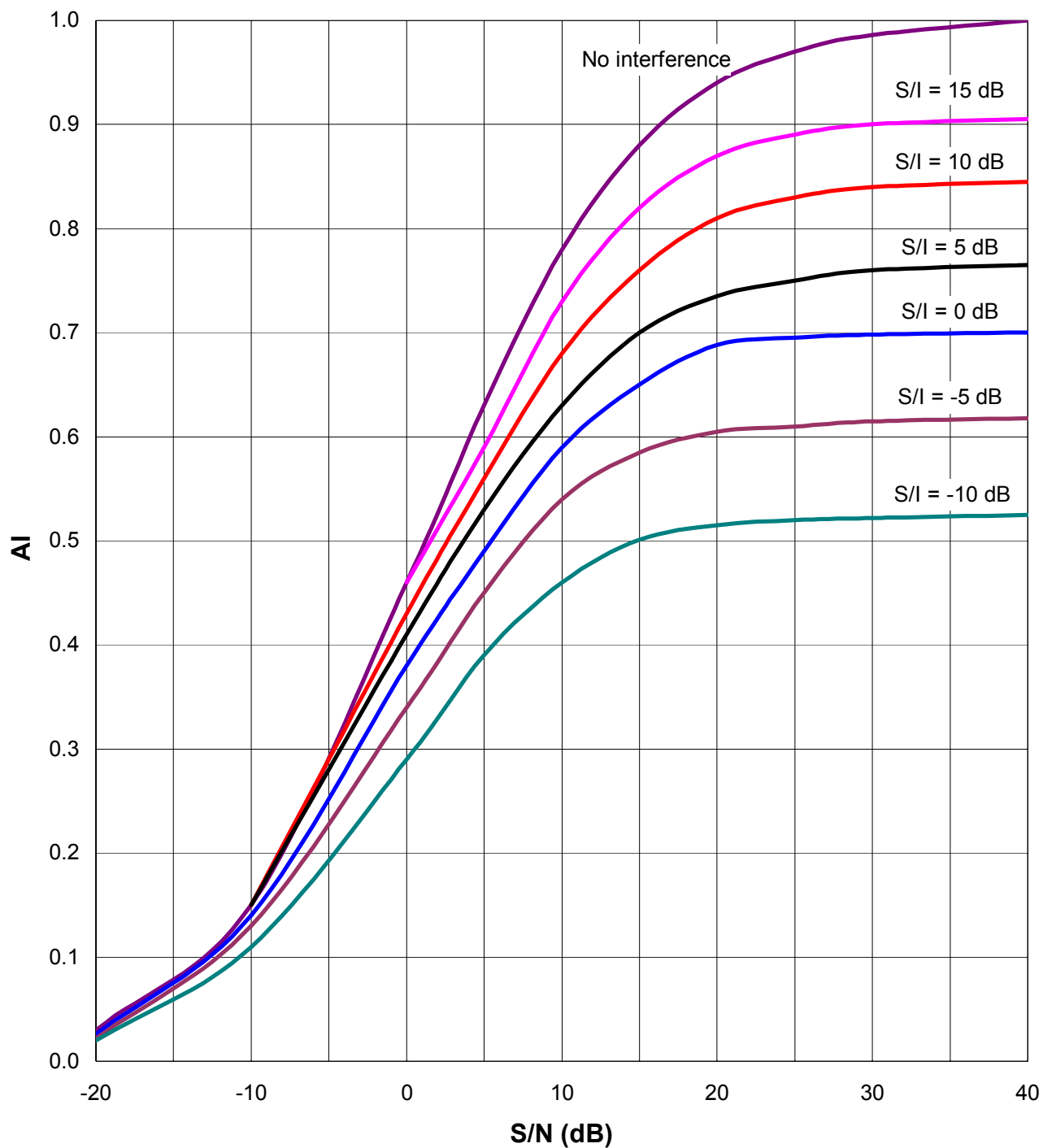
Figure 5-4. AI vs. (S/N)$_{in}$ Curves for SSB Voice Receiver with FSK Interference

5.5 FM VOICE

5.5.1 Description

An FM signal has an RF carrier whose instantaneous frequency depends on the amplitude of the modulating waveform. More precisely, the deviation of the RF carrier frequency from its unmodulated state f_c is made to be proportional to the amplitude of the modulating waveform $m(t)$. For FM voice, $m(t)$ is the voice waveform. However, properties of FM are more easily understood if the modulation is assumed to be sinusoidal. When the modulating waveform is a sinusoidal tone with frequency f_m, the FM signal may be written:

$$v(t) = A \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

(5-6)

where $v(t)$ = waveform magnitude at time t, in V
  t = time, in s
  A = constant signal amplitude, in V
  f_c = carrier frequency, in Hz
  β = modulation index (a unitless constant)
  f_m = frequency of modulation tone, in Hz

The instantaneous frequency is the time derivative of the argument of the cosine function:

$$2\pi f = 2\pi f_c + \beta \, 2\pi f_m \cos(2\pi f_m t)$$

(5-7)

where f = instantaneous frequency, in Hz

The maximum frequency deviation, which occurs when the modulating signal amplitude is at its maximum, is given by:

$$\Delta f = \left(f - f_c \right)_{\max} = \beta f_m$$

(5-8)

where Δf = maximum (peak) frequency deviation, in Hz

The bandwidth of the FM signal is found to be:

$$B = 2(\beta+1)f_m = 2(\Delta f + f_m)$$

(5-9)

This equation states that the FM signal bandwidth is twice the sum of the maximum frequency deviation and the modulating frequency. This equation is called *Carson's Rule*. For FM voice, the modulation frequency f_m is the maximum baseband (audio) frequency.

5.5.2 Performance

5.5.2.1 FM Threshold

The *FM threshold* is a point on the curve of output S/N vs. input S/N at which the curve changes dramatically. Above threshold, the input vs. output S/N relationship is linear. Below threshold, the output S/N falls off faster than the input S/N.

The threshold value is arbitrarily defined to be the value of input S/N for which the output S/N is 1 dB lower than the linear value. When the input S/N falls below this point, the system should be considered unusable. The threshold increases with an increase in the modulation index. Even though a large modulation index results in improved performance above threshold, this improvement comes with the price of a relatively higher threshold.

Clearly, it is desirable for an FM receiver to have as low a threshold as possible. However, lowering the threshold implies a reduction in modulation index, which in turn deprives the system of the improved output S/N that a larger modulation index provides. System designers exploit these dependencies to tailor FM systems for particular applications. A system with a low modulation index – β equal to 0.5 or 0.6, for example – will have a low threshold, enabling it to be used in conditions of marginal reception. This property may be important for tactical communications, where the low threshold is more important than high fidelity. On the other hand, a system with a larger modulation index will be capable of higher fidelity, but at the expense of a higher threshold. This trade-off may make sense for commercial FM systems that compete for listeners based, in part, on high-fidelity programs.

The threshold for a particular system depends on more than the modulation index. There are several techniques available to designers for implementing FM demodulators. These implementations have some bearing on the point at which threshold occurs. The most reliable way to determine the threshold of an FM demodulator is by measurement. Along with the sensitivity, the threshold of an FM receiver should be part of the system specifications.

FM systems that are designed for low threshold operation are often called narrowband systems. Since these systems have relatively low modulation indices, they also require relatively small operating bandwidths. Systems designed for high-fidelity operation are often called wideband systems. These systems have relatively large modulation indices, and require correspondingly large operating bandwidths.

5.5.2.2 Narrowband Interference and Narrowband FM

Narrowband interference may simply be due to signals from narrowband transmitters in the receiver environment. The self-interference due to multipath propagation in built-up areas – commonly referred to as fading – also tends to be narrowband in nature.

Narrowband FM addresses these interference types in two ways. By maintaining a low threshold, narrowband FM offers more operating margin than wideband FM. Moreover, since narrowband FM means less channel bandwidth, there are likely to be more available channels from which a non-degraded channel can be selected.

5.5.2.3 Pre-emphasis and De-emphasis

An FM system makes the best use of its operating bandwidth when the spectrum of the modulating signal is flat. An FM system can make it flat by applying a filter with the appropriate transfer function. This process is referred to as *pre-emphasis*. The demodulator cancels the effect of pre-emphasis by applying a filter with the complementary transfer function. This process is called *de-emphasis*. The success of this technique depends on a prior knowledge of the modulating signal spectrum.

Pre-emphasis and de-emphasis can improve the output S/N by 5 or 6 dB in some cases. This improvement should be interpreted as bringing a system closer to its optimum operating specification. If an FM system uses this technique, it is very likely that the sensitivity specification already includes the adjustment.

5.5.3 Transfer Function

For an FM demodulator operating above threshold with noise (or noise-like interference), the output S/N is related to the input S/N as follows:

$$\left(\frac{S}{N}\right)_{\text{out}} = \left(\frac{S}{N}\right)_{\text{in}} + 10\log\left(\frac{3}{2}\beta^2\right)$$

(5-10)

where $(S/N)_{out}$ =   output signal-to-noise power ratio, in dB

$(S/N)_{in}$ =   input signal-to-noise power ratio, in dB

β =   modulation index of the demodulator, unitless

This expression demonstrates a fundamental property of FM – the ability to improve the output S/N by increasing the modulation index. Since an increase in modulation index corresponds to an increase in maximum frequency deviation, this improvement necessitates an increase in bandwidth.

For a narrowband FM demodulator operating above threshold with narrowband FM interference, simulations show the transfer function to be:

$$\left(\frac{S}{N} \right)_{out} = \left(\frac{S}{N} \right)_{in} + 5$$

(5-11)

5.5.4 AI Curves

Figures 5-5 and 5-6 show AI curves for an FM voice receiver. In these graphs, the term "interference" and the variable *I* refer to non-noise-like interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: an ASK signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the ASK interfering signal power, and the x-axis variable S/N is the ratio of the desired signal power to the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated by time-domain simulation. The IF bandwidth was 16 kHz. The audio (baseband) bandwidth was from 300 Hz to 3.5 kHz. The peak frequency deviation was 5 kHz. These parameters imply a modulation index of $\beta = 5/3.5 = 1.4$. Note that the IF bandwidth is slightly smaller than the 2 (5 + 3.5) = 17 kHz specified by Carson's rule. Pre-emphasis and de-emphasis were not included in the receiver model. In the last part of the simulation, the signals were transformed to the frequency domain for calculating the AI, as described in Section 5.2.1.

Figure 5-5 shows AI curves for an FM voice receiver with ASK interference. In the simulation, the ASK interfering signal had a bit rate of 100 bps and was on-tune with the receiver. There are six curves displayed. Each curve is a plot of AI vs. S/N. The top curve (labeled "No interference") applies to the case in which there is no ASK interference. The other five curves are for cases with ASK interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the intelligibility (as indicated by the AI) improves as the S/N increases. As also expected, for a given S/N the intelligibility improves as the S/I increases.
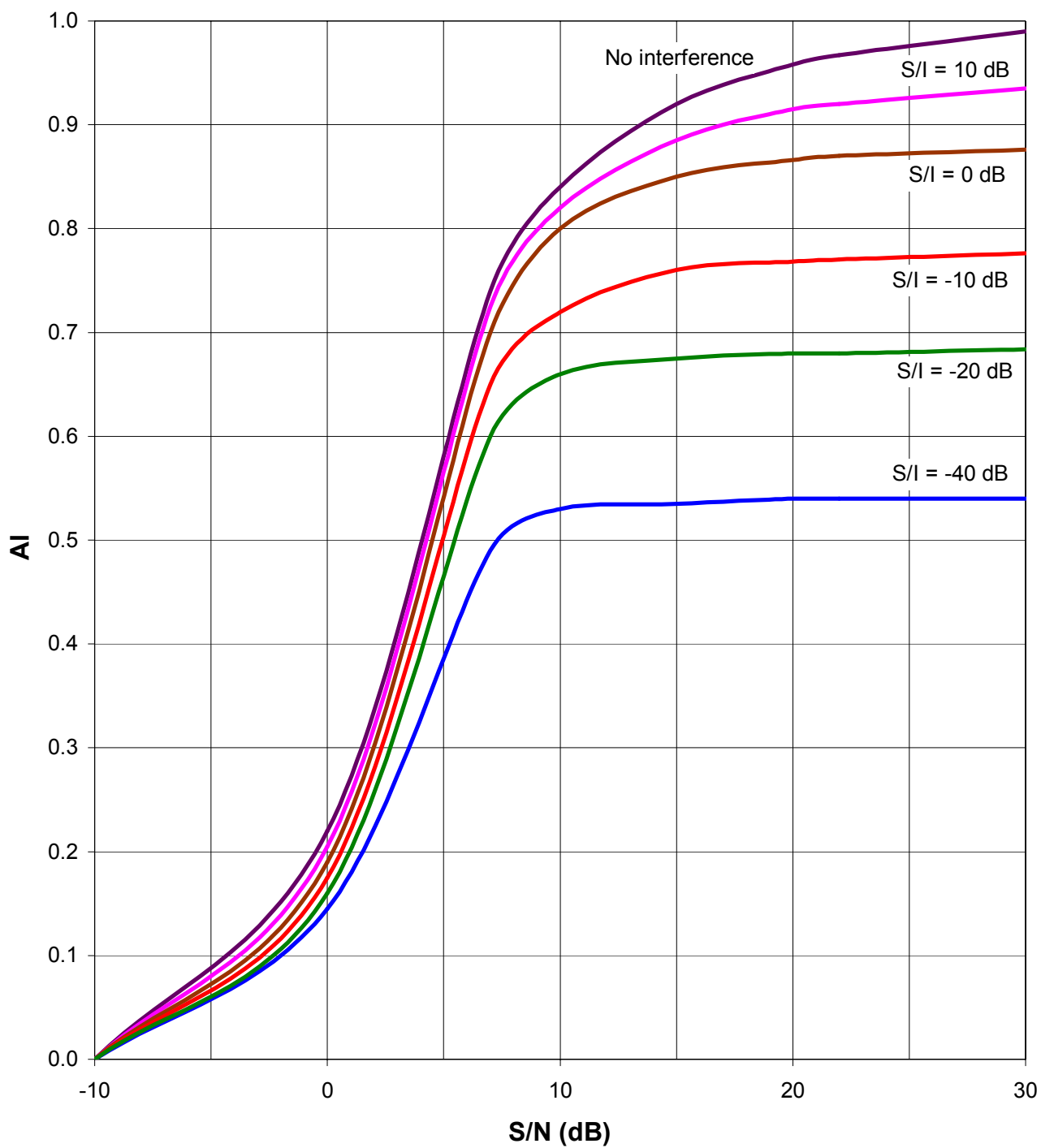
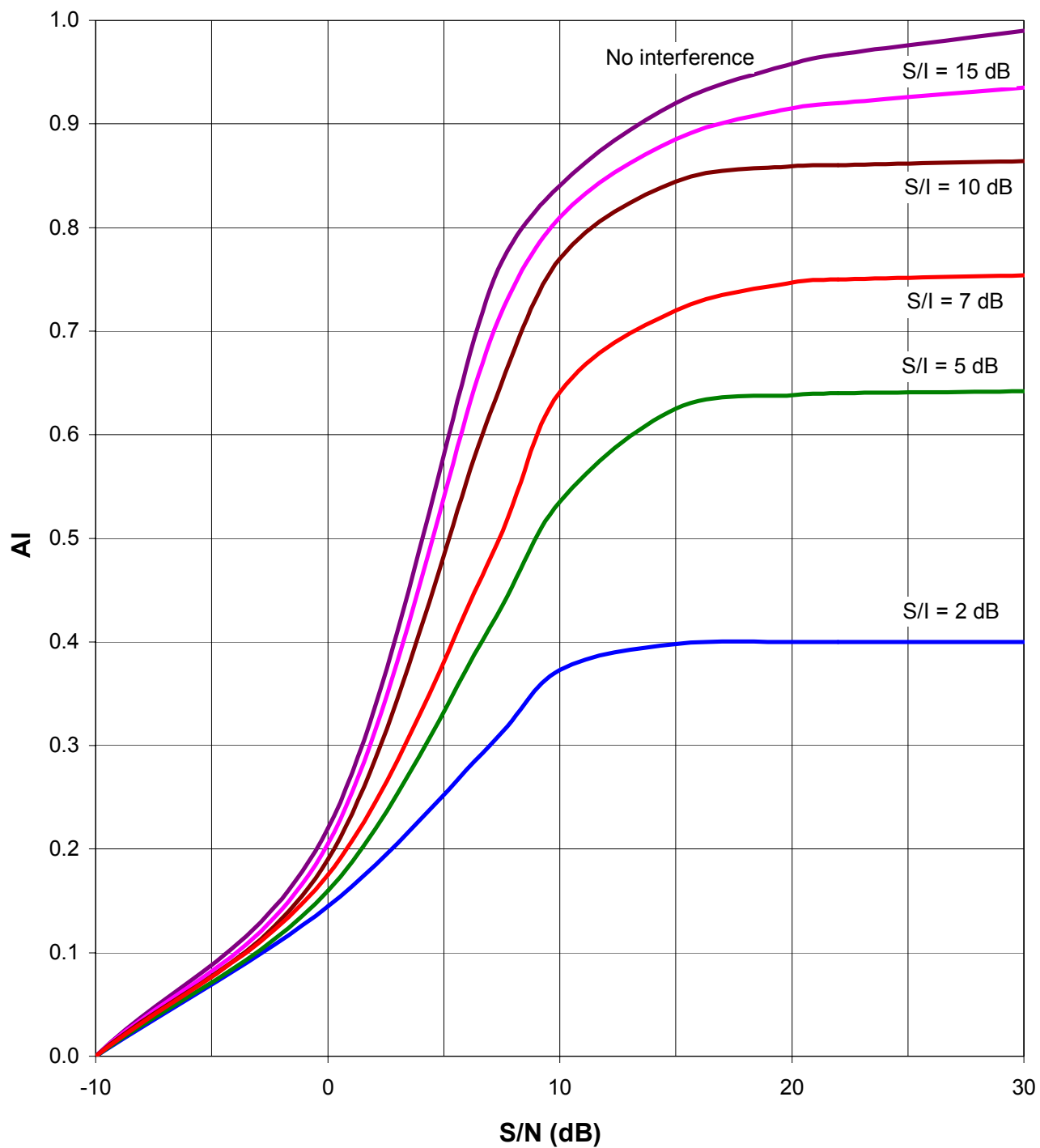**Figure 5-5. AI vs. (S/N)ᵢₙ Curves for FM Voice Receiver with ASK Interference**

Figure 5-6. AI vs. (S/N)ₐₙ Curves for FM Voice Receiver with FSK Interference

Figure 5-6 shows AI curves for an FM voice receiver with FSK interference. In the simulation, the FSK interfering signal had a bit rate of 50 bps and an off-tuning of 500 Hz, but other simulations showed the results to be insensitive to the bit rate and to variations of off-tuning within the same frequency channel. The frequency deviation of the FSK signal was ± 200 Hz.

5.6 BROADCAST TELEVISION

5.6.1 Description

Vestigial sideband AM (with the carrier, one complete sideband, and a small portion of the other sideband) is used exclusively for broadcast television. Since the output is a combination of sound and picture, a single S/N value is inadequate to measure the receiver performance. For television receivers, it is common to specify the required performance subjectively. By experimentation, the subjective measurements can be related to the input S/N. In the TASO scoring procedure, observers are asked to rate picture quality on a scale of 1 to 6 as shown in Table 5-1.

Table 5-1. TASO Score Definition

TASO Score	Name	Description
1	Excellent	The picture is of extremely high quality, as good as you could desire.
2	Fine	The picture is of high quality providing enjoyable viewing. Interference is perceptible.
3	Passable	The picture is of acceptable quality. Interference is not objectionable.
4	Marginal	The picture is poor in quality and you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	The picture is very poor but you could watch it. Definitely objectionable interference is present.
6	Unusable	The picture is so bad that you could not watch it.

5.6.2 TASO Curves

Figure 5-7 shows TASO curves for an analog broadcast TV receiver. There are three curves, one for each type of interference: ASK, noise-like, and TV interference. Each curve is a plot of TASO score vs. input S/I. As expected, each curve shows that the picture quality (as indicated by the TASO score) improves as the S/I increases.
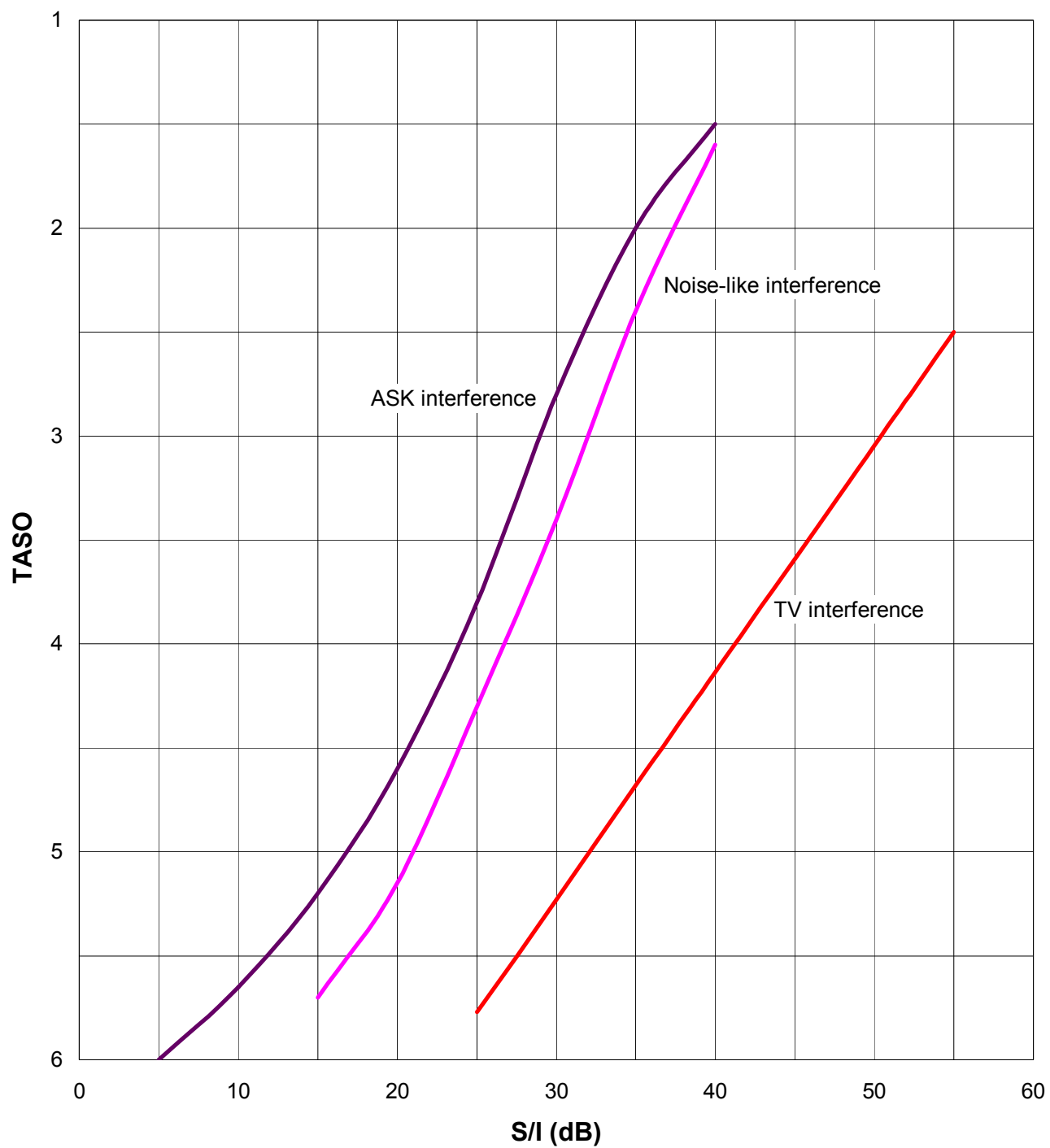
Figure 5-7. TASO Score vs. (S/I)$_{in}$ Curves for Broadcast TV Receiver

The curves were generated by collecting responses from a viewer panel.[5-1] The receiver was a standard US commercial TV with an IF bandwidth of 5 MHz. The ASK interfering signal had a bit rate of 100 bps and was on-tune with the receiver. The interfering TV signal was also on-tune with the receiver.

[5-1]   H. Fine, *A Further Analysis of TASO Panel 6 Data on Signal-to-Interference Ratios and Their Application to Description of Television Service*, T.R.R. Report No. 5.1.2, Washington, DC: FCC, 1 April 1960.

SECTION 6 – DIGITAL DEMODULATOR

This section describes the demodulator model for digital receivers. Performance measures for digital demodulators are discussed. The ratio of bit energy to noise power density (E_b/N_o) is defined. Transfer functions that relate output error rate to input E_b/N_o for noise-like interference are presented. Curves that specify output BER as a function of E_b/N_o and S/I are also displayed.

6.1 INTRODUCTION

The purpose of the digital demodulator is to reconstruct the digital data that was used to modulate the carrier. In some cases, this digital data is a digitized audio or video waveform. The three carrier properties most commonly modified in the modulation process are phase, frequency, and amplitude.

There are several properties of digital RF systems that affect system performance:

Power Spectral Density and Bandwidth. The bandwidth required for a digital communication system depends primarily on the data rate and the modulation type. It may be assumed that the data modulating the carrier is random.

Since the data is random, the modulated signal constitutes a stochastic process. This process can be characterized in the time domain by the autocorrelation function, which is calculated from the statistics of the modulated signal. The Fourier transform of the autocorrelation function is called the *power spectral density* (PSD) of the modulated signal. The PSD at a given frequency is the power per unit bandwidth (e.g., in W/Hz) at that frequency.

Measurements and calculations of the PSD for digital signals show that the signal bandwidth is nominally equal to the bit rate or symbol rate. However, the precise bandwidth and spectral shape depends on the type of modulation. Some modulation types are quite spectrally efficient. The indiscriminate use of bandpass filtering to reduce the signal bandwidth is not practical, because a narrow filter bandwidth will distort the signal.

Susceptibility to Noise and Interference. The susceptibility depends on the modulation type. A coherent demodulator generates a local waveform that is in phase with the received signal. Although it is more difficult and costly to implement than the noncoherent demodulator, it decreases susceptibility to noise and interference. Other modulation characteristics, such as the carrier property modulated (e.g., phase), also affect the susceptibility. As expected, performance is best for any modulation type when the desired signal power is significantly higher than the noise and interference power. Unlike analog

demodulators, digital demodulators exhibit a transition from unacceptable to virtually error-free performance over a relatively small range of input values.

Synchronization and Threshold. Digital demodulators require some level of synchronization to operate properly. At the very least, the demodulator must be able to identify the data bit boundaries. Additional levels of synchronization may also be required, since digital data is often organized into symbols, frames, packets, or other multi-bit structures.

Synchronization algorithms in the receiver are designed to be particularly robust. Therefore, the required S/N for message transmission (in steady interference conditions) is usually also adequate for maintaining synchronization. If synchronization is lost for some reason – perhaps due to an interference pulse – it may be necessary to estimate the time needed to regain synchronous operation. If this time is not provided as part of the system specification, it is best determined by measurement or simulation.

Soft Decision Demodulation. For a given received digital symbol waveform, a demodulator may make a definite decision as to which symbol is associated with that waveform. The output of this *hard-decision* demodulator is a sequence of symbol values. In contrast, the output of a *soft-decision* demodulator is a sequence of likelihood values. Each soft-decision value indicates the likelihood that a given symbol was sent, and a subsequent error correction device uses that likelihood to make a decision. Section 6 considers only hard-decision demodulators. For soft-decision systems, the demodulator and FEC decoder are treated as one combined module in Section 7.

6.2 BINARY SYSTEMS

In binary digital communications systems, the data can be represented as an information sequence consisting of binary digits (bits). Prior to modulation, additional bits may be added for error correction, synchronization, etc. Each bit has one of two possible values (0 or 1) and is associated with a distinct waveform. Consequently, bits have several properties that derive from their waveform representation:

- The bit duration T_b is the duration of the waveform associated with each bit.
- The bit rate (or data rate) R_b is the number of bits transmitted per second.
- The bit energy E_b at a given point in the system is the energy in the bit waveform at that point.

The bit rate and bit duration are related by:

$$R_b = \frac{1}{T_b}$$

(6-1)

where R_b =   bit rate, in bits/s or Hz

          T_b =   bit duration, in s

The bit energy is related to the signal power *S* by:

$$E_b = ST_b$$

<div align="right">(6-2)</div>

where E_b =   bit energy, in J

         S =   signal power, in W

Noise within the receiver is assumed to have, at its point of origin, a constant power density at all frequencies. At the demodulator input, the total noise power is given by:

$$N = N_o B$$

<div align="right">(6-3)</div>

where N =   noise power at the demodulator input, in W

        N_o =   one-sided noise power density at the demodulator input, in W/Hz

        B =   narrowest filter bandwidth preceding the demodulator, in Hz

Based on the previous definitions, the ratio of bit energy to noise power density is given by:

$$\frac{E_b}{N_o} = \frac{ST_b}{N/B} = \frac{S}{N}\frac{B}{R_b}$$

<div align="right">(6-4)</div>

Just prior to making a decision on a given bit, the demodulator integrates all the power in the bit waveform. This action equivalently filters the signal with a bandwidth R_b, which is the signal bandwidth. If *B* exceeds R_b, then this operation improves the S/N by the factor B/R_b. Thus E_b/N_o can be viewed as an improved S/N (at the input to the decision circuit). In many cases, *B* is equal to the bit rate and E_b/N_o is equal to the S/N at the demodulator input.

For each bit the demodulator must decide, based on the received signal, which of the two possible waveforms was transmitted. The waveforms are designed to be different enough that the correct decision will be made most of the time, even though the received signal has been degraded by the propagation environment. The standard measure of performance for the demodulator is the probability

of bit error, commonly referred to as the BER. Analytical expressions for the BER as a function of E_b/N_o are available for many digital demodulators, and are included in this section.

