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# Use of Computers for Environmental Engineering Related to Buildings

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Standards

UNITED STATES DEPARTMENT OF COMMERCE • Maurice H. Stans, *Secretary*
NATIONAL BUREAU OF STANDARDS • Lewis M. Branscomb, *Director*

# Use of Computers for Environmental Engineering Related To Buildings

## Proceedings of a Symposium Sponsored by the National Bureau of Standards, the American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., and the Automated Procedures for Engineering Consultants, Inc.

Held at the National Bureau of Standards
Gaithersburg, Maryland
November 30 - December 2. 1970

Edited by

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## Abstract

These proceedings of the First Symposium on the Use of Computers for Environmental Engineering Related to Buildings contain all of the technical papers and invited addresses presented at the symposium, which was held November 30 - December 2, 1970, at the National Bureau of Standards.

The fifty-nine papers deal with the application of the computer to such environmental engineering problems as building heat transfer calculations, heating and cooling load calculations, system simulations, energy usage analyses, computer graphics, air and smoke movement inside buildings, and weather data analyses for load and energy usage calculations.

Key Words: Building heat transfer analysis, energy usage, environmental engineering, heating and air conditioning, use of computers

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# Foreword

For a number of years the National Bureau of Standards has been a leader in the development and use of computers in scientific and engineering fields. We strongly believe that the effective application of computers to the problems of the building industry will be of significant benefit to that industry. In what we hope to be a helpful step in this direction, we were pleased to be able to join with the American Society of Heating, Refrigerating, and Air Conditioning Engineers and the Automated Procedures for Engineering Consultants, Incorporated, in sponsoring this First Symposium on the Use of Computers for Environmental Engineering Related to Buildings.

In recent years the use of computers has had a rapidly increasing impact on the design, performance analysis, and control of environmental systems related to buildings. The purpose of the Symposium was to provide a forum for exchange of the latest information and ideas among engineers, architects, and planners who use computers. The Symposium attracted leading authorities in the field of environmental engineering not only from all parts of the United States but also from many parts of the world. Over 400 architects and engineers representing the building industry, governments, universities, and utilities participated. Applications of computer programs and calculation methods covering several topics in environmental engineering are presented in these proceedings in a form useful to consulting firms, government agencies, research organizations, and industrial firms.

Lewis M. Branscomb, Director
National Bureau of Standards

# Preface

The use of computers is now widespread among environmental engineers working with buildings. Subjects ranging from routine heating and cooling load calculations to sophisticated computer graphic display systems are being handled. Although the environmental engineers have been slow in adapting the computer to their needs, at least until the middle of the 1960's, their use of the computer is now increasing rapidly. This development is in fact taking place so fast that no major coordinated activities for exchanging ideas and disseminating information have been undertaken except those of the APEC (Automated Procedures for Engineering Consultants). While the APEC is active mainly in the area of programs for practicing engineers, needs are also recognized for advanced techniques or new procedures--such as the calculation of accurate room temperature change under realistic climatic conditions, simulation of air conditioning system dynamics, optimization of the system and component selection based upon relatively advanced mathematical concepts, and effective use of graphic displays or data structuring. The First Symposium on the Use of Computers for Environmental Engineering Related to Buildings was to serve this need by providing opportunities for creative environmental engineers to meet each other and exchange new ideas. Because this was the first symposium of its kind and because heating and cooling load calculations are currently the most popular subject among the environmental engineers, the largest percentage of the papers presented dealt with the temperature and the thermal load calculations for buildings. The papers presented illustrated that there exists much duplication of effort in many parts of the world as well as in the United States. Although the program committee's selection of papers led to some redundancy, the purpose of their inclusion was to encourage the participation of as many of the first line investigators in the field who have been active in the use of computers for environmental calculations. The symposium gathered 59 papers from 12 countries and was attended by approximately 400 engineers, scientists, and architects--firmly justifying this type of conference. The papers presented include those which are highly theoretical as well as those which describe popular programs. Nine sessions were required during three days to present all of these papers. In addition, a technical forum was held one evening to exchange informal opinions on computerized controls. This was well attended. It is hoped that this symposium made a major contribution to environmental engineering design and these proceedings will be useful to all using computers in this field. The program committee will welcome reactions and suggestions as an aid to planning future conferences of this kind.

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# Contents

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## COMPUTER GRAPHICS
Co-Chairmen: J. B. Chaddock
E. M. Barber

ENERGY LOAD CALCULATIONS
Co-Chairmen: E. Stamper
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## AIR CONDITIONING CALCULATIONS AND WEATHER DATA

Co-Chairmen: M. Lokmanhekim
D. L. Richardson

## WALL CONDUCTION AND THERMAL LOAD SIMULATION

Co-Chairmen: D. G. Stephenson
T. Kusuda

# WELCOME ADDRESS

F. K. Willenbrock
National Bureau of Standards

Good morning.

Welcome to the National Bureau of Standards. I am substituting for the Director, Dr. Branscomb, who unfortunately will be unable to greet you in person.

The Bureau has a deep interest in this First Symposium on the Use of Computers for Environmental Engineering Related to Buildings. We are pleased to be one of the three sponsors; we are happy to be your host. While you are here we hope that you will find time to meet and talk with our staff, and that you will take advantage of the tour of <u>your</u> facilities. I said <u>your</u> facilities to the American taxpayers present because the Bureau is a tax-supported public institution with a goal to "strengthen and advance the Nation's science and technology, and to facilitate their effective application for the <u>public benefit</u>".

The National Bureau of Standards from its inception in 1901 has been closely involved with building research and technology. In the early days buildings were not viewed from a systems standpoint and the work was organized in response to specific, recognized needs for technical information on the properties of building <u>materials</u>.

As early as 1905, the Bureau had a 100-thousand pound testing machine which was used to measure the strength of structural materials, such as steel and concrete. Later, the Bureau joined with the National Fire Protection Association and the Underwriters' Laboratory in a program from which flowed a large amount of data on the fire resistance of materials. These data were subsequently incorporated in fire and electrical codes throughout the country.

In 1921, these scattered activities were combined by the Secretary of Commerce, at that time Herbert Hoover, into a Division of Building and Housing. The functions of the Division were to coordinate scientific, technical, and economic research on building; to aid in the revision of state and municipal codes, and to engage in the simplification and standardization of building materials. Despite these broad goals, the primary effort remained in the materials evaluation area, and the application of the findings to building codes and standards based on materials specifications. During this period, however, there was a small but growing program concerned with the environmental conditions in housing. The first attempts to study the habitability of housing could also be considered as the exploratory examinations of buildings from a systems standpoint.

During the 1920's, the emphasis was on a "Better Homes Program". In the depression of the 1930's, the Better Homes Program became low-cost housing; in World War II, the conservation of scarce building materials was the major effort. After the war, for fifteen or so years, the building research programs again stressed the properties of various kinds of building materials, with the exception of the environmental work which continued in the direction of a systems approach to building problems.

But the winds of change have influenced even the Bureau, and today in our building research and technology program we talk primarily about building systems; we are challenged by the problems of evaluating the function and performance of buildings as they satisfy the user. We are still concerned with materials, but view them as components which are part of a system. Our efforts are toward the development of performance requirements and performance evaluation techniques for building components and systems. Such efforts are compatible with the national trend toward industrialized building construction.

The development of standards based on performance, and the consideration of buildings as systems requires the evaluation of masses of data which are orders of magnitude larger than required for the earlier materials evaluation or specifications studies. It is clear that the computer has influenced our investigations in many fundamental ways and it is my prediction that it will have an increasing impact on our thinking about building systems in the future.

A good Ph.D. subject for a student of the history of technology would be to determine how much the change in our perceptions of buildings has been influenced by the availability of the computer as a data and information-handling device.

Even today we are well past the relatively simple use of computers for the analysis of masses of data. During this symposium we shall hear how computers are used in modeling or design studies related to the environment of buildings; how they are used for evaluating the non-steady thermal behavior of office buildings, how they are used for the design of heat/air-conditioning installations.

Our speakers in this symposium come from 11 foreign countries and from all across the United States. They represent industry, the research community, the universities, and government. We shall hear from architects, engineers, computer specialists, systems analysts, and from those in other disciplines. Indeed, this symposium program is indicative of how computers are stimulating a "quiet revolution" in the building technology field. What is being done in this segment of the building process points the way to what must inevitably be the norm for the entire building process.

So welcome once again to the Bureau, and to this First Symposium on the Use of Computers for Environmental Engineering Related to Buildings. It is our hope this symposium will provide an effective forum for the exchange of ideas in the field. It is our hope that these sessions will stimulate others to explore how computers may be used throughout all parts of the building process.

## KEYNOTE ADDRESS

### Some Objectives for the Technology Man

Bruce J. Graham

Skidmore, Owings and Merrill
Chicago, Illinois

A recent article in Time Magazine pointed out that the Unisex Society developing in the United States is a symptom of the decay of our civilization. As ultimate proof, it indicated that out of two thousand previous civilizations - fifty five which suffered of this same symptom - such as the Greek and the Roman eventually disappeared. I would propose that the other nineteen hundred and forty five also disappeared or at least there is no evidence of their existence today.

The rise and fall of civilization has very little to do with the morality of those civilizations. It becomes important to define what we mean by civilization. In Webster's dictionary definitions read: "the condition of being civilized; social organization of a high order, marked by advances in the arts, sciences, etc.; the total culture of a people, nation, period, etc: as, the civilization of the Occident differs from that of the Orient". And finally, "the countries and peoples considered to have reached a high stage of social and cultural development.". I contend that the American people do not fall under any of those definitions. We certainly do not have order nor have we reached a high stage of social and cultural development. In fact, there may never be an American civilization. I believe that we are engaged in the process of developing a single civilization throughout the world - one in which America will play a very important and, I hope, responsible role. Other nations have contributed in great measure and will continue to do so. We are in fact children of past civilizations from Greco-Roman, from African, Mayan and Chinese ancestries. It is, therefore, nonsense to talk gloom and doom when we are barely participating in the dawn of this emerging culture.

The technological equipment today is breath-taking in scope. We have been able in the last thirty years to break through barriers of exploration which did not exist one hundred years ago. Yet, we have failed miserably on this earth in the efforts that deal with the problems of an ever-expanding population. Recently man has created the first reproducing cell, but he has been unable to control the reproduction of man. We have created a completely antiseptic environment that can hurdle into space at unbelievable speed - returning to earth with a relative safe and healthy human specimen, but we have been unable to provide even the most basic of housing needs for the great majority of people of the world.

It matters little what political system we support, what nations we swear allegiance to - singly or jointly all nations have failed. The promise in America that capitalistic democracy would achieve individual freedom is a myth. The much touted equality of communism is a fiasco and the self-serving smugness of Scandinavian countries exists only at the expense of suffering millions around the world. We are all aware of the usual capability of nations to wage war, regardless of financial stability. Starving millions find no food, but plenty of guns with which to serve militarist demagogues.

Of paramount importance in our time is not the search for the secret new technology or the wonderful do-all material, but the philosophical leadership which will redirect the great energy being expended for the benefit - rather than the detriment - of people. There is hardly a technical problem existing that cannot be solved, but equally there is hardly a solution in sight for the sufferings in the world. No leaders plead the case for civilization.

The City today is hell bent on disaster. This phenomenon exists, by no means, only in America. It matters very little whether we speak about a socialistic or capitalistic nation. Technocratic achievement and production have become the paramount value. Other values are secondary. The cries and, in fact, the screams of a few have had very little effect on the relentless progress of production for the sake of production. It matters little what we produce, so long as we feed labor and raw materials to the machine. As a result other values cannot be served. The typical urban center is plagued with a series of fantastic problems - pollution, not only of the air and the water, but pollution of sound - of vision - of taste - and of mind. Transportation is a story book of failures. Tokyo, like New York City and London are reaching the point of standstill. The customary jokes about traffic jams in Rome and Paris are no laughing matter to the Romans and Parisians. Tempers have risen on this subject alone to a point of no return. Transportation has created and fostered economic segregation; the poor in the cancerous center, the middle class in the greenbelt.

The university - once a sacred place - is in complete disarray everywhere. It matters little whether we speak of disillusionment of the student at the Sorbonne, Kent University or the University of San Marcos in Lima. The academy is no longer believable. Academic isolation has led to irrelevance. Yet we know, or have faith that solutions could be found and that these solutions will depend heavily on our technological baggage. This premise has been held for some time, but our credibility has lapsed. The philosophical evaluation of priorities has eluded our grasp.

Much has been said about the expanding population of the world, and this is a problem. Much has been said about the depleting resources of the world, and this is a problem; but little recognition exists that it is not expansion in numbers alone, but rather the accelerated increase in ambitions which cause the confrontations we now experience. The wandering Arab is no longer happy to wander. The potato-growing Quechua Indian is no longer happy with a diet of potatoes. The millions of India are no longer happy with an 18-year life span. In fact, not even the people of Wales are satisfied with 2nd-class citizenship.

We do not need any more automobiles from General Motors or from Volkswagon or from Toyota. The people need a healthy environment first, and it is not up to the leadership to deny it. The advertising campaigns which are used to sell unnecessaries should now be used to sell the necessaries.

Poll-taking as an excuse for leadership is the instrument of the present political scientist. This method will lead to continued mediocrity and worse. Ask a drug addict what he wants, he will say drugs. Ask a hunter, he will say guns, but we need neither drugs nor guns. It seems inconceivable that in this day we can produce models of the human body in a computer and measure the good and bad effect of environmental inputs. Yet we cannot decide once and for all what is a good diet - what kind of air we should breathe - what kind of noise we can bear or what kind of environment we can survive in. The priorities of the modern economist do not recognize the primacy of human life.

It may become important at last that we begin to make value commitments, for the survival of existing governments, universities and intellectual leadership will depend upon their ability to commit the energies of human kind immediately towards the needs for survival. The pressure towards such commitment will not come from the fickle and easily converted popular movements. Those are easily swayed by Madison Avenue advertising or Latin American demagogues. Each and every educated man must mold his well trained efforts towards the simple realities that face the world. This individual effort can have tremendous influence since it is the technocrat who controls the valves of cornucopia. It wasn't Hitler or Churchill or Roosevelt or Stalin who invented the atom bomb, but it is their outmoded heritage that is rattling that frightening instrument.

Architects and planners of the last twenty years have been pre-occupied with their profession. They are designers of objects. It appears today that they have proven without a doubt their own irrelevancy. Walter Gropius many years ago decried the lack of involvement by architects epitomized by the Ecole Beaux Arts in Paris - the Aesthetic of the Renaissance was but a symptom of the selfish role the professional had carved unto himself. The Bauhaus movement of the 20's in Germany was a successful attempt to convert the industrial machinery into a viable architectonic language. Mies van der Rohe epitomizes that success. In his hand the products of modern man became a poetry of space. However, his lingo combined with the virile language of Corbusier - has been converted into a substitute for the cliques of the renaissance. We are now extremely capable modern temple builders, except that we care little what gods dwell in our temples. Our works are terribly important and, since they serve the images of false gods, they curse the life of the urban dweller around them.

Architects and planners have to turn about and realize that we are but transitory instruments in the evolution of cities. Instant civilization is not about to happen - we are just barely defining the kind of civilization we expect to create. We know that in such a civilization national boundaries do not exist. Isolation from the dynamic world forces is impossible and existing political systems are obsolete. All people of the world must participate, for in exclusion we seed discontent, and in segregation moral decay.

The larger picture of the world affects the life of every individual and we must be prepared to meet both ends of this candle. Some glimpses of such a society are possible. We know that the individual citizen must participate in those decisions that affect his immediate life and that of his family. He must, therefore, have something to say about where he lives - the school that his children attend - the work he does, but on the other hand, he must enjoy the fruits of international medical research, the writing of poets - the art of painters and scultors - the pleasures of travel - free air - all these which cannot come about through micro systems, but that belong to larger structures.

Computer technology, if it has any promise, is this: it can make available to an individual the knowledge of all; and the ability to make decisions at the most personal of levels under that larger umbrella of knowledge. It is that promise which must direct the efforts of your conference.

I would propose to this conference that the papers presented here and at future conferences should concentrate on the problems that face the urban centers of the world:

1. On Transportation - not how to move people, but how a city can exist, expand and grow without the convulsion of movement we now enjoy. How can man live near his place of work - near his children - breathe free air? Today that freedom of choice is denied.
2. What is a house - what kind of a house does a family need - what kind of environment and air should children breathe - what kind of neighborhood does this house belong to - how can a man move from one stage of life to a later one without the loss of ties to his family and to his tribe?
3. What kind of diet does man need - how can this be distributed equitably from the farmer to the dinner table?
4. Medicine - should not be the hunting ground for doctors. How do we provide medical care for all, but more important, preventative medical care so that healthy lives can be a backbone for fulfillment. As an example - humidified air is now the privilege of machine environment, but shouldn't the delicate nasal passage of children be protected?
5. In the integrated community how do we distribute the benefits of culture - music - dance - theater - art - and all the other fulfilling human experiences, so that they become a part of all peoples' lives - rather than the privilege of the few?

6. How do we maximize the fruits of this earth - preservation of forest - clean rivers and lakes - in fact clean oceans? How should the resources be protected?
7. Education is an integral part of all the prior values, but how do we expand, elaborate and create a meaningful civilization so that the recognized values of the intellectual become the every day values of all citizens?

I propose that a conference such as yours should address itself to what end you work. It is not important to develop a new program of heat transfer or of the design for sophisticated duct systems unless that program is a meaningful part of the value set which makes up the fiber of our emerging civilization.

The fracturized construction industry in America with its multiplicity of goals is only matched in disarray by the even more fracturized industry of construction in other parts of the world. Self interest is the motivating force in construction. This force rules everyone connected with our labors from bankers and land owners to government and labor unions. I was told five years ago by a high government official that if architects do not respond to the crying needs of society, the government would step in and solve it. At that time it seemed a ludicrous statement. Today, government's failure to respond is even more obvious. The housing stock in America is depleting at a faster rate than anyone will recognize. We have even gone so far as to substitute trailer and trailer parts for units of housing. The trailer is not a viable housing - it is sub-standard by anybody's definition.

For your work to become meaningful we must learn to make it part of a larger whole, we must recognize that we are in the childhood of an emerging world civilization. For myself I find being a part of this transition much more satisfying than believing we could be in the Golden Age.

# An Insight into Three Dimensional Graphics

Arthur R. Paradis

Dynamic Graphics, Inc.

Computer graphics can be used to relieve much of the tedium and time associated with the production of perspective drawings. It frees the architects for more creative aspects of the design process. It enables the architect to work closer with his client through a constant flow of perspective drawings. There are problems associated with implementing such a graphics system. First, the formatible image of the computer must be overcome. Then, a simple project description process must be implemented. It must be simple enough to use and flexible enough to make the system worth using. Ideally, there would be a common data structure for the graphics programs and the various engineering packages. Finally, there must be an efficient hidden line removal technique to make the system feasible. Techniques are now developed which can make such a system possible. Preliminary work done for Skidmore, Owings & Merrill indicates that such a system can be an economical and time saving tool. This paper will present the technical aspects of three dimensional computer graphics: the basic tools; the structure necessary; and a comparison of hidden line removal techniques.

Key Words: Architectural Graphics, computer graphics, data structure, hidden line removal, perspective drawings, projective geometry.

## 1. Introduction

Three dimensional computer graphics is becoming a cost-effective and time saving tool for architects and designers. Computer graphics allows architects and designers the freedom to study their layouts with perspective drawings from more vantage points and to try more design variations than would be otherwise possible with conventional means. This paper will introduce the basic tools of three dimensional computer graphics, both software and hardware; discuss the various components of the structure necessary for a three dimensional graphics system; and compare techniques for producing perspective line drawings.

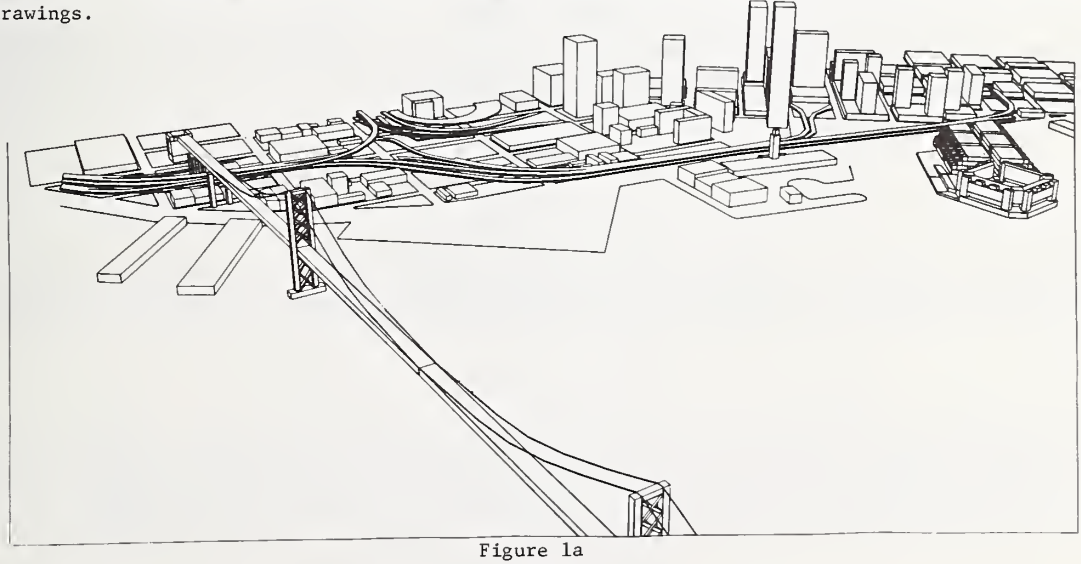


Figure 1a

Two views of San Francisco waterfront area
produced for Skidmore, Owings & Merrill

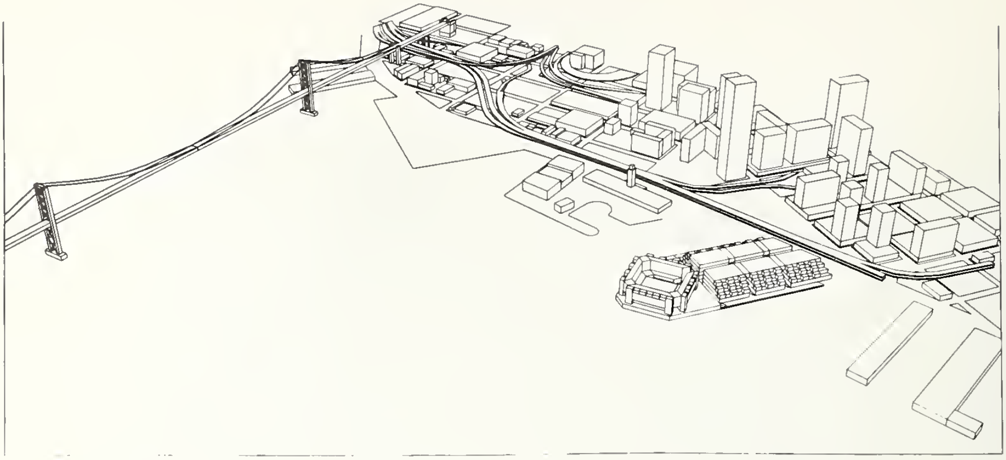


Figure 1b

A computer generated perspective plot of the San Francisco waterfront area showing a proposed waterfront project

## 2. Basic Tools of Three Dimensional Computer Graphics

### 2.1 Software Tools

There are three basic software tools which are combined to provide a flexible system for producing perspective drawings: the projection of lines in space, the representation of surfaces (topography), and the portrayal of complex solid objects. Each area will be presented as current capabilities and as advanced features which are being developed or are considered feasible.

### a. Projection of Lines in Space

Lines in space may be represented by connecting a series of projected points with straight line segments. More advanced features allow the line to be represented by a smooth curve through the projected points and permit the line to pierce surfaces or solid objects.

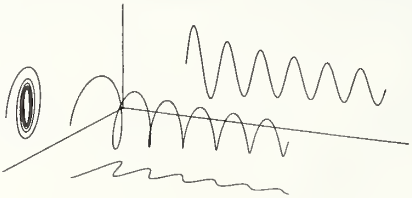


Figure 2

A projected line in space

### b. Surfaces

Topography may be represented as a rectangular gridded mesh which may be displayed as a projected mesh (fig. 3) or as a projected contour. It is not difficult to have either regular or irregularly spaced grid lines. Thus, flat areas need not contain the same information density as rougher terrain. It is more difficult to handle missing grid points. These may be computed by some interpolation process or left as holes in the grid. Finally, there exists a whole series of functions which may operate on either gridded data or randomly spaced data points.

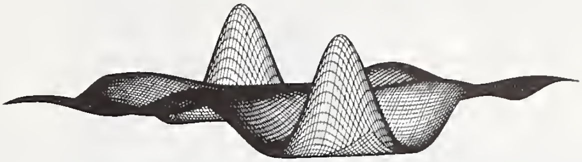


Figure 3

A surface defined by a gridded mesh

### c. Solids

Complex solid objects are generally represented by a series of bounded planar surfaces. The visible portions of the planar boundaries are drawn with solid lines which the non-visible portions are generally either blanked or drawn with dashed lines. For added flexibility, boundary lines can be specified as non-visible and additional lines or patterns can be drawn on the face of any surface. Within the framework of the basic system, curved surfaces must be approximated by a series of small planar surfaces.

More advanced features could include the ability to specify curved surfaces. Also, solid objects could pierce each other. The amount of detail shown could be a function of the final viewing size such that buildings or trees in the far distance would not be drawn to the same degree of detail as buildings very close to the observation point.

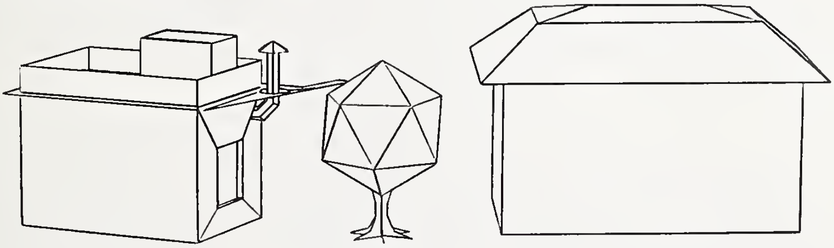


Figure 4

Representations of solid objects defined by planar surfaces

## 2.2 Graphics Hardware

There is a wide range of graphical display equipment available which can be used at computer service bureaus or purchased for in-house usage. The features, application areas, and price ranges for various types of graphics equipment will be given below.

### a. Pen Plotters

Pen plotters are computer driven pen and ink plotting devices. They are the most inexpensive and most common graphics devices used. Pen plotters are available in a wide range of sizes from small drum plotters to large flat bed plotters. Optional extra pens for different colors or line widths are also available. Perspectives, plan views, PERT charts, etc. can be produced using pen plotters when used with the appropriate software.

Price Range: $8,000 to $100,000 (including input device)

### b. CRT Displays

CRT (Cathode Ray Tube) displays are becoming more popular. There are two basic types -- the raster scan CRT, which works much like a normal television set; and a vector CRT which draws lines in any sequence. The vector CRT's are much easier to program for general graphics work as lines can be displayed as they are calculated. Some CRT's use a mini-computer for picture refreshing and local editing, thus reducing the computer load and special software requirements of the main computer. Keyboards, light pens, moveable cursors, and Rand Tablets are available as input devices for CRT displays. CRT displays are valuable for providing quick results and effective data editing capabilities. They are capable of providing general graphics output for applications which do not require high resolution, large display area or hard copy (hard copy devices may be connected to a CRT).

Price Range: $10,000 to $250,000.

### c. Microfilm Plotters

A microfilm plotter is basically a CRT display with a camera attached for producing hard copy. They are ideal for creating computer generated movies. Hard copy can be directly produced or can be made from the 16mm or 35mm film. The film is convenient for long term storage.

Price Range: $50,000 to $250,000.

### d. Electrostatic Plotters

Electrostatic plotters produce a grid of dots. This type of plotter can produce either line plots or render areas with a halftone effect. It has the potential for effectively displaying shadows.

Price Range: $12,000 to $50,000 (including input device)

### e. Halftone Displays

The University of Utah has done a great deal of research into producing computer generated color halftone pictures. These spectacular pictures are for the time being more of a laboratory tool and not economical for most applications.

## 3. Structure of Three Dimensional Graphics

The structure of three dimensional graphics may be divided into four areas -- Application Language, Data Structure, Projective Geometry, and the Hidden Line Problem.

## 3.1 Application Language

The value of a graphics system, in this case an architectural system, lies with economic factors and convenience. For an architectural graphics system to embody both flexibility and convenience, it must be carefully interfaced with the architect in mind. Skidmore, Owings and Merrill are currently working on this problem with encouraging results. The following shortcuts have proved very helpful in simplifying the data description.

### a. Implicit Relatsionships

The planes which define a rectangular block can be defined in more than one way. The easiest and most cumbersome way is to define the coordinates (X,Y,Z) triplets for each of the six planes. This would require the definition of twenty-four points (72 numbers) and would win few friends. By using the implicit relationships of the orientation of the six planes of the rectangular box, it can be defined by a height, width, length, location and orientation (the orientation can be implicitly defined in many cases). This is defined by six numbers and is much more liveable.

### b. Repetitive Definitions

A single window definition can be repeated to provide a whole face of windows or the windows for the whole building. Similarly, the definition of a building can serve for similar buildings in the site.

### c. Predefined Objects

Trees, vehicles, people, surface textures, building complexes, and even areas of large cities may exist as predefined objects in the architect's library.

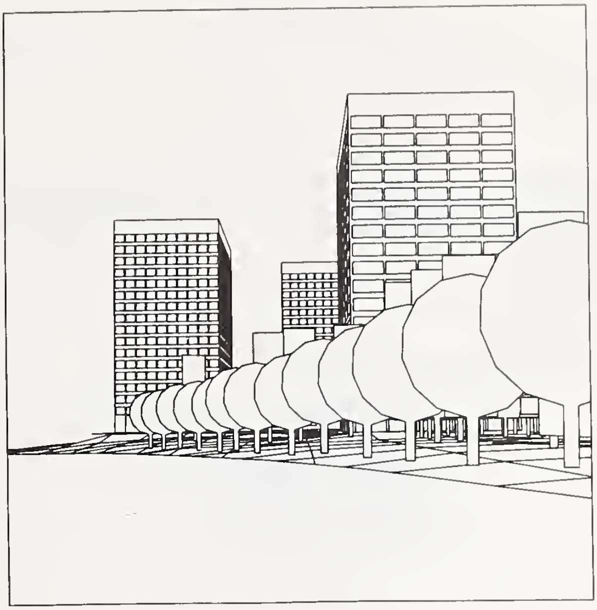


Figure 5

Windows are defined by patterns and the trees
are predefined objects

### 3.2 Data Structure

The resulting data structure should contain more information than just the definition of lines and planes. Information about the logical groupings and any hierarchial structure will allow more powerful editing and manipulation capabilities. The data structure should be flexible enough to interface with engineering programs such as duct layout, space allocation programs, or heating and cooling load calculation programs with minimal additional information. Plan views and elevations can also use the same data structure.

### 3.3 Projective Geometry

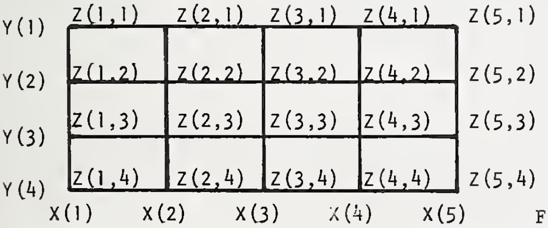
Both perspective projections and parallel projections are easily implemented. Perspective projections add realism to the drawings and the required mathematics is clearly presented by Kubert, Szabo and Giulieri [1].[1]

### 3.4 Hidden Line Removal

Determining by computer which lines are "hidden" when viewing from a specific point is a very challenging and frustrating problem. There exist various solutions, each tailored to a specific purpose, such as surface algorithms, planar solid algorithms, etc., and these may be combined to efficiently solve complex problems, but the resulting system is far from simple. Much work and possibly larger computers are required before simple general algorithms can be developed which will process in a reasonable time and at a reasonable cost.

## 4. Comparison of Surface Algorithms

Three different algorithms for solving the hidden line problem for surfaces will be compared. The advantages and disadvantages of each will be explored and general statements describing the relative efficiency of the algorithms will be presented.



Example of structure of gridded mesh used in surface definitions. In FORTRAN terms the structure is comprised of an X array, a Y array and a doubly dimensioned Z array; and the mathematical relationship between the X, Y, and Z arrays is

Z(I,J) = f(X(I),Y(J))

where f is a single valued function.

Figure 5

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[1] Figures in brackets indicate the literature references at the end of this paper.

## 4.1 Aerospace Algorithm

This algorithm was developed by Kubert, Szabo and Giulieri [1] at the Aerospace Corp.

### a. Definitions

The point to be tested for visibility will be called the test point; the line between the observation point and the test point will be called the test line and the plane perpendicular to the X-Y plane containing the test line will be called the test plane.

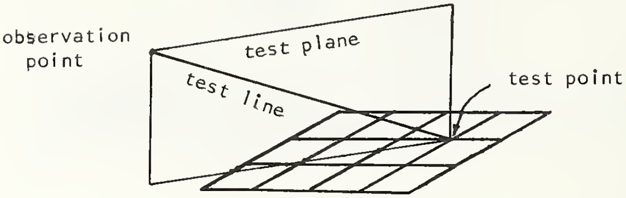

Figure 6

Test point, test line, and test plane

### b. Basis of the Method

For a point to be non-visible, the test line has to pierce the surface. A brief step by step method will be given below:

### c. The Basic Algorithm

STEP 1: A series of test criteria points are calculated from the intersection of the test plane and the gridded mesh. (Note: only the points between the test point and the observation point are calculated.)

STEP 2: The test line divides the test plane into two sections. The test point is declared non-visible if there is at least one test criteria point in each of the two sections of the test plane; otherwise, the test point is visible.

STEP 3: When two adjacent grid mesh points are visible, the connecting line is drawn.

STEP 4: When a grid point is visible and the adjacent grid mesh point is non-visible, the visibility, non-visibility transition point is calculated by using a binary search and using the above steps to determine the visibility of the successive midpoints.

### d. Advantages

This algorithm is easy to implement and requires a relatively small program.

### e. Disadvantages

The execution time rises exponentially as the size of the defining grid mesh increases. There are more test points and each test point requires the calculation of more test criteria points for the visibility testing. (This exponential relationship became painfully clear when it was discovered that a surface defined by 150 by 150 mesh points costs over $400.00 to compute.) This method also requires the whole grid mesh to reside in memory at all times. Finally, the method does not always produce the exact solution to the hidden line problem as steps 3 and 4 do not catch all changes of visibility.

## 4.2 Warnock Algorithm

This algorithm was developed by Dr. John Warnock [2] at the University of Utah. The following description does not do this algorithm justice as its real power lies in its ability to easily produce halftone pictures when coupled with the appropriate plotting equipment.

### a. Definitions

Picture resolution will refer to the smallest distance between two adjacent points on the given display device.

### b. Basis of the Method

This algorithm uses an interesting method for solving the hidden line problem. An area of the projection plane is examined. If the method determines that the area is "simple" then it contains no visible line so processing is finished on that area; otherwise the problem is simplified by subdividing the area into smaller sub-areas. The process is then applied to each of the sub-areas and reapplied until the sub-area is either simple or the picture resolution is reached. If the picture resolution is reached the square contains a visible line and the resolution sized area can be displayed as a dot.

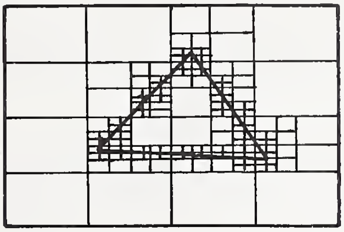

Figure 7

Subdivision Process

### c. The Basic Algorithm

STEP 1: For a given sub-area of the projection plane, determine the proper classification (out of three) for each plane in the surface.

Case 1) The projected boundary of the plane surrounds the area of the projection plane being considered.

Case 2) Part of the area of the projected surface overlaps with the area of the projection plane being considered.

Case 3) The projected boundary of the plane lies totally outside of the area of the projection plane such that the two areas do not overlap.

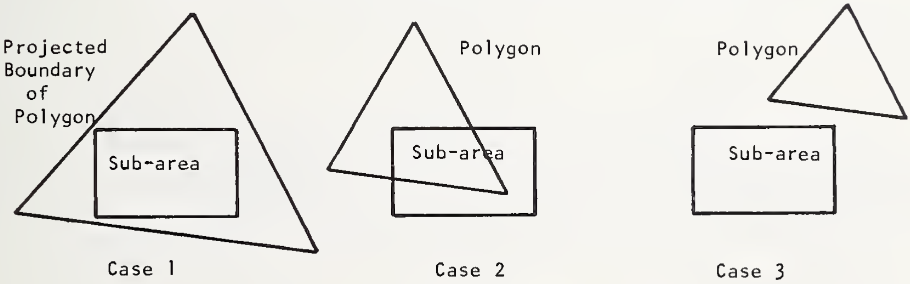

Figure 8

Three possible relationships betweeen sub-area and projected boundary of a polygon

STEP 2: For each case 1 or case 2 plane, determine the distance from the observation point to the plane at all four corners of the surface plane.

STEP 3: Determine whether the sub-area of the projection plane is simple. The sub-area is simple if:

1) The sub-area contains no planes of case 1 or case 2. It is blank.

2) There exists a case 1 plane which is clearly closer to the observation point than all other case 1 or case 2 planes. A plane will be clearly the closest if the plane is closest to the observation point at all four corners.

STEP 4: If the sub-area is not simple then subdivide it into four equal sub-areas and depending on the size of the new sub-area either:

1) If the sub-area is larger than a the picture resolution, start with step 1 and process the first new sub-area.

2) If the new sub-area is not larger than the picture resolution, then it contains a portion of a visible line on the surface, so add the point to the display file.

If the sub-area is simple, it implies that no visible lines in the surface are contained in that sub-area, so go on and process any of the other of the four sub-areas which remain to be processed, or then process any of the sub-areas remaining in the next higher level until the processing is finished.

### d. Advantages

The algorithm works well with both surfaces and planar solids. Intersecting solids present no problem. Time for solving the hidden line problem is reasonable, although it could get excessive with a large quantity of data. This is a good method for producing half tone pictures.

### e. Disadvantages

This method works well only with CRT type displays as pen and ink devices use an extreme amount of excess pen motion. Also, computer storage rises rapidly for large problems because each sub-area contains two list of planes associated with it.

## 4.3 Horizon Method

This method was developed by the author at the University of California at Berkeley [3].

### a. Definitions

Horizons -- An upper and lower horizon delineate a closed opaque region.

Grid Line -- The line connecting any two adjacent grid mesh points.

Mesh Element -- Any four grid lines which form a closed rectangular box.

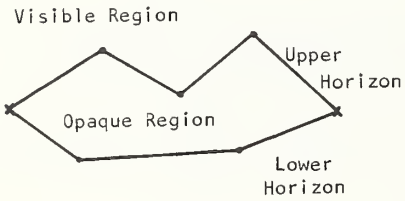

Figure 9

Upper and lower horizons delineate visible region from opaque region

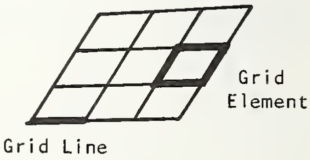

Figure 10

Example of a grid line and grid element

### b. Basis of the Method

This algorithm uses the basic property that portions of a surface closer to the observation point cannot be covered by portions of the surface more distant.

### c. The Basic Algorithm

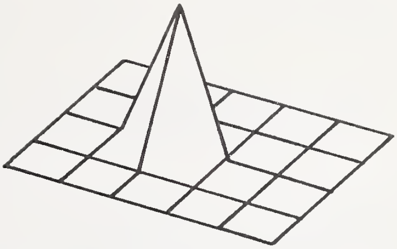

Figure 11

Sample grid to be processed

The surface is processed from near to far. The method presented is for viewing the surface from a corner area. Other viewing areas require an additional step.

STEP 1: The edge row closest to the observation point will always be visible. It is plotted and the projection of the edge row is used to define the opaque region.

Figure 12

Upper and lower horizons after first row has been processed (identical)

STEP 2: The grid lines not already processed in the first mesh element (the bottom grid line would have already been processed) of the next row are now processed. The lines are compared with the opaque region defined by the horizons. The portions of the projected grid lines visible are plotted, and the closed visible portions expand the definition of the horizons.

Figure 13

Upper and lower horizons after first grid element of second row has been processed

STEP 3: The mesh element adjacent to the element just processed by step 2 is processed by comparing, plotting, and expanding the opaque region as in step 2. This process is continued until each mesh element in the row has been processed. Steps 2 and 3 are continued for the remaining rows.

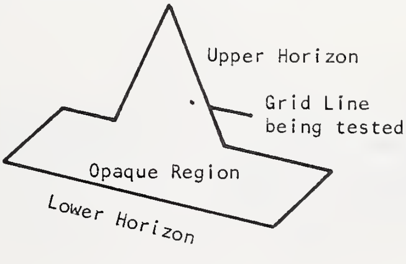

Figure 14

Grid Lines compared with horizons and produced one visible segment for this example

### c. Advantages

Processing time is nearly a linear functions of the number of mesh points. The method also produces the exact solution to the surface hidden line problem. Also, the whole surface need not reside in memory at any one time.

### d. Disadvantages

The algorithm is more difficult to implement and the actual program requires a larger computer.

## 4.4 General Observations for the Surface Algorithm Comparisons

a. To be economically feasible, solution times should not increase exponentially as the size of the problem increases. This implies that the time required to test the visibility of a point is independent of the size of the problem.

b. By using projected points for the hidden line removal, the last two methods were significantly faster.

c. Using any pre-knowledge is also helpful for a faster solution, i.e., the inherent ordering of a mesh surface can be used to advantage and further increase processing speeds.

## 5. Extensions into Solid Algorithms

Basically the same principles apply for the solid case that apply for the surface case. The three surface algorithms each have their counterparts in a solid algorithm. The solid case is generally harder since the implicit ordering of the gridded mesh is missing.

### a. Extension of the Aerospace Algorithm

The basic test of visibility is modified to test whether a test line pierces any of the other planes of the solid object. This can produce a tremendous number of tests and is definitely not feasible for data representations produced from large quantities of data.

### b. Extension of the Warnock Algorithm

The Warnock algorithm basically works equally well for both surfaces and solid representations. Any solid program will process gridded surfaces with minor modifications as a surface can be represented by a series of planes. However, since they do not take advantage of implicit ordering they are not as fast as specialized surface programs.

### c. Extension of the Horizon Algorithm

If the planes defining the solid object are ordered from near to far, then a series of small opaque regions are defined as the planes are processed. Methods are being developed which minimize the effect of having a large number of small opaque regions necessary for testing.

## 6. Conclusion

It is now possible to use computer graphics to produce perspective line drawings for a limited number of design applications which are cost competitive and produce drawings in a fraction of the time of conventional methods. The sphere of feasible applications is growing rapidly and it will now be up to the architects and designers to learn how to use this powerful new tool and to guide future developments.

## 7. References

[1] B. Kubert, J. Seabo, S. Giulieri, The Perspective Representation of Functions of Two Variables, JACM, Vol. 15, 1968, pp. 193-204.

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[3] A. Paradis, An Algorithm for the Efficient Removal of Hidden Lines from Projected Surfaces, Tech. Report 34, University of California, Berkeley, California, June 1969.

# The Use of Graphics in the Development of Computer Aided Environmental Design for Two Storey Houses

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The Architecture Research Unit (ARU) is working on a two year research project to develop the use of computers in the field of housing design and production. This research is sponsored jointly by the Scottish Special Housing Association and the Ministry of Public Building and Works. The ARU's task is to develop a convenient technique for generating a description of the fabric of a building, within a computer. This must convey the geometric information which is traditionally contained in architects' drawings, in such a way that it remains intelligible to the user and is also suited to the further attachments of topological relationships associated with a variety of design considerations. Current use of graphics by designers is being studied, to prepare for new and acceptable conventions which are suitable for computer graphics input and output. It is now possible to use the computer to design a house plan on a cathode ray tube display, and introduce modifications to shape, size and building elements. This information can be fed into a program to check for consequences on construction, thermal environment, daylighting and other design properties which may be stored in the computer's data structure. This paper considers the relevance of graphics in an existing context of house design and production, and shows how this relevance is maintained through the application of a computer aided design system. Computer equipment currently being used on this project include a DEC PDP7 and 340 display with light pen, linked to an Elliott 4130 with disc backing. Hard copy output is obtained from a Calcomp 563 incremental plotter. Application of this research will be directed at two storey house production by the Scottish Special Housing Association; and benefits may be expected in subsequent improved ability to meet evolving environmental design requirements, to make greater use of scarce professional services, and to facilitate costing and construction of houses.

Key Words: Computer graphics, design practice, design process, geometry, graphic conventions, housing, information structures, man machine interaction, problem description, production information, topology.

## 1. Introduction

Any benefit from the use of computers in assisting the solution of a problem is dependent on an appropriate and clear description of that problem. The problem description needs to be unambiguous and intelligible to the machine, whilst also remaining recognisable to the person who is using the machine. In problems concerning environmental design relating to buildings, satisfactory solutions are dependent on suitable means for describing buildings.

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[1] Research Architect

[2] Architect/programmer and mathematician/programmer, respectively.

Prior to the availability of interactive computer graphics, building description for input to computers required a lengthy process of identifying co-ordinate reference points relating to a building's geometry. This information had to be compiled into long lists of numbers, unfamiliar to the designer. The task of translating the building description into a form suited to computer input (1)[1] required the skills and dedication of a specialised designer/programmer. This difficulty is a principal cause of the slow and reluctant acceptance of computers by designers, in the building industry.

The present object is to discover whether the opportunities provided by computer graphics facilities are suited to closing the comprehension gap between designers and the machine; to see whether designers may benefit from using computers as a general design aid, and so be encouraged to accept its use in practice.

## 2. Design Functions

The process of designing buildings is sometimes described as a linear sequence of activities, from inception of a new design through to completion of building (table 1) (2) and could continue throughout the useful life of a building to the time of its demolition.

Table 1. Stages in Design Process (based on the RIBA Outline Plan of Work)

| Stage | Usual Terminology (and intended sequence) |
|---|---|
| A. Inception<br>B. Feasibility | Briefing |
| C. Outline Proposals<br>D. Scheme Design | Sketch Plans |
| E. Detail Design | Working Drawings |
| F. Production Information<br>G. Bills of Quantities<br>H. Tender Action | |
| J. Project Planning<br>K. Operations on Site<br>L. Completion<br>M. Feed-Back | Site Operations |

The linear sequence of these activities is readily questioned when considering the evidence of practice, and observing the return loops and the lateral deviations which actually occur. But the linear description is useful as a scale by which to refer to the particular levels of operation in any system, to produce relevant indications of the kind of information which will need to be processed, and the appropriate manner of presenting and conveying this information.

Using the scale A to M of table 1, and by reference to the work of others in the field of computer aided design, it becomes possible to define the scope of the ARU's work. Some of the work undertaken in Britain can be regarded as dealing primarily with production information after design decisions have been taken (3), producing bills of quantities, ordering schedules and references to standard construction details; operating from E to H. The other end of the scale is represented by work on analytical processes which lead to early design decisions, relating the results of computer

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[1] Figures in brackets refer to the bibliography at the end of this paper.

analysis to single line design representations on a c.r.t. (4); and operating from A to C.

The field of application offered by the Scottish Special Housing Association, with its commitment to build, places a bias on the ARU's work towards achievement of benefit at the production information end of the design process. However, having the precedent of work produced by others in this field and seeing difficulties in bridging the gap in operation between the design and production ends of the scale, the ARU decided that it should attempt to operate within this gap and work outwards towards both ends. Thus the ARU is currently operating from C to F, with the intention of allowing a designer to build up a problem description in various ways, to respond to property analysis by the computer relating to design decisions, and leading gradually towards specific and detailed production information.

### 2. 1. Graphics related to Design Functions

Existing precedent in design practice indicates a relationship between the levels of specificacy relating to stages in the design process and the form of graphics used to convey information (table 2). The relationship of graphics to stages in the design process will vary, in response to varying fields of application. Where the building type leads to repetition of relatively stable information, as in housing, the link between a and c will occur early in the design process. Where complex and non-repetitive building forms are involved, as in schools or hospitals, the progression from a to c is likely to be more gradual. This is illustrated in table 3, which relates the use of different forms of graphics to the applications fields of new housing, modification to standard housing and more complex buildings.

Table 2. Association of Graphics with stages in the Design Process

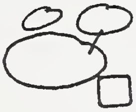
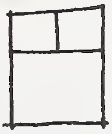
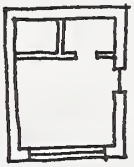
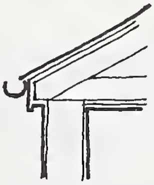
| Form of Graphics | | RIBA Stages |
|---|---|---|
| a. Schematic |  | A to B |
| b. Diagramatic single line |  | B to C |
| c. General Arrangement double line (locational reference for detail information) |  | C to H |
| d. Detail Representation complex graphics (assembly information) |  | E to H |
| e. Component Information |  | E to H |

Table 3. Relationship of Graphics to stages in the Design Process effective in different Design Fields

| Stage | Design Fields : New House Design | Std. House Modification | Schools etc. |
|---|---|---|---|
| Inception | a | c | a |
| Feasibility | b c | c | a b |
| Outline Proposals | c | c | b c |
| Scheme Design | c | c | c |
| Working Drawings | c d | c d | c d |
| Details | d e | d e | d e |

The alpha characters refer to the forms of graphics given in table 2.

The use of graphics represented by c under new and standard housing closely resembles practice at the SSHA and is used to refer to the more variable information being accessed and generated during design.

### 2. 2. Communication of Information

In building design practice a number of particular circumstances exist which influence the way in which information can be conveniently handled. The functions of storage and recall of information are affected by the large variety of people with diverse motives and ability, who are involved in building. The interdependence and interaction of a great variety of interests present during design requires that any system cannot depend on a linear sequence of functions and must be capable of entry at various points.

A designer's presentation of information normally consists of an assembly of previously known bits of information, which make up a proposal, or instructions, for a new building. The newness and relevance of a particular presentation exists in the relationship of one bit of information to another; its presence, location and physical fit (5). In detail considerations this may include shape; a new relationship of one surface to another which encloses a specified material. This amounts to the geometric or topological information of or between objects or activities.

The different bits of known information contained in the assemblage are identified by the use of conventions which are familiar to all the people involved. The convention enables each person to recall the particular information which is being referred to.

The general predominance of geometric or topological information, as the meaningful content in a designer's presentation of information, has formed the basis for extensive use of graphics. This is true of the past, and if people are to continue being involved in building design and be in control of their environment, then this dependence on graphics is likely to continue into the future.

### 2. 3. Computer Graphics

In order to devise new and acceptable conventions which are suitable as computer input and output, it is necessary to consider first the current use of graphics by designers and relate this to alternative vehicles for conveying information i. e. numeric or verbal descriptions.

Verbal or numeric representations are built up by stringing together many characters or numerals, either singly or in groups; and the association between characters or numerals is governed by the operation of laws i. e. grammar or mathematics discipline. Each character alone is meaningless, the combination of characters is made to be meaningful. This structure is absent from most conventional graphic modes of presenting information; and it is this difference which has led to the discrepancy between the use of machine aids for alphanumeric information and the lack of use of machine aids for graphic information.

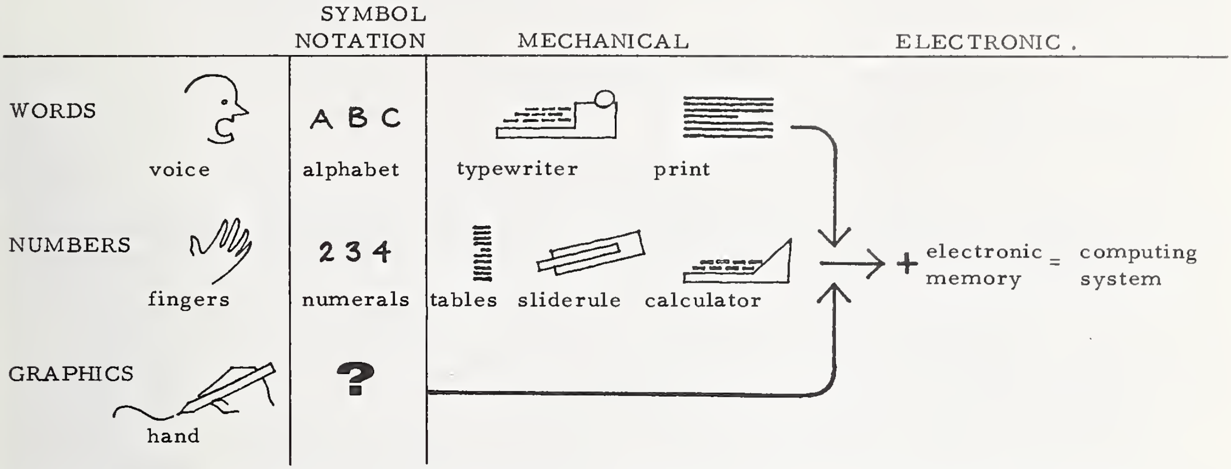


figure 1. The use of symbols or characters in combination to represent information, affecting development of appropriate machine aids.

Computers must receive information in bits, each with prespecified relevance, which can be compiled within a system to represent a whole assembly of meaningful data. In current design practice information is presented by drawing lines by hand; each bit of line on its own conveying little information to anybody other than the person doing the drawing. It is only as the drawing develops that its information content becomes more meaningful. The hand drawn information does not have to make sense until the drawing is complete. New graphic conventions need to be developed which consist of separate elements, or bits, of prespecified significance, which can be assembled to convey new and complex data. In this way a useful "grammar" for graphics should begin to grow.

## 3. Field of Application

A two year research project has been undertaken by the Architecture Research Unit (ARU) of the University of Edinburgh, which is being sponsored jointly by the Ministry of Public Building and Works and the Scottish Special Housing Association (SSHA). The initial two years of research is aimed at establishing the feasibility of applying computer graphics within an existing building design organisation, to serve as a useful aid to the production of new buildings.

A narrow field of application has deliberately been chosen, to maximise the opportunity for establishing principles of computer operation. If a satisfactory form of problem description can be achieved, which is applicable to a narrowly defined design environment, then the principles of operation which will have been developed should be capable of subsequent expansion to suit a wider range of more complex applications.

The field of application is provided by the SSHA. This is an organisation which builds approximately 5,000 houses per year and is one of the largest house producers in Scotland. Most of this housing consists of two-storey terraces, built of "No-fines" concrete, though the total output includes single and multi-storey houses and flats and includes brick construction.

The SSHA designs, manages and maintains the houses which it builds, usually on behalf of local borough or city authorities. It provides all the professional, constructional and managerial services associated with the entire life of its houses, within the one organisation. As such it already has an exceptionally large store of information which should become readily available to designers through the application of computers, to lead to informed decisions relating to new designs.

The great majority of house forms consist of simple rectangles with rectilinear internal subdivision, on two floors of equal and constant storey height. The roofs are usually pitched, with tile cladding. The range of materials and details used for construction are limited and there is little variation in the required environment within houses. These small and simple building forms appear to be ideally suited to standardization, both of design requirement and building product; but the amount of variation which actually occurs at a detail level of specificacy, relating to construction information, is extensive. The permutation of these detail variations within a whole house or between one house and another gives rise to lengthy manual search procedures to check that all consequences are accommodated in new construction information

### 3.1. Graphic Requirements

The function of graphics is to convey geometric or topological descriptions; to provide locations for bits of information; to identify the spaces which may contain material specifications.

Where the function of graphics is to describe a building to a computer, such problem description should not anticipate or predetermine a solution. The graphics alone should not automatically indicate a particular form of construction, but should allow free and gradual opportunity for subsequent decisions leading to a specific design solution.

In providing geometric information graphics will tend to indicate relative size. This has to be accommodated and controlled by the graphic conventions which are developed for the applications context; the implied size accuracy should not be finer than person's ability to read off the viewed image.

Given the context of SSHA houses, together with current national moves to co-ordinate all height dimensions occurring within housing, it is possible to interpolate much of the three-dimensional information required for building design and production from plans. In this context the need for three-dimensional or animated computer graphic projections receives a low priority and it is possible to concentrate effort on purely orthogonal projections.

A convenient form of building description input to computers could provide quick access to computer analysis routines, which check the design for compliance with design standards or regulations. Design alterations could be fed into a program to check for consequences on construction, thermal standards, daylighting and other environmental properties.

Suitable computer input should enable cost information to be accessible at all stages during design and this information could be continually updated by new information received from building operations. Such use of computers should further provide output in the form of printed bills of quantities, ordering schedules and intelligible working drawings.

### 3.2. Equipment

The project team at the ARU has access to computing facilities in other University departments. This consists of a DEC PDP7 and 340 interactive graphics display terminal with light pen, and a Calcomp 563 incremental plotter.

The graphics terminal is connected by high speed link to an Elliott 4130 central processor, with 64K word core and magnetic disc backing store.

In a design environment such as that of the SSHA, which does not yet practice the general application of computers, fully interactive graphics may initially prove too expensive. The ARU is therefore considering alternative cheaper and less sophisticated graphics facilities; and a parallel research programme has been started which aims to develop the application of an ARDS direct view storage tube, linked by delay line to an ICL System 4/75. The possibility of using a d.v.s.t. has been taken into account in writing the program for the fully interactive graphics facilities.

The ARU has its own on-line Teletype terminal linked by voice grade line to a remote time sharing bureau service, which is being used as a convenient form of computer access for interactive program development.

## 4. Development of a Computer Graphics Application

Research work by the ARU on the application of computer graphics techniques to the work of the SSHA is described in the following paragraphs and illustrations.

### 4.2. Information Structures

The general data handling capability of computers is usually dependent on a precise and predetermined logic structure, so that it will make sense of any data it receives. In design practice a similar methodical approach to handling information is sometimes attempted; but rules are often broken. Where information passes between understanding people the method may appear to survive, but where information passes to a computer any violated rules will cause a failure of the system. In applying computers to the work of the SSHA, it is necessary to reassess the use of familiar information structures, so that these may be modified to fit a computer's data structure; specifying those areas of design activity which can best be handled by user interaction with a computing system.

In order to prepare for the need to process SSHA information through a computing system, the ARU's approach to data structures has been to distinguish between different principal computer functions. These differences are used to distinguish between the requirements of different data structures. Each separate structure is developed to interrelate with the others but each is suited to its own particular function. So far work has been based on distinctions between a graphics data structure (GDS), an applications data structure (ADS), and a file handling system (LIBRARY).

The GDS notes the way in which points and lines come together on the screen, to represent meaningful information to the user. It stores the relationships between the points and lines, and the walls, windows, doors, rooms and surfaces which these represent; to which the user may want to attach other non-graphic information.

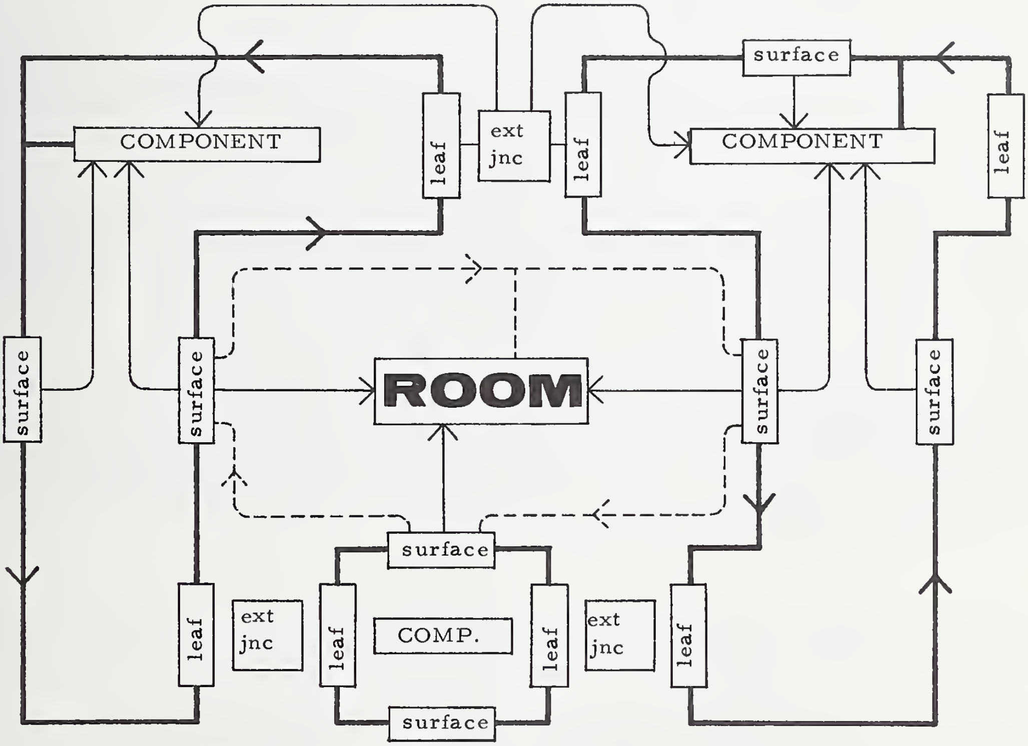


figure 2. Example of an Applications Data Structure referring to a Room

The ADS holds the computer's pool of information which is received from the user and is interpreted by reference to a permanent file of information stored on magnetic tape or disc. This pool of information, which grows as the user builds up a design, is structured in terms of accommodation zones (floors), spaces (rooms), components (walls, windows, doors), surfaces and junctions (fig. 2). The ADS has to note the relationships which exist between these items and has to relate incoming information from the user to a corresponding stored item or group of items.

The computer has constantly to compare information received from the user with that already stored in its LIBRARY, e.g. comparing component junctions with known working detail specifications. It also sends information taken from the LIBRARY and qualified by the ADS to the user, e.g. ranges of options for material specification displayed on the c.r.t. screen.

A request by the user to give or receive information is usually initiated by the user indicating a point on the display. The computer uses the GDS to identify which item, or group of items, in the ADS is being referred to. The computer then uses its immediate experience (the ADS) and its LIBRARY to interpret the request, and supply or store the information relevant to the request.

### 4.3. The Application

The representation of house plans on the c.r.t. is achieved by selecting graphic symbols which can be used to build up graphic elements depicting walls, doors or windows. These elements then serve as locating devices within the computer, for insertion of components of information.

The symbols are the basic graphic bits, rather like individual characters in an alphanumeric presentation, which are used to assemble the graphics. Individually each symbol carries very little information, other than an approximate indication of relative size and direction. A limited range of five symbols is found to be sufficient for representing the building fabric of houses (fig. 3). A simple square is used to represent external or party walls and main internal loadbearing walls. The same square bisected represents windows through such walls. The single bisecting line without the square represents doors in the same walls. A smaller square is used to represent partition walls, and a short straight line represents partition doors.

The first three symbols are used to fill 300 mm. square zones on a house plan and the last two symbols fill 100 mm. zones. This corresponds to the nationally adopted incremental system of 300 mm. and 100 mm. for house building, accompanying the change to metric measures and the introduction of dimensional co-ordination. These two dimensions are used in the computer application to provide the basic order by which more complex graphics may be assembled.

Graphic elements are built up from symbols on the c.r.t. Each element (fig. 5) carries information on the location, form, length and approximate width of a building element, e.g. wall. The design environment may further allow interpolation of overall height, and the subdivision into parts, e.g. window cill and head height. A number of elements can be assembled, changing the symbol for windows, doors and partitions, until a complete house plan is produced.

A component of information refers to the data which the user wishes to associate with the graphic element, which the computer receives into its ADS, and which may be filed in the LIBRARY. Such a component may refer to conceptual properties or performance characteristics of the design, e.g. the intended heat transference through a wall, or the required structural stability to withstand given loading. A component may refer directly to a material specification, or partial specification, for an element which constitutes a part of the building fabric. A graphic element does not necessarily have to carry a component of information, it can be empty.

The figures 3 to 9 generally illustrate the procedure for assembling the graphic representation of house plans on to the c.r.t. Plans may be modified, by deleting and rebuilding one or more elements (figs. 10 to 12); and plans can be stored by the computer on disc or paper tape for subsequent retrieval and further modification. Hard copy output is provided by the digital plotter.

The facility for materials specification is considered to be a necessary part of the procedures available to the user for describing a problem to a computer. Materials specification, as with graphics representation, is optional to the user, depending on the particular computer analysis which is to be performed on the problem description (fig. 17).

The user can build up a materials specification for a symbol or an element by selecting options which appear as computer controlled menus on the c.r.t. He is guided through the process of selection by messages which appear over the menus, which inform him of the stage of specification which has been reached. In the case of walls the specification is made in three stages i.e. primary material, external cladding and internal cladding (figs. 14 to 16). As each selection is made the computer ADS references the LIBRARY in order to generate an appropriate subsequent menu, for display and further selection.

If a symbol is selected for materials specification, the computer will generate an appropriate menu of general primary options, followed by appropriate general internal and external claddings. These tend to be short menus including only those materials which can be used whenever the symbol is used to build up elements throughout a plan. When the user indicates a particular element for materials specification the computer will generate appropriate menus containing specific options; and these menus tend to be longer, containing the wider range of materials suited to specific locations in a plan.

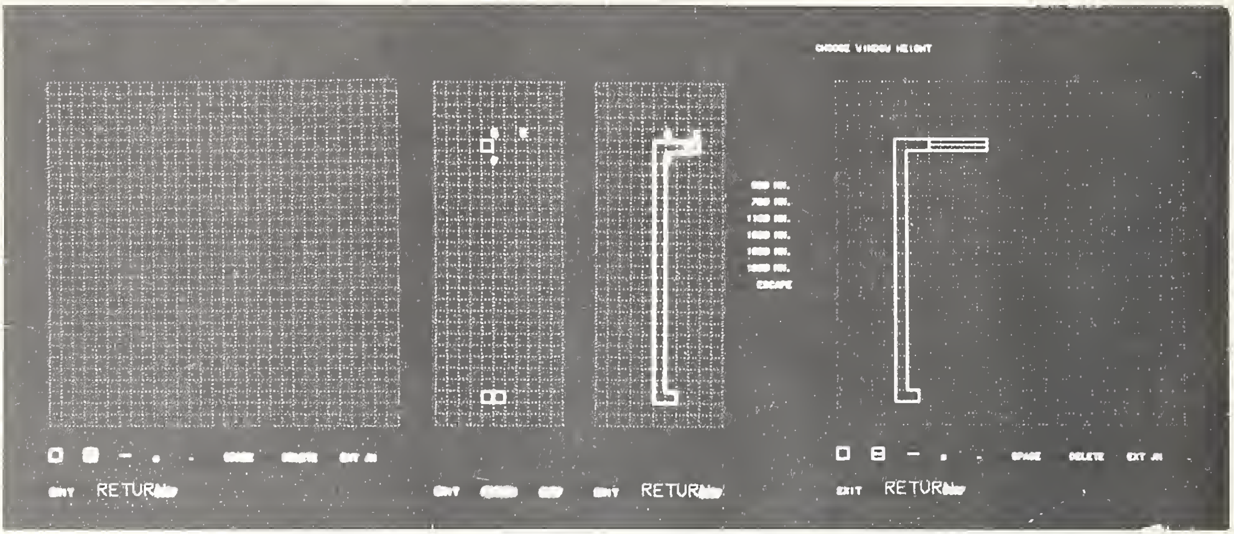
The specification for any symbol or element does not need to be complete; the user may select the SKIP option in any menu to call the next menur, or select EXIT if he wishes to terminate the specification (fig. 15). At any stage of the development of the problem description the user may subsequently return materials specification to modify or add to previous information.

The materials specification built up through user selection from menus displayed on the c.r.t. assigns materials to a graphic representation of a house plan which is viewed at a scale of 1 : 50. As soon as the specification refers to a number of adjacent elements the computer can assemble associated data leading to information on junctions between components. The computer can then identify construction details and recognise fit or misfit conditions. In the ARU application this information is cross-referenced with manually prepared standard construction details which back up the computer's store of information to allow the output of practical production information.

Operation of the system which allows the user to prepare and interact with the problem description is illustrated in figure 17 and the various stages, or modes of operation, are explained in the accompanying table 4. Lines linking the stages indicate a sample of possible routes through the system; the continuous line showing an entry through the general specification of materials to symbols, leading on to graphics; the dotted line showing an entry through graphics, leading directly on to some analytical function or passing through specific materials specification; and the dashed line showing an entry through modification of an existing problem description.

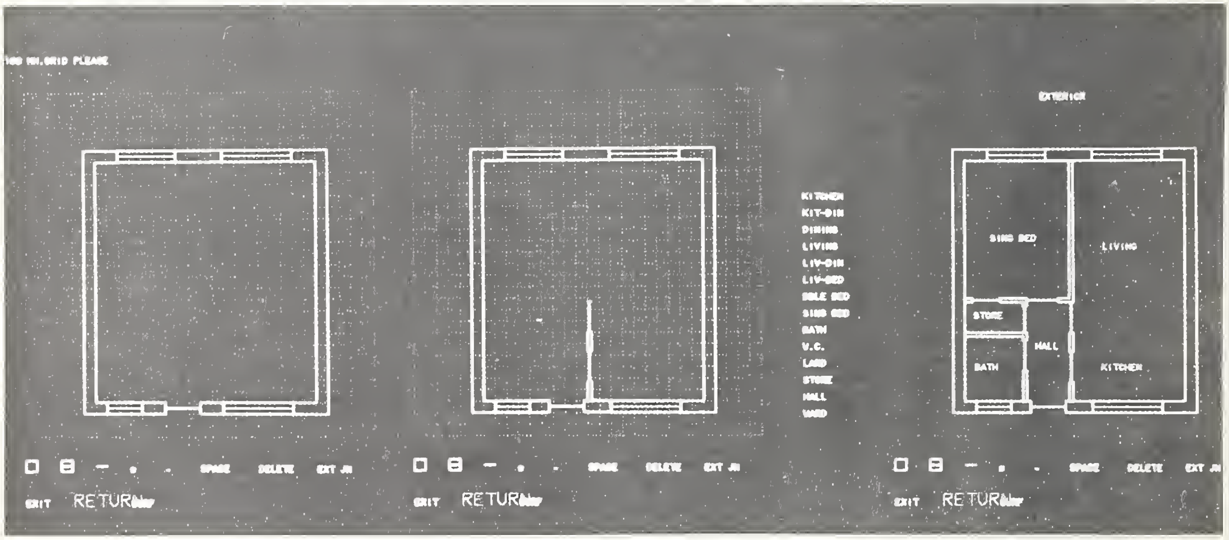
Figure 18 and the accompanying table 5 illustrate an example of one analysis function which can be performed. This example is concerned with heating, and the analysis is structured to allow the user to select alternative start points, and by varying the input data, to arrive at computed information on either the temperature levels which will be maintained or the heat input which is required. Where the desired result cannot be obtained by manipulating the variables ( D E G or J) in this function, the user can return to modify the main problem description.

The major part of the ARU's research effort has concentrated on the development of an operating system (fig. 17) which allows the designer to describe a building to a computer, introduce modifications and build up information at various levels of specificacy; to prepare for the operation of a wide range of computer analysis functions. This technique for problem description needs to be tested for a wider range of applications, involving different and more complex building forms and including the arrangement of grouped buildings.



figures 3 4 5 6

Initially the display shows a planning grid representing 300 mm. squares, with five graphic symbols and a number of light buttons displayed along the bottom. The user selects a symbol with a light pen and tracking cross, and this is used to locate the extremities and corner positions of a graphic element. If a graphic element representing a window is built up on the display, the user is given an opportunity to specify particular height information.

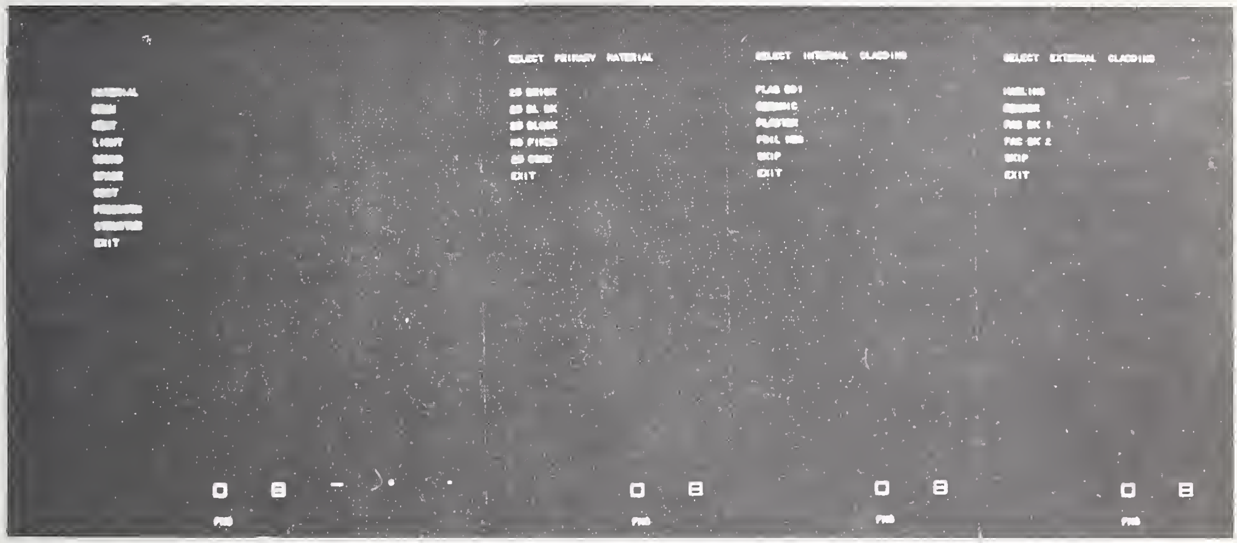


figures 7 8 9

The user proceeds to construct elements to represent a house plan, completing the perimeter boundary walls and proceeding with the internal subdivision. When the graphics is complete the user can call up the space function which causes a display of room labels and he can proceed to identify the spaces bounded by elements as rooms.

figures 10 11 12

The user can modify completed house plans, by deleting elements and selecting new symbols in order to build up new and different graphic elements.



figures 13 14 15 16

Material specification is carried out by selecting MATERIAL from a list of functions displayed on the screen. The user identifies a symbol, or an element, with the light pen, and a range of appropriate primary material options appear in the menu area, together with a message calling on the user to make a selection. The menus are paged under computer control, for internal and external claddings, until the specification is complete.

figure 17 CAAD OPERATING SYSTEM

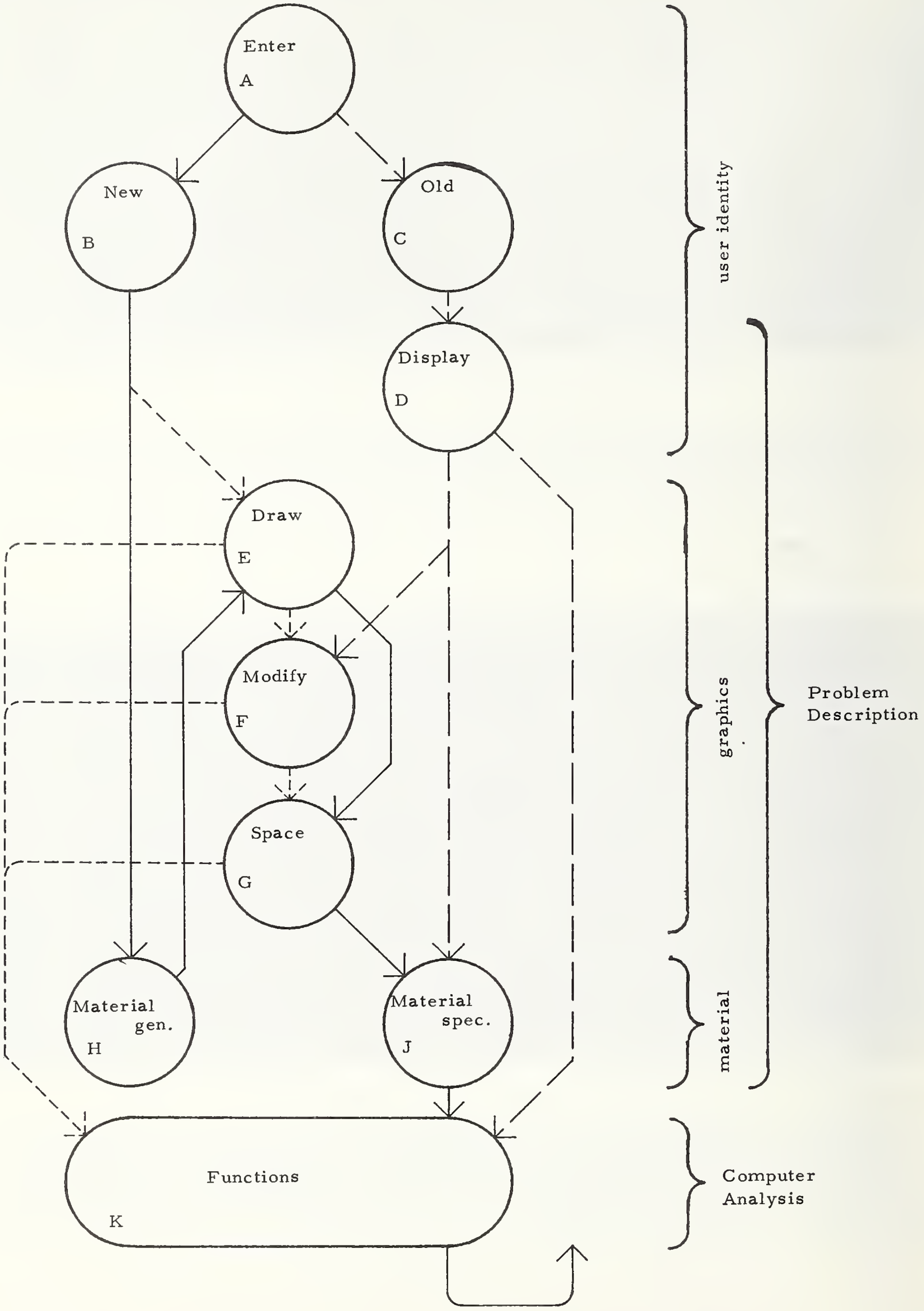
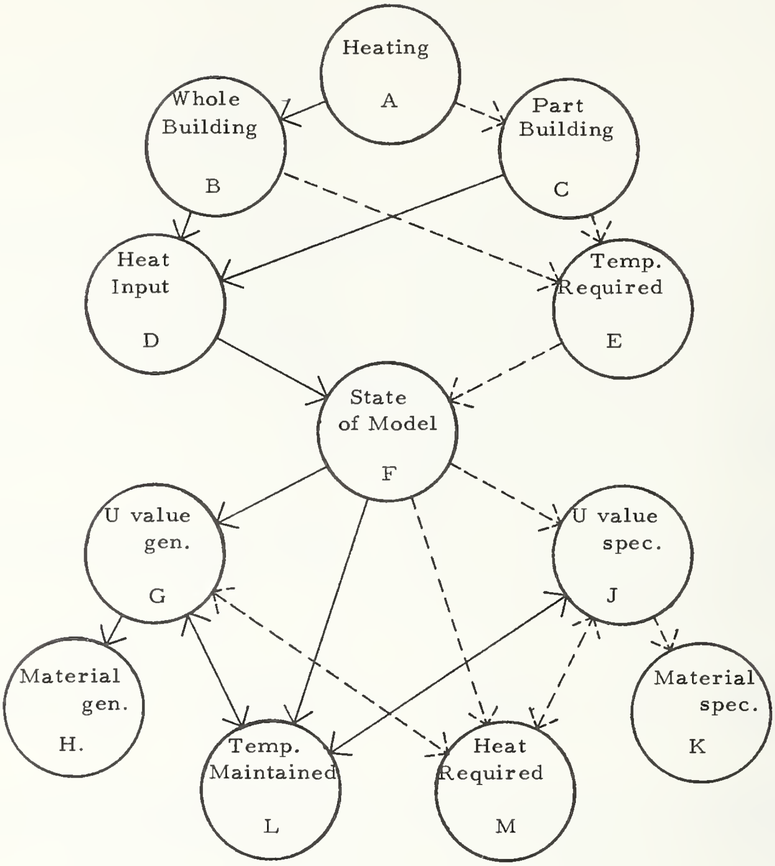


Table 4. MAN MACHINE INTERACTION - CAAD OPERATING SYSTEM

| USER PARTICIPATION | STAGE LABEL | COMPUTER FUNCTION |
|---|---|---|
| User enters job reference code | A. Enter | Computer uses user identity as control on further information which will become available, identifies existing project files and is ready to create new ones, calls for description of problem: new or old? |
| User indicates new problem | B. New | Computer is ready to create new information file; to receive new problem description for translation into computer model. |
| User indicates old problem | C. Old | Computer retrieves existing model from store, deposits this in core; ready for display, modification or analysis. |
| User selects problem description for display | D. Display | Problem description is displayed on c. r. t. |
| User defines geometry and topology of components of the problem description, by assembling graphic elements on the c. r. t. | E. Draw | Computer begins to assemble associative model by restructuring elements into components, surfaces and junctions. |
| User modifies existing graphics on the c. r. t. by deleting and adding new graphic elements | F. Modify | Computer modifies existing model and checks consequences on affected data already contained in the ADS. |
| User labels spaces which are described by graphic elements | G. Space | Computer defines the space by forming a ring of surfaces to the components which form the boundaries of the space. |
| User can assign material specifications to each or any of the symbols before elements are assembled on the c. r. t.; the specification is made by user selection from menus, which appear on the screen | H. Material general | As each symbol is indicated, the computer searches the library file in order to build up appropriate menus and store the selected specification for subsequent entry into components as graphic elements are input on the c. r. t.; this information can serve as a control on later decisions by the user as the graphic description proceeds. |
| User can assign material specifications to displayed graphic elements on the c. r. t. | J. Material specific | Computer enters data into the ADS and will overwrite data which may previously have been used to describe the affected components; this information can be made subject to controls, i. e. recognition of fit between components. |
| User can indicate particular analytical function which is to be performed by the computer on the problem description | K. Functions | Computer calls the appropriate analytical routines into core, to operate on the model contained in the ADS, and the first function is to check whether the model is complete for purposes of executing the required analysis; at this point, or at any stage of analysis, the computer can indicate the need or opportunity to return to any of the stages D to J. |

figure 18 Function : HEATING



The arrows indicate a few of many possible routes through the system.

Table 5. MAN MACHINE INTERACTION - FUNCTION : HEATING

USER PARTICIPATION

| | | |
|---|---|---|
| User selects HEATING from list of functions displayed on the c.r.t. | A. function: HEATING | Computer checks request against appropriate completion of building model (problem description) already in the ADS, informs user if not complete and awaits input of required further data. |
| User indicates heating evaluation relating to all the space within external wall boundary. | B. Whole Building | Computer extracts information from ADS on state of external walls. |
| User indicates heating evaluation relating to a specific space | C. Part building | Computer extracts information from ADS on the state of the boundaries to the specific space. |
| User specifies amount of heat to be supplied to the space | D. Heat input | |

| | | |
|---|---|---|
| User specifies temperature range to be maintained in the space (against a given external environment) | E. Temperature required | |
| User calls for information on previous decisions relating to the problem | F. State of model | Computer tells user whether materials have been specified for the boundary elements and whether there are restraints on exercising further options. |
| User can assign U values to each or any of the symbols | G. U value general | Computer enters data into the ADS at the appropriate locations indicated by the graphic elements on the c. r. t. |
| User can assign material specifications to each or any of the symbols | H. Material general | Computer searches the library file for specifications which provide the required U value appropriate to each symbol indicated by the user; in order to build up appropriate menus, and relate the selected specification to the corresponding elements already existing in the problem description; newly selected material specifications will replace previous specifications. |
| User can assign U values to each or any of the elements | J. U value specific | Computer enters data into the ADS; and overwrites previously specified corresponding data for the same locations with this new data. |
| User can assign material specifications to each or any of the elements | K. Material specific | Computer searches the library file in order to build up menus which provide the required U value (as for H above). |
| User calls for information on the heating levels which will be maintained, in response to previously entered data; the user may return to modify data until satisfactory heating levels are achieved. | L. Temperature maintained | Computer checks whether input data is complete and then proceeds to perform heating analysis taking account of:<br>a) exterior temperatures<br>b) surface area of space<br>c) U values of boundaries<br>d) amount of heat input<br>to arrive at figures which describe the heating levels which will be maintained. |
| User calls for information on the amount of heat input which is required, in response to previously entered data; the user may return to modify data until a satisfactory figure for the amount of heat input is obtained. | M. Heat required | Computer checks whether input data is complete and then proceeds to perform heating analysis taking account of:<br>a) exterior temperatures<br>b) surface area of space<br>c) U values of boundaries<br>d) range of temperature to be maintained<br>to arrive at figures which describe the amount of heat input which is required. |

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# Anticipatory Techniques for Enhancing Remote Computer Graphics

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Techniques for enhancing the performance of graphical display terminals located remotely from a central computer system and connected by limited communication lines are discussed, with emphasis on the system requirements of environmental engineering applications. A set of mechanisms that anticipate a user's needs is presented, including related techniques that have been used to support computer graphics terminals and new ideas for optimizing display operation.

Factors considered in this study of anticipatory techniques include the effect of communication line loading and central system response to requests from a local display-driving computer. Also of interest are various ways for deciding what to anticipate and for considering multiplexed communication lines. Extension into a computer networking environment is also discussed. A few potential application areas for anticipation, including some in environmental engineering, are described to illustrate the possible use of these techniques.

Key Words: Computer-aided design, computer graphics, interactive graphics, remote graphics terminals.

## 1. An Engineering Problem

There has been much discussion the past few years concerning the use of graphical displays attached to supporting computer systems. The load placed on such systems by highly interactive display usage has demanded a large percentage of central system resources and has led to the use of local logic and in some cases small computers associated with displays to relieve this burden from the central system [1], [2], [3].[2]

It is desirable to have access to large computer systems which have large, high-speed main memory, powerful instruction repertoires, and large backup file systems. The nature of the interactive activity associated with display usage is such that these powerful resources are required only for short periods at relatively infrequent intervals. It is, therefore, economically advantageous to attach several display terminals to a large computer system and to control their operation with a time-sharing executive.

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[1] Chief, Computer Systems Section.

[2] Figures in brackets indicate the literature references at the end of this paper.

The problems involved in supporting a number of graphical displays are greater than for supporting slower teletypewriter devices, and early systems have served only a few displays, along with a much larger number of teletypewriters [4], [5]. The use of intermediary computers to assist each display terminal or a group of terminals promises to increase the number of displays that can be serviced by one large computer.

Problems that arise when using a local display computer to drive either one or many displays are accentuated when the local computer is located at a considerable distance from the central system. When located near the central system it is assumed that a very high bandwidth communication line can be established between the local computer and the central system. At longer distances this may not be possible. Even when it is possible, the cost of doing so may be unbearably high.

It is desirable, then, to utilize a restricted bandwidth communication line to interconnect the local and central systems. For instance, it would be convenient if successful operation could be obtained utilizing a single voice-grade line with a capacity of 2400 bits per second.

The limitations imposed on system operation with such a restricted line are immediately obvious, since the nature of the data transmitted to and from the display is such that large amounts of data are involved in the transfer of a complete picture. A picture with 1000 elements, each requiring 20 to 50 bits per element, requires 20, 000 to 50, 000 bits. With a single phone line, 8 to 20 seconds would be required for transmission of a complete picture. If pictures are frequently required, this time interval is too great for satisfactory man-machine interaction at the graphical terminal.

The question of what is done locally versus what is done at the central system acquires a new meaning with a restricted communication line. The possibility of transmitting parts of pictures and assembling them locally looks more appealing. Means for compressing data on the communication line may also be useful, and any additional techniques that can be developed to enhance user response time or system performance for graphical applications are of interest. The justification for a local display-driving computer, rather than a buffer plus some simple editing logic, is even greater than for displays adjacent to the central system.

It appears desirable to maintain an image of the general data structure stored in the central system with a smaller, simpler one in the local system. Changes initiated by the display user can thus be used to update the local image and immediately change the displayed picture as well as to update the complete data structure in the central system at the same time. Minor changes made by a display applications program in the central system to the complete data structure may be transmitted incrementally to change the appropriate portion of the simplified local display structure. The changes are immediately incorporated in the displayed image. Only when major changes are made to the central data structure is it necessary to recreate from scratch the complete local structure. It is only at such times that long delays will be experienced when using a restricted communication line.

One simple means for imaging a complex data structure is useful when an over-all picture is composed of sub-pictures and perhaps multiple levels of sub-pictures. The basic sub-picture elements can be stored locally and the composition of these sub-pictures into display images can be directed from the central system. Sub-pictures, such as schematic representations of coils, fans, and diffusers, may be identified by a short code and given a position for each "instance" in the displayed picture. It is unnecessary to transmit the detailed display generation information for each sub-picture every time that sub-picture is used.

Despite use of such techniques, there will still be times when entirely new pictures are to be transmitted or when such a sub-picture strategy is not useful. At times, a lengthy delay in responding to a user's request may be inevitable. At other times, however, the technique described in this paper, anticipating a user's needs, may be employed to minimize this delay.

## 2. The Anticipation Concept

Although several systems utilize anticipatory methods in one way or another, to the author's knowledge there has not been a general exposition of these methods as a whole. Various anticipatory techniques can be unified and applied in general to remote computer graphics. In some cases, use of anticipation can lead to substantially improved terminal operation.

The concept is essentially the prediction of a remote terminal user's needs and the preloading of programs and/or data that he may soon require. If, at any given point in a user's interaction with a system, the number of alternatives for major display changes are minimal and the probability of choosing one or a few of these alternatives is high, then it is possible to anticipate his needs. While the user is performing local interaction, or when the terminal is idle while the user is thinking, the local computer can request one of the high probability pictures or programs. The request can be transmitted and the requested pictures returned via the communication line, which is normally idle during this interval between direct central system requests on the part of the user.

One example of anticipation has already been given. The prestoring of sub-pictures locally in preparation for their assembly into complete pictures is the anticipation of the use of these parts of larger pictures. Transmitting them to the local processor before they are needed makes the transmission of full pictures shorter, and can decrease the over-all system response time to a user's request which requires such transmission.

Another example of anticipation is the pre-loading of an entire set of programs and data for a local computer from a central system in preparation for a particular application or class of applications. It has been suggested that for each application the larger computer could assemble a package of programs for the local computer that will enable it to operate as independently as possible and to require minimal service from the central system.

In both of these examples the success of the anticipation is dependent on the high probability of usage of the pre-loaded programs and data. It is assumed that if a user requests an air distribution system design program, for instance, then he will make use of this program, and therefore use the prestored component sub-pictures, before calling another program having a different set of components. It is likewise assumed that once a user has called for an application program, or has designated an application class, that he will then be working in this application area for a reasonable period before switching to another one. He may, however, have made a mistake; or he may change his mind. So the proability of using the pre-loaded programs and data is not unity.

In general, anticipation is successful when the estimated probability of using data or a program is sufficiently high. For any given collection of data and programs stored in the central computer system, there is a probability of usage in the near future associated with each item in the collection. There must be adequate local storage for that activity requiring immediate attention by the local computer. To take advantage of anticipation, there must be some additional storage for data and programs of slightly lower usage probability. If this storage is large enough to hold the entire collection, then the need for central system storage is eliminated. This extreme represents a substantial local investment and will not usually be practical. It is for a local storage size larger than a few items, but less than adequate for the entire collection that it is useful to anticipate.

When the probability of need for an item is unity, it shall be considered essential for immediate display terminal operation. If the item is not located locally, it must be requested from the central computer system, and the user must wait for transmission of the request to the central system and for receipt of the requested item. The usage probabilities of all presently loaded items, data and programs, the probabilities of needing items still located in the central system, and the amount of unused local storage must all be considered in determining possible anticipatory requests by the local computer to the central system.

A request should not be made unless there is an item not located locally that has a high near-future usage probability. The communication line must not be needed for direct activity in support of immediate display operation. Storage must be available for the item locally. In some cases, items may reside in local storage that are not needed immediately and which have a near-future usage probability lower than these that can be requested. This space may be considered reclaimable for high probability items as they are received.

Depending on the response time of the central computer to anticipatory requests, it may be desirable to have several requests pending simultaneously. If a probability indicator is attached to such requests, the central system might give them appropriate priority. Since assigned probabilities are relative, it is possible for the probability of a requested item to change after the request has been sent to the central system because of continuing display-user interaction. Depending on the system, it may be desirable, if a significant change occurs, to send an addendum request or even an entirely new request to change the priority given to the previous request. To do this, the local computer should keep a list of pending requests. One important instance of such a change is if the probability of an item changes from moderately high to unity, i.e., it is immediately needed. If it has already been requested, and if the central system does not vary service based on priority, then the local computer just waits for the reply. If priority is adjustable, then it might ask the central system to increase the priority of the prior request or might submit a new, high priority request.

The notion of priority in submitting and servicing anticipatory requests can be extended to take into account more than just the expected probability of usage in the near future. It can also include some measure of estimated size of items being requested, thereby considering transmission and service delay in obtaining the item and local storage that will be consumed by it after it is received. These delays may be a function of current central system and communication line loading, which might be measured by the local computer dynamically by noting the response times from the central system. These times may vary according to type as well as length of requested items. All of these considerations may be included in the priority-determining algorithms used for submitting anticipatory requests as well as by the central system in servicing requests.

If the prime objective of system design is to optimize the display/user interface, taking into account the limited communication line, but not caring about the burden placed on it or on the central system, then the primary concern in anticipating is to make sure the various system resources are available for the highest probability requests when they occur, even if this means abandoning lower priority requests in progress. Under some conditions, the added burden on the central system of servicing anticipatory requests may be enough to limit the rate of input of such requests and may be used to limit the generation of these requests by the local computer.

With respect to the communication line from the remote terminal to the central system, two kinds of configurations may be considered: a dedicated line and a shared line.

### 3. Anticipation with a Dedicated Communication Line

If a single communication line connects the display terminal with the central system, then conflicts of line usage may be resolved in favor of optimum user service. This selfish operation on the part of the local computer may have to be tempered when central system loading requirements are taken into account.

The nature of typical interaction between terminal and central system is such that the communication line is used normally only in bursts and is idle during relatively long intervals between bursts. Here is a major system resource going unused--a situation which can be used to advantage in some cases by anticipatory techniques.

The resultant higher average usage of the communication line must not disturb the unity probability item requests. Items needed before man/terminal interaction can continue must be given highest priority. One way of accomplishing this is to give the local computer control over the communication line in such a way that it can interrupt transmission in either direction. This would ensure that high priority messages are transmitted immediately. Another mechanism is to assign priorities to requests and to ensure that all transmissions on the communication line are short. Short transmissions can be achieved by selecting short message formats or by segmenting longer messages. In either case, a highest priority message would be guaranteed of having the line within the maximum transmission time of a short message or message segment. Highest priority transmissions in both directions need not be as short as lower probability transmissions.

For the dedicated line the average usage should be higher than without anticipation, and there will be some increased load on the central system to service the additional anticipatory requests.

### 4. Anticipation with a Shared Communication Line

Several graphical display users may share a communication line, either by multiplexing through a common local display-driving computer or through a communications concentrator, even though each has his own local computer. With such shared activity on the line the average usage will likely be higher even without anticipation, so there may be less unused capacity for anticipatory use.

The same techniques for message priorities, interruptable low-priority messages or short message segments, will allow highest priority requests to take over the line. With the shared line, however, the effect of the low bandwidth line can be more evident if high priority requests are made by two display terminals simultaneously. While in the worst case for one terminal a delay of 5 to 10 seconds might occur, this would double to 10 or 20 seconds with just two terminals. Of course, the probability of exactly simultaneous requests is low, and the probability of worst case occurrence is usually low with shared line usage.

Sharing of a communications line is a means for increasing the average utilization of an expensive resource. Since effective anticipation thrives on unused resources, it does not do as well as the load on the line becomes heavier. The same effect is evident as low priority anticipatory requests compete for central system service. It is necessary to give higher priority requests from all communication lines better service; otherwise, the system could be saturated servicing a large number of requests that really should be given only unused central system resources.

## 5. Potential Applications

A few application areas appear very likely for the use of anticipation techniques. It is not expected that all work in these particular application areas can benefit from these techniques, but anticipation is a tool to be used as appropriate.

Suppose a user is scanning a large drawing, the entire detail of which is stored in the central system. He observes the drawing, and may modify it, by looking through a "window" at some part of the complete picture. In general, the greater the magnification, the more detail shown in the window and the less area of the full picture that can be observed at one time.

Also assume that the local storage is adequate for what is currently being shown in the window and for the necessary programs to manipulate it and to communicate with the central system. In addition, suppose that some additional local storage is available, but not enough to store the entire picture with full detail.

The user is scanning the over-all picture. This process consists of a combination of scanning horizontally and vertically at a given magnification as well as switching to other magnifications as needed. Suppose that a user has been observing a particular area for some time and that no prior history is available to predict what he might do next. Equal probability may be given all sides of the window--unless it is at or near an edge of the over-all picture. It is possible to assemble those picture elements just outside the window in a band as shown in figure 1. The elements in this predictor band are at the same magnification as the current window. Thus, if the user starts moving in one direction, say to the right, the anticipation program can immediately handle movement the width of the predictor band without communication with the central system.

Once such movement has been initiated, however, the local computer must request additional picture elements to fill out a new band surrounding the current window. It may be desirable, depending on user scan speed, available storage, and nearness to the over-all picture's edge, to bias the anticipation band in the direction of movement, as shown in figure 2. When scan movement slows or halts, estimated probabilities for movement in each direction based on the last and possibly earlier movements can be used to determine any desired predictor band bias.

The operation of combining part of the predictor band with the present window during the scanning process may be difficult. The effect of scissoring at the window edge, of viewing parts of individual display elements, must be maintained as the window moves over the picture. A similar problem exists on the opposite window boundary, where display elements or parts of elements must be removed from the window.

This anticipatory scan at one magnification level may improve system response to the user. It will not be adequate, however, if frequent magnification changes are requested. If the probability of changing magnification is high, and if particular zoom levels are more probable than others, all or part of windows at these levels may be anticipated and stored locally. If both single and multi-level magnification scanning is possible and likely, then the anticipation routines must take both into account--perhaps doing so dynamically depending on the user's operation. For the first few minutes of a session the anticipatory routines can either use prior data for this user or use some universal initial parameters. The anticipator can, thus, be made to adapt to particular conditions. It can directly sense how well it is doing, since it can measure response times apparent to the user and can adjust anticipation parameters to optimize some measure of response time.

Another potential application of anticipation is for scanning text. Such text might be descriptive notes associated with working drawings within an interactive graphics application or it could stand alone as separate documentation. Scrolling up or down continuous text is an operation similar to, but simpler, than the full graphics scanning described above. Figure 3 shows a predictor band above and below a window of viewed text. Each band might include several lines of text and variable predictor band bias can be applied as above once scrolling has begun.

The boundary problem is much simpler, since lines of text can appear and disappear as entities without disturbing the window presentation. Structural text, that is, text having a hierarchial structure with respect to detail, can be anticipated in a manner similar to that used for multiple magnification levels, as described above.

Figure 4 shows the possible use of multiple predictor bands for text in various stages of preparation for viewing. Those lines in band A might be ready for immediate viewing, while those in band B are being retrieved from the central system.

Other potential application areas for anticipation include information retrieval from a highly structured data base and browsing through a collection of documents. If the retrieval process is gradual, such as working down a tree while narrowing in on an area of interest, then nodes may be reached at which one or more branches have a very high probability of selection compared to the others at that node. Especially in the case when selection of a likely branch requires substantial transmission to the display terminal, anticipation could lead to considerably improved average response times.

These few potential applications of anticipatory techniques will hopefully suggest many others for which anticipation can be a valuable tool for improving system response. While looking forward to the day in which high bandwidth communications lines will be widely available, this paper proposes some ideas as to how to live with reality for some time to come. Even as such communications capability is realized, the cost of a high capacity line will still be higher than a low one, and system design trade offs will take into account the difference at any point in time.

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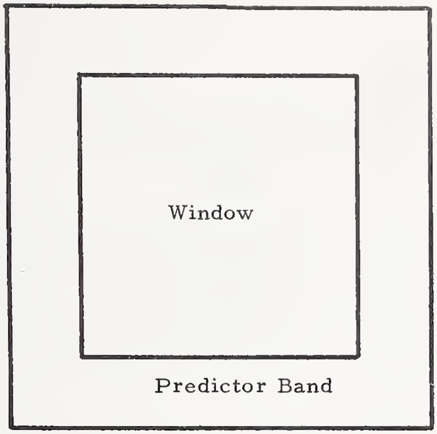

Figure 1. Uniform Predictor Band

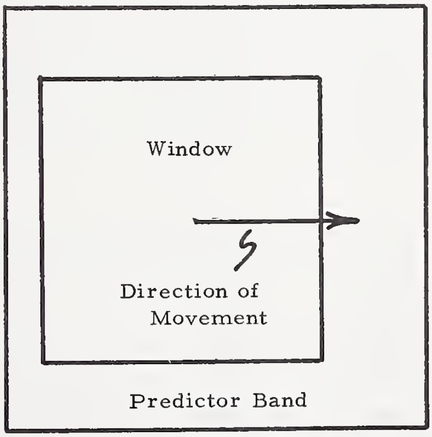

Figure 2. Biased Predictor Band

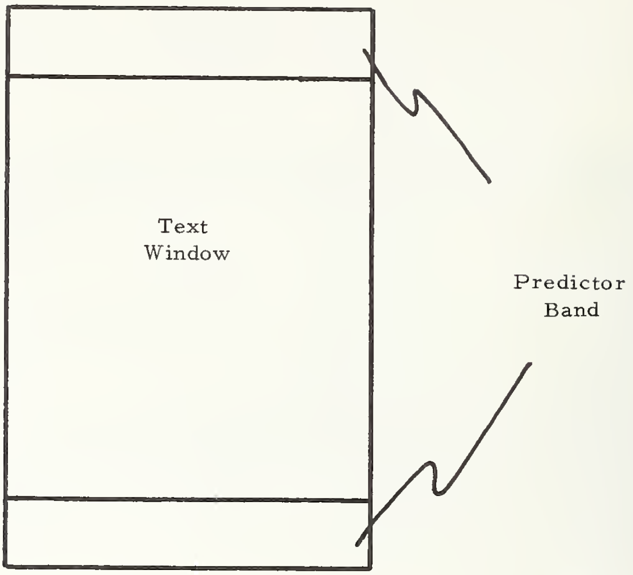

Figure 3. Text Anticipation

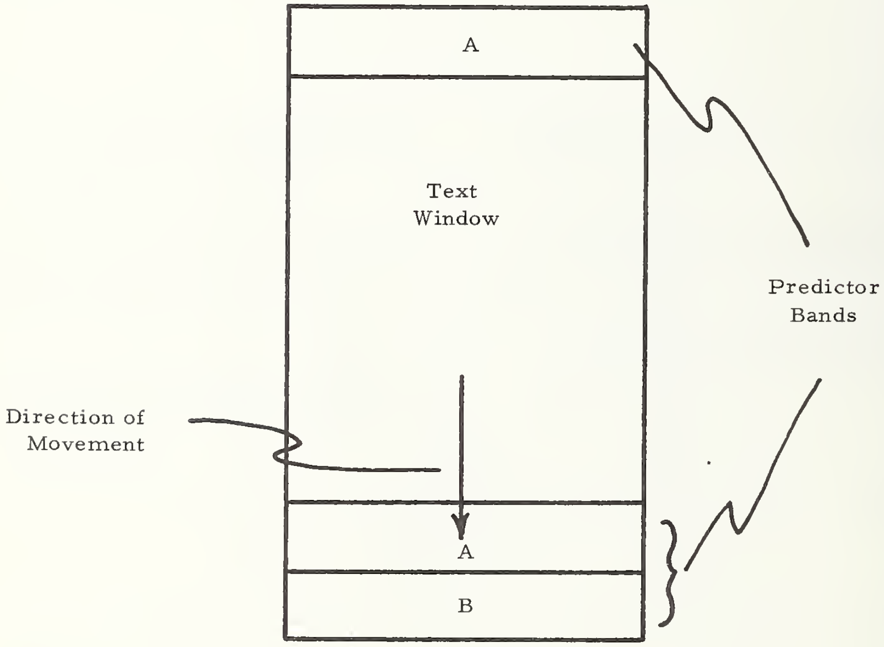

Figure 4. Multi-Level Bands

# Computer Graphic Data Structures for Building Design

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The data structures employed in computer graphics are studied with the objective of discovering the common aspects of structures now in use. A general graphic data structure is developed for its educational value and employed to represent simple display items. The use of list processing languages is discussed; an example of a special-purpose structure is given

Key Words: Data structure, computer graphics, list processing, pointers, hash coding, subpicture, associative memory, multi-level storage.

## 1. Introduction

"Computer graphics" is the general term applied to the use of a digital computer to form an internal model representation of an externally perceived graphical entity. The objective of such modeling is to extract information from the graphical entity so that it may be modified, manipulated, or otherwise processed.

After graphical information has been digitized, an organization must be provided for the storage and retrieval of this data within the memory of the computer system. The linear array is the simplest scheme available, but it is not of interest within the scope of this report even though it enjoys wide use in certain classes of applications. Rather, this report will be directed to a discussion of those organizations which represent the relationships among the components of the graphical entity. The relationships which must be represented within this subset of computer graphics include topology and dependency relationships. It is most convenient to represent such information in a hierarchical data structure which, in some abstract way, models the external graphical entity.

Historically, this subset has been restricted to the study of line drawings, but gray-scale and color representations are currently under investigation. The purpose of the data structure is to facilitate the extraction of intelligence and manipulation of both the image and the information it represents.

A graphical image is a preferred medium of human communication because of the possibilities inherent for maximum information transfer with minimum effort. Graphical communication is often highly stylized, requiring significant training for both the generation and interpretation of images. Conventions are established and propagated through architectural education, which enable precise communication with a minimum effort and non-graphical information.

Since graphical communication among humans is such an easy and effective technique, effort has been expended to extend this communication to digital systems. As with other languages which are close to human language and far from machine language, considerable resources must be allocated to store and manipulate the graphical communication within the digital system.

## 2. Overview of Graphical Data Structures

Both medium and conventions present difficulties associated with the digital representation of graphical information. The significant attribute of the medium problem is dimensionality. Graphics deals with two dimensions, often representing three dimensions. High speed primary memory is usually addressed in a piece-wise linear fashion, therefore a transformation is required to map the graphical structure into a one-dimensional frame. The information describing a graphical entity must be stored in such a way that it can be retrieved, manipulated, edited, and used to produce the desired graphical image.

The pertinent information associated with a graphical entity frequently consists of the geometrical description of the graphical item: scaling, position, and orientation data; relationships and connections to other items; name and identification of the item; graphical constraints on the item itself and on its relationship to other items; and non-display (textual) data intimately connected with the graphical entity.

One of the first decisions to be made in designing a graphical data structure is whether it should be completely general, or tailored to a specific application. The general fixed structure format is usually inefficient of storage since it must provide for unused options[1] [1]. Furthermore, no matter how general the fixed structure was designed to be, there always exists the pathological case which exceeds the capability of the structure. In such a case, one has no choice but to redesign the structure, hopefully maintaining upward compatibility.

The tailored structure meets all of these objections, but necessitate the effort of construction. In fact, the existence of general-purpose structures is extremely helpful to the user interested in tailoring a structure to his application. The intellectual effort implicit in the general structure may simply be transferred to the tailored structure, simultaneously modifying the structure to meet the current objectives. Since most graphical data structures are pointer-type structures, with such pointers being explicit or associatively addressed, the presence of a language designed to work with such pointers greatly facilitates the construction of a data structure.

While it is certainly possible to build the entire graphical data structure up from scratch, the use of a list processing or associative processing language greatly simplifies the work. In fact, most of the literature ostensibly devoted to computer-aided design is in fact concerned with data structure. Particular attention is called to references [1], [2], [3], [9] and [11] where the salient features of new and more established "service" languages are discussed.

The next problem to be considered involves the communication of data to and from the graphical data structure. This data will often be involved in the process of drawing a picture, in modifying an existing picture, or in re-drawing a picture from the data structure. The use of computer graphics in facilitating the convenient use of computers requires that this information transferal process not be a burden on the human user [25]. Delay that is annoying to the user should be avoided; a measure of tolerable delay is the user's concept of the difficulty of the task. While the design of graphical processors is not within the scope of this survey, one cannot ignore the hardware requirements forced by the desire for rapid response.

The mode of operation is for the user to communicate his desires to the digital system, often using graphical input devices such as light pen,joystick, and mouse; and for the system to respond by displaying the desired picture on his CRT display. Simultaneously the data structure needs to be updated to reflect the changes resultant from the CRT activity. Rapid response criteria require that at least part of the data structure must be instantaneously accessible to the user at the terminal [5], [6]. The size of the local processor and the capacity of the communication line with the main system determine the extent of the local image of the complete data structure.

A final problem is concerned with storage capacity in both the display driving computer and in the central system. The portion of the data structure represented in the local memory is usually less than that stored in the central system and may reasonably be restricted to that portion of the structure being displayed. If additional storage is available, this may be augmented by logically adjacent substructures possibly selected by an anticipator mode algorithm.

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[1] Figures in brackets indicates the literature references at the end of this paper.

Storage restrictions in the central system require greater attention and careful study. At the time that a structure is created or modified, it is necessary that the storage used not be the limitation upon the process. Thus, high speed core storage is required. Considering the possible extent of graphical data structures and the core use limitations imposed by the operating system environment, it is quite reasonable to expect and provide for the possibility of insufficient core storage being available. The solution exists in the form of a paging scheme, but careful attention and re-examination must be paid to the design and handling of the paging [1[, [2], [3], [12].

## 3. General Graphic Data Structure

The concepts and techniques of graphical data structures will be introduced here in the form of an example structure. This structure will purposely be kept at a level comprehendable to human users and modifiable by them. It is not intended that this presentation be exhaustive, but rather typical and hopefully educational. The organization of the data structure presented here is an explicit referencing structure, similar to that of Cotton and Greatorex [5], the GRAPHIC-2 system [4], [7] and GRASP [8]. The structure is certainly not exclusive; additional examples are given the surveys by Gray [9] and Dodd [21]. The structure will be presented in the form of a directed graph; the mechanism of representing such a structure in a computer memory will be discussed subsequently.

In an effort to minimize the amount of graphics terminal machine language programming required, especially by users that are not interested in such a level of detail, the commonly used picture-building elements are provided as building blocks called "basic subpictures". A basic subpicture may be a single command to draw (display) a point, line, or conic section; it may also be a sequence of such commands to draw a commonly used geometric entity. Such basic subpictures are often in the form of the "frame" or "skeleton" of an open subroutine, because the essential positioning information must be supplied by each reference to (use of) the basic subpicture. The basic subpicture data block must also contain the identification and relative location of the externally accessible terminals of the subpicture, such terminals being the parts of points of connection to the subpicture by the greater, outside world. If there are to be constraints on the use of the basic subpicture, these constraints must also be contained within a block pointed to by the basic subpicture data block.

Within the context of basic subpictures lie all of the characters and geometric figures directly presentable with a single machine level command, these constitute the "hardware character set". Commonly used graphical entities may also be coded in graphical control language as a short program, and provided as a time-saving service. Such basic subpictures provided might include walls, pipes, conduits, doors, windows, etc. In addition it is desirable for a user to be able to define his own subpictures which are useful for his application. The definition of a subpicture is usually not within the scope of the graphical language-data structure being herein described, but is at the level of graphical control language.

Thus, a picture is the highest level, encompassing all lower constituent levels. These lower levels being collectively referred to as subpictures. The lowest level is termed a basic subpicture in that it is the only level which references the display.

Using graph terminology, each basic subpicture is a node in the directed graph which is the graphical data structure. Although it is quite possible for the node block to be of variable size, herein pointers will be used to reference variable sized data segment blocks. Under these conventions, a basic subpicture node can appear as in figure 1.

Since a basic subpicture does not possess any absolute frame of reference, it cannot in and by itself cause any display. It must be referenced by a higher node to be used as a display item. These higher nodes will in general reference multiple lower nodes, thus building a picture out of previously created components. The terms "higher" and "lower" relate only to position in a directed graph drawn to describe the structure and might instead be termed "subsequent" and "prior" respectively.

The construction of a picture is described by a directed graph wherein the top node is the picture, the intermediate nodes are subpictures, and the terminal nodes are the basic subpicture. A sample graphical data structure is represented in figure 2.

While it is fairly obvious that when expressed in computer code the picture and subpicture nodes need to be represented by data blocks, it may not be immediately clear that the same is true for the branches connecting the nodes. There is of course a trade-off between the information associated with the node block and that associated with the branch block. The following selections are somewhat arbitrary although typical [7]. The subpicture node block will essentially consist of pointers to other information containing blocks. Among these blocks is the branch block, which deserves special mention.

The branch block contains the necessary transformation on the lower nodes to incorporate them into the subpicture defined by the subpicture node. Certain blanks in the skeleton frame of the lower level picture must be completed, and other parameters may require systematic modification. The transformation information consists essentially of displacement, scale, and rotation information.

Since all nodes except the highest "picture" node are intrinsicly referenced to zero, a new reference displacement must be provided for each instance of use of subpicture and basic subpicture node. This displacement may occur in a virtual display space having no size relationship to the display viewing area; therefore, an additional displacement calculation may be required in the process of display generation when the segment of the structure is selected for display.

Scale and rotation information must also be provided when constructing the present subpicture out of lower level items. In most applications it is unlikely that the rotation information will require changing after picture-generation time, but scale may be a continuously varying parameter. The analogy of photographic enlargement is accomplished by a modification of the scale parameter. In certain applications this operation could be the critical item in operation of the facility. This technique was first introduced as "homogenious coordinates" in Sketchpad [26].

The connectivity of the subpicture constituting the present level subpicture must be separately treated. It is not sufficient to build a picture by appropriately placing subpictures so that their terminals coincide. Subsequent parameter change in the branch blocks, or minor malfunction in the hardware, could easily destroy such coincidence. Thus, there must be explicit provision for an ordered relationship among the terminals of subpictures associated within a higher level picture.

For this reason, another block is provided, this being the "connector" block. The connector block is a specific example of a constraint. It is presented as a separate block by virtue of its prevalence. The connector block must identify the lower level terminals to be connected, and it must point to the constraints on such connection. These constraints might include a requirement for coincidence. If the terminals being connected were not coincident, the connector block would be required to provide a line to form the connection. This line in turn could have attributes such as intensity and rate of blink. These blocks may be represented by figures 3, 4, 5 and 6.

The design of the pointer scheme is a critical part of any data structure. An excellent discussion is given by Dodd [21], to which the reader is referred. The most simple pointer structure is the single linked list wherein each block contains a pointer to the succeeding block in the list. The pointer field in the terminal block contains a special symbol known as the "null pointer" indicating the termination of the list.

The major shortcoming of the single linked list is the inability to return to the head of the list without having previously saved the location of this head in a well known location to which reference might be made.

The single linked list is rarely employed simply because a ring structure may be obtained by having the end of the list point back to the head. Of course, the head and tail must be suitably flagged to avoid endless ring-chasing. Another way of returning to the head of the list is to use a doubly-linked list possessing a backward as well as a forward pointer, but this involves twice as many pointers. It is, on the average, twice as fast as the single linked list in returning to the head of the list.

Since the purpose of rings or doubly-linked lists is to be able to return to the head of the list, or the list pointer, when a success or failure has occurred, the second pointer which the doubly-linked list requires is often replaced by a back pointer to the head of the list. To conserve space, the pointer to the head of the list may not occur in every block, but rather in strategically placed blocks. Such a scheme is similar to, but simpler than, the CORAL structure [9], [13], [14], [21]. No common name exists for this pointer scheme. It is suggested that it be called a "pie" structure, based on the diagram of figure 7.

A great deal of effort has gone into the development of pointer arrangements, this being the critical decision in designing a data structure. The structures examined by Gray [9] appear more different than similar, yet they are all concerned with related problems.

## 4. Constraints

One of the most important valuable aspects of the data structure for building design application is the utilization of constraints. As has been mentioned, the terminal block is a form of constraint. Additional constraints particular to this application are perpendicularity of planar surfaces and inclusion of subpictures. The perpendicularity constraint assures that as subpictures are manipulated that they retain the desired form. It is not sufficient to draw such perpendicularity without also constraining the data structure to preserve it under transformation.

Another valuable constraint is inclusion of one subpicture within another. By constraining windows, doors, pipes and electrical services to remain within a wall it is possible to move the wall during the design process and assure that loose ends are not left dangling.

## 5. Associative Addressing

The pointers used in the sample structure above are "explicit" pointers in that they address direct access media [21]. For large classes of problems wherein the data base is subject to random access, and is possibly stored in multiple levels of secondary and primary memory, content addressability offers certain advantages. The essence of content-addressable, or associative, memory is that it does not employ explicit addresses. A stored item is addressed by a partial description of its contents. Current implementations [1], [2], [3], [15] are accomplished by software, there being serious problems with hardware associative memories [1].

Interactive processors in general, and interactive graphics in particular, can benefit from the use of associative languages. The programming techniques possibly may require a re-orientation on the part of the user, but the popularity of associative language is attested to by several sources [1], [3]. In part, the decision to use an explicit pointer or associative pointer language depends on the availability, support, ease of utilization, and prevailing attitude at the installation where the user is to work. The only a priori advantage of associative processing seems to be in the area of utilization of secondary storage, which is discussed below.

## 6. Calculated Addressing

Since hardware associative memories have not economically arrived, associative systems are currently implemented using a calculated addressing mechanism [1]. Calculated addresses are not restricted to schemes which explicitly involve content addressability, but even when they do not, the underlying concept remains the same: namely, that the address of the memory location to be accessed is determined by an algorithmic operation upon the contents of the pointer. In this usage, the pointer is more aptly termed a "key" which may be part of information content of the data structure rather than a separate item devoted strictly to pointing. Calculated addressing works by treating the symbolic information as a set of numeric items which are to be manipulated by a known and well-defined algorithm to produce a memory location address [1], [21], [22]; such an operation is often called "hash coding".

One of the disadvantages of hash coding is that the calculated address is not necessarily unique. When two keys are calculated to point to the same memory location an ambiguity, or collision, occurs. To provide for the advent of collisions, the pointer-chasing mechanism must check that the contents of the calculated address matches the key used to calculate that address. One strategy is to treat the contents of the calculated address as a pointer, usually a direct address pointer, to a location within a block area where all the collision items may be found. It is perhaps safest if a disjoint area is reserved for this purpose.

### 7. Multiple Levels of Data Structure Storage

The requirements of fast response to operationally complex requirements and possible large data structures necessitate that the structure be simultaneously maintained in more than one level of storage, at least in part. First, consider the question of storage in the display.

Early graphical displays required the exclusive service of a large computer system. Today the trend is to provide a small computer as the local service to each display and to service this local processor-graphical terminal from the central system only when necessary. There is a whole spectrum of capabilities of local processor attached to graphical processors. We shall not go into the evolution of such dedicated processors here; the situation has been stated elsewhere [16]. Needless to say, however, the extent and kind of representation of the graphical data structure in the local computer is highly dependent upon the kind(s) and amount of storage available, the instruction repertoire, and the speed of the local processor.

The minimum information to be kept in the local processor is the display list which directly controls the picture presented. The display list is extremely machine-dependent, containing the necessary machine instructions to generate the display. If the computing capability of the local processor is non-existent or extremely minimal it may be necessary to construct the display list in the main system for transmission to the graphical display. In such a case the local processor fulfills only the function of refreshing. In this situation it is impossible to reference the graphical data structure via the display image because the display list has been generated only for display purposes.

In systems with minimum local processing ability, or even in more substantial systems, it seems a waste of an expensive resource to store the display list in randomly addressable core storage. It appears a better allocation of resources to use rotating storage for the display list. Not only does this free core storage for programs, but it makes it possible to carry on display refresh as a parallel process. Recent [17], [18], [19] and not so recent systems [20] have used this approach[2].

The next step is to provide an association between the display list and the graphical data structure. Such an association requires a referencing technique from the display list back to the data structure. A pointer scheme can be implemented, but difficulty occurs as to the subpicture level to be pointed to. Under various conditions the user at the graphical terminal might be interested in pointing to a picture, a level of subpicture, or a basic subpicture. An automatic safe technique is to have the pointer go to the highest level of subpicture being referenced, with the user being able to initiate pointer chasing under his control to reach the desired lower level.

If memory and speed allow, part or all of the graphic data structure may be contained in the local processor. If sufficiently large and fast, the local processor could contain the entire data structure, generate its own display list, and reference the main system only for archival purpose or for linking to other subsystems. In this form of operation the graphical subsystem can be considered a "sketchpad" on which various trial drawings are made. When an acceptable one is produced it can be preserved by referring it to the central computer.

In general, the data structure will be too vast to be contained completely in the graphical terminal. A compromise is then effected wherein part of the data structure might be transported as needed between the central system and the graphical subsystem. The degree of compromise is a function of the processing capability of the graphical terminal, a subject well discussed by Myer and Sutherland [16]. For convenient operation the transmission must occur within the user's wait tolerance. When only part of the data structure is resident in the graphical subsystem, extreme care must be taken with the pointer to the non-resident parts of the structure. There must be a mechanism for flagging references to non-resident items; there must be a mechanism for enlarging or contracting the size of the portion of the data structure available. These problems are quite akin to the multiple-level storage problem in the central system, which shall be discussed next.

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[2] But a cycling display carries with it three prices to pay: (1) it is usually slow to access, (2) it is fixed in size (but the size can be very large), and (3) it can be quite difficult to respond to light pen interactions.

The adage of "a picture being worth a thousand words" is magnified in computer representation. It can easily require many thousand words to store a moderately complex picture. Such storage requirements can easily consume available high-speed primary memory.

It is certainly possible to design the driving and service program, the "resident system", into minimally-interacting modules. These modules can be brought into core as pages [1] or overlays [12], thus reducing the core storage which must be devoted to the system.

For user-created programs and data structures the situation is different. It is desirable that as few restrictions as necessary be placed on the programmer. Therefore, systems are written which automatically assign program and data to storage pages [1], [3], [12]. However, there is not total rigidity in these page assignments; variable-sized pages [1] and partial user control [3] helps to adapt the system to its current use. For the storage of the graphical data structures it is even more important that the system be given as much information as is available. With complete information it is possible to implement valid anticipation of program needs [3].

## 8. Sample Use of General Graphic Data Structure

Representing a graphical data structure on paper is an awkward necessity; awkward because the confines of standard paper size makes it an exercise in topological ingenuity on the part of the writer and parallax error elimination on the part of the reader; necessary because the expository approach alone generally produces an incomplete information transferal.

The first illustration, in figure 8, is of the structure representing a triangle. This is a trivially simple structure involving only one node and one basic subpicture. The one node, which is automatically the top (picture) node points to three rings: branch, terminal, and connector. Each block in the branch ring contains a pointer to the basic subpicture used, namely "point", the X, Y coordinates of the instance of that point, and a pointer to the next branch block and back to the node block. Each terminal block contains the coordinates of the terminal relative to the origin of the subpicture (which in this case are selected to be all the verticies), and the forward ring pointer. Each connector block contains a pair of pointers to the terminals being connected, and the forward ring pointer. All of the remaining fields contain zeroes interpreted as null pointers. The organization of the blocks is in conformance with figures 1, 3, 4, 5, and 6.

Let us now use this triangle as a subpicture in building a larger picture. As an example, consider the hexagon shown in figure 9(a). The triangle used of the subpicture is assumed to have been drawn in the position shown in figure 9(b). Note that external terminals are denoted by small circles in these figures.

The data structure of the hexagon is drawn in figure 10. Included is the terminal block ring of the triangle data structure which is necessarily referenced by the connector ring of the hexagon. Note the dotted lines representing pointers from the connector ring of the hexagon to the terminal ring of the triangle.

These dotted lines from connector blocks to terminal blocks associated with another node block are representations of an amazingly complex pointer chasing mechanism required. The connector block must point to the branch ring which it accesses by pointing to the branch pointer in the node block. From the appropriate branch block it obtains a pointer to the node block of the subpicture references. Also from the branch block it obtains the displacement, rotation, and scale data which is necessary for the calculation of location of the desired terminal in the particular instance of use. From the node block pointed to by the branch block it obtains the pointer to the terminal ring associated with that level of subpicture. Finally, that terminal ring is traversed until the particular terminal desired has been acquired.

Since such pointer chasing is not an abnormality in graphic data structures, there must be a mechanism easing such constructions. The pointer concatenation facility of $L^6$ [23], recently implemented by R. A. Siegler in conversational form as CL6 [24], is one technique which facilitates such pointer chasing.

## 9. The Tailored Graphical Data Structure

As discussed in the overview, it is frequently convenient to construct a data structure which is tailored to the graphical image to be modeled. The tailored structure can eliminate those features of the general graphic data structure which do not apply to the problem at hand. The structure of the graphical entity may be taken into account in designing the tailored structure; conditions which were provided for in the general case may not occur. Therefore, the space reserved for the eventualities in the general structure could be released for other use in a tailored structure.

Also as discussed in the overview, the use of a list processing language greatly simplifies the work of creating a tailored data structure. Rather than continue with the abstract rendering of geometric figures, the illustrative example of a tailored data structure will be concerned with computer program flowcharts with which the author has been working.

## 10. Sample Tailored Graphical Data Structure

Our illustrative example will be a data structure used for the (internal) representation of flowcharts. The data structure is created using the list processing language CL6 [24], a version of $L^6$ [23]. The defining portion of the program is exhibited as figure 11. Since the reader is most probably not conversant in $L^6$, the operations which are pertinent to the creation and use of the data structure will be discussed in detail.

One useful feature in $L^6$ is the ability to define the location of a "field" within a "block" of consecutive computer words. A field, once defined, may contain a pointer to another block, an arithmetic value, or anything else the programmer desires. A field is designated by a single letter name. The complete specification of a field includes the word in the block in which the field is to exist, the name of the field, and the inclusive bit boundaries constituting the field within the computer word.

The format of the field definition command is

```
( <word>, D, <field name>, <bit bound 1>, <bit bound 2>)
```

where <word> , <bit bound 1> , and <bit bound 2> are all integers indicating the relative work in the block, and the inclusive bit boundaries within that word. <field name> is the single letter name by which the field is symbolically referenced.

Like most programming languages, $L^6$ provides the programmer with a means for inserting comment lines for internal documentation. In $L^6$ such comment lines must contain an asterisk in column one and are ignored by the $L^6$ translator. In figure 11 the first six lines numbered 1 are comment lines which explain the usage of the fields defined in line 0.

In the program, line 0 defines three fields in word zero: field I, the block number, bits 31 through 36; field B, the first forward pointer or message pointer, bits 1 through 15; and field C, variously used as the back pointer, the input block pointer, or the count of the number of words in the message, bits 16 through 30. The meaning attributed to field contents is the programmer's responsibility. Note also that field definitions are non-unique, for line 4 defines field J to be bits 1 through 36 of word 0.

The ability to define those fields appropriate to the specific application is only one advantage of the tailored data structure. It is used in this example to define fields in words 0 through 2 of the block. Another advantage of this tailored data structure is the ability to define variable size blocks, the size being determined during program execution. This feature permits optimum memory utilization.

In $L^6$, the procedure of defining a block is performed by the "get" operation having the format

```
( <block name> , GT, <block size> )
```

where <block name> is the single letter, called a "bug", which points to and thereby identifies the block, and <block size> is the number of words in the block which is being gotten.

In line 5 of the program, two fixed length blocks are obtained; block E being 3 words long and block B being 65 words long. Skipping a few details, in line 12 a line of up to 65 characters is read into B. This block is scanned for the end of line character, the carriage return, in lines 13 through 16, keeping count of the number of characters in the line. In line 17 a new block D is gotten having length C, where C was determined by the counting of lines 13 through 16. In essence, block B was used as a fixed size input buffer from which the contents are transferred to the custom sized block D.

We could, if we chose, continue this detailed analysis of $L^6$ as used for this application program. The author does not believe that to do so would be of further educational benefit. Those interested in following the workings of $L^6$ are referred to the defining paper by Knowlton [23].

In addition to the main pointer structure an auxiliary structure is provided. This auxiliary structure consists of one word blocks, the fields of which are perforce identical to those of the zeroth word of the main structure as shown in figure 12. This second string forms a linear chain most easily searched and is only used for retrieving blocks in the main structure. This second string is created on line 18 of figure 11. An example of the application of this structure is given in figure 13. The flowchart segment represented is drawn in figure 14.

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---

| |
|---|
| Identification of Node as Basic Subpicture<br>Name of Basic Subpicture<br>Pointer to Terminals<br>Pointer to Non-Display Information<br>Pointer to Display Instructions |

Figure 1. Basic Subpicture Node

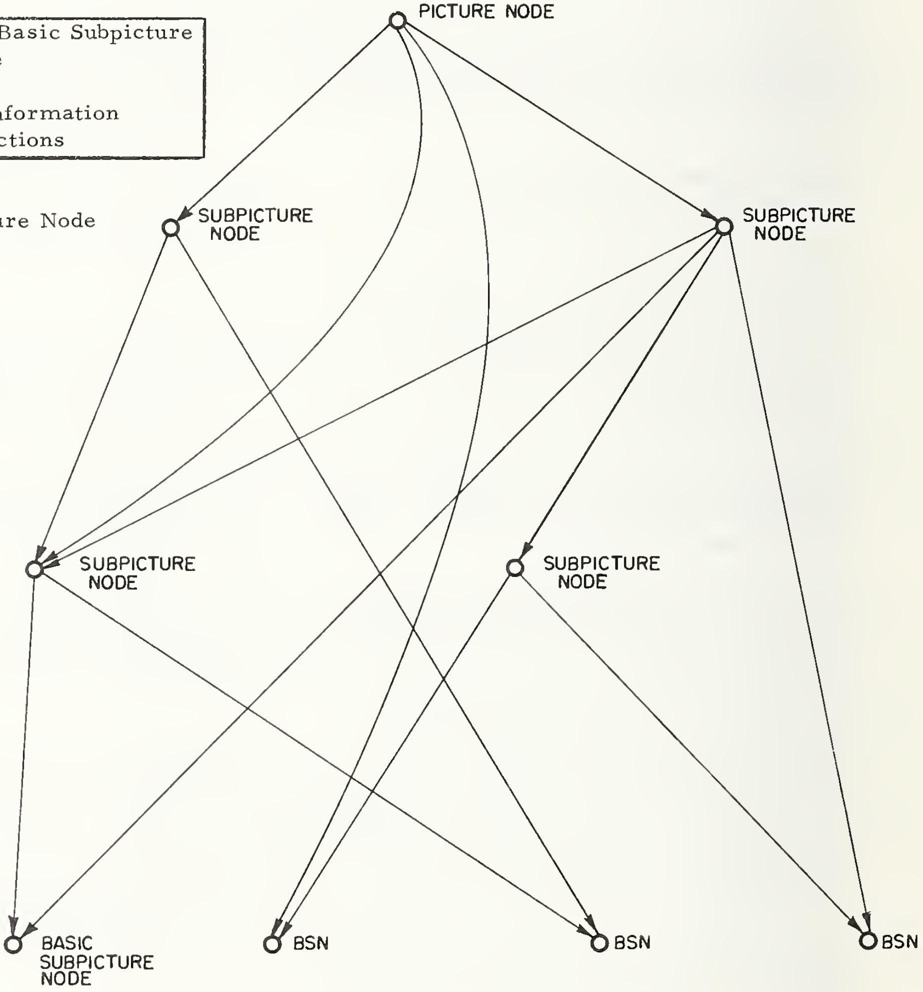

FIGURE 2. SAMPLE GRAPHICAL DATA STRUCTURE

Identification as Connector Block
Pointers to Terminals
Blink Rate, Constraints
Pointer to Next Connector Block

Figure 3. Connector Block

Identification as Branch Block
Name of Branch
Pointer to Lower Node
Displacement, Rotation and Scale of Lower Node
Pointer to Non-display Information
Pointer to Next Branch Block

Figure 4. Branch Block

Identification as Node Block
Name of Node
Pointer to Terminal Block
Pointer to Branch Block
Pointer to Connector Block
Pointer to Non-display Information

Figure 5. Node Block

Identification as Terminal Block
Relative Location of Terminal
Pointer to Next Terminal Block

Figure 6. Terminal Block

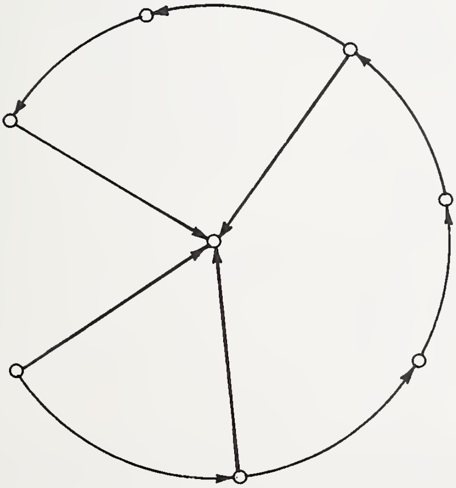

FIGURE 7. "PIE" DATA STRUCTURE

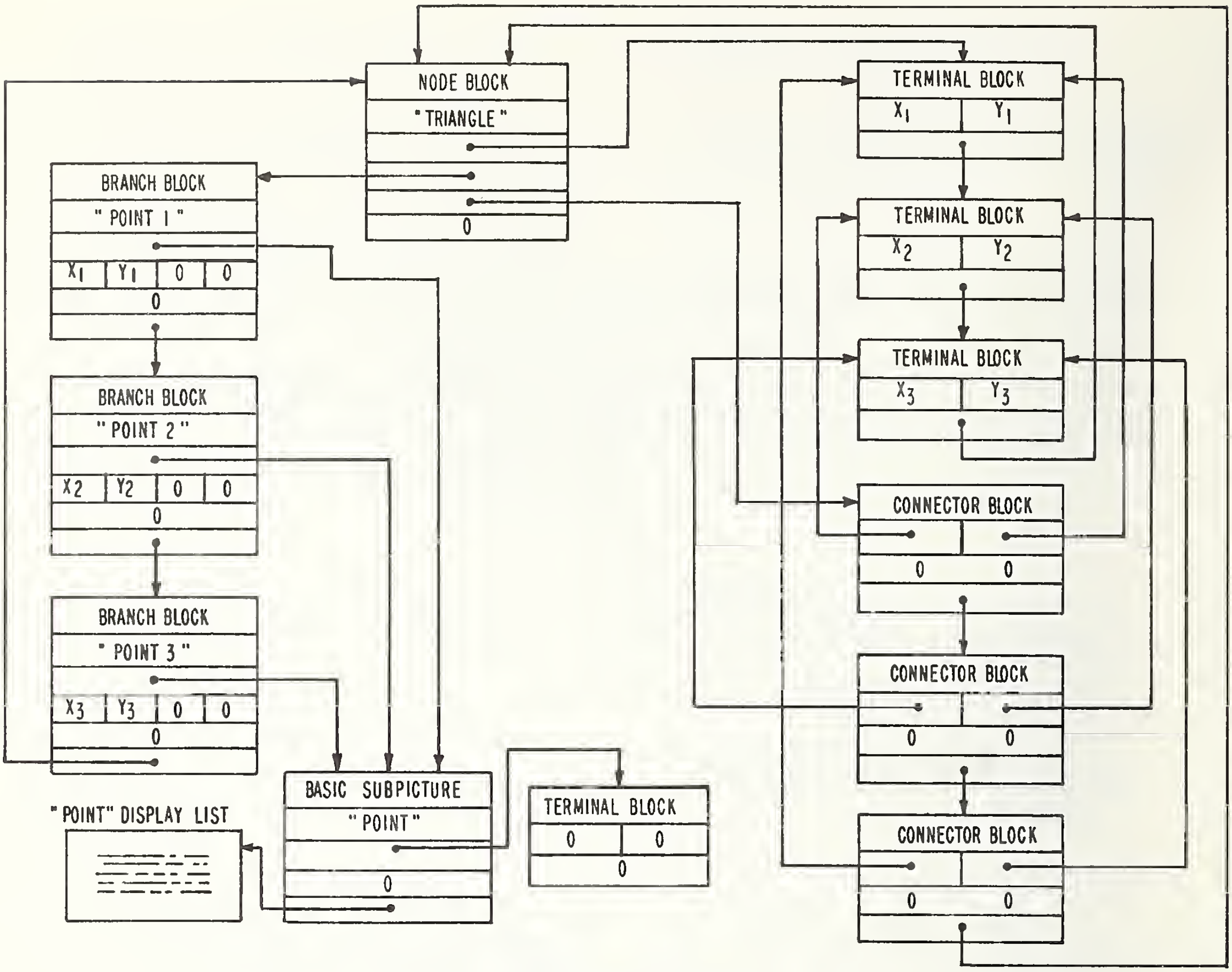

FIGURE 8. DATA STRUCTURE OF A TRIANGLE

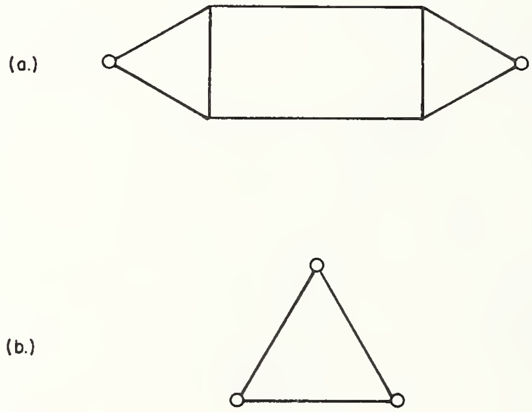

FIGURE 9. HEXAGON. (a) COMPOSED FROM TRIANGLE. (b) TERMINALS DENOTED THUS: (—○)

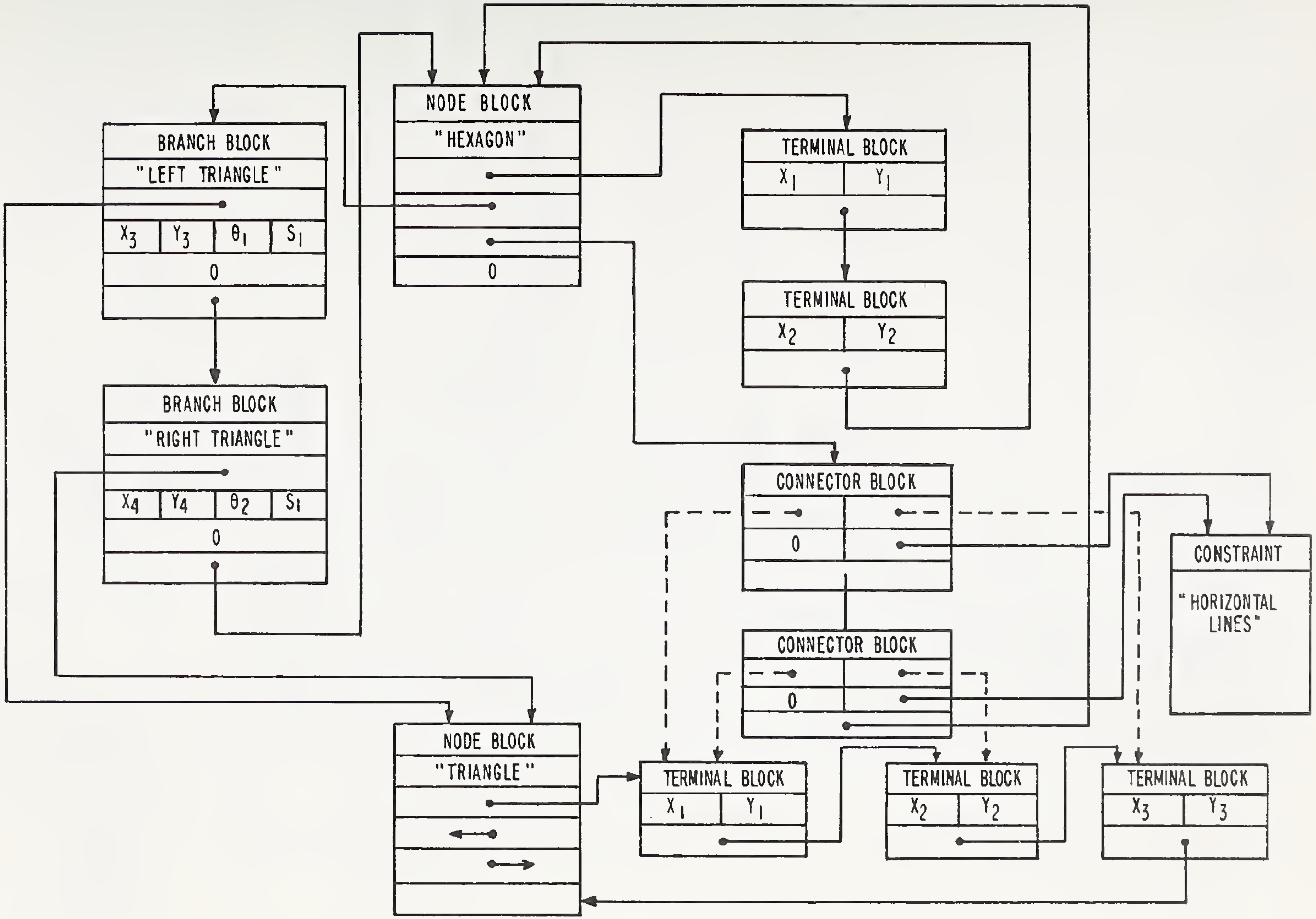

FIGURE 10. DATA STRUCTURE OF HEXAGON

```
0     SETUP1 :(0,D,C,16,30)(0,D,B,1,15)(0,D,I,31,36)
1     * I = BLOCK NUMBER
1     * B = FORWARD PR #1
1     * B = MESSAGE POINTER
1     * C = # WDS IN MESSAGE
1     * C = BACK PR
1     * C = INPUT BLOCK PR
1      :(1,D,A,1,30)(1,D,D,31,36)
2     * A= BOX TYPE
2     * D = # FWRD PR
2      :(2,D,E,1,15)(2,D,F,16,30)(2,D,G,31,36)
3     * E = FWRD PR # 2
3     * F = FWRD PR # 3
3     * G = # CHARS
3      :(3,D,H,1,36)
4     * H = ASCII WORD
4      :(0,D,J,1,36)
5      :(E,GT,3)(B,GT,65)(X,GT,4)
6     * X IS SUBR C<M BLOCK
6     * E = 3 CHAR COMMAND BUFFER
6     * B = INPUT STRING BUFFER
6     * N = MESSAGES
6     * M = POINTERS TO MESSAGES
10    FOUND3 :(Z,E,5)(DO,MSG)(65,RL,B)(EJ,CP,3,B)
11     :(FOUND3,DO,BOXTYP)
12    * C IS LOCAL POINTER
12    CONTNT :(Z,E,7)(DO,MSG)(65,RL,B)(Y,P,B)(C,E,2)
13    LOOP1 :(C,A,1)(Y,A,1)
14     IF(YJ,EO,432000000000) : FOUND1
15     IF (C,G,67) : STOP
16     : LOOP1
17    FOUND1 :(C,A,1)(D,GT,C)(DD,E,1)(DO,FORPTR)(DA,E,XD)
18     :(DI,E,G)(HC,P,D)(HI,E,G)(H,GT,1,HB)(HBB,P,H)(C,S,3)
19     :(DG,E,C)(C,E,0)(Z,P,D)(Y,P,B)
20     IF(DA,E,4):(DD,E,3)
21    * H IS BLOCK OF POINTER STRING
21    LOOP2 :(ZH,E,YJ)
22     :(Z,A,1)(C,A,1)(Y,A,1)
23     IF(C,G,DG):(G,A,1)NEXT?
24     : LOOP2
25    FINDF :(I,P,JB)
26    * I IS TEMPORARY POINTER
26    LOOP3 IF(II,E,F):(D,E,0)FORPTR
27     IF(II,E,0):(Z,E,3)(DO,MSG)FAIL
28     :(I,P,IB)LOOP3
```

Figure 11. Portion of CL6 Program

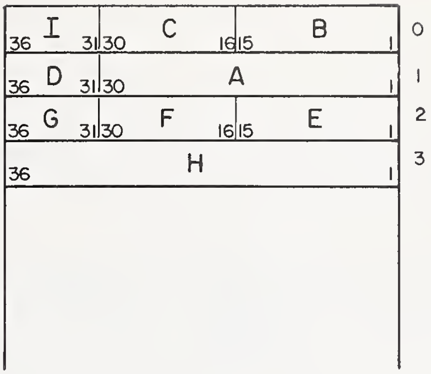

FIGURE 12. DEFINED FIELDS FOR FLOWCHART DATA STRUCTURE

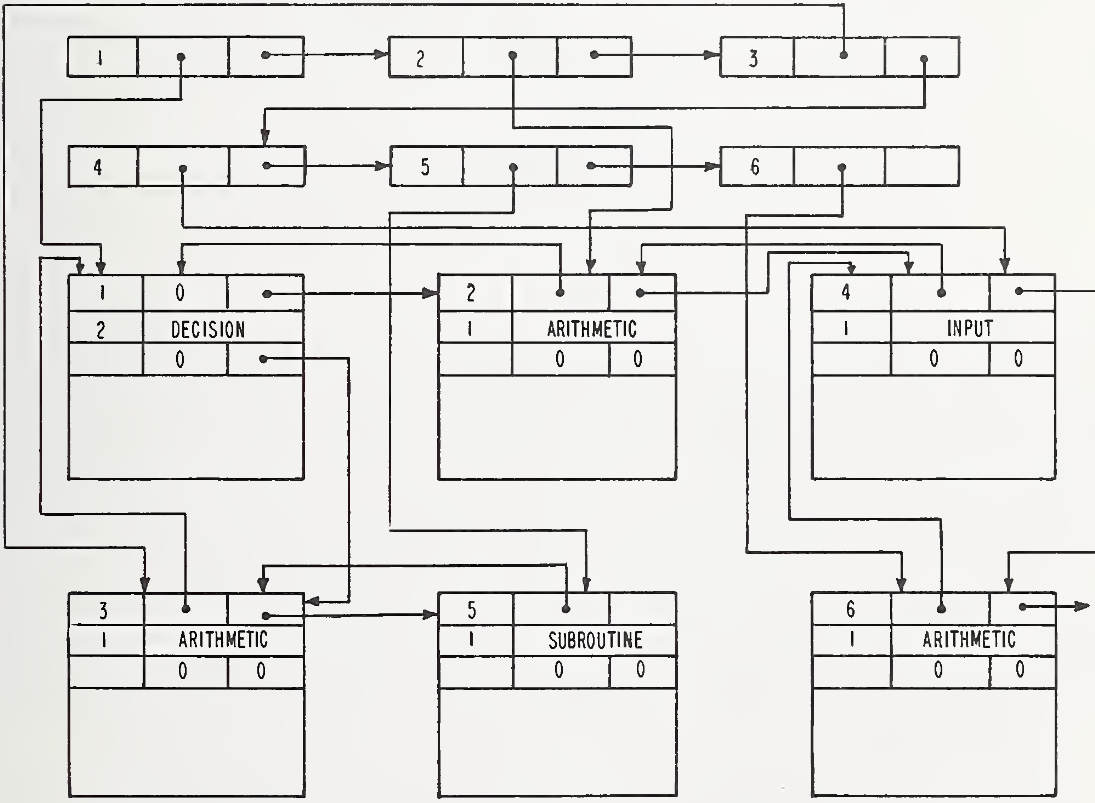

FIGURE 13. FLOW CHART GRAPHICAL DATA STRUCTURE

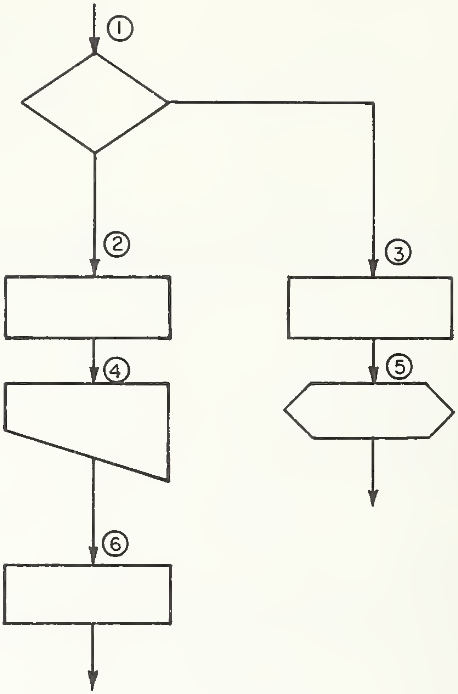

FIGURE 14. FLOWCHART REPRESENTED BY DATA STRUCTURE

# A Systems Model for Environmental Design of Buildings

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This paper presents a computer-oriented design model for evolving optimized building performance specifications to achieve satisfactory indoor environment. The interactive aspects of environmental design with respect to thermal and lighting sub-systems are considered in terms of fifteen design variables specifying insulation, inertia, glazing, shading, siting, and surface treatments. The criterion of optimality is chosen to be minimum design cooling load.

The method consists in coupling an optimization program based on a simplex search type technique to an analysis program for buildings subjected to variable climatic and use conditions. Provision has been made for varying the environmental performance criteria and for imposing constraints on the design variables.

An open plan office is chosen as an example for application of this model. Results obtained on a CDC 3600 computer are presented.

Key Words: Air-conditioning, building design, computer, daylighting, environment, indoor climate, load calculation, optimization, performance specifications, thermal behavior.