6.3 M-ARY SYSTEMS

In some digital communications systems, the bits are converted into multi-bit symbols prior to modulation. Each symbol has one of M possible values (0, 1, 2, …, M-1) and is associated with a distinct waveform. Consequently, symbols have several properties that derive from their waveform representation:

- The symbol duration T_s is the duration of the waveform associated with each symbol.
- The symbol rate R_s is the number of symbols transmitted per second.
- The symbol energy E_s at a given point in the system is the energy in the symbol waveform at that point.

The number of possible symbols is given by:

$$M = 2^k$$

(6-5)

where M =   number of possible symbols
k =   number of bits per symbol

A system in which $k = 1$ and $M = 2$ is referred to as binary (Section 6.2). If $k > 1$, the system is referred to as *M-ary*. For example, an 8-ary system has three bits per symbol and $M = 2^3 = 8$. The possible symbol values are 0, 1, 2, 3, 4, 5, 6, 7 and each of those values is associated with a distinct waveform.

Equations 6-1, 6-2, and 6-4 can be generalized to the M-ary case:

$$R_s = \frac{1}{T_s}$$

(6-6)

$$E_s = ST_s$$

(6-7)

$$\frac{E_s}{N_o} = \frac{ST_s}{N/B} = \frac{S}{N}\frac{B}{R_s}$$

(6-8)

Just prior to making a decision on a given symbol, the demodulator integrates all the power in the symbol waveform. This action equivalently filters the signal with a bandwidth R_s, which is nominally equal to the bandwidth of the desired signal. If B exceeds R_s, then this operation improves the S/N by the factor B/R_s. Thus, as in the binary case, E_s/N_o can be viewed as an improved S/N (at the input to the decision circuit). In many cases, B is equal to the symbol rate and E_s/N_o is equal to the S/N at the demodulator input.

For each symbol the demodulator must decide, based on the received signal, which of the M possible waveforms was transmitted. The standard measure of performance for the demodulator is the probability of symbol error, commonly referred to as the symbol error rate (SER). Analytical expressions for the SER as a function of E_s/N_o are available for many digital demodulators, and are included in this section.

6.4 M-ARY SYSTEM PERFORMANCE IN BINARY TERMS

It is often convenient to express the performance of an M-ary system in binary terms; i.e., in terms of BER and E_b/N_o instead of SER and E_s/N_o. This facilitates the performance evaluation of a demodulator by always expressing the input and output parameters in terms of the fundamental data unit – the bit.

Because there are k bits per symbol, the duration of a symbol (T_s) is k times the duration of a bit (T_b). Equations 6-2 and 6-7 can then be combined to give:

$$\frac{E_s}{N_o} = \frac{ST_s}{N_o} = \frac{SkT_b}{N_o} = k\frac{E_b}{N_o}$$

(6-9)

The E_b/N_o and E_s/N_o ratios are usually given in dB, in which case Equation 6-9 takes the form:

$$\left[\frac{E_s}{N_o} = \frac{E_b}{N_o} + 10\log(k)\right]_{dB}$$

(6-10)

The relationship between BER and SER is not so simple. Because there are k bits per symbol, a particular symbol error may result in only one bit error or as many as k bit errors (depending on which incorrect symbol was chosen). There are two common situations that occur.

In the first situation, the M waveforms are orthogonal in signal space. An example is 8-ary FSK, for which 8 waveforms can be treated as 8 orthogonal vectors in an 8-dimensional vector space. Each dimension is an independent frequency channel. This means that a random noise burst can occur with equal likelihood in any channel. Therefore, all incorrect symbols are equally likely to be chosen. In this case, the average number of bit errors (averaged over all possible incorrect symbol choices) can be calculated. The relationship between BER and SER is given by:

$$BER = \left(\frac{M/2}{M-1} \right) SER$$

(6-11)

For example, the BER for an 8-ary FSK system would be 4/7 of the SER.

In the second situation, the M waveforms are not orthogonal in signal space. An example is 8-ary PSK, for which 8 waveforms exist in a 2-dimensional vector space. In this situation, incorrect symbols adjacent to the correct symbol are more likely to be chosen. In many such cases, the assigning of waveforms to symbols can be done so that adjacent symbols differ by only one bit. In these cases, a symbol error corresponds to one bit error and k-1 correct bits, so that

$$BER = \frac{SER}{k}$$

(6-12)

For example, the BER for an 8-ary PSK system would be 1/3 of the SER.

If the noise and interference levels are high, then non-adjacent symbols may be chosen. Figure 6-1 shows an analytical performance curve for 8-ary PSK, obtained by calculating the SER with an analytic equation and then using Equation 6-12. The corresponding simulation curve is also shown. Note that when the BER exceeds 0.2, Equation 6-12 is inaccurate (because of non-adjacent symbol choices).

Equations 6-11 and 6-12 constitute upper and lower bounds, so that in general

$$\frac{SER}{k} \leq BER \leq \left(\frac{M/2}{M-1} \right) SER$$

(6-13)
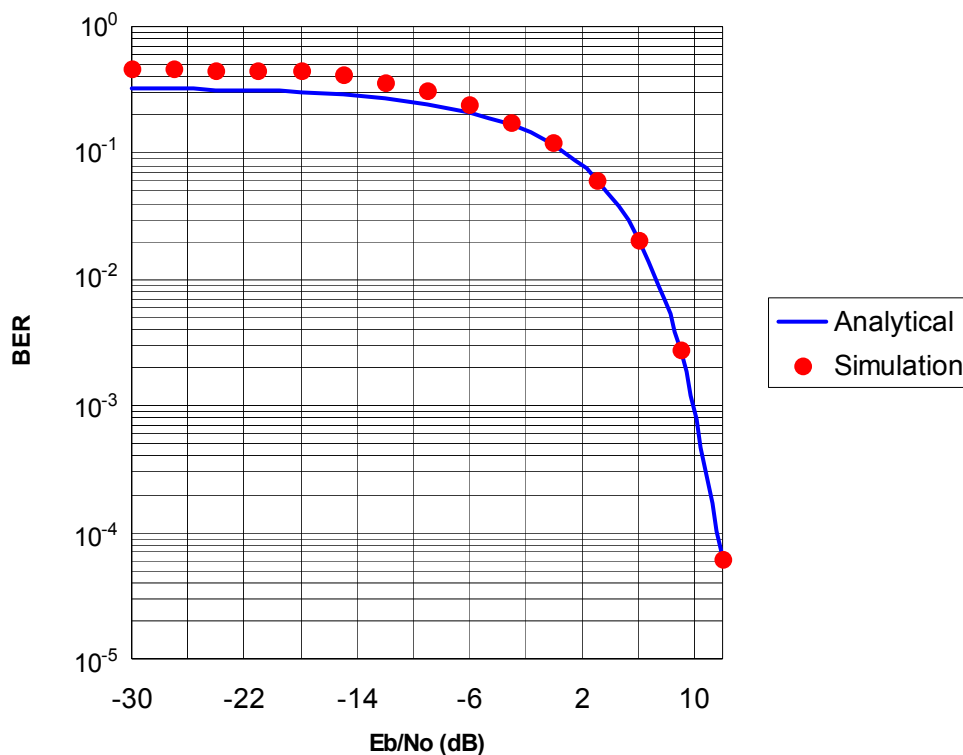
Figure 6-1. Analytical and Simulation BER Curves for PSK Receiver with *M* = 8

6.5 TYPES OF INTERFERENCE

Section 3.3.3 presents a case in which interference becomes noise-like when it passes through the RF/IF section. In that case, the interfering signal bit rate or chip rate exceeds that of the receiver, and the interfering signal bandwidth exceeds the receiver bandwidth. Thus broadband interference may become noise-like interference. To analyze its effect on performance, the noise-like interference power can simply be added (in W) to the other noise power within the receiver.

At the other extreme, a signal that theoretically has zero bandwidth is the single-frequency sinusoid, or CW signal. Narrowband interference may often be modeled as CW interference, because the performance is not sensitive to the specific details of the interference modulation. Because the CW signal is a simple, unmodulated signal, analytical expressions for BER for the case of noise and CW interference are available for some digital demodulators. In other cases for which a simulation must be performed, the use of a CW interfering signal simplifies the simulation.

For all performance curves presented in this section, only two types of interference are considered – noise-like and CW. This assumption permits the curves in the handbook to be used in many practical

cases with broadband and narrowband interference. If a continuous interfering signal is neither broadband nor narrowband, the effect on the BER will generally fall between that of CW interference and that of noise.

If an interfering signal is intermittent, the SER or BER will vary, and will therefore be less useful as a simple measure of performance. In this case, an analysis approach that considers the temporal properties of the interference should be devised (Section 3.4).

6.6 PSK

The PSK characterized here assumes coherent reception. In coherent reception, the receiver regenerates the carrier from the received signal and then uses the regenerated carrier as a reference for determining the phase of the modulating signal. Noncoherent PSK is presented in Section 6.7.

6.6.1 Description

6.6.1.1 Binary PSK

Binary PSK (BPSK) is PSK with one bit per symbol ($k = 1$ and $M = 2$). The BPSK modulation represents logical 1 and logical 0 with two waveforms of the same frequency that are 180° out of phase. The voltage waveforms for 1 and 0 are:

$$v_1(t) = A \cos(2\pi f_c t)$$
$$v_0(t) = A \cos(2\pi f_c t + \pi) = -A \cos(2\pi f_c t)$$

$$(6\text{-}15)$$

where A =   amplitude, in V
f_c =   carrier frequency, in Hz
t =   time, in s

Alternatively, the waveforms may be expressed as the product of the modulating (baseband) signal and the RF carrier:

$$v(t) = b(t)\, A \cos(2\pi f_c t)$$

$$(6\text{-}16)$$

where $b(t) = 1$ for logical 1 and $b(t) = -1$ for logical 0.

For noise and noise-like interference, the BER is given by:

$$BER = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

(6-17)

where erfc(x) is the complementary error function.

6.6.1.2 Quadrature PSK

Quadrature PSK (QPSK) is PSK with two bits per symbol ($k = 2$ and $M = 4$). One of four possible waveforms is transmitted:

$$v(t) = A \cos\left(2\pi f_c t + i\frac{\pi}{2}\right)$$

(6-18)

where $i = 0, 1, 2,$ or 3. The waveforms differ only in their phase. The phase angles are specified to have a uniform spacing of $\pi/2$ radians. Since there are four possible waveforms, it is possible to represent two bits of data per symbol.

The QPSK signal is essentially two binary PSK signals in phase quadrature. QPSK is a special M > 2 case in which an exact expression for the BER can be given. For noise and noise-like interference, the BER for QPSK is the same as for BPSK:

$$BER = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

(6-19)

The SER is given by (Reference 4-2):

$$SER = \left[\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)\right]\left[1 - \frac{1}{4}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)\right]$$

(6-20)

6.6.1.3 *M*-ary PSK for *M* > 4

This is an extension of 2-phase and 4-phase modulation to the general case of *M*-phase modulation with *k* bits per symbol. One of *M* possible waveforms is transmitted:

$$v(t) = A \cos\left(2\pi f_c t + i\frac{2\pi}{M} \right)$$

$$(6\text{-}21)$$

where $i = 0, 1, 2, \ldots, M-1$. The waveforms differ only in their phase. The phase angles are specified to have a uniform spacing of $2\pi/M$ radians.

For noise and noise-like interference, the SER for *M* > 4 is approximated by (Reference 4-2):

$$SER = \text{erfc}\left[\sqrt{k\frac{E_b}{N_o}} \sin\left(\frac{\pi}{M}\right) \right]$$

$$(6\text{-}22)$$

Equation 6-12 can be used to determine the corresponding BER.

6.6.2 BER Curves

Figures 6-2, 6-3, 6-4, and 6-5 show BER curves for a PSK receiver with $M=2$, $M=4$, $M=8$, and $M=16$, respectively. In these graphs, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated from analytic expressions, including Equations 6-12, 6-17, 6-19, and 6-22. Analytic expressions for cases involving CW interference required numerical integration.

Each figure displays six curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other five curves are for cases with CW interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.
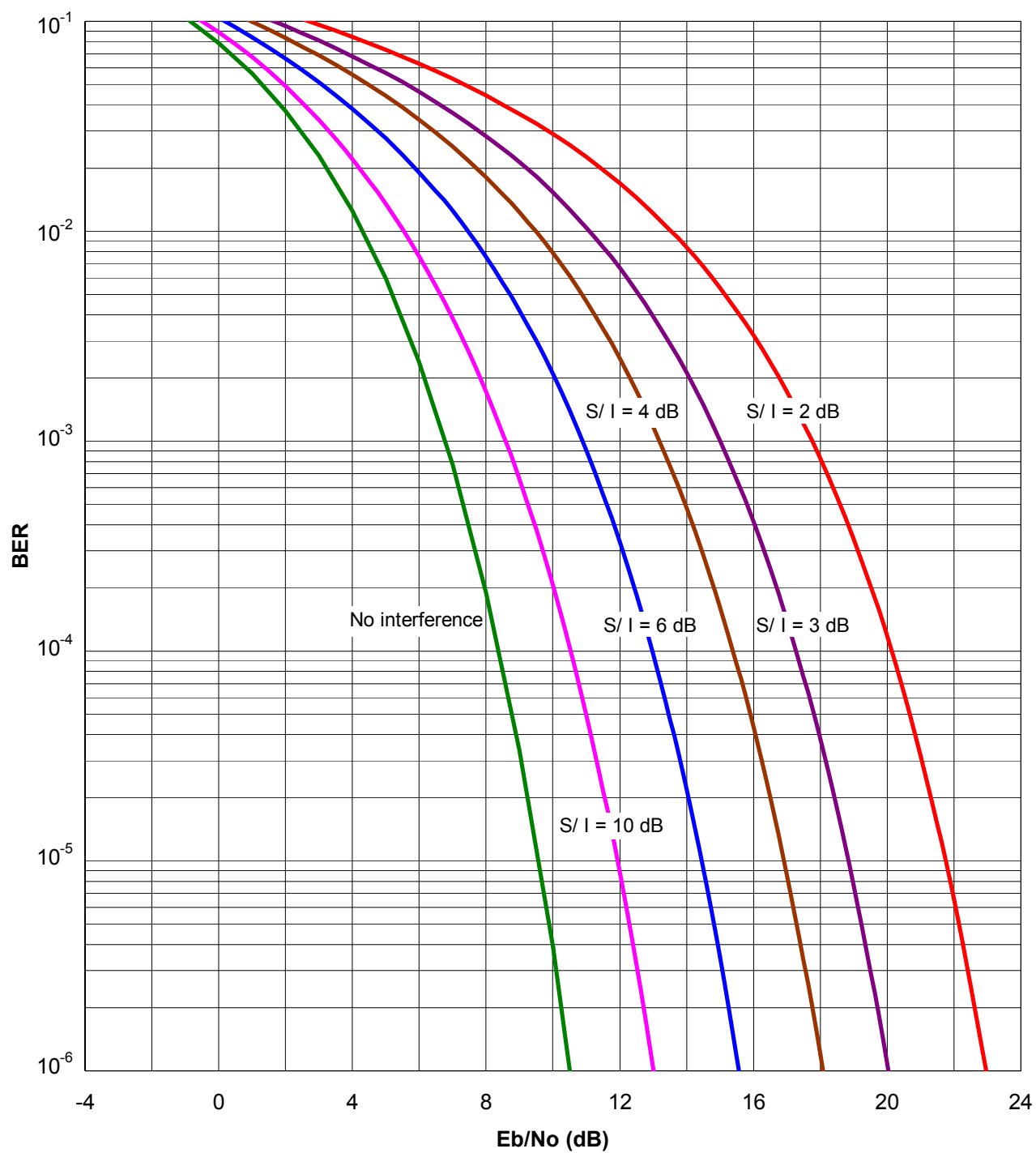
Figure 6-2. BER vs. E_b/N_o Curves for PSK Receiver ($M = 2$) with CW Interference
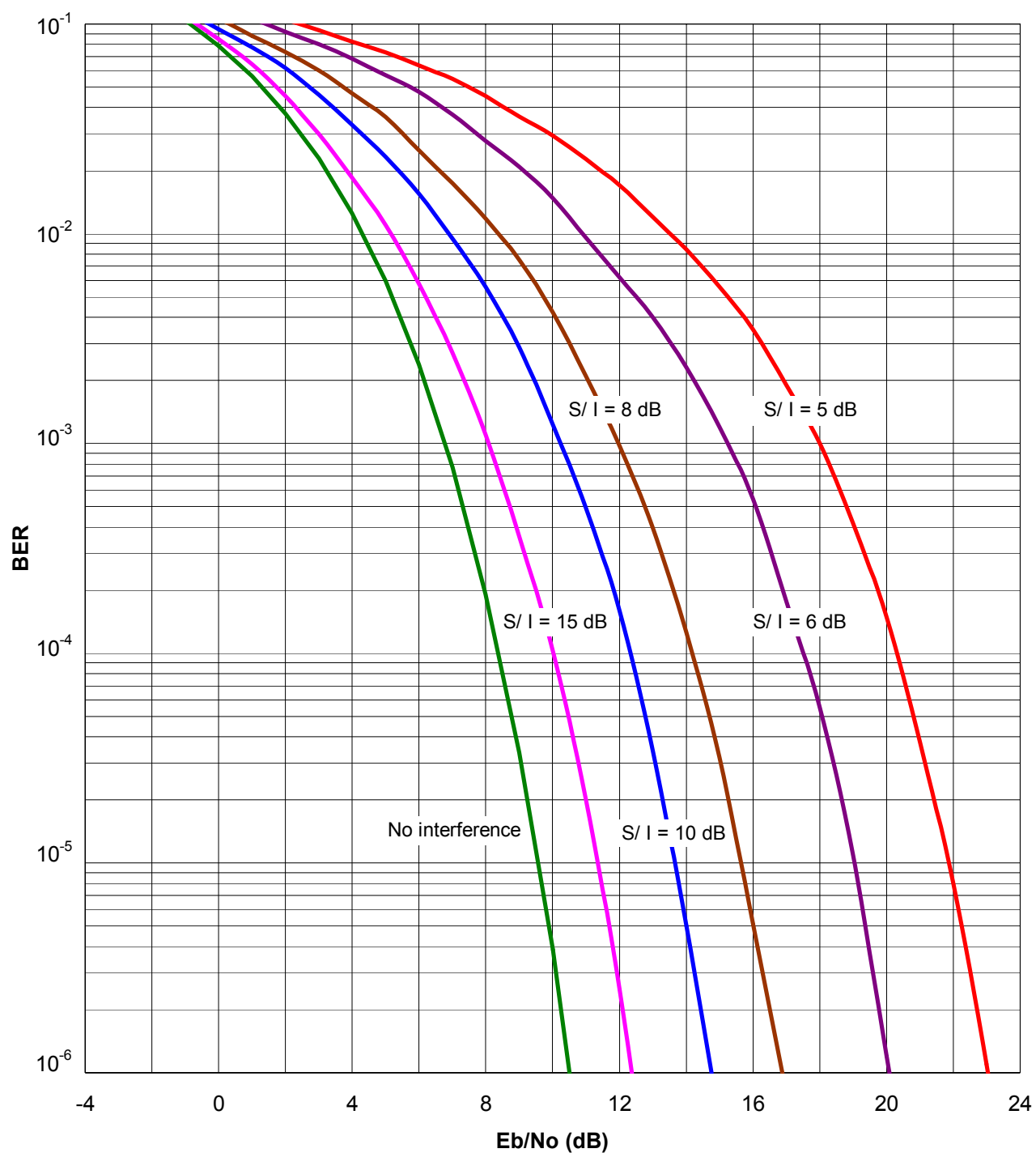
Figure 6-3. BER vs. E_b/N_o Curves for PSK Receiver (M = 4) with CW Interference
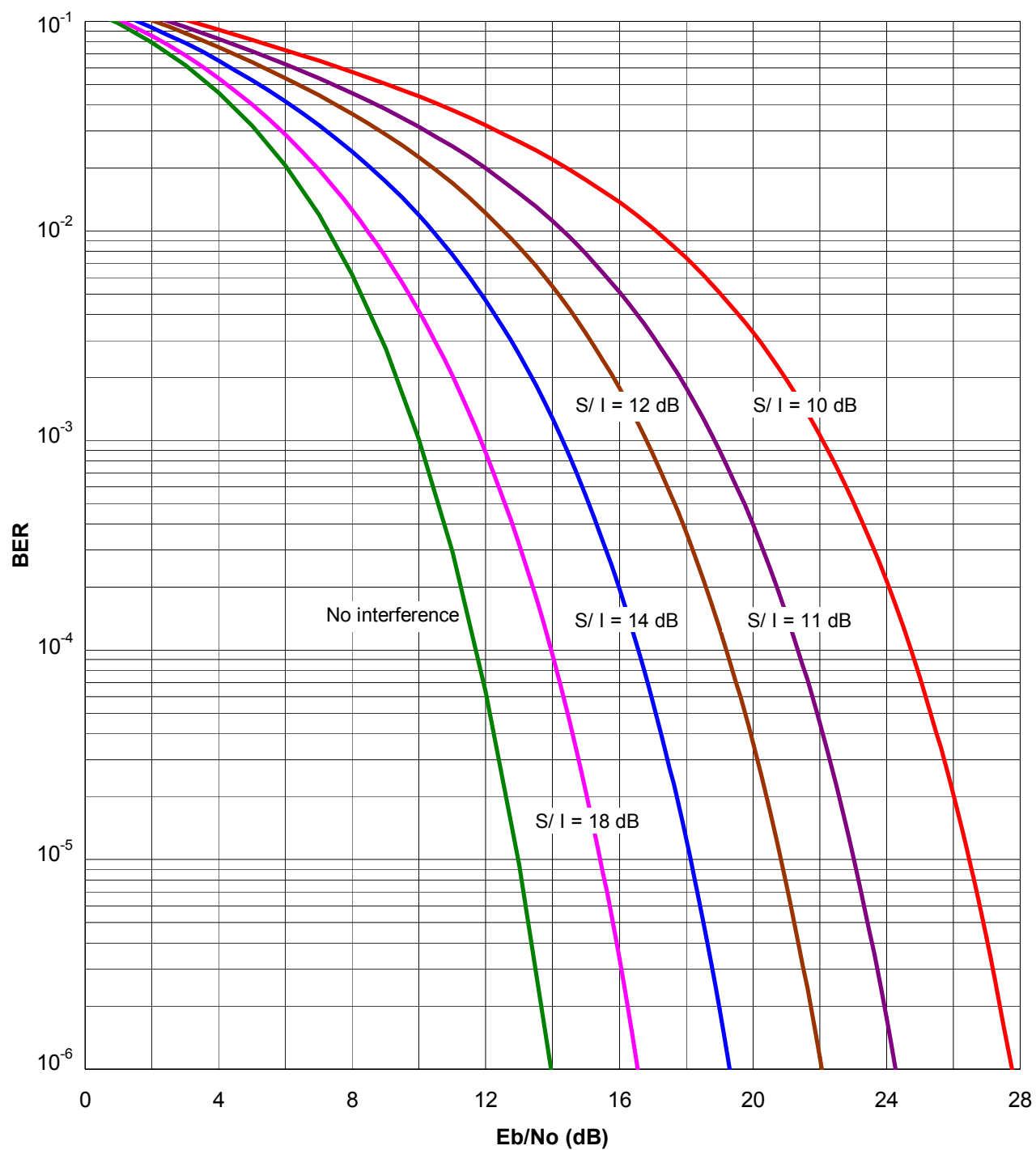
Figure 6-4. BER vs. E_b/N_o Curves for PSK Receiver ($M = 8$) with CW Interference
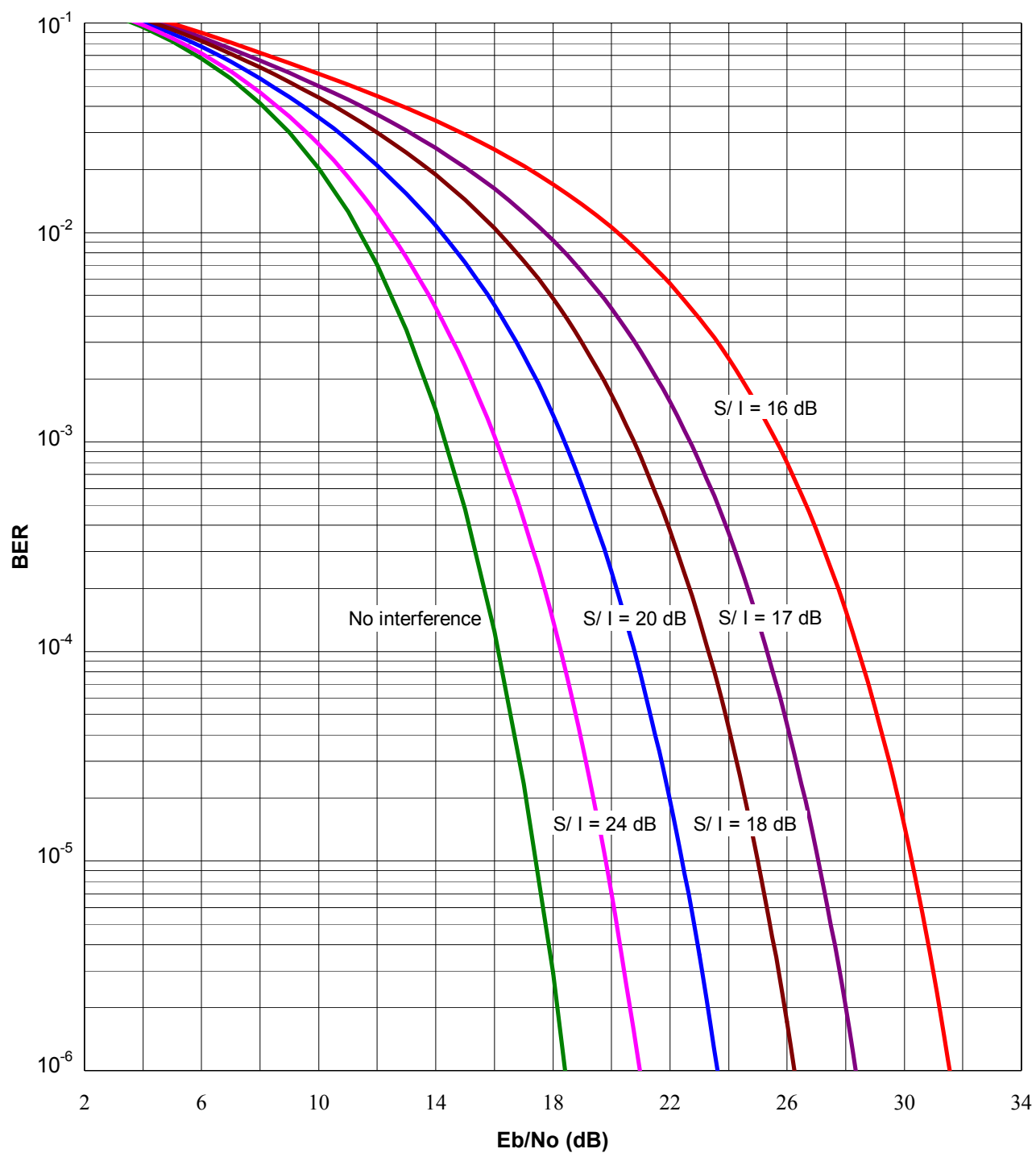
Figure 6-5. BER vs. E_b/N_o Curves for PSK Receiver ($M = 16$) with CW Interference

6.7 DPSK

DPSK is a noncoherent modulation scheme. In coherent reception, the receiver regenerates the carrier from the received signal and then uses the regenerated carrier as a reference for determining the phase of the modulating signal. In DPSK, the receiver does not regenerate the carrier. The demodulator compares the phase of the current bit waveform to the phase of the previous bit waveform rather than to the carrier. Although DPSK permits a simpler receiver implementation, the error performance is not as good as the coherent case. Coherent PSK is presented in Section 6.6.

6.7.1 Description

The waveforms for DPSK are the same as for PSK (Equations 6-15, 6-18, and 6-21). However, the phase term in the equation has a different interpretation. In coherent PSK, a waveform has the phase $i2\pi/M$ when the corresponding symbol value is i. In DPSK, two succeeding waveforms have the phase difference $i2\pi/M$ when the corresponding symbol value is i.

For noise and noise-like interference, the BER for binary DPSK is given by:

$$BER = \frac{1}{2} \exp\left(-\frac{E_b}{N_o} \right)$$

(6-23)

For DPSK with $M > 2$, the SER for noise and noise-like interference is given by (Reference 4-2):

$$SER = \text{erfc}\left[\sqrt{k\frac{E_b}{N_o}}\, \sin\left(\frac{\pi}{\sqrt{2}M} \right) \right]$$

(6-24)

Equation 6-12 can be used with Equation 6-24 to estimate the BER for M-ary DPSK.

6.7.2 BER Curves

Figures 6-6, 6-7, and 6-8 show BER curves for a DPSK receiver with $M = 2$, $M = 4$, and $M = 8$, respectively. In these graphs, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is

calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated by simulation. Each figure displays six curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other five curves are for cases with CW interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

6.8 FSK

6.8.1 Description

FSK uses M different frequencies to transmit the digital data. These signaling frequencies are equally spaced and centered about the RF carrier frequency. The waveform of the FSK signal may be expressed:

$$v_i(t) = A \cos(2\pi f_i t)$$

(6-25)

where f_i = symbol frequency, in Hz.

The separation between adjacent frequencies ($\Delta f = f_i - f_{i-1}$) affects the required bandwidth and the BER. The optimum (smallest) BER occurs when the signaling frequencies are orthogonal. The frequencies are orthogonal when $2\Delta f$ is an integer multiple of the symbol rate R_s.

Coherent reception means that the demodulator is phase-locked to the received signal. For noise and noise-like interference, the BER for coherent FSK (CFSK) is (Reference 4-2):

$$BER = \frac{M-1}{2} \operatorname{erfc}\left(\sqrt{\frac{k}{2}\frac{E_b}{N_o}}\right)$$

(6-26)

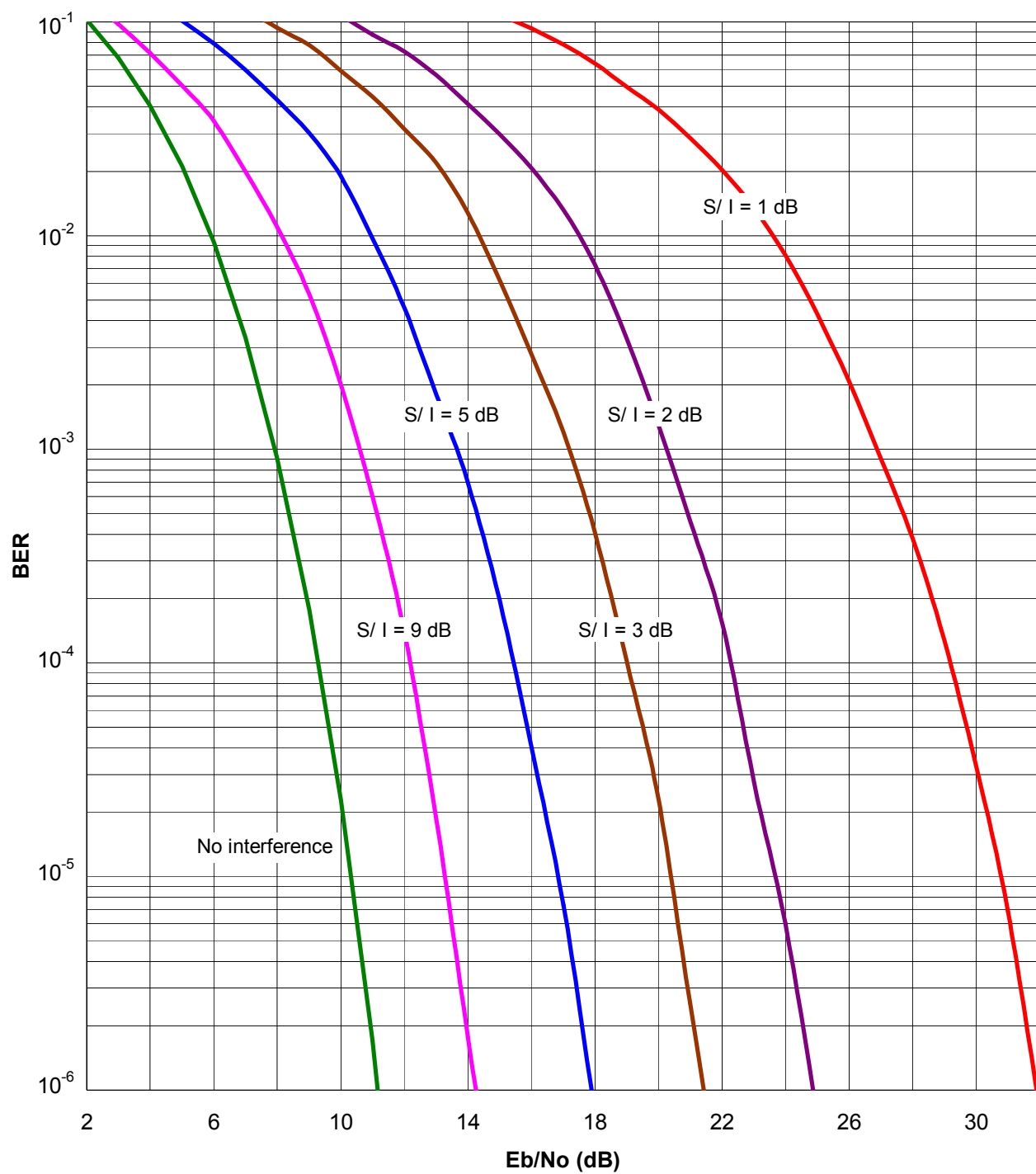This equation assumes orthogonal signaling frequencies.

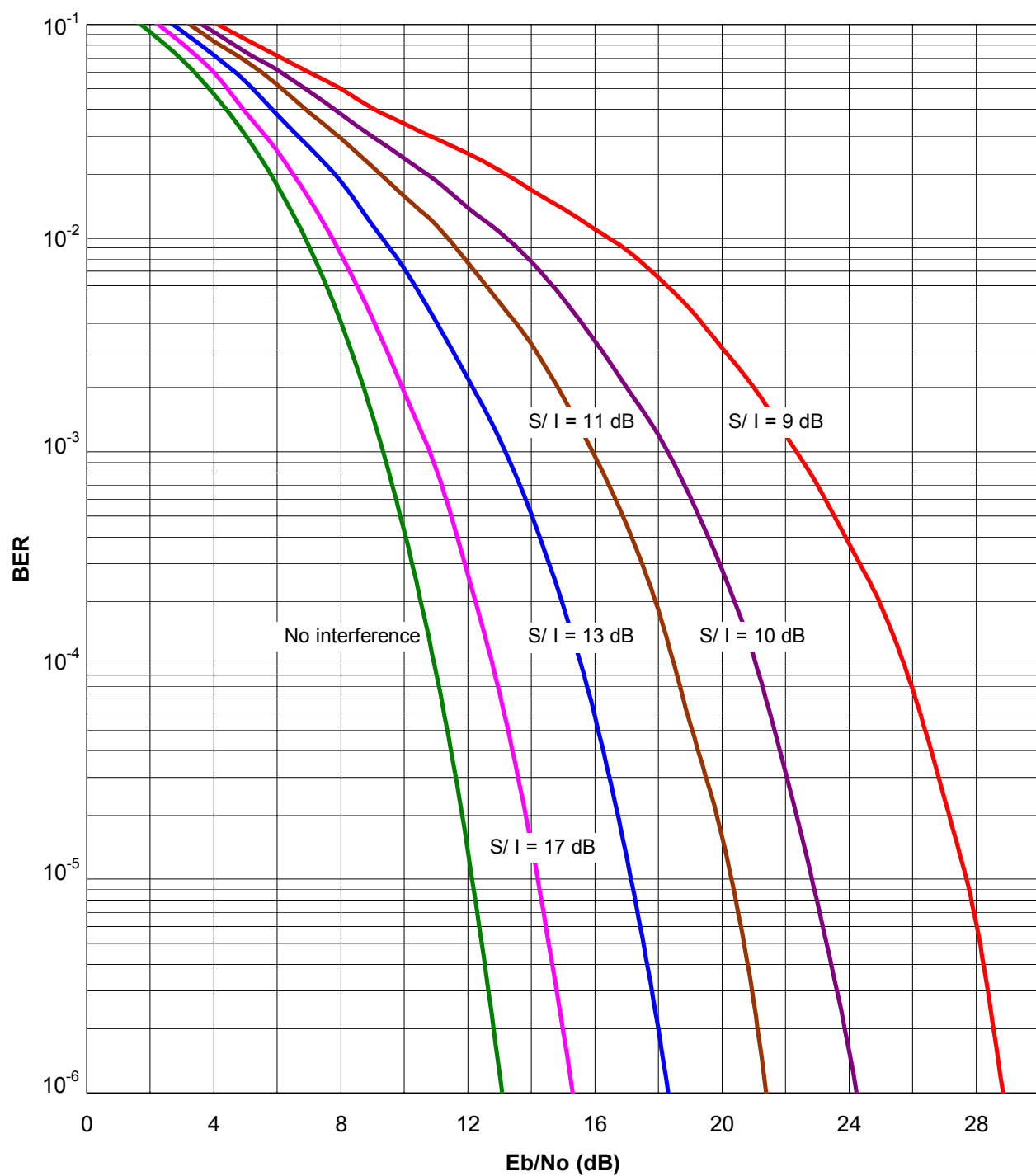**Figure 6-6. BER vs. E_b/N_o Curves for DPSK Receiver ($M = 2$) with CW Interference**

Figure 6-7. BER vs. E_b/N_o Curves for DPSK Receiver (M = 4) with CW Interference
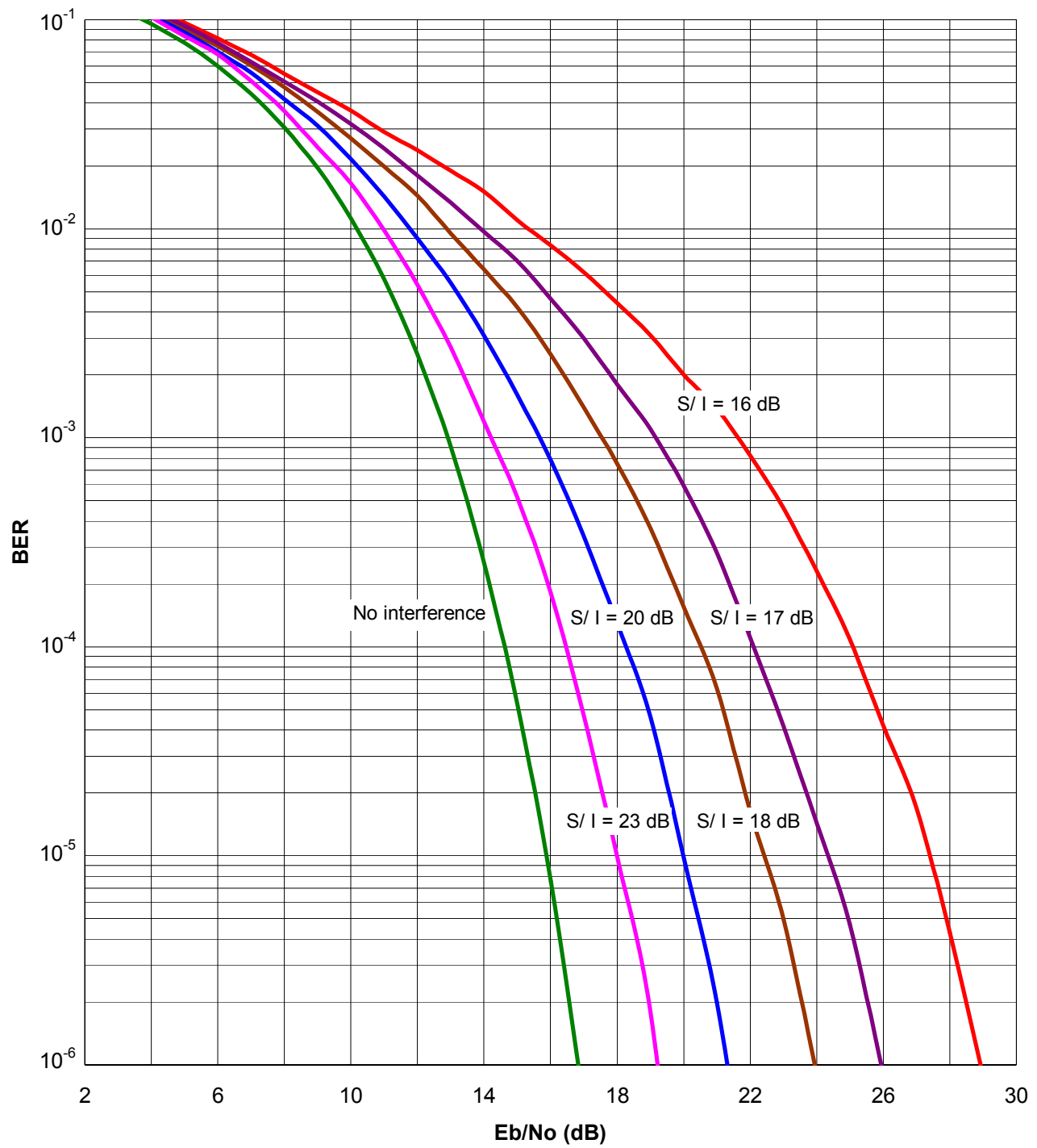
Figure 6-8. BER vs. E_b/N_o Curves for DPSK Receiver ($M = 8$) with CW Interference

The FSK demodulator can be simplified by employing two bandpass filters to noncoherently detect the signaling frequencies. Such a scheme results in degraded performance relative to coherent detection. For noise and noise-like interference, the SER for noncoherent FSK (NCFSK) is (Reference 4-2):

$$SER = \sum_{j=1}^{M-1} (-1)^{j+1} \frac{(M-1)!}{j!(M-1-j)!} \frac{1}{j+1} \exp\left(-\frac{jk}{j+1} \frac{E_b}{N_o}\right)$$

(6-27)

Equation 6-11 can be used with Equation 6-27 to estimate the BER for noncoherent FSK. For the binary case ($k = 1$ and $M = 2$), Equation 6-27 reduces to:

$$BER = \frac{1}{2} \exp\left(-\frac{1}{2} \frac{E_b}{N_o}\right)$$

(6-28)

6.8.2 BER Curves

Figure 6-9 shows BER curves for a binary coherent FSK receiver. Figures 6-10, 6-11, and 6-12 show BER curves for a noncoherent FSK receiver with $M = 2$, $M = 4$, and $M = 8$, respectively. In these graphs, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated from analytic expressions. The expressions involving CW interference required numerical integration. Each figure displays six curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other five curves are for cases with CW interference. Each of those five curves is labeled with the S/I for that curve. In all cases, the CW interference was close in frequency to one of the M signaling frequencies. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

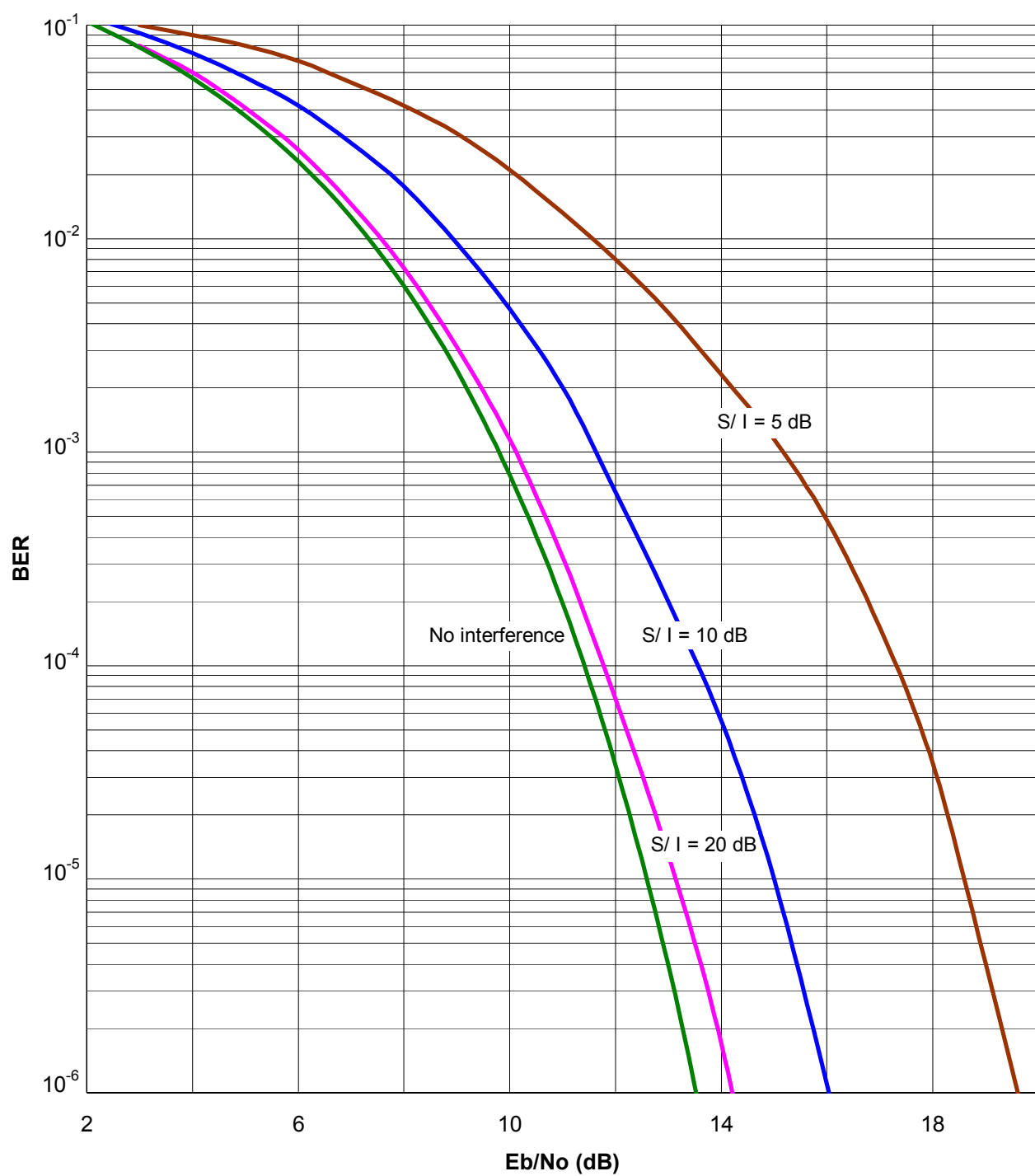**Figure 6-9. BER vs. E_b/N_o Curves for Coherent FSK Receiver ($M = 2$) with CW Interference**
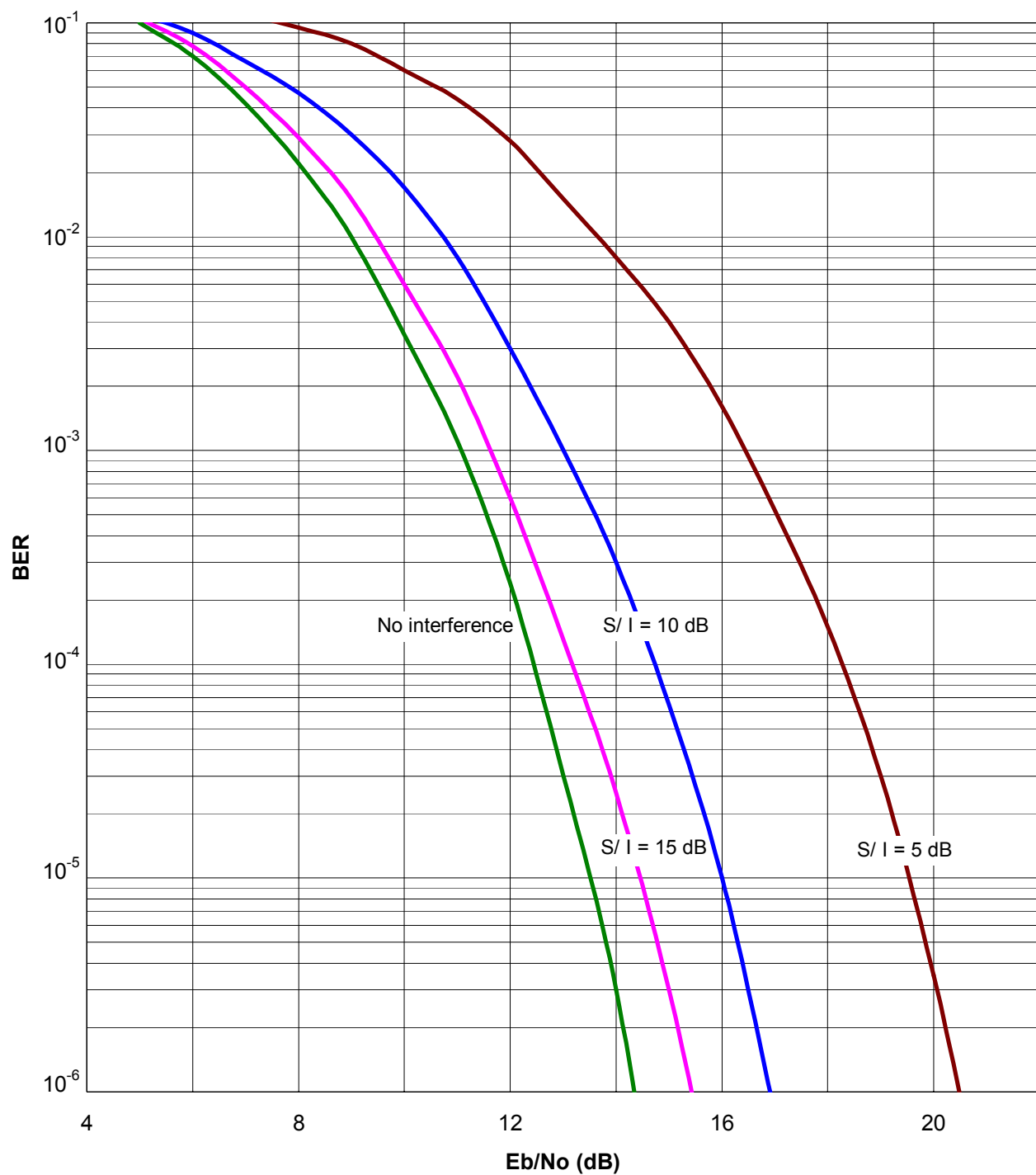
Figure 6-10. BER vs. E_b/N_o Curves for Noncoherent FSK Receiver ($M = 2$) with CW Interference
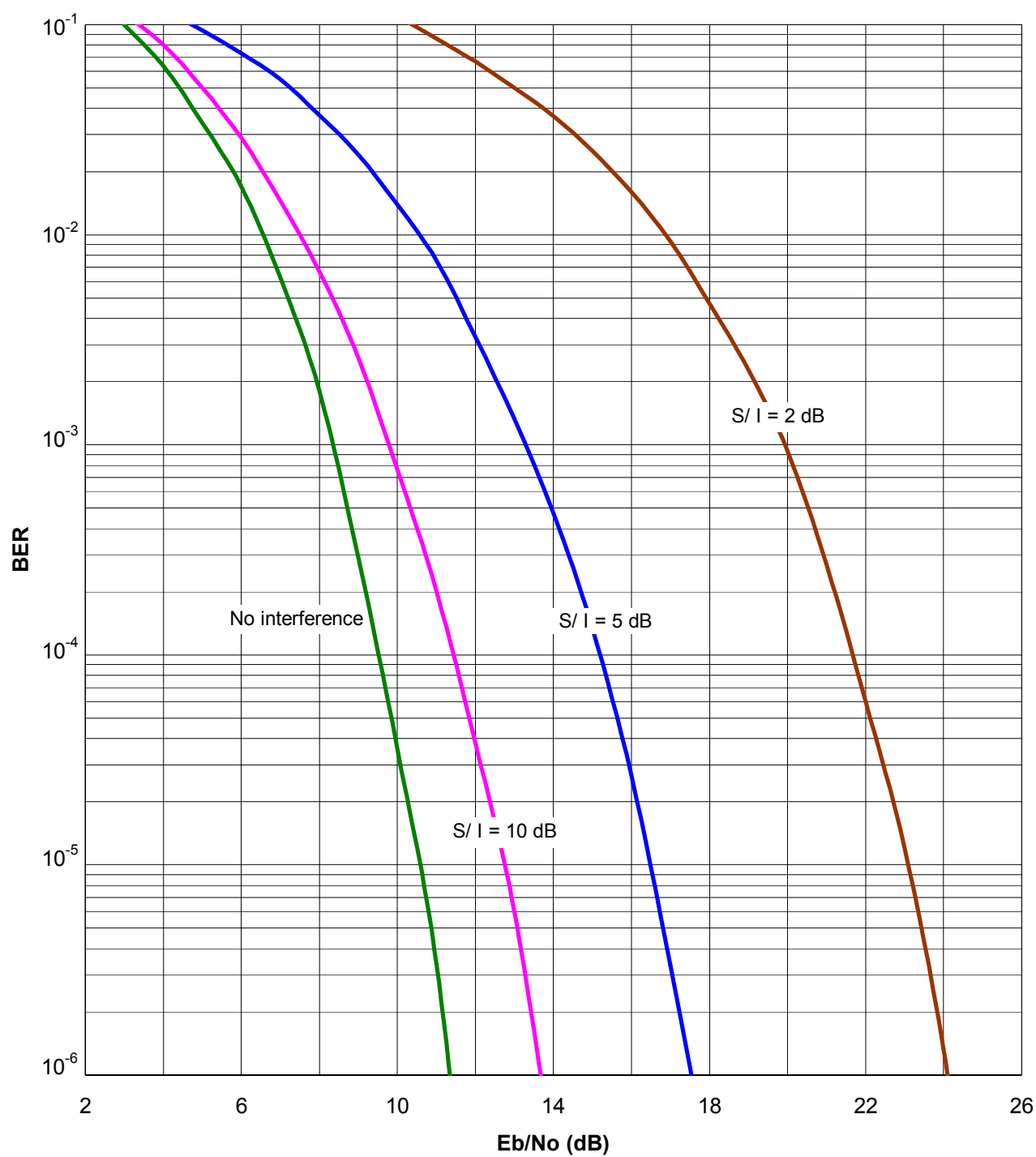
Figure 6-11. BER vs. E_b/N_o Curves for Noncoherent FSK Receiver ($M = 4$) with CW Interference
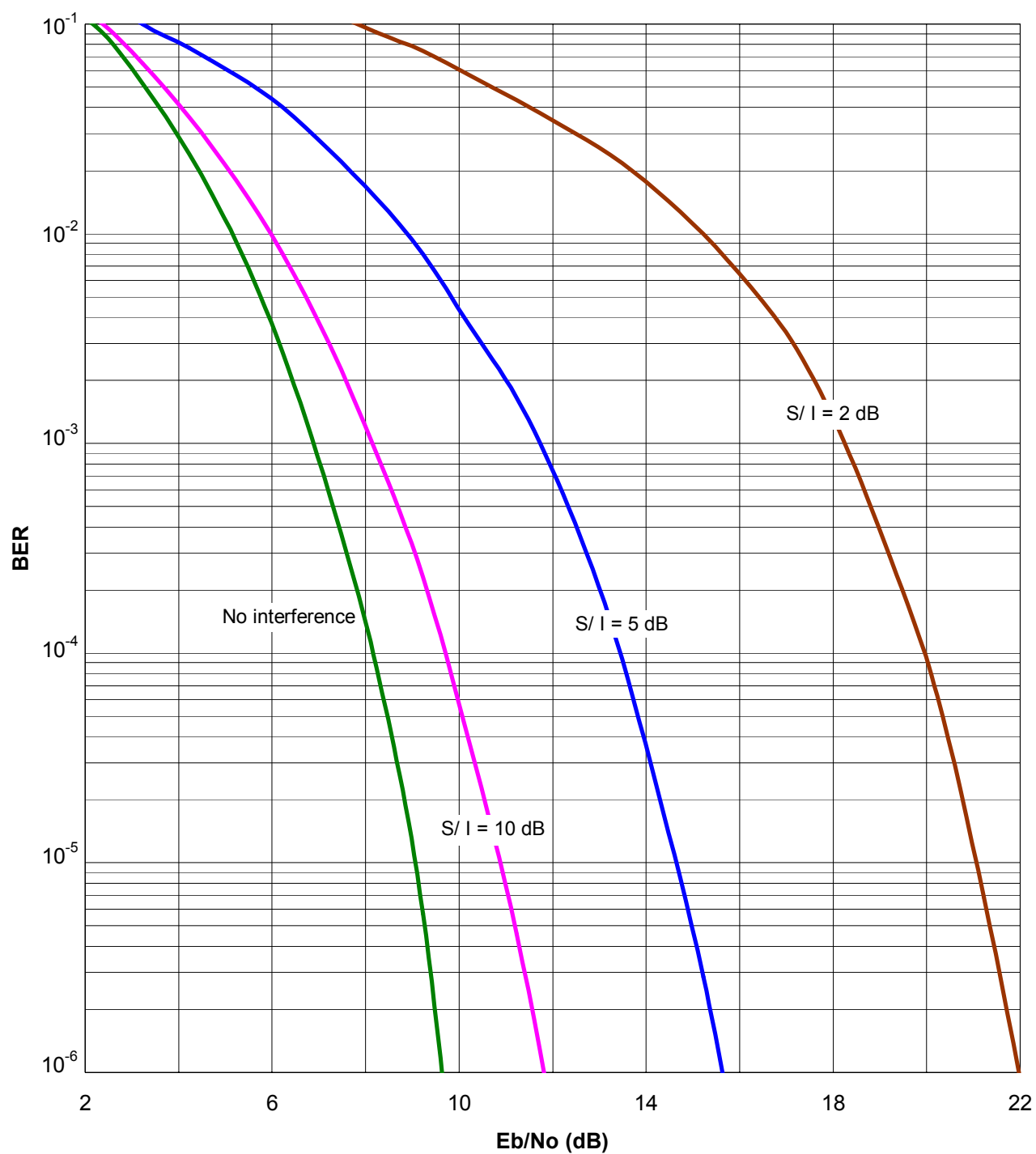
Figure 6-12. BER vs. E_b/N_o Curves for Noncoherent FSK Receiver (M = 8) with CW Interference

6.9 MINIMUM-SHIFT KEYING

6.9.1 Description

The minimum-shift keying (MSK) waveform can be represented as an FSK waveform with the two signaling frequencies separated by $R_b/2$. This is the minimum separation that allows orthogonality. MSK can also be represented as a form of quadrature-modulated PSK (i.e., QPSK), with the additional specifications that

- The quadrature carriers are offset in time by T_b (offset QPSK or OQPSK)
- Even bits appear on the in-phase channel and odd bits appear on the quadrature channel
- The baseband (modulating) pulses are sinusoidal, rather than rectangular.

Because of the waveform complexity, it is necessary to give the waveform of the entire bit sequence rather than the waveform for a single bit. The entire MSK waveform is (Reference 4-2):

$$v(t) = A\left[\sum_{n=-\infty}^{\infty} I_{2n} u(t - 2nT_b)\right]\cos(2\pi f_c t) + A\left[\sum_{n=-\infty}^{\infty} I_{2n+1} u(t - [2n+1]T_b)\right]\sin(2\pi f_c t)$$

$$(6\text{-}29)$$

where $\displaystyle\sum_{n=-\infty}^{\infty} I_{2n}$ represents the sequence of even-numbered information bits ($I_{2n} = \pm 1$)

$\displaystyle\sum_{n=-\infty}^{\infty} I_{2n+1}$ represents the sequence of odd-numbered information bits ($I_{2n+1} = \pm 1$)

The sinusoidal pulse-shaping function is:

$$u(t) = \sin(\pi t / 2T_b) \qquad\qquad 0 \le t \le 2T_b$$

$$(6\text{-}30)$$

Note that $u(t)$ extends over two bit intervals. These MSK properties eliminate phase discontinuities that would otherwise occur when the waveform changes state in accordance with the modulating signal. For this reason, MSK is more spectrally efficient than either standard QPSK or OQPSK with rectangular modulating pulses.