## 1. Introduction

In any building design it is necessary to consider the interaction of environment and building, for the primary role of the latter is to modify the environment. In the most general case, this would require an understanding of how the outdoor environment affects and is affected by a building or a complex of buildings, and their interaction to produce the indoor environment. Not much is known about how the buildings affect their external environment, and so it is usually considered to be specified for a given design problem. However, the provision of feasible solutions to the creation of a desired indoor environment generates many inter-related problems, which require careful handling of a number of inter-dependent systems such as thermal, visual, and aural and a complex array of interacting variables defining building materials and components, shape and orientation, occupancy, fenestration, user requirements, and similar factors for their solution. The existing design practice cannot handle these interactions adequately because of lack of integration and generality in its structure. In addition, there are too many empirical assumptions based on rationalization of past experience, which may no longer be valid for significant design problems faced in the context of radical and rapid changes in technology and society. On the other hand, availability of large computers and development of system engineering techniques specially suited to deal with interactions, strongly suggest the formulation of a system model for the environmental design of buildings. Such a model can be further linked to other aspects of building such as structural and economic, and is ideally suited for efficient evolution of an optimum environmental design with respect to some chosen criterion or a set of allied criteria.

A systems approach to optimum thermal design has been developed by Gupta {1}[1]. It has been utilized by Gupta and Spencer {2} to optimize the thermal design of panelized constructions for residential use. As a further step towards the objective of optimum integrated environmental design, interactive aspects of thermal and lighting sub-systems as they affect the design of building envelope have been incorporated into the systems model outlined in the present paper. The objective of the formulation is to provide optimum performance specifications and the effect of perturbation in these on the chosen criterion of optimality at the sketch plan stage.

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[1] Figures in brackets indicate the literature references at the end of this paper.

## 2. Basic Design Philosophy

Indoor environment in a building is primarily influenced by four basic elements and their interactions : outside climate, the building structure, the indoor space and contents, and the occupancy. For environmental evaluation of architectural design alternatives and optimization of design variables with respect to performance, it is advisable to perform unsteady-state analyses for a cycle of representative design climatic data. Three-hourly values of air temperature, wind velocity, and solar radiation for a minimum period of three days, and sunlight and clear sky illumination values for a specified solar altitude are used for the outdoor climatic data relevant to the building location.

For the thermal sub-system, three-hourly values of indoor environmental temperatures {3} and cooling loads are computed by employing a distributed thermal network representation of the building and using the harmonic method of analysis {4,5}. For load estimation, allowance is made for considering either constant indoor environmental temperatures or a known variation permitting temperature swings. The particular method of analysis chosen is more accurate and economic in use of computer time than the finite difference methods {6}, but it cannot consider temperature-dependent properties or variable network representations. The design variables are considered in section 3.2 and the calculation method is discussed in section 4.1.

For the lighting sub-system, calculations are made of the distance from windows at which a given level of natural illumination will be available on different sides of the building for a reference solar altitude of $30^{\circ}$ and a clear design sky proposed by Kittler {7}. These distances determine the floor area over which artificial lighting is required in the interior of the building. In this way, relative contribution of natural and artificial lighting to cooling loads can be taken into account. Direct sky component and sunlight, internal reflected components, ground-reflected sunlight and daylight are taken into account, but obstructions due to other buildings are not considered in any of the sub-systems. The computational scheme is oriented to office buildings with relatively large window areas, and as yet does not include the computation of glare index. The design variables are considered in section 3.3 and the calculation method is discussed in section 4.2.

For the purposes of system optimization, some of the system parameters are kept fixed as specifications for a given problem and the rest are allowed as design variables within explicity stated upper and lower constraints. These constraints are usually governed by economic and aesthetic considerations and by requirements imposed by bye-laws, materials, land availability, and such other factors not directly taken into account. The limits imposed by mutual interactions of these design variables, if any, are also taken into account in terms of implicit constraints. The design alternatives are generated within the computer program and the objective function based on performance criterion or criteria is evaluated. In this design model the objective function is chosen to be degree of discomfort {1,2} for unconditioned buildings and peak cooling load for the conditioned ones. A cost function is currently under development. As compared to an intuitive process or exhaustive enumeration scheme {8,9}, an optimization scheme generally makes it possible to evolve an optimal design solution in a much smaller number of evaluations. The technique adopted for the present paper is discussed in section 5. A schematic diagram showing a building as an environmental system is shown in figure 1.

## 3. System Design Variables

### 3.1 General Considerations

The system provides for optimizing up to fourteen design variables, any one or more of which can be fixed if desired. For simplicity, a rectangular plan with specified floor area on an exposed site is considered. As such, the model is applicable to a residential building as a whole, a single storey of an open plan multi-storey office building, or any one air-conditioning zone of big buildings. In detailed estimation of design loads or lighting, the calculations can be done room by room, if desired.

### 3.2 Thermal Design Variables

#### a. Materials

The thermal penetration coefficient, P, which is defined as the product of thermal conductivity, k, density, p, and specific heat, c, is utilized as the basic thermophysical property defining the materials. Values for the insulating materials in the roof and walls and for the outer skin material of the walls are considered as design variables. The remaining materials such as floor, roof type, and inner lining of walls are initially specified for the type of building being optimized. The outer skin of the wall is considered in particular since it provides the major variable component of thermal inertia in constructions popular in Australia. The model is flexible and can consider materials other than these.

### b. Components

For the three materials specified in section 3.2(a), thicknesses are also the design variables. To convert these thicknesses into thermal resistances, a predefined linear relationship between penetration coefficient and conductivity is used. The locations of these components in the building elements are fixed initially. The final specifications list thermal resistance as well as thicknesses, so that departure of individual materials from the P-k relationship can be taken into account.

### c. Spatial Envelope

The design variables fixing the configuration define the areas of various elements constituting the envelope and the volume enclosed therein. The variables are orientation, aspect ratio (defined as equal to length/width), and ceiling height, and these determine availability and extent of natural lighting and incidence of solar radiation. For a given floor area, the aspect ratio and ceiling height determine the exposed envelope area, and the volume of air required for ventilation or air-conditioning. The minimum number of air changes corresponding to lower ceiling heights is governed by health requirements and also by the effect of ceiling height on the flow jets from inlet diffusers or grills {10}.

### d. Glazing and Shading

The area of glazing per unit wall area and its shading coefficient are considered as design variables. All four walls are considered to have the same percentage of window area unless specified otherwise. The total glazed area as a percentage of floor area is checked so as to remain within the constraints imposed by bye-laws or the psychophysical requirements of view and vision {11}.

### e. Surface Treatments

Two design variables specify the solar absorptivity of the outdoor surface of opaque elements such as roof and walls. These determine the contribution of solar radiation to the design outdoor environment denoted by sol-air temperatures.

Internal furnishing, furniture, and partition areas are not considered as design variables and their resistance-capacitance values are prespecified. However, their areas are fixed relative to the floor area and ceiling height, and although these ratios may be fixed the actual values would vary within an optimization run. These masses are very important in relation to the internal lighting loads and direct sunshine through windows. Internal convective loads due to occupancy are specified per unit floor area for any given problem.

## 3.3 Lighting Design Variables

The design variables common to thermal and lighting sub-systems are orientation, shape, ceiling height, and window area. Internal luminous reflectance for ceiling, walls, and floor and for outside ground can be fixed as problem specifications. The design daylight factor beyond which artificial lighting would be required and the design solar altitude can also be specified. The only design variable considered is luminous transmittance of windows. Since this is usually related to the shading coefficient, check constraints are imposed so that the ratio of the two remains within the limits for available types of shading arrangements.

## 4. Methods of Analysis

A hierarchy for environmental performance of buildings comprising thermal performance, heating or cooling, and daylighting branches, is shown in figure 2. In the present system simulation program, the design variables are considered from the lowest level and the optimization is performed on the basis of a definitive part of the building, as indicated in section 3.1. The following methods of calculation for thermal and lighting sub-systems have been programmed and are used as components of the system simulation on a digital computer.

## 4.1 Thermal Sub-system

The basic method of calculation used is due to Muncey {4}. For sinusoidal heat flow and temperatures, the indoor environmental temperature, $t_B$ is represented by

$$t_B = \frac{\sum A\ t_A\ (1/P_{12}) - W}{\sum A\ (P_{11}/P_{12}) + \sum B\ (Q_{21}/Q_{22})} \qquad (1)$$

where $t_A$ exp (jwt) is air or sol-air temperature on a typical external heat flow path of area A and transmission matrix (P), W exp (jwt) is the heat flow to the inside air, B is the area of an internal path having transmission matrix (Q) for half the thickness, and the summation is over the parallel heat paths constituting the building. Transmission matrices (P) and (Q) are of the type

$$[P] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{2}$$

These are obtained by matrix multiplication of the matrices for individual layers in the relevant heat path, which are of the form

$$\begin{bmatrix} \cos \text{ƶ} & -\{(R \sin \text{ƶ})/\text{ƶ}\} \\ \{(\text{ƶ} \sin \text{ƶ})/R\} & \cos \text{ƶ} \end{bmatrix}$$, where R is the thermal resistance and C is the thermal capacity per unit area of the layer.

The complex argument $\text{ƶ}$ is given by

$$\text{ƶ} = (jwCR)^{\frac{1}{2}}$$

Internal radiant loads can be handled by using a heat flow multiplier developed by Muncey and Spencer {5}. The internal environmental temperature $t_{ea}$, under practical situations, is obtained by superposing eq. 1 for a reasonable number of frequencies to get

$$t_{ea} = \sum_{m=o}^{m=n} t_B \exp(jwt) \tag{3}$$

where w = 2πm/T, and T is the number of hours in the design climatic cycle.

The degree of discomfort, D, can be computed by using the equation in {2}.

$$D = \frac{1}{N}\left[ \sum_{\substack{\text{day} \\ \text{hours}}} \left\{ \frac{|t_{ea} - t_c|}{\Delta D} -1 \right\}^{+} + \sum_{\substack{\text{night} \\ \text{hours}}} \left\{ \frac{(t_{ea} - t_{un})^{+}}{\Delta N} \right\} \right] \tag{4}$$

where N = number of ordinates in the design climatic cycle at which the temperatures $t_{ea}$ are known in a cycle.
$t_{ea}$ = indoor environmental temperature at some hour
$t_c$ = daytime preferred temperature for comfort
$t_{un}$ = upper limit for night time comfort
ΔD = half the variation allowed in daytime
ΔN = variation allowed in the night time
\+ (only positive values to be considered)

For load calculations to maintain an indoor environmental temperature $t_{Ra}$, the component for the frequency w, $(L)_w$, is obtained by the equation

$$(L)_w = -\left[ \{\sum At_A (\frac{1}{P_{12}}) -W\} -t_R \{\sum A(^P11/^P12) + \sum B(^Q21/^Q22)\} \right] \tag{5}$$

If the indoor temperature $t_{ea}$ has already been computed, eq (5) reduces to

$$(L)_w = -(t_B - t_R) \{\sum A(^P11/^P12) + \sum B(^Q21/^Q22)\} \tag{6}$$

where $t_{Ra} = \sum_{m=o}^{m=n} t_R \exp(jwt)$

The sign convention is such that negative values of load mean cooling loads and positive values mean heating loads.

## 4.2 Lighting Sub-system

As in the thermal sub-system, clear day conditions are assumed for design. The analysis tackles the inverse problem of determining the distance, D, from the window at which a prespecified level of natural illumination, $E_o$, is available in the work plane and beyond which artificial lighting would be required. A flux method similar to the lumen method {12} is used except that tabulated coefficients of utilization cannot be used in optimization studies. This problem is resolved by treating the internal reflected component and the direct illumination from the sun and the clear sky separately. The intensity of sunlight normal to the beam, $E_n$, the design diffuse illumination on a horizontal surface from the clear sky, $E_h$, and visible reflectances are specified for the problem.

Instead of considering it as a time-varying problem like the thermal sub-system, a design solar altitude with respect to the horizon is fixed for lighting. This is a reasonable assumption, since it has been shown by Hopkinson, Petherbridge, and Longmore {13} that for lighting in clear sky areas, where reflected sunlight is very significant, the daylight and reflected sunlight components so adjust during the working day that the sum of their contributions to internal lighting is practically constant.

To calculate natural lighting, the first step is to determine the amounts of direct and reflected sunlight and skylight on any wall surface. The following equations have been used for the purpose:

$$E_V = E_h [ A + \{B + C \cos (\alpha - \alpha_s)\}^+ ] \tag{7}$$

$$E_{SV} = E_n \cos \theta \cos (\alpha - \alpha_s) \tag{8}$$

$$E_{RV} = 0.5 \, R_G . E_h \tag{9}$$

$$E_{RSV} = 0.5 \, R_G . E_n \, \mathrm{Sin} \, \theta \tag{10}$$

$$E_W = E_V + E_{SV} + E_{RV} + E_{RSV} \tag{11}$$

and

$$\alpha_s = \cos^{-1} \{(\sin \delta - \sin L \sin \theta) / (\cos L \cos \theta)\} \tag{12}$$

where $E_V$ is the direct sky illumination, $E_{SV}$ is the direct sunlight, $E_{RV}$ is the ground-reflected skylight, and $E_{RSV}$ is the ground-reflected sunlight, $\alpha$ is the bearing of the normal to the wall, $\alpha_s$ is the solar azimuth, $\theta$ is the specified solar altitude, $\delta$ is solar declination, L is the latitude of the place, $R_G$ is the uniformly reflecting ground reflectance, and + means only positive values to be taken. The reflections from neighboring facades have not been taken into account but direct sunlight has been allowed for walls having $(\alpha - \alpha_s)$ less than $60^o$. For solar altitude of $30^o$, values of A,B,C are found to be 0.40, 0.16 and 0.67 respectively for the clear sky {7}. These are based on computations by Krochman {14}, and the values of $E_V$ so computed are in good agreement with those recommended for summer in the IES Lighting Handbook {12}.

The internal reflected component (I.R.C.) of natural illumination is computed by using the split-flux principle {14} applied to clear sky conditions. The computation is oriented to an open plan office so that the average internal reflectance is determined for the whole internal surface area under consideration. The actual component is determined for windows on each wall separately, and with no constraints imposed by partition walls and no contributions from windows on other walls.

With these assumptions,

$$\mathrm{I.R.C.} = T.\ K A_w [(E_V + E_{SV}) R_F + (E_{RV} + E_{RSV}) R_C] / [A_R (1 - R_R)] \tag{13}$$

where T is the diffuse luminous transmittance of the windows (a design variable), K is the maintenance and frame reduction factor, usually taken as equal to half, $A_w$ is the total window area in any wall, $R_F$ is the floor reflectance, $R_C$ is the ceiling reflectance, $A_R$ is the total internal surface area of the building envelope, ceiling, and floor, and $R_R$ is the area weighted average internal reflectance.

The required depth, D, is now obtained by solving the following transcendental equation by Newton Raphson's method {15}

$$E_o = \mathrm{I.R.C.} + T.KE_w \left[ \tan^{-1} (\tfrac{D}{W}) - \{ (D)(H^2 + D^2)^{-\frac{1}{2}} \} \tan^{-1} \{ W (H^2 + D^2)^{-\frac{1}{2}} \} \right] \tag{14}$$

where W is half the window width and H is the window height above the working plane.

From these values of D, one for each side, the area for artificial lighting is calculated. The uniform luminance assumptions {13} implied in taking the bracketed expression in eq (14) are fairly justified when reflected sunlight makes a dominant contribution to indoor lighting. It has not been considered worthwhile to do more sophisticated lighting calculations for these optimization studies at the present stage, but it may be done later at the detailed plan stage or when the groundwork and procedure for the systems model proposed in this paper have been established in practice.

## 5. System Design and Optimization

### 5.1 Design Problem Formulation

The environmental design of buildings has been formulated in terms of the independent systems design variables $P_i$, as described in section 3. There are also dependent system variables, $Q_j$ such as the ratio of luminous transmittance to the shading coefficient of windows, and total wall thickness, which have to be considered in evolving practicable design solutions. As stated in section 1, the dependence of external microclimate on design has not been considered in this model and is prespecified as input. The response variables, $R_k$, such as indoor environmental temperature and daylight illumination determine the performance level. Constraints are usually placed on the variables $P_i$ to satisfy the requirements of bye-laws and the client's brief, on $Q_j$ to ensure practicality and economy, and on $R_k$ to satisfy predefined environmental performance criteria. The mathematical formulation of the problem is as follows:

Let P, Q, R be the column vectors defined by equations:

$$P = \{P_i\} \ ; \quad i = 1,2,\text{---}m_1 \tag{15}$$

$$Q = \{Q_j\} \ ; \quad j = 1,2,\text{---}m_2$$

$$R = \{R_k\} \ ; \quad k = 1,2,\text{---}m_3$$

$$\text{where} \quad Q = Q\ (P)$$

$$R = R_1\ (P,Q,F)$$

$$\equiv R\ (P)$$

for a given outdoor climate vector $\{F\}$ and $m_1$, $m_2$, $m_3$ are the number of independent, dependent, and response variables. If the lower and upper bounds vectors L and U be such that

$$\{L\} \leq (P,\ Q,\ R) \ \leq \{U\} \tag{16}$$

where P, Q, R is the complete set consisting of vectors P, Q, and R, the design problem is to find a vector P consistent with eq (16), which defines the feasible design space. Obviously, there are many possible design solutions corresponding to the multitude of points enclosed in this space and in conventional practice; only a very few intuitively selected alternatives are evaluated and one of these is chosen.

### 5.2 System Optimization

The object of formulating a systems model and optimizing it is to select the best or near best of an infinite number of possible designs without having to evaluate too many of them. First of all, an objective function, S, has to be defined which is design dependent and is related to the merit of the system. For environmental design, this may be minimum overall cost or minimum design cooling load for satisfactory environmental performance when artificial control systems are available, or minimum degree of discomfort for unconditioned buildings. The optimum design problem consists in selecting a vector, P, so that S is optimized subject to eq (16).

The choice of optimization procedures is generally governed by the nature of the function S and the constraint vectors L and U. The environment design problem formulated in sections 2 to 5.1 is a constrained optimization problem with a non-linear objective function and linear inequality constraints. Also, it is desirable not to have to calculate the derivatives of the objective function to suit the methods of analysis adopted. Further, the sensitivity of the optimum solution with respect to perturbations in the design variables is more significant design information at the sketch plan stage rather than the attainment of a global optimum. On account of these considerations, a sequential simplex type search technique {16} has been selected. The search proceeds from an initial point, which may represent the best judgment for the values of design variables in the absence of optimization, or may be generated pseudo-randomly in the feasible space. According to Mitchell and Kaplan {17}, a simplex of points is generated around this initial point and the values of objective function are determined for each of these points by a simulation program incorporating methods of analysis of section 4. The optimization procedure continues through successive changes of the simplex position so that the worst vertex is replaced by another one in a favorable direction {16} in any single move. The process is continued until three successive changes do not modify the value of the objective function at the simplex centroid by more than a desired amount governed by precision. The best point is obtained after a specified number of such iterations, which use the best point from previous iteration as the initial point for the next run. The result of optimization consists in the best value obtained for the objective function, the corresponding optimum design solution comprising a set of values for the design variables and the values of the objective function at the vertices of the simplex around this point.

It is to be noted, however, that the search methods do not guarantee the evolution of a global optimum.

In physical terms, the optimum design values correspond to the desired performance specifications, and variations in the objective functions at the vertices indicate the sensitivity of this performance to the largest permissible perturbations in the specifications, varied singly around the best design solution.

## 6. Demonstration Example

As an example of application of the systems model formulated in sections 2 to 5, the top floor of a multi-storeyed building located in Sydney (latitude 33.8$^{o}$ south, longitude 151.2$^{o}$ east) has been considered for optimization. The objective function has been chosen to be minimum peak cooling load (sensible part only) for a typical climatic design cycle during summer {2}. Criteria for satisfactory indoor environment specified that the indoor environmental temperature be maintained at the preferred temperature for Sydney, 73$^{o}$F with a permissible rise of 3 deg F in the afternoons and an artificial lighting intensity of 75 lumens ft$^{-2}$ to be available on the horizontal work plane in all areas where the design daylight intensity is less than 30 lumens ft$^{-2}$.

An open plan office is considered, with a central service core occupying 10 percent of the floor area. The roof is designed to be a six layered structure with provisions for an acoustic ceiling, 1 ft thick air space, 6 in concrete deck, insulation to be designed, waterproofing layer, and 2 in of concrete topping. The floor is a similar structure except that there is a carpet instead of the three top layers of insulation, waterproofing, and topping. The walls are specified to be rendered inside and have three layers, two of which are design variables. The conditions of occupancy provide for 30 persons on each floor and a fresh air supply rate of three air changes per hour including infiltration. Windows are considered to be provided on all four sides of the building, which is assumed to be located on an exposed site. Nominal values of the other parameters which constitute the set of design variables, and their upper and lower limits, are shown in Table 1.

Table 1. Input values for design variables of an open plan office building

| | Independent Design Variables ($P_i$) | Nominal value | Lower limit | Upper limit |
|---|---|---|---|---|
| 1 | Wall, thickness of outer skin (in) | 4.0 | 3.0 | 7.5 |
| 2 | Wall, penetration coefficient of outer skin (P*) | 300.0 | 180.0 | 330.0 |
| 3 | Wall, thickness of insulation (in) | 1.0 | 0.01 | 2.0 |
| 4 | Wall, penetration coefficient of insulation (P*) | 0.15 | 0.07 | 0.25 |
| 5 | Roof, thickness of insulation (in) | 3.0 | 2.0 | 5.0 |
| 6 | Roof, penetration coefficient of insulation (P*) | 6.4 | 3.0 | 24.0 |
| 7 | Roof, absorptivity | 0.7 | 0.5 | 0.8 |
| 8 | Wall, absorptivity | 0.7 | 0.6 | 0.8 |
| 9 | Window shading coefficient | 0.5 | 0.4 | 0.6 |
| 10 | Aspect ratio (north wall/east wall) | 0.71 | 0.5 | 2.0 |
| 11 | Window, light transmittance | 0.1 | 0.1 | 0.8 |
| 12 | Glazing ratio (glazed area/wall area) | 0.5 | 0.3 | 0.6 |
| 13 | Orientation (true bearing) | 082 | 082 | 082 |
| 14 | Floor area (ft$^2$) | 3264 | 3264 | 3264 |
| 15 | Ceiling height (ft) | 9 | 9 | 9 |
| | Dependent Variables ($Q_j$) | | | |
| 1 | Wall thickness ($P_1 + P_3$) | 5.0 | 2.9 | 8.0 |
| 2 | Window area/floor area | 0.32 | 0.1 | 0.5 |
| 3 | Light/heat ratio ($P_{11}/P_9$) | 0.2 | 0.2 | 2.0 |

* P. Penetration coefficient = thermal conductivity x specification x density (BTU$^2$ in ft$^{-5}$ hr$^{-1}$ deg$^{-2}$F)

The convergence criterion for the optimization program 'Design' is fixed so that if three consecutively occurring design alternatives indicate peak cooling loads within 0.02 tons, the iteration is terminated. Two such iterations have been provided for in the case of this example. The optimized specifications and the sensitivity analysis are arrived at after about 150 design alternatives have been examined by the computer. The CDC 3600 computer uses about 20 minutes of machine time for a complete run of this type. Table 2 contains a set of optimized specifications and values for peak cooling load ratios when each of the variables is successively put equal to the lower and upper limits (Table 1), the others being kept fixed at the optimum level. The peak cooling load for initial nominal design is 6.6 tons and it is reduced by 37 percent for the optimum design. Peak cooling load ratios are the actual peak cooling loads divided by the value for optimum design.

Table 2. Optimization results for an open plan office building

| No. | Performance specifications | Optimum value | Sensitivity analysis (Peak cooling load ratio) | |
|---|---|---|---|---|
| | | | Value at lower limit | Value at upper limit |
| 1 | Wall, thermal resistance of outer skin (R*) | 0.56 | 1.02 | 0.98 |
| 2 | Wall, penetration coefficient of outer skin (P) | 2.81 | 1.00 | 1.00 |
| 3 | Wall, thermal resistance of insulation (R*) | 0.89 | 1.01 | 1.00 |
| 4 | Wall, penetration coefficient of insulation (P) | 0.15 | 1.00 | 1.00 |
| 5 | Roof, thermal resistance of insulation (R*) | 4.6 | 1.06 | 0.98 |
| 6 | Roof, penetration coefficient of insulation (P) | 6.25 | 0.99 | 1.04 |
| 7 | Roof, absorptivity | 0.6 | 0.99 | 1.03 |
| 8 | Wall, absorptivity | 0.68 | 0.99 | 1.01 |
| 9 | Window shading coefficient | 0.53 | 0.96 | 1.16 |
| 10 | Aspect ratio (north wall/east wall) | 1.41 | 0.99 | 1.00 |
| 11 | Window, light transmittance | 0.42 | 1.00 | 1.00 |
| 12 | Glazing ratio (glazed area/wall area) | 0.57 | 1.01 | 1.00 |
| 13 | Orientation (true bearing) | fixed | - | - |
| 14 | Floor area (ft$^2$) | fixed | - | - |
| 15 | Ceiling height (ft) | fixed | - | - |

* R = Thermal resistance = thickness/thermal conductivity (ft$^2$ hr degF Btu$^{-1}$)

## 7. Acknowledgments

Thanks are extended to Mr. J.W. Spencer for help in the computer work and to Mr. E.R. Ballantyne and Dr. R.W.R. Muncey for discussions. All computations were carried out on a CDC 3600 digital computer which is part of CSIRO computer network.

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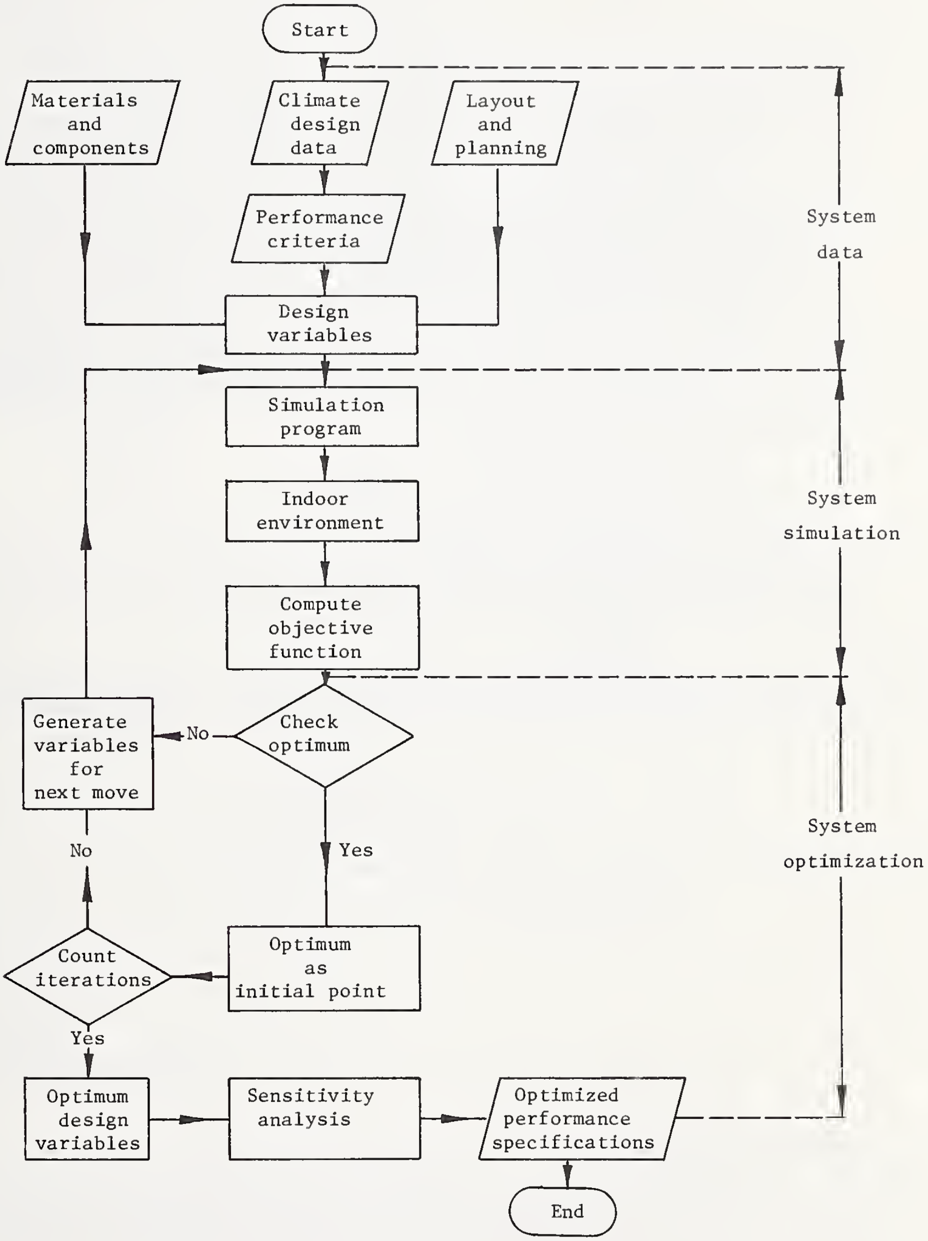

Figure 1 - Building as an environmental system

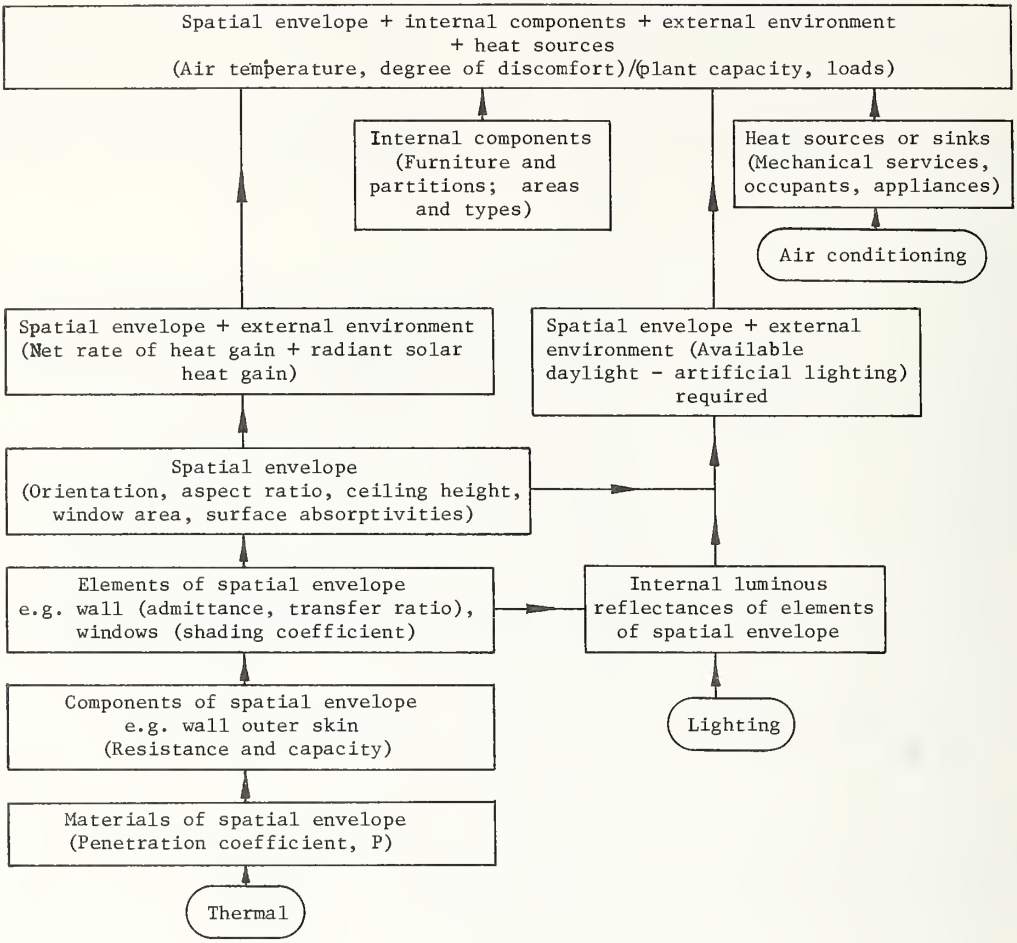

Figure 2 - Hierarchy for environmental performance simulation program

# Design Considerations for a Practical Heat Gain Computer Code

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A digital computer program for heat gain computation is described. Emphasis is placed on the development of engineering and programming design criteria to ensure practicality, flexibility and ease of usage. Fenestrated and opaque surfaces, internal loading, plenum usage, duct losses, ventilation, et cetera are considered. Computational questions are encountered which may form the basis for future analysis and research. Results from the implemented code include the determination of such design conditions as apparatus dew point, mixing and entering air temperatures, leaving and supply air temperatures, design and return air quantities, number of air changes and tonnage.

Key Words: Heat gain computation, design condition computation, cooling load, apparatus dew point, air quantities, system design, zone design.

## 1. Introduction

This paper describes the steps involved in the development of a particular digital computer program to perform heat gain and resultant design condition computations. The program is structured to be of value to the practicing engineer but is not a substitute for engineering experience and knowledge.

Evolved as a part-time project over a period of some three years, the program utilizes much of the presently available knowledge, provides a framework wherein new developments may be quickly implemented and has been thoroughly tested. There are, however, areas where it is felt additional research is needed or where additional capability should be included. These areas form the basis for the future evolution of the code.

## 2. Design Objectives

Before initiating any development of the code, the following design objectives were stipulated:

1) Engineering computations should include methods for the determination of:

   a) fenestration heat gain.
   b) wall or opaque surface heat gain.
   c) internal heat gain.
   d) plenum heat gain.
   e) duct heat gains and losses.
   f) ventilation and exhaust requirements.
   g) leakage.

   The computations should be valid at any site in either the northern or southern hemisphere.

2) The methods employed should be "standard practice" except where formulas might be developed which would represent tabular data to some degree of accuracy in the least squares sense. New developments should only be considered in unusual circumstances.

3) The code structure should be sufficiently flexible to permit the determination of the design conditions within a zone or system given the applicability of any or all of the above heat gain computations.

4) The input should be minimal consistent with the desires of the user for various capabilities and should be structured to reduce the chances of error.

5) The results obtained from the code should be sufficiently detailed to permit hand computation for checking purposes and to augment engineering experience.

6) The code structure should be sufficiently flexible to permit rapid changes, additions, or deletions with a minimum of impact on the code, thereby enabling the code to keep pace with new developments and techniques.

In addition a guideline was established whereby the criteria for selection between various methods was practicality, i.e., if a particular technique was simpler or easier to implement and did not possess any distinct advantage with respect to the final result obtained, then this technique was to be preferred.

## 3. Engineering Methods

Because of the number of algorithms employed to perform the engineering computations, only the more significant techniques will be mentioned. A more detailed presentation will be found in reference 8.

### 3.1 Preliminary Computation

The code employs the U.S. Standard Atmosphere, 1962 [7][1] to determine atmospheric pressure and density at a given altitude. Solar data and sol-air temperatures are determined using the methods of references 6 and 9.

Surface heat transfer film coefficients are determined from the following relations:

Internal film coefficient [4]

$$H_i = 0.0826 \cos^2 \tau + 0.2954 \cos \tau + 1.078 \quad , \tag{1}$$

where $H_i$ is the internal film coefficient (Btu $hr^{-1}$ $ft^{-2}$ $°F^{-1}$),
$\tau$ is the tilt angle of the surface (radians from vertical).

External opaque surface film coefficient [1,2]

$$H_e = 2 + 4 W/15 \tag{2}$$

where $H_e$ is the external opaque surface film coefficient (Btu $hr^{-1}$ $ft^{-2}$ $°F^{-1}$),
$W$ is the wind velocity (miles $hr^{-1}$).

External fenestration film coefficient [6]

$$H_f = - 0.00125 W^2 + 0.262 W + 1.45 \quad , \tag{3}$$

where $H_f$ is the external fenestration film coefficient (Btu $hr^{-1}$ $ft^{-2}$ $°F^{-1}$),
$W$ is the wind velocity (miles $hr^{-1}$).

It will be noted that the selection with respect to the external opaque surface film coefficient does not include a factor for surface roughness. On investigation it was found that the best available data for this coefficient [1,6] involved a subjective judgement on the part of the user which

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[1] Figures in brackets indicate the literature references at the end of this paper.

could cause wide variation in the determination of the coefficient[2]. In addition the data did not cover the full spectrum of today's surfaces. It is strongly recommended that future research be undertaken which will more exactly quantify this term.

### 3.2 Heat Gain Computation

#### 3.2.1 Fenestrated Surfaces

Heat gain through a fenestrated surface is found from the relation

$$H_g = A_g \left| U(t_o - t_i) + C[S_A S_L D_N (T_D + N_o A_{D_o} + N_i A_{D_i}) + D_f (T_d + N_o A_{d_o} + N_i A_{d_i})] \right| , \qquad (4)$$

where $H_g$ is the fenestration heat gain (Btu $hr^{-1}$),

$A_g$ is the fenestration area ($ft^2$),

$U$ is the fenestration heat transmission coefficient (Btu $hr^{-1}$ $ft^{-2}$ $°F^{-1}$),

$t_o$ is the outdoor dry full temperature (°F),

$t_i$ is the indoor dry full temperature (°F),

$C$ is a composite correction factor for haze, altitude, and internal shading [3],

$S_A$ is a sash or frame correction factor,

$S_L$ is the sunlit fraction of the fenestration area,

$D_N$, $D_f$ are the direct and diffuse incident solar intensity respectively (Btu $hr^{-1}$ $ft^{-2}$),

$N_o$, $N_i$ are the inward flowing fractions of the absorbed radiation through the outer and inner panes respectively,

$A_{D_o}$, $A_{D_i}$ are the incident absorption coefficients for the outer and inner panes respectively,

$A_{d_o}$, $A_{d_i}$ are the diffuse absorption coefficients for the outer and inner panes respectively,

$T_D$, $T_d$ are the incident and diffuse transmission coefficients respectively.

The determination of the absorption and transmission coefficients is via a table look-up procedure utilizing the data of reference 2.

The determination of the sunlit fraction of the fenestration area utilizes a generalization of the procedure of Tseng-Yao Sun [9] to permit the use of tilted window surfaces. Adopting the same notation as Sun, the shadow area depth is computed from the relation

$$T = P \frac{\tan\beta - \cos\gamma \cot\delta}{\cos\gamma + \tan\beta \cot\delta} , \qquad (5)$$

where T is the shadow area depth (in.),

P is the projection outward normal to the window surface (in.),

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[2] However, preliminary analysis of the data showed that two of the coefficients were related by the relation

$$A = .004 C - .0078 ,$$

and that the coefficient B might be related to the emissivity E by the expression

$$B = .45787 E^{5.03367} .$$

Here A, B, and C are as defined in reference 6.

$\beta$ is the solar altitude (radians),

$\gamma$ is the wall-solar azimuth (radians),

$\delta$ is the counter-clockwise angle between the horizontal and the outward normal to the surface (radians),

In addition provision has been incorporated in the code to eliminate the possibility of overlapped shadow areas.

### 3.2.2 Opaque Surface Heat Gain

An extension of the method of H. A. Johnson [5] is utilized which enables application of the technique to heterogeneous structures. To make the extension, the homogeneous conditions postulated by Johnson were approximated by computing "equivalent parameters" for specific heat, wall thickness, thermal conductivity and specific weight as follows:

Specific heat

$$C_e = \left[ \left( \sum_i W_i X_i \right) \sum \frac{1}{C_i W_i X_i} \right]^{-1} \quad . \tag{6}$$

Wall thickness

$$X_e = \sum_i X_i \quad . \tag{7}$$

Thermal conductivity

$$K_e = \frac{X_e}{\sum_i (X_i/K_i)} \quad . \tag{8}$$

Specific weight

$$W_e = \left( \sum_i W_i X_i \right) \Big/ X_e \quad . \tag{9}$$

where $X_i$ is the thickness of material i (ft),

$W_i$ is the specific weight of material i (lbs $ft^{-3}$),

$C_i$ is the specific heat of material i (Btu $lb^{-1}$ $°F^{-1}$),

$K_i$ is the specific conductivity of material i (Btu $ft^{-1}$ $hr^{-1}$ $°F^{-1}$).

Hence the thermal diffusivity, $A_e$, of the wall is given by

$$A_e = \frac{K_e}{C_e W_e} = \frac{\left( \sum_i X_i \right)^2 \sum_i (X_i W_i C_i)^{-1}}{\sum_i (X_i/K_i)} \quad . \tag{10}$$

The heat gain is then given by

$$H_\omega = A \left\{ U(T_{S_m} - T_i) + H_i \sum_n T_{S_n} B_n \cos(\omega_n t - \beta_n - \delta_n) \right\} \quad , \tag{11}$$

where A is the wall surface area ($ft^2$),

U is the overall heat transfer coefficient (Btu $ft^{-1}$ $hr^{-1}$ $°F^{-1}$),

$T_{S_m}$ is the mean sol-air temperature obtained from the Fourier analysis (°F),

$T_i$ is the room temperature (°F),

$H_i$ is the interior air surface film transfer coefficient (Btu $ft^{-2}$ $hr^{-1}$ $°F^{-1}$),
$T_{s_n}$ is the nth Fourier coefficient (°F),
$\omega_n$ is the nth harmonic frequency from the Fourier analysis (cycles $hr^{-1}$),
$\beta_n$ is the nth harmonic phase angle from the Fourier analysis (radians),
t is the time measured from midnight (hr).

$B_n$ and $\delta_n$ are factors determined from the expressions

$$B_n = d_n \Big/ \sqrt{f_n^2 + g_n^2} \quad , \tag{12}$$

$$\delta_n = \tan^{-1}(g_n/f_n) \quad , \tag{13}$$

where

$$d_n = \frac{K_e}{H_i} \sqrt{\frac{\pi n}{24A_e}} \qquad n = 1, 2, \ldots \quad , \tag{14}$$

$$\begin{aligned} f_n &= a_1 \cosh\eta \cos\eta + \frac{1}{2} (\sinh\eta \cos\eta + \cosh\eta \sin\eta) \\ &\quad + a_2 (\sinh\eta \cos\eta = \cosh\eta \sin\eta) \quad , \end{aligned} \tag{15}$$

$$\begin{aligned} g_n &= a_1 \cosh\eta \cos\eta - \frac{1}{2} (\sinh\eta \cos\eta - \cosh\eta \sin\eta) \\ &\quad + a_2 (\sinh\eta \cos\eta + \cosh\eta \sin\eta) \quad . \end{aligned} \tag{16}$$

Here

$$a_1 = d_n + \frac{K_e}{H_o} \sqrt{\frac{\pi n}{24A_e}} \quad , \tag{17}$$

$$a_2 = d_n \frac{K_e}{H_o} \sqrt{\frac{\pi n}{24A_e}} \quad , \tag{18}$$

$$\eta = X_e \sqrt{\frac{\pi n}{24A_e}} \quad . \tag{19}$$

### 3.2.3 Internal Heat Gain

The internal heat gain is computed from the input sensible and latent heat data according to the relation

$$H_L = \sum_i d_{V_i} H_{M_i} \quad , \tag{20}$$

where $H_L$ is the internal heat load (Btu $hr^{-1}$),
$d_{V_i}$ is the diversity factor for load i,
$H_{M_i}$ is the maximum internal heat for load i.

### 3.2.4 Plenum Heat Gain

The code provides for the computation of heat gain from that portion of the internal load entering the plenum directly and from the transfer of heat from either the space or through the exterior building structure. The impact of the air movement through the plenum is not accounted for at present.

### 3.2.5 Duct Heat Gain

Two methods are provided for the user. The first employs percentages of room sensible and room latent loads to estimate the duct heat gains or losses. The second involves actual physical data concerning the duct's construction and location as may be seen from the relation utilized to determine the heat gain or loss, namely

$$H_D = UPX(\Delta t) \frac{28.8\ AV}{28.8\ AV\rho + PX} , \qquad (21)$$

where $H_D$ is the duct heat gain (Btu $hr^{-1}$),

$U$ is the heat transfer coefficient (Btu $ft^{-1}$ $hr^{-1}$ $°F^{-1}$),

$P$ is the duct perimeter (ft),

$X$ is the duct length (ft),

$\Delta t$ is the temperature difference between the surrounding environment and the air entering the duct (°F),

$A$ is the cross sectional area of the duct ($ft^2$),

$V$ is the average duct air velocity (ft $min^{-1}$),

$\rho$ is the density of the air (lbs $ft^{-3}$).

### 3.2.6 Ventilation and Exhaust Requirements

Ventilation requirements are imposed via the relations

$$O_s = 1.08\ A(t_o - t_i) , \qquad (22)$$

$$O_L = 0.68\ A(w_o - w_i) , \qquad (23)$$

where $O_s$ is the outdoor sensible air load (Btu $hr^{-1}$),

$O_L$ is the outdoor latent air load (Btu $hr^{-1}$),

$A$ is the amount of outside air required (cfm),

$t_o$ is the outdoor dry bulb temperature (°F),

$t_i$ is the indoor dry bulb temperature (°F),

$w_o$ is the outdoor moisture content (grains $lb^{-1}$),

$w_i$ is the indoor moisture content (grains $lb^{-1}$).

Utilizing this information as well as other information, an initial estimate is made of the return air required. The air loads are then modified to reflect the percent (input) of the return air which is to be exhausted.

### 3.2.7 Leakage

At the present time the amount of leakage is specified as an infiltration quantity via input. An extension to accommodate a more exact evaluation of this quantity based on crack length, door usage, shaft and stack effects, et cetera, is under development.

### 3.3 Design Condition Computation

To determine the zone and system design conditions, the program utilizes a mathematical formulation whose exact structure is proprietary. However, because any such formulation must have an analog with the more traditional graphical technique, it is convenient to describe the process in these terms.

Briefly, referring to figure 1, the apparatus dew point, $T_A$, is found at the intersection of the saturation curve with the effective sensible heat factor (ESHF) line, the mixture point is found at the intersection of the grand sensible heat factor (GSHF) line with the line joining the indoor and outdoor design points, and the supply air point is found at the intersection of the GSHF line with the room sensible heat factor (RSHF) line. Note, however, that even under normal conditions the temperature of the supply air, $T_s$, will not be coincident with that of the leaving air, $T_L$, and hence a separate computation for this point is included.

The program treats several abnormal conditions which may be categorized as follows:

a) $T_i - T_s > \Delta T_{max}$ - the amount of sensible reheat is determined such that $T_i - T_s \leq \Delta T_{max}$ .

b) The ESHF line fails to intersect the saturation curve - a value of $T_A$ is selected such that $T_i - T_s \leq \Delta T_{max}$ and the amount of reheat required is computed.

c) The RSHF line fails to intersect the saturation curve - the amount of dehumidification required to make the ESHF line tangent to the saturation curve is determined and then if further refinement is required, the amount of reheat is computed so that $T_i - T_s \leq \Delta T_{max}$.

Obviously the above actions in response to the conditions mentioned will not satisfy all designers but printout of the reheat and dehumidification required will serve as an indication of the trouble encountered. The designer can then take whatever action he considers best.

## 4. Program Structure

In order to meet the design objective of minimal input consistent with conditions and to provide the maximum flexibility in use, modification, and extension, a modular approach was adopted not only for the computational sequence but also for the input. To accomplish the latter a set of 25 key words were defined which when coded on cards enable a structuring or blocking of the input stream. A particular block can then be included or omitted as conditions dictate. At the present time these words are as follows:

Basic Data

| | |
|---|---|
| TITLE | Titling information data block |
| ENVIRON | Exterior environment data block |
| DESIGN | Interior environment data block |
| ORIENT | Building orientation data block |
| GLASS | Fenestration data block |
| SASH | Sash data block |
| PROJECT | Window projection data block |
| MATERIAL | Building material data block |
| CONSTRUCT | Building construction data block |
| DUCT | Duct construction data block |
| DIVERSITY | Diversity schedule data block |

Configuration Control

| | |
|---|---|
| SYSTEM | System definition block |
| ZONE | Zone definition block |

Space Data

| | |
|---|---|
| AIR | Outside air load data block |
| INTERN | Space internal loads data block |
| WINDOW | Window data block |
| BUILD | Building surface data block |
| PLENUM | Return plenum data block |
| RETURN | Return duct data block |
| SUPPLY | Supply duct data block |

Execution Control

| | |
|---|---|
| COMPUTE | Initiate computation |
| UPDATE | Data update or parametric analysis |
| NEXT | Next building analysis |
| STOP | Termination of computation |

Referring now to figure 2 we see that after initialization the program scans for key words during the input process, placing these key words and their associated data on auxiliary storage until a COMPUTE card is encountered. This set of data forms the base line data case against which all subsequent updating or parametric studies may be conducted. Since updating is accomplished dynamically during execution, the base line data case is preserved until a NEXT or STOP card is encountered.

It will also be noted in figure 1 that the input data stream is segregated into basic data, space data and the two types of control functions permitting a preliminary scan of the data ordering to minimize chances of computational failure. It will be further noted that the actual data input is accomplished via selection of a subroutine and its execution. Each subroutine reads and prints the input data, performs whatever preliminary computation may be required and places the resulting data on auxiliary storage.

Figure 3 shows the computational control sequence for the base line data case. The sequence is designed to further ensure that the proper ordering of the input data has been accomplished. However, it will be noted that it is possible to make a change in the basic data during execution of the case, providing, for example, the capability to alter the interior design conditions within a particular system or within a particular zone. Note also that the supply or return ducts require special treatment, this treatment being necessitated by the fact that the data input for these data blocks may contain percentages of room heat to determine the various heat gain or loss quantities.

The subroutines selected perform fenestration heat gain, opaque surface heat gain, plenum heat gain, internal heat gain, et cetera. Following the determination and printout of the relevant heat gain contributions, a table of the various sensible and latent heat factors is generated and printed for the user's information. A design point is selected which reflects the maximum heat gain and the design conditions determined using a proprietary psychrometric process. Printout of these conditions is given for each individual zone, if a multiple zone system, and for the total system.

## 5. Application

The program has been applied to a variety of structures, the following being typical:

A three story department store is located at 47.5° N. latitude, 122.3° W. longitude and at an altitude of 300 feet above mean sea level. The building consists of a basement with an exposed loading dock, a first floor sales area, and a second floor sales, stock, phone and office areas. The orientation of the structure is as shown in figure 4. The construction is principally brick and concrete with a single glass entrance shaded by an overhang.

The building was divided into four systems with the basement and first floor being treated as single zoned systems S1 and S2, respectively, and the second floor being treated as two multi-zoned systems, S3 and S4. Input to the program specified not only the location and orientation but the weather conditions for the design day of August 21, the interior design condition of 74°F dry bulb and 62°F wet bulb, the detailed cross sections of the construction, the diversity schedules, and the internal peak load conditions. Duct factors, plenum conditions and required ventilation were also input as required by the designer.

Typical of the output is that shown in figures 5 and 6 for system S2. Note that the various sensible and latent loads are given on an hourly basis. The design point occurred at 5:00 P.M. when the maximum grand total heat was reached. Since the system was of the draw-through type, the mixture and entering coil temperatures were the same. Note also that the difference between the supply air temperature and the space design temperature is less than the 23°F maximum difference which the designer specified.

The results, in general, agreed with the hand computations, especially with respect to the computations pertinent to the psychrometric process where it is felt that the code produced a more reliable result than the traditional graphical technique.

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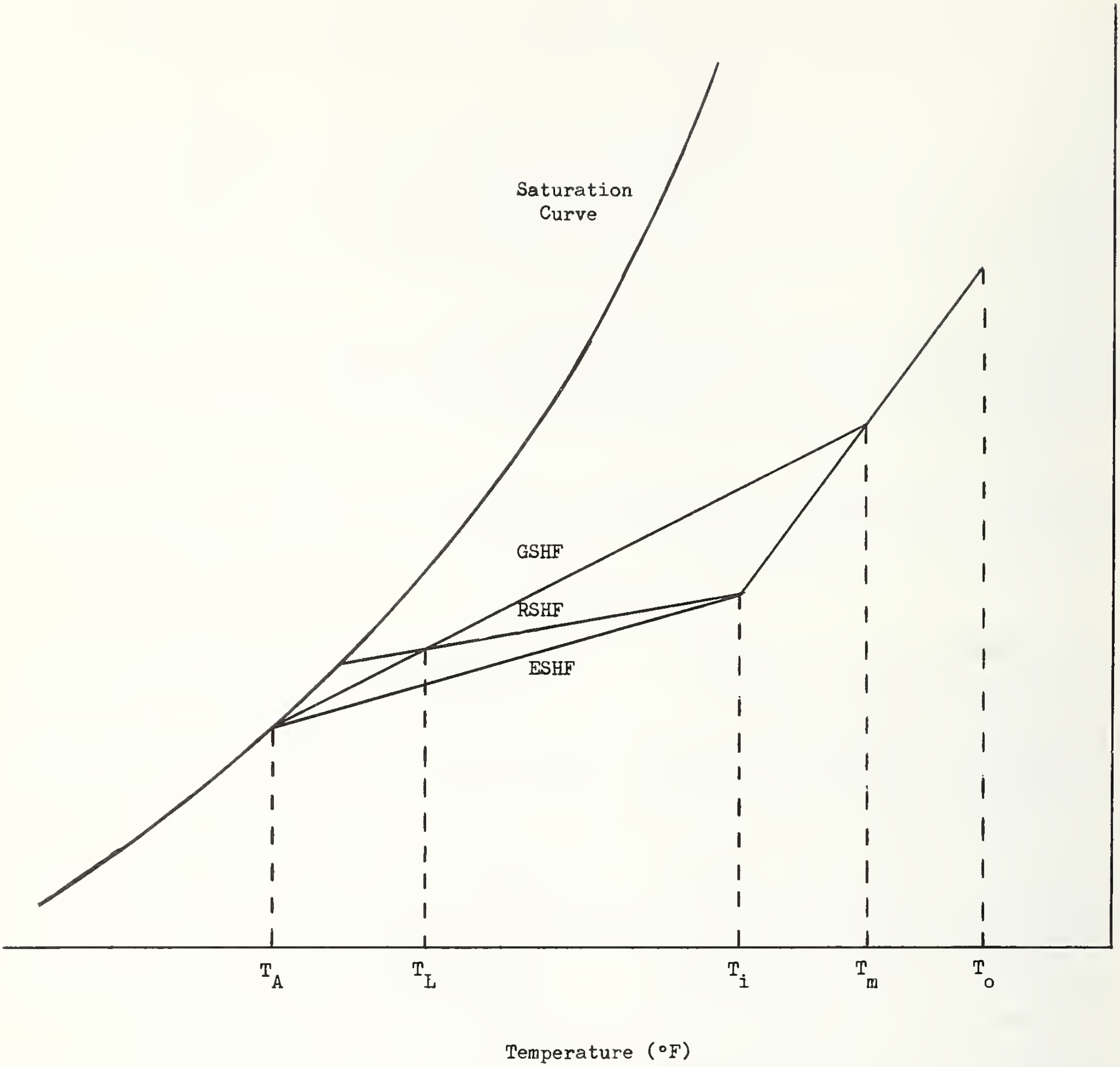

Figure 1

Typical Psychrometric Process

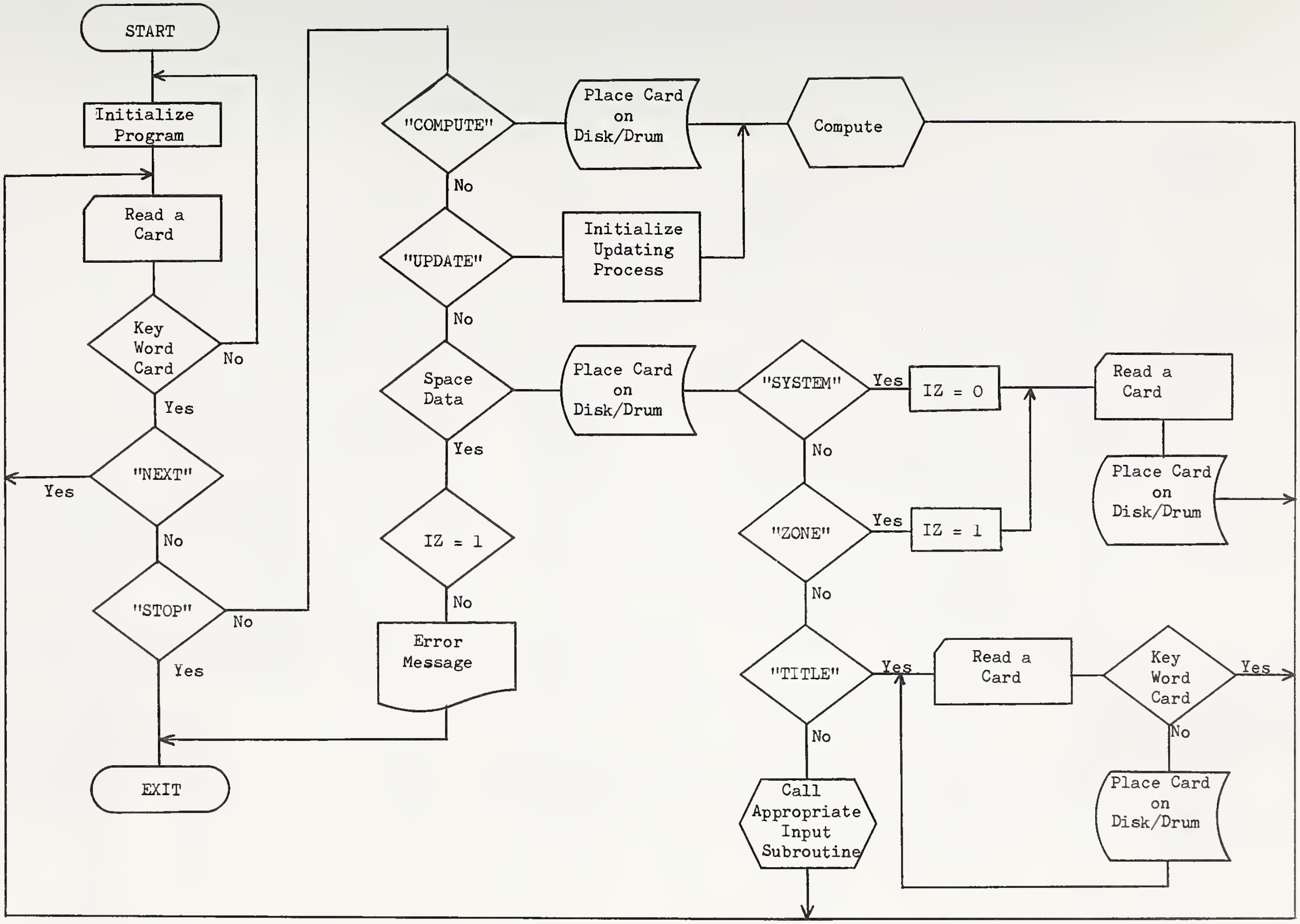

Figure 2
Input Control

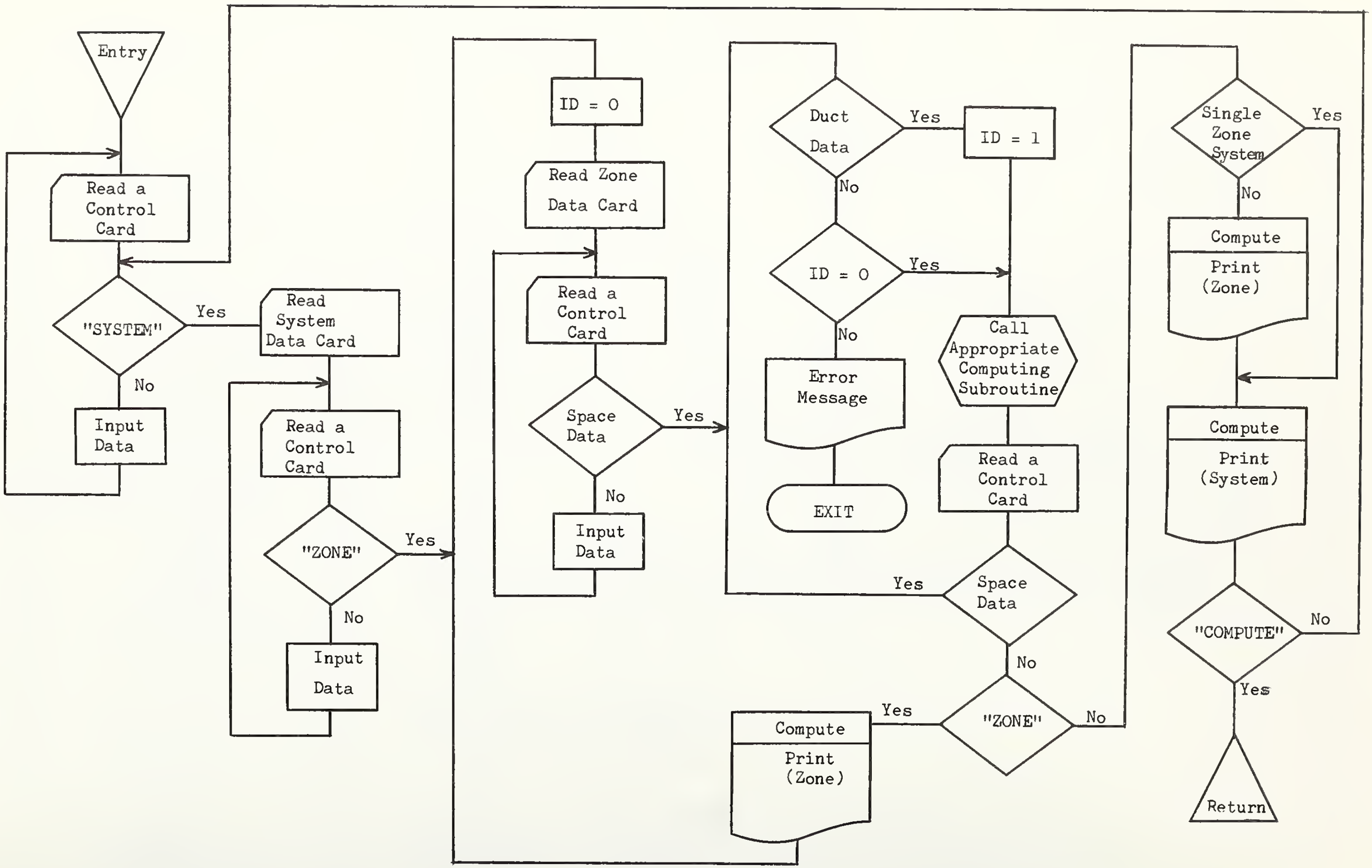

Figure 3

Computational Processing of a Basic Data Case

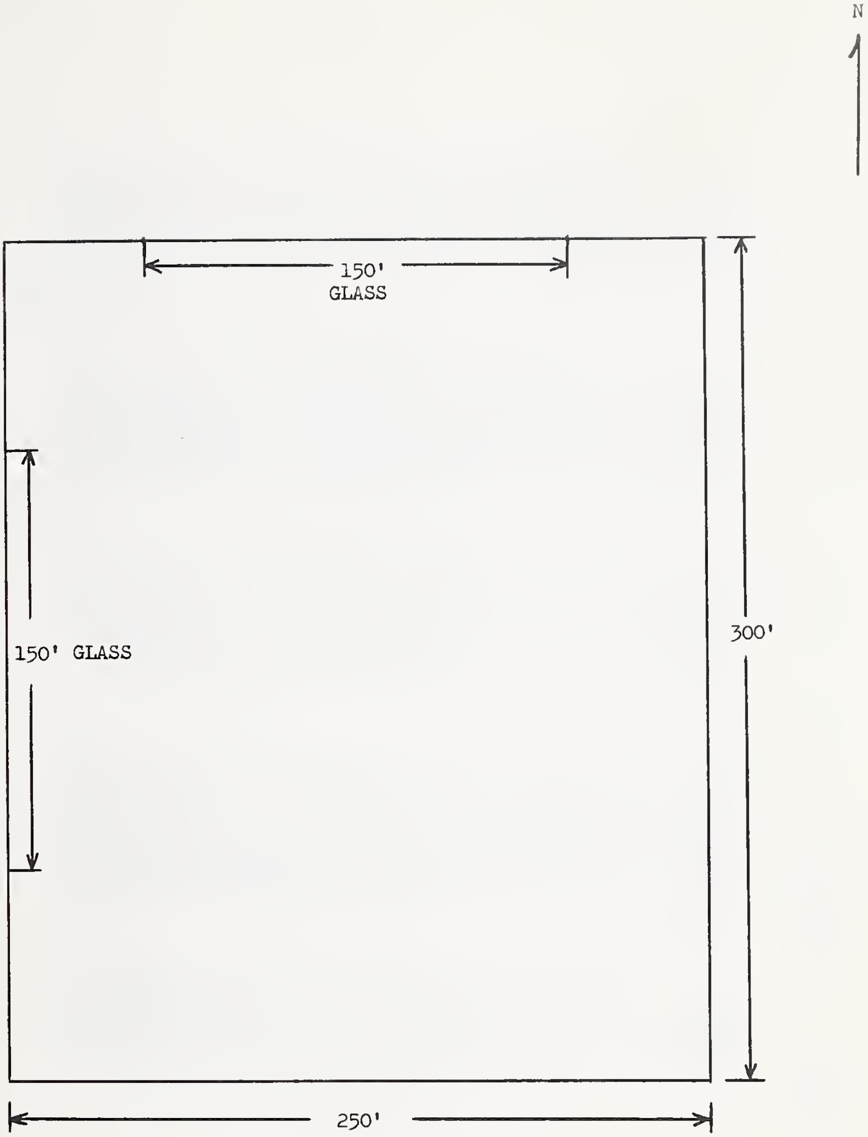

Figure 4

Building Orientation

```
Output for System S2          Volume =     1008000.000

     System Heat Conditions

          Hour     Room             Room      Effective      Effective       Total          Grand
                 Sensible          Latent       Room           Room        Sensible         Total
                   Heat             Heat      Sensible        Latent         Heat           Heat
                                                Heat           Heat

           1     131256.313          0.0     146475.688          0.0      152514.563     152514.563
           2     106979.313          0.0     119528.250          0.0      124477.125     124477.125
           3      88458.375          0.0      98970.000          0.0      102957.375     102957.375
           4      73709.938          0.0      82599.250          0.0       85960.688      85960.688
           5      58718.156          0.0      65958.438          0.0       68826.875      68826.875
           6      57360.254          0.0      64451.164          0.0       66554.875      66554.875
           7     231609.875     37642.449     257868.188     38771.719     261985.813     300757.500
           8     585627.000     75284.813     643051.125     85511.875     583041.375     740270.750
           9    1138608.000    112927.250    1262691.000    118304.938    1266320.000    1402533.000
          10    1294804.000    188212.313    1439956.000    197610.500    1483022.000    1714399.000
          11    1424106.000    263497.125    1589313.000    269185.750    1689260.000    1938499.000
          12    1580504.000    338782.063    1764859.000    350603.750    1887446.000    2252973.000
          13    1621577.000    376424.688    1814338.000    391388.813    1975023.000    2399454.000
          14    1675015.000    376424.688    1877542.000    387410.750    2076550.000    2461200.000
          15    1723229.000    376424.688    1933004.000    385423.813    2152354.000    2517135.000
          16    1764271.000    376424.688    1976616.000    387410.750    2180872.000    2565522.000
          17    1788126.000    376424.688    2003095.000    387410.750    2209062.000    2593712.000
          18    1758902.000    376424.688    1968713.000    383413.500    2156769.000    2501447.000
          19    1688848.000    376424.688    1887064.000    387386.813    2037291.000    2421702.000
          20    1643076.000    376424.688    1832369.000    391365.875    1944309.000    2368510.000
          21    1536892.000    338782.063    1710618.000    356579.000    1783348.000    2208628.000
          22    1169106.000    112927.250    1298488.000    116315.063    1327536.000    1443851.000
          23     321216.563     37642.449     356553.938     39567.992     360743.813     407478.188
          24     156258.438          0.0     174228.063          0.0      181380.000     181380.000
```

**Figure 5**

System S2 Heat Quantities

```
Output for System S2                     Volume = 1008000.000

System Design Conditions

     Time of Peak Load  1700 Hours

     Outdoor Dry Bulb Temperature =                    83.000

     Grand Total Heat =                           2593712.000

     Design Air (Variable Volume System) =          92747.563

     Return Air (Variable Volume System) =          74747.563

     Design Air (Constant Volume System) =          92747.563

     Return Air (Constant Volume System) =          74747.563

     Apparatus Dew Point Temperature =                 51.781

     Mixture Temperature =                             76.285

     Entering Coil Temperature =                       76.285

     Leaving Coil Temperature =                        54.231

     Supply Air Temperature =                          56.149

     Number of Air Changes Per Hour =                   5.521

     Tonnage =                                        216.143
```

Figure 6

System S2 Design Conditions

# Solving the Communication Problem in A Computer-Controlled Environmental System

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The use of computers in the control and supervision of environmental and other systems for large buildings and complexes is just getting under way.

A fundamental problem in this type of application is the transmission of data to and from the computer. The traditional methods of hardwiring, frequency multiplexing and electro-mechanical multiplexing are not satisfactory in a computer environment, or in any complex, large-scale dynamic environment.

Both digital and analog data are needed in the control system. Digital data can be handled faster and more accurately than analog data in both the communication system and the computer. Analog-digital and digital-analog converters are needed, with all information being transmitted in digital form. Then each data item becomes a series of bits in a stream of digital bits, and can be transmitted over any type of digital communication system.

This paper describes a communication system utilizing "time-division" in which each discrete sensor and control element is assigned a unique time address in a high-speed digital bit stream. By this means, the problem of addressing and communication is greatly simplified.

It is noted that this approach is made possible by the recent advances in microcircuit technology which makes it economically possible, for example, to use individual A/D converters for each analog sensor.

The concept of approaching the "control problem" as a "communication problem" should make it easier to analyze and design large computer-operated systems.

Key Words: Control, digital communication system, environmental system, multiplexing, supervisory control, time-division address.

## 1. Introduction

There is a growing consideration of the use of computers for control of large and/or complex systems, such as environmental systems in buildings or complexes. A few such systems have been installed and others are being designed.

Central supervisory systems with or without computers become virtually essential for adequate monitoring and control in large institutional or commercial complexes. They are justified economically on the basis of improved visibility and control as well as reduction in personnel requirements and lower operating costs.

The use of computers with supervisory systems increases the speed of data acquisition and simplifies data reduction. If the computerized system is properly designed, much of the start-up, shut-down and reset programming can be done automatically through computer-executed programs.

Since most large building complexes are dynamic and expandable the supervisory system must also be dynamic and expandable.

A careful analysis of such systems indicates that a basic difficulty is the need to deal quickly, accurately and efficiently with large quantities of data. For example, in a typical industrial complex with 2,000,000 square feet of air-conditioned floor space, proper monitoring and control requires communication with about 3000 sensor and control devices. Some of these must be monitored regularly, others only intermittently.

This paper considers various traditional methods of data communication, and concludes that a digital system using time-division addressing may be the best approach.

## 2. History of Supervisory Control Systems

Central supervisory controls for environmental systems are a comparatively recent development. Design changes have been evolutionary, rather than revolutionary, dictated by the increasing size and complexity of the buildings.

The first such systems were simply extensions of the control wiring (or piping) from a few systems to a central point (fig. 1). Since separate, permanent connections are required for each control function, this is usually termed a "hardwired" system. This approach is satisfactory so long as the subsystems to be coordinated are few in number and closely related geographically.

As the number of subsystems increases and they become more widely separated, the cost and complexity of the "hardwired" approach makes it necessary to look for alternatives. Multiplexing, in one form or another, is such an alternative. The basic idea of multiplexing is to use a common bus or a set of "common function" wires which serve all the subsystem control devices. Some means of "addressing" is necessary to select each station in turn for monitoring and control operations. The devices which perform the addressing and selecting functions are called multiplexers (fig. 2). Multiplexers take various forms depending on the concept and the type of data being handled. The more common forms are frequency multiplexing and electro-mechanical and solid state multiplexing.

The frequency multiplexer uses a set of discrete frequency carrier waves, one frequency for each station to be addressed, on which the signals to be sent and received are superimposed. This is, of course, subject to error due to variations in frequency or voltage at the power source, and extraneous "noise" due to frequencies from other sources.

Another type of scheme uses a series of pulses as the carrier system. The number of pulses per unit time, or the elapsed time between pulses can be varied to correspond with the signal value. These pulse groups can be combined to form a continuous stream, with divisions between individual signals indicated by some sort of a coding system.

Electro-mechanical and solid state multiplexers use relays which will, in response to a proper address signal, connect the common function wires to the subsystem desired. This operates reliably, but is comparatively slow.

The first central supervisory systems to use computers simply patched an analog computer into the manual control board and used the computer for monitoring and data acquisition (fig. 3). It was obvious from the beginning that the analog computer could also be used for control. However, the digital computer is preferable for control purposes, although there are many software problems, including those associated with multiplexing and demultiplexing.

## 3. The Nature of Control Data

Data in an environmental control system are both analog and digital in nature. Analog signals are associated with temperature, humidity, flow, pressure and modulating position, while digital signals are associated with two-position, on-off, start-stop, go-no go, and similar functions. If these data are handled on a manual basis, as they are in most presently installed supervisory systems, then speed is not important and data can be transmitted in either form. If, however, we wish to use a computer-controlled communication system, then digital data should be used. Digital data can be transmitted at much higher speeds, and with greater accuracy, than analog data. It then becomes necessary to provide analog to digital (A/D) or digital to analog (D/A) converters for the analog-type sensors and controllers; thus the communication system need handle only digital data.

Only a small amount of data is derived from any single function, and this can be handled at rates as low as 120 bits per second (BPS) or less. The computer, however, operates at speeds up to several million BPS. It is therefore necessary to provide hardware to make these bit speeds operate together. One such system which is used is called time-division multiplexing.

## 4. Time-Division Multiplexing

Time-division multiplexing is defined by its name. A high-speed, repeated, digital bit stream is generated by the computer and fed into the main communications bus (usually a coaxial cable). At each device, or group of devices, a coupler is connected to the main bus in such a way that it interfaces with a small portion of this bit stream (fig. 4). A typical "slice" is one word, and consists of the same word each time. That is, the coupler is identified by a particular time segment (or slot) in the repeated bit stream. Another word is identified with a second coupler and so on. In one system, the bit rate of the coupler is 4800 bits per second. This is derived by repeating the 32-bit data word 150 times a second. The minimum bit rate required on the main bus is then 4800 BPS times the number of couplers on the bus. The actual bus bit rate is usually greater than this minimum to provide for buffering and synchronization.

In this system the coupler address is said to be "strapped," that is, a particular time slot is permanently assigned to that coupler and the computer identifies the data as coming from that coupler by its location in the repeated bit stream. Strapping is essentially a hardware function and is used to reduce software. We noted earlier that most control devices require 120 BPS or less. Therefore, it is not necessary to use a separate 4800 BPS coupler for each device. But we can serve several control devices from this one coupler by using a secondary bus with low speed device connectors to reduce the bit rate still further (fig. 5). If these connectors are also strapped, no further addressing is necessary.

Many control devices require regular monitoring and a permanently strapped connection. But some devices may require only infrequent service and a permanent connection is not required. By assigning a group of connectors to a larger group of devices and providing a "switching" program, these devices can be connected (addressed) as required with a reduction in hardware, but with an increase in software to take care of the switching. A certain amount of overhead is necessary to provide addressing, but it is much less where part of the addressing capability is inherent in the hardware. Providing switching/addressing routines in the operating programs increases the overhead and slows down the operation. A trade-off study must be made in each case to determine the most economical method.

In a similar manner, a single A/D or D/A converter can be made to serve several devices by multiplexing techniques. This is feasible because these converters now operate at high speeds. It is possible to provide a "free-running" A/D conversion system which continuously scans several analog signals and stores the present values of those signals in a set of digital storage registers. The computer operating program then provides for reading the contents of the registers at timed intervals.

While time-division multiplexing may be programmed into almost any general-purpose computer, it would be very wasteful of software and storage capability. It is preferable and desirable to use a computer system which has the time-division addressing and data handling included as part of the basic hardware-software package. The operating programs then become much simpler to write and debug.

The operating program must be written for each specific application. However, this too can be simplified by the use of a high-level language, such as ATLAS, which uses a fairly simple English syntax and covers a broad spectrum of test and control functions. Of course, any high-level language requires a compiler resident in the computer. This, in turn, implies a fairly large computer, but greatly reduces the cost of the software, both for the original program and for changes and additions.

The time-division multiplex system thus described is made possible by the recent advances in microcircuit technology, which allow the design of complex circuits in small packages and at low cost. We expect to soon see most of the necessary hardware contained in small MOS/LSI chips at a cost of less than $100 each. By using this approach, the computer is relieved of complex addressing operations and is free to concentrate on the business of control.

## 5. Conclusion

We believe that the effectiveness of a computer-controlled system depends largely on fast and efficient communication. A time-division multiplex system designed to be integral with the operating computer system provides the best, simplest and most reliable communication available today. This system also allows for connection by a simple coaxial cable, and is easy to expand, requiring only additional interface hardware, extension of the coax bus, and small additions to the operating program as more devices are added.

## Appendix

Time-division addressing systems may be "bit-interlaced" or "word-interlaced." Figure 6 shows a typical main bus "frame" for a bit-interlaced system. This contains 16 channels (bits) per frame, with the first bit being amplitude-coded for synchronization. A coupler may be strapped to read one specific bit (time slot) from each frame as in figure 7. When 36 frames have passed the coupler, 36 bits have been extracted to form a complete word. (This number 36 is arbitrary, and varies to suit the manufacturer. A 36-bit word, as shown, provides four supervisory bits and 32 bits of data. The supervisory bits identify the type of data being transmitted.)

Figure 8 shows a word-interlaced system, in which the coupler is strapped to identify and read one specific word out of a frame which contains 256 words or channels. (Again, the number 256 is arbitrary.)

With either system, it can be seen that the time slot is identified with a specific coupler and that all channels in the frame are serviced essentially simultaneously. Thus the system is ideal for real-time, on-line control.

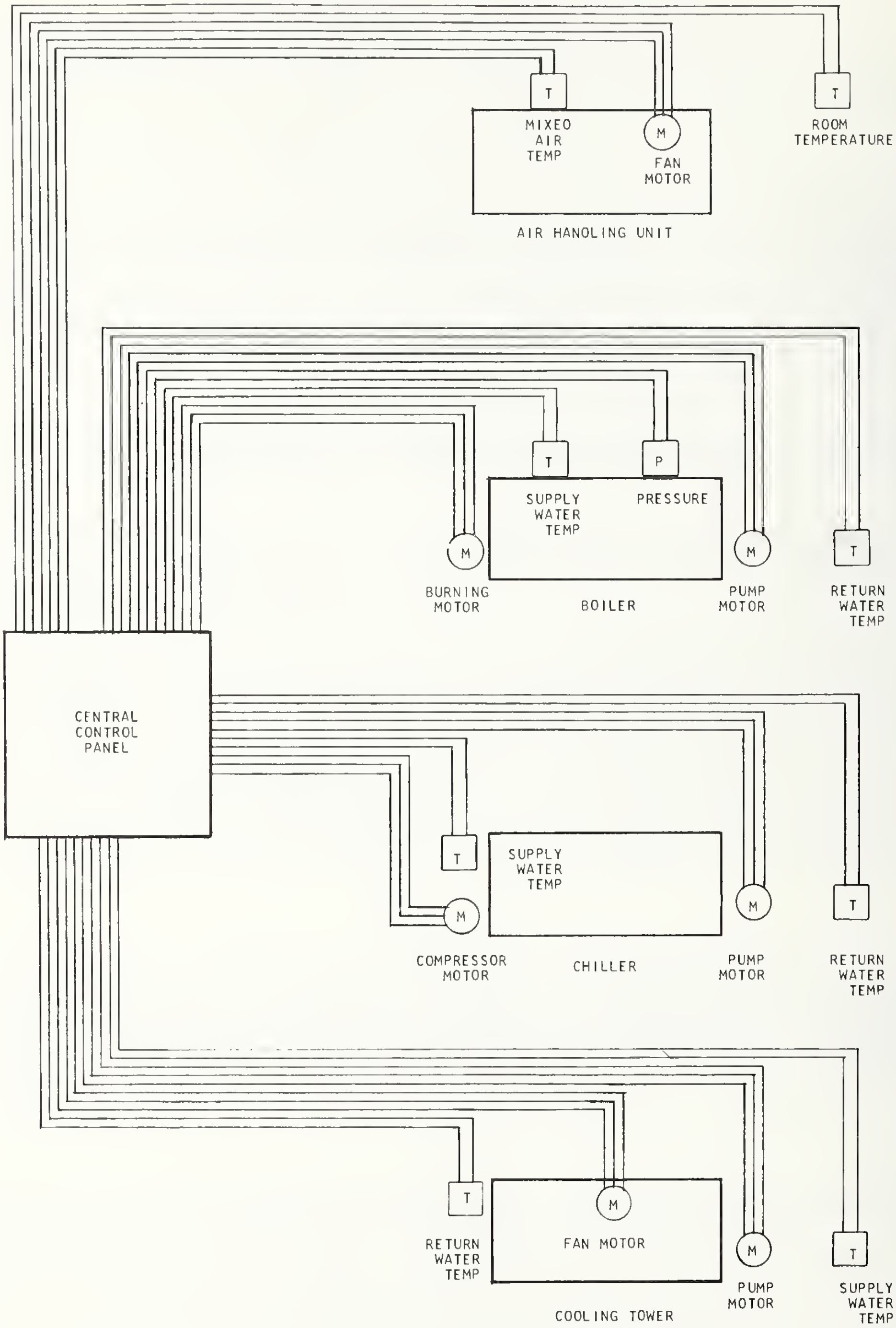

Figure 1. Hardwired Central Control

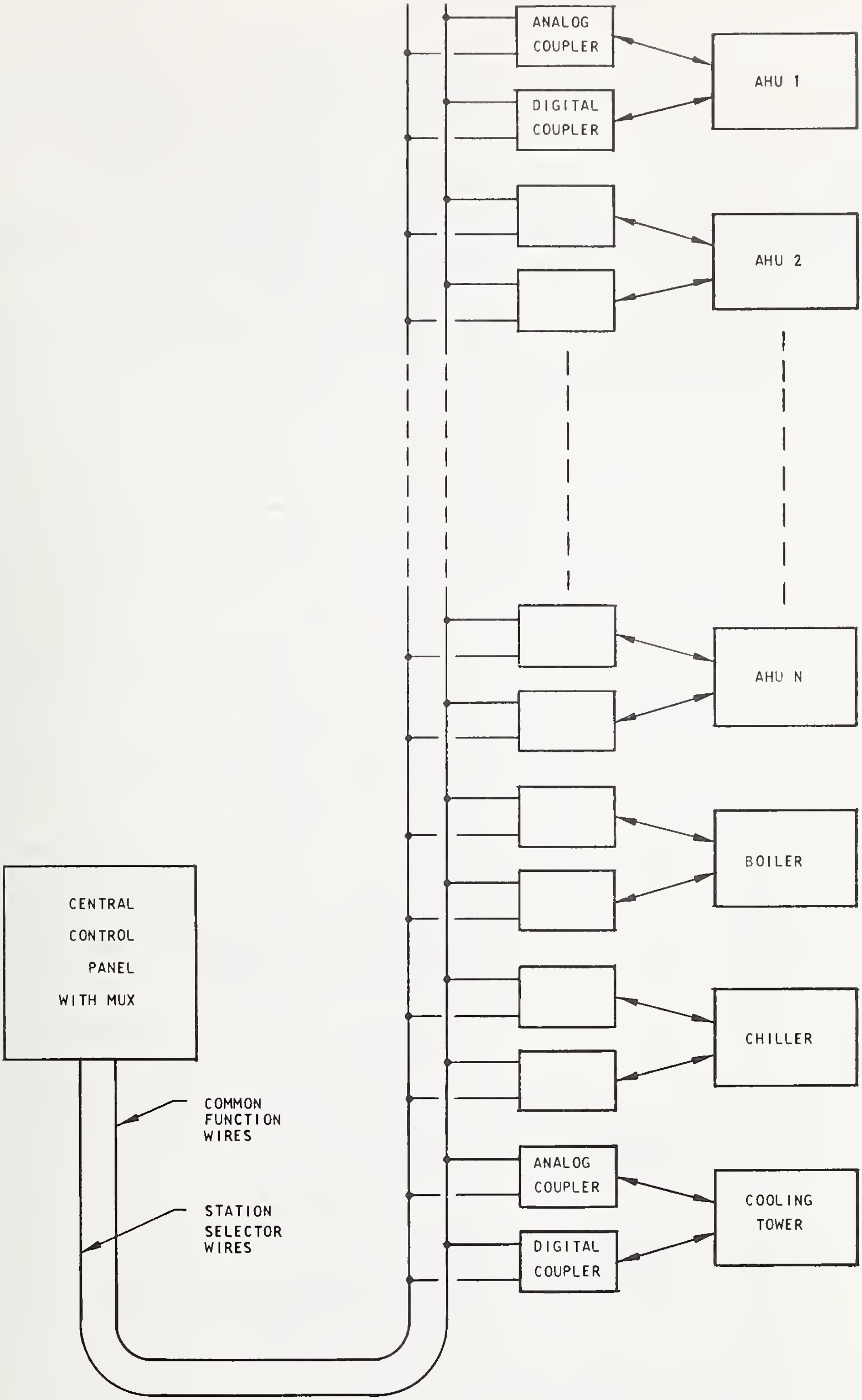

Figure 2. Central Control with Mux

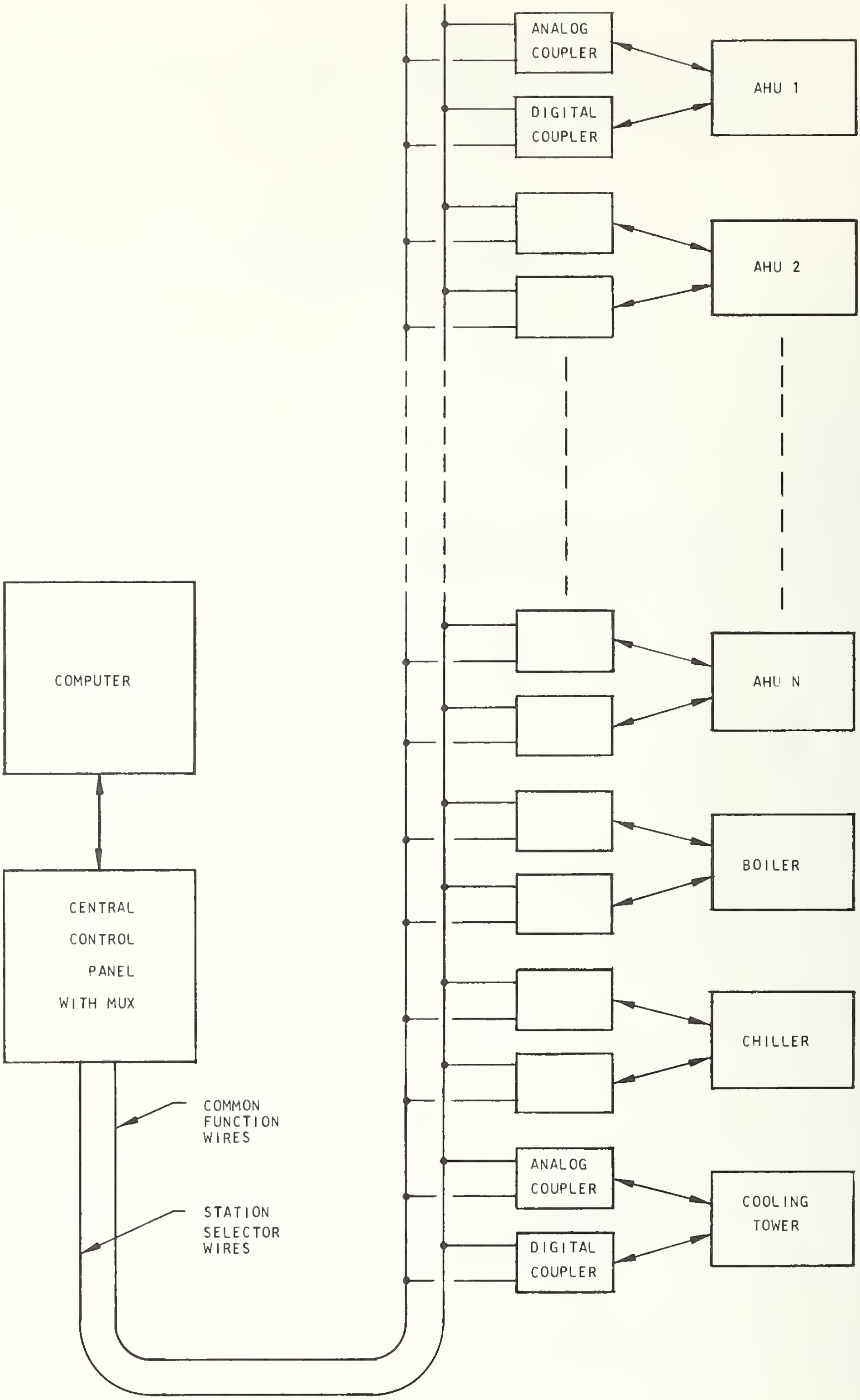

Figure 3. Central Control with Mux and Computer

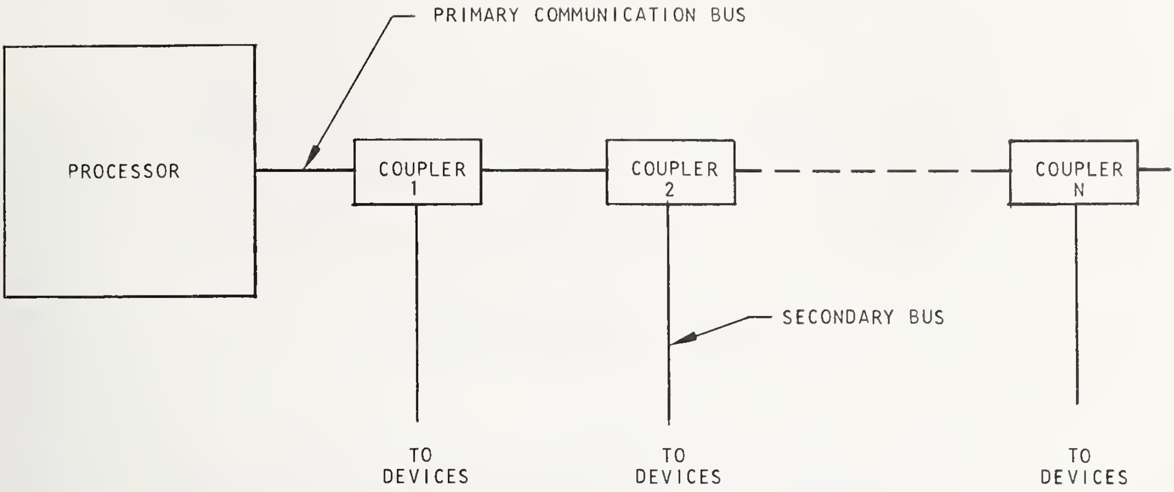

Figure 4. TDM Bus with Mux

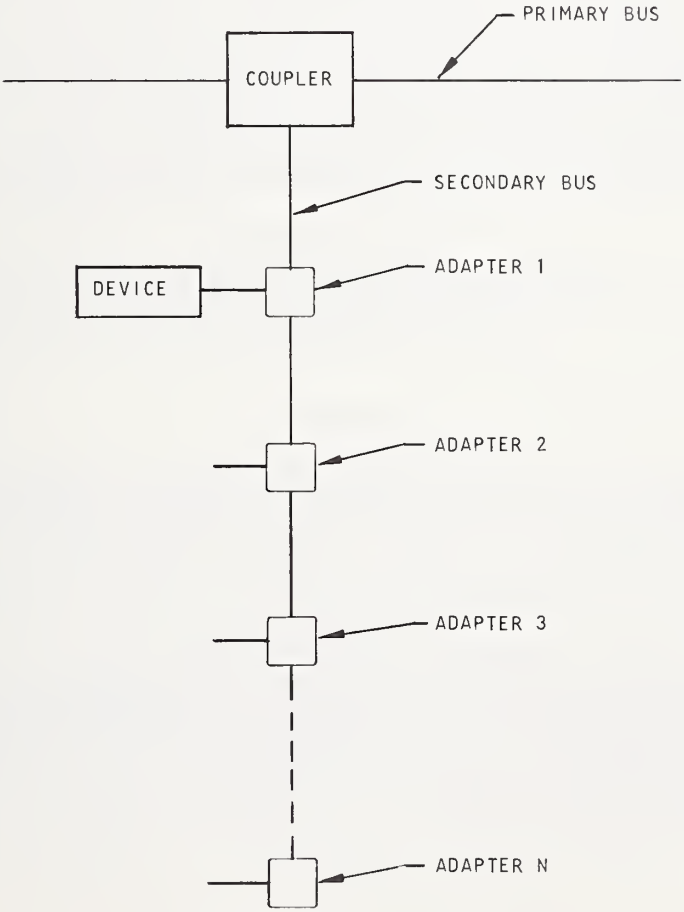

Figure 5. Mux with Secondary Bus and Device Adapter

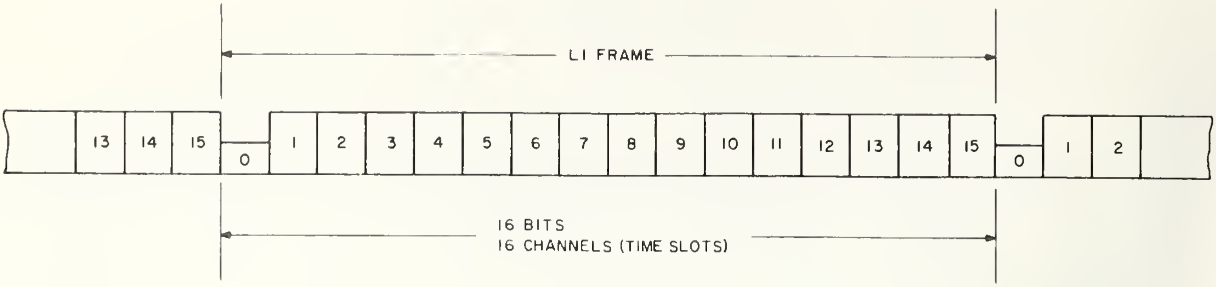

Figure 6. L1 Frame Format

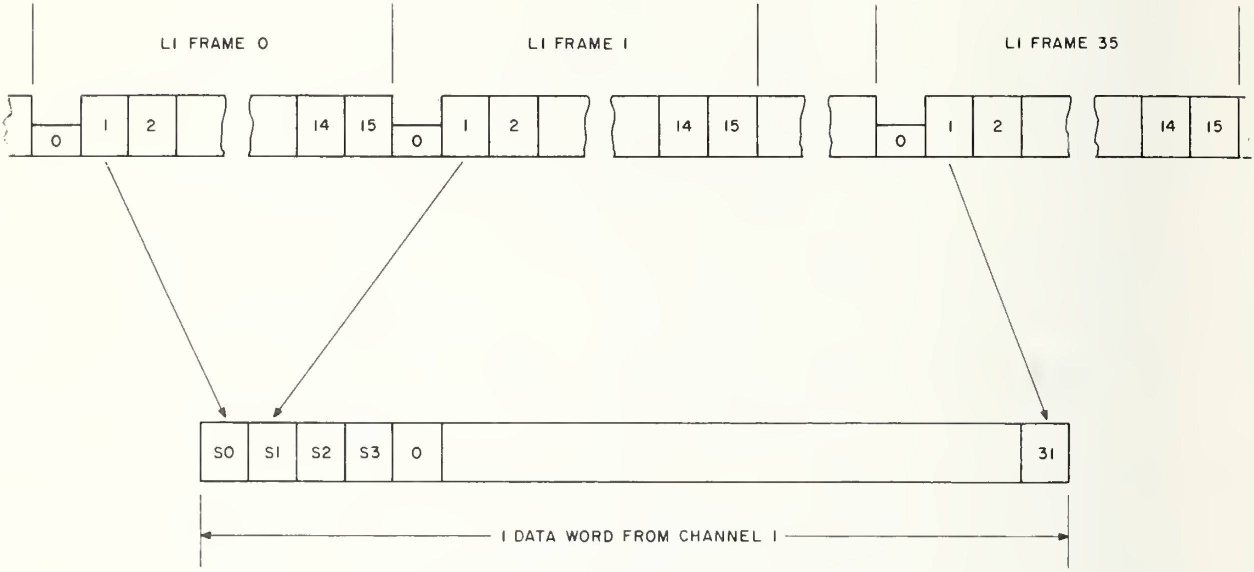

Figure 7. Channel/Data Word Relationship

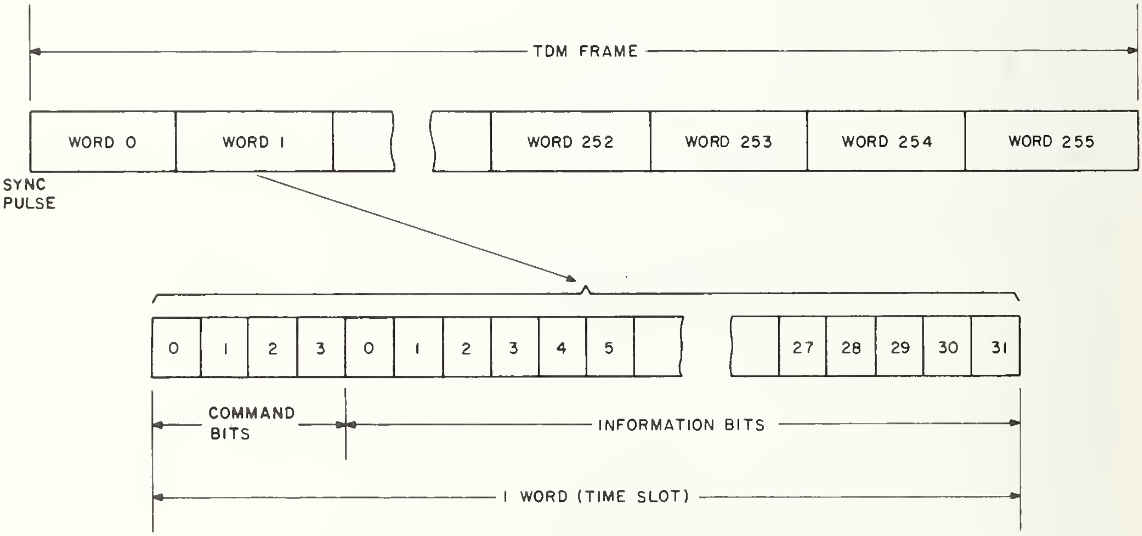

Figure 8. TDM Frame Format

# A Linear Programming Model for Analyzing Preliminary Design Criteria for Multizone Air Distribution Systems

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A linear programming model has been developed to analyze design criteria affecting multizone air distribution systems and to provide information for making design decisions during the preliminary, or conceptual, phase of building design. The location of potential primary mechanical equipment spaces; physical constraints for the air distribution systems; zone data, including preliminary air requirements and single-point zone distribution coordinates, and basic system configurations are fed into an IBM 1130 computer to develop a mathematical model of the building multizone air distribution system. Linear programming is then applied to determine the "least first cost" multizone air distribution system. Postoptimal reports are developed to show the effects of price changes in the air distribution systems or primary equipment selections, the physical size of mechanical equipment spaces, and changes in zone requirements (and the ranges for which the effects would be valid) on the least first cost system selection. Parametric reports are also developed to show the effects of utilizing alternate primary mechanical equipment spaces as well as other system changes in which several variables are changed simultaneously. Examples of applications of linear programming in preliminary multizone system design situations are presented.

Key Words: Air conditioning systems, air distribution systems, equipment selection, linear programming, mathematical programming, multizone systems, optimization, postoptimal analysis.

## 1. Introduction

The conceptual development of design criteria affecting mechanical systems is a phase of building design that must be reevaluated before optimal design of these systems can be accomplished. Computer-aided design systems and new mathematical techniques offer designers new opportunities to optimize the conceptual and ultimate final design and to provide additional tools for maintaining cost controls over the project. The need for additional tools for analysis of mechanical system design criteria is becoming of greater importance as the costs of building continue to rise and as the costs of the mechanical system continue to consume an ever-increasing portion of the project budget.

In this paper, we will discuss an application of linear programming as a component of a design system to assist in evaluating design criteria effecting a multizone air distribution system. Some basic requirements for a system of this scope are: a data base permitting up-dating of the original design criteria with a minimal amount of data manipulation by the designer, the ability to analyze changes in design criteria in terms of effects on the total system and capability of providing the results in the form required for the decision-making activities of the designer, performance of design calculations, optimizing the design and, to some extent, estimating costs, material quantities and equipment selection. All these must be accomplished with an economic advantage over the traditional methods of the conceptual design.

## 2. Linear Programming

It is difficult to define the class of problems which linear programming can solve. In general, these problems include a variety of different resources to be distributed in a variety of ways. A number of constraints may be applied. Some or all of the items may be available in limited quantities, or are tolerable only up to certain limits, or some may be parceled out only in integral units. Under these constraints, an overall measure, such as cost or profit, is to be minimized or maximized.

For a precise formulation of the general linear programming problem [2]*, we assume $a_{ij}$, $b_i$, and $c_j$ are sets of constants ($i = 1, \ldots, m$; $j = 1, \ldots, n$) and $X_j$ ($j = 1, \ldots, n$) is a set of decision variables. We seek solutions $X = (x_1, x_2, \ldots, x_n)$ which satisfy the inequalities

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i \qquad i = 1, \ldots, m \quad , \tag{1}$$

$$x_j \geq 0 \qquad j = 1, \ldots, n \tag{2}$$

and at the same time minimize the linear functional

$$\lambda = \sum_{j=1}^{n} c_j x_j \quad . \tag{3}$$

The linear programming problem is to obtain such a solution.

Equation (3) defines the objective function $\lambda$ and this function is linear in each set $x_j$. The value of $\lambda$ is a function of the vector $X = (x_1, x_2, \ldots, x_n)$, and hence we may express (3) as

$$\lambda = f\ (x_i, x_2, \ldots, x_n) \quad . \tag{4}$$

The function is defined for all values of X with finite components; however, our consideration is limited to those values whose components satisfy restrictions (1) and (2).

We see that in eq (1), we require that the components satisfy m linear inequalities, and in eq (2), we require that all components be non-negative. We commonly refer to eq (2) as the non-negativity restrictions, and to the inequalities (1) as the functional constraints. In matrix terminology, we may represent the functional constraints by the equation

$$A = X \geq B \tag{5}$$

in which $A = (a_{ij})$ and $B = \text{col}\ (i)$.

While the non-negativity restrictions merely limit the set of admissible vectors to vectors X with positive or zero components, the functional constraints further limit this set to those vectors satisfying the matrix inequality (5). The restrictions and the functional $\lambda$ characterize that part of linear programming in which we attempt to minimize an objective function.

The linear programming problem may be stated in an equivalent form in which a linear functional is to be maximized and the functional constraints are $\geq$ relations rather than $\leq$ relations. Since this problem is obtained from the one stated by multiplying eq (1) by -1 and minimizing $-\lambda$, this case can be covered adequately by discussions of the minimizing problem.

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*Number in the bracket refers to the reference.

# 3. Development of Preliminary Design Criteria

## 3.1 Assumptions

In the development of conceptual design system involving the application of linear programming as a design tool, some initial ground rules must be defined. The reason for selecting the multizone system out of all mechanical systems available is simply because data regarding packaged units are readily available. Also, the distribution systems for each zone are much easier to identify and thus lend themselves more readily to this type of a system solution.

Those portions of multizone air distribution system representing the primary cost variables are used in this model. Supply registers and other hardware, will remain fairly constant in the basic system configurations. However, the return and exhaust systems are actually separate systems somewhat similar in scope to the supply distribution systems. As such, they could very easily be included as additional elements of this same model. Their presence would lengthen this presentation so they have been omitted. Other discrepancies resulting from the assumptions made so far can be partially explained since the design system being discussed is used to define conceptual design criteria. Hence, only the variable cost factors that make it difficult to determine optimal system design are included.

## 3.2 Preliminary Design Criteria

Preliminary design computations are performed within the limitations of conceptual design criteria through a series of subroutines described very briefly herein.

### a. Air Volume Computation

In the early phases of design development, a method of estimating the cfm requirements for supply zones, is required to establish realistic bounds for the mathematical model being developed.

The amount of ventilation air required is computed on the basis of any one of the following criteria:

1. Number of air changes per hour,
2. Volume of air required per occupant,
3. Volume required per square foot of floor space.

For the first method, $C_v = n \times V_r$; for the second, $C_v = A \times N$; and for the third, $C_v = B \times S$.

$C_v$ = volume of ventilation air flowing, cubic feet per hour.

$n$ = number of air changes per hour.

$V_r$ = volume of the room in cubic feet.

$A$ = cubic feet of air per hour per occupant.

$N$ = the number of occupants.

$B$ = air volume required per square foot of floor space.

$S$ = the area of the floor in square feet.

The method selected and used depends upon the judgment of the designer.

### b. Duct Sizing

After computation of the zone air volumes, we can determine the size of ducts that are required to transport the air. The subroutine presently used for this purpose computes the equivalent duct diameter using a constant friction loss of 0.1 inches w.g. per 100 feet of equivalent duct length for air volumes less than 2130 cfm, and a constant velocity of 1200 feet per minute for air volumes in excess of 2130 cfm.

For the volume of air less than 2130 cfm, the equivalent duct diameter [1] is:

$$D_c = (2.7(Q/250\pi)^{1.82})^{1/1.40} \quad . \tag{6}$$

For Q greater than 2130 cfm,

$$D_c = (4Q/V\pi)^{1/2} \quad . \tag{7}$$

In eqs (6) and (7),

$D_c$ = equivalent duct diameter, feet,

V = air velocity, feet per minute,

Q = air flow rate, cubic feet per minute.

The general form of the equation for conversion of the circular duct diameter to the equivalent rectangular duct is:

$$d_c = 1.30(ab)^{0.25}/(a+b)^{0.250} \tag{8}$$

where

a = length of one side of rectangular duct, inches,

b = length of adjacent side of rectangular duct, inches,

and

$d_c$ = circular equivalent of a rectangular duct for equal friction and capacity, inches.

In the conversion, it is necessary to consider both the aspect ratio and any space restrictions in which the ducts are to be routed.

The aspect ratio (AR) is the ratio of the long side to the short side. An increase in the AR increases both the installation cost and operating cost of the system. Therefore, it is desirable to maintain an AR as near unity as practical. This is accomplished using the following algorithm.

For the case in which the aspect ratio is 1, let a = b in eq (8) and compute $d_c$. If $d_c$, as previously computed, is less than the limiting $d_c$, then set a = b and recompute 'a' using the computed value of $d_c$:

$$a = (1.46D_c)^{1/2.375} \quad . \tag{9}$$

If $d_c$ is greater than the limiting value for $d_c$, set be equal to the maximum depth and compute 'a'.

Interpreting 'a' as the length of the longest dimension of the rectangular duct, the gage of galvanzied steel required is then selected from Table 1.

Table 1. Galvanized steel sheet metal gage for rectangular low pressure ducts [1].

| Dimension 'a' Inches | Gage | $Lb/ft^2$ |
|---|---|---|
| Through 12 | 26 | 0.906 |
| 13 - 30 | 24 | 1.156 |
| 31 - 54 | 22 | 1.406 |
| 55 - 84 | 20 | 1.656 |
| 85 and greater | 18 | 2.156 |

This information is contained within the duct sizing subroutine so that all information about the duct run required to make a cost analysis has been determined except for the length of duct run. The method of obtaining the length of duct run for each zone is briefly described in the next section.

### c. Spacial Description

Determination of lengths of duct runs is assumed to be based upon a single point of delivery to each zone. In most projects during preliminary design, this assumption is presumed sufficient.

Similarly, assuming that the discharge plenum from the multizone units may also be adequately described as a single point, it is easy to describe the multizone unit in the three dimensional space.

Using the three dimensional coordinate system, the points of supply and distribution are then uniquely described by sets of coordinates. By comparing coordinates, it is then a simple task to compute the length of ductwork from each zone to each multizone unit. With this final data, the cost per unit volume of air for the ductwork for all possible system configurations is easily obtainable.

### d. Multizone Unit Costs

In Table 2, the incremental costs for multizone units are listed.

Table 2. Incremental costs for horizontal blow through, heating and cooling multizone units with insulated coil and fan section, drain pan, forward curved wheels, DWDI, Class 1, motor with vari-drive and belt guard and heating and cooling zone dampers, with a coding coil face velocity of 550 fpm, and a system of 2 inches [4].

| ASU No. | cfm | | First Cost $ | +80% | Total Cost $ | $/cfm | | |
|---|---|---|---|---|---|---|---|---|
| | min | max | | | | min | ave | max |
| 1 | 2,500 | 3,500 | 850 | 680 | 1,530 | .612 | .525 | .437 |
| 2 | 3,500 | 4,500 | 1,000 | 800 | 1,800 | .514 | .457 | .400 |
| 3 | 4,500 | 6,500 | 1,200 | 960 | 2,160 | .480 | .406 | .332 |
| 4 | 6,500 | 8,800 | 1,420 | 1,136 | 2,556 | .393 | .342 | .290 |
| 5 | 8,800 | 11,000 | 1,700 | 1,360 | 3,060 | .347 | .313 | .278 |
| 6 | 11,000 | 14,000 | 2,080 | 1.664 | 3,744 | .340 | .303 | .267 |
| 7 | 14,000 | 17,000 | 2,480 | 1,984 | 4,464 | .318 | .290 | .262 |
| 8 | 17,000 | 21,000 | 3,000 | 2,400 | 5,400 | .317 | .287 | .257 |
| 9 | 21,000 | 23,500 | 3,300 | 2,640 | 5,940 | .282 | .267 | .252 |
| 10 | 23,500 | 30,000 | 3,970 | 3,176 | 7,146 | .304 | .271 | .238 |

To determine the appropriate cost factors to use in the objective function of the LP model, the cost per cfm of air volume for the recommended maximum cfm and the minimum cfm, are averaged. As indicated in Table 2, these values include consideration for the installation costs.

### e. Duct Costs

Table 3 represents the installed costs of galvanized sheet metal ductwork [4]. The costs also include the sheet metal contractor's profit.

To determine the total installed costs of low pressure, straight rectangular ducts on a lineal foot basis, add the width in inches and the depth in inches and multiply by the appropriate multiplier from Table 3.

Table 3. Cost factors for galvanized steel sheet metal for low pressure rectangular ducts [4].

| Duct Gage | Multiply by |
|---|---|
| 26 | 0.140 |
| 24 | 0.162 |
| 22 | 0.195 |
| 20 | 0.217 |
| 18 | 0.234 |

To determine the cost per cfm for the duct from each multizone unit, the total cost is divided by the volume of air being supplied. This figure is then used in the LP model in the objective function.

## 4. Formulation of the Multizone LP Model

A linear programming model for multizone air distribution systems can now be developed from the previous discussions. Given 'n' competing activities consisting of the volume of air required for each zone, Zk, and the volume of air available from each source, ASUs, the decision variables $x_1, x_2, \ldots, x_n$, in (2), represent the levels of these activities. The general form of the model is illustrated in figure 1.

In our model, the volume of air required for the $k^{th}$ zone is formulated as

$$\sum_{\ell=1}^{r} Z\ell.K = ZK \qquad k = 1, \ldots, p \tag{10}$$

for a system with 'r' multizone units and 'p' zones.

Similarly, the volume of air supplied by the $r^{th}$ multizone unit is formulated as

$$\sum_{\ell=1}^{r} ASU\ell.s \leq TCAP \qquad s = 1, \ldots, t \tag{11}$$

where there are 't' possible selections for the $r^{th}$ multizone unit.

The summation of the individual zone air volume requirements supplied by a particular multizone unit and the total capacity of the multizone units in the solution base must be zero, or

$$\sum_{\ell=1}^{r} \left(\sum_{k=1}^{p} Z\ell.k - \sum_{s=1}^{t} ASU\ell.s\right) = 0 \quad . \tag{12}$$

Furthermore, the volume of air supplied to each zone from a particular multizone unit must not exceed the total volume of air required by the zone, or for the $k^{th}$ zone,

$$Z\ell.k \leq Zk \qquad \ell = 1, \ldots, r \tag{13}$$

and each multizone unit cannot supply a volume of air in excess of ASUs, or for the $s^{th}$ multizone unit,

$$ASU\ell.s \leq ASUs \qquad \ell = 1, \ldots, r \quad . \tag{14}$$

Finally, the objective function, COST, can now be written as

$$\sum_{\ell=1}^{r} (\sum_{k=1}^{p'}(CO\ell.k)(Z\ell.k) + \sum_{s=1}^{t}(CA\ell.s)(ASU\ell.s)) = \min \mathrm{COST} \qquad (15)$$

Since, by our previous discussions, each of the $Z\ell.k$ and the $ASU\ell.s$ are assumed linear over the range for which they appear in the solution base, our model then performs according to the general linear programming problem.

## 5. Results

The experimental values obtained during testing of the computer-aided design model thus far substantiate the possible economical use of the model in analyzing conceptual design criteria for multizone air distribution systems.

A test building for which design data has been recorded, has been used to verify the results obtained from the model. Typically, input data for 30 zones has required about one hour for preparation. Using an IBM 1130 computing system, about 15 minutes are required for computation of the bounds for the LP model and the cost coefficients for the objective function. The LP model, using the IBM LPMOSS [3] program, requires about 30 minutes to obtain the first optional solution.

Output from the computer-aided design model includes the following:

1. The minimal possible first cost for the multizone air distribution system using the given set of design criteria,

2. The determination of the multizone unit from which each zone must be supplied to obtain the minimal system first cost,

3. The multizone unit selection for each subsystem required to minimize the first costs,

4. Cost per unit of air volume for all duct runs and multizone units in the range for which the solution is applicable,

5. Cost reduction or increase possible per unit volume of air within the vicinity of the optimal solution achieved by changes on the constraints.

After the initial optimal solution has been determined, the output data is extremely useful in indicating the directions in which to proceed to improve upon the solution. This may be as simple as changing one or two bounds, or as complicated as changing the location of multizone units or a number of zones. The constraints are automatically modified, as well as the coefficients in the objective function. Since the initial optimal solution is maintained by the program and used as the new initial solution, the analysis of the new design criteria is accomplished in a substantially reduced time and reduced design cost.

Additional modifications to the model will include printout of the optimal air distribution system using a plotter, modeling of additional basic air distribution systems, the use of graphical display devices, and the consideration of return and exhaust air systems.

## 6. References

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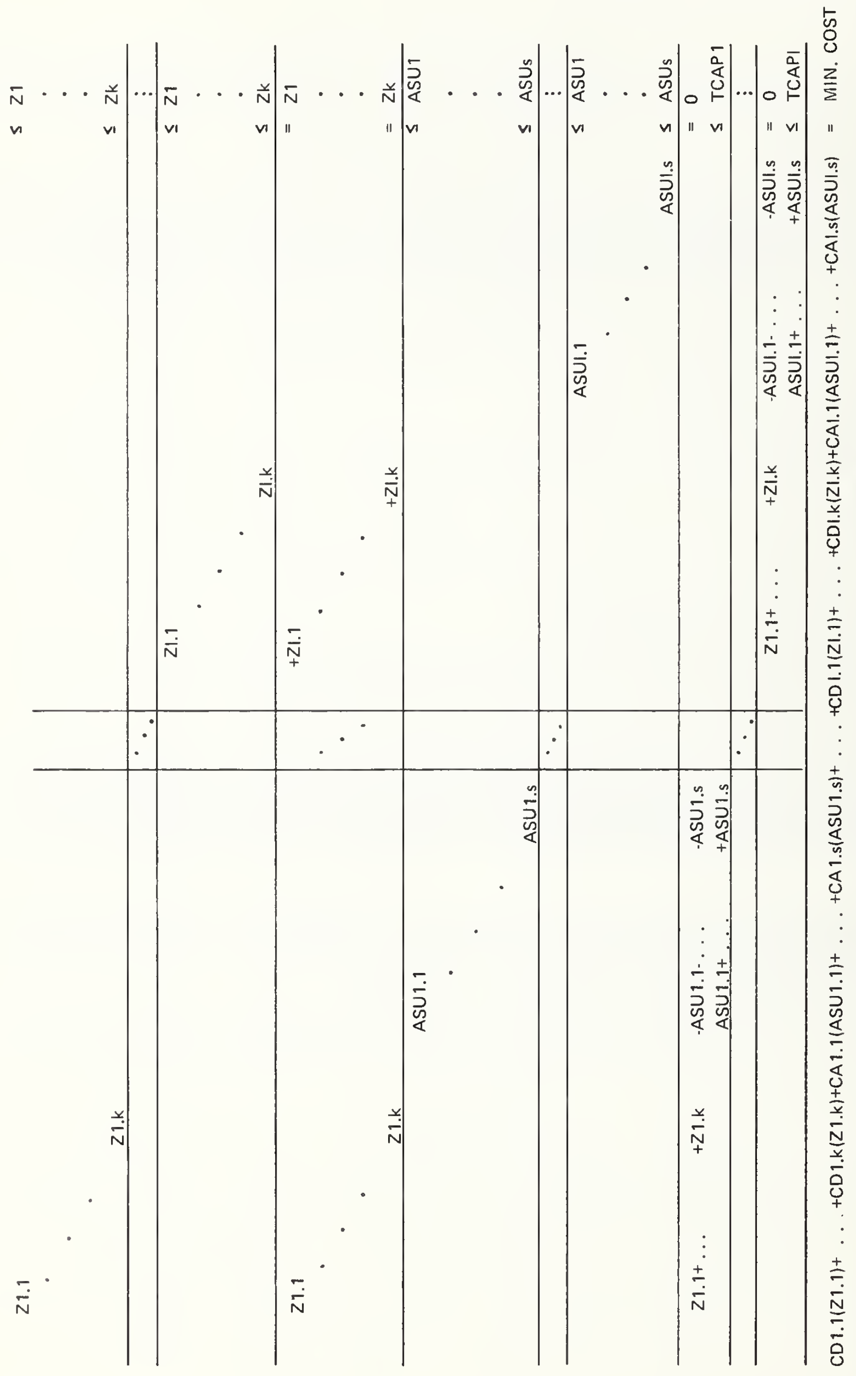

Figure 1. General linear programming model for the multizone air distribution problem.

# A Conceptual Survey of Computer-oriented Thermal Calculation Methods

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This paper surveys computer-oriented methods used for calculating cooling or heating loads and/or for determining indoor space temperatures in buildings. The methods have been classified into groups depending upon the way they tackle the underlying heat conduction problem. Limitations and merits of each of these techniques are then discussed in terms of the ease of computation and degree of exactness with which they handle considerations such as heat conduction through multilayer walls and roofs, heat capacity of the enclosing fabric and of the contents, internal radiant loads and radiative-convective exchanges, ability to provide for an internal temperature swing and thermostatic operation, variable ventilation, shading, surface coefficients of heat transfer, and heat flow through the ground. Harmonic or matrix methods and response factor methods are discussed in greater detail.

Key Words: Air conditioning, computer, indoor climate, load calculation, survey, thermal performance.

## 1. Introduction

The importance of designing buildings as environment modifiers and the economic necessity of accurately estimating the loads for correct sizing of air conditioning devices has led to considerable development of dynamic thermal calculation methods. The advent of high speed electronic computers has shifted the emphasis from development of simplified handbook methods, necessarily based on very restrictive assumptions, towards making more routine use of sophisticated methods already available and to develop more exact ones. An ideal method should permit the designer to know accurately how the thermal performance or air conditioning loads on a building vary with time of day and time of year and the influence of variations in the building structure, the control conditions within the enclosed space, the capacity and operating schedule of the plant, and the behaviour of the sensing and thermostatic control devices.

The thermal environment of any confined space is the result of interaction between the outdoor climate, the enclosing structure and the energy sources or sinks within. The commonly required data for thermal calculations are design climatic conditions, thermophysical properties of the structure, heat inputs due to occupancy, appliances, ventilation, and lighting, and the desired indoor conditions required for comfort. To obtain design climatic conditions, a selection criterion is used to pick out time sequences for external air temperatures, wind velocity and solar radiation from meteorological records. Alternatively, solar radiation can be computed from an assumption of atmospheric conditions and a knowledge of solar position. The selection criterion is independent of thermal calculation methods and has not been discussed in this paper. It has to be noted, however, that simplified handbook methods, which form the basis of the majority of current computer programs in routine use, do not permit as wide a selection of design solar radiation values as they do for air temperatures. Similarly, the design values for internal heat input and thermal comfort conditions are independent of the calculation method adopted, even though they may be handled differently by different methods.

In regard to heat flow through building elements, most computer-oriented methods assume it to be unidirectional and thus neglect corners and heat bridges such as studs and rafters. The thermal conductivity and heat capacity of homogeneous layers are considered to be constant even though their values may be taken with reference to moisture conditions likely to occur

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in use. Thus the governing differential equation for heat conduction through a building element is the one dimensional, linear heat conduction equation. The corresponding simplification in boundary conditions, obtained by neglecting the non-linear character of surface coefficients of heat transfer or by treating them as constant, is no longer universally adopted. Air in the enclosed space is still considered to be at a uniform temperature and non-absorbing to radiation.

## 2. Types of Thermal Calculation Problems

In the field of thermal design of buildings and load estimation, four main types of problems are encountered:

(a) Calculation of indoor air temperature in the absence of artificial cooling or heating.

(b) Calculation of indoor air temperature when some heat is being removed or added but the indoor temperature is still variable.

(c) Calculation of heat gains or losses and cooling or heating loads when the indoor air temperature is kept constant at a known value.

(d) Calculation of heat gains or losses and cooling or heating loads when the indoor air temperature is variable and specified - the case of "temperature swing".

Further, the variable inputs may be of periodic type or of any general type. The former yield to exact steady periodic analyses and are usually sufficiently representative of design conditions but the latter are necessary when energy usage is being considered over a large period of time or actual comparisons are being made in the field between observed and computed values.

Significant concepts to be discussed in relation to the various thermal calculation problems are:

(a) Heat conduction through multilayer elements.

(b) Converting the solar radiation transmitted through windows and the internal radiant loads to cooling loads or changes in indoor air temperature.

(c) The ability to handle variable networks representing either a variable amount of ventilation or variable coefficients of heat transfer at the surfaces of walls and roof.

(d) Automatic controls such as thermostatic control of temperature and operation of blinds.

(e) Interactions with daylighting, latent load calculations, shading due to other buildings, more complicated considerations of multidimensional heat flow through heat bridges and ground, and coupled heat and mass transfer problems.

## 3. Established Design Methods

A limited comparison of the established design load estimation methods, which form the basis of most computer programs in current use, has been carried out by Milbank and Harrington-Lynn(1)[3]. A detailed discussion of the mechanism of heat transfer in buildings has been reported by Gupta(2). To provide a proper perspective for the methods reported in later sections of this paper, major concepts in relation to three methods, which are widely used and represent basically different approaches, are discussed in this section.

The ASHRAE method(3) is limited to load estimation for conditioned spaces held at constant air temperature. Specified values of surface coefficients of heat transfer are assumed in the exact analytical treatment for steady periodic flow, which forms the basis of determining the equivalent temperature differences used in this method. Heat conduction through multilayer elements is treated rather empirically. Sol-air temperature allows for the effect of solar radiation on opaque elements and solar heat gain factors and shading coefficients allow for the amount of solar radiation entering through windows of different types. Average clear days are used for estimating solar radiation. The instantaneous cooling load due to internal radiant loads and transmitted solar radiation is obtained by averaging these over a period of time governed by the weight of the structure.

In the Carrier method(4), the equivalent temperature differences for opaque elements are calculated for sunlit as well as shaded conditions by using numerical methods. The internal radiant loads and transmitted solar radiation are considered to be absorbed by the structure. Based on field measurements, hourly storage factors are tabulated, which when multiplied by the peak solar heat gain through ordinary glass give hourly cooling loads corresponding to different weights of structure, different shading conditions and different hours of plant operation. Allowance is made for reduction in peak values of load for different amounts of temperature swings permitted in internal space temperatures. The incident solar radiation values can be adjusted for a haze factor.

The method due to Boeke(5) is widely used in Scandinavian countries and seems to be very general in its approach. It calculates loads for a specified indoor air temperature which may be constant or variable or else determines the indoor

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[3] Figures in brackets indicate the literature references at the end of this paper.

air temperature for a given plant capacity and also in the absence of artificial sources and sinks. Computations on an hourly basis are done for cycles of clear as well as cloudy days in every month of the year by using hourly heat balance equations for similar modules in the building. The modules are supposed to have only one wall exposed. Heat transmitted through opaque portions, shaded as well as sunlit, are calculated by using Carrier's tables(4). Internal radiant loads and transmitted solar radiation through windows are considered to be absorbed by a hypothetical element representing the ceiling, partitions and floor in a two lump system corresponding to the core and surface. The special features of the method are its capacity to tackle variable shading due to other buildings, variable ventilation, environmentally controlled operation of blinds and lighting, thermostatic control of indoor air temperatures and intermittent plant operation in a straight forward operation. The inaccuracies involved in the handling of conduction through multilayered slabs are of the same order as in other design methods. The main limitation seems to be the provision for only one exposed element.

There are many other proprietary programs such as ARTHUR and BASIL in France, WESTINGHOUSE, APEC and GATE in the USA. Also there are a large number of easily programmable desk calculation methods, using unsteady state heat flow considerations, which are very widely used in the USSR and European countries. By and large, these do not use any significantly different concepts and differ only in detail. A discussion of these is omitted for lack of space.

## 4. Numerical Methods

It is not always convenient to obtain exact analytical solutions of the one-dimensional heat conduction equation under the varied boundary conditions of interest in building heat transfer problems. In fact, it is not possible to obtain these by purely analytical means if the boundary conditions are non linear or thermal properties become temperature or time dependent. Also, for multilayered slabs, the exact number of layers have to be specified in advance, thereby restricting the generality of application. It is, therefore, of interest to consider numerical methods, which essentially involve the conversion of derivatives into finite differences. If only the space derivatives are converted, the equation reduces to a set of simultaneous ordinary differential equations at a grid of points or nodes. These have been solved by using R-C network analysers, analog computers and thermal analysers, which require a specific type of equipment and involve experimental errors. Large size analogs, which can handle thermal problems of buildings as a whole, have been reported in recent literature by Korsgaard and Lund(6), Button and Owens(7) and Euser(8). The basic principles of these have been reviewed by Stephenson(9) and these will not be discussed further in the present paper.

For purely numerical methods, which form the basis of digital computer programs of this type, both space and time derivatives are converted into finite differences. This results in a set of algebraic equations which can be easily handled by matrix algebra. In physical terms, this approximation amounts to representing a distributed thermal system by a lumped thermal network consisting of resistances between nodes and capacitances at the nodes and to calculating step by step with respect to time. The larger the number of lumps and steps, the nearer to the actual system this representation would be but the computer time required would be increased.

A vast body of literature(10) exists for numerical methods of solving the heat conduction equation, but the following discussion should be sufficient for outlining the concepts and defining the terms.

The one dimensional heat conduction equation for a homogeneous medium with constant thermal properties is represented by

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \qquad (1)$$

where T is the temperature, x is the space dimension, t is the time and α is thermal diffusivity. Considering h to be the incremental step in x and ε in time t and using the notation $T(m,n)$ to represent the temperature at position mh at time nε, a general finite difference representation of eq 1 would be

$$M \begin{bmatrix} \beta(T(m+1,n+1) - 2\,T(m,n+1) + T(m-1,n+1)) \\ +(1-\beta)\ (T(m+1,n) - 2\,T(m,n) + T(m-1,n)) \end{bmatrix} = T(m,n+1) - T(m,n) \qquad (2)$$

where $M = \alpha\varepsilon h^{-2}$ is the modulus and β is the interpolation parameter. The relative magnitudes of ε and h are to be chosen such that the scheme is computationally stable, i.e. the rounding off errors do not go on increasing and their actual magnitudes govern the accuracy of finite difference representation. The conditions of stability(10) are that

$$M > 0, \qquad \text{if } 0.5 \leqslant \beta \leqslant 1$$
$$0 < M \leqslant 0.5 \qquad \text{if } \beta = 0 \qquad (3)$$

If β = 1, an implicit form is obtained, which is stable for all values of M. It involves solving a set of simultaneous equations for every time increment, which may, however, be large. If β = 0, an explicit form is obtained, which is stable only for $M \leqslant 0.5$. This means that even though only one equation has to be solved for each time increment at each node, the increment has to be very small, being necessarily less than half the smallest time constant of all the nodes for stability.

Two well known cases are obtained as follows:

If $\beta = 0$, $M = 0.5$, eq (3) becomes

$$T(m,n+1) = 0.5\ \{T(m+1,n) + T(m-1,n)\} \tag{4}$$

If $\beta = 0.5$, $M > 0$, eq (3) becomes

$$\frac{M}{2}\{(T(m+1,n+1) + T(m+1,n)) - 2(T(m,n+1) + T(m,n)) + (T(m-1,n+1) + T(m-1,n))\} = T(m,n+1) - T(m,n) \tag{5}$$

Equation (4) corresponds to Schmidt's method and eq (5) corresponds to Crank-Nicholson's method (quoted in 10). The former is simple but accurate only up to second order differences and can give oscillating errors if the initial estimates are not chosen properly. The latter is more accurate and is stable for all positive values of M.

All numerical methods require initial values to be specified and generate temperature data at all nodes even when only air temperature and surface temperatures are required. There are also lumping errors due to nodal approximation of a distributed network by a discrete system, which can be made quite small at the cost of considerable increase in computer time. Nevertheless, the arithmetic is very simple, and multilayered structures, non-linear boundary conditions and variable networks can all be handled. Programs are general in nature and are not necessarily limited to buildings(11). Climatic data inputs are completely open ended data sequences and any length of period can be considered. Some of the well known methods of this class, which treat the whole building and use digital computers are discussed hereunder:

Kusuda and Achenbach(12) have used an explicit technique to solve three dimensional heat and mass transfer problems associated with underground shelters. Space increments of one foot and time increments of the order of two hours have been taken for predicting the temperatures for a fourteen-day period. Buchberg et al(13) and Sheridan(14) have developed similar types of explicit techniques for predicting loads or temperatures in a room which is characterised by the finite difference representation of its elements combined in parallel in the form of a lumped network system. The maximum value of time increment is fixed by stability considerations to be less than the minimum value of the product

$$\{C_i/\sum_j(1/R_{ij})\}$$

which denotes the thermal time constant for the node i connected with nodes j. The temperature and time dependent boundary conditions incorporating the radiative and convective modes of heat transfer are taken into account in the form of a delta network. Buchberg et al(13) have also included heat exchange with the ground and sky at the outdoor surfaces and solar radiation penetration through windows is considered to impinge on the floor. For calculations over long periods of time, say a year, Buchberg and Roulet (15) have also developed a very fast implicit type technique and applied it to compute loads for a structure of homogeneous construction with no direct solar radiation transmission and for constant indoor air temperature. Wheway and Vahl Davis(16) have specifically developed a method for rooms in intermediate storeys of air conditioned multistoreyed buildings having only one side exposed to outside conditions and with constant indoor air temperature. Variable shading and reflected radiation due to adjacent buildings, reflected radiation from ground and a variable coefficient of heat transfer at the external surface are taken into account. Radiation transmission through the windows has been considered to be a cooling load and a constant value of the combined coefficient of heat transfer at the indoor surfaces taken. Beckman(11) has developed a multimode, multinode model for systems with combined conduction, radiation and convection heat transfer, which can be applied to buildings. It assumes a semi-gray enclosure and a specular-diffuse model(17) for internal radiation exchange and uses fifth order Hamming's method(18) to solve the set of first order non-linear ordinary differential equations obtained for a lumped system. It is specifically coded to provide data for variations in temperatures caused by changes in the physical parameters. Non-linear boundary conditions, variable networks and switching inputs can be easily handled. There is difficulty in obtaining surface temperatures as nodes with zero capacity cannot be handled. Two major advantages are that the time increment can be large and the initial temperature estimates are not a critical part of the solution.

## 5. Harmonic Methods

When the climatic data can be considered as periodic cycles, which is usually the case for design studies, the methods surveyed in this section have a special merit on account of their mathematical elegance and speed and ease of computation. The input data sequences are harmonically analysed into a steady state term and a sufficient number of harmonics with frequencies in integral multiples and then each one of the pure sine waves is considered as a separate input. The responses are then synthesised to give the desired loads or temperatures. The main restrictions on the applicability are that the building system parameters have to be time invariant, linearisation approximations have to be used for the convective and radiative boundary conditions and switching inputs are subject to Gibb's phenomena (quoted in 19) as they have to be harmonically analysed.

One of the earliest analyses was done by Mackey and Wright(20) for estimating heat gains into buildings maintained at constant indoor air temperature. Exact heat conduction solutions were obtained for homogeneous elements and an equivalent homogeneous wall(21) was defined for multilayered constructions. Air temperature and solar radiation impinging on an opaque element were combined into a single input known as the sol-air temperature(22). Internal masses were not considered and transmitted solar radiation and other internal radiant loads were considered as cooling loads directly. Degelman(23) has computerised this method for year round usage from first principles so as to obtain greater flexibility in the choice of material properties, surface coefficients of heat transfer and the computation of solar radiation data input as compared to the ASHRAE method(3), the earlier version of which was based on this theory. Nottage and Parmelee(24,25) removed the limitations of constant indoor air temperatures and the internal radiant loads not being linked to internal masses by applying the harmonic inputs to a lumped network representation of the building. Periodic types of inputs necessitated the solution of only one set of simultaneous algebraic equations for each of the harmonics as against a very large number for numerical methods, even though lumping errors were introduced as before.

Van Gorcum(26) obtained an exact solution for homogeneous slabs subjected to harmonic inputs and showed that by analogy with passive four terminal networks of electrical circuits, the analysis could easily be extended to composite slabs in series. By considering any building as a combination of heat paths in parallel, each of which may consist of a number of homogeneous slabs in series, Muncey(27) devised a technique quoted as the matrix method to predict variable internal air temperatures, which did take into account the internal masses. For a specified indoor air temperature, which may be constant or variable, this method could also be used to predict heat gains. Pipes(28) reformulated the method in terms of hyperbolic functions using an electrical analogy. Gupta(29) introduced the delta network representation of indoor radiative convective exchanges into the method due to Muncey(27) so as to obtain indoor air and surface temperatures simultaneously and to allow the transmitted radiation to be linked to internal masses. Muncey and Spencer(30) showed that the errors caused by taking a combined surface coefficient of heat transfer at the indoor surface instead of introducing a delta network were not more than those caused by neglecting the furniture. Depending upon the value of this coefficient used, the calculation gave a mean radiative-convective space temperature rather than the air temperature. This temperature is akin to the environmental temperature proposed by Loudon(31). Rao(32) devised a set of thermal system functions to calculate cooling loads by the matrix method, which provided for temperature swings and internal radiant loads. Gupta(33) extended his earlier (29) scheme to take into account multidimensional heat flow through the ground so as to provide suitable inputs for floors laid on ground. Muncey and Spencer(34) developed an alternative technique to take into account internal radiant loads without having to introduce a delta network for indoor radiative exchange. This extension(34) also showed how heat flow within paths and various parallel branches in the individual heat flow paths could be taken into account. Gupta(35) has also interlinked the matrix method with daylight requirements so as to assign suitable values to internal radiant loads due to artificial lighting during daytime as these acquire critical significance in determining peak loads for open plan office buildings of large floor areas.

The matrix type of harmonic method after incorporating the extensions outlined above, can handle all types of thermal problems stated in section 2. The requirements of system linearity and invariability are to be observed and as such variable networks and non-linear boundary conditions cannot be handled. Further, the length of the periodic design climatic cycle should be at least twice the thermal time constant of the enclosure(36) or more simply twice the largest thermal time constant value amongst the heat flow paths(37) and not merely a day if a steady periodic regime has to be obtained inside the enclosure.

## 6. Response Factor Methods

When energy requirements over a fairly long period of time are to be assessed, the climatic data are expected to be non-periodic and harmonic methods are no longer applicable. Numerical methods can still be used, but these must introduce lumping errors. Response factor methods have been devised so as to handle periodic, non-periodic and intermittent inputs equally well without necessarily being subject to lumping errors. The essential strategy is to determine the system response to a unit excitation under identical boundary conditions as for the actual inputs. Numerical integration of the convolution integral(10) is then carried out and the system response is determined by superposing the unit responses or their scalar multiples over a significant period of time prior to the time in question such that the actual excitations are approximated by a succession of scalar multiples of unit excitations. The unit response may be characterised by a set of numbers giving the response at equally spaced points of time or by an influence function. These numbers or response factors depend only on the construction and not on the climate and can even be tabulated for different types of constructions for handbook type calculations. Since the principle of superposition has to be used, the requirements of system linearity and invariability are still to be met. Step by step calculation, however, makes it possible that the implications of these requirements are not so stringent in the actual applications as in harmonic methods. It is usual practice to take hourly or half-hourly intervals for load estimation but shorter intervals may be desirable for control systems evaluation.

The earliest of such techniques, due to Nessi and Nissole(38), calculated two influence functions corresponding to heat flow at the internal surface of a wall when there is a unit step change either in the external or in the internal air temperature. For a complete room, the heat flows are added after being multiplied by appropriate areas. This gives the total heat flow at the inside surface corresponding to a unit rise in external air temperature when the internal air temperature is constant or else to maintain a unit rise in internal air temperature when the external air temperature is constant. The former case is used to calculate cooling loads for constant internal air temperature and the latter by a process of inversion to determine the rise in internal air temperature for a constant heat flow indoors. Multilayered constructions are approximated by a lumped system and there is no provision for internal radiant loads. Recently, Pratt and Ball(39) and Choudhury and Warsi(40) have derived unit response functions by exact analytical procedures for enclosures having heat flow paths containing up to a maximum of three layers.

Brisken and Reque(41) were the first to consider response factors as a set of numbers denoting values of a unit response function at equally spaced intervals of time. They took the unit excitation function as a rectangular pulse of unit amplitude and unit time step duration and treated the individual paths of the building as double lump networks. Combined surface coefficients of heat transfer were taken at the indoor surfaces and included as part of the networks. Heat balance methods were used at each node to derive transfer heat admittance and control point heat admittance parameters similar to the influence functions of Nessi and Nissole (40). Provision was made for internal temperature swings but the transmitted solar radiation and internal radiant loads were linked directly to the indoor air. The calculations were done in two steps namely determining the heat gains for constant indoor air temperature and then determining the change in indoor air temperature for a given plant capacity or for a different control setting. The climatic data sequences were approximated by a succession of rectangular pulses. The method, however, cannot handle temperature and time dependent boundary conditions as these form part of the lumped network representation of the heat flow paths.

Mitalas and others have presented an improved version of the response factor method in a series of papers (42,43,44,45). The major points of difference from Brisken and Reque(41) as enumerated in (42) are that the individual layers constituting heat flow paths are treated by exact analysis as distributed systems, the unit excitations are triangular pulses of unit amplitude and twice the time step duration and the heat transfer at the indoor surfaces of the enclosure is represented by a delta network. The first improvement removes lumping errors and the computation for multilayered constructions involves Laplace inversion of the transmission matrix for a composite slab(43). This matrix is the same as used for harmonic methods(26) except for the presence of the transform parameter S. These time series are truncated when a desired degree of precision is obtained. Unit triangular pulses approximate the external climatic cycles and internal convective flows much better than rectangular pulses as the former are equivalent to trapezoidal approximations. However, a switching type of input such as an artificial lighting load could be better approximated by rectangular pulses. The linking of internal surfaces by a radiative network makes the surface temperature response factors dependent upon enclosure geometry as a simultaneous set of heat balance equations are to be solved. However, the surface temperatures are calculated as part of the computations, the internal radiant loads are distributed to the internal surfaces and non-symmetric elements can be handled more easily. Also non-linear boundary conditions(44) due to condensing or evaporative heat transfer or due to temperature dependence of the radiative component or due to variable wind affecting the convective component of surface coefficients of heat transfer can be taken into account in this method.

The number of sets of surface temperature response factors is equal to the number of excitations plus one for the room air temperature. Cooling load response factors can be calculated from the surface temperature response factors both for constant air temperatures and variable air temperatures. Once the response factors are known, they can be combined with any set of excitations to obtain cooling loads, air temperatures and surface temperatures by simple arithmetical processes. An example showing how the time series method can be used to compute cooling loads and to predict temperature swings for a given capacity or for intermittent running of the plant or to determine indoor air temperatures is given by Stephenson and Mitalas (45). The conditions of system linearity and invariability still require the thermal properties of the materials to be constant. However, variable ventilation can be handled as the calculations of air temperature are done step by step.

Kusuda(46) has recently extended the response factor method due to Mitalas, Stephenson and Arsenault to multilayer structures with various curvatures of finite thicknesses such as spherical and cylindrical systems and to semi-infinite systems, such as ground. Formulae for evaluating interfacial temperatures and heat fluxes in multilayer constructions have been derived and the evaluation of response factors for multilayered constructions has been described in detail.

Muncey(47) proposed an alternative approach to the computation of thermal response factors of multilayered slabs and their application to the determination of the transient thermal response of enclosures. Instead of finding numerically the roots of a complicated transcendental equation for the entire composite structure(43,46), he computed the matrix elements for composite structures at prespecified frequencies and used a precalculated matrix to determine the coefficients of a large series of exponential terms with prespecified exponents. By selecting suitable values and a sufficient number of the frequencies and exponents, any desired degree of precision can be obtained. Thus, the time consuming procedures used for the Laplace inversion in the other methods are avoided by making use of the fact that frequency response curves in the case of buildings are smooth and stable and point by point matching is in order. This is because there is no thermal analog of series capacitance or inductance in electrical circuits.

All the previous methods use individual response factors to obtain separate heat flows for constant indoor air temperature and then add them to determine cooling loads. Changes in indoor air temperature or cooling loads permitting temperature swings need to use another set of response factors for the indoor air temperature variation. Muncey et al(47,48) have developed a procedure, which determines the response factor for the total building pertaining to each of the climatic sequences and internal heat loads or air conditioning flows. This is done by determining the indoor air temperature first and using a combined coefficient of surface heat transfer at the internal surfaces. However, provision exists for linking internal masses to internal radiant loads and accounting for heat flows occurring internally to any path, as in the harmonic case (37). All four types of problems mentioned in section 2 can be handled. Both triangular and pulse types of unit excitations are used for the appropriate types of inputs(48). In this method, however, it is not possible to consider non-linear boundary conditions or variable networks. Experience has shown that if the calculation for sol-air temperatures takes into account the variable outdoor coefficient of heat transfer, using a time averaged constant value for it in the calculation of response factors is sufficient. Further, if the internal radiant loads are linked to internal surfaces and not to the air temperature, using a combined and constant value of surface coefficient of heat transfer at the internal surfaces is expected to be satisfactory for building problems. Variable ventilation can be included with certain restrictions but only at the cost of analytical rigour.

## 7. Conclusions

A wide variety of computer oriented thermal calculation methods pertaining to buildings have been considered in relation to the concepts they use, the assumptions they employ and the limitations in regard to their applicability. No attempt has been made to compare their validity with respect to actual buildings or their efficiency in terms of computer time. This can only be done by using all of them for the same large size actual building and comparing the estimates with the experimentally observed data and the actual computation costs incurred, i.e. by instituting some sort of round robin test. In a fast developing discipline, like the subject of this survey it is very likely that some conceptually significant methods may have escaped the notice of the authors and the omissions, if any, are not intended to reflect on the methods.

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# Method for Thermal Calculations using Total Building Response Factors

by R. W. R. Muncey*, J. W. Spencer+ and C. L. Gupta+

Thermal calculations for buildings may be conveniently undertaken by multiplication of the time sequence of climate parameters and the response factor of the building for each parameter. The true response factor is the sum of an infinite number of exponential terms which may be approximated by truncation directly or by matching the response with a chosen number of exponential terms having prespecified time constants.

Computationally the latter method is attractive because it may use the response values for the building to sinusoidal changes of a number of prespecified frequencies. The combination of the behaviour of the various heat paths is then relatively simple irrespective of the number of layers in any one path and even if branches or heat flows occur within some of the paths.

The process involves calculation of the thermal response of the separate heat paths relevant to the climate parameters at the steady state and at a set of 18 frequencies, the combination of these responses to determine the total building response to any one climate excitation and multiplication by a precalculated matrix to give the exponential series for the response factor. It has been found that the errors introduced in the matching process are insignificant when compared with the inaccuracy in knowledge of the building's thermal properties and of climatic data.

Because, in the normal heavy building, the response factor even at 10 days is not completely negligible, some method is desirable to reduce the data bank necessary to store the total building response factor. This is achieved by calculating and retaining the values at hourly intervals to 6 hr and at times in the ratio of 1:√2 upwards from 0.177 days (and including ¼, ½, 1, 2, 4 ....days).

Results will be shown as obtained by use of a Control Data 3600 computer and an indication given of approximate means for overcoming the inherent shortcomings of this and comparable methods.

Key words: Building, computer, exponential series, harmonic, indoor temperatures, matrix, response factors, step function, thermal.

## 1. Introduction

The growing desire to understand the internal thermal environment of buildings and the greater need to tailor the capacity of air conditioning devices have led to notable improvements in the calculation methods available. With the advent of electronic digital computers giving improved speed, complexity and reliability in comparison with earlier methods, it is no longer necessary to restrict investigation to simple cases or to adopt simplifying assumptions of doubtful validity.

The data commonly available for use in specific cases consist of a knowledge of the structure and its orientation, the dimensions and thermal properties of its components and the climatic variables expressed as time sequences, generally at hourly intervals, of the parameter values. These inevitably relate to past cycles and the calculation may use a set derived from an earlier specific occasion or a set representative of earlier occurrences averaged by a selected method not relevant at the moment. Sequences for external air temperature, sol-air temperatures of various surfaces, sunshine penetration of windows and internal heat loads are the most suitable and will be used hereunder although other series defining comparable climatic variables could be used.

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Several assumptions are implicit in even the most sophisticated methods presently of interest. It is almost universally assumed that the transmission through the various paths (e.g. walls, floor, roof) is unidimensional and that the effects of corners and lumped construction such as wall studs and roof rafters may be ignored. Constancy of thermal values of conductivity and heat capacity is assumed even although these are known to vary somewhat with temperature and moisture content. Film resistances are commonly also assumed constant and current work by the authors suggests this assumption does not introduce errors of an unacceptable magnitude. Muncey and Spencer[1]* showed that the transfer of heat between the bounding surfaces of a room could be treated adequately by a star network connecting each surface to a "mean convective-radiative temperature" for the errors thereby introduced are less than those caused by neglecting the presence or the location of furniture.

## 2. Overall Strategy

A convenient method, when one has available sequences describing climatic data, is to determine response factors which connect the temperature or heat flow to be calculated with the climatic data by a relation

$$\text{internal temperature} = \Sigma(\text{response factor} \times \text{climatic data sequence}) \qquad \ldots\ldots(1)$$

A well known method (Nessi and Nissole[2], Brisken and Reque[3], Stephenson and Mitalas[4] Kusuda[5]) determines the response factor relating the heat flow (with a constant internal temperature) for each path separately and thence evaluates the total heat flow. By finding the response of the internal temperature to a step function or unit pulse heat flow to the inside, it is readily possible by an inversion process to evaluate the internal temperature conditions within or following a given climatic data sequence.

This paper will describe a method which evaluates the response factor for the total building, there being a particular set of response factors corresponding to each external climatic sequence and to internal heat loads or air conditioning heat flows. The response factor for a total building derives from the sum of several sets of an infinite series of exponential terms, the number of sets being $n!$ if there be a total of n "slabs" within the several paths for heat flow within the structure. As the value of n might easily reach 20, and since the exponential decrements are related to the solutions of transcendental equations with values dependent on the thermal properties of the structure, the complexity of the exact solution needs no emphasis.

One method that might be used in a search for simplicity is to truncate the series described and it is common to find that, except for very short intervals (i.e. soon after the initiating pulse) appropriate accuracy can be achieved with only one or two terms. An alternate method is used here. In this the response is calculated for the steady state and for cases where the external driving stimulus is sinusoidal. The building can readily be treated as a whole (i.e. the effect of the several paths may be combined) even if parallel paths occur as arms within a particular identified path or heat flows occur at points internal to the path (Muncey and Spencer[6]). By suitable choice of the frequency of the stimulus and by using an adequate number of frequencies, the response may be characterised with any desired degree of precision. It will then be shown that, from these sinusoidal responses, by multiplication by a precalculated matrix, the coefficients of a large series of exponential terms with prespecified time constants can be evaluated. Again, any desired degree of precision can be achieved by using sufficient exponential terms. In the work being described, 18 terms are used with the (angular) frequency of the sinusoidal variations ranging from 1 in 768 hr to 170 2/3 per hr and the time constants ranging from 768 hr to 3/512 hr. The total errors introduced by the use of only 18 terms are of the order of 0.01 per cent. for cases where the time constants are of the order of 1 hr to 1 day.

## 3. Harmonic Response

An individual homogeneous slab of infinite area with sinusoidal temperatures on and heat flows across the faces can be considered using the same mathematics as for an electrical "four pole" (Van Gorcum[7], Vodicka[8]). The surface temperatures $T_1 \exp(j\omega t)$ and $T_2 \exp(j\omega t)$ and the heat flows $W_1 \exp(j\omega t)$ and $W_2 \exp(j\omega t)$ are related as follows:

$$\begin{vmatrix} T_2 \\ W_2 \end{vmatrix} = \begin{vmatrix} \cos H & -(R \sin H)/H \\ (H \sin H)/R & \cos H \end{vmatrix} \begin{vmatrix} T_1 \\ W_1 \end{vmatrix} \qquad \ldots.(2)$$

wherein R is the thermal resistance of the slab per unit area, C is the thermal capacity of the slab per unit area and $H = (j\omega CR)^{\frac{1}{2}}$.