This expression for the MSK waveform clearly shows the information bits, the pulse-shaping function, and the quadrature carriers. A variety of modulation types can be derived from OQPSK by varying the

pulse-shaping function. Some communications systems are capable of varying the pulse-shaping function in response to operational conditions.

The BER for MSK is the same as for standard QPSK:

$$BER = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_o}}\right)$$

(6-31)

6.9.2 BER Curves

Figure 6-13 shows BER curves for an MSK receiver. In this graph, the term "interference" and the variable *I* refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where *N* is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves are the same as the QPSK curves, which were generated from analytic expressions. The expressions involving CW interference required numerical integration. The figure displays six curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other five curves are for cases with CW interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

6.10 QUASI-BANDLIMITED MSK

6.10.1 Description

Quasi-bandlimited MSK (QBL-MSK) is a variation of MSK. In the MSK waveform (Equations 6-29 and 6-30), a pulse shaping function $u(t)$ is applied to the normally rectangular pulses that represent the input bit sequence. The pulse-shaping function is used to provide smooth transitions between bit periods, as seen in the time domain. The result of this pulse-shaping, as seen in the frequency domain, is that more of the signal power is concentrated around the RF carrier. This in turn allows for greater *bandwidth efficiency*, which is the ratio of the bit rate to the bandwidth required for transmission at that bit rate.
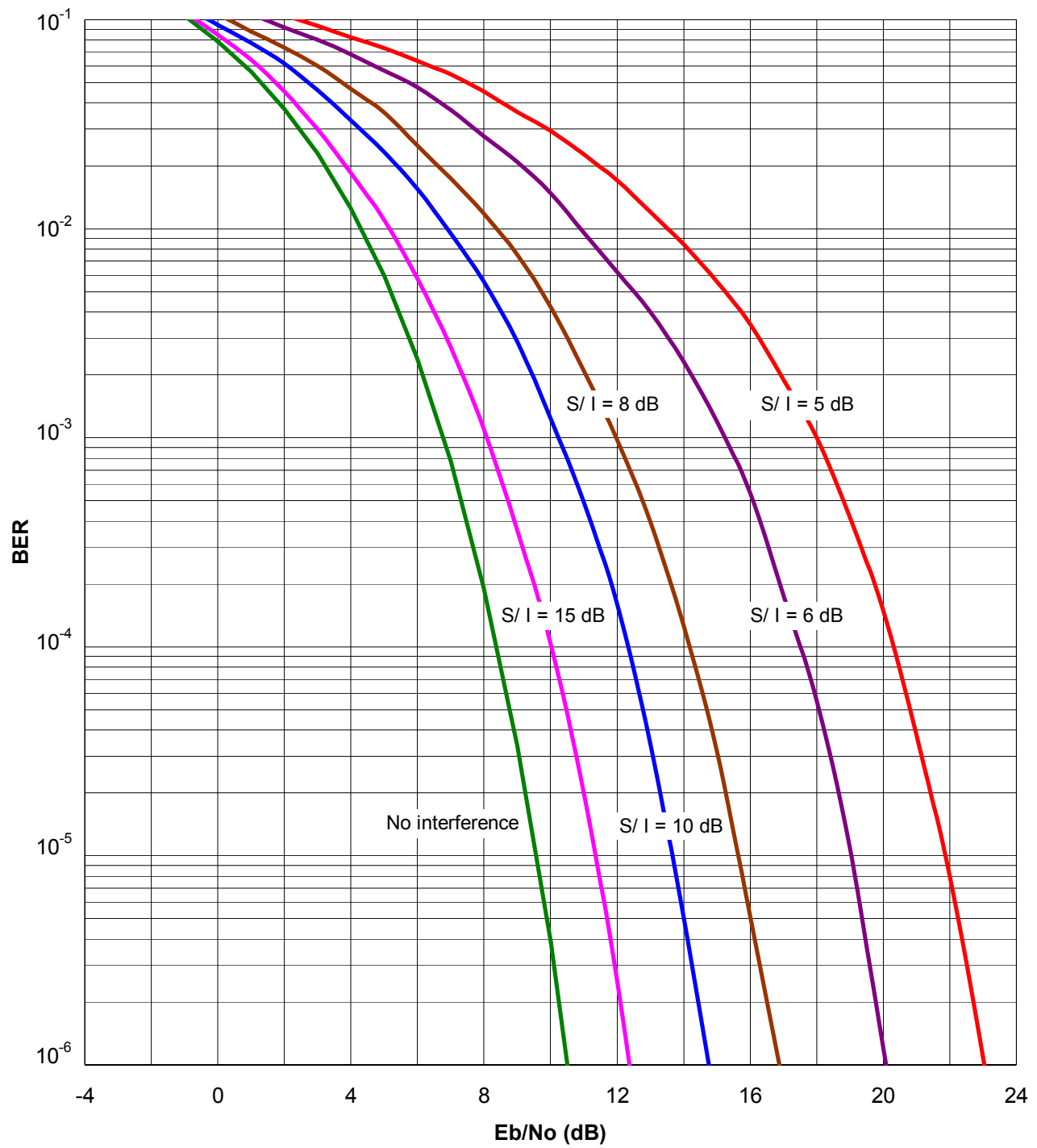
Figure 6-13. BER Curves for an MSK Receiver with CW Interference

Bandwidth efficiency is indirectly related to error performance. While it may be useful to compare the error performance (BER vs. E_b/N_o) of modulation types assuming no band-limiting, any practical system must operate within a predetermined bandwidth. This constraint favors modulation types that have high bandwidth efficiency. For a given bit rate, bandwidth efficiency can be adjusted by employing different pulse-shaping functions.

The wide variety of pulse-shaping functions that have been studied can be classified as constant-envelope or varying-envelope, referring to the modulated RF signal. The type of envelope affects the *power efficiency* of the communications system. A signal with a non-constant envelope requires linear amplification. If the amplification is not linear, the bandwidth of the signal will be increased. Linear amplification is not required for signals with a constant-envelope. A typical amplifier is linear over only part of its operating range. Consequently, a signal with a non-constant envelope can utilize only a portion of the available operating range of an amplifier.

Certain pulse-shaping functions that exhibit high bandwidth efficiency result in modulated signals that have non-constant envelopes. This property reduces the power efficiency of the transmitter. Either a more linear or a more powerful amplifier will be required, or else the transmitted power will be reduced.

Bandwidth efficiency and power efficiency are competing interests for signal designers. Consideration of these interests has led to the development of modulation types that offer a compromise. QBL-MSK is an example of such a modulation type. It offers relatively good bandwidth efficiency, while maintaining the constant-envelope property. Thus, power efficiency is not compromised.

The waveform of the MSK signal includes a sinusoidal pulse-shaping function $u(t)$. For QBL-MSK, the pulse-shaping function is:

$$u(t) = \left(\frac{\sin(\pi t / T_b)}{\pi t / T_b} \right)^n \qquad -2T_b \leq t \leq 2T_b$$

(6-32)

where typically $n = 3$.

The result of employing this pulse-shaping technique to bandlimit the signal is a modest (2 dB or less) improvement in error-performance with respect to standard MSK. The precise improvement depends on the precise bandwidth.

6.10.2 BER Curves

Figure 6-14 shows BER curves for a QBL-MSK receiver with $n = 3$ (Equation 6-32). In this graph, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated by simulation. The figure displays three curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other two curves are for cases with CW interference. Each of those two curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

6.11 ASK

6.11.1 Description

ASK uses M different equally spaced amplitudes to transmit the digital data. Each M-ary symbol represents k bits of information. The waveform of the ASK signal may be expressed:

$$v_i(t) = A_i\, u(t) \cos(2\pi f t)$$

$$(6\text{-}33)$$

where A_i =   amplitude for symbol i, in V

$u(t)$ is a pulse-shaping function. If the pulses are rectangular, then $u(t) = 1$. Other pulse shapes may be employed to reduce the required bandwidth of the transmitted signal.

Coherent reception means that the demodulator is phase-locked to the received signal. If the system is coherent, the detector can distinguish phase reversals. In that case, A and −A are two distinct amplitudes. A *unipolar* system uses only non-negative amplitudes, whereas a *bipolar* system uses positive and negative amplitudes. For example, a 4-ary unipolar system has amplitudes 0, a, 2a, 3a. A 4-ary bipolar system with the same amplitude spacing (a) has amplitudes -3a/2, -a/2, a/2, 3a/2.
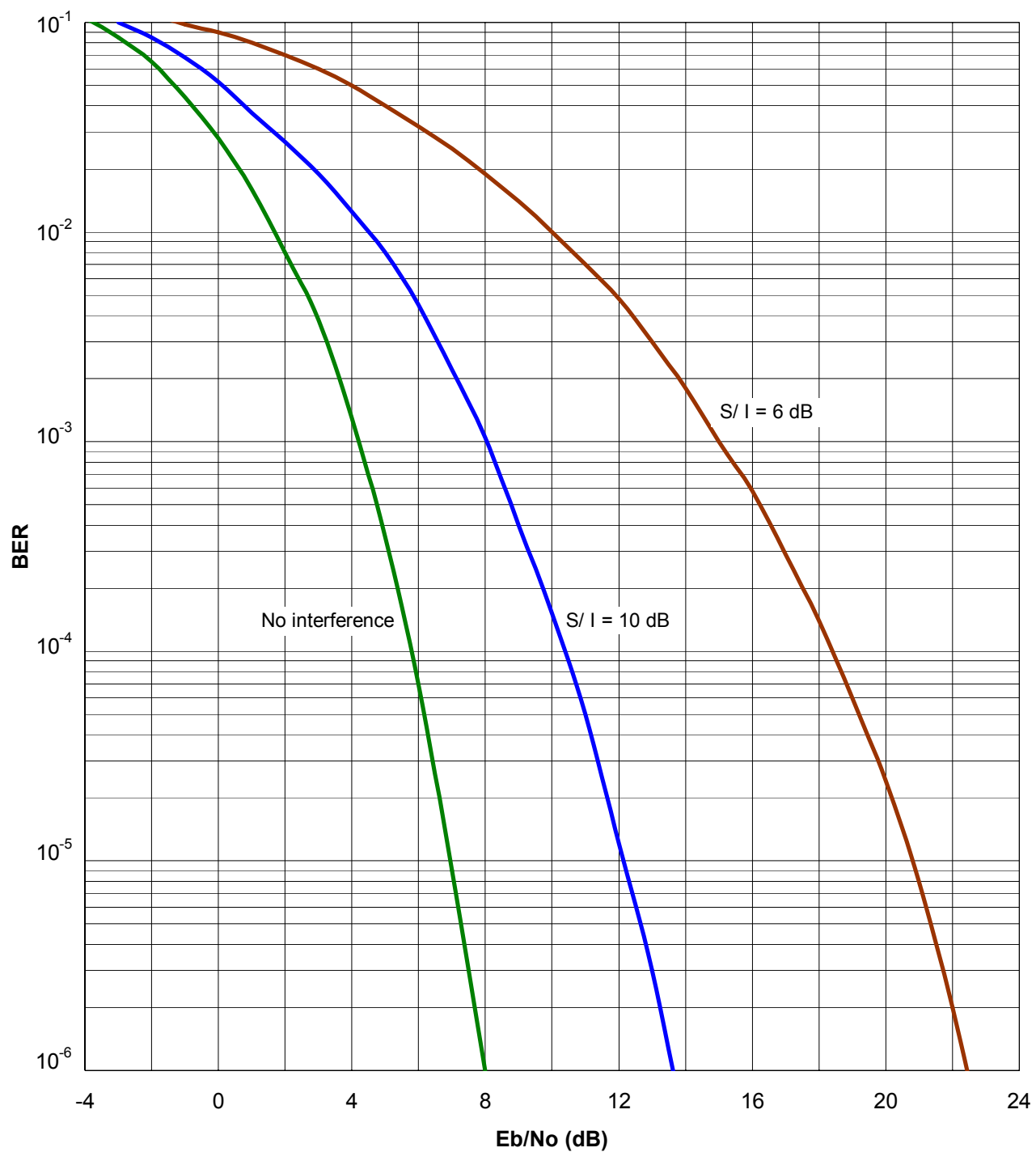
Figure 6-14. BER vs. E_b/N_o Curves for QBL-MSK Receiver with CW Interference

For noise and noise-like interference, the SER for coherent unipolar M-ary ASK is (Reference 4-2):

$$SER = \frac{M-1}{M} \text{erfc}\left(\sqrt{\frac{3k}{2(M-1)(2M-1)} \frac{E_b}{N_o}}\right)$$

(6-34)

For noise and noise-like interference, the SER for coherent bipolar M-ary ASK is (Reference 4-2):

$$SER = \frac{M-1}{M} \text{erfc}\left(\sqrt{\frac{3k}{(M^2-1)} \frac{E_b}{N_o}}\right)$$

(6-35)

Equation 6-12 can be used with either Equation 6-34 or 6-35 to determine the corresponding BER.

Coherent unipolar binary ASK (with amplitudes 0 and *a*) is also known as *on-off keying* (OOK). Note that coherent bipolar binary ASK (with amplitudes –*a*/2 and *a*/2) is the same as binary PSK and Equation 6-35 reduces to Equation 6-17 in the binary case.

ASK can also be detected noncoherently. In this case, the waveforms must be unipolar. The noncoherent demodulator implementation is simpler, but its performance is not as good as that of coherent detection.

6.11.2 BER Curves

Figures 6-15 and 6-16 show BER curves for a coherent unipolar ASK receiver with $M = 2$ and $M = 4$, respectively. Figures 6-17 and 6-18 show BER curves for a coherent bipolar ASK receiver with $M = 4$ and $M = 8$, respectively. Figure 6-19 shows BER curves for a noncoherent ASK receiver with no CW interference. In these graphs, the term "interference" and the variable *I* refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where *N* is the total noise-like power (including the receiver noise and the noise-like interfering signal).

Figure 6-15. BER vs. E_b/N_o Curves for Coherent Unipolar ASK Receiver

(M = 2) with CW Interference

Figure 6-16. BER vs. E_b/N_o Curves for Coherent Unipolar ASK Receiver

($M = 4$) with CW Interference

Figure 6-17. BER vs. E_b/N_o Curves for Coherent Bipolar ASK Receiver
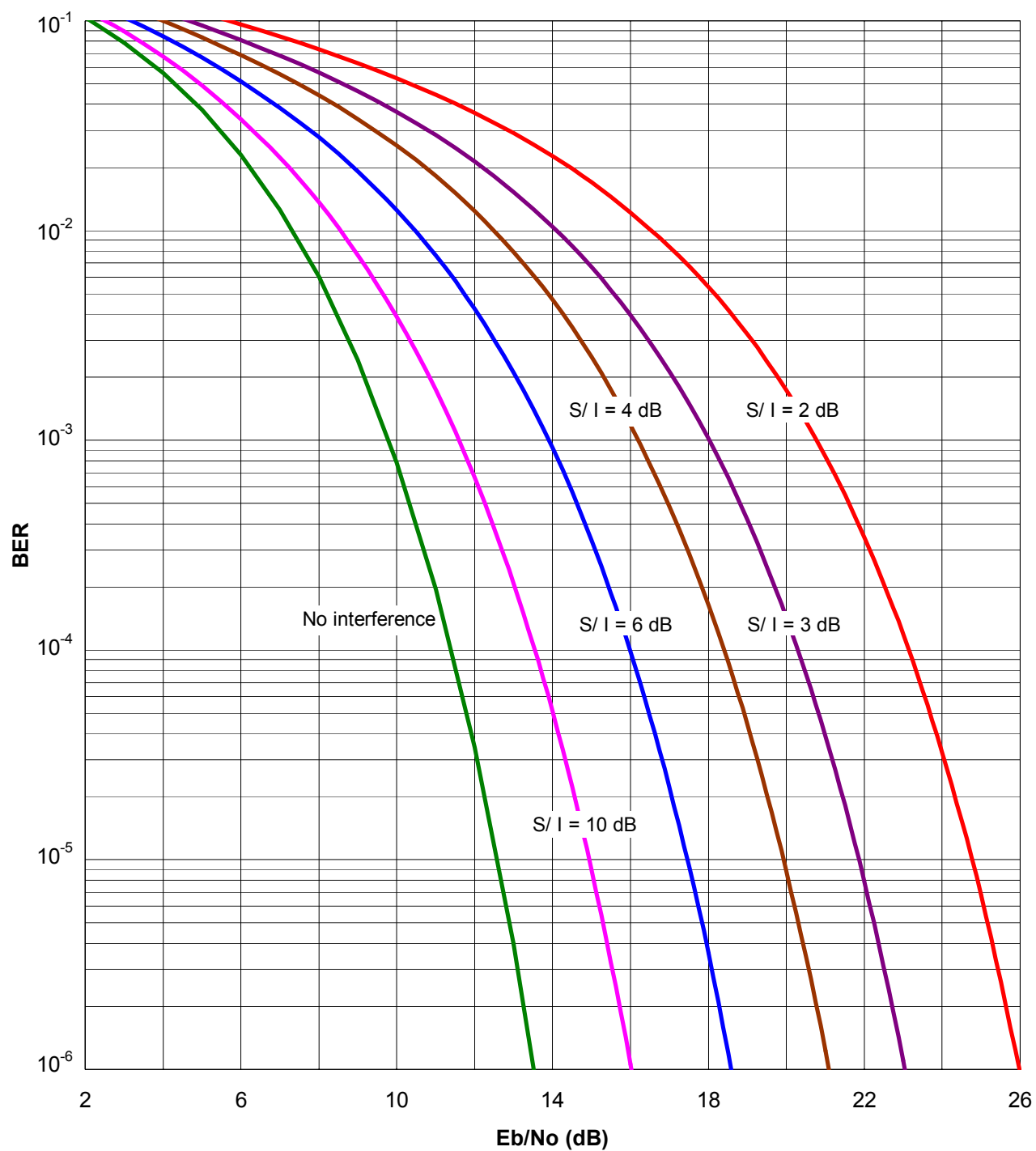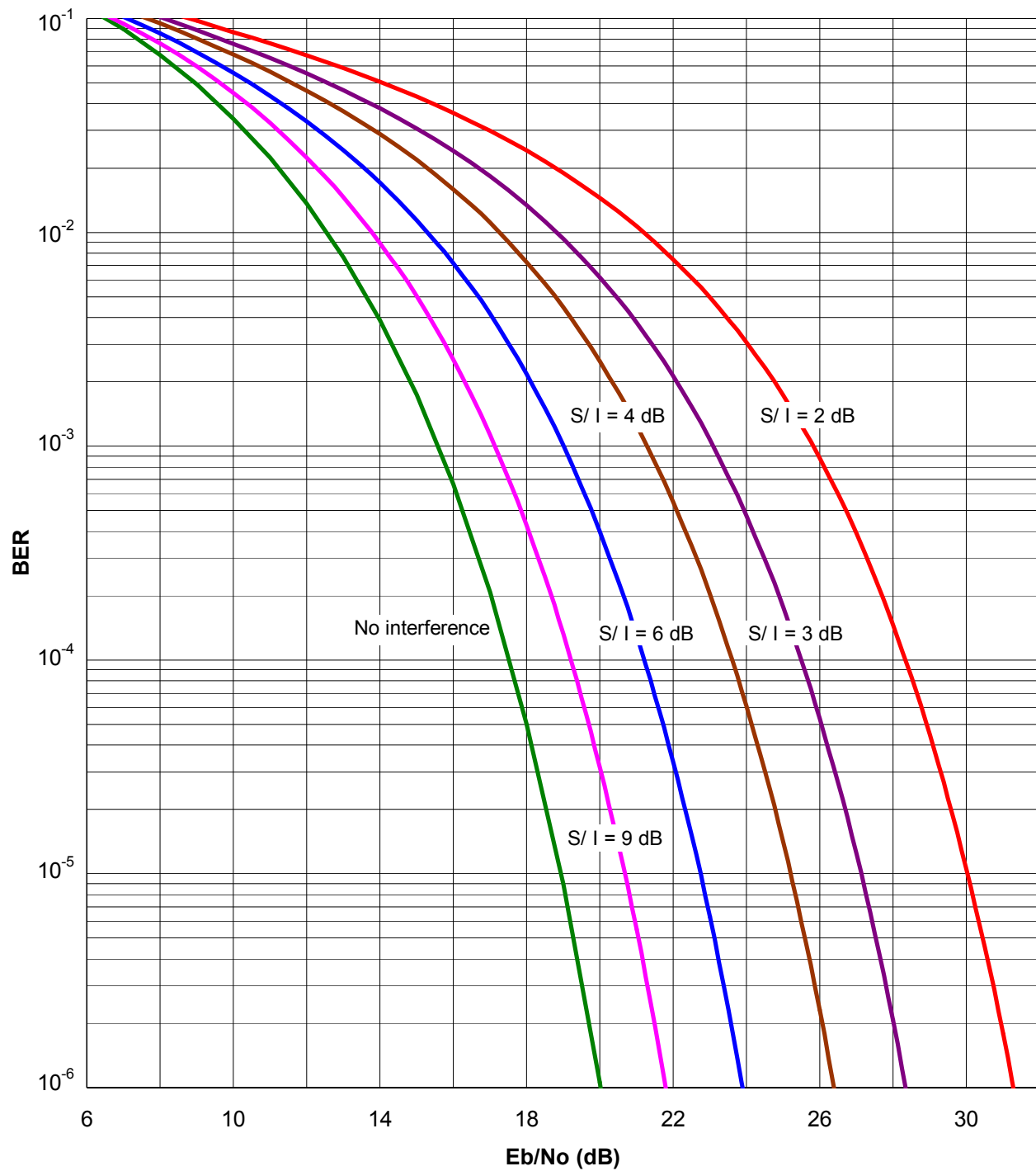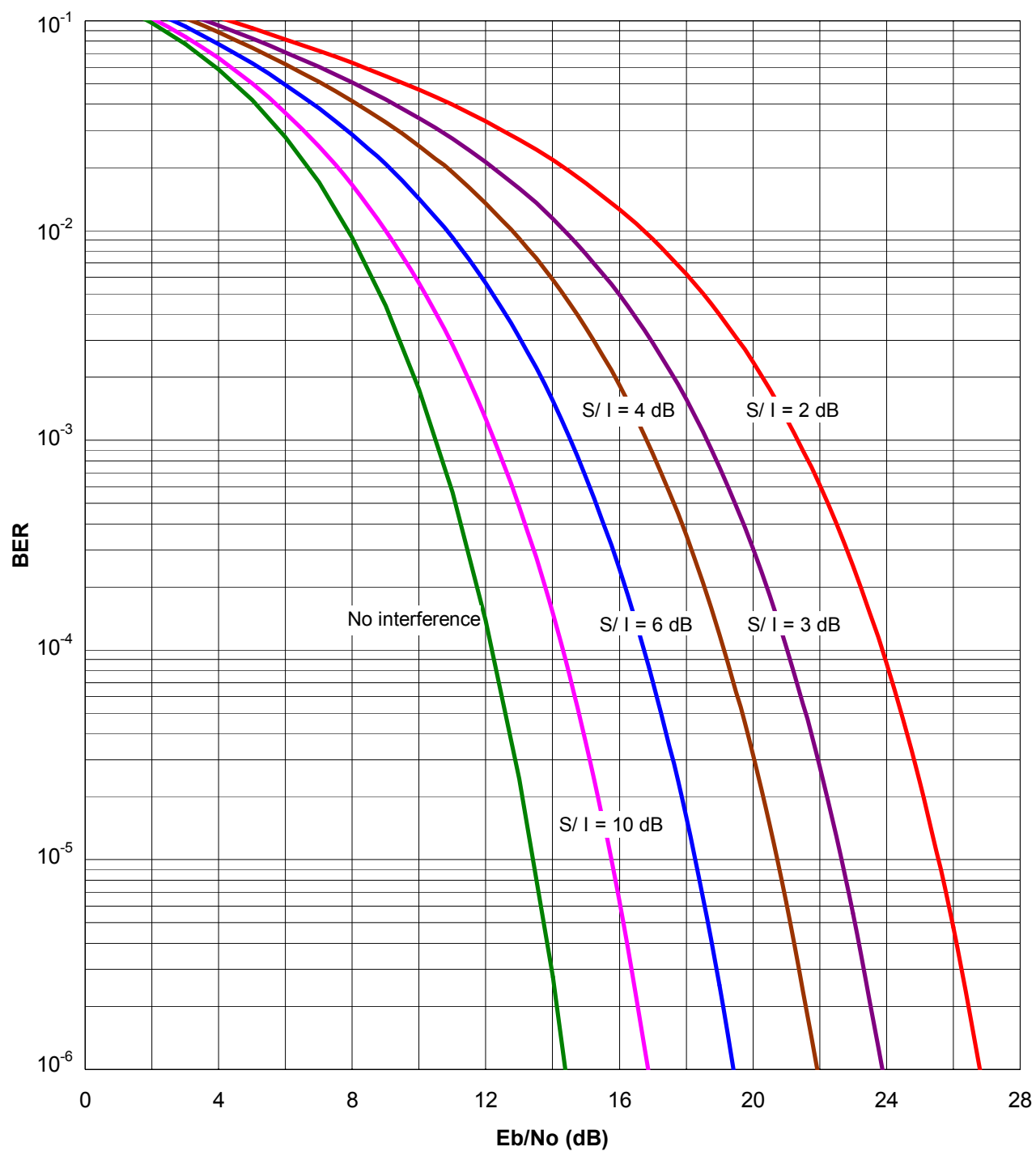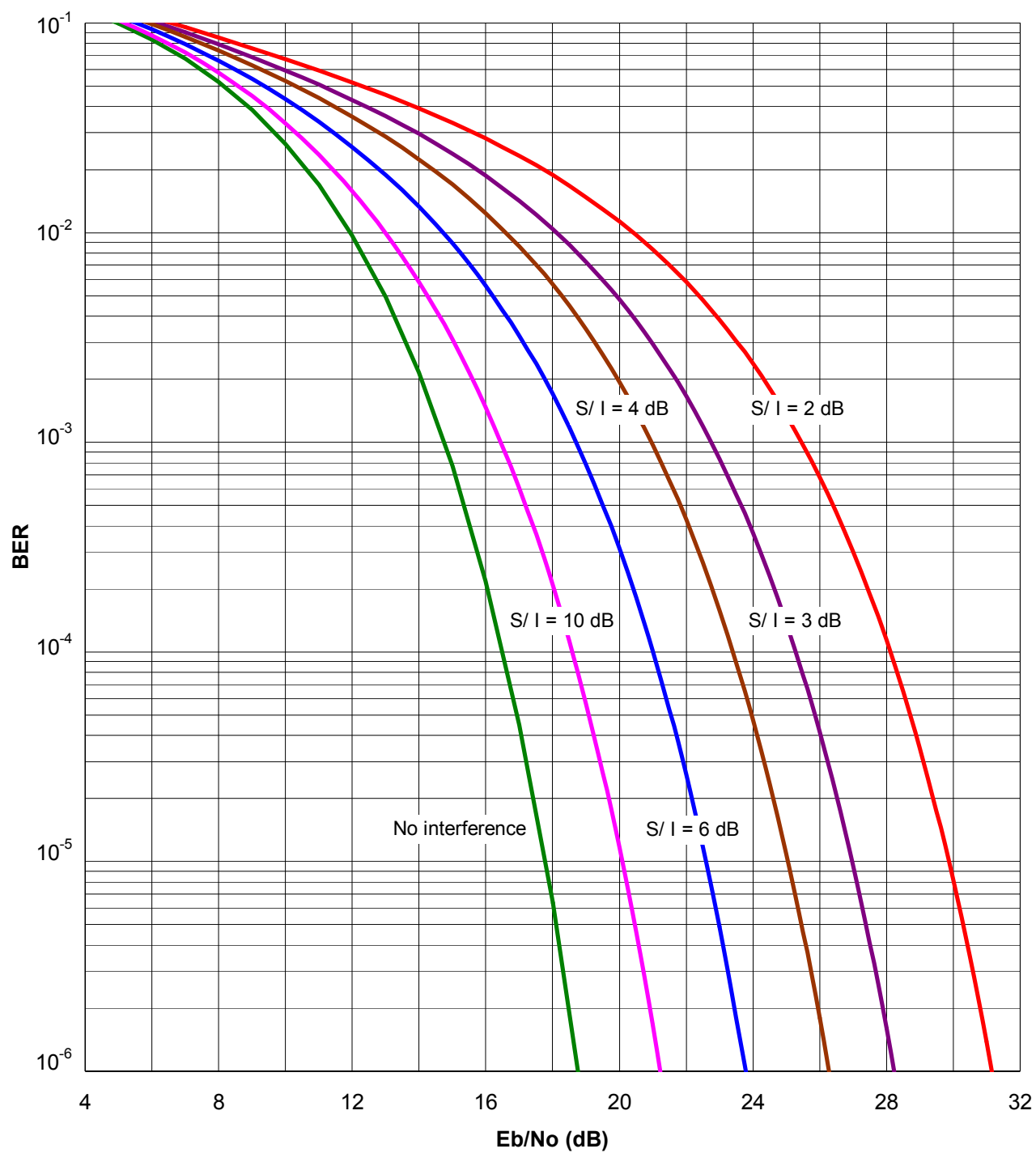
($M = 4$) with CW Interference

Figure 6-18. BER vs. E_b/N_o Curves for Coherent Bipolar ASK Receiver

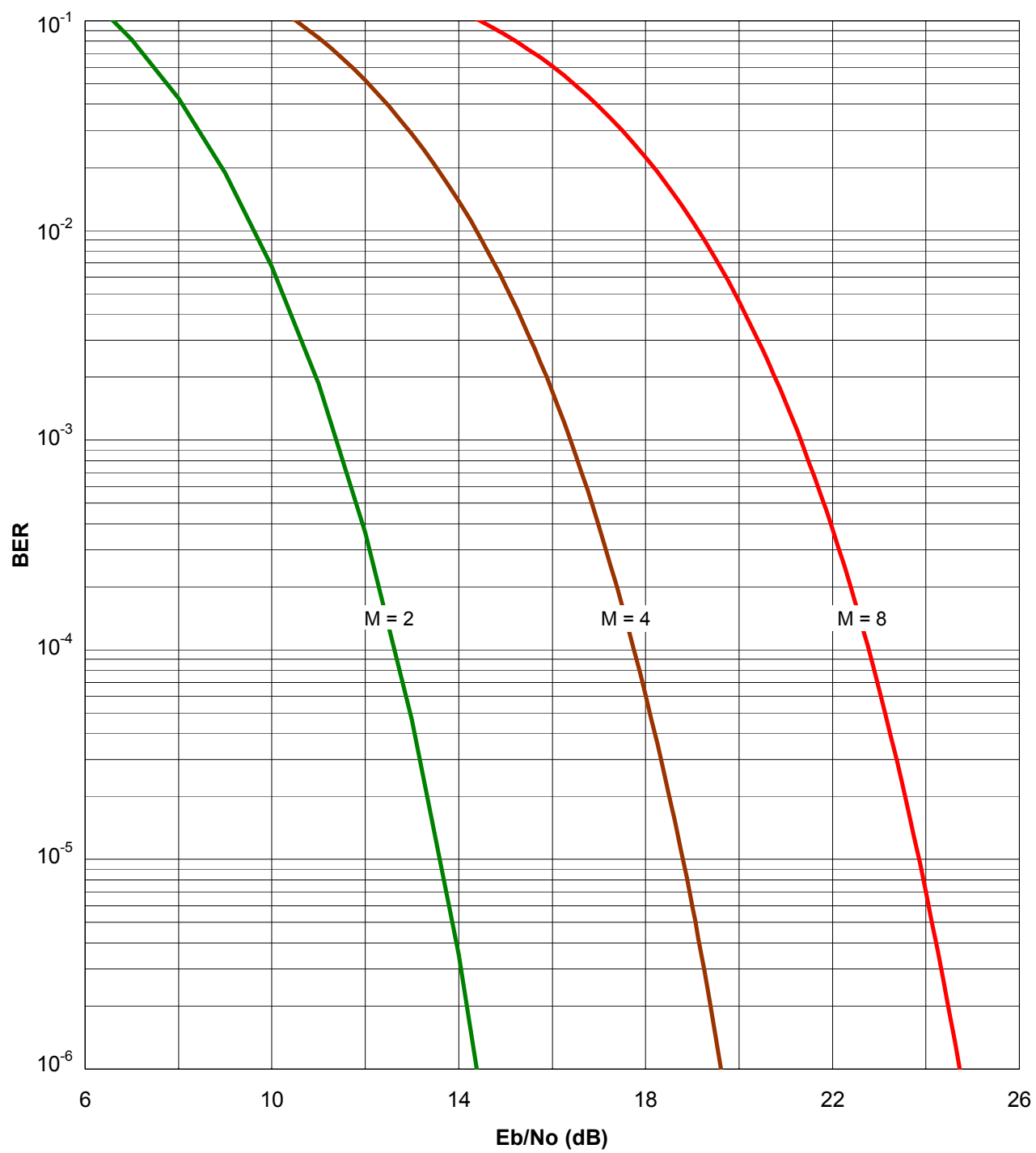**($M = 8$) with CW Interference**

Figure 6-19. BER vs. E_b/N_o Curves for Noncoherent ASK Receiver

The curves were generated from analytic expressions. The expressions involving CW interference required numerical integration. Figures 6-15 through 6-18 display six curves of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other five curves are for cases with CW interference. Each of those five curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

6.12 QUADRATURE AMPLITUDE MODULATION

6.12.1 Description

The quadrature amplitude-modulation (QAM) technique employs two carrier signals in phase quadrature. The QAM-modulated waveform can be expressed:

$$v_i(t) = A_i u(t) \cos(2\pi f_c t) + B_i u(t) \sin(2\pi f_c t)$$

(6-36)

where A_i, B_i = in-phase and quadrature amplitudes for symbol i, in V

$u(t)$ is a pulse-shaping function. If the pulses are rectangular, then $u(t) = 1$. Other pulse shapes may be employed to reduce the required bandwidth of the transmitted signal. Usually, there are 2^m values of A_i and 2^m values of B_i, where m is an integer. Thus, the value of M might be $(2^2)(2^2) = 16$, $(2^3)(2^3) = 64$, $(2^4)(2^4) = 256$, etc. Equation 6-5 ($M = 2^k$) is still applicable, so $M = 256$ when the number of bits per symbol (k) is 16. 4-ary QAM is the same as QPSK.