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*Figures in brackets indicate the literature references at the end of this paper.

Van Gorcum further showed that, if slabs were placed in series with intimate contact over their surfaces, the matrix connecting the temperatures and heat flows over more than one slab could be found by multiplication of the individual matrices.

An individual heat path in a building can be represented by a number of slabs in series and the total building with its several heat paths by several of these groups in parallel. If the matrix for a typical heat path of area A connected to an external climatic element be

$$P = \begin{vmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{vmatrix}$$

and that of a typical heat path of area B connected to a face over which no heat flows (e.g. half a symmetrical wall exposed on each side to the same temperature) be

$$Q = \begin{vmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{vmatrix}$$

and if the value of the sinusoidal temperature be $T_A \exp(j\omega t)$ externally and $T_B \exp(j\omega t)$ internally and the heat flow to the inside (air) be $W \exp(j\omega t)$ then

$$T_B = \frac{\Sigma AT_A(1/P_{12}) - W}{\Sigma A(P_{11}/P_{12}) + \Sigma B(Q_{21}/Q_{22})} \qquad .....(3)$$

as shown by Muncey[9,10] and Pipes[11].

## 4. Step Function Response

Consider a thermal circuit as shown in figure 1 with a driving stimulus at D and a response at R. The stimulus may be an imposed temperature or a heat flow and connecting circuits may have any configuration (series, parallel or series-parallel) and some may not be present in the specific case. The steady state response $x_0$ at R due to a unit stimulus at D and the periodic response $x_0 + (x_k + jy_k) \exp(j\omega_k t)$ at R due to a stimulus $\exp(j\omega_k t)$ at D can be found for relevant $\omega_k$ by the method outlined in the previous section. It is desired to find the response $B_0 + \sum_{n=1}^{\infty} B_n \exp(-b_n t)$ at R corresponding to a unit step function at D. Consideration of the response at large values of t will show that $B_0 = x_0$. The periodic stimulus $\exp(j\omega_k t)$ may be considered as the result of innumerable infinitesimal step-function stimuli $j\omega_k \exp(j\omega_k t)\delta t$ occurring from infinite time past.
Applying the step-function response to each stimulus and adding, the harmonic response is

$$\int_{-\infty}^{t} j\omega_k \exp(j\omega_k t') \left[B_0 + \sum_{n=1}^{\infty} B_n \exp(-b_n(t-t'))\right] dt'$$

i.e. $B_0 \exp(j\omega_k t) + \sum_{n=1}^{\infty} B_n \frac{j\omega_k}{b_n + j\omega_k} \exp(j\omega_k t)$

Equating the response from the two methods and removing $B_0$ and $x_0$, a series of equations remain for evaluation of the values of $B_n$. Each equation has the form

$$\sum_{n=1}^{\infty} \frac{\omega_k^2 + jb_n\omega_k}{b_n^2 + \omega_k^2} B_n = x_k + jy_k$$

and the whole may be represented in matrix form

$|R|.|B|=|x|$ and $|S|.|B|=|y|$

Presuming that the number of n's and k's are the same, the values of the B's may be found from either set of equations as

$|B| = |R|^{-1}.|x|$ or $|B| = |S|^{-1}.|y|$

The same problem is handled by an alternate method by Muncey[12].

## 5. Practical Evaluation

The method outlined would be merely theoretically interesting if a number of conditions are not fulfilled. It is desirable that, to achieve an acceptably accurate result in representing the thermal change;

(a) the number of harmonics and exponential terms is not excessive

(b) the range of frequencies and time constants adequately covers the area of interest

(c) the elements in the inverted matrix $|R|^{-1}$ or $|S|^{-1}$ are not excessive

(d) the exponential coefficients are not large compared with the thermal change.

An early choice, which has been found to satisfy these conditions very satisfactorily, is as follows:

(a) the values of $b_1$ and $\omega_1$, i.e. the inverse time constant of the exponential and the angular frequency, to be equal

(b) the ratio between successive b's and ω's to be 2:1

(c) the total number of harmonics (and exponential terms) to be between 10 and 20.

In the presently used calculation the number is chosen as 18. This choice gives an angular frequency from 1/768 hr to 170 2/3 per hr and time constants from 768 hr (32 days) to 3/512 hr (21 sec) which adequately covers the range of interest. The $|S|$ matrix is symmetrical and $|S|^{-1}$ matrix has values up to 30.07. The quarter matrix is given in Table 1. It should be noted that by calculating the matrix inverse for increasing sizes, it can easily be seen that each row in the infinite matrix would have the values given in Table 2 in order from the diagonal to the left and right and thereafter the ratio from one element to the next is -0.5. In a matrix of large order the elements close to the top left and bottom right corners (within say 6 rows or columns) are very close to those given in Table 1. All this implies that the coefficient b is largely fixed by the values y of the harmonic response at frequencies ω close to b, a result that is not really surprising.

The above treatment relates to the response to a step function excitation. It is more suitable, in representing the external climate, to assume a linear change to occur between successive values of the climatic data. This can be achieved by transforming the step function response to give the response to one of the excitation patterns of figure 2 for temperature or heat flow.

It can readily be shown by integrating the response at time t from t-1 to t (fig.2(a)), by differencing the responses at times t and t-1, each being integrated from t-1 to t (fig.2(b)), or by differencing the responses at times t and t-1 (fig.2(c)) that the factors by which the term $B_n \exp(-b_n t)$ must be multiplied are

| | t = 1 | t > 1 |
|---|---|---|
| Figure 2(a) | $(\exp(b_n)-1)/b_n$ | $(\exp(b_n)-1)/b_n$ |
| Figure 2(b) | $(\exp(b_n)-1)/b_n$ | $-(\exp(b_n)-1)^2/b_n$ |
| Figure 2(c) | 1 | $1 - \exp(b_n)$ |

Table 1. One Quarter of the Symmetrical 18 x 18 Inverse Matrix

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7.75597 | | | | | | | | |
| -10.34130 | 21.54437 | | | | | | | |
| 6.89420 | -19.53356 | 27.67254 | | | | | | |
| -3.72068 | 11.85510 | -22.84083 | 29.45742 | | | | | |
| 1.89682 | -6.24977 | 13.54116 | -23.75077 | 29.92131 | | | | |
| -0.95304 | 3.16754 | -7.09692 | 13.99835 | -23.98384 | 30.03842 | | | |
| 0.47710 | -1.58918 | 3.59163 | -7.32579 | 14.11503 | -24.04247 | 30.06776 | | |
| -0.23862 | 0.79527 | -1.80129 | 3.70610 | -7.38415 | 14.14435 | -24.05713 | 30.07507 | |
| 0.11932 | -0.39772 | 0.90133 | -1.85853 | 3.73528 | -7.39879 | 14.15166 | -24.06074 | 30.07679 |
| -0.05966 | 0.19887 | -0.45075 | 0.92994 | -1.87311 | 3.74258 | -7.40241 | 14.15338 | -24.06143 |
| 0.02983 | -0.09943 | 0.22538 | -0.46504 | 0.93721 | -1.87671 | 3.74430 | -7.40309 | |
| -0.01491 | 0.04971 | -0.11268 | 0.23250 | -0.46863 | 0.93893 | -1.87740 | | |
| 0.00745 | -0.02484 | 0.05631 | -0.11620 | 0.23421 | -0.46932 | | | |
| -0.00372 | 0.01240 | -0.02810 | 0.05799 | -0.11688 | | | | |
| 0.00185 | -0.00615 | 0.01394 | -0.02877 | | | | | |
| -0.00089 | 0.00298 | -0.00676 | | | | | | |
| 0.00039 | -0.00131 | | | | | | | |
| -0.00012 | | | | | | | | |

Table 2. Major Elements in the Infinite Inverse Matrix

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 30.07756 | -24.06204 | 14.15414 | -7.40371 | 3.74507 | -1.87801 | 0.93969 | -0.46993 | 0.23498 |

It should be noted that these factors can range in magnitude from $10^{-3}$ to $10^{146}$ and that the steady state term $B_0$ remains at all times when using the pulse shape of figure 2(a), but cancels at times later than t = 1 for the pulse shapes of figures 2(b) and 2(c). The authors have found it most convenient to use the pulse type of figure 2(a) for temperature changes, that of figure 2(b) for solar heating through windows and that of figure 2(c) for "air-conditioning" and internal heat flows.

With exponential time constants ranging up to 32 days it is obvious that the steady state may not be approximated satisfactorily even at 50 days. Storage of behaviour at hourly intervals would require 1200 memory cells for each heat path in each building and a similar storage for either each temperature sequence or future temperature accumulation. This huge store demand has been reduced by the following device. The response factor for each path of the building and the accumulation of future temperatures is undertaken for 1, 2, 3, 4, 5 and 6 hr and for $3 \times 2^{\frac{1}{2}}$, $3 \times 2^{2/2}$, $3 \times 2^{3/2}$, ..... $3 \times 2^{m/2}$ ..... $3 \times 2^{18/2}$ hr and the future accumulator for $3 \times 2^{19/2}$ and $3 \times 2^{20/2}$ hr (90.5 and 128 days) set equal to that at $3 \times 2^{18/2}$ hr (64 days). Accumulation at each future time is made by adding the product of the climate parameter for each path by the response factor for the relevant path and time.

Following suitable "thermostat" procedures, the time marker is moved one hour and the new accumulator values are found as follows:

| | |
|---|---|
| 1 to 5 hr | by substitution from previous 2 to 6 hr values |
| $3 \times 2^{2/2}$ hr to 64 days | by interpolation as described later |
| $64\sqrt{2}$ and 128 days | by copying the 64 day value |
| 6 hr | by copying from $3 \times 2^{2/2}$ hr |
| $3 \times 2^{\frac{1}{2}}$ hr | by second order Bessel interpolation using linear time and the values for 3, 4, 5 and 6 hr. |

In the series from 6 hr to 64 days the value for $(3 \times 2^{m/2} + 1)$ hr old time (i.e. $3 \times 2^{m/2}$ hr new time) is found by second order Bessel interpolation with a logarithmic time scale. Calculations for cases recognized to be difficult to match with the chosen exponential series and operation of the repeated interpolation process have been shown to introduce errors of the order of 0.01 per cent. in conditions normally of interest in buildings.

## 6. Implementation

Computer programs to enable such calculations to be undertaken have been written for an Elliott 803 computer and more recently a Control Data 3600 computer. Since only time units are implicit in the program, it is capable of operation in any consistent system of units. Refinements allowing detailed data checking, calculation of solar position, sol-air temperatures and heat flows consequent on radiation transmitted by windows have been included. Very lengthy climatic sequences can be handled and the total memory required for the program is of the order of 28K cells. Allowing for heat path groups and 12 heat paths plus thermostat ventilation, heating and cooling paths, 10 buildings can be accommodated at one time. The whole calculation for one building with say 10 heat paths and a 21 day sequence considered hourly requires about 40 seconds computational time.

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Figure 1. Generalised thermal circuit, driving stimulus at D, response at R.

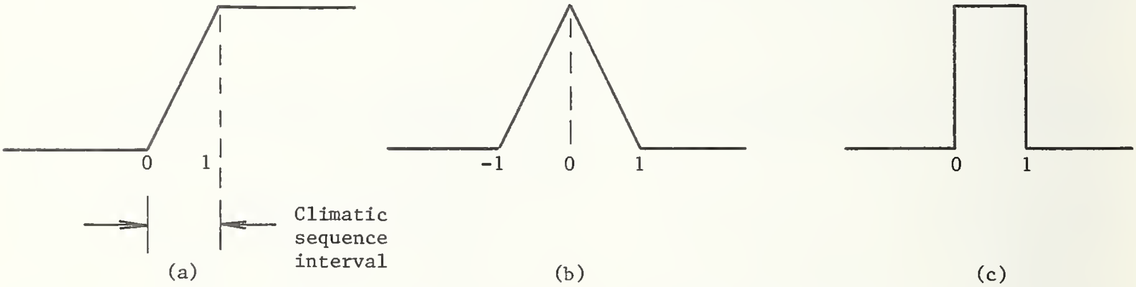

Figure 2. Temperature or heat flow excitation pulse shapes.

# Calculation of Building Thermal Response Factors (BTLRF) as Wiener Filter Coefficients

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Recent advances in the application of computers for environmental engineering problems have brought forth a number of sophisticated computer programs for simulating the hour by hour thermal performance of buildings. These programs not only calculate hourly thermal load of the building spaces, but also simulate the operation of energy distribution systems and mechanical equipment. When applied to a large building, however, the amount of computations to be performed become formidable even for the modern high speed and large memory computers. One way to reduce the computational requirement and to save the computer time (and cost) is to use building thermal response factors, (BTLRF), which are secondary sets of numbers derived from the limited amount of detailed calculations which are obtained from the exact thermal analysis. Presented in this paper is a preliminary attempt to apply time series analysis for obtaining BTLRF of a single room building. It is pointed out that BTLRF could also be obtained from measured thermal performance data or energy consumption data.

Key Words: Building thermal load response factors, energy requirements, heating and cooling load calculation, Wiener filters

## 1. Background

Calculations to determine the heating and cooling load for use in predicting building energy requirements can now be done by many digital computer programs. Although differing in minor technical details, most of the current computer programs for energy calculation obtain the hourly thermal load in conjunction with hourly weather tape data as provided, for example, by the National Weather Record Center.

This hour by hour calculation of energy requirements, based upon detailed simulation of building thermal response, has been considered more accurate for a wider type of buildings than other simplified techniques, commonly known as the "degree day method", the "equivalent load factor method" and the "bin method"[**]. These simplified methods are based upon the assumption that the building thermal performance can be calculated by a simple linear function of outdoor air temperature, particularly the temperature difference between the out- and indoor air. The temperature difference concept of the simplified techniques ignores the fact that the building thermal load is also dependent upon the other factors, such as solar radiation, moisture content of air, internal heat generation, and heat storage of the building structure. The simplified methods based upon the linear temperature difference concept have been, however, considered relatively accurate for use in residential applications, mainly due to the fact that the effect of solar radiation is relatively small and the internal heat generation is small and relatively constant as compared with commercial or industrial buildings.

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[**] Although numerous references are available for these traditional methods, the most convenient one will be the ASHRAE Guide and Data Book, Systems 1970, Chapter 40, pp. 619-634.

The factors that have justified the use of the simple temperature difference concept become increasingly inappropriate as the building becomes larger and operational and occupancy characteristics grow complex. For example, the solar radiation effect becomes extremely important when an exterior wall of a modern office building is largely glass. For another case, internal heat generation due to the heavy lighting power per square foot of floor area tends to eclipse the indoor-outdoor temperature difference effect on the thermal load. Added to the complexity of these characteristics of the modern large scale building is the sophisticated nature of the heating and cooling system and its controls, for distributing excess internal heat of the building core to the periphery to limit the heating requirements during the winter.

The hourly load simulation method with the use of computers performs an algorithmic operation which is designed to follow the actual thermal performance of buildings under realistic or randomly fluctuating outdoor weather conditions. One of the difficulties involved in using the sophisticated and exact hourly calculation of building thermal performance is that a large amount of computer and memory time is needed. For example[1], the computer program developed by the U. S. Post Office Department requires a 100 K core storage computer (if applied to large postal facilities) and approximately 2 minutes of UNIVAC 1108 time to obtain an energy requirement estimate for heating and cooling of one room for a period of 365 days. If the calculation is to be performed for a large building consisting of, say 100 different rooms, the total computation time becomes prohibitive. This is particularly so when the building characteristics under consideration are complex, and when the accuracy requirement is such that the simplification of the computational efforts may be risky. The reduction of the computational effort is usually accomplished in two different ways. The first is to simplify the algorithms such as to delete refined calculation routines (thermal storage effect and infiltration effect). The second method is to simplify the building structure such as to treat a multi-room building as a single room building by ignoring the heat exchange among rooms and by ignoring details of the building structure. However, it has not been well-established under what conditions these computational shortcuts are justified. But in addition to these two, also presented in this paper is a preliminary attempt to study a third alternative for the reducing computational requirement, with a reasonable accuracy. Its objective is to obtain a secondary set of numbers called the building thermal load response factors (BTLRF) from the results of a limited number of detailed calculations.

## 2. Building Thermal Response Factors (BTLRF)

These building response factors are basically regression coefficients as it becomes clear in the later discussion. The primary assumption imposed upon this technique is that the building thermal loads are a linear function of various excitation parameters such as outdoor temperature, solar radiation, internal heat generation and as well as the indoor temperature[*].

It is also assumed that the stochastic characteristics of the thermal load as well as the excitation time parameters are stationary, meaning that their basic means and standard deviation do not change with respect time.

As a matter of fact, it is important to point out that the basic technique used to derive these building response factors can be applicable to any time series relationship, whether it be the heating/cooling load, energy requirement, space thermal load or building thermal load. The time series, therefore, could be the observed energy usage values rather than the calculated values as mentioned previously. The idea is to derive regression coefficients from any input and the output time series by a suitable regression technique.

For example, the hourly room thermal load may be calculated by a detailed computer program for a predetermined period (say N hours). The calculated hourly thermal load is then the desired output time series whereas the dry-bulb temperature, solar radiation and the internal heat generation may be considered input time series or the excitation time series.

Denoting the hourly values of the thermal load, outdoor dry-bulb temperature, room temperature, solar radiation and the internal heat generation by q, DB, T, SOL and LT, respectively, it is assumed that the following linear relationship exists among them.

$$q_t = \sum_{s=o}^{M} (f_1(s),\ f_2(s),\ f_3(s)) \begin{pmatrix} DB_{t-s} - T_{t-s} \\ SOL_{t-s} \\ LT_{t-s} \end{pmatrix} \quad ,\ t = 1, 2 \ldots N \qquad (1)$$

---

[*] The constant factor relating the degree days data to the energy requirement is a simplified BTLRF when the temperature difference is the major contributor to the energy requirement.

In this expression $f_1(s)$, $f_2(s)$ and $f_3(s)$ for s = 0, 1, 2 ... M are the regression coefficients for the excitation parameters, temperature, solar radiation and the internal heat generation respectively. Subscript t in eq. (1) refers to the hour at which q is calculated and t-s refers to DB, T, SOL and LT evaluated at t-s hour.

These regression coefficients are called the Wiener filters [1] if they are determined in such a way that

$$\delta = \sum_{t=1}^{N} (q_t - q_t')^2 \tag{2}$$

in minimum, whereby the q' is the value obtained by the exact calculations taking into account the building details and, or the desired output time series, whereas $q_t$ is the value approximated by eq. (1) solely on the basis of time series analysis of participating variables.

In eq. (2) N is the total number of data points to be analyzed to arrive at the least squares regression coefficients or Wiener filters. For example, if the two weeks data were used for the hourly thermal load calculations, N should be 336.

A computer program to obtain the Wiener filters coefficients has been developed and published by E. A. Robinson [2]. The program utilizes a recursion type solution of multi-channel normal equations of the data to be processed. Given in the following section are examples of the application of the Robinson's computer program to the heating and cooling load calculation by the thermal analysis program [2] of the U. S. Post Office Department (USPOD).

## 3. Sample Calculations

In order to examine the feasibility of the use of the Wiener filter routine to obtain BTLRF as the least square regression coefficients, hourly heating and cooling loads of a one-room building was first computed for 336 hours by the USPOD program. The weather data used for the calculation were for January 1949 of Washington, D. C.

Figure 1 shows the trend of the excitation functions, namely the dry-bulb temperature, solar radiation and internal heat generation during the computation periods.

In order to simplify the calculation, the room temperature, $T_t$, in eq. (1) was assumed constant at 75 °F. When the calculated thermal load was plotted against the outdoor temperature and against the solar radiation, they showed very much scatter as shown in figures 2 and 3 respectively. Figure 2, for example, suggests a danger of estimating hourly thermal load by a linear relationship with outdoor air temperature alone.

The Wiener filtering technique was applied to the calculated thermal load regressed with $(DB-75)_{t-s}$, $SOL_{t-s}$ and $LT_{t-s}$ for eq. (1) for s = 0, 1, 2, ... M.

The value M in equation (1) is called the filter length and is related to the delayed reaction of the thermal load $q_t$ with respect to the excitation parameters. A satisfactory value for M may be determined by letting M = o, 1, 2 ... in eq. (1) until further increase does not significantly decrease the value of δ. In this particular example, values of M up to 20 have been tried and it was found that the optimum value is 3 for all the practical purposes.

In order to illustrate building response factors for M = 3, the filter coefficients for a one-room building are listed as follows:

| | | |
|---|---|---|
| $f_1(0) = 31.913$ | $f_2(0) = 3.807$ | $f_3(0) = 4.308$ |
| $f_1(1) = -.426$ | $f_2(1) = -.056$ | $f_3(1) = 1.809$ |
| $f_1(2) = -.267$ | $f_2(2) = 1.777$ | $f_3(2) = 1.762$ |
| $f_1(3) = -.245$ | $f_2(3) = 1.110$ | $f_3(3) = 2.639$ |

Normalized values[1/] of $\delta$ for $M = 0, 1, 2 \ldots 10$ respectively for a similar analysis are 0.219, .137, .093, .067, .062, .057, .054, .053, .050, and .047, which show the diminishing return for M beyond 3. It should be pointed out that it is difficult to draw physically meaningful conclusions from these coefficients, since they were derived solely by numerical data manipulation. Nevertheless, they simulate thermal load very accurately for the period where the original data were analyzed. Also to be pointed out is the reduction of mathematical operation manifested in a simple algebraic formula of equation (1) against a detailed thermal analysis program consisting of approximately 2000 Fortran statements.

It is, however, to be expected from the theory of heat conduction equation that the absolute values of BTLRF should start to decrease steadily[3/] as the value of s increases beyond a certain value, say $S_{max}$, such that

$$\ldots\ \left| f_1\ (S+3) \right| < \left| f_1\ (S+2) \right| < \left| f_1\ (S+1) \right| < \left| f_1\ (S) \right|$$

when $S \geq S_{max}$

This decreasing trend was not observed for this sample calculation even when M was carried up to 20, although it is possible that filter coefficients of more physically consistent nature might have been obtained, had a suitable smoothing technique been applied to the input data.

Although these response factors did reproduce the original data very well, a true test of the response factors would be when they are applied in a predictive manner. Figure 4 shows the same response factors applied to eq. (1) for the climatic data beyond the period when the original thermal load was calculated. Figure 5 is in turn the thermal load calculated by the USPOD program for the same weather record period. If the response factors are ideal, figures 4 and 5 should match each other well for the entire period.

By overlaying figure 4 on figure 5 it can be shown that the two curves match almost perfectly for the first 336 hours during which period the response factors were generated. The same two curves, however, begin to differ considerably as the time goes beyond the first 336 hours and particularly during the summer period, although general trend of the increase of the mean thermal load is obtained by the response factor calculation. The increase of the diurnal amplitude of the thermal load during the summer, however, was not well represented by the calculation using BTLRF.

The similar calculation repeated for 336 hours (two weeks period) data of thermal load and accompanying weather data during the last week of June yielded another set of building response factors such as:

$f_1(0) = 39.497$
$f_1(1) = 15.488$
$f_1(2) = -50.894$
$f_1(3) = 38.594$

$f_2(0) = 11.558$
$f_2(1) = -4.362$
$f_2(2) = -4.177$
$f_2(3) = 10.768$

$f_3(0) = 7.206$
$f_3(1) = 1.668$
$f_3(2) = 1.789$
$f_3(3) = 1.650$

These values were in turn used again to calculate the hourly building thermal load from January to June by eq. (1), results of which are shown in figure 6.

The agreement between the thermal loads obtained by the detailed calculation with use of USPOD program and those approximated by eq. (1) is poor during the winter this time. The decrease of the average values and amplitudes of the building thermal load during the winter is not well reproduced.

These dwo sets of calculations and figures 4 and 6 suggest that the BTLRF can be made a function of time.

It is assumed that they will change from set (3) to (4) by a linear fashion such that:

$$[f(t)] = [f_w]\ \xi_t + [f_s]\ (1 - \xi_t) \tag{5}$$

where $[f_w]$ and $[f_s]$ represent the winter and summer building response factors and $[f]$ is those adjusted with time.

The value of $\xi_t$ in eq. (5) was assumed to be a step time function representing:

$$\xi_t = \frac{\text{Integer part of } (t/336)}{12} \qquad (6)$$

for the 12 bi-weekly periods spanning the beginning of January through the near end of June.

The result of this calculation is shown in figure 7, and indicates a better agreement with the detailed calculations (figure 3) obtained by the USPOD program throughout the period than figures 4 and 6. The agreement should be further improved if the values of BTLRF were made a more complex function of time than a simple linear function.

## 4. Summary

A possible new approach to enhance the use of computers for calculating building thermal load is the application of Wiener-type filter coefficients which are called in this paper the BTLRF or the building thermal load response factors. It is pointed out in this paper that BTLRF can be obtained either from the heating/cooling load calculated by the very comprehensive computer program (simulating entire building heat transfer processes) or from the experimentally observed values for a limited period of time, say two weeks. Once determined, these BTLRF can permit the calculation of the thermal load by one simple linear algebraic equation. This results in drastic reduction of the computational effort as well as the core requirement on the computers, from a computer program needing a few thousand Fortran statements and 100 K core storage computer to a program of a few Fortran statements that can be executed on a mini-computer. A rough estimate of computer time reduction is from 2 minutes per room of a building to a few seconds per room for a computation covering 365 days.

This paper presents one result of an exploratory investigation to derive BTLRF by the use of Wiener Filter Technique to the heating and cooling load calculated by the U. S. Post Office Energy Analysis Computer Program.

The BTLRF were found to be dependent on time if they were to be applicable for the calculation of hourly building thermal load over as long as a half year's period. This consideration is necessary because building thermal load characteristics cannot be considered stationary if the time span is as long as a half year.

The time span of the hourly data used to determine the BTLRF was 336 hours for the calculation illustrated in this report, although it could most possibly have been shortened to 168 hours or even less. A satisfactory length of the filter appeared to be 4 terms ($j = 0, 1, 2, 3$).

Although BTLRF provide a relatively good estimate in load calculation by a very simple algebraic operation, the coefficients obtained by the Wiener filtering technique did not follow the expected trend that the absolute value would eventually start decreasing steadily. Further work is being performed at the Environmental Engineering Section of the National Bureau of Standards to obtain building thermal load response factors which do follow this expected trend and which are therefore more amenable to physical interpretation.

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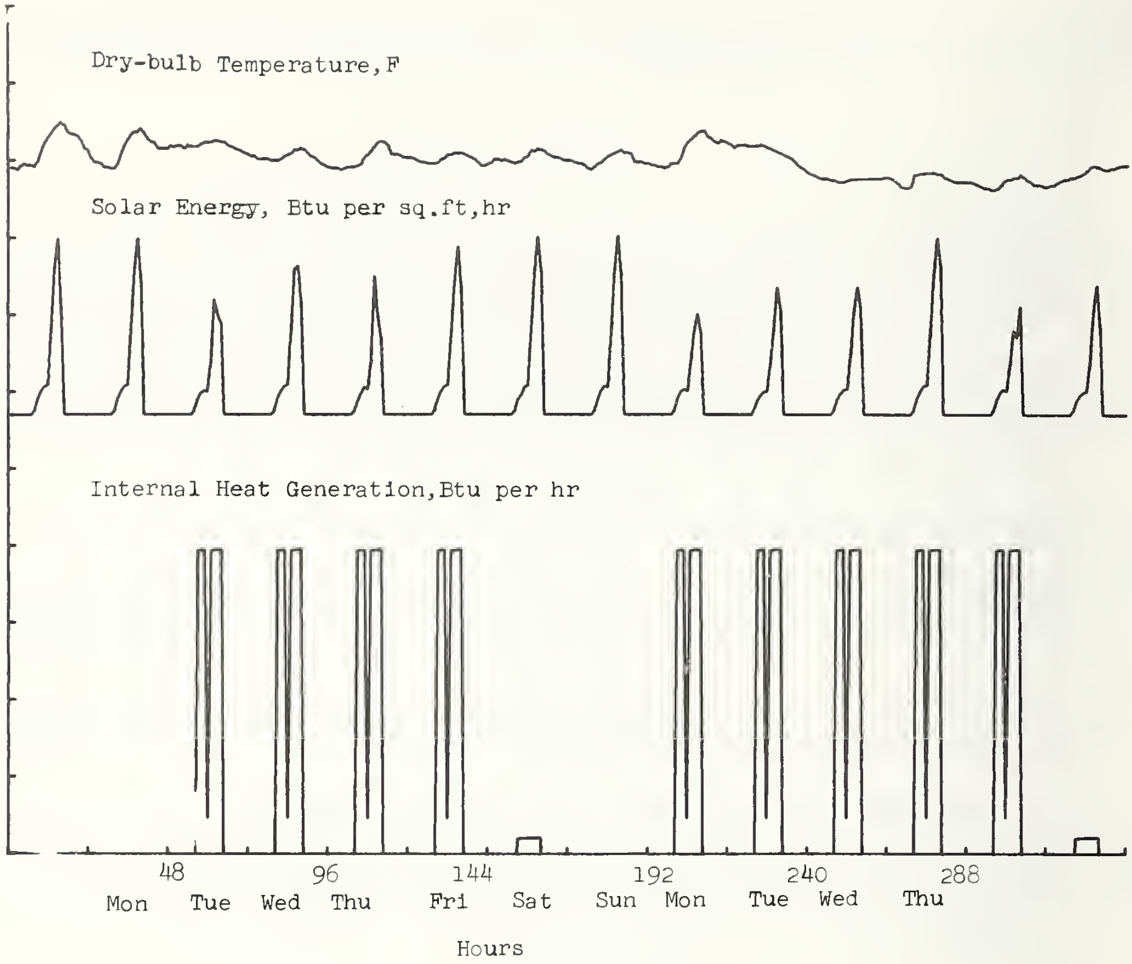

Figure 1. Excitation functions used for the thermal load calculation by USPOD computer program for the first two weeks of January.

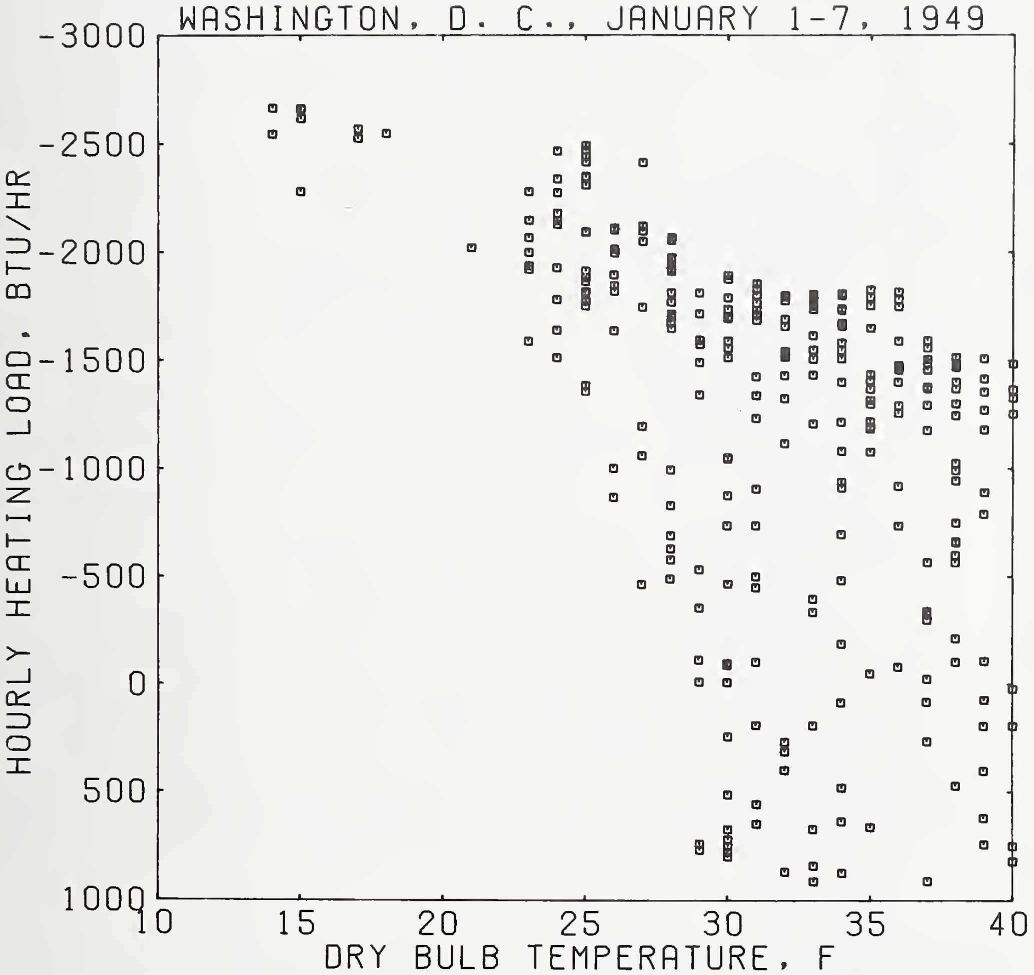

Figure 2. Relationship between the calculated hourly thermal load and outdoor air dry-bulb temperature.

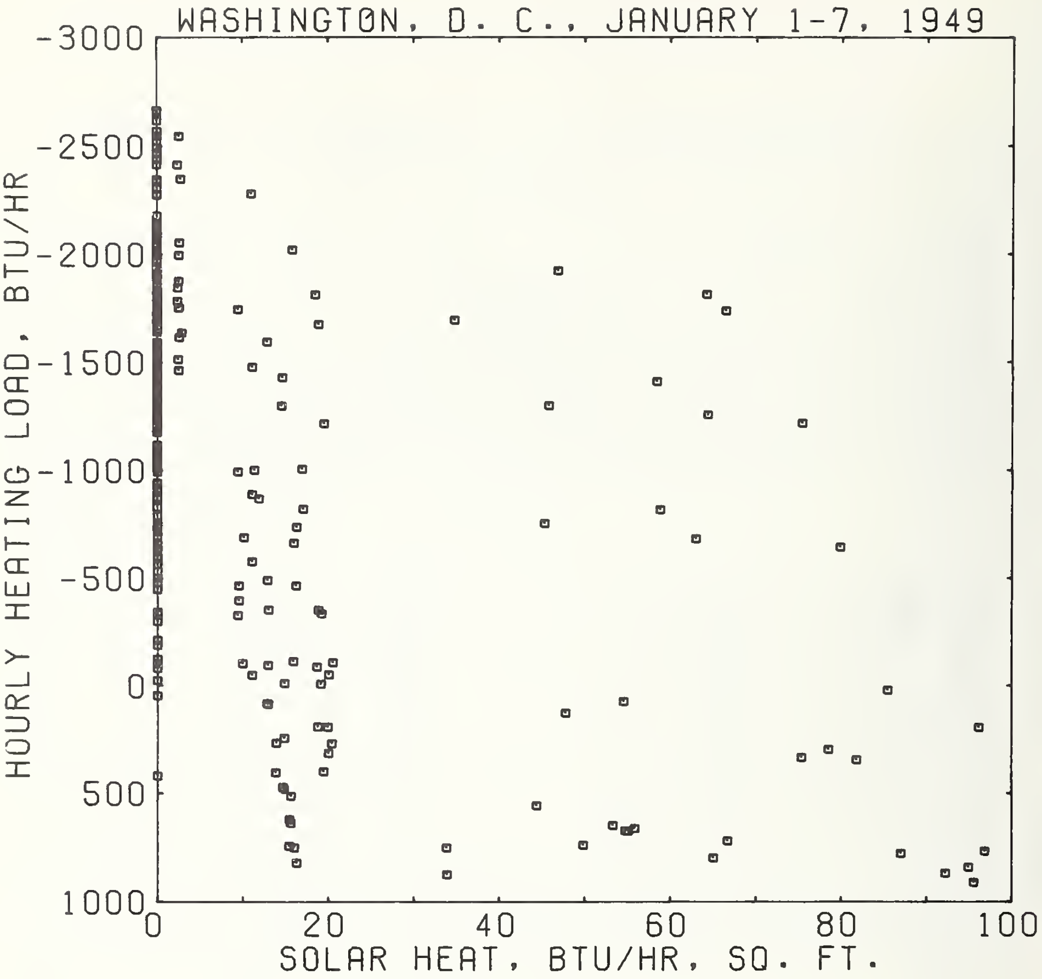

Figure 3. Relationship between the calculated hourly thermal load and solar radiation over the south facing wall.

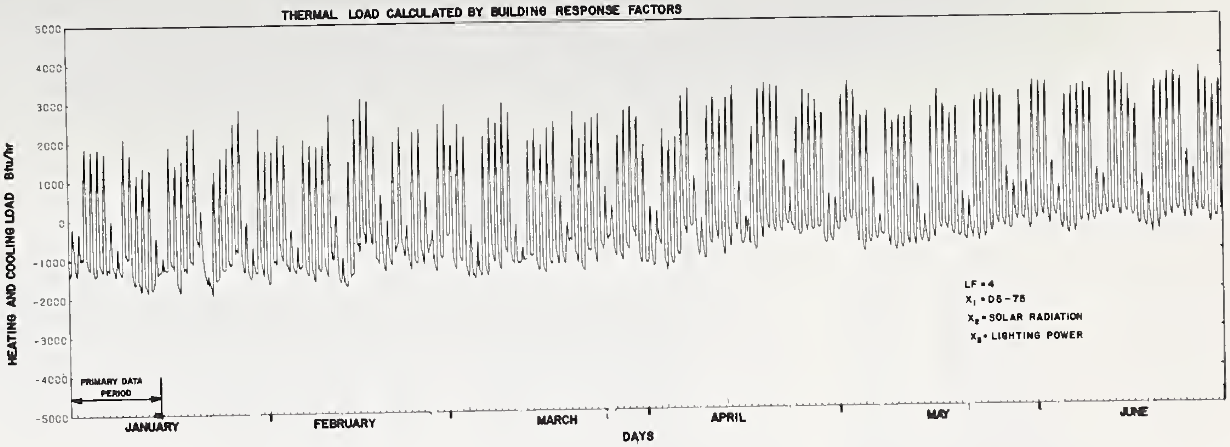

Figure 4. Thermal load calculated by the winter BTLRF for January through June.

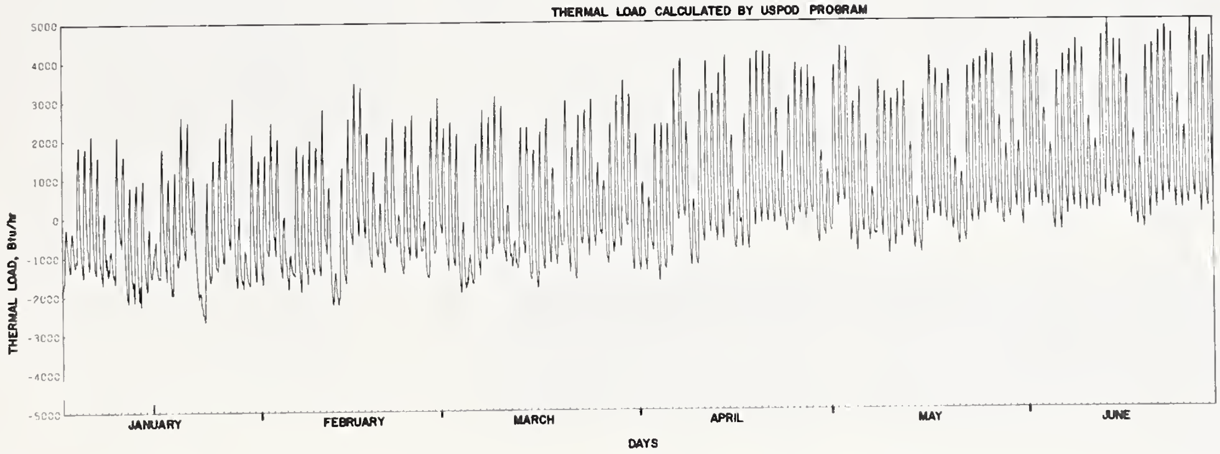

Figure 5. Thermal load calculated by the detailed computer program of USPOD for January through June.

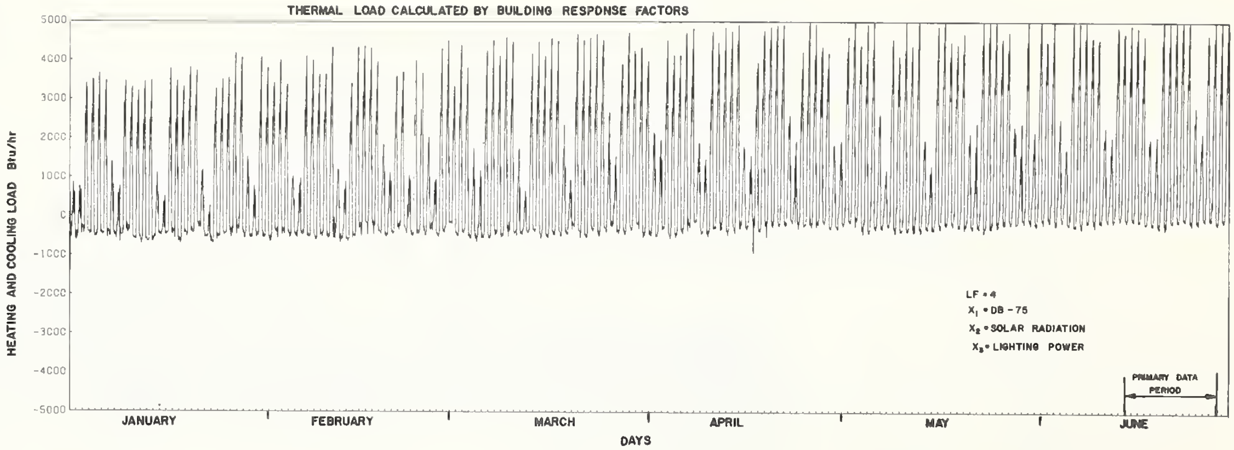

Figure 6. Thermal load calculated by the summer BTLRF for January through June.

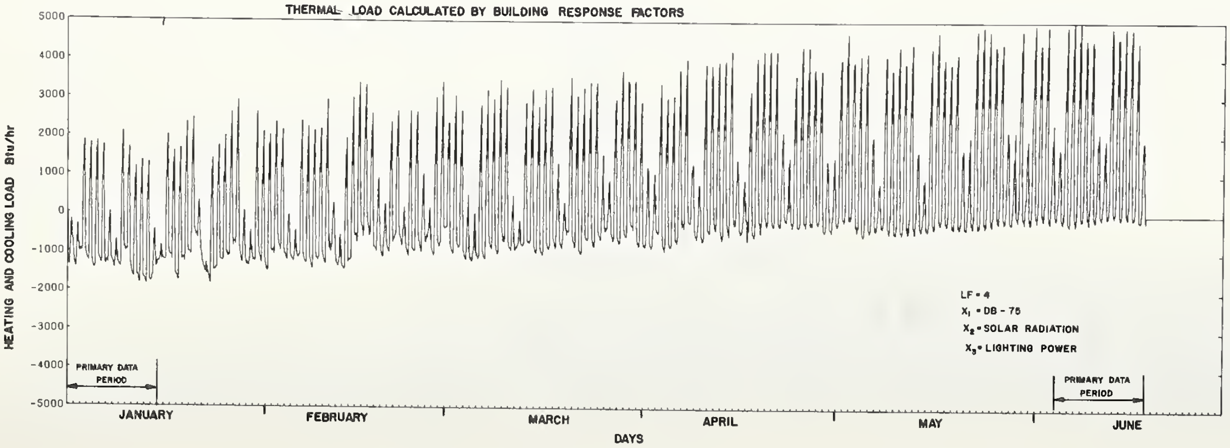

Figure 7. Thermal load calculated by the BTLRF which is a linear function of time.

# Thermal studies by electrical simulation. Application example to the study of the heating equipment of an apartment building heated by electricity.

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For the study of trasient heat flow problems we dispose of a simulateur composed of an electrical model with resistors ant capacitors, a direct current analog computer fitted with a logical console and the necessary input and output devices.

The electrical model allows to represent a group of three rooms. The time contraction on this model is in the ratio of 0,2 second for 24 hours.

The direct current analog computer allows to :
- feed on the model the climatic or occupancy data,
- represent heating and its control.

The input device allows to store for instance the climatic or occupancy data throughout an entire heating season.

The output device includes a fast recorder and a group of digital counters allowing an analysis of results.

The following problems can be treated :
In artificial climatization (heating or summer air conditionning) :
- Studies on the power required in the course of sequences of the hottest or coldest day.
- Studies on the consumption in the course of a season of heating or air conditionning.
- Studies on the efficiency of a control system during typical sequence of several days or months.

In natural climatization :
- Studies on the variation of the interior temperature in a room during typical sequence of several days or months.
- Studies on the frequency curve of the temperature in a room throughout a season.

As an exqmple are given some results relating to the study of the heating device in an partment building heated by electricity.

The heating system selected for the study includes a base heating by storage in the solid concrete floors where the energy is supplied principally during the night hours and a additional forced air heating which is controlled by a thermostat in each room.

We have studied the influence of the heating device and its control on the comfort condition and the energy for heating (base temperature, power capacities, etcetera).

Key Words : Electric heating, accumulation heating in the floor, blowed air heating, consumptions, power, control, comfort.

## 1, Introduction

To solve conduction heat transfer problems in unsteady state conditions we make use of an analog method to represent thermal characteristics of walls and floors by networks of resistors and capacitors. The simulator we have developed on this basis is specially suited for buildings problems. Below we give a brief description of this simulator and as an exemple the study for the electrically heated system of an apartment building.

## 2, Description of the simulator

The simulator is composed of three basic section :
- RC network
- a direct current analog computer fitted with a Logical console,
- the data input and output devices.

The photograph of figure 1 shows the overall view of the simulator and also indicates names of the major components of all the sections described below.

### 2.1, RC network

The model is based on the analogy between heat transfer in wall having thermal inertia and propagation of electricity in a circuit having distributed resistance and capacitance. In pratice it is difficult to design such an electric medium. So the distributed constant equivalent circuit representating a wall is obtained by combining quadripoles in serie, each of them representing a slice of the equivalent circuit, therefore a layer of the wall.

To determine the thickness of the slices, the heat transfer of a sinusoïdal signal in an homogeneous wall has been studied. The mathematical resolution of this problem being known, it is possible to calculate the temperature and the heat flux in each plane parallel to the faces of the wall. It is also possible to calculate, for the same signal, the temperature and the heat flux at the interface of each section of the lumped circuit equivalent wall. By comparing these two calculations the error introduced by the slicing is determined : this error is mainly a function of the frequency of the signal. Thus, knowing the highest frequency present in the problem and the acceptable error, the thickness of each slice can be determined. In case the highest frequencies present in the problem are introduced by a on-off thermostat for instance (the cycle of the transient phenomenon being of the order of fractional hour) and with an acceptable error of on per cent, the thickness of the slice must be about one centimeter. Thus it requires a great number of slices. Still hight frequency signals transmitted through the wall are quickly damped, and it is not necessary to keep the same slicing over all the thickness. In practise, after four of the thinest slices, it is possible to double the thickness and so on.

To allow an easier operation, fifty walls, each one including six sections have been prewired, thus the coupling between plug-in type resistors and capacitors were made once for all.

The surface resistances network in a room is also pre-wired. In this network the convective heat transfer between each wall and air and the radiative heat transfer from each wall to the others have been distinguished. Three networks, allowings to represent a group of three rooms, are pre-wired in this way. The value of the convection coefficients on the horizontal walls changing with the sense of the heat flux, these coefficients are represented in the model by circuits of the type shown on figure 2.

The ratio of electrical time to thermal time used in the model being of the order of 2 to 4 x $10^{-6}$, the duration of one day is 0.2 to 0.4 second. The ratio of electrical resistance to thermal resistance is of the order of $10^{-6}$ ohms for 1°C $W^{-1}$but can be changed. Taking into account the time ratio above, the ratio of electrical capacitance to thermal capacity is of the order of 2 to $4.10^{-6} \mu F$ for 1 $J°C^{-1}$.

### 2.2, The direct current analog computer.

Fifty operational amplifiers are used to perform the following functions :
- Impedance matching of the generators supplying the climatic and occupancy data to the network and also impedance matching of the network to the device recording the voltage at several nodes.
- Data summation : for instance summation versus time of voltages representing outside temperature and solar radiation on a wall to obtain a voltage representing sol-air temperature.
- Current generators ; for instance current generator representing solar heat flux entering in a room by openings.
- Representation of heating, air conditionning control systems.

The logical console is composed of logical modules (AND, OR, NOR, NAND, comparators, etcetera) and electromechanical relays or electronic switches, this system, supplied with pulses or square signals recorded on the input device (magnetic recorder) described in the section below, allows to control at predetermined times the following functions, for instance :
- generation of a heat supplied by the heating or air conditionning equipment or by the occupancy,
- change of a ventilation rate.

### 2.3, Data input and output devices.

The data input device is composed of a punched tape reader, a digital to analog converter and a fourteen tracks magnetic tape recorder. Each climatic data is recorded on a punched tape in fifteen minutes steps. The same operation is made for the occupancy data if they are functions of the time only. During the reading of the punched tape the digital data is converted to analog signal and recorded on a track of the magnetic tape. All the data relating to a building in a given locality outside dry bulb temperature, solar radiation on the walls, humidity rates of the outside air, etcetera, are stored in this manner.

Two output devices can be used. The first one is a twelve tracks ultraviolet photographic recorder fitted with two types of galvanometers ; the three decibels bandpass of which are four hundred of fifteen hundred cycles per second respectively. The second one is a system of digital counters allowing an analysis of the results : consumption statements, statements of the number of times a temperature is reached or exceded, etcetera.

## 3, Scope of the simulator.

The design of the simulator has been made to match at best the study of the following problems :

### 3.1, Natural climatization.

- study on the variation of room temperature during a typical hot or cold spell, for instance.
- study on the variation of temperature in a room throughout a heating or cooling season. In such a case, real climatic data of a given locality are used and the maximum daily temperatures frequency curve, or the curve giving the total time during which the temperature stays at a given value may be determined.

### 3.2, Artifical climatization
(heating or summer air conditionning)

- Analysis of the power required in a room throughout the coldest or hottest days sequences, to determine the heating or air conditionning equipment.

- Analysis of the power consumption in a room throughout an entire heating or air conditionning season, to determine the energy requirement or the frequency curves of the room temperature (air temperature, floor temperature in the case of floor heating or the air relative humidity in humidifying climatization systems, for instance).

- Analysis of the efficiency of a control system throughout the season or a sequence of clear but sunny days in mid-season heating. The efficiency of the control system may be judged by taking into account the energy requirement and comfort conditions obtained by the system.

## 4, Study of the heating device of an apartment building heated by electricity

Reported here is a study of a heating device of an apartment building, the power source being electricity.

The building had a large thermal inertia and a good thermal insulation.

The system analyzed includes :
- A base heating by storage in the solid concrete floors, the power being supplied principally during the night hours.
- An additional forced air heating controlled in each room by a thermostat. The air taken from outside, is pre-heated to a temperature Tp before supplied to the room ; this principally is to allow sufficient amount of humidification is necessary.

The heating system and its control system (capacities, base and preheating temperatures, ...) were studied to determine the best balance comfort conditions - energy consumption.

### 4.1, Climatic data

From the records obtained by the national Meteorology Office in Le Bourget station, near Paris, we have chosen two sequences (Fig 3) :

$1^{st}$ sequence (february 1963. 13 th to 28 th)
this is a typical sequence of cold and sunny days, the outside air mean temperature being near the base temperature of the place (- 7°C) for several days.

$2^{nd}$ sequence (march 29 th to april 11 th, 1964)
this is a typical mid season sequence, the outside air mean temperature being rather high, the diurnal variation and the direct solar radiation being also high.

For these two sequences :
- The outside air temperature is the real one recorded in the station.
- The solar radiation has been computed by means of the curves giving the intensities of the direct normal radiation and of the diffuse radiation on an horizontal plane with clear atmosphere, of the sunshine hours and of the cloud cover factor. The shading effect created by the balcony has been taken into account.
- The long wave radiative exchange balance is a linear approximation of the following formula :

$$B = a \quad \rho_o \left( T_{re}^4 - \Theta_s^4 \right)$$

b : heat balance (W $m^{-2}$)
a : absorption coefficient of the wall
$\Theta_s$ and Tre : respectively wall surface temperature and environment radiant temperature (°K)
$\rho_o$ : Stefan - Boltzmann constant (W $m^{-2}$ ° $K^{-4}$)

Tre is approximated only as a function of the cloud cover factor. We have, for a vertical wall :
Tre = Tae - 2°C for a cloud cover factor greater than 3
Tre = Tae - 6°C for a cloud cover factor lower or equal to 3,
(cloud cover factor is in the range of 0 to 8)
Tae being the outside temperature.

### 4.2, Description and characteristics of the studied rooms (fig.4)

The building includes about 200 apartments of a single surface exposure. It has a symmetry plane parallel to the façades and its orientation is E.W.

We consider a slice of the building bounded on two sides by the west frontage and the symetry plane and on the other sides by the adjacent rooms.

We assume that the slice being studied and the adjacent ones have the same operation characteristics therefore, the same inside conditions.

1, Inside Wall Composition (1)
- Horizontal walls : they are made of heavy aggregates solid concrete, 15 cm thick ; the floor can be covered with a velvet pile with or whitout coarse haire cloth.

- Vertical walls : they are either of heavy aggregates solid concrete 15 cm thick as the horizontal walls or, plaster slabs 7 cm thick.

### 2, Façade Composition

The façade panel is of the light type. Its mean thermal transmission coefficient $K_m$ is 1.68 W m$^{-2}$ °C$^{-1}$.

The opening is fitted with double glazed windows (1), externally screened by shutters and internally by a light-coloured blind.

The shutters are always closed during night from 9 p.m. to 8 a.m. and on occasions during the day. The opening thermal transmission coefficient is dependent of the position of the shutters :

Shutters open : $K = 3$ W m$^{-2}$ °C$^{-1}$
Shutters closed : $K = 2.4$ W m$^{-2}$ °C$^{-1}$

### 3, Occupancy

The heat generated by the bed-room occupancy has been fixed at 90 W from 9 p.m. to 8 a.m. next morning.

### 4. Exchange coefficients.

The radiative exchange coefficients were calculated by the standart formulas while the convection exchange assumed the following values :

- Vertical walls : 5.4. W m$^{-2}$ °C$^{-1}$
- Horizontal walls, upward flux : 6.3 W m$^{-2}$ °C$^{-1}$
- Horizontal walls, downward flux : 0.6. W m$^{-2}$ °C$^{-1}$

## 4.3, The heating device (Fig 4 bis)

1, A preheating of the force circulated ventilation air to a temperature Tp. The primary purpose this preheating is to be able to maintain the water content in the ventilation air above 6 g for 1 kg of dry air. The forced air rate is constant at 30 m3 h$^{-1}$.

2, A base heating by a cable embelled in the concrete floors allowing a storage. The energization of the embelled heater is made during the off peak usage hours, at the maximum during night hours. Heating and preheating alone vould give a mean temperature Tf of 10 to 18°C according to the time of the heating season taken into consideration.

3, A supplementary heater installed in the air supply system to the individual room complements the base heater system to provides 20 +o 22°C with a control thermostat in each room (thermostat threshold ± 0,5°C, response time 10 minutes).

The power capacity of this supplementary heater is fixed at 500 w, much higher than the room needs, to take into account losses by the adjacent rooms and to allow a greater flexibility to the heating system.

The air is exhaused in the passage-room, where the heat losses by the adjacent rooms and to allow a greater flexibility to the heating system.

The air is exhaused in the passage-room, where the heat loss from the forced air supply duct is taken into account.

The power consumpted in the supplementary heater can be billed individually and its rate structure is different from that of the base heating, which is billed collectively.

## 4.4, Résults

The control system of the base floor heating is an open-loop system (no feedback from the air temperatures obtained inside). The night hours are divided in a number of equal intervals of time. In each of these intervals the connection of the heating resistances is commended in such a way that the ratio $\frac{\text{time of connection}}{\text{length of interval}}$ is a function f $(0 \leqslant f \leqslant 1)$ of outside weather parameters : this is approximately equivalent to supply continuously a heat power $\Phi$ equal to f. $\Phi_o$ ( $\Phi_o$ = installed power).

Here we limit ourselves to the comparison of two control systems :
- one taking into account the instantaneous value of the outside temperature during the time when the baseheating system is energized,
- the other including the mean values of the outside temperature and the solar radiation on the room façade.

The comparison will made on the maximum temperatures attained in the room and on the energy consumption.

We will consider only the case where the power is supplied to the floor heater during night hours, that is to say from 10 p.m. to 6 a.m.

### a, Control from the instantaneous value of the outside air temperature.

Assume first that the power heat supplied to the base floor heater was controlled to the following expension :

$$\Phi \ (t) = k_1 \ Tf \ (t) - k2 \ Tae \ (t) - k3 \ Tp \ (t) \qquad (2)$$

- Coefficients k1, k2, k3 are defined as functions of the thermal characteristics of the room and those of the heating and ventilating equipment.
- The preheating is set on a particular temperature Tpo :

$$\text{if } T_{ae} \leqslant T_{po} \ , \quad T_p = T_{po}$$

$$\text{if } T_{ae} > T_{po} \ , \quad T_p = T_{ae}$$

The results which are presented are those obtained when the preheating is not switched off during the peak hours.

From the first it is know that the control mode cannot be satisfactory, on the one hand because it does not take into account the solar radiation contribution and on the other hand, because the outside air mean temperature measured between 10 p.m. and 6 a.m. may be quite different from that of the day. This leads to a base temperature drift, this drift becomes more significant as the preheating is low and the diurnal variation of air temperature becomes large.

During the february sequence, this drift is very low, as shown on the air temperature records of figure 5 where the base heating is working alone. Otherwise it is of about 2,5° at the end of the april sequence, for Tf = 16°C and Tp = 6°C, where the solar contribution are not taken into account by the control ; this drift is increased due to the fact that hight solar heat gain are coincident with hight diurnal variation of the outside temperature.

Minimization of this drift is possible only by lowering the base temperature during mid season.

Figures 6,7 and 8 show the inside air temperature when the thermostat is set at 20°C.

In february, and in the case the solar, contribution in the room is less important because of balcony structure, the inside temperature, 21°C mean value, is still acceptable for a base temperature of 16°C, preheating to a temperature greater than 6°C improves the comfort conditions (air and floor surface temperatures in the morning) and lowers very appréciably heating energy requirement during the night hours ; so when Tf = 16°C, Te = - 5°C and if 22°C inside temperature is wanted by heating during the off peak hours, one must have :

if $T_p$ = 6°C : Power capacity = 1500 W
floor = 25.5°C
ceiling = 30°C

if $T_p$ = 16°C : Power capacity = 1100 W
floor = 24.2.°C
ceiling = 27,5°C

At the end of the april sequence and for Tf = 14°C, are still recorded very incomfortable air temperatures, of the order of about 24°C mean value (without balcony). In fact, windows would be opened to restore more acceptable comfort conditions, but the consumption waste would be increased.

### b, Control from mean outside air temperature and solar radiation.

The heat flux supplied is of the form :

$$\Phi\ t = k_4\ (Tf)t - k_5 \frac{1}{t2-t1} \int_{t-t1}^{t-t2} Tae\ d\,\Theta - k6\ (Tae)t - k7 \frac{1}{t4-t3} \int_{t-t3}^{t-t4} R\ d\,\Theta$$

$$- k8\ (Tp)t$$

- The time intervals $t_1 - t_2$and $t_3 - t_4$ being included in a cycle which may be daily.
- Coefficient $k_4$to $k_8$ are defined as functions of the thermal characteristics of the room and those of the heating and ventilating equipment.
- R is the solar radiation on the façade.
- $T_p$ is defined as in 4.4, a, but the preheating is switched off during the peak hours.

The improvement attained by this type of regulation over the former one is very distinct for the april mid-season sequence.

On the fig.9 some results are reproduced, the base heating working alone, with and without controls taking into account the solar.radiation.
- In case the controls does not take into account the solar radiation, the drift is slightly greater than 0,5°C at the end of the sequence with solar factor zero and,
- In the case the controls takes into account the solar radiation, the drift stays sensibly in a bracket of ± 0,5°C.

On figures 10 and 11 one can read the inside air temperature variation for different values of Tf, the thermostat being set at 20°C.

At the end of the april sequence, heat gain from the solar and occupancy contributions are greater than the losses (Tf being between 10 and 16°C), inside air temperature does not change much, staying in mean value around 21 to 22°C, the peaks not exceeding 24°C.

The difference is much significant in relation to the relative values of the power consumed ; with a balcony in the case corresponding to figures 10 and 11, the consumptions recorded during the fourteen last days for base and supplementary heatings are the following :

$T_f$ = 16°C ( Base : 60 kWh
(Supplementary : 8.1 kWh

$T_f$ = 10°C (Base : 22.1 kWh
(Supplementary : 48.9. kWh

For comparison with the control scheme defined in 4.4, a, the energy requirement recorded in the some conditions are :

$T_f$ = 16°C (Base : 82 kWh
(Supplementary : 4 kWh

The results for the february sequence are given in fig.12.

## 5, References

(1) The thermal characteristics of the building walls (thermal conductivity, mass per unit surface, K coefficient, etcetera) are extracted from the Document Technique Unifié : "Computation rules for the serviceable thermal characteristics of the building walls and off the basic losses of the buildings".

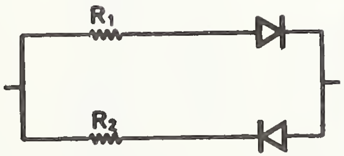


Figure 2, Horizontal Walls Surface Exchange Coefficients Representation.

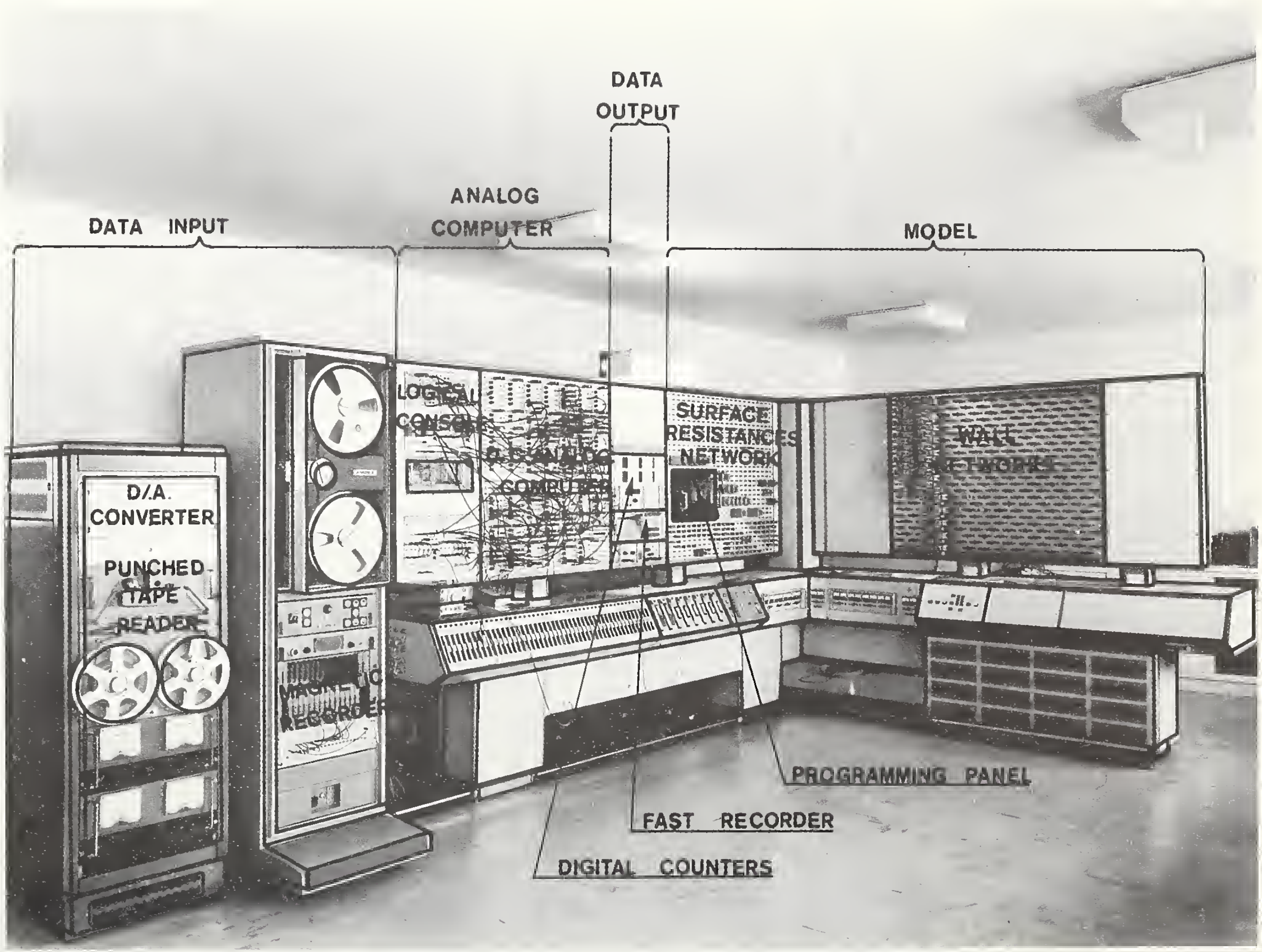


Figure 1, General view of the simulator.

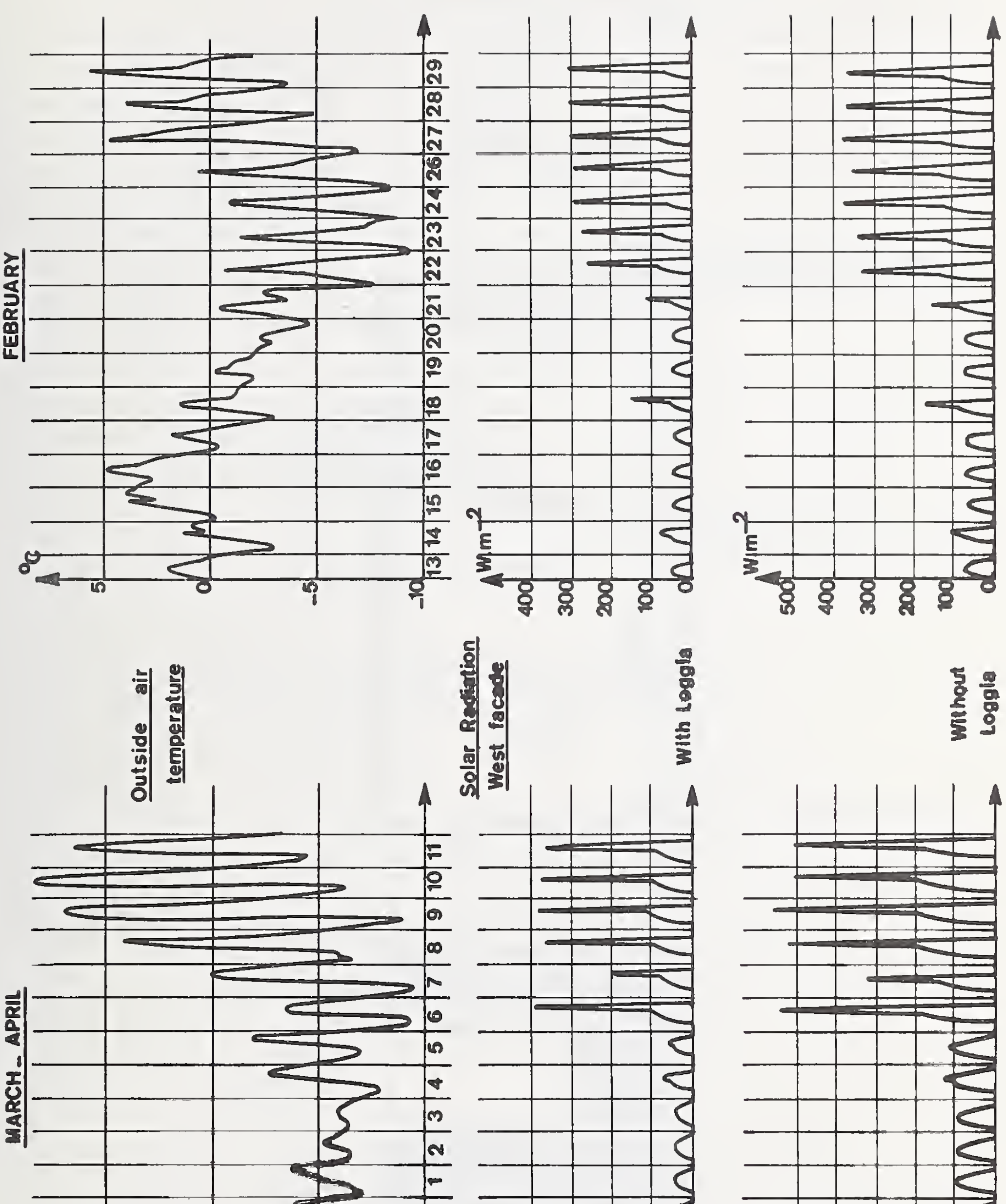


Figure 3, Climatic Data

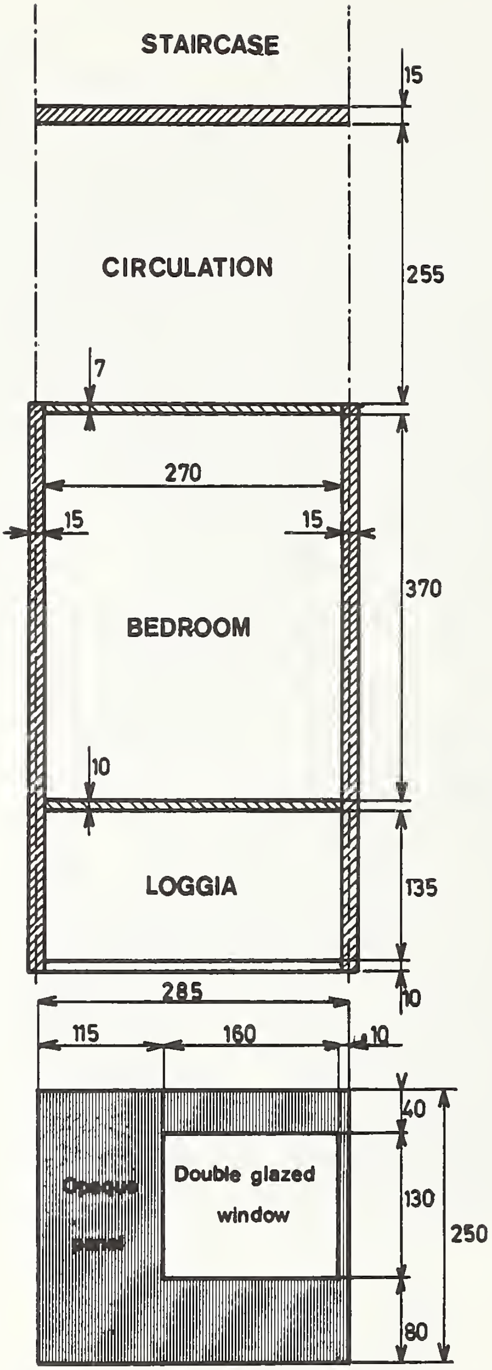


Figure 4, Studied Rooms and Façade Drawings

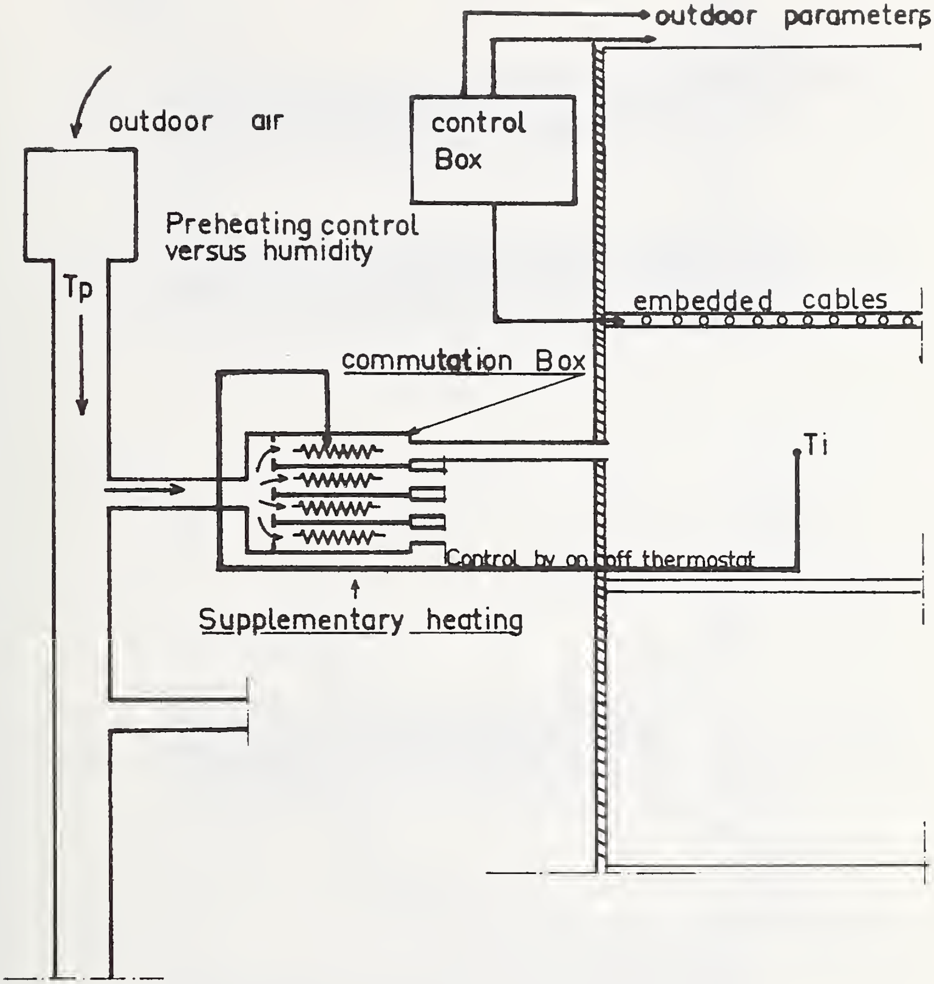


Figure 4bis - System schematic

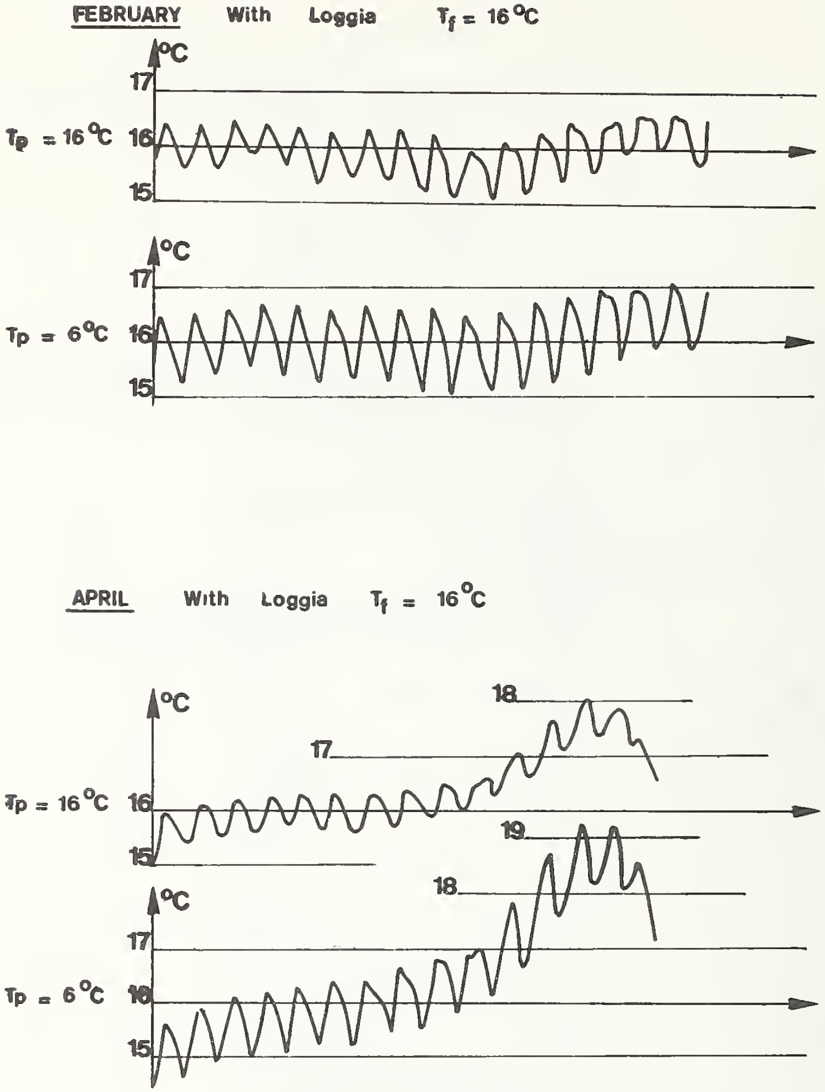
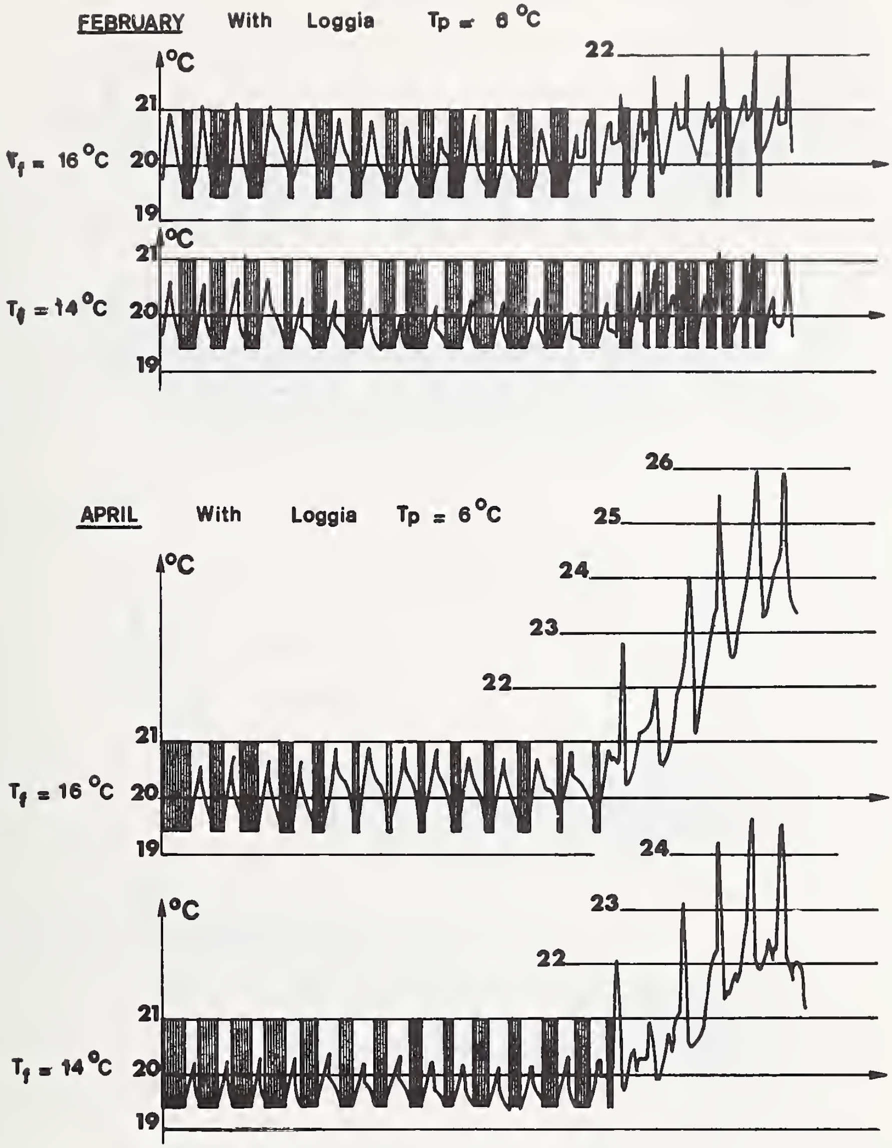


Figure 5, Inside Air Temperature.
Base heating only, regulated as a function
of the outside air temperature.



**FEBRUARY** With Loggia Tp = 6 °C

**APRIL** With Loggia Tp = 6 °C

Figure 6, Inside Air Temperature. Base heating regulated as a function of outside air temperature. Additional heating thermostat setting : 20°C

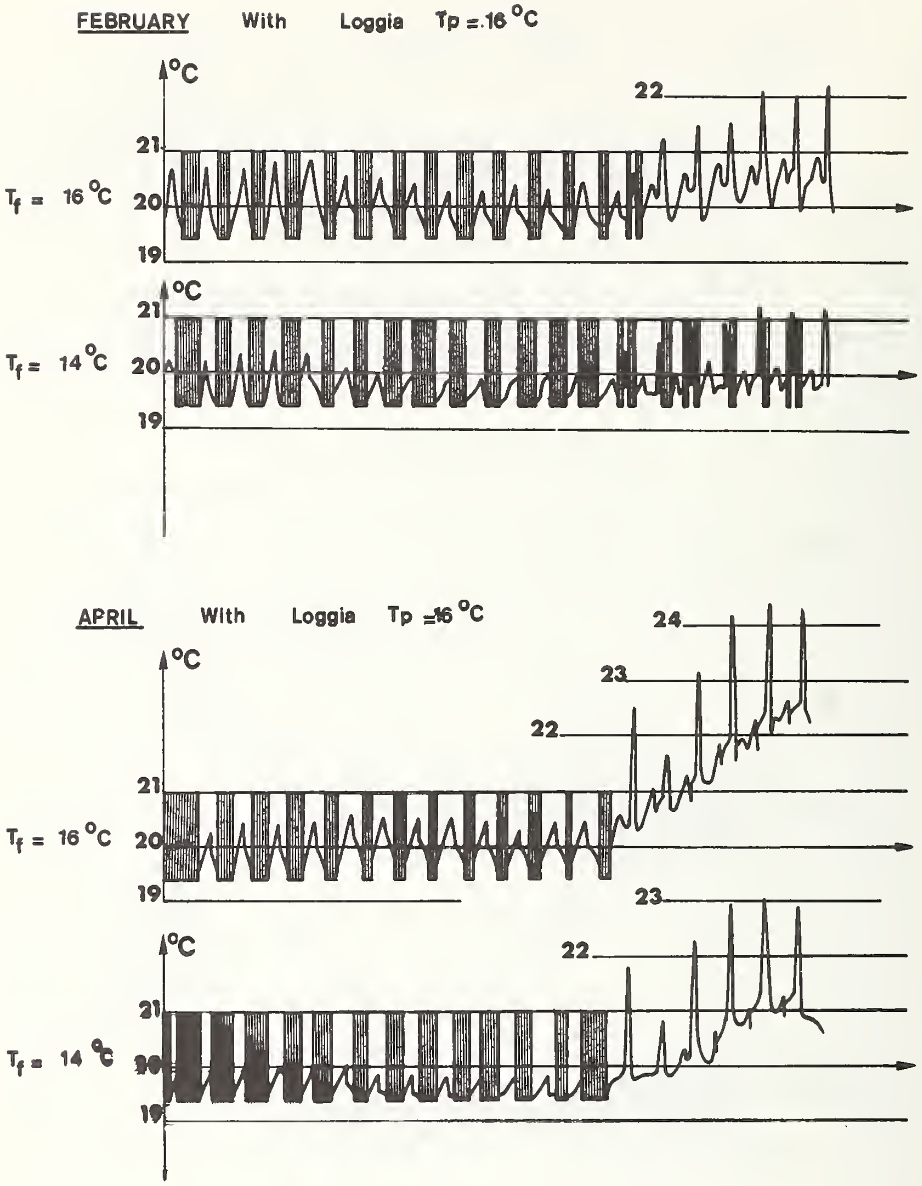


Figure 7, Inside Air Temperature. Base heating regulated as a function of outside air temperature. Additional heating thermostat setting : 20°C

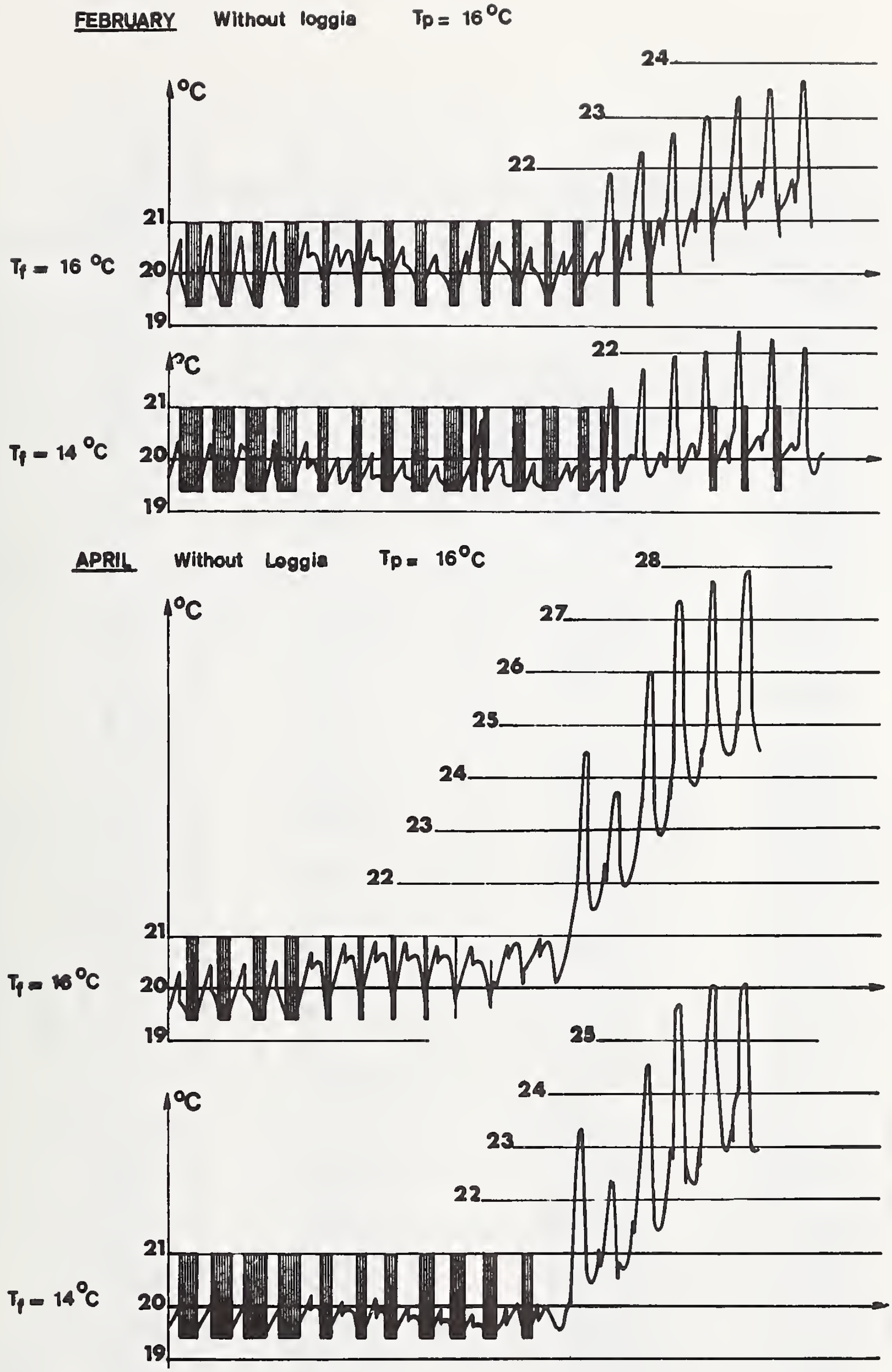


Figure 8, Inside Air Temperature. Base heating regulated as a function of outside air temperature. Additional heating thermostat setting : 20°C

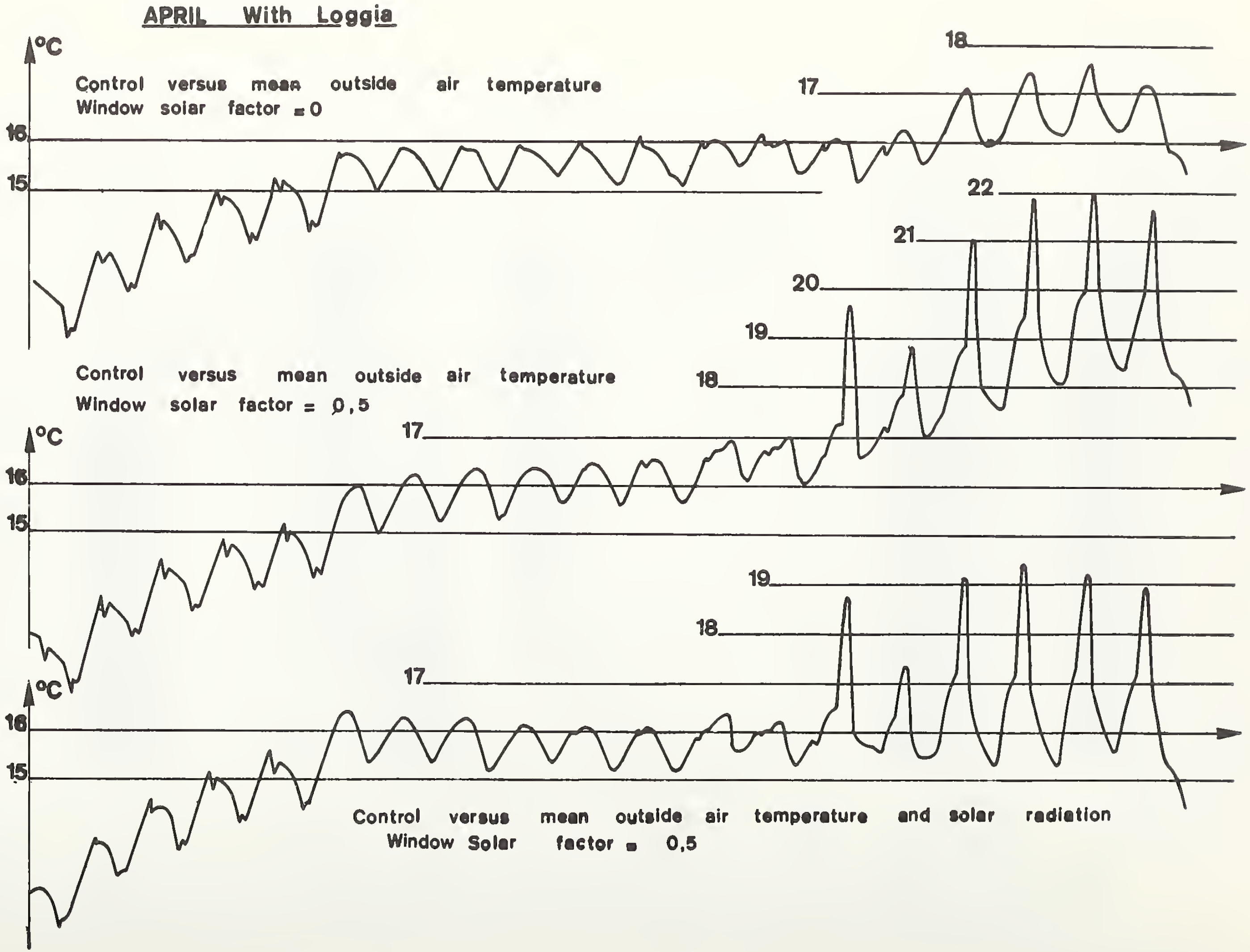


Figure 9, Inside Air Temperature. Base heating only, outside of peak hours. Pre-heating temperature Tp = 6°C, base temperature Tf = 16°C.

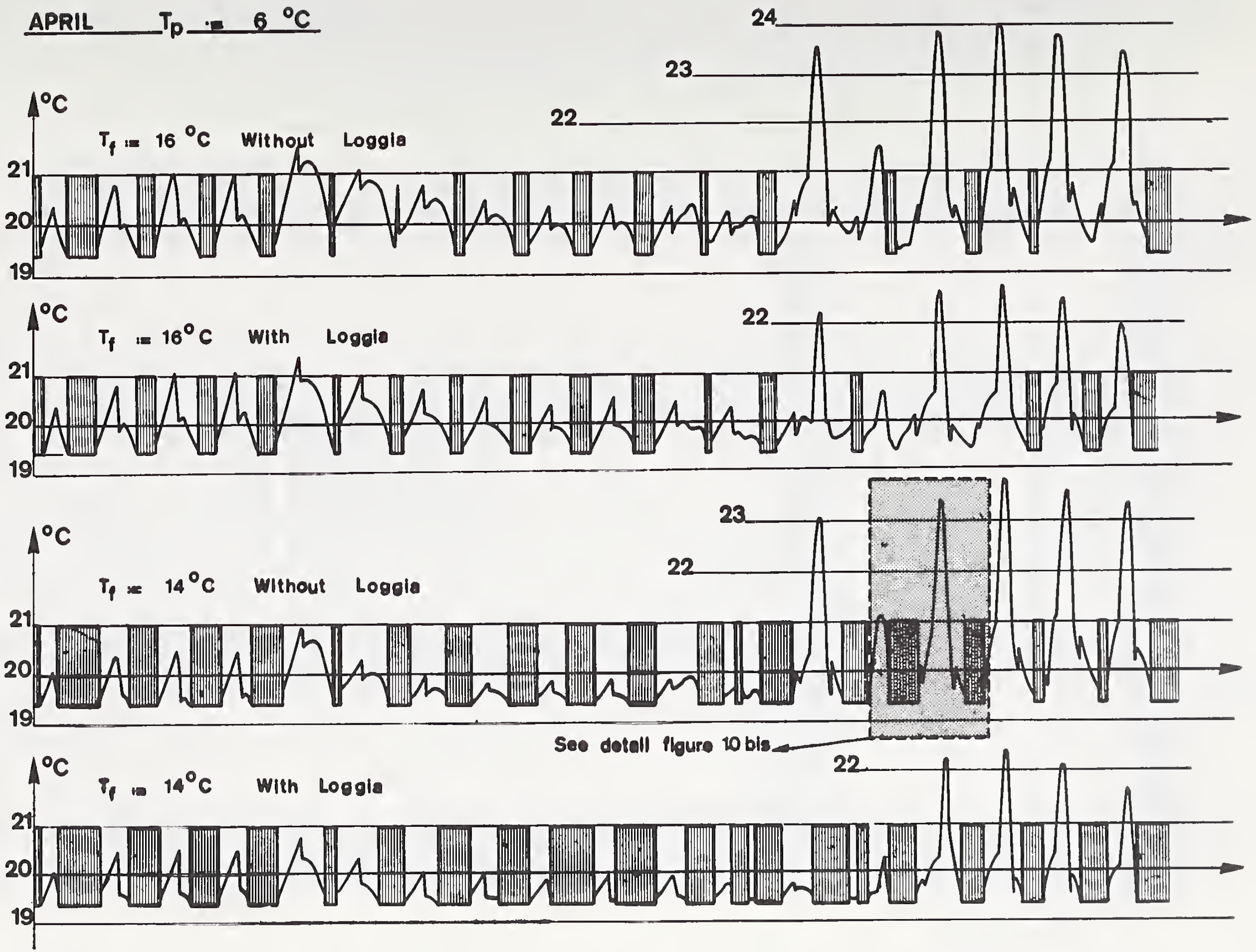


Figure 10, Inside Air Temperature. Base heating, outside of peak hours, regulated as a function of outside air temperature mean value and of the sunning. Additional heating thermostat setting : 20°C

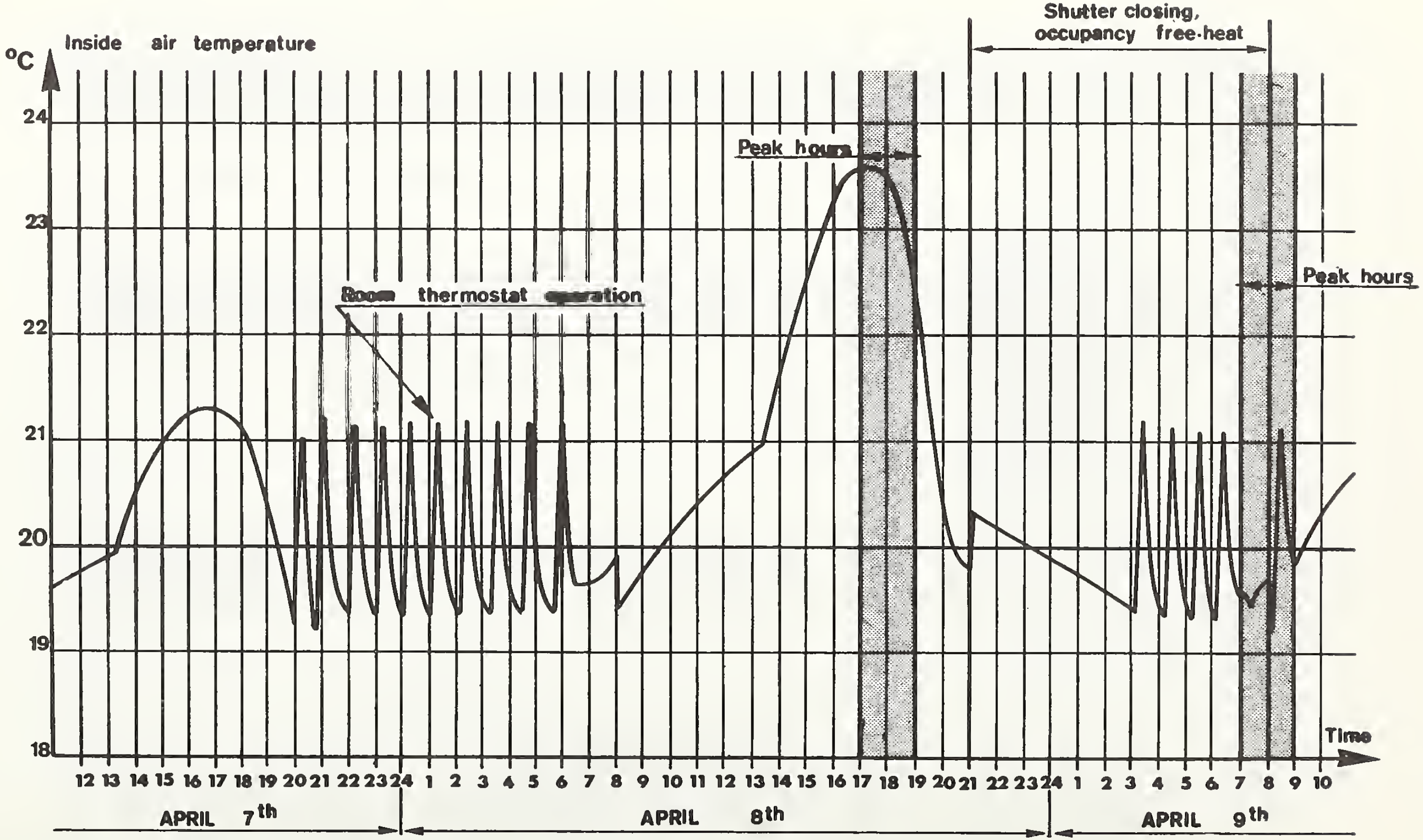


Figure 10 bis, Figure 10 Shaded area detail.

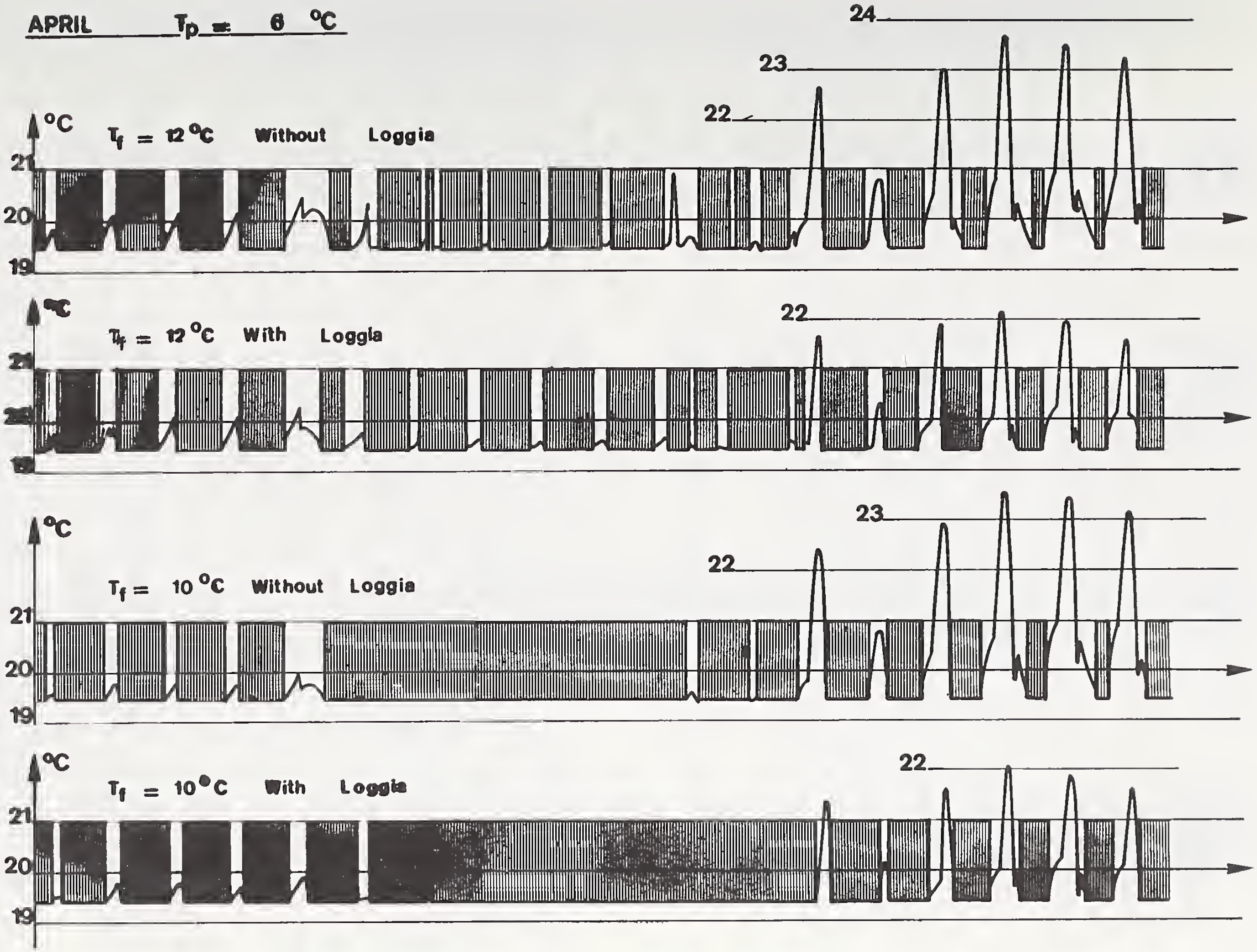


Figure 11, Inside Air Temperature. Base heating, outside of peak hours, regulated as a function of outside air temperature mean value and of the sunning. Additional heating thermostat setting : 20°C

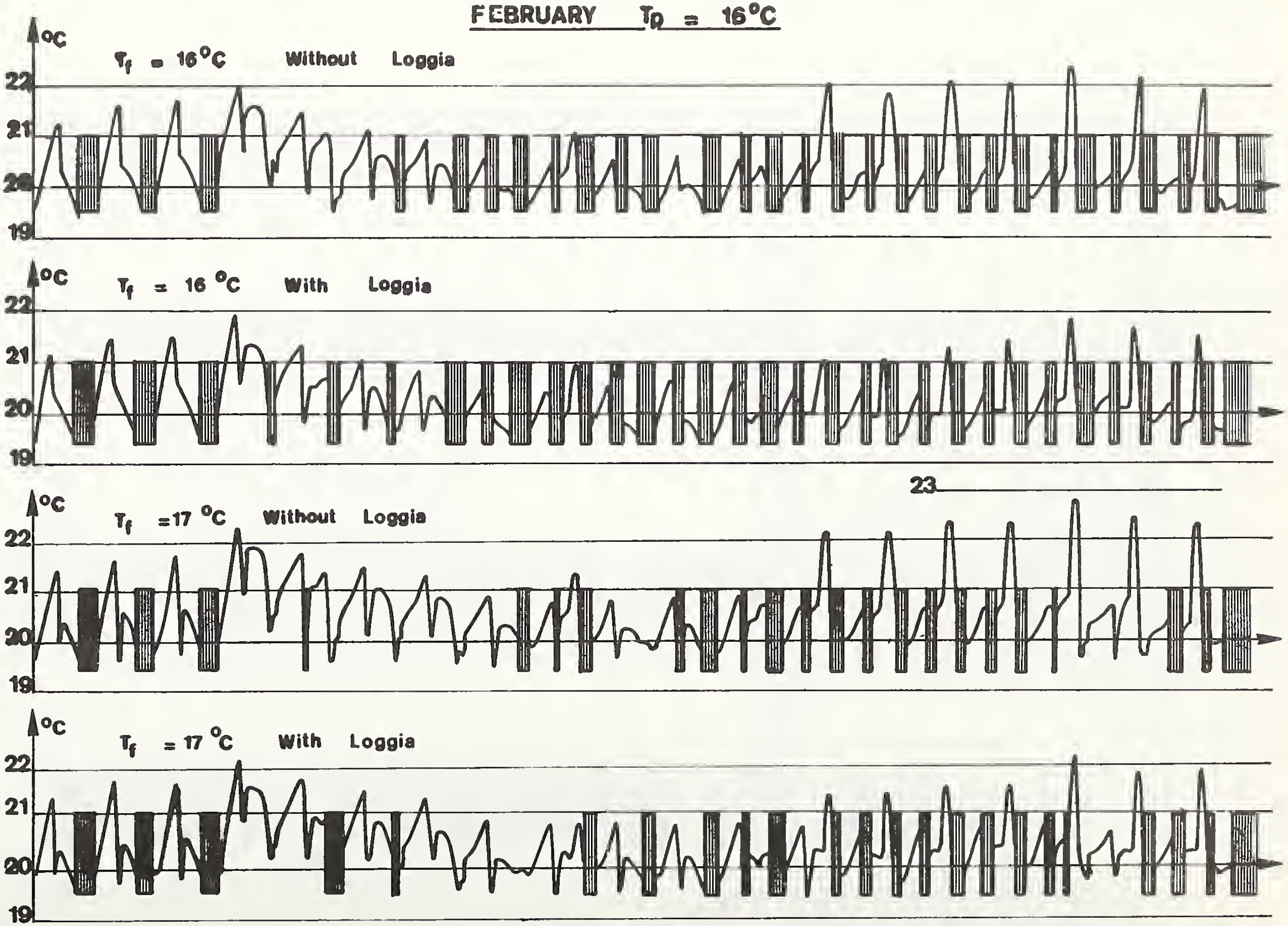


Figure 12, Inside Air Temperature. Base heating, outside of peak hours, regulated as a function of outside air temperature mean value and of the sunning. Additional heating thermostat setting : 20°C

# Analog Computer Simulation of an Air Conditioning System in a Commercial Building Incorporating Yearly Weather Data

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Estimated operating costs of various air-conditioning systems is an important economic consideration in the design and selection of equipment for a new building. Building construction, the heating/cooling system and the controls must all be considered to minimize these operating costs and provide comfort control. To analytically accomplish this task, a small commercial building was simulated on an analog computer so that the building orientation, construction and number of zones could be easily varied. Yearly weather data for various U.S. cities was programmed on a 13-channel tape recorder according to recommended ASHRAE procedures to simulate realistic environmental conditions. Programming the analog computer for various heating/cooling plants and control systems provides a quick analysis of initial cost, operating cost and comfort performance.

Key Words: Analog, control system, digital, heat transfer, modeling, simulation, solar radiation, system analysis, thermal capacity.

## 1. Introduction

Company sales and profitability can be significantly affected by employees' working environment. Maintaining the optimum environment at the least cost should yield increased productivity and higher profits. The main factors influencing operating cost and thermal comfort are the building's construction, location and orientation, the heating/cooling system and the control system. To analyze all these factors creates a complex problem requiring special tools such as analog and digital computers. Specifically, analog computers provide the dynamic results essential to complete analysis of control system stability and performance. The analog computer can simulate an actual heating/cooling system and illustrate the anticipated performance under various environmental conditions, building construction and controls. Cost performance data may be graphically displayed in real time, permitting rapid dissemination and comparisons of various systems.

An optimum environmental system maintains a comfortable environment at the lowest operating cost. This paper shall present a method for analytically determining system operating costs. The method utilizes an analog computer simulation of a small commercial building. Cost and performance data from this simulation are then used to accurately define optimum system performance.

## 2. Cost Analysis

Heating/cooling system costs are composed of initial cost, operating cost and maintenance cost. Generally initial cost includes hardware such as the compressor, condenser, furnace, boiler, duct work, piping, controls, etc., plus the cost of system installation. Once the system is installed, however, operating and maintenance costs are the main concern of the owner. This paper will be limited to determining the operating costs.

The financial return of a building is highly dependent on the efficient operation of the heating/cooling system. In dollars, this means the lowest possible total energy consumption necessary to maintain comfort conditions. Operating costs are generally those derived from operation of the condenser fan, delivery system, furnace and controls.

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An accurate calculation of yearly heating/cooling operation can be a simple summation of the heating/cooling plant's operating time if it is strictly on/off. If a modulating system is used, integration of the heat/cool delivered energy must be made. The energy requirements, together with the efficiency of the heating/cooling plant, can predict fuel consumption. Efficiency defined as the ratio of output in BTU/hr. to input energy provides a functional relationship (between energy required and energy consumed) to incorporate into the simulated system. System operation can then be calculated since the system is responsive to the outdoor conditions affecting efficiency, such as dry-bulb temperature (for an air cooled condenser) and wet-bulb temperature (for water cooling towers). All of these calculations may be made with an analog computer.

## 3. Computer Design of Temperature Control Systems

Numerous authors have proclaimed the advantages of computers, particularly in comparison to field testing. For to rely on field testing only leads to added costs, development delays and inaccurate results because of an uncontrollable environment. A computer-aided design, however, enjoys the benefits of a controllable environment, accurate definition of the effects of a single variable, accurately defined results and hence lower total development costs.

Of particular concern in determining operating cost is the solution of dynamic problems. An analog computer was selected for this analysis because of the ease of simulating the transient behavior of a heating/cooling system and the inherent dynamic problem solving ability.

The analog computer complements the passive electrical circuit simulating thermal properties of the small commercial building, automatically incorporating nonlinearities involved in total system analysis. It thereby completely calculates the transient behavior of the heating/cooling system as the system responds to demands of the conditioned space, created by anticipated internal as well as external environment loads. For the simulation, the external environment was modeled using data from the U.S. Weather Bureau.

## 4. Simulated Commercial Building

The simulated commercial building is a 90' x 30' structure, 8' in height per story with a slab floor and a flat ceiling. Exterior walls are all opaque material, 50% opaque and 50% transparent, or all transparent. A floor may be divided up to 12 separate areas, each 15' on a side, to create separate 225 sq. ft. zones. An analysis of a multiple story building may be made by rerunning the simulation with appropriate adjustments for the thermal inputs to the floor and ceiling, changing the thermal characteristics of the floor and ceiling, and eliminating all heat flow between stories by assuming the same control temperature for all floor sections. Thus the structure of the building, the number of zones and the type of exterior walls all may be varied.

The analysis of dynamic control performance and operating costs requires that the simulation include all thermal characteristics of the structure. Therefore all three modes of heat transfer - conduction, convection, radiation, and the thermal capacity of the structure must be included in the simulation. However, a given wall, floor or ceiling can never be perfectly simulated to produce an exact duplicate of the temperature profiles in the structure or the heat transferred. Therefore assumptions are needed to bring the problem within the practical capability of present-day techniques and technologies and be solvable with a realistic expenditure of funds.

In this simulation the lumped nodal point method of analysis was used. This analysis assumes that for each wall, a uniform temperature is maintained on any surface, the wall is of uniform construction, and it has a linear temperature gradient between any two surfaces.

In an actual installation heat does not flow in a straight path through the wall but rather flows along the path of least resistance. There is also a discontinuity between wall material surfaces. In this simulation a single temperature is assumed for the active wall and an average discontinuity is assumed between any two members (i.e. a uniform discontinuity is assumed between the studs and the plasterboard that is fastened to the studs). Assuming an average discontinuity an average thermal contact resistance may be calculated. The thermal contact resistance is the resistance to heat transfer between two items fastened together.

From past experience, these assumptions should present a deviation less than 10 per cent between actual and predicted heat flow characteristics of the wall, so that the dynamic effects on analysis and total energy loss will be minimized. The anticipated difference is further minimized by using one "T" section for each wall material and then building up the several "T" sections for an entire wall (as shown in Figure 2) rather than just one "T" for the entire wall. The equation:

$$1 = 44\sqrt{a/f}$$

(where $l$ = thickness of material to be treated by one "T" section for an accuracy of 5%, ft. a = thermal diffusivity, $ft^2$/hr., f = frequency of the disturbance, cph), may be used to define the maximum allowable thickness of material that may be represented by a single "T" to achieve a maximum deviation of 5% in a 24 hour variation of temperature (i.e. as would occur over an entire day). For example, plasterboard may be 4" thick before more than one "T" is required to prevent a 5% deviation of the response of an oscillating heat flow through the plasterboard.

The electrical circuit to model one zone (Figure 1) includes resistive elements representing heat transfer due to radiation between the walls, floor and ceiling, convection heat transfer between the room air and the walls, floor and ceiling, and conductive heat transfer through the walls, floor and ceiling. Figure 1 also shows the heat transmitted through the windows (this element is removed if glass is not present) and the capacitive elements simulate heat storage in the room air and walls, floor and ceiling. For a typical exterior wall (Figure 2) the resistive elements represent heat conduction through the various portions of the wall and capacitors represent the heat storage of these parts. Values of resistors and capacitors are dependent upon the scaling used to model the structure. For this simulation the dynamic effects occurring during 24 hours of actual time are calculated in 14.4 seconds.

Air movement between sections is simulated by adding a special resistive network to the basic circuit (Figure 1). The resistance network is connected to points representing room air temperature. Thermal capacitance of each air space is represented by a single capacitor.

Figures 3 and 4 show the cabinet that contains the physical circuitry used to make this simulation. The resistors and capacitors that form the "T" sections shown in Figure 2 are placed in a plug-in container. One container is used for each wall section (i.e. the container for an interior wall would include the resistors and capacitors for an 8' x 15' area). These containers are visible on the front of the cabinet (Figure 3). The containers on the lower panel of the cabinet are for the walls and windows; above these are the containers for the floor and ceiling sections. These containers may be easily changed so that different types of wall construction may be simulated. For example, if concrete blocks were used instead of brick and plaster on the exterior walls, the exterior wall containers would be changed to those that include the resistor and capacitors sized for a concrete block wall. Since these are plug-in containers, changes can be made quickly and the analysis continued to determine the effects of the new wall construction.

## 5. The Simulated External Environment

To calculate the annual cost of operating a heating/cooling system, the external environment is simulated using weather data from the U.S. Weather Bureau. Weather variables are changed every hour to closely approximate actual dynamic changes. Ten years of data are contained on a single reel of magnetic tape. The data are provided in a digital form from which the characteristics pertinent to the thermal simulation were converted into analog signals and recorded on an analog magnetic tape.

Environmental factors used were dry- and wet-bulb temperatures, relative humidity, wind velocity and direction and solar radiation. Hourly values for the factors necessary to calculate the solar radiation (cloud type and height, amount of cloud cover and the time of day and day of the year) were read from the Weather Bureau's digital magnetic tape and then used according to ASHRAE procedures outlined by the Task Group on Energy Requirements, providing a programmed method of calculating both direct and diffuse solar radiation intensities for any wall. Additional statements were added to the digital computer program to read information from the Weather Bureau's digital tape. The resulting digital computer program read and calculated values for the solar radiation intensities and 5 other pertinent environmental factors. A total of 13 variables - dry- and wet-bulb temperature, relative humidity, wind velocity and direction, the year, hour and week, solar radiation intensities for an east- west- north- and south- facing wall, and a direct normal radiation intensity were either calculated or read from the digital weather data tape. Hourly values for a 4-year period for these 13 variables were then recorded on a 13-channel analog magnetic tape. The analog tape is an FM recording, although the output from the tape recorder is a voltage or analog signal. The outdoor ambient temperature, reproduced as an analog or voltage signal, is connected to the appropriate points of the electrical circuit (Figure 1). Since solar radiation intensities are directionally oriented, the simulated building may be oriented in any direction by simply changing analog signals in the cabinet (Figure 3).

Inside the cabinet (Figure 4) is the pull-out printed circuit board on the right hand side which contains all the interior radiation and convection heat paths. Figure 5 illustrates the board used for the one zone application. The six panels near the bottom of the cabinet (Figure 4) contain the resistive, capacitive network that simulates the heat transfer through the ground. Points from this circuit are connected to the underside of the building's slab floor. This two-dimensional circuit effectively simulates heat transfer to 16' where typically only 5% of the yearly outdoor ambient temperature oscillation is found. The circuit automatically provides for the complex heat loss from the building to outdoor ambient conditions through the earth by incorporating the dynamic nonlinear effects of the ground, effects that would otherwise be next to impossible to solve by either digital or analytical solutions.

Air infiltration is accounted for by a direct heat transfer path between each point representing the space temperature for a 225 sq. ft. area and an equivalent outdoor ambient temperature, simulated by a representative resistance. Wind velocity defines the value of the equivalent outdoor ambient temperature. Special analog circuits to achieve the correct direction for air flow and duplicate wind direction heat gain or loss.

## 6. Simulation of the Heating/cooling System

Analog simulation of the building structure combined with the heating/cooling plant provides a system approach to analyzing the complete control loop (Figure 6). To simulate the heating/cooling system, response characteristics of the physical components -- the furnace, conveyance, cooling coils, -- must be known. Once the response characteristics are defined - either by sinusoidal inputs (frequency response method) or step inputs (step response method) - the control loop may be established (Figure 6).

Furnace time response may be found by measuring plenium air temperature from a step input, simulated by a transfer function of the form

$$T/Q = K/(\tau s +1)$$

where T = air temperature rise in the plenum, Q = heat output of the furnace, K = steady state plenum temperature rise per unit Q, s = the Laplace operator, and = the single order time constant characteristic of the furnace. Various types of furnaces - electric, gas, oil, coal, etc., may be modeled using different time constants ( $\tau$ ). Radiant panels or baseboard heaters may be simulated by similar transfer functions, but with heat added directly to the ceiling surfaces for radiant panels and to the walls and floors in addition to the air for baseboard heaters.

The cooling plant may be modeled similarly with the extent or complexity of the simulation depending on the type of plant used, i.e., cap tube, absorption, reciprocating or centrifugal chiller. Since the sensible-latent heat removal relationship is continually varying, the dynamics of operation are more complex than the furnace simulation. Latent heating effects are necessarily included to calculate realistic operating costs, since the efficiency of the air-conditioning unit is dependent not only on the outdoor ambient conditions but also on the latent heat load across the cooling coils. The latent heat introduced by infiltration of outdoor air as well as that generated by occupants must be considered.

Transient moisture storage of various materials found in typical furnishings was based on actual field measurements of step-response tests from a humidity source. The data obtained from these tests defined the transfer functions used in the computer simulation. Dry-bulb and relative-humidity sensors were modeled and added to the control system circuit through appropriate transfer functions, taking into account the respective time constants. If an Air Economizer provides free cooling by outside air, another block must be added in Figure 6 and additional circuitry added to the simulation.

The heating and cooling conveyance, if present, should also be modeled because its relative time response may be significant to that of the total system, depending on its location and the time constants of other system components. A typical metal duct transfer function might be a lead-lag term such as

$$T_O/T_i = K ( \tau_1 s+1)/( \tau_2 s +1)$$

where $T_O$ = outlet temperature rise, $T_i$ = plenum temperature rise or duct inlet temperature rise, K = steady state ratio of $T_O/T_i$, $\tau_1$ = lead time constant, and $\tau_2$ = lag time constant. The time constants reflect the relative duct length and the heat exchange between the air, the duct and the environment.

Last is modeling the control system. A simple on/off control is shown in the control loop (Figure 6). The basic control system consists of a sensor to detect the current state or condition a logic device to differentiate what is sensed from a preset or desired condition, and an actuator to trigger desired action from the appropriate equipment after receiving a command signal from the logic device. Various auxiliary control components may be added to this basic model as demanded by the application. Controls simulated may be electric, electronic, pneumatic, fluidic, mechanical or any combination.

## 7. Computer Operation and Data Acquisition

The computer combines the simulated heating/cooling system with the commercial building in a control loop with the external environment provided by the taped weather data. Since these data form the load on an unoccupied building, occupancy effects were added separately.

Solar radiation intensity adds heat to the structure. Since current is analogous to heat in the simulation, the voltage signal from the weather data tape must be transformed into current. These current signals are then sent to the simulated building by connecting to the appropriate point in the simulated circuit using special analog current generators. Distribution of this absorbed heat (or current) is proportional to the voltages supplied from the taped weather data. Separate current generators are used for the roof and each side of the building and to simulate heat transmitted through the glass windows.

The voltage signal from the tape recorder representing the solar radiation intensity is a combination of direct and diffuse components, adjusted to account for average transmissivity or absorptivity of incident surfaces. This is obtained by time-averaging values for each direction (north, south, east, west, or perpendicular to the earth). This averaging tends to smooth the daily cyclic pattern somewhat; however, the difference between hourly changes and average transmitted heat is always less than 10% for any given day.

Sensible heating effects of lighting and occupancy in each zone are accounted for by injecting convective and radiant heat into the simulated structure at the point that represents the room air temperature and surrounding surfaces of the particular zone in question.

Output data is recorded by several instruments: A digital voltmeter with BCD (binary-coded decimal) output capability, a counter timer with BCD output, an 8-channel oscilloscope, an X-Y plotter or an analog tape recorder. In the present case a counter timer recorded yearly operating costs by integrating total on time of an on-off system by pulsing a gate to allow the timer to count on its internal calibrated time base during the permitted period representing the system on time. If the system modulates according to demand, the heating/cooling requirements must be integrated and the counter is then used to accumulate the number of integrations to a given value.

Dynamic performance of the system was recorded on an oscillosgraph where a graphic representation of zone air temperature, along with other variables, was obtained, permitting dynamic temperature swings of the zone to be easily determined. The performance and operational characteristics for a complete year are obtained in only 87.6 minutes, meaning several building types and various heating/cooling systems can be examined in a single day.

## 8. Illustrative Example

For example, consider a single-story single-zone building 30' wide by 90' long by 8' in height with no internal walls or partitions, and the thermostat is mounted on one of several support columns.

All exterior walls have plate glass along the top half, opaque material on the bottom half. The building, located in Houston, is oriented so the long wall faces south.

A simple one-stage heat, one-stage cool system was simulated for the heating/cooling plant. To this was added a heat/cool space thermostat for control and an Air Economizer to use outside air for free cooling whenever possible. The Air Economizer has two temperature sensors: One which senses outdoor air temperature and one which senses combined (or mixed) air temperature obtained from return air and outside air temperatures. Two setpoints (one for each sensor) let the Air Economizer pull in outside air when outside dry-bulb temperature is below its setpoint and regulate the amount of outside air entering through a damper according to the mixed air temperature (channel E, of Figure 7). The damper responds to loads on the structure and the control setpoint. For the illustration shown, the mixed air setpoint was 60°F and the outdoor air permit temperature setpoint was 70°F. The minimum position of the damper was set to provide 10% outside air for ventilation purposes.

Figure 7 presents the typical oscillograph output of such a simulation. The time period shown is the last 2 days of January and the first day of February, 1955. The two timing channels, D and H, are identical. Channel H is included only as a reference. Time is recorded in 1-hour steps from midnight through 24 hours, then resets. The cyclic pattern of outdoor air temperature, channel A, varies from day to day. To approximate this pattern with an average condition that would produce the same response in space air temperature (channel B) would be extremely difficult as would trying to achieve an average condition for the directional solar radiation intensities displayed by channels I, J, and K.

The type of cloud cover (if any), the amount of cloud cover, haze in the atmosphere, ground reflectivity and sky diffisuivity are accounted for in the values of these solar radiation intensities. Infiltration effects of outside air are regulated by wind velocity, (channel F) and wind direction, (channel G). The sharp swings on channel G are because of the scale used for the trace, from due north through a full rotation of 360 degrees. As the wind direction changes from north to south the oscillograph pen must traverse approximately one-half the trace. Outdoor relative humidity is recorded on channel L.

Variations in the cyclic space air temperature swings, channel B, coincide with the cycling rate of the thermostat and the heating/cooling plant (channel C). These variations are in response to changes in the internal occupant and lighting load (present from 8 A.M. to 5 P.M.) and to changes in the outdoor weather conditions.

On time of the heating/cooling plant may be accumulated by a counter-timer to calculate the operating costs for the time period and system in question. The effect of different components on the total system operating cost and performance may be easily determined by changing the simulated part and rerunning the same weather data.

## 9. Conclusions

With this simulated small commercial building and weather data, yearly system operating costs may be determined for a number of different types of building construction, composition, orientation, location and use. A clear concise distinct analysis providing specific information on the effects of a single variable may be made using analog computation. Effect of a single variable on total system performance may be readily defined, as may various conceptual control ideas such as using outside air for free cooling through an Air Economizer. Multizone and single-zone systems may be readily examined by changing from a 12-zone to a 1-zone structure. The number of zones may be easily changed. Various systems may be examined not only over long-term but also over short periods. Occupancy and lighting loads, as well as other internal loads, may be readily incorporated and the dynamic effects examined. Geographical effects of a particular building and system may be determined for any location in the United States for which a magnetic weather data tape has been produced. The type of application a given heating/cooling system is to be subjected to may be readily evaluated. The type of control may be easily changed and the dynamic performance as well as the yearly operational cost resulting from the particular control defined. Knowledge of what the optimum control should consist of, and knowledge that the control parameter values defined are indeed the optimum values, may be graphically illustrated with this engineering approach. The information obtained here is not easily obtained using field tests or other analytical solutions, especially not within a controlled environment that provides a convenient and economical method of comparison.

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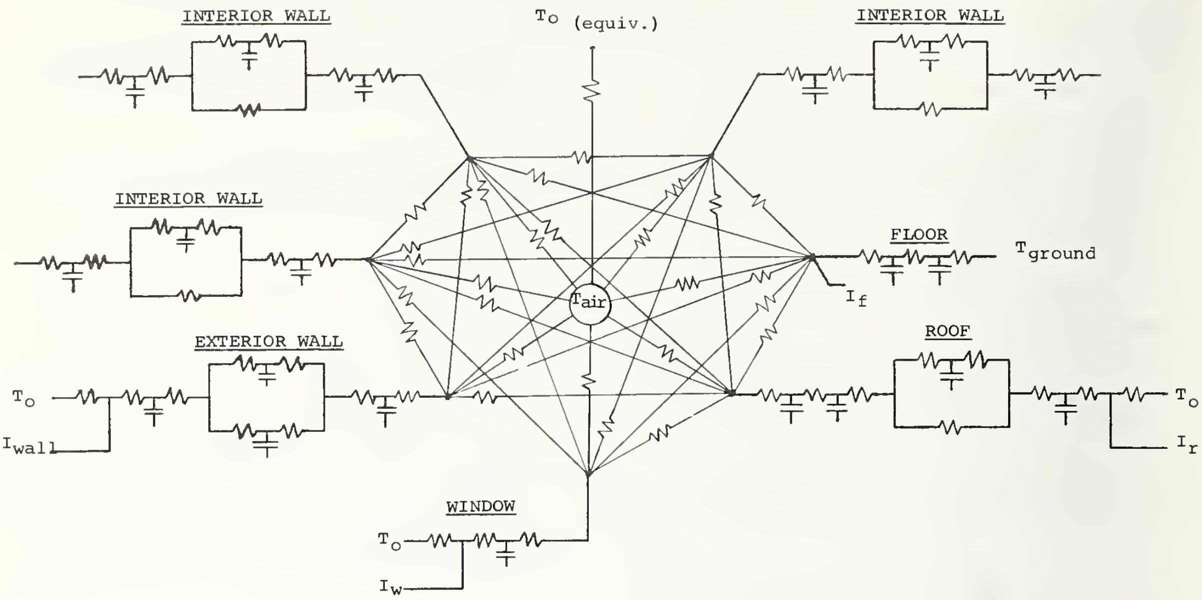

Figure 1. Simulated thermal circuit of one zone in a multizone building

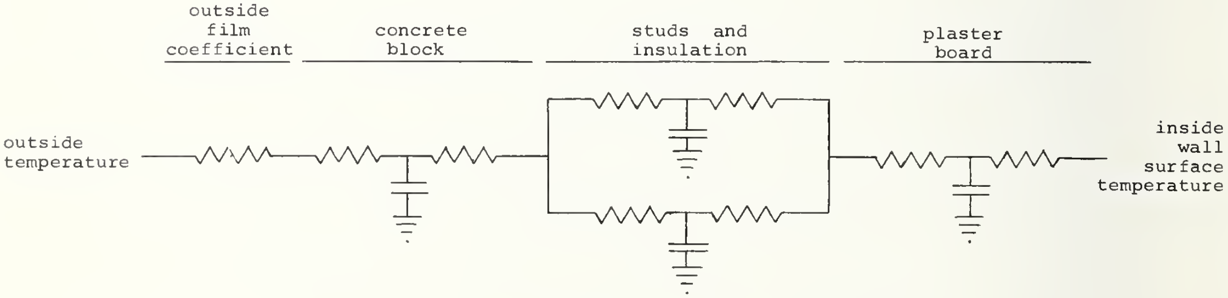

Figure 2. External wall simulated thermal circuit

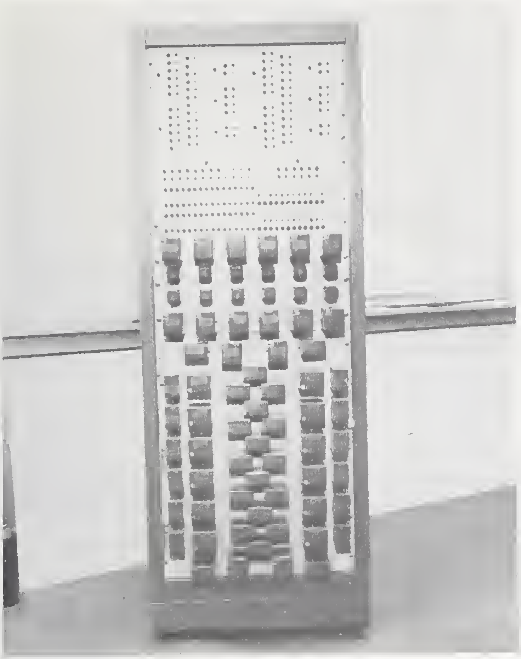

Figure 3. Simulated Building Cabinet

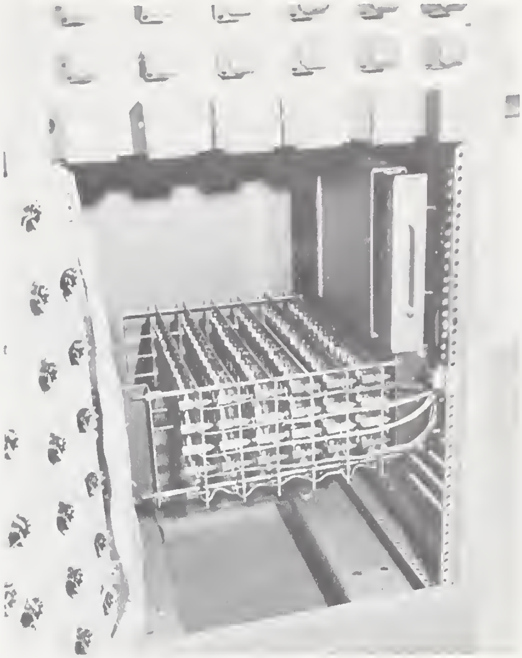

Figure 4. Interior of Cabinet Showing Ground Circuit and Radiation Convection Circuit Board

Figure 5. Simulated Radiation and Convection Circuit Board

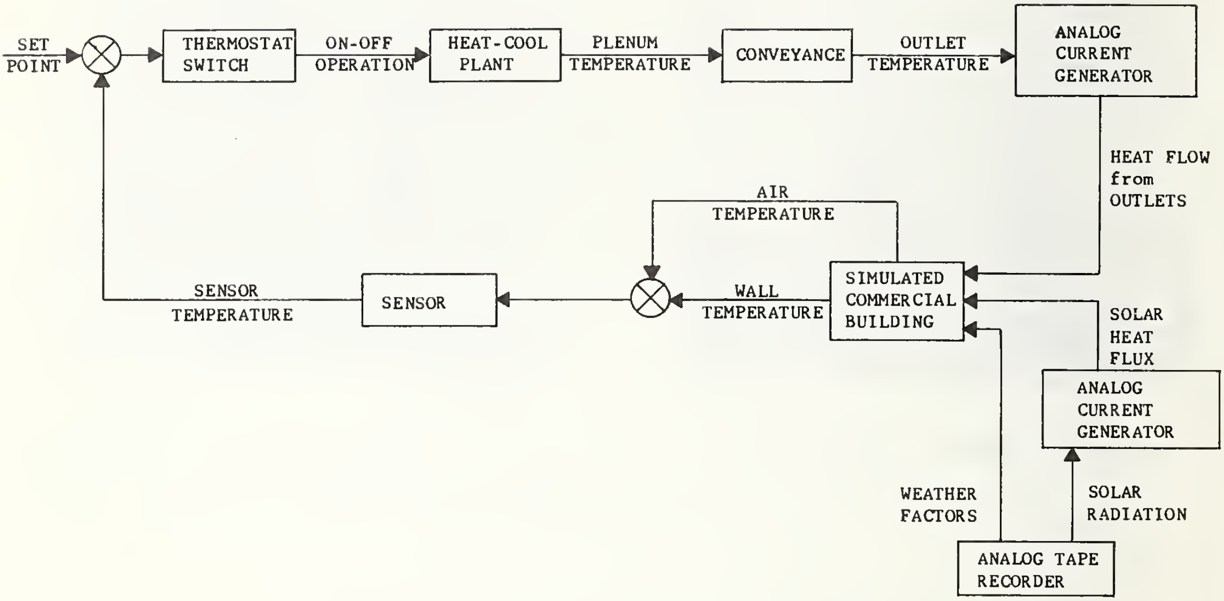

Figure 6. Temperature Control Loop

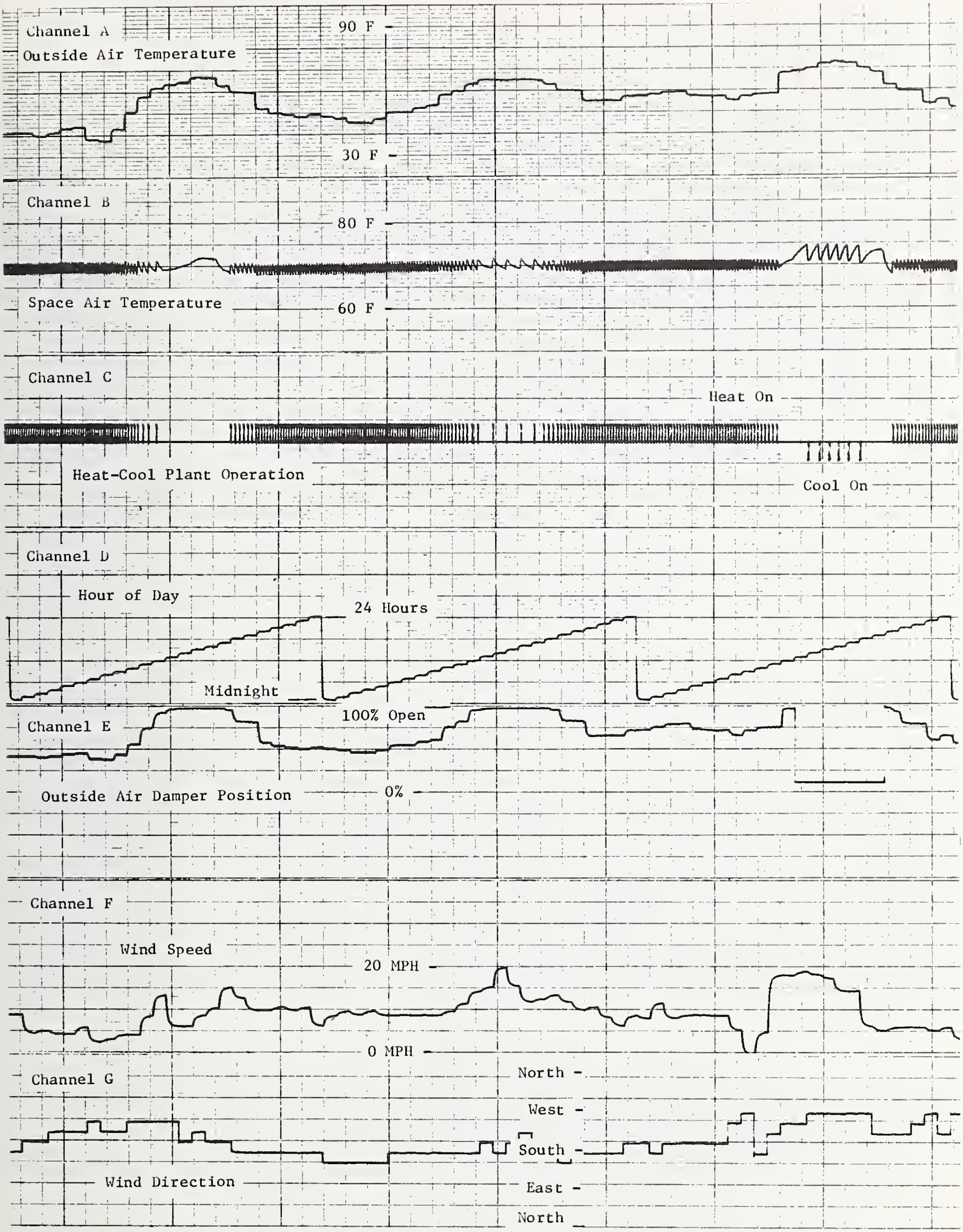

Figure 7: Oscillograph Recording of System Variables

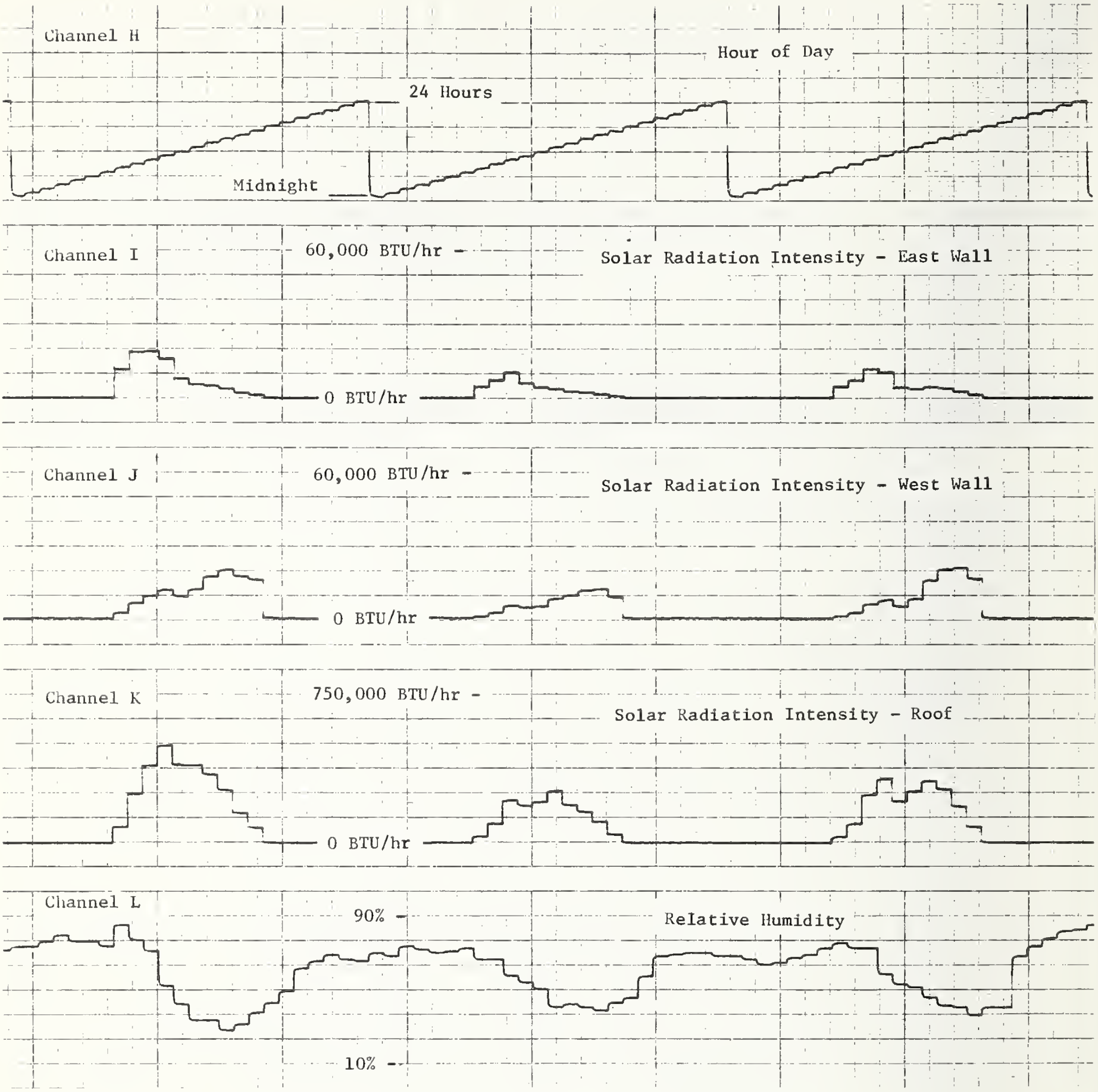

Figure 7: Continued

# Experience with a thermal network analysis programme applied to heat flow in buildings

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After a brief discussion of the general features of thermal models for buildings, the paper describes the general purpose network analysis programme that has been modified for use in building heat flow calculations. The input data includes the dimensions and thermal properties of the heat flow paths, the building orientation, the solar radiation on a horizontal plane, the ambient air temperature, the wind velocity and the rate of air infiltration. Sub-routines allow calculation of variable convective and radiative resistors. Calculated output includes inside surface temperatures and heat flow into the building interior. As an example, calculated values of the diurnal and daily heat flows and the maximum wall temperatures are given for a simple enclosure with walls of various materials and different thickness. Factors affecting the alignment of the mathematical model with the prototype are discussed with relation to a simple enclosure. The advantages and limitations of the method are critically examined and some comparisons made with the van Gorcum matrix method, which has also been programmed for similar problems. It is concluded that the mathematical simplicity and the versatility of the general purpose thermal analyser give considerable advantage for University type research. On the other hand, the long computing time of this programme is seen as a limiting factor in its use for routine investigations.

Key Words: Air conditioned buildings, building heat flow, lumped parameter network, optimum insulation, thermal design, thermal model, thermal network analysis.

## 1. INTRODUCTION

As a thermal shelter, a building acts to reduce the daily temperature range and to permit adjustment of the temperature level by heating or cooling. Heat flow in the building shell is transient as a result of the periodic nature of the ambient temperature and the radiation received at the building surface. Thus, predictions of the thermal performance need to allow for the unsteady flow and to account for the thermal capacitance of the shell as well as its resistance.

For a naturally ventilated building, the problem is usually one of predicting the inside temperatures, both of the surface and the air. On the other hand for air conditioned buildings, the net heat flow and the wall surface temperatures will be needed.

Justification for a detailed thermal study of a building is usually made on economic grounds. For the naturally ventilated case, the aim may be to decide the cheapest of various alternative ways of reducing unwanted heat gains. For the air conditioned case, the aim may be to determine the most economical structural system which will result from the lowest annual cost of owning and operating the building.

The unsteady flow analysis can be extended to cover the case of unsteady energy input into the air conditioner, such as is the case in the solar air conditioned buildings which have been the subject of research at the University of Queensland.[1] Here, the problem is one of distributing the thermal storage between the solar collecting system, the air conditioning system and the building in the most economical way.

## 2. THERMAL MODELS

Thermally, a building consists of a number of different heat flow paths in parallel, each subjected to boundary conditions that may vary from path to path. A typical path through a homogeneous wall (Fig. 1) has distributed capacitance and resistance. It is affected externally by solar radiation, long-wave reradiation to the surroundings and convective heat exchange with the ambient temperature and internally by long-wave radiative exchange within the interior and by convective heat exchange with the room air.

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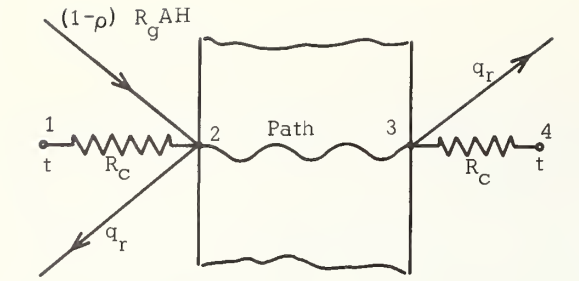

Fig. 1. Homogeneous wall with boundary conditions

It is assumed that heat will flow normally to plane surfaces the dimensions of which are large compared to the thickness, i.e. the flow will be one-dimensional. Thus, each plane surface of uniform construction is considered to be a single path with the same set of boundary conditions.

a. Boundary Conditions.

Short-wave radiation entering the surface can be calculated from the insolation on a horizontal plane, a factor to allow for surface orientation to the sun and the surface reflectance.

$$q = (1 - \rho) R_g A H. \qquad \ldots \qquad (i)$$

External long-wave radiative exchange will be governed by the usual Stefan-Boltzmann law with the assumption that some effective or average temperature can be found for the surroundings, $T_s$.

$$q = \varepsilon_2 F_{s2} A (T_s^4 - T_2^4)$$

This equation can be modified to the form:

$$q = \frac{(T_s - T_2)}{\dfrac{1}{\varepsilon_2 F_{s2} A (T_s + T_2)(T_s^2 + T_2^2)}} = \frac{(T_s - T_2)}{R_r} \qquad \ldots \qquad (ii)$$

where $R_r$ is a temperature dependent resistance for a particular wall.

Convective heat transfer will depend on the film resistance $R_c$ which will in turn be some function of the velocity in the case of forced convection or of temperature in the case of natural convection. The temperature difference is between the ambient temperature and the wall temperature. Thus:

$$q = \frac{T_1 - T_2}{R_c} \qquad \ldots \qquad (iii)$$

Internal radiative exchange could be treated as above or alternatively by the matrix equation:

$$|B| \; |W| = |bT^4| \qquad \ldots \qquad (iv)$$

$|W|$ is a matrix of leaving flux densities representing the response,

$|bT^4|$ is a matrix with terms of the form $\frac{A_2 \varepsilon_2 \sigma}{\rho_2} T_2^4$ representing the excitation,

$|B|$ is a matrix of system properties and is thus the transfer matrix.

For some studies, the boundary conditions can be simplified by the sol-air temperature concept.[2] (The sol-air temperature $t_e$ is that temperature of the outdoor air which, in the absence of all radiation exchanges, will give the same rate of heat entry into the surface as would exist under the combination of heat exchanges in the model above (Fig. 1). A combined resistance $R_{cr}$, with temperature difference taken to the ambient temperature, is usually used as:

$$q = \frac{1}{R_c}(T_1 - T_2) + \frac{1}{R_r}(T_s - T_2) + \alpha R_g A H = \frac{1}{R_{cr}}(T_1 - T_2) + \alpha R_g A H = \frac{1}{R_{cr}}(T_e - T_2) \quad \ldots \quad (v)$$

These boundary conditions may be available as a continuous record though more usually they will consist of a time-series with values at one hourly or three hourly intervals.

b. Thermal Response of Heat Transfer Path.

If the convective and radiative resistances can be considered as constants, the response of the system to the excitation can be calculated by superposition of the known response to simple components of the excitation function. Components that have been used are Fourier harmonics,[3] rectangular pulses,[4] and triangular pulses.[5]

Each excitation function is resolved into components. It is sufficient to determine the response of the system to unit values of the excitation components since, for the assumed linear system, the magnitude of the response will be linearly related to the magnitude of the excitation. The response function will be determined by adding the components of the response.

Mathematical manipulation can be conveniently handled in matrix form in which a transfer matrix containing fixed properties of the system is post-multiplied by a column matrix of the response variable and equated to a matrix containing the remaining terms of the heat balance.

Application of this approach depends upon the ability to determine the transfer matrix and/or the unit response for the distributed parameter conduction path. These methods are detailed elsewhere.

c. Lumped Parameter Approximation.

The conduction path can be approximated by a lumped parameter network of thermal resistances and capacitances which gives a finite difference approximation to the partial differential conduction equation. The one-dimensional heat flow path of building problems is imagined as being divided into a number of slabs which have their heat capacity concentrated at their midpoints. The path for heat flow is formed by the thermal resistance of each slab which is lumped to connect appropriate midpoints or nodes. For a reasonable approximation from 3-5 divisions of each conduction path should be made.[6]

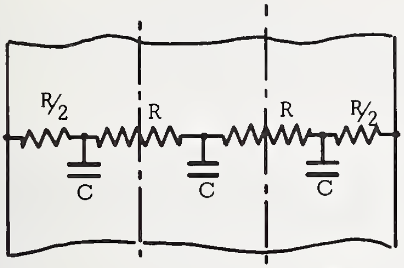

Fig. 2. Thermal network of conduction path

Heat flows into or out of the path can be made by adding or subtracting heat from boundary nodes or by allowing heat to flow through convective or radiative resistances connected to the boundary nodes. The interconnected circuit of thermal resistances and capacitances is the thermal network.