For noise and noise-like interference, the SER for a QAM system is approximated by (Reference 4-2):

$$SER = 2 \operatorname{erfc}\left(\sqrt{\frac{3k}{2(M-1)} \frac{E_b}{N_o}} \right)$$

(6-37)

QAM can be viewed as two coherent bipolar ASK signals impressed on the quadrature channels. Therefore, the SER with CW interference can be approximately determined by using Equation 6-35 (with $M = \sqrt{M_{QAM}}$) to determine the SER for each coherent bipolar ASK signal. Then the SER for the QAM signal is given by:

$$SER = 1 - [1 - SER(ASK)]^2$$

(6-38)

where   *SER(ASK)*   =   SER from Equation 6-35.

Equation 6-12 can be used with either Equation 6-37 or 6-38 to determine the corresponding BER.

6.12.2 BER Curves

Figures 6-20 and 6-21 show BER curves for a QAM receiver with $M = 16$ and $M = 64$, respectively. In these graphs, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).

The curves were generated from analytic expressions. Each figure displays four curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other three curves are for cases with CW interference. Each of those three curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.
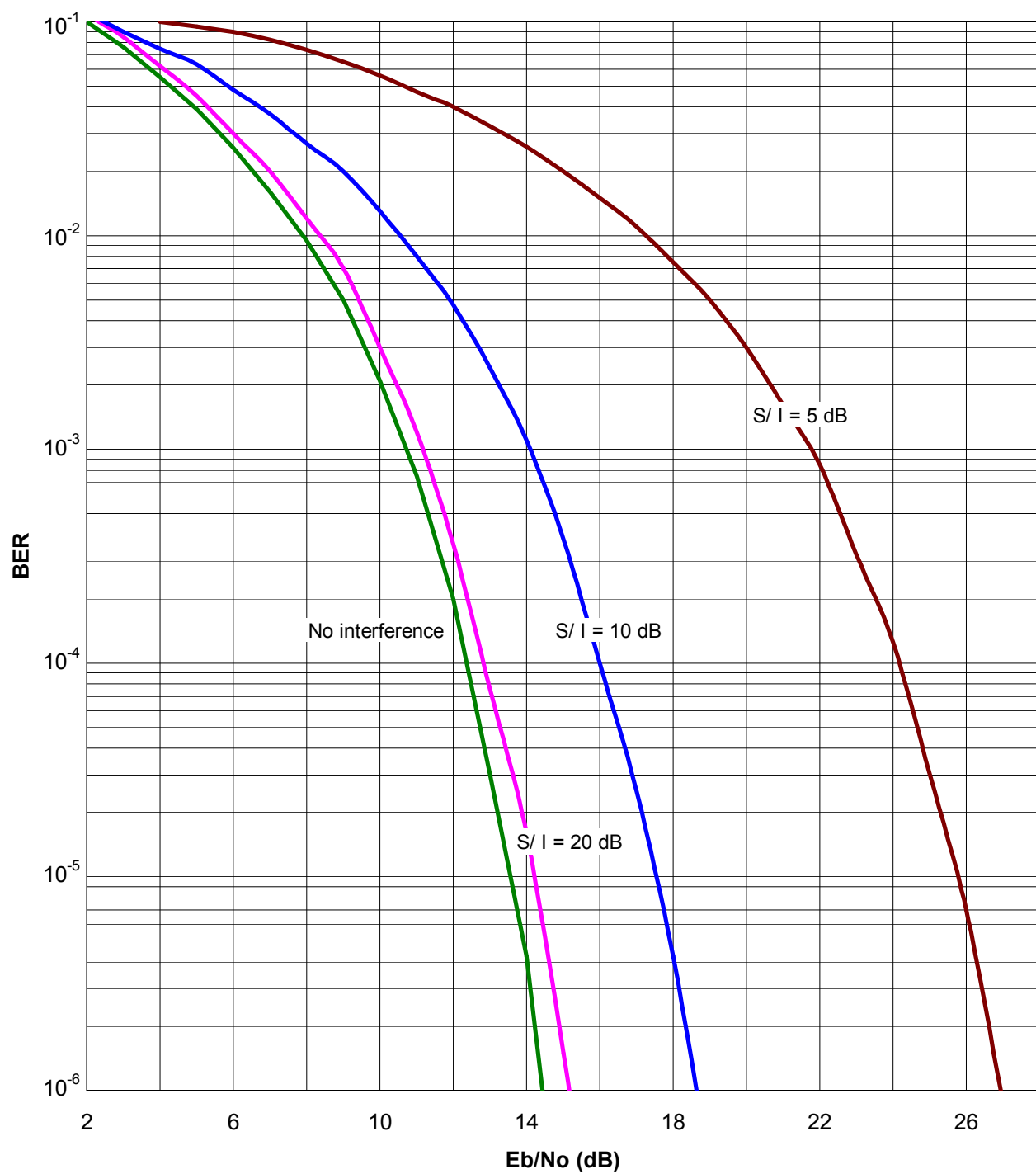
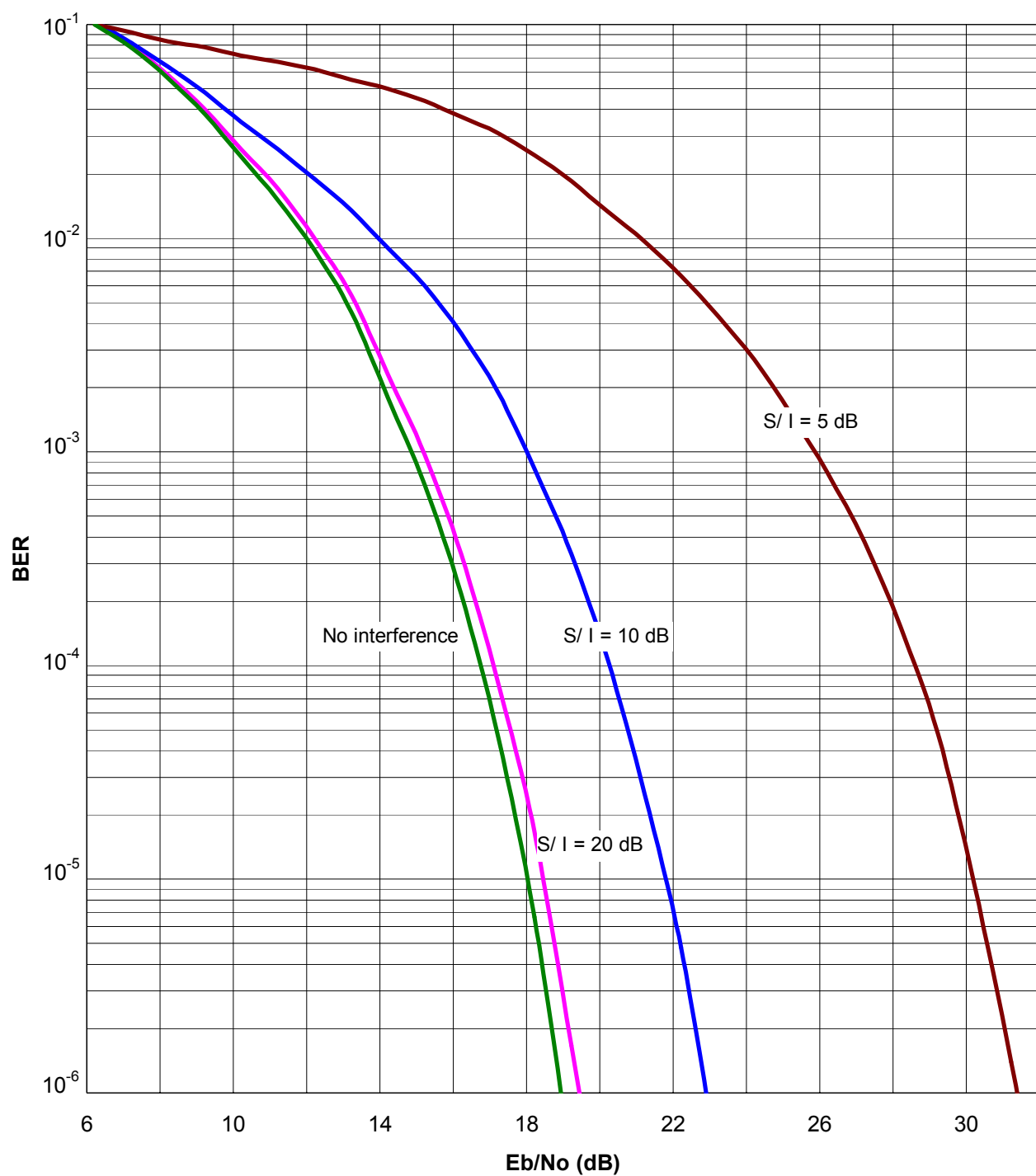**Figure 6-20. BER vs. E_b/N_o Curves for QAM Receiver ($M = 16$) with CW Interference**

Figure 6-21. BER vs. E_b/N_o Curves for QAM Receiver ($M = 64$) with CW Interference

SECTION 7 – FEC DECODER

This section describes the FEC decoder model for digital receivers. Decoder inputs and outputs are described for both hard-decision and soft-decision schemes. The properties of various block and convolutional decoders are specified. Some general procedures for calculating output BER are presented, including the case in which a receiver has concatenated FEC decoders. For hard-decision systems, curves that specify output BER as a function of input BER are displayed. For soft-decision systems, the demodulator and FEC decoder are treated as one combined module, and curves that specify output BER as a function of input E_b/N_o are displayed.

7.1 INTRODUCTION

The purpose of the FEC decoder is to correct bit errors that are present in the demodulated data. These errors are the result of noise and interference in the channel. The errors are corrected by applying the decoding algorithm associated with the encoding algorithm used by the transmitter. Ideally, the bit sequence at the FEC decoder output is identical to the original bit sequence at the FEC encoder input.

FEC encoding involves adding redundancy, in the form of extra bits, to the information data. The FEC decoder removes this redundancy during the decoding process. There are two basic types of FEC code: *block codes* and *convolutional codes*.

There are limits to the capabilities of error-correction codes. In general, the reduction in BER is directly related to the amount of redundancy added. This section provides transfer functions and analysis techniques that quantify the performance of FEC decoders. This section also introduces some of the terminology that is specific to FEC technology and analysis.

7.2 HARD DECISION VS. SOFT DECISION

The FEC decoder gets its input from the digital demodulator. A *hard-decision* demodulator makes a definite decision on each input bit waveform (which may be degraded with noise and interference) and then outputs that decision as a 0-bit or a 1-bit. This output bit waveform may use two voltage levels, say -V and +V, to represent a logical 0-bit or 1-bit. There is nothing in the output data that indicates how reliable the demodulator decisions are.

A *soft-decision* demodulator does not make a definite decision on a given input bit waveform. Instead, it outputs a *bit likelihood indicator*; i.e., a value that indicates the likelihood that a 0-bit or 1-bit was sent. For example, the bit likelihood indicator for a given bit may be an integer value between 0 and 7,

where 0 represents a very high likelihood that a 0-bit was sent and 7 represents a very low likelihood that a 0-bit was sent. The intermediate values from 1 to 6 represent intermediate likelihoods that a 0-bit was sent. For example, a value of 6 means that a 1-bit was probably sent but that there is some unreliability in that decision.

FEC decoders can be designed to work with either type of input. The decoder examines bit sequences for errors and, if it detects any, it makes correcting adjustments to the data. If the decoder input is a soft-decision input, the decoder can use the reliability information to improve its correction algorithm. Soft-decision demodulation and decoding can significantly improve the performance of a system that employs FEC coding, while adding very little to the system complexity and cost.

7.3 DECODER PERFORMANCE

For a hard-decision system, the input to the FEC decoder is a sequence of bits, including information bits and redundancy bits. Because the decoder removes the redundancy bits, the output bit rate is lower than the input bit rate. Typically, the decoder corrects some bit errors and the output BER is less than the input BER. The measure of performance in this handbook for such an FEC decoder is the output BER expressed as a function of the input BER. The output BER is sometimes referred to as the *residual error rate*.

For a soft-decision system, the input to the FEC decoder is a sequence of bit likelihood indicators, one for each information bit and redundancy bit. As in hard decision, there is one decoder output bit for each information bit, and a bit error occurs whenever the decoder output bit is not equal to the corresponding original information bit. In this case, there is no BER at the decoder input because definite bit decisions have not been made at that point. For such a system, the most direct measure of performance combines demodulator and decoder effects. It specifies the decoder output BER as a function of E_b/N_o and S/I at the demodulator input.

7.4 TEMPORAL FLUCTUATIONS

If an undesired signal is intermittent, bit errors tend to be more numerous during the interference dwells, and less numerous between the dwells. If the interference is of sufficient strength and duration, the FEC decoder may be overloaded and the BER may approach ½ during a dwell, which means the decoder output is completely random. Over the long term, this condition is likely to manifest itself as error bursts, separated by periods of relatively error-free operation.

However, the overload status of the decoder depends on how the interference dwell duration "matches up" with decoder properties such as code word duration. Interleaving (described in Section 7.5.8) can

be used with hard-decision or soft-decision, block or convolutional codes to effectively convert long interference pulses into shorter, more uniformly distributed pulses. It should be noted that some FEC codes are designed specifically to handle burst errors. Such a code may be concatenated with a second code designed to combat random errors, in order to provide more complete error protection.

In both block and convolutional decoding, output bit errors tend to appear in clusters. However, the output BER, which is a long-term average quantity, is still a valid and useful measure of performance.

7.5 BLOCK CODES

7.5.1 General Concepts and Terminology

A block code is a mapping between two sets of bits: the information bits and the coded bits. The information bits are grouped into blocks with k bits per block. These information blocks are sometimes referred to in technical literature as *messages*. There are 2^k possible messages. The coded bits are grouped into blocks with n bits per block, where $n > k$. These code blocks are usually called *code words*. There are 2^n possible code words. The set of code words is called the *code set*, or simply the *code*.

The code mapping is the algorithm that associates every possible message to a unique code word. Thus, block encoding involves the addition of $n - k$ bits of redundancy to each message. Such a code is referred to as an (n, k) code. The ratio defined as $R = k / n$ is the *code rate*. The mapping is one-to-one, which means that there are only 2^k valid code words, even though there are 2^n possible code words. The decoder exploits the fact that only some of the possible n-bit code words are valid, through a process called *maximum likelihood decoding*.

The *weight* of a code word is defined as the number of 1-bits in the code word. If all of the code words in the set have the same weight, the code is called an *equal-weight code*.

The *distance* between two code words is the number of bit positions in which the two code words differ. The *minimum distance* of a code set, which is designated *d*, is the smallest distance between all code word pairs in the set. In general, a larger minimum distance implies a more powerful error-correction capability.

A block code is *linear* if the modulo-2 sum of any two code words is also a code word. This property reduces the complexity of the FEC encoder.

A *systematic* code is one in which each codeword is a concatenation of the message it represents with $r = n - k$ additional parity bits. This property reduces the complexity of the FEC encoder.

An example of a (7, 4) linear systematic code is shown in Table 7-1.

Table 7-1. Linear, Systematic (7, 4) Block Code

Message	Code Word
0000	0000000
0001	0001011
0010	0010110
0011	0011101
0100	0100111
0101	0101100
0110	0110001
0111	0111010
1000	1000101
1001	1001110
1010	1010011
1011	1011000
1100	1100010
1101	1101001
1110	1110100
1111	1111111

The set of code words in Table 7-1 may be generated from the set of messages by means of a matrix modulo-2 multiplication:

$$\mathbf{v} = \mathbf{uG}$$

(7-1)

where $\mathbf{v}$ = a code word $(v_7, v_6, v_5, v_4, v_3, v_2, v_1, v_0)$

$\mathbf{u}$ = the corresponding message (u_3, u_2, u_1, u_0)

$\mathbf{G}$ = the *generator matrix* $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

It may be confirmed by inspection that every pair of code words in Table 7-1 differs in at least three bit positions, so $d = 3$ for this code.

7.5.2 BER Calculation for Block Coding FEC

The output BER for a block FEC decoder depends on the number of bit errors per code word that the code can correct. This quantity is related to the minimum distance for the code as follows:

$$t = \text{Int}\left(\frac{d-1}{2}\right)$$

(7-2)

where t =   number of correctable bit errors per code word

$\text{Int}(x)$ =   function that truncates x to an integer

In the (7, 4) linear block code example in Table 7-1, the minimum distance is 3, so the code can correct one bit error per code word.

For a hard-decision decoder, calculating the output BER is a two-step process.

The first step is to calculate the probability of a code-word error, which is also called the *word error rate* (WER). This depends on the input BER and the parameters n, k, and t (which depends on d). If it is assumed that input bit errors are randomly distributed, and that the bit errors are statistically independent events, then the WER can be determined with a simple equation. The code word will be correct if it has from 0 to t bit errors (because the decoder can correct that many errors). Consequently, the WER is:

$$WER = 1 - \sum_{j=0}^{t} \frac{n!}{(n-j)!\,j!} BER_{\text{in}}^{j} (1 - BER_{\text{in}})^{n-j}$$

(7-3)

where BER_{in} =   input BER

The second step, assuming a code word error has been made, is to estimate the number of erroneous bits in the output message. Assuming that all of the possible erroneous messages are equally likely, the output BER is (Reference 4-2):

$$BER_{\text{out}} = \frac{2^{k-1}}{2^k - 1} WER$$

(7-4)

The results of applying Equations 7-3 and 7-4 to the (7, 4) code example in Table 7-1 (with $t = 1$) are shown in Table 7-2.

Table 7-2. Performance of (7, 4) Linear Block Code With *t* = 1

Input BER	Output BER	Improvement
0.1	0.08	Negligible
0.01	0.0011	Order of magnitude
0.001	0.000011	Two orders of magnitude

For soft-decision block decoders, the decoding algorithms do not yield simple equations such as Equation 7-3. However, simulations show that soft-decision systems are about 2 dB better than hard-decision systems; i.e., E_b/N_o must be about 2 dB greater in a hard-decision system to get the same BER as in the corresponding soft-decision system.

As the above example illustrates, to calculate the output BER from the input BER for a hard-decision decoder, it is necessary to know the following:

- The number of bits per code word (*n*)
- The number of bits per message (*k*)
- The number of bits per code word that can be corrected (*t*)

A wide variety of FEC codes are available. These are typically specified by name and by the above parameters.

7.5.3 Bose-Chaudhurie-Hocquenghem Codes

An important class of block codes is the class of Bose-Chaudhurie-Hocquenghem (BCH) codes. These codes have the following properties:

- $n = 2^m - 1$, where *m* is an integer greater than 2
- $d \geq 2t + 1$
- $r = n - k \leq mt$, where r is the number of redundancy bits in a code word

The parameters for several BCH codes are listed in Table 7-3. A much more complete table with n extending to 1023 that can be used to obtain the number of correctable bit errors t is available in the literature.[7-1]

Table 7-3. BCH Code Parameters for $n \leq 63$

n	k	t
7	4	1
15	11	1
	7	2
	5	3
31	26	1
	21	2
	16	3
	11	5
	6	7
63	57	1
	51	2
	45	3
	39	4
	36	5
	30	6
	24	7
	18	10
	16	11
	10	13
	7	15

Figure 7-1 shows the output BER vs. input BER curve for a hard-decision (255, 239) BCH decoder, which is implemented in some communications systems. This decoder has $r = 16$, $m = 8$, and $t = 2$. The curve was generated by applying Equations 7-3 and 7-4.

[7-1]   Shu Lin and Daniel Costello, *Error Control Coding: Fundamentals and Applications*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1983.
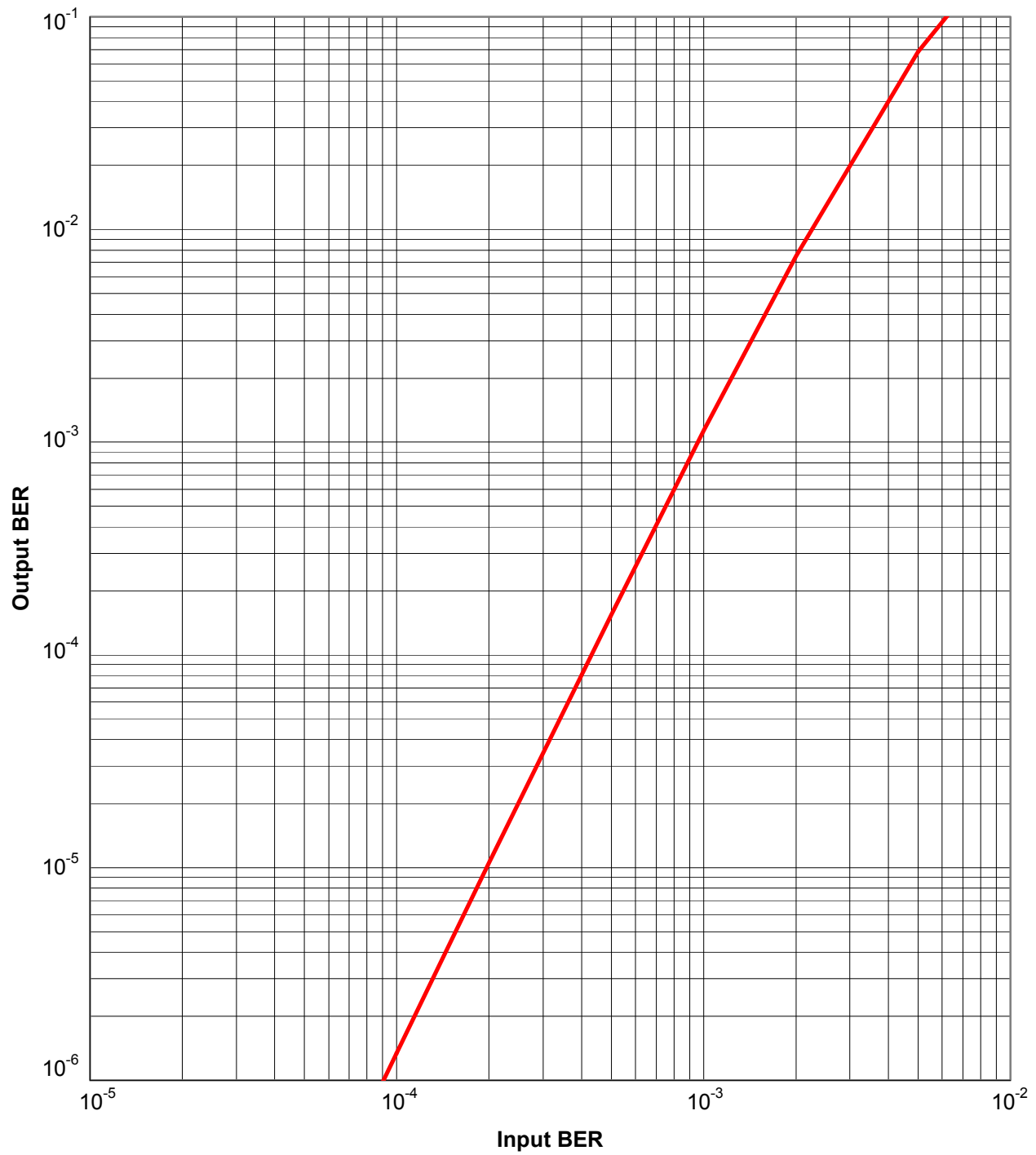
Figure 7-1. Output BER vs. Input BER Curve for Hard-decision (255, 239) BCH Decoder

7.5.4 Hamming Code

The Hamming code is a relatively simple single-error correcting code. It is a special case of the BCH code with $r = m$. Therefore, it has the following properties:

- $n = 2^r - 1$, where r is the number of redundancy bits in a code word
- $d = 3$
- $t = 1$

The (7, 4) Hamming code is sometimes *extended* to (8, 4) by the addition of another redundancy bit. This does not affect the number of correctable errors, but it is more convenient for implementing a system with the code rate $R = 1/2$.

Figure 7-2 shows the output BER vs. input BER curve for a hard-decision extended (8, 4) Hamming decoder. This decoder has $r = m = 3$. The curve was generated by applying Equations 7-3 and 7-4. Figure 7-3 shows BER vs. E_b/N_o curves for a soft-decision extended (8, 4) Hamming decoder and for various demodulator types. For these curves, the demodulator and FEC decoder are treated as one combined module.