## 3. SOLUTION OF THERMAL NETWORK

Electric analogues of the thermal network, essentially special purpose passive-network computers, have been built.[7] A large number of components are necessary since all parallel paths must be in operation at the same time. However, the solution time is short since the time constant of the electrical path is several decades below the equivalent thermal path. Generally, the machines lack versatility since the set-up time is long and inevitably special components are needed for most new applications.

Digital computer solution of the thermal network is a sequential process which, at each succeeding time, calculates for each node in turn a new node temperature that results from the heat flows in the paths connected to the node in the preceding time step. Thus a temperature history of each node is obtained.

There are two types of node - those with capacity and those without.

a. Nodes with Thermal Capacity.

In the general case, node i will be connected by resistances $R_{ij}$ to a number of surrounding nodes $j_1$, $j_2$, etc. The node will have a thermal capacitance $C_i$, a heat input due to mechanisms other than conduction or equivalent conduction $q_i$ and a temperature $T_\theta$ at the time $\theta$ .

Quantity of heat entering the node in the interval $\Delta\theta$ is given by:

$$Q = q_i\Delta\theta + \frac{T_{\theta,j_i} - T_{\theta,i}}{R_{ij_1}} \Delta\theta + \frac{T_{\theta,j_2} - T_{\theta,i}}{R_{ij_2}} \Delta\theta + \ \ldots$$

This heat causes a rise in temperature of the node such that:

$$Q = C_i \left(T_{\theta + \Delta\theta,i} - T_{\theta,i}\right)$$

Thus,

$$C_i \left(T_{\theta + \Delta\theta,i} - T_{\theta,i}\right) = q_{i\Delta\theta} + \frac{T_{\theta,j_i} - T_{\theta,i}}{R_{ij_1}} \Delta\theta + \ \ldots$$

$$\frac{C_i}{\Delta\theta}\left(T_{\theta + \Delta\theta,i} - T_{\theta,i}\right) = q_i + \sum_{j=1}^{n} \frac{T_{\theta,j}}{R_{ij}} - T_{\theta,i} \sum_{j=1}^{n} \frac{1}{R_{ij}}$$

whence

$$T_{\theta + \Delta\theta,i} = \frac{\Delta\theta}{C_i}\left[q_i + \sum_{j=1}^{n} \frac{T_{\theta,j}}{R_{ij}} - T_{\theta,i} \sum_{j=1}^{n} \frac{1}{R_{ij}}\right] + T_{\theta,i} \qquad \ldots \qquad \text{(vi)}$$

b. Nodes without Thermal Capacity.

These will frequently be surface nodes. The heat that enters the node must leave under the action of the temperature potential of the node.

$$q_i\Delta\theta = \frac{T_{\theta + \Delta\theta,i} - T_{\theta,j_1}}{R_{ij_1}} \Delta\theta + \frac{T_{\theta + \Delta\theta,i} - T_{\theta,j_2}}{R_{ij_2}} \Delta\theta + \ldots$$

$$q_i = T_{\theta+\Delta\theta,i} \sum_{j=1}^{n} \frac{1}{R_{ij}} - \sum_{j=1}^{n} \frac{T_{\theta,j}}{R_{ij}}$$

$$T_{\theta+\Delta\theta,i} = \frac{q_i + \sum_{j=1}^{n} \frac{T_{\theta,j}}{R_{ij}}}{\sum_{j=1}^{n} \frac{1}{R_{ij}}} \quad \ldots \quad \text{(vii)}$$

To ensure stability of the calculation, i.e. to ensure that the finite difference solution is convergent, the time increment $\Delta\theta$ must be less than the minimum time constant for the circuit, i.e.

$$\Delta\theta < (RC)_i = \frac{C_i}{\sum_{j=1}^{n} \frac{1}{R_{ij}}} \quad \ldots \quad \text{(viii)}$$

Heat flows can be found from the heat flow in appropriate resistance paths for each time step. When the heat leaving a conduction path is needed, it can be found from the heat flowing in the conductive resistance connected to a boundary node. For the heat entering the room air, the convective resistance can be used.

For any given time interval, $m\,\Delta\theta$, the average rate of heat transfer towards the node i will be given by:

$$q = \frac{\sum_{m=1}^{m} \frac{T_{i,m\Delta\theta} - T_{j,m\Delta\theta}}{R_{ij}}}{m} \quad \ldots \quad \text{(ix)}$$

The computation of new temperatures by the node equations is simple and requires very short computing time but it must be repeated for each node at each time increment. Since with practical building systems, large networks of several hundred nodes may be involved and the allowable real time increment is small, maybe of order several seconds, the computing time for a run of several days is necessarily long, and several days must be run for each case since the error resulting from the assumption of initial temperatures takes two to three days to become negligible.

## 4. THE COMPUTER PROGRAMME

Though this programme is described as a general purpose thermal network analyser, it has been written with the aim of allowing easy modification for the addition of special functions. Thus it consists of a simple main programme as shown on the flow chart (Fig. 3) with many of the operations representing subroutines which are on call.

The input data for the programme is as follows:

| | |
|---|---|
| Resistances | - Identification by resistance number; nodes to which resistance connected; dimensions of resistance element; thermal conductivity. |
| Capacitances | - Identification by node number; dimensions of capacitance element; specific thermal capacity. |
| Solar Factors | - Latitude. Time of year. Node number; inclination and azimuth; area; insolation on a horizontal surface for each hour. |
| Cathode follower | - Pairs of nodes i and j. (Temperature of node i will be replaced with temperature of node j). |
| Radiative resistance | - Resistance number; value of $\epsilon F_{12}$. |
| Convective resistance | - Resistance number; area A; exponent m or n for free or forced convection (Fig. 3). |
| Table Association | - Table number and associated node or resistance number. |
| Problem constants | - Print interval. Problem cut off time. Initial time. Number of tables, etc. |
| Heat flow calculations | - Node number at which heat flow required; resistance number. |
| Temperature output requirements | - Node numbers at which temperature data required. |
| Tables | - For each table: Table number; argument type; variable type; argument list; corresponding variable list. (Possible arguments: time, temperature. Possible variables: temperature, resistance, capacitance, heat input). |

The output consists of the temperature of specified nodes at required intervals of time together with the rate of heat flow, during the previous interval, through separately specified nodes, e.g. the hourly temperatures of any desired nodes with the heat flows for a separate list of nodes can be obtained.

## 5. TYPICAL PROBLEM

Calculations were made on a simple enclosure (Fig. 4) of fixed internal dimensions.[8] The wall temperatures and total heat flow into the enclosure were determined for wall thicknesses in the range 1-6 inch. Properties of the two construction materials used, viz. concrete and polystyrene foam, are given (Table I) as are the heat transfer coefficients and surface absorptance (Table II).

Table I Thermal Properties of Construction Materials

| Property | Concrete | Polystyrene foam |
|---|---|---|
| Density, lb ft$^{-3}$ | 140 | 4 |
| Thermal conductivity, Btu h$^{-1}$ft$^{-1}$F$^{-1}$ | 1.0 | 0.022 |
| Specific Heat, Btu lb$^{-1}$F$^{-1}$ | 0.21 | 0.27 |
| Volumetric heat capacity, Btu ft$^{-3}$ | 29.4 | 1.08 |
| Thermal diffusivity, ft$^{2}$ h$^{-1}$ | 0.0340 | 0.0204 |
| Overall heat transfer coefficient, Btu h$^{-1}$ft$^{-2}$F$^{-1}$ (3" thick, vertical wall) | 4.0 | 0.088 |

Table II Heat Transfer Coefficients and Radiation Absorptance

| | | | |
|---|---|---|---|
| Heat transfer coefficient, Btu h$^{-1}$ft$^{-2}$F$^{-1}$ | Vertical wall | Inside | 2.18 |
| | | Outside | 4.0 |
| | Horizontal wall | Inside | 2.44 |
| | | Outside | 4.0 |
| Absorptance | | Inside | 1.0 |
| | | Outside | 0.8 |

As this work was part of a study of air conditioned buildings for tropical Australia, ambient conditions were chosen to be representative of a typical location. The data (Table III) is for an average sunny December day in Cloncurry, Australia, an inland town at approximately 20$^{o}$S latitude.

Table III

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time of day, h | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| Ambient Temp, $^{o}$F | 81.5 | 78.6 | 76.7 | 76.1 | 76.1 | 76.8 | 78.5 | 80.6 | 83.2 | 85.8 | 89.3 | 93.0 | |
| Insolation, Btu h$^{-1}$ ft$^{-2}$ | - | - | - | - | - | - | 32 | 93 | 167 | 230 | 276 | 305 | |
| Time of day, h | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Ambient Temp, $^{o}$F | 97.4 | 100.0 | 100.0 | 98.2 | 97.2 | 96.5 | 96.0 | 93.5 | 90.2 | 87.7 | 85.6 | 83.3 | 81.5 |
| Insolation, Btu h$^{-1}$ft$^{-2}$ | 315 | 300 | 264 | 208 | 141 | 72 | 25 | - | - | - | - | - | - |

The thermal network (Fig. 5) consists of six circuits in parallel representing the paths through the walls, roof and floor. These circuits are connected to a common node through a resistance network which models the internal convective transfer. This common node, held at a constant temperature of 78$^{o}$F, stands for the indoor air. Other resistances connect the indoor surface nodes to allow for radiative transfer.

Outdoor surface nodes receive heat flows (Fig. 6) that have been calculated from the insolation and are connected through convective resistances to nodes which receive an input of the ambient temperature. (It will be noted that long-wave radiation from the outdoor nodes is neglected for simplicity).

The floor circuit is connected to a node held at 74$^{o}$F and representing the constant earth temperature.

The output from the analyses is the hourly temperature of each surface node, the hourly heat transfer rates from each inside wall to the inside air node and the daily heat transfer rate for each wall.

A typical result giving the diurnal heat flow per square foot is shown (Fig. 7). It will be seen that the roof gives by far the highest rate and that the West wall is next. The North wall has no direct sunshine but is affected by diffuse radiation. The South wall shows an effect due to the direct component of radiation when compared to the North.

Integration of the heat flows on a daily basis was performed for each case (Table IV). For each wall thickness, the heat flow through each wall was expressed as a fraction of the roof flow, called the relative heat flow. It will be noticed that, even though the roof flow ranges from 36 to 1080 Btu day$^{-1}$ft$^{-2}$, the relative flows for each orientation do not vary significantly.

When wall flows are expressed as a percentage of the total daily flow (Table V), it will be noticed again that the proportion through each orientation does not depend significantly on the thickness or the insulation of the wall. (There is a significant increase in the effect of the floor for the well insulated cases, Polystyrene 4 inch and 6 inch, but the total flows involved at this level of insulation are relatively small.).

Total heat flows per day, appearing in this table (Table V), have been plotted against the overall heat transfer coefficient U (Fig. 8). The result is an almost linear increase in heat transfer with increase in the coefficient, as might be expected.

Table IV
Heat Flow per day per square foot for each orientation

| Material | Thickness -inch | Heat Flow - Btu $day^{-1}ft^{-2}$ | | | | | | Relative Heat Flow | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | N | S | E | W | R | F | N | S | E | W | R | F |
| Concrete | 1 | 370 | 540 | 670 | 620 | 1080 | 100 | .34 | .50 | .62 | .57 | 1.0 | .09 |
| | 2 | 340 | 490 | 590 | 550 | 990 | 89 | .34 | .50 | .62 | .56 | 1.0 | .09 |
| | 3 | 320 | 460 | 550 | 510 | 910 | 78 | .35 | .51 | .61 | .56 | 1.0 | .09 |
| | 4 | 300 | 430 | 510 | 480 | 850 | 69 | .35 | .51 | .60 | .56 | 1.0 | .08 |
| | 6 | 270 | 380 | 460 | 430 | 750 | 55 | .36 | .51 | .61 | .57 | 1.0 | .07 |
| Polystyrene | 1 | 67 | 93 | 114 | 107 | 173 | -3 | .39 | .54 | .66 | .62 | 1.0 | -0.02 |
| | 2 | 37 | 51 | 62 | 59 | 95 | -12 | .39 | .54 | .65 | .62 | 1.0 | -0.12 |
| | 3 | 25 | 35 | 43 | 41 | 66 | -17 | .38 | .53 | .65 | .62 | 1.0 | -0.26 |
| | 4 | 19 | 27 | 33 | 31 | 51 | -20 | .37 | .53 | .65 | .61 | 1.0 | -0.39 |
| | 6 | 13 | 18 | 23 | 22 | 36 | -23 | .36 | .50 | .64 | .61 | 1.0 | -0.64 |

From the network analysis, wall temperatures are also available. The maximum daily temperature and the time of occurrence have been tabulated for each orientation and two thicknesses of each material (Table VI). The advantage of insulation is obvious as the temperature is reduced as much as 42°F when comparing roofs of the same thickness but different material. A further point is the significantly higher temperatures of the roof, west and east walls when compared with the north and south walls.

Table V
Wall Heat Flows as a percentage of Total Flow

| Material | Thickness - inch | N | S | E | W | R | F | Total Flow - Btu $day^{-1}$ x $10^{-3}$ | % | % | U Btu $h^{-1}ft^{-2}$ $F^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Concrete | 1 | 12.3 | 17.6 | 16.1 | 15.0 | 35.6 | 3.3 | 125 | - | 100 | 1.26 |
| | 2 | 12.4 | 17.7 | 16.2 | 15.1 | 35.5 | 3.2 | 114 | - | 91 | 1.14 |
| | 3 | 12.5 | 17.8 | 16.2 | 15.2 | 35.4 | 3.0 | 105 | - | 84 | 1.07 |
| | 4 | 12.5 | 17.8 | 16.2 | 15.2 | 35.4 | 2.9 | 99 | - | 79 | .96 |
| | 6 | 12.6 | 17.8 | 16.3 | 15.3 | 35.5 | 2.6 | 87 | - | 70 | .83 |
| Polystyrene | 1 | 13.7 | 18.9 | 16.9 | 15.9 | 35.1 | -0.6 | 20.3 | 100 | 16 | .22 |
| | 2 | 14.2 | 19.7 | 17.7 | 16.6 | 36.6 | -4.8 | 10.7 | 49 | 9 | .12 |
| | 3 | 14.8 | 20.5 | 18.5 | 17.4 | 38.5 | -9.6 | 7.1 | 35 | 6 | .08 |
| | 4 | 15.5 | 21.5 | 19.5 | 18.3 | 40.8 | -15.6 | 5.2 | 26 | 4 | .06 |
| | 6 | 17.2 | 24.1 | 22.0 | 20.7 | 46.6 | -30.0 | 3.2 | 16 | 3 | .04 |

Table VI
Maximum Internal Surface Temperature

| Material | Concrete | | | | Polystyrene | | | |
|---|---|---|---|---|---|---|---|---|
| Thickness | 1 inch | | 4 inch | | 1 inch | | 4 inch | |
| | Time | Temp | Time | Temp | Time | Temp | Time | Temp |
| North | 14 | 93.7 | 16 | 89.3 | 14 | 80.8 | 15 | 78.8 |
| South | 17 | 97.5 | 18 | 93.1 | 14 | 81.4 | 17 | 79.0 |
| East | 8 | 110.0 | 10 | 98.9 | 8 | 83.3 | 9 | 79.5 |
| West | 16 | 113.7 | 18 | 103.4 | 16 | 84.0 | 17 | 79.7 |
| Roof | 13 | 128.3 | 14 | 113.9 | 13 | 86.0 | 14 | 80.4 |
| Floor | 15 | 81.5 | 17 | 80.3 | 14 | 78.2 | 16 | 77.7 |

This data was subsequently used in an economic analysis of the cost of air conditioning buildings of 1000 $ft^2$ floor area.[9] Cost data for Australian conditions was estimated as follows:

Capital cost of building For U = 0.3 Cost = \$ 7.8 $ft^{-2}$
U = 0.2 Cost = \$ 8.6 $ft^{-2}$
U = 0.1 Cost = \$10.8 $ft^{-2}$
U = 0.05 Cost = \$13.4 $ft^{-2}$

Owning cost of building = 8% per annum
Capital cost of air conditioner = $940\,t^{-0.28}\,\$\,\text{ton}^{-1}$ (t - refrigerator capacity in tons)
Owning cost of air conditioner = 10% per annum
Operating cost of air conditioner = $220\,t^{-0.35}\,\$\,\text{ton}^{-1}$

The average cost levels thus obtained are designated levels $A_2$ and $B_2$ (Fig. 9). Levels $A_1$ and $A_3$ are approximately $\pm 25\%$ $A_2$ while levels $B_1$ and $B_3$ are approximately $\pm$ 45% $B_2$. Thus the range of likely costs is spanned. The results show that the minimum cost occurs for values of U between 0.1 and 0.2.

## 6. ALIGNMENT OF THE MODELS

Some experimental work and computation has been performed with the aim of proving the models. Factors that have been investigated include:

a. One dimensional approach.

It is obvious from the different temperatures that can occur in adjacent areas, e.g. the roof and north wall, that considerable heat will be transferred in directions other than normal to the wall surface. Thus the assumption of one dimensional flow must be evaluated. Other factors that can modify the approximation to one dimensional flow are discontinuities in the structure, such as with stud and panel wall construction, and the geometrical effect of corners. The effect is dependent, among other things, on the size of the building and is greater in scale models of buildings especially where the material thickness is not scaled. Errors greater than 5% can result.

b. Fineness of the mesh.

Studies with electric analogues have indicated that dividing homogeneous conduction paths into four to five lumps gives sufficiently accurate results. Our studies, using material properties as in Table I, and sinusoidal inputs for which the theoretical solutions can be obtained, indicate that using even three lumps will enable calculation of heat flow within 5%. There may be greater inaccuracy in the amplitude ratio which varied up to -7% for the network with three lumps.

c. Heat Transfer Coefficients.

The outside heat transfer coefficient is usually considered as a function of the wind velocity which varies with height and time. If wind velocity is taken as $V_\infty$ for the surface, it will have different effects on surfaces of different orientation.

The value of the coefficient has been adjusted within limits when attempting to align calculated and measured results.

d. Size of the time step.

Since the time step can be of any value less than the stability limit, some results were taken to determine the improvement in accuracy for time steps as small as 0.1 of the stability limit. It was shown that for the system considered, reducing the step to 0.125 of the stability limit improved the accuracy of amplitude ratio by a factor of 4. Practically, this would also increase the computing time by almost eight times and make such small steps uneconomic.

e. Damping of initial value transient.

Initial values of node temperature are usually assumed at some constant value though in practice some distribution of temperature resulting from the previous variable input will remain. The transient from this incorrect assumption takes several cycles to become ineffective. The error in the amplitude ratio is reduced by 44% between the first and second cycle and by only 8% between the second and third cycles when it is approaching the long term value. Thus it would seem that only two to three cycles need be calculated to remove this error.

f. Comparison with the van Gorcum matrix method.

The van Gorcum method was compared with the network analyser for some simple problems. Being a superposition method, it must be used with constant values of the resistances but, for most building problems, separate averaging of these resistance values will usually give adequate accuracy. The calculation method does not suffer inaccuracy due to lumping since the distributed properties are used.

Since Fourier components of the input are required, it is somewhat less easy to deal with actual weather data input over a long period.

## 7. CONCLUSIONS

A basic inaccuracy arises from the many approximations in modelling a real situation and this applies to the mathematical model used to calculate the heat flows in a structure. Thus absolute accuracy in the calculation method is not of paramount importance as long as the calculation error does not unduly increase the overall expected error.

The thermal response methods accurately model the conduction path of one dimensional systems and can produce heat flows for periodic boundary conditions with a short computing time. Since superposition is involved, variable convection coefficients and material non-linearities cannot be accommodated. Perhaps, pulse methods are more flexible in their handling of boundary conditions than Fourier methods.

Lumped parameter networks, analysed by solution of node heat balance equations at finite time steps, are simple in concept. They are not restricted to one dimensional flow, can handle variable resistances and internal radiative exchange. But, since sequential solution for each node is required at each time step, the computing time is long. While they can approximate space-wise variations as accurately as desired by decreasing the spatial increments, computing time is increased as the space increment is decreased.

It would seem that the thermal response method may be more suitable for routine investigations with programmes adapted for a particular class of work, e.g. routine calculation of heat flow into air conditioned buildings.

On the other hand, the thermal network analyser is suitable for investigational work on a wide variety of problems. It is particularly useful for University type research due to its conceptual simplicity, its adaptation to parametric studies and its ability to model complex situations.

## SYMBOLS

$A$ - area, ft$^{2}$
$C$ - thermal capacity, Btu ft$^{-3}$
$F$ - shape factor
$H$ - insolation on a horizontal surface, Btu h$^{-1}$ ft$^{-2}$
$q$ - heat transfer, Btu h$^{-1}$
$Q$ - heat quantity, Btu
$R$ - thermal resistance, h F Btu$^{-1}$
$R_g$ - orientation factor, surface to sun

$t$ - temperature, $^{o}$F
$T$ - temperature, $^{o}$R
$W$ - flux density, Btu h$^{-1}$ft$^{-2}$
$\alpha$ - absorptance
$\Delta\theta$ - time increment, h
$\epsilon$ - emittance
$\theta$ - time, h
$\rho$ - reflectance
$\sigma$ - Stefan-Boltzmann constant, $0.173 \times 10^{-8}$ Btu h$^{-1}$ ft$^{-2}$ R$^{-4}$.

## SUBSCRIPTS

c - convective
e - sol -air

i - any node
j - other node connected to i

s - surroundings
$\theta$ - at particular time
$\Delta\theta$ - with time increment

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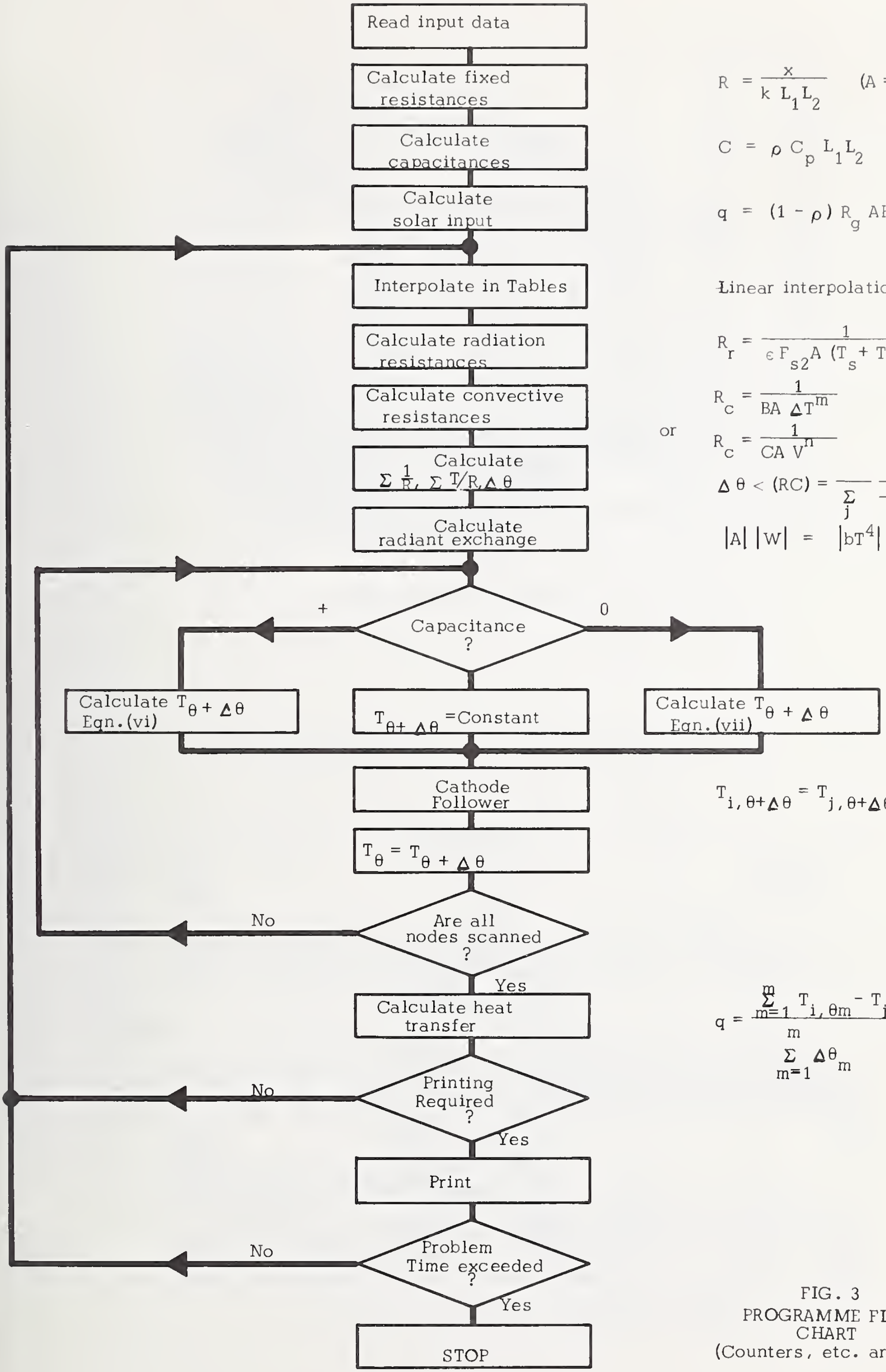

$$R = \frac{x}{k\ L_1 L_2} \qquad (A = L_1 L_2)$$

$$C = \rho\ C_p\ L_1 L_2 \qquad (A = L_1 L_2)$$

$$q = (1 - \rho)\ R_g\ AH \qquad \ldots \qquad (i)$$

Linear interpolation

$$R_r = \frac{1}{\epsilon F_{s2} A\ (T_s + T_2)(T_s^2 + T_2^2)} \qquad (ii)$$

$$R_c = \frac{1}{BA\ \Delta T^m}$$

or

$$R_c = \frac{1}{CA\ V^n}$$

$$\Delta\theta < (RC) = \frac{C_i}{\sum_j \frac{1}{R_{ij}}} \qquad \ldots \qquad (viii)$$

$$|A|\ |W| = |bT^4| \qquad \ldots \qquad (iv)$$

$$T_{i,\theta+\Delta\theta} = T_{j,\theta+\Delta\theta}$$

$$q = \frac{\sum_{m=1}^{m} T_{i,\theta m} - T_{j,\theta m}}{\sum_{m=1}^{m} \Delta\theta_m} \qquad \ldots \qquad (ix)$$

FIG. 3
PROGRAMME FLOW
CHART
(Counters, etc. are not shown)

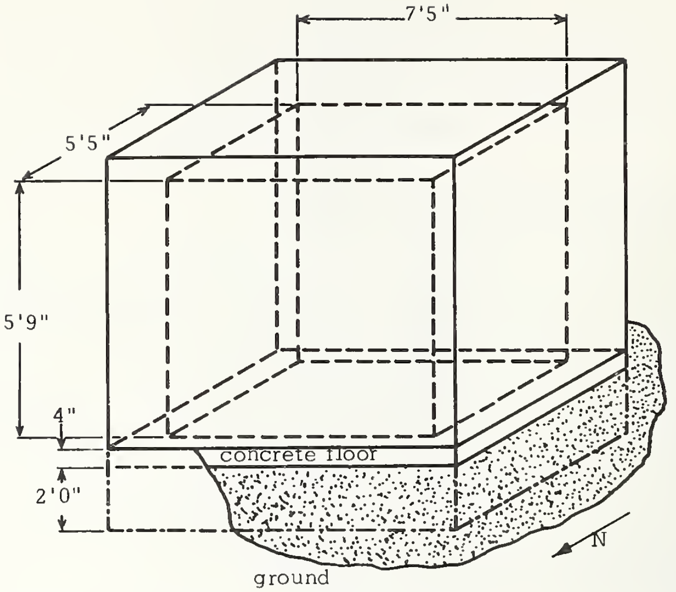

FIG. 4 VIEW OF SIMPLE ENCLOSURE

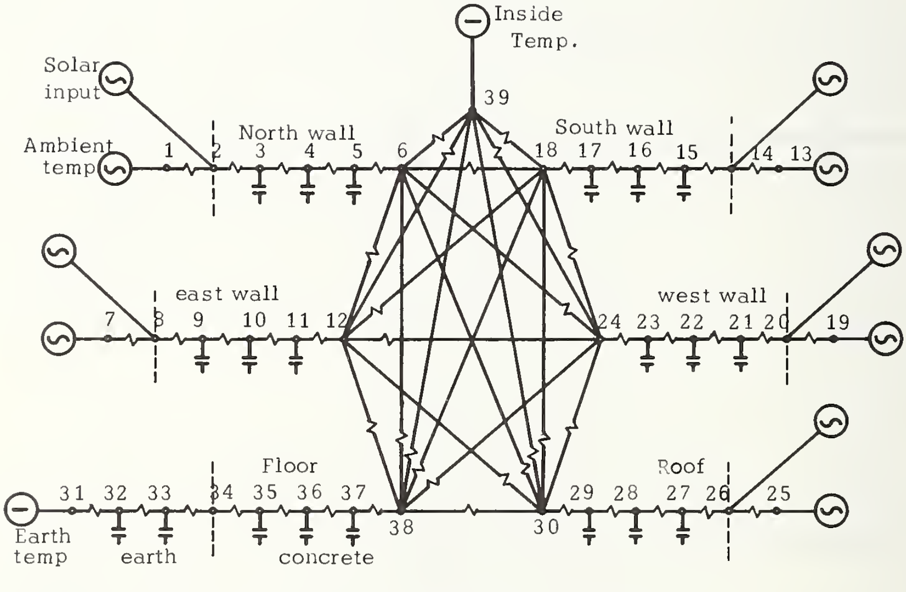

FIG. 5 THERMAL NETWORK OF SIMPLE ENCLOSURE

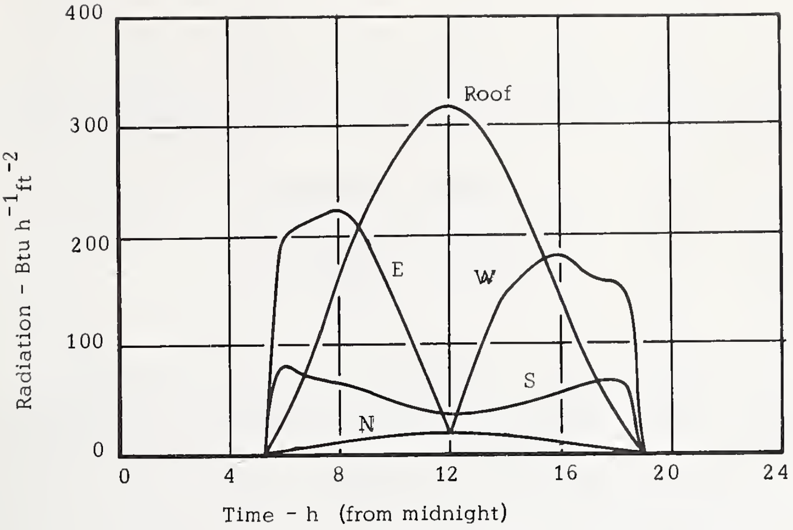

FIG. 6 RADIATION ON WALLS OF DIFFERENT ORIENTATION

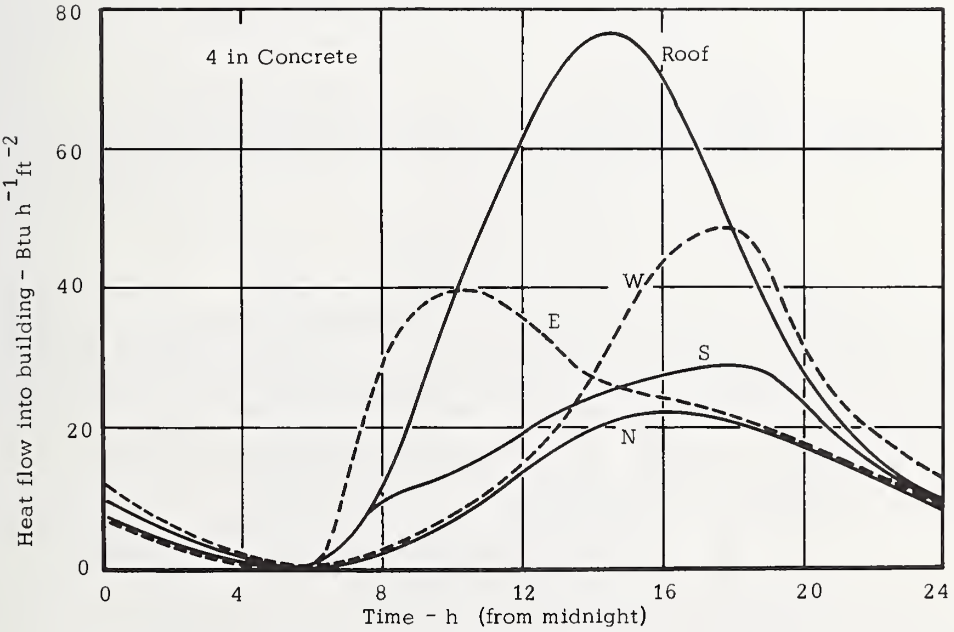

FIG. 7 HEAT FLOW IN BUILDING WALLS

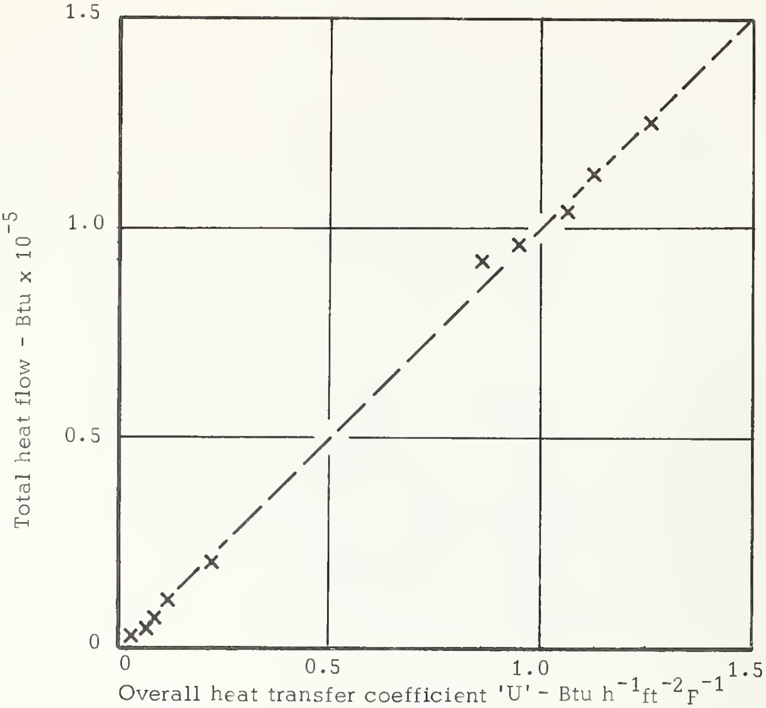

FIG. 8 TOTAL HEAT FLOW IN STRUCTURE

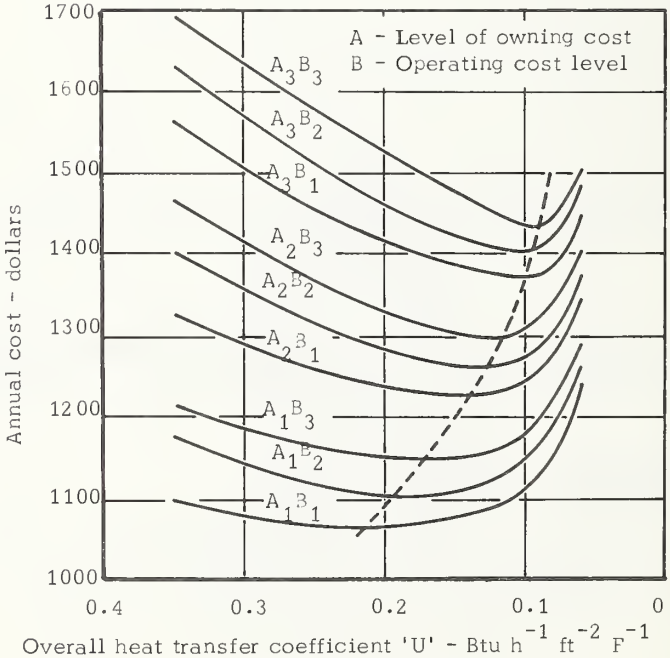

FIG. 9 OPTIMUM INSULATION THICKNESS

# A Method of Computer Simulation through Modified Signal Flow Graphs and Operator Concepts and its Application to Synthesis of Heating-Equipment Capacities

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In order to facilitate simulation of a physical system, direct simulation on an analog computer through a signal flow graph obtained directly from a schematic diagram in the physical system is used in this paper. Physical meanings of the method are confirmed and modified in view of algorithm as follows: (1) By creating a summing point which defines a signal, a wrong signal flow graph resulting from two definitions of a signal is avoided and an inversion law and interconnection of subgraphs are clarified. (2) A scaling method in s-domain is studied only by the use of a translation of a modified signal flow graph; It is made possible to obtain the completed program on an analog computer in which mutual relations between a signal and a scaled signal (a machine variable) are elucidated. (3) An operator concept is developed into the digital domain and a physical meaning in a closed loop is confirmed, so that simulation on digital computers by same methods as in the above-mentioned simulation on analog computers is made possible.
As an application, a synthesis of heating-equipment capacities is performed together with the confirmation of troublesome points in the actual operation, where not only a building but also automatically controlled heating equipments are simulated.

Key Words: Algorithm, closed loop, definition point, digital operation, dynamic balance, initial value, integral operator, modified signal flow graph, operator concepts, warming up load, scaling in s-domain, space series.

## 1. Introduction

An environmental design related to buildings would be an optimization of the ways of combination of components (said to be a structure of system) and their values in a buildings system containing equipments, which adjusts its entire balance under certain specified conditions. Considering the use of a computer from the point of view of design, therefore, it is desirable to use it synthetically rather than analytically, that is, it is necessary to be able to talk with the computer. As one of the useful means for it, there exists such simulation as correspondence of components in a system one-to-one, because a building system becomes large-scaled, complicated, high-priced and made-to-order, so that experiments of actual systems are impossible. For the sake of its usefulness, the technique of simulation has been widely used in fields of electronics, automatic-control, chemical process and so on.

The types of computers used in simulation are analog type, digital type and hybrid type (which is the combination of previous 2 types). Considering them from the standpoint of simulation, there, it shows the problems as follows: (1) There are differences of the models (expressions) as languages and ways of thinking depend on the kind of computers. (2) Advanced knowledges and techniques are required for simulation. (3) As the analog computer has the limited usages except for a differential analyzer or

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a simulator, it is necessary to consider simulation on the digital computer. However, the large-scaled and high-speed machine is required in its case.

As a countermeasure for the above mentioned, it is necessary to consider the next points: (1) Investigating programming-rules through models in use of the same concept which is independent of the kind of computers. (2) Using symbolism with sufficient informations in expression of system and its description. (3) A symbolism with algorithm which leads automatically from description to program. (4) In synthesis, it is necessary that a program one-to-one corresponds to a system in parts and the program is newly made by interconnection and division according to changing of the structure in system by interconnection and division, and also values on the program can be easily changed. (5) Finding out physical meanings and investigating calculation rules which calculation accuracy is not less than it was before even if simple procedures are used for the purpose of using a small machine such as a mini computer or a desk calculator.

This paper deals with a new method of computer simulation with algorithm which automatically gets to a simulation program through a model from an object system.

## 2. Method of Computer Simulation

It is well known that phenomena of system should be expressed in use of elements and a pair of across variable and through variable with time. For instance, heat conduction phenomena can be expressed as simultaneous differential equations using thermal resistance and thermal capacity as elements, and using temperature and heat flow as across variable and through variable with time. In these equations, relations among components such as wall and boiler which actually construct the system, namely, system structure is not clear.

There are graphical symbolisms as a way of expressing this structure, namely a physical network model, a block diagram, a signal flow graph, an analog computer diagram and so on. These are diagrammed to emphasize different aspects respectively. It is performed to symbolize many informations as to relations of actual components and each element. However, informations for the casual relation in each variable are not diagrammed. The signal flow graph and the block diagram are expressed in regard to the casual relation, but the relation to the actual object becomes weak rather than the physical network model. In the above expressions, direct informations of time are lost. The analog computer diagram has a nature of emphasizing element itself, its own function in itself and connection with other elements.

Observing these expressions from a view of simulation programming on the computer, with respect to the analog diagram, simulation programming seems to have been accomplished at one sight. However, as the analog computer diagram is usually introduced in terms of an expression of simulataneous differential equations, the system structure is lost and the correspondence of system one-to-one in parts can not be found. Simulation in use of this procedure requires to supplement informations through thoughts of the structure. Therefore, it is very useful to obtain a simulation diagram by making the best use of the characteristics of each graphical symbolism. That is; at first, the physical network model is introduced by a schematic graph [ 1][2] which indicates actual system; and then, it is transformed into the signal flow graph [ 2] or the block diagram; and finally, the simulation diagram is obtained. However, it is not always said to accomplish algorithm of final processes. Symbolism applicable to both computers is, therefore, developed under considerations of terms of physical concepts as follows:

### 2.1. Concepts of Signal and Operator

Observing the relations between variables and elements from a new standpoint, variables are signals which transmit in a system and the signal is modified by the element, so that it becomes the next signal in succession. Elements should be thought as operators. Under such thoughts, what is diagrammed in s-domain is a signal flow graph. However, it should be noted that descriptions corresponding to a system have many equivalent signal flow graphs, but physical meaning of the graph is clear only when the graph is described in the form of 1/s concerning time, namely, in the form of an integral operator, because physical phenomena may be said in general to depend on the past and the conservation of energy principle.

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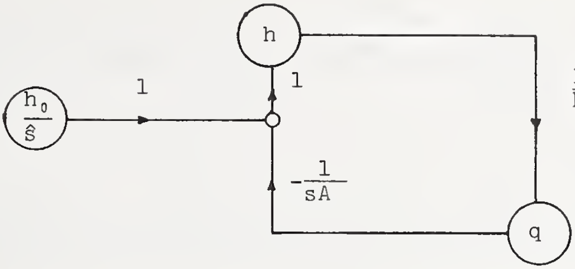
[2] Figures in brackets indicate the literature references at the end of this paper.

## 2.2. Modification of a Signal Flow Graph

As a simple example to clarify the above mentioned, it is considered that water is discharged from water tank (across sectional area A) using a pipe (resistance R). This is applied also to the case in which heat is discharged only by ventilation out of the room. When the water level is h, its initial value is $h_0$ and outgoing water flow is q, the modified signal flow graph expression of this system is given as eq (1).

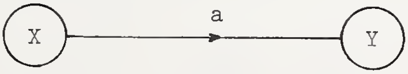


(1)

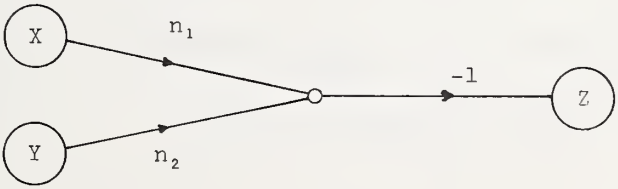
where, the signal flow graph is modified as follows: The signals are enclosed by a large circle to be distinguished from transmittance (operator) and the new summing points shown by small circles are made, they are also definition points in signals. Observing the definition as to h, it is equivalent to the next equation.

$$h = -\frac{q}{sA} + \frac{h_0}{\hat{s}} \tag{2}$$

By the modifications, the next merits may occur. By means of separating and symbolyzing definition points in such a way as each a signal has only one definition point (by elimination of signals on the way, it is not prevented that the signal has sequentially two more definition points), misses in two definitions of a signal, when the flow graph is drew, can be prevented. Physical meaning of interconnections in system is in a concordance with across variables and a continuity of through variables. An interconnection in sub-graphs (which correspond to sub-system) attending an interconnection of sub-system requires that in any interconnection the across variables are connected by a branch with transmittance 1 and the through variable is defined by another through variables, connected by each branch so that continuity conditions may be held. In this case, if a signal has two definition points through the interconnection, here, a branch of either definition point is inversed (in which 1/S is left as it is) as a definition point for a different signal, and two definitions resulting newly from it is inversed in succession until reaches a signal having no a definition point. Furthermore, by creating this summing point, the analog computer diagram can be easily expressed by modified signal flow graphs. Namely, a potentiometer, a summing amplifier and a summing integrator are expressed as eqs (3), (4) and (5). By using them, an analog simulation diagram corresponding to a system one-to-one can be made only by equivalent transformations of the graph.



(3)



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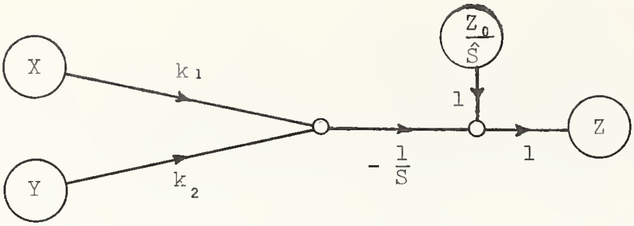


(5)

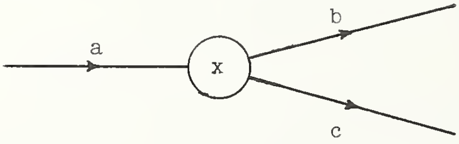
However, if a scaling change is not done in the analog computer programming, it is not that the program is perfect. A perfect analog simulation in only a signal flow graph is not always done. It means that there remains problems of algorithm. Next, a scaling in s-domain is considered.

## 2.3. Scaling in S-domain

A scaling has two kinds, first is to transform variables in a system into machine variables which are non-dimension and smaller than 1. Transforming them in s-domain by considering a magnitude scale factor $\alpha$ with dimensions, it becomes as follows:

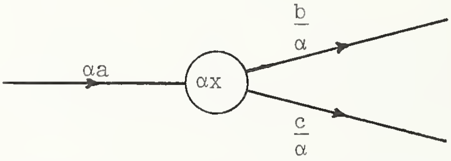
$$X = \alpha x \tag{6}$$

For the purpose of representing this relation on a graph it is necessary to add a new rule, that is, by multiplying a signal by $\alpha$ , the procedure in such a way that a transmittance of incoming branch is multiplied by $\alpha$ and that of outgoing branch is multiplied by $1/\alpha$ is required so that the graph is equivalent to the original graph.



(7)

is equal to



(8)

Next, it is thought to transform concerning time in use of time scale factor $\beta$ for simulation within time or frequency adapted to the machine.

At first, in t-domain

$$\tau = \beta t \tag{9}$$

transforming (4) into s-domain

$$\frac{1}{S^2} = \beta \frac{1}{s^2} \tag{10}$$

where t, s correspond to real time and $\tau$ , S to machine time. And according to Mixnsky's expression [ 3]

$$\tau = \frac{1}{S^2} \tag{11}$$

$$t = \frac{1}{s^2} \tag{12}$$

Observing both sides of these equations from the point of view of a dimension, they do not coincide because 1/s is said to have a dimension of t.

The next points are considered to clarify this problem. Observing a signal as the result of operators acting on a unit impulse, in a usual description of function in s-domain, operators alone are expressed and the unit impulse is not expressed. Therefore, this unit impulse having value 1 and the dimension 1/t (because the unit impulse is thought as a limit of a pulse in the width $\Delta t$ and the height $1/\Delta t$) should be affixed to the right hand of eqs (11) and (12) and both equations coincide also in the dimension. Hereafter, to distinguish clearly a signal from a group of operator, the signal is written in the form of affixing I having a dimension of 1/t. Considering physical meaning of time scale change in eq (9), the phenomena is extended by $\beta$ in time. As an area of a unit impulse must be always 1, the unit impulse $\mathbb{1}$ in machine time is given as:

$$\mathbb{1} = \frac{I}{\beta} \tag{13}$$

A unit step function which results in one integral operator acting on the unit impulse is expressed as, I/s, $\mathbb{1}$/S, respectively. As this unit step function is an infinite step without concerning time scale change, they must be equal. And show them as $1/\hat{s}$.

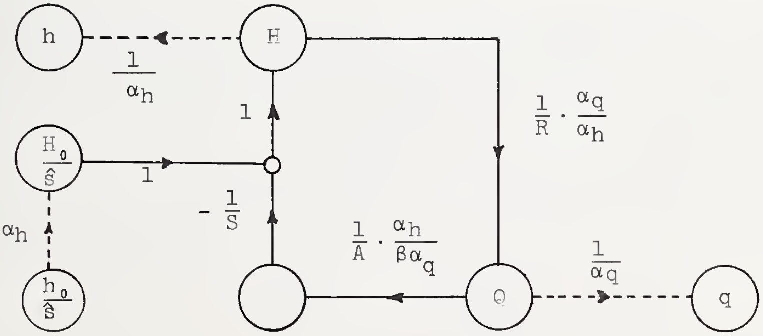
$$\frac{1}{s}1 = \frac{1}{S}\mathbb{1} \equiv \frac{1}{\hat{s}} \tag{14}$$

accordingly

$$\frac{1}{s} = \frac{1}{\beta S} \tag{15}$$

From the above investigation, if time scale change is done directly in s-domain, it will be done by affixing 1 to input signal and also by adapting eqs (14) and (15).

For the purpose of showing a simple example of analog simulation procedures, the graph of the tank model shown already in eq (1) is transformed equivalently in such a way as constructed by analog computer elements given in eqs (3), (4) and (5). And when magnitude and time scale change are done with regard to the above mentioned, the analog simulation program can be accomplished automatically and successively as follows:



(16)

where, the values of H and Q are non-dimension and machine variables smaller than 1 and $1/R \cdot \alpha_q / \alpha_h$, $1/A \cdot \alpha_h/\beta\alpha_q$ are non-dimension and values smaller than 1 and indicate potentiometer values. The newly added branches shown as dotted line indicate relations between machine variables and variables of the original system (these do not become the object of the simulation). Next, consider the case of digital operation.

## 2.4. Digital Operation

In the case of an operation on a digital computer, various numerical methods have been developed in regard to information which can be obtained when it is sampled. That is, information related to the structure is weak, so that they don't always adapt to system simulation. For the purpose, the "time series" method [ 4] and the "thermal response factor" method [ 5] were published. But it is difficult to adapt the methods to the next cases; namely (1) when systems are interconnected, (2) when the system has non-linearity, (3) when the system has the initial value which represents the past effect, (4) when the problem having non-periodic intermitting heating including off days is solved.

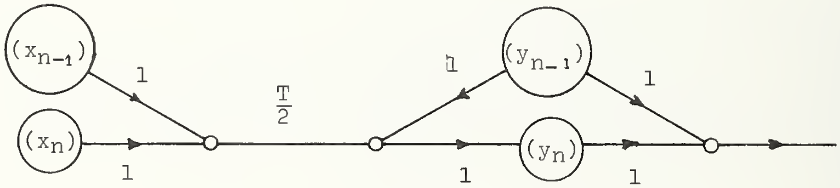
A digital operation of an integral operator is considered, observing that time is represented only by the integral operator 1/S in the simulation diagram corresponding to a system one-to-one. When values of a signal at t=0, T, 2T, ...., etc. (T is time interval) are $x_0$, $x_1$, $x_2$, $x_3$, ....., etc., the signal x is approximated by straight line segments at each interval. It is expressed as eq (17), which is named "space series".

$$[\ (x_0),\ (x_1),\ (x_2),\ (x_3),\ldots\ldots\ ] \qquad (17)$$

The signal resulting from the integral in eq (17) is expressed by space series as follows:

$$\frac{T}{2}\left[(0),\ (x_1 + x_0),\ (x_2 + x_1 + x_1 + x_0),\ (x_3 + x_2 + x_2 + x_1 + x_1 + x_0),\ldots\ldots\right] \qquad (18)$$

As mentioned above, y resulting from the digital operation of 1/s from (n - 1)T to nT is expressed as follows:



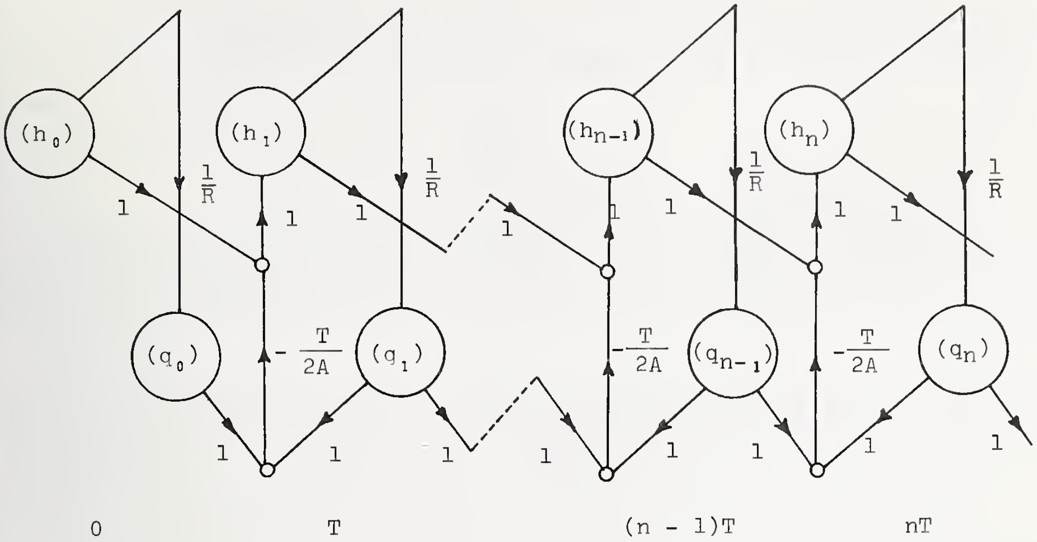
(19)

Therefore, it is proper to replace all 1/s of the graph with the above equation and calculate it step by step at each interval. However, as exceptions, when the signal value is always zero in the previous interval and it rises to the value $(x_0)$, an operation of $0 + (x_0) = (0)$ must be used, and in a unit impulse, the calculation is such that instantaneously (1) will be preserved.

Next, consider adaptation of the digital operation in eq (1). When eq (1) is solved theoretically with Mason's rule [ 6], eq (20) is given as:

$$\frac{h}{h_0} = \frac{ST_c}{1 + ST_c} = e^{-\frac{t}{T_c}} \qquad (20)$$

where, $T_c$ = AR is time constant. Equation (1) is expressed in the form of the digital operator using sampling interval T as follows:



(21)

Solving it in accordance with the graph rule at time T

$$\frac{(h_1)}{(h_0)} = \frac{2 - \frac{T}{T_c}}{2 + \frac{T}{T_c}} \tag{22}$$

at time 2T = $2\mu T_c$, where $T/T_c = \mu$

$$\frac{(h_2)}{(h_1)} = \frac{2 - \mu}{2 + \mu} \tag{23}$$

or

$$\frac{(h_2)}{(h_0)} = \left(\frac{2 - \mu}{2 + \mu}\right)^2 \tag{24}$$

and at time nT = $n\mu T_c$

$$\frac{(h_n)}{(h_0)} = \left(\frac{2 - \mu}{2 + \mu}\right)^n \tag{25}$$

Table 1 indicates the calculation result with $\mu = 1/4$ and it agrees to theoretical values very well.

Table 1. Comparison between calculation values in use of $\mu = T/T_c = 0.25$ and theoretical values.

| Time | Time(by $T_c$) | Calculation Value | Theoretical Value |
|---|---|---|---|
| 0 | 0 | 1.0000 | 1.0000 |
| T | $0.25T_c$ | 0.7777 | 0.7788 |
| 2T | $0.5\ T_c$ | 0.6049 | 0.6065 |
| 3T | $0.75T_c$ | 0.4705 | 0.4723 |
| 4T | $1.0\ T_c$ | 0.3659 | 0.3678 |
| 5T | $1.25T_c$ | 0.2846 | 0.2865 |
| 6T | $1.5\ T_c$ | 0.2213 | 0.2331 |
| 7T | $1.75T_c$ | 0.1721 | 0.1737 |
| 8T | $2.0\ T_c$ | 0.1339 | 0.1353 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 12T | $3.0\ T_c$ | 0.0490 | 0.0497 |
| 16T | 4.0 | 0.0179 | 0.0183 |

When $\mu = 2$ (namely the interval of 2 $T_c$), eqs (22) and (23) become zero (these exact values are 0.135335 and 0.018316). And when $\mu > 2$, they oscillate in -, +, -, +, ........, etc., having values smaller than 1. Therefore, it is seen that $\mu$ is an index for modelizing a distributed system into a lumped system. That is, time constants in each part should be divided to coincide as much as possible. In order to clarify correspondence to the system, if divisions are done in such a way that time constants in each part have considerable differences , the interval in calculation of each part should be modified in such a way that $\mu$ becomes equal in parts. For the purpose of rough calculations, when the calculations are tried with large intervals, it is proper to neglect heat capacities in the part of $\mu > 2$.

## 3. The Application to Synthesis of Heating Equipment Capacitance

In use of the methodology mentioned above, it is reported to simulate hot water heating in a building on an analog computer. A one-story house (100 $m^2$) having concrete walls of thickness of 15 cm affixed with glasswool 5 cm is heated by hot-water radiator and the system is represented in figure 1 using physical network model. As the used computer is small, the building is one-room model with one boiler (with hot-water-supply tank inside) having one radiator, and a burner is controlled ON-OFF by room temperature and water temperature in the boiler.

### 3.1. Warming up load

As the results of simulation, figure 2 indicates an intermittent operation in which an operation is sixteen hours and a stoppage is eight hours. In this case, an average outside air temperature is -10°C and a calculation load in steady state is 10000 kcal $hr^{-1}$.

Observing the results, at night the room temperature in stoppage of operation falls from 20°C to 6°C, therefore, it seems as if fuel is saved in general. But judging from figure 2, it is said that the sum of outgoing heat flow falls only a little. The reason is that heat stored in the wall is discharged at night and the heat is compensated during warming up time. It requires about three hours until it reaches 20°C even when a burner of 20000 kcal $hr^{-1}$ (two times of calculation load) is used. In this example, the intermittent operation has not saved even 10 %, compared to the continuous operation and it is clear that the burner output from 2 to 4 times larger than the steady state load, would be required, corresponding to the interval of warming up time. Therefore, considering initial cost, the continuous operation is profitable rather than the intermittent operation.

### 3.2. The Need of Dynamic Balance of System

Figure 3 indicates ON-OFF of the burner and the boiler water temperature, there, the ratio k of the radiator capacity in steady state to the burner capacity is changed to 1.0, 1.1, 1.2.

As the result, in spite of having no troubles in steady state, it is seen that in transient state of warming up time, the boiler water temperature reaches a limit and the ON-OFF operation begins before the room temperature reaches 20°C. This ON-OFF operation means that the burner output becomes smaller. Therefore, it is necessary to consider not only static balance of system in steady state but also balance in transient state.

### 3.3. The Drop in Hot Water Supply Temperature and Additional Load in Hot Water Supply

Figure 4 indicates the drop of the hot water supply temperature and an influence on the room temperature when the hot water is supplied in thirty minutes at the rate of 10 ℓ every minute. At that time, there are two cases such as the intermittent operation with the burner output in 20000 kcal $hr^{-1}$ and the continuous operation in 10000 kcal $hr^{-1}$.

From the results of these simulations, it is seen that when limit design of equipment capacities and so on is done, each simulation should be done case by case because the characteristics are different because of the differences of the systems and therefore limit design should be determined after confirmation and investigation of problems.

## 4. Other Considerations

By means of the concept of the operator (the concept of the very system element itself which is the operator) and the modified signal flow graph (where it indicates that signals are modified by the operators), algorithm was reported where simulation will take place from the environmental system related to building to its simulation automatically and continuously without regard to the type of computer. It will be thought that the description method is also convenient for common expressions of phenomena in fields of environmental engineering such as electricity, electronics, dynamics, fluid dynamics, process and so on.

As the example of synthesis only the methodology using the small analog computer was indicated. If a large-scaled analog computer is used, it is possible to indicate each room. As digital computers occupy the major parts in general, simulation in use of the digital computer should be indicated. Languages oriented conversations with computers are in the stage of development in our laboratory. It will be discussed on another occasion.

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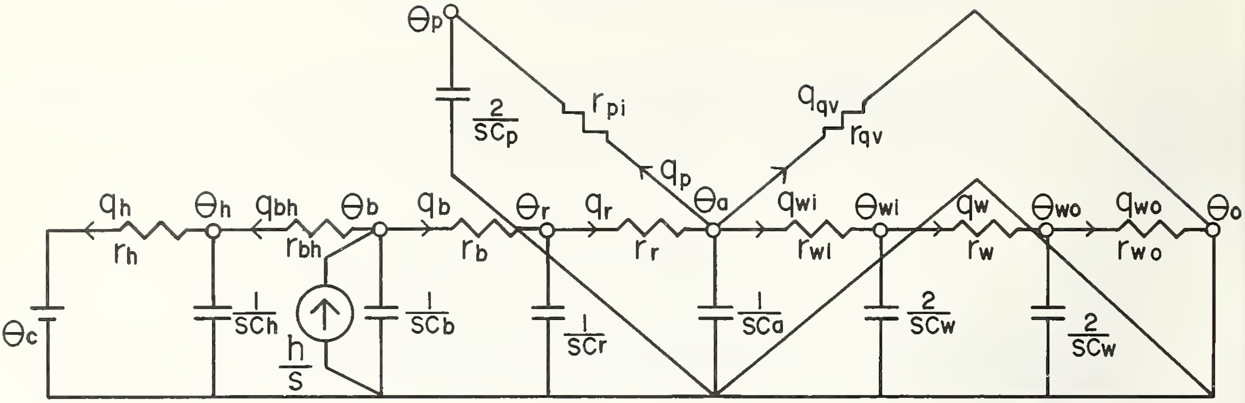

Fig 1 Physical Network Model
$\theta$: temperature, C: heat capacitance, h: burner output,
q: heat flow, r: resistance
(Subscripts)
a: room, b: boiler, c: cold water, g: glass, h: heat water supply,
i: inside, o: outside, p: room wall, r: radiator, v: ventilation,
w: wall

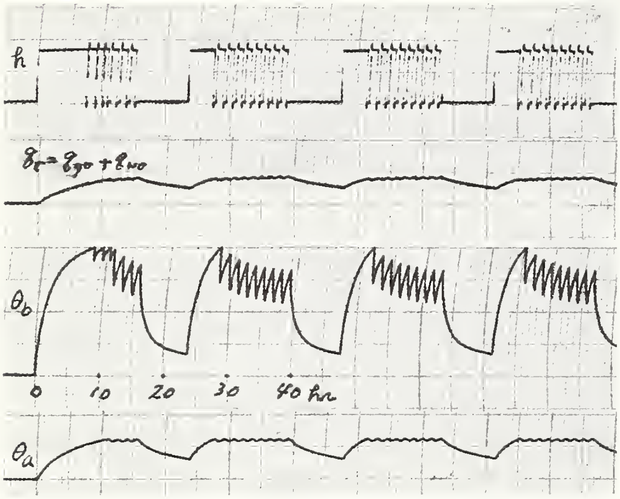

Fig. 2 Response concerning h, $Q_{qr}$, $\theta_b$ and $\theta_a$ for Intermittent Operation

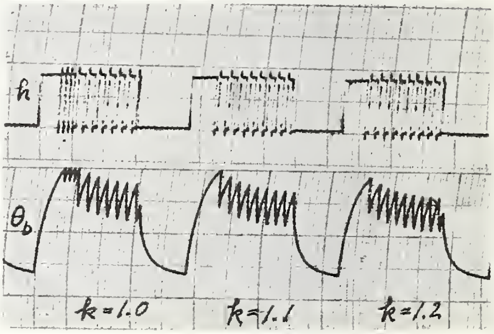

Fig. 3 Response concerning h and $\theta_b$ for Intermittent Operation when k = 1.0, 1.1 and 1.2

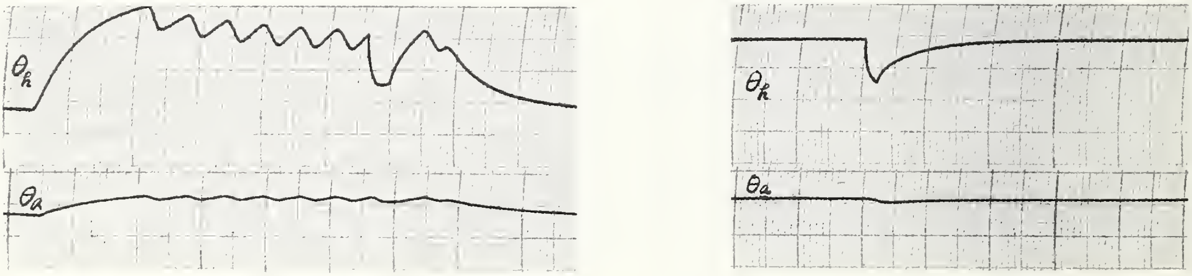

Fig. 4 (a) Response concerning $\theta_h$ and $\theta_a$ for Intermittent Operation with $h = 20000$ kcal $hr^{-1}$
(b) Response concerning $\theta_h$ and $\theta_a$ for Continuity Operation with $h = 10000$ kcal $hr^{-1}$

# Shared Time System Computer Programs for Heating and Cooling Energy Analysis of Building Air Conditioning Systems

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Heating and cooling Energy Calculations are made by shared time computer programs using Weather Data taken from Air Force Manual 88-8 and U. S. Weather Bureau Climates of the States covering 218 areas in the United States.

Three basic programs are used, Reheat, Heat-Cool-Off, and Multizone or Double-Duct to produce net requirements of ton-hours and BTU x $10^5$. In addition to the weather file, 91 numbers are required to describe building gains and losses, heating and air conditioning system and building use.

The output file of these programs are processed in another program to convert the ton-hours and BTU x $10^5$ building requirements to KW, KWH and BTU x $10^6$ input to equipment by additional data of 32 numbers describing the equipment and efficiencies.

In addition to evaluation of the three basic systems, analysis may be made of the effects of many variations of each system and programs schedule, such as:

Economiser system with or without reset of mix air temperature.
Hot deck temperatures on cooling cycle.
Perimeter heating loads.
Reduced temperature in unoccupied hours; intermittent operation in unoccupied hours.
Fuel conversion efficiencies; electrical demand.

Modifications and combinations of these basic programs may be used to evaluate Variable-Volume; Variable Volume with reheat; Fan-coil units in exterior and Multizone in interior; etc. The programs have also been used to analyze energy requirements of all electric and total energy systems. Mechanical and electrical systems for schools, office buildings, hospital operating suites, hospital patient rooms, apartment buildings, shopping centers and even a bicycle shop have been analyzed with the use of these programs.

This system of calculation by computer is an outgrowth of many years of experience using manual calculations. Programming, using Basic Language, was started in March 1968 and improvements continue to the present day.

Charles J. R. McClure and Associates, Inc., the developer of the system, has been making practical use of the information provided for several years. Medsi's customers have been using the programs on their own terminals since November 1969.

Approximately 25 seconds of processor time are used with about 30 minutes connect time for each run; programs are available from a restricted library on SBC, Call/360 system.

Key Words; Energy, heating, cooling, air conditioning systems, shared time programs, evaluation, gas, oil, electric, dollars.

## 1. Objective

The Engineers associated with developing these programs have made many estimates of heating and cooling energy requirements for building mechanical systems by manual calculations using degree days, full load hours, average temperatures, typical 24-hour weather profiles for each month, etc. The last manual calculation in 1967 consumed over 2,000 man hours, most of which was spent in determining the net ton-hours and BTU x $10^5$ required by the heating and air conditioning system, without regard to efficiencies of machinery. About this time, a computer time sharing system became available and management decided to exploit the computer's ability to make many calculations in a very short time.

The objective was a monthly tabulation of net ton-hours and BTU x $10^5$, as shown in Table 1, with high accuracy and requiring a minimum of repetitive manual calculation.

TABLE 1. - Ton-hours and BTU x $10^5$

```
SAMPLE: OFFICE BLDG, MINNEAPOLIS              DEC 1,1969
     FIN TUBE RADIATION AT EXTERIOR, CONVENTIONAL RETURN
     SYSTEM #1 WITH ECONOMISER
   MULTIZONE OR DOUBLE-DUCT SYSTEM

MONTH PERIOD       TON HOURS          BTUX10'5 INT       BTUX10'5 EXT

 JAN NIGHT               0.0                476.6               782.1
 JAN DAY                 0.0                739.8               639.0
 JAN EVNG                0.0                370.2               751.3
          SUB TOT              0                        3759
 FEB NIGHT               0.0                417.3               656.1
 FEB DAY                35.0                644.9               494.0
 FEB EVNG                0.0                326.8               614.7
          SUB TOT             34                        3153
 MAR NIGHT               0.0                414.5               602.8
 MAR DAY               249.1                671.2               433.4
 MAR EVNG                6.3                333.5               549.5
          SUB TOT            255                        3004
 APR NIGHT             139.1                333.8               402.6
 APR DAY              3184.5                491.3               225.2
 APR EVNG              343.0                256.7               338.3
          SUB TOT           3666                        2047
 MAY NIGHT             832.5                249.6               255.2
 MAY DAY              7645.9                318.9               100.9
 MAY EVNG             1047.2                176.2               192.8
          SUB TOT           9525                        1293
 JUN NIGHT            1680.7                167.4               123.1
 JUN DAY             10422.3                212.5                29.1
 JUN EVNG             1676.9                117.6                86.6
          SUB TOT          13779                         736
 JUL NIGHT            2076.0                138.7                81.0
 JUL DAY             11669.4                176.9                 8.6
 JUL EVNG             1912.0                 99.4                45.5
          SUB TOT          15657                         550
 AUG NIGHT            2092.2                146.9                88.1
 AUG DAY             11842.8                180.9                10.1
 AUG EVNG             1947.0                105.3                50.7
          SUB TOT          15882                         582
 SEP NIGHT            1012.2                229.3               207.0
 SEP DAY              8376.4                263.6                61.5
 SEP EVNG             1071.9                164.2               169.8
          SUB TOT          10460                        1095
 OCT NIGHT             369.7                311.1               342.1
 OCT DAY              5296.5                425.7               144.1
 OCT EVNG              533.0                240.6               301.6
          SUB TOT           6199                        1765
 NOV NIGHT              22.2                370.5               528.7
 NOV DAY               592.2                639.0               396.7
 NOV EVNG               43.0                304.5               500.9
          SUB TOT            657                        2740
 DEC NIGHT               0.0                441.0               696.0
 DEC DAY                 0.0                727.6               574.8
 DEC EVNG                6.1                350.5               665.4
          SUB TOT              6                        3455
 TOTAL NIGHT          8224.7               3696.7              4764.8
 TOTAL DAY           59314.0               5492.2              3117.5
 TOTAL EVNG           8586.5               2845.4              4267.1

 ANNUAL TOTAL        76125.1              12034.3             12149.4
```

## 2. Weather

Previous experience indicated that Air Force Manual 88-8, "Engineering Weather Data" (1), should be the weather source, since it was available, compact, included 8,760 observations per year, and covered many areas throughout the world. Weather observations in AFM 88-8 are grouped into 3 periods of the day, 1 A.M. to 8 A.M. (night), 9 A.M. to 4 P.M. (day) and 5 P.M. to 12 midnight (evening). The accumulated monthly observations are also grouped into 5 degree dry bulb segments together with the mean coincident wet bulb. In addition to the hours of dry bulb and wet bulb there is stored in the weather-file the hours of sunshine for each period of each month together with solar heat gain factors for each period as compared to the maximum hour in July for 9 exposures (N, NE, E... NW, Horizontal). Hours of sunshine for each locality are taken from U. S. Weather Bureau's "Climates of the States" (2) and solar intensity is calculated by computer using the method outlined in "ASHRAE". Medsi has weather on file for instantaneous call for over 10 locations in the United States. Each weather file takes 3 or 4 units of storage on Call/360. To prepare a weather-file for another location costs about $100. During each run the first part of the weather-file is read into a two-dimensional array, 45 by 125 maximum, and the second part covering solar gains is read into ten one-dimensional arrays of 36 factors each. Review of many projects leads to the conclusion that these weather-files are the most accurate input of all the data that enters into energy calculations.

## 3. Input Data

The input data required is shown on Medsi form No. 1 (Rev. 12-1-69) (Table 2). The data is entered into the program in the form of data statements together with job identification in the form of print statements. These are best prepared off-line by punch tape and then entered into the system. These are entered, given a name (program file) and saved. It is then possible to 'WEAVE' this program of data statements with other programs such as a reheat program, multizone program, heat-cool-off program etc., to readily make comparisons.

### 3.1 Occupancy Schedule

Lines 1 through 4 of form No. 1 (Table 2) are the occupied hours expressed as a decimal part of the total hours of each period. These numbers can be changed for each month so that vacations and holidays may be considered. The first number of line 1 is .208 and tells the computer that 20.8% of the January night time hours are occupied, that the outdoor CFM (3000) shown on line 16 is to be used, that the room temperature is 75 degrees as shown on line 5 and that the first number of lines 24 (BTUH lighting gain), 26 (BTUH other sensible gain) and 28 (BTUH latent gain) are to be used for these hours. Medsi form No. 2 (Table 3) is used to determine these numbers. Schedule C, occupancy, is filled in with x in the occupied hours of the week. The number of x (12) divided by 56 (8 hours per day x 7 days per week) equals .215. Multiplying this by 30/31, to allow for 1 holiday in January, gives .208.

### 3.2 Temperatures, Humidities and Enthalpies

Temperatures, humidities and enthalpies of the air throughout the system are entered on lines 5 through 14, and line 18. The room dry bulb (line 5) is used in both winter and summer calculations during occupied hours. To simplify calculations, enthalpies (BTU/# AIR) are used instead of Wet Bulb Temperatures and specific humidities (Grains/# AIR) are used instead of relative humidities. Lines 9, 10 and 11 are summer conditions of air in the supply duct of a terminal reheat system or the cold deck of a multizone or double duct system. Lines 12, 13 and 14 are winter conditions at the same points when cooling by refrigeration is used in winter.

### 3.3 Air Quantities

The total air quantity circulated is entered on line 15 and should be the actual air quantity circulated. The actual minimum fresh air is entered on line 16. The use of outdoor air for cooling in winter, instead of using refrigeration, is called an economiser in this set of programs and is considered later. Line 17 is unoccupied CFM or infiltration and is applicable to all unoccupied hours. However, since the refrigeration is usually turned off in unoccupied hours, the unoccupied CFM should be typical of the heating season.

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(1) Figures in parenthesis indicate the literature reference at the end of this paper.

TABLE 2 - Medsi Form No. 1

MEDSI
Form No. 1 *(REV. 12-1-69)*

Project SAMPLE OFFICE BUILDING

DATA STATEMENTS *(91 NUMBERS)*

Location MINNEAPOLIS, MINN Date 12/1/69

OCCUPANCY SCHEDULE

| | | Night | Day | Evng. | | Night | Day | Evng. | | Night | Day | Evng. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Jan. | .208 | .79 | .172 | Feb. | .215 | .803 | .178 | Mar. | .215 | .803 | .178 |
| 2 | April | .215 | .803 | .178 | May | .208 | .79 | .172 | June | .215 | .803 | .178 |
| 3 | July | .208 | .79 | .172 | Aug. | .215 | .803 | .178 | Sept. | .208 | .79 | .172 |
| 4 | Oct. | .215 | .803 | .178 | Nov. | .208 | .79 | .172 | Dec. | .208 | .79 | .172 |

DESIGN CONDITIONS:

| | | | |
|---|---|---|---|
| S | Room Dry Bulb *(WINTER & SUMMER)* | 75 | |
| 6 | Room Enthalpy *(SUMMER)* | 28.2 | |
| 7 | Room Minimum Humidity, Grains / lbs. | 0 | *(ZERO IF NO WINTER HUMIDIFICATION)* |
| B | Outdoor Enthalpy Max. *(SUMMER DES.)* | 38.4 | |
| 9 | Summer Supply Dry Bulb | 55 | |
| 10 | Summer Supply Humidity, Grains / lbs. | 58 | |
| 11 | Summer Supply Enthalpy | 22 | |
| 12 | Winter Supply Dry Bulb | 55 | *USED WHEN THERE IS NO ECONOMISER CYCLE* |
| 13 | Winter Supply Humidity, Grains / lbs. | 58 | |
| 14 | Winter Supply Enthalpy | 22 | |
| 1S | Total CFM | 31000 | |
| 16 | Outdoor CMM Occupied | 3000 | |
| 17 | Outdoor CFM Unoccupied | 1000 | |
| 18 | Summer Mode Hot Deck Temp. | 95 | *(NOT LESS THAN ROOM DRY BULB)* |
| 19 | Max. Trans. Loss in BTUH | 582000 | |
| 20 | Heat Loss Design Temp. Diff. | 95 | |
| 21 | Max. Trans. Gain in BTUH | 76000 | |
| 22 | Heat Gain Design Temp. Diff. | 14 | |

SOLAR GAINS (Maximum in July)

| | N | NE | E | SE | S | SW | W | NW | Horiz. |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 11900 | 0 | 64000 | 0 | 395000 | 0 | 64000 | 0 | 69000 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | LIGHTING GAIN | | | | | | |
| 24 | Occupied | Night | 172000 | Day | 328000 | Evening | 170000 |
| 2S | Unoccupied | Night | 51000 | Day | 51000 | Evening | 116000 |
| | OTHER SENSIBLE GAIN | | | | | | |
| 26 | Occupied | Night | 18800 | Day | 42300 | Evening | 30000 |
| 27 | Unoccupied | Night | 0 | Day | 0 | Evening | 0 |
| | LATENT GAIN | | | | | | |
| 2B | Occupied | Night | 15000 | Day | 33900 | Evening | 24000 |
| 29 | Unoccupied | Night | 0 | Day | 0 | Evening | 0 |

| | | DECIMAL OF TRANS. HEAT LOSS | DECIMAL OF LIGHT HEAT APPLICABLE |
|---|---|---|---|
| 30 | EXTERNAL AREA | .804 | 0 |
| 31 | INTERNAL AREA | .196 | 1.0 |
| 32 | RETURN AIR | 0 | 0 |
| | | TOTAL 1.000 | TOTAL 1.000 |

| | | |
|---|---|---|
| 33 | DECIMAL OF HORIZONTAL SOLAR GAIN TO RETURN AIR | 0 |
| 34 | DECIMAL OF TRANSMISSION GAIN TO RETURN AIR | 0 |
| 3S | SUPPLY FAN HEAT IN BTUH | 60 000 |
| 36 | RETURN FAN HEAT IN BTUH | 0 |

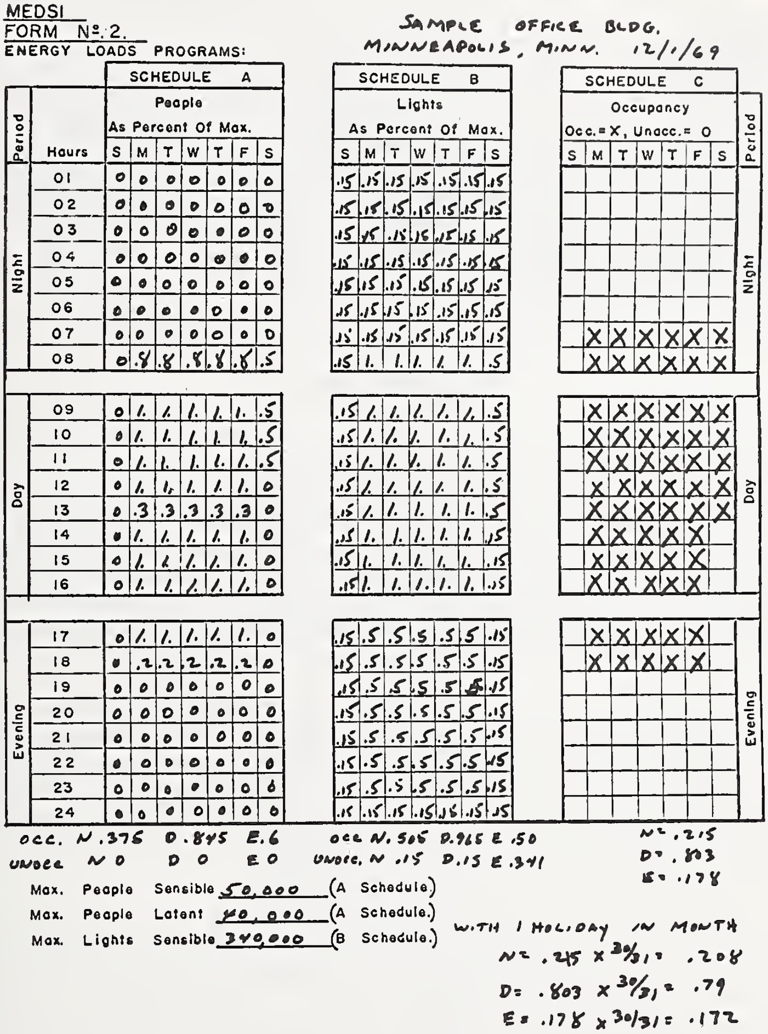
## 3.4 Building Gains and Losses

The transmission (conduction) loss is put in line 19 and the temperature difference used in the calculation is put in line 20. The transmission gain, not including solar gains, and the temperature difference are entered in lines 21 and 22. Solar gains are placed on line 23. They are not coincident solar gains but rather, are maximum gains for the peak hour for each exposure. For example, solar gain for East occurs about 8 A.M. and for West about 4 P.M. but the values would be the same if they were similar in glass area, shade factor, etc. These gains and losses should not include any safety factors or pick-up allowance that might be normal considerations for apparatus sizing.

## 3.5 Internal Gains

Lines 24 through 29 are internal gains applicable in night, day and evening periods when occupied and when unoccupied. These values (BTU/Hr.) are calculated manually using the schedule charts shown in Table 3. as follows. The schedules are filled in with percentages of maximum for each hour for each day of the week. The occupancy schedule C shows 12 x 's representing 12 occupied hours in the night period. The percentages of lighting for these 12 hours shows 6 hours of 15%, 5 hours of 100% and 1 hour of 50%. The average occupied night hour has 50.5% of the lights on and if the maximum heat gain from lights is 340,000 BTU/Hr., the heat gain for each night occupied hour is 172,000 (the first number on line 24 of Figure 2). People gains, sensible and latent are determined in a similar manner. Miscellaneous gains, if the energy source is electrical energy, such as an air cooled electric refrigerator in the space, should be included in with lighting gains since these loads will later be connected to KWH. Otherwise, include appliance and intermittent equipment loads in other sensible and/or latent gains.

TABLE 3 - Medsi Form No. 2



MEDSI
FORM No. 2.
ENERGY LOADS PROGRAMS:

SAMPLE OFFICE BLDG.
MINNEAPOLIS, MINN. 12/1/69

| | | SCHEDULE A | | | | | | | SCHEDULE B | | | | | | | SCHEDULE C | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | People As Percent Of Max. | | | | | | | Lights As Percent Of Max. | | | | | | | Occupancy Occ. = X, Unocc. = 0 | | | | | | | |
| Period | Hours | S | M | T | W | T | F | S | S | M | T | W | T | F | S | S | M | T | W | T | F | S | Period |
| Night | 01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | Night |
| | 02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |
| | 03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |
| | 04 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |
| | 05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |
| | 06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |
| | 07 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | X | X | X | X | X | X | |
| | 08 | 0 | .8 | .8 | .8 | .8 | .8 | .5 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | |
| Day | 09 | 0 | 1. | 1. | 1. | 1. | 1. | .5 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | Day |
| | 10 | 0 | 1. | 1. | 1. | 1. | 1. | .5 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | |
| | 11 | 0 | 1. | 1. | 1. | 1. | 1. | .5 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | |
| | 12 | 0 | 1. | 1. | 1. | 1. | 1. | 0 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | |
| | 13 | 0 | .3 | .3 | .3 | .3 | .3 | 0 | .15 | 1. | 1. | 1. | 1. | 1. | .5 | | X | X | X | X | X | X | |
| | 14 | 0 | 1. | 1. | 1. | 1. | 1. | 0 | .15 | 1. | 1. | 1. | 1. | 1. | .15 | | X | X | X | X | X | | |
| | 15 | 0 | 1. | 1. | 1. | 1. | 1. | 0 | .15 | 1. | 1. | 1. | 1. | 1. | .15 | | X | X | X | X | X | | |
| | 16 | 0 | 1. | 1. | 1. | 1. | 1. | 0 | .15 | 1. | 1. | 1. | 1. | 1. | .15 | | X | X | X | X | X | | |
| Evening | 17 | 0 | 1. | 1. | 1. | 1. | 1. | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | X | X | X | X | X | | Evening |
| | 18 | 0 | .2 | .2 | .2 | .2 | .2 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | X | X | X | X | X | | |
| | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | | | | | | | |
| | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | | | | | | | |
| | 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | | | | | | | |
| | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | | | | | | | |
| | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .5 | .5 | .5 | .5 | .5 | .15 | | | | | | | | |
| | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .15 | .15 | .15 | .15 | .15 | .15 | .15 | | | | | | | | |

OCC. N .375 D .845 E .6 OCC N .505 D .965 E .50 N = .215
UNOCC N 0 D 0 E 0 UNOCC. N .15 D .15 E .341 D = .803
E = .178

Max. People Sensible 50,000 (A Schedule.)
Max. People Latent 40,000 (A Schedule.)
Max. Lights Sensible 340,000 (B Schedule.)

WITH 1 HOLIDAY IN MONTH
N = .215 x 30/31 = .208
D = .803 x 30/31 = .79
E = .178 x 30/31 = .172

## 3.6 Division of Building Heat Losses and Light Heat Gains

Figure 1 represents the floor plan of a building with area 5 being an office with a window and area 7 being an interior office. It is apparent that the lights in area 7 cannot be expected to aid in the heating of area 5; neither can the cold glass in area 5 help cool area 7. Consider also, figure 2 which represents a store with only the window and roof exposed to the weather. The lights in this instance can be expected to help offset heating load. However, if fin tube radiation is added to offset heat losses of the glass (Fig. 3) then the light can only be applied to the heat loss of the roof. It is necessary that some consideration be given to this problem and lines 30 through 34 have been provided for entering percentages of heat losses, and applicable light heat gains. Lines 32, 33 and 34 are used primarily for ceiling return systems.

### 3.7 Fan Heat

Heat of the supply fan and return fan are entered on lines 35 and 36. The brake horsepower times 2545 should be used unless the motor is in the air stream in which case KW input to the motor times 3400 should be used.

## 4.0 The System Programs

The programs that process the input data to produce ton-hours and BTU x $10^5$ are:

*HCE1RH - Reheat System
*HCE2HCO - Heat-Cool-Off System
*HCE3MZ - Multizone or Double Duct System

Basic procedure common to all three programs is illustrated in the logic Diagram, Figure 4. As indicated in the diagram, each of the 36 time periods that make up the weather year are printed after the influence of each weather incident is calculated with the input data. The programs determine solar heat gains, external heat losses and gains, and internal gains and losses, process this raw building load in the unique features of the selected system program, calculate part load hours and accumulate the totals of the various out put information.

### 4.1 Reheat Program

Figure 5. Logic Diagram, Reheat Program (*HCE1RH) illustrates the considerable complexity of analysis used to reflect the performance of this system in meeting the building loads. When the outdoor temperature is lower than room dry bulb, the program accounts for the effect of economiser cycle. A determination of the mix air temperature is made, based on input data concerning reset range of mix air if used, and, if the O.A. temperature is lower than the adjusted mix air temperature then the cooling effect of the O.A. is calculated. The quantity of reheat is calculated for conditions when the space needs additional heat and added to the heat required to preheat O.A. and to humidify. This program allows for 3 degree shift in room temperature before the reheat load is figured to account for thermostat throttling range. These calculations are done for each different weather condition.

If there is no economiser cycle, the program calculates required reheat for the mix air temperature resulting from the introduction of the minimum quantity of ventilation air. The refrigeration load required to cool the supply air down to the design supply temperature is adjusted to reflect the cooling effect of minimum O.A. If the space does not require the full available cooling effect, then the program calculates the necessary reheat and determines required heat for humidification. A separate loop is provided to account for a special control cycle that will use 100% O.A. when the outdoor wet bulb temperature is below return wet bulb.

When the outdoor temperature is above room dry bulb, the program calculates the reheat needed in the same manner as with the economiser cycle. However, the refrigeration requirement is determined as the sum of the minimum O.A. sensible cooling, return air heat gain, supply fan heat, space heat gains and the heat added as reheat plus the latent heat load from internal loads and outdoor air dehumidification. The same loop as above is available to reflect 100% O.A. when outdoor wet bulb is below return wet bulb. The program also accounts for the scheduled mode of operation during unoccupied hours.

### 4.2 Heat - Cool- Off System

Logic Diagram Heat-Cool-Off (*HCE2HCO), Figure 6, illustrates the calculation procedure for this system. When the outdoor air is below room dry bulb temperature, and the building internal gains exceed the heat losses, the program accounts for the cooling, heating and humidification required for minimum O.A., and then determines if the system is in occupied mode and if there is an economiser cycle. The heating needs for humidification and building refrigeration loads are calculated. When the outdoor temperature is below room temperature and the building internal gains are less than the heat losses, the program calculates the heat required for treating the minimum fresh air and supplying heat to offset the losses. When the dry bulb is not lower than room dry bulb, the computer determines the cooling required to offset sensible and latent gains, including minimum O.A., and then adjusts this figure to reflect a reduction in latent load proportional to the ratio of total load to the size of the cooling system. A loop of computer operations will take account of a special control cycle that provides 100% O.A. when outdoor wet bulb is below return wet bulb. The program also can determine the refrigeration load in unoccupied cycle if desired.

### 4.3 Multizone or Double Duct System

Figure 7. Logic Diagram, Multizone or Double Duct System shows the analysis employed to account for special considerations inherent with this system. Separate modes of calculation are used to reflect the system performance when the relationship of outdoor air temperature to room temperature changes as in *HCE1RH and *HCE2HCO. By-pass factor is the percentage of supply air that goes through the heating coil and this value is determined for each weather condition. The influence of the economiser cycle on cooling and heating energy use is calculated; heat required for humidification is determined for each new condition of outdoor air quantity, enthalpy and internal latent gain. The influence of 100% O.A. when outdoor wet bulb is lower than return wet bulb is calculated. Refrigeration required is calculated using the adjusted by-pass factors and including the latent heat load of outdoor air. (The factor .633 is a constant converting grains per pound of air to BTU per CFM). Use of the recalculated by-pass factor reflects the changing conditions of face and by-pass control and is valid when the by-pass is merely untreated mix air, as when no heat is added to a hot deck in the summer cycle, and when there is heat added to the by-passed air.

## 5. Running the Program

As a check on number of entries and to provide a permanent record of the input data used for the run, it is good practice to weave *HCEDATA program with the data entered. If the number of entries checks, the Form 1 data will be listed in full. After this check, the data program is then weaved with the appropriate system program, *HCE1RH, *HCE2HCO or *HCE3MZ. An outfile must be established to receive the output and, when the "Run" command is given, the program will ask the following series of questions.

"Enter Input File Name": Response is the name of the desired weather file.

"Enter Output File Name": Response is the name assigned for the output data.

"Do you want hours and part load": If the response is negative, the program by-passes this portion of calculation and some computer time is saved. A positive reply causes the program to calculate the percentage of boiler and chiller load required for each weather incident and to accumulate the number of hours of each part load increment.

"What is the tons of refrigeration machine": Respond with actual machine size selected.

"What is the MBH output of boiler": Respond with actual boiler size selected.

"What is the unoccupied winter room temperature setback": Response is the net difference from occupied room temperature (Form 1, Item 5). There is no allowance built in for "Spindown" or "Pick-up". It is assumed that adequate controls are provided to prevent establishing peak demand for heat pick-up by staging ventilation loads or similar control over load segments.

"Is there economiser cycle": Response indicates whether outdoor air will be used to remove excess heat gain during the heating cycle. In Heat-Cool-Off program the amount of outdoor air above the input minimum ventilation air is determined by room heat gains only. In both the Reheat and Multizone programs, the quantity of additional outdoor air is the amount required to maintain the input mix air temperature.

"What is the economiser mix temperature at 0° outdoors": Response is to indicate the upper limit of mix temperature reset if a variable control is used.

"What is the economiser mix temperature at 55° outdoors": Response must be 55° or higher. If variable mix temperature is used, the program will calculate the specific temperature for each weather condition, as a linear function.

"Is CFM all outdoor air on cooling cycle when outdoor wet bulb is below return air wet bulb": Response should indicate if this control feature is used.

"Is system off in unoccupied hours when outdoor dry bulb is above room dry bulb": Response should indicate if system is stopped in unoccupied mode during cooling season.

When the last question is answered, the computer will print out the ton hours and BTU x $10^5$ as shown in Table 1. These values are the net load requirements of the building for the system selected and operational program used.

## 6. *HCENERGY Program

The outfile created by the *HCE System program may be used in a supplementary program, *HCENERGY, along with additional input, to produce the total fuel and electrical energy input required for the apparatus selected for the project,Table 5. Output of this supplementary program is illustrated in Table 4 and is arranged to facilitate comparative analysis of several alternatives of equipment selection and fuel source.

Additional input information describing the characteristics of the apparatus to be used in serving the building loads must be entered on Medsi Form 3, Table 5. The reduced load performance characteristics, capacity and quantity of boilers, chillers, towers and pumps is related to the part load hour calculations made in the HCE System program to account for variable energy conversion efficiency. Additional data concerning domestic hot water loads other electrical loads that are not considered in heating-cooling calculations and some further operational schedule data as included in the 32 entries on Form 3. Methods of loading the data and running the programs are the same as described above.

### 6.1 Part Load Hours

*HCENERGY has another option that will print out the list of hours of part load of the refrigeration plant and the heating plant as illustrated in Table 6. The information printed shows the number of hours the plant will operate at, for instance, 50% and 66% of full capacity and may be used to select increments of plant size in multiple machine installations.

### 6.2 Dollars

An additional supplement in the Medsi library will read the output of *HCENERGY into a cost of energy program. This routine is developed to permit use of utility and fuel rate features peculiar to the project location. The local data must be written into the program by the user, or it can be programmed by Medsi.

## 7. Alternatives and Variations

Since one complete run as outlied above uses so little computers time, it is economical to compare other systems and combinations of systems to evaluate alternatives available. One such combination, illustrating the flexibility of these programs, might be a multizone system in the interior with fan coil units at the exterior. This is easily run by separating the data into two segments as though each system were serving a separate building and running the appropriate data with *HCE2HCO and *HCE3MZ.

TABLE 4 - Output of KW, KWH and BTU x $10^6$

```
SAMPLE: OFFICE BUILDING, MINNEAPOLIS            DEC 1,1969
     FIN TUBE RADIATION AT EXTERIOR, CONVENTIONAL RETURN
     5Y5TEM #1 WITH ECONOMI5ER
     MULTIZONE OR DOUBLE-DUCT 5Y5TEM

       LIGHTING      BOILER ACC      5UPPLY &    MI5CELANEOU5
                    & HTG PUMP5   EXHAU5T FAN5   ELECTRICAL
       KW     KWH    KW     KWH    KW     KWH    KW     KWH
JAN   100   34376    0      557    18    5280    10    3355
FEB   100   31363    0      503    18    4875    10    3083
MAR   100   34724    0      557    18    5398    10    3413
APR   100   33604    0      536    18    5224    10    3303
MAY   100   34376    0      538    18    5280    10    3355
JUN   100   33604    0      477    18    5224    10    3303
JUL   100   34376    0      458    18    5280    10    3355
AUG   100   34724    0      469    18    5398    10    3413
5EP   100   33267    0      511    18    5110    10    3247
OCT   100   34724    0      553    18    5398    10    3413
NOV   100   33267    0      539    18    5110    10    3247
DEC   100   34376    0      557    18    5280    10    3355

TOT        406783          6262          62863         39846

    AB5ORPTION   ELECTRIC                     TOTAL          TOTAL
      REFRIG      REFRIG      ELECTRIC     AB5ORPTION      ELECTRIC
    PUMP5,FAN5  PUNP5,FAN5     REFRIG        REFRIG         REFRIG
      & AUX       & AUX       MACHINE        SYSTEM         5Y5TEM
    KW    KWH   KW    KWH    KW     KWH    KW     KWH    KW      KWH
JAN  0      0    0      0     0       0   128   43570   128    43570
FEB  0      6    0      3     0      34   128   39833   128    39865
MAR  7     45    4     28    46     255   136   44139   180    44378
APR  7    634    4    403    70    3666   136   43308   203    46738
MAY  7   1619    4   1029    70    9525   136   45169   203    54105
JUN  7   2124    4   1350    70   13779   136   44733   203    57740
JUL  7   2250    4   1431    70   15657   136   45721   203    60559
AUG  7   2290    4   1456    70   15882   136   46296   203    61344
5EP  7   1696    4   1078    70   10460   136   43832   203    53675
OCT  7   1069    4    680    70    6199   136   45159   203    50969
NOV  7    119    4     75    46     657   136   42283   180    42897
DEC  0      1    0      0     0       6   128   43571   128    43577

TOT     11857        7540         76125        527613         599421

           FUEL INPUT BTU X 10:6          RE5I5TANCE KWH AT 100% EFF
        HTG   H.W. AB5ORP.      TOTAL          HEATING  TOT HTG & H.W.
                                         KW        KWH              KWH
JAN     575    8      0          583    262     110515           112136
FEB     492    7      1          501    262      92727            94192
MAR     485    8      9          503    216      88343            89965
APR     355    7    136          500    198      60207            61776
MAY     242    8    345          595    192      38027            39649
JUN     143    7    455          607    126      21644            23214
JUL     109    8    499          616    126      16170            17791
AUG     115    8    509          632    126      17111            18732
5EP     207    7    361          577    140      32204            33774
OCT     317    8    228          553    192      51895            53516
NOV     447    7     25          481    216      80567            82136
DEC     539    8      0          547    262     101585           103207

TOT    4031   96   2573         6701            711000           730093
```

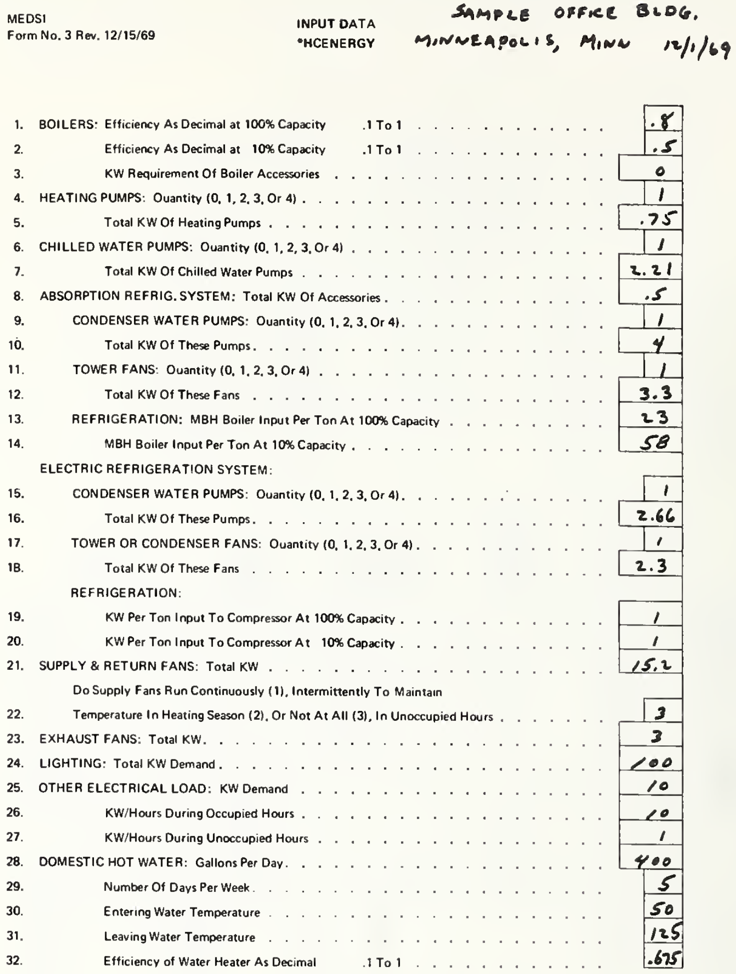
As the maximum cooling or heating calculated by the programs during any period is not limited to the cooling capacity of the total CFM, the reheat program may be used to evaluate an induction system by letting the total CFM be equal to the primary air CFM and using the proper temperature, outdoor CFM etc.

Medsi's library contains program variations for evaluating:

Variable Volume systems
Variable Volume with reheat
Internal Source Heat Pump

These programs have been used as a base for evaluating simultaneous energy requirement for Total Energy Plants and heat with light systems, and recently, one user is estimating the air pollution caused by fuels for various systems.

TABLE 5 - Additional Input Data to produce KW, KWH and BTU x $10^6$



MEDSI
Form No. 3 Rev. 12/15/69

INPUT DATA
*HCENERGY

SAMPLE OFFICE BLDG.
MINNEAPOLIS, MINN 12/1/69

| | | |
|---|---|---|
| 1. | BOILERS: Efficiency As Decimal at 100% Capacity .1 To 1 | .8 |
| 2. | Efficiency As Decimal at 10% Capacity .1 To 1 | .5 |
| 3. | KW Requirement Of Boiler Accessories | 0 |
| 4. | HEATING PUMPS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 5. | Total KW Of Heating Pumps | .75 |
| 6. | CHILLED WATER PUMPS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 7. | Total KW Of Chilled Water Pumps | 2.21 |
| 8. | ABSORPTION REFRIG. SYSTEM: Total KW Of Accessories | .5 |
| 9. | CONDENSER WATER PUMPS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 10. | Total KW Of These Pumps | 4 |
| 11. | TOWER FANS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 12. | Total KW Of These Fans | 3.3 |
| 13. | REFRIGERATION: MBH Boiler Input Per Ton At 100% Capacity | 23 |
| 14. | MBH Boiler Input Per Ton At 10% Capacity | 58 |
| | ELECTRIC REFRIGERATION SYSTEM: | |
| 15. | CONDENSER WATER PUMPS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 16. | Total KW Of These Pumps | 2.66 |
| 17. | TOWER OR CONDENSER FANS: Quantity (0, 1, 2, 3, Or 4) | 1 |
| 18. | Total KW Of These Fans | 2.3 |
| | REFRIGERATION: | |
| 19. | KW Per Ton Input To Compressor At 100% Capacity | 1 |
| 20. | KW Per Ton Input To Compressor At 10% Capacity | 1 |
| 21. | SUPPLY & RETURN FANS: Total KW | 15.2 |
| 22. | Do Supply Fans Run Continuously (1), Intermittently To Maintain Temperature In Heating Season (2), Or Not At All (3), In Unoccupied Hours | 3 |
| 23. | EXHAUST FANS: Total KW | 3 |
| 24. | LIGHTING: Total KW Demand | 100 |
| 25. | OTHER ELECTRICAL LOAD: KW Demand | 10 |
| 26. | KW/Hours During Occupied Hours | 10 |
| 27. | KW/Hours During Unoccupied Hours | 1 |
| 28. | DOMESTIC HOT WATER: Gallons Per Day | 400 |
| 29. | Number Of Days Per Week | 5 |
| 30. | Entering Water Temperature | 50 |
| 31. | Leaving Water Temperature | 125 |
| 32. | Efficiency of Water Heater As Decimal .1 To 1 | .675 |

## 8. Future

At the present time, utilities are doing most of the energy calculations as a part of utility sales promotion. Because of the competitive nature of fuel supplies, design engineers will be required to take more responsibility and to perform more of these calculations and evaluations. Medsi is available to assist engineers in this work and will be improving and adding to its library of shared time computer programs. The opportunity to evaluate other details of design with computer accuracy and speed require the progressive designer to develop skills in this area.

## 9. Summary

The primary objective in developing these programs has been achieved. Medsi programs give accurate answers with minimum manual calculation. The program and an application manual are available now to qualified subscribers.

TABLE 6 - Part Load Hours

```
SAMPLE: OFFICE BUILDING, MINNEAPOLIS           DEC 1,1969
        FIN TUBE RADIATION AT EXTERIOR, CONVENTIONAL RETURN
        SYSTEM #1 WITH ECONOMISER
        MULTIZONE OR DOUBLE-DUCT SYSTEM

                        HOURS OF PER CENT OF 70   TONS
             80     75     66     50     33     25     20     10             TOT
JAN      0      0      0      0      0      0      0      0      0           0
FEB      0      0      0      0      0      0      0      0      0           0
MAR      0      0      0      5      0      0      0      0      0           5
APR      1      5     22     51      0      0      0      0      0          81
MAY     16     22     53    114      0      0      0      0      0         207
JUN     92     34     61     83      0      0      0      0      0         272
JUL    136     31     43     77      0      0      0      0      0         288
AUG    134     33     42     82      0      0      0      0      0         293
SEP     46     26     52     92      0      0      0      0      0         217
OCT      8      0     47     80      0      0      0      0      0         137
NOV      0      0      0     14      0      0      0      0      0          15
DEC      0      0      0      0      0      0      0      0      0           0

TOT    436    155    324    603      0      0      0      0      0
             TOTAL HOURS OF REFRIG. REQ'D. 1520.23

                        HOURS OF PERCENT OF 893  MBH
             80     75     66     50     33     25     20     10             TOT
JAN    127     23     74    160    280     70      6      1      0         743
FEB     98     16     48    131    250    105     12      7      0         671
MAR     79     19     40    101    231    189     43     34      3         743
APR     30     22     28     60     92    151     96    176     56         715
MAY      3      8     27     16     48     59     73    283    196         717
JUN      0      0      9      2     31     35     45    182    330         637
JUL      0      0      1      0     16     41     36    118    396         610
AUG      0      0      2      0     18     43     37    125    397         626
SEP      0      5     25      9     45     36     59    275    223         681
OCT     15     18     37     28     85     88    101    267     96         738
NOV     66     20     36    106    172    197     57     56      5         719
DEC    108     15     73    130    266    124     14     10      0         743

TOT    530    150    404    748   1541   1144    584   1538   1706
             TOTAL HOURS OF HEATING REQ'D 8350.59
```

## 10. References

(1) Engineering Weather Data, Department of the Air Force Manual AFM 88-8, Chapter 6, Superintendent of Documents, U. S. Government Printing Office, Washington D.C., 20402

(2) U. S. Department of Commerce - Climates of the States, for the State involved, Superintendent of Documents, U. S. Government Printing Office, Washington D. C., 20402

(3) ASHRAE Handbook of Fundamentals, 1967, Chapter 28. American Society of Heating, Refrigeration and Air Conditioning Engineers, Inc., 345 East 47th Street, New York 10017

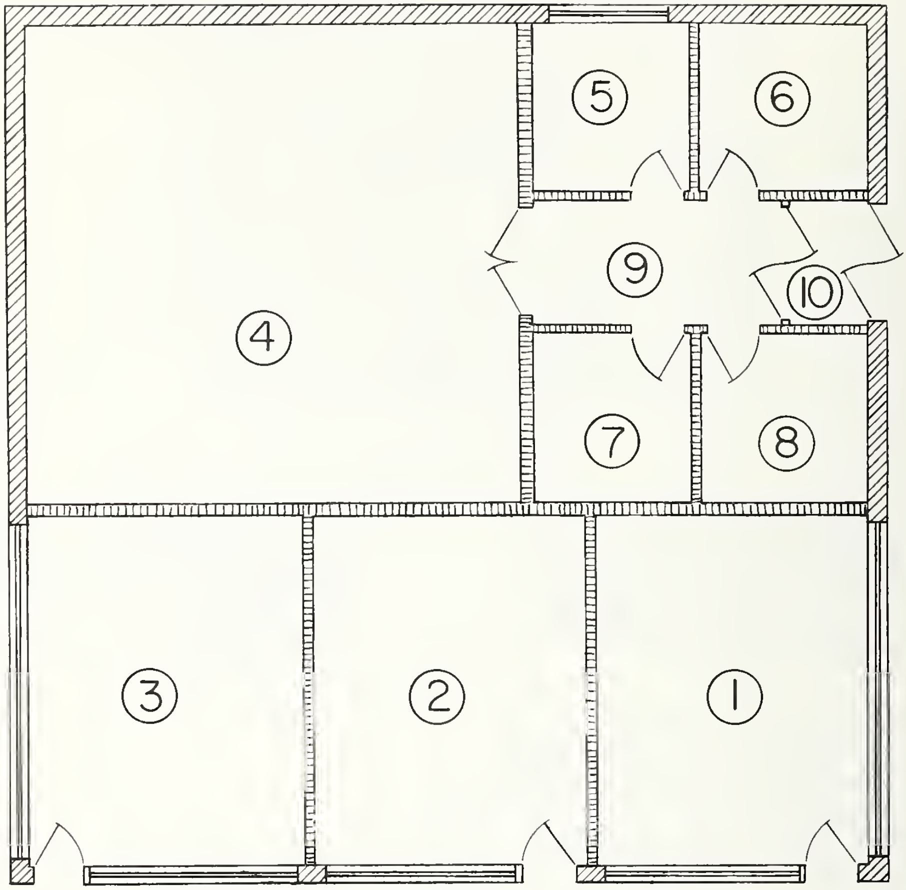


Figure 1. Floor plan showing application flight heat to heat loss.

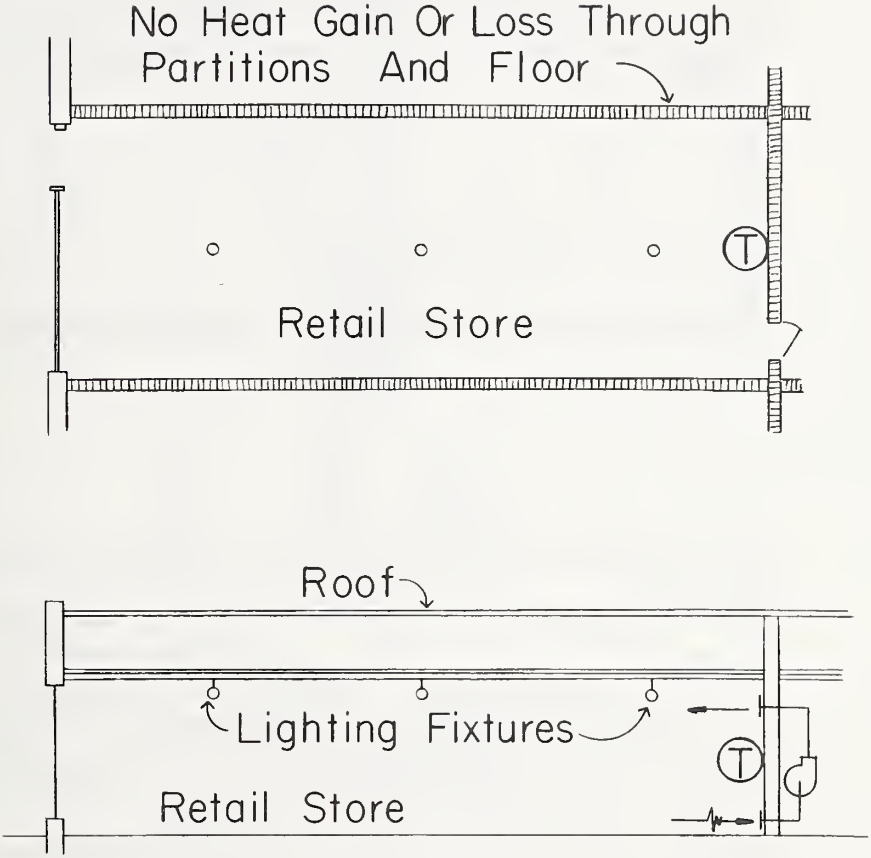


Figure 2. Retail store with lights applying to all of heat loss.

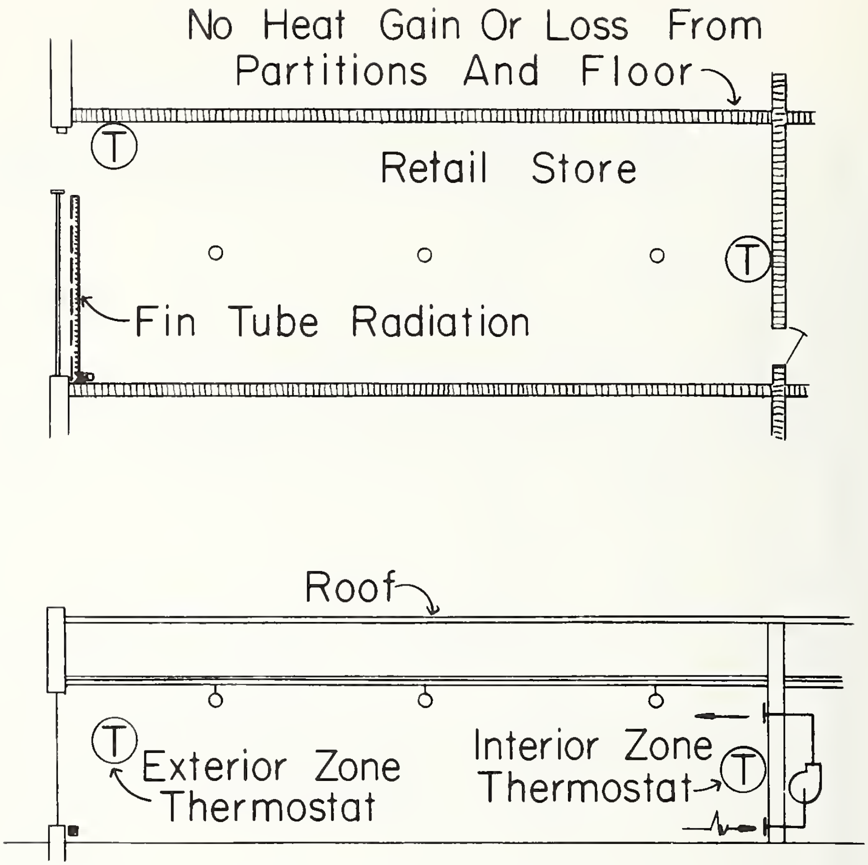


Figure 3. Retail store with lights applying to some of heat loss.

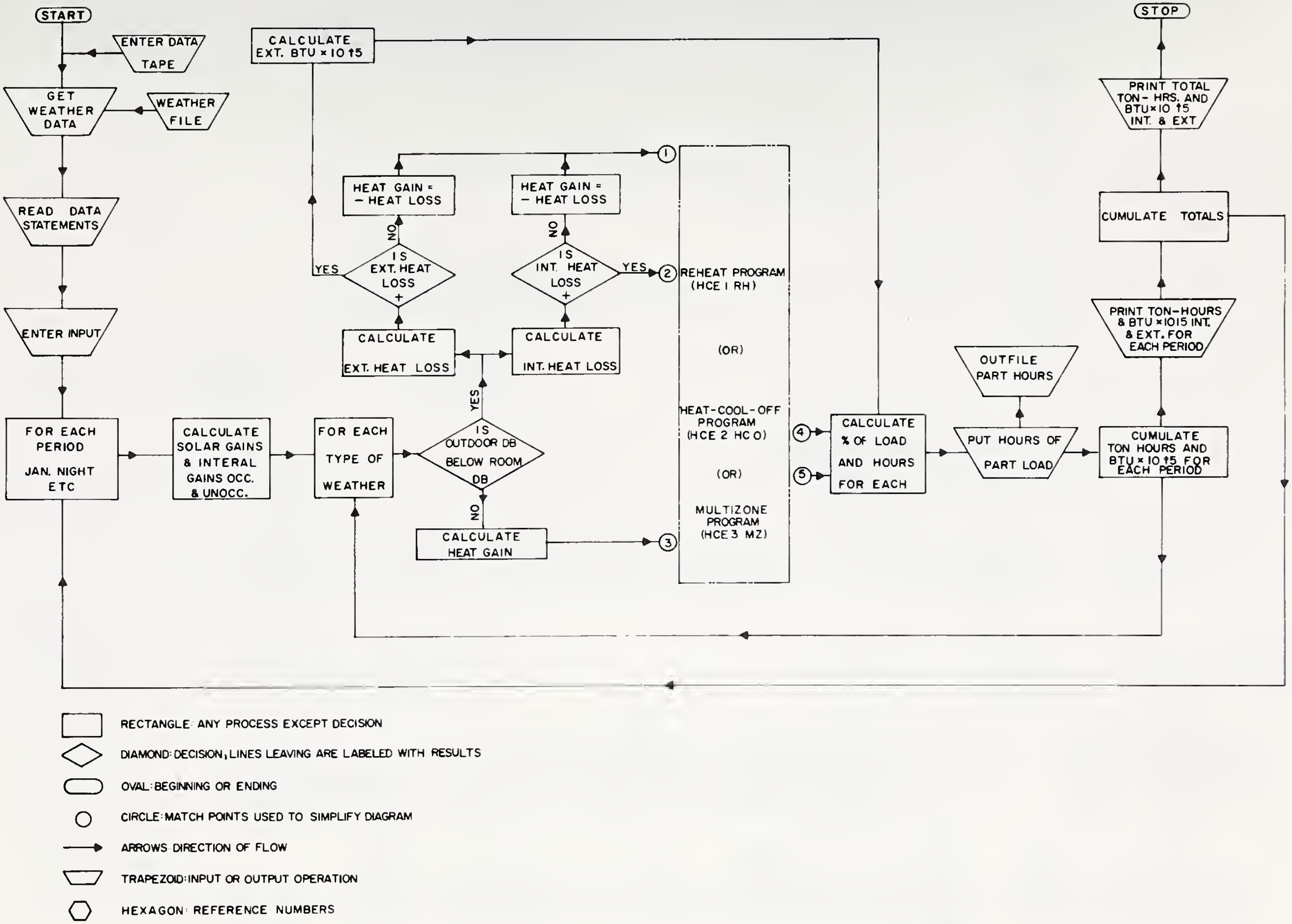


Figure 4. Basic logic diagram common to all programs.

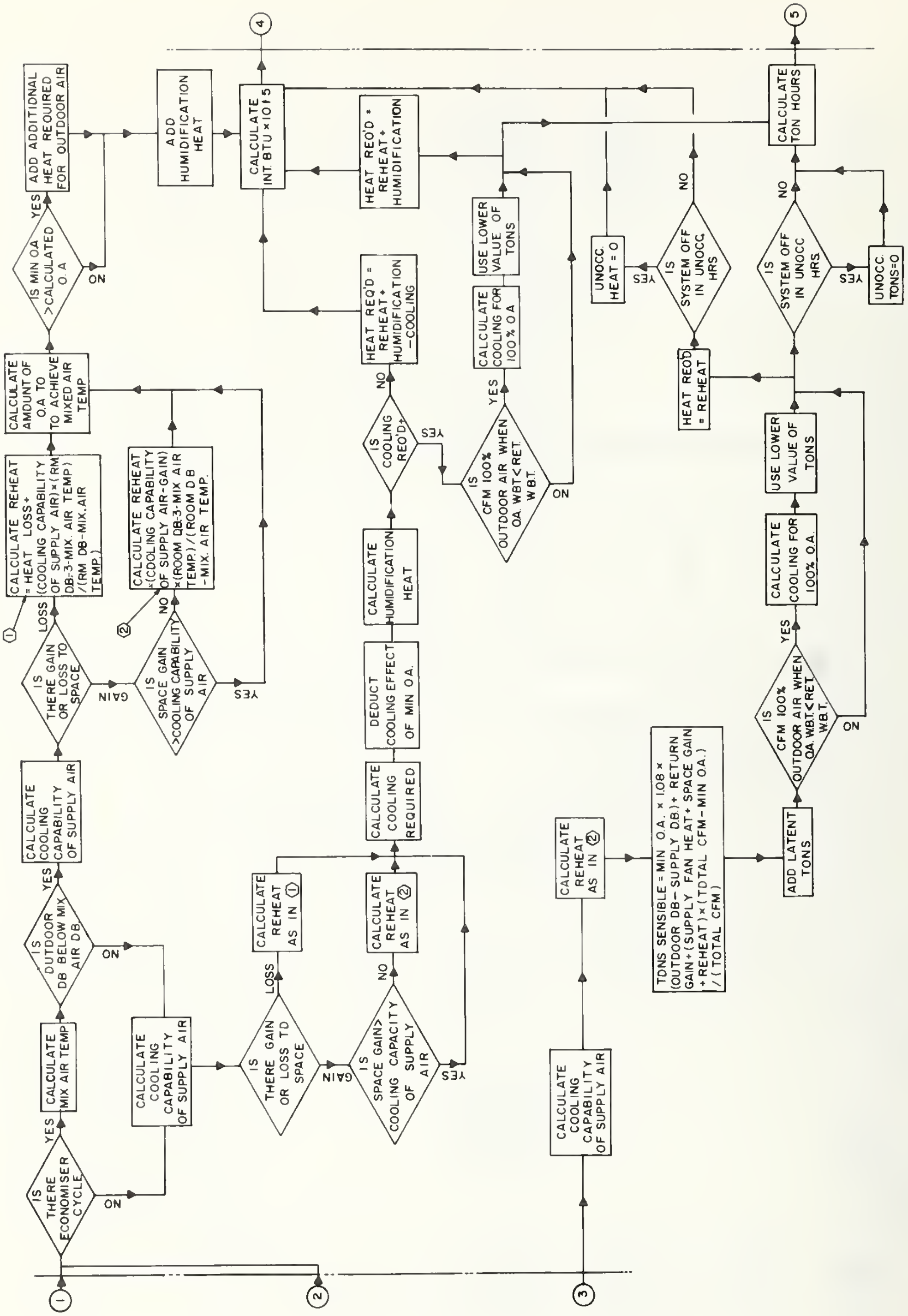


Figure 5. Logic diagram for reheat program.

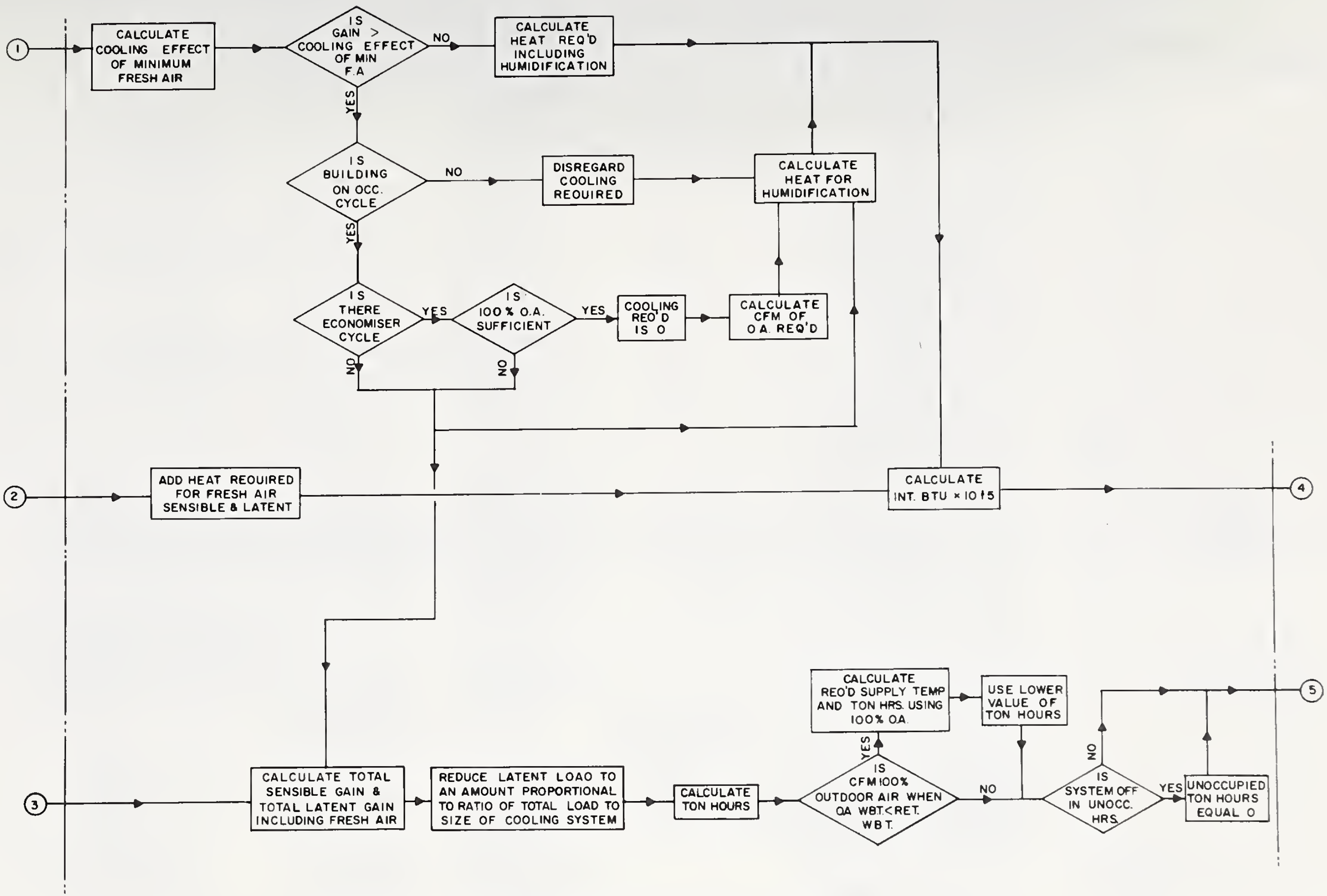
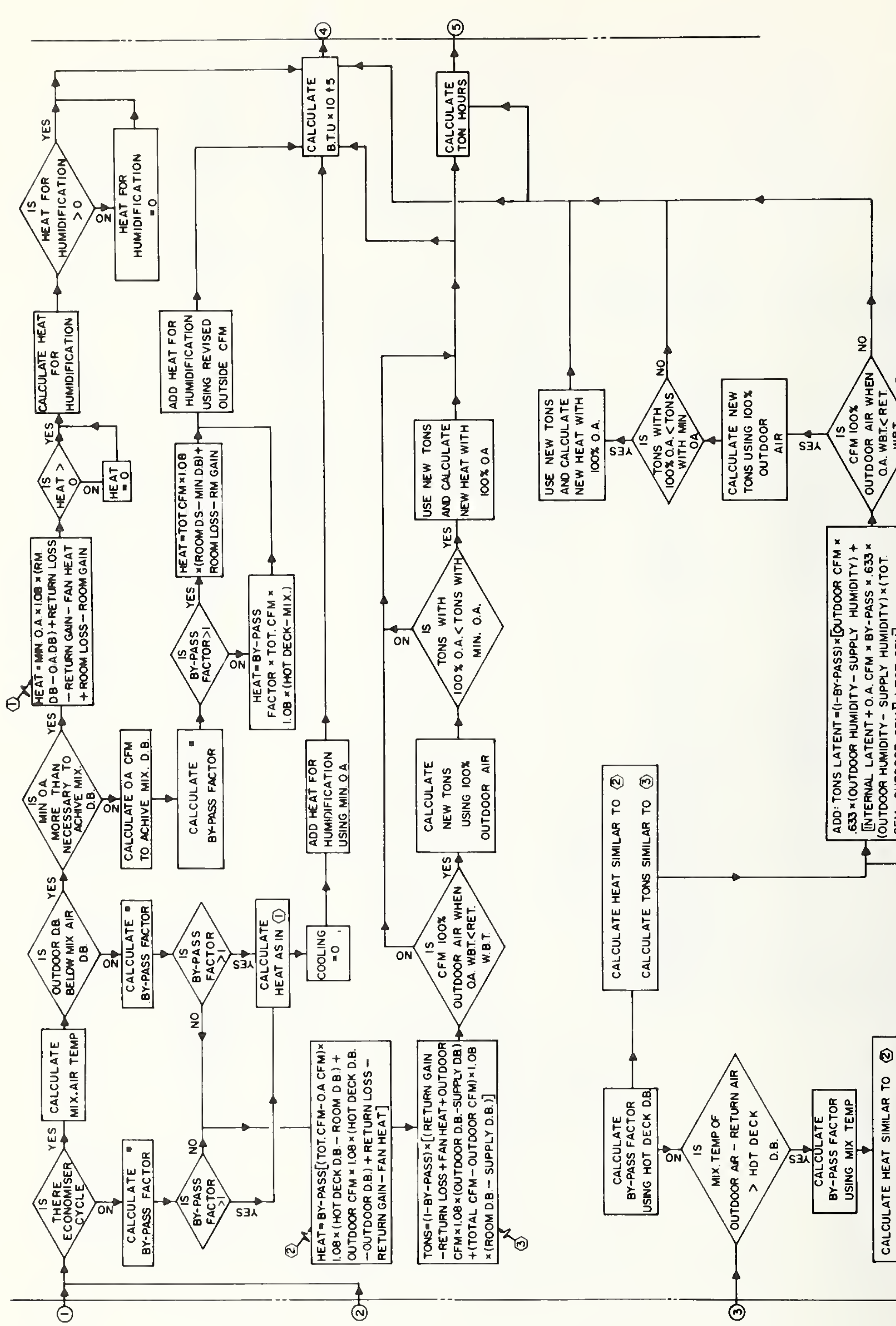


Figure 6. Logic diagram for heat-cool-off program.



* NOTE: BY-PASS FACTOR:
(1 − BY-PASS FACTOR) × TOTAL CFM × 1.08 × (ROOM D.B. − SUPPLY D.B.) = ROOM SENSIBLE GAINS + BY-PASS FACTOR × 1.08 × (HOT DECK TEMP − ROOM D.B.)

Figure 7. Logic diagram for multizone program.

# The Program of the ASHRAE Task Group
# on the
# Determination of Energy Requirements
# for
# Heating and Cooling Buildings

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Consulting Engineer

A description is given of the work being carried out by the ASHRAE Task Group on Energy Requirements for Heating and Cooling of Buildings. The Task Group's work is being done by four subcommittees. Subcommittee #1 is responsible for developing methodology and calculation procedures for hour by hour determination of heating and cooling loads. Subcommittee #2 has the task of developing a new calculation technique which will apply the heating and cooling loads to the equipment components and determine the energy requirements. Subcommittee #3 has the job of combining load calculation and system energy determination with weather data, building operating schedule and other factors affecting system performance to develop overall annual energy requirements of the building. Subcommittee #4 is responsible for instrumenting buildings to measure energy requirements and refine the calculation procedures.

A summary of the progress made to date is presented.

Key Words: Task Group on Energy Requirements, building energy requirements, calculating energy requirements.

## 1. Introduction

At the present time, for most engineers, the calculation of the energy required for heating and cooling a building is more of an art than a science. The ASHRAE Guide and Data Book in Chapter 54 of the 1968 Applications [1][2] volume describes calculation procedures which are not claimed to be exact but which, based on experience and the application of good judgement, can give reasonably accurate estimates for <u>residential</u> buildings. Not even this limited claim is made for any calculation procedure for commercial and industrial installations. A single paragraph on page 656 covers the subject and plainly states:

"To properly evaluate the energy requirements for commercial and institutional buildings, it is necessary to establish the character of all thermal load sources, the resultant magnitude of each of these specific heat release mechanisms, and their relationship to the most effective method of load removal. A thorough analysis of both the total energy balance and the character of the system operating cycle must be made in order to accurately establish the energy requirements for each specific building". This statement may be a good general description of the problem, but it provides little help to the engineer faced with determining the energy requirement for <u>his</u> specific building.

Yet the accurate determination of the energy required for heating and cooling a building is one of the most important, and also one of the most difficult problems for the air conditioning engineer. It is important because the energy cost is an essential and significant element of the building's overall owning and operating cost. Accurate or not, it may be the determining factor in the selection of the air conditioning system or the energy source for a new structure. The problem is difficult because of it's complexity. It involves not only an accurate determination of the heating and cooling loads, taking into account the varying influences of the weather and the building operating schedule, but the even more complex problem of determining the performance of the heating and cooling system under varying conditions of partial load. The complexities of the problem have led to solutions based on approximation,experience, judgement, rules of thumb, judge factors, or just plain guess.

In recognition of the need for ASHRAE to develop better engineering information on this subject, a

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[1] Chairman of ASHRAE Task Group on Energy Requirements for Heating and Cooling Buildings.
[2] Figures in brackets indicate the literature references at the end of this paper.

Presidential Committee on Energy Consumption was established in 1965. This committee reviewed the problem in considerable detail and then, typical of Presidential Committees, recommended the appointment of another committee, a special Task Group, under the Research and Technical Committee, to "develop accurate methods for determining annual or seasonal energy requirements for heating and cooling, taking into account all energy sources and all sizes and types of buildings".

The original Task Group carried the analysis of the problem further until the resignation of the chairman at the end of 1966. The Task Group as presently organized met first in March 1967. It has been meeting on a schedule of about two days every two months since them.

At our first meeting it was recognized that other engineering groups were already working on new load and energy calculation methods based on computer techniques. It was decided to take advantage of this work, in so far as possible, in the development of the ASHRAE calculation procedures. Subsequently, several organizations, particularly the National Bureau of Standards, the Post Office Department and the National Research Council of Canada, have contributed to the Task Group's program.

The work of the Task Group is carried on by four sub-committees. A description of their assignments will outline the plan of operation being followed.

Subcommittee #1 on Heating and Cooling Load Requirements is responsible for developing the methodology and calculation procedures for determining Heating and Cooling Loads for energy calculations.

Subcommittee #2 on System and Equipment Energy has the task of developing a new calculation technique which will apply these heating and cooling loads to the equipment components of an air conditioning system and determine the corresponding energy requirements.

Subcommittee #3 on the Overall Logic Pattern has the job of combining the load calculation and the corresponding system energy determination with the weather data, the building operation schedule, the requirements of auxiliaries and any other factors affecting the system performance, to develop the overall annual or seasonal energy requirements of the bulding.

Subcommittee #4 on Field Validation Studies is responsible for plans to instrument one or more buildings which will be used to refine and validate the energy calculation procedure.

The whole Task Group periodically reviews the work of each subcommittee to provide coordination and direction for the total program.

## 2. Calculation Procedures

The first step in such an energy calculation program is obviously the development of an accurate heating and cooling load calculation procedure. Taking advantage of computer capabilities it is now possible to work with more sophisticated calculation procedures and gain in basic accuracy as well as speed. It was decided at the beginning of the program to take fullest advantage of the newest calculation concepts, which require the use of computers, in order to make the ASHRAE procedure the most advanced possible.

For accurate energy calculations a continuous, or hour-by-hour calculation of the building heat transfer, instead of the conventional, single point, design load calculation, is necessary. The heating and cooling energy requirement responds to the everchanging dynamic heat loss and gain of the building as it is influenced by the continually varying outdoor air conditions, the position of the sun, the cloud pattern and wind effect, and by the operation schedule and the heat generating characteristics of the building. The air conditioning system responds to this changing load as it appears inside the building and is picked up by the heating and cooling distribution system.

The current methodology of design heat load calculation presented in the ASHRAE Book of Fundamentals (1967) [2] has to be modified for the hour-by-hour load calculations, especially with reference to the transient thermal response of the building structure to the outdoor weather conditions and changes in internal space temperatures. The total-equivalent-temperature-difference concept employed in the Book of Fundamentals is only applicable as long as the hour-by-hour pattern of weather conditions repeats prescribed design cycles. The actual weather pattern, however, is puite random, or non-steady periodic, so that the design equivalent-temperature-difference concept is not valid for a real weather situation.

A new methodology, called the Thermal Response Factor Technique is better suited to the calculation of non-steady periodic heat transfer. The application of this technique to cooling load calculations was proposed by Stephenson and Mitalas [3]. The ASHRAE Task Group adopted this new methodology for calculating the transient heat transfer of exterior walls and roofs, and to some extent, the heat storage effect of the internal mass of the building. The subcommittee on Heating and Cooling Loads has completed development of a new calculation procedure based on this technique which will be used as the basis for our energy requirement calculation.

The details of this load calculation procedure were first released for limited distribution to engineers working in this field in June 1968 at the ASHRAE annual meeting at Lake Placid [4]. Comments, criticism and suggestions were requested and these were reviewed in a Forum at the January 1969 ASHRAE meeting in Chicago. On the basis of these comments, and some further work by the subcommittee, some revisions have been made and a revised version has been prepared for general release. This revision includes an interesting and valuable short cut procedure which will considerably reduce the computation time involved. Instead of calculating each category of load for all 8760 hours of the year, it has been found practical to develop combined response factors for the whole building, or for a zone, based on approximately 100 hours of the year. Then the combined response factors can be used for a very rapid calculation for the total 8760 hours. This technique has been tested at the Bureau of Standards.

The output of this load calculation will be the hour-by-hour heating and cooling loads to which the air conditioning system responds. We know of no present calculation procedure that attempts to accurately translate this load, through the system performance characteristics, into the system energy requirements. At this stage all calculations resort to some approximation procedure to relate the heating and cooling load to the partial-load performance characteristics of the system. Such an approximation has been essential because of the tremendous complexity of any more rigorous determination.

Here again the availability of computer capability and new calculation techniques presents a challenge and an opportunity to develop a new methodology. Rather than attempt to refine present approximation procedures, the Task Group decided to take a more fundamental approach. The subcommittee on System and Equipment Energy is developing a new system-simulation technique which will make it possible to calculate the response of the complete system to the hour-by-hour load changes. It is believed that this new technique will open the way to improved methods of evaluation of heating and air conditioning systems and their controls. This work is in the early development stage. Sample simulation techniques have been developed for some typical systems. Other systems and system variations and a generalized simulation procedure are now being developed.

A bulletin covering the preliminary work of this subcommittee was released for limited distribution in June 1969 [5] at the ASHRAE meeting in Denver. Comments on this procedure were reviewed at a Forum in January 1970. Further work on this technique is now underway looking toward the release of a revised and more complete bulletin.

## 3. Overall Logic Subcommittee

The work of the subcommittee on the Overall Logic Pattern will begin in earnest when the system simulation procedure is well in hand. We can, however, already visualize some of the problems involved in bringing this whole program together into a unified calculation procedure. One of the major problems is related to weather data. The heating and cooling load calculation for this procedure requires the coincident hourly readings of dry-bulb temperature, relative humidity or dewpoint or wet-bulb temperature, wind velocity and direction and direct and diffuse solar radiation. Further, the performance of many system components is related to these same outdoor weather parameters. Existing ASHRAE weather data, which covers only Design Load weather conditions, are clearly unsuitable for the needs of this procedure. The Task Group is working with the ASHRAE Technical Committee on Weather Data to obtain the necessary data in the form needed for this program.

Another weather problem is the determination of a typical year for use in calculating predicted energy requirements. Out of ten or twenty years of weather data, how does one determine what a typical year would be for calculating energy requirements and operating costs? ASHRAE has sponsored two research projects aimed at eventually providing Typical Year Weather Data for a number of locations. One of these projects follows what might be called "conventional, meteorological approaches". Loren Crow has made a study of the weather at one location and by the "scientific and careful use of meteorological factors" has developed a typical year's weather data made up of 12 typical months, each selected to bring it within acceptable tolerances to the long-term average condition and the total climatic range for that particular month.

In a parallel project with the same ultimate objective, Z. O. Cumali is developing a mathematical analysis of the weather data at the same location to determine if it is possible to develop mathematical relationships whereby a typical year's weather data, including all the variables needed in the energy calculation can be generated within the computer as a part of the load calculation procedure. This unconventional approach shows distinct possibilities and indicates long range benefits in other areas of weather information.

Subcommittee #3 is made up of engineers who are regularly working with and operating computer programs for determining energy requirements. It is in effect our user's committee and provides a very valuable and practical viewpoint and criticism of the theoretical work of the Task Group. Since this subcommittee represents at least seven of the most advanced computer programs used in the United States today, it provides a unique opportunity to check the calculations of these programs against each other. To do this a project called "Operation Cross Check" is being carried on in which all of the programs calculate the same building using the same input information. The results are then analyzed in considerable detail to evaluate program differences. The understanding gained from this project will provide an invaluable input in helping to standardize the ASHRAE calculation procedure.

We can see ahead that this energy calculation procedure is going to require more information on the partial-load performance of some system components than is ordinarily available now. Moreover, we can see that this information will be needed in forms suitable for use by the computer. It is commonly recognized that all heating and air conditioning systems only operate at their design of full-load condition for a very few hours each season. Most of the operating hours are at varying partial-load conditions, significantly different from full-load. To determine the system response to these partial-load conditions we must have data on the partial-load performance of the system components. Moreover, for optimum use in the computer program, this information is needed in equation form suitable for use by the computer. A preliminary step in setting up this kind of performance information was taken by subcommittee #2 in the bulletin released in June 1969 [5]. This bulletin included examples of the equation forms proposed for expressing performance information of the major components of the system.

The Task Group is now requesting manufacturers to provide such information. At the ASHRAE meeting in San Francisco, the Task Group requested the Research and Technical Committee and the Board of Directors of the Society to approve a statement setting forth this future requirement for component partial-load performance information. Responding to that request the attached statement was approved. The need for this information has also been recognized by the members of APEC and that organization is also urging manufacturers to provide this kind of performance information.

We see this as the beginning of a long range and long term program to revise the performance information issued by manufacturers on the system components they manufacture. We believe that the ready availability of such information is essential for the future, more accurate, calculation of building energy requirements.

## 4. Field Validation

Subcommittee #4 on Field Validation Studies has the responsibility for setting up and conducting one or more field tests on actual buildings to validate and, or, refine the overall calculation procedure. This program is considered too complex and too vital to the interests of the Society and to the engineering field to be released as an ASHRAE approved procedure without a well supervised field check.

The field study plan proposes to set up instrumentation in one or more buildings which will measure the local weather and other inputs required for the calculation. Each building will then be set up on a computer by the local research contractor, using the calculation procedures developed by the Task Group. Then each month the measured input data will be processed and the calculated energy requirement will be checked against the actual, measured energy use of the building. It will be the responsibility of the local research organization to analyze the monthly data and, working with the Task Group refine and validate the calculation as indicated.

As a first step in this program, four test sites were selected and preliminary studies were made to develop the costs of instrumentation and carrying out the proposed two year test program. At the ASHRAE meeting in San Francisco, in January 1970, the Research and Technical Committee recommended, and the Board of Directors approved, going ahead immediately with the field test program at one site, the one at Ohio State University in Columbus, Ohio.

The work of the Task Group has already indicated areas where present ASHRAE engineering information is inadequate. Through the Director of Research, the Task Group has requested that research studies be undertaken in the following areas:

1. An up-to-date determination of the energy distribution from lights, including the effects of thermal storage.
2. A study of the effects of moisture absorption within the conditioned space on the cooling loads; i.e., the effect of latent heat storage on the latent cooling load.
3. A study of the transient heat transfer response of walls, ceilings and floors, including non-homogenous sections.
4. Guidance from weather experts in establishing typical year weather data.
5. A study to relate reported cloud cover to solar radiation.

## 5. Summary

The work of this Task Group is considered by many responsible members of the Society to be one of the most important and far-reaching undertakings of the Society. Based on it's work we now can make heat-loss and heat-gain calculations which adequately and accurately reflect the actual transient heat flow due to the varying outdoor weather and also the varying indoor space temperature and load conditions. Through the thermal response and weighting factor technique we can take into account the heat storage effects of the structure and determine the hourly, actual heating or cooling load imposed on the heating and air conditioning system. By means of a general simulation technique we then expect to be able to calculate the system performance based on the performance characteristics of the system components. This will not only allow us to more accurately predict the operation of a given system and determine it's operating cost, it will provide a calculation basis for design optimization studies of both the structure and the heating and air conditioning system. Other forward looking engineers see the work of the Task Group as laying the foundation for more accurate and comprehensive computerized control of the heating and air conditioning systems of large buildings, with resultant significant savings in operating cost.

As for the Task Group, we have our work cut out for us for sometime ahead. The revised bulletin on load calculations has been released within the last few months [3]. Within a year we expect to release a revised bulletin covering the work on system-simulation and component and system energy calculations [5]. Our work will then continue on the field test programs and in efforts to improve, simplify and refine the procedure. We expect this program to generate other research activities, within and outside Society, as it has already done, to develop the engineering information needed for the program.

Hopefully, as a result of all this effort on the part of a dedicated group of Society members, we can look forward to the day when air conditioning engineers can precalculate the performance of their system designs and determine the energy requirements with confidence and accuracy, based on standard ASHRAE calculation procedures.

## Appendix A

A prediction of the energy required to operate the heating and air conditioning system of a building is essential to a complete and realistic evaluation of a heating and air conditioning system design. Such a prediction requires not only an accurate calculation of the heating and cooling loads but also a determination of the response of the H & AC system to those loads as they vary with the weather and with the changing conditions of building operation.

The ASHRAE Task Group on Energy Requirements for Heating and Cooling Buildings is developing calculation procedures to accurately determine and predict such system energy requirements. The Task Group program is being carried out under the supervision of the Research and Technical Committee in response to a specific authorization of the Board of Directors of the Society.

An essential element of this calculation procedure is a determination of the energy requirements of the various system components in response to the partial load conditions under which they operate. The performance information presently available on many system components is inadequate for such a determination. In addition to the performance information normally provided for equipment selection at design load conditions, performance data are required covering the partial load conditions under which the components normally operate. To facilitate the use of this information in computer calculations, it is desireable for these data to be expressed in equation rather than tabular form.

The Task Group on Energy Requirements is requesting that equipment manufacturers move as rapidly as possible to provide such information in this form. It further requests all ASHRAE Technical Committees to work with the manufacturers and the Task Group in developing this information.

## 6. References

1) ASHRAE Guide and Data Book (1968 Applications)

2) ASHRAE Handbook of Fundamentals (1967)

3) Stephenson, D. G. and Mitalas, G. P., "Cooling Load Calculations by Thermal Response Factor Method". ASHRAE Semi-Annual Meeting, Detroit, Michigan, January 1967, Paper No. 2018.

4) Proposed Procedure for Determining Heating and Cooling Loads for Energy Calculations, Edited by M. Lokmanhekim. ASHRAE Task Group on Energy Requirements for Heating and Cooling, 1968.

5) Proposed Procedure for System Simulation, Edited by W. Stoecker. ASHRAE Task Group on Energy Requirements for Heating and Cooling, 1969.

# Successful Applications of Energy Analysis Programs

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The lessons learned in bringing several environmental computer programs to a successful operating state can be used to make this process easier for others. Several suggestions such as attempting to make operations as foolproof as possible, allowing for development time, documenting data and creating a set of accessory packages should be beneficial to others when they first implement their new programs. The mistakes of oversimplifying and over complicating several different environmental programs have proven that a good engineering assessment of the time context and data available are essential in the successful application of these programs. In the successful use of several in-house developed programs, G.A.T.E. programs, and A.P.E.C. programs, each has always required considerable initial effort. The use of approximation techniques is essential as experience is gained. The use of the "Sol-Air" method used in the A.P.E.C.-H.C.C. program with the G.A.T.E. energy analysis is one example of such approximating techniques. These experiences, which have been gained over a period of years, should prove valuable to anyone involved in making an environmental program operationally successful.

Key Words: A.P.E.C. programs, energy analysis programs, environmental computer programs, G.A.T.E. programs, Sol-Air, approximating techniques.

The key to most successful operations is experience. We all want to be immediately successful in all things, but unfortunately we are not. This is especially true with new computer programs in the field of environmental control. The first few times one works with any new computer program might be compared to the first few times a laborer attempts to work a new kind of hand tool. It takes time to learn how to be effective. The amount of time depends upon many variables including the knowledge available from others who have experienced a similar process.

In the last few years, we have had the opportunity to analyze and work with many computer programs associated with environmental control. As a result of this experience, the following suggestions can be made:

1. Make all operations as simple and foolproof as possible. Most people are short tempered when trying to understand someone else's work. When something doesn't work easily, people naturally assume it is someone else's fault. Programs will not be successful if you assume that everyone else has your intelligence and patience. No one really cares that the reason your program is considered useless is because people were stupid in the way they applied it. You will be the one blamed simply because the program was unsuccessful. It is up to you to insure against this. Use charts and graphs plus plenty of redundant information whenever possible. Keep input data and the calculations for input data to a minimum. You'll be surprised that often when you aim the operating procedures so that they can be performed by idiots, people will claim that you are a genius. If you aim your operating procedures toward geniuses, many people may claim that you are an idiot. We will see later how charts and graphs can be used to easily collect what might otherwise amount to rather extensive data.

2. Allow for plenty of time. Experience indicates that if a time limit is imposed that might be restrictive that that time limit will almost always be exceeded. In fact, it seems the more desperately something is needed, the more certain you can be that a good printout cannot be obtained on time. There is a good scientific explanation for this phenomenon. The successful application of a computer program requires many successive operations. Each of these operations like correctly filling in the data sheets or having a computer in running condition when you are ready, will not cause any problems nine times out of ten, but when taken in successive steps Baye's Rule of Probability takes effect. We know from Baye's Rule

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[1] Energy Systems Sales Supervisor

that if only seven steps were required, each being 90% certain of being done correctly, that the overall probability would favor an incorrect computer run. This is because the total probability is the product of each step's probability. In your enthusiasm to successfully apply a new program, always allow for plenty of time. Nothing else but time and experience can be used to reduce the number of operations and the chances of error to a minimum. It is far better to do without than to try a program in a critical situation which has to be resolved in a limited time. People can become very bitter with a program which fails them in their first involvement. Early users must understand that some experience is necessary before definite time commitments can be made. Even then it is wise to allow for plenty of insurance time when making your time commitments.

3. Be certain of your data. So many different numbers are used in an environmental program that it is extremely easy to forget which units are being used and how these numbers and units relate to the calculations in a program. One of our first steps when working with new programs is to add units and explanatory remarks right into the printout. Often the originating programmer is so familiar with the program that he forgets that these numbers can be very difficult for others to understand. Another device which adds certainty to your data is intermediate printout of calculations. In other words, before summing key calculations, provide an intermediate printout. This is a great help in the successful application of a program because usually the user does not have the programmer's ability to dump the whole program in order to look for why a calculation went wrong. A user also is more confident in a final answer if all the component parts of that answer appear in the printout and seem to be of the right magnitude. If they are not of the right magnitude, it makes it easier for him to trace down which input data applies so that it may be reanalyzed. These precautions may seem like a lot of drudgery compared to developing the logic which is the heart of the program, but the greatest program in the world isn't worth anything to others if it can't be successfully applied to solve their problems. Make the input and output data easy to understand.

4. Provide a good program package. A real key to any program's successful application is in having a good package for it to operate in. All sorts of provisions which can help a user successfully apply a program are often neglected when a program is given to a user. A good documentation in both common english and the language of the program can save the user much anguish. The user should have good records of the control cards needed and other data which relate directly to the use of the program on his particular computer. As soon as is practical, at least two copies of the program should be made for insurance purposes. Plenty of clearly labeled folders and covers for input data and printouts also need to be provided. Without these labeled devices, soon all the input data and printout data from various projects can get mixed. This can lead to some frustrating situations, especially in multiple runs of the same project. A little effort to insure that the program will be operated within a well managed and equipped package is well justified.

One of the ways we learn is by mistakes. Some of our mistakes were costly and time consuming. It is hoped that a discussion of them will help others in avoiding these pitfalls. The first environmental energy analysis program we worked with was probably one of the world's worst applications of the computer. This program was basically a set of imperical formulas and rules of thumb that had come from various sources. Some of these formulas were obtained from generalized experience, some from prejudice or pure myth. Instead of capitalizing on the computers capabilities to handle large amounts of data and many calculations, we simply had a program which made the same mistakes we made in hand calculations, only faster. This program was very unsatisfactory. Rules of thumb have their place when related to human judgment but some very strange answers can develop when they are unmercifully applied by a machine. This program was discarded soon after it was developed.

Our next program was more scientific and in fact was a series of programs. The first in this series was a program which determined the energy requirements of a facility hour by hour for one year. Once this data had been obtained, it was given to a second program which contained equipment characteristics and calculated the amount of gas and electricity various systems would require in order to heat and cool a building. The third program was simply a rate cost program which solved for the cost of gas and electricity used. This annual cost data could then be compared to the initial cost for an economic analysis of which system offered the best investment to a potential owner.

We made many mistakes with this program also, almost all of which were related to the input data for the first program in the series, the energy requirements program. We found that we had limited data which could be given to the program. As a result, the program did not accurately reflect the real requirements of many facilities. Basically we tried to obtain the input for this program from the heat load calculations done manually by the consulting engineer. For the entire facility, we broke the calculated heat load into time dependent loads and temperature dependent loads. The computer could then take each of these loads plus a time schedule and temperature schedule and produce an approximation of hour by hour energy requirements.

The time dependent loads were basically the solar component, the people component and light component of the heat load. Each of these loads described in a profile format of how they varied hour by hour for different day types was provided as input data. (See fig. 1 and fig. 2)

The temperature dependent loads were transmission, outside air (both sensible and latent) plus infiltration. Each of these loads were given at at least two different temperatures in order to provide a method of linear interpretation for loads at other temperatures. (See fig. 3)

As you might guess, this type data was far better than a rule of thumb approach but still was rather gross. In most facilities, a significant load is found from the effect of zoning. Many times both heating and cooling are going on simultaneously creating an artificial internal heat load. Some distribution systems deliberately mix heating and cooling energy under certain conditions. We call this artificially produced load balance or trim heat. Transmission through walls of any real thickness is not strictly linear. Latent outside air loads are wet bulb dependent, which was not being accounted for. The solar load is affected by clouds and adjacent buildings which had also not been accounted for.

We made our biggest mistake at this point. We decided to have a program written which took into account every possible factor which we thought would effect a building's energy requirements. This awesome task was undertaken by Southwest Research Institute for 26 member gas utilities and the result, after many years work, was known as the G.A.T.E.[2] long form program. This program was fantastic. It took into affect temperature of street water, leakage around air handler coils, every control setting in the building, wind direction and speed, and every other conceivable piece of data which might affect the building's energy requirements.

Although this program was technically excellent, operationally it was a great mistake. We found that the data needed by the program was not usually available until the building was well under construction. By that time, the answers it provided were of no practical use as most of the decisions which these answers would effect had already been made. If we assumed and guessed at input data at an earlier date in construction, we found that so many variables had such wide error margins that the resulting data in many cases was meaningless.

Upon discovering this dilemma, we gave a lot of thought to how we might ever obtain a successful Energy Analysis Program. There seemed to be a paradox. By the time accurate data was available, the decisions which might be affected by that data had already been made.

We now realize that our mistake was in the degree of accuracy which we were trying to obtain. The computer can be so very accurate, it is hard not to try to obtain all the accuracy that is available. But practical engineering tells us that ground temperature and air-handler leakage usually have little effect compared to major loads like outside air temperature and solar radiation. We can usually determine the following major loads to within plus or minus ten percent at a relatively early stage of construction:

1. Transmission heat load
2. Outside air sensible heat load
3. Outside air latent heat load
4. People sensible and latent heat load
5. Lighting heat load
6. Solar heat load
7. Distribution system balancing heat load

These loads are sufficient for us to obtain data upon which practical decisions can be made. A program starting from this data coupled with the data available from a year's weather data from the U.S. Weather Bureau provides a pretty good picture of the facilities energy requirements. The Weather Bureau data use has hour by hour records of dry bulb temperature, wet bulb temperature, cloud cover and other factors which might affect these basic loads. This data like the other data may only be within ten percent or so of what any future weather year will be like.

When we had settled upon a ten percent or so allowable error, we found that the program became very much more workable. Users could provide answers that were in this range from rough calculations and experience. We also limited ourselves in most cases to the outside shell of the building which greatly simplified matters. The effect of zoning and other internal imbalances was then loaded as a

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[2] Group to Advance Total Energy, Inc.

function of the type distribution system plus whatever the designer's experience indicated. This saved many very complicated calculations that otherwise would have been required for this relatively small load. We also carried loads forward as stored heat when equipment capacity had been exceeded. A simple calculation allowed us to also determine the drift from inside design temperature when stored heat was in effect. Once we had settled upon a lesser degree of accuracy in our input data, we found that we actually had not given up much at all. When compared on the same building, we found one program gave answers within five percent of the other. In addition, when it came to using this data to select systems, every system compared in exactly the same relationship in either program.

This program was initially known as the G.A.T.E. APPROX program. For the last year, the old long form program has been dropped and the newer program is now the official G.A.T.E. program. We have run this program well over 100 times in the last year and found that it can provide very meaningful data concerning the use of various energy systems in all types of buildings.

We took this program and worked hard at making it as operationally successful as was possible. We simplified the input data and printout into an easily understood format. We allowed for plenty of time when making studies. We also built up a good package of accessories to insure good management of the program. For example, rather than collect data directly on key punch sheets, we used the forms in the figures at the end of this paper for ease of understanding. Later we transferred this graphical data to input sheets as numbers and reproduced it in that format in the printout.

We still had an occasional timing problem. Often decisions are made concerning a facility even before the basic heat load calculations have been done. The consulting engineer usually will not start his calculations to determine the component heat loads needed by the G.A.T.E. program until plans are fairly firm. Yet decisions whether to use a central plant or to use gas or electricity as an energy source might be made before that time. Sometimes a consulting engineer has not even been retained when these basic decisions are being made. We found that we were having to do many heat load calculations ourselves. This caused us to become interested in an organization known as A.P.E.C.[3] We joined A.P.E.C. and began using their H.C.C. program which provides heat load calculations from the basic plans of a building. Again we had many start up problems, but each time we found it a little easier to bring a program to an operational state.

We now are in a position to obtain the data necessary for decisions on building energy systems at a very early stage. We can even do a fair job of approximating energy requirements from as little data as a plot plan and an artist rendering. These give enough physical data in conjunction with other data we have gained from past experiences to begin a preliminary study. As new data becomes available at later dates, the study can be updated as required.

A surprising bonus developed out of our work with the A.P.E.C.-H.C.C. program. We found that this program could not only describe the hour by hour load from solar radiation quite accurately, it could also calculate the non-linear portion of transmission and handle the effect of hour averaging. These calculations are quite sophisticated, but can be done by the computer. The results provide far greater accuracy than is usually provided by manual calculations. This fallout has provided us with a solution to a very perplexing problem. We knew that a better heat load calculation method would be possible if we used the "Sol-Air Method" as described in the 1967 A.S.H.R.A.E.[4] Guide. The use of the "Sol-Air Method" allows for the consideration of the dynamic nature of heat transfer in a building. This method is used in the A.P.E.C.-H.C.C. program for a 24-hour day but it is very difficult to use for a 8760 hour year-long energy analysis. We were neglecting this effect in our energy analysis and using only linear relationships for our temperature dependent heat loads. Through a modification to the A.P.E.C.-H.C.C. program, we are able to extract the non-linear portions calculated when using the Sol-Air Method for a typical day for each month of the year and then we created a time profile of these non-linear components of transmission loads in an hour by hour printout. What we actually did was add a step to the A.P.E.C.-H.C.C. program which calculated what the transmission would be on a straight linear basis and then subtracted this quantity from the more sophisticated quantity calculated using the "Sol-Air Method" and hour averaging. This represented the hour by hour differences for a typical day in one month. The A.P.E.C.-H.C.C. program was also looped so that we obtained an hourly table of typical days differences for each month of the year. In other words, we are approximating a very complex temperature dependent load into the G.A.T.E. program as a more simply handled time dependent load. This is only an approximation of what is really happening but it has proven to be a satisfactory way to handle this complex load. In the energy analysis program, we adjust the hour by hour data calculated without using the Sol-Air Method by the total correction factor derived for that hour by the modified A.P.E.C.-H.C.C. program for a typical day of each month. In practice, since the G.A.T.E. program requires an hour by hour solar table for typical days of each month, we simply have the A.P.E.C.-H.C.C. program add the solar and non-linear component of the transmission load together before printing our a yearly table of these values. (See fig. 2)

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[3] Automated Procedures for Engineering Consultants, Inc.

[4] American Society for Heating, Refrigeration and Air Conditioning Engineers

With continuing experience, our capabilities continue to grow. We have learned how to make haste slowly and not to expect too much too soon with new programs. We have also learned that in order to be successful, we need to truly have a good understanding and feeling for the data that we are dealing with and have a system to manage and contain our work. The overall result is the successful application of these programs, but we had to learn many lessons first. It is hoped that some of these lessons will be helpful to you in your applications of environmental control and energy analysis programs.

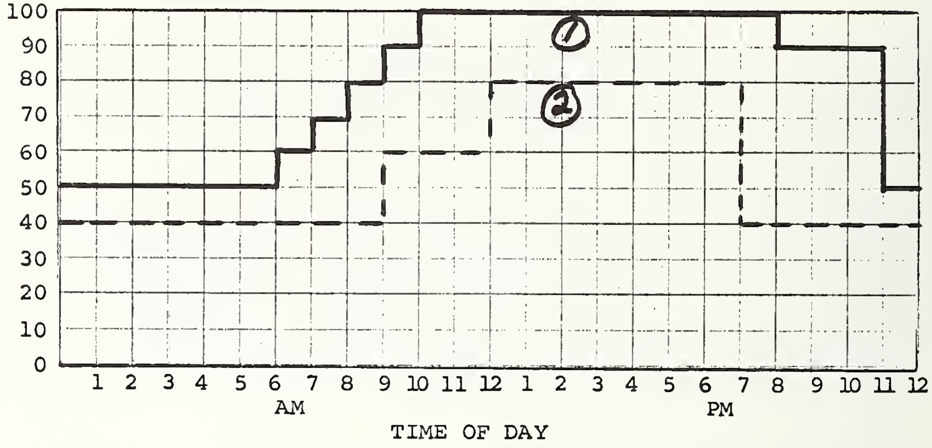
Thermal Loads:

(Each day type must be described. Number functions in each graph by day type number.) 1= WEEKENDS, 2= WEEKDAYS

Heat from electrical appliances (primarily lights):

@ 100% = 2850 MBtu @ 5 WATTS/FT² = 835.8 KW

@ 3.412 MBTU/KW = 2850



Heat from people:

@ 100% = 200 MBtu

250 PATIENTS @ 350 = 87,500
100 NURSES @ 750 = 75,000
75 VISITORS @ 500 = 37,500
200 MBTU

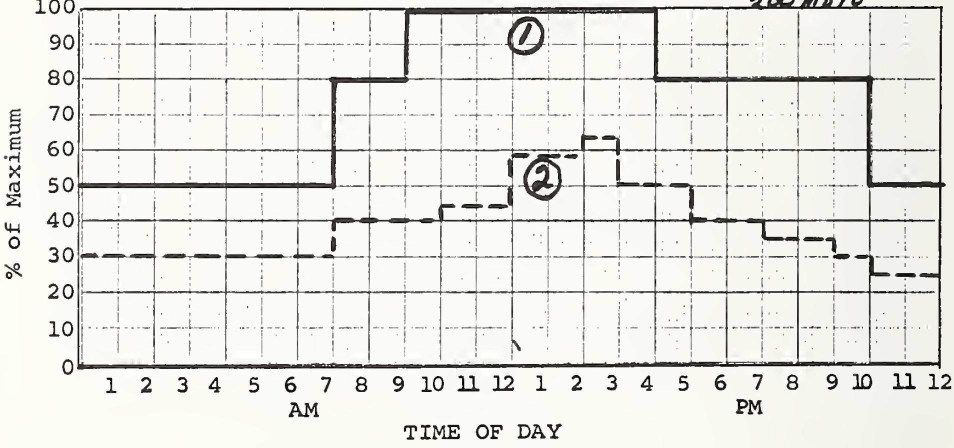
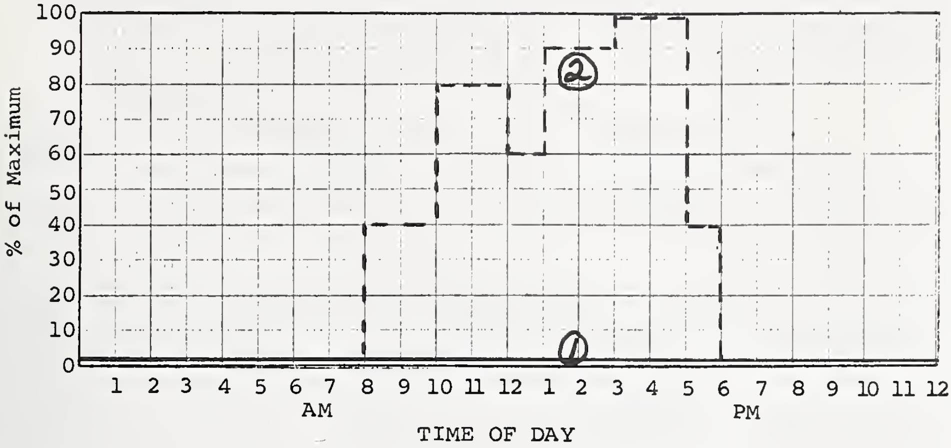


Figure 1. The form used for collecting data regarding time dependent heat loads from people and from lights with example data filled in.

Heat from Other internal sources: (LABORATORY BURNERS + FURNACE)

@ 100% = 1 MBtu



Heat from solar radiation:

@ 100% = 756 MBtu

(The percentage solar radiation by hour at 34$^{o}$ latitude for a facility of uniform sides has already been included in the program. If actual conditions will be greatly different from this, please note below.)

| Time | Dec | Jan-Nov | Feb-Oct | Mar-Sep | Apr-Aug | May-Jul | June |
|---|---|---|---|---|---|---|---|
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.435 | 0.662 | 0.733 |
| 7 | 0.000 | 0.086 | 0.495 | 0.689 | 0.867 | 0.959 | 0.980 |
| 8 | 0.581 | 0.698 | 0.857 | 0.966 | 1.000 | 1.012 | 0.990 |
| 9 | 0.857 | 0.906 | 0.988 | 1.001 | 0.959 | 0.905 | 0.887 |
| 10 | 0.970 | 0.980 | 0.989 | 0.918 | 0.824 | 0.710 | 0.680 |
| 11 | 0.999 | 0.989 | 0.963 | 0.815 | 0.653 | 0.507 | 0.457 |
| 12 | 0.984 | 0.979 | 0.929 | 0.778 | 0.573 | 0.368 | 0.353 |
| 1 | 0.999 | 0.989 | 0.963 | 0.815 | 0.653 | 0.507 | 0.457 |
| 2 | 0.970 | 0.980 | 0.989 | 0.918 | 0.824 | 0.710 | 0.680 |
| 3 | 0.857 | 0.906 | 0.988 | 1.001 | 0.959 | 0.905 | 0.887 |
| 4 | 0.581 | 0.698 | 0.857 | 0.966 | 1.000 | 1.012 | 0.990 |
| 5 | 0.000 | 0.086 | 0.495 | 0.689 | 0.763 | 0.959 | 0.980 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.435 | 0.662 | 0.733 |

Figure 2. The form used for collecting data regarding time dependent heat loads from sources other than people and lights plus data regarding the solar heat load with example data filled in.

Temperature Dependent Variables:

All temperature dependent variables are assumed to be linear. At least two points are needed to define a straight line and therefore to describe any single temperature dependent variable. Up to five different points may be used to describe all the variables being considered.

| | @ Design Heating | * | @ Inside Design | * | @ Design Cooling |
|---|---|---|---|---|---|
| Dry Bulb Temp. ($^{o}$F) | 34° | | 75° | | 94° |
| Dew Point Temp. ($^{o}$F) | ( )** | ( )** | 55° | | 64° |
| Transmission (MBtu) | -1630 | | 0 | | 756 |
| Outside Air Sensible (MBtu) | -1859.8 | | 0 | | 861.84 |
| Outside Air Latent (MBtu) | 0 | | 0 | | 724.21 |
| Balance or Trim Heat (MBtu) | 0 | | 504 | | 0 |
| Other Heat Loads (MBtu) | 0 | | 0 | | 0 |

* If an economy cycle is used, each temperature point where the outside makeup air percentage is changed must be described.

** Leave blank and make outside air latent equal to zero in these columns if humidity control equipment will not be used when heating.

Air Handling System Size = 168,000 CFM @ 400 C.F.M./TON

Outside Air Makeup Rate = 25 % = 42,000 C.F.M.

Installed Heating Capacity = 4200 MBtu @ 25 BTU/FT$^2$

Installed Cooling Capacity = 420 Tons

Heating System Shutoff Temperature = N/A $^{o}$F

Cooling System Shutoff Temperature = N/A $^{o}$F

Figure 3. The form used to collect data regarding temperature dependent heat loads and data regarding the equipment system used with example data filled in.

# Comparison of a Short Form Load and Energy Program with the Detailed Westinghouse Load and Energy Programs

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Comparisons have been made between two heating and air conditioning load and energy programs. The purpose of this comparison was to determine the proper applications and usefulness of these programs. Using the same input data necessary for each program, typical buildings were analyzed by both programs. The results of these programs were then compared to determine the accuracy of the short form program. Each building was divided into the required number of zones. The requirement being that each zone had the same basic physical and operating characteristics. The short form program is basically a computerized *A.S.H.R.A.E. Guide* [1][2] method of load calculation. The energy calculations are handled with a modified heating degree day method and an equivalent full load hour method for air conditioning. These modified methods allow for the evaluation of internal heat gains from lights, people and equipment. The load section of the Westinghouse Electric Corporation's program uses the thermal response factor method of calculating the loads on the structure. These loads are determined on an hourly basis. The energy program can use any defined weather year and will calculate the energy requirements for all the building functions for each hour of the year. The results of these hourly calculations are then summarized to give the annual energy requirements of the structure. The final results indicate that the short form program can be a useful tool for the engineer in properly evaluating building environmental systems. The results also indicated the advantages of using the more detailed program for larger structures.

Key Words: Electric heat, load profiles, control set points, ASHRAE method, degree day approach, equivalent full load hours.

## 1. Introduction

Only in recent years since electric heat has made an impact as a practical method of heating a building has it been necessary to properly estimate the annual energy requirements of a building. Many times the need for an accurate estimate must be made long before final plans of the building are completed. In fact, in order to properly utilize the various advantages of the available fuels, the decision as to which fuel is to be used must be made while the architect is doing his early stage planning.

Very accurate and elaborate computerized methods of predicting the energy consumptions for a proposed building have been developed in the past five or six years. One of these methods is the Westinghouse Electric Company's energy program. The program is divided into three sections. Section one is a load model into which you feed the building characteristics, hourly environment specifications, and actual hourly weather records. From this input data the program prints out the hourly zone load profiles. Part two is the mechanical supply system design. From data obtained from part one the

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[2] Figures in brackets indicate the literature references at the end of this paper.

engineer sizes and selects the equipment he intends using in part three. Part three is the systems operation simulation. It simulates the performance of the system selected in part two. The simulation takes into account such items as equipment performance curves at partial loads, control set points, lighting system interface with the space conditioning system, etc. Various systems may be inserted in part two and their operation simulated in part three. The final print out of all three sections is, hourly electrical requirements, hourly fuel requirements, maximum percent heating plant load, maximum monthly fuel use, and monthly maximum electric demand.

For large buildings the accuracy of this type of program is necessary. However, for small commercial buildings this type of program is not normally used.

In this study the results of a smaller, less sophisticated program were compared to the Westinghouse program. This program is a computerized manual calculation which took approximately 540 man-hours to develop. The program was designed to analyze the small commercial buildings which were being done manually. The program uses the ASHRAE method of load calculation and combines this with a modified degree day approach to arrive at the estimated heating energy consumptions. The air conditioning energy consumption is based on equivalent full load hours for the equipment.

The final print out of the Duquesne Light Company program gives the design heating capacity for each zone, the estimated air conditioning capacity at two hour intervals from 8:00 a.m. to 8:00 p.m., the monthly electrical requirements and associated costs based on Duquesne Light Company's rates for all the electrical usages in the building.

## 2. Description of Input

For the same building, the Westinghouse program has about 40 sheets of input data with approximately 1,000 pieces of input information. It takes about 12 man-hours to fill out the input sheets. On the other hand, the Duquesne Light Company program only has 3 pages of input sheets per zone (maximum of nine zones) with approximately 500 pieces of input information. It requires up to 8 man-hours to fill out all of the sheets.

## 3. Description of Cases Studied

In order for a valid comparison to be made, three buildings were used in the comparison. The same input data was supplied to both programs. The first building studied was a 10 story office building with about 60,000 square feet. The second building was also an office building. This building had a floor area of approximately 300,000 square feet. The third building was a large two, and in some places, three story structure, with over 700,000 square feet of floor area.

No energy consumption data was available for building number one. The second building was operated on an 8 to 10 hour per day schedule. The third building was occupied 24 hours per day, except for a small portion of it which was used as offices.

The buildings are larger than what the short form program was designed to handle. However, these three buildings were the only ones available for comparison.

Since the Westinghouse program has the capacities of performing hourly calculations, building operation simulation was straight forward. However, judgment had to be used when using the Duquesne Light Company program, since it simulates the operation as either day or night operation, and average conditions must be assumed.

The accuracy of the Westinghouse program has been established in several comparisons apart from this paper. In one case where actual weather data and building characteristics were put into the computer after the building had been in operation for a year, the computer was within 0.3% (three-tenths of one percent) of the actual consumption. In other studies similar results were obtained. Therefore, for the purposes of this paper, the short form program was compared to the Westinghouse program.

## 4. Results

Tables 1, 2, 3 and 4 show the results of the short form program as compared to the Westinghouse program.

Table 1. Comparison of short form capacities to the Westinghouse heating and cooling capacities for building number 1.

| | Heating | | | Cooling | | |
|---|---|---|---|---|---|---|
| | Westinghouse | Duquesne Light Company | % Difference | Westinghouse | Duquesne Light Company | % Difference |
| 1. | 44,425 | 42,763 | -3.9 | 112,191 | 120,390 | 7.3 |
| 2. | 102,807 | 98,240 | -4.4 | 184,552 | 157,726 | -14.5 |
| 3. | 44,425 | 42,763 | -3.7 | 100,204 | 100,429 | 0.2 |
| 4. | 137,573 | 131,340 | -4.5 | 204,214 | 175,473 | -14.1 |
| 5. | 1,394,776 | 1,311,305 | -6.0 | 2,366,416 | 2,245,000 | - 5.1 |
| 6. | 205,137 | 199,605 | -2.7 | 309,110 | 296,203 | - 4.2 |
| 7. | 1,354,725* | 1,264,409* | -6.7 | 1,077,100 | 802,800 | -25.4 |
| 8. | - | - | - | - | - | - |
| 9. | - | - | - | - | - | - |
| Total | 3,283,868 | 3,090,425 | -5.9% | 3,927,000 | 3,812,000 | -2.9% |

* Ventilation load of building

Table 2. Comparison of short form capacities to the Westinghouse heating and cooling capacities for building number 2.

| | Heating | | | Cooling | | |
|---|---|---|---|---|---|---|
| | Westinghouse BTUH | Duquesne Light Company BTUH | % Deviation | Westinghouse BTUH | Duquesne Light Company BTUH | % Deviation |
| 1. | 1,079,488 | 1,068,852 | -1 % | 1,069,377 | 1,328,687 | +24 % |
| 2. | 498,806 | 336,823 | -32 % | 562,590 | 894,505 | +59 % |
| 3. | 1,194,213 | 1,178,269 | +1.3% | 1,228,649 | 1,890,470 | +55 % |
| 4. | 402,907 | 369,908 | +8.2% | 491,790 | 534,139 | + 8.5% |
| 5. | 1,413,635 | 1,119,166 | -21 % | 1,923,085 | 2,123,450 | +10.5% |
| 6. | 65,801 | 693,781 | +11 % | 686,005 | 477,152 | -30 % |
| 7. | 528,107 | 567,818 | + 7.5% | 573,527 | 870,900 | +52 % |
| 8. | 206,400 | 256,383 | +24 % | 264,581 | 289,649 | + 9.4% |
| 9. | 3,020,613 | 3,203,347 | + 6 % | 4,045,156 | 4,781,861 | +18.3% |
| Total | 9,701,524 | 8,794,347 | -9.4% | 11,257,705 | 11,820,000 | + 5 % |

Table 3. For building number 3.

| | Heating | | | Cooling | | |
|---|---|---|---|---|---|---|
| | Westinghouse B.T.U. | Duquesne Light Company B.T.U. | % Deviation | Westinghouse Tons | Duquesne Light Company Tons | % Deviation |
| 1. | 2,116,178 | 2,267,999 | + 7.2% | 365 | 376 | + 3 % |
| 2. | 6,164,014 | 6,559,231 | + 6.4% | 823 | 845 | + 2.7% |
| 3. | 2,507,617 | 2,724,041 | + 8.6% | 172 | 187 | + 8.8% |
| 4. | 3,246,680 | 3,484,565 | + 7.3% | 237 | 246 | + 3.8% |
| 5. | 2,925,500 | 3,074,979 | + 5.1% | 332 | 344 | + 3.6% |
| 6. | 3,547,782 | 3,731,313 | + 5.1% | 336 | 360 | + 7.1% |
| 7. | 795,753 | 940,095 | +18 % | 36 | 42 | +16.5% |
| Total | 21,303,500 | 22,782,176 | + 7 % | 2,301 | 2,362 | + 2.6% |

Table 4. Comparisons of annual consumptions.

| | HEATING | | COOLING | | TOTAL | | |
|---|---|---|---|---|---|---|---|
| | Westinghouse | Duquesne Light Co. | Westinghouse | Duquesne Light Co. | Westinghouse | Duquesne Light Co. | % Difference |
| Bldg. 2 | 9,598,000 | 1,912,000 | 2,507,000 | 11,593,000 | 41,725,000 | 47,675,000 | +14 % |
| Bldg. 3 | 6,920,000 | 3,155,000 | 2,386,000 | 3,185,000 | 22,449,000 | 19,064,000 | -12.5% |

For large complex buildings with complicated environmental systems it is felt that the accurate results obtained from the Westinghouse program make it far superior.

The comparative tests show that for capacity comparisons the short form program is fairly accurate for total building capacity. The biggest difference of the three test cases was only -9.4% and the smallest +2.6%. This comparison did point out, however, that on certain zones, due to their orientation, the short form program can be as much as 59% high on cooling. This value, one for 55% and one that is 51% high are shown on table 2. These errors resulted from all of these zones peaking early in the morning. The short form program assumed maximum temperature differential at this time. Due to the ability of the Westinghouse program to look at each hour of the year, it was able to determine more accurately the number of people in the zone and the ventilation for that hour. The Duquesne Light Company short form program has only the ability to decide between day occupancy and its ventilation rate or night occupancy and its ventilation rate.

The results of the annual energy consumption were much different. Although the total consumption was 12.5% low for building number two and 14.0% high for building number three, the Kwh usages for the various components were way out of line. The heating consumption was estimated as much as 90% low. The cooling consumption was as much as 300% high (see table 4).

These large errors are due to the equations used for estimating the energy consumption which are:

Annual Heating Consumption

$$\text{Day Kwh} = \frac{(\text{Heat loss day - Internal load day})}{\begin{array}{l}\text{Temperature differential at which}\\ \text{heat loss was calculated}\end{array}} \times \begin{array}{c}\text{Degree day at change}\\ \text{over temperature}\end{array} \times \begin{array}{c}\text{\% day}\\ \text{operation}\end{array} \times 24 \text{ hours per day}$$

$$\text{Night Kwh} = \frac{(\text{Heat loss night - Internal load night})}{\begin{array}{l}\text{Temperature differential at which}\\ \text{heat loss was calculated}\end{array}} \times \begin{array}{c}\text{Degree days at night}\\ \text{temperature}\end{array} \times \begin{array}{c}\text{\% night}\\ \text{operation}\end{array} \times \begin{array}{l}\text{24 hours}\\ \text{per day}\end{array}$$

Annual Cooling Consumption

Cooling Kwh = Tons cooling x Kwh/ton x Effective full load hours**

$$\text{**Effective full load hours} = \frac{24 \times D \times C}{tm - td}$$

D = Cooling degree days on base temperature equal to change over temperature

$tm = (\text{outdoor design temperature} - \frac{\text{Daily Range}}{2})$

td = change over temperature

c = % time space is air conditioned

The above equations do not adequately handle the large internal zones of major buildings with their high internal heat gains, and the short form method appears to work best in smaller exterior zones having more standard heating requirements.

The hourly analysis is far more accurate in estimating annual consumption for large complicated buildings than these empirical formulas. These formulas were developed from test data obtained from samplings of offices and commercial buildings located in large metropolitan areas. For small offices and commercial buildings these approximations are reasonable. For the larger building with large interior zones, known also as core areas, these formulas do not yield accurate estimates.

## 5. Conclusions

This study shows that for estimates of heating and cooling, the short form, when used with judgment, can be fairly accurate.

However, this study pointed out that for an annual estimate of energy consumption, a program that analyzes the building systems hourly is far more accurate than one using the old empirical equations. This becomes even more important on very large buildings with many large internal zones.

## 6. References

[1] *American Society of Heating, Refrigerating and Air-Conditioning Engineers*, New York, (1968).

# Energy Estimating - How Accurate?

Robert Romancheck, P.E.

Pennsylvania Power & Light Company
Allentown, Pennsylvania

The Heating and Air Conditioning Engineer has, during the past five years, endeavored to utilize the vast capabilities of the computer to synthesize highly sophisticated mathematical models for use in problem-solving. This paper is an attempt to relate (1) How we designed a building load and energy estimating computer program and (2) Propose the hypothesis that greater programming sophistication does not necessarily mean better estimating of energy consumption.

Work to computerize heating load and energy calculations was initiated in 1964 and the implementation of computer studies began in 1965. The complete system as it has evolved to date includes five programs:

1. A sort routine.
2. A design heat load and an energy estimate based on degree day data.
3. A design cooling load with an hourly synthesis of the projected thermal requirements integrated with U. S. Weather Bureau Data.
4. Hourly energy summation routine
5. An on-site generation, total energy analysis, hourly computation.

Engineering studies relating to energy costs are usually undertaken in order to determine the most economical fuel to use. Many computer programs have been written and will be written in an attempt to make this complex task easier. We often wonder where does one choose to end their program development and what "K" factor is used to account for this decision.

Key Words: Air conditioning, computer methodology, computer program evaluation, energy determination, heat loss, heat pumps, solar effect, Weather Bureau data.

## 1. Introduction

The ability to predict, within predetermined accuracy limits, design loads and energy consumption of a building's environmental system can now be reliably accomplished on a dynamic, rather than static basis, through the use of computers. Complexities of conditioning systems, capital costs and the competitiveness of energy suppliers have become so interrelated that the designer must now have this capability. At the same time, however, computer routines must be realistic in the amount of input data required and the relative accuracy of the results generated. The computer programs that are discussed here are written in both Fortran II and PL/1 programming languages, and are operational on IBM 7074, 10K storage, or an IBM 360, Model 60 525K storage computer.

## 2. Program Methodology

### 2.1 Philosophy

In 1964 studies were undertaken to determine the feasibility of using an IBM 7074 computer to provide estimates of energy needs for space heating installations. This was necessitated because engineers in the sales group were spending most of their time preparing owning and operating cost analyses. This severely limited their ability to penetrate the bulk of the space heating market which, in turn, led to our basic philosophy with respect to this computer application -- development of a routine that didn't need an engineer to supply the input data, but which would still yield results that were more accurate than any method available. This concept would allow preparation of the greatest number of operating cost studies, thereby effecting maximum market penetration.

The initial reasoning was to write a computer program capable of integrating building heat losses and gains, and heating and cooling energy requirements. This idea was soon discarded because of the complex logic and the time delay required for debugging a program of this magnitude. A sequential program system was then investigated and subsequently proved to be the best solution to our problem. In selecting this method the first program would be written to generate design load calculations and energy use by the degree day method. Calculations from this program could then be stored on a tape file for use as input data in subsequent routines. It is believed that program debugging was considerably reduced by this sequential system. It was also determined that routines must be compatible for residential, commercial, and industrial estimating use.

## 2.2 Heating Program

Program #1, Design Heat Loss Determination, has as input the various building design characteristics such as:

1. Wall, roof and floor constructions
2. Window and door constructions
3. Building orientation
4. Design temperatures
5. Wall, roof and floor areas by zone
6. Internal heat gains
7. Ventilation rates
8. Occupancy schedules

Figure 1 illustrates the input data form used for this routine. All heat losses are calculated by zone, according to the methods outlined in the "ASHRAE Guide." Internal heat gains are also calculated for both the occupied and unoccupied periods and are used to adjust the building heating load requirements. Cooling load determination was not part of the original operating system design. The occupancy schedule input sheet, figure 2, can be as complex or as simple as the application requires. A file of resistance values for the more common building materials is also on call. Simply by inputting a 3 digit number and the material thickness, coefficients of transmission (U values) can be generated. The user also has the option of developing his own "R" or "U" value and inputting this number on the data sheet. Building ventilation needs can be determined by one of three methods, number of air changes, cfm per occupant or infiltration by zone. Infiltration values are based on the most representative data currently available. It is a personal observation that additional work needs to be done in the area of infiltration rates based on varying wind speeds and the newer types of windows and doors currently used in construction. One other element of Program #1 is the calculation of an energy requirement based on the degree-day method. Initially, this was the quickest way to get our system operational and since studies were at that time calculated by using degree days, accuracy could only improve. The use of this routine was placed into production in the spring of 1965. Computer outputs (fig. 3 and 4) were generated for residential, commercial and industrial installations at the rate of 20-30 per day.

## 2.3 Air conditioning program

Program #2, Air Conditioning Load Determination, was designed to complement the first routine by providing cooling load information. Calculated values from Program #1 were written on a tape file for use in this program and without any additional input, air conditioning design loads were provided. This was an interim procedure, however, since the programming efforts to provide this information was minimal compared to hourly energy requirement determination.

In order to provide the optimum result it was of course necessary to integrate weather data into the system to obtain a reliable synthesis or projected hourly thermal requirements. We had at our disposal 5 years of data from the U. S. Weather Bureau. An attempt was made at averaging the data (discarded this method because of the loss of temperature extremes) or creating a typical year (discarded because of definition, what is "typical"). The final decision was to use the year that was most in line with the normal number of degree days for the area. The hourly observations used, temperature, specific humidity (obtained by calculation from the dew point temperature) and cloud cover (50% cloud cover or less constituting a sunny hour), were recorded on a tape file. Because of computer core limitations of the 7074 computer, it was necessary to average two hours of observations into one hour.

This use of weather data will probably prove interesting and hopefully will provoke some deep thought and your consideration. First, each building zone is analyzed hourly, to determine whether it requires heating or cooling. Solar load is an important consideration at this point. If the sun is shining, (50% or less cloud cover) a calculation of the solar effect is made. The design solar load, as calculated by the sol-air temperature difference method developed by Messrs. C. O. Mackey and L. T. Wright, Jr. with additional work by Mr. J. P. Stewart, is used to predict the hourly BTU gain on the various building zones. The energy gain is then calculated to equal, the design solar load multiplied by the ratio of the temperature occurring at that hour, divided by the cooling design temperature.

$$\text{Solar Heat Gain (BTU's)} = \text{Design Solar Load} \times \frac{\text{Ambient Temperature}}{\text{Cooling Design Outdoor Temperature}}$$

This solar effect is applied to the building energy needs in the heating as well as cooling season.

Since input energy to a refrigeration machine varies with load, outdoor temperature, design of conditioning system and manufacturer, some assumptions had to be made with respect to equipment energy use. Our entire problem-solving system, as we previously mentioned, is based on a production basis philosophy. It was therefore necessary to provide an equation, obtained from a regression analysis, to recognize the different characteristics of various manufacturers' equipment. This does not preclude the ability to input a curve based on a specific piece of equipment and generate results relative to its use. An output was then generated (fig. 5) which contained summations of hourly heating and cooling energy requirements. Additional information was included in the output as an aid to the system designer.

### 2.4 Heat Pump Option

Since the determination of a heat pump Coefficient of Performance (C.O.P.) is obtainable only by an hourly analysis, the routine posseses this facility also. We do not compute heat pump feasibility studies on a production basis, but rather on an individual need, using a specific manufacturer's input-output energy relationships.

Residential dwelling unit studies are not included in the hourly energy analysis. Much time and effort was spent in trying to correlate known operating energy usage with the hourly routine projection. Constant internal heat gains, variable gains, various occupancy schedules, set back temperature conditions were all tried without success. The NEMA equation, in our estimation, still yields the best solution of heating energy determination for residential units.

### 2.5 Total Energy Routine

Since we are constantly developing and overlaying hourly BTU building thermal needs, it is a relatively simple procedure to create a file storing these values and use them as input to a so called, "Total Energy," isolated generation routine. The BTU values are initialized positive, if heating, and negative, if cooling, for identification purposes, before being placed on this file.

Additional input required for this run consists of the specified engine or turbine fuel rate curve, waste heat availability curve, hourly electrical demands (weekly, monthly, etc.) and hourly process steam loads, if any. This data is then merged on an hourly basis with the building thermal requirement and an output generated (fig. 6) which lists hourly KWH generated, cooling or heating BTU requirements, process heating BTU requirements, if any, and cubic feet of gas or gallons of oil required. A summary is also developed which includes annual KWH, gas or oil needs and an overall thermal efficiency of the isolated generation system. The hourly output is generated in order that any doubter would have the ability to verify the calculated quantities.

## 3. Program Evaluation

### 3.1 Industry Interest

The accuracy of these programming routines, we believe, has been demonstrated not only by actual billing records, but also by the acceptance of electric heating in the residential, commercial and industrial markets. The present total-electric customer breakdown within our Company area includes 54,000 residential units, 4,000 commercial units and 240 industrial installations out of a total of 820,000 billed accounts.

There are numerous organizations that have heating and cooling design and energy calculation computer programs available and in use. Some of these include: The Electric Heating Association, American Electric Power, Westinghouse, American Gas Association, Automatic Procedures for Engineering Consultants (APEC), Post Office Department (TACS) and numerous consultants and utilities. The degree of complexity varies widely and in all probability the results also. This leads into the next discussion.

### 3.2 The Unknown Factors

In 1967 after making a presentation to an ASHRAE Task Group on the procedures we had in use, a letter was received which in part stated, "the Task Group is attempting to develop a very sophisticated calculation program that will take into account all significant factors affecting the heating and cooling loads." The initial reaction to this statement was to recall our own experience when we began to define and analyze the many variables in this problem. Which of these variables do you choose to exclude and which do you include and a directly related question, how long will it take to provide the input data and obtain the results?

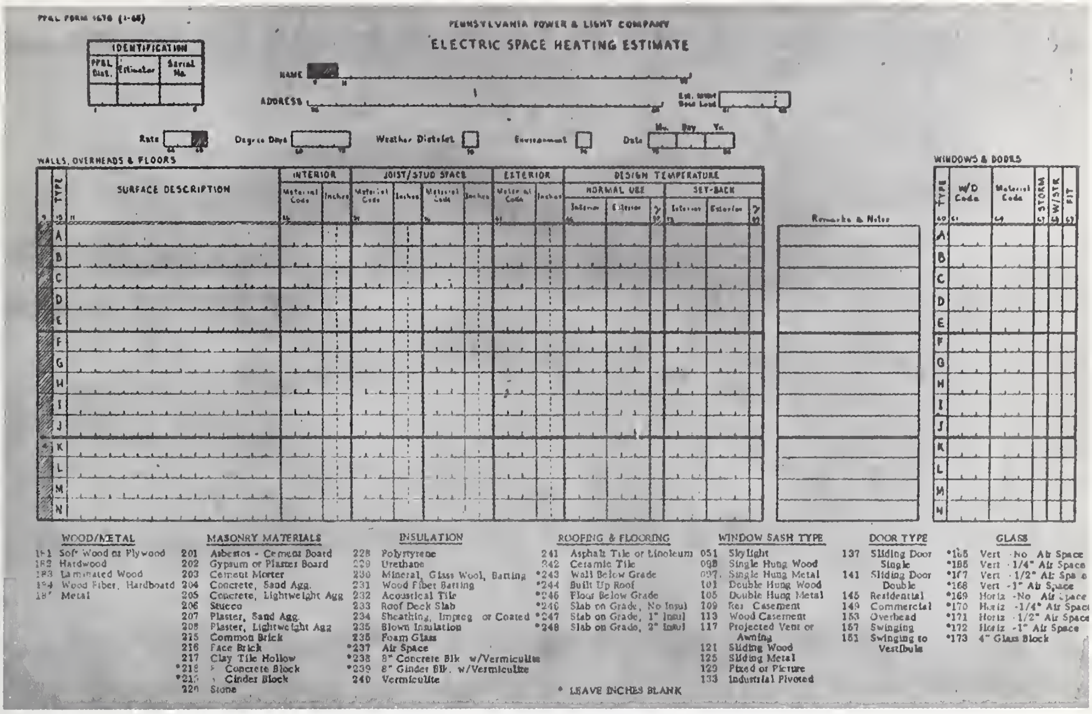
Who will decide what is significant and what isn't? Where will new data be supplied for infiltration values or isn't it significant? U. S. Weather Bureau data is recorded at the airport. Must all new buildings be built on a runway for the data to be applicable or isn't this significant. Some weather data, such as cloud cover, are only observations. What factors do we use to account for this? In the preliminary design state of a mechanical system all components, ducts, etc. must be engineered for the various systems in order to be able to select the best. Who will do this or "Isn't it significant?" Contractors build structures in varying degrees of soundness. What "K" factor is used to adjust the estimate, particularly since we don't even know who the low bidder will be. Building occupancy schedules in actual operation rarely, if ever, agree with the preliminary objectives. How do we adjust the energy estimate. Do we continually adjust the resistance values of building materials relative to outdoor temperatures or do we neglect this? How do we compensate for thermostat settings ranging from 69°F to 78°F or isn't this important? Control systems must be designed to properly monitor the system. But control system contracts are given to the low bidder as are most other contracts, what's the "K" factor relationship here?

The basic problem being introduced should be obvious. What real value is there in computing the sun's exact angle relative to a new building when someone comes along and builds a structure adjacent to it. The building unit is a dynamic living entity not a static dormant box. The complexities of any program input must be justified by the accuracy of the program output. It is also the moral obligation of the industry to honestly evaluate <u>all</u>, not just some, building energy needs, whether the structure contains 200 square feet or 2,000,000 square feet. It is seriously doubted that these evaluations can be made for all clients unless the cost and program input are reasonable.

### 3.3 A Comparison

We recently compared the results of a study with one of those complex routines, which hourly computes the solar altitude, azimuth and incidence angles, and requires a various assortment of other input data. In no case did a zone heat loss, heat gain, heating or cooling energy requirement vary by more than 5% and in most cases, the difference was negligible.

Energy analyses are necessary in the preliminary planning stages with preliminary design data since this is when decisions are made. Deeper analyses of systems and designs are necessary, and should be done on operating systems, in order to determine what is the optimum system design. The ability of computer programs to generate reliable and accurate results by using complex relationships with numerous unknown inputs must at least be questioned.



PP&L FORM 1670 (1-68)

PENNSYLVANIA POWER & LIGHT COMPANY

ELECTRIC SPACE HEATING ESTIMATE

| IDENTIFICATION | | |
|---|---|---|
| PP&L Dist. | Estimator | Serial No. |
| | | |

NAME

ADDRESS

Est. Winter Base Load

Rate

Degree Days

Weather District

Environment

Date Mo. Day Yr.

WALLS, OVERHEADS & FLOORS

| TYPE | SURFACE DESCRIPTION | INTERIOR | | JOIST/STUD SPACE | | | | EXTERIOR | | DESIGN TEMPERATURE | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | NORMAL USE | | | SET-BACK | | |
| | | Material Code | Inches | Material Code | Inches | Material Code | Inches | Material Code | Inches | Interior | Exterior | γ | Interior | Exterior | γ |
| A | | | | | | | | | | | | | | | |
| B | | | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | | | |
| D | | | | | | | | | | | | | | | |
| E | | | | | | | | | | | | | | | |
| F | | | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | | | |
| H | | | | | | | | | | | | | | | |
| I | | | | | | | | | | | | | | | |
| J | | | | | | | | | | | | | | | |
| K | | | | | | | | | | | | | | | |
| L | | | | | | | | | | | | | | | |
| M | | | | | | | | | | | | | | | |
| N | | | | | | | | | | | | | | | |

Remarks & Notes

WINDOWS & DOORS

| TYPE | W/D Code | Material Code | STORM | W/STR | FIT |
|---|---|---|---|---|---|
| A | | | | | |
| B | | | | | |
| C | | | | | |
| D | | | | | |
| E | | | | | |
| F | | | | | |
| G | | | | | |
| H | | | | | |
| I | | | | | |
| J | | | | | |
| K | | | | | |
| L | | | | | |
| M | | | | | |
| N | | | | | |

WOOD/METAL

| | |
|---|---|
| 181 | Soft Wood or Plywood |
| 182 | Hardwood |
| 183 | Laminated Wood |
| 184 | Wood Fiber, Hardboard |
| 18' | Metal |

MASONRY MATERIALS

| | |
|---|---|
| 201 | Asbestos - Cement Board |
| 202 | Gypsum or Plaster Board |
| 203 | Cement Mortar |
| 204 | Concrete, Sand Agg. |
| 205 | Concrete, Lightweight Agg |
| 206 | Stucco |
| 207 | Plaster, Sand Agg. |
| 208 | Plaster, Lightweight Agg |
| 215 | Common Brick |
| 216 | Face Brick |
| 217 | Clay Tile Hollow |
| *218 | 8" Concrete Block |
| *219 | 8" Cinder Block |
| 220 | Stone |

INSULATION

| | |
|---|---|
| 228 | Polystyrene |
| 229 | Urethane |
| 230 | Mineral, Glass Wool, Batting |
| 231 | Wood Fiber Batting |
| 232 | Acoustical Tile |
| 233 | Roof Deck Slab |
| 234 | Sheathing, Impreg or Coated |
| 235 | Blown Insulation |
| 236 | Foam Glass |
| *237 | Air Space |
| *238 | 8" Concrete Blk w/Vermiculite |
| *239 | 8" Cinder Blk. w/Vermiculite |
| 240 | Vermiculite |

ROOFING & FLOORING

| | |
|---|---|
| 241 | Asphalt Tile or Linoleum |
| 242 | Ceramic Tile |
| *243 | Wall Below Grade |
| *244 | Built Up Roof |
| *245 | Floor Below Grade |
| *246 | Slab on Grade, No Insul |
| *247 | Slab on Grade, 1" Insul |
| *248 | Slab on Grade, 2" Insul |

WINDOW SASH TYPE

| | |
|---|---|
| 051 | Skylight |
| 093 | Single Hung Wood |
| 097 | Single Hung Metal |
| 101 | Double Hung Wood |
| 105 | Double Hung Metal |
| 109 | Met Casement |
| 113 | Wood Casement |
| 117 | Projected Vent or Awning |
| 121 | Sliding Wood |
| 125 | Sliding Metal |
| 129 | Fixed or Picture |
| 133 | Industrial Pivoted |

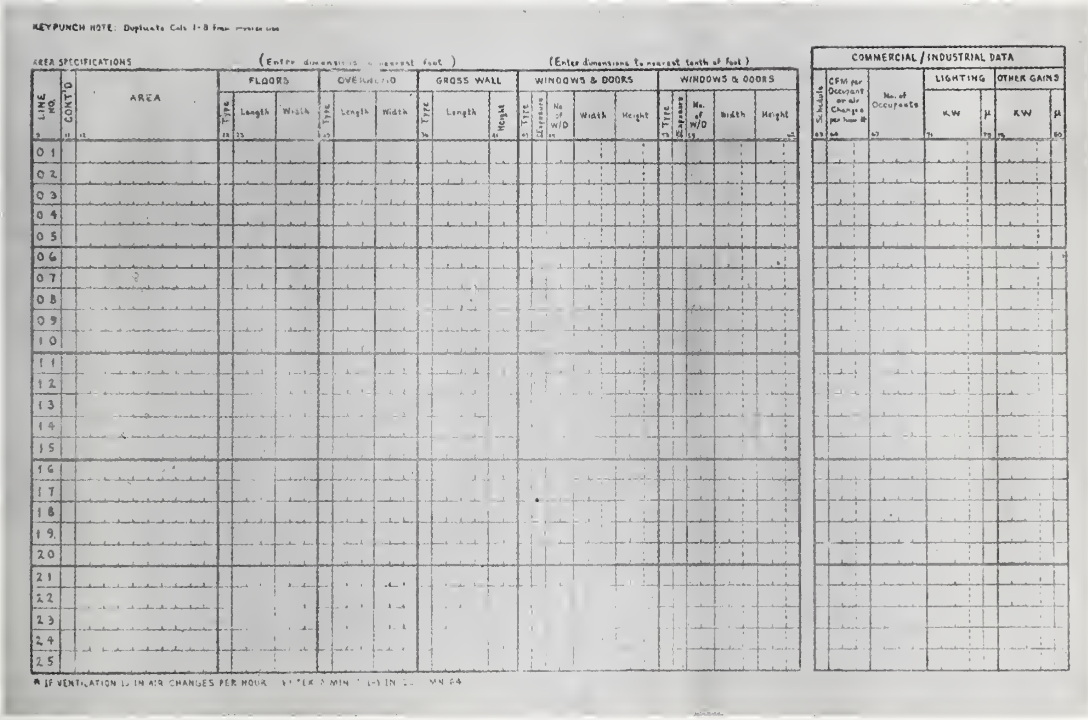
DOOR TYPE

| | |
|---|---|
| 137 | Sliding Door Single |
| 141 | Sliding Door Double |
| 145 | Residential |
| 149 | Commercial |
| 153 | Overhead |
| 157 | Swinging |
| 161 | Swinging to Vestibule |

GLASS

| | |
|---|---|
| *165 | Vert -No Air Space |
| *166 | Vert -1/4" Air Space |
| *167 | Vert -1/2" Air Space |
| *168 | Vert -1" Air Space |
| *169 | Horiz -No Air Space |
| *170 | Horiz -1/4" Air Space |
| *171 | Horiz -1/2" Air Space |
| *172 | Horiz -1" Air Space |
| *173 | 4" Glass Block |

* LEAVE INCHES BLANK



KEYPUNCH NOTE: Duplicate Cols 1-8 from master use

AREA SPECIFICATIONS (Enter dimensions to nearest foot) (Enter dimensions to nearest tenth of foot)

| LINE NO. | CONT'D | AREA | FLOORS | | | OVERHEAD | | | GROSS WALL | | | WINDOWS & DOORS | | | | | WINDOWS & DOORS | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Type | Length | Width | Type | Length | Width | Type | Length | Height | Type | Exposure | No of W/D | Width | Height | Type | Exposure | No. of W/D | Width | Height |
| 01 | | | | | | | | | | | | | | | | | | | | | |
| 02 | | | | | | | | | | | | | | | | | | | | | |
| 03 | | | | | | | | | | | | | | | | | | | | | |
| 04 | | | | | | | | | | | | | | | | | | | | | |
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| 08 | | | | | | | | | | | | | | | | | | | | | |
| 09 | | | | | | | | | | | | | | | | | | | | | |
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| 12 | | | | | | | | | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | | | | | | | | | | |
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| 21 | | | | | | | | | | | | | | | | | | | | | |
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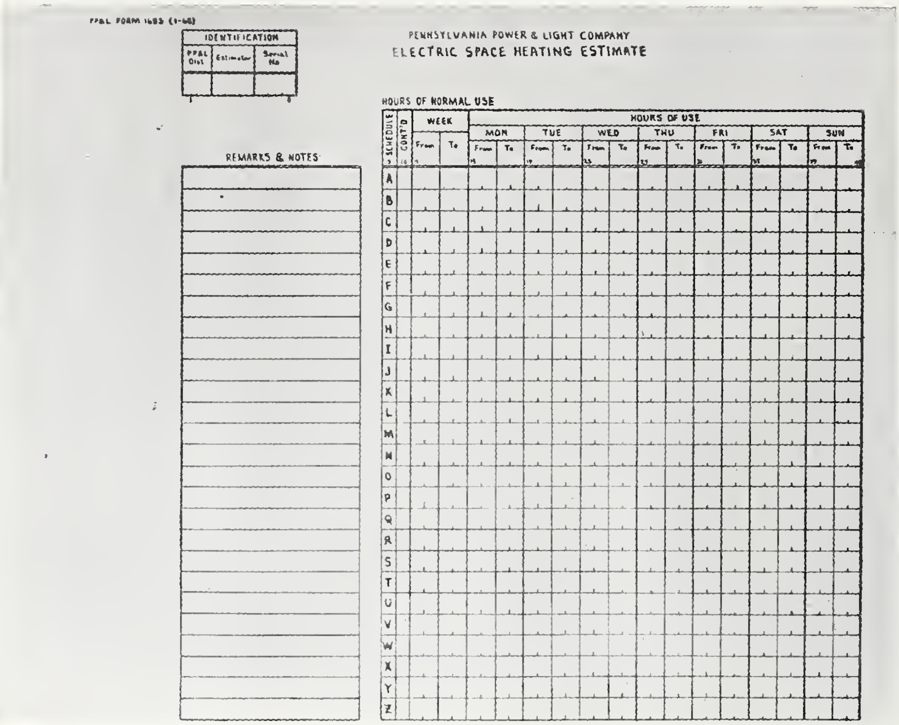
| 23 | | | | | | | | | | | | | | | | | | | | | |
| 24 | | | | | | | | | | | | | | | | | | | | | |
| 25 | | | | | | | | | | | | | | | | | | | | | |

COMMERCIAL / INDUSTRIAL DATA

| Schedule | CFM per Occupant or air Changes per hour * | No. of Occupants | LIGHTING | | OTHER GAINS | |
|---|---|---|---|---|---|---|
| | | | KW | μ | KW | μ |
| | | | | | | |

* IF VENTILATION IS IN AIR CHANGES PER HOUR ENTER A MINUS (-) IN COLUMN 64

Figure 1



PP&L FORM 1683 (1-68)

| IDENTIFICATION | | |
|---|---|---|
| PP&L Dist | Estimator | Serial No |
| | | |

PENNSYLVANIA POWER & LIGHT COMPANY

ELECTRIC SPACE HEATING ESTIMATE

REMARKS & NOTES

HOURS OF NORMAL USE

| SCHEDULE | CONT'D | WEEK | | HOURS OF USE | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | MON | | TUE | | WED | | THU | | FRI | | SAT | | SUN | |
| | | From | To | From | To | From | To | From | To | From | To | From | To | From | To | From | To |
| A | | | | | | | | | | | | | | | | | |
| B | | | | | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | | | | | |
| D | | | | | | | | | | | | | | | | | |
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| N | | | | | | | | | | | | | | | | | |
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| P | | | | | | | | | | | | | | | | | |
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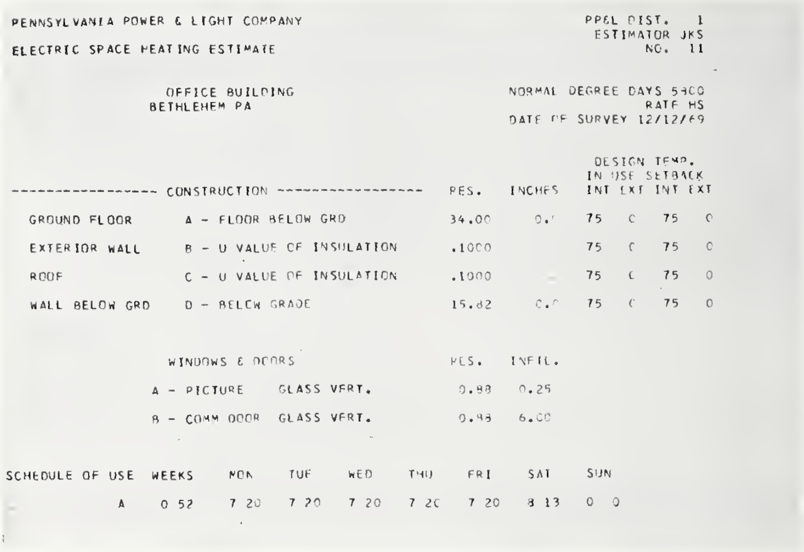
| R | | | | | | | | | | | | | | | | | |
| S | | | | | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | | | | | |
| U | | | | | | | | | | | | | | | | | |
| V | | | | | | | | | | | | | | | | | |
| W | | | | | | | | | | | | | | | | | |
| X | | | | | | | | | | | | | | | | | |
| Y | | | | | | | | | | | | | | | | | |
| Z | | | | | | | | | | | | | | | | | |

Figure 2



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PENNSYLVANIA POWER & LIGHT COMPANY                                         PP&L DIST.   1
                                                                           ESTIMATOR JKS
ELECTRIC SPACE HEATING ESTIMATE                                                  NO.  11

                         OFFICE BUILDING                            NORMAL DEGREE DAYS 5300
                       BETHLEHEM PA                                               RATE HS
                                                                    DATE OF SURVEY 12/12/69

                                                                          DESIGN TEMP.
                                                                         IN USE SETBACK
----------------- CONSTRUCTION -----------------   RES.    INCHES     INT EXT INT EXT

  GROUND FLOOR       A - FLOOR BELOW GRD           34.00      0.0     75   0   75   0

  EXTERIOR WALL      B - U VALUE OF INSULATION     .1000              75   0   75   0

  ROOF               C - U VALUE OF INSULATION     .1000              75   0   75   0

  WALL BELOW GRD     D - BELOW GRADE               15.82      0.0     75   0   75   0


                         WINDOWS & DOORS           RES.    INFIL.

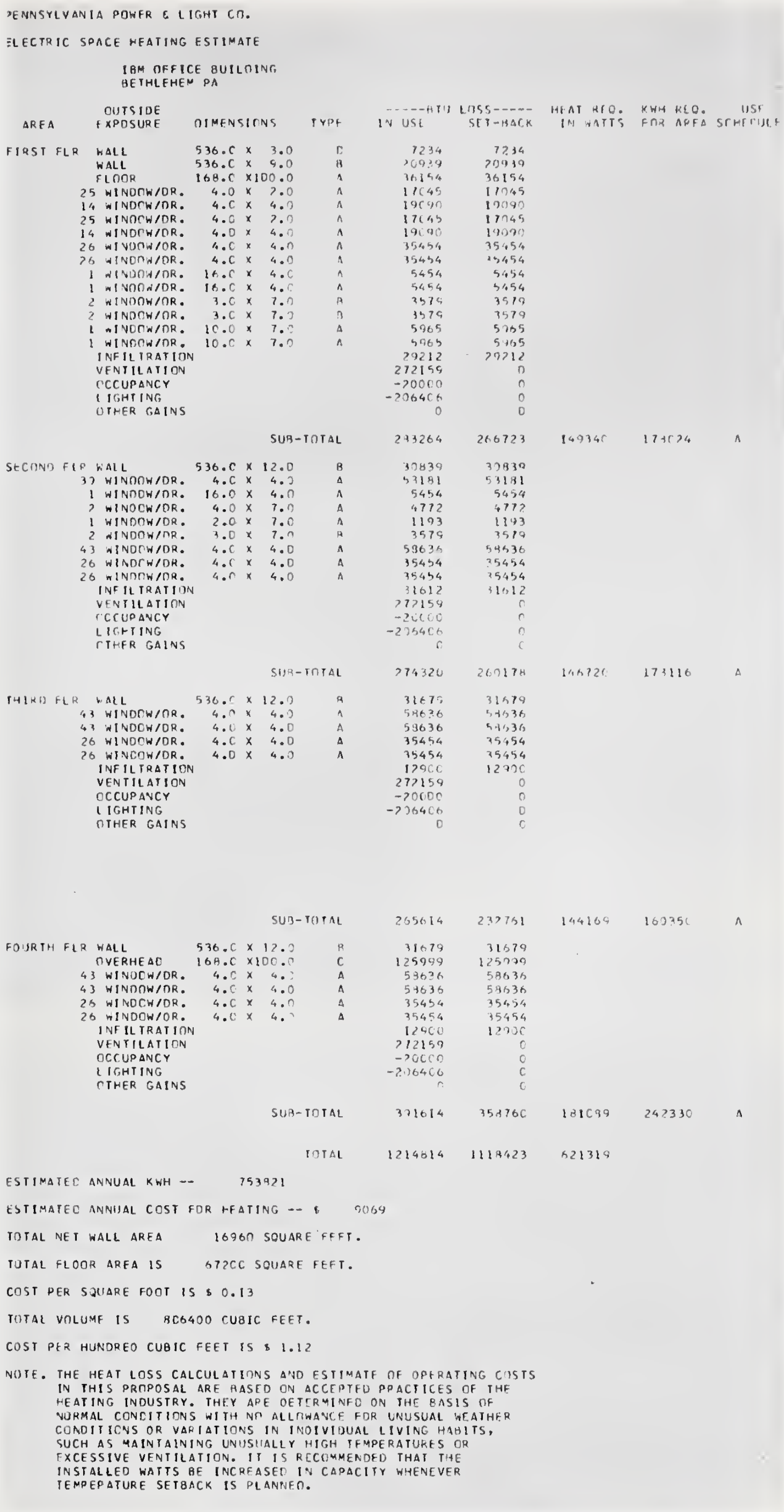
                       A - PICTURE    GLASS VERT.   0.88    0.25

                       B - COMM DOOR  GLASS VERT.   0.88    6.00


SCHEDULE OF USE  WEEKS    MON    TUE    WED    THU    FRI    SAT    SUN

              A   0 52    7 20   7 20   7 20   7 20   7 20   8 13   0  0
```

Figure 3



```
PENNSYLVANIA POWER & LIGHT CO.

ELECTRIC SPACE HEATING ESTIMATE

              IBM OFFICE BUILDING
              BETHLEHEM PA

             OUTSIDE                               -----BTU LOSS-----  HEAT REQ.  KWH REQ.    USE
  AREA       EXPOSURE     DIMENSIONS     TYPE      IN USE    SET-BACK  IN WATTS  FOR AREA SCHEDULE

FIRST FLR  WALL          536.0 X  3.0      D         7234      7234
           WALL          536.0 X  9.0      B        20939     20939
           FLOOR         168.0 X100.0      A        36154     36154
        25 WINDOW/DR.      4.0 X  2.0      A        17045     17045
        14 WINDOW/DR.      4.0 X  4.0      A        19090     19090
        25 WINDOW/DR.      4.0 X  2.0      A        17045     17045
        14 WINDOW/DR.      4.0 X  4.0      A        19090     19090
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
         1 WINDOW/DR.     16.0 X  4.0      A         5454      5454
         1 WINDOW/DR.     16.0 X  4.0      A         5454      5454
         2 WINDOW/DR.      3.0 X  7.0      B         3579      3579
         2 WINDOW/DR.      3.0 X  7.0      B         3579      3579
         1 WINDOW/DR.     10.0 X  7.0      A         5965      5965
         1 WINDOW/DR.     10.0 X  7.0      A         5965      5965
           INFILTRATION                             29212     29212
           VENTILATION                             272159         0
           OCCUPANCY                               -20000         0
           LIGHTING                               -206406         0
           OTHER GAINS                                  0         0

                              SUB-TOTAL            293264    266723    149340    174024        A

SECOND FLR WALL          536.0 X 12.0      B        30839     30839
        32 WINDOW/DR.      4.0 X  4.0      A        53181     53181
         1 WINDOW/DR.     16.0 X  4.0      A         5454      5454
         2 WINDOW/DR.      4.0 X  7.0      A         4772      4772
         1 WINDOW/DR.      2.0 X  7.0      A         1193      1193
         2 WINDOW/DR.      3.0 X  7.0      B         3579      3579
        43 WINDOW/DR.      4.0 X  4.0      A        58636     58636
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
           INFILTRATION                             31612     31612
           VENTILATION                             272159         0
           OCCUPANCY                               -20000         0
           LIGHTING                               -206406         0
           OTHER GAINS                                  0         0

                              SUB-TOTAL            274320    260178    146720    173116        A

THIRD FLR  WALL          536.0 X 12.0      B        31679     31679
        43 WINDOW/DR.      4.0 X  4.0      A        58636     58636
        43 WINDOW/DR.      4.0 X  4.0      A        58636     58636
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
           INFILTRATION                             12900     12900
           VENTILATION                             272159         0
           OCCUPANCY                               -20000         0
           LIGHTING                               -206406         0
           OTHER GAINS                                  0         0




                              SUB-TOTAL            265614    232761    144169    160350        A

FOURTH FLR WALL          536.0 X 12.0      B        31679     31679
           OVERHEAD      168.0 X100.0      C       125999    125999
        43 WINDOW/DR.      4.0 X  4.0      A        58636     58636
        43 WINDOW/DR.      4.0 X  4.0      A        58636     58636
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
        26 WINDOW/DR.      4.0 X  4.0      A        35454     35454
           INFILTRATION                             12900     12900
           VENTILATION                             272159         0
           OCCUPANCY                               -20000         0
           LIGHTING                               -206406         0
           OTHER GAINS                                  0         0

                              SUB-TOTAL            391614    358760    181099    242330        A

                                  TOTAL           1214814   1118423    621319

ESTIMATED ANNUAL KWH --      753921

ESTIMATED ANNUAL COST FOR HEATING -- $    9069

TOTAL NET WALL AREA      16960 SQUARE FEET.

TOTAL FLOOR AREA IS     67200 SQUARE FEET.

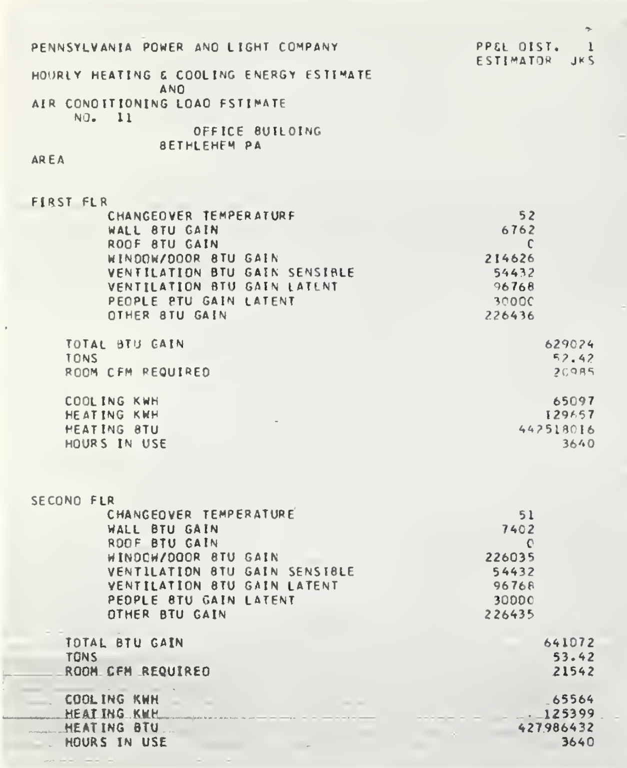
COST PER SQUARE FOOT IS $ 0.13

TOTAL VOLUME IS    806400 CUBIC FEET.

COST PER HUNDRED CUBIC FEET IS $ 1.12

NOTE. THE HEAT LOSS CALCULATIONS AND ESTIMATE OF OPERATING COSTS
      IN THIS PROPOSAL ARE BASED ON ACCEPTED PRACTICES OF THE
      HEATING INDUSTRY. THEY ARE DETERMINED ON THE BASIS OF
      NORMAL CONDITIONS WITH NO ALLOWANCE FOR UNUSUAL WEATHER
      CONDITIONS OR VARIATIONS IN INDIVIDUAL LIVING HABITS,
      SUCH AS MAINTAINING UNUSUALLY HIGH TEMPERATURES OR
      EXCESSIVE VENTILATION. IT IS RECOMMENDED THAT THE
      INSTALLED WATTS BE INCREASED IN CAPACITY WHENEVER
      TEMPERATURE SETBACK IS PLANNED.
```

Figure 4



```
PENNSYLVANIA POWER AND LIGHT COMPANY                    PP&L DIST.   1
                                                        ESTIMATOR  JKS
HOURLY HEATING & COOLING ENERGY ESTIMATE
             AND
AIR CONDITIONING LOAD ESTIMATE
    NO.  11
                    OFFICE BUILDING
             BETHLEHEM PA
AREA


FIRST FLR
        CHANGEOVER TEMPERATURE                         52
        WALL BTU GAIN                                6762
        ROOF BTU GAIN                                   0
        WINDOW/DOOR BTU GAIN                       214626
        VENTILATION BTU GAIN SENSIBLE               54432
        VENTILATION BTU GAIN LATENT                 96768
        PEOPLE BTU GAIN LATENT                      30000
        OTHER BTU GAIN                             226436

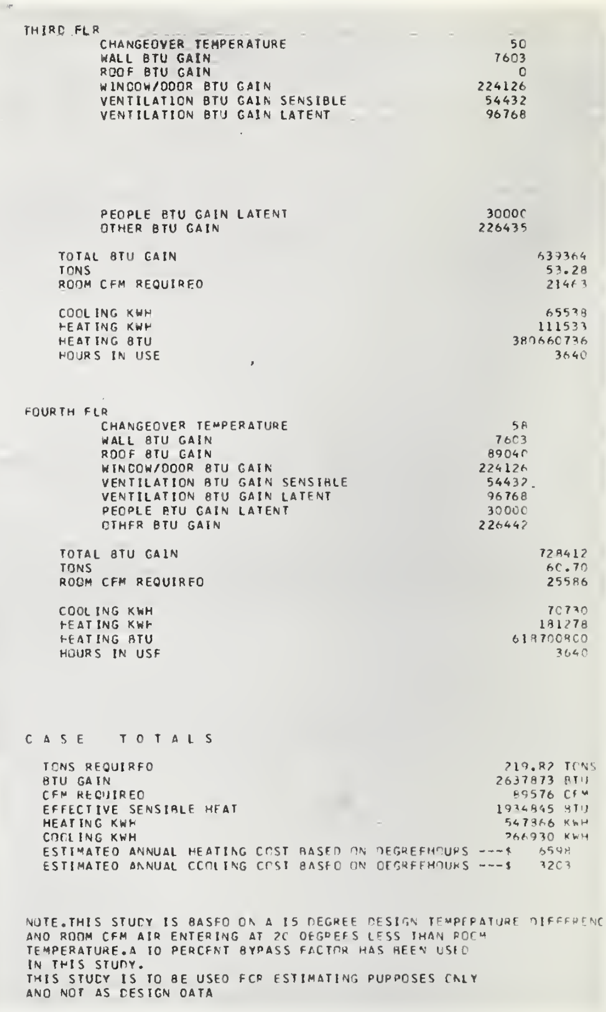
    TOTAL BTU GAIN                                        629024
    TONS                                                   52.42
    ROOM CFM REQUIRED                                      20985

    COOLING KWH                                            65097
    HEATING KWH                                           129657
    HEATING BTU                                        442518016
    HOURS IN USE                                            3640


SECOND FLR
        CHANGEOVER TEMPERATURE                         51
        WALL BTU GAIN                                7402
        ROOF BTU GAIN                                   0
        WINDOW/DOOR BTU GAIN                       226035
        VENTILATION BTU GAIN SENSIBLE               54432
        VENTILATION BTU GAIN LATENT                 96768
        PEOPLE BTU GAIN LATENT                      30000
        OTHER BTU GAIN                             226435

    TOTAL BTU GAIN                                        641072
    TONS                                                   53.42
    ROOM CFM REQUIRED                                      21542

    COOLING KWH                                            65564
    HEATING KWH                                           125399
    HEATING BTU                                        427986432
    HOURS IN USE                                            3640
```



```
THIRD FLR
        CHANGEOVER TEMPERATURE                         50
        WALL BTU GAIN                                7603
        ROOF BTU GAIN                                   0
        WINDOW/DOOR BTU GAIN                       224126
        VENTILATION BTU GAIN SENSIBLE               54432
        VENTILATION BTU GAIN LATENT                 96768



        PEOPLE BTU GAIN LATENT                      30000
        OTHER BTU GAIN                             226435

    TOTAL BTU GAIN                                        639364
    TONS                                                   53.28
    ROOM CFM REQUIRED                                      21463

    COOLING KWH                                            65538
    HEATING KWH                                           111533
    HEATING BTU                                        380660736
    HOURS IN USE                                            3640


FOURTH FLR
        CHANGEOVER TEMPERATURE                         58
        WALL BTU GAIN                                7603
        ROOF BTU GAIN                               89040
        WINDOW/DOOR BTU GAIN                       224126
        VENTILATION BTU GAIN SENSIBLE               54432
        VENTILATION BTU GAIN LATENT                 96768
        PEOPLE BTU GAIN LATENT                      30000
        OTHER BTU GAIN                             226442

    TOTAL BTU GAIN                                        728412
    TONS                                                   60.70
    ROOM CFM REQUIRED                                      25586

    COOLING KWH                                            70730
    HEATING KWH                                           181278
    HEATING BTU                                        618700800
    HOURS IN USE                                            3640

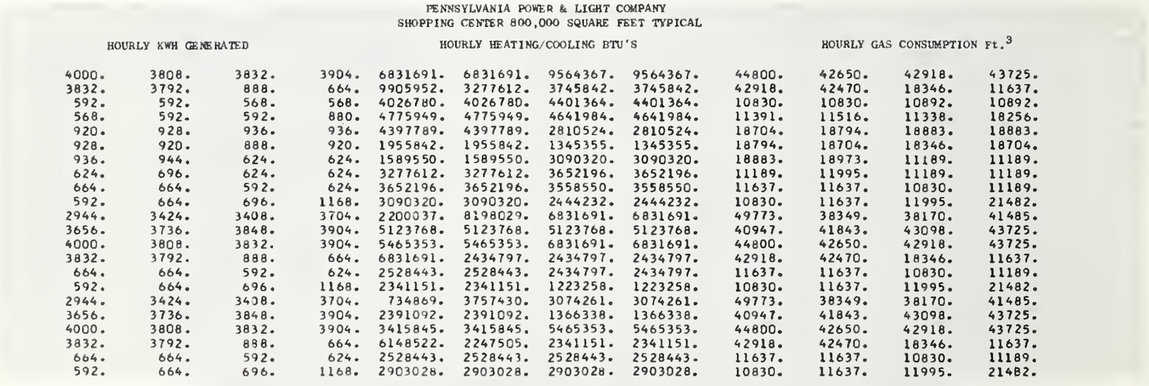


C A S E   T O T A L S

  TONS REQUIRED                                         219.82 TONS
  BTU GAIN                                             2637873 BTU
  CFM REQUIRED                                           89576 CFM
  EFFECTIVE SENSIBLE HEAT                              1934845 BTU
  HEATING KWH                                           547366 KWH
  COOLING KWH                                           266930 KWH
  ESTIMATED ANNUAL HEATING COST BASED ON DEGREEHOURS ---$   6598
  ESTIMATED ANNUAL COOLING COST BASED ON DEGREEHOURS ---$   3203


NOTE.THIS STUDY IS BASED ON A 15 DEGREE DESIGN TEMPERATURE DIFFERENC
AND ROOM CFM AIR ENTERING AT 20 DEGREES LESS THAN ROOM
TEMPERATURE.A 10 PERCENT BYPASS FACTOR HAS BEEN USED
IN THIS STUDY.
THIS STUDY IS TO BE USED FOR ESTIMATING PURPOSES ONLY
AND NOT AS DESIGN DATA
```

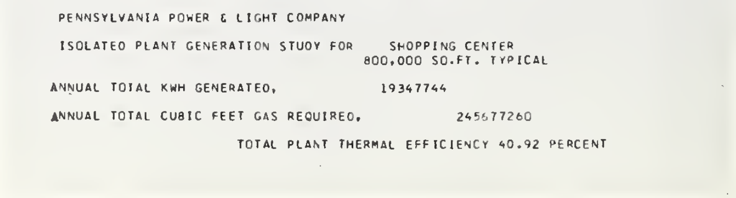
Figure 5



```
                                  PENNSYLVANIA POWER & LIGHT COMPANY
                               SHOPPING CENTER 800,000 SQUARE FEET TYPICAL

      HOURLY KWH GENERATED                  HOURLY HEATING/COOLING BTU'S                    HOURLY GAS CONSUMPTION Ft.³

 4000.   3808.   3832.   3904.  6831691.  6831691.  9564367.  9564367.   44800.   42650.   42918.   43725.
 3832.   3792.    888.    664.  9905952.  3277612.  3745842.  3745842.   42918.   42470.   18346.   11637.
  592.    592.    568.    568.  4026780.  4026780.  4401364.  4401364.   10830.   10830.   10892.   10892.
  568.    592.    592.    880.  4775949.  4775949.  4641984.  4641984.   11391.   11516.   11338.   18256.
  920.    928.    936.    936.  4397789.  4397789.  2810524.  2810524.   18704.   18794.   18883.   18883.
  928.    920.    888.    920.  1955842.  1955842.  1345355.  1345355.   18794.   18704.   18346.   18704.
  936.    944.    624.    624.  1589550.  1589550.  3090320.  3090320.   18883.   18973.   11189.   11189.
  624.    696.    624.    624.  3277612.  3277612.  3652196.  3652196.   11189.   11995.   11189.   11189.
  664.    664.    592.    624.  3652196.  3652196.  3558550.  3558550.   11637.   11637.   10830.   11189.
  592.    664.    696.   1168.  3090320.  3090320.  2444232.  2444232.   10830.   11637.   11995.   21482.
 2944.   3424.   3408.   3704.  2200037.  8198029.  6831691.  6831691.   49773.   38349.   38170.   41485.
 3656.   3736.   3848.   3904.  5123768.  5123768.  5123768.  5123768.   40947.   41843.   43098.   43725.
 4000.   3808.   3832.   3904.  5465353.  5465353.  6831691.  6831691.   44800.   42650.   42918.   43725.
 3832.   3792.    888.    664.  6831691.  2434797.  2434797.  2434797.   42918.   42470.   18346.   11637.
  664.    664.    592.    624.  2528443.  2528443.  2434797.  2434797.   11637.   11637.   10830.   11189.
  592.    664.    696.   1168.  2341151.  2341151.  1223258.  1223258.   10830.   11637.   11995.   21482.
 2944.   3424.   3408.   3704.   734869.  3757430.  3074261.  3074261.   49773.   38349.   38170.   41485.
 3656.   3736.   3848.   3904.  2391092.  2391092.  1366338.  1366338.   40947.   41843.   43098.   43725.
 4000.   3808.   3832.   3904.  3415845.  3415845.  5465353.  5465353.   44800.   42650.   42918.   43725.
 3832.   3792.    888.    664.  6148522.  2247505.  2341151.  2341151.   42918.   42470.   18346.   11637.
  664.    664.    592.    624.  2528443.  2528443.  2528443.  2528443.   11637.   11637.   10830.   11189.
  592.    664.    696.   1168.  2903028.  2903028.  2903028.  2903028.   10830.   11637.   11995.   21482.
```



```
PENNSYLVANIA POWER & LIGHT COMPANY

ISOLATED PLANT GENERATION STUDY FOR    SHOPPING CENTER
                                    800,000 SQ.FT. TYPICAL

ANNUAL TOTAL KWH GENERATED,           19347744

ANNUAL TOTAL CUBIC FEET GAS REQUIRED,          245677260

                      TOTAL PLANT THERMAL EFFICIENCY 40.92 PERCENT
```

Figure 6

# Instantaneous Cooling Loads by Computer based on ASHRAE'S time averaging method

R. V. THOMAS[1]

Naval Facilities Engineering Command
Washington, D.C. 20390

Variables affecting cooling load calculations are numerous, often difficult to define precisely, and always intricately interrelated. Many of the components of the cooling load vary in magnitude over a wide range during a 24 hour period, and as the cyclic changes in load components are not usually in phase with each other careful analysis is required to establish the resultant maximum cooling load for a building or zone. There may be an appreciable difference between the net instantaneous rate of heat gain and the total cooling load at any instant. This differene is caused by the storage and subsequent release of heat by the structure and its contents. This thermal-storage effect may be quite important in determining an economical cooling equipment capacity. This computer program uses the ASHRAE method of calculating cooling loads. It takes the instantaneous heat gain and breaks it into radiation and convection components. Convection components are added directly to the room space and instantaneous cooling load. The radiation components are summed and averaged over a given time period (dependent on building construction) up to and including the time of the desired load. They are then changed into convected components and added back into the room space as part of the instantaneous cooling load (See figure 1). Therefore, a cooling load calculation at any given time is actually taking into account the radiation build up that has been taking place several hours earlier. The program will print out a building's cooling load, at any location, room by room, for any hour between 8 AM and 5 PM and contains a psychrometric routine for calculating moisture content, latent heat and relative humidity. The building itself can also be rotated to find the optimum orientation. Built into the program are eleven types of roofs, eight types of walls, six activity levels of people and six room lighting levels.

Key Words: Convection, direction cosines, fenestration, heat lag, latent heat, radiation, relative humidity, sensible heat, solar heat gain factor, thermal-storage, total equivalent temperature differentials, "U" factor.

## 1. Introduction

Calculation of a cooling load by hand using ASHRAE'S methods becomes very tedious and lengthy, especially when the process has to be repeated to find the hour of the maxium load or the orientation for minimum load. Lengthy and repetitious calculations such as these lend themselves very well to solution by computer and in this case the problem was programmed in fortran for a Burroughs 5500 time sharing computer.

## 2. Basic Considerations

Heat lag should be carefully considered in the cooling load calculations. In certain types of buildings the effect of solar radiation is still apparent several hours after the sun has shifted from that exposure. In other types having a much lighter construction, the heat gain due to solar radiation decreases markedly with the passing of the sun. Some walls, warmed by the sun, may radiate heat long after the passing of the sun. In these cases the greater mass may be used to your advantage by pre-cooling it below the room design conditions prior to the hour of peak load and much of the sun's radiant energy will go into raising the temperature of the walls. (see figure 1).

---

[1] Mechanical Engineer, mechanical design.

## 2.1 Fenestration Areas

Solar heat gain factors were used in calculating heat gain through fenestration areas. These solar heat gain factors were developed from the equations found in Chapter 28 of [1][2] which locate the sun's position in the sky with respect to the surface in question. These equations express the radiation from the sun on a surface as a function of the date, time of day, latitude, and direction cosine of the surface receiving the radiation. To further simplify the input and eliminate the need for putting in direction cosines of building surface, direction cosines for the most commonly used directions (N, NE, E, SE, S, SW, W, and NW) were built in to the program so the user need only supply the direction ("N", "NE", ...etc) the surface is facing. By using a looping method and advancing the hour each loop, the sun's position is advanced across the sky and the resulting solar heat gain to each building surface is calculated. In the early stages of development, the solar heat gain factors thus calculated were then checked with the standard ASHRAE solar heat gain tables for various latitudes.

The program then time averages these factors (length of time lag average depends on building construction)and breaks them into convected and radiant components. It then multiplies them by the fenestration area and adds the resultant heat gain to the room load. Shading devices such as drapes, blinds, sun screens, etc. are handled by inputing the proper shade factor.

## 2.2. Walls and Roof

Heat gain through the walls and roof is calculated by using total equivalent temperature differentials (heat gain divided by "U" value). These quantities have dimensions of temperature and take into account the solar radiation heat gain and heat gain from the difference between inside and outside design temperatures.

$$\left(\begin{array}{l}\text{Total heat transmission from}\\ \text{solar radiation and temperature}\\ \text{difference between outside and}\\ \text{room air, Btu hr}^{-1}\text{ ft}^{-2}\end{array}\right) = \left(\begin{array}{l}\text{Total equivalent}\\ \text{temperature differential}\end{array}\right) * \left(\begin{array}{l}\text{Heat transmission coefficient}\\ \text{Btu hr}^{-1}\text{ ft}^{-2}\ {}^{o}\text{F}^{-1}\end{array}\right)$$

Total equivalent temperature differentials for eight types of walls and eleven types of roofs from [1][2] have been stored in permanent files in order to reduce the amount of input data required. Since these differentials are based on a daily outdoor temperature range of 20 degrees, a routine for correcting these equivalent temperature differentials for daily ranges other than 20 degrees has been built in.

Heat transmitted through the walls and roof is broken down into components similar to the method used above for glass. The convected portion is obtained by multiplying the total temperature differential of the time of the desired load by the "U" value and the area. The radiant portion is obtained by averaging the total equivalent temperature differential several hours leading up to and including the hour of the desired cooling load (the exact number of hours time lag is determined by the program depending on the type of construction). These values are then multiplied by the "U" value and the area.

Following the methods of ASHRAE, 40% of the convected and 60% of the radiant heat gain are added to the instantaneous cooling load.

## 2.3. Latent Heat

Heat and moisture are given off by humans at different rates depending on their level of activity and these sensible and latent heats can, in many instances, become a large fraction of the total load. Six different activity levels of occupants with their associated lighting Watts $ft^{-2}$ and ventilation requirements were built into the program so the user need only specify the type and number of persons who will be occupying the space. An exception code was also built in which allows the user to insert any values of sensible and latent heat, lighting or ventilation CFM which differ from the standard values.

A subroutine using computer developed equations from [3] was then written to express relative humidity and moisture content as a function of the dry bulb and wet bulb temperature. This routine is used to calculate the moisture content of the ventilation air, the resulting latent load, and the relative humidities of the air at indoor and outdoor design conditions.

The daily outdoor temperature is varied from morning to night by a routine based on a "time" versus "outdoor temperature" chart from [2].

---

[2] Figures in brackets indicate the literature references at the end of this paper.

## 3. Output

A sample of the program's output (see figure 2) shows a typical search for a building's peak load. The following is an explanation of headings over the printout:

RM ID - This is the ID. No. of the various rooms for which the cooling load is being solved. The entire building may be looked at as one room for a quick analysis of a building cooling load.

RSH - This is the room sensible heat. It is composed of radiation and convected heat through the glass, doors, wall, roof, lights and the sensible gain from the people plus the sensible load from the ventilation air that by-passes the coil plus any miscellaneous load.

RTH - This is the room Total heat. It is composed of RSH plus the latent load from the people and the latent load from the ventilation air that by-passes the coil.

OA SH - This is sensible heat of the outside air that is not by-passing the coils. It is calculated for the given time of day.

OA TH - This is the OA SH plus the latent heat of the outside air that is not by-passing the coils. It is calculated for the given time of day.

OA CFM- This is the amount of outside air that is being supplied to the room. Usually based on local codes and ordinances on ventilation requirements.

GTH - This is the grand total heat and is the sum of RTH plus the OA TH.

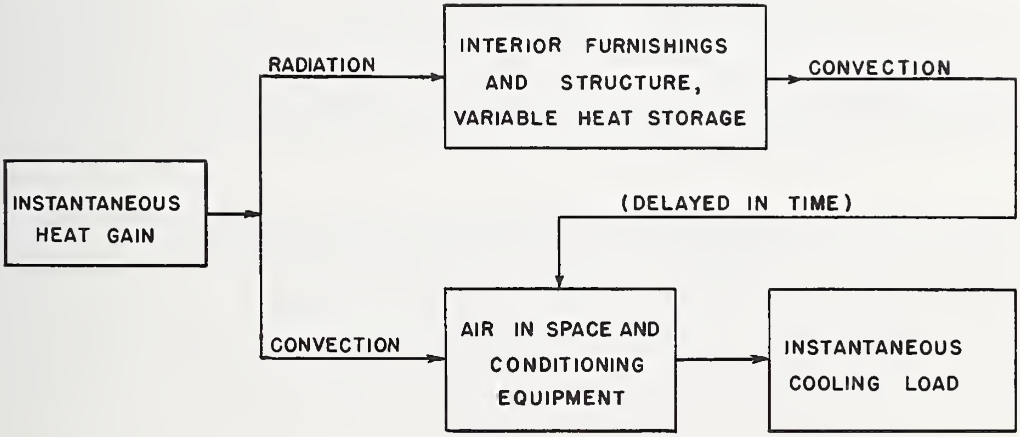
BUILDING TOTAL - This is the sum of all the GTH's from each room. In the case where the building is treated as a single room, GTH will equal BUILDING TOTAL.

## 4. References

1 ASHRAE, Handbook of Fundamentals 1967.

2 Trane, Solar Tables for heat gain calculations, 1966.

3. Fairfax, J. P. and Tung, J. S-T "Construction of a psychrometric table by use of a digital computer", ASHRAE JOURNAL, March 1968, 59-61.



**THE RADIATION ABSORBED BY THE INTERIOR FURNISHINGS AND STRUCTURE REACHES THE CONDITIONING EQUIPMENT AFTER A CONSIDERABLE TIME DELAY**

Figure 1. Origin of the Difference Between the Magnitudes of the Instantaneous Heat Gain and Instantaneous Cooling Load

```
***BUILDING COOLING LOAD AT TIME= 1500
OUTSIDE DSGN DB & WB TEMP= 95.0   78.0   RH%= 47.4
INSIDE DSGN DB & WB TEMP=  75.0   60.5   RH%= 43.2


 RM ID       RSH       RTH        OA SH        OA TH        OA CFM       GTH

    1       10798.    12117.        935.        2741.        50.       14859.
    2       16563.    18718.        748.        2193.        40.       20911.
    3       13588.    15698.       1496.        4386.        80.       20084.
    4       10442.    11761.        935.        2741.        50.       14503.

BUILDING TOTAL=    70357.

WHAT TIME NEXT
?1600


***BUILDING COOLING LOAD AT TIME= 1600
OUTSIDE DSGN DB & WB TEMP= 95.0   78.0   RH%= 47.4
INSIDE DSGN DB & WB TEMP=  75.0   60.5   RH%= 43.2


 RM ID       RSH       RTH       OA SH        OA TH        OA CFM       GTH

    1       11450.    12769.       907.        2713.        50.       15483.
    2       16625.    18780.       726.        2171.        40.       20950.
    3       13774.    15884.      1451.        4341.        80.       20225.
    4       10815.    12134.       907.        2713.        50.       14847.

BUILDING TOTAL=    71505.

WHAT TIME NEXT
?1700


***BUILDING COOLING LOAD AT TIME= 1700
OUTSIDE DSGN DB & WB TEMP=  95.0  78.0   RH%= 47.4
INSIDE DSGN DB & WB TEMP=   75.0  60.5   RH%= 43.2

 RM ID       RSH       RTH       OA SH        OA TH        OA CFM       GTH

    1       11493.    12812.       842.        2648.        50.       15460.
    2       16325.    18480.       673.        2118.        40.       20598.
    3       13810.    15920.      1346.        4237.        80.       20157.
    4       10973.    12292.       842.        2648.        50.       14939.

BUILDING TOTAL=    71154.
```

Figure 2. Building Cooling Load at Time = 1500

# Computer Method for Estimating Net Energy Requirement for Heating Buildings

Nathaniel E. Hager, Jr.

Armstrong Cork Company
Research and Development Center
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The net energy needed to maintain constant building temperature is estimated by subtracting cost-free heat gains from external losses. This quantity is integrated over the heating season by a computer programmed to combine data on climate, structure, and occupancy. The method is based on straightforward application of the energy conservation principle, and does not depend on arbitrary choice of a degree-day base or other such factor requiring previous experience with similar situations.

Key Words: Architectural heating, building heating, computer, environmental engineering, fuel consumption, heat gains, net energy balance, residential heating, space heating.

## 1. Introduction

The degree-day method [1, 2, 3][1] has long been used for computing annual heat loss from buildings and estimating resultant fuel consumption. It seemed practical to use this method when fuel was plentiful and homes were not well insulated. Because the total energy use was so large, cost-free internal gains were relatively unimportant, and there was little incentive for handling them in a sophisticated way. Lack of adequate weather data and computer facilities added further discouragement. Thus, these small internal heat gains were handled by using an arbitrary degree-day base of 65°F, which amounted to assuming that the heating plant kept the building temperature up to 65°F while the internal gains did the rest. For buildings with higher gains, lower base temperatures were selected on the basis of experience with similar buildings.

With the growth of electric heating it became apparent that the degree-day mentioned gave results too high for electric heating estimates [4, 5, 6]. Consequently, the National Electrical Manufacturers Association (NEMA) recommended that electric heating specialists adjust the degree-day results downward by using a multiplying factor determined from previous experience [7]. This recommendation served a practical purpose, but, from the analytical viewpoint, compounded the arbitrariness of the degree-day method. Both the degree-day method and the NEMA recommendation were unsatisfactory for handling unconventional situations, and failed to provide a sound basis for theoretical evaluation of new heating and insulation methods.

In a 1962 publication [8] the writer showed that the growing inadequacy of these methods rose from two important trends in residential structure. First, houses were becoming better insulated and tighter, reducing the total energy used in heating. Second, more appliances and higher lighting levels were being used, causing an increase in available internal heat gains which did not need to be supplied by the heating plant. As a result, it was no longer practical to ignore these gains, nor was it enlightening to handle them with indirect and arbitrary approximations. It was then shown that the gains could be taken into account by direct application of the law of conservation of energy. A procedure was described which only counted internal heat gains during periods when they are profitably used to offset heat losses.

Several subsequent workers recognized the need for taking into account cost-free heat gains in order to obtain better estimates. Thomas [9] subtracted the gross annual heat gain during the heating

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[1] Figures in brackets indicate literature references at the end of this paper.

season from the gross annual loss during the same season, without taking into account whether or not these gains were profitably employed. Later, Billington [10] described a method for modifying the degree-day method to account for "miscellaneous sources of heat other than the heating system". In 1967 an ASHRAE proposal [11] suggested that a modification of the degree-day method be used for homes and that a "heat balance" method be used for commercial and industrial buildings.

One purpose of the present paper is to show how straightforward inclusion of estimated internal gains in the overall heating picture produces reasonable results without the use of arbitrary degree-day factors, and without having previous experience with similar buildings. A second purpose is to show how the previously described procedure has been extended to take advantage of high-speed computers and improved weather data. The present procedure makes it feasible to predict the economic consequences of using new components or systems, and to design heating plants for unconventional structures. The method is sufficiently general to be applied to any type of structure including homes, commercial or industrial buildings, mobile homes, and computer facilities.

Present emphasis on minimizing pollution should ultimately lead to spending more money on insulation in order to minimize consumption of fuel. Thus, the effort to reduce pollution from heating plants will cause internal heat gains to assume even greater relative importance in the overall space heating picture, and will add further emphasis to the need for properly including these gains when estimating heating requirements.

## 2. Theory

At first glance, it might appear that the net heat loss for the heating season can be estimated by subtracting the total heat gain for the season from the total heat loss. Brief reflection makes it clear that this approach is incorrect because it assumes that all heat gains are profitably employed. Actually, these gains are only of value when they offset losses and help maintain the desired indoor temperature. Usually, when the gains exceed the losses, windows are opened or cooling is employed; potential heat gains are thus discarded, and are not used profitably. It is recognized that some portion of the excess gains can be stored by heating of massive parts of the structure, but this process is not included within the scope of this analysis. This omission is of little importance when dealing with light weight structures where there is relatively little heat storage.

The following analytical procedure is developed for the purpose of taking into account cost-free heat gains when, and only when, they offset losses. The procedure consists of selecting an interval of temperature, say 5 degrees wide, and computing the rates of loss and gain when the outdoor temperature falls within that interval. If the resultant net rate of heat loss is negative it is discarded. If there is a net loss, this is multiplied by the number of hours during the heating season when the temperature falls within this interval. This is done for all intervals of outdoor temperature encountered, and the results are summed to get the total loss which must be replaced by the heating system during the heating season.

Therefore, data are needed describing the number of hours per year when the temperature falls within each defined interval. Such data are available for over 200 locations in the United States [12]. An example of these data is given in table 1 which shows the statistical distribution of dry-bulb temperatures for Pittsburgh, Pennsylvania. The distribution is given for 5-degree temperature intervals. The distribution is further broken down into three equal periods of the day corresponding to the periods when people normally sleep, work, and engage in recreation, and the following procedure is especially tailored to make good use of this additional breakdown.

If $\Delta t_j^{(S)}$ designates the aggregate time per heating season during the sleeping period, when the indoor-outdoor temperature difference falls within the interval centering on $\Delta T_j$, then the rate of heat loss during these hours is given by

$$H_j^{(S)}\ (\text{LOSS}) = u\Delta T_j \Delta t_j^{(S)} , \tag{1}$$

where u is a summation of all losses expressed in the form

$$u = U_1A_1 + U_2A_2 + U_3A_3 + \dots = \sum_i U_iA_i . \tag{2}$$

It is assumed that each loss item is expressible as a product of fixed factors which, when multiplied by the indoor-outdoor temperature difference $\Delta T_j$, will give the rate of heat loss during those periods included within the aggregate time period

$$\Delta t_j^{(S)}.$$

Table 1. Mean frequence of occurrence of dry bulb temperature in hours per year for Pittsburgh, Pennsylvania; data obtained from reference [12].

| Temperature Range °F | Sleeping (S) 1:30 a.m.-9:30 p.m. | Period of Day Working (W) 9:30 a.m.-5:30 p.m. | Recreation (r) 5:30 p.m.-1:30 a.m. |
|---|---|---|---|
| 95/99 | 0 | 1 | 0 |
| 90/94 | 0 | 15 | 0 |
| 85/89 | 0 | 107 | 11 |
| 80/84 | 6 | 264 | 57 |
| 75/79 | 34 | 317 | 141 |
| 70/74 | 151 | 286 | 276 |
| 65/69 | 291 | 239 | 319 |
| 60/64 | 323 | 204 | 296 |
| 55/59 | 277 | 174 | 240 |
| 50/54 | 263 | 163 | 217 |
| 45/49 | 225 | 166 | 192 |
| 40/44 | 219 | 192 | 213 |
| 35/39 | 242 | 215 | 223 |
| 30/34 | 291 | 217 | 276 |
| 25/29 | 218 | 152 | 171 |
| 20/24 | 141 | 93 | 112 |
| 15/19 | 91 | 57 | 78 |
| 10/14 | 74 | 37 | 52 |
| 5/9 | 37 | 15 | 30 |
| 0/4 | 22 | 5 | 13 |
| -5/-1 | 11 | 1 | 3 |
| -10/-6 | 4 | 0 | 0 |
| -15/-11 | 0 | 0 | 0 |
| -20/-16 | 0 | 0 | 0 |

The heat gain during this same time period is calculated from the equation

$$H_j^{(S)}\ (\mathrm{Gain}) = q^{(S)}\Delta t_j^{(S)} , \tag{3}$$

where q is the sum of the contributing sources of heat gain during the sleeping period, given by the expression

$$q^{(S)} = q_1^{(S)} + q_2^{(S)} + q_3{}^{(S)} + \dots \; . \tag{4}$$

Therefore, the net loss is obtained by subtraction of eq (3) from eq (1), giving the equation

$$H_j^{(S)}\ (\mathrm{NET}) = (u\Delta T_j - q^{(S)})\ \Delta t_j^{(S)} \; . \tag{5}$$

Similar expressions can be written for the working period (W) and the recreation period (R). The net heat loss H for the heating season is obtained by summing over all three periods of the day, and over all temperature intervals in accordance with the following equation:

$$H = \sum_j (u\Delta T_j - q^{(S)})\Delta t_j^{(S)} + \sum_j (u\Delta T_j - q^{(W)})\Delta t_j^{(W)} + \sum_j (u\Delta T_j - q^{(R)})\Delta t_j^{(R)} \; . \tag{6}$$

It is understood that only positive terms from each of the summations are to be included in obtaining H.

### 2.1 Proposed Extension of Method

The form of the above method was influenced by the form of presently available tabulations on the statistical distribution of climate conditions. That component of u accounting for infiltration loss, represented above by a constant average value, would be better described if the distribution of wind velocities were taken into account. It has been shown by others [13, 14] that the infiltration loss from a building can be correlated with wind velocity. The U-factors for the various parts of the structural shell also depend on wind velocity. Likewise, the component of heat gain $q^{(S)}$ which accounts for solar gain, would be better described if the frequency with which the solar energy level falls within arbitrarily selected ranges were taken into account. The attached appendix gives further details on how these modifications can be accomplished.

## 3. Application to Representative Home

### 3.1 Insulation Systems Defined

The above method is applied to a representative 1-1/2-story house insulated to four different degrees described in table 2. Insulation System I represents the case where the house is virtually uninsulated; no materials are used which involve cost exceeding that required for structural strength. System II represents a typical contemporary insulation installation usually considered adequate when conventional heating plants are used. System III is a superior installation of the type specified for electrically-heated houses. System IV is an even tighter installation which is slightly futuristic but, nevertheless, technically feasible.

Table 3 lists areas $A_i$ and air-to-air conductance factors $U_i$ pertaining to the various components of each system. The U-factors are determined in accordance with usual handbook procedures [15]. For simplicity, so that the heat loss associated with the floor can be computed using the same temperature difference used for computing the other losses, the $U_iA_i$ values for the floor are taken to be one-fourth of the actual value when the floor is not insulated, and one-half of the actual value when it is insulated. The $U_iA_i$ values listed for infiltration are fictitious values which may be treated like the real values for walls, ceiling, etc., in that multiplying them by the same indoor-outdoor temperature difference gives an appropriate estimated value for the infiltration heat loss.

Table 2. Definition of thermal insulation systems for representative house.

| Heat Loss Item | I | II | III | IV |
|---|---|---|---|---|
| Ceiling | Plaster on lath; no insulation; attic vented. | Same as (I) with 2-in. glass wool insulation. | Same as (I) with 6-in. insulation. | Same as (I) with 6-in. insulation. |
| Walls | Plaster on lath; no insulation; gypsum sheathing; brick or wood siding. | Same as (I) except 2-in. glass wool. | Same as (I) except use 3-5/8-in. glass wool. | Same as (III). |
| Floors | 2 layers of wood; no insulation. | Same as (I). | Same as (I) with 2-in. glass wool. | Same as (III). |
| Doors | Wood. | Wood with storm doors. | Wood with storm doors. | 2-in. cellular-plastic-insulated core in laminated door. |
| Windows | Single glass weather stripped. | Same as (I) with storm windows or thermopane. | Same as(II). | Triple glazed; or double-glazed with storm windows. |
| Infiltration | 2 air changes per hr. | 1-1/2 air changes per hr. | 1 air change per hr. | 0.5 air change per hr; air processed. |
| Description of Insulation System | No insulation. | Good contemporary insulation. | Superior contemporary insulation. | Futuristic insulation. |

Table 3. Heat loss factors for representative house; dependence on insulation system. $A_i$ is in ft$^2$, $U_i$ is in Btu/hr-ft$^2$-°F, and u is in Btu/hr-°F.

| Heat Loss Item | $A_i$ | Insulation System I |  | II |  | III |  | IV |  |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $U_i$ | $U_iA_i$ | $U_i$ | $U_iA_i$ | $U_i$ | $U_iA_i$ | $U_i$ | $U_iA_i$ |
| Ceiling | 900 | 0.70 | 630 | 0.14 | 126 | 0.05 | 45 | 0.05 | 45 |
| Walls | 1800 | 0.34 | 612 | 0.10 | 180 | 0.07 | 126 | 0.07 | 126 |
| Floor | 900 | 0.35 | (79) | 0.35 | (79) | 0.10 | (45) | 0.10 | (45) |
| Doors | 40 | 0.50 | 20 | 0.35 | 14 | 0.35 | 14 | 0.10 | 4 |
| Windows | 180 | 1.13 | 203 | 0.55 | 99 | 0.55 | 99 | 0.40 | 72 |
| Infiltration | --- | ---- | (260) | ---- | (195) | ---- | (130) | ---- | (65) |
| Total $u = \sum_i U_iA_i$ |  |  | 1804 |  | 693 |  | 459 |  | 357 |

### 3.2 Internal Heat Gains Described

Table 4 shows estimated values for the most important internal heat gains during each of the three periods of the day. Because the solar heat gain through the roof depends on the degree of insulation, separate estimates must be made for each insulation system.

The solar heat gains for roof and windows are estimated on the basis of data given for Pittsburgh, Pa. They are estimated by the usual method shown in the ASHRAE Guide; the values shown are estimates of averages for the entire heating season. Note that the solar gains are estimated to be zero during the sleeping and recreation periods; it is assumed that significant solar radiation is spread evenly over the 8-hour working period of the day. Solar heat gain due to sunlight falling on vertical walls is neglected; this gain is probably less important than the others because of the presence of shrubbery, because of the common use of light-colored paints, and because a good fraction of the vertical wall area is devoted to windows taken into account separately.

At the present time, the average home uses electrical energy at an average rate close to 0.5 kw; this figure is used for the representative house with Insulation Systems I and II. The electrical energy usage is distributed unevenly during the three periods of the day such that the average value for the entire day, expressed in Btu/hr, is equivalent to the 0.5-kilowatt average. When it is assumed that the representative house is insulated in accordance with System III, which is a superior contemporary insulation system, it is assumed that there are more heavily powered electrical devices in the house, and that the electrical power is used at an average rate of about 1.0 kw. Similarly, it is assumed that homes of the future, ones with insulation systems like System IV, will use even more power for purposes other than space heating; therefore, it is assumed that the representative house with Insulation System IV uses power at a rate of about 1.5 kw. Figures given in the table are obtained by noting that 1.0 kw is approximately equal to 3,4000 Btu/hr.

The average human radiates body heat at a rate ranging from approximately 400 Btu/hr when at rest to as much as 1500 Btu/hr when exercising. For purposes of estimating heat gain from human occupants, it is assumed that there are four people at rest during the sleeping period, one person rather active during the working period, and four people moderately active during the recreation period.

## 4. Results

The IBM 360 computer is programmed to perform the calculations indicated by eq (6). Punched cards are prepared containing climate data such as shown in table 1 for each of 60 locations in the United States. Once these cards are prepared there is no further need for the estimator to be concerned with climate data. The heating energy requirement for the representative house is determined for any location by running the program using the appropriate climate card. To make estimates for the representative house values of u are taken from table 3 and values of $q^{(S)}$, $q^{(W)}$, and $q^{(R)}$ from table 4. This completes the description of the insulation system and the conditions of occupancy. The program also permits entering any desired indoor temperature (thermostat setting).

Table 4. Estimated heat gains in Btu/hr representative house in Pittsburgh, Pennsylvania.

| Insulation System | Period of Day | Solar Roof | Solar Windows | Electrical Gas | Human | Total (Btu/hr) | Symbol |
|---|---|---|---|---|---|---|---|
| I | S | - | - | 300 | 1600 | 1900 | $q^{(S)}$ |
| | W | 7500 | 2400 | 2000 | 1000 | 12900 | $q^{(W)}$ |
| | R | - | - | 3000 | 2800 | 5800 | $q^{(R)}$ |
| II | S | - | - | 300 | 1600 | 1900 | $q^{(S)}$ |
| | W | 1500 | 2400 | 2000 | 1000 | 6900 | $q^{(W)}$ |
| | R | - | - | 3000 | 2800 | 5800 | $q^{(R)}$ |
| III | S | - | - | 600 | 1600 | 2200 | $q^{(S)}$ |
| | W | 600 | 2400 | 4000 | 1000 | 8000 | $q^{(W)}$ |
| | R | - | - | 6000 | 2800 | 8800 | $q^{(R)}$ |
| IV | S | - | - | 900 | 1600 | 2500 | $q^{(S)}$ |
| | W | 600 | 2400 | 6000 | 1000 | 10000 | $q^{(W)}$ |
| | R | - | - | 9000 | 2800 | 11800 | $q^{(R)}$ |

The printout shows the annual energy requirement for heating the house for each of the four insulation systems. It also prints out, for each insulation system, estimates calculated by the degree-day method and by the NEMA method using the 18.5 C-factor commonly employed in electric heating estimates.

## 4.1 Comparison With Other Methods

Assuming that the representative house is located in Pittsburgh, Pa., which has a 5356 degree-day heating season, and that it is occupied by four people with the thermostat set at 70°F, calculations are run for the four described insulation systems. The results are shown in figure 1, the annual energy requirement being plotted as a function of u, the insulation system parameter. Also plotted are theoretical curves obtained by the degree-day and NEMA methods for the same insulation systems. The net energy method gives higher estimates for poorly insulated (high-u) houses. When the house is moderately insulated, however, the net energy method gives results very close to those obtained by the degree-day method; it is this type of home for which the degree-day method has been most successfully employed. For heavily insulated homes, the type designed for electric heating, using Insulation System III, the net energy method produces an estimate lower than the degree-day method and more nearly in agreement with the NEMA method, using a C-factor of 18.5. In other words, the net energy method agrees with the degree-day method where the latter is most valid, and agrees with the NEMA method in situations where it gives the best results. With insulation systems similar to System IV, the net energy method predicts that the annual energy requirement is approximately half of the value predicted by the degree-day method and substantially lower than the value predicted by the NEMA method. Therefore, the net energy method agrees with the traditional methods where it should agree, and gives a rational basis for predicting requirements under conditions where neither of the traditional methods should be applied.

The correlation between the net energy method and the traditional methods is further substantiated by running the calculation for over 60 different locations. Fig. 2 shows a comparison between points calculated by the net energy method and lines calculated by the traditional methods. The upper curve shows how the annual energy requirement varies with climate according to the degree-day method. This is done for the representative house with Insulation System II, a situation in which the degree-day method should give valid results. The circles plotted in the graph represent calculations made by the net energy method for the same house with the same insulation system. Each circle represents the annual energy requirement, when the house is located in a particular city, plotted against the degree-day climate factor for the same location. Similarly, the lower curve shows results predicted by the NEMA method, using a C-factor of 18.5. This curve is for the same house, but with Insulation System III. This system is the type used for electrically heated homes for which the NEMA method is ordinarily used. The triangular points represent calculations made by the net energy method for the same situation.

It is evident that the net energy method consistently gives the same result as the degree-day method for moderately insulated homes, and the same result as the NEMA method for heavily insulated homes. The agreement covers a wide range of climate conditions ranging from 1,000 to 10,000 degree-days.

### 4.2 Effect of Thermostat Setting

Next, calculations are run for the representative house, the conditions of occupancy being as described in table 4, in order to estimate the effect of changing the thermostat setting. Results are shown in figure 3 for three of the insulation systems described. The results show clearly that estimating methods can lead to very large errors if they ignore the possibility that occupants might enjoy keeping the house at 80°F.

### 4.3 Effect of Number of Occupants

Calculations are run for the representative house having Insulation System III located in the Pittsburgh area. The results, shown graphically in figure 4, predict the effects of varying thermostat setting and number of occupants. With 8 occupants, the heat gain values are taken to be twice as large as those shown in table 4. With 2 occupants, the values are taken to be half as large as those shown in the table.

The results shown in figure 4 indicate that two elderly people who like the thermostat set at 80°F will use twice as much heat as a family of 8 occupying the same house with the thermostat at 70°F. This may be a rather extreme comparison, but it serves to emphasize the important effect that occupancy conditions can have in modifying the heating requirements for a building.

## 5. Summary

Because the net energy method is based largely on using conventional estimating methods for taking individual factors into account, it cannot be claimed to be more accurate than traditional methods in cases where extensive experience has established reliable correction factors. However, the net energy method has a considerable number of advantages enumerated as follows:

1. It produces results in agreement with the degree-day method for moderately insulated homes, and with the NEMA method for well-insulated homes.

2. It is free of arbitrary "experience" factors. It should, therefore, give valid results for unconventional structures, or structures with unconventional insulation or occupancy factors, for which insufficient experience is available.

3. It is useful for theoretical or economic studies of the space heating problem. For example, it can be used to compute the most economical insulation thickness, or how fuel usage is affected by a number of appliances, thermostat settings or other occupancy factors. For another example, suppose one wants to decrease pollution by increasing thermal insulation (to reduce fuel use) and adding a cleaning system to the chimney: the net energy method makes it possible to design such a system in the most economical way.

4. The net energy method shows how badly traditional methods can be in error under slightly unusual occupancy conditions, and makes clear the need for explicitly accounting for internal gains.

5. Finally, the computerized net energy method, is, for all practical purposes, as easy to use as the computerized degree-day method. In one case, when the energy requirement was computed for the representative house in over 60 different locations, the IBM 360 took only a minute or two for the whole series of computations.

## 6. Appendix

If a constant average value for solar radiation is assumed, but k different intervals of wind velocity are assumed, leading to k different values of u for each of the three periods of the day, eq (6) expands to

$$H = \sum_j \sum_k (u_k^{(S)}\Delta T_j - q^{(S)})\Delta t_{jk}^{(S)} + \sum_j \sum_k (u_k^{(W)}\Delta T_j - q^{(W)})\Delta t_{jk}^{(W)} + \sum_j \sum_k (u_k^{(R)}\Delta T_j - q^{(R)})\Delta t_{jk}^{(R)} , \qquad (7)$$

where $\Delta t_{jk}$ now represents the number of hours during which the wind velocity falls in the kth interval, while the temperature falls within the jth interval.

Variations in the solar radiation level are taken into account by assuming that the degree of cloud cover or the normal-incident solar intensity falls within l different intervals. It is assumed that solar radiation is negligible except during the working period of the day. Equation (7) is modified to include variations in the solar heat gain by writing an equation as follows:

$$H = \sum_j \sum_k (u_k^{(S)}\Delta T_j - q^{(S)})\Delta t_{jk}^{(S)} + \sum_j \sum_k (u_k^{(R)}\Delta T_j - q^{(R)})\Delta t_{jk}^{(R)} + \sum_j \sum_k \sum_l (u_k^{(W)}\Delta T_j - q_l^{(W)})\Delta t_{jkl}^{(W)} . \qquad (8)$$

U. S. Weather Bureau data are available [11] giving hourly values for temperature, percentage cloud cover, and wind velocity. Using such data, analysis can be done to fit the needed values for $\Delta t_{jkl}$ and $\Delta t_{jk}$. If, in addition to the 15 temperature intervals, one uses three wind velocity intervals, and three cloud coverage intervals, the frequency of 135 different sets of conditions must be taken into account in order to evaluate the net heat loss during the working period.

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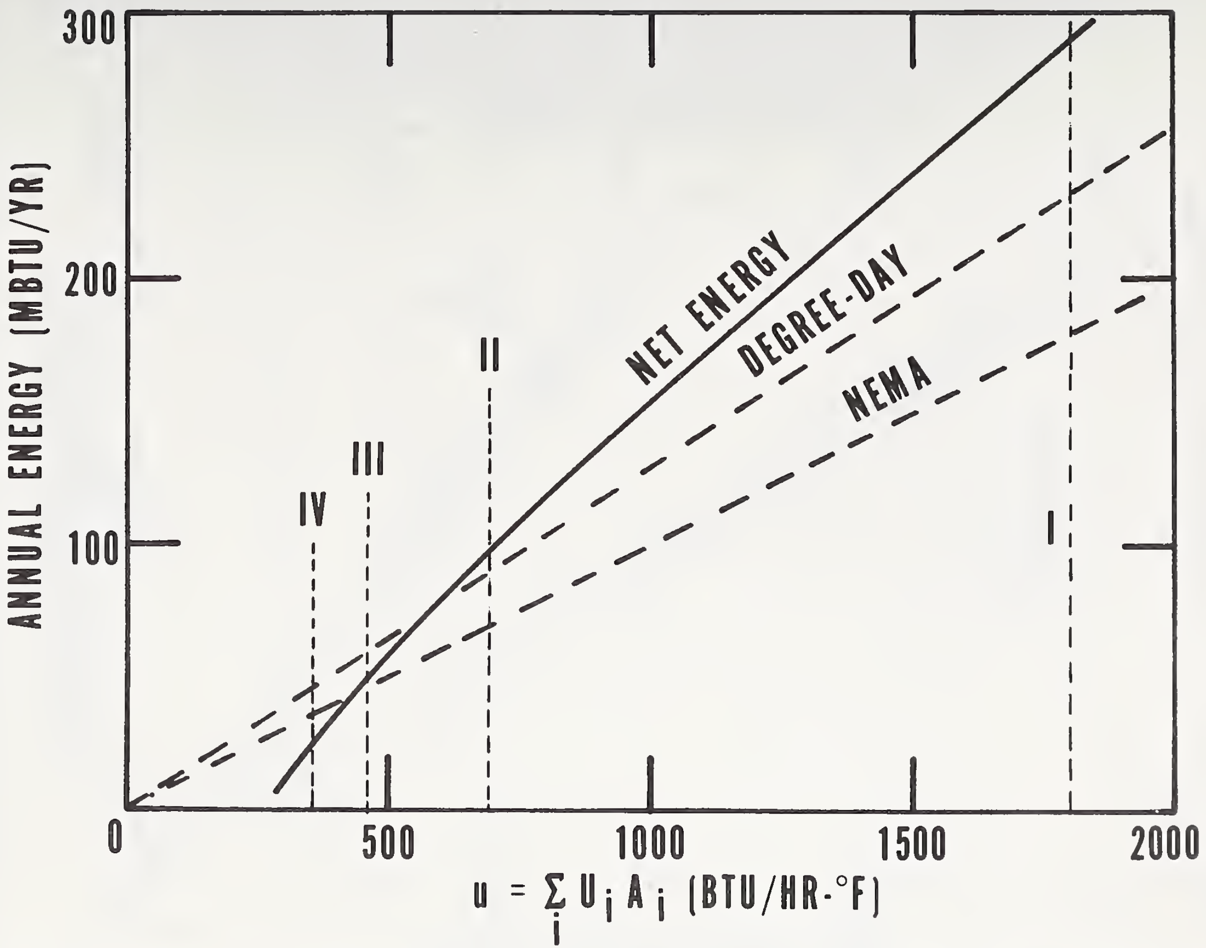

Figure 1. Effect of variation of insulation system parameter u on annual energy requirement. Representative house, located in Pittsburgh, Pennsylvania, kept at 70°F. Annual energy requirement estimated by net energy method compared with degree-day estimate taking 5356 degree-days for Pittsburgh, and with NEMA estimate taking C = 18.5. Value of u, for insulation systems described in text, indicated by vertical lines labeled with corresponding Roman Numerals.

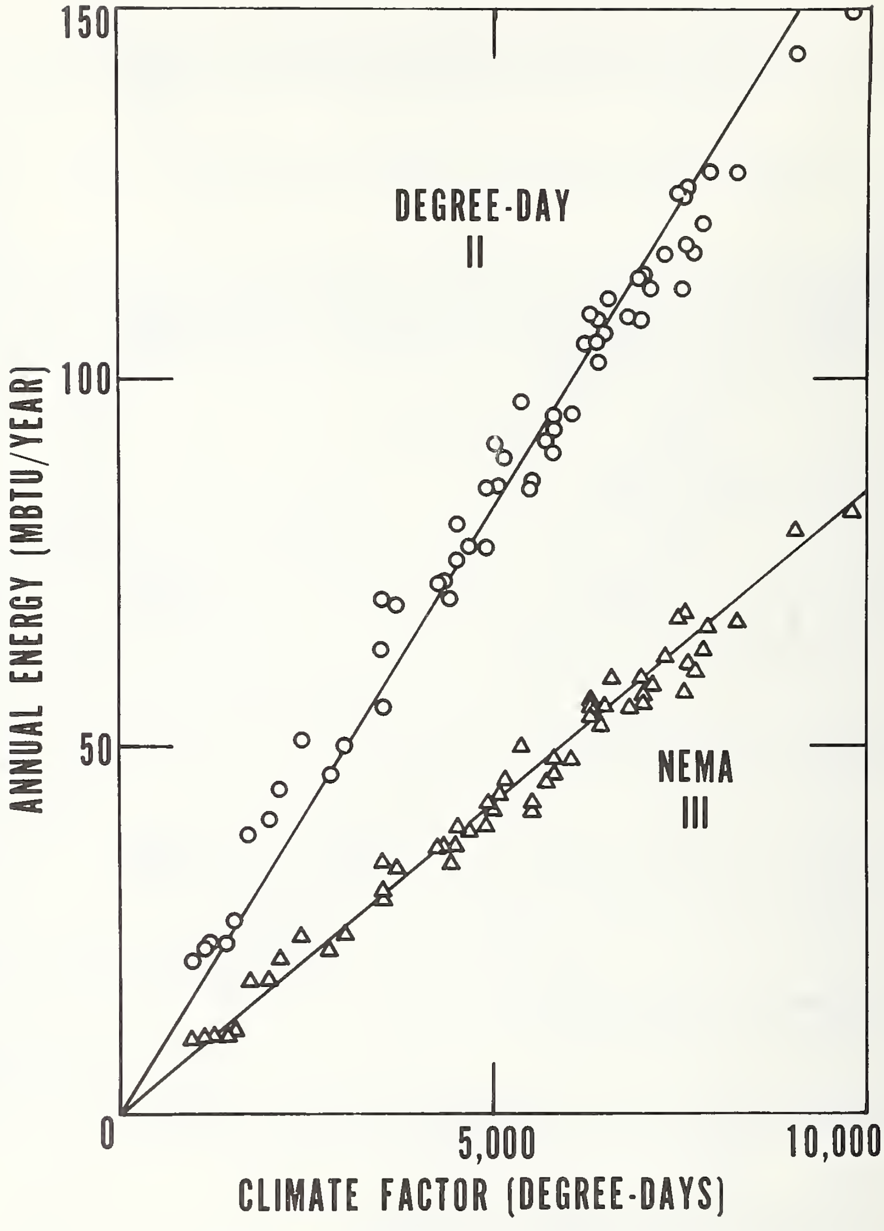

Figure 2. Correlation between estimates made by net energy method with estimates made by degree-day and NEMA methods for representative house under different climate conditions. The upper line represents the degree-day estimate with Insulation System II, and the circles represent net energy estimates with the same insulation system. The lower line represents NEMA estimates with Insulation System III, and the triangles indicate net energy results with this insulation system. All estimates assume 70°F indoor temperature.

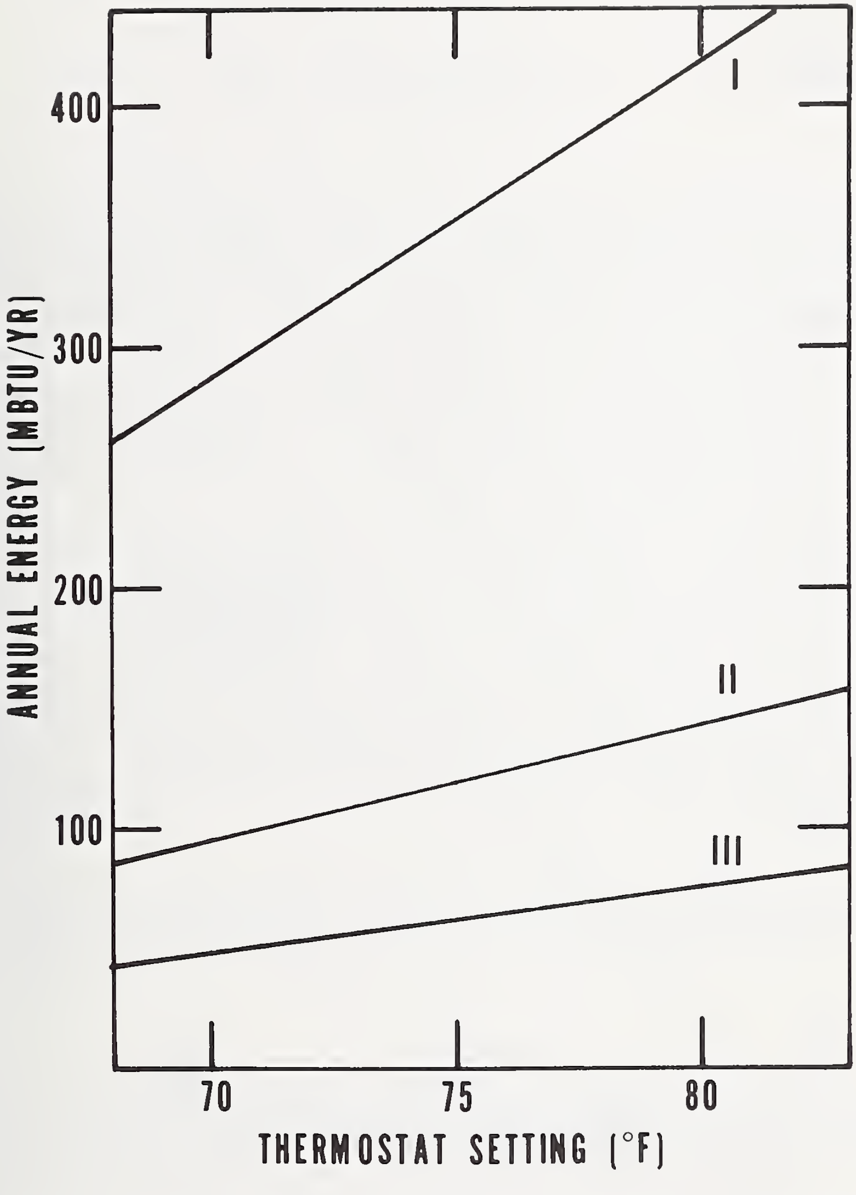

Figure 3. Effect of thermostat setting on annual heating requirement for representative house with Insulation Systems I, II and III. House situated in Pittsburgh climate, occupied by four people.

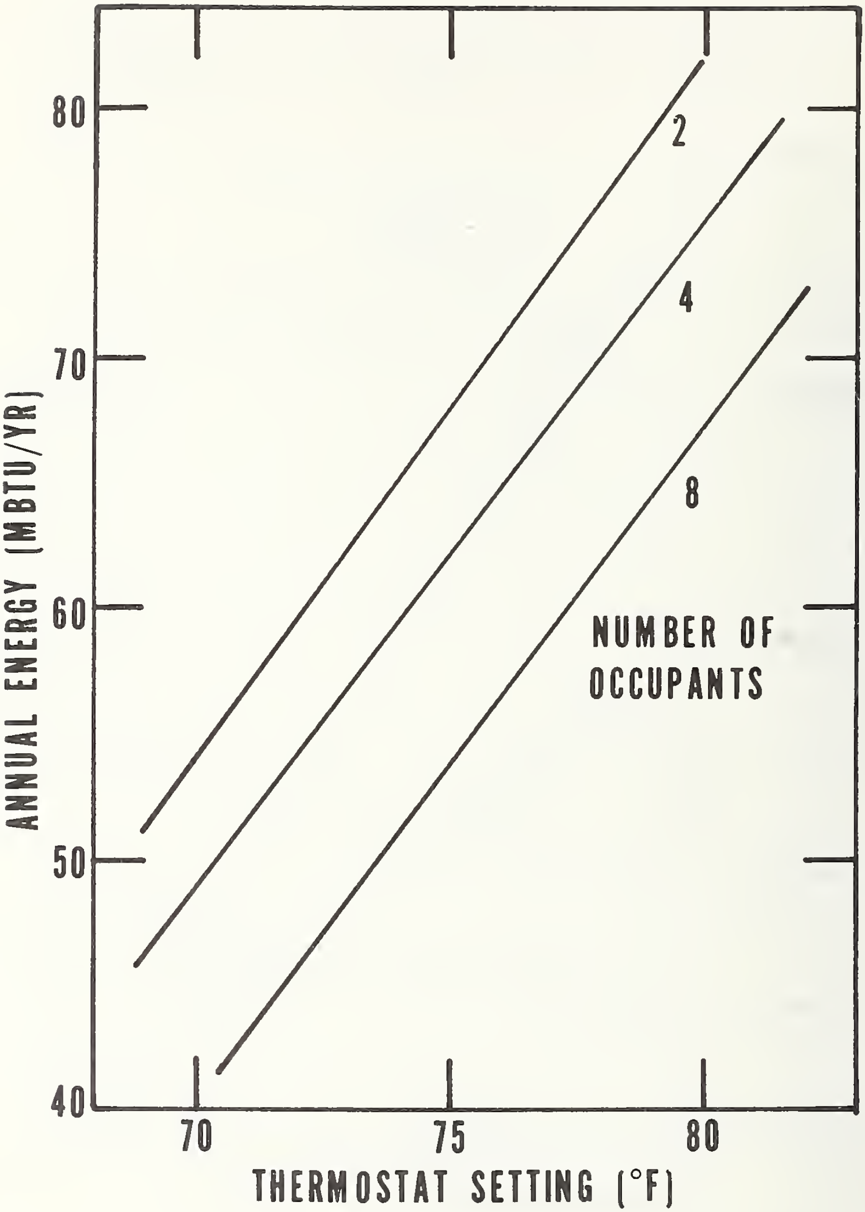

Figure 4. Effect of number of occupants and thermostat setting on annual heating requirement for representative house with Insulation System III. House in Pittsburgh location.

# The Practical Application of Small Computers for Heating and Air Conditioning Load Evaluation

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Although digital computers are ideal for heating and air conditioning load evaluation, their practical application for the majority of engineers involved requires establishment of appropriate restrictive parameters, for consideration of a wide range of load types and sizes, but with small to medium-sized installations. Minimum requirements include orientation to current ASHRAE methodology, utilization of basic formulae versus tabular or approximated data, sophisticated evaluation of time-variable factors, controlled flexibility for various system types, load accumulation into designated sub-systems with varying conditions and diversities, simplified input data, calculation of "end-use" data for equipment selection, and printout of input factors and results in complete, concise form for record purposes plus immediate engineering use. Limitations include tailoring software to readily available hardware with limited core storage, compressed overall time-frame per load calculation, and availability in terms of low acquisition and operating costs. The APEC HCC program, with certain modifications, solves this particular problem superlatively. It operates efficiently on an IBM 1130 8K single disk system, and is available to APEC members for under 2% of development costs. It determines simultaneous peak loads from 24 complete hourly calculations, each involving time lag and thermal storage factors, calculated solar shading variables, hourly temperature and humidity profiles, time-variable internal loads, external and internal surface loads, ventilation and infiltration, interrelation of varying space requirements, etc. A single "Building" calculation is limited to 39 individually variable "Zones" (air systems), and 1000 "Rooms" (basic areas). Exterior and interior load surfaces are limited to 20 "Types" each, accessible to any "Room" being evaluated. Output presents detailed cooling, heating, and humidification totals and supply air cfm for each Room, Zone, and Building level, arranged for "stand-alone" $8\frac{1}{2}$" x 11" permanent record of the load evaluation, and adaptable as input for various planned systems analysis programs.

Key Words: APEC, calculated shade ratios, design load calculation, diversity data, HCC, hourly evaluation, input simplification, minimum personnel qualifications, operational output variations, output self-explanation, practical considerations, small computers.

## 1. Introduction

The application of digital computers to the solution of heating and air conditioning load evaluation problems is ideally suited to the needs of concerned practicing engineers. In today's glass buildings and with the transient nature and varying requirements of building occupancies, there is simply no other practical way to arrive at a true simultaneous peak load condition.

The difficulty is obviously not one of technique, since the basic heat transfer calculation methods have been worked out for some years and found to be both reliable and easy to use. These procedures are also continually being polished and honed by the standard of the industry, ASHRAE, which carries out the parallel benefit of physical testing for theoretical correlation.

The only real problem of course is time, which from the standpoint of the consulting engineer in particular is all that he has to sell and therefore all important. Any good engineer, given enough time, could come up with just as valid and as accurate a load evaluation as the most sophisticated computer program on the largest installation. Both his design and the building itself would have become obsolete in the meanwhile, however.

## 2. Problems & Prospects

### 2.1. Pre-Computer Compromises & Pitfalls

The practice in the past, therefore, has of necessity been one of educated compromise. The experience and judgment of the engineer have been the controlling factors, without benefit of a myriad of comparative calculations, each taking into account a variety of changing conditions. Pre-calculated tabular data has taken the place of detailed formulae. Single-run load calculations for an assumed peak load period, with liberal safety factors to stay out of trouble, have been the order of the day. Time-lag, storage-effect, and variations in effectiveness of external shading on glass have been mistrusted, guessed at, or ignored. Only rarely has the importance of the project - and the size of the fee - been such that any significant amount of detailed comparative calculation could be considered. Since most systems do work, it therefore must be concluded that the building owner historically has bought, unknowingly, an environmental control system to some degree larger and more expensive than the circumstances actually justify.

Another continuing problem for the consulting firm, or for anyone else with a necessity to calculate heating and air conditioning loads, has been that the very personal nature of this type engineering task is extremely vulnerable to both individual and parochial habits - not always to the best interests of either the firm doing the calculating or the ultimate client. Even with standardization of procedures and careful checking of results, undesirable deviations are bound to creep in undetected from time to time. Thus we find the second major practical benefit of using digital computers, where the approved procedures can be "frozen" in a program and thereby remain beyond the misapplication or forgetfulness of the individual user.

Not to be ignored is the aspect of accuracy. No engineer has failed to experience the misplaced decimal, the incorrect addition, or the inverted slide rule operation. Too often the ultimate detection of an error of this type results in a vast amount of lost time and duplicated effort for its correction, not to mention those errors which remain hidden until after they are constructed and in unsatisfactory use. If just one error of the latter variety can be eliminated, a considerable amount of money and effort toward a computerized operation can be justified.

### 2.2. Computer Use Deterrents

If all this be true, then why not an immediate conversion by all engineers to automated procedures? The controlling factor is really the same, in a slightly different form - overall cost, or perhaps the assumed probable cost.

It is obvious of course that many, if not most, engineers would find it difficult or impossible to afford even the smallest computer installation at today's market prices. Similarly, even if acquisition cost weren't a factor, only the largest firms could keep even a small installation busy enough to justify the day to day expense of its existence. It therefore follows that in most cases the average engineer will have to rely initially on service bureaus, time-sharing terminals, or time-rental on available installations in nearby offices. The probable result is that experienced, capable, computer-oriented personnel will not be continuously available as staff members.

A parallel factor is that, since most computer use by engineers for HVAC load evaluation will represent only a relatively small amount of his total work time regardless of the circumstances, he normally will have neither the opportunity nor the inclination to become more than superficially involved in either computer operation or computer programming. Similarly, his use of computers in any form is likely to be less than constant or day-to-day, which reduces the complexity of details he can be expected to master and retain for such operation.

### 2.3. Computer Use Requirements

Still another aspect is that of the short supply of technically proficient and experienced manpower in general. Since experience can only be gained as a direct function of time, it follows that the most desirable program will have much of the necessary "experience factors" built in, so that less experienced personnel can be used in "feeding" basic and easily understood data to the computer. If successful, this approach frees the most experienced men for other more pressing duties, while expanding the overall work-producing capability of the firm.

The type and arrangement of output information is also a critical factor. The very nature of engineering, as related to the construction of systems from engineering design, requires the engineer to be an inveterate record-keeper. Limits of legal liability for an engineer's work vary from place to place and from a few years to forever, which requires that old records be clear and self-explanatory regardless of the passage of time. Another and happily more frequent need for good design records is for the intelligent and efficient future expansion of projects. A third factor arises on those projects where a design analysis must be presented to the client as part of the engineer's service. All of these however are secondary to the basic first need for the data, which is to present to the engineer the answers required to complete his design, in a form directly usable by him without extensive further manual manipulation, and arranged in such a way as to minimize the possibility of data misinterpretation by the engineer or his subordinates. In this same vein, the engineer should have some degree of control over the volume of output for any given calculation, consistent with the importance of the project and the necessity for in-depth data presentation.

The problem of available software cannot be overlooked. Most engineers know exactly the type and amount of design data they must create and retain for any project, and largely because of ASHRAE's influence over the years, these details are reasonably similar from office to office. Most HVAC engineers however are not programmers and most programmers are not HVAC engineers. The costs associated with developing an adequate program in-house are completely out of the question for any but the largest firms. The average engineer must therefore obtain his software from some outside source, and must search out the available programs for the one most suited to his taste, budget, and method of operation. Once found, of course, any program must be sufficiently studied and tested by the engineer to make him comfortable in its use and confident of its results.

From this combination of benefits to be obtained versus overall costs and other limitations there evolves a pattern, within which any automated procedure must fit before it can be considered a practical application for general use in heating and air conditioning load evaluation by the majority of engineers. It will be helpful at this point to summarize these factors.

### a. The System Must:

(1) Be based on the latest proven procedures and techniques, preferably ASHRAE oriented.

(2) Concentrate on the use of accurate basic formulae, versus tabular data or shortcut approximations.

(3) Be capable of analyzing simultaneous load factors, both internal and external, at time intervals not exceeding one hour throughout a typical day, and determining by the use of such data the peal load condition.

(4) Be sufficiently flexible to permit the engineer a wide range of options in establishing type of system, space relationships, time variables, load diversity at various levels of load accumulation, and special internal and external factors, in addition to the physical properties of the environmental surfaces.

(5) Be fast, not so much from the standpoint of program run time (although this is important) as from that of overall time required from beginning of input data assembly to receipt of final printout.

(6) Be sophisticated in operation, but simple to input, to make maximum use of the capabilities of lesser trained personnel for input purposes, with automatic elaborate evaluation of basic information by the program.

(7) Be complete but concise in output data, with basic input factors as well as final results presented in a self-explanatory mannor, and with optional additional output of detailed backup data where desired.

(8) Be available, both hardware and software, at a reasonable cost and in a form capable of producing useful results from the moment of acquisition and/or access.

## 2.4. A Practical Solution

Fortunately, these very problems have been substantially solved by one of the organizations sponsoring this symposium - APEC, or Automated Procedures for Engineering Consultants. This group, with cooperative effort and financing, commissioned the writing of a unique heating and cooling load calculation program by Bruce E. Birdsall, William B. Deeming, and Tseng-Yao Sun (1)[1]. The program, known as HCC, is excellent, easy to use, sophisticated, ASHRAE oriented, designed to run on as small an installation as the readily available IBM 1130, and is distributed to members at approximately 1% of the program development costs. Released in source deck form with complete documentation, it obviates the "black box" onus sometimes associated with canned programs which may be used for a fee but which are not revealed in detail. It is this program, with numerous user-oriented modifications developed by the author, that is presented herein as a practical application of small computers for heating and air conditioning load evaluation (similarly limited to APEC members).

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[1] Figures in parentheses indicate references at the end of this paper.

## 3. Description of HCC

### 3.1. Generally

The technical authenticity of HCC is its almost total dependence upon ASHRAE for methodology. All of the basic load calculation algorithms are based on the ASHRAE 1967 "Handbook of Fundamentals" as the latest and most complete generally accepted and available source of reliable information. Tabular data is scrupulously avoided except for empirical values used as input in various formulae, such as values of transmittance and absorptance for different glass types, pre-calculated "U" factors for combinations of building and insulation materials, sensible and latent heat generation of occupants for different types of activities, and the like.

The program is design oriented for peak load determination, as opposed to an energy analysis tool. Hourly cooling calculations are made for all 24 hours of a typical design day for a selected month, with varying solar loads, temperatures, ventilation, infiltration and internal loads, and with peak load selection and printout. Heating calculations are made and output for a single given set of conditions. Loads are calculated and presented in a form primarily suited to some type of constant volume air system.

### 3.2. Input Capabilities

Flexibility and simplicity of use in practice can best be explained by reference to the program input data as follows:

#### a. Permanent Data

Permanent data is input one time, and stored in protected disk files for use by all calculation runs. Its use is geared to the input simplification philosophy of the program, in that while remaining readily available for possible future revision, this data can normally be taken for granted after initial input. Included in this category are:

(1) Solar Radiation data including Equation of Time, Solar Declination, apparent solar irradiation at air mass = 0., atmospheric extinction coefficient, and a surface to sky radiation constant, for the 21st day of each month, for use in calculating hourly values for such days of direct normal intensity, direct intensity, and diffuse intensity on horizontal and vertical surfaces (2).

(2) Absorptance and transmittance coefficients for up to 20 different types of single and double glazing (3).

(3) Transmittance and shading coefficients for four profile angles of each of four representative types of solar screen material.

#### b. Semi-Permanent Data

Data in this group is permanent in the sense of its relationship to a specific geographical location, and may optionally be left on disk file or read anew for each different project. In practice, the data requires preparation only once for any given site with its own unique conditions, and the cards retained on file for re-input on subsequent runs in the manner of control cards. Input includes:

(1) Project Latitude, in degrees, for solar angle calculations.

(2) Ground Reflectivity Factor, for use in calculating solar heat gain factors.

(3) Inward Flow Fractions, for use in calculating overall heat transfer coefficients for glass, as affected by interior air motion. Fractions are input for still air, an increased factor for air being supplied under the glass at the sill line, and a further increased factor for sill supply located between the glass and a drapery.

(4) Clearness Factor, for calculating solar intensities in various atmospheres; ranging from 0.8 in smoggy industrial areas to 1.2 in very clear regions.

(5) Maximum Solar Heat Gain Factor, in Btu/hr/sq. ft., as an arbitrary limit for consideration of interior shading of glass surfaces. When interior shading devices are specified (with window data, later) such as venetian blinds, draperies, etc., the program assumes they will be left open when their use is not required for interior comfort. Only when the solar heat gain factor reaches this pre-set limit does the program assume the occupants will close the shading device, and thereafter takes it into account until the heat gain without it is once again less than the limit.

(6) Degrees from Standard Meridian, to allow a true sun time and hour angle to be calculated.

(7) Daylight Saving Time input factors, as signals as to which months, if any, are so affected, for calculation of true sun time values.

(8) Temperature and Humidity Ratio Data, in deg. F. and lbs. moisture/lb. dry air respectively are 24 hour cooling load outdoor weather profiles for a typical design day in each of the 12 months.

### c. Master Data

Master Data is input once for each job, and stored in temporary disk files. It is similar to the permanent data in that, as its name implies, it is geared to input data simplification by establishing master factors for an individual project which can either be accessed by a space unit being analyzed, or which will apply automatically in all cases unless specifically over-ridden.

(1) Job Identification Data, in alphameric form, is used to prepare a title sheet for the printout, and to identify each printout page as to job number, job description, and date of calculation.

(2) General Building Design Data (fig. 1).

(a) Hours for Calculation. Any 12 hours may be selected, inputting numbers from 01 to 24. Although load calculations are made for each of the 24 hours to account properly for hour averaging, only the results for the 12 hours selected will be stored and used in final load determinations. This is a compromise to permit the program to run on a small installation.

(b) Hour Average, as the number of hours over which all radiant portions of the heat gain are to be averaged, due to storage effect. Convective load components are considered as instantaneous loads.

(c) Heating Design Month, to designate a heating month for calculation of solar gain so as to indicate when such heat gain can exceed heat losses.

(d) Winter Outside-Inside Temperature in deg. F. and Humidity Ratios in lbs. moisture/lb. dry air are for heating and humidification design calculations in the conventional sense.

(e) Cooling Design Month, to cause the program to use that temperature and humidity ratio profile stored for such month, as originally input as semi-permanent data (par. 3.2.b).

(f) Cooling Outside-Inside Temperature and Humidity Ratios are design values, with inside conditions considered to be fixed. The data input for outside conditions however are considered to be peak values of the respective profiles for the selected month, and such profiles will be raised or depressed by the difference between the input peaks and the stored peaks. This permits minor adjustments of stored data without re-input of the entire weather file.

(g) Rotate Building Off South. To facilitate data input, all building exposures are taken off as though "Plan North" were true north, in terms of azimuth deviation from south as $0^o$; i.e., north = $180^o$, east = $-90^o$, west = $+90^o$, etc. This master correction factor is then input so as, when added to the assumed take-off azimuth, to result in the true azimuth, and is used by the program automatically to correct all related data. This is particularly useful in site-adapting buildings for various orientations.

(h) Master Wall Height is a data take-off simplification factor, to permit the input of wall lengths only for the various areas to be considered, and allow the program to calculate areas and volumes automatically with the master height. This factor can be over-ridden in several ways for non-typical situations.

(3) Master Internal Data (fig. 1).

(a) Occupancy Load Factors, as Sensible and Latent BTU/ Hr./Person (5).

(b) Hour In, "From-Through". The numeric hour designation, 01 to 24, of the first and last hour in which the occupancy load is to be included. Hours used do not have to fall within the selected calculation hours. Also applies to lighting and appliance loads.

(c) Occupant Loading. An optional factor to assign occupants to Rooms on an area basis, and an optional over-ride limit for total simultaneous occupancy in the project. The program will pyramid occupancy data in accordance with other restraints, but will reduce the aggregate to this limit if exceeded.

(d) Lighting. An optional input reducing factor, to assign lighting load on the basis of area, and an optional factor to relegate the input percentage of the lighting load to return air, rather than have it become a part of the room load and thus affect the room cfm calculation. The deferred load is re-added after the affected room loads and cfm's have been summed into zone groups as later discussed.

(e) Infiltration. An optional factor for inclusion of infiltration in the load calculation for each room, both heating and cooling, based on the calculated room volume, and a multiplier which will be used to diversify a pyramid total of infiltration loads at the building summary level.

(f) Ventilation. A series of factors, each used to calculate an outdoor air cfm value for a specified air system. The program will compare the results, and use the largest value for load purposes.

1. Air Changes Per Hour, Cfm Per Sq. Ft., Cfm Per Person. Conventional ventilation rate factors.

2. Maximum Sq. Ft. Per Person. A form of diversity factor which can be used to reduce overall ventilation on a cfm/person basis, by limiting the density of occupancy for purposes of ventilation calculation only. The result of this calculation, if less than that resulting from the calculated occupancy on any other basis times the cfm/person factor, will supersede the latter.

(g) Diversity Factors. A series of optional decimal multipliers which will be applied to the respective building totals for lighting, people, and appliances, to prevent unrealistic pyramiding of loads for the entire project.

(h) Supply Air Master D.T.R. A fixed dehumidified air temperature rise in degrees F. for supply air Cfm calculations. Admittedly, one of the practical compromises in the program in its present form, but effective for most applications.

(i) System Type. An optional factor, to differentiate between system types as to chilled water or direct refrigerant expansion. If the former, the program will add 2.0% of the total project sensible load to account for pump horsepower; otherwise, no effect.

### d. Master Building Shell Data

Data for the various environmental surfaces, or the building "shell", are input once for each type as master data and stored on disk. Up to a maximum of twenty (20) different types for each shell classification can be considered, each accessible by assigned type number as needed. Alphameric descriptive data is input for each shell data type for identification listing in the output (fig. 1).

(1) Exterior Walls. Any vertical opaque fixed surface between the conditioned space and the outside weather.

(a) Decrement Factor; Time Lag. Data for calculation of equivalent temperature differentials (6).

(b) Color. Color of exterior surface, as "L" (light), "M" (medium), or "D" (dark), for varying effect of solar intensity on wall load.

(c) "U" Values. Conventional heat transmission coefficients in BTU/Hr./Sq. Ft./Deg. F. for the particular wall construction involved, winter and summer (7).

(d) Below Grade BTU/Hr./S.F. Direct heat loss factor for sub-grade walls, used in lieu of "U" factor and temperature differentials (8). Not used in cooling load calculations.

(2) Exterior Roofs. All data similar to that for walls, without of course any sub-grade factors.

(3) Interior Partitions. For the purpose of evaluating heat transfer to or from adjacent interior unconditioned spaces. Doors in interior partitions are considered insignificant, and included as a part of the gross wall area.

(a) "U" Value. As for walls and roofs.

(b) Temperature of Non-Conditioned Space. The arbitrarily selected temperature, winter and summer, of the adjacent space, in deg. F. Different temperatures for the same partition construction in different locations would be considered as different partition types.

(4) Floors and/or Ceilings. Any horizontal surface between the conditioned space and an adjacent unconditioned area.

(a) Air Space Beneath, Winter and Summer. "U" values are conventional for the floor and/or ceiling construction. Temperature inputs are those of the unconditioned areas, above and/or below the conditioned space, assumed as outside design values if left blank.

(b) Slab Below Grade. Direct heat loss factor in Btu/Hr./Sq.Ft. as for sub-grade walls (8), in lieu of a "U" value and temperature differential calculation.

(c) Slab on Grade. Direct heat loss factor for exposed slab edges in Btu/hr./lineal ft. (9).

(d) Both Floor, and Ceiling Exposed. An indicator signal that the same exposure occurs both above and below the conditioned space, causing the calculated load to be included twice.

(5) Exterior Doors. "U" value and door area input are used with exterior design temperature values for load calculations. Areas are also used to facilitate calculation of net exposed wall areas as discussed later. Cfm infiltration input is optional, for heating calculations only, and will supersede other infiltration values if greater (10).

(6) Windows. Any vertical glass surface between the conditioned space and the outside weather. Each unique combination of glass size, material, and interior and exterior shading factors constitutes a separate window type.

(a) Glass Type. Input is a type number which calls up Alpha and Tau coefficients for a particular glass material, thickness, and/or multi-pane combination from storage in permanent disk data files. For glass types other than those on file, the Special Glass Shading Coefficient is input alternatively as a compromise method of approximating glass solar characteristics (11).

(b) Height and Length dimensions, in feet and tenths, are for the gross window size and are used for transmission loads and for the mathematical framing of the glass area for calculation of sunlit and shaded portions thereof. Net glass area for solar gain consideration is calculated by use of the Percent Open Area input factor applied against the gross area.

(c) "U" values are for transmission calculations for winter losses (12) and for summer gains with and without interior shading devices (13). Values are increased for wiping effect of air stream (14) on inward flow fraction of transmission coefficient, if input indicates supply air will be admitted below the glass or between the glass and an interior shading device.

(d) Inside Shade Coefficient. The decimal effectiveness of the interior shading device, if any, in reflecting solar radiation (15).

(e) Exterior Shading effectiveness of overhangs and attached side fins or projections is a calculated value. Sizes of such devices and their dimensional relationships to the glass surface are simple input distances in feet and tenths, and are used to complete the three-dimensional mathematical representation of the window and shading surfaces. This model is then used with the various glass orientations and hourly solar angles for the selected load month to calculate the actual shade pattern under any condition, and to reduce the solar radiation load component representatively (16).

### e. Space Relationships and System Definition

As a program oriented to a constant volume air system, loads are accumulated and individually time-evaluated in three levels, "Room", "Zone", and "Building".

(1) The "Room" is the basic space unit for which specific input is defined as to size, internal loads, and number, type, and orientation of any exposed surfaces.

(2) The "Zone" is a group of "Rooms", input specified, to be considered as a single air system, at which level ventilation rates and loads are calculated and considered.

(3) The "Building" is a grouping of all "Zones" for overall consideration.

During initial planning the sponsoring organization, APEC, determined after considerable study of available hardware, that the best interests of most engineers would be served by a program which could be run on a relatively small installation (and with relatively simple modifications on larger systems as well). The IBM 1130 was found to be the machine most frequently being installed or considered in most engineering offices, and one which could accommodate most of the desired and all of the necessary features of HCC. A minimum configuration was thus established to include an 8K memory, single disk drive, card reader-punch, and 1132 line printer. This decision dictated the principal program restrictions to a maximum of 39 Zones, 1000 Rooms, and 320 Rooms per Zone.

The assignment of "Room" areas to specific "Zones" or air systems is also master data in the sense of pre-defining the space framework, and placing the information on disk file for later access at the time of Room load calculation. Individual characteristics of each Zone are included in such access for application to Room conditions.

(1) Zone Definition. Each Zone is given an individual number from 1 to 9999, as is each Room. Desired groupings of Rooms into specific Zones is accomplished by a series of input cards (fig. 1), each listing a Zone number and up to five Room numbers. Each Room number is followed by a group of 10 alphameric characters to serve as a Room name, which data is stored on disk for output purposes whenever the

corresponding number is printed out. The principal value of room names is of course to enhance the "stand-alone" characteristics of the output, both for engineering use and for permanent record, by eliminating the necessity of referring to plans or other related documents for identification.

(2) Duplicate Rooms. All Rooms must be defined as being included in some Zone. Input, run time, and output are all conserved however, in the case of two or more Rooms which are duplicates in size, exposure, orientation, and other load factors, by designating one of the group as a "Master" Room, and listing all others as "Duplicates" of that Master. Having done this, it is necessary only to input Room data for the Master Room, and the resultant load values will be assigned to the respective Zones for each duplicate Room as well. This procedure is reflected in the output in several meaningful ways.

(3) Zone Diversity Data. In most projects, such as hospitals, schools, office buildings, etc., the various air systems (Zones) are normally subject to some deviation in individual requirements from the master data input. To account for these differences, each Zone is separately listed by number and identification name, and any deviations are noted.

(a) Zone Load Diversity Factors are optional decimal multipliers to be used on pyramid totals of lighting, occupancy, and appliance loads for all Rooms in that Zone. If left blank, no diversity will be taken at the Zone level.

(b) Zone Ventilation Factors, with regard to Air Changes per Hour, Cfm/Sq.Ft., or Cfm/Person, are optionally input if it is desired to over-ride the building master factors for that Zone. If desired, a fixed cfm value can be input for ventilation purposes, or the ventilation cfm may be set equal to the supply cfm, either of which will supersede any other ventilation data.

(c) Zone Inside Design Conditions. Master data input for Summer and/or Winter Inside temperature and/or humidity ratio may be over-ridden for any Zone as desired, and will apply to each Room in that Zone.

(d) Maximum Zone Occupancy may be input if desired, as an upper limit to the number of occupants accumulated from the individual Room loads. Final zone totals, on whatever basis, will be used in the initial Building summary.

(e) Zone Dehumidified Temperature Rise, in deg. F, may be optionally input if desired to supersede the master data value. Cfm values for each Room in any Zone will be calculated on the controlling DTR basis.

(f) Miscellaneous Factors. Indicator signals may be optionally input for the purpose of omitting either the heating or the cooling calculation for any Zone, which will apply to any Room in such Zone. A Safety Factor percentage may also be input for any Zone, should load data uncertainty warrant it, which will be applied to the Room load totals. A System Type indicator signal is used for recognition of duct heat gains and leakage and fan Hp load, as "1" for a low pressure system (adds 4% to Room Sensible Heat), "2" for a high pressure system (adds 12% to Room Sensible Heat), or "0" for a room unit system (adding no additional load, and omitting cfm calculation) for all Rooms in that Zone.

### f. Room Data

The "Room", as previously indicated, is the basic load building block upon and around which all other load data is built. It is the final phase of input, in which the specific load factors applying to each basic space unit of the project are defined. As such, while complete and normally the most voluminous part of the input, Room data is strongly oriented to input simplfication.

One set of Room data is required for each Master Room defined, numerically identified, and consisting of one or more input cards depending upon the complexity of exterior exposure and extent of deviation from the master load factors. Input cards fall into three categories; Type 1 for Basic Room Data, Type 2 for Special Internal Load Data, and Type 3 for Exposure Data (fig. 1). An interior Room, with no vertical exterior exposure and which conforms to the master internal load factors, requires only a Type 1 card for complete input. Exterior Rooms with non-standard internal loads would require a Type 2 and at least one Type 3 card as well. Although there is no actual limit to the number of Type 3 cards which may be used for a given Room, most Rooms need just one, and only unusual exposure circumstances require more than two or three.

(1) Basic Room Data includes the minimum information required for spacial definition of the Room to be evaluated, and to establish the calculation framework.

(a) Exposure quantity and Special Internal Load card indicator signals are input as required, for the purpose of specifying the number and format of additional input cards which are to be read for this room.

(b) Room Dimensions are for area and volume calculations, and require lineal input only in ft. and hundredths. Room height need not be input unless it deviates from that in the Master Data.

(c) Roof, Floor, and Partition exposure is indicated by Type Number, if any, thus calling out of file the stored thermal characteristics and/or related data for the material or materials in question. Area inputs for Roof and/or Floor exposures are required only if such data is not equal to the product of Room length and width. Partition length input calculates area with room height.

(d) Cfm Infiltration is an optional direct load factor superseding any other input. Cfm Exhaust does not affect load calculations as such, but is accumulated at the Zone level and compared with ventilation calculation results, causing output of a warning message if exhaust exceeds ventilation.

(e) Number of People, Lighting Watts/Sq.Ft. and Special Hour Average are optional input factors, to over-ride Master Data if desired.

(2) Special Internal Load Data is optional throughout, and necessary only to over-ride various Master Data factors for non-typical situations.

(a) Lighting Data. In addition to over-riding master values for hours of operation and/or percent of light heat to return air, the exact Room wattage may be input if available which will supersede all other data. If lighting is incandescent, rather than the normal fluorescent predicated by the program, the indicator signal causes 80% of the lighting load to be considered radiant heat for hour averaging rather than 50% (17).

(b) Appliance Loads, in addition to "appliances" in the conventional sense, can represent any form of sensible and/or latent heat generation within the conditioned space. Input is directly in Btu/hr., with specified percentages of the load to be considered as radiant for time averaging, or convective as instantaneous room load. Hours of operation may be input if varying from Master Data.

(c) Occupancy over-ride values may be input for non-typical load periods or metabolism rates.

(d) Minimum Room Circulation Rate may be input in Minutes/Air Change, which will increase the calculated Cfm value accordingly if Room sensible heat requirements are inadequate for the desired air movement. If no input, the program assumes 30 minutes per air change to be the minimum acceptable rate.

(3) Exposure Data takes into account all vertical exterior exposures. Nominally, one card per specific orientation is used, except that complicated walls with a multiplicity of shell types may require several cards for complete input.

(a) Wall Azimuth Angle is the exposure orientation in degrees variance from "Plan South" as $0^{\circ}$ (west = $+90^{\circ}$, east = $-90^{\circ}$, north = $180^{\circ}$). The program will correct this input for actual orientation of the surface by adding the Master Data deviation factor, and will use the corrected value for calculation of all solar angles for this exposure.

(b) Length of Exposure in ft. and hundredths is used with Wall Height to calculate gross area of the exposed surface. It is also used directly to calculate slab edge losses, if so indicated by Floor type input.

(c) Walls are input by Type number, to access stored thermal transfer data. Two different wall types may be input per exposure card, but if so must also include individual net wall areas of each in sq. ft. and tenths. If only a single wall type is involved, as is normal, area input is not required.

(d) Windows and Doors are input by Type number (which includes area in stored data), and quantity count. Two different window types may be input per exposure card. The effect of external shading on windows may be approximated by input of time period during which complete shading is to be assumed in the calculations.

### 3.3. Output Capabilities

For best overall response to the goals of efficient immediate design use, presentation in design analysis form, and permanent record keeping, all printout is formatted to fit on standard $8\frac{1}{2}$" x 11" paper with binding edge allowances. A summary of Master Data (fig. 2) and Master Building Shell Data (fig. 3) input, suitability identified, appears at the beginning of the printout. This is followed by a summary of Zone Diversity Data (fig. 4), indicating the factors to be used by each Zone, whether specifically input for it or the Master Data value if not over-ridden. All Room numbers are sorted in ascending order, and so listed along with the Master Room and Zone number associated with each.

The amount and nature of printed output for Room, Zone, and Building calculations is subject to considerable control by the engineer. Room calculation results for example may be printed out on a single line per Room, including Room number and name, hour of peak load, peak cooling values of sensible and total heat gain in Btu/hr., supply air cfm and minutes/air change check figure, and heating losses in Btu/hr. This type printout, although comparatively rapid, is somewhat limited in usefulness since the associated input data and load component breakdown is not presented.

The normal Room printout (fig. 5) is considerably more useful, giving a complete record of all input factors (specific input and/or Master Data) contributing to the calculation as well as a thorough breakdown of the results. The underlying concept of practical, "stand alone" output format is perhaps best illustrated by this single sheet record of the basic load calculation unit.

(1) Complete Identification. The page heading identifies the firm or engineer responsible for the work, job number, job name or other related information, date of calculation, and page number in the overall data. Room number and name appears both at the top of the sheet as a conventional heading and at the lower right corner for rapid location in bound form. Zone identification and the total number of Rooms for which this is a Master Room appears in the lower left corner.

(2) Complete Record of Load Factors. All input data from Room cards is printed out with appropriate headings, together with calculated results where standard factors or other parameters were utilized.

(a) Areas are mirrored when input, or calculated as appropriate when not. Room area for example represents the product of input length times width, as do roof and floor areas if not specifically input otherwise, and if a roof and/or floor type number was input. Partition areas are the product of input length times wall height (input or master) as are exposed wall areas, except that related window and/or door areas are deducted from the latter leaving a net value. All areas are represented in sq.ft.

(b) People are listed as the input quantity or the calculated quantity by Master factor if not input. Lighting is presented both as total watts and watts/sq.ft., with values deriving from whatever specific input or Master data was permitted to control by the engineer.

(c) Hourly load factors for people, lights, and appliances are indicated as the input values or Master data if no specific input. The value used for hour averaging purposes is indicated, reflecting either a specific input or the Master value, including in the latter case a reduction of one hour for each exposure more than a single exposure (but never less than 1 hour).

(3) Complete Breakdown of Load Components. The peak cooling hour is indicated, and load values for that hour listed for window, wall, roof, partition, floor (and/or ceiling), door, infiltration, lights (% to room), people, and appliance load components, together with fan Hp and safety factor allowances if any. The percent of light heat to return air, if any, is deferred to the appropriate Zone load as later discussed. Heating load components are similarly listed for exterior exposure losses, with separate compilation of internal gain factors for comparative purposes. Winter solar gain is also presented to permit complete evaluation as to when internal gains may exceed winter losses.

(4) Cfm supply air calculation is based on satisfaction of Room sensible heat with Zone dehumidified air temperature rise (DTR). Result is compared with Room volume and minimum minutes/air change factor (input or master), with the larger selected and rounded off to the nearest 10. cfm.

(5) Check Figures, for Load Verification. Cooling data includes Btu/sq.ft., Btu/cu.ft., and sq.ft./Ton of refrigeration, as well as supply air checks by Minutes/air change and Cfm/sq.ft. Heating data includes Btu/sq.ft., Btu/cu.ft., and Sq.ft./MBtuh.

In addition to this data, an option is available at the Room level for printout of hourly load values, by load components, for each of the 12 load hours selected (fig. 6).

After all Room calculations have been completed and hourly load totals assigned to the appropriate Zones, the Zone printout occurs. Each Zone is presented on a single sheet (fig. 7), identified in a manner similar to that of the Room sheet. Since Zone diversity data is already an output record it is not repeated here, but is used to perform load evaluation as a complete air system serving a designated group of basic load units.

(1) Ventilation Calculations. As an air handling system, the introduction of ventilation is assumed to take place at the air handling unit. Accumulated area, volume, and diversified people quantities are utilized with the appropriate input factors to calculate ventilation cfm values. In the latter case, a comparative occupancy density is also calculated from the Master input limit value and used with the cfm/person factor to produce an upper limit cfm for this basis. The highest cfm value is then selected as the ventilation rate for the Zone, unless over-ridden by a fixed Zone input value as zero, some finite amount, or 100% of the accumulated Room supply air cfm. The ultimately selected cfm quantity is then used for calculation of sensible and latent heat gain values for each of the specified load evaluation hours, and for winter heat loss.

(2) Load component diversity input factors for this Zone are next applied to the accumulated hourly load totals for lights (both Room load and return air components), people, and appliances.

(3) Hourly Zone cooling load totals are then created as summaries of Room load components (including diversification effect, if any), ventilation load values, and the return air segment of light heat. The peak total value is selected and printed out in complete load breakdown form. For reference purposes, the accumulated load values for lights, people, and appliances before application of diversity factors are also printed out.

(4) Heat loss load components are similarly printed out, including a separate accumulation of internal heat gains. Btu/hr. equivalent of humidification requirements is calculated to offset effects of ventilation and infiltration air, and added to the heating subtotal for a comparative comprehensive value. As with the Room loads, further comparative totals reflect the effects of internal heat sources and external winter solar gain for the Zone.

(5) Check figures for both cooling and heating values are presented in the same form as for Room loads, but at two load levels; the Room total accumulation, and again after the inclusion of ventilation and return air heat loads. Total tons of refrigeration is also indicated at the lower left sheet corner, for rapid comparative evaluation by the engineer.

(6) Total supply air cfm to be provided by the system fan is indicated, representing a pyramid total of supply air requirements for all Rooms in that Zone. A "Coil Air" cfm figure is also printed, representing the ideal cfm required to offset the sensible heat requirements of the Zone at its peak load condition, using the same dehumidified air temperature rise value.

As for the Room level, an optional printout is available for hourly values of all cooling load components, for each of the selected load hours (fig. 8).

An additional optional Zone printout is an "Air Side Analysis, by Rooms" for that Zone (fig. 9). As such, it is principally geared to the practical use of the output data in the production of engineering design. Each Room in the Zone, whether Master or Duplicate, is listed on consecutive print lines by Room number, name, Room size in sq. ft. area and cu. ft. volume, supply air cfm, and minutes/air change check figure. Zone totals for area, volume, supply air, and cfm check figures are indicated following the final Room listing. Space is also provided for manual insertion of the inevitable Room number change, as well as any desired change in supply air value. This sheet, together with a one-line duct routing indication on a floor plan, is all the information required by the duct designer to produce his work.

As each Zone is printed out, its significant physical data and hourly load component values are summed into the Building level for final diversification and evaluation. The type of data and presentation format (fig. 10) is very similar to that for the Zone level. Building master diversity factors and/or over-ride limits are applied to Zone totals, whereas cfm values for supply air, exhaust air, and ventilation air are direct summaries of Zone results without modification. Infiltration totals are diversified or not according to the input data. Check figure data are similar to Zone figures, and based on the true simultaneous total loads. An hourly printout similar to that at the Zone level is available as an option for presentation of Building cooling loads.

## 4. Future Potential

The entire thrust of HCC is to solve the design load calculation problem in a comprehensive manner, with output geared to direct and immediate use by the engineer. In this achievement however it is obvious that the present useful results, no matter how valuable, are only a weak distillate of the powerful data within the program not yet directly usable.

The physical space definitions for example need little else to be suitable for lighting calculations, with resultant wattage data then available for original load calculations or the refinement of pre-calculated data. With a minimum of supplemental space relationship definition and operating parameters, the Room, Zone, and Cfm data can be used for design of duct systems, as can the Zone and Building load data for design of piping systems. Spacial comparisons are a logical next step, with prediction and/or avoidance of interferences between ducts, piping, lighting fixtures, beams, etc. From this point, the useful extension of interrelated data to embrace all building design disciplines is inevitable, and the benefits to be obtained virtually limitless.

The bridge to this development is the creation of a comprehensive computing system with a common data base, with entry available at a number of different points, each having access to data created by other disciplines and with the capability to modify and/or supplement such data appropriately with its own output. Just such an approach, known as the Computer Aided Building Design System (CABDS) is currently in initial development by APEC, and descriptions are appearing in current literature. Although of necessity geared to considerably larger hardware for full implementation, the system will also be capable of partial operation on the small 8K installations and thus retain a practical availability to the entire user spectrum.

Along parallel lines, the new ASHRAE response factor algorithms present an improved technical approach to both design load calculations and thermal energy analysis. This approach is under continuous evaluation by APEC, and will without question be incorporated into HCC as it is adapted toward use in CABDS. In the meanwhile, the current usefulness of HCC as a practical design tool is a substantial step forward in the development of the art, and a reassuring plateau upon which to regroup forces for the next breakthrough.

## 5. References

(1) Birdsall, Bruce E., Mechanical Engineer, Technical Director, APEC, Dayton, Ohio; Deeming, William B., Architectural Engineer and Programmer, Skidmore, Owings & Merrill, San Francisco, California; Sun, Tseng-Yao, Mechanical Engineer & Vice President, Ayres, Cohen & Hayakawa, Los Angeles, California.

(2) Table 8, p. 476, and intensity formulae, p. 479, ASHRAE Handbook of Fundamentals (1967).

(3) Table 10 and solar heat gain formulae, p. 479, ASHRAE Handbook of Fundamentals (1967).

(4) Table 20, p. 486, ASHRAE Handbook of Fundamentals (1967).

(5) Table 30, p. 497, ASHRAE Handbook of Fundamentals (1967).

(6) Table 27, p. 492, ASHRAE Handbook of Fundamentals (1967).

(7) Tables 4, 5, 7, 8, p. 435-440, ASHRAE Handbook of Fundamentals (1967).

(8) Table 1, p. 460, ASHRAE Handbook of Fundamentals (1967).

(9) Table 2, p. 460, ASHRAE Handbook of Fundamentals (1967).

(10) Fig. 5, 6, 7, p. 412, 413, ASHRAE Handbook of Fundamentals (1967).

(11) Table 12, p. 480, ASHRAE Handbook of Fundamentals (1967).

(12) Table 18, p. 453, ASHRAE Handbook of Fundamentals (1967).

(13) Table 9, p. 477, ASHRAE Handbook of Fundamentals (1967).

(14) Discussion, p. 477-480, ASHRAE Handbook of Fundamentals (1967).

(15) Tables 15, 16, 17, p. 482; fig. 7, 8, 9, p. 482, 483; Table 19, p. 485, ASHRAE Handbook of Fundamentals (1967).

(16) Sun, Tseng-Yao: Shadow Area Equations for Window Overhangs and Side Fins and Their Application in Computer Calculation. (ASHRAE Transactions Volume 74, 1968).

(17) Table 29, p. 496, ASHRAE Handbook of Fundamentals (1967).

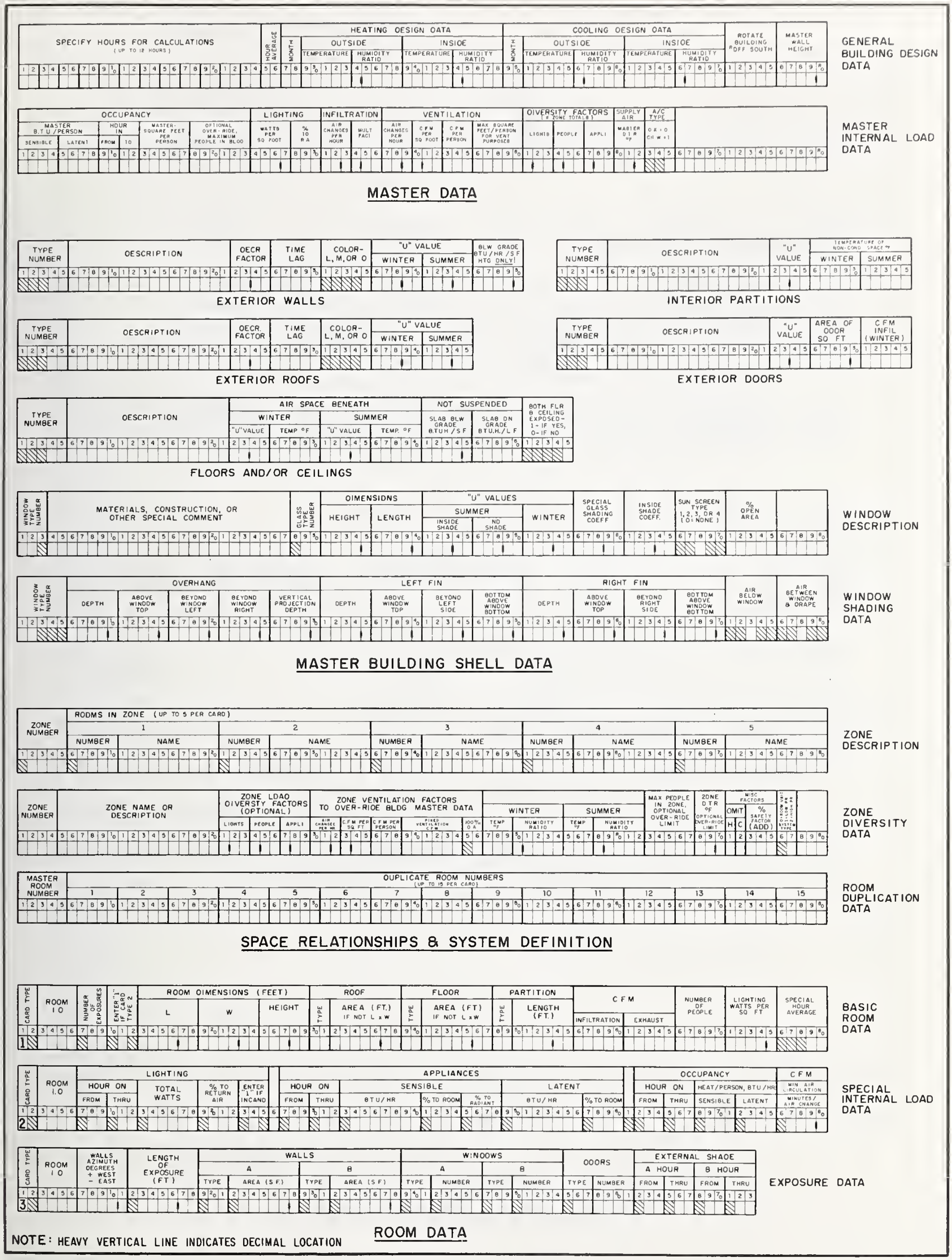
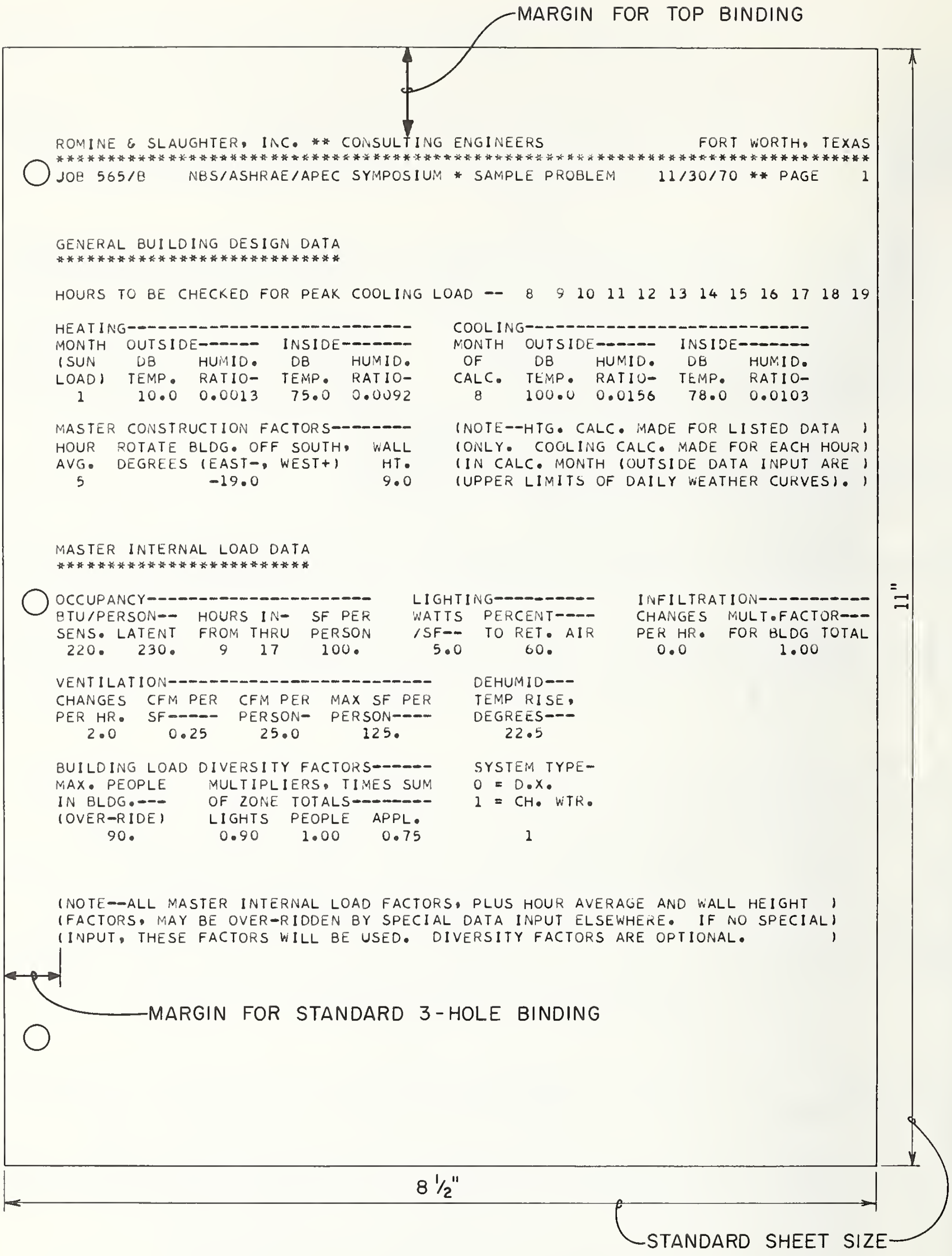


Figure 1. Room Data



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********************************************************************************
JOB 565/B    NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM    11/30/70 ** PAGE    1


GENERAL BUILDING DESIGN DATA
****************************

HOURS TO BE CHECKED FOR PEAK COOLING LOAD --  8  9 10 11 12 13 14 15 16 17 18 19

HEATING----------------------------    COOLING----------------------------
MONTH  OUTSIDE------  INSIDE-------    MONTH  OUTSIDE------  INSIDE-------
(SUN    DB    HUMID.   DB    HUMID.     OF     DB    HUMID.   DB    HUMID.
LOAD)  TEMP.  RATIO-  TEMP.  RATIO-    CALC.  TEMP.  RATIO-  TEMP.  RATIO-
  1     10.0  0.0013   75.0  0.0092      8    100.0  0.0156   78.0  0.0103

MASTER CONSTRUCTION FACTORS--------    (NOTE--HTG. CALC. MADE FOR LISTED DATA  )
HOUR  ROTATE BLDG. OFF SOUTH,  WALL    (ONLY.  COOLING CALC. MADE FOR EACH HOUR)
AVG.  DEGREES (EAST-, WEST+)    HT.    (IN CALC. MONTH (OUTSIDE DATA INPUT ARE )
  5            -19.0            9.0    (UPPER LIMITS OF DAILY WEATHER CURVES). )


MASTER INTERNAL LOAD DATA
*************************

OCCUPANCY----------------------    LIGHTING----------    INFILTRATION-----------
BTU/PERSON--  HOURS IN-  SF PER    WATTS  PERCENT----    CHANGES  MULT.FACTOR---
SENS. LATENT  FROM THRU  PERSON    /SF--  TO RET. AIR    PER HR.  FOR BLDG TOTAL
 220.   230.    9   17    100.       5.0      60.          0.0         1.00

VENTILATION--------------------------    DEHUMID---
CHANGES  CFM PER  CFM PER  MAX SF PER    TEMP RISE,
PER HR.  SF-----  PERSON-  PERSON----    DEGREES---
   2.0     0.25     25.0      125.          22.5

BUILDING LOAD DIVERSITY FACTORS------    SYSTEM TYPE-
MAX. PEOPLE    MULTIPLIERS, TIMES SUM    0 = D.X.
IN BLDG.---    OF ZONE TOTALS--------    1 = CH. WTR.
(OVER-RIDE)    LIGHTS  PEOPLE  APPL.
     90.        0.90    1.00    0.75          1


(NOTE--ALL MASTER INTERNAL LOAD FACTORS, PLUS HOUR AVERAGE AND WALL HEIGHT  )
(FACTORS, MAY BE OVER-RIDDEN BY SPECIAL DATA INPUT ELSEWHERE.  IF NO SPECIAL)
(INPUT, THESE FACTORS WILL BE USED.  DIVERSITY FACTORS ARE OPTIONAL.        )
```
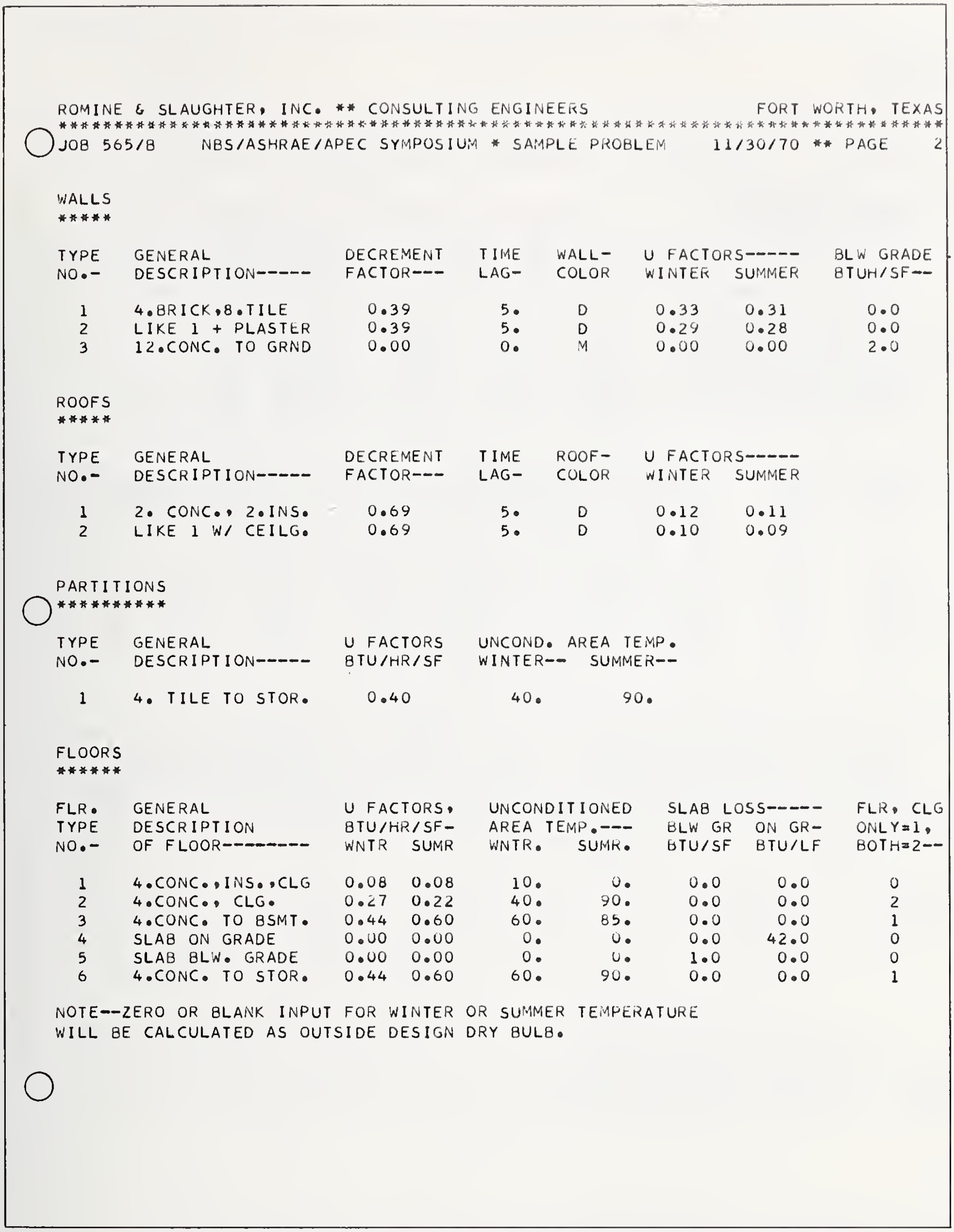