7.5.5 Hadamard Code

This (*n*, *k*) block code is based on the Hadamard matrix, which is an $n \times n$ matrix with the property that all of the rows (except one) have an equal number of 1-bits and 0-bits. The code words are simply the rows of the matrix. Other properties of the Hadamard code are

- $n = 2^k$
- $d = n/2$
- $t = \text{Int}[(d-1)/2] = \text{Int}[(n-2)/4]$

A typical example is the (64, 6) Hadamard code, which can correct up to 15 bits per code word. This capability is made possible by the 58 bits of redundancy that are added to each 6-bit message. Hadamard matrices may be used to define code sets for direct-sequence spread-spectrum systems. The equal-weight property is desirable for this application.
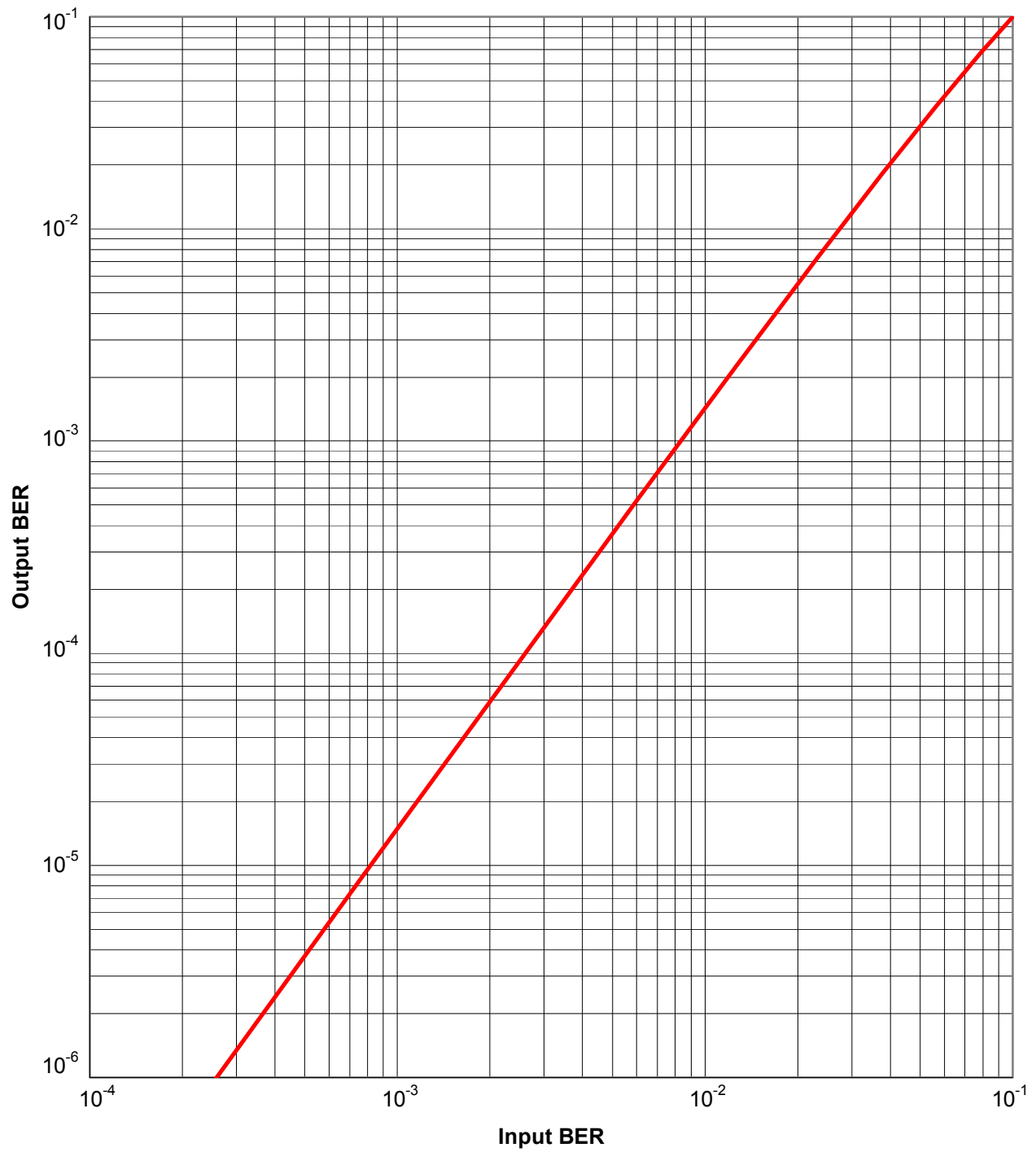
Figure 7-2. Output BER vs. Input BER Curve for Hard-decision (8, 4) Hamming Decoder
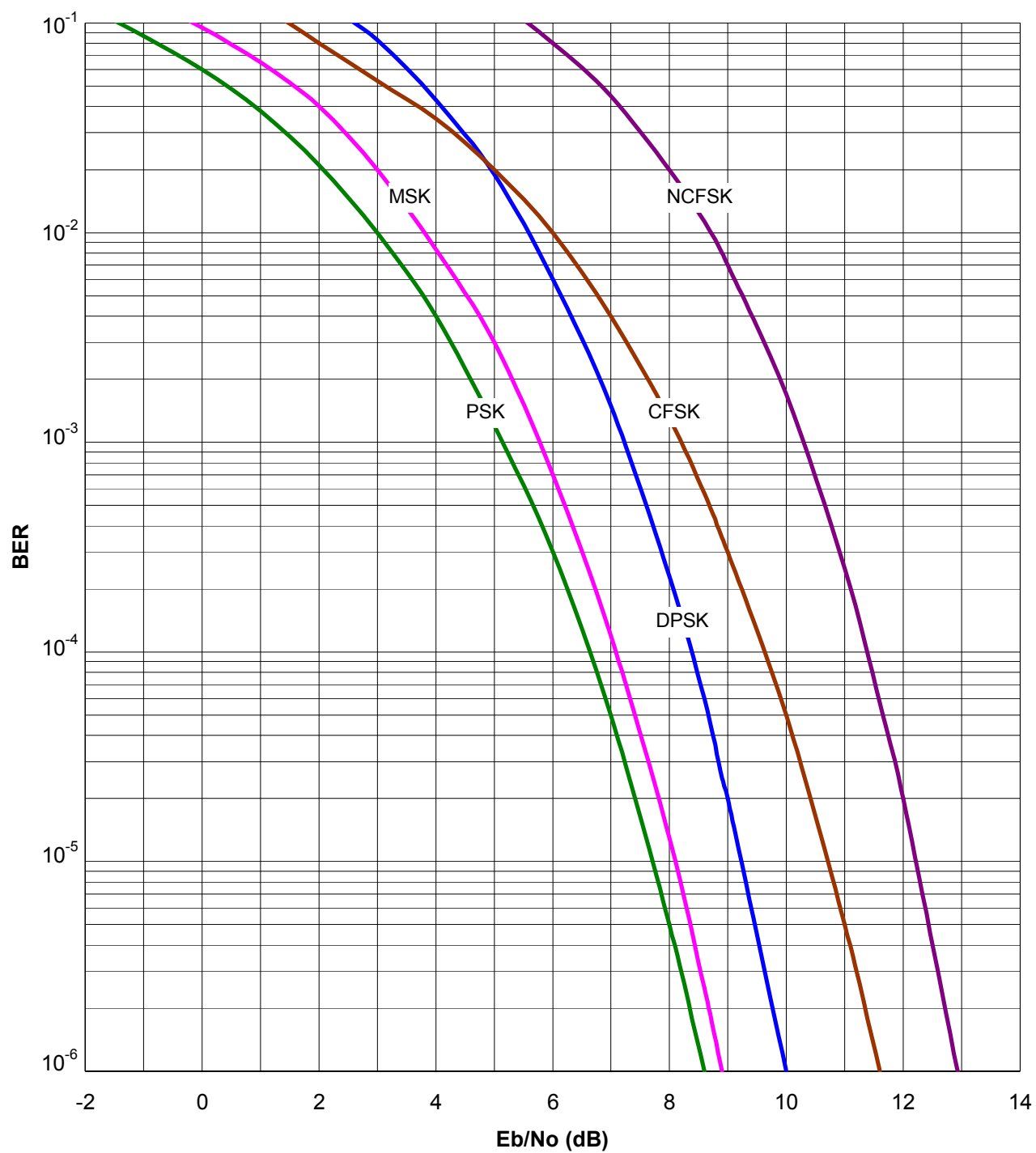
Figure 7-3. BER vs. E_b/N_o Curves for Soft-decision (8, 4) Hamming Decoder

Figure 7-4 shows the output BER vs. input BER curve for a hard-decision (64, 6) Hadamard decoder. The curve was generated by applying Equations 7-3 and 7-4. Figure 7-5 shows BER vs. E_b/N_o curves for a soft-decision (64, 6) Hadamard decoder and for various demodulator types. For these curves, the demodulator and FEC decoder are treated as one combined module.

7.5.6 Golay Code

The Golay code is an (n, k) block code with the following properties:

- $n = 23$
- $k = 12$
- $d = 7$
- $t = 3$

The Golay code is sometimes extended to (24, 12) by the addition of another redundancy bit. This does not affect the number of correctable errors, but it is more convenient for implementing a system with the code rate $R = 1/2$.

Figure 7-6 shows the output BER vs. input BER curve for a hard-decision extended (24, 12) Golay decoder. The curve was generated by applying Equations 7-3 and 7-4. Figure 7-7 shows BER vs. E_b/N_o curves for a soft-decision extended (24, 12) Golay decoder and for various demodulator types. For these curves, the demodulator and FEC decoder are treated as one combined module.

7.5.7 Reed-Solomon Codes

In a Reed-Solomon (RS) encoder, the input bits are grouped to form *RS symbols* (not to be confused with *M*-ary demodulator symbols). Then the RS symbols are grouped into messages, and redundancy RS symbols are added to form code words. The code word length and the message length are expressed in terms of RS symbols rather than bits. RS codes have the following properties:

- $n = 2^m - 1$, where n is the number of RS symbols per code word and m is the number of bits per RS symbol
- $d = n - k + 1$, where d is the minimum distance measured in RS symbols and k is the number of RS symbols per message
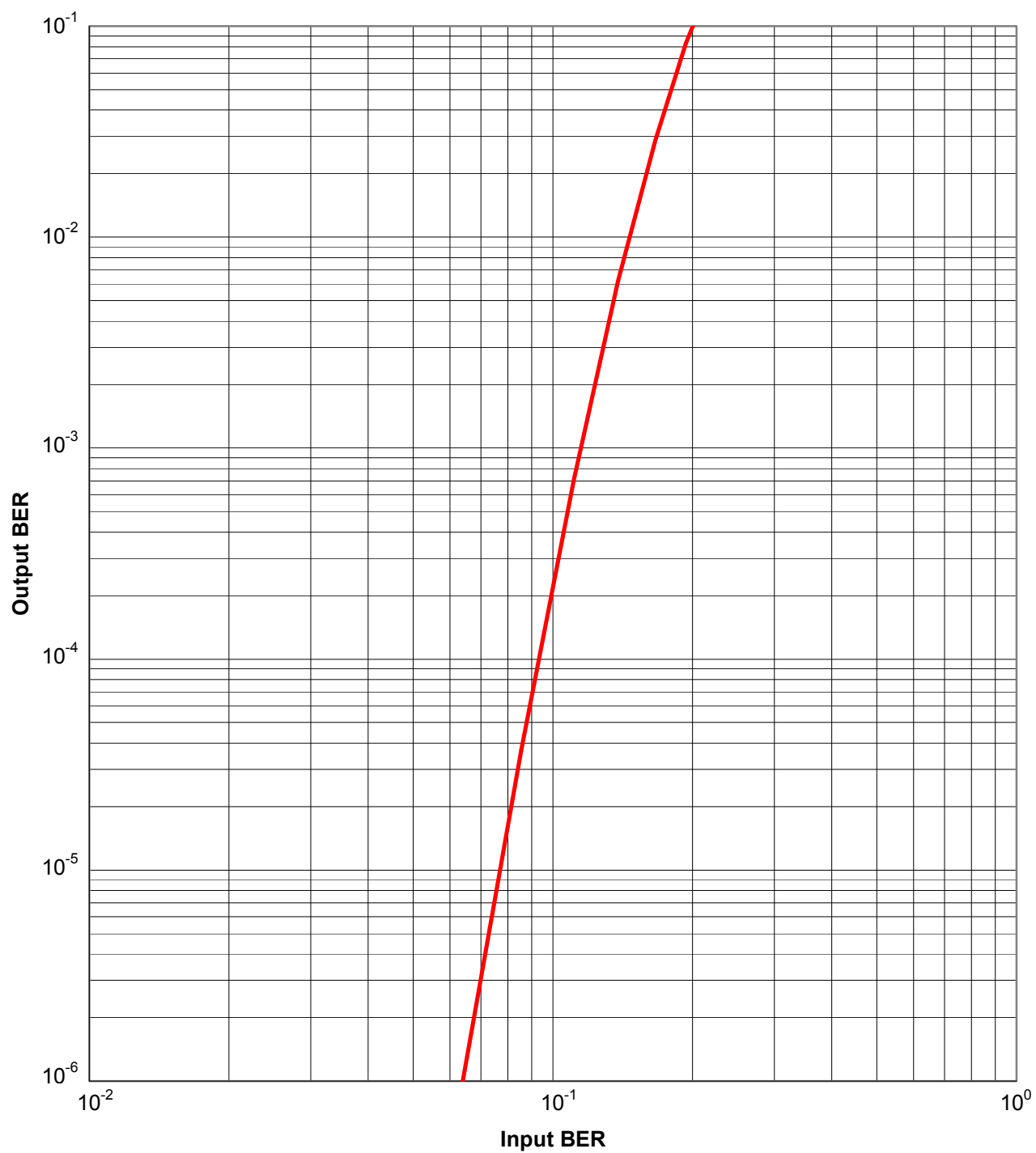
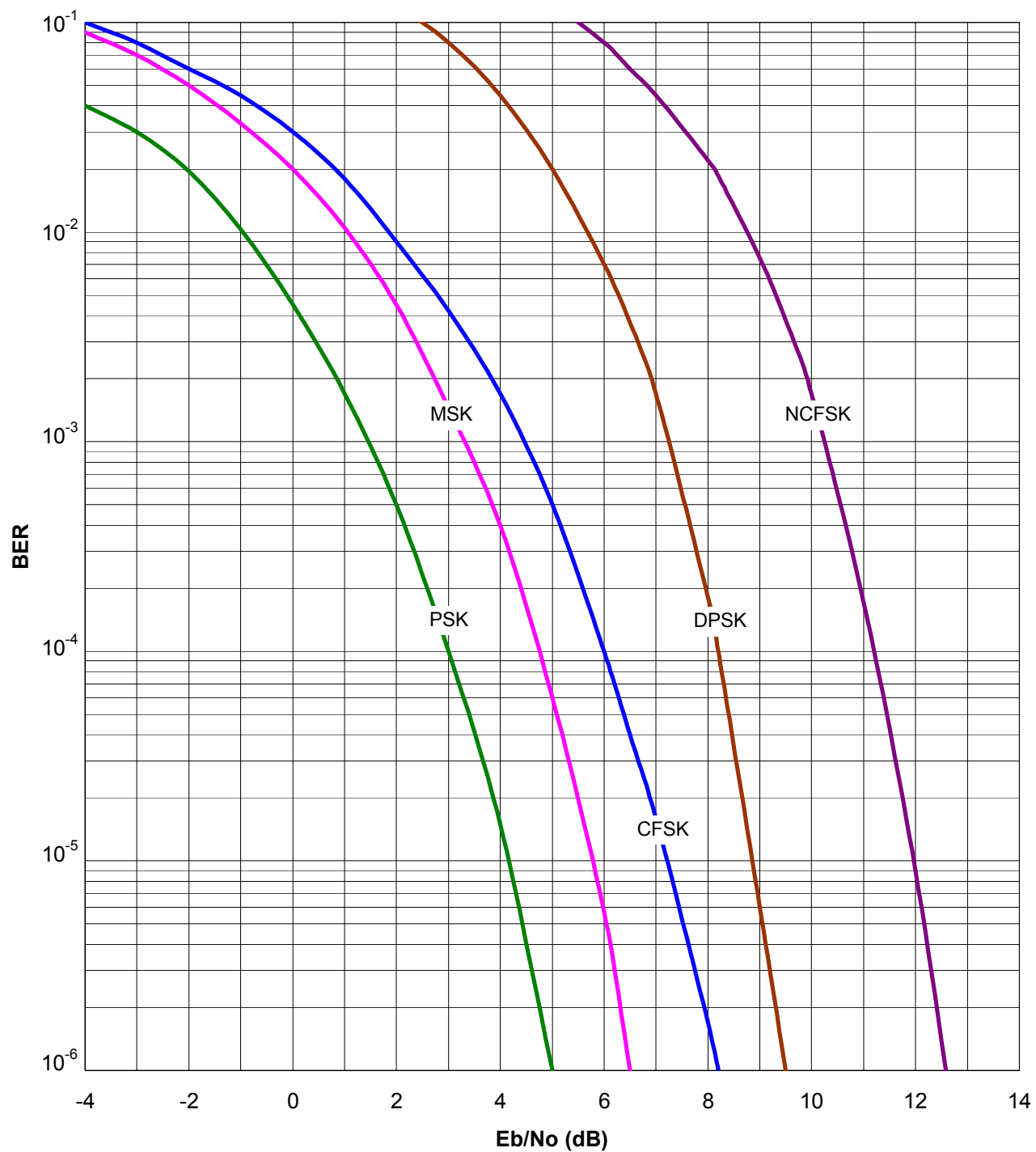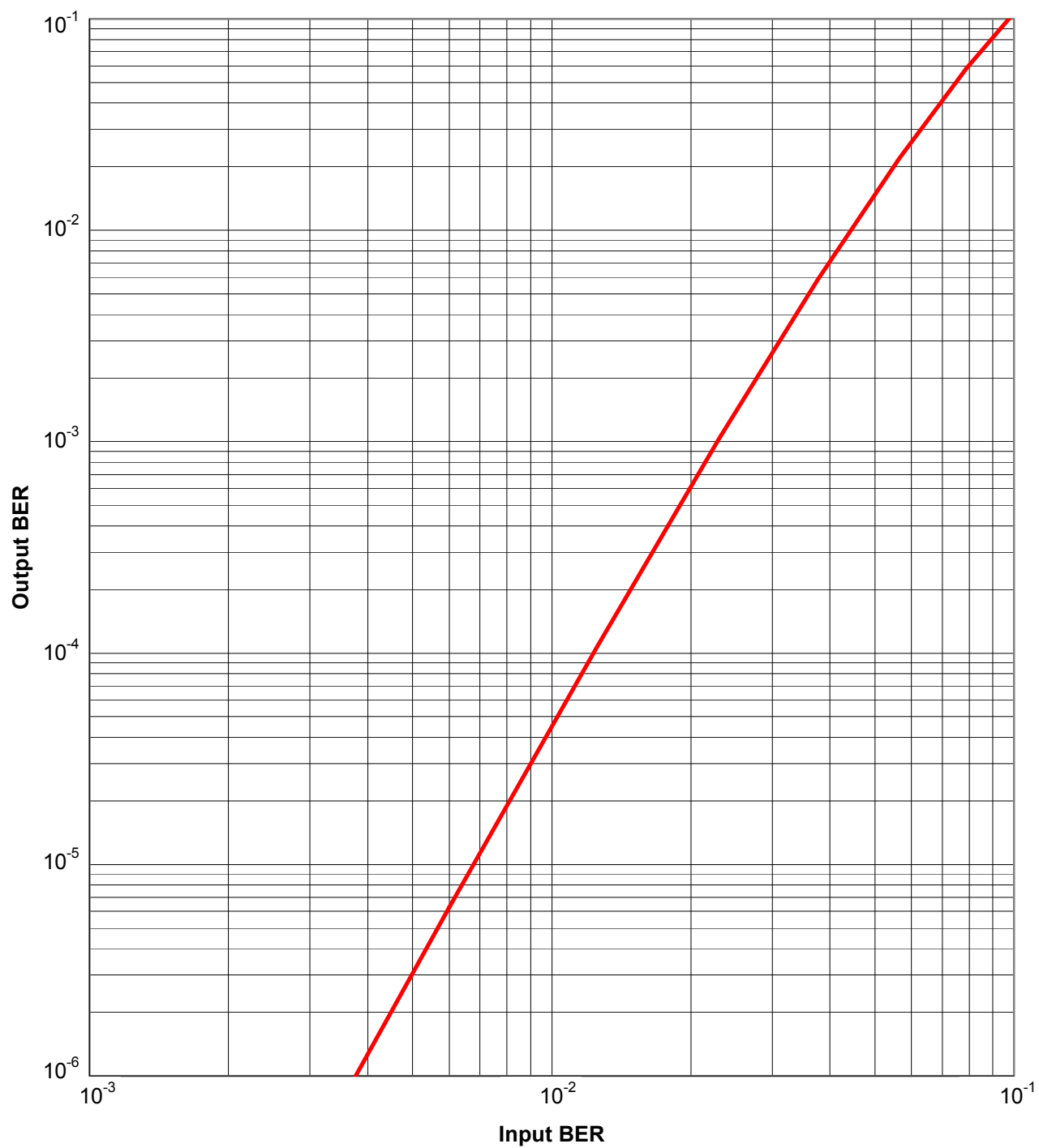**Figure 7-4. Output BER vs. Input BER Curve for Hard-decision (64, 6) Hadamard Decoder**

Figure 7-5. BER vs. E_b/N_o Curves for Soft-decision (64, 6) Hadamard Decoder

Figure 7-6. Output BER vs. Input BER Curve for Hard-decision (24, 12) Golay Decoder
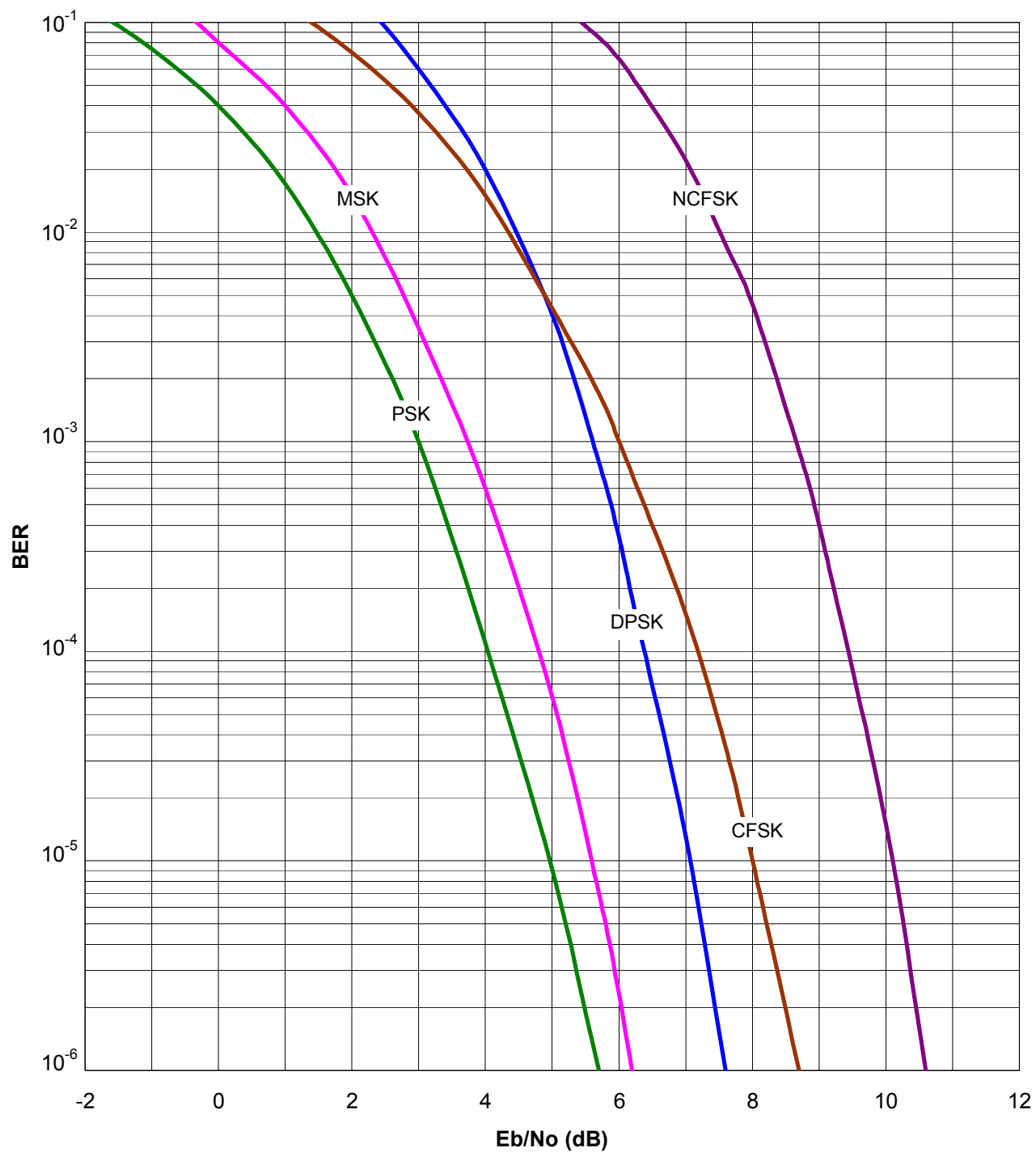
Figure 7-7. BER vs. E_b/N_o Curves for Soft-decision (24, 12) Golay Decoder

Equation 7-2 can still be used for RS codes, but *d* and *t* are expressed as numbers of RS symbols rather than bits. For example, a (15, 13) RS code has 4 bits per RS symbol, 13 RS symbols per message, and 15 RS symbols per code word. The minimum distance for the (15, 13) RS code is 3 RS symbols and the number of correctable RS symbols in a code word is 1.

The symbol-based structure of RS codes can be used to protect a system from burst errors, which are often caused by intermittent interference. For example, a (15, 13) RS code can correct one symbol per 15-symbol code word, regardless of the number of erroneous bits in the erroneous symbol.

Because a given RS symbol is formed by grouping *m* bits, it is correct only if all *m* bits are correct. Therefore, the RS SER is given by:

$$SER_{in} = 1 - (1 - BER_{in})^m$$

(7-5)

where SER_{in} =   RS symbol error rate at RS decoder input
          BER_{in} =   bit error rate at RS decoder input

Because the RS decoder operates on RS symbols rather than bits, Equation 7-3 is modified to show the dependency on RS SER (Reference 4-2):

$$WER = 1 - \sum_{j=0}^{t} \frac{n!}{(n-j)!\,j!} SER_{in}^{j} (1 - SER_{in})^{n-j}$$

(7-6)

Figure 7-8 shows the output BER vs. input BER curve for a hard-decision (255, 223) RS decoder. In this case, *m* = 8 and *t* = 16. The curve was generated by applying Equations 7-5, 7-6, and 7-4.

7.5.8 Interleaving

Every FEC block code has the ability to correct *t* erroneous bits per *n*-bit code word. If a code word has more than *t* erroneous bits, the code word is decoded incorrectly. Such a code is very effective at protecting against the occasional random errors that result from noise and noiselike interference. Intermittent interference, which can be modeled as a sequence of pulses, is another matter. Intermittent interference often results in a very high bit-error probability during a pulse. If the pulse duration T_i is long enough that it can corrupt more than *t* bits in a particular codeword, the FEC decoder will be unable to correct the errors. This will occur when $T_i > tT_b$, a condition referred to as overloading.
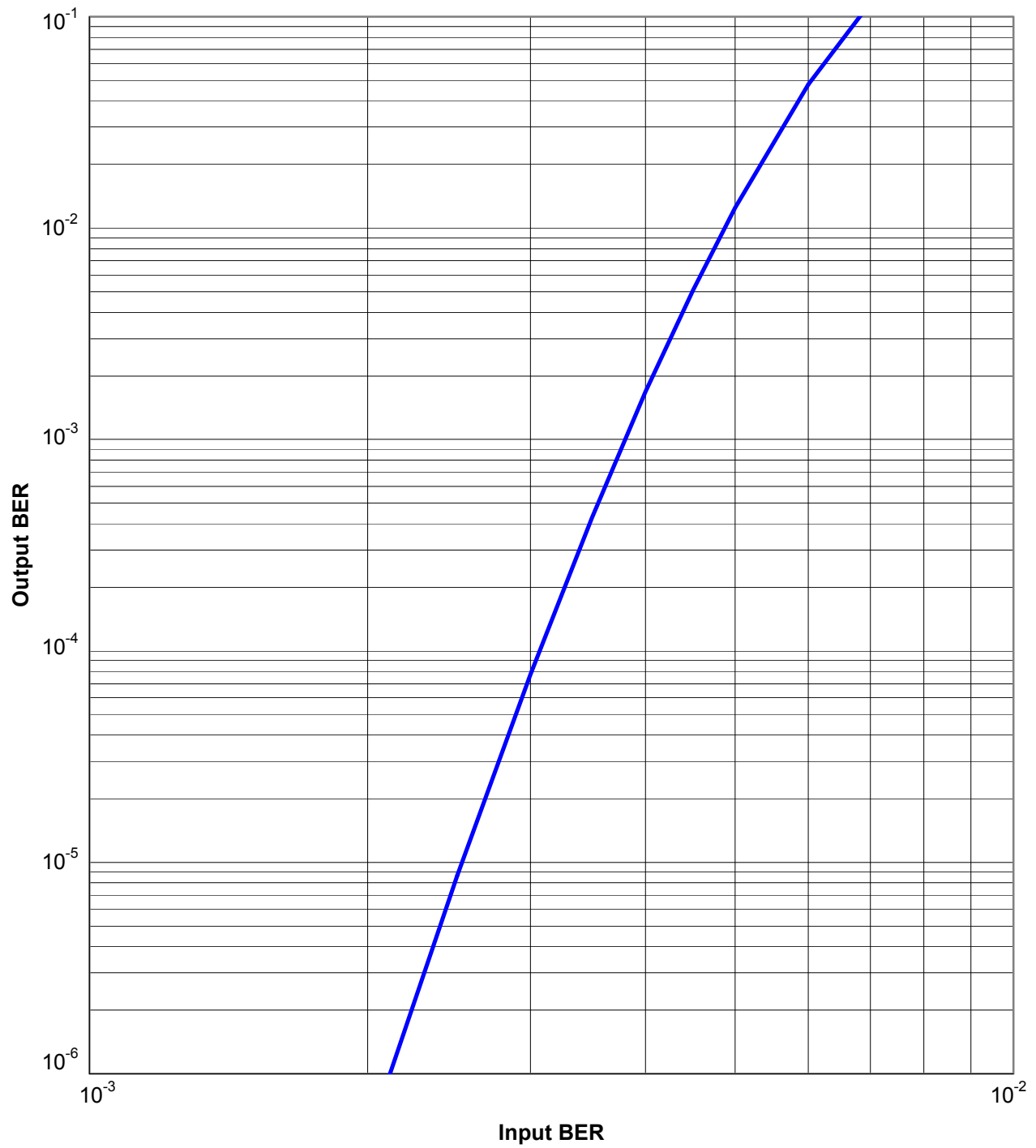
Figure 7-8. Output BER vs. Input BER Curve for Hard-decision (255, 223) RS Decoder

Figures 7-9 and 7-10 illustrate pulsed interference to a digital system. Figure 7-9 illustrates overloading for a sequence of four code words (W_1, W_2, W_3, W_4), assumed to be part of a longer transmission for a system using a (7, 4) code. This code can correct one bit error per code word. Because the interference pulse is longer than one bit, the decoder is overloaded.
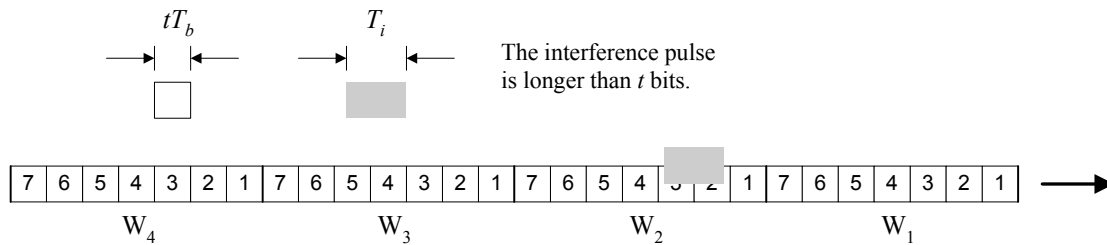
Figure 7-9. Pulsed Interference Without Interleaving
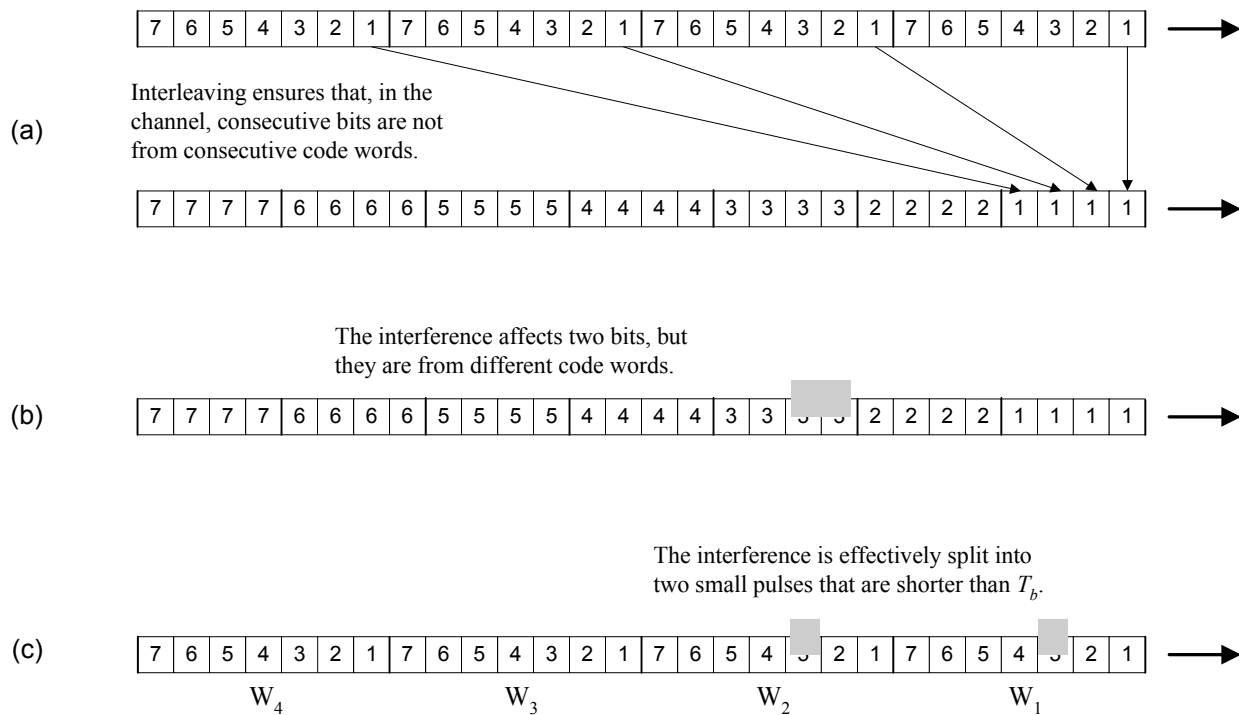
Figure 7-10. Pulsed Interference With Interleaving

Interleaving the source data prior to transmission addresses this problem. Interleaving reorganizes the source data, so that consecutive bits in the channel are not from the same code word. With interleaving, bits corrupted by an interference pulse are distributed across multiple code words.