Figure 2. Typical Printout - Master Data



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JOB 565/B     NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE    2

WALLS
*****

TYPE    GENERAL                 DECREMENT    TIME     WALL-    U FACTORS-----    BLW GRADE
NO.-    DESCRIPTION-----        FACTOR---    LAG-     COLOR    WINTER  SUMMER    BTUH/SF--

  1     4.BRICK,8.TILE            0.39         5.       D       0.33    0.31        0.0
  2     LIKE 1 + PLASTER          0.39         5.       D       0.29    0.28        0.0
  3     12.CONC. TO GRND          0.00         0.       M       0.00    0.00        2.0

ROOFS
*****

TYPE    GENERAL                 DECREMENT    TIME     ROOF-    U FACTORS-----
NO.-    DESCRIPTION-----        FACTOR---    LAG-     COLOR    WINTER  SUMMER

  1     2. CONC., 2.INS.          0.69         5.       D       0.12    0.11
  2     LIKE 1 W/ CEILG.          0.69         5.       D       0.10    0.09

PARTITIONS
**********

TYPE    GENERAL                 U FACTORS    UNCOND. AREA TEMP.
NO.-    DESCRIPTION-----        BTU/HR/SF    WINTER--  SUMMER--

  1     4. TILE TO STOR.          0.40         40.       90.

FLOORS
******

FLR.    GENERAL                 U FACTORS,    UNCONDITIONED    SLAB LOSS-----    FLR, CLG
TYPE    DESCRIPTION             BTU/HR/SF-    AREA TEMP.---    BLW GR   ON GR-   ONLY=1,
NO.-    OF FLOOR--------        WNTR  SUMR    WNTR.    SUMR.   BTU/SF   BTU/LF   BOTH=2--

  1     4.CONC.,INS.,CLG        0.08  0.08     10.       0.      0.0      0.0       0
  2     4.CONC., CLG.           0.27  0.22     40.      90.      0.0      0.0       2
  3     4.CONC. TO BSMT.        0.44  0.60     60.      85.      0.0      0.0       1
  4     SLAB ON GRADE           0.00  0.00      0.       0.      0.0     42.0       0
  5     SLAB BLW. GRADE         0.00  0.00      0.       0.      1.0      0.0       0
  6     4.CONC. TO STOR.        0.44  0.60     60.      90.      0.0      0.0       1

NOTE--ZERO OR BLANK INPUT FOR WINTER OR SUMMER TEMPERATURE
WILL BE CALCULATED AS OUTSIDE DESIGN DRY BULB.
```

Figure 3. Typical Printout - Master Building Shell Data

```
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JOB 565/B    NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM    11/30/70 ** PAGE     7

ZONE DIVERSITY DATA
*******************

ZONE ZONE NAME-------   MULTIPLIERS, TIMES SUM   VENTILATION FACTORS-----------
NO.- OR--------------   OF ROOM TOTALS--------   CHGS  CFM  CFM-  FIXED---  ALL-
---- DESCRIPTION-----   LIGHTS  PEOPLE  APPLI.   /HR.  /SF  /PER  VENT-CFM  O.A.

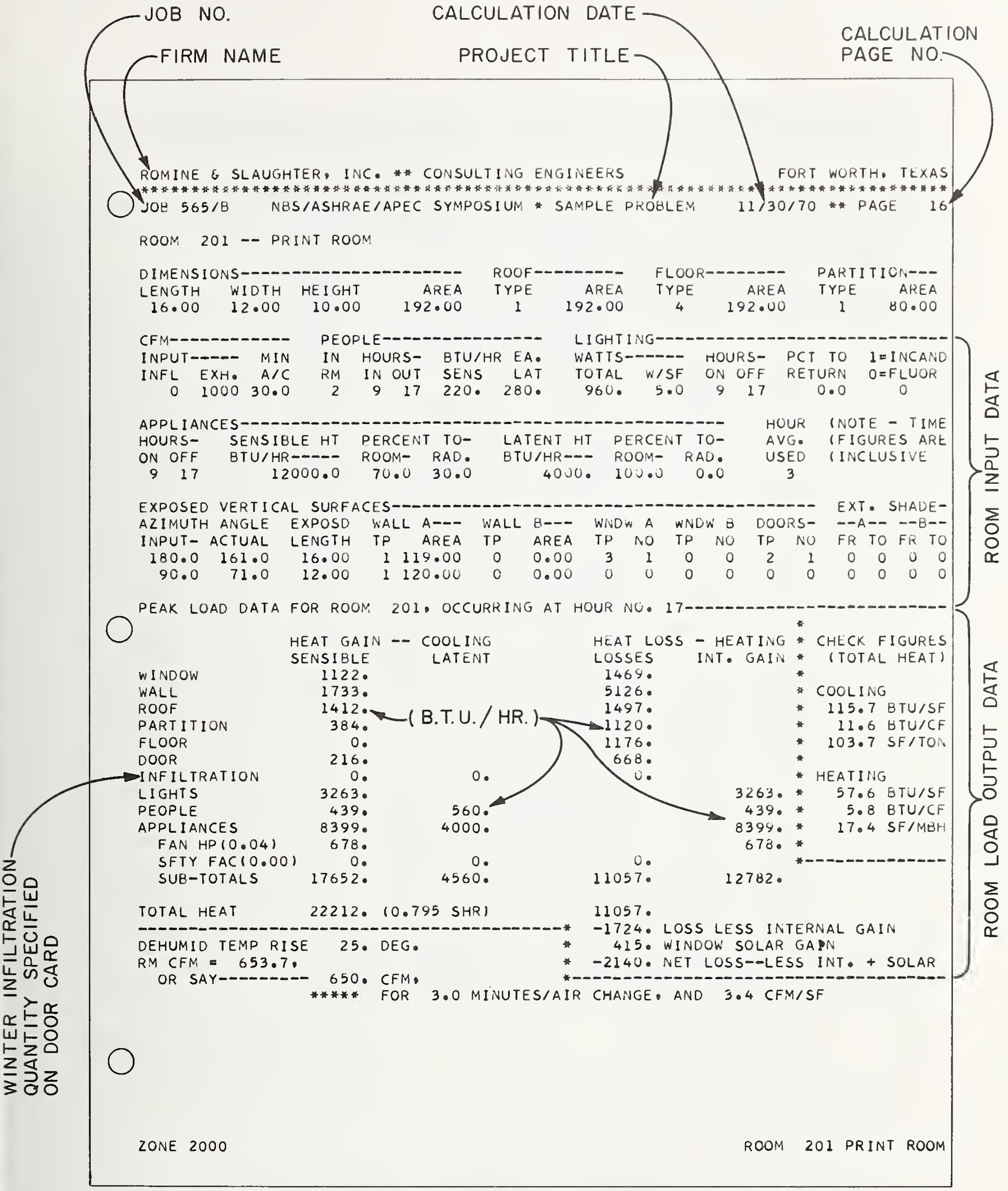
1000 ENGINEERING AREA    1.00    0.90    1.00     2.0 0.25  30.0
2000 REPRO. & MAILING    1.00    1.00    0.75     2.0 0.25  25.0             X
3000 PERSONNEL MNGMNT    0.95    1.00    1.00     2.0 0.25  25.0       0.
4000 SERVICE & STORES    1.00    1.00    1.00     2.0 0.25  25.0
5000 EXECUTIVE SUITE     0.85    1.00    1.00     2.0 0.25  25.0    2000.
6000 DISTRIBUTION        1.00    0.80    1.00     2.5 0.30  25.0

ZONE   MAXIMUM   ZONE INSIDE DESIGN COND.--   ZONE   AIR SYSTEM   MISCELLANEOUS-
NO.-   PEOPLE-   WINTER------  SUMMER------   DTR-   0=RM UNIT,   OMIT---  PERCT
----   IN-----   TEMP  HUMID.  TEMP  HUMID.   DEG.   1=LOW PR.,   CALC.--  SAFT.
----   ZONE---   DEG.  RATIO-  DEG.  RATIO-   F.--   2=HIGH PR.   HTG CLG  FACT.

1000      35.     75.  0.0092   75.  0.0092   22.5       1                 10.0
2000       5.     75.  0.0092   78.  0.0103   25.0       1                  0.0
3000       *.     75.  0.0092   78.  0.0103   22.5       0         X        0.0
4000       *.     60.  0.0013   78.  0.0103   22.5       0             X    0.0
5000      60.     77.  0.0090   72.  0.0100   22.5       2                 15.0
6000      18.     75.  0.0092   78.  0.0103   22.5       2                  0.0


(NOTE--ALL ZONE DIVERSITY DATA IS OPTIONAL INPUT, TO OVER-RIDE BUILDING MASTER )
(DATA.  AIR SYSTEM TYPE SHOULD ALWAYS BE SPECIFIED, TO ACCOUNT FOR APPROPRIATE )
(AMOUNT OF FAN HEAT, DUCT GAIN, + DUCT LEAKAGE -- 0 FOR ROOM FAN AND COIL UNIT,)
(4 PERCENT FOR LOW PRESSURE DUCT SYSTEM, AND 12 PERCENT FOR HIGH PRESSURE DUCT )
(SYSTEM, ADDED TO ROOM SENSIBLE LOADS.  SAFETY FACTOR IS A PERCENT FIGURE TO BE)
(ADDED TO ROOM SENSIBLE, LATENT, AND HEATING LOADS.  * = SUM OF ROOM TOTALS.   )
(MASTER DATA LISTED IF NO OVER-RIDE INPUT.                                      )
```

Figure 4.  Typical Printout - Zone Diversity Data



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JOB 565/B     NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE   16

ROOM  201 -- PRINT ROOM

DIMENSIONS----------------------    ROOF---------    FLOOR--------    PARTITION---
LENGTH    WIDTH  HEIGHT       AREA   TYPE      AREA   TYPE      AREA   TYPE     AREA
 16.00    12.00   10.00     192.00     1     192.00     4     192.00     1     80.00

CFM-----------    PEOPLE----------------    LIGHTING----------------------------------
INPUT-----  MIN   IN   HOURS-   BTU/HR EA.   WATTS------    HOURS-  PCT TO   1=INCAND
INFL  EXH.  A/C   RM   IN OUT   SENS   LAT   TOTAL   W/SF   ON OFF  RETURN   0=FLUOR
   0  1000 30.0    2    9  17   220.   280.   960.    5.0    9  17    0.0       0

APPLIANCES------------------------------------------------------     HOUR  (NOTE - TIME
HOURS-    SENSIBLE HT   PERCENT TO-    LATENT HT   PERCENT TO-       AVG.  (FIGURES ARE
ON OFF    BTU/HR-----   ROOM-  RAD.    BTU/HR---   ROOM-  RAD.       USED  (INCLUSIVE
 9  17        12000.0   70.0   30.0       4000.    100.0   0.0         3

EXPOSED VERTICAL SURFACES------------------------------------------------  EXT. SHADE-
AZIMUTH ANGLE  EXPOSD  WALL A---   WALL B---   WNDW A   WNDW B   DOORS-    --A-- --B--
INPUT- ACTUAL  LENGTH  TP   AREA   TP   AREA   TP  NO   TP  NO   TP  NO    FR TO FR TO
 180.0  161.0   16.00  1 119.00    0    0.00    3   1    0   0    2   1     0  0  0  0
  90.0   71.0   12.00  1 120.00    0    0.00    0   0    0   0    0   0     0  0  0  0

PEAK LOAD DATA FOR ROOM  201, OCCURRING AT HOUR NO. 17------------------------------
                                                                     *
                    HEAT GAIN -- COOLING       HEAT LOSS - HEATING   * CHECK FIGURES
                    SENSIBLE     LATENT        LOSSES     INT. GAIN  *  (TOTAL HEAT)
WINDOW                 1122.                   1469.                 *
WALL                   1733.                   5126.                 * COOLING
ROOF                   1412.                   1497.                 *  115.7 BTU/SF
PARTITION               384.                   1120.                 *   11.6 BTU/CF
FLOOR                     0.                   1176.                 *  103.7 SF/TON
DOOR                    216.                    668.                 *
INFILTRATION              0.          0.          0.                 * HEATING
LIGHTS                 3263.                               3263.     *   57.6 BTU/SF
PEOPLE                  439.        560.                    439.     *    5.8 BTU/CF
APPLIANCES             8399.       4000.                   8399.     *   17.4 SF/MBH
  FAN HP(0.04)          678.                                678.     *
  SFTY FAC(0.00)          0.          0.          0.                 *-------------
  SUB-TOTALS          17652.       4560.      11057.      12782.

TOTAL HEAT            22212. (0.795 SHR)      11057.
------------------------------------------*   -1724. LOSS LESS INTERNAL GAIN
DEHUMID TEMP RISE   25. DEG.              *     415. WINDOW SOLAR GAIN
RM CFM =  653.7,                          *   -2140. NET LOSS--LESS INT. + SOLAR
  OR SAY---------   650. CFM,             *-----------------------------------------
                   *****  FOR  3.0 MINUTES/AIR CHANGE, AND  3.4 CFM/SF




ZONE 2000                                                       ROOM  201 PRINT ROOM
```
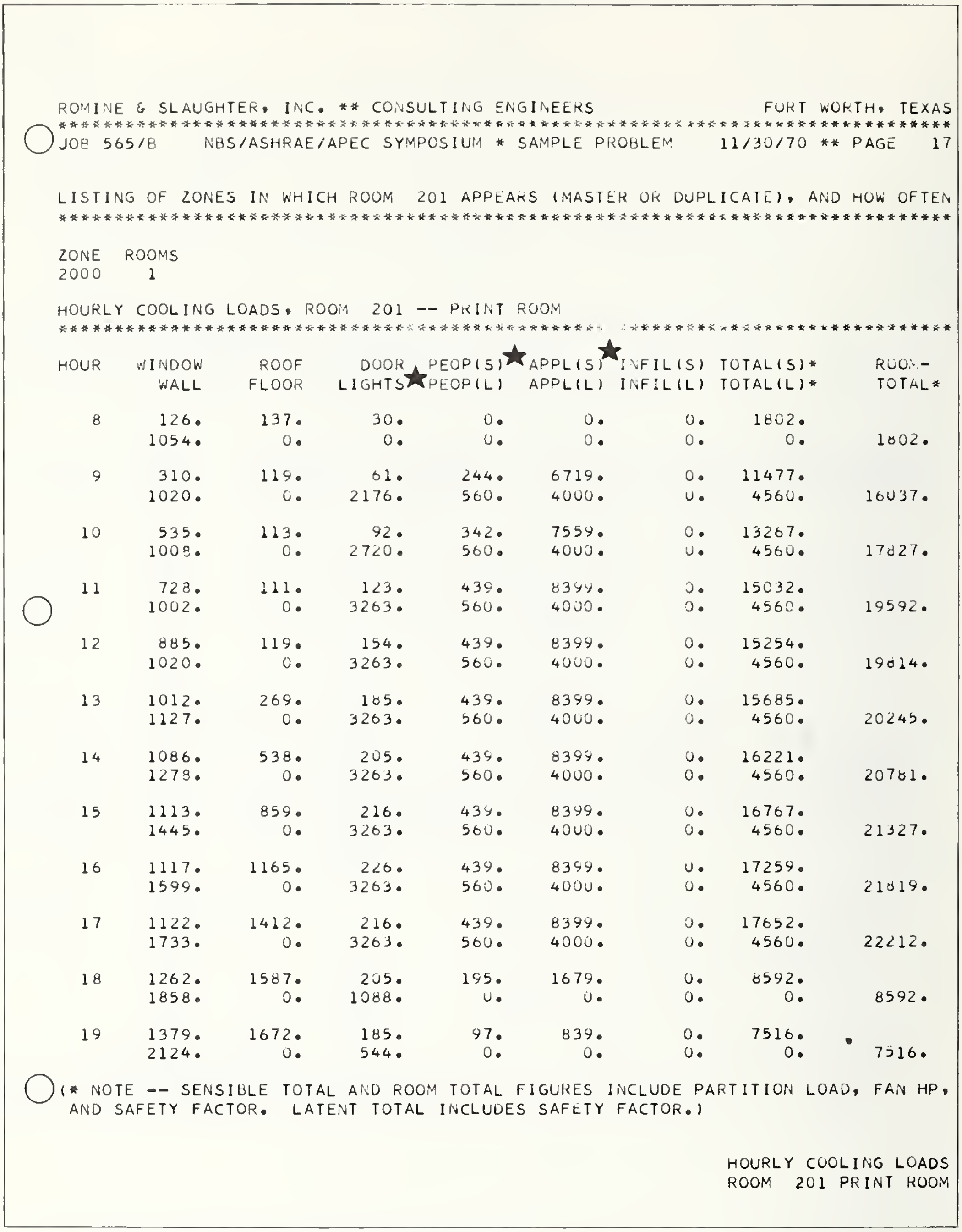

Figure 5. Typical Room Load Printout



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JOB 565/B     NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE   17

LISTING OF ZONES IN WHICH ROOM  201 APPEARS (MASTER OR DUPLICATE), AND HOW OFTEN
*******************************************************************************

ZONE   ROOMS
2000     1

HOURLY COOLING LOADS, ROOM  201 -- PRINT ROOM
*******************************************************************************

HOUR    WINDOW     ROOF      DOOR  PEOP(S)★APPL(S)★INFIL(S) TOTAL(S)*    ROOM-
          WALL    FLOOR   LIGHTS★PEOP(L)  APPL(L)  INFIL(L) TOTAL(L)*    TOTAL*

   8      126.     137.      30.      0.       0.       0.     1802.
         1054.       0.       0.      0.       0.       0.        0.     1802.

   9      310.     119.      61.    244.    6719.       0.    11477.
         1020.       0.    2176.    560.    4000.       0.     4560.    16037.

  10      535.     113.      92.    342.    7559.       0.    13267.
         1008.       0.    2720.    560.    4000.       0.     4560.    17827.

  11      728.     111.     123.    439.    8399.       0.    15032.
         1002.       0.    3263.    560.    4000.       0.     4560.    19592.

  12      885.     119.     154.    439.    8399.       0.    15254.
         1020.       0.    3263.    560.    4000.       0.     4560.    19814.

  13     1012.     269.     185.    439.    8399.       0.    15685.
         1127.       0.    3263.    560.    4000.       0.     4560.    20245.

  14     1086.     538.     205.    439.    8399.       0.    16221.
         1278.       0.    3263.    560.    4000.       0.     4560.    20781.

  15     1113.     859.     216.    439.    8399.       0.    16767.
         1445.       0.    3263.    560.    4000.       0.     4560.    21327.

  16     1117.    1165.     226.    439.    8399.       0.    17259.
         1599.       0.    3263.    560.    4000.       0.     4560.    21819.

  17     1122.    1412.     216.    439.    8399.       0.    17652.
         1733.       0.    3263.    560.    4000.       0.     4560.    22212.

  18     1262.    1587.     205.    195.    1679.       0.     8592.
         1858.       0.    1088.      0.       0.       0.        0.     8592.

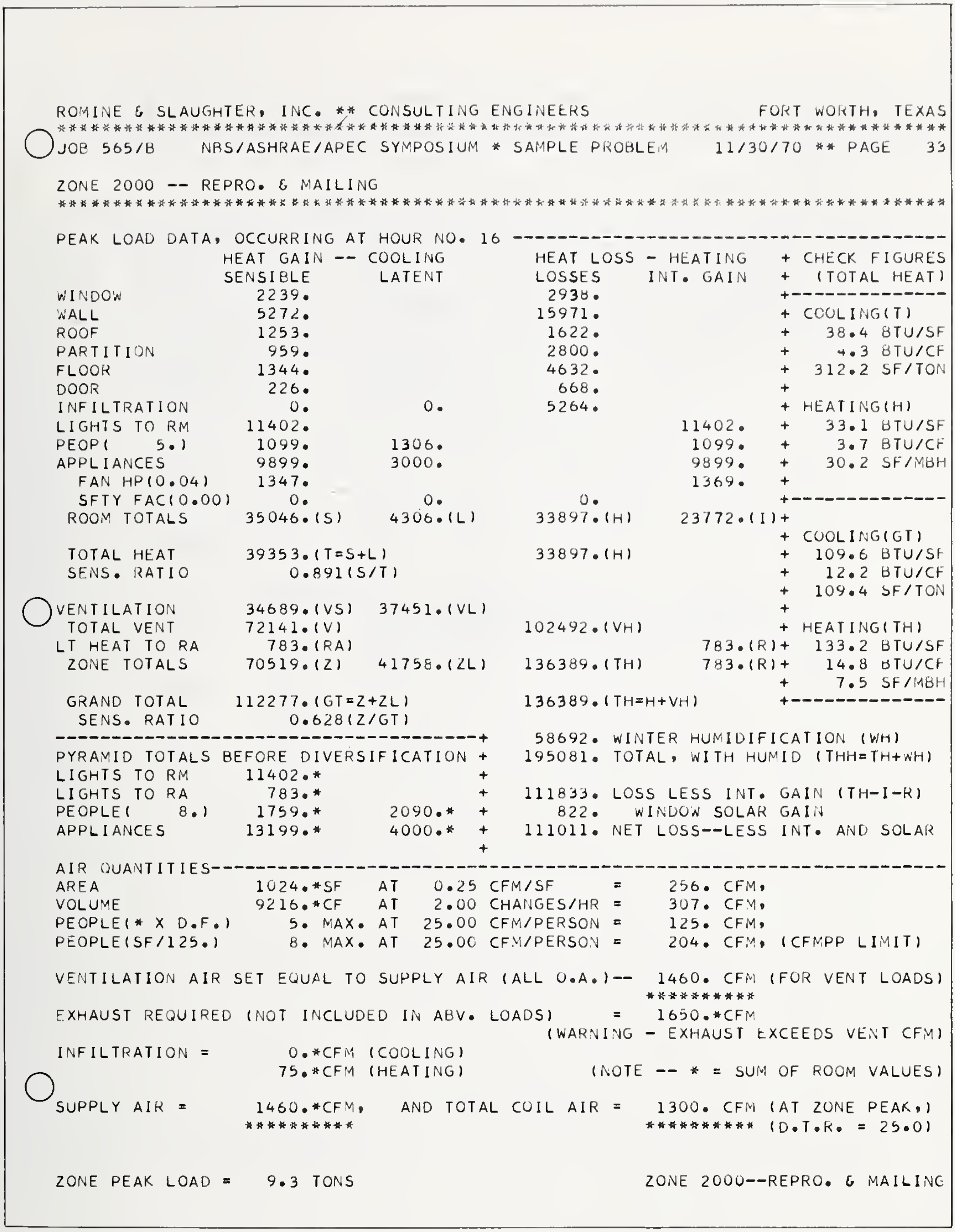
  19     1379.    1672.     185.     97.     839.       0.     7516.
         2124.       0.     544.      0.       0.       0.        0.     7516.

(* NOTE -- SENSIBLE TOTAL AND ROOM TOTAL FIGURES INCLUDE PARTITION LOAD, FAN HP,
 AND SAFETY FACTOR.  LATENT TOTAL INCLUDES SAFETY FACTOR.)

                                                          HOURLY COOLING LOADS
                                                          ROOM  201 PRINT ROOM
```

(★ NOTE HOUR AVERAGING EFFECT)

Figure 6. Optional Cooling Load Hourly Printout



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ROMINE & SLAUGHTER, INC. ** CONSULTING ENGINEERS                FORT WORTH, TEXAS
*********************************************************************************
JOB 565/B    NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE   33

ZONE 2000 -- REPRO. & MAILING
*********************************************************************************

PEAK LOAD DATA, OCCURRING AT HOUR NO. 16 ----------------------------------------
               HEAT GAIN -- COOLING         HEAT LOSS - HEATING   + CHECK FIGURES
               SENSIBLE      LATENT         LOSSES    INT. GAIN   +  (TOTAL HEAT)
WINDOW             2239.                      2938.               +--------------
WALL               5272.                     15971.               + COOLING(T)
ROOF               1253.                      1622.               +   38.4 BTU/SF
PARTITION           959.                      2800.               +    4.3 BTU/CF
FLOOR              1344.                      4632.               +  312.2 SF/TON
DOOR                226.                       668.               +
INFILTRATION          0.         0.           5264.               + HEATING(H)
LIGHTS TO RM      11402.                                 11402.   +   33.1 BTU/SF
PEOP(    5.)       1099.      1306.                       1099.   +    3.7 BTU/CF
APPLIANCES         9899.      3000.                       9899.   +   30.2 SF/MBH
  FAN HP(0.04)     1347.                                  1369.   +
  SFTY FAC(0.00)      0.         0.              0.               +--------------
 ROOM TOTALS      35046.(S)   4306.(L)       33897.(H)   23772.(I)+
                                                                  + COOLING(GT)
 TOTAL HEAT       39353.(T=S+L)              33897.(H)            +  109.6 BTU/SF
 SENS. RATIO          0.891(S/T)                                  +   12.2 BTU/CF
                                                                  +  109.4 SF/TON
VENTILATION       34689.(VS) 37451.(VL)                           +
 TOTAL VENT       72141.(V)                 102492.(VH)           + HEATING(TH)
LT HEAT TO RA       783.(RA)                                783.(R)+  133.2 BTU/SF
 ZONE TOTALS      70519.(Z)  41758.(ZL)     136389.(TH)     783.(R)+   14.8 BTU/CF
                                                                  +    7.5 SF/MBH
 GRAND TOTAL     112277.(GT=Z+ZL)           136389.(TH=H+VH)      +--------------
  SENS. RATIO         0.628(Z/GT)
--------------------------------------+      58692. WINTER HUMIDIFICATION (WH)
PYRAMID TOTALS BEFORE DIVERSIFICATION +     195081. TOTAL, WITH HUMID (THH=TH+WH)
LIGHTS TO RM      11402.*             +
LIGHTS TO RA        783.*             +     111833. LOSS LESS INT. GAIN (TH-I-R)
PEOPLE(    8.)     1759.*     2090.*  +        822.   WINDOW SOLAR GAIN
APPLIANCES        13199.*     4000.*  +     111011. NET LOSS--LESS INT. AND SOLAR
                                      +
AIR QUANTITIES-------------------------------------------------------------------
AREA              1024.*SF   AT   0.25 CFM/SF      =    256. CFM,
VOLUME            9216.*CF   AT   2.00 CHANGES/HR  =    307. CFM,
PEOPLE(* X D.F.)     5. MAX. AT  25.00 CFM/PERSON  =    125. CFM,
PEOPLE(SF/125.)      8. MAX. AT  25.00 CFM/PERSON  =    204. CFM, (CFMPP LIMIT)

VENTILATION AIR SET EQUAL TO SUPPLY AIR (ALL O.A.)--  1460. CFM (FOR VENT LOADS)
                                                      **********
EXHAUST REQUIRED (NOT INCLUDED IN ABV. LOADS)      =    1650.*CFM
                                             (WARNING - EXHAUST EXCEEDS VENT CFM)
INFILTRATION =       0.*CFM (COOLING)
                    75.*CFM (HEATING)            (NOTE -- * = SUM OF ROOM VALUES)

SUPPLY AIR =      1460.*CFM,   AND TOTAL COIL AIR =    1300. CFM (AT ZONE PEAK,)
                 **********                           ********** (D.T.R. = 25.0)


ZONE PEAK LOAD =   9.3 TONS                           ZONE 2000--REPRO. & MAILING
```
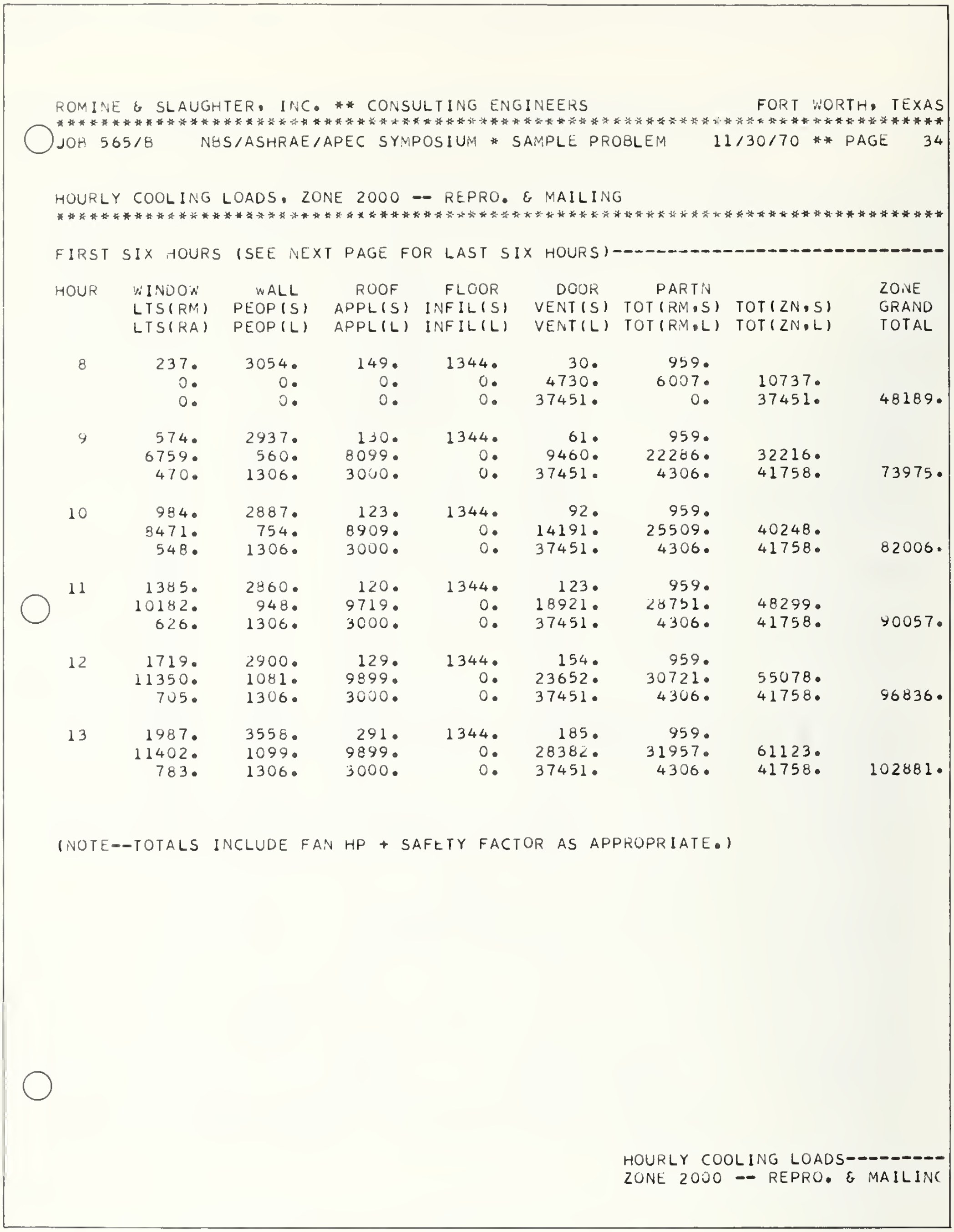

Figure 7. Typical Zone Load Printout



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*********************************************************************************
JOB 565/B     NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE    34

HOURLY COOLING LOADS, ZONE 2000 -- REPRO. & MAILING
*********************************************************************************

FIRST SIX HOURS (SEE NEXT PAGE FOR LAST SIX HOURS)-------------------------------

HOUR     WINDOW     WALL      ROOF     FLOOR       DOOR      PARTN                  ZONE
         LTS(RM)  PEOP(S)  APPL(S)  INFIL(S)    VENT(S)  TOT(RM,S)  TOT(ZN,S)       GRAND
         LTS(RA)  PEOP(L)  APPL(L)  INFIL(L)    VENT(L)  TOT(RM,L)  TOT(ZN,L)       TOTAL

  8        237.     3054.     149.    1344.        30.       959.
             0.        0.       0.       0.      4730.      6007.     10737.
             0.        0.       0.       0.     37451.         0.     37451.       48189.

  9        574.     2937.     130.    1344.        61.       959.
          6759.      560.    8099.       0.      9460.     22286.     32216.
           470.     1306.    3000.       0.     37451.      4306.     41758.       73975.

 10        984.     2887.     123.    1344.        92.       959.
          8471.      754.    8909.       0.     14191.     25509.     40248.
           548.     1306.    3000.       0.     37451.      4306.     41758.       82006.

 11       1385.     2860.     120.    1344.       123.       959.
         10182.      948.    9719.       0.     18921.     28751.     48299.
           626.     1306.    3000.       0.     37451.      4306.     41758.       90057.

 12       1719.     2900.     129.    1344.       154.       959.
         11350.     1081.    9899.       0.     23652.     30721.     55078.
           705.     1306.    3000.       0.     37451.      4306.     41758.       96836.

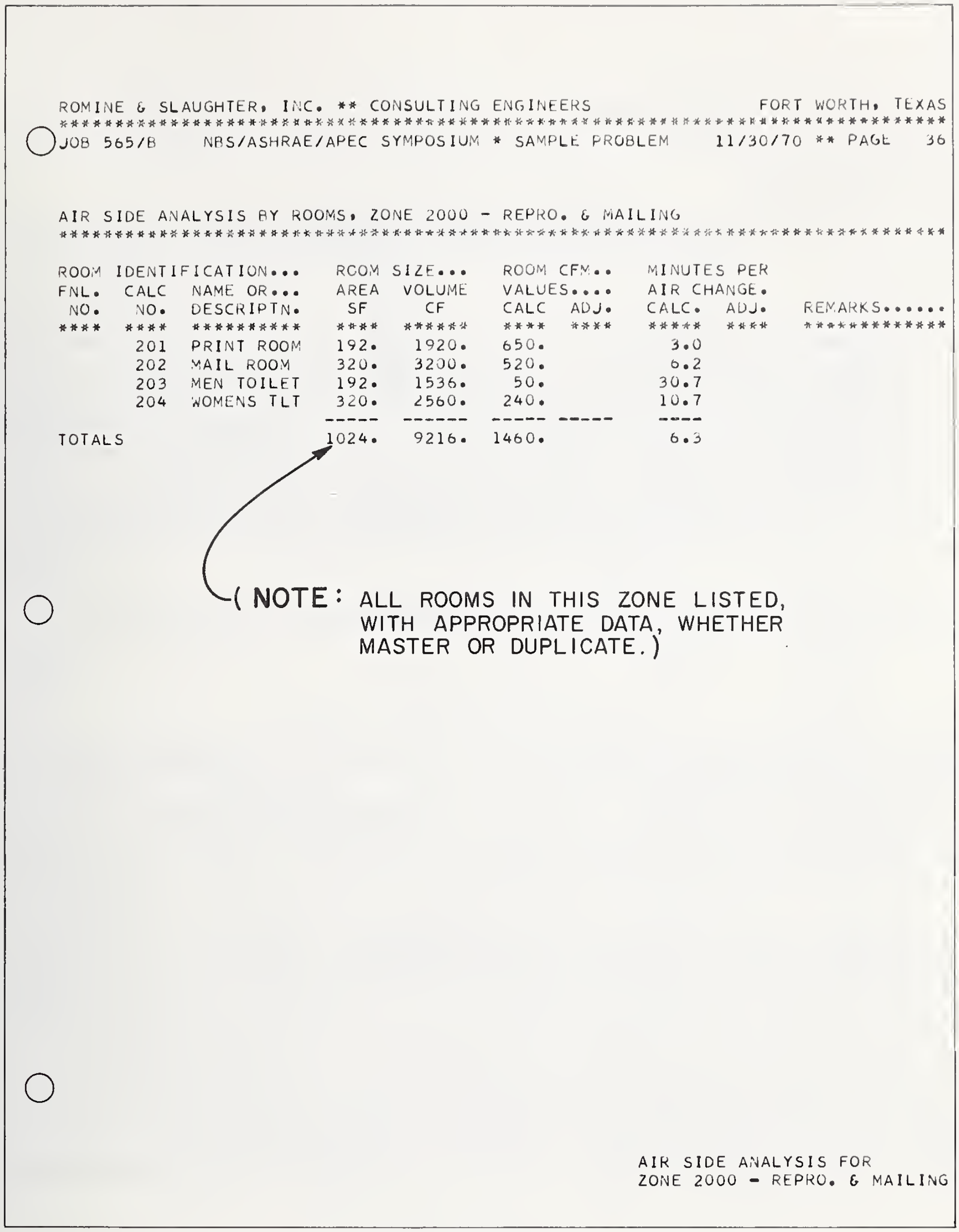
 13       1987.     3558.     291.    1344.       185.       959.
         11402.     1099.    9899.       0.     28382.     31957.     61123.
           783.     1306.    3000.       0.     37451.      4306.     41758.      102881.


(NOTE--TOTALS INCLUDE FAN HP + SAFETY FACTOR AS APPROPRIATE.)




                                                      HOURLY COOLING LOADS---------
                                                      ZONE 2000 -- REPRO. & MAILIN
```

Figure 8. Optional Zone Cooling Load Hourly Printout



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ROMINE & SLAUGHTER, INC. ** CONSULTING ENGINEERS                FORT WORTH, TEXAS
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JOB 565/B     NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM     11/30/70 ** PAGE   36


AIR SIDE ANALYSIS BY ROOMS, ZONE 2000 - REPRO. & MAILING
*********************************************************************************

ROOM IDENTIFICATION...    ROOM SIZE...    ROOM CFM..     MINUTES PER
FNL.  CALC  NAME OR...    AREA  VOLUME    VALUES....     AIR CHANGE.
 NO.   NO.  DESCRIPTN.     SF     CF      CALC  ADJ.     CALC.  ADJ.    REMARKS......
****  ****  **********    ****  ******    ****  ****     *****  ****    ************
       201  PRINT ROOM    192.   1920.    650.             3.0
       202  MAIL ROOM     320.   3200.    520.             6.2
       203  MEN TOILET    192.   1536.     50.            30.7
       204  WOMENS TLT    320.   2560.    240.            10.7
                          -----  ------  ----- -----       ----
TOTALS                    1024.   9216.  1460.             6.3
```
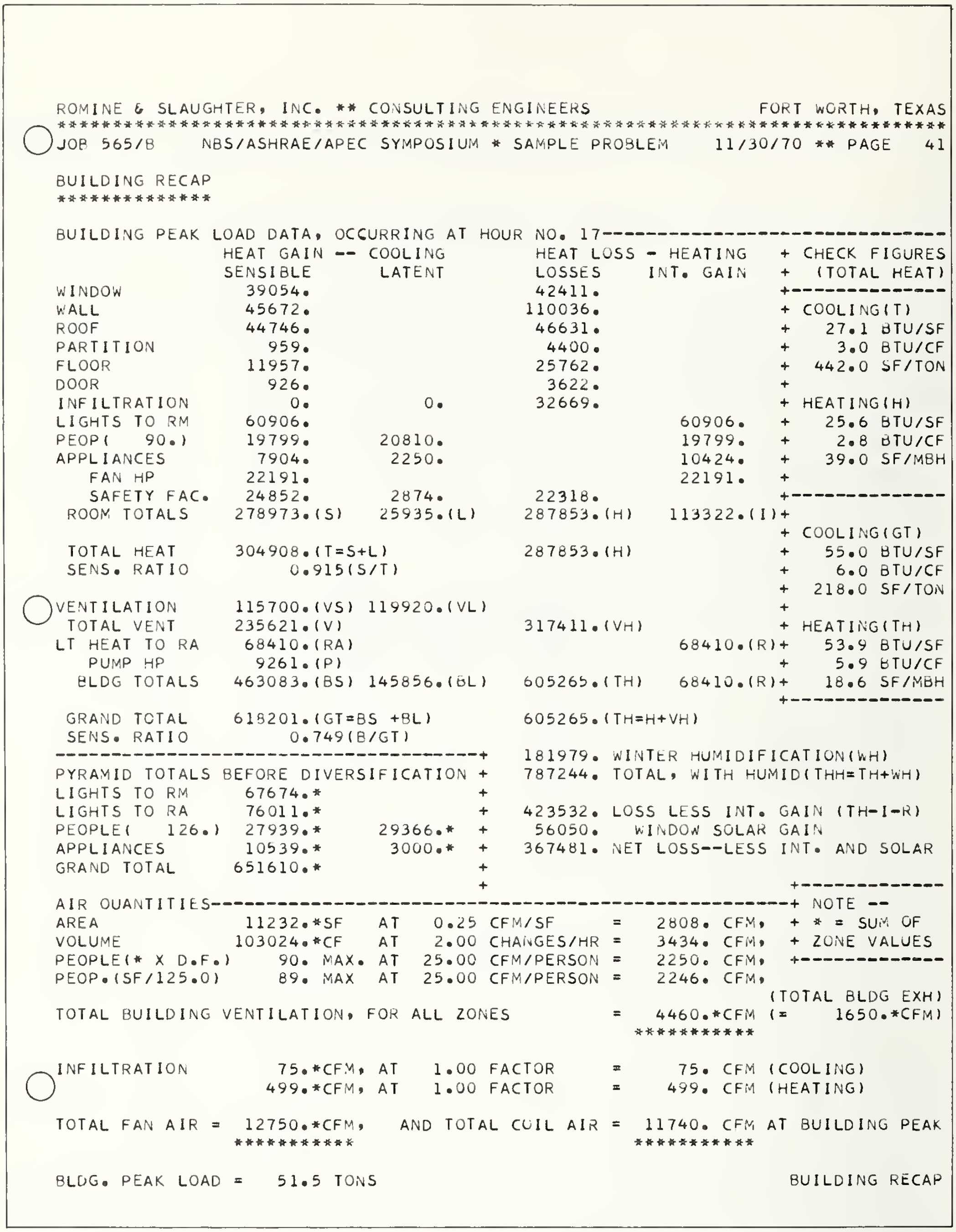

( **NOTE**: ALL ROOMS IN THIS ZONE LISTED, WITH APPROPRIATE DATA, WHETHER MASTER OR DUPLICATE.)

```
                                                    AIR SIDE ANALYSIS FOR
                                                    ZONE 2000 - REPRO. & MAILING
```

Figure 9. Optional Additional Zone Printout



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*********************************************************************************
JOB 565/B    NBS/ASHRAE/APEC SYMPOSIUM * SAMPLE PROBLEM    11/30/70 ** PAGE   41

BUILDING RECAP
**************

BUILDING PEAK LOAD DATA, OCCURRING AT HOUR NO. 17--------------------------------
               HEAT GAIN -- COOLING        HEAT LOSS - HEATING   + CHECK FIGURES
               SENSIBLE      LATENT        LOSSES    INT. GAIN   +  (TOTAL HEAT)
WINDOW           39054.                    42411.                +---------------
WALL             45672.                   110036.                + COOLING(T)
ROOF             44746.                    46631.                +   27.1 BTU/SF
PARTITION          959.                     4400.                +    3.0 BTU/CF
FLOOR            11957.                    25762.                +  442.0 SF/TON
DOOR               926.                     3622.                +
INFILTRATION         0.          0.        32669.                + HEATING(H)
LIGHTS TO RM     60906.                                 60906.   +   25.6 BTU/SF
PEOP(   90.)     19799.      20810.                     19799.   +    2.8 BTU/CF
APPLIANCES        7904.       2250.                     10424.   +   39.0 SF/MBH
   FAN HP        22191.                                 22191.   +
   SAFETY FAC.   24852.       2874.        22318.                +---------------
 ROOM TOTALS    278973.(S)   25935.(L)    287853.(H)   113322.(I)+
                                                                 + COOLING(GT)
 TOTAL HEAT     304908.(T=S+L)            287853.(H)             +   55.0 BTU/SF
 SENS. RATIO         0.915(S/T)                                  +    6.0 BTU/CF
                                                                 +  218.0 SF/TON
VENTILATION     115700.(VS) 119920.(VL)                          +
 TOTAL VENT     235621.(V)                317411.(VH)            + HEATING(TH)
LT HEAT TO RA    68410.(RA)                             68410.(R)+   53.9 BTU/SF
   PUMP HP        9261.(P)                                       +    5.9 BTU/CF
  BLDG TOTALS   463083.(BS) 145856.(BL)   605265.(TH)   68410.(R)+   18.6 SF/MBH
                                                                 +---------------

 GRAND TOTAL    618201.(GT=BS +BL)        605265.(TH=H+VH)
 SENS. RATIO         0.749(B/GT)
--------------------------------------+   181979. WINTER HUMIDIFICATION(WH)
PYRAMID TOTALS BEFORE DIVERSIFICATION +   787244. TOTAL, WITH HUMID(THH=TH+WH)
LIGHTS TO RM     67674.*              +
LIGHTS TO RA     76011.*              +   423532. LOSS LESS INT. GAIN (TH-I-R)
PEOPLE(   126.)  27939.*     29366.*  +    56050.   WINDOW SOLAR GAIN
APPLIANCES       10539.*      3000.*  +   367481. NET LOSS--LESS INT. AND SOLAR
GRAND TOTAL     651610.*              +
                                      +                           +--------------
AIR QUANTITIES----------------------------------------------------+ NOTE --
AREA             11232.*SF   AT   0.25 CFM/SF      =   2808. CFM,  + * = SUM OF
VOLUME          103024.*CF   AT   2.00 CHANGES/HR =   3434. CFM,  + ZONE VALUES
PEOPLE(* X D.F.)    90. MAX. AT  25.00 CFM/PERSON =   2250. CFM,  +--------------
PEOP.(SF/125.0)     89. MAX  AT  25.00 CFM/PERSON =   2246. CFM,
                                                                (TOTAL BLDG EXH)
TOTAL BUILDING VENTILATION, FOR ALL ZONES         =   4460.*CFM (=    1650.*CFM)
                                                    ***********

INFILTRATION        75.*CFM, AT   1.00 FACTOR      =     75. CFM (COOLING)
                   499.*CFM, AT   1.00 FACTOR      =    499. CFM (HEATING)

TOTAL FAN AIR =  12750.*CFM,   AND TOTAL COIL AIR =  11740. CFM AT BUILDING PEAK
                ***********                         ***********

BLDG. PEAK LOAD =   51.5 TONS                                     BUILDING RECAP
```

Figure 10. Typical Building Load Printout

# Accuracy Requirements For Computer Analysis of Environmental Systems

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Westinghouse Electric Corporation
East Pittsburgh, Pa. 15112

The difference between the actual energy requirement for an environmental system and the energy requirement as calculated by a computer program is termed error. The usefulness of such a computer program in selecting between alternative systems is a function, in part, of the magnitude of the error. The total error is considered in two parts - bias error that has the same percentage effect on all systems considered, and random error that is unpredictable from one system to another. The effect of error is quantified by assuming a uniform probability distribution of random error between limits, and then calculating the probability that the lower total cost system has been identified by the computer calculation. The evaluation of this probability is accomplished by means of a simple decision tree analysis.

Key Words: Accuracy, competitive analysis, computer program, decision, economic analysis, energy, environmental system, error.

## 1. Introduction

Most computer programs that calculate energy requirements of environmental systems in buildings have been developed to aid in the choice between alternative competing systems. Important examples include the choice between energy sources, i.e., gas, oil, or electricity, and the choice between energy conservation systems. Certainly there are other uses for energy calculation programs, such as the study of physical properties of a system, and perhaps the use of energy calculations as an integral portion of a computerized control system. The accuracy requirements developed in this paper, however, apply only to the use of the programs to select between alternative systems or subsystems and do not apply directly to other uses.

In the past, little consideration has been given to the relationship between the accuracy of energy programs and the cost of using them. This paper presents such a consideration and should give some insight into the answers of the following two questions: What is the expected accuracy? Does this accuracy, combined with the result, justify the cost of the computer run? Precise answers to these questions applicable to a specific situation are difficult to obtain. It would be necessary to calculate the energy requirements for the various systems, construct a building with each of the systems and measure the results over a period of years. Obviously, this is not practical. The purpose of this paper, therefore, is not to permit the absolute determination of accuracy requirements, but rather to permit some quantification of accuracy, based on estimated calculation errors, and the resulting value of studies. Then, at least, users of building energy computer programs can begin to develop adequate evaluations of their own use of such programs. In addition, and perhaps more important, it is hoped that developers of new programs can gain more of a feel for the accuracy required of their computer programs.

## 2. Types of Errors

The errors that occur in the computer calculation of building energy requirements may be categorized as either bias or random errors. Bias errors are those that have identical percentage effects on all alternatives and, therefore, on the differences between alternatives. For example, if the calculations result in energy requirements that are 10 percent high for all alternatives, then the difference between any two alternatives will also be 10 percent high. Bias errors tend to occur in the portions of the calculations that are the same for all alternatives, such as heat flow and weather factors. Random errors are those that are unpredictable (within limits) between alternatives. For

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[1] Engineering Section Manager and Electrical Engineer, respectively.

example, the calculated energy requirements for one system may be five percent high, and for another alternative, five percent low. Obviously for random errors, the percentage error in the difference between alternatives can be vastly greater than the percentage random error in each alternative.

Neglecting bias errors, the actual value of energy consumption for a building system alternative will be the calculated or expected value, plus or minus some random error. The probability distribution of the actual energy consumption about the expected value is some type of curve whose shape is unknown at this time, but which probably peaks near or at the expected value. In other words, if the random error is plus or minus five percent, then the probability of the actual value being five percent above the calculated or expected value most likely is less than the probability of the actual value being the expected value. The limited experience to date, however, indicates that within the expected limits of random error, the curve of probability distribution is relatively flat. For the purposes of this paper and to facilitate analysis, the probability distribution of random errors is assumed to be flat between the random error limits, and equal to zero outside the random error limits.

## 3. Effects of Errors

The effects of random and bias errors can be visualized by means of Figure 1. "A" is the energy cost for system A and "B" is the energy cost for system B. The lines about A and B represent the actual values, which equal the calculated values plus or minus a random error of 10 percent, with no consideration of the bias error. The calculated values of A and B are 1.0 and 0.6 respectively. Thus, the calculated value of (A-B) is 0.4. Considering only random errors, the actual value of (A-B) can be any value within $(1 \pm 0.1) - (0.6 \pm 0.06)$ with the entire population of (A-B) being contained in the parallelogram in Figure 1. It should be noted that the breakeven differential cost between A and B (BEΔC) is defined as the annual cost (excluding energy) of B minus the annual cost (excluding energy) of A. If (A-B)(calculated) is greater than BEΔC, then B is the apparent choice. The probability that the actual value of (A-B) is greater than BEΔC is equal to the shaded area of the parallelogram divided by the total area. In Figure 1, BEΔC is 0.3, and the probability that (A-B)(actual) is greater than 0.3 is 92 percent. Thus the probability that B is the correct choice is 92 percent. Similarly, if (A-B) (calculated) is less than BEΔC, then A is the apparent choice.

The effect of bias error is assumed to move the parallelogram to the right or left by an amount equal to the product of the bias error and (A-B)(calculated). This is equivalent to adding the same product to BEΔC. In Figure 1, if the bias error is expected to be plus 10 percent, then BEΔC becomes 0.34 instead of 0.3. Thus the probability that B is the correct choice becomes 77 percent instead of 92 percent.

## 4. Probability Curves

The relationships described by Figure 1 were calculated for a wide range of values of the difference in annual energy costs of alternatives A and B, shown as $(\frac{A-B}{A})$, breakeven difference in energy costs (BEΔC), and random errors. The results are plotted in Figure 2 through 11. Note that each figure contains a different family of curves which applies to one value of the relative closeness of the calculated annual cost differential (A-B) and the breakeven cost differential (BEΔC). The different curves within a family apply to different values of the relative closeness of the annual energy costs of each alternative. The abscissa is the random error of the energy calculations and the ordinate is the probability that the more economical choice has been correctly identified by the calculation.

The proper curve is chosen by calculating $(\frac{A-B}{A})$ and BEΔC, and finding the closest applicable parameters on a curve in the available families of curves. Then, knowing the approximate random error of the energy calculations, one can find the probability that his choice is correct. This probability, of course, is one measure of the relative usefulness of energy calculation computer programs in selecting between systems. However, there is a need to relate the probability to economic factors, as will be shown in the next section.

## 5. Maximum Acceptable Study Cost

Figure 12 is an elementary decision tree analysis illustrating the decision that must be made as to whether or not to use an energy analysis computer program, and the chance events that occur after the decision is made. For simplicity, it is assumed that there are only two alternative systems (A and B), but the same analysis could be expanded to include any number of alternatives, chance events and decisions. In order to choose between two alternative systems, it is necessary to utilize either a computer evaluation, at some expense, or an individual's own judgement which is assumed to be free. In either case, a decision is made, presumably in favor of the less expensive system, and with the decision is associated a certain probability that the correct choice was selected. When only judgement is used, it is assumed that the individual making the decision is inexperienced in choosing between these two systems, so there is a probability of 50 percent that he will select either system. This probability could be adjusted to any expected value.

The total cost of each decision is the cost of making the decision plus the equivalent cost of each chance event, using the calculated expected value costs. The equivalent cost of a chance event is equal to the summation of the products of the probabilities and the costs of all possible outcomes. The total cost of an outcome must include both the energy and the capital (including all other costs). Obviously, all costs must be on either an annual basis, or a present worth or capitalized basis. Such a decision tree analysis shows that the maximum allowable expense of a computer study is that expense which results in the two decisions being of equal total cost.

Unfortunately, the decision tree analysis of Figure 12 cannot be used to evaluate a computer study that is yet to be done. The reason for this is that the computer study results are required for the decision tree analysis. Its use, instead, is to evaluate the worth of past computer studies which, in turn, should help in developing policies for future situations.

In order to account for variations in the accuracy of the calculations for energy costs, it is necessary to add an additional term in the total decision cost. This is the "Evaluation of Risk of Loss" in Figure 12. Its value is equal to zero, if the calculations are absolutely precise, and of such a value that the acceptable computer cost is zero, if the computer results are very imprecise. Analysis will reveal that this is equivalent to replacing the expected total cost of the computer selected system with a chance event. This chance event has a probability (P) of selecting the less expensive system with its expected total costs, and (1-P) probability of selecting the higher cost system with its expected total costs. (P) is the probability determined from the families of curves.

## 6. Hypothetical Example

The analysis approach developed in this paper can be illustrated by means of the following example:

| Alternative | Calculated Annual Energy Cost | Capital Cost |
|---|---|---|
| A | $100,000 | $400,000 |
| B | 80,000 | 535,000 |
| A - B | $ 20,000 | - $135,000 |

In addition to the above parameters, assume a capital recovery factor of 0.1175 (based on a 20 year recovery period with a 10 percent interest rate), a bias error of (+) five percent, random error of (±) five percent and a study cost of $5,000.

The breakeven differential cost between A and B (BEΔC) is $15,900, on an annual basis. The apparent choice is B, since BEΔC is less than (A-B)(1-Bias Error) or $19,000. The use of Figure 9 (since $\frac{BE\Delta C}{A-B}$ + Bias error = 0.85) shows that there is a 75 percent probability that if we choose system B, we have chosen the more economical system. The use of a decision tree analysis, adjusted for risk of loss, with this probability of 75 percent, indicates that the breakeven cost for the computer study would have been $9,000, on a first cost basis. This means that the expenditure of $5,000, in this situation, for the computer study has resulted in a net savings of $4,000, and thus, the decision to use the computer analysis was correct. It also indicates that the accuracy of the computer analysis was adequate.

## 7. Appendix-Equations

Definitions:
A - annual energy cost for alternative A
B - annual energy cost for alternative B
BEΔC - annual costs (excluding energy) of B less same for A
F - capital recovery factor to convert capital costs to annual costs
RE - random error in energy cost calculations - per unit
BE - bias error in energy cost calculations - per unit
P - probability that calculations will identify more economical alternative
SC - maximum allowable study cost (that cost which will result in no basis for using computer calculations compared to the toss of a coin to choose between alternatives A and B)

Equations:

If:

$$BE\Delta C \leq (A-B)(1-BE) - (A+B)\ RE \tag{1}$$

Then:

$$P = 1 \text{ that B is economic choice}$$

If:

$$(A-B)(1-BE) - (A+B)\ RE \leq BE\Delta C \leq (A-B)(1-RE-BE) \tag{2}$$

Then:

$$P = 1 - \frac{\left[BE\Delta C - (A-B)(1-BE) + RE(A+B)\right]^2}{8\ RE^2\ (A)(B)} \tag{3}$$

that B is economic choice

If:

$$(A-B)(1-RE-BE) \leq BE\Delta C \leq (A-B)(1-BE) \tag{4}$$

Then:

$$P = 1 - \frac{BE\Delta C - (A-B)(1-BE) + RE(A)}{2\ RE(A)} \tag{5}$$

that B is economic choice

If:

$$(A-B)(1-BE) \leq BE\Delta C \leq (A-B)(1+RE-BE) \tag{6}$$

Then:

$$P = 1 - \frac{(A-B)(1-BE) - BE\Delta C + RE(A)}{2\ RE(A)} \tag{7}$$

that A is economic choice

If:

$$(A-B)(1+RE+BE) \leq BE\Delta C \leq (A-B)(1-BE)+(A+B)\ RE \tag{8}$$

Then:

$$P = 1 - \frac{\left[(A-B)(1-BE) - BE\Delta C + RE(A+B)\right]^2}{8\ RE^2\ (A)(B)} \tag{9}$$

that A is economic choice

If:

$$BE\Delta C \geq (A-B)(1-BE)+(A+B)\ RE \tag{10}$$

Then:

$$P = 1 \text{ that A is economic choice}$$

$$SC = \frac{(P-0.5)}{F} \left| (A-B)(1-BE) - BE\Delta C \right| \tag{11}$$

## 8. References

(1) Hertz, D. B., Risk Analysis in Capital Investment, Harvard Business Review, Jan-Feb., 1964, p. 95.

(2) Hammond, J. S., III, Better Decisions with Preference Theory, Harvard Business Review, Nove-Dec., 1967, p. 123.

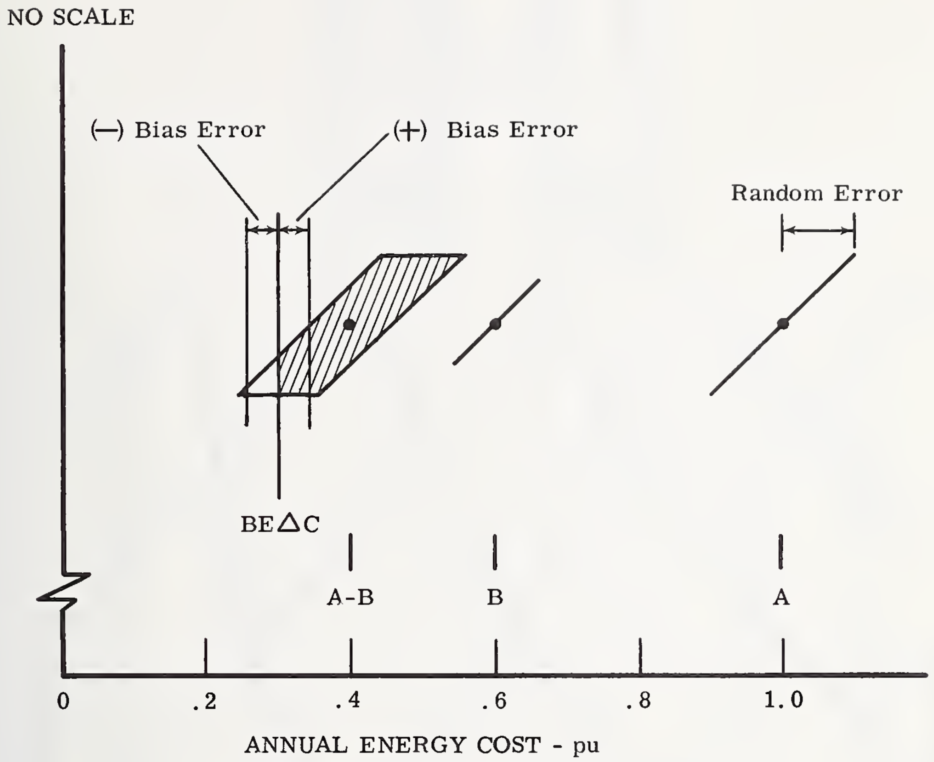


Figure 1. Illustration of the effect of random and bias errors on differential (A-B) energy cost.

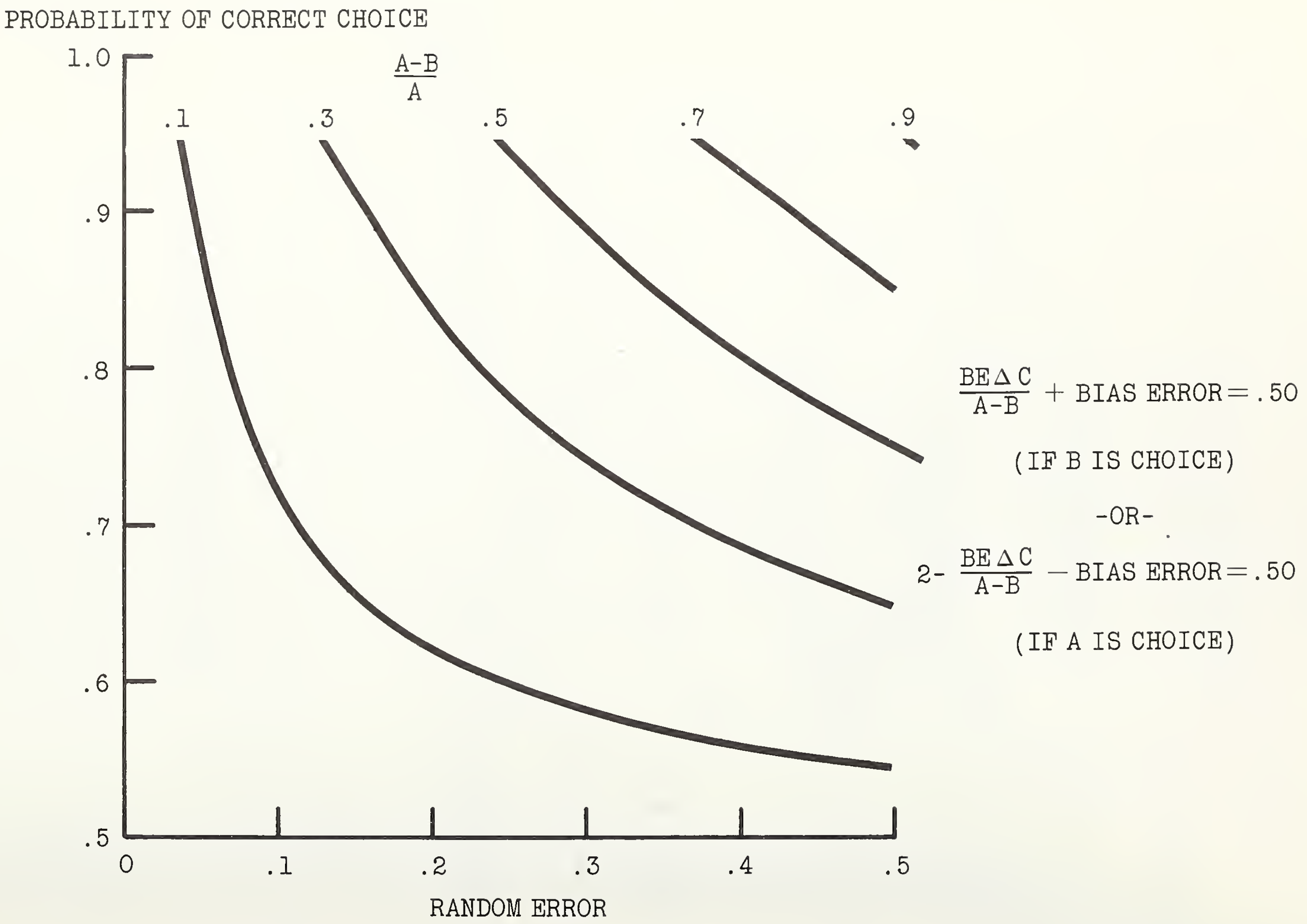


**Figure 2.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .50.

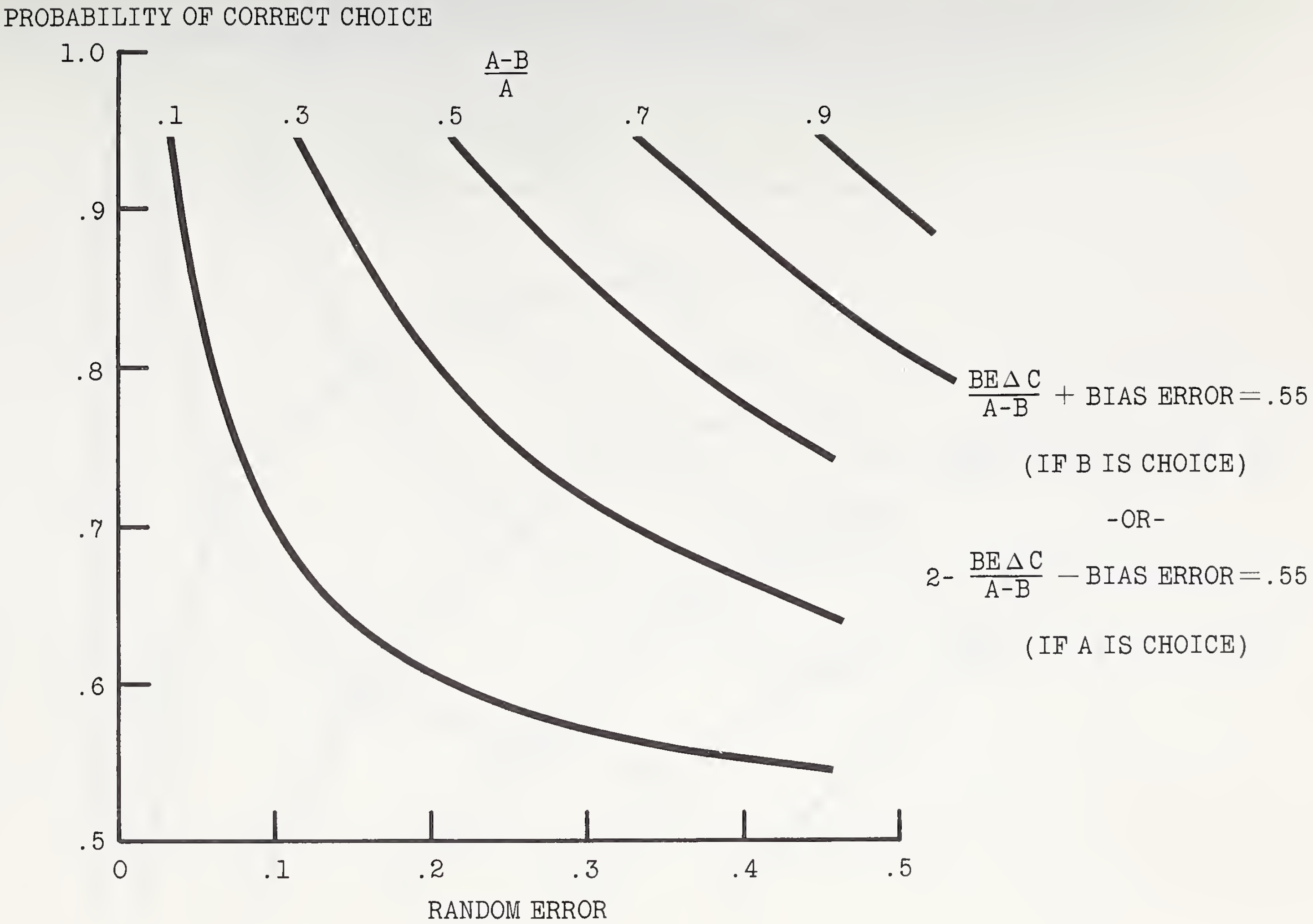


**Figure 3.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .55.

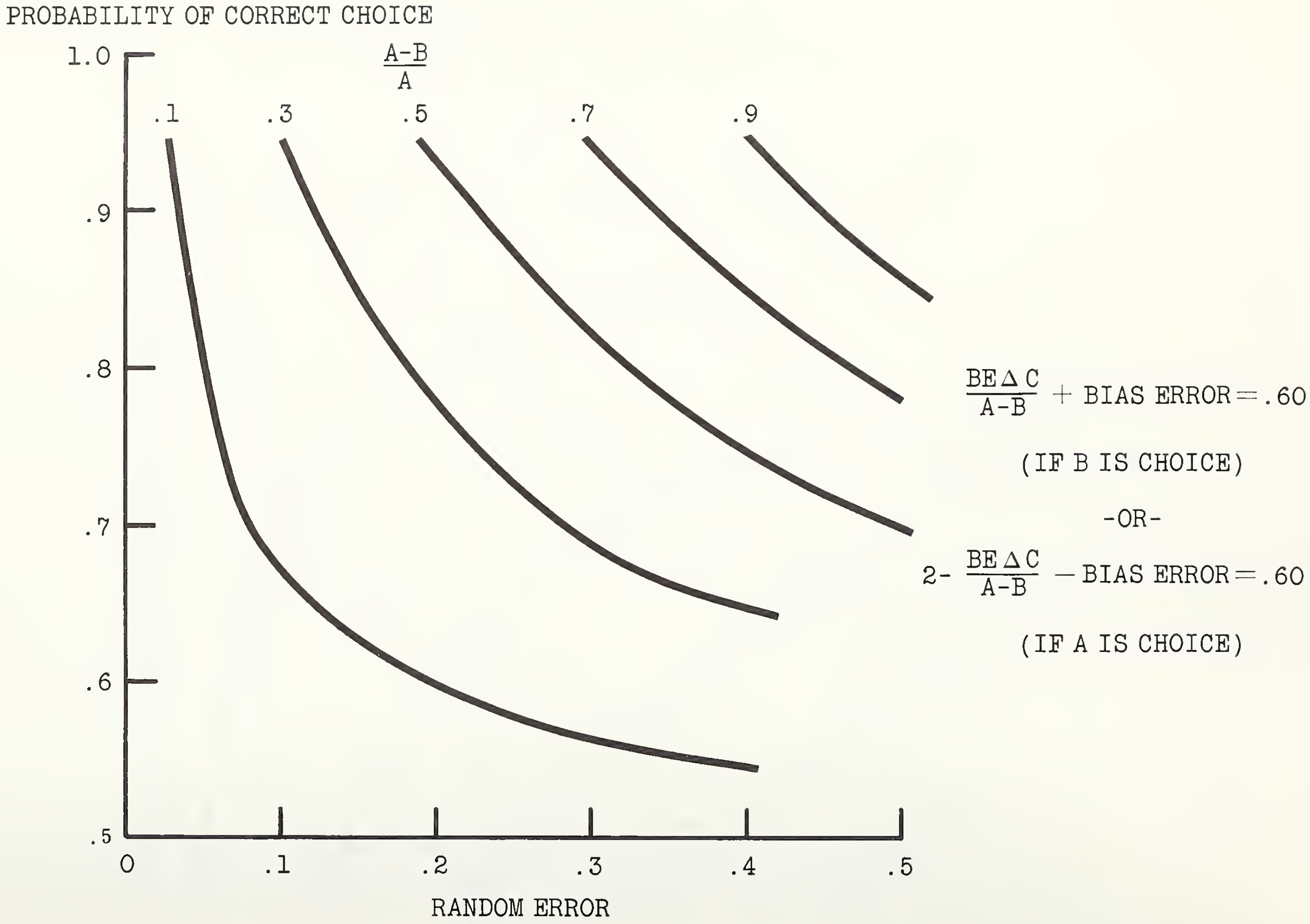


**Figure 4.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .60.

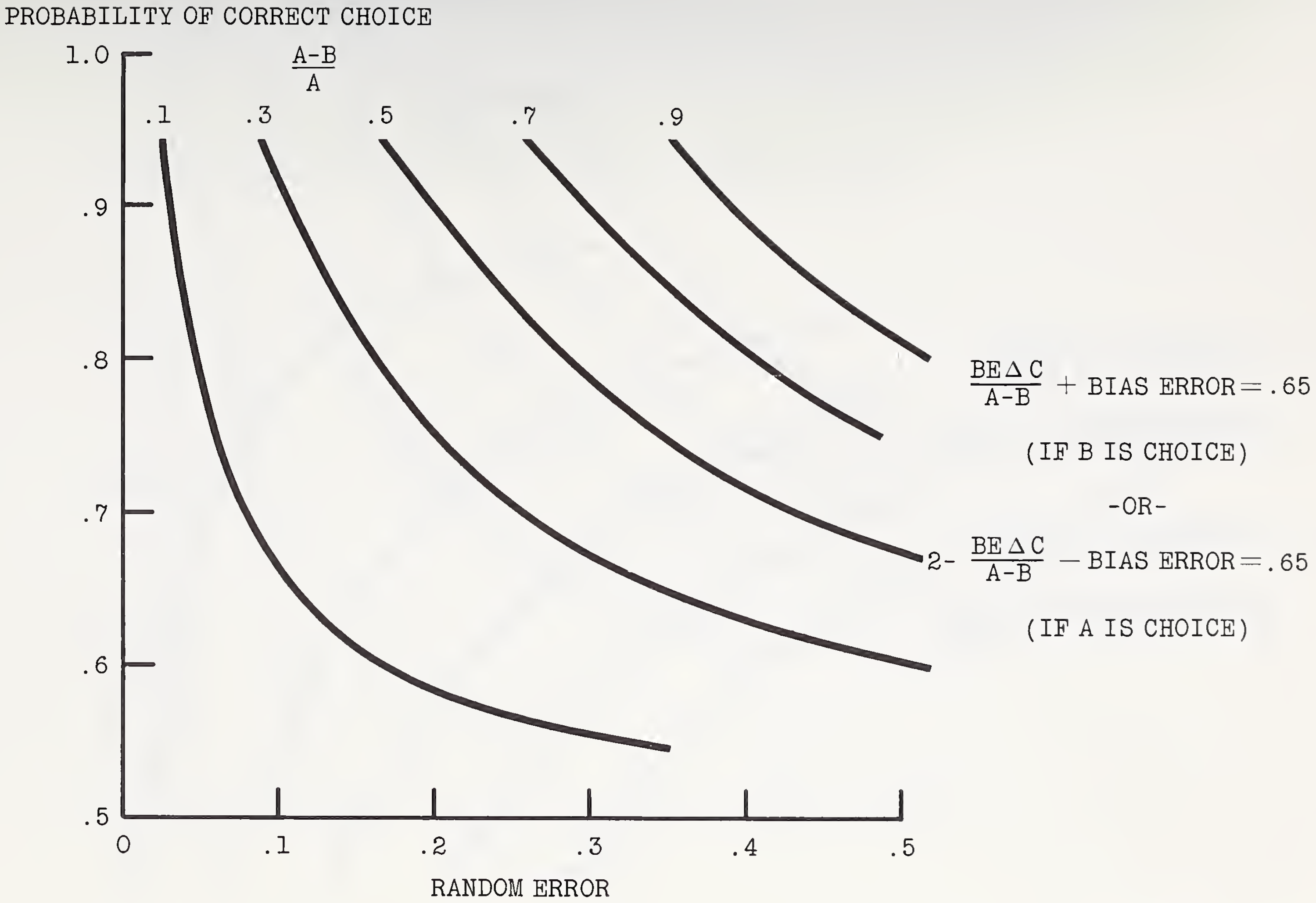


Figure 5. Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .65.

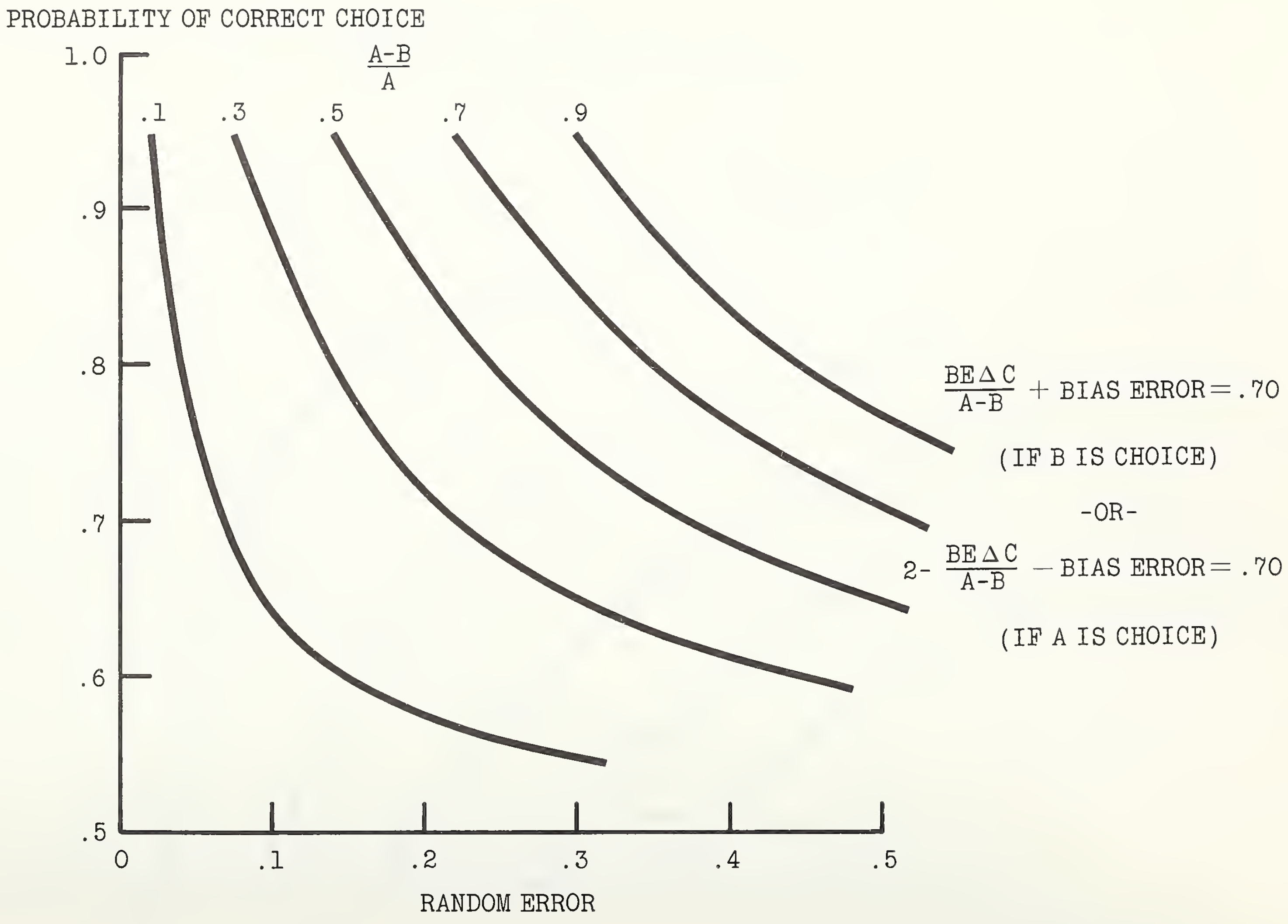


**Figure 6.** **Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .70.**

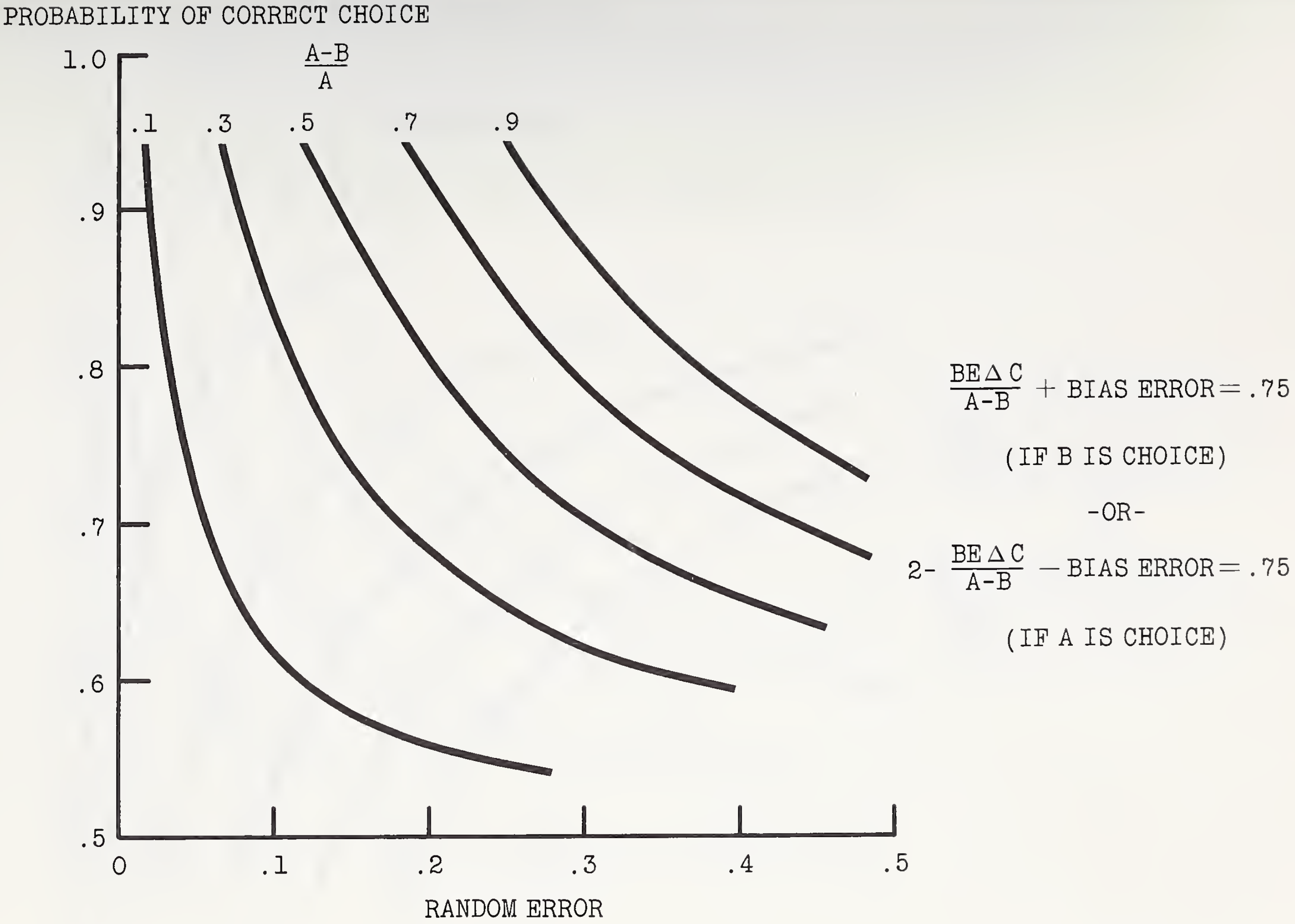


**Figure 7.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .75.

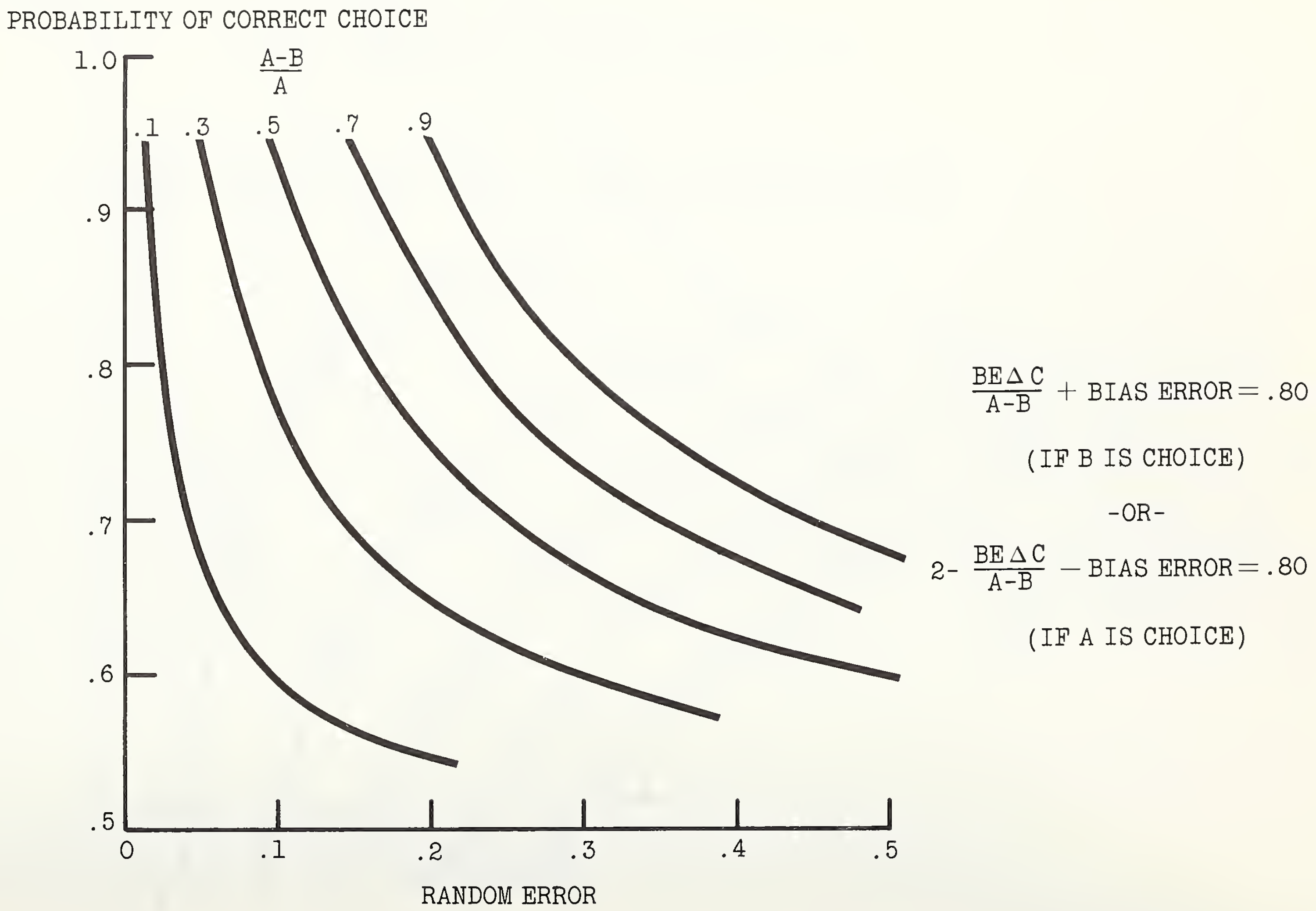


Figure 8. Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .80.

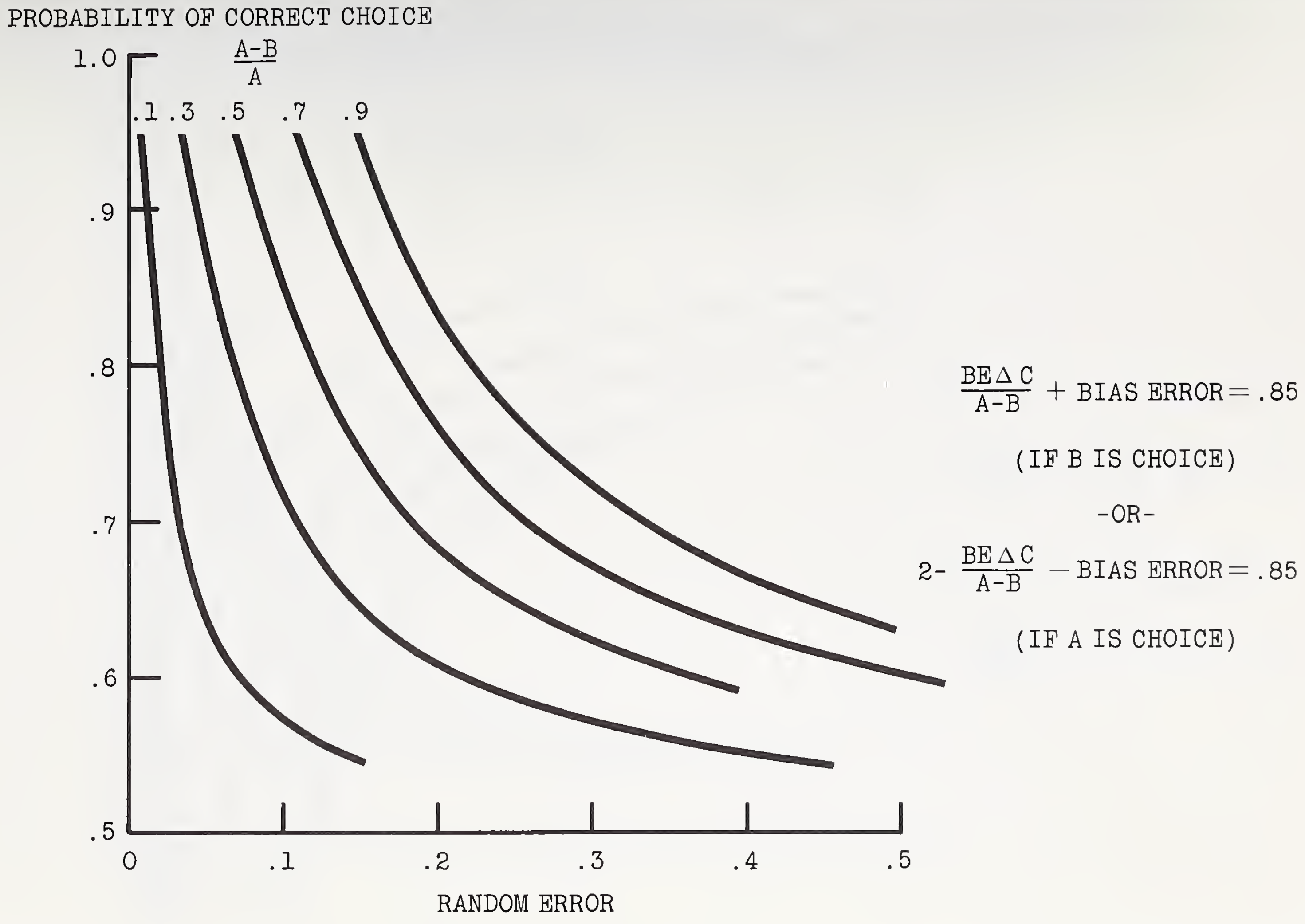
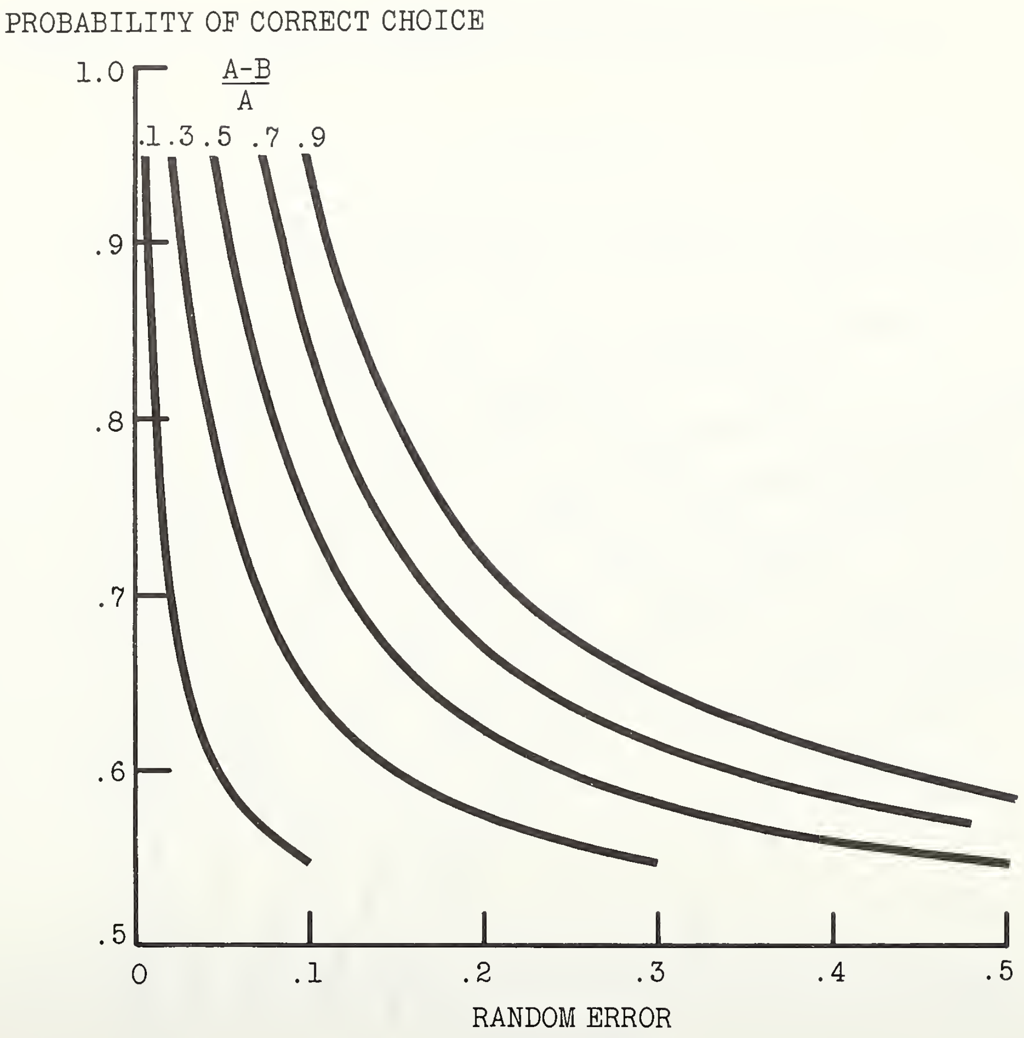


**Figure 9.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .85.



$$\frac{BE\Delta C}{A-B} + \text{BIAS ERROR} = .90$$

(IF B IS CHOICE)

-OR-

$$2 - \frac{BE\Delta C}{A-B} - \text{BIAS ERROR} = .90$$

(IF A IS CHOICE)

**Figure 10.** Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .90.

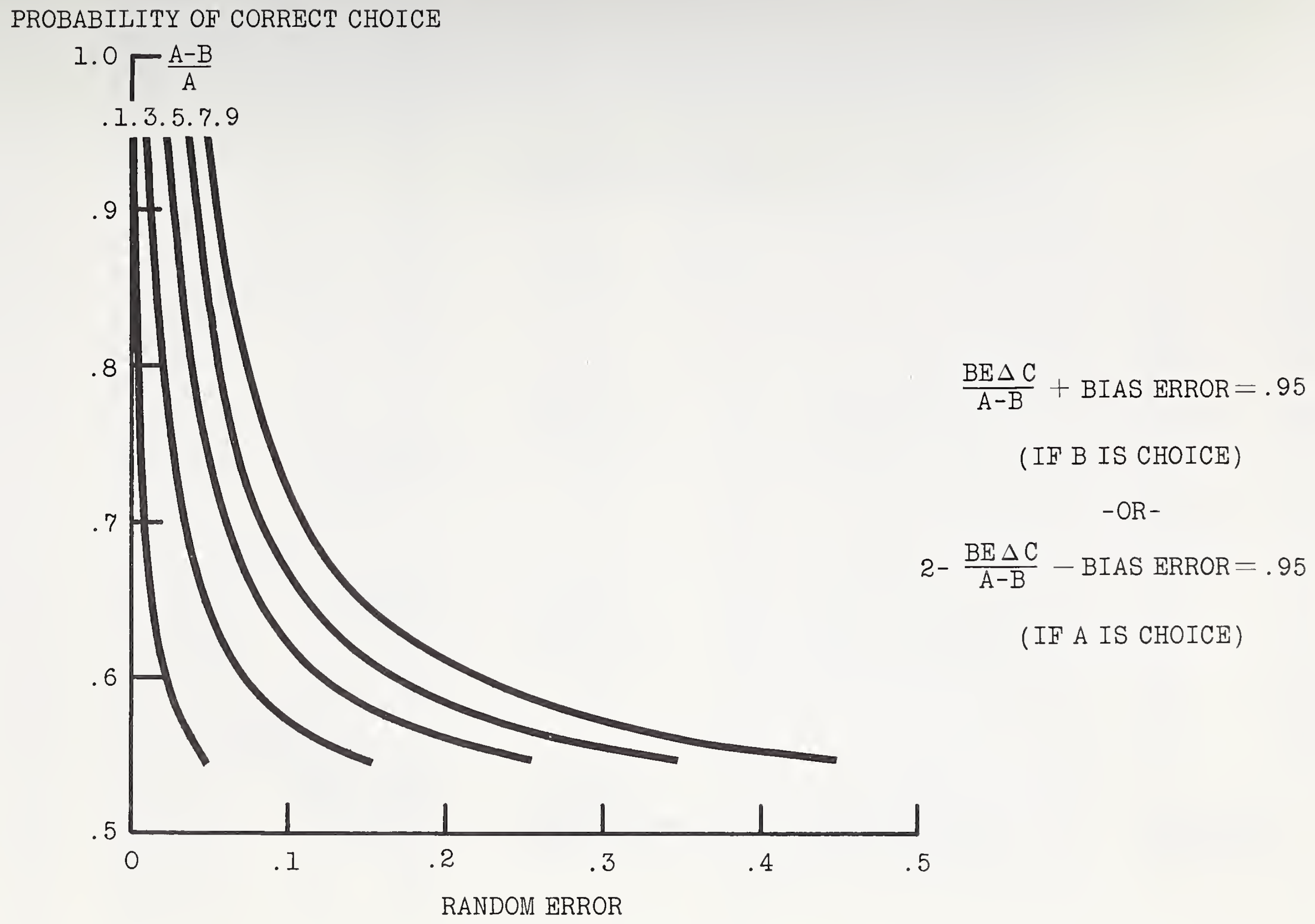


Figure 11. Probability of correct choice curves for relative closeness between calculated and breakeven cost differentials equal to .95.

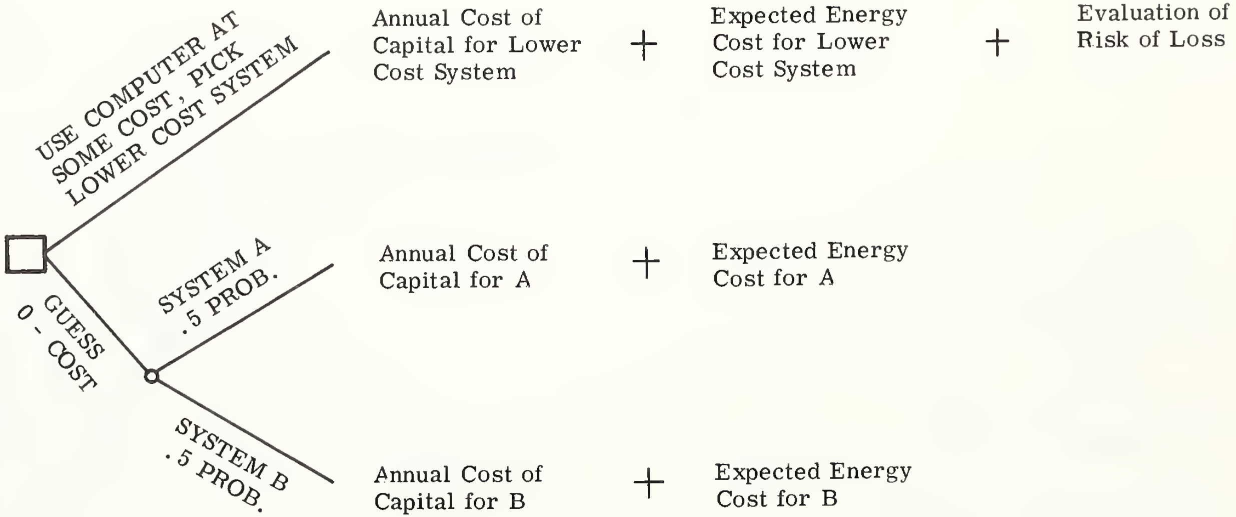


**Figure 12.** Elementary decision tree analysis illustrating the decision as to whether or not to use an energy analysis computer program.

# Calculation of Energy Requirements with the Proposed ASHRAE Algorithms for U. S. Postal Buildings[1]

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The accurate calculation of the energy requirements and heating and cooling equipment sizes for buildings is one of the most important, as well as one of the most difficult, problems facing the air conditioning engineer. It is important because energy cost is an essential and significant element of the building's overall owning and operating cost and very likely could be the determining factor in selection of the air conditioning system for a new structure. The problem is difficult because of its complexity. It not only requires accurate determination of the heating and cooling loads, taking into account the varying influences of weather and operating schedules, but also determination of the performance of heating and cooling systems under varying conditions of partial load. The application of digital computers for solution of these complex problems has made more accurate and thorough treatment of a total building analysis possible and has made practical the comparison of all types of systems and energy sources that an engineer wishes to consider. An analysis of the type discussed above would normally take from a week to many months if performed by hand. Now it can be done in a matter of a few hours or a few days, depending upon the size of the building and the degree of analysis desired. The computer program described in this paper utilizes proposed ASHRAE algorithms and accomplishes all of the above items along with economic analysis.

Key Words: ASHRAE algorithms, digital computers, economic analysis, energy requirements, heating and cooling equipment sizes, operating schedules, partial load, performance of heating and cooling systems, varying influences of weather.

## 1. Introduction

Calculations of energy requirements and heating and cooling equipment sizes for buildings by conventional methods is and always has been more art than science. The most important element in any calculation has been the experience factor, and the man whose experience permitted him to realistically select an appropriate value is perhaps as much an artist in his own right as Picasso is in his.

Unfortunately, art and truth are not necessarily the same. With good reason, heating and cooling equipment has generally been specified substantially oversized as a hedge against a poor guess. What's more, energy requirement calculations in themselves are little more than guesswork, irrespective of the effect of any experience factor.

Such inexactness in determining energy requirements and heating and cooling equipment sizes for buildings is rapidly becoming a problem. First, rising construction costs and the increasing value of floor space are driving building owners to insist that equipment be no larger than absolutely necessary. Second, competition between energy suppliers is stiff and their claims are often conflicting, which means that more refined methods of comparing energy sources are needed. Third, energy utilization equipment and methods are more varied now than at any time in the past. Again, accurate cost

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[1] The work described in this paper was sponsored by the Bureau of Research and Engineering of U. S. Postal Service (Contract No. RE 49-67) under the Technical Monitorship of James M. Anders. The details of the complete project are given in Reference 5.

[2] Manager, Thermal, Mechanical and Computer Systems.

comparisons are necessary. Fourth, architects and building owners would like the flexibiltiy of knowing quickly and accurately the effects of changes in such factors as design, materials, and building orientation on heating and cooling costs. Not having this information inhibits progress in building design. Last, and perhaps most important, many experts are predicting that the United States is heading for an energy shortage which will boost energy costs substantially. Energy requirements, which entail the greatest amount of guesswork in conventional calculations, probably will become the dominant factor in the selection of heating and cooling equipment.

## 2. Fundamentals of the Computer Program

The solution to the present dilemma is to upgrade and refine heating and cooling load calculations by making them more reflective of the actual conditions taking place in the building. This entails switching from present calculation methods based upon the physical impossibility of steady-state or steady periodic heat transfer to a new calculation method which evaluates transient heat transfer "like it really is".

The computer program developed by General American Research Division (GARD) of General American Transportation Corporation (GATX) for the United States Postal Service (USPS) does exactly this. Further, it doesn't stop at heating and cooling load calculations; it carries right on through to system simulation and final economic analysis. Known as the Computer Program for Analysis of Energy Utilization in Postal Facilities, the program specifically will:

(1) Evaluate total building design and the effects of size, shape, orientation, wall and roof constructions, and window design on the heating and cooling demands.

(2) Evaluate system selection and the effects of equipment capacity, schedule of operation, and choice of components on the ability to maintain design requirements in every space of the building for each hour of the year.

(3) Evaluate owning and operating costs and the effects of the type of equipment, the type of energy source, and maintenance and overhaul costs on maximizing return on investment.

The USPS desired such a program so that it could determine exactly which energy source/equipment combinations provide the lowest overall owning and operating cost for each one of many thousands of new postal facilities planned for construction throughout the country.

To account for the thermal storage effect of a building, the heating and cooling load calculation portion of the computer program utilizes the most advanced method of load determination, the "Convolution Principle". The mathematical theory behind the Convolution Principle is not new in itself. It has been known and applied in different engineering problems for a number of years. However, its expansion into a practical and workable computer program which provides a wealth of design information for building heating and cooling equipment selection has resulted from the research and development carried out by the ASHRAE Task Group on Energy Requirements for Heating and Cooling [1, 2][3], the National Research Council of Canada [3], the National Bureau of Standards of USA [4], and GARD/GATX [5, 6, 7, 8, 9]. The computer program developed by GARD/GATX for the United States Postal Service utilizes the full analysis outlined in the ASHRAE booklet [1] and applies the convolution principle in two different places. First, the transient heat conduction through a multi-layer wall (or roof) is calculated by convolving the outside and inside surface temperatures with the wall (or roof) "response factors", and secondly, the space cooling load is calculated by convolving the instantaneous heat gain with its "weighting factors".

In heating and cooling load calculations, the hourly weather data obtained from the U. S. Weather Bureau is used. The weather data utilized includes dry and wet-bulb temperatures, wind velocity, barometric pressure, cloud type and amount. At this point, it is worthwhile to emphasize that in addition to the utilization of the Convolution Principle, use of coincident dry and wet-bulb temperatures, modification of calculated solar radiation data by cloud type and amount, and calculation of time-dependent outside film coefficient as a function of wind velocity and type of surface are further features of the computer program.

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[3] Figures in brackets indicate the literature references at the end of this paper.

## 3. Description of the Computer Program

The computer program consists of four main sub-programs performed in sequence, with the output of one becoming the input to the next. The function of each sub-program is summarized below.

(1) Load Calculation Sub-program. Calculates sensible and latent components of hourly heat losses and heat gains, for each space in the building for a desired period of time. This program is supported by wall and roof selection sub-programs.

(2) Thermal Loads Plot Sub-program. With the support of the Punch Sub-program, plots the load profile of any space for any period of time. Comparisons between plots permit the grouping of compatible spaces into fan system control zones. This is achieved by the use of Editing Support Sub-program.

(3) Systems Simulation Sub-program. Simulates the operation of each fan system by combining the heating or cooling requirement of each zone with ventilation air requirements, thus obtaining the hourly heating and cooling requirements imposed on the heating and cooling equipment. These thermal requirements are then converted into hourly energy requirements based upon partial load machine characteristics. Depending upon building floor area, Central or Packaged Systems Simulation Sub-programs may be utilized.

A. Central Systems Simulation Sub-program: The thermal distribution systems which this program analyzes are:

- single zone system
- multi-zone system
- dual-duct system
- unit ventilator
- unit heater
- single zone reheat system
- radiant floor panel heating.

Heating and Cooling Plant combinations that the program can handle include:

a) Conventional Systems

- Hermetic Reciprocating Chiller with Gas, Oil, or Steam Heat
- Hermetic Centrifugal Chiller with Gas, Oil, or Steam Heat
- Open Centrifugal Chiller with Gas, Oil, or Steam Heat
- Steam Absorption Chiller with Electric or Steam Heat

b) Total Electric Systems

- Hermetic Reciprocating Chillers with Electric Heat
- Hermetic Centrifugal Chiller with Electric Heat
- Open Centrifugal Chiller with Electric Heat

c) Total Gas Systems

- Steam Absorption Chiller with Gas Heat
- Steam Turbine Driven Open Centrifugal Chiller with Gas Heat

d) Total Oil Systems

- Steam Absorption Chiller with Oil Heat
- Steam Turbine Driven Open Centrifugal Chiller with Oil Heat

e) On-Site Generation Systems

- Total Gas with Steam Absorption Chiller, Gas Heat or Gas Engine-Generation Sets
- Total Oil with Steam Absorption Chiller, Gas Heat or Diesel Fuel Engine-Generation Sets

B. Packaged Systems Simulation Sub-program: The systems which this program analyzes are:

- Electric Air Conditioning (DX coil) with Oil Heat
- Electric Air Conditioning (DX coil) with Gas Heat
- Reversible-cycle Heat Pump with Electric Resistance Heating
- Gas Air Conditioning with Gas Heat

(4) Economic Analysis. Calculates the annual owning and operating costs of various combinations of heating and cooling plants.

## 4. Sequence of Sub-programs Use

The sequence of using sub-programs depends upon the information the engineer wishes to obtain and if he can initially break the buildings into fan system control zones rather than just spaces. Figure 1 illustrates the paths of sub-program sequencing that can be taken as a function of the engineer's decisions. As the engineer becomes more adept at breaking a building directly into control zones, the need for space load plots and/or re-grouping of spaces will diminish.

The card input data required by some of the sub-programs cannot be prepared until the output of another sub-program has been examined. For example, the proper grouping of spaces into control zones, and then control zones into fan systems required for the Systems Simulation Sub-program probably cannot be done until the results of the Thermal Load Plot Sub-program are reviewed to establish which spaces have similar load profiles. Neither can monthly energy costs be calculated and inputted to the Economics Analysis Sub-program until the monthly energy consumption summary has been received from the Systems Simulation Sub-program. Nor can the engineer collect the necessary equipment cost data required by the Economics Analysis Sub-program until the Systems Simulation Sub-program tells him the quantity and capacity of the chillers, boilers, cooling towers, etc. Figure 2 illustrates this dependence of input upon output for a large Post Office building analysis. Figure 3 illustrates the same for a small Post Office building (less than 35,000 square feet of floor space) analysis where use of all sub-programs is probably not required. For small buildings, the engineer can elect to use only the Load Calculation Sub-program, the Packaged Systems Simulation Sub-program, and the Economics Analysis Sub-program. If experience indicates no need for economic evaluation of fuels, energy and equipment variations, then the engineer can elect to run only the Load Calculation Sub-program for the peak heating and cooling months. This would indicate maximum and minimum heating and cooling requirements for proper capacity equipment selection.

## 5. Inputs and Outputs of Each Sub-program

### Load Calculation Sub-program

The inputs to the Load Calculation Sub-program are the geometry of the building and its surroundings, thermal and physical properties of wall and roof constructions, operating schedules of the building, i.e., occupants, lighting equipment, etc., schedules, hourly weather data from U. S. Weather Bureau tapes, and internal loads.

The mandatory output on paper is the facility identification, weather data and station identification, building thermal and infiltration loads summary, and spaces maximum and minimum heating and cooling loads summary. The mandatory output on magnetic tape is the hour of the year, sun index, dry and wet-bulb temperatures, wind velocity, humidity ratio, pressure, enthalpy, and density of outside air and for each space, space number, space sensible load, space latent load, plenum return air lighting load, and space lighting and equipment power.

Optional outputs on paper are the wall and roof specifications, shadow pictures, and if desired, the same information on magnetic tape.

### Thermal Loads Plot Sub-program

The inputs to the Thermal Loads Plot Sub-program are the space number, the number of days to be plotted, the starting date, and the loads data.

The output is a strip chart graph detailing the heat gain or loss for each hour plus the maximum heat loss and gain for the period.

### System Simulation Sub-program

The inputs to the System Simulation Sub-program are the output tapes of the Load Calculation (or Editing Support) Sub-program, fan system characteristics, chiller operating characteristics, and types of heating/cooling combinations and energy sources to be analyzed.

The output on paper includes a summary of fan system characteristics, a summary of zone air flows, a summary of equipment sizes, and energy consumption analysis.

### Economic Analysis Sub-program

The inputs to the Economic Analysis Sub-program are the monthly energy costs, the installed equipment costs, equipment life, maintenance and overhaul costs, and anticipated annual percentage increase of labor and material.

The output on paper gives the monthly and yearly energy, machine, and equipment costs and total owning and operating annuity.

## 6. Closing Remarks

As a result of different experiments performed, it can be stated that the Convolution Principle used in heating and cooling load calculations gives more realistic values of the peak loads and the associated times of occurrence. Because the effects of heat storage are included in the calculations, the program clearly shows that peak loads occur several hours after the hottest time of the day, at which time generally there are no occupants in the building. For practical purposes, this means that cooling and heating equipment can often be specified smaller than would normally be selected, and elusive demand figures for utility services can be pegged accurately, which is vital to achieving realistic energy costs. It is also worthwhile to mention that the computer program can be utilized in different ways, such as:

(1) Pre-design selection of the basic system and energy source. This includes total electric systems, total gas or oil systems (on-site generation), purchased steam-absorption systems, and combinations of the above systems and energy sources.

(2) During the design stage, utilize various subroutines or sub-programs to evaluate specific design concepts and modifications.

(3) During the construction stage, evaluate contractor proposals for modification or deviation from the construction plans and specifications.

(4) After completion of the building, use the "ideal building", as finalized by the computer program, to evaluate the maintenance and operation of the facility and the utilities systems. This will completely optimize total owning and operating costs and provide the greatest return on utility dollars.

At the present time, the computer program has different versions to run on Control Data 3600, 6400 and 6600, Univac 1108 and IBM 360 computer systems. Input data preparation and computer running times depend on the complexity of the building under consideration. Both data preparation and computer running times decrease as the engineer gains experience. Depending upon computer system used and the complexity of the building, the running time on the computer varies between 1/50 to 1/200 second per space per hour.

## 7. References

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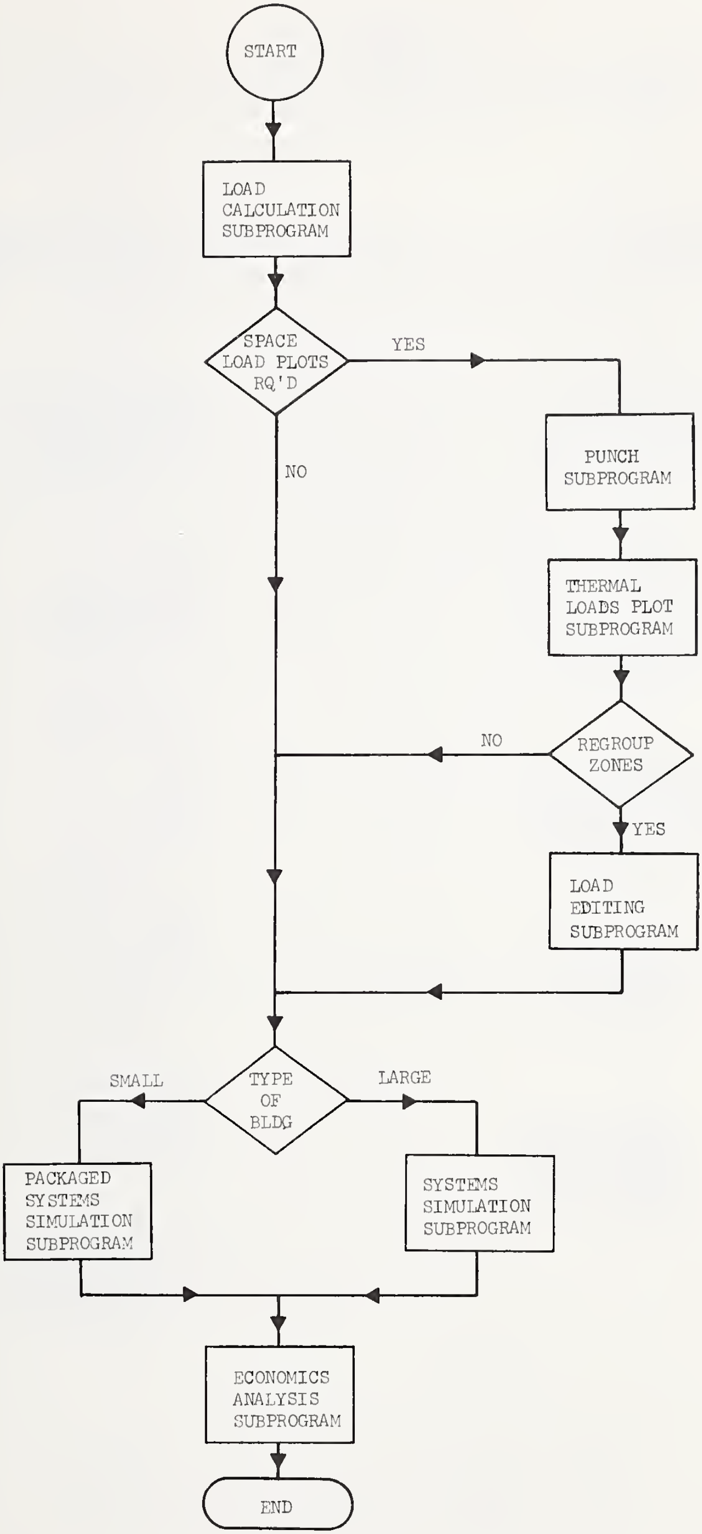

Figure 1 The paths of sub-program sequencing

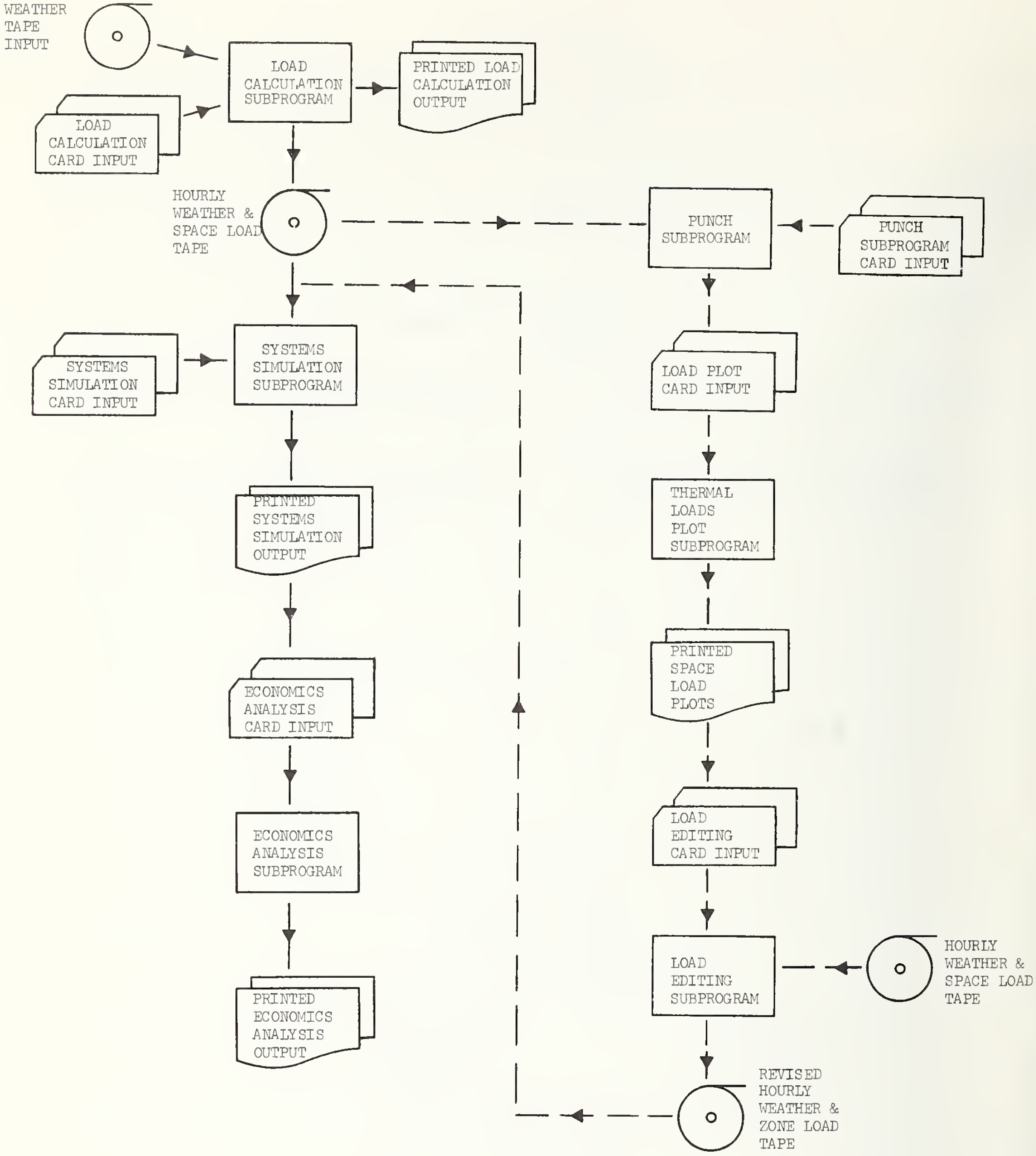

Figure 2 Analysis of Large Post Office Buildings

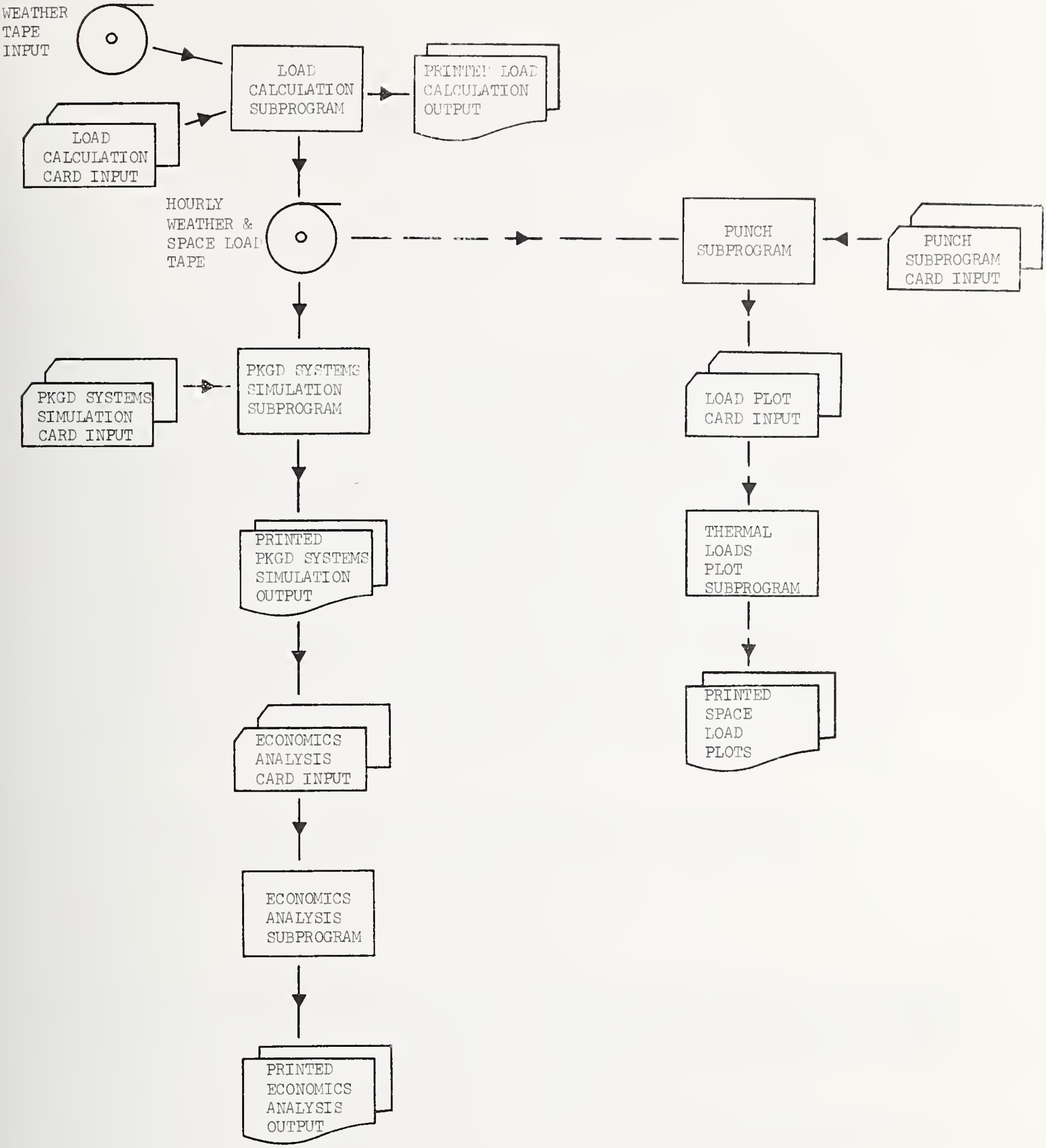

Figure 3 Analysis of Small Post Office Buildings

# An Accurate Computing Method for the Analysis of the Non-Steady Thermal Behaviour of Office Buildings

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For the calculation of room temperatures and cooling loads in buildings a computer program has been made that meets certain special requirements in respect of accuracy and flexibility. The room model chosen has five inner walls and one facade. Each inner wall may consist of several layers, the possible cavity in the ceiling can be ventilated. The facade has a non-transparent part and a transparent part with glazing and sun shading. The transparent system may consist of three planes, each of which may be a glass plane (common, tinted, coated), a venetian blind (inside, in between, outside) or a screen (inside, in between, outside). It is possible to ventilate the cavities between these planes. The computing procedure is based on an implicit difference method with time and space discretisation. The number of nodes with both resistive and capacitive heat flow is 71, distributed over inner and outer walls. Input data refer to solar radiation, outdoor temperature, inner heat gain from occupants, lighting and apparatus, furthermore air and water inlet temperatures as well as air flow. The incident solar radiation, divided into direct, sky and ground radiation, is calculated depending on latitude, date, orientation, time and sky dust factor. The solar radiation fractions absorbed in glazing and shading planes are calculated from reflection, absorption and transmission property data, and thus also the transmitted solar radiation. Indoor air temperature may be free or controlled at a prescribed curve. Output data are indoor air temperature, indoor surface temperatures and cooling load. When the discrete time interval is a quarter of an hour the computing time will be about 5 minutes on a IBM 360/65.

Key Words: Conduction, convection, discretisation, glazing, implicit difference method, solar radiation, solar absorption, solar reflection, solar transmission, sun shading, thermal behaviour, thermal radiation.

## 1. Introduction

Some years ago the accurate calculation of room temperatures and cooling loads in buildings could only be made by using electrical simulation methods. References are [1][1], [2], [3], [4]. Afterwards computer programs were developed for air conditioning plant calculations [5], [6] in which however the heat transfer process in the rooms, expecially the heat conduction in the walls, had to be approximated because of the extent of these installation programs and the frequency of the computations for each project. Meanwhile some programs for the analytical calculation of the non-steady heat transfer in separate composite wall constructions became available [7,8,9].

For the calculation of the non-steady heat transfer in a room we developed a program by means of which accurate and extensive parameter computations are possible, especially as regards the solar transmission, the heat transfer at the glazing and the heat accumulation in the walls.

In this paper a description is given of the program, involving the possibilities of this program for research purposes and for plant capacity calculations.

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[1] Figures in brackets indicate the literature references at the end of this paper

## 2. Description and Formulation of the Heat Transfer Processes in Rooms exposed to Sun Radiation

When the glass planes of a room are sun radiated various planes of this room will be warmed up through absorption of sun radiation and heat transfers occurs by conduction in the walls, by convection between surfaces and room air, and by radiation between the surfaces of walls and planes. We refer to more detailed descriptions, for instance [1], and shall confine ourselves here to the formulation of the heat transfer equations to be solved.

### 2.1. Heat Transfer by Conduction

Non-steady heat conduction in a homogeneous medium is described by the partial differential equation of Fourier. When T(x,t) is the temperature at place x and time t, then

$$\lambda \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} \tag{1}$$

where $\lambda$ is the thermal conductivity and $\rho c$ the volumetric heat capacity. To calculate the temperature T(x,t) and the heat flow $-\lambda \frac{\partial T}{\partial x}(x,t)$ the boundary and initial conditions must be known:

$$T(x,0) = f(x) \tag{2}$$

$$a_o \frac{\partial T}{\partial x}\bigg|_{x=x_o} + a_1 T(x_o,t) = g(t) \tag{3}$$

where $a_o$ and $a_1$ are constants.
When the medium is not homogeneous but consists of different layers, then eq (1) is valid for each layer and additional conditions occur on account of contineous heat flow at the internal boundaries. For the numerical evaluation of the partial differential equations of type eq (1) in case of composite wall constructions, place and time discretisation is applicable by using explicit or implicit difference methods. We chose an implicit method on account of its total stability. A homogeneous layer is supposed to be divided in M segments. Let the temperature in segment k at time n$\Delta$t be $T_{k,n}$, then:

$$\frac{\partial T}{\partial t} = \frac{T_{k,n} - T_{k,n-1}}{\Delta t} \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{k-1,n} - 2T_{k,n} + T_{k+1,n}}{(\Delta x)2} \tag{5}$$

Substitution of eqs (4) and (5) into eq (1) gives the following implicit difference equation:

$$\frac{\Delta x}{\lambda} T_{k-1,n} - \left(\frac{2\Delta x}{\lambda} + \frac{\rho c \Delta x}{\Delta t}\right) T_{k,n} + \frac{\Delta x}{\lambda} T_{k+1,n} = - \frac{\rho c \Delta x}{\Delta t} T_{k,n-1} \tag{6}$$

for k = 1,2,3......., M,
with $T_{o,n}$ and $T_{M+1,n}$ as boundary conditions,
and with $T_{o,o}$, $T_{1,o}$, $T_{2,o}$, ........., $T_{M,o}$ as initial conditions.
With eq (6) it is possible to set up a computation model as shown in fig. 1, where

$$R_1 = R_2 = \frac{\Delta x_1}{F\lambda_1} \quad \text{and} \quad R_3 = \frac{\Delta t}{\rho c F \Delta x_1}$$

with F being the area of the segment.
The advantage of this model is that $R_1$ and $R_2$ truly represent the heat resistances. Resistance $R_3$ is a fictitious heat resistance representing the heat capacity of the segment divided by $\Delta$t. Other heat resistances (see par 2.2 and 2.3) and the various heat sources (see par 3) may easily be joined imaginarily with this model.

Eq (6) may also be written as a recurrent matrix equation:

$$A\ \underline{z}(n) + B\ \underline{u}(n) = C\ \underline{z}(n-1) \qquad (7)$$

where $\underline{z}(n)$ is a vector with temperatures $T_{1,n}$; $T_{2,n}$; ........., $T_{M,n}$,

$\underline{u}(n)$ is a vector with boundary conditions $T_{o,n}$ and $T_{M+1,n}$,

A, B and C are coefficient matrices of equation (6),

z(o) is the initial condition.

The solution of eq (7) is obtained by applying matrix inversion:

$$\underline{z}(n) = A^{-1}[C\ \underline{z}(n-1) - B\ \underline{u}(n)] \qquad (8)$$

By repeated substitution eq (8) passes to

$$\underline{z}(n) = A^{-1}C\ \underline{z}(o) - \sum_{l=1}^{n} (A^{-1}C)^{l-1}A^{-1}\ \underline{u}(l) \qquad (9)$$

Thus the solution $\underline{z}(n)$ only depends on the initial condition $\underline{z}(o)$ and the boundary conditions $\underline{u}(l)$.

## 2.2. Radiation Heat Transfer

The radiation heat transfer between two grey surfaces is described by the Stefan-Boltzmann law, written in its technical form:

$$\Phi_{r12} = \sigma F_\epsilon A_1 F_{12}(T_1{}^4 - T_2{}^4) \qquad (10)$$

where $\Phi_{r12}$ is the heat radiation flux from surface 1 to surface 2,
$\sigma$ is the Stefan-Boltzmann constant, $F_\epsilon$ is an emissivity factor dependent on emissivities $\epsilon_1$ and $\epsilon_2$ of both surfaces and on the geometrical arrangement, $A_1$ is the area of surface 1, $F_{12}$ is the configuration or geometric factor defined as the radiation fraction leaving surface 1 which falls on surface 2, $T_1$ and $T_2$ are the absolute temperatures of surfaces 1 and 2.
If $(T_1-T_2) = \Delta T$ is small, eq (10) can be approximated by:

$$\Phi_{r12} = 4\sigma F_\epsilon A_1 F_{12}\overline{T}^3\Delta T \qquad (11)$$

where $\overline{T} = \frac{T_1+T_2}{2}$. Then the heat resistance for radiation $R_r$ for two surfaces 1 and 2 follows from:

$$R_r = \frac{\Delta T}{\Phi_{r12}} = \frac{1}{4\sigma F_\epsilon A_1 F_{12} T^3} \qquad (12)$$

When the temperature range is sufficiently small as by approximation in most cases of radiation transfer in rooms, $R_r$ will be a constant. Moreover for most room wall surfaces $F_\epsilon \approx 1$, so that then $R_r$ = constant x $\frac{1}{A_1F_{12}}$.

The geometric factor $F_{12}$ for heat radiation between the wall surfaces of a room is calculated applying well-known formulae. For perpendicular planes, indicated by 1 and 2, use is made of (fig 2):

$$F_{12} = \frac{1}{\pi L}(E_1+E_2+E_3) \qquad (13)$$

with $E_1 = L \text{ arctg } (\frac{1}{L}) + N \text{ arctg } (\frac{1}{N}) - \sqrt{N^2+L^2} \text{ arctg } (\frac{1}{\sqrt{N^2+L^2}})$

$$E_2 = \tfrac{1}{4} \ln \left[ \frac{(1+L^2)(1+N^2)}{1+N^2+L^2} * \frac{L^2(1+L^2+N^2)}{(1+L^2)(1^2+N^2)} \right]^{L^2}$$

$$E_3 = \left[ \frac{N^2(1+L^2+N^2)}{(1+N^2)(L^2+N^2)} \right]^{N^2}$$

and for parallel planes (see fig 3):

$$F_{12} = \frac{2}{\pi xy} (E_4+E_5+E_6) \tag{14}$$

with $E_4 = \frac{1}{2} \ln \left[ \frac{(1+x^2)(1+y^2)}{1+x^2+y^2} \right]$

$$E_5 = y \sqrt{1+x^2} \text{ arctg } \left( \frac{y}{\sqrt{1+x^2}} \right) + x \sqrt{1+y^2} \text{ arctg } \left( \frac{x}{\sqrt{1+y^2}} \right)$$

$$E_6 = -y \text{ arctg } y - x \text{ arctg } x$$

Radiation at the blades of venetian blinds is a special case. For both heat radiation and the reflection, absorption and transmission of solar radiation, several geometric factors were to be calculated. They are indicated in fig 4 and formulated below:

$$F_4(z,\psi,m) = \tfrac{1}{2} - \frac{1}{2mz} \sqrt{1+z^2+2z+2z \sin \psi} + \frac{1}{2mz} \sqrt{1+(1-m)^2z^2+2z(1-m) \sin \psi} \tag{15}$$

where z = W/S, m is the part of W which is radiated by the sun, $\psi$ is the position of the blades. If the blades are fully radiated by the sun then m = 1.

Furthermore:

$$F_5(z,\psi,m) = \frac{1}{2mz} \sqrt{1+z^2+2z \sin \psi} + \frac{1}{2mz} \sqrt{1+m^2z^2-2zm \sin \psi} - \frac{1}{2mz} \sqrt{1+(1-m)^2z^2+2(1-m)z \sin \psi} - \frac{1}{2mz} \tag{16}$$

$$F_6(z,\psi,m) = \tfrac{1}{2} - \frac{1}{2mz} \left[ \sqrt{1+m^2z^2-2mz \sin \psi} - 1 \right] \tag{17}$$

The factor $F_1$, $F_2$ and $F_3$ are special cases of $F_4$, $F_5$ and $F_6$. When the whole blade is sun radiated then $F_1$, $F_2$ and $F_3$ must be applied (m=1):

$$F_1(z,\psi) = F_4(z,\psi,1) \tag{18}$$

$$F_2(z,\psi) = F_5(z,\psi,1) \tag{19}$$

$$F_3(z,\psi) = F_6(z,\psi,1) \tag{20}$$

It is to be noted that

$$F_4(z,\psi,m) + F_5(z,\psi,1) + F_6(z,\psi,1) = 1 \tag{21}$$

### 2.3. Convection Heat Transfer

At the various wall surfaces of a room convection heat transfer occurs. The heat flow between wall and room air is, as usual, described by:

$$\Phi_c = a_c A(T_w - T_a) \tag{22}$$

where $a_c$ is the convection heat transfer coefficient, A is the area of the wall surface, $T_w$ is the wall surface temperature, $T_a$ is the room air temperature. Thus the heat resistance for the heat transfer between the wall surfaces and the room air may be presented by

$$R_c = \frac{\Phi_c}{\Delta T} = \frac{1}{a_c A} \tag{23}$$

The convection heat transfer coefficient $a_c$ generally depends on the air velocity v and the temperature difference $\Delta T$. In case of room wall surfaces a useful approximation for $a_c$ is:

$$a_c = 2 + 6\sqrt{v} \quad \text{for} \quad v \leqslant 5 \text{m/s}$$
$$a_c = 6{,}5 v^{0{,}8} \quad \text{for} \quad v > 5 \text{m/s} \tag{24}$$

v in m/s and $a_c$ in W/($m^2.^oC$).

## 3. The Room Model

In paragraph 2 the simulation of conduction, radiation and convection heat transfer by a computation scheme or network consisting of heat resistances and fictitious heat capacity resistances has been explained. In figure 5 the computation scheme of the applied unit room is shown schematically. In this figure the resistances drawn represent the convection resistances between the (numbered) nodes for the surface and air temperatures, and some of the radiation resistances between the surface temperature nodes. The other radiation resistances has been left out for clearness' sake. The blocks represent the capacitive walls or layers. The figures in these blocks correspond to the segments in which the walls are discretised. The complete room model consists of 71 nodes. Some detailed parts of this model are given in figures 6, 7 and 8 referring to the schemes for the facade wall and for one of the various possible glazing and sun shading constructions. The various resistance symbols are explained below fig 6.

For each node the heat balance equation can be written, containing the occurring heat fluxes, each of which as a quotient of temperature difference and resistance, as described in par 2. The 71 nodes equations together form the matrix equation (7).

## 4. Boundary Conditions

In this section the boundary conditions are described, such as the outdoor air temperature and the solar heat sources in the room derived from the incident solar radiation.

### 4.1. Outdoor Air Temperature

The outdoor air temperature may be approximated by a single sinus function or by the sum of some sinus functions. For summer conditions in The Netherlands a useful approximation is:

$$T_{ao} = 23.0 + 5.0 \cos \frac{2\pi}{24} (t_s - 14) \tag{25}$$

where $T_{ao}$ is the outdoor air temperature in $^oC$ and $t_s$ is the solar time in hours.
If sufficient meteorological data are available more accurate functions for $T_{ao}$ can be obtained, for instance the following form computed by Halahyja [10], which is valid for Hurbanovo (Czechoslovakia), July:

$$T_{ao} = 22.06+7.38 \cos 2\pi(\frac{t_s-14.6}{24})+0.45 \cos 2\pi(\frac{t_s-16.3}{12})+0{,}96 \cos 2\pi(\frac{t_s-9.0}{8})+0.11 \cos(\frac{t_s-13.2}{6}) \tag{26}$$

### 4.2. Direct Solar Radiation

The direct solar radiation on earth, normal to the radiation direction, $S_{dr}$, is generally considered as a function of the altitude of the sun (h) and the sky turbidity (T). Using Nehring [13] the following relation we derived:

$$S_{dr}(h,T) = S_{dr\ max}(T)(\sin h)^{0.09T+0.14} \tag{27}$$

with

$$S_{dr\ max}(T) = \exp[\ln 10(\log S_{dr\ o} - T/24.06)] \tag{28}$$

where $S_{dr\ o}$ is the (direct) extra terrestrial solar radiation, normal to the radiation direction. The altitude of the sun (h) is given by:

$$\sin h = \sin \delta \ \sin \varphi + \cos \delta \ \cos \varphi \ \cos \frac{2\pi}{24}(t_s - 12) \tag{29}$$

where $\delta$ is the declination of the sun and $\varphi$ is the latitude on earth. If k is the $k^{th}$ day of the year then:

$$\delta = \frac{23^o22'}{360} \; 2\pi \; \sin \left[\frac{2\pi}{365} (k-80)\right] \tag{30}$$

For vertical surfaces $S_{dr}$ must be multiplied by cos $\Phi$, where $\Phi$ is the angle of incidence.

### 4.3. Diffuse Radiation

The diffuse sky radiation is caused by the scattering of direct solar radiation in the atmosphere. After numerous measurements for the diffuse sky radiation on a horizontal plane Bernhardt and Philipps [14] derived:

$$S_{df,h} = 0.33(S_{dr\ o} - S_{dr}) \sin h \tag{31}$$

The diffuse sky radiation on the vertical plane ($S_{df,v}$) depends, of course, on $S_{df,h}$, but also on the position of the sun in respect of the vertical plane in question. Threlkeld [15,16] derived the latter relation by investigation [16]. We approximated this relation by two polynoms:

$$\frac{S_{df,h}}{S_{df,v}} = \begin{cases} 0.560 + 0.436 \cos \Phi + 0.35 \cos^2\Phi & , \text{for } \cos \Phi \geqslant -0.3 \\ 0.473 - 0.043 \cos \Phi & , \text{for } \cos \Phi < -0.3 \end{cases} \tag{32}$$

($\Phi$ is angle of incidence)

In fig 9 the computed diffuse sky radiation on the vertical plane is given as a function of the altitude of the sun, with the azimuth of the sun with regard to the facade, as a parameter. Here the turbidity factor T = 4.0.

Another part of the diffuse radiation on facades is caused by reflection of solar radiation by the ground surface and by the adjacent buildings. The solar radiation reflected by the ground which falls on a facade can be approximated by:

$$S_g = \tfrac{1}{2} R_g (S_{dr,h} + S_{df,h}) \tag{33}$$

where $R_g$ is the reflection factor of the ground surface for solar radiation.

### 4.4. Absorption and Transmission of Solar Radiation at Glass Planes and Sunshading Surfaces

At the glazing and the sun shading devices the direct solar radiation and the diffuse radiation fluxes are partly reflected, absorbed and transmitted. Thus the solar heat sources in the room are activated (see fig 8). In order to calculate the various absorbed fractions, the properties of the glass planes and sun shading elements must be known as a function of the so called profile angle [12]. For this purpose the basic calculation method is taken from Parmelee [11,12].

Besides the properties of the separate glass planes etc , also the absorption, the transmission and the reflection factors of the combined glazing and sun shading systems must be calculated. Here a distinction must be made between direct solar radiation, diffuse sky radiation and diffuse radiation from the ground and any surrounding buildings.

In the calculation of the (spread) solar heat sources in the various solar radiated surfaces, all the above mentioned effects are taken into account.

Till now the calculation of seven combinations of ordinary glass with different sun shading devices have been carried out. They are:
a) single glass
b) single glass with indoor blinds
c) single glass with outdoor blinds
d) double glass

e) double glass with indoor blinds
f) double glass with blinds in between
g) double glass with outdoor blinds

Glazings with sun absorbing planes are provisionally calculated in another way. For each plane the absorption, reflection and transmission fractions for a mean incidental angle of 45° is applied. If necessary these quantities are determined experimentally with the aid of a spectrometer.

Glazings with reflecting coatings, which are however as a rule slightly tinted, are calculated while taking into account the dependence of the reflection and transmission on the angle of incidence.

The part of the incident solar radiation, which is transmitted through the glazing and sun shading system which enters the room is spread over the various wall surfaces, in general homogeneously, if necessary unhomogeneously.

### 4.5. Internal Heat Sources

In an office room in general heat is produced by occupants, the lighting (light flux, equipment heat) dissipation) and any apparatus present. The heat fluxes produced are partly transferred to the air or when the air temperature is controlled, they are added to the cooling load. Another part of the internal heat load, however, is radiated to the walls and can partly be accumulated. Therefore the internal heat fluxes must always be separated into a convection and a radiation part.

## 5. Survey of the Program

### 5.1. Possibilities

The units of the calculation model discussed in the former paragraphs are so built up to permit murhvariants can be carried through. These variants refer to both the composition of the inner walls and the facade and sun shading elements. Besides the diversity of glazing and sun shading systems, it may for instance, also be assumed that the blinds may be drawn up during a certain period of the day. Furthermore it is possible to calculate the point of time at which artificial illumination should be switched on, on account of the computed visuable solar radiation flux that enters the room. Shading effects caused by surrounding buildings or by overhangs and side fins may also be taken into account [17].

Between the various capacitive layers of the walls cavities may occur, which may be ventilated. The temperature of the ventilation air may either be prescribed or free. The room air may be ventilated with outdoor air or with air of another (prescribed or free) temperature.

The temperature of the room air may follow a precribed curve, in which case the cooling load results as a function of time. On the other hand the capacity of the cooler may be prescribed; then the room air temperature is calculated as a function of time.

Besides the mean room air temperature respectively the cooling load, all the temperatures in all the nodes are calculated. The heat fluxes between the nodes may be calculated if necessary. For indoor climate analysis the radiant temperature at certain points in the room may be calculated. It is of course possible to combine these quantities with the room air temperature, in order to calculate the so-called dry resulting temperature.

### 5.2. Flow Diagram

In fig 10 a simplified flow diagram of the program is given.

The computing time of the program on a IBM 360/65 computer for one situation (room, facade, sun shading, date, orientation, latitude) is about 5 minutes, subdivided in 2 minutes compilation time, 1 minute matrix inversion and 1 minute temperature and heat flux calculations when $\Delta t$ = 15 minutes.

The core memory required is about 300 K oktades.

### 5.3. Test Results

In fig 11 and 12, some test results are given. In the case under consideration, the glazing consisted of two normal glass planes, with venetian blinds between these planes. As to the incident solar radiation, the following data were chosen: latitude 52°, orientation SW, date July 23, sky turbidity factor T=4. The internal load amounted to 800 W. The ventilation rate was three room changes per hour.

These results, and others, will be checked by means of an RC. network simulator.

## 6. Further Development

Several other variants as the above mentioned may be included in the program through relatively small modifications, such as a second facade with glazing, a separate ceiling part for lighting equipment, a second non-transparant outer wall or roof.

For technical calculations, which must as a rule be carried out frequently, a reduced program will be made, based on the same concept, with about 20 in stead of 71 nodes. Since the computing time is nearly proportional to the third power of the number of nodes, then a reduction of the computing time of about 30 times will be obtained.

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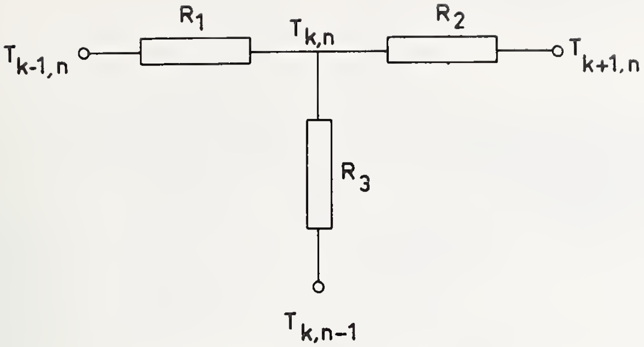


Figure 1. Base scheme for the used heat conduction difference equation

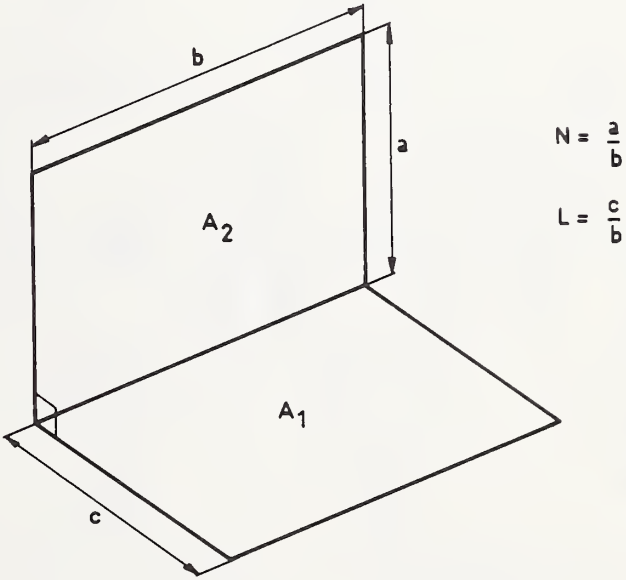


Figure 2. The configuration quantities in case of perpendicular walls, used in eq(13)

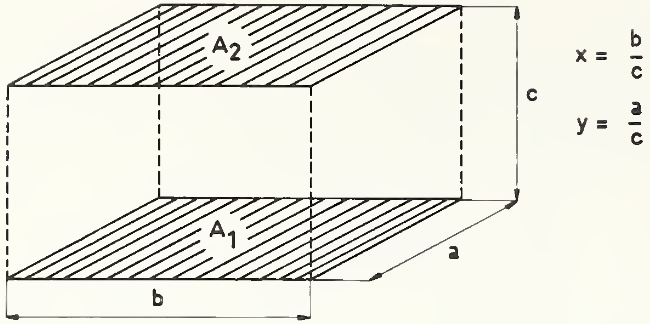


Figure 3. The configuration quantities in case of parallel walls, used in eq(14)

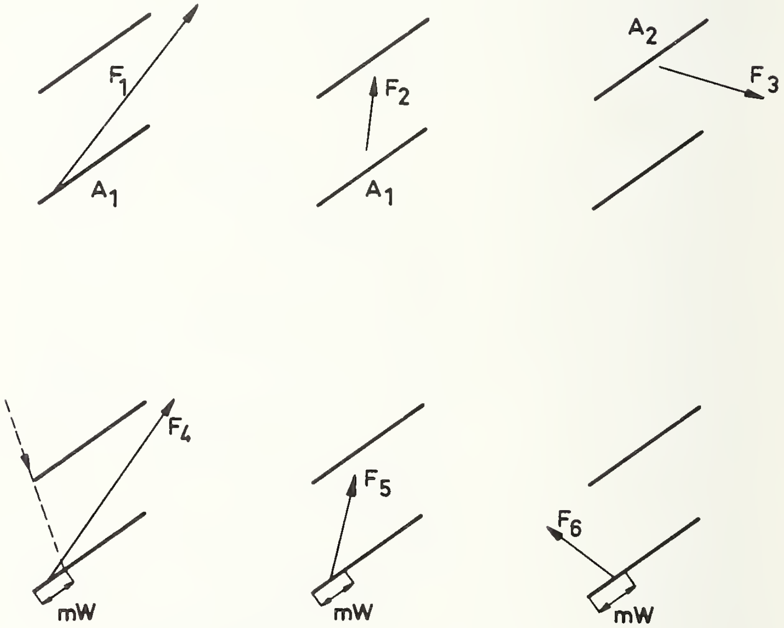


Figure 4. The configuration quantities used with the calculation of the heat and solar radiation transport at venetian blinds

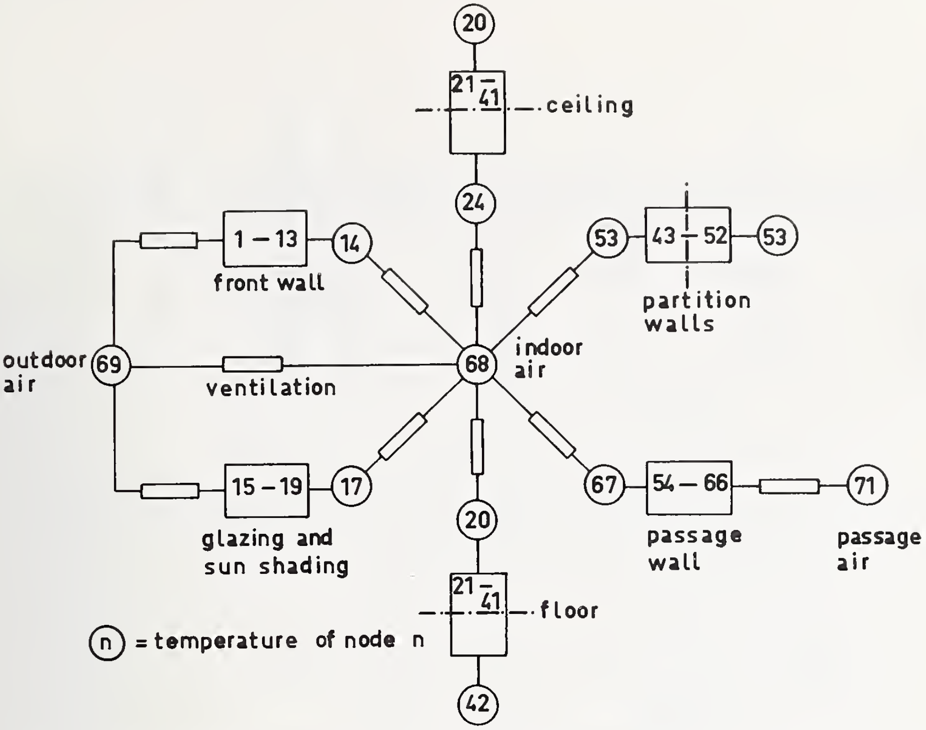


Figure 5. The thermal model of the unit room (for clearness the radiation resistances are omitted)

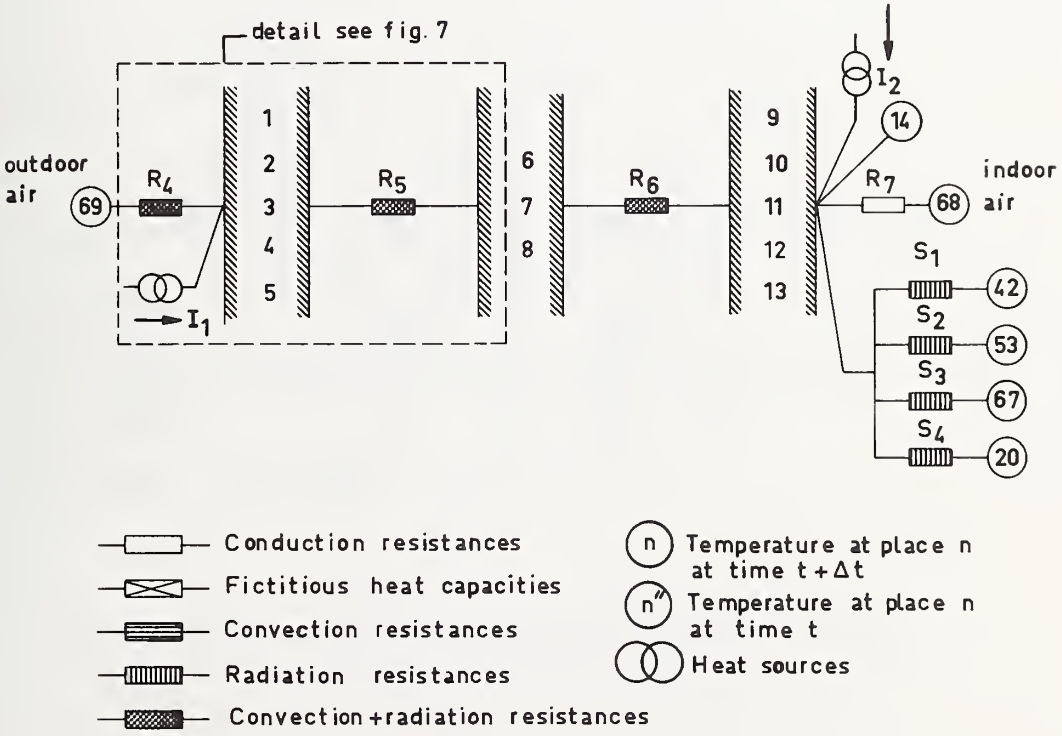


Figure 6. The thermal model of the front wall

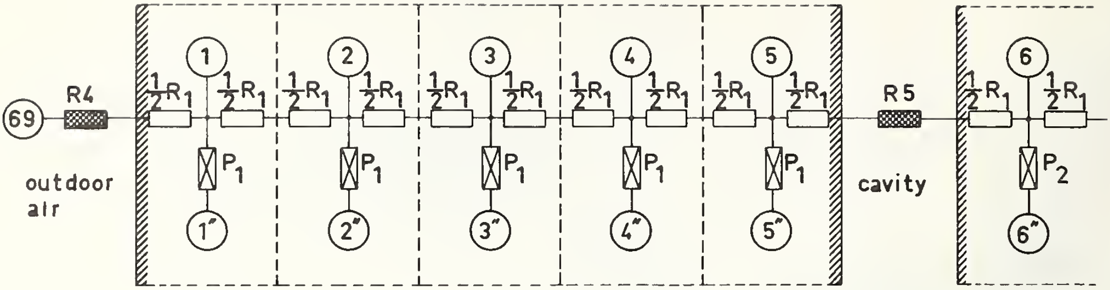


Figure 7. The computation scheme for a part of the front wall (for symbols see fig. 6)

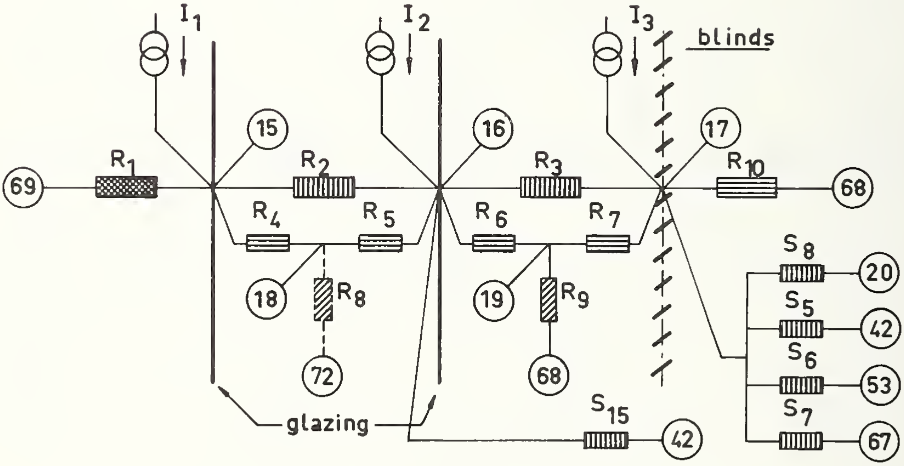


Figure 8. The computation scheme for a glazing and sun shading system (for symbols see fig. 6)

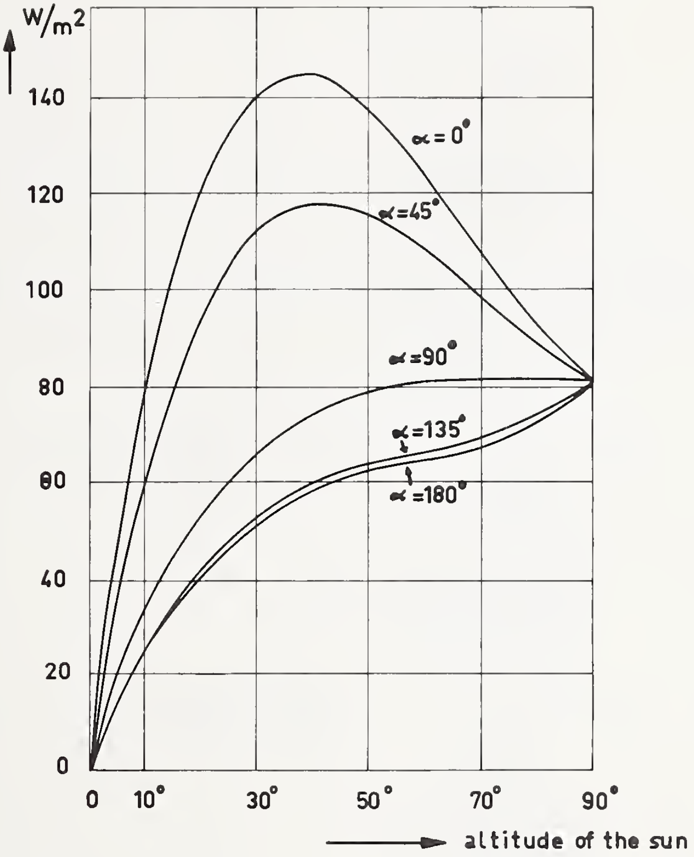


Figure 9. The diffuse sky radiation on a vertical plane ($S_{df,v}$) as a function of the altitude of the sun for different horizontal positions of the sun (α) with regard to the facade. Turbidity factor T=4.0

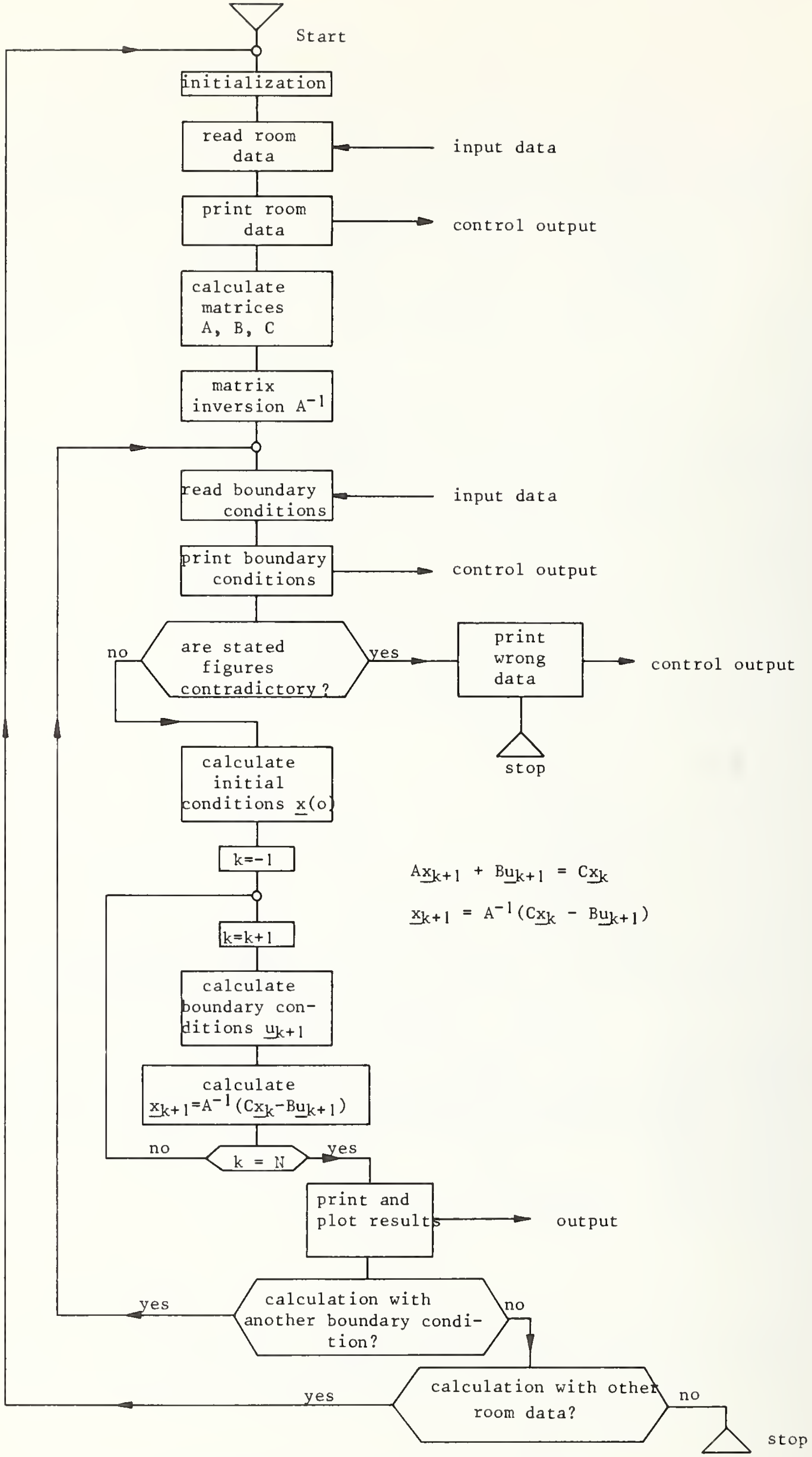
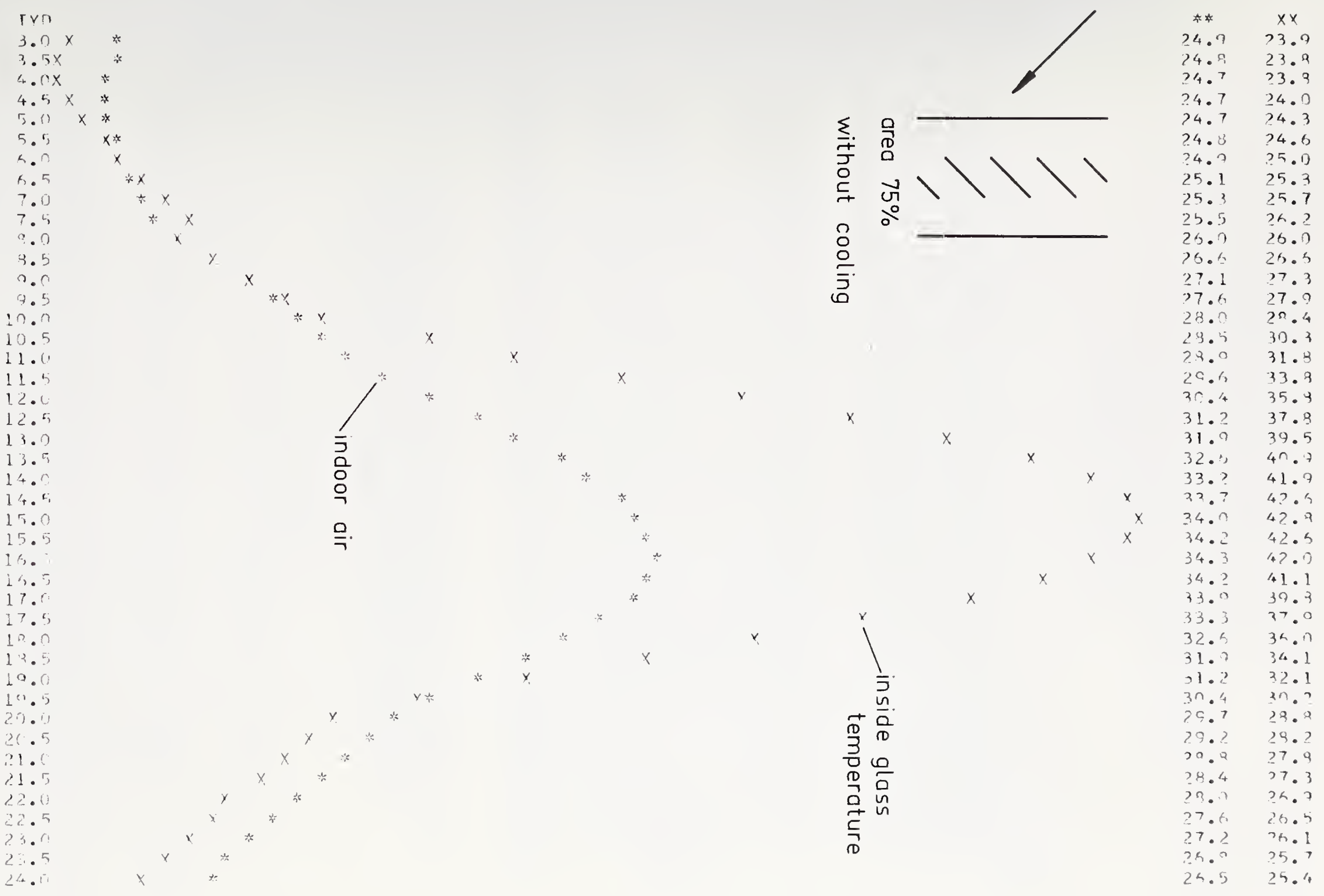


Figure 10. Simplified flow diagram of program arrangement

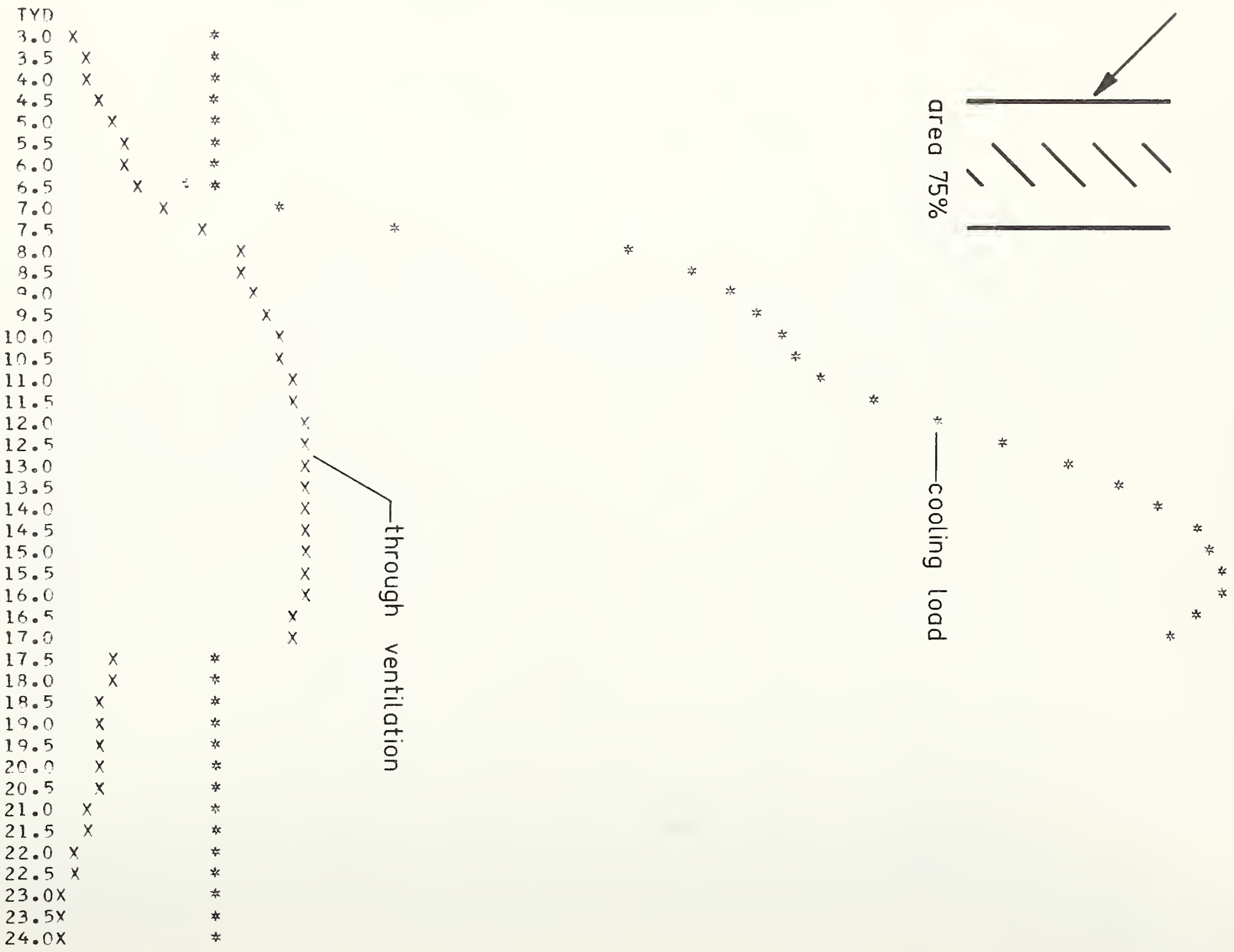
DE BUITENLUCHTTEMPERATUUR (*) EN HET BINNENSTE GLASVLAK OF DE JALOEZIE (X)



| TYD | ** | XX |
|---|---|---|
| 3.0 | 24.9 | 23.9 |
| 3.5 | 24.8 | 23.8 |
| 4.0 | 24.7 | 23.8 |
| 4.5 | 24.7 | 24.0 |
| 5.0 | 24.7 | 24.3 |
| 5.5 | 24.8 | 24.6 |
| 6.0 | 24.9 | 25.0 |
| 6.5 | 25.1 | 25.3 |
| 7.0 | 25.3 | 25.7 |
| 7.5 | 25.5 | 26.2 |
| 8.0 | 26.0 | 26.0 |
| 8.5 | 26.6 | 26.5 |
| 9.0 | 27.1 | 27.3 |
| 9.5 | 27.6 | 27.9 |
| 10.0 | 28.0 | 28.4 |
| 10.5 | 28.5 | 30.3 |
| 11.0 | 28.9 | 31.8 |
| 11.5 | 29.6 | 33.8 |
| 12.0 | 30.4 | 35.8 |
| 12.5 | 31.2 | 37.8 |
| 13.0 | 31.9 | 39.5 |
| 13.5 | 32.6 | 40.9 |
| 14.0 | 33.2 | 41.9 |
| 14.5 | 33.7 | 42.6 |
| 15.0 | 34.0 | 42.8 |
| 15.5 | 34.2 | 42.5 |
| 16.0 | 34.3 | 42.0 |
| 16.5 | 34.2 | 41.1 |
| 17.0 | 33.9 | 39.3 |
| 17.5 | 33.3 | 37.0 |
| 18.0 | 32.6 | 36.0 |
| 18.5 | 31.9 | 34.1 |
| 19.0 | 31.2 | 32.1 |
| 19.5 | 30.4 | 30.2 |
| 20.0 | 29.7 | 28.8 |
| 20.5 | 29.2 | 28.2 |
| 21.0 | 28.8 | 27.9 |
| 21.5 | 28.4 | 27.3 |
| 22.0 | 28.0 | 26.9 |
| 22.5 | 27.6 | 26.5 |
| 23.0 | 27.2 | 26.1 |
| 23.5 | 26.9 | 25.7 |
| 24.0 | 26.5 | 25.4 |

Figure 11. The indoor air and inside glass temperature computed for a test case, without cooling. Further conditions are stated in par. 5.3

DE KOELING (*) EN DE VENTILATIE (X)



| TYD | ** | XX |
|---|---|---|
| 3.0 | 0.0 | -208.3 |
| 3.5 | 0.0 | -196.8 |
| 4.0 | 0.0 | -183.4 |
| 4.5 | 0.0 | -169.9 |
| 5.0 | 0.0 | -156.4 |
| 5.5 | 0.0 | -142.2 |
| 6.0 | 0.0 | -127.5 |
| 6.5 | 0.0 | -112.4 |
| 7.0 | 107.1 | -76.3 |
| 7.5 | 299.2 | -18.6 |
| 8.0 | 648.9 | 39.6 |
| 8.5 | 748.3 | 55.1 |
| 9.0 | 804.7 | 70.3 |
| 9.5 | 849.8 | 85.0 |
| 10.0 | 891.9 | 98.9 |
| 10.5 | 914.4 | 111.8 |
| 11.0 | 956.2 | 123.5 |
| 11.5 | 1038.5 | 133.7 |
| 12.0 | 1136.0 | 142.3 |
| 12.5 | 1235.4 | 149.2 |
| 13.0 | 1328.9 | 154.2 |
| 13.5 | 1411.4 | 157.2 |
| 14.0 | 1479.5 | 158.2 |
| 14.5 | 1530.6 | 157.2 |
| 15.0 | 1563.0 | 154.2 |
| 15.5 | 1575.8 | 149.2 |
| 16.0 | 1568.7 | 142.3 |
| 16.5 | 1542.4 | 133.7 |
| 17.0 | 1498.2 | 123.5 |
| 17.5 | 0.0 | -149.6 |
| 18.0 | 0.0 | -160.7 |
| 18.5 | 0.0 | -165.4 |
| 19.0 | 0.0 | -166.7 |
| 19.5 | 0.0 | -166.7 |
| 20.0 | 0.0 | -168.4 |
| 20.5 | 0.0 | -177.6 |
| 21.0 | 0.0 | -188.2 |
| 21.5 | 0.0 | -198.7 |
| 22.0 | 0.0 | -208.6 |
| 22.5 | 0.0 | -217.3 |
| 23.0 | 0.0 | -224.5 |
| 23.5 | 0.0 | -230.0 |
| 24.0 | 0.0 | -233.5 |

Figure 12. The cooling load in the case of fig. 11, when the indoor air temperature is prescribed from 6.00 am to 8.30 am linear downwards to 22°C, and from 8.30 am to 5.00 pm sinusoidally with a maximum value of 24 °C at 2.00 pm

# A Successive Integration Method for The Analysis of the Thermal Environment of Building

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This method is to calculate the room air temperature or the heating (cooling) load variations for each $\Delta t$ step with repetition of simple multiplications and additions by utilizing both the nature of an exponential function which decreases by equal ratio for each $\Delta t$ step and the fact the indicial response of the room, wall and/or the heating equipment to a thermal input of unit step function is approximate to the sum of exponential functions.

The method is effective not only for the ordinal transient heating (cooling) load calculations but for the simulations of such cases as ; when the system has multiple rooms of different conditions, when the ventilation rate of the room or the heat transfer coefficient of the wall varies, etc..

As another distinctive feature of this method, it is easy to change $\Delta t$ in the way of calculation whenever it is necessary, therefore when the heat capacity of the building is quite large and the actual outdoor conditions (including solar radiation) should be considered, it is possible to calculate with fewer times of calculations with high accuracy by this method.

In the report the authors deal with the principle of this method, method to change $\Delta t$, some considerations for setting the initial conditions to minimize the time of calculations and some examples of calculations.

Key Words : Thermal environment, Successive integration method, Indicial response, Temperature excitation, Heat flow response, Duhamel's integration formula, Heating load, Non-linear factor, Heat transfer coefficient, Radiation, Ventilation, Change of $\Delta t$ .

## 1. Introduction

The factors related to precise analysis of the thermal environment of a building are numerous as follows.

- Outside conditions ; fluctuations of temperature, solar radiation, atmospheric radiation, wind velocity, movement of sunlit and shaded area, etc.
- Inside conditions ; regulation of temperature and zoning, intermittent heat supply, effect of unconditioned space, distribution of air temperature and radiant heat transfer in a room, rate of ventilation or infiltration and its change, etc.

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[1] Associate Prof. (M.E.), M.E. (Takasago Netsugaku Co.) and Research Assistant (M.E.) respectively

- Others ; two or three dimensional heat flow at the beams, columns or wall corners, thermal capacity of furnitures and room furnishings, dynamic performance of heating equipment, etc.

Detailed studies on the weighting function for the analysis of thermal environment have been carried on for a long time by Dr. T. Maeda, Dr. S. Fujii, Dr. F. Hasegawa and others.

This successive integration method is one application of these earlier investigations, the intention is to make analysis more flexible and make calculation easier. This method would be applicable for many kinds of problems in which the factors listed above are concerned and depending on the purpose or the requisit accuracy of the analysis various conbinations and simplifications would be possible.

## 2. Duhamel's integration formula

Consider a system (such as a room, a wall or a piece of equipment) as illustrated in figure 1 and assume the heat flow response of the system to a unit step function of temperature $\theta_{u(t)}$ is given by $h(t)$ , then the heat flow response consequent on the arbitrary temperature excitation $\theta(t)$ is obtained by the use of Duhamel's integration formula as follows ;

$$H(t) = \int_0^t \theta'(\tau) \cdot h(t-\tau)\, d\tau + \theta(t=0) \cdot h(t) \qquad (1)$$

where $\tau$ : variable of integration

$\theta'(t)$ : the first derivative of $\theta(t)$

$h(t)$ : indicial response of heat flow to a unit step function of temperature

however when $t<0$ $\left.\begin{array}{l}\theta(t) = 0 \\ h(t) = 0\end{array}\right\}$

For example, heat flow at the inner surface of the wall under the conditions of arbitrary inside and outside air temperature variations $\theta_i(t)$ and $\theta_o(t)$ are expressed by the sum of the responses which are excited by both excitation of $\theta_i(t)$ and $\theta_o(t)$ as follows ;

$$H(t) = H_i(t) - H_o(t)$$

$$= \int_0^t \theta_i'(\tau) \cdot h_i(t-\tau)\, d\tau + \theta_i(t=0) \cdot h_i(t) - \int_0^t \theta_o'(\tau)\, h_o(t-\tau)\, d\tau - \theta_o(t=0) \cdot h_o(t) \qquad (2)$$

where $H_i(t)$ : heat flow at the inner surface of the wall when the inside air temperature is $\theta_i(t)$ and the outside air temperature is kept at 0°C

$H_o(t)$ : heat flow at the inner surface of the wall when the outside air temperature is $\theta_o(t)$ and the inside air temperature is kept at 0 °C

$h_i(t)$, $h_o(t)$ : indicial response of heat flow at the inner surface of the wall as seen in figure 2 ( Watt $\deg^{-1}$ )

## 3. Approximation of an indicial response of heat flow

The indicial response of heat flow of the system to a unit step function of temperature can be obtained by several ways, and in so far as the linear characteristics of the system are kept, it would be expressed by the sum of the infinite series of exponential functions. And in actual use it can be sufficiently approximated by several terms as follows ;

$$h(t) = B_0 + \sum_{m=1}^{j} B_m \cdot e^{-\beta_m \cdot t} + g\ \delta(t) \qquad (3)$$

$B_o$ : the term for steady state heat flow ( $Watt \cdot deg^{-1}$ )

$\delta(t)$: delta function

$g$ : imaginary thermal capacity of the system ( $Watt \cdot h \cdot deg^{-1}$ )

$\beta_m$ : $\beta_m$ becomes larger in order of suffix $m$

The imaginary thermal capacity $g$ is the amount of heat to be supplied to the system instantly when the temperature of excitation is raised suddenly from 0 °C to 1 °C at $t = 0$. However, for the approximation of the indicial response of heat flow at the inner surface $h_o(t)$, it would be advisable to approach it so as to satisfy the following conditions from the nature of thermal response to an outside excitation.

$$h_o(t=0) = 0$$

$$\therefore \quad g = 0 \quad , \quad B_o + \sum_{m=1}^{j} B_m = 0$$

There are many studies concerning the approximation and simplification of the indicial response as shown in the reference. 4), 5), 6)

## 4. Approximation of temperature variation by linear equation and the successive calculation method of heat flow

By substituting eq.. (3) for (1), the heat flow response of the system $H(t)$ to an arbitrary temperature excitation $\theta(t)$ is expessed as ;

$$H(t) = \int_0^t \theta'(\tau) \cdot h(t-\tau) \ d\tau + \theta(t=0) \cdot h(t)$$

$$= \underbrace{B_o \ \theta(t)}_{Y(t)} + \underbrace{\sum_m \left[ \int_0^t \theta'(\tau) \ B_m \cdot e^{-\beta_m (t-\tau)} d\tau + \theta(t=0) \cdot B_m \ e^{-\beta_m t} \right]}_{\Sigma Z_m(t)} + \underbrace{\int_0^t \theta'(\tau) \cdot g \cdot \delta(t-\tau) \cdot d\tau}_{D(t)} \qquad (4)$$

The heat flow $H(t)$ is expressed as the sum of steady state term $Y(t)$, transient terms $Z_m(t)$ and impulsive term $D(t)$.

Now, let's assume the temperature variation $\theta(t)$ is approximated by a linear equation within the time of $t_n \leqslant t \leqslant t_n + \Delta t$ as seen in figure 3 and expressed as ;

$$\theta(t) = \theta_1(t) + \theta_2(t) \qquad (5)$$

where

$$\left. \begin{aligned} \theta_1(t) &= \theta(t) \\ \theta_2(t) &= 0 \end{aligned} \right\} \ t \leqslant t_n$$

$$\left. \begin{aligned} \theta_1(t) &= \theta_n = \text{const.} \\ \theta_2(t) &= A_{(n+1)} \ (t - t_n) \end{aligned} \right\} \ t_n \leqslant t \leqslant t_n + \Delta t$$

$A_{(n+1)}$ : temperature gradient ( $deg \cdot h^{-1}$ )

Substituting eq. (5) for (4) the heat flow at the time ( $t_n + \Delta t$ ) will be ;

$$\begin{aligned} H(t_n+\Delta t) = \ & B_o \ \theta(t_n+\Delta t) + \sum_m \left[ \int_0^{t_n+\Delta t} \theta_1'(\tau) \cdot B_m \cdot e^{-\beta_m (t_n+\Delta t-\tau)} d\tau + \theta(t=0) \cdot B_m \ e^{-\beta_m (t_n+\Delta t)} \right] \\ & + \sum_m \left[ \int_0^{t_n+\Delta t} \theta_2'(\tau) \cdot B_m \cdot e^{-\beta_m \cdot (t_n+\Delta t-\tau)} d\tau \right] \\ & + \int_0^{t_n+\Delta t} \{ \theta_1'(\tau) + \theta_2'(\tau) \} \cdot g \cdot \delta(t_n+\Delta t-\tau) \cdot d\tau \end{aligned} \qquad (6)$$

defined as follows ;

$$\theta_1'(t) = 0 \qquad t_n \leqq t \leqq t_n + \Delta t$$

$$\theta_2'(t) = \theta_2(t) = 0 \cdot \cdot \qquad t \leqq t_n$$

$$\int_0^\infty f(\tau) \cdot \delta(t-\tau) \, d\tau = f(t) \qquad t > 0$$

Equation (6) becomes

$$H(t_n+\Delta t) = B_0 \left\{ \theta_{(n)} + A_{(n+1)} \cdot \Delta t \right\} + \sum_m \Big[ \underbrace{\int_0^{t_n} \theta_1'(\tau) \cdot B_m \cdot e^{-\beta_m (t_n - \tau)} d\tau + \theta(t=0) \cdot B_m \cdot e^{-\beta_m \cdot t_n}}_{Z_{m(n)}} \Big] \cdot e^{-\beta_m \cdot \Delta t}$$
$$+ \sum_m \Big[ \int_{t_n}^{t_n+\Delta t} A_{(n+1)} \cdot B_m \cdot e^{-\beta_m (t_n + \Delta t - \tau)} d\tau \Big] + A_{(n+1)} \cdot g \qquad (7)$$

In equation (7) the underlined portion is the same as the transient term $Z_m$ at the time of $t_n$ . Thus the following simple calculation method is obtained ;

$$H(t_n+\Delta t) = H_{(n+1)} = Y_{(n+1)} + \sum_m Z_{m(n+1)} + D_{(n+1)}$$

$$= B_0 \cdot \theta_{(n)} + A_{(n+1)} \, B_0 \cdot \Delta t + \sum_m \left\{ Z_{m(n)} \cdot E_m + A_{(n+1)} \cdot X_m \right\} + A_{(n+1)} \cdot g \qquad (8)$$

where $$E_m = e^{-\beta_m \cdot \Delta t} = \text{const.} \qquad (9)$$

$$X_m = \frac{B_m}{\beta_m} \cdot (1 - e^{-\beta_m \cdot \Delta t}) = \text{const.} \qquad (10)$$

If we take the time interval of each section $\Delta t$ to be the same as seen in figure 4 then the coefficient of $E_m$ and $X_m$ becomes constant and the calculation of eq. (8) becomes very simple. It is also easy to change $\Delta t$ by only changing $E_m$ and $X_m$ whenever necessary.

This eq. (8) gives the rate of heat flow at the end of each interval so that we are able to analyze the heating load or the temperature variations by substituting this for eq. (16) and it may be somewhat easier to understand but the following method (which uses the quantity of heat flow during the interval instead of the rate of heat flow at the end of the interval) would be more accurate in calculation.2)

## 5. Integration of heat flow in each interval

The quantity of heat flow during an interval of $t_n \sim (t_n + \Delta t)$ can be obtained by integrating eq. (7) with respect to $\Delta t$ as follows (by using a variable of integration $\xi$ instead of $\Delta t$ ) ;

$$\Delta H_{(n+1)} = \int_{t_n}^{t_n+\Delta t} H(t) \, dt = \int_0^{\Delta t} H(t_n + \xi) \cdot d\xi$$

$$= \int_0^{\Delta t} B_0 \left\{ \theta_{(n)} + A_{(n+1)} \cdot \xi \right\} d\xi + \sum_m \Big[ Z_{m(n)} \cdot \int_0^{\Delta t} e^{-\beta_m \cdot \xi} d\xi + A_{(n+1)} \cdot \frac{B_m}{\beta_m} \int_0^{\Delta t} (1 - e^{-\beta_m \xi}) \, d\xi \Big] + A_{(n+1)} \cdot g \cdot \int_0^{\Delta t} d\xi$$

$$= B_0 \cdot \theta_{(n)} \cdot \Delta t + \sum_m \Big[ Z_{m(n)} \cdot \frac{1}{\beta_m} (1 - e^{-\beta_m \cdot \Delta t}) \Big] + A_{(n+1)} \cdot \Big[ \frac{B_0 \cdot \Delta t^2}{2} + \sum_m \frac{B_m}{\beta_m} \left\{ \Delta t - \frac{1}{\beta_m} (1 - e^{-\beta_m \Delta t}) \right\} + g \cdot \Delta t \Big]$$

$$= B_0 \cdot \theta_{(n)} \cdot \Delta t + \sum_m \Delta Z_{m(n)} + A_{(n+1)} \cdot \Delta X_0 \qquad (11)$$

where $$\Delta X_0 = \Big[ \frac{B_0 \cdot \Delta t^2}{2} + \sum_m \frac{B_m}{\beta_m} \left\{ \Delta t - \frac{1}{\beta_m} (1 - e^{-\beta_m \cdot \Delta t}) \right\} + g \cdot \Delta t \Big] = \text{const.} \qquad (12)$$

$$\Delta Z_{m(n)} = Z_{m(n)} \cdot \frac{1}{\beta_m} (1 - e^{-\beta_m \cdot \Delta t}) \qquad (13)$$

If the time interval $\Delta t$ is invariable after the time of $t_n$ as seen in figure 4 then eq. (12) becomes constant and from eqs.(13) and (8) the following equation results ;

$$\Delta Z_{m(n+1)} = Z_{m(n+1)} \frac{1}{\beta_m}(1-e^{-\beta_m \cdot \Delta t})$$

$$= \left\{ Z_{m(n)} \cdot E_m + A_{(n+1)} \cdot X_m \right\} \cdot \frac{1}{\beta_m}(1-e^{-\beta_m \cdot \Delta t})$$

$$= \Delta Z_{m(n)} \cdot E_m + A_{(n+1)} \cdot \Delta X_m \tag{14}$$

where $\Delta X_m = \frac{B_m}{\beta_m^2}(1-e^{-\beta_m \cdot \Delta t})^2$ (15)

$= \text{const.}$

If the temperature gradients $A_{(n+1)}$ , $A_{(n+2)}$ , ……… are known, each $\Delta Z_{m(n+1)}$ , $\Delta Z_{m(n+2)}$ ………… can be easily calculated in succession by eq. (14), and each $\Delta H_{(n+1)}$ , $\Delta H_{(n+2)}$ , ……… is also obtained from eq. (11) by the repetition of simple calculations.

## 6. Equation of heat balance

( when the heat transfer coefficients of the room do not change )

Consider room $k$ which is adjacent to rooms $K = 1, 2, 3, \cdots\cdots$ where room air temperatures are different from each other as seen in figure 5. The following equation of heat balance can be given ;

$$Q_k \cdot \frac{d\theta_{k(t)}}{dt} = W_{(t)} - H_{k(t)} + \sum_K \left[ H_{K(t)} + C_p \cdot V_{K(t)} \cdot \left\{ \theta_{K(t)} - \theta_{k(t)} \right\} \right] \tag{16 }$$

where $Q_k$ : thermal capacity of the air of room $k$ ( $\mathrm{Watt} \cdot h \cdot deg^{-1}$ ) including that of furnishings of which the temperature change is considered the same as that of the air temp.

$\theta_{k(t)}$ , $\theta_{K(t)}$ : temperature of the room and adjacent rooms

$W_{(t)}$ : heating rate supplied to room air ( Watt ) including auxiliary heat from human bodies and equipment

$H_{k(t)}$ : heatloss through the surrounding walls of the room, where the air temp. is $\theta_{k(t)}$ and the adjacent room air temp. is $\theta_{K(t)} = 0$

$H_{K(t)}$ : inflow of the heat from the inside surface of the partitions adjacent to the room $K$ under the condition of $\theta_{k(t)} = 0$ adjacent room $\theta_{K(t)}$

$V_{K(t)}$ : air volume infiltrated from room $K$ (outflow air is not related)

$C_p$ : specific heat of the air for unit volume ( $\mathrm{Watt} \cdot h \cdot deg^{-1} \cdot m^{-3}$ )

Assume the divisions of time are the same as in figure 4 and assume the temperature variations within the time interval $t_n \sim (t_n + \Delta t)$ are as follows ;

$$\theta_{(t)} = \theta_{(n)} + A_{(n+1)} \cdot (t - t_n)$$

$$\theta_{(t_n+\Delta t)} = \theta_{(n+1)} = \theta_{(n)} + A_{(n+1)} \cdot \Delta t \tag{17}$$

And assume the heating rate $W_{(t)}$ and the infiltration rate $V_{(t)}$ are constant during the time interval of $\Delta t$ . Then by integrating eq. (16) with respect to $t$ from $t_n$ to $( t_n + \Delta t )$ and substituting eq. (11), the following relation is obtained ;

$$\int_{t_n}^{t_n+\Delta t} \left[ Q_k \cdot \frac{d\theta_{k(t)}}{dt} \right] dt = \int_{t_n}^{t_n+\Delta t} \left[ W_{k(t)} - H_{k(t)} + \sum_K \left[ H_{K(t)} + C_p \cdot V_{K(t)} \left\{ \theta_{K(t)} - \theta_{k(t)} \right\} \right] \right] dt$$

$$Q_k \cdot \Delta t \cdot A_{k(n+1)} = W_{k(n+1)} \cdot \Delta t - B_{ok} \cdot \theta_{k(n)} \cdot \Delta t - \sum_m \Delta Z_{mk(n)} - \Delta X_{ok} \cdot A_{k(n+1)}$$ (continue)

$$+ \sum_{K} \left[ B_{oK} \cdot \theta_{K(n)} \cdot \Delta t + \sum_{m} \Delta Z_{mK(n)} + \Delta X_{oK} \cdot A_{K(n+1)} + C_p \cdot V_{K(n+1)} \cdot \Delta t \cdot \left\{ \theta_{K(n)} - \theta_{k(n)} + \frac{\Delta t}{z} (A_{K(n+1)} - A_{k(n+1)}) \right\} \right] \tag{18}$$

### 6.1 When the temperature is known

In eq. (18) the temperature gradients $A_{k(n+1)}$ and $A_{K(n+1)}$ are known so the total heating load $W_{k(n+1)}$ is obtained easily. By subtracting the auxiliary heat from this result, the net heating load is obtained.

### 6.2 When the temperature is not known

As for unconditioned space or when intermittent heating or cooling is a factor, the temperature gradient $A_{k(n+1)}$ is obtained from the following equation.

$$A_{k(n+1)} = \Big[ W_{k(n+1)} \cdot \Delta t - B_{ok} \cdot \theta_{k(n)} \cdot \Delta t - \sum_{m} \Delta Z_{mk(n)} + \sum_{K} \Big\{ B_{oK} \cdot \theta_{K(n)} \cdot \Delta t + \sum_{m} \Delta Z_{mK(n)} + A_{K(n+1)} \cdot \Delta X_{oK}$$

$$+ C_p \Delta t \cdot \left( \theta_{K(n)} - \theta_{k(n)} + \frac{\Delta t}{z} \cdot A_{K(n+1)} \right) \Big\} \cdot V_{K(n+1)} \Big] \div \left[ \Delta t \cdot Q_k + \Delta X_{ok} + \frac{C_p \cdot \Delta t^2}{z} \cdot \sum_{K} V_{K(n+1)} \right] \tag{19}$$

### 6.3 When the temperatures of adjoining rooms are not known

When the building has many rooms or spaces of which temperature variations are not given, we have to solve equation (19) as a simultaneous equation of unknown temperature gradients.

However, from the nature of the thermal response to a outside excitation, the influence of the temperature variation of the adjoining room is gradual as seen in figure 2. Therefore the accuracy of the temperature gradient of the adjoining room $A_{K(n+1)}$ is not too important for the calculation of $A_{k(n+1)}$ and this is a very fortunate characteristic of this calculation method.

In this case it would be good to assume that the temperature variation of the adjoining room is the same as that at the time $\Delta t$ before. That is to use $A_{K(n)}$ which was already calculated instead of the unknown quantity $A_{K(n+1)}$ in eq. (19).

## 7. When the heat transfer coefficient varies with time or temperature

Actually a fairly large part of the heat from a human body, electricity, radiators and window sunlight, transfers by radiation to the surrounding walls and moreover the convection heat transfer coefficient varies with the wind velocity or with the temperature difference between the wall surface and the air, so that for the precise analysis of the problem it is necessary to treat the heat transfer coefficient as a variable.

In this case, it is alse possible to use the same method as above by considering each surface of the wall, of which the heat transfer coefficient is a variable, as a kind of a room where the temperature is not known. The order of the calculation is as follows ;

Step 1. heat balance of the surface of the wall

An equation of heat balance of the surface of the wall at time $t$ and during the time interval of $\Delta t$ would be written as follows ;

$$\alpha_{k\ell}(t) \left\{ \theta_k(t) - \theta_{k\ell}(t) \right\} - H_{k\ell}(t) + H_{K\ell}(t) + J_{k\ell}(t) = 0 \tag{20}$$

$$\int_{t_n}^{t_n + \Delta t} \alpha_{k\ell(n+1)} \cdot \left\{ \theta_k(t) - \theta_{k\ell}(t) \right\} dt - \Delta H_{k\ell(n+1)} + \Delta H_{K\ell(n+1)} + J_{k\ell(n+1)} \cdot \Delta t = 0 \tag{21}$$

where $\alpha_{k\ell(n+1)}$ : heat transfer coefficient of wall "$\ell$" of room $k$ and which is constant during the time of $t_n \sim (t_n + \Delta t)$

$J_{k\ell(n+1)}$ : effective radiation to the surface of wall "$\ell$"

$\theta_k(t)$ : air temperature of room $k$

$\theta_{k\ell}(t)$ : temperature of wall surface " $\ell$ " of room $k$

$$H_{k\ell}(t) = \int_0^t \theta'_{k\ell}(\tau)\cdot h_{k\ell}(t-\tau)\,d\tau \quad , \quad \Delta H_{k\ell(n+1)} = \int_{t_n}^{t_n+\Delta t} H_{k\ell}(t)\,dt$$

$$H_{K\ell}(t) = \int_0^t \theta'_{K\ell}(\tau)\cdot h_{K\ell}(t-\tau)\,d\tau \quad , \quad \Delta H_{K\ell(n+1)} = \int_{t_n}^{t_n+\Delta t} H_{K\ell}(t)\,dt$$

$h_{k\ell}(t)$ : indicial response of heat flow of the inside surface of wall " $\ell$ " to a unit step function of temperature of the same surface

$h_{K\ell}(t)$ : indicial responce of heat flow of the inside surface of wall " $\ell$ " to a unit step function of temperature of the outside surface

$\theta_{K\ell}(t)$ : temperature of outside surface of wall " $\ell$ "

By substituting the following relations to the eq. (21)

$$\left.\begin{aligned} \theta_{k\ell}(t) &= \theta_{k\ell(n)} + A_{k\ell(n+1)}\cdot(t-t_n) \\ \theta_k(t) &= \theta_{k(n)} + A_{k(n+1)}\cdot(t-t_n) \\ \theta_{K\ell}(t) &= \theta_{K\ell(n)} + A_{K\ell(n+1)}\cdot(t-t_n) \end{aligned}\right\}$$

whereby

$$\alpha_{k\ell(n+1)}\cdot\left[\left\{\theta_{k(n)} - \theta_{k\ell(n)}\right\}\cdot\Delta t + \left\{A_{k(n+1)} - A_{k\ell(n+1)}\right\}\cdot\frac{\Delta t^2}{2}\right]$$

$$- \theta_{k\ell(n)}\cdot B_{ok\ell}\cdot\Delta t - \sum_m Z_{k\ell m(n)} - \Delta X_{ok\ell}\cdot A_{k\ell(n+1)}$$

$$+ \theta_{K\ell(n)}\cdot B_{oK\ell}\cdot\Delta t + \sum_m Z_{K\ell m(n)} + \Delta X_{oK\ell}\cdot A_{K\ell(n+1)} + J_{k\ell(n+1)}\cdot\Delta t = 0 \qquad (22)$$

If the temperature gradients $A_{K(n+1)}$ , $A_{K\ell(n+1)}$ are not known, assuming that $A_{k(n+1)}$ is $A_{k(n)}$ , and $A_{K\ell(n+1)}$ is $A_{K\ell(n)}$ , the temperautre gradient of the wall surface $A_{k\ell(n+1)}$ is obtained from eq. (22).

Step 2. Equation of heat balance of the room air

After the temperature gradients of the surrounding wall surfaces $A_{k\ell(n+1)}$ are obtained the heat balance of the room air will be expressed as follows ;

$$Q_k\cdot A_{k(n+1)}\cdot\Delta t = \sum_\ell S_\ell\cdot\int_{t_n}^{t_n+\Delta t}\alpha_{k\ell}(t)\cdot\left\{\theta_{k\ell}(t) - \theta_k(t)\right\}dt$$

$$+ \sum_K\cdot\int_{t_n}^{t_n+\Delta t} C_p\cdot V_K(t)\left\{\theta_K(t) - \theta_k(t)\right\}dt + W_{k(n+1)}\cdot\Delta t$$

$$= \sum_\ell S_\ell\cdot\alpha_{k\ell(n+1)}\cdot\Delta t\cdot\left\{\theta_{k\ell(n)} - \theta_{k(n)} + \frac{\Delta t}{2}\left(A_{k\ell(n+1)} - A_{k(n+1)}\right)\right\}$$

$$+ \sum_K\left[C_p\cdot V_{K(n+1)}\cdot\Delta t\left\{\theta_{K(n)} - \theta_{k(n)} + \frac{\Delta t}{2}\left(A_{K(n+1)} - A_{k(n+1)}\right)\right\}\right] + W_{k(n+1)}\cdot\Delta t \qquad (23)$$

From eq. (23) the unknown value of the temperature gradient $A_{k(n+1)}$ or the heating load $W_{k(n+1)}$ is obtained.

## 8. Initial conditions of calculation

Figure 6 shows an example of the room air temperature variation of a flat house which is built with reinforced concrete and is heated intermittently. As seen in the example, if the thermal capacity of the system is large it will take many calculations to eliminate the influence of inadequate initial conditions. Therefore, to minimize the number of calculations, the setting of initial condition is important and the following methods can be used.

### 8.1 To assume a steady state condition

One of a simple method is to make the initial condition of temperature of each room as close to the mean daily temperature of each room as possible and assume a steady state as in case 2 in figure 6,then the initial value of $\Delta Z_{m(n)}$ becomes 0.

### 8.2 To assume periodical variation of temperature

When the temperature variation is approximated to a sine function as seen in figure 7 then the value of $\Delta Z_{m(n)}$ becomes as follows ;

$$Z_{m(n)} = \int_{-\infty}^{t_n} \theta'(\tau) \cdot B_m \cdot e^{-\beta_m (t-\tau)} d\tau$$

$$= \frac{\omega}{\beta_m^2 + \omega^2} \cdot B_m \cdot \theta_a \cdot \left[ \beta_m \cdot \cos \omega \cdot t_n + \omega \cdot \sin \omega \cdot t_n \right] \tag{24}$$

$$\Delta Z_{m(n)} = \frac{1}{\beta_m} (1 - e^{-\beta_m \Delta t}) \cdot \frac{\omega}{\beta_m^2 + \omega^2} \cdot B_m \cdot \theta_a \cdot \left[ \beta_m \cdot \cos \omega \cdot t_n + \omega \cdot \sin \omega \cdot t_n \right] \tag{25}$$

By using eq. (25) for the initial value of $\Delta Z_{m(n)}$ at the time of $t_n$ , high accuracy of analysis can be obtained with fewer calculations.

### 8.3 To change the time interval of calculation

One of the distinctive features of this method is the easiness of changing $\Delta t$ . Thus by at first using a large $\Delta t$ and rough calculations, much detailed analysis can be done later as seen in figure 8.

To change the time interval $\Delta t$ to $\Delta t'$ , we have to recalculate the new constants $\Delta X_0'$ , $E_m'$ and $X_m'$ and replace the terms $\Delta Z_{m(n)}$ with $\Delta Z_{m(n)}'$ as follows

$$\Delta Z_{m(n)}' = \Delta Z_{m(n)} \times \frac{(1 - e^{-\beta_m \cdot \Delta t'})}{(1 - e^{-\beta_m \cdot \Delta t})} \cdot \frac{E_m}{E_m'} + A_{(n)} \cdot \left\{ X_m \times \frac{(1 - e^{-\beta_m \cdot \Delta t'})}{(1 - e^{-\beta_m \cdot \Delta t})} - X_m' \right\} \times \frac{1}{E_m'} \tag{26}$$

Figure 9 shows a comparison of two cases A and B.

A : calculated by $\Delta t = 0.25(h)$ from the bigining to the end (a dotted line)

B : calculated by $\Delta t = 1.0$ till the 13th day and change to $\Delta t = 0.25(h)$ thereafter (a broken line)

The results agree perfectly after the 14th day. It will be also seen that if there is a sudden change in heat supply and the time interval $\Delta t$ is large then the calculated temperature fluctuates for a while. However, from the nature of this integration method the calculated temperature approaches an accurate value rapidly and even when it is fluctuating the average value or the integrated value of the results over the several intervals is always accurate. And of course when the variation of excitation is contenuous or when an interval $\Delta t$ is short there is no trouble like that.

For the system in which thermal capacity is quite large, such as for the underground structure, or when a synthetic effect of the systems is in consideration, such as a heating system and a room in which time constants are very different for each other, the considerations of this sub-section are very important.

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(Papers above are written in Japanese)

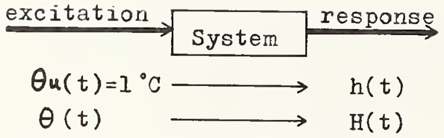

Fig. 1 Thermal system

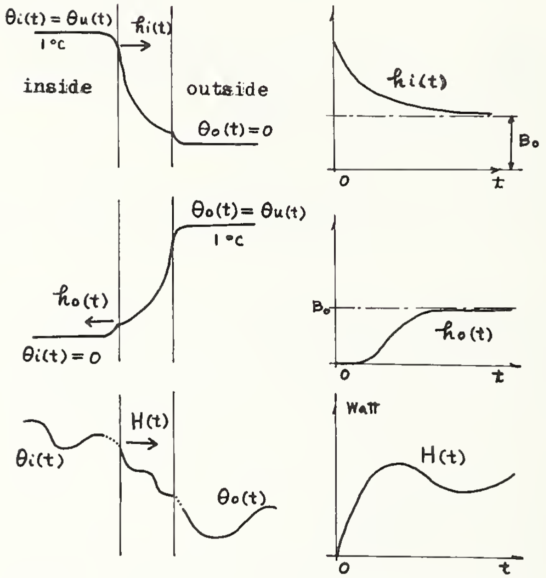

Fig 2. Indical response of heat flow

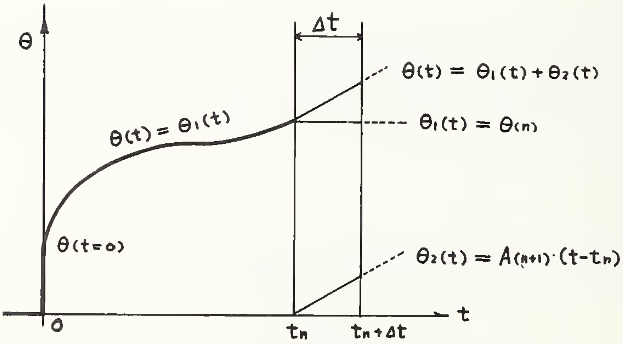

Fig. 3 Approximation of temperature

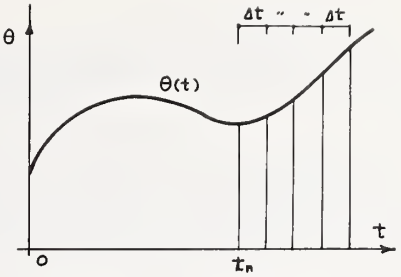

Fig. 4 Division of time

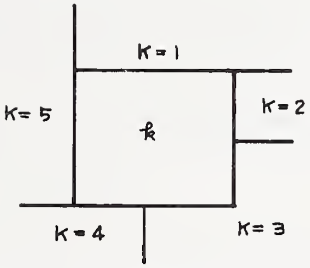

Fig. 5 Room k and adjoining room K

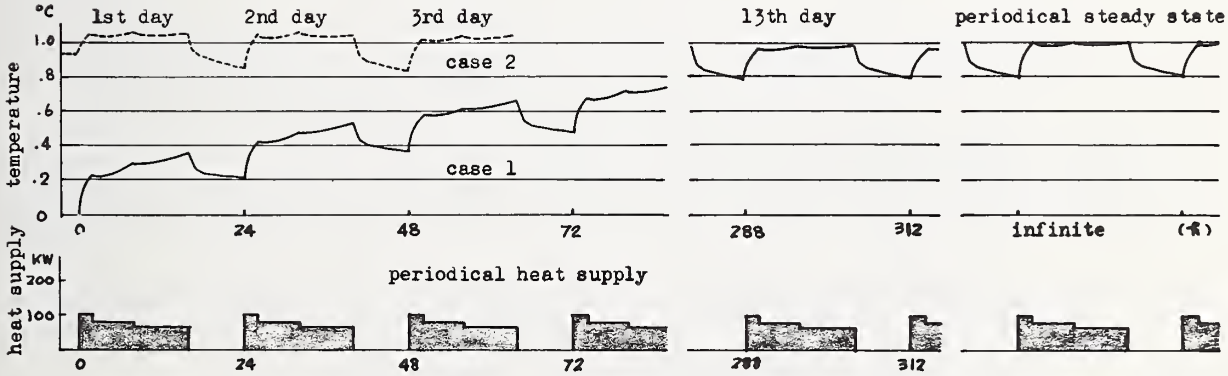

Fig. 6 Intermittent heat supply and calculated room air temperature

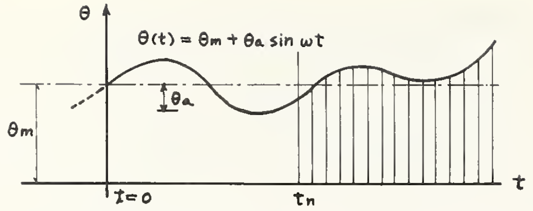

Fig. 7 Assuming of periodical steady state

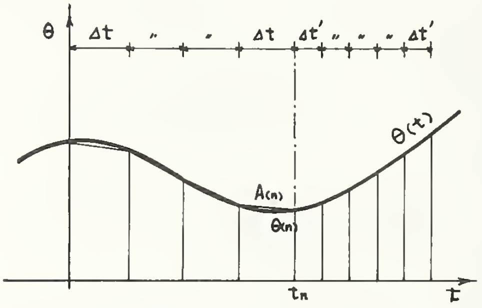

Fig. 8 Change of time interval

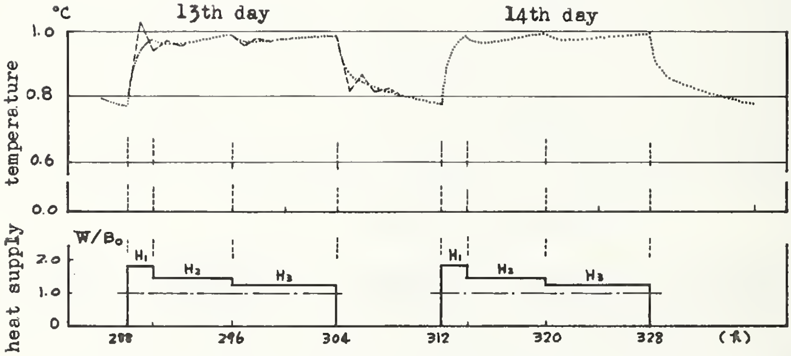

Fig. 9 Examples of calculation when Δt is changed

# Digital Simulation of Building Thermal Behaviour

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The mathematical modelling of building thermal behaviour to predict transient temperatures and humidities within it is presented in this paper The subdivision of the building into thermally-independent regions, each comprising up to five zones, each of uniform air temperature, is the initial step. Heat and moisture transfer to or within these zones is then described mathematically. In particular, the heat transfer through walls is traced by using finite element techniques. The solution of the assembled equations has been programmed in Fortran IV. Boundary conditions for each heat transfer path within a zone are compounded within the calculation from the basic input data of external air temperature and solar radiation (external boundaries) or internal radiative loads and solar radiation penetration (internal boundaries). Occupancy, direct convective-to-air internal loads, ventilation and infiltration variations may all be included as functions of time. The program may be used to calculate either internal zone air temperature from a knowledge of cooling or heating effect supplied, or zone cooling or heating load required to maintain a specified internal air temperature profile. Test results from two buildings are compared with post test simulations and reasonable agreement between measured and calculated temperatures is demonstrated.

Key Words: Building, digital simulation, heat transfer, internal air temperature, cooling or heating load, test results, thermal behaviour.

## 1. Introduction

The creation of a mathematical model for whole or for part of a building in order to study its thermal behaviour is a task now commonly practised in the air conditioning field [1, 2, 3] [2]. The model may be relatively simple, amenable to hand calculation, or more complex, requiring digital computing aids. That there is a use for such models is emphasized by the fact [4] that in the U. S. A., for example, which itself accounts for 35% of the world's annual power consumed, a third of this is expended on providing environmental comfort for people. Such considerations justify work aimed at

(a) generating more confident estimates of building cooling/heating/air-conditioning requirements,
(b) allowing quantitative assessments of the more effective use of building properties,
(c) enabling more effective programming of installed plant.

The task of creating such a model was commenced during 1969 utilizing the digital computing facilities of CSIRO. It was divided into two phases: initially a description of the heat transfer aspects of building performance was attempted and secondly an extension to humidity prediction was incorporated. The first phase is complete, the second is in progress and may be commented upon during the Symposium.

Part of the material contained in this paper was presented [5] to the 1970 Annual Conference of the Australian Institute of Refrigeration, Air Conditioning and Heating to whom acknowledgement is made.

## 2. Modelling Technique and Assumptions

### 2.1 Building Subdivision

Any part, or the whole, of a building may be divided into a number of discrete zones not

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[2] Figures in brackets indicate the literature references at the end of this paper.

necessarily thermally isolated from one another, but possessing different distinguishing thermal characteristics or boundary conditions. For example, inner and perimeter modules of a large building, or rooms associated with north and south walls, would possess sufficiently different boundary conditions to make them identifiable as zones. The choice of zones, in which the air temperature is assumed uniform, is made by the user, and is, to a large extent, arbitrary. Such zones form the coarse subdivision of the model: in each there are numerous heat transfer paths, e.g. wall, window, between some source temperature and room temperature. These paths form the basic elements of the heat transfer model.

The subdivision of a building into such zones is shown schematically on figure 1. This central and peripheral configuration is the type of breakdown which might be employed for a storey of a multi-storey building. On the other hand, a residential structure would require different subdivision for analysis, such as is shown on figure 2. It is emphasized that, although it is often convenient to do so, it is not mandatory to use zones identical to those of a particular air conditioning system.

It is the aim of the calculation techniques that, having established the model for any particular building, the input of desired boundary conditions as a function of time may permit the output of either air conditioning sensible load for a prescribed indoor temperature profile or the converse. Later phases of the work will incorporate the moisture transfer processes and generate a more complete air conditioning solution.

## 2.2 Model

The equation representing the balance of sensible heat transfer to the air within a zone may be described thus :-

Cooling or Heating Effect from plant
+ Direct-to-air convective load (e.g. equipment, people)
+ Fresh air load
+ Surface-to-air convective load from all exposed surfaces within zones
= 0.

Using symbols defined in the list this equation may be written

$$QLS + QID + QSFA + \sum_{j=1}^{JJ} h_j A_j (T_{w_j} - T_i) = 0 \tag{1}$$

Figure 3 shows the situation schematically.

The calculation of building sensible load requires that QLS be made the subject of eq (1), thus

$$- QLS = QID + QSFA + \sum_{j=1}^{JJ} h_j A_j (T_{w_j} - T_i)$$

Considering each term on the right hand side individually, the first is derived directly from input data on the number of people and their work rating and on the operation of equipment within a zone. These are both permitted as functions of time in the input information.

The second, fresh air, term may be re-expressed

$$QSFA = HFA (T_o - T_i) \tag{2}$$

where HFA is the product of a constant x fresh air ventilation and infiltration rates, and $T_o$ and $T_i$ outdoor and indoor air temperatures respectively. Again all parameters in this expression are permitted as functions of time.

The third term is more complex to derive: finite difference numerical procedures are used to solve the diffusion equation for each path from the individual boundary conditions for that path; hence the inner wall temperature and convective flux is obtained.

The establishment of boundary conditions for each path at each integration interval is performed within the program from input data of outside air temperature, solar radiation, shading and of internal radiation and air temperature. It is in this fashion that radiation is introduced; that is, radiation, be it long wave or short, is allowed to fall on the appropriate surface, be absorbed and ultimately appear as a convective load to the indoor air. In this way both the thermal capacity of each heat transfer path is accommodated and radiation is handled in the manner in which that mode of heat transfer occurs in practice Thus the radiant contribution to load does not appear explicitly in the results of the load calculation, nor is any factoring of radiant heat required.

In order to predict indoor air temperature, given a specified heating or cooling effect, $T_i$ must be made the subject of eq (1). Thus

$$T_i = \frac{QLS + QID + HFA \, . \, T_o + \sum h_j \, A_j \, T_{W_j}}{HFA + \sum h_j \, A_j} \tag{3}$$

Latent loads are also handled within the program. A simple calculation of latent heat gain due to equipment, people and fresh air is calculated from the input data according to the practice of the ASHRAE Guide [6]. If internal humidity is to be calculated simultaneously with indoor dry bulb temperature, the increase in moisture level over the integration interval is assessed and combined with the internal air dry bulb temperature to calculate relative humidity.

The whole calculation proceeds on a step-by-step basis in time, each time increment being equal and determined from the building properties, which do not normally change with respect to time. There are always an integral number of time steps to the hour and results are normally output every hour. The maximum integration increment is 15 minutes.

The facility has been provided for the user to include the characteristics, including thermostat operation, of an air-conditioning system when they cannot be expressed independently as input data in the form of cooling effects.

### 2.3 Assumptions

Both diffuse and direct solar radiation (incident upon any surface) are calculated as a function of time, either from total and diffuse radiation for a horizontal surface, which must be input in tabular form, or from the expressions of reference [7]. A test is made at each time interval to determine whether any path is in shadow: those in shadow are exposed to the diffuse and reflected direct components, whereas those in the sun receive all components.

Long wave radiation is assumed to be incident on all external paths according to the product of a coefficient $h_R$ and the difference between surface temperature, $T_{W_j}$ and the mean radiant temperature, $T_R$. $(T_R - T_o)$ may be input as a function of time.

Any radiation passing through the windows of a zone is assumed to be incident on and wholly absorbed by the floor of the zone. Internally generated radiation is apportioned to each path of a zone according to an input form factor.

Thus we may create a forcing or source temperature on the path boundary remote from the room and a sink temperature on the room side boundary

$$T_{source} = T_o + \frac{Q_s + h_R (T_R - T_o)}{(h_o + h_R)} \tag{4}$$

$$T_{sink} = T_I + \frac{Q_I}{h_i} \tag{5}$$

These temperatures are analogous to the sol-air temperature [6] and are derived from a defining equation of the type

$$(h_o + h_R)(T_{source} - T_W) = h_o (T_o - T_W) + Q_s + h_R (T_R - T_W)$$

where $T_W$ is the wall temperature of any path.

Internal paths either wholly within a zone or separating two adjacent zones are permissible. Both source and sink temperatures are defined by equations of type (4) above for such paths.

All paths are assumed homogeneous in the direction of heat transfer so that they possess the properties of a uniform medium in that direction. Later stages of the study will enable non-uniformities to be modelled more exactly.

The building thermal model, including its boundary heat transfer coefficients, is not permitted to vary as a function of time. Again, later stages may permit some variation of these coefficients.

## 3. Applications

### 3.1 Test Building

Experimental data from two different test periods has been taken to illustrate comparisons of temperatures measured and predicted within a fairly heavy two storey office building. Curves are included on figure 4 illustrating these comparisons. Further work on this aspect is continuing, but agreement in these tests is considered encouraging.

### 3.2 Examples of Use

Two hypothetical buildings in Sydney have been chosen to illustrate the calculation of air conditioning sensible load, under normally imposed design climatic conditions and under measured severe climatic environments. The normally-used design dry bulb of 32°C has been adopted to perform a load calculation by the ASHRAE Guide method for both an 18 square brick veneer residence and for a typical storey of a city office building. Maximum sensible space loads (excluding fresh air) of 8.5 kW and 83 kW respectively, for a constant indoor dry bulb temperature of 24°C, were deduced from this approach. The computer model for each building was fed with climatic data from the three hourly Commonwealth Bureau of Meteorology readings on magnetic tape for Sydney. Heatwave periods were input for the hottest spell in the ten years (1955-1965) on record and for the spell containing a maximum dry bulb exceeded only 10 times during that period. Clear day solar radiation was input for these spells being calculated by a method [7], previously described by the author. Figures 5 and 6 show how the sensible space load of the office storey varied with time during these spells. The difference between the once-in-10-year maximum and the once-in-a-year maximum is significant, the latter amounting to 93% of the former. A comparison is summarised in table 1 below for both the residence and the office buildings.

There are substantial differences between the cooling calculated from the transient thermal model and from the ASHRAE Guide quasi-steady-state approach. However, it is not advocated that actual climatic sequences be used for design purposes: that would require far too much analysis. It is proposed to develop design sequences using a transient mathematical model to test for frequency of load occurrence, but this will be the subject of a separate paper. At this time, these examples are included to illustrate uses of the computational model.

Table 1. Maximum sensible coolings loads for 3 different external environments (kW)

| Sydney Building | ASHRAE Guide | PRESENT METHOD Once in 10 years | PRESENT METHOD Once in a year |
|---|---|---|---|
| | Max. d. b. t. 32°C | Max. d. b. t. 42.2°C | Max. d. b. t. 38.2°C |
| 18 sq. Residence (167 m$^2$) | | | |
| Space load | 8.5 | 6.4 | 5.0 |
| People, Equipment and fresh air | 2.0 | 2.3 | 2.2 |
| TOTAL | 10.5 | 8.7 | 7.2 |
| Storey of Multi-storey block | | | |
| Space load including people, equipment | 83.2 | 70.6 | 65.7 |
| Fresh Air | 14.4 | 31.4 | 24.3 |
| TOTAL | 97.6 | 102.0 | 90.0 |

An alternative calculation on the above office building illustrates the use of the program in predicting air temperature. Indoor air temperature has been estimated over a three day design period for three levels of constant cooling effect, at 50, 75 and 100% of maximum required by the calculation for constant room temperature of 24°C. Figure 7 illustrates the temperature profiles obtained: from these it may be observed that, standby and pull down considerations aside, a somewhat smaller plant could still generate comfortable conditions. It is, of course, easy to examine the indoor temperature profile under any occupancy loading and after periods of shut down over weekends, say, so that better informed planning of standby and pull down requirements is possible.

## 4. Conclusions

It has been the aim of this paper to present a broad outline of the mathematical modelling procedure for the prediction of building thermal performance now set up at the CSIRO Division of Mechanical Engineering, and to illustrate some of the validation and uses.

While the experimental validation is continuing on larger buildings, the results obtained so far are sufficiently promising to suggest that the model is reliable.

Results of applications attempted so far indicate that, by being able to simulate correctly the heat diffusion process and by inputting radiation so that it is first of all absorbed, the part of a building thermal load transmitted via the structure to the air may amount to only some 75% of that predicted by conventional quasi-steady state methods.

The power of the modelling technique in being able to estimate indoor temperature excursions for many varieties of input enables an investigation of the effects of different building properties or of modes of operation of cooling or heating plant.

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6 ASHRAE Handbook of Fundamentals, Chapter 28 (1967).

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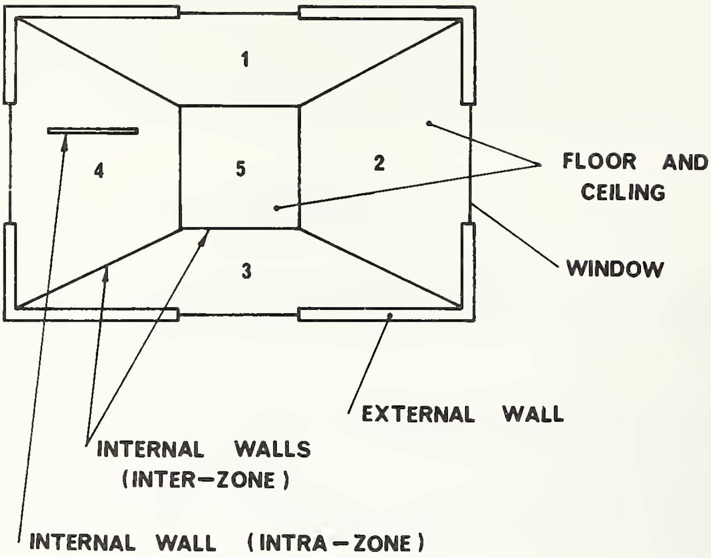
## 6. Notation

Symbols

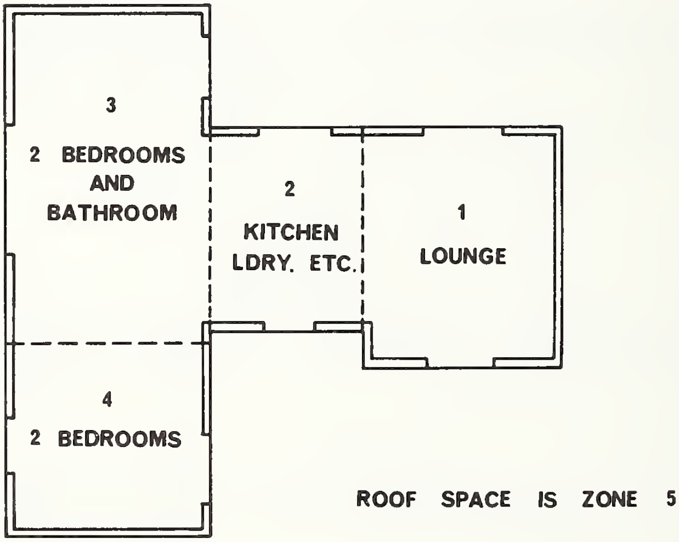
A - area
h - heat transfer coefficient
HFA - = constant x fresh air flow rate from both ventilation and infiltration
JJ - Total number of heat transfer paths per zone of building
$Q_S$ - Short wave radiation x path absorptivity
$Q_L$ - Long wave radiation x path absorptivity
$Q_I$ - internally generated radiation intensity x path form factor
QLS - sensible load for cooling and heating
QID - internally generated direct convective load
QSFA - fresh air load
T - Temperature
$\tau$ - glass transmissivity

Subscripts

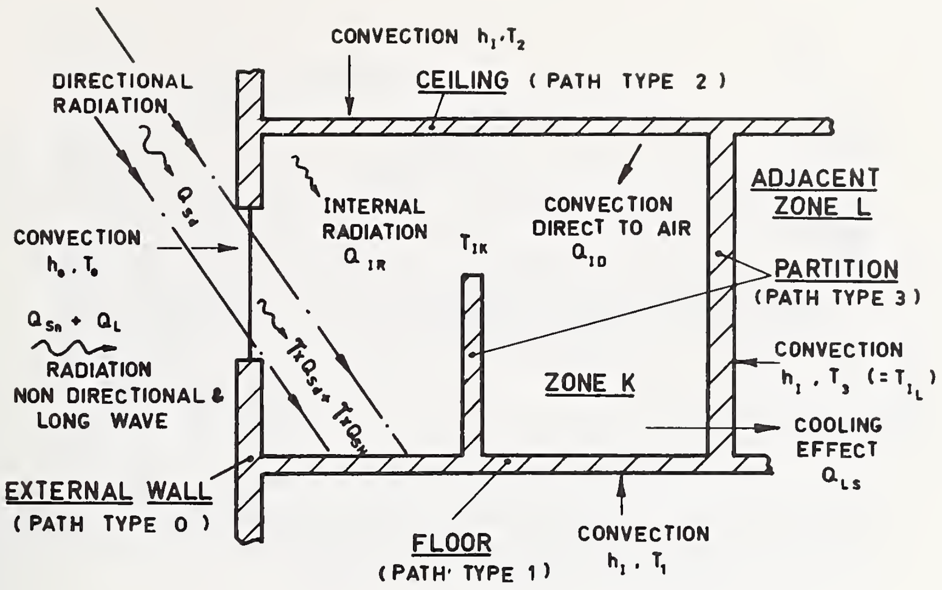
j - path identifier
i - indoor condition
o - outdoor condition
R - long wave radiation
W - wall
Source - external boundary layer forcing value
Sink - internal boundary layer forcing value
d - directional
n - non-directional



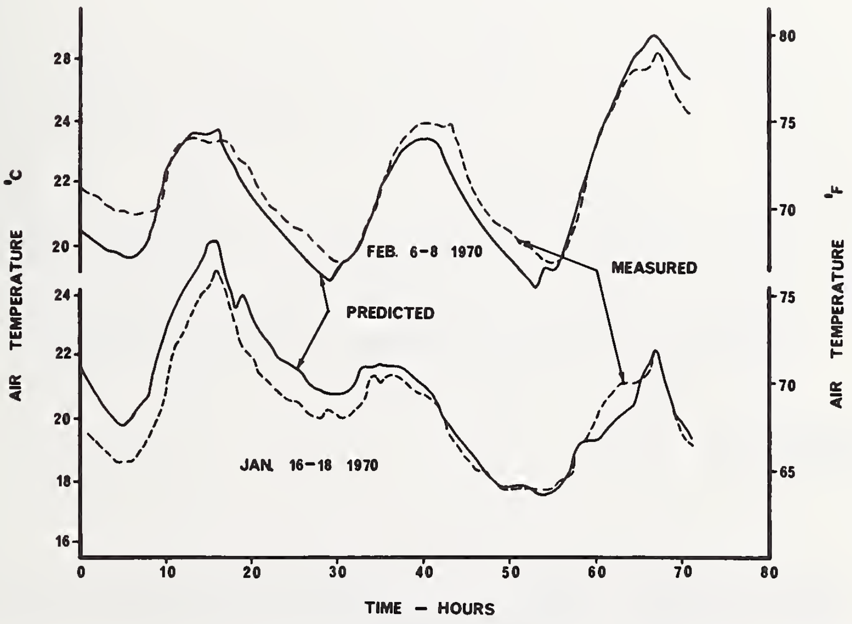
1. Schematic of Building Plan Subdivision into zones.



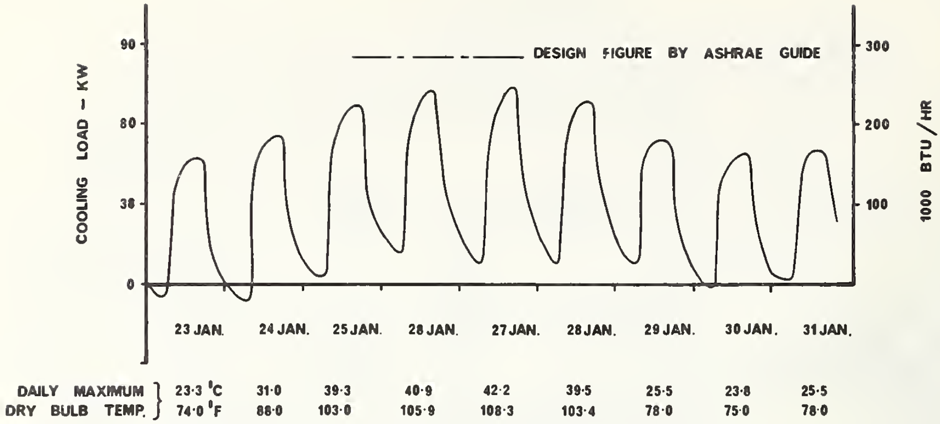
2. Typical Zonal Subdivision for Residential Building Analysis.



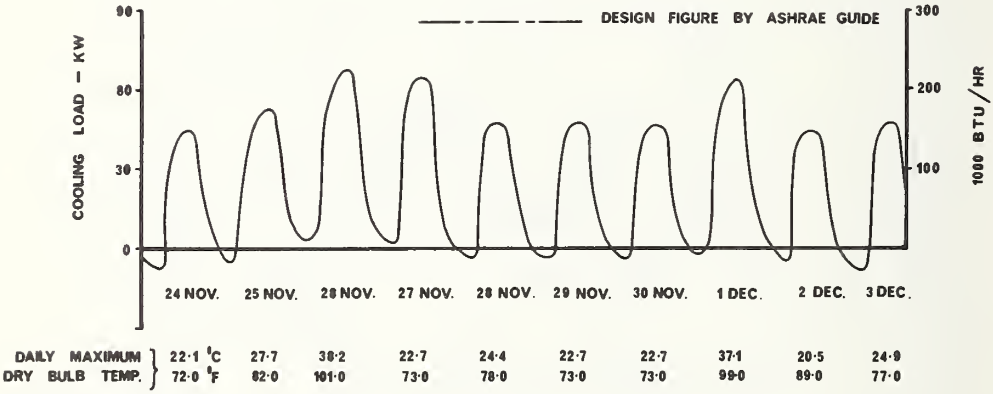
3. Schematic of Incident Heat Fluxes for Typical Building Zone.



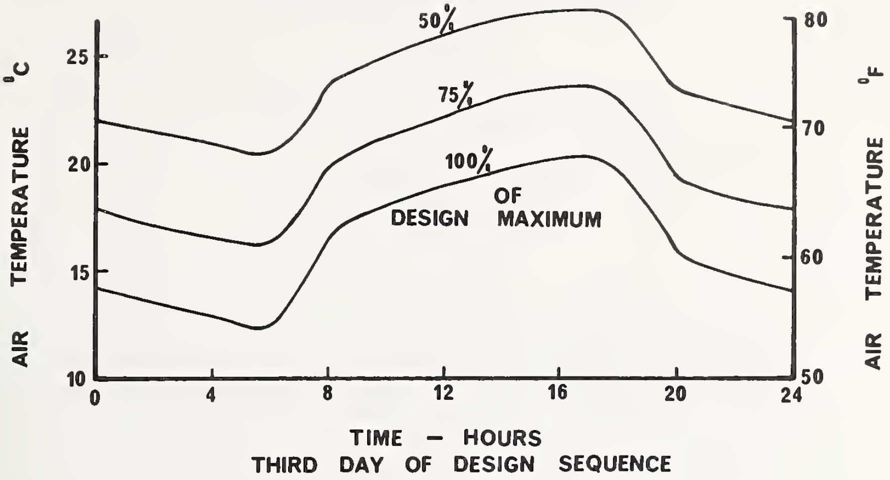
4. Measured and Predicted Air Temperature Profiles for an office building in Melbourne.



5. Sydney Office Block - Typical Storey : Estimated Sensible space cooling load for period 23 - 31 Jan. 1960. (Maximum dry bulb temperature on 27 Jan. 1960 never exceeded during 1955 - 65)



6. Sydney Office Block - Typical Storey : Estimated Sensible space cooling load for period 24 Nov. - 3 Dec. 1964. (Maximum dry bulb temperature on 26 Nov. 1964 exceeded 10 times during 1955 - 65)



7. Sydney Office Block - Typical Storey : Estimated Mean Air Temperature for 3 different (constant) applied cooling effects.

# A Computer Programme for the Calculation of Individual Room Air Temperature of Multi-Roomed Buildings

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In recent years the use of computers for the calculation of heating and cooling loads and for the prediction of indoor air temperatures of buildings has been well established and is in the increase. Many organisations in a number of countries have developed suitable computer programmes in accordance with their specific requirements and the computer facilities available. Most of the programmes which are meant for the calculation of indoor air temperatures of the unconditioned buildings, treat the whole building as a single enclosure. However it is also of practical interest to determine the individual room air temperatures within a multi-roomed building. For this purpose a computer programme which is suitable for a small machine like IBM 1620 digital computer with 60K memory has been developed. This programme takes into account the orientation of the rooms, ventilation, internal heat sources, internal mass and furnishings and solves a set of linear simultaneous equations for obtaining the indoor air temperatures of the individual rooms. Provision has also been made in the programme to determine the inside surface temperatures of walls, roofs and floors, mean radiant temperatures and the heating and cooling loads of individual rooms, if desired.

The main programme utilizes a good deal of precalculated data obtained by a number of separate programmes developed for the calculation of sol-air temperatures, shade patterns of overhangs and fins, solar heat transmission through glass and frequency response characteristics of building sections.

Key Words: Computer programme, driving point admittance function, Fourier analysis and synthesis, heating and cooling loads, heat transmission coefficient, matrix equation, non-structural heat gains, room air temperature, shade patterns, sol-air temperature, surface temperature, transfer admittance function.

## 1. Introduction

The problem of determining the thermal response of buildings under any given climatic conditions has drawn the attention of many research and industrial organisations the world over. With the ever increasing use of digital computers, in all most all branches of building science, the computational methods, which were earlier considered as unmanageable, are now finding wide

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[1] Scientist and Senior Scientific Assistant, respectively.

application. This trend is more evident for the problem of the calculation of inside air temperatures and heating and cooling loads of buildings exposed to variable weather conditions.

In the last decade several computational techniques, which are specifically suitable for high speed machine calculations, have been evolved by various researchers in the field of Heat transfer(1)[2] (2) (3) (4). A number of computer programmes for the accurate estimation of heating and cooling loads and the indoor air temperatures have also been reported (5) (6) (7) (8). However, almost all of these programmes treat the building as a whole as a single enclosure and do not account for the variations of temperatures of the individual rooms within the building. The present paper deals with a method and a computer programme developed for the calculation of individual room air temperatures of multi-roomed buildings.

## 2. Factors Involved in Room Air Temperature Calculations

In order to determine the thermal behaviour under a given climate, a large number of factors are to be taken into account. These factors can conveniently be grouped into three categories.

### 2.1 Climatic Factors

(a) Diurnal variations of shade air temperature
(b) Solar radiation (direct and diffuse)
(c) Net low temperature radiation exchange
(d) Wind speed and direction
(e) Humidity

### 2.2 Design Factors

(a) Thermal characteristics of building sections i.e. walls, roof, ceiling, floor, doors, partitions etc.
(b) Surface radiation characteristics i.e. absorptivity, emmissivity and transmitivity of building sections
(c) Shape and Orientation of the building and internal layout
(d) Window design, number, location and type of glass areas
(e) External and Internal shading devices such as overhangs, fins, louvres and venitian blinds, curtains respectively

### 2.3 Utilization Factors

(a) Ventilation
(b) Internal heat sources and sinks
(c) Density of occupancy
(d) Living habits which influence (a), (b) and (c).

The complexity of the problem increases not only due to the large number of variables that come into the picture but also due to the interactions between them.

## 3. Computational Method

As buildings are subjected to periodic variations of air temperatures, solar radiation and other climatic elements, the inside air temperatures of buildings would also fluctuate periodically. Several methods have been developed for determining the thermal response of building elements (9) (10) (11). Of these the transfer Martix method of Van Gorcum (12) for the determination of periodic heat flow through a homogeneous slab has gained popularity being well suited for high speed digital compu-

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[2] Figures in brackets indicate the literature references at the end of this paper

tations. The Matrix method was further extended by Muncey (13) for predicting indoor air temperature variations of a single room. In this approach, the bounding elements of the enclosure are taken as parallel heat paths and the internal temperatures are computed by applying the principle of heat balance for steady as well as harmonic parts.

The equations for indoor air temperature variations (14) can readily be expressed in terms of areas, sol-air temperatures of exposed surfaces, steady state heat transmission coefficients (U values), transfer and driving point admittance functions of building sections, ventilation rate and internal heat sources and are given below.

$$\breve{t}_{ia} = \bar{t}_{ia} + \sum_{1}^{h} \breve{t}_{ia,h} \; \mathrm{Cos} \; (w_h \tau \; - \phi_h) \qquad (1)$$

where t is the temperature at any instant, $\bar{t}$ and t are the steady state and periodic components of the temperature waveform respectively, w is the angular frequency, $\phi$ is the phase lag and $\tau$ is the time and the subscripts, ia, and h stand for internal air and number of harmonics respectively.

$$\bar{t}_{ia} = \left[ \sum_{1}^{m} A \; U \; \bar{t}_{sa} + \bar{t}_{oa} \; C.V + \sum_{1}^{\ell} \bar{W}_{\ell} \right] \Big/ \left[ \sum_{1}^{m} A \; U + C \; V \right] \qquad (2)$$

where A and U are the area and the steady state heat transmission coefficient of a building section respectively, C is the specific heat of air at constant pressure, V is the ventilation rate, $\bar{W}$ is the steady state component of the variable heat source, m and $\ell$ are the number of bounding elements and sources respectively and the subscripts, sa, oa and i stand for sol-air, shade-air and internal respectively.

$$\breve{t}_{ia,h} = \left[ \sum_{1}^{m} A \; TY_h \; \breve{t}_{sa,h} + \breve{t}_{oa,h} \; CV + \sum_{1}^{\ell} \breve{W}_{\ell} \right] \Big/ \left[ \sum_{1}^{m} ADY_h + \sum_{1}^{k} A \; DY_h + C \; V \right] \qquad (3)$$

where TY and DY are the transfer and driving point admittance functions of the building section respectively, $\breve{W}$ is the periodic component of the variable heat source, k is the number of internal masses in the enclosure.

In the above case the internal elements such as partitions and intermediate floors are treated as slabs for which the heat flow at the central plane is zero. The assumption involved here is that the temperature fluctuations on either side of the internal elements are symmetrical. In other words the building as a whole is taken as a single enclosure and the variations of air temperatures within individual rooms are neglected. However, there can be situations where significant differences in air temperatures from room to room within the same building occur. This is specially so in the case of multi-storey buildings.

In order to make the solution of the problem more general, it is imperative that the heat balance for each room should be considered separately. In a multi-roomed building, the air temperature of any room will not only be a function of the exposed elements, but also of the air temperatures of the surrounding rooms. As the air temperature of a room is in turn depends on the air temperatures of other rooms these are required to be determined simultaneously.

The heat balance equations for steady and periodic parts of any room in a multi-roomed building are given below.

$$\bar{t}_{ia} \left[ \sum_{1}^{m} AU + CV \right] + \sum_{1}^{e} \bar{t}_{sa} \; AU + \sum_{1}^{i} \bar{t}_{i} \; AU + \sum_{1}^{\ell} \bar{W}_{\ell} + \bar{t}_{oa} \; CV = 0 \qquad (4)$$

where e and i are the number of exposed and internal elements of the room and

$$\breve{t}_{ia,h}\left[\sum_{1}^{m} ADY_h + \sum_{1}^{k} ADY_h + CV\right] + \sum_{1}^{e}\breve{t}_{sa}\; ATY_h + \sum_{1}^{i}\breve{t}_{i,h}\; ATY_h + \sum_{1}^{\ell}\breve{W}_{\ell,h} + t_{oa,h}\, CV = 0 \qquad (5)$$

On the total 'n' such equations for steady state and '2 n' equations for each harmonic component (being complex quantities) of the periodic part will result for a building having 'n' rooms.

For high speed computer applications it is more convenient to express these equations in a matrix form. A typical matrix equation (for steady state component) that would result from a set of 'n' simultaneous equations is given below.

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & - & - & - & - & - & x_{1n} \\ x_{21} & x_{22} & x_{23} & - & - & - & - & - & x_{2n} \\ x_{31} & x_{33} & x_{33} & - & - & - & - & - & x_{3n} \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ x_{n1} & x_{n2} & x_{n3} & - & - & - & - & - & x_{nn} \end{bmatrix} \times \begin{bmatrix} \bar{t}_{ia,1} \\ \bar{t}_{ia,2} \\ \bar{t}_{ia,3} \\ - \\ - \\ \bar{t}_{ia,h} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ - \\ - \\ C_n \end{bmatrix} \qquad (6)$$

Similar matrix equations for real and imaginary parts for each harmonic component of room air temperatures can be formed. The Fourier components of the air temperatures of individual rooms will then be obtained by solving these matrix equations. The hourly air temperatures for each room can also be obtained by Fourier synthesis.

## 4. Computer Programme

A generalised computer programme for the determination of individual room air temperature variations in a building, should naturally take all the factors involved and their mutual interactions into account. A schematic diagram of the stages in the calculation is given in Figure 1.

A flexible computer programme has been developed in FORTRAN II language for the above said purpose. The main programme consists of a number of sub-programmes which would calculate the sol-air temperatures of differently oriented surfaces, shade patterns of overhangs and fins, solar heat transmission through glass and thermal system functions of the building sections from the climate and design factor input data. A separate programme has also been prepared for the precalculation of solar radiation data for differently oriented vertical and sloping surfaces, which is required for the calculation of sol-air temperatures. Non structural sensible heat gains are computed by the hourly variation of the loads due to ventilation, internal heat sources, (lights, fans and other appliances) and room occupancy. The directly transmitted solar radiation through glass area as modified by the internal shading devices, is considered as an internal heat source in these calculations.

The programme is made flexible and will determine the surface temperatures (inside and outside) of wall and roof sections and the air conditioning loads of individual rooms, if desired. As a small machine (IBM 1620, with 60K memory) is only available, all the sub-programmes mentioned above are used as separate programmes to calculate the required input data for the main programme. However where larger machines are accessible all these sub-programmes can be clubbed with the main programme as CALL type sub-routines.

## 4. 1. Flow chart

The sequential steps in the computation of room air temperatures, surface temperatures and air conditioning loads are shown in Figure 2 in the form of a flow chart.

In this programme the number of rooms (N) in a building, significant harmonics (L) to be considered and hourly shade air temperatures are first read. The following operations required for determining the heat flow through various parallel paths into each room are performed.

(a) Shade air temperature waveform is Fourier analysed.

(b) Number of exposed (NE), internal (NI) elements and number of internal masses (NK) for each room are read.

(c) Hourly sol-air temperatures for each exposed element (precalculated) are read and Fourier analysed.

(d) Shaded and sunlit areas of exposed elements due to the presence of overhangs and fins, for each hour of the day, (which were precalculated) are fed.

(e) Steady and periodic heat flow characteristics for each harmonic, which are obtained by a separate program, are read and the corresponding harmonics of the heat flow into the room are computed.

(f) The non-structural heat gains viz., lighting, ventilation, occupancy loads etc., which were also precalculated are added harmonic wise.

(g) Driving point room admittance coefficients at the internal surfaces, due to the periodic parts of the indoor air temperature fluctuations, are computed.

(h) Transfer and driving point room admittance coefficients for the internal elements and masses of the room concerned are then calculated.

(i) The elements of the matrices for the steady and periodic (in terms of real and imaginary) parts are generated.

(j) The steady state and harmonic components of the air temperatures for all the rooms in the building are obtained by solving the matrix equations with the help of a call subroutine SOLEQN.

(k) The hourly temperatures of the rooms are obtained by a call subroutine FSS.

The inside surface temperatures of the building elements of a room are required in cases where radiant heat gains to the human body is to be estimated. For such situations additional computations are required and a provision has been made in the program to calculate these quantities by a call subroutine SURTEM.

If the estimation of hourly variation of cooling and heating loads of each roomis also required, the call subroutine LOAD provided in the program would perform the necessary computations.

## 5. Conclusions

(a) The limitations imposed by the assumption of considering a large multi-roomed building as a single enclosure in calculating the indoor air temperatures and air conditioning loads are removed by the method and the program presented in this paper.

(b) It is now possible to determine the temperature variations within the individual rooms and thus to evaluate the effect of various design factors on the thermal performance of buildings on room by room basis.

(c) These long felt refinements in thermal calculations for buildings are only possible because of the speed and accuracy provided by the digital computers. However, it may be stressed that the accuracy of the computer estimation basically depends upon how accurate the input data is.

(d) The approach of using considerable precalculated data obtained by a number of subprograms is not the inherent limitation of the method but was only adopted to suit a small machine like IBM 1620. The program can be made more versatile by linking all the subprograms as call subroutines of the main program, if large sized computers are easily accessible.

## 6. Acknowledgement

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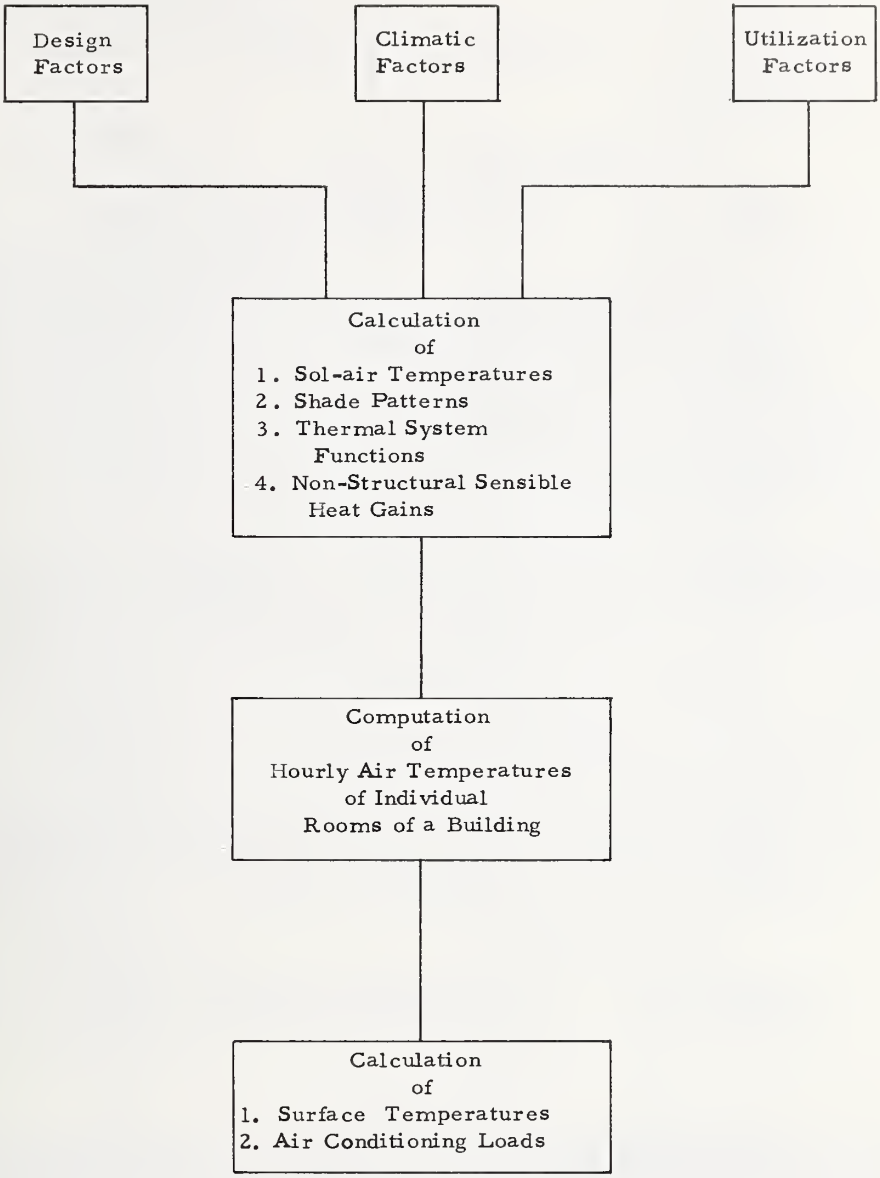

Figure 1 Schematic Diagram for Digital Computer Determination of Temperatures within Buildings

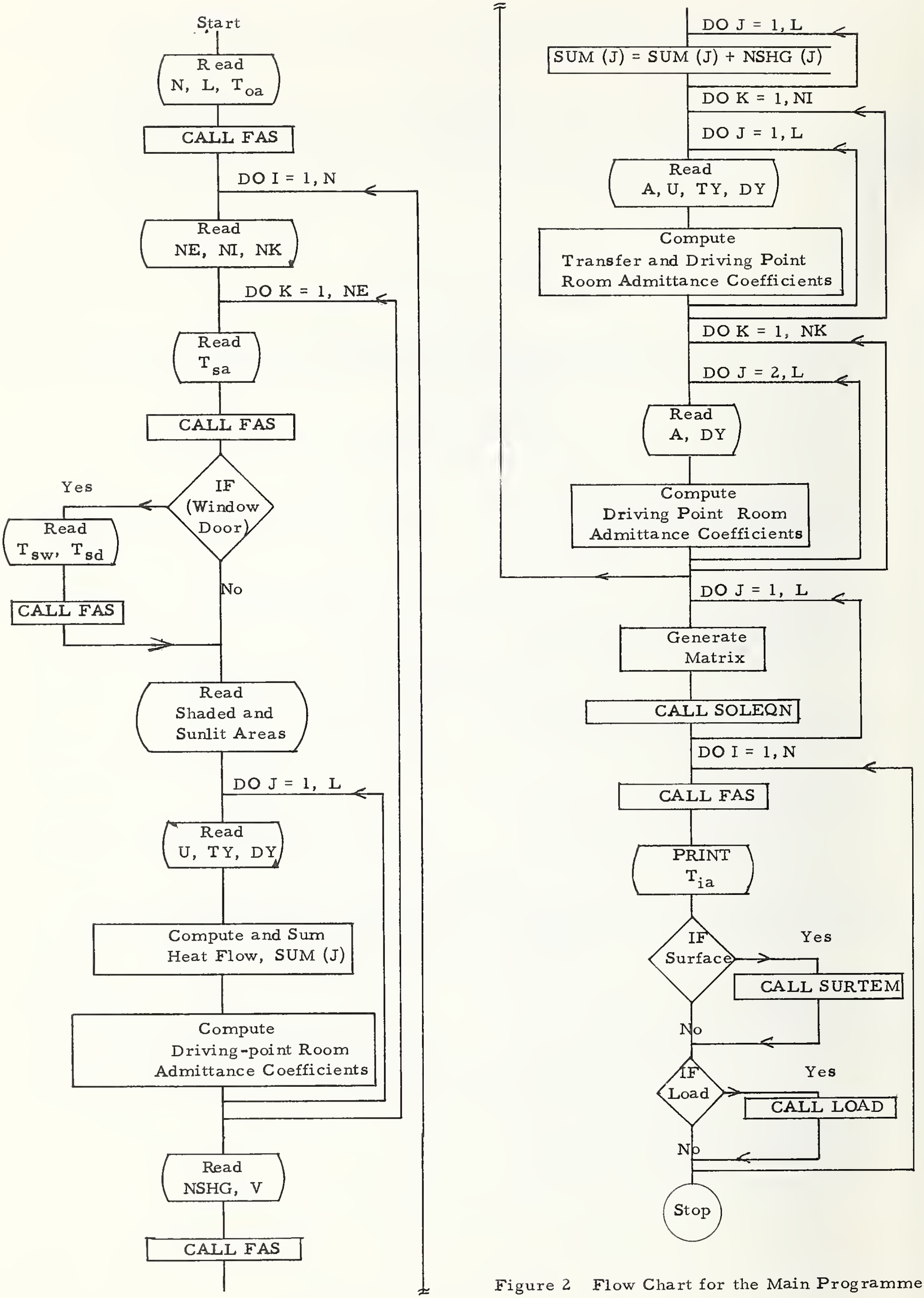

Figure 2 Flow Chart for the Main Programme

# A Practical Method for Calculating Room Temperature, Heating Load and Cooling Load of Multiroom

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It is possible to set up heat balance equations representing the dynamic thermal characteristics of a multiroom by using weighting functions, if the system is linear and invariable. With the advent of the digital computer, it is now possible to solve them directly, even if there are numerous rooms with different thermal behavior. However, as the number of rooms increases, the cost of calculation and the capacity of the computer required increase rapidly. Therefore, it may not be practical to solve the balance equations directly, when the number of rooms is large. This paper proposes a practical method for calculating room temperature, heating load and cooling load of a multiroom by means of a digital computer of relatively small capacity at reasonable cost. Rooms constituting a multiroom system may be divided into two groups; one has a stronger influence on the relation between the excitation and the system's response, the other has a lesser influence. Then, the response to the system consisting of only the former group may be good approximation to the response to the original undivided system. The accuracy is not known exactly at this stage, but it is shown that the effect of neglecting the latter group of rooms can be easily investigated on the principle of superposition or Thevenin's theorem. Also the improvement of accuracy can be made to any desired degree by a relaxation method on the same principle. Many examples for obtaining the room temperature of a multiroom by this method were studied and it has been found that, with an increasing number of rooms, the usefulness of the present method for analysis and synthesis increase rapidly.

Key Words: Multiroom digital computer, room temperature heat source, Thevenin's theorem.

## 1. Introduction

It is possible to set up heat balance equations representing the dynamic thermal behavior of a multiroom system by using the linear theory. Actually, it is difficult to obtain analytical solutions for room temperature because of the complexity. With the advent of the digital computer, it is now possible to obtain numerical solutions with any desired accuracy, even if there are numerous rooms with different thermal performances. It may not be worthy to get the solutions more accurate than required for practical problems. To avoid such worthless calculations, it is necessary to simplify the system's elements and the mathematical models. However, as the number of rooms increases, the cost of calculation and the capacity of the computer required increase rapidly. Therefore, it may not always be practical to solve the balance equations directly when the number of rooms with different thermal behavior is large. Our method is practical for engineering and application problems because calculations can be made by hand, by desk calculation or the digital computer of relatively small capacity. Also, it can be done at a reasonable cost, depending on the amount of calculations needed.

This method has been applied to obtain the room temperature of a multiroom system consisting of two rooms, five rooms in parallel, and fifteen rooms in series. To explain the present method clearly, the mathematical models are simplified by assuming a one dimentional heat flow, by uniform room air temperature, and by a combined heat transfer coefficient which would approximate the radiant heat interchange, between the inside surfaces, and the convective heat interchange, between the inside surfaces and the room air. Attempt is made to calculate the indoor temperature variation caused by air conditioning, taking the initial temperature 0 ℃ and assuming the outdoor temperature always being 0 ℃. These calculations are carried out using an electronic digital computer with the capacity of 2-K words.

## 2. Practical Method

### 2.1 Connection and Division of Systems

There are two systems; one has some heat sources, and the other has none. Therefore, there can be temperature variation in the former. Now, we attempt to connect the latter to the former at any boundary position with a constant temperature of 0 ℃. Consequently, different responses will occur because of the effect of the connection. To investigate the effect, an artificial heat source is supplied to the combined position. The magnitude of this heat source is equivalent to the heat fluxes at the same position of the original system, which is caused by the original heat sources. The response of this artificial heat flow to the combined system represents the difference of the responses between the original system and the combined one, that is the effect of the connection. This will be described here in terms of the superposition principle as follows.

First, a couple of artificial heat sources, heating and cooling with the same thermal quantity, is supplied to the combined position of the total combined system, as shown in figure 1-A. Then, there is no change in the thermal performance, since their total heat fluxes supplied artificially become zero.

Second, a couple of heat sources is decomposed by the superposition principle. Two cases result; one is an artificial heat source with the original heat sources, the other is only an artificial heat source, as shown in figure 1-B,C. If the temperature of the divided position in the former case is kept at 0 ℃ at all times, this system should be equivalent to the original one shown in figure 1-D. The magnitude of the artificial heat source producing such a condition is equal to the heat fluxes at the marked position in the original system. Therefore, the effect of the connection is obtained by calculating the temperature distribution in the latter case shown in figure 1-D. When the artificial heat source is less than the original one, it can be very practically ignored. Thus, the response to the original system can be used as an approximation of the response of the combined one. This approximation can be directly applied to the system in which addition of rooms is made.

Furthermore, we can attempt to divide a system into two parts by the principle mentioned above. It is possible to simulate the divided system to the original one described above, so that the effect of the division can be obtained by the same method of the connection, as shown in figure 1.

### 2.2 Practical Method

Rooms constitution a multiroom are divided into two groups; one has a stronger influence on the response, the other has a lesser influence. Then, the response to the divided system of the former may be an approximation to the response to the total undivided system. The effect of neglecting the latter can be easily obtained by the method given in the foregoing paragraph. If the effect is negligible, the accuracy at this stage is sufficient for engineering purposes. In order to obtain an accurate solution, the improvement of accuracy must be made by calculating the response to the total system caused by the artificial heat source supplied for releasing another one at the divided position, as shown in figure 1-E. This system is also divided into two groups different from the previous ones. Then, the response to the divided system consisting only of the group having a stronger influence on the thermal behavior can be an approximation to the response to the total undivided system. The second approximation is obtained by adding the improved result to the first one. It is then decided whether or not to continue the improvement of accuracy. It is necessary to continue successively the process of division and connection, until the accuracy becomes good. Then, the final solution is obtained by adding the result of each process. If the improvement is continued to infinity, the complete solution is obtained.

The divided system should be as simple as possible. It is convenient to take a unit room, since there will be no trouble in how to divide a system, and since the thermal characteristics required for this method are only of a unit room, and not of a multiroom system.

Summarizing, calculation by this method may be performed in some very distinct stages as follows.

1. First stage

   a. The divided system is taken to be the heating or cooling room. The air temperature caused by the original heat source is calculated under the condition of the air temperature of 0 °C for all adjoining rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 °C must be calculated for every adjoining room. It is then decided whether or not to improve the accuracy.

2. Second stage

   a. The divided system is taken to be each adjoining room. The air temperature caused by the heat source mentioned in 1-b is calculated under the condition of the air temperature of 0 °C for all neighboring rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 °C must be calculated for every neighboring room. It is then decided whether or not to improve the accuracy.

3. Third stage

   a. The divided system is taken to be each neighboring room. The air temperature caused by the heat source mentioned in 2-b is calculated under the condition of the air temperature of 0 °C for all adjoining rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 °C must be calculated for every adjoining room. It is then decided whether or not to improve the accuracy.

4. Fourth stage

It is necessary to continue the procedure successively until the accuracy is sufficient for engineering purposes. Then, the final solution is obtained by adding up the result of each process.

The frequency of calculation required in this procedure depends upon the structure of a system and usually it may be a small number. The temperature caused by the artificial heat source may be negligible at early stages of the procedure, since the magnitude of the heat flow decreases radpidly with increase of the distance from the source.

### 2.3 Procedure of 5-Rooms in Parallel

In the case that the heat is supplied only to the center room as shown in figure 3, we will attempt to obtain the air temperature of 5 rooms in parallel by the method described above.

1. First stage

   a. Air temperature of heating room.

$$T_1(1) = G_1 \times Q \qquad (1)$$

   b. Magnitude of the artificial heat source at every adjoining room.

$$Q_2(1) = F_{1\cdot 2} \times T_1(1) \qquad (2)$$

$$Q_3(1) = F_{1\cdot3} \times T_1(1) \tag{3}$$

$$Q_4(1) = F_{1\cdot4} \times T_1(1) \tag{4}$$

$$Q_5(1) = F_{1\cdot5} \times T_1(1) \tag{5}$$

2. Second stage

a. Air temperature of every adjoining room

$$T_2(1) = G_2 \times Q_2(1) \tag{6}$$

$$T_3(1) = G_3 \times Q_3(1) \tag{7}$$

$$T_4(1) = G_4 \times Q_4(1) \tag{8}$$

$$T_5(1) = G_5 \times Q_5(1) \tag{9}$$

b. Magnitude of the artificial heat source at every neighboring room.

$$Q_1(2) = F_{2\cdot1} \times T_2(1) + F_{3\cdot1} \times T_3(1) + F_{4\cdot1} \times T_4(1) + F_{5\cdot1} \times T_5(1) \tag{10}$$

$$Q_2(2) = F_{3\cdot2} \times T_3(1) + F_{5\cdot2} \times T_5(1) \tag{11}$$

$$Q_3(2) = F_{2\cdot3} \times T_2(1) + F_{4\cdot3} \times T_4(1) \tag{12}$$

$$Q_4(2) = F_{3\cdot4} \times T_3(1) + F_{5\cdot4} \times T_5(1) \tag{13}$$

$$Q_5(2) = F_{2\cdot5} \times T_2(1) + F_{4\cdot5} \times T_4(1) \tag{14}$$

3. Third stage

a. Air temperature of every neighboring room.

$$T_1(2) = G_1 \times Q_1(2) \tag{15}$$

$$T_2(2) = G_2 \times Q_2(2) \tag{16}$$

$$T_3(2) = G_3 \times Q_3(2) \tag{17}$$

$$T_4(2) = G_4 \times Q_4(2) \tag{18}$$

$$T_5(2) = G_5 \times Q_5(2) \tag{19}$$

b. Magnitude of the artificial heat source at every adjoining room.

Final results.

$$T_1 = T_1(1) + T_1(2) + T_1(3) + - - - - - \tag{20}$$

$$T_2 = T_2(1) + T_2(2) + T_2(3) + - - - - - \tag{21}$$

$$T_3 = T_3(1) + T_3(2) + T_3(3) + - - - - - \tag{22}$$

$$T_4 = T_4(1) + T_4(2) + T_4(3) + - - - - - \tag{23}$$

$$T_5 = T_5(1) + T_5(2) + T_5(3) + - - - - - \tag{24}$$

Where;

Ti(J) : air temperature of No.i room at J stage of calculation.
Qi(J) : magnitude of the artificial heat source of No.i room at J stage of calculation.
Gi : transfer function between the magnitude of No.i room heat source and No.i room air temperature.
Fik : transfer function between No.i room air temperature and the heat fluxes from No.i to No.k room.
i,k : subscript denoting room number. No.1 is the center room, No.2, 3,4,5 are the others.
J : subscript denoting the frequency of calculation in the procedure given in the foregoing paragraph.
Q : magnitude of the original heat source at No.1 room.
T,Q,G,F : these functions are represented by the Laplace transformation.

## 3. Calculation and Solution of Examples

### 3.1 Description of Examples

This method is applied to some multirooms; two rooms, five rooms in parallel, and fifteen rooms in series. The module of each room is an office, 10×10×4 meters, without any windows and any furniture. It is constructed of concrete with a thickness of 0.15 meters. The thermal condition and structure of every story are similar, with a ceiling and floor slub which seems to be insulated. The heat source with a unit step function is supplied to the corner of two rooms, fifteen rooms, and the center of five rooms. The inside combined heat transfer coefficient for these calculations is taken to be 8 Kcal in $hr^{-1}m^{-2}$deg C and the outside one to be 20 Kcal in $hr^{-1}m^{-2}$deg C.

### 3.2 Results

The results of these calculations are given in figure 2,3 and 4, where the room temperature is plotted versus time at every stage of this method. In figure 2 and 3 the curves are represented for the grade of improvement for every room, and in figure 4 it is represented for the frequency of calculation by this method described in foregoing paragraph.

The results for two rooms shown in figure 2 indicate that as the frequency of successive improvement increases, the temperature degree to be improved becomes smaller remarkably. For instance, the second improvement ratio between the true temperature of each room and the improved supplementary one, X3/X and Y3/Y in figure 2, is less than one per cent. In the first stage of improvement, good accuracy is obtained. It is thus possible to finish the calculation when it reaches the fourth stage described in foregoing paragraph. The approximation is given by summing up the first and second temperatures, X1 and X2 or Y1 and Y2 in figure 2.

The computer capacity required for approximation by this method is about one-half as small as the one required for the complete solution by working out the balance equations directly. In the case of a multiroom with just a few rooms, the time necessary for calculation does not differ greatly from that of the complete solution.

The results for five rooms in parallel shown in figure 3 indicate that the approximation with good accuracy is obtained when the frequency of calculation reaches the fourth stage, so that it is given by summing up the first and second temperatures, X1 and X2 or Y1 and Y2 in figure 3.

The computer capacity required for approximation by this method is about two-fifths as small as the one required for the complete solution by working out the balance equations directly. The time necessary for calculation of this method is about one-fiftieth as short as that of the balance equations. It is possible to calculate them by hand or by a desk computer.

The results for fifteen rooms in series shown in figure 4 indicate that as the frequency of successible calculation increases, the temperature degree to be improved decreases remarkably. For instance, the temperature at the fourth stage, curve 4 in figure 4, is about one-hundredth as small as that at the first stage, curve 1 in figure 4. Therefore, the temperature seems to be zero for rooms with a room number larger than 5, and the approximation of the rooms numbering from 1 to 4 is obtained by summing up two curves representative of each room.

The computer capacity required is about one-fifteenth as small as that for solving the balance equations, and the time necessary for calculation may be about one-two hundredths as short as the latter. It is also possible to calculate them by hand or by a common desk computer.

## 4. Summary

The practical method for obtaining room temperature of a multiroom has been proposed. Many examples; two rooms, five rooms in parallel, and fifteen rooms in series, were studied, and it has been shown that the present method is excellent for engineering and application purposes because these calculations can be made by hand, by a desk computer, or by a digital computer of relatively small capacity. This can be down at a reasonable cost.

It has also been demonstrated that with an increasing number of rooms, the usefulness of the present method increases remarkably. For instance, in the case of fifteen rooms in series, the time necessary for calculation of this method is about one-two hundredths as short as that of the balance equations method. The computer capacity for the former is about one-fifteenth as small as the latter.

It has also been found that this method is useful to obtain the effect produced by an addition of rooms.

It is obvious that this method is also useful for obtaining the heating load and cooling load.

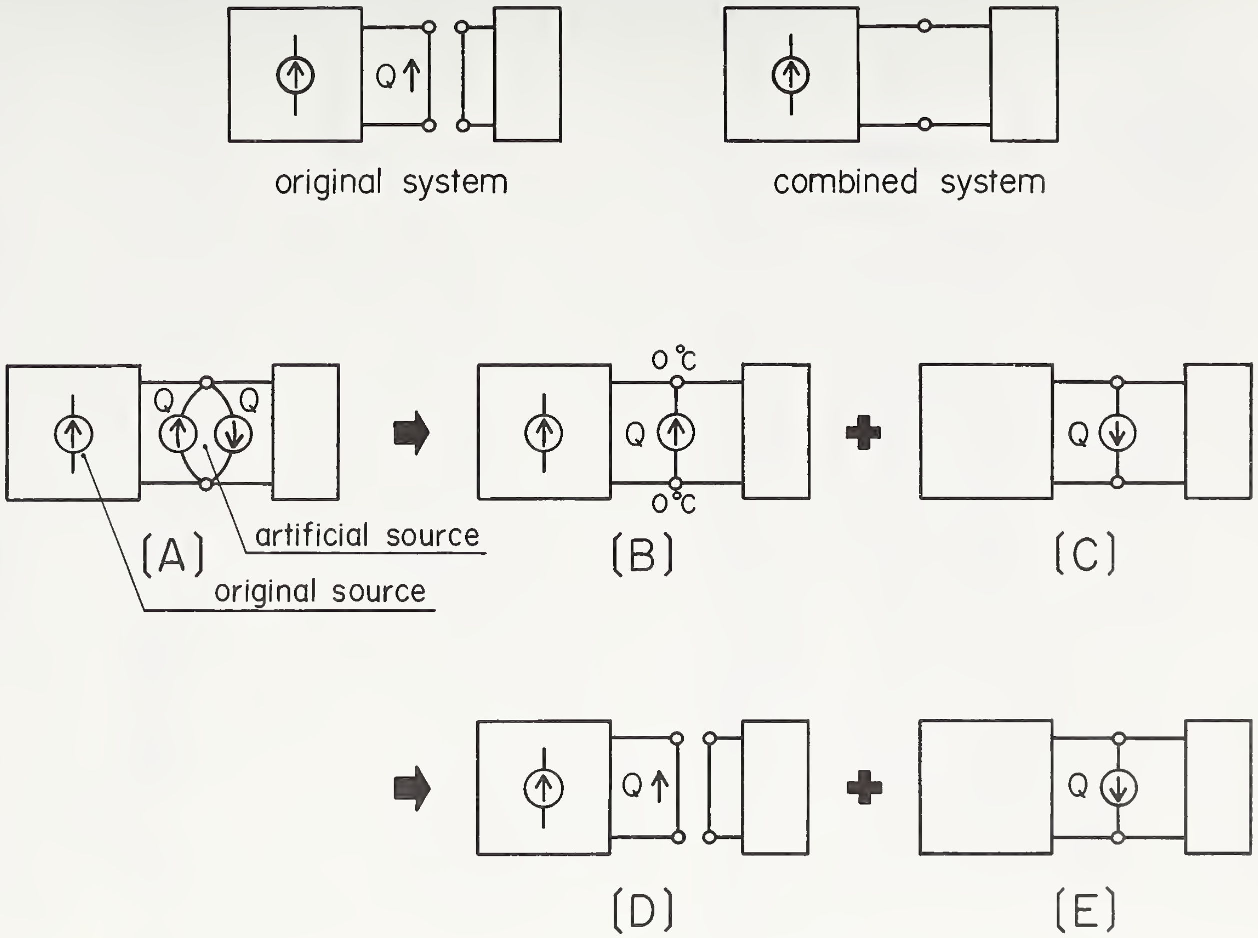

Figure 1 Schematic interpretation of connection and division of systems.

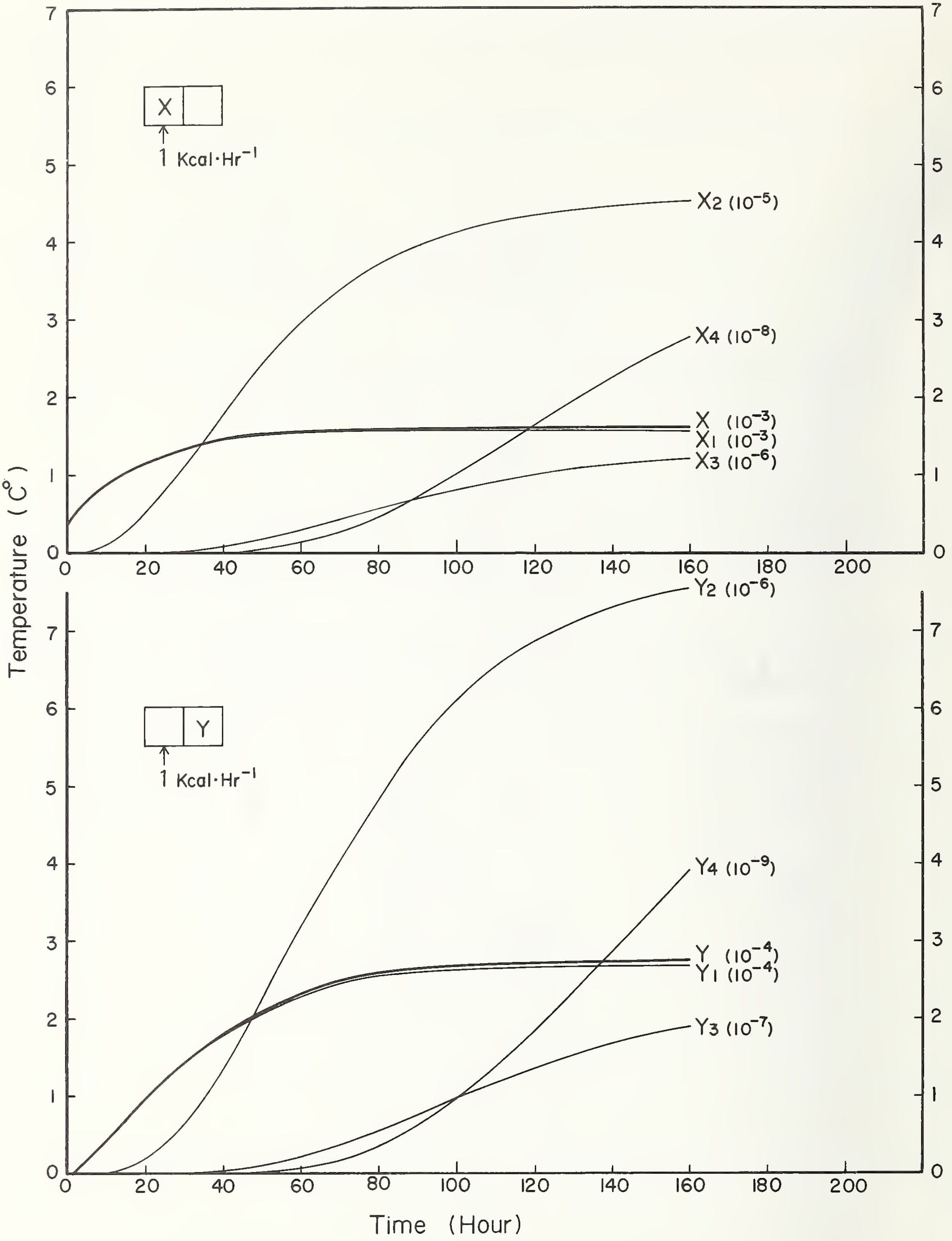

Figure 2 Room air temperature of 2-rooms resulted from heating in the form of a unit step function at X-room.

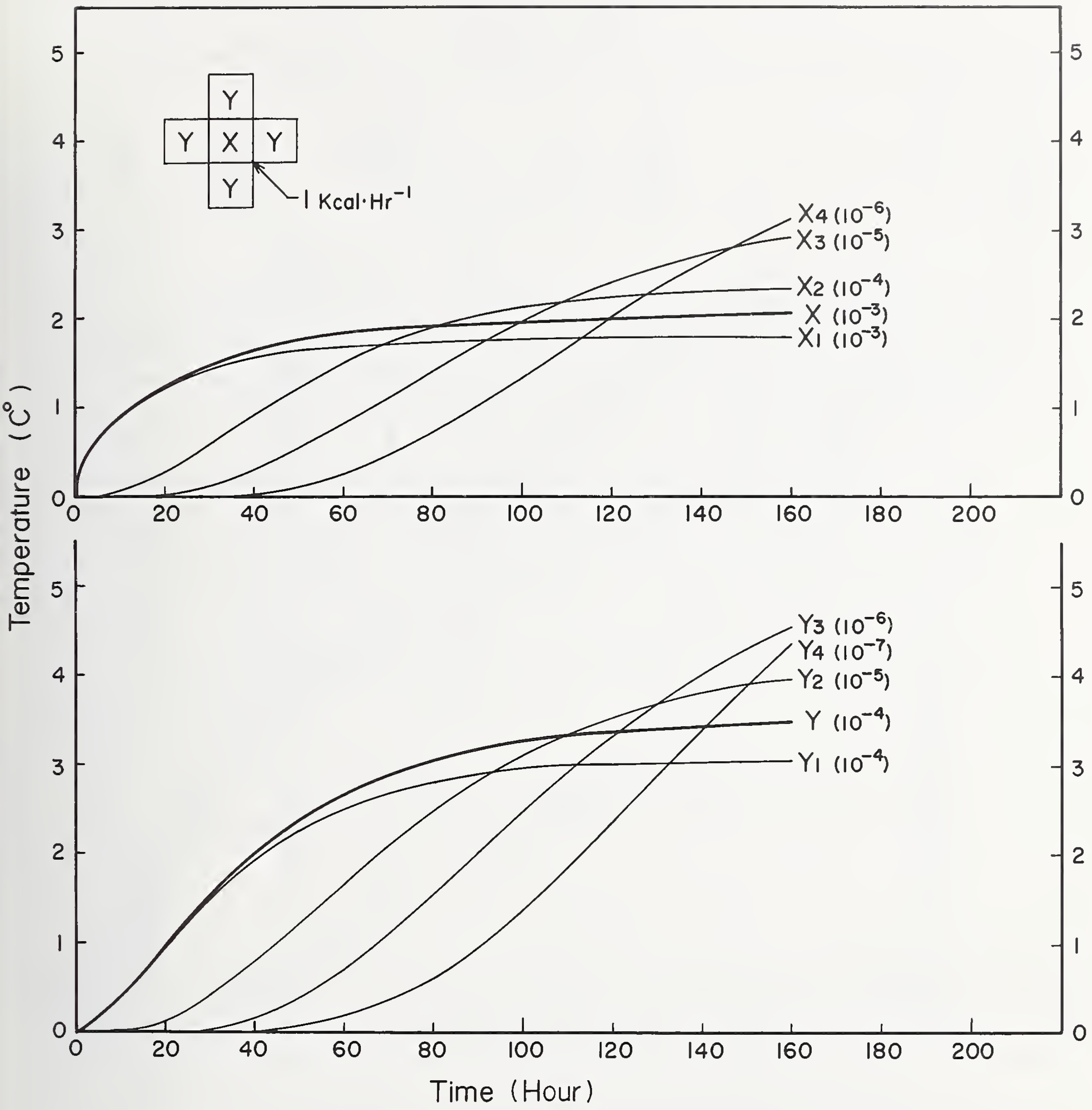

Figure 3 Room air temperature of 5-rooms in parallel resulted from heating in the form of a unit step function at X-room.

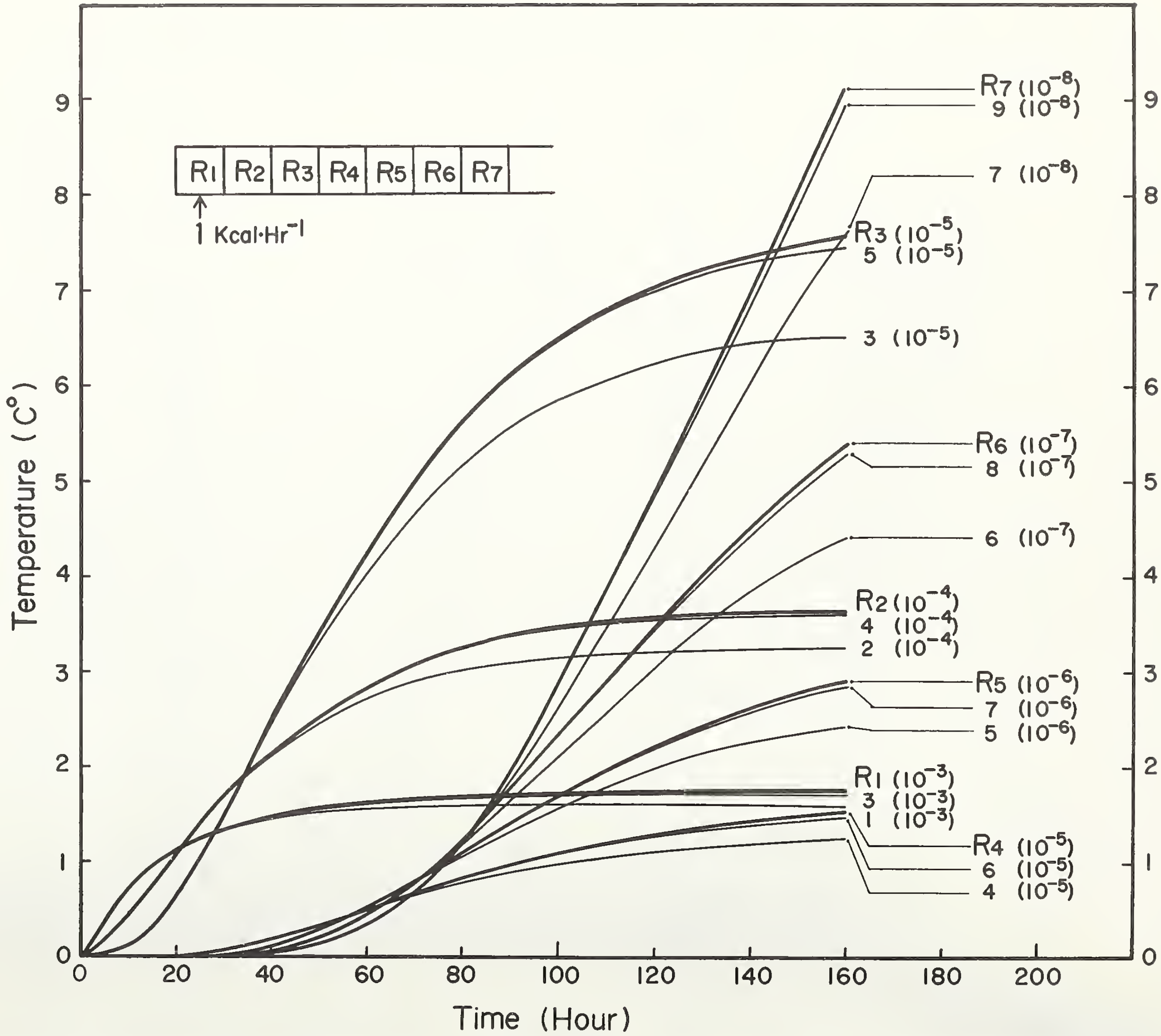

Figure 4 Room air temperature of 15-rooms resulted from heating in the form of a unit step function at R1-room.

# Simulation by Digital Computer Program of the Temperature Variation in a Room

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A computer program has been developed at the Royal Institute of Technology, Stockholm, Sweden, capable of calculating the temperature variations that occur in a room subject to variable internal and external heat gains. The program offers a very flexible choice of the quantities that are to be calculated. The air temperature in the room, the temperature and supply volume of the ventilating air, and the capacity of the heating or cooling units in the room can either be calculated separately, or in pairs when the limiting values of both quantities are known.

The program solves a system of heat balance equations which take into effect surface heat transfer by radiation between the room surfaces, by convection between the room air and room surfaces, and by conduction between the different material layers of the walls and floors. Not only is the temperature of the room air calculated, but also always that of the room surfaces and the material layers and outside surfaces of those walls and floor slabs which enclose the room. This practice has proved to be valuable when using the program in connection with studies of different types. When an operative temperature is to be determined, the temperatures of the different surfaces in the room must be given, in addition to the room air temperature. The calculated temperatures on the outside surfaces of the walls and floor slabs which enclose a room come into play in connection with studies of the function of temperature control systems in adjacent rooms having different temperatures due to different heat gains. An example of this is given in the article.

Key Words: Absorption factors, ALGOL, choice of unknown variables, daylight distribution in rooms, digital computer program, finite-difference method, heat balance equations, operative temperature, temperature control, temperature variation in a room.

## 1. Introduction

Use of computers in the study of non-uniform temperature variation in buildings began rather early in Sweden. The first example of such use dates from 1957 when a method was described for computer calculation of the temperatures in an external wall exposed to solar radiation /1/. In the same year another investigation was presented dealing with design outdoor temperatures for heating load calculations /2/. The temperatures proposed in the survey were determined on the basis of computer calculated temperature variations in different types of buildings with temperature variation out of doors. In both these cases, the calculations were made on the Swedish computer BESK which was installed in 1953.

The equations on which the present program is based were first published in 1963 /3/. Since this date, the program has been developed in order to make it more versatile, cheaper, and easier to use. It has thus been supplemented in such a way that the quantity which is to be calculated can be chosen from among several different variables. By revising the program, which is chiefly written in ALGOL (except for a lesser part in internal computer code), and by modernizing the computer itself, i.e. the Swedish-made TRASK, the cost of computer time has been decreased. For a normal case, it costs, at present, about US $ 10 for the calculation of a 24 h period of variation in one room.

A special alternative for easy use of the program has been developed in the form of the "engineer's version" and enables the calculations to be made with less work, especially concerning the presentation of data since the data need not be presented on special forms. The complete program version described in this article allows, however, greater flexibility concerning the choice of variables.

The calculation method, which is an iterative, finite-difference method, has been developed with respect to its application for the dimensioning of plant for cooling and heating rooms, but also for the study of how buildings should be constructed in order to provide a comfortable thermal environment at the lowest cost. It should be possible to use the program both for design and for research and investigation work. In view of the fact that it is essential to be able to explore the effect of all factors which may arise, the complete program version is so written that no material constants or values for temperature and incoming radiation are stored in the computer memory. Furthermore, there are no limitations regarding the definition of the surroundings of the room, i.e. its position in the building.

To date, the program has been used in a fairly large number of cases in connection with the design of hospitals, hotels and office buildings. Further, the program has been used in a survey concerning a design guide for school buildings in the study of the effects of different factors on classroom temperature. The investigation resulted, among other things, in a proposal for a standard for calculation of the temperature in classrooms for design of school buildings throughout Sweden. According to the proposal, such calculations will involve the use of the program presented here, or a program giving the same result /4/.

## 2. Input Data

Data for the calculations is recorded on eight different types of forms: 1 Room, 2 Wall or floor, 3 "Facade", 4 Window, 5 Lighting unit, 6 Heating unit, 7 Time dependent data, and 8 Constants. Time dependent and time independent quantities are differentiated, see table 1. The time dependent quantities can be assigned variable values during the calculation period, i.e. the time period under which the calculation is considered. They are identified on forms 7 and 8 by numbers assigned to them from forms 1-6. A form of type 7 can be used to record the values of a given quantity at different time points, while a form of type 8 is used to record data for such quantities which are in principle time dependent but which for the present case do not vary with time.

The room is assumed to have parallelepiped shape. Forms of type 1 contain a sketch of a room including a coordinate system enabling the addition of sub-surfaces. Such sub-surfaces are windows and lighting units and even heating units which emit thermal radiation and therefore need to be included as surfaces in the room.

Values of outside air temperature and radiation from the sun and sky are found from tables or diagrams. The tables used here are computer calculated and based on Finnish measurements /5,6/. They give sun and sky radiation against vertical facades and horizontal roofs as well as radiation transmitted through vertical and horizontal windows with double-glazing of ordinary window glass. The radiation reflected at the ground is included in the radiation values. In cases where a facade falls within the shadow of surrounding buildings a separate program is used to calculate the portion of the day when the different windows in the facade are sunlit. Computer drawn diagrams are used in determining the incoming radiation through windows which are shaded by overhangs or which are set back from the external plane of the facade.

The shading coefficients (tab.1) give the transmission through the fenestration in question in relation to the total radiation energy transmitted through a double-glazed window of ordinary window glass /7/. These coefficients are determined under the assumption that the surface heat transfer coefficient for the exterior side of a double-glazed window is 16 and 8 $Wm^{-2}deg^{-1}$ for the interior side.

The thermal resistance of the interior side of a window is not included in the calculations since the temperature of the window surface towards the room is included in the programmed heat balance equations.

Shading coefficients and thermal resistance of windows are listed in table 1 as time dependent quantities since these values are not constant for windows having drapes or Venetian blinds which are used for only part of a 24 h period.

The word "facade" is taken to mean the exterior side of each wall or floor for which incoming radiation, surface heat transfer coefficient, and air temperature are known. Thus a "facade" is not necessarily the outside of an exterior wall. It can just as well be the outer surface of a roof or of a wall giving onto a corridor.

A form of type 6 is used to designate a heating or cooling unit whose surface radiation or thermal capacity must be taken into consideration. Furniture can be considered as such heating units of effect 0. For convective heating units with no thermal capacity, the data is instead listed under "heat direct to room air". Heat from persons present in the room is often listed under this variable.

Radiation from people can be introduced in the calculations in the form of radiation from a lighting unit which is located on the floor. The amount of body heat given off decreases with an increase of temperature. The program can simulate lower convective heat radiation when the calculations indicate a higher air temperature.

Table 1. List of quantities in the computer program.

| Type of form on which data or identification number is given | Time independent quantities | Time dependent quantities |
|---|---|---|
| Type 1<br>Room | Length, width and height of room. Sub-surface dimensions in x-and y-directions and coordinates for their lower left hand corners. Reflectance for room surfaces and sub-surfaces. | Outside air temperature, volume of air leakage, room air temperature, ventilating air temperature, volume of ventilating air, heat direct to room air. Unknowns. |
| Type 2<br>Wall or floor | Thickness, coefficient of thermal conductivity, density and specific heat of different layers of materials of the wall. Thermal resistance of air layer in the wall. Number of partial layers of each material layer. | ——— |
| Type 3<br>"Facade" | Absorptance of "facade" surface. Cloudiness. | Radiation towards "facade". Air temperature. Surface heat transfer coefficient. |
| Type 4<br>Window | ——— | Radiation transmitted through double-glazed windows. Shading coefficient for total and for direct transmitted radiation. Thermal resistance. Outside air temperature. |
| Type 5<br>Lighting unit | Percentage of radiation from lighting units. | Lighting effect. |
| Type 6<br>Heating unit | Volume of heating or cooling unit, density and specific heat of the material of the unit. Surface enlarging factor of the unit. | Effect of heating or cooling unit. |

## 3. Unknowns

Those variables which may be unknown are room air temperature, ventilating air temperature, volume of ventilating air, heat direct to room air, and effect of heating unit. The variable which is to be calculated is determined in the program by the code number given to "unknowns". Code numbers are listed on the forms in the same way as the values for other time dependent variables. The code number gives the different positions in the program where the sought variable is solved from the heat balance equation for the room air.

It is sometimes desirable to seek a variable given the conditions that it lies within certain limiting values. If the calculated value is larger than a maximum value or less than a minimum value, then the value for another variable must be found. Thus the program first tests a variable against its limit value (which is not necessarily constant during a 24 h period). If this limit is crossed, the position of another variable is found, the result is calculated, and the first variable is given its limit value.

## 4. Thermodynamic basis

### 4.1 Angle and Absorption factors

The equations for heat transfer between room surfaces take into consideration reflexion of short wave radiation (from the sun and sky) but not long wave radiation. Thus it is assumed that all long wave radiation which is emitted from a surface $A_i$ and which meets a surface $A_j$ is totally absorbed by $A_j$. Emission is furthermore assumed to occur diffusely and transmitted energy is determined by the angle factor $_{ij}$, which is calculated using generally known equations and methods, see e.g. /8/ and /9/.

Transmitted radiation energy for short wave radiation is found by the equation system

$$\psi_{ij} = \varphi_{ij} a_j + \sum_{p=1}^{m} \varphi_{ip} r_p \psi_{pj}$$

Here $\Psi_{ij}$ is the absorption factor for radiation between $A_i$ and $A_j$, $a_j$ is the absorptivity of $A_j$, $r_p$ is the reflectivity of some surface $A_p$ of the m room surfaces, and $\Psi_{pj}$ is the absorption factor for radiation between $A_p$ and $A_j$. The first term of the expression for $\Psi_{ij}$ is that portion of radiation from $A_i$ which is absorbed at $A_j$ and which has not been reflected by any intermediary surface. The sum term is that radiation absorbed by $A_j$ which has first been reflected by the room surfaces.

In cooling load calculations it is useful to know at which time daylight is so weak that lighting units must be turned on. The equation system above has been used in the study of daylight distribution in rooms in this context, the program having first been extended so that the absorption factors for radiation from windows towards the top side of a small horizontal surface placed in different parts of the room can be calculated. A condition for these calculations is that the radiation from the windows is spread diffusely. Such is the case for windows with drapes or Venetian blinds as shown from previous measurements. These measurements have also shown that a calculation of the distribution of light from a window can be correctly performed only if room surfaces perpendicular to the window surface are divided into sub-surfaces when angle and absorption factors are determined.

## 4.2 Heat Balance Equations

Time variation of temperature in a room is calculated from the heat balance equations for the material layers on the surface and in the walls and floors, for the interior side of the windows, and for the room air.

Walls are considered to be divided into layers which are parallel to the wall surface. The equation for a layer yields the magnitude of temperature change in a layer of thickness $\Delta x$ during a time interval of $\Delta t$ (ordinarily 15-60 minutes) caused by the surface heat transfer of both of the limiting surfaces of the layer.

Crank-Nicolson's equation is used for an individual layer inside the wall in the form

$$K_1\theta_{n-1,t} + K_2\theta_{n,t} + K_3\theta_{n+1,t} = K_4\theta_{n-1} + K_5\theta_n + K_6\theta_{n+1} \qquad (A)$$

where the temperatures at the beginning of the time period are written on the right side and the unknown temperatures on the left side. The index n is used to denote the layer being treated while n-1 and n+1 are the adjacent layers.

If the three layers are of the same material, then $K_1 = K_3 = -1$, $K_4 = K_6 = 1$, $K_2 = 2\ \Delta x^2/a\ \Delta t + 2$, $K_5 = 2\ \Delta x^2/a\ \Delta t - 2$. The constant a here denotes the material's thermal diffusivity.

Equation (A) is also used for the limiting surface between two different materials as well as the surface of an air layer inside the wall. The coefficients then include the thicknesses, the thermal conductivities and the thermal diffusivities for the layers of material on both sides of the limit surface, and, in the latter case, the thermal resistance of the air layer /3/.

The following relation is valid for a layer n at the surface of a wall:

$$-F_1\theta_{n\pm1,t} + F_2\theta_{n,t} - F_3\theta_{r,t} - \Sigma F_4\theta_{u,t} = \qquad (B)$$

$$= F_1\theta_{n\pm1} + F_5\theta_n + F_3\theta_r + \Sigma F_4\theta_u + F_6$$

The index $n \pm 1$ indicates the adjacent layer, r the room air, and u a room surface from which long wave radiation reaches the surface.

Equation (B) is applicable not only for surfaces of walls, floors and ceilings but also for the surfaces of heating units, interior sides of windows, and facade surfaces. Contrary to the K values of eq (A), the F values of eq (B) are not all constant. The reason is that they include variables such as radiation from windows and lamps, effect of heating units, shading coefficients, thermal resistances of windows and the outside air temperature. For outdoor facade surfaces, $F_6$ includes the sol-air temperature which contains the variables outside air temperature and radiation from sun and sky. It can be pointed out that the equation for the sol-air temperature is so formed that the dependence of long wave radiation upon the degree of cloudiness is taken into consideration at roof surfaces /10/.

The convective surface heat transfer coefficient $h_c$ for a room surface is dependent upon the temperatures of the surface and air as given by the calculations. With each repeated application of the equations in which $h_c$ appears, the most recently found value for $h_c$ is inserted, during the calculation, in the F coefficients.

Windows are assumed to lack thermal capacity. When eq (B) is used for the interior surface of a window, the temperatures $\theta_{n\pm1}$, $\theta_n$, $\theta_r$ and $\theta_u$ which are valid at the beginning of the time interval $\Delta t$ are all set equal to zero, since the surface temperature at the end of the interval is then independent of these temperatures.

A third type of heat balance equation, type (C), is used for the room air. This equation states that the heat which is introduced convectively from heating units in the room and by ventilating air and possible leakage air is removed by the exhaust air and by convective surface heat transfer at the room surfaces.

## 5. Calculation Procedure. Steps and Tolerances

The temperatures in the material layers, at the interior surfaces of windows, and of the room air are all calculated simultaneously from a system of equations of the types (A), (B), and (C). The calculation begins with assumed values for these temperatures at time point 0 o'clock if a 24 h period is to be studied, and then gives the temperatures after a first time interval of the calculation step's length $\Delta t$. These temperatures, in their turn, make up the start values for the calculation of the temperatures at the end of the following time interval of the same length. The same equation system as for the first step is used here. However, the coefficients may have received new values. The calculation is continued step by step until the variation for the whole calculation period has been found.

In general, the length of the period is 24 hours. It can however have other lengths, e.g. four times as long. Stationary as well as non-periodic cases, where the start values are known and the input data then varies at a known rate, can be studied.

Solution of the equation system which normally contains some fifty equations is done by relaxation. A calculation referring to one and the same time point is repeated time after time with increasing accuracy. The difference between the two values for the same variable from two consecutive calculations are compared each time with a predetermined tolerance, the relaxation tolerance. The calculations for the time point in question continue until this difference is less than the tolerance for all variables. This tolerance is dimensionless and is expressed in such a way that it can be used for temperature as well as for air volumes and heating and cooling effects.

After the calculations have been carried out for all the time points during the period as determined by $\Delta t$, they are repeated for the same time points during a new period. The difference between values for one and the same variable at the end of this period and the previous period are compared with a predetermined tolerance, the period tolerance. The calculations are now repeated for more periods until the differences for all the variables fall below the prescribed tolerance. The same expression is used here as for the relaxation tolerance. The value for the period tolerance must, however, be larger than that for the relaxation tolerance.

The values of the calculated variables are printed out at time points determined by the print step. The print out also includes the average values for the variables during the period. The pring step can often be made longer than the calculation step which is determined by technical factors. The normal value for the calculation step is 0.5 hours, for the print step 1 hour, for the relaxation tolerance 0.1 % and for the period tolerance 0.2 %. These values are used if no others are motivated.

## 6. Application Examples

### 6.1 The Effect of Wall and Floor Insulation upon Temperature Variation in a Room

During a series of clear days with equal sunshine and outside temperature, the mean room air temperature for a 24 h period is not dependent upon an exterior wall being insulated from the outside or inside. If the room is surrounded by similar rooms with the same temperature, this mean air temperature is also unaffected by the presence of a layer of insulation on interior walls and floors.

Both these measures, however, affect the magnitude of temperature fluctuation about the mean temperature. Heat storage in the structure has a dampening effect. Insulation of the heavier layers decreases storage and results in stronger temperature fluctuation in the room. In this respect, insulation on the inside of exterior walls produces the same effect as the insulation of interior structures.

For an illustration of this, consider the curves in figure 1 which are drawn on the basis of calculations using the computer program. These refer to a clear July day in Stockholm (latitude 59°21'N) for an empty room without artificial lighting. The room was 2.7 m high and 5 m deep. It had a 3.8 m long window wall and was ventilated over the whole daily cycle by 250 kg/h of outside air. The window had a glazed area of 4 $m^2$ with double glazing of ordinary window glass with light colored drapes on the inside, and faced S 60° E. The temperature in the corridor was a constant 24°C.

The dash-dotted curves in the figure indicate a room having a reinforced concrete exterior wall insulated on the outside with mineral wool. The floor is also of reinforced concrete and partition and corridor walls are of cellular concrete. Dotted curves indicate a room with the same exterior wall but where the mineral wool insulation is located on the inside. The upper surface of the floor slab has been covered by a wooden floor over mineral wool insulation, while the under surface has a suspended ceiling with a thin layer of mineral wool insulation. The partition walls and corridor wall have also been insulated with thin slabs of mineral wool mounted on studs. All wall insulation is protected by 1/2 inch thick plaster board. The thickness of the concrete and cellular concrete has been decreased sufficiently so that the weights of walls and floors including the insulation are the same.

In the second case described, the quality of sound insulation between the rooms has been improved, but the thermal conditions have deteriorated. Mean air temperature under a 24 h period for both cases is 24°C, but because of the insulation, the maximum temperature for the second case rises by 2.4° from 25.9° to 28.3°C. Mean temperature here during the period 8-17 o'clock rises by 1.6° from 25.7° to 27.3°C.

Figure 1 also shows how the mean temperature for the room surfaces varies during the period 8-17 o'clock. The value is a weighted mean value with the surfaces as weights (the floor surface temperature is not included). Because of the insulation, the maximum value rises by 3.2° from 27.5° to 30.7°C, while the mean value for the period 8-17 o'clock increases by 2.2° from 27.0° to 29.2°C.

The example shows that the weight of the interior walls and floors and the weight and thermal transmittance of the exterior walls are not sufficient data on the building structure for calculation of the temperature variation in rooms.

## 6.2 The Temperature Variation in a Room as Affected by Heat Gains in Adjacent Rooms and the Type of Temperature Control (Individual or Central)

Given a building with several rooms of the same type along a facade and assume that a system is to be installed for regulating the temperature of ventilating air in such a way that the room air temperature during working hours has a desired value independent of whether the room is being used or not. If certain deviations from the desired value can be permitted, the problem is then to find out whether it is necessary, under summer conditions, to provide controls in every room (individual temperature control), or whether it is sufficient to feed cooled ventilating air of the same temperature to both unoccupied and occupied rooms (central temperature control).

Figure 2 presents the results of calculations for similar rooms of the same size as in the preceding example. Whereas the walls and floors were of a different construction, the window had the same orientation and incoming radiation is thus the same. Outside air temperature was also the same. The ventilating air was supplied at a rate of 300 kg/h during the whole 24 h period. It was cooled only during the period 7-18 o'clock, and was 1°C warmer than the outside air during the rest of the time. Occupied rooms were used by two persons during working hours (8-17), and lighting units were turned on during the afternoon producing an effect of 25 W/m$^2$ of floor surface. The windows in these rooms were double-glazed and had roller blinds which were kept down except for during the afternoon; there were also drapes inside the windows which were kept drawn during the whole 24 h period. The unoccupied room was assumed to be surrounded by occupied rooms, and the blinds in this room were not used. The desired air temperature in the rooms during working hours was 22°C. However, the ventilating air temperature was not allowed to fall below 15°C.

Under these conditions, according to figure 2a, the air temperature in the occupied rooms can be maintained at 22°C except for a slight increase in the afternoon. The ventilating air is then insufficiently cooled to wholly compensate for the heat release from people and lighting units.

Whereas the ventilating air to the occupied rooms need not be cooled so low as to 15° during the morning, it is necessary for the ventilating air to the unoccupied room (see fig.2b). This is based upon the fact that the roller blinds between the panes in this room were not let down.

In cases of central temperature control where the ventilating air for the unoccupied room has the same temperature as that for the occupied rooms the temperature during the morning and through the lunch hour exceeds the desired value by ca. 1°. Contrary to this, the temperature towards the end of the working day is about 1.5° too low, as shown in figure 2c. On the basis of these deviations from the desired value, it is possible to decide whether the cheaper alternative of central temperature control can be accepted.

In the calculations for the unoccupied room the heat conduction through walls and floors between this room and the occupied rooms was taken into consideration. This was accomplished by using the previously determined surface temperatures of the occupied rooms as the outside air temperatures at the external surfaces of the walls and floors of the unoccupied room. These surfaces were treated as "facades" in the program. The surface heat transfer coefficient for them, were given a very high, fictive value. This procedure involves approximations which, however, can be reduced by repeated calculations. In this case, they proved to be insignificant.

Room air and ventilating air temperatures were considered as unknown quantities during the larger part of the 24 h period in the calculations concerning the occupied room. Maximum values were then assigned to them for a part of this period and minimum values for another part. During the night, the room air temperature was considered as the unknown.

During the calculations for the unoccupied room with individual temperature control, minimum values were assigned to the temperatures of the room air and ventilating air during the whole 24 h period. In the case of the central temperature control alternative the room air temperature was treated as the unknown during the whole 24 h period.

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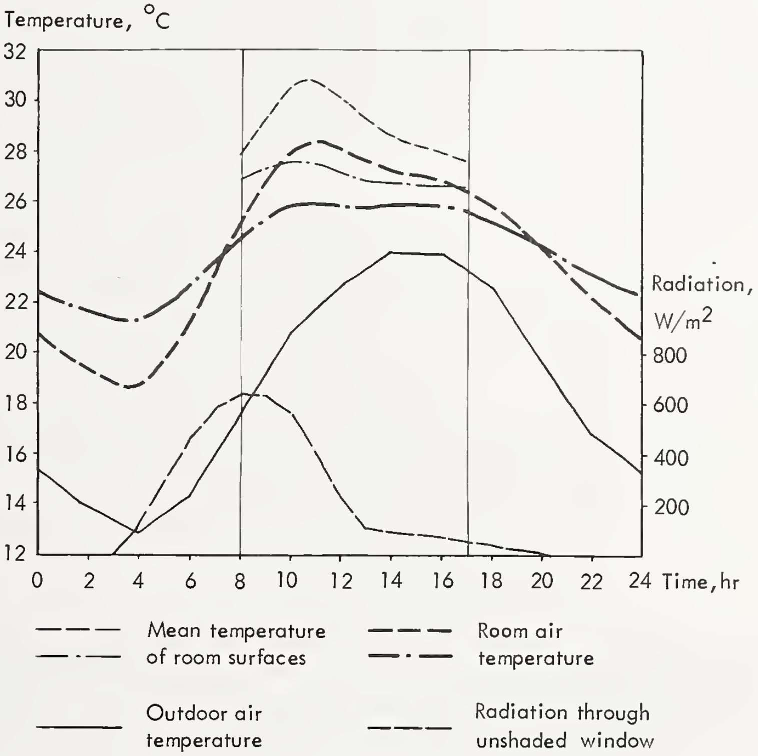
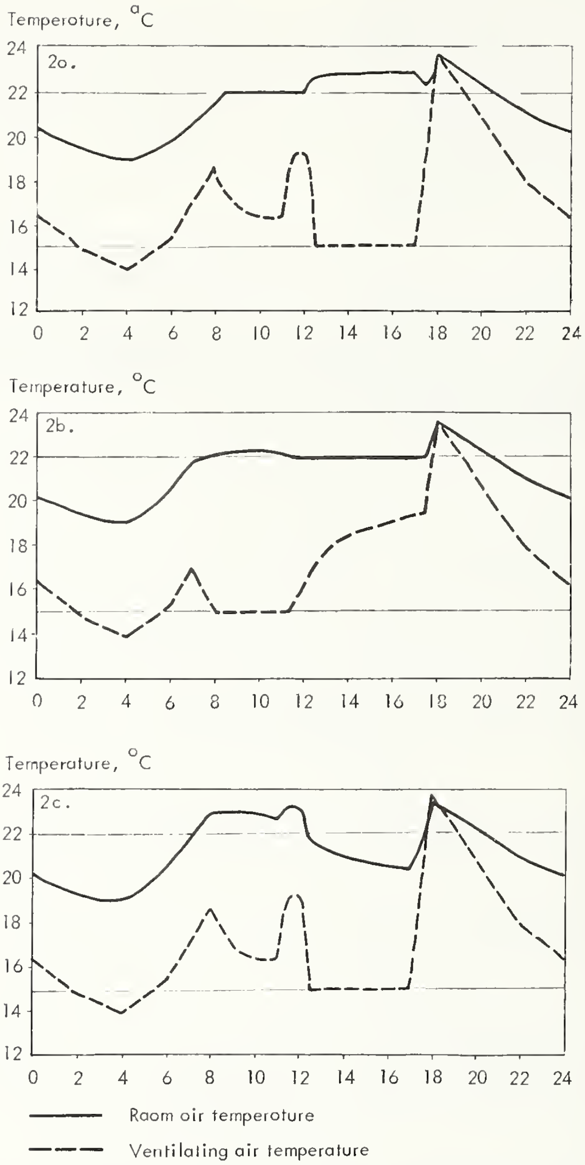


1. Calculated temperature of air and room surfaces in two rooms whose walls and floors have the same weight but different insulation:
-.-.-.-.-. Exterior wall insulated on the outside,
non-insulated interior walls and floors
---------- Exterior wall insulated on the inside,
insulated interior walls and floors



2. Calculated air temperatures in occupied and unoccupied rooms.
   a. Occupied room
   b. Unoccupied room, individual control
   c. Unoccupied room, central control

# Optimization of an Air-Supply Duct System

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The design of a multi-branch air supply system is complex because there are as many decisions to make as there are sizes of duct sections to specify. A further complication is that a decision on the size of one section affects the performance of the entire system. If an optimal duct system is sought, then, the design process becomes a multi-variable optimization.

The premise on which the mathematics of the procedure explained in this paper is based is that all of the available static pressure is dissipated in the duct and fittings with no artificial pressure drop due to dampers. Furthermore, of the multitude of systems that would be balanced without the addition of dampers, the computer program selects the one with minimum cost.

The objective function is the total cost of the system based on a fixed cost per pound of metal. The constraints appear because the pressure drop from the fan discharge to each outlet must equal the available static pressure. Thus, there are as many constraints as outlets. The forms of both the objective function and the equality constraints are preditable, so the optimization yields itself to the method of Lagrange multipliers.

This paper presents the basis of a computer program which performs this optimization for a circular-duct system. The user of the program provides the following information:

(a) available static pressure at the fan outlet,

(b) the geometrical coordinates of each outlet, branch, and elbow,

(c) the flow rate at each outlet, and

(d) the cost of the metal per pound.

The printout includes the diameter, duct surface area, velocity, pressure drop, and cost for each section of duct for the optimum system.

Key Words: Air supply, duct system, optimization.

## 1. Introduction

An air-supply system is a requirement for practically every environmental control unit for a building. The design of the supply system is a complex one because the system has many components--each section of straight duct and each fitting is a component--and as is true of classical systems, component performance is interrelated. The question might be raised, "How can such a complex system be designed and constructed in practice by methods that are usually unsophisticated?" There are two answers

(a) The most popular design procedures afford a systematic way of arriving at a workable system,and

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(b) Most duct systems are not optimum.

The two answers are related. The first task of the designer is to specify a system that will meet the requirements, which for duct systems is to provide the specified flow rate at designated locations. Furthermore, the designer must perform his task relatively quickly--he cannot make a career of one or two duct systems. To his aid come the standard procedures for duct design, such as the "velocity-reduction method" and the "equal-friction method." The merits of these methods are

(a) that they result in workable systems, and

(b) that they provide methodical procedures for the designer to follow which simplify his task.

A further merit of the velocity reduction method is that it provides velocity limits which acknowledge noise considerations. The equal-friction method, on the other hand, while it has no built-in procedures for limiting velocities does come closer to achieving a duct system of minimum first cost.

One of the coverups for a non-optimum design is a high fan static pressure. Given an unlimited static pressure at the fan and a generous supply of dampers, almost any duct system will provide the required flow at designated locations. Unfortunately, the economies realized in reduced design time are sometimes dwarfed by unnecessarily high first and operating costs.

This paper presents the basis for a procedure for optimizing the first cost of an air-supply system. The procedure has been incorporated into an operating computer program. Specifically, the optimization problem can be formulated as an objective function with equality constraints which is soluble by the method of Lagrange multipliers. The computer program executes the optimization, solving the set of non-linear simultaneous algebraic equations by the Newton-Raphson technique.

## 2. Air-Supply System

A sample duct system (shown schematically in fig. 1) must supply $Q_I$, $Q_{II}$, and $Q_{III}$ cfm, respectively, at the positions shown. The optimization procedure presented in this paper determines the minimum-cost duct system for a given available static pressure at point 1. The minimum cost system is the one which requires the minimum weight of metal, a proposition which conforms to the conventional method used by contractors in making bid estimates.

An immediate question that emerges is whether the total system can truly be optimum if the static pressure at point 1 is arbitrarily assigned. Figure 2 shows that when the static pressure is extremely low, the duct cost will be excessive due to its large size regardless of the design procedure used. On the other hand, when a high static pressure is selected, the lifetime operating cost becomes the controlling quantity. There is a certain static pressure that results in minimum lifetime cost. For any static pressure there will be a minimum cost duct system. The program described in this paper designs such a system. To obtain the information required to construct figure 2, this program must be used to find the optimum duct system pressures.

## 3. Premise of Optimum Design

From physical reasoning the optimum duct system is one where the available static pressure is dissiptated completely in each run. A "run" is defined as the path from the fan discharge to an air outlet. The basis for this premise is that if a run should exist where the flow is too high, the flow should be reduced not by throttling with a damper, but by reducing the size and thus the cost of some duct section in that run. The consequence of applying this premise is that the resulting duct system uses all of the available static pressure to overcome friction in the straight ducts and fittings, and no static pressure is dissipated in dampers.

Obedience to the foregoing premise, however, does not describe a unique duct system. This fact could be shown schematically as in figure 3. Assume that the ducts shown by solid lines are of such size that they deliver the desired flow rates of air to the two outlets when a certain static pressure exists at the fan outlet. Another duct system that has a smaller size trunk section and larger size in the branches and will also deliver the desired flow rates can be found (shown by the dashed lines). One of those two duct systems undoubtedly will have a lower cost than the other. A conclusion, then, is that there is an infinite number of duct systems that consume the available static pressure, but only one of these will have minimized cost.

## 4. Cost Function

The mathematical form of the duct optimization problem consists of two parts:

(a) the cost function which is the objective function to be optimized, and

(b) the constraints.

For circular ducts the cost of a section is

$$\text{Cost of section} = \pi DLW(CCP) \qquad (1)$$

where

$D$ = diameter of the duct, ft

$L$ = length, ft

$W$ = effective weight per square foot of duct surface (includes allowance for fittings, seams, hangers, and scrap), $(lb)(ft)^{-2}$

$CCP$ = cost per pound of duct, $(dollars)(lb)^{-1}$

The allowance for fittings, seams, hangers and scrap in W is 20 percent of the weight of duct surface. The weight of the duct per $ft^2$ conforms to recommended construction for low-pressure ductwork (less than 2 in. of water static pressure) as recommended in Tables 11 and 15, Chapter 3 [1][4]. The gauge of the metal is thus a function of the duct diameter. The total cost, COST, of the example system in figure 1 is

$$COST = C_{1-2} + C_{3-4} + C_{5-6} + C_{4-5} + C_{6-7} + C_{8-9} + C_{10-11} \qquad (2)$$

When the costs of each section from eq (1) are substituted into eq (2), the variables for optimization become the diameters. The other terms L, W, and CCP are dictated by costs and layout and are not variables of optimization.

Rather than choosing the diameters as the variables, it is more convenient for later manipulations to express the diameter in terms of a pressure drop, $\Delta p$. The pressure drop in a section of straight duct can be expressed by

$$\Delta p = 12f \frac{L}{D} \frac{V^2}{2g_c} \frac{\rho_a}{\rho_w} \qquad (3)$$

where

$\Delta p$ = pressure drop, in. of water

$f$ = friction factor dimensionless

$V$ = velocity, fps

$g_c$ = gravitational constant 32.2 $(ft)(lb_m)(lb_f)^{-1}(sec)^{-2}$

$\rho_a$ = density of air, $(lb)(ft)^{-3}$

$\rho_w$ = density of water, $(lb)(ft)^{-3}$

Further, the velocity V can be expressed

$$V = \frac{Q}{60(\pi D^2/4)} \qquad (4)$$

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[4] Figures in brackets indicate the literature references at the end of this paper.

where

Q = flow rate, cfm

Substituting eq (4) into eq (3) and solving for D results in

$$D = \text{constant}/\Delta p^{0.2} \tag{5}$$

Substituting eq (5) into eq (1) and then into eq (2) results in the objective function which is the total cost,

$$\text{COST} = \frac{K_{1-2}}{(\Delta p_{1-2})^{0.2}} + \frac{K_{3-4}}{(\Delta p_{3-4})^{0.2}} + \frac{K_{5-6}}{(\Delta p_{5-6})^{0.2}} + \frac{K_{4-5}}{(\Delta p_{4-5})^{0.2}} + \frac{K_{6-7}}{(\Delta p_{6-7})^{0.2}} + \frac{K_{8-9}}{(\Delta p_{8-9})^{0.2}} + \frac{K_{10-11}}{(\Delta p_{10-11})^{0.2}} \tag{6}$$

The constants K incorporate the length, flow rate, friction factor and effective weight per $ft^2$ for each section.

Equation (6) is the function that must be optimized subject to the constraints that will be presented in the next section.

## 5. Constraints

The constraints essentially state that all of the available static pressure will be dissipated in friction in the fittings and straight sections. The number of constraints is the same as the number of runs which also equals the number of outlets. The constraint equations for the system in figure 1 are:

$$\Delta p_{1-2} + \Delta p_f + \Delta p_c + \Delta p_{2-3} + \Delta p_{3-4} + \Delta p_{4-5} + \Delta p_{5-6} + \Delta p_I = SP \tag{7}$$

$$\Delta p_{1-2} + \Delta p_f + \Delta p_c + \Delta p_{2-4} + \Delta p_{4-5} + \Delta p_{5-6} + \Delta p_{6-7} + \Delta p_{II} = SP \tag{8}$$

$$\Delta p_{1-2} + \Delta p_f + \Delta p_c + \Delta p_{2-4} + \Delta p_{4-5} + \Delta p_{5-8} + \Delta p_{8-9} + \Delta p_{9-10} + \Delta p_{10-11} + \Delta p_{III} = SP \tag{9}$$

where

$\Delta p_f$ = pressure drop in filter, in. of water

$\Delta p_c$ = pressure drop in coil, in. of water

$\Delta p_I$ = pressure drop in terminal at outlet I

SP = static pressure at the fan, in. of water

The optimization procedures consist of minimizing eq (6) subject to the constraints of eqs (7-9).

## 6. Lagrange Multiplier Solution

The objective function, eq (6), and the constraints, eqs (7-9), are soluble by the method of Lagrange multipliers. If an objective function y is to be minimized, where

$$y = y(x_1, \cdots x_n)$$

subject to the constraints

$$\phi_1(x_1, \cdots x_n) = 0$$

$$\cdots$$

$$\phi_m(x_1, \cdots x_n) = 0$$

the method of Lagrange multipliers [2] states that the optimum occurs where the following equations are satisfied:

$$\nabla y - \lambda_1 \nabla \phi_1 - \cdots - \lambda_m \nabla \phi_m = 0 \tag{10}$$

where

$$\nabla y = \frac{\partial y}{\partial x_1} \bar{i}_1 + \frac{\partial y}{\partial x_2} \bar{i}_2 + \cdots + \frac{\partial y}{\partial x_n} \bar{i}_n$$

and $\lambda_1 \cdots \lambda_m$ are constants called Lagrange multipliers.

Equation (10) is a vector equation and in order for it to equal zero the coefficients of each unit vector, such as $\bar{i}_1$ must equal zero. Equation (10), therefore, represents n scalar equations which, along with the m constraints, is a system of equations for the n + m unknowns, namely, $x_1, \cdots x_n$, $\lambda_1, \cdots \lambda_m$.

Solving the equations represented by the vector equation, eq (10), along with the constraint equations poses special requirements because the equations are often nonlinear. An iterative method for solving nonlinear simultaneous equations is the Newton-Raphson iteration [2]. In this process, trial values are assumed for each of the unknowns. These trial values are substituted into the n + m equations. If all of the equations are satisfied the problem is solved. Usually, however, the equations will not be satisfied with the first trials and the values of the variables need to be corrected. To determine how much each of the trial values should be changed, the Newton-Raphson technique prescribes a set of simultaneous linear equations. The solution of these linear equations gives the corrections which must be applied to the previous trial values to provide more correct values of the variables.

The fortunate circumstance which makes the method of Lagrange multipliers so applicable to the duct optimization problem is that the form of eqs (6-9) is dependable--the objective function is a sum of terms, all of which have the same form, and the constraints are linear in the variables which are to be optimized, $\Delta p_i$. After the optimal values of the $\Delta p$'s have been determined, the optimal diameters for each section can be computed.

## 7. Computer Program

Robert C. Winn [3] developed a computer program to optimize duct systems automating the foregoing mathematical procedure. The listing, flow diagram, and documentation of this program are found in [3]. To anticipate the eventual use of this program under coordinate geometry, the duct system is described in terms of the coordinates of all elbows, branches and outlets. Once this information is entered, the computer determines the angle of turn of an elbow or branch. The complete list of input information required is:

(a) static pressure developed by the fan,

(b) type of each joint and its geometrical coordinates,

(c) pressure drop through filters, coils and outlets,

(d) upstream and downstream joint numbers of each outlet,

(e) air flow rate at each outlet,

(f) temperature of the air at the fan, and

(g) current cost of ductwork per pound.

The output, as example of which is shown in figure 4, consists of the following information for each section: diameter, velocity, flow rate, length, duct surface area, sheet metal gauge, pressure drop, and cost. In addition, the total cost of the entire duct system is provided. The output corresponds to the duct system shown in figure 5.

## 8. Strengths and Limitations of the Program

While the application of a standard mathematical tool and the computer program in which it is utilized are intended to be a contribution to improved procedures in the design of environmental systems, the work is not to be considered complete. One of the obvious extensions would be to include the capability to optimize a system of rectangular duct in addition to circular duct which the program now can accommodate. It would be realistic to encounter a dimensional limitation, such as the depth of the duct, and in such a case the program would determine the optimum width within this depth limitation.

The method for calculating cost used in this program multiplies the weight of metal by a factor. This practice oversimplifies the influences which contribute to the cost since the weight of metal alone is not an accurate indicator of, for example, the labor cost. On the other hand, the most commonly used method of estimating the cost of a duct system for bidding purposes is first to evaluate the weight of metal and then multiply by a factor which varies somewhat depending upon the number and type of fittings and ease or difficulty of installations. As long as bids are determined on this basis, the weight of metal is the preferred quantity to minimize.

In its present status the program imposes no limits on velocity. In other words, a short run of ductwork with the outlet close to the fan will optimize at an extremely small diameter because all of the available pressure must be dissipated in the duct itself. Standard practice calls for enlarging the duct and installing a damper to accomplish the pressure drop. It should not be difficult to incorporate in the computer program the provision for checking the velocity against some prescribed maximum, enlarging the duct if necessary to abide by the velocity restriction, and including on the output a statement that dampering will be needed. Such a refinement of the program is planned. This practice, however, raises a more basic question. Since the limitation of velocity is particularly to reduce the noise level, which process would generate the most noise for a given drop in pressure, flow through a damper or flow in a length of small diameter duct? There is evidence that a detached boundary layer (as in flow past a damper) generates more noise than an attached boundary layer (as in flow through a small duct). A more thorough study of this question seems warranted.

The pressure drop through branch takeoffs and the straight-through portion of branch are computed using data from [1]. The graphical data have been converted to equation form for convenience in the computer program. In air-duct systems the pressure drops in fittings will be of the same order of magnitude as that experienced in the straight sections of duct so accuracy in the fitting calculation is important. The calculation of pressure drops in straight sections is probably reliable, but the prediction of pressure drops in fittings is highly dependent upon the quality of craftsmanship of the fitting. The use of fittings manufactured in factories under controlled conditions and carefully tested will add to the reliability of optimization efforts such as this.

The principal claim made for this method of duct design is that it results in the duct system of minimum first cost for a specified available static pressure. A further benefit is that it results in a balanced system. No one would be so foolhardy as to install a system designed with this computer program and omit dampers in each branch. The accuracy of the basic pressure drop data and the quality of workmanship is just too variable. It is likely, however, that the system will be more nearly balanced than systems designed by other methods, so that the cost of balancing should be reduced.

## 9. Acknowledgments

During his graduate studies the work of R. C. Winn on this project was supported by an Industrial Research Assistantship of Skidmore, Owings and Merrill.

## 10. References

[1] American Society of Heating, Refrigeration and Air Conditioning Engineers, ASHRAE Guide and Data Book, Systems and Equipment Volume, New York, 1967.

[2] Stoecker, W. F., "Design of Thermal Systems," McGraw-Hill Book Co., New York, (in press).

[3] Winn, R. C., "Development of a Computer Program for the Optimum Design of Air Supply Duct Systems," M.S. Thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois 1969.

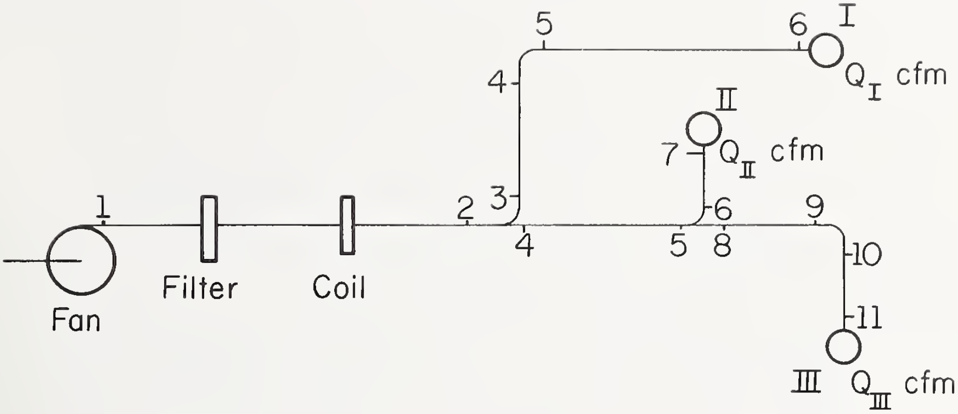

Figure 1. A multiple-branch duct system.

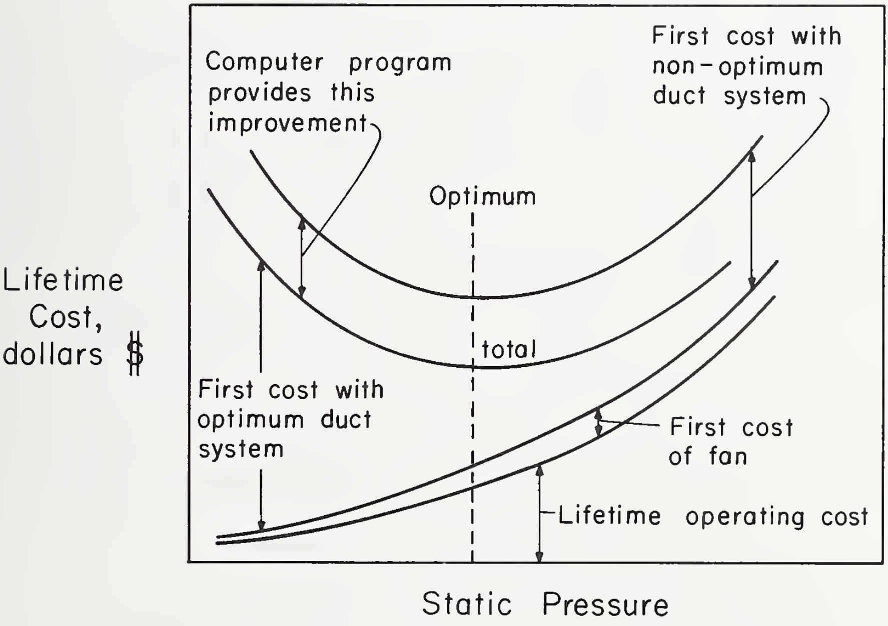

Figure 2. Effect of fan static pressure on lifetime cost of combined fan and duct system.

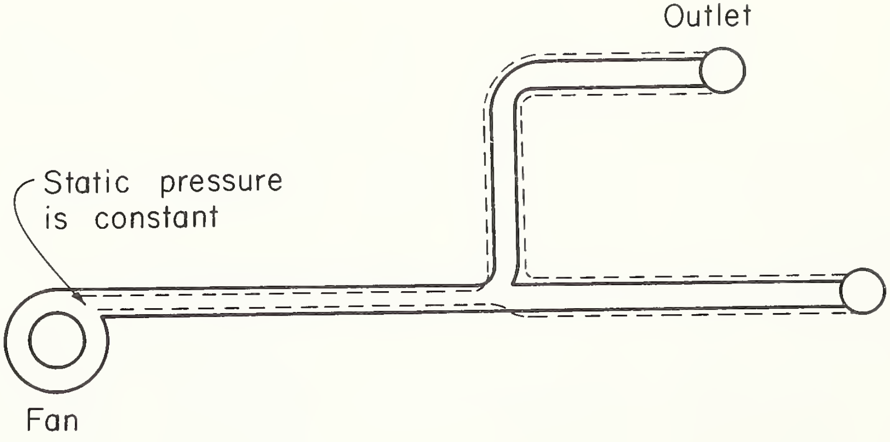

Figure 3. Two different duct systems, both of which use all available static pressure without artificial dampering.