Figure 7-10 is for the same example as in Figure 7-9, but with interleaving. The interleaving process is illustrated in Figure 7-10(a). In Figure 7-10(b), the interference pulse is shown corrupting two bits, as before. When the data is demodulated and de-interleaved in the receiver, however, the corrupted bits are seen to reside in two different code words. This is shown in Figure 7-10(c). The effect is the same as splitting the interference pulse into two shorter – and therefore less harmful – pulses.

In this example, the interleaving process is applied to a group of four codewords. The process would be repeated for every group of four code words for the duration of the transmission. By increasing the number of code words in the interleave group (referred to as the interleaver depth L), the protection is extended to longer pulses. When the data is interleaved with depth L, an interference pulse will overload the decoder only if $T_i > LtT_b$.
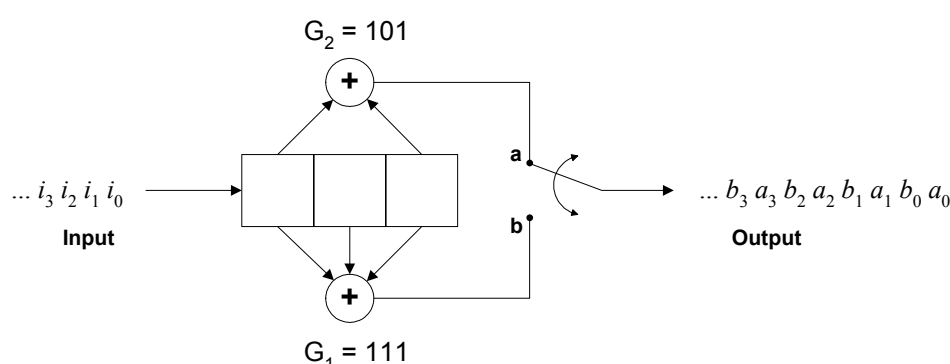
7.6 CONVOLUTIONAL CODES

7.6.1 Description

Convolutional coding is the other major approach to FEC coding. Convolutional coding and decoding techniques are very briefly described here. A more detailed discussion of convolutonal codes can be found in the literature (for example, Reference 4-2).

In contrast to block coding, which transforms k-bit messages into n-bit code words, convolutional coding is applied to a continuous stream of data to produce another continuous stream of data. Redundancy is added in the process, so the encoder output data rate is higher than the encoder input (information) data rate. The ratio of input to output data rates is called the *rate* of the code, denoted R.

Convolutional encoding is implemented with a shift register of several stages, and two or more modulo-2 adders. The number of stages is called the *constraint length* of the code, denoted k. Certain stages are connected to the adders, so that at any given time the output of each adder is the modulo-2 sum of the shift register stages to which it is connected. The input to the shift register is the sequence of information bits, so the clock rate of the shift register is the data rate of the information sequence. The output is taken from the adders. Each adder is sampled in turn, such that all of the adders are sampled once per clock cycle.

Convolutional codes are given the designation (R, k), in recognition of the dependence on the rate and constraint length. In addition to these parameters, a particular convolutional code is characterized by the specific connections between shift register stages and adders. These connections may be designated graphically, using a diagram of the sequential logic circuit. Alternatively, the connections may be specified in terms of code tap connections. A code tap connection is represented by a k-bit symbol $G = (g_{k-1}, \ldots, g_1, g_0)$ that designates the connections of a particular adder. If the ith stage is connected to the adder, then $g_i = 1$, otherwise $g_i = 0$. There will be one code tap connection symbol for each adder in the encoder circuit. A (1/2, 3) convolutional encoder is illustrated in Figure 7-11.

Figure 7-11. A (1/2, 3) Convolutional Encoder

Several convolutional codes are listed in Table 7-4. These are all rate-1/2 codes, so there are two code tap connections for each code.

Table 7-4. Rate R = 1/2 Convolutional Codes

Constraint Length k	Code Tap Connections G_1, G_2
3	111, 101
4	1111, 1101
5	11101, 10011
6	111101, 101011
7	1111001, 1011011
8	11111001, 10100111

7.6.2 Decoding Convolutional-Coded Data

The convolutional decoder examines an incoming sequence of data to determine the likelihood that such a sequence could have actually been transmitted. If the decoder decides that the sequence contains errors, it makes the necessary adjustments to the data. Unlike the block decoder, the convolutional decoder is not limited to a particular sequence length. Typically, convolutional decoders examine sequences that are several times longer than the constraint length. This process is called *finite-memory decoding*.

The decoding process is based on the state-transition properties of the encoder, which ensures that only certain coded sequences are possible. A convolutional code can be represented by a *code tree*, which is a diagram that indicates the transitions – and therefore the sequences – that are possible. One of the most important and universal convolutional decoders is called the Viterbi algorithm. This algorithm is typically applied to convolutional codes with rates of ½ to ¾, constraint lengths of 3 to 8, and memories of 3 or 4 times the constraint length. Viterbi decoders generally employ soft-decision decoding.

7.6.3 BER Calculation for Convolutional Coding

There is an analytical approach for estimating the output BER for a convolutional decoder. An explanation of the technique and of the parameters involved is beyond the scope of this handbook. The objective is to simply show how the BER can be calculated.

The technique involves two parameters D and N and three functions $T(D)$, $T(D,N)$, and $P(D)$. The *transfer function $T(D)$* of the convolutional code is derived from the state-transition representation of the code. It is a polynomial:

$$T(D) = \sum_{j=d}^{\infty} a_j D^j$$

(7-5)

where d =   *minimum free distance* of the code
         a_j =   polynomial coefficient for the code

The transfer function is extended to form $T(D,N)$ by adding a parameter N, which is also based on the state-transition representation. Then that function is differentiated with respect to N, and evaluated at $N = 1$. The new polynomial function $P(D)$ may be called the *derivative polynomial*:

$$P(D) = \frac{d}{dN} T(D,N) \bigg|_{N=1} = \sum_{j=d}^{\infty} c_j D^j$$

(7-6)

where c_j =   derivative polynomial coefficient for the code

Lists of the functions $T(D)$ and $P(D)$ for various codes are published in the literature, and are often provided with system descriptions. A few examples are given in Tables 7-5 and 7-6.

Table 7-5. Transfer Functions for R = 1/2 Convolutional Codes

k	$T(D)$
3	$D^5 + 2D^6 + 4D^7 + 8D^8 + 16D^9 \cdots$
4	$D^6 + 3D^7 + 5D^8 + 11D^9 + 25D^{10} \cdots$
5	$2D^7 + 3D^8 + 4D^9 + 16D^{10} + 37D^{11} \cdots$
6	$D^8 + 8D^9 + 7D^{10} + 12D^{11} + 48D^{12} \cdots$
7	$11D^{10} + 38D^{12} + 193D^{14} + 1331D^{16} + 7230D^{18} \cdots$

Table 7-6. Derivative Polynomials for R= 1/2 Convolutional Codes

k	$P(D)$
3	$D^5 + 4D^6 + 12D^7 + 32D^8 + 80D^9 \cdots$
4	$2D^6 + 7D^7 + 18D^8 + 49D^9 + 130D^{10} \cdots$
5	$4D^7 + 12D^8 + 20D^9 + 72D^{10} + 225D^{11} \cdots$
6	$2D^8 + 36D^9 + 32D^{10} + 62D^{11} + 332D^{12} \cdots$
7	$36D^{10} + 211D^{12} + 1404D^{14} + 11633D^{16} + 76628D^{18} \cdots$

The first step is to identify the derivative polynomial $P(D)$ for the code. The lowest degree is d, the minimum free distance of the code. The next step is to identify the hard-decision demodulator transfer function. This equation expresses the BER at the demodulator output as a function of E_b/N_o:

$$BER_{\text{hard}} = F\left(\frac{E_b}{N_o}\right)$$

(7-7)

where BER_{hard} =   BER for a hard-decision demodulator

        $F(x)$ =   function for a given demodulator type

The BER at the decoder output is approximated by the following upper bound:

$$BER = \sum_{j=d}^{\infty} c_j F\left(jR\frac{E_b}{N_o}\right)$$

(7-8)

where R =   code rate of the convolutional code

As an example, consider a BPSK system with soft-decision (1/2, 7) Viterbi decoding. The derivative polynomial $P(D)$ is the last one listed in Table 7-6. Equation 6-17 gives the BPSK function $F(x)$:

$$F(x) = \frac{1}{2}\,\text{erfc}\left(\sqrt{x}\right)$$

(7-9)

Substituting that function into Equation 7-8 gives:

$$BER = \sum_{j=d}^{\infty} c_j \frac{1}{2}\,\text{erfc}\left(\sqrt{jR\frac{E_b}{N_o}}\right)$$

$$= \frac{36}{2}\,\text{erfc}\left(\sqrt{\frac{10}{2}\frac{E_b}{N_o}}\right) + \frac{211}{2}\,\text{erfc}\left(\sqrt{\frac{12}{2}\frac{E_b}{N_o}}\right) + \frac{1404}{2}\,\text{erfc}\left(\sqrt{\frac{14}{2}\frac{E_b}{N_o}}\right) + \cdots$$

(7-10)

Of the five $P(D)$ terms shown in Table 7-6, only three are explicitly shown in Equation 7-10. More terms can be included for greater accuracy.

These equations apply to soft-decision decoding. If hard-decision convolutional decoding is encountered (which is unlikely), its performance will be roughly 2 dB worse.

7.6.4 BER Curves

Figure 7-12 shows the BER vs. E_b/N_o curves for a soft-decision (1/2, 7) Viterbi decoder and for various demodulator types. For these curves, the demodulator and FEC decoder are treated as one combined module.

7.7 CPM

7.7.1 Description

The term continuous-phase modulation (CPM) refers to signals that are designed to maintain continuity of phase for the duration of the transmission. This constraint results in a phase-modulated or frequency-modulated signal that has memory. This memory can be used to improve the BER of the demodulated signal, but at the cost of an increase in receiver complexity. The complexity is due to the fact that the receiver must perform recursive processing of the received signal to exploit the memory. One approach is to employ convolutional encoding and decoding in addition to the CPM. The receiver can incorporate the Viterbi decoding algorithm into the demodulation process. This algorithm is designed to consider signal history when determining the most probable value of the current demodulated symbol.

CPM generally encompasses a broad class of modulation types. The curves presented here are for continuous-phase FSK signals with Viterbi decoding. The modulation index is the ratio of the spacing of adjacent tones to the symbol rate

7.7.2 BER Curves

Figure 7-13 shows the BER vs. E_b/N_o curves for a soft-decision CPM receiver with $M = 2$ and a modulation index of 1/2. Figure 7-14 shows the BER vs. E_b/N_o curves for a soft-decision CPM receiver with $M = 4$ and a modulation index of 1/6. For these curves, the demodulator and FEC (Viterbi) decoder are treated as one combined module.

In these graphs, the term "interference" and the variable I refer to CW interference. Any noise-like interference power is simply assumed to be added (in mW) to the noise power. For example, suppose there are two interfering signals: a CW signal and a noise-like signal. The S/I parameter is the ratio of the desired signal power to the CW interfering signal power, and the x-axis variable E_b/N_o is calculated with Equation 6-4, where N is the total noise-like power (including the receiver noise and the noise-like interfering signal).
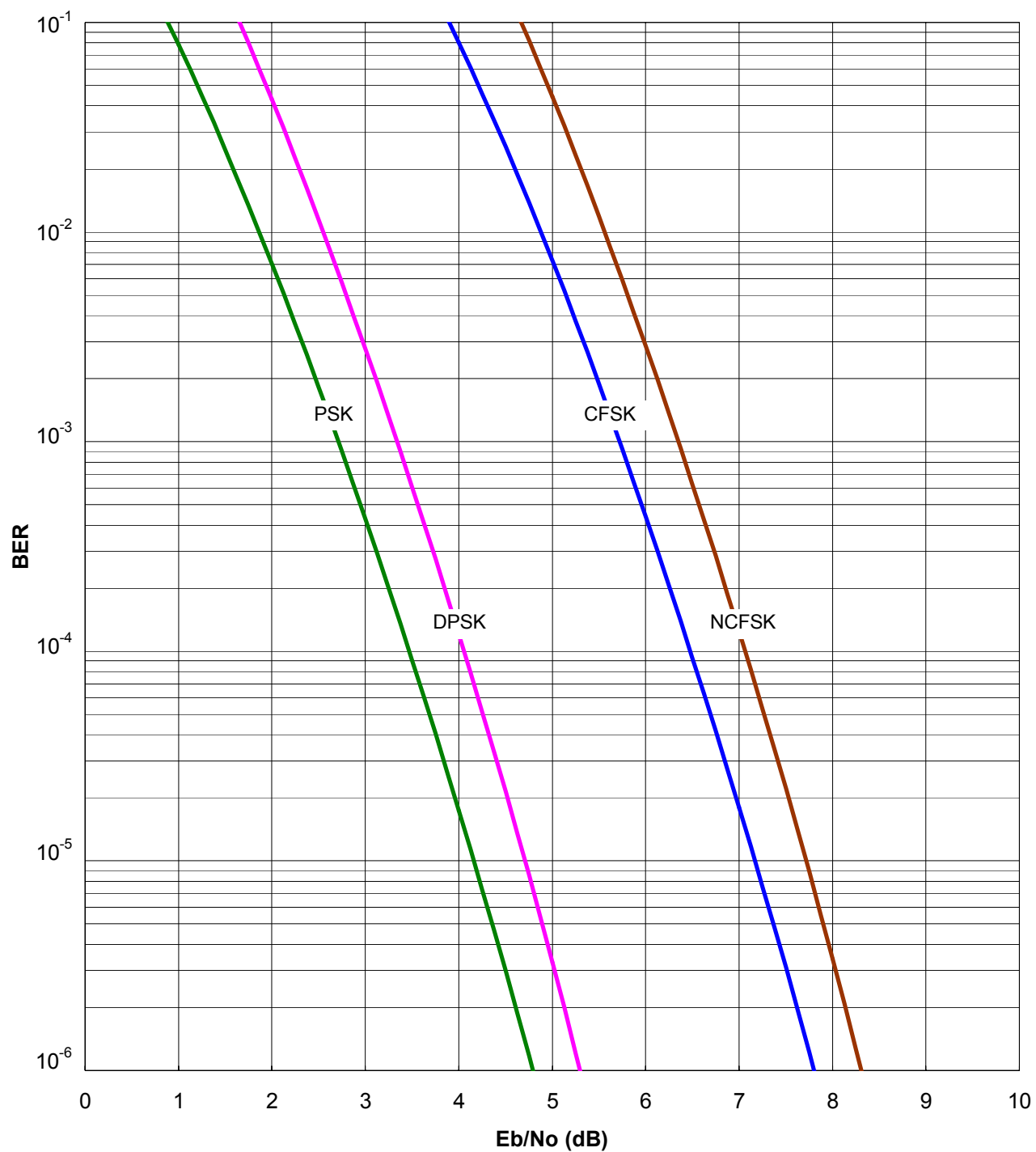
Figure 7-12. BER vs. E_b/N_o Curves for Soft-decision (1/2, 7) Viterbi Decoder
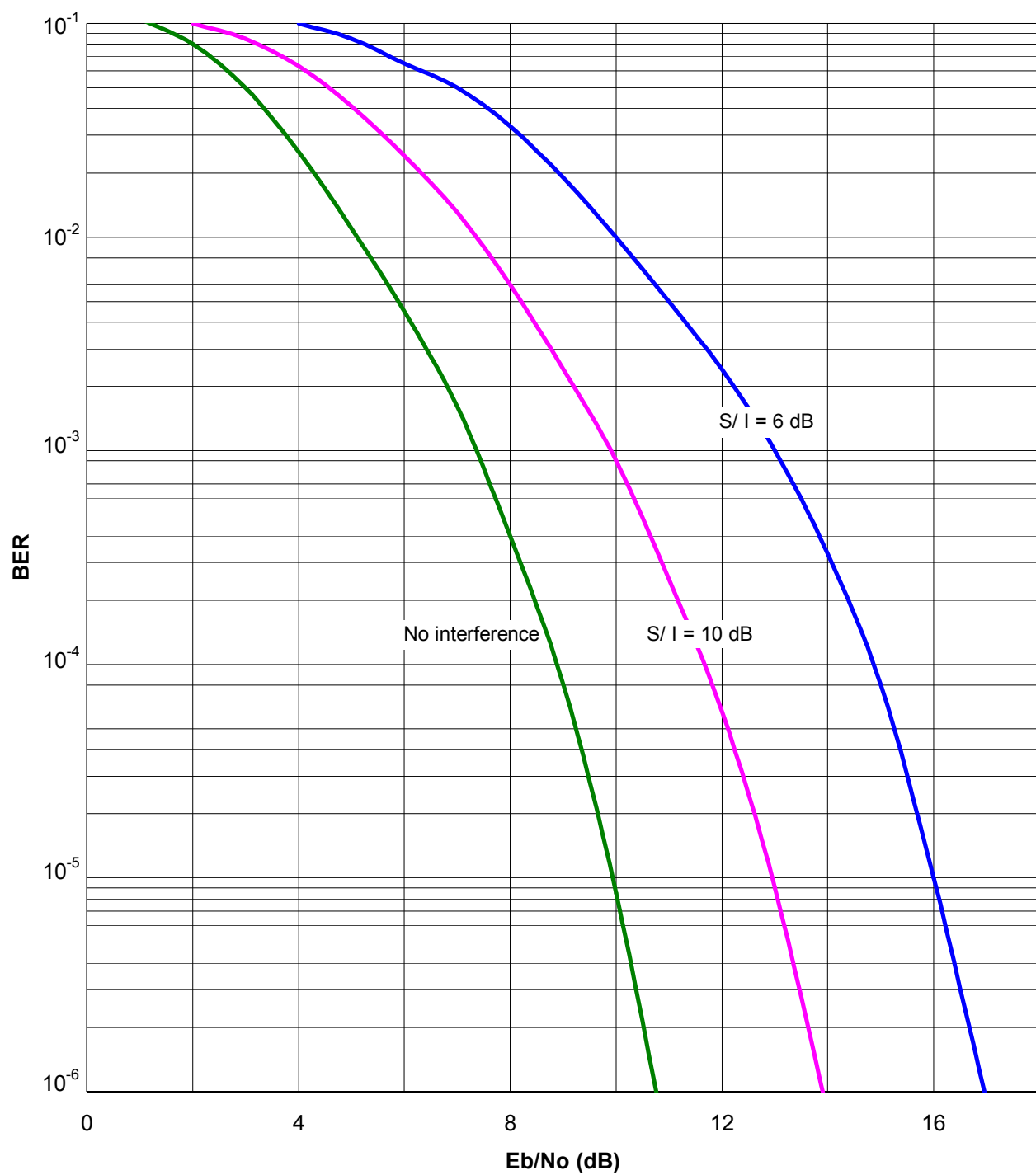
Figure 7-13. BER vs. E_b/N_o Curves for Binary CPM Decoder (Modulation Index 1/2)
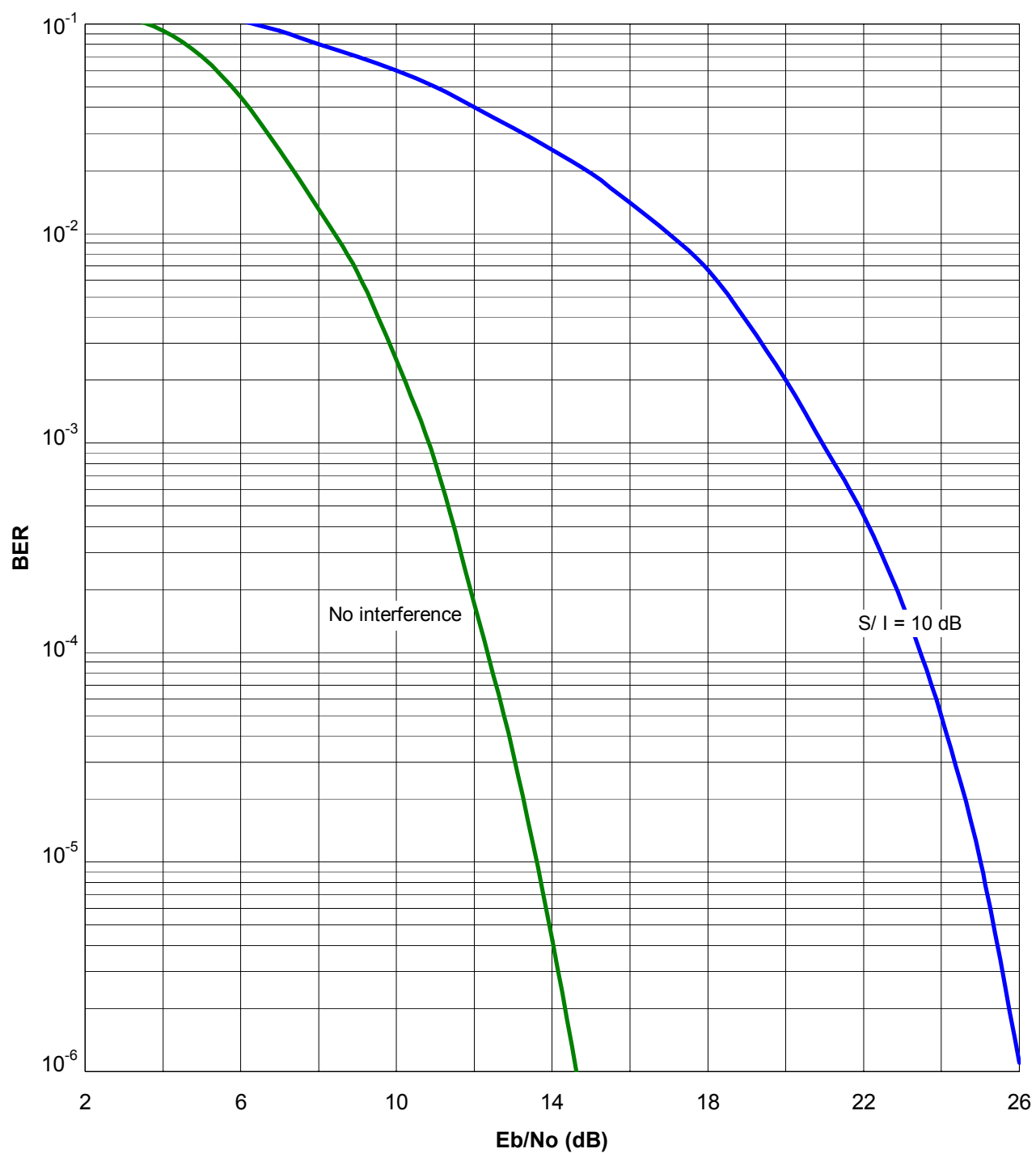
Figure 7-14. BER vs. E_b/N_o Curves for 4-ary CPM Decoder (Modulation Index 1/6)

The curves were generated by simulation. Each figure displays multiple curves. Each curve is a plot of BER vs. E_b/N_o. The curve labeled "No interference" applies to the case in which there is no CW interference. The other curves are for cases with CW interference. Each of those curves is labeled with the S/I for that curve. As expected, each curve shows that the BER decreases as the E_b/N_o increases. As also expected, for a given E_b/N_o the BER decreases as the S/I increases.

7.8 CONCATENATED CODES

An RF communications system may employ two FEC coding systems to increase the protection against errors. The two codes are referred to as the inner code and the outer code, as shown in Figure 7-15. A typical application is a system that must guard against both random and burst errors. Suppose an (n, k) RS code capable of correcting two symbol errors per code word is specified as the outer code. If an interference burst can cause at most two symbol errors, the RS decoder can correct those errors. However, if in addition to those two errors there is another (random) error somewhere else in the code word, the decoder will be overloaded. The probability of random (non burst-related) errors occurring can be reduced by incorporating an inner code. This code would not be able to correct errors due to bursts, leaving that task to the outer code. The inner code would simply minimize the BER between bursts. A convolutional code is often used as the inner code in a concatenated coding system.
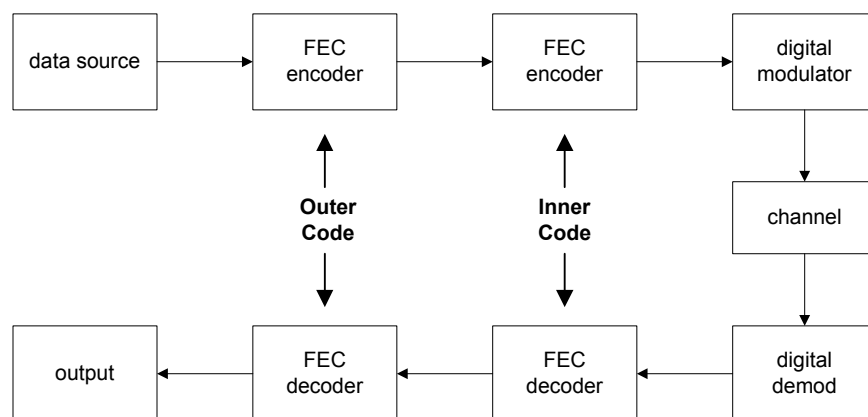
Figure 7-15. Concatenated Codes

The inner decoder may be either a soft-decision or hard-decision device, and the outer decoder is a hard-decision device. To analyze a concatenated coding scheme, the transfer functions for the individual decoders are computed and then combined. For example, if the inner decoder is a soft-decision decoder, then the demodulator and inner decoder are treated as one combined module. For a given E_b/N_o and S/I,

the output BER for that combined module is obtained from the appropriate curve. Then viewing that BER value as the input BER for the outer decoder, the output BER for the outer decoder is obtained from the curve for that device.

SECTION 8 – SOURCE DECODER

In a transmitter, the source encoder converts the source information into a sequence of bits at a certain data rate. If the source is analog, an ADC samples and encodes the analog waveform. If the source is digital, the source encoder reformats and retimes the input data if necessary. The output of the source encoder is a sequence of information bits, at the information data rate.

In the receiver, the source decoder converts the information bit sequence to the format required by the user of the receiver. If this format is analog, a DAC will be employed at this stage. The output of the source decoder is the output of the receiver. In general, it will differ somewhat from the original information signal due to noise and interference. This difference can be quantified, and is a measure of the overall system performance.

A DAC, along with its associated ADC, is used in a hybrid communications system. A hybrid system uses digital processing and transmission techniques to communicate analog information. When used to transmit speech, hybrid systems usually provide duplex capability; in such an application the ADC/DAC pair are often combined in a single unit referred to as a *codec*. There are various techniques available for performing the analog-to-digital and digital-to-analog conversion; the particular technique employed has an effect on the fidelity of the reconstructed analog signal as well as the sensitivity of the system to channel-induced bit errors.

8.1 MULTI-BIT SAMPLING

8.1.1 Description

Pulse-code modulation (PCM) is a straightforward conversion algorithm. The analog waveform is sampled at regularly spaced intervals established by the sampling frequency, and each sample is coded into a k-bit digital word according to its amplitude. Because there are multiple bits associated with each sample, PCM is also called *multi-bit sampling*. The analog waveform is thus represented by a sequence of k-bit digital words. The resulting bit stream is transmitted; channel impairments typically result in a number of errors in the received demodulated bit stream. In the receiver, the DAC decodes the digital words and produces a sequence of samples. Signal conditioning operations such as interpolation and lowpass filtering are then performed on the samples, resulting in a reconstructed analog waveform that should be a close approximation to the original information signal. The block diagram for a PCM ADC and DAC appears in Figure 8-1.
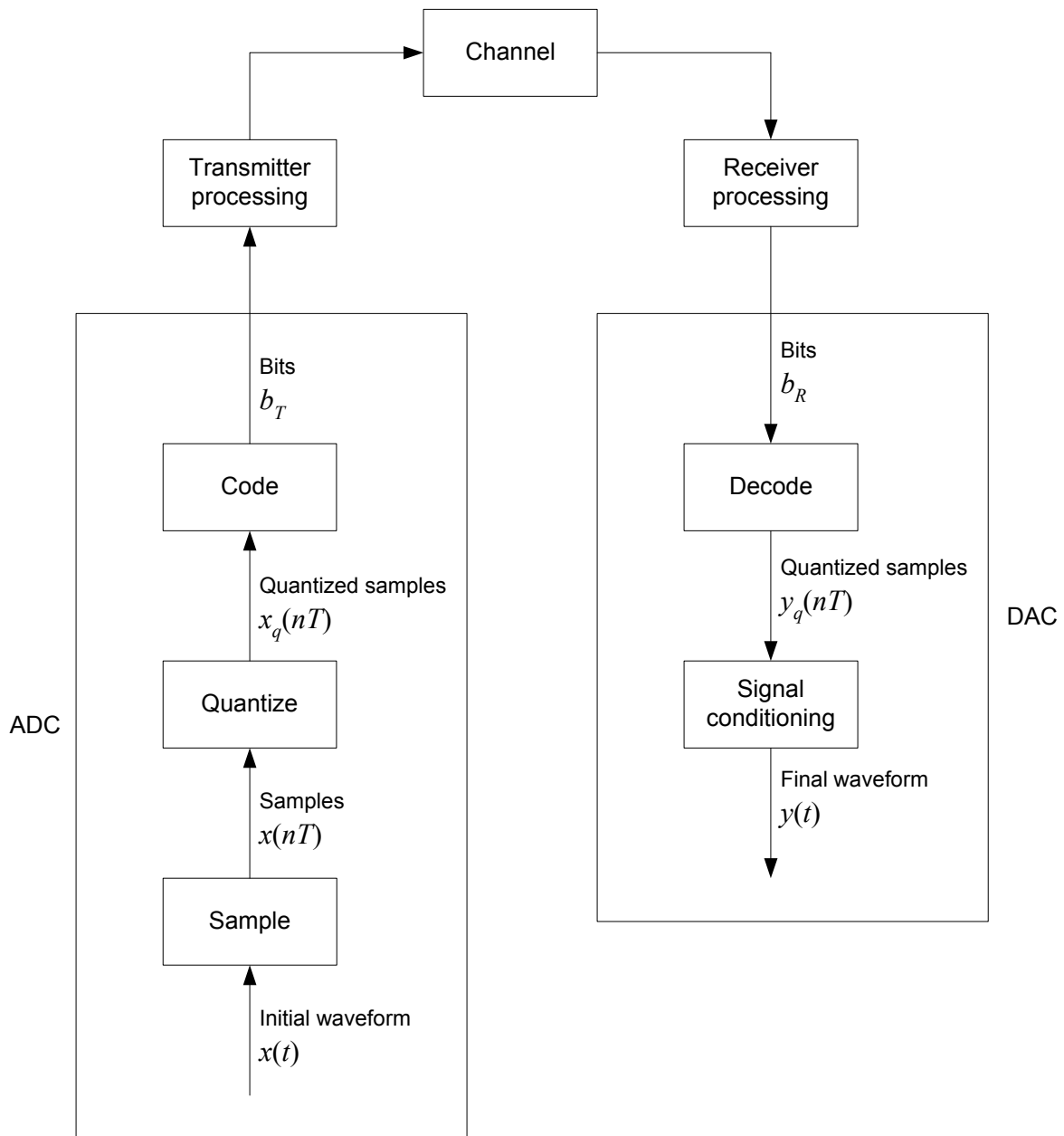
Figure 8-1. Block Diagram for PCM ADC and DAC

The input signal to the transmitter is denoted $x(t)$ in Figure 8-1. The Sample operation produces a sample sequence denoted $x(nT)$, where T is the sampling interval and $n = 1, 2, \ldots, L$. These samples are then quantized. For k-bit PCM, the number of quantization levels is 2^k. Each sample $x(nT)$ is replaced by the nearest quantization level $x_q(nT)$. The Code operation then replaces each quantized sample with a sequence of k bits. For example, in a 6-bit ADC there are $2^6 = 64$ quantization levels. These levels in decimal form are 0, 1, 2, …, 63, where the 0 value represents the minimum signal and the 63 value

represents the maximum signal. If a particular sample is quantized to the decimal level 5, then the Code operation replaces the decimal level 5 by its binary form 000101. Thus, a sequence of L samples becomes a sequence b_T of kL bits.

Then the bit sequence is processed in the transmitter to create the channel waveform. This processing includes modulation and perhaps error correction coding and spreading. The channel waveform travels across the channel to the receiver, where it is processed to recover the received bit sequence b_R. The receiver processing includes demodulation and perhaps error correction decoding and despreading. If the transmission is error-free, then $b_R = b_T$.

When the received bit stream is decoded, the result is again a sequence of quantized samples. This sequence is denoted $y_q(nT)$ in Figure 8-1. If $b_R = b_T$, then $y_q(nT) = x_q(nT)$. Then the sequence of samples is conditioned (e.g., interpolated and lowpass filtered) to convert it to a continuous analog signal, denoted $y(t)$ in Figure 8-1. In good conditions, $y(t) \approx x(t)$.

Note that even in the absence of bit errors, the DAC signal conditioning block may not reproduce $x(t)$ with perfect fidelity, because of information loss in the quantization process. The resulting distortion, i.e., the difference between $x(t)$ and $y(t)$ in error-free transmission, is referred to as *quantization noise* and is a performance-limiting factor in any hybrid communications system. It should be noted that quantization noise can be made arbitrarily small by increasing the number of bits per sample. The trade-off, of course, is an increase in data rate and required bandwidth.

8.1.2 Output Signal-to-Noise

A fundamental assumption in a performance degradation analysis of hybrid communications systems is that the noise due to quantization and the noise due to channel-induced bit errors are additive. It is appropriate, therefore, to specify a source decoder transfer function that gives output S/N (including both types of noise) as a function of input BER.

The source decoder performance curves for a multi-bit (PCM) decoder [8-1] are given in Figure 8-2. The S/N vs. BER relationship is shown for k-bit PCM with $k = 4, 5, 6, 7,$ and 8. The sampling rate in each case is 8 kHz.

[8-1]   L. Rabiner and R. Schaefer, *Digital Processing of Speech Signals*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1978.
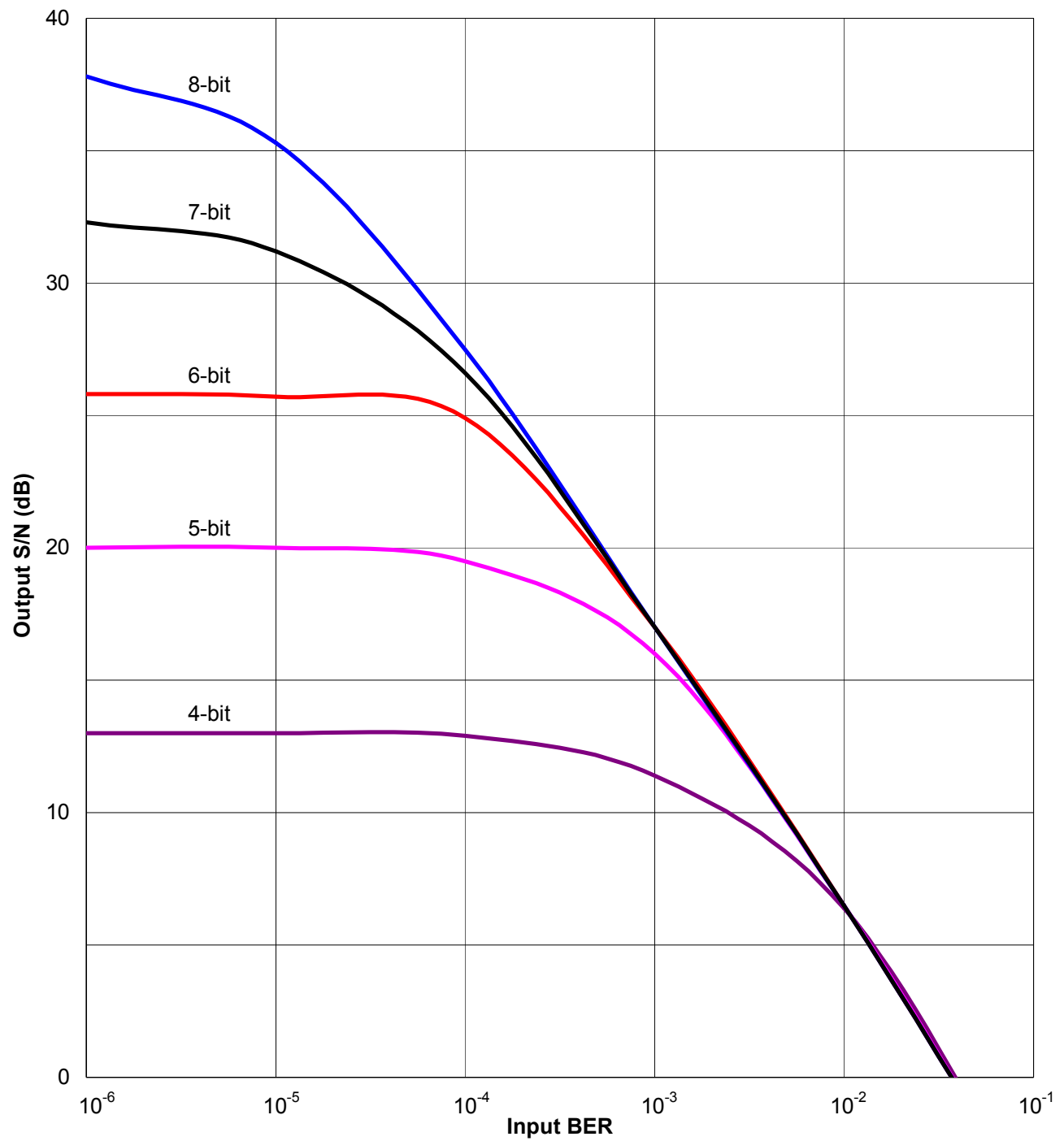
Figure 8-2. Output S/N vs. Input BER for PCM DAC

8.2 SINGLE-BIT SAMPLING

8.2.1 Description

Differential PCM (DPCM) is a signal conversion scheme similar to PCM in that the ADC samples an analog waveform. Rather than sampling, quantizing, and coding the actual input waveform, the ADC performs these operations on the difference between the waveform and an estimate of the waveform. A DPCM encoder is shown in Figure 8-3. The input waveform, estimate of the waveform, and difference signal are denoted $x(t)$, $x_E(t)$, and $d(t)$, respectively. The estimation algorithm is designed so that the estimate $x_E(t)$ closely resembles the actual waveform $x(t)$. The difference signal $d(t)$, therefore, tends to be relatively low in amplitude. Consequently, the difference signal can be coded with relatively few bits per sample, resulting in a more efficient system than strict PCM coding. Also, the estimation algorithm itself can serve as the receiver DAC.

Perhaps the simplest DPCM estimation algorithm involves a single-sample delay. The underlying assumption is that the input waveform varies slowly enough so that the present sample is very close to the previous sample. Delta modulation (DM) is an example of such a technique, with the further simplifying assumption that the difference signal can be adequately represented by a one-bit code. In other words, a DM ADC simply transmits a single bit according to whether the input waveform sample has made a positive or negative excursion with respect to the previous sample. If no difference is detected, the system will typically idle, transmitting a sequence of alternating 1-bits and 0-bits. This technique is also called *single-bit sampling* or *oversampling* (because the sampling rate must be high to ensure that the waveform varies slowly between samples).

Signal conversion algorithms involving estimation techniques, such as DPCM, often permit the estimation procedure the flexibility to adapt in response to certain conditions. If the DPCM difference signal exceeds some specified value, for example, the estimation procedure may change in such a way as to reduce the difference in the future. This technique has been successfully applied to DM systems resulting in what are known as adaptive DM (ADM) systems. One variation of ADM that is currently popular, owing to its simplicity and resistance to channel errors, is continuously variable slope delta (CVSD) modulation. CVSD systems are capable of adequate performance at bit error rates as high as 10%. For this reason, CVSD systems are appropriate for use in communications applications where

systems must function properly under adverse channel conditions. The CVSD algorithm and the other signal conversion techniques referred to in this section are described more thoroughly in many texts.[8-2]
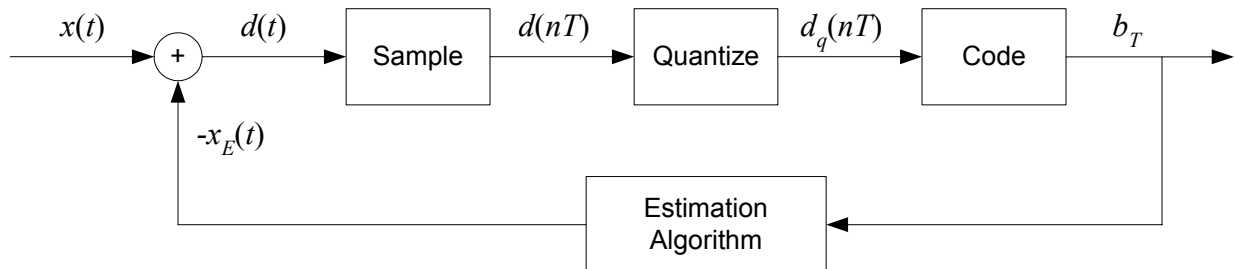
Figure 8-3. Block Diagram for DPCM Encoder

8.2.2 Output Signal-to-Noise

Figure 8-4 shows a BER vs. S/N curve for 16-kbit CVSD.[8-3] Comparing the curve with the curves in Figure 8-2, note that 6-, 7-, and 8-bit PCM outperform CVSD at low values of BER, but CVSD outperforms PCM when the BER is large. Note that CVSD achieves this performance at a data rate between one-fourth and one-third the data rate of the comparable PCM system.

[8-2]   N. Jayant and P. Noll, *Digital Coding of Waveforms*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1984.

[8-3]   E. Harras and J. Preusse, *Communication Performance of CVSD at 16/32 Kilobits/second*, Communications-Electronics System Integration Office, US Army Electronics Command, Fort Monmouth, NJ, 1974.
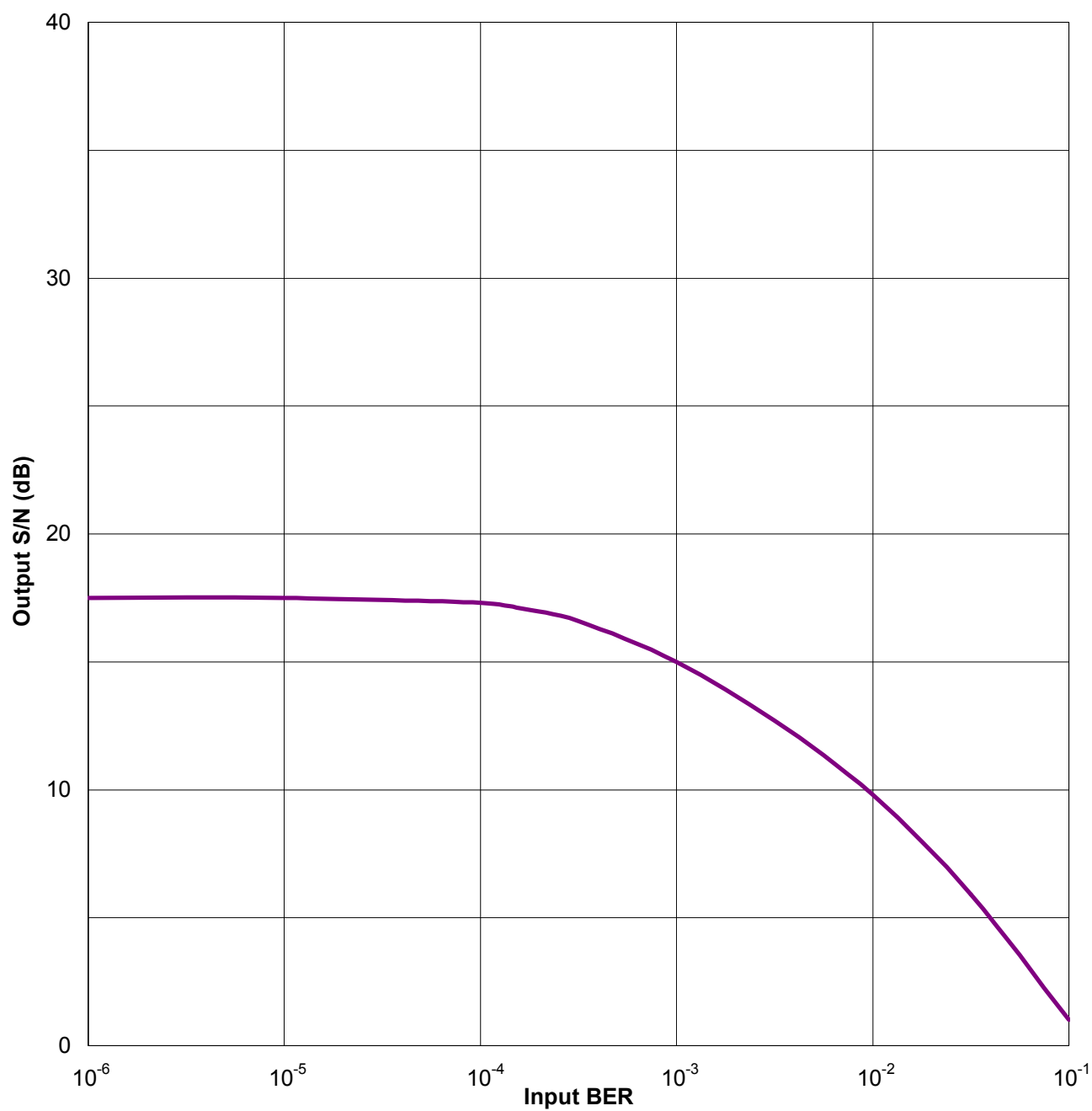
Figure 8-4. Output S/N vs. Input BER for 16-kbit CVSD

SECTION 9 – SAMPLE RECEIVER ANALYSIS PROBLEMS

This section presents several examples of using this handbook to perform a receiver analysis. Section 2.4.5 lists the specific steps required to perform an analysis.

9.1 RECEIVER PERFORMANCE ANALYSIS EXAMPLE 1

A two-way radio system is being planned to support digital (speech) communications. BPSK modulation will be used. There is some flexibility in the system design regarding transmitter power and whether or not FEC is used. The S/N of the output voice signal is required to be greater than 15 dB.

An RF analysis is required to

- Estimate input signal requirements without FEC
- Estimate input signal requirements with FEC
- Compare the two implementations

9.1.1 Approach

The following approach will be taken:

- Model each system
- Work backwards for each system to calculate input signal requirement
- Compare the two implementations

9.1.2 Models

The characteristics of System 1 are as follows:

- DAC: 48 kBit PCM, 8000 samples/s at 6 bits/sample
- FEC: none
- Demodulator: BPSK
- IF Bandwidth: 100 kHz

The characteristics of System 2 are as follows:

- DAC: 48 kBit PCM, 8000 samples/s at 6 bits/sample

- FEC: Extended Golay (24, 12) soft-decision
- Demodulator: BPSK
- IF Bandwidth: 200 kHz

System 2 requires twice the bandwidth because the coded data rate is twice the uncoded data rate.

9.1.3 Calculate Input Signal Requirement for System 1

The following analysis steps apply for System 1:

- The receiver has the RF/IF section, demodulator, and source decoder modules.
- There is no interference – the analysis will consider only the effects of noise.
- The required performance measure is a minimum output S/N of 15 dB.
- Using the Input BER vs. Output S/N curve for 48 kBit PCM (Figure 8-2), an output S/N = 15 requires approximately BER = 10^{-3} at the input.
- Using the BPSK demodulator curve (Figure 6-2), an output BER = 10^{-3} requires an input E_b/N_o = 7 dB. Using Equation 6-8 with a 100 kHz bandwidth and a 48 kbit/s bit rate, the corresponding S/N is 3.8 dB.

9.1.4 Calculate Input Signal Requirement for System 2

The following analysis steps apply for System 2:

- The receiver has the RF/IF section, demodulator, FEC decoder, and source decoder modules.
- There is no interference – the analysis will consider only the effects of noise.
- The required performance measure is a minimum output S/N of 15 dB.
- Using the Input BER vs. Output S/N curve for 48 kBit PCM (Figure 8-2), an output S/N = 15 requires approximately BER = 10^{-3} at the input.
- Using the BER vs. E_b/N_o curve for the soft-decision Golay decoder (Figure 7-7), for PSK an output BER = 10^{-3} requires an input E_b/N_o = 3 dB. Using Equation 6-8 with a 200 kHz bandwidth and a 96 kbit/s coded bit rate, the corresponding S/N is -0.2 dB.

9.1.5 Comparing the Two Implementations

System 2 requires 4 dB less S/N than System 1 to achieve the required output. However, the bandwidth of System 2 is twice that of System 1, so it will have twice the noise (i.e., 3 dB more). Thus, System 2

has a 1-dB power advantage. It can transmit slightly more power than System 1 and achieve the same performance goal. If transmitting at the same power level, System 2 will have a slightly greater communications range, at the expense of greater receiver complexity.

9.2 RECEIVER PERFORMANCE ANALYSIS EXAMPLE 2

This example has wider scope than the previous example. To illustrate the context in which a receiver analysis frequently occurs, a realistic problem is presented and solved with a combination of techniques, including techniques from Sections 3 and 5 of this handbook.

A two-way FM voice radio system is being planned to support communication between a mobile radio and a fixed base station. The operational area extends to 100 km from the base station. The preferred operating frequency is at or near 160 MHz. The base station transmitter has 5 W of power and employs an omnidirectional antenna with 2 dBi of gain. The mobile antenna has not been specified, pending the outcome of the analysis. The system will be operated in an area with a moderate density of commercial and industrial development. In this area, it is known that there is a digital communications link operating in the same frequency band.

Focusing on the performance of the mobile receiver, an RF analysis is required to

- Validate the basic design decisions
- Determine the optimum mobile antenna
- Minimize the impact of the interfering digital communications link

9.2.1 Approach

The following approach will be taken:

- Summarize the operating environment
- Specify the equipment characteristics
- Specify or summarize the required performance goals
- Use link-budget analysis to estimate the received-signal power
- Utilize this handbook to estimate the system performance and answer the questions about the antenna and the interference.

9.2.2 Operating Environment

- Size: Radius of mobility is 100 km, so the maximum distance for the desired link is 100,000 meters.
- Type: "Moderate density of commercial and industrial development" indicates the possibility of signal fading. It will be assumed that previous measurements at a similar site determined that a 20-dB fade margin is required.
- Interference: Digital (FSK) modulation, operating in a single 25-kHz channel in the VHF band, at 5 W.

9.2.3 Equipment Characteristics (for the Desired Signal)

It is assumed that the following information has been determined:

- Transmitter power: 5 W = 5000 mW, 10 log(5000) = 37 dBm
- Base station antenna: omni, gain = 2 dBi
- Mobile antenna: unspecified
- Modulation: FM voice, frequency deviation ±5 kHz
- Receiver IF bandwidth: 16 kHz
- Receiver noise figure: 4 dB
- Adjacent Channel Rejection: 65 dB
- Frequency: VHF band, preferably at or near 160 MHz
- Receiver sensitivity: 0.3 μV for output S/N = 12 dB

9.2.4 Required Performance

The required performance is specified in terms of the AI. The interpretation of AI is subjective and is generally context-dependent. For this example, the following interpretation is assumed:

- Good: AI > 0.9
- Marginal: 0.7 < AI < 0.9
- Unacceptable: AI < 0.7

Note: The receiver sensitivity can also be used to evaluate performance, but it does not address signal intelligibility and it does not address non-noise-like interference.

9.2.5 Received Signal Power Calculation

Equation 2-1 is used to estimate the minimum received-signal power. The distance is assumed to be the maximum operating distance of 100 km. The mobile antenna is unspecified at this point, so its gain is arbitrarily set to 0 dBi.

$$
\begin{aligned}
P_R &= P_T + G_T + G_R - 20 \ \log(d) - 20 \ \log(f) + 27.6 \\
&= 37 + 2 + 0 - 20 \ \log(100{,}000) - 20 \ \log(160) + 27.6 \\
&= -77 \ \text{dBm}
\end{aligned}
$$

9.2.6 Comparison of Received-Signal Power With Receiver Sensitivity

The receiver sensitivity is 0.3 uV (= 3 x 10^{-7} V). The following equation is used to convert this quantity to power:

$$
P = 10 \ \log\left(\frac{V_{rms}^2}{R}\right) + 30 = 10 \ \log\left(\frac{9 \cdot 10^{-14}}{50}\right) + 30 = -117 \ \text{dBm}
$$

where

P	=	signal power, in dBm
V_{rms}	=	RMS voltage of signal, in V
R	=	antenna impedance (50 ohms)

Thus a received-signal power of -117 dBm is required to produce an output S/N of 12 dB. The estimated minimum received-signal power is -77 dBm, which provides a considerable margin – more than enough to accommodate the 20-dB fade margin. This initial result is encouraging, but the handbook will be used to develop a more complete picture that includes the noise and interference.

9.2.7 Noise Calculation

The receiver noise power referenced to the receiver input is given by:

$$
\begin{aligned}
N &= F_R - 144 + 10 \ \log(B_R) \\
&= 4 - 144 + 10 \ \log(16) \\
&= -128 \ \text{dBm}
\end{aligned}
$$

where

F_R = receiver noise figure, in dB

B_R = receiver IF bandwidth, in kHz

In this equation, the term -144 dBm/kHz is the thermal noise power density at standard room temperature (290 K). Depending on the frequency range of operation, it may be appropriate in some cases to include an additional factor for man-made noise contributions.

9.2.8 Interference Calculation

The worst-case assumption is that the interference is on tune; i.e., in the desired-signal channel. The interfering signal bandwidth is about the same as the desired signal bandwidth since they both occupy a 25-kHz channel. In this case, Section 3 indicates that there is no FDR.

The undesired received-signal power can be estimated just like the desired signal by means of Equation 2-1. However, both the desired and the undesired received-signal powers would have to be calculated at every point within the radius of mobility. The calculation of desired signal power that was done previously for maximum distance does not necessarily give the worst-case S/I. The S/I will vary as the mobile moves throughout the area. If the location of the interference were known, it would be possible to use propagation-modeling software to compute the area coverage in terms of S/I. Instead of performing those detailed calculations, the interference effects will simply be characterized here in general terms by using the handbook transfer functions.

The relevant transfer function is Figure 5-6, which displays multiple AI vs. S/N curves. One curve is for the case with no interference. Each of the other curves is for a particular S/I value. Because the minimum received-signal power is -77 dBm and the noise power is -128 dBm, the worst-case S/N is (-77) - (-128) = 51 dB. Including the 20-dB fade margin, the worst-case S/N is 31 dB.

It is clear from the plot that, in the absence of interference, this is more than enough S/N for proper operation. Since noise is not a problem, the operation is said to be interference-limited. The curves show that an S/I of less than about 12 dB puts the system in the marginal range (AI < 0.9), and an S/I of less than about 6 dB puts the system in the unacceptable range (AI < 0.7). Because the desired and interfering transmitters have the same output power and both lie in the area of interest, there will be a region of points close to the interfering transmitter antenna at which the mobile receiver performance will be unacceptable (S/I < 6 dB). Propagation modeling is required to determine the size and shape of the "denied" region.

9.2.9 Remediation

There are several ways to remediate the problem:

- Increase the desired transmitter power. This may not be possible for practical, economic, or regulatory reasons. For example, it may degrade the performance of other systems in the environment. If it is possible, the action may shrink the denied region to an acceptable size.

- Employ a directional mobile antenna. Such an antenna would have to be steerable to ensure that it always pointed toward the desired antenna. However, there will be places where the mobile antenna cannot point toward the desired antenna without including the undesired antenna in the capture area. If the undesired antenna is directional and its pattern and orientation are known, it might be possible to operate in spite of being co-channel, but it would be difficult to perform the necessary geographic analysis. This action may also shrink the denied region to an acceptable size.

- Select a channel that takes the interference out of the receiver passband. This is the most practical and effective solution. Since the adjacent channel rejection is 65 dB, the handbook transfer functions show that the mobile receiver will then operate as required in most locations. (Because of nonlinear (cosite) interactions, there is usually a small denied region that cannot be eliminated.) The decision regarding the mobile antenna can then be based on other considerations.