```
SECTION   DIAMETER   VELOCITY   FLOW RATE   LENGTH   SURFACE AREA   GAUGE   PRESSURE DROP   COST
NO.       INCHES     FPM        CFM         FEET     SQ. FEET       NO.     IN. OF WATER    DOLLARS
 UP   DN
  1    2   18.62      3436.      6500.       30.00     146.27        24         0.231        273.92
  2    3   19.02      1521.      3000.       20.00      99.57        24         0.031        186.47
  3    4   11.17      1471.      1000.       25.00      73.08        26         0.071        107.26
  3    6   14.77      1682.      2000.       65.00     251.28        24         0.168        470.58
  2    7   15.99      2509.      3500.       20.00      83.74        24         0.101        156.83
  7    8   10.75      3175.      2000.       20.00      56.27        26         0.259         82.58
  7   10   11.41      2113.      1500.       35.00     104.53        26         0.192        153.43

THE TOTAL FAN STATIC PRESSURE IS 1.500 INCHES OF WATER.
THE TOTAL COST IS   1431.06 DOLLARS BASED ON A DUCTWORK COST OF 1.35 DOLLARS PER POUND.
THE CALCULATIONS WERE BASED ON AN AIR TEMPERATURE OF  75. DEGREES.
THE SYSTEM IS CONSTRUCTED OF SPIRAL LOCK SEAM, GALVANIZED STEEL DUCTS.
```

Figure 4. Computer output.

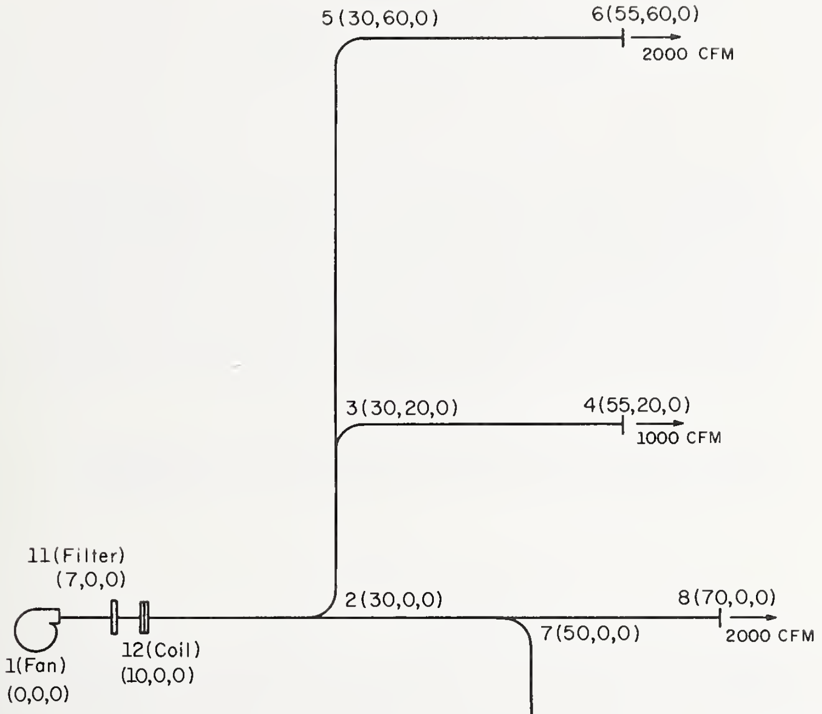

Figure 5. Duct layout example.

# Computerized Calculation
# of
# Duct Friction

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With the duct friction factor known the pressure drop due to friction in a pipe can be readily calculated by the Darcey-Wiesbach equation. Colebrooks' equation, which relates the friction factors, Reynolds number, diameter and material roughness factors, requires an iterative solution. This paper presents the Newton-Raphson method for solving Colebrooks' equation, and all of the pertinent subroutines to calculate the pressure drop in straight air ducts. Presented also is an air friction chart for an altitude of 5000 feet.

Key Words: Colebrook, computer, Darcey, density, elevation, friction chart, friction factor, viscosity.

## 1. Introduction

Duct system simulation and design on the digital computer is becoming more and more common in the Consulting Engineer's office. At Sargent and Lundy we have modeled exhaust ventilation systems in such a manner so as to obtain inherently balanced systems. In these systems the branch and main duct networks are designed so that both networks have the same total pressure at the junction of the tee. In addition to modeling systems we have generated friction charts for (1) air less dense than standard air, and (2) fluids other than air.

A basic requirement for any modeling computer program is the ability to readily calculate friction factors. As is well known the friction factor is a function of Reynolds number, the material roughness factor, and the pipe diameter. The relationship between these parameters is illustrated by the Moody diagram (1)[1] which is presented in figure 1. Colebrook (2) developed the mathematical relationship, eq (1) below, for these parameters. It is not the intent to discuss the development of this equation, but rather, its solution, since Vennard (3) and others have excellent discussions on its development.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{12}{3.7}\frac{E}{D} + \frac{2.51}{R_N\sqrt{f}}\right] \qquad (1)$$

where:

$f$ = friction factor, dimensionless

$E$ = material roughness factor, feet

$D$ = pipe diameter, inches

$R_N$ = Reynolds Number ($\rho D V \mu^{-1}$), dimensionless

---

[1] Figures in parenthesis indicate the literature references at the end of this paper.

$\mu$ = fluid viscosity, $lbs\text{-}ft^{-1}\text{-}hr^{-1}$

$\rho$ = fluid density, $lbs\text{-}ft^{-3}$

V = fluid velocity, $ft\text{-}min^{-1}$

It is recognized that many organizations have in their computer program library a method for solving Colebrook's equation and calculating the pressure drops of fluids in a pipe. Nevertheless, this paper is being presented for those who wish to initiate modeling of duct and piping systems to meet their needs. The only limitation to modeling various types of systems is the fittings in your program library. More effort needs to be expended in the correlation of fitting data. To assist in the correlation of data, Dr. Inoue has an excellent summary of fitting data in his "Duct Design Handbook"(4).

In addition to presenting the program listings for determining friction factors and calculating the pressure drop in a pipe the pertinent subroutines for calculating the properties of moist air are presented for your convenience. Also illustrated in section 3 is a friction chart for air at a density of 0.062 $lbs\text{-}ft^{-3}$, which is equivalent to an altitude of 5000 feet.

## 2. Pressure Drop Calculations

### 2.1 Friction Factors

Of the numerical techniques analyzed to solve Colebrook's equation it was found that the Newton-Raphson method requires the least computer execution time. This section presents an analysis of this method for solving the friction factor equation.

Rearranging eq (1) and setting the function of the friction factor, F(f), equal to zero yields eq (2). For fixed values of Reynolds number ($R_N$), material roughness factor (E), and pipe diameter (D), the value of F(f) can be calculated for incremental values of friction factors (f) and the results plotted as illustrated in figure 2. In this example Reynolds number and the dimensionless ratio E/D (relative roughness) are 600,000 and 0.001, respectively. The methodology is best illustrated by following the numerical solution on this specific graphing of the function. The iteration process is initiated by assuming 0.01 as a value for the friction factor f. Values less than 0.01 are unlikely since the Reynolds number would have to exceed 2.5 x $10^6$ (refer to fig 1). At the initial value of f (0.01) the value of the function, F(f), is 2.988 as calculated by eq (2). After calculating the derivative of function by eq (3), the value of the intercept to the f-axis is calculated by eq (4) to be 0.0159. Using this value of f (0.0159) in the second iteration process the value of F(f) is 0.892, and the value of the intercept to the f-axis of a line tangent to the curve at 0.892 is 0.0194. At this point the difference between the last two values of f is 0.0035. This difference is greater than the desired accuracy of 1/2 percent or difference of 0.001, therefore the iteration process continues and f is reset to 0.0194. Using this value of f in the next iteration the value of the next intercept to the f-axis is 0.0201. Since the difference between this value (0.0201) and the previously calculated value of f (0.0194) is 0.00067, which is less than the difference desired, the iteration process is terminated. The solution for the friction factor, therefore, is 0.0201. By referring to

$$F(f) = \frac{1}{\sqrt{f}} + 2\log_{10}\left[\frac{12\ E}{3.7(D)} + \frac{2.51}{R_N\sqrt{f}}\right] = 0 \qquad (2)$$

$$F'(f) = \frac{-1}{2\ f^{3/2}}\left[1 + \left(\frac{0.868590}{\frac{12\ E}{3.7(D)} + \frac{2.51}{R_N\sqrt{f}}}\right)\left(\frac{2.51}{R_N}\right)\right] \qquad (3)$$

$$f_{NEW} = f_{OLD} - \left[\frac{F(f)}{F'(f)}\right] \qquad (4)$$

figure 1 the correlation of Colebrook's equation to Moody's diagram is readily demonstrated. To calculate a friction factor three iterations are usually required, however in a few cases, four iterations will be necessary to yield a solution. A listing of the computer program for calculating friction factors is presented in figure 3. Figure 4 is a flowchart for this program.

## 2.2 Pressure Drop in Straight Ducts

The pressure drop in a pipe may be calculated by the Darcey-Wiesbach eq (5). This relationship, used with the expression for velocity pressure (eq 6), may be used directly in your mainline programs, or as a subroutine. A Fortran listing of the pressure drop subroutine, named DUCT, is presented along with the friction factor subroutine (FRICT) in figure 3. These programs, in addition to your modeling programs, may be readily used to develop friction charts for any fluid, fluid density and viscosity.

$$\Delta P = f \left[\frac{L}{D}\right] H_V \tag{5}$$

$$H_V = \rho \left[\frac{V}{1097}\right]^2 \tag{6}$$

where:

$\Delta P$ = pressure drop, inches water gage pressure (iwgp)

$f$ = friction factor, dimensionless

$L$ = pipe length, inches

$D$ = pipe diameter, inches

$H_V$ = velocity pressure, iwgp

$\rho$ = fluid density, $\text{lbs-ft}^{-3}$

$V$ = fluid velocity, $\text{ft-min}^{-1}$

## 2.3 Properties of Moist Air

To use the friction factor and pressure drop programs, the density and viscosity of the fluid must be known. For your convenience in modeling air duct systems, or generating other air friction charts (see section 3), the pertinent moist air subroutines are presented in the following sections.

### a. Barometric Pressure

Altitude and barometric data by NACA (5) was correlated linearly by the method of least squares between sea level and 5000 feet elevation. The resulting relationship, eq (7), has a coefficient of determination of 0.999. Refer to figure 5 for the function subroutine of this equation.

$$\overline{BP} = 29.841 - (0.9935)(10^{-3})(\overline{ELEV}) \tag{7}$$

where:

$\overline{BP}$ = barometric pressure, inches mercury

$\overline{ELEV}$ = altitude, feet

### b. Moist Air Density

Function subroutines for the properties of moist air are presented in figure 5 and are based on the "Algorithms For Psychrometric Calculations" by the National Bureau of Standards (6). As may be noted the subroutine for calculating the air density (RHO) in turn calls the absolute humidity (W), the partial pressure of water vapor (PV), and the partial pressure of water vapor in moisture saturated air (PVS) subroutines.

### c. Viscosity of Moist Air

Temperature-viscosity data for dry air at 14.7 psi from Jakob (7) is approximately linear, and therefore the data was fitted by the least squares method to the regression line between -60°F and +300°F. The resulting eq (8) has a coefficient of determination of 0.991.

$$\mu = 0.03948 + (6.213)(10^{-5})T \qquad (8)$$

where:

$\mu$ = dynamic viscosity, lbs mass-$ft^{-1}$-$hr^{-1}$

$T$ = dry-bulb temperature, °F

Since the viscosity of moist air varies little from that of dry air at normal atmospheric pressure (8), the above equation for viscosity is used to calculate the Reynolds number of air flowing through the duct. The function subroutine for viscosity is presented in figure 5.

## 3. 5000-Foot Friction Chart

An air friction chart for an altitude of 5000 feet using the subroutines in this paper is presented in figure 6. Comparing these values to the standard air friction chart in the ASHRAE Guide (9) the pressure drop for the less dense air at the higher elevation ranges from 20 to 32 percent less than that at sea level ($\rho$=0.075 lbs-$ft^{-3}$). For example, for a flow of 1000 cubic feet of air per minute the loss in 100 feet of 6-inch diameter pipe at sea level is 6.9 inches of water (refer to fig 6), while at 5000 feet ($\rho$=0.062) the loss is 5.5 inches of water. This decrease in air density results in a 20.3 percent (based on standard air) reduction in resistance to air flow. As stated previously, other charts can be readily generated by developing a mainline program utilizing the friction factor (FRICT) and pressure drop (DUCT) subroutines.

## 4. References

(1) L. F. Moody, "Friction Factors for Pipe Flow", ASME Transactions, Vol. 66, 1944, p. 671.

(2) C. F. Colebrook, "Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws", Jour. Inst. Civil Engrs, London, Feb. 1939, p. 133.

(3) J. K. Vennard, "Elementary Fluid Mechanics", 3rd Edition, Wiley, New York, 1959, p. 193.

(4) U. Inoue, "Duct Design Handbook", Waseda University, Japan, pp. 116-137.

(5) "Tables and Data for Altitudes to 65,800 Feet", NACA Report 1235, Washington, D.C., 1955, pp. 66-81.

(6) T. Kusuda, "Algorithms for Psychrometric Calculations", National Bureau of Standards, U.S. Department of Commerce, Washington, D.C., Report No. 9818, March, 1969.

(7) M. Jakob and G. A. Hawkins, "Elements of Heat Transfer", 3rd Edition, Wiley, New York, 1959, p. 10.

(8) "Handbook of Fundamentals", American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE), New York, 1967, figure 12, p. 109.

(9) "Guide and Data Book, Equipment", ASHRAE, New York, 1969, figs 2 & 3, pp. 26-27.

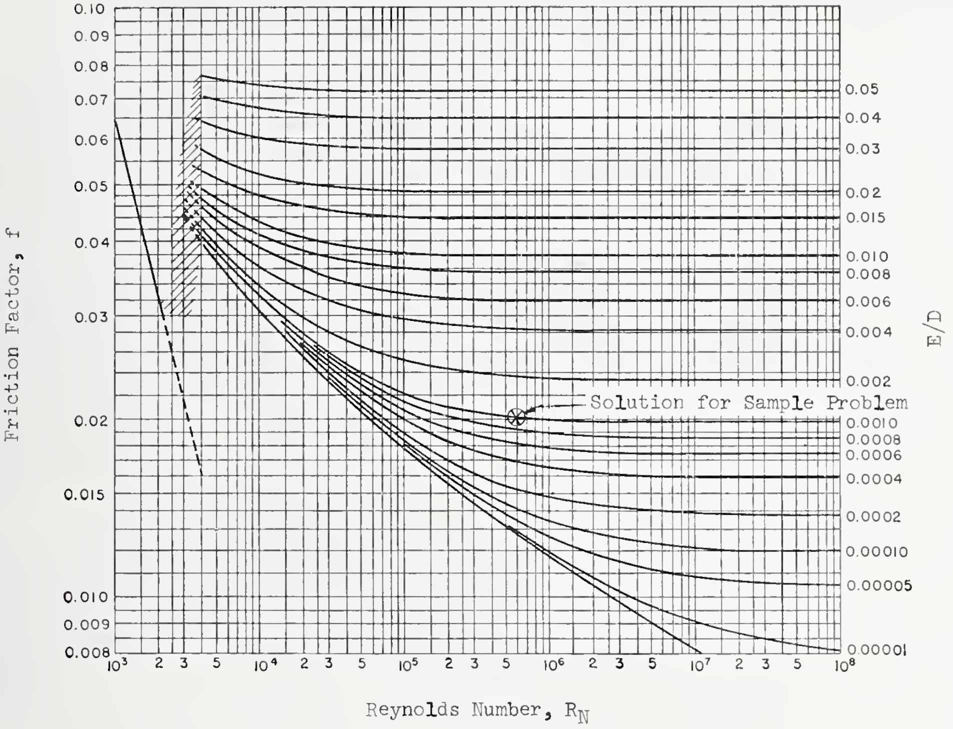


Figure 1. Moody Diagram (from ref 8, p. 87, fig 12)

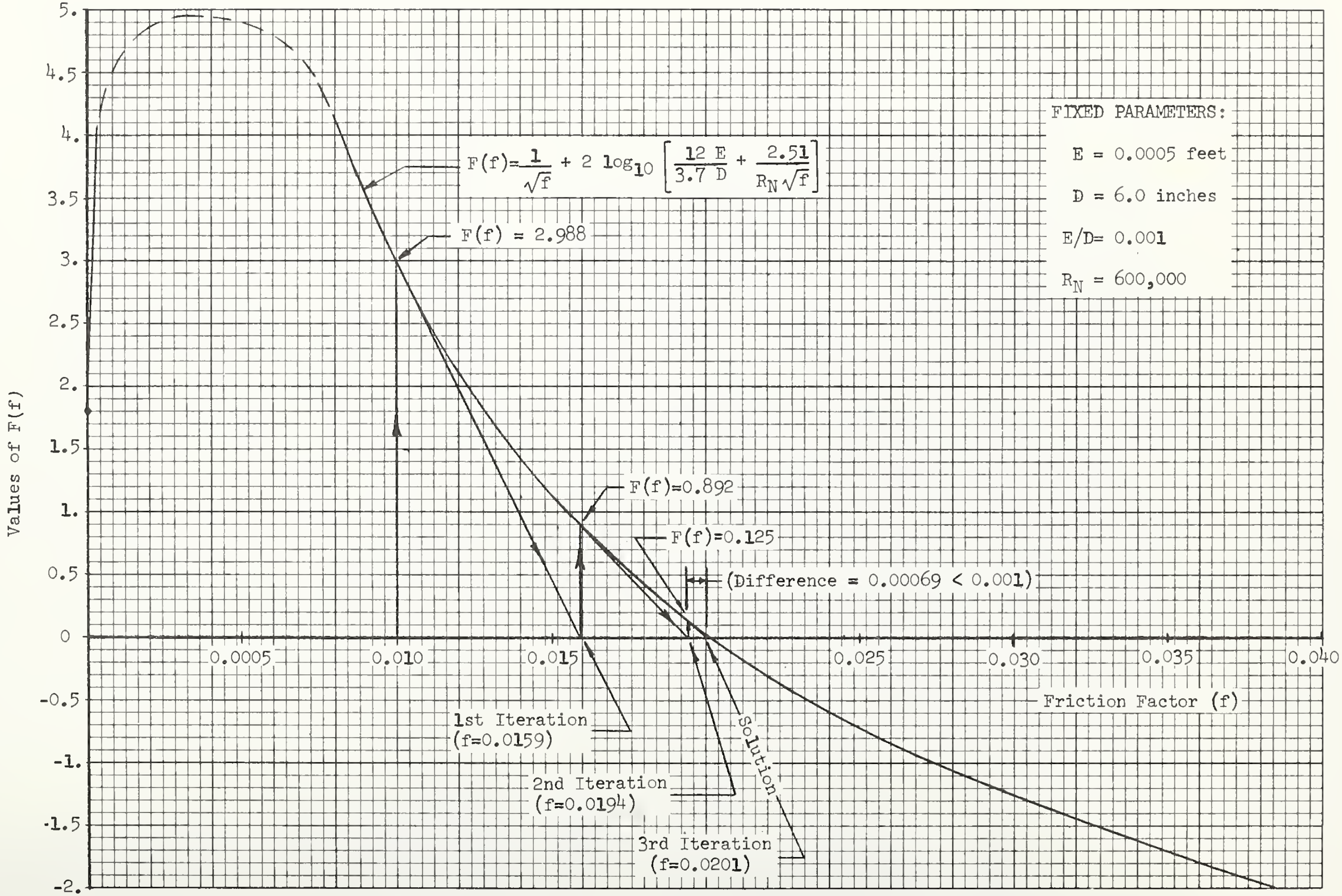


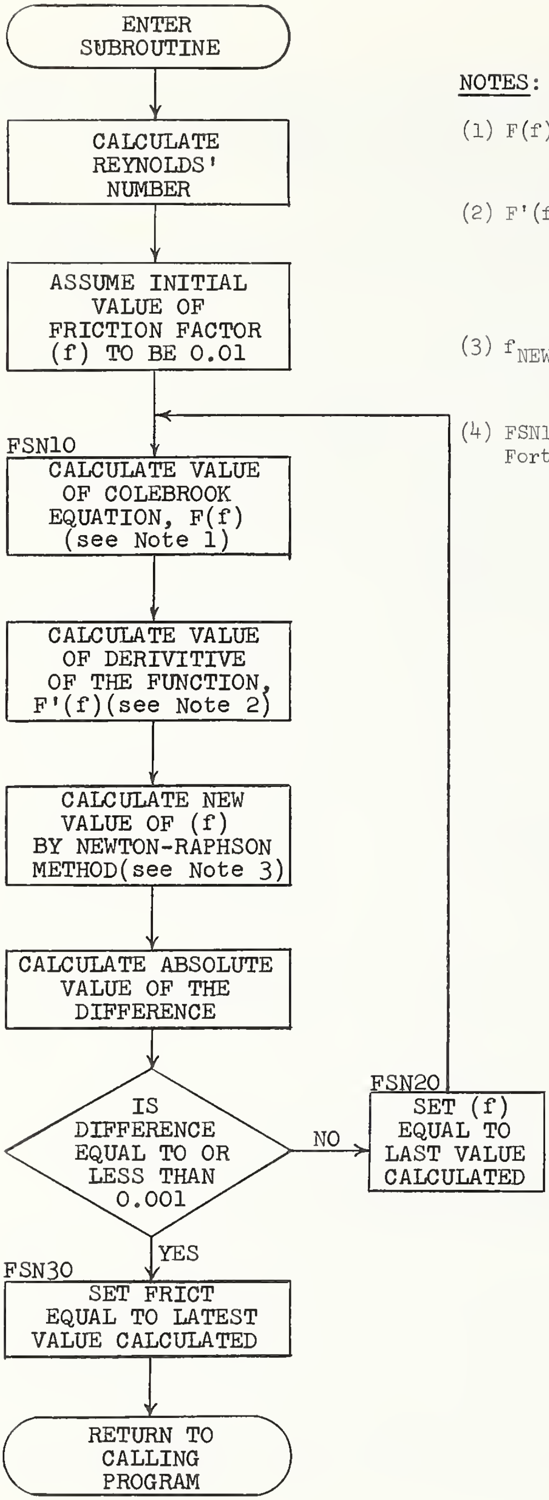
Figure 2. Example Problem Illustrating the Solution of Colebrook's Equation

```
C NOMENCLATURE:
C     E IS THE PIPE MATERIAL ROUGHNESS FACTOR, FEET (SEE REF. 8, P. 88, TABLE 4)
C     DIA IS THE DIAMETER OF THE PIPE, INCHES
C     CFM IS THE VOLUMETRIC RATE OF FLUID FLOW, CUBIC FEET PER MINUTE
C     RHO IS THE DENSITY OF THE FLUID, LBS PER CUBIC FEET
C     AMU IS THE VISCOSITY OF THE FLUID, LBS PER FT-HR
C     ALGT IS THE LENGTH OF PIPE, FEET
C     FRICT IS THE FRICTION FACTOR, DIMENSIONLESS
C     DUCT IS THE PRESSURE FOR THE LENGTH OF PIPE (ALGT) INPUTED, INCHES OF WATER


C FRICTION FACTOR (FRICT) FUNCTION SUBROUTINE
      FUNCTION FRICT(E,DIA,CFM,RHO,AMU)
      RENNO=(916.733*RHO*CFM)/(AMU*DIA)
      F=0.01
   10 C1=12.*E/(3.7*DIA)
      C2=SQRT(F)
      C3=2.51/(RENNO*C2)
      C4=C1+C3
      FF=1.0/C2+0.868589*ALOG(C4)
      FDF=(-0.5/(C2*F))*(1.+(2.1801/(C4*RENNO)))
      FNEW=F-(FF/FDF)
      DIF=ABS(FNEW-F)
      IF(DIF-0.001)30,30,20
   20 F=FNEW
      GO TO 10
   30 FRICT=FNEW
      RETURN
      END


C PRESSURE DROP (DUCT) FUNCTION SUBROUTINE
      FUNCTION DUCT(DIA,ALGT,CFM,RHO,FRICT)
      VP=RHO*(CFM*0.1671/DIA**2)**2
      DUCT=FRICT*ALGT*VP*12./DIA
      RETURN
      END
```

Figure 3. Subroutine Listings for Calculating Friction Factors and Pressure Drop



NOTES:

(1) $F(f) = \frac{1}{\sqrt{f}} + 2 \log_{10} \left[\frac{12*E}{D*3.7} + \frac{2.51}{R_N \sqrt{f}}\right]$

(2) $F'(f) = - \frac{1}{2f^{1/2}f} \left[1 + \left(\frac{0.868509}{\frac{12*E}{3.7*D} + \frac{2.51}{R_N \sqrt{f}}}\right)\left(\frac{2.51}{R_N}\right)\right]$

(3) $f_{NEW} = f - \left[\frac{F(f)}{F'(f)}\right]$

(4) FSN10, FSN20, FSN30 are Fortran Statement Numbers, see listing, figure 3

Figure 4. Flowchart of the Friction Factor Program

```
C NOMENCLATURE:
C     DBT IS THE DRY-BULB TEMPERATURE, DEGREES F
C     WBT IS THE WET-BULB TEMPERATURE, DEGREES F
C     ELEV IS THE ALTITUDE ABOVE SEA LEVEL, FEET

C DENSITY OF MOIST AIR (RHO), POUNDS PER CUBIC FOOT OF DRY AIR
      FUNCTION RHO(DBT,WBT,ELEV)
      VOL=(0.754*(DBT+459.688)*(1.+7000.*(W(DBT,WBT,ELEV)/4360.)))/BP
     1(ELEV)
      RHO=1./VOL
      RETURN
      END

C BAROMETRIC PRESSURE AS A FUNCTION OF ALTITUDE (BP), INCHES MERCURY
      FUNCTION BP(ELEV)
      BP=(0.298411E02)-(0.993523E-03)*ELEV
      RETURN
      END

C HUMIDITY RATIO OF MOIST AIR (W), LBS OF WATER VAPOR PER POUND OF DRY AIR
      FUNCTION W(DBT,WBT,ELEV)
      W=0.622*(PV(DBT,WBT,ELEV)/(BP(ELEV)-PV(DBT,WBT,ELEV)))
      IF(W)10,20,20
   10 W=0.
   20 RETURN
      END

C PARTIAL PRESSURE OF WATER VAPOR IN MOIST AIR (PV), INCHES MERCURY
      FUNCTION PV(DBT,WBT,ELEV)
      PV=PVS(WBT)-(0.000367*BP(ELEV)*(DBT-WBT)*(1.+(WBT-32.)/1571.))
      IF(PV)10,20,20
   10 PV=0.
   20 RETURN
      END

C PARTIAL PRESSURE OF WATER VAPOR IN MOISTURE SATURATED AIR (PVS), INCHES MERCURY
      FUNCTION PVS(TEMP)
      DIMENSION A(6),B(4)
      DATA A/-7.90298,5.02808,-1.3816E-7,11.344,8.1328E-3,-3.49149/
      DATA B/-9.09718,-3.56654,0.876793,0.0060273/
      T=(TEMP+459.688)/1.8
      IF(T-273.16)10,20,20
   10 Z=273.16/T
      P1=B(1)*(Z-1.)
      P2=B(2)*0.43429*ALOG(Z)
      P3=B(3)*(1.-1./Z)
      P4=0.43429*ALOG(B(4))
      GO TO 30
   20 Z=373.16/T
      P1=A(1)*(Z-1.)
      P2=A(2)*0.43429*ALOG(Z)
      P3=A(3)*(10**(A(4)*(1.-1./Z))-1.)
      P4=A(5)*(10**(A(6)*(Z-1.))-1.)
   30 PVS=29.921*10**(P1+P2+P3+P4)
      RETURN
      END

C VISCOSITY OF MOIST AIR (AMU), LBS MASS PER FT-HR
      FUNCTION AMU (DBT)
      AMU=0.03948+0.6213E-4*DBT
      RETURN
      END
```

Figure 5. Subroutine Listings for Determining the Properties of Moist Air

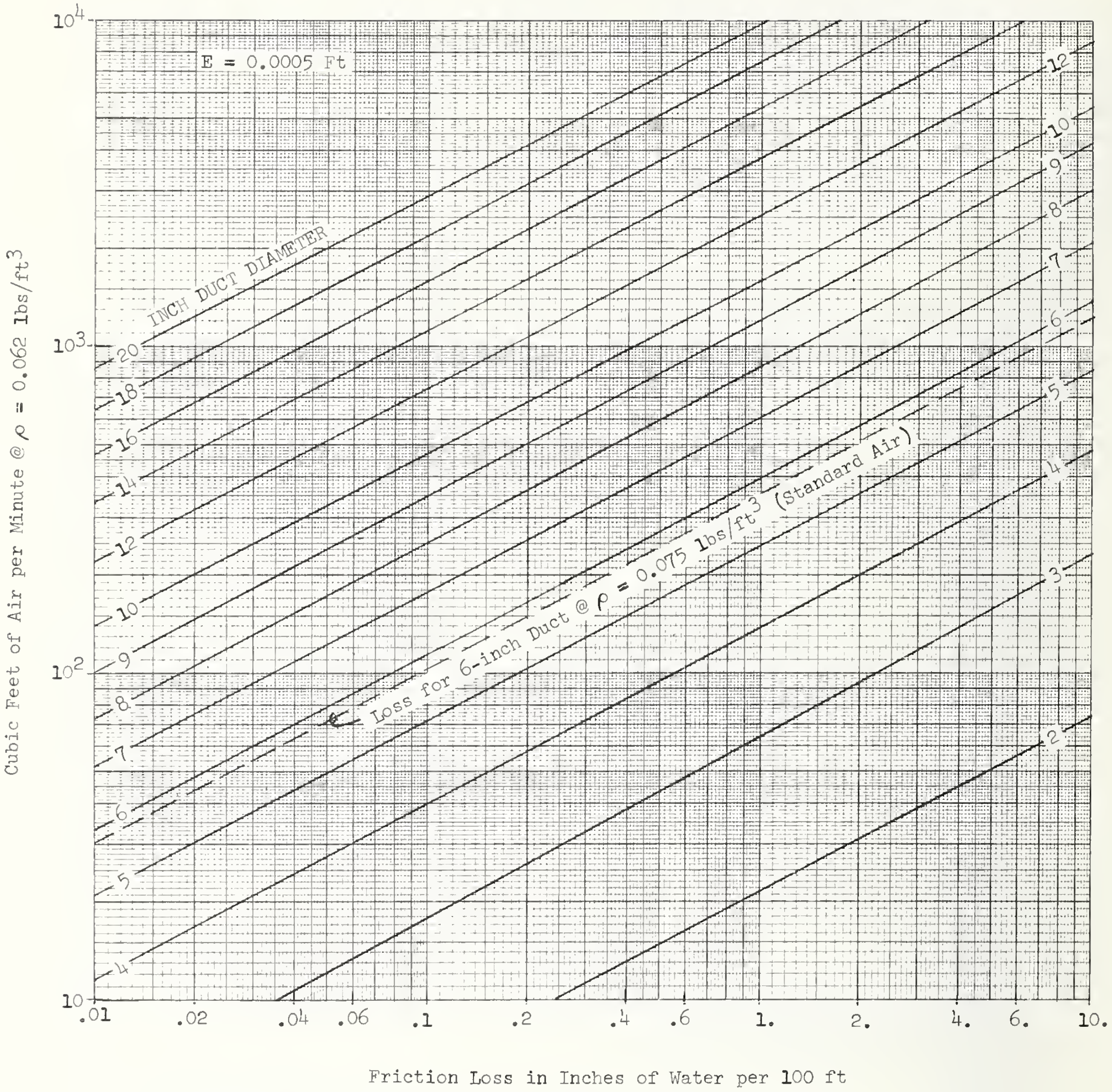


Figure 6. Friction of Air at an Altitude of 5000 Feet

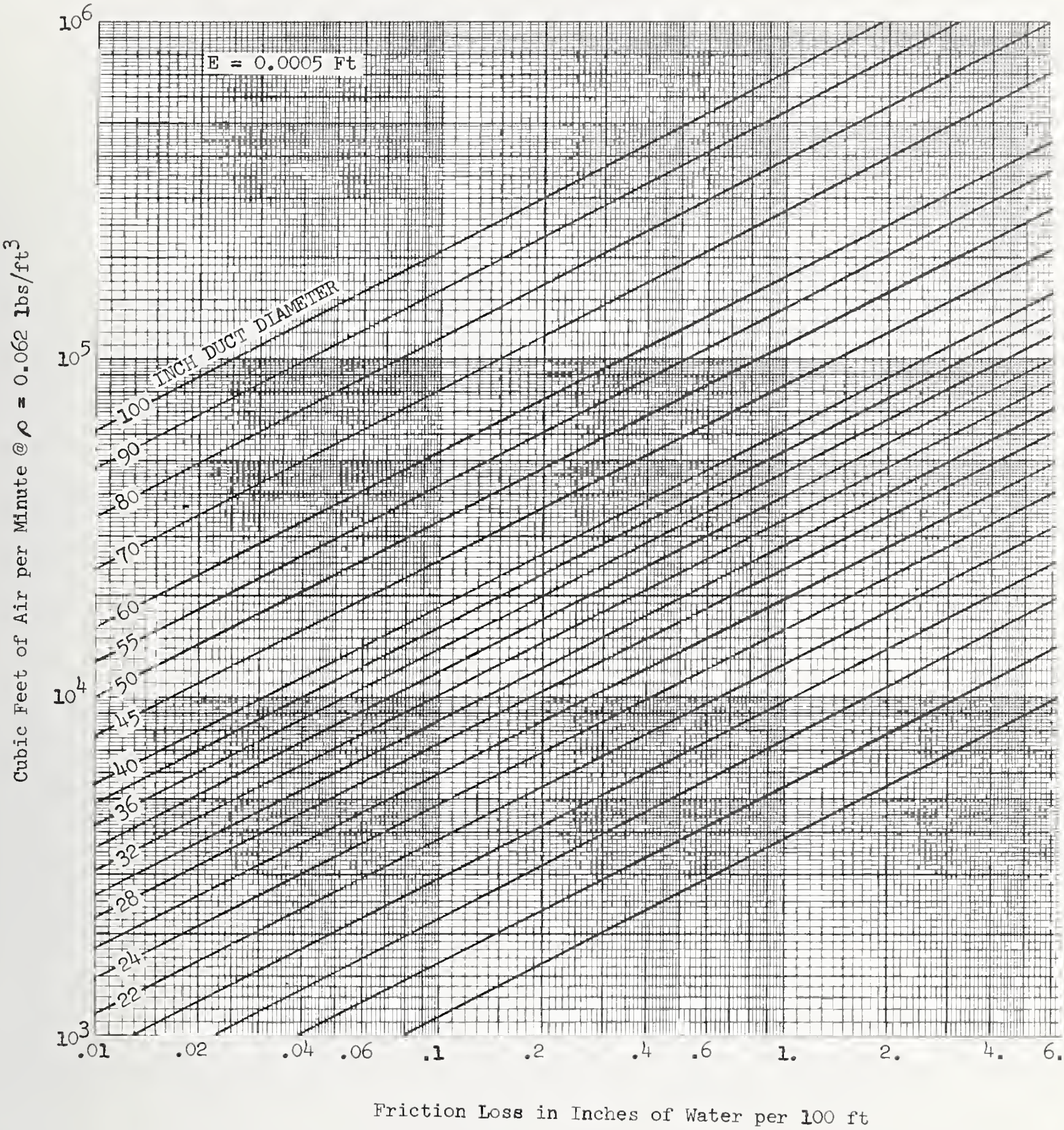


Figure 6. (cont'd) Friction of Air at an Altitude of 5000 Feet

# Pressure Loss Coefficients
# for the
# 45-Degree Return Air Tee

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For good design of duct system, the losses of the comcomponents in the system network must be known. For the tee there exists an infinite number of combinations of through-flow and branch loss coefficients. Data for both the through and branch coefficients have been correlated for the 45-degree return air tee and the resulting family of curves presented. For those who wish to develop their own computer programs, the subprogram listings are provided. In addition, the significance of the negative loss coefficients is discussed.

Key Words: Computer, design, exhaust, loss coefficients, pressure drop, return air, tee.

## 1. Introduction

The tee is the focal point in any duct system analysis because the resistance of the main duct network, including the losses in the through portion of the tee, should match at design air quantities the resistance of the branch network; otherwise the system will not be in balance. Heretofore little emphasis was placed on the losses which occur concurrently through both the main and branch sections of tees. For both good design practice and modeling of duct systems on the computer the losses occurring at the same time must be known. This paper presents the correlation of experimental data for the 45-degree return air tee which has a circular cross-section, equal inlet and outlet areas in the main, and a branch area equal to or less than the main. For those who wish to develop their own duct system models for computer design, the program listings for the tee-through and tee-branch loss coefficients are included herein.

## 2. Tee Loss Coefficients

Forty-five degree return air through-flow and branch to main loss coefficients have been determined experimentally by Petermann(1)[1] and the coefficients summarized by Dr. Inoue in his handbook for duct design(2) as presented in table 1.

Although this discussion pertains to the data correlation of the 45-degree tee with a round cross-section, other data are available, primarily the 90-degree return air tee (3, 4), and the supply air tees listed in table 2. These data should also be correlated so that more diversified duct systems may be modeled.

### 2.1 Tee-Through Flow Loss Coefficients

Petermann's tee-through loss coefficients are best represented (least standard deviation from the original data) by a family of parabolas as shown in figure 1. As

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[1] Figures in parenthesis indicate the literature references at the end of this paper.

Table 1. 45-Degree Return Air Tee Loss Coefficients

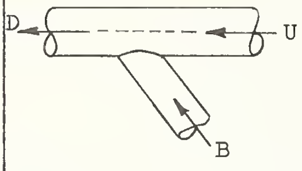
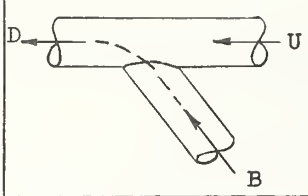
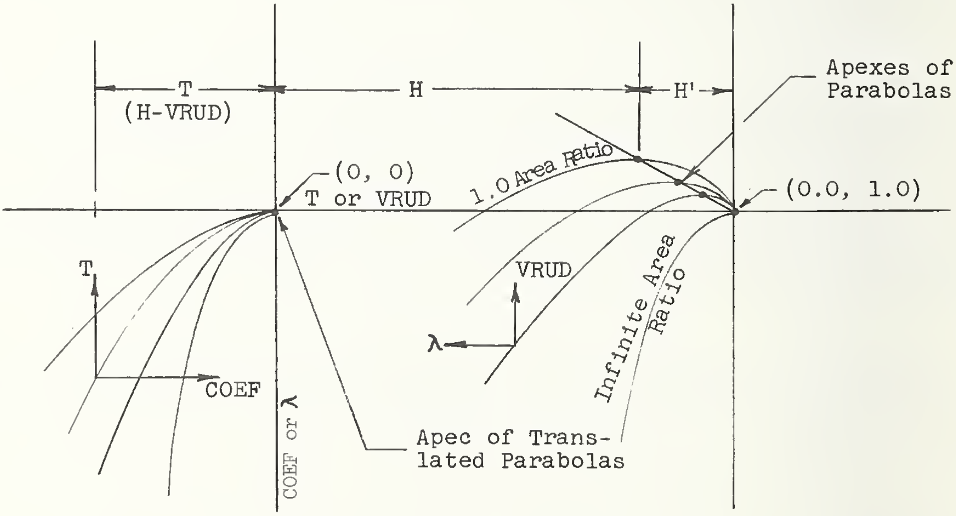
| Tee Configuration | Flow Path | Loss Coefficient (λ) | | | | | |
|---|---|---|---|---|---|---|---|
| | | Area Ratio (D/B) | Velocity Ratio (U/D) | | | | |
| | | | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|  | Main | 1.0 | -0.17 | 0.06 | 0.19 | 0.17 | 0.04 |
| | | 3.0 | -1.50 | -0.70 | -0.20 | 0.10 | 0 |
| | | 8.2 | -5.70 | -2.90 | -1.10 | -0.10 | 0 |
| | | Area Ratio (D/B) | Velocity Ratio (B/D) | | | | |
| | | | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 |
|  | Branch | 1.0 | 0 | 0.22 | 0.37 | 0.37 | 0.20 |
| | | 3.0 | -0.36 | -0.10 | 0.15 | 0.40 | 0.75 |
| | | 8.2 | -0.56 | -0.32 | -0.05 | 0.24 | 0.55 |

Table 2. Sources of Tee Data

| Angle | Type of Take-Off | Author | Reference |
|---|---|---|---|
| 45° | Straight | Petermann | (1) |
| 90° | Straight | Ashley | (5) |
| 90° | Conical | Ashley | (5) |

illustrated below, these parabolas can be correlated to the zero coordinates since the location of their apexes is related to area ratio. This correlation is a straight line going through the point (1.0, 0.0) with the area ratio approaching infinity as the velocity ratio approaches 1.0. The resulting equations for the displacement of the apexes from the (1.0, 0.0) coordinates are as follows:



$$H'(HPRIM) = 0.3015\ ARDB^{-0.6566} \quad (1)$$

$$YK = 0.5978 - 0.5926\ H \quad (2)$$

where:

$H = 1.0 - H'$

ARDB = area ratio from the tee main to the tee branch, dimensionless

HPRIM = acronym used in computer program

The family of parabolas at the (0.0, 0.0) axis are represented by eq (3). It should be noted that the parabolas approach the negative Y-axis as the area ratio approaches infinity.

$$COEF = \left[-1.461\ ARDB^{0.9306}\right] T^2 \quad (3)$$

where:

$T = H - VRUD$

VRUD = velocity ratio from upstream of the tee main to downstream of the tee main, dimensionless

With the equation of the parabolas and their displacement known, the tee-through loss coefficients can be readily calculated by eq (4). Using these relationships, the data may be represented as shown in figure 2, or calculated utilizing the computer listing presented in figure 3. For comparison purposes the parabolas for area ratios of 1.0, 3.0 and 8.2 are superimposed on figure 1 to show the correlation to Petermann's original data. The coefficient of correlation of the curves to the original data is 0.987.

$$\lambda_M(TEETH) = COEF + YK \quad (4)$$

where:

$\lambda_M$ = main loss coefficient, dimensionless

TEETH = acronym used in computer program

## 2.2 Tee-Branch Flow Loss Coefficients

Along with data by Brown (6), Pettermann's tee-branch coefficients are shown in figure 4. Since both sources of data correlate exceptionally well, confidence in the data is high and Petermann's data was fitted to the general form of $[\lambda_B + 1 = VR^2]$ as suggested by Healy (7). The resulting correlation, with a coefficient or correlation of 0.962, is given by eq (6), and using this relationship the family of curves in figure 5 were generated. It should be noted that as the area ratio increases the loss coefficient approaches the ideal loss coefficient given by $[\lambda_B + 1 = VR^2]$.

$$\lambda_B + 1 = \left[\frac{1.0276}{\sqrt{ARDB}}\right] - \left[\frac{0.3405\ (VRBD)}{\sqrt{ARDB}}\right] + \left[VRBD^{2.0}\right] \quad (5)$$

where:

$\lambda_B$ = branch loss coefficient, dimensionless

ARDB = area ratio from the tee main to the tee branch, dimensionless

VRBD = velocity ratio from the branch to downstream of the tee main, dimensionless

Figure 5 can be readily utilized by those who have a need for expedient data. For those who wish to simulate and design their systems by use of the computer the program listing for determining the branch loss coefficients is presented in figure 6.

## 2.3 Total Pressure Drop

With the loss coefficient known the total pressure loss in the through or branch portions of the tee may then be calculated by the following expression.

$$\Delta P = \lambda \left[ V_D / 1097 \right]^2 \rho \qquad (6)$$

where:

$\Delta P$ = main or branch total pressure drop, inches of water gage

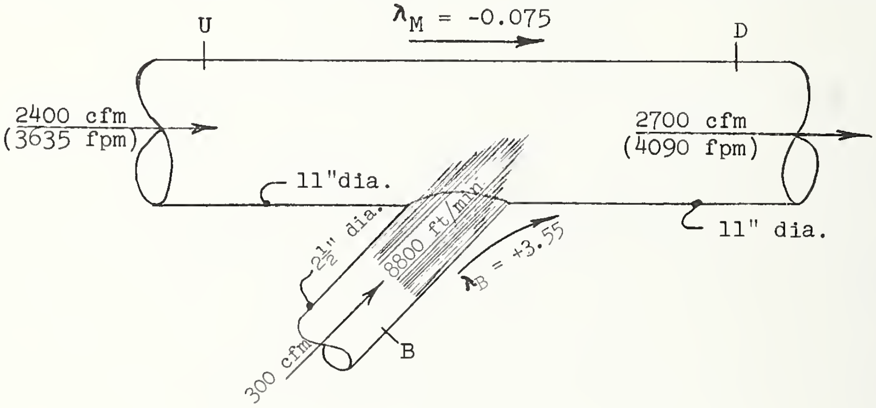
$V_D$ = downstream velocity, feet-min$^{-1}$

$\rho$ = moist air density, lbs-ft$^{-3}$

$\lambda$ = main or branch loss coefficient, dimensionless

## 2.4 Negative Loss Coefficients

By investigating figures 2 and 5 it may be noted that one or the other coefficient may be negative. This can be best illustrated by the following example and sketch. For an area ratio of 19.3 from the main to the branch, and a velocity ratio of 0.89



from upstream of the main (U) to downstream (D) the tee-through loss coefficient is -0.075; while the tee-branch coefficient is +3.55 for a velocity ratio from (B) to (D) of 2.15. In effect, the branch jet at 8800 ft/min coming into the mainstream, which is moving at 3635 ft/min, has an asperating affect or acts like an ejector. This aspiration phenomenon, also noted by Healy (7), becomes predominant as smaller branch ducts are connected to relatively large mains.

This illustration actually occurred in one of our industrial power plants where the noise generated due to the high velocities is not a problem. These relatively high velocities are a result of our system design philosophy, which decreases the size of the branch until the total pressure for the main and branch duct systems are approximately the same. For systems in which noise would be a problem the branch diameter would not be decreased, but rather, resistance added in the branch by other means such as obstructions. In most applications high velocities would not be encountered, and both the main and branch coefficients most likely would be positive.

## 3. References

(1) F. Petermann, "Der Verlust in Schiefwinkligen Rohrverzweigungen", MHITHM (Mitteilungen des Hydraulischen Instituts der Technischen Hochschule München), Heft 3, 1929, pp. 98-117.

(2) U. Inoue, "Duct Design Handbook", Waseda University, Japan.

(3) G. Vogel, "Untersuchungen über den Verlust in rechtwinkligen Rohrverzweigungen", MHITHM, Heft 1, 1926, pp. 75-90.

(4) G. Vogel, "Untersuchungen über den Verlust in rechtwinkligen Rohrverzweigungen", MHITHM, Heft 2, 1928, pp. 61-64.

(5) C. M. Ashley, et al, "Branch Fitting Performance at High Velocity", ASHAE Transactions, Vol. 62, 1956, pp. 279-294.

(6) W. K. Brown, Jr., et al, "Friction Loss Characteristics of Branch Duct Fittings with a Fixed Duct Configuration", ASHRAE Transactions, Vol. 72, Part I, 1966, p. 346.

(7) J. H. Healy, et al, "Pressure Losses Through Fittings Used in Return Air Systems", ASHRAE Transactions, Vol. 68, 1962, pp. 281-295.

## 4. Acknowledgement

We are grateful to Mr. Y. S. Chen of the University of Kansas for indicating the availability of Petermann's data as summarized by Dr. Inoue and the discussions with respect to the significance of the negative loss coefficients.

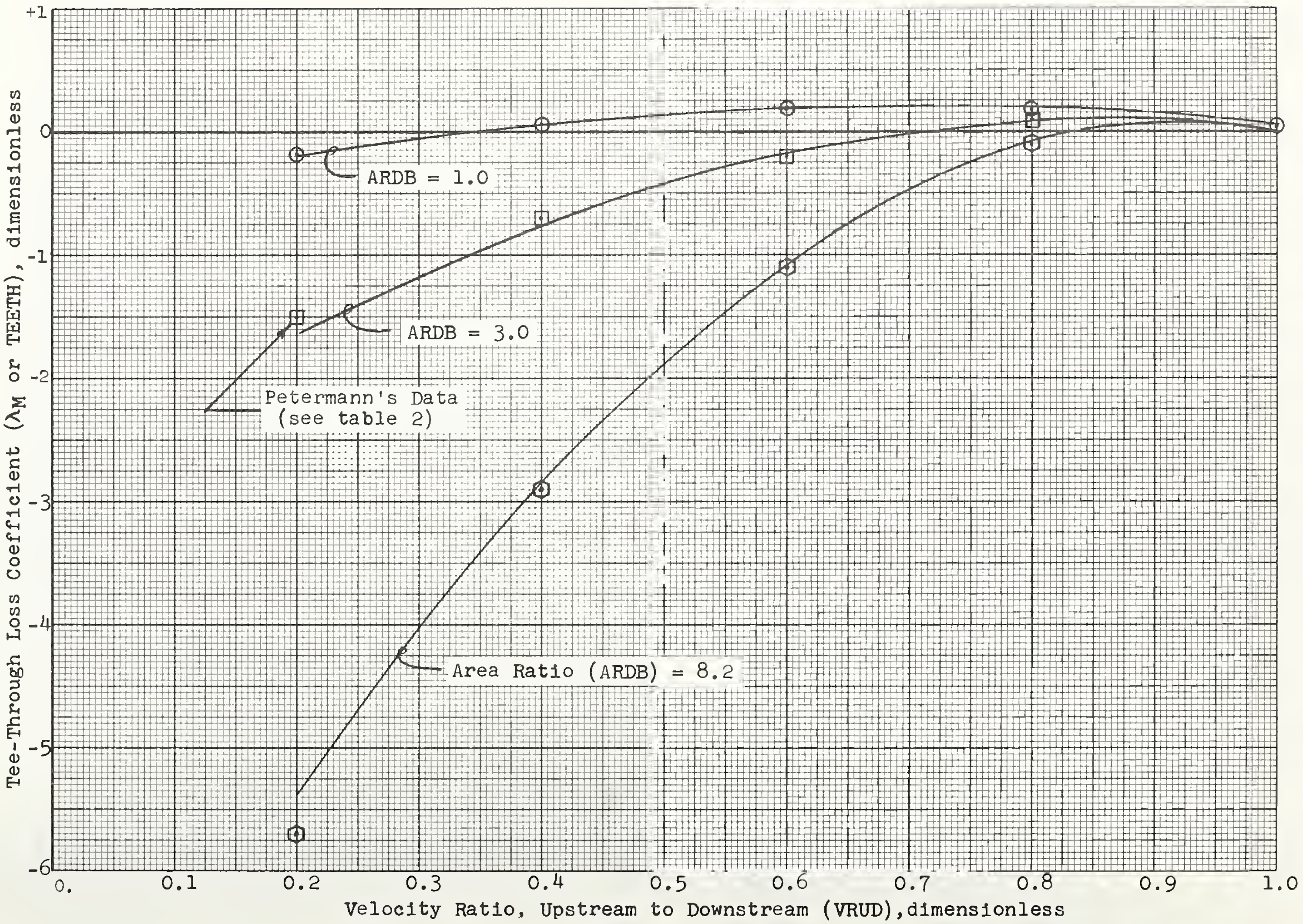


Figure 1. Analysis of Petermann's Tee-Through Pressure Loss Coefficients

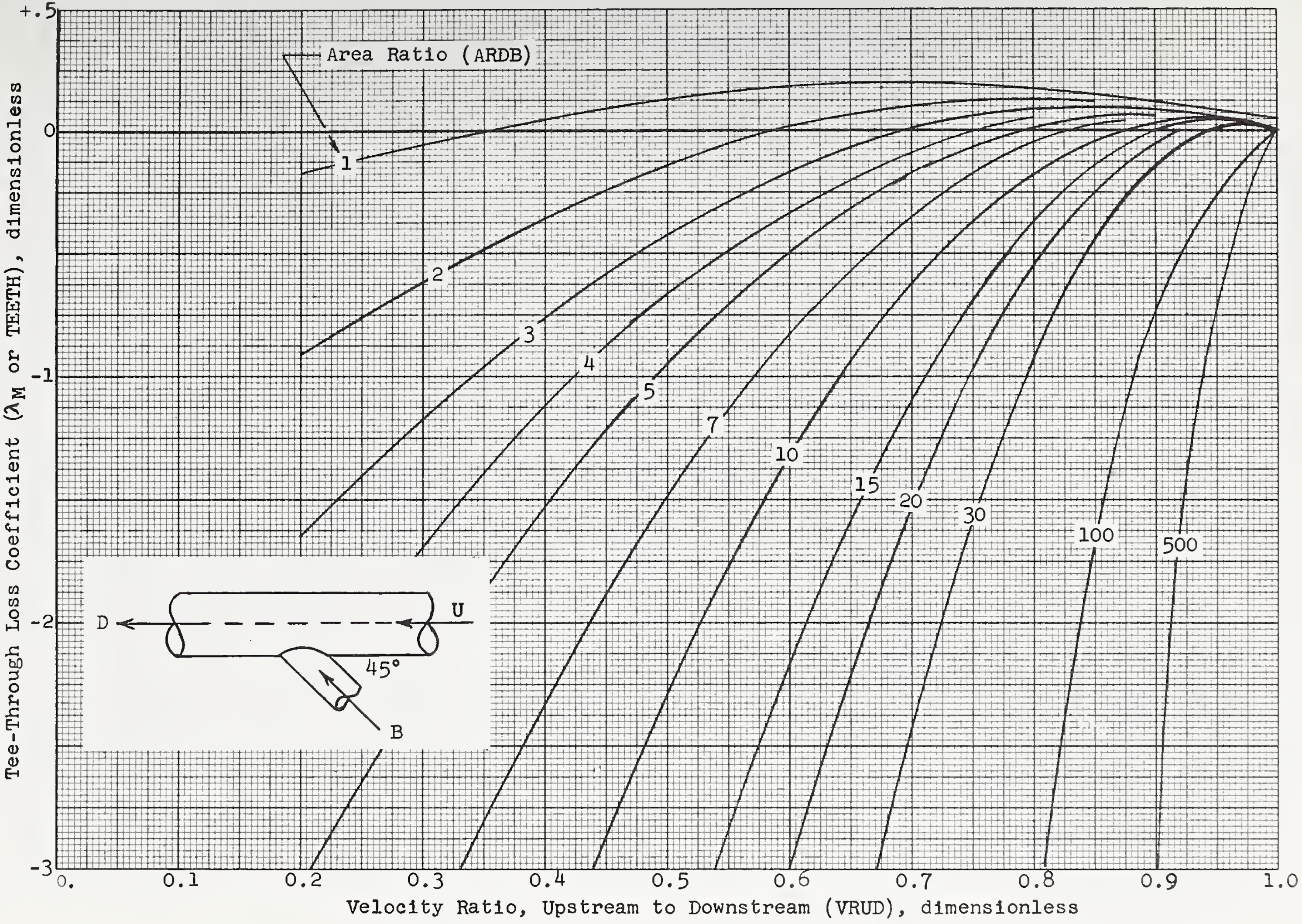


Figure 2. Return Air Tee-Through Flow Loss Coefficients

```
C TOTAL PRESSURE LOSS COEFFICIENTS IN THE THROUGH SECTION OF A RETURN AIR
C 45-DEGREE TEE (TEETH), DIMENSIONLESS
C
C ARDB IS THE AREA RATIO OF THE TEE MAIN TO THE TEE BRANCH, DIMENSIONLESS
C
C VRUD IS THE VELOCITY RATIO FROM UPSTREAM OF THE TEE MAIN TO DOWNSTREAM OF THE
C TEE MAIN, DIMENSIONLESS
      FUNCTION TEETH(ARDB,VRUD)
      HPRIM=0.3015/ARDB**0.6566
      H=1.0-HPRIM
      T=H-VRUD
      YK=0.5978-0.5926*H
      C=-1.461*ARDB**0.9306
      COEF=C*T**2
      TEETH=COEF+YK
      RETURN
      END
```

Figure 3. Fortran Function Subprogram Listing for Determining Tee-Through Loss Coefficients

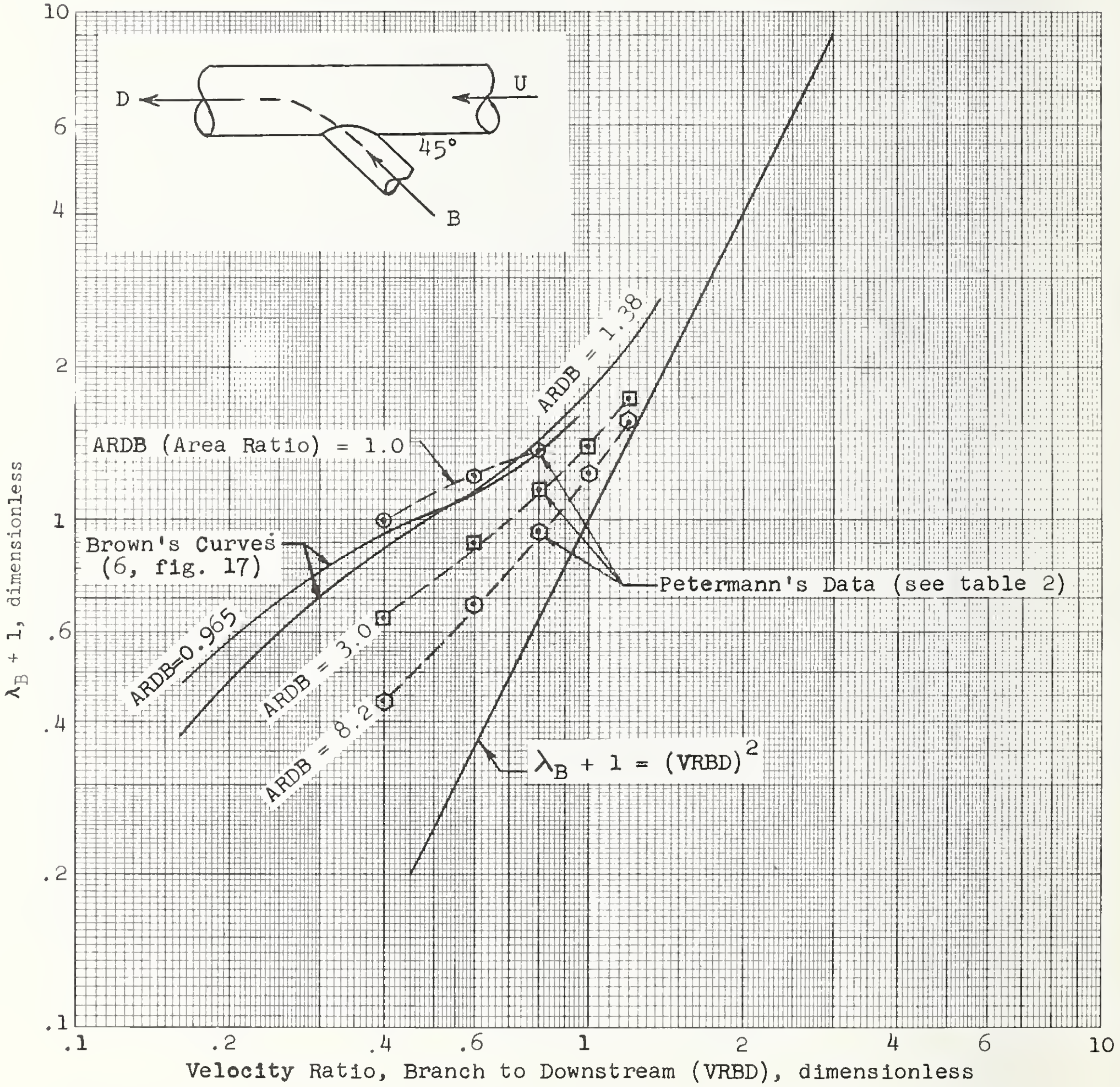


Figure 4. Comparison of Brown's and Petermann's Branch Coefficients

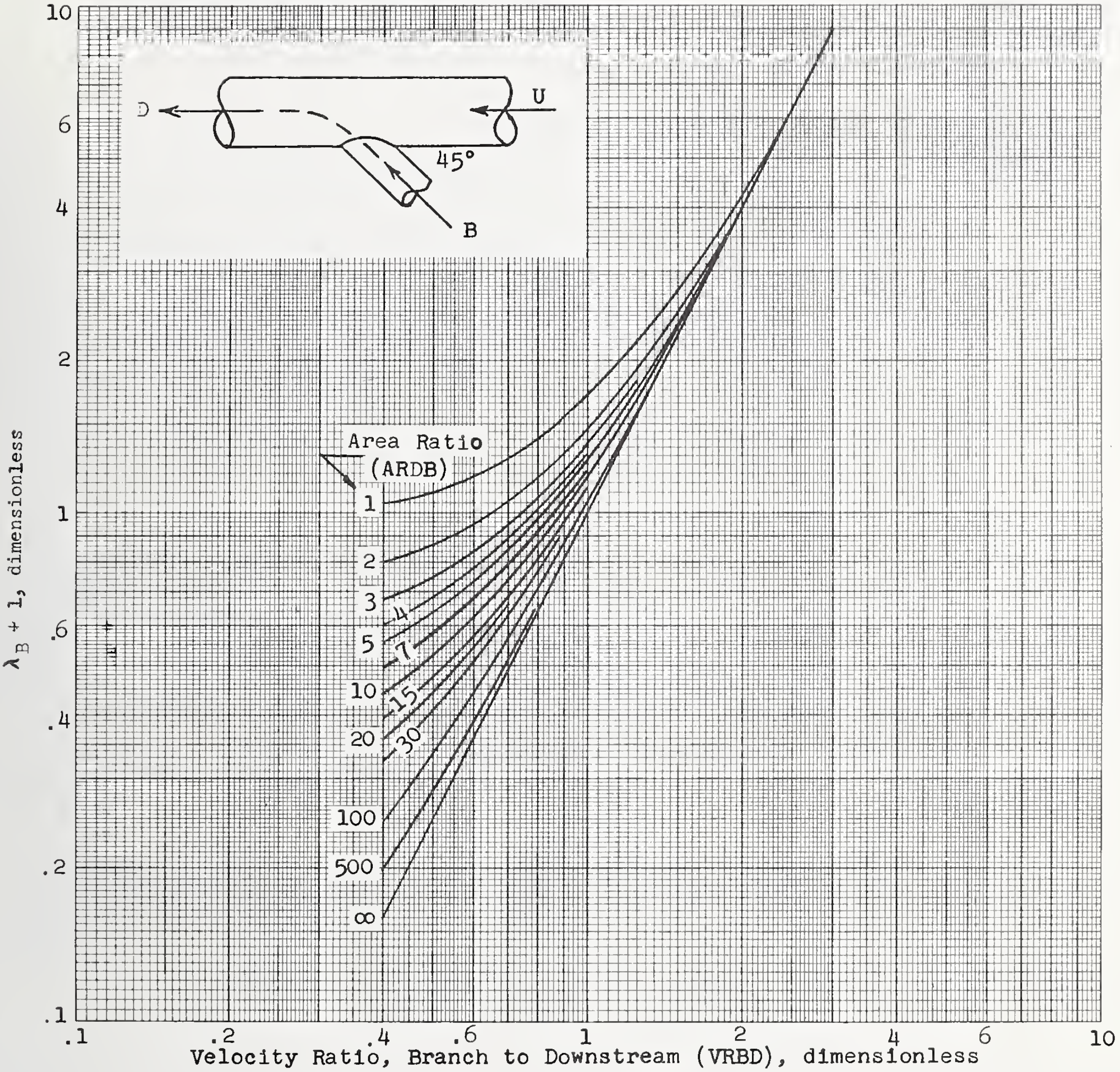


Figure 5. Return Air Tee-Branch Loss Coefficients

```
C TOTAL PRESSURE LOSS IN THE BRANCH OF A RETURN AIR 45-DEGREE TEE (TEEBR),
C DIMENSIONLESS
C
C ARDB IS THE AREA RATIO OF THE TEE MAIN TO THE TEE BRANCH, DIMENSIONLESS
C
C VRBD IS THE VELOCITY RATIO FROM THE BRANCH TO DOWNSTREAM OF THE TEE MAIN,
C DIMENSIONLESS
      FUNCTION TEEBR (ARDB, VRBD)
      A=1.0276
      B=0.3405
      R=(A/ARDB**0.5)-(B*VRBD/ARDB**0.5)+(VRBD**2)
      TEEBR=R-1.
      RETURN
      END
```

Figure 6. Fortran Function Subprogram Listing for Determining Tee-Branch Loss Coefficients

# Automatic Design
of Optimal Duct Systems

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A system of ducts for the delivery of air in an air-conditioning installation subject to pressure-balance conditions is found to be indeterminate and thus capable of satisfying additional conditions. Upper bounds are imposed on the air velocities in some sections and the problem of economically optimal diameters is formulated as a constrained non-linear programming problem. This is transformed into an unconstrained problem capable of solution by known numerical methods. A computer program based on the concept of Lagrange multipliers and the Newton-Raphson iterative method is outlined. Techniques for acceleration of convergence are stated. Properties of resulting optimal solutions are discussed.

Key Words: Ducted flow, friction losses, annual cost, capital cost, running cost, kinetic energy, potential energy, static regain.

## 1. Scope and Definitions

The duct system under discussion consists of several sections, each of a uniform equivalent diameter along its entire length. If a section contains bends, it will be represented by a straight section of an equivalent length, according to conversion formulas summarised by Carrier and others [1][1]. Each section terminates either in a divided flow fitting or in a terminal outlet. Each divided flow fitting has one inlet and two or more outlets. One of these outlets may be a terminal outlet, others connect to the inlets of further downstream sections. There is only one entrance section. From the description, it follows that there are at least as many sections as terminal outlets.

A run is a set of duct sections commencing at a terminal outlet and continuing upstream to the inlet of the entrance section. The number of runs is equal to the number of terminal outlets.

## 2. Balancing Conditions

To maintain the required flow of air at each terminal outlet, it is necessary that the sum of pressure drops along a run, plus the pressure at its outlet $p_o$ is equal to the inlet pressure $p_i$ for all runs:

$$p_i = \sum \Delta p_{jk} + \sum \Delta p_l + p_o \tag{1}$$

The summations extend over all sections l of a run and over all divided flow fittings connecting section j to section k along the same run. The singly subscripted pressure drops result from friction losses, doubly subscripted ones from the application of Bernoulli equation at the divided flow fittings, as shown in Appendix 1. For given section lengths, flow quantities and fixed shapes of divided flow fittings, the pressure drops $\Delta p$ of eq (1) are functions of equivalent diameters of the corresponding sections.

For a system with M + 1 terminal outlets, there are M + 1 equations of type (1), one for each run. By subtracting the first of these from each of the remaining ones, the inlet pressure is eliminated and M equations are obtained in the following form:

---

[1] Figures in brackets indicate the literature references at the end of this paper.

$$\sum \Delta p_{jk} + \sum \Delta p_l - \left( \sum \Delta p_{mn} + \sum \Delta p_r \right) = 0 \qquad (2)$$

These M simultaneous equations will be written as

$$H(D) = 0 \qquad (2a)$$

where H is an M x 1 matrix of the left hand side of eqs (2) and the argument D is an N x 1 matrix of the equivalent diameters of all the N sections. As the number of sections is at least equal to M + 1 and each section may have a different equivalent diameter, there are N independent variables available to satisfy $M < N$ simultaneous equations. Thus the balancing equations have no unique solution. Consequently, a duct system corresponding to the description of Section 1 is always free to satisfy at least one condition in addition to the balancing conditions.

## 3. The Degree of Freedom

The difference between the number of sections N and the number of balancing equations M will be called the degree of freedom of a duct system. In systems of one degree of freedom, only one section diameter or velocity may be freely chosen, all others are determined by the balancing equations. Where the degree of freedom exceeds one, the design problem is complicated and while different feasible designs may be evaluated by comparison, it is difficult to estimate how much better the best possible design might be. In the following, the problem of optimal duct system will be posed, analysed, and a computer program for the determination of optimal dimensions will be described.

## 4. Criterion of Optimality

A duct system which delivers required quantities of air at the required outlet pressures may suffer from only two major shortcomings: excessive costs and noise. Standard reference literature [3] suggests empirical rules for maximum permissible air velocity in various duct sections. The computer program to be described provides means of incorporating arbitrary upper bounds on any air velocity in the system.

Having satisfied requirements relating to noise, a duct system may be optimised with relation to costs. For given layout, section lengths, air quantities, duty period, types of divided flow fittings and unit cost factors, the total cost of the system depends on the transversal dimensions. It will be analysed on the basis of equivalent diameters.

## 5. Costs

All capital costs will be reduced to their annual equivalents, following standard accounting practice [2]. The annual capital cost of section j of length $X_j$ and diameter $D_j$ is

$$C_j = q \cdot X_j \cdot (a_o + a_1 S_1 D_j + a_2 S_2 D_j^2) \qquad (3)$$

where q is the capital recovery factor and the a's are unit cost factors. The first of these, $a_o$, reflects costs dependent on the length alone, $a_1$ represents the component of cost related to the duct surface area, $a_2$ stands for the rental value of the space occupied by the duct section. Any two of these three factors may be simultaneously zero. The factors $S_1$, $S_2$ relate the equivalent diameter to circumference and $D^2$ to the cross-section area; in a circular duct, they are $\Pi$ and $\Pi/4$ respectively. For rectangular ducts, analogous factors follow from Ref. [4].

The annual capital cost of the duct system is the sum

$$C(D) = \sum C_j \qquad (4)$$

over all the sections. According to eq (3) it is a quadratic scalar function of the equivalent diameter matrix D.

The running cost of a duct system is the cost of energy necessary to maintain the airflow for the required time. The power applied at the entrance section is

$$P = Q_1 \cdot p_1(D) \tag{5}$$

where $p_1$ is the total pressure at the entrance of the system and $Q_1$ is the air flow rate. The derivation of $p_1$ is in Appendix 1. For an overall motor-fan-diffuser efficiency e, the running cost is

$$r_{(D)} = e^{-1} a_3 t P(D) \tag{6}$$

where $a_3$ is the cost per unit of energy at the motor and t the total time of running per year.

The total annual cost is then

$$C_T(D) = C(D) + r(D) \tag{7}$$

As the pressure increments which combine to determine the running cost are proportional to velocity to a power of approximately 2 (see Appendix), r(D) is a function of diameter raised to a power of approximately -4. The capital cost is a quadratic function of diameter and both C(D) and r(D) are, of course, positive. Thus $C_T(D)$ of eq (7) is a convex function and satisfies second order necessary conditions in the sense of Ref. [5] for the local uniqueness of the optimal solution.

## 6. The Constrained Problem

The problem of optimal duct system design may now be formulated:

$$\text{minimise} \quad C_T(D) \tag{8a}$$

$$\text{subject to} \quad H(D) = 0 \tag{8b}$$

$$\text{and} \quad D_i - B_i \geq 0 \tag{8c}$$

where $B_i$ is the lower bound of diameter $D_i$ corresponding to the prescribed upper bound on velocity in section i.

This is a non-linear programming problem with accessory conditions (8b) and unequality constraints (8c). Techniques of solution are summarised by Fiacco and McCormick [5], who refer to this form as Problem A. A particular technique embodied in a working computer program for iterative solution of optimal duct diameters will be described.

## 7. The Unconstrained Problem

The constraints (8c) are represented by ficticious costs; if, during the computation, any diameter $D_i$ becomes smaller than the corresponding constraint $B_i$, the cost of section i is increased by a quantity proportional to the square of the excess $B_i - D_i$. This increase is applied with sufficient weight, so that the resulting optimal diameter is within a prescribed tolerance interval of the applicable bound. Thus, the constrained problem is transferred into an unconstrained problem with accessory conditions:

$$\text{minimise} \quad C_F(D) \tag{9a}$$

$$\text{subject to} \quad H_j(D) = 0, \quad 1 \leq j \leq M \tag{9b}$$

where $C_F$ is the sum of actual and ficticious costs:

$$C_F = C_T + \sum k_i (B_i - D_i)^2 \tag{9c}$$

$k_i$ being the ficticious cost factor

$$k_i = 0 \qquad \text{if } B_i \leq D_i \tag{9d}$$

$$k_i > 0 \qquad \text{if } B_i > D_i \tag{9e}$$

The solution of problem (9) is a set of N diameters $D_i$ constituting an N x 1 matrix D which satisfies eq (9b) and the condition for extreme value

$$\frac{\partial W}{\partial D_i} = 0 \tag{10}$$

for all i's. Here, W is the Lagrangian

$$W = C_F + U^T H \tag{11}$$

consisting of the cost and the M balance condition $H_j(D) = 0$;. $U^T$ is the M x 1 matrix of Lagrange multipliers, transposed. (For a brief outline of the Lagrangian technique, see reference [6])

The design problem now consists of finding the N values of $D_i$ and M values of $U_j$ which simultaneously satisfy eqs (10) and (9b); these are M + N independent conditions binding an equal number of variables.

The numerical solution commences by choosing a tentative initial set of values for the Lagrange multipliers $U_j$, solving eq (10) for the corresponding diameters $D_i(U)$ and substituting these into eq (9b). Due to an error in U, the left hand side of eq (9b) is not equal to zero, but to an error matrix E dependent on U:

$$H(D(U)) = E(U) \tag{12}$$

A better set of multipliers is obtained from the initial one by Newton-Raphson method [7]. A correction $\delta U$ in the values of U which reduces the error E is calculated from the first order approximation:

$$E + \frac{\partial H}{\partial U} \cdot \delta U = 0 \tag{13}$$

This equation is solved by inversion of the M x M matrix $\partial H / \partial U$:

$$\delta U = - \left( \frac{\partial H}{\partial U} \right)^{-1} \cdot E \tag{14}$$

The original values of U are corrected by $\delta U$ and the procedure is iterated until the residual error E is reduced within prescribed bounds. The physical significance of E is the amount of pressure unbalance in the duct system.

The matrix $\partial H / \partial U$ is obtained as the derivative of a composite function:

$$\frac{\partial H}{\partial U} = \frac{\partial H}{\partial D} \times \frac{\partial D}{\partial U} \tag{15}$$

All elements of the M x N matrix $\partial H / \partial D$ are available in analytical form following substitution into eq (2a) from eq (2) and from the pressure drop expressions in Appendix 1. The N x M matrix $\partial D / \partial U$ is obtained from the Lagrangian eq (10) by the rules for derivatives of an implicit function. The typical element is

$$\frac{\partial D_i}{\partial U_j} = - \frac{\frac{\partial Z_i}{\partial U_j}}{\frac{\partial Z_i}{\partial D_i}} \qquad (16)$$

where $Z_i = \frac{\partial W}{\partial D_i}$ , from eq (10).

The denominator $\partial Z_i / \partial D_i$ in eq (16) is the second partial derivative of the Lagrangian W in respect of $D_i$.

## 8. Convergence

The iterative process converges slowly and it is necessary to use special precautions to ensure stability while the initially chosen values of U are far from the correct ones. The program limits the step size and restricts, in the initial stages, all diameters by cost penalties based on a rough estimate of optimal diameters. These penalties are cancelled when stability is reached and have no effect on the result.

## 9. Properties of Optimal Duct Systems

The duct system resulting from the above procedure differs to some extent from systems designed by current methods. The difference is minimal in systems of one degree of freedom, i.e. those where the number of sections is equal to the number of terminal outlets. The static regain method [8] uses physical concepts employed in the setting up of balancing conditions, eq (2a) and eq (2c), Appendix 1 of the present study. With one degree of freedom, the system is determined by any single variable being fixed; it follows that any such system is optimal for some set of design parameters. However, these may equal the parameters actually applicable only by coincidence.

TABLE 1

SYSTEM OF ONE DEGREE OF FREEDOM

DESIGN A: $a_1 = 1.50$ \$/ft$^2$ DESIGN B: $a_1 = 16.70$ \$/ft$^2$

| DUCT SECTION | LENGTH ft | Q, CFM | DIAM. INCH | VELOCITY ft/min | DIAM. INCH | VELOCITY |
|---|---|---|---|---|---|---|
| 1 | 50 | 10,000 | 38.1 | 1,261 | 24.0 | 3,185 |
| 2 | 40 | 8,000 | 36.3 | 1,115 | 23.4 | 2,680 |
| 3 | 20 | 6,000 | 32.6 | 1,036 | 21.4 | 2,412 |
| 4 | 20 | 4,000 | 27.8 | 949 | 18.6 | 2,127 |
| 5 | 20 | 2,000 | 20.9 | 837 | 14.3 | 1,795 |

| | | |
|---|---|---|
| CAPITAL COST, at $a_1 = 1.50$ \$/ft$^2$ | \$1,957 | \$1,263 |
| ANNUAL CAPITAL COST | \$184.74 | \$119.30 |
| RUNNING COST at 8750 hrs/year 0.02 \$/kWh | \$42.11 | \$294.50 |
| TOTAL ANNUAL COST | \$226.85 | \$413.80 |

System of one degree of freedom, for Table 1

Table 1 compares two optimal systems of one degree of freedom, differing only in the capital cost factor $a_1$, the cost per unit area of the duct surface. Both systems have been obtained as results of the procedures described above. Design A is optimal for a realistic value of $a_1 = 1.50$ \$/ft$^2$; design B would cost 82% more to own and operate in the same cost environment. It happens to be optimal for duct cost 16.70 \$/ft$^2$. Design B consists of sections 10 - 14 of an example given in Ref. [9]. The example quoted may, of course, be optimal for a lower capital cost and a shorter duty period.

For systems of degree of freedom higher than one, the current design methods offer no unique solution. Thus, there is no basis for economic comparisons between them and the present method. It is true, however, that while static regain systems of degree one are inevitably optimal, even if perhaps for some unrealistic cost factors, higher degree systems may only be optimal by an unlikely multiple coincidence.

To understand the shortcoming of existing methods, it is necessary to realise that a duct system supplies two things to each divided flow fitting: the air to be distributed and the energy to drive it through the subsequent sections. This energy exists in two forms: kinetic and potential (static pressure). The static regain method, consisting in a "procedure in which the duct is sized so that the increase of static pressure or regain at each take off offsets the pressure loss of the succeeding section of the run" [10] transports all this energy as kinetic. This involves friction losses upstream whereas transport in the form of potential, or pressure, is free from losses in systems without leaks. On the other hand larger ducts are required to reduce the kinetic energy content. The present procedure optimises the proportion of kinetic to potential energy according to the criterion of minimal costs, subject to balancing conditions and velocity constraints.

## 10. References

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[2] Ibid. p. 513.

[3] ASHRAE Guide and Data Book, Systems and Equipment for 1967, p. 37 (American Soc. of Heating, Refrigeration and Air-Conditioning Engineers)

[4] Ibid. p. 29, eqs (6).

[5] Fiacco, A.V. and McCormick, G.P. Non-linear Programming: Sequential Unconstrained Minimisation Techniques (Wiley 1968)

[6] Korn, G.A. and Korn, T.M. Mathematical handbook for Scientists and Engineers (McGraw-Hill 1968)

[7] Ibid. section 20.2-8.

[8] Ref. 1 , p. 261.

[9] Ref. 3 , pp. 42 - 44, Table 9, sections 10 - 14.

[10] Ibid. p. 39, Col. 2.

# APPENDIX 1

## Pressure Drop Formulas

The following expressions are based on the ASHRAE Guide [3]. Equation (1) is the SI equivalent of formula 25, p. 39. Equation (2) is a generalised form corresponding to formulas 14, 16 - 20 of page 35. Any of the guide expressions may be obtained from eq (2) by a suitable choice of parameters $C_U$, $C_D$.

The remaining equations of this Appendix are derived from eqs (1) and (2).

### Symbols and Units

| | | |
|---|---|---|
| $\rho$ | air density | (kg $m^{-3}$) |
| $v$ | air velocity | (m $s^{-1}$) |
| $D_i$ | equivalent diameter of section i | (m) |
| $L_i$ | equivalent length of section i | (m) |
| $Q_i$ | flow rate in section i | ($m^3$ $s^{-1}$) |
| $\Delta p_i$ | pressure drop in section i | (N $m^{-2}$) |
| $\Delta p_{kl}$ | pressure drop due to a divided flow fitting connecting section k to section l | (N $m^{-2}$) |

In a duct of circular cross-section the pressure drop resulting from friction is

$$\Delta p_i = 0.0202 \,.\, L_i \,.\, Q_i^{1.82} \,.\, D_i^{-4.86} \qquad (1)$$

The pressure drop corresponding to a transition from section j to section k would be, in a loss free system

$$\Delta p_{jk} = \frac{\rho}{2} \,( v_k^2 - v_j^2 ) \qquad (2a)$$

Introducing coefficients $C_U$ and $C_D$ to represent losses occurring upstream and downstream of the divided flow fitting, we have

$$\Delta p_{jk} = \frac{\rho}{2} \,.\, ( C_D \, v_k^2 - C_U \, v_j^2 ) \qquad (2b)$$

Expressing the velocities by flow rates and diameters

$$v_i = S_2^{-1} \,.\, Q_i \, D_i^{-2} \qquad (2c)$$

where $S_2$ is the proportionality factor between $D^2$ and cross-section area for the particular shape employed

$$\Delta p_{jk} = \frac{\rho}{2} \,.\, S_2^{-2} \, ( C_D \, Q_k^2 \, D_k^{-4} - C_U \, Q_j^2 \, D_j^{-4} ) \qquad (2d)$$

The total pressure at the entrance of the system consists of the velocity head and of all the pressure drops along any run,

$$p_1 = \frac{\rho}{2} \,.\, v_1^2 + \sum \Delta p_i + \sum \Delta p_{jk} + p_o \qquad (3)$$

and the necessary fan power is

$$P = Q_1 \,.\, p_1 \qquad (4)$$

which leads to eq (5) of Section 5.

The first and second partial derivatives of the running cost in respect of any diameter follow after substitution into eq (3) for $v_1$ and all the pressure drops from eqs (1), (2a) and (2d).

# A System of Computer Programs Widely Used in Europe for Designing, Selecting and Analysing Different Air Conditioning Systems

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The ultimate object of using computer programs in air conditioning design is to make feasible an optimal choice and integration of building features and A.C.-equipment that complies with indoor environment requirements. To this end it is necessary not only to compute heating and cooling loads in all individual rooms for specified room temperatures and outdoor conditions, but also to obtain the data required for directly selecting all main central and peripherial components of the A.C.-plant from manufacturers' catalogues and to assess the average yearly energy consumption. These items are substantially influenced by the type of plant chosen and the interaction of the different loads handled simultaneously in all zones. Further, heat storage capacities cooperating with tolerated uncontrolled variations of room temperatures within specified limits, moving shadows from other buildings, control systems and setting values, causing annihilation and/or recuperation of energy etc., play an essential part. In this paper a set of computer programs is presented which yield all the information mentioned for different kinds of A.C.-systems in addition to many other valuable data, such as temperature curves occurring in rooms without air conditioning. There is also a program for the calculation of complete duct systems. Manuals and take-off-sheets are available in different languages and units systems. Computer results are printed in corresponding varieties. The programs are used in many countries in Europe and several hundreds of projects have hitherto been calculated. As a result of these programs there is now a growing practice for architects, together with civil, electrical, acoustical and A.C.-engineers, etc., to form a team as early as the financial stage of the planning, thus obtaining an optimal total design without individual dominance of any part concerned.

Key Words: Air conditioning, A.C. central systems, A.C. load calculations, A.C. plant design, computer calculations.

## 1. Introduction

In articles and discussions about the application of computers in A.C.-design work one often meets the opinion that this application would be identical with and confined to the computerized calculation of maximum heating and cooling loads per room - and possibly the dimensioning of duct systems.

Without denying the importance and extensiveness of these basic calculations, every A.C.-designer knows very well that they constitute only a minor part of the entire work. They do not give him any answers to questions such as total cooling and heating energy consumption in the entire building - consisting of many different rooms - during a normal year, determination of size of room units and central plant in such a way that room temperatures are allowed to glide freely within certain specified limits, setting values of different control functions, etc.

These, and many similar items, are dependent on the way in which the calculated net heating and cooling demands are satisfied, and on the thermo-dynamic losses and regain possibilities inherent to the A.C.-system chosen. A complete package of application programs within the A.C.-field should therefore incorporate not only the necessary load computing routines, but also the digital simulation of most current A.C.-systems.

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[1] Professor and Engineer, respectively

For this reason, in 1964 we started to develop such a set of programs (see fig. 1), the second generation of which have already been in extensive daily use throughout Europe for several years and a third generation are now being completed and are already partly operative.

## 2. Survey of Computer Programs Developed

When planning a new building in moderate European climate areas the first question posed by architects and principals is often whether an A.C.-installation is in fact necessary, or whether an acceptable indoor climate could be maintained by a simple ventilation system. The prime decisive factor in this case is the air temperature attained in the rooms during the hottest part of the year. Therefore a special program has been established which calculates the course of daily room temperature variations without air-conditioning, and without or with mechanical ventilation at a given rate and temperature. Of course all given structural characteristics and specified conditions as regards the use of the building and the local outdoor climate are taken into consideration in this calculation.

If the need for an A.C.-installation is found to exist, the necessary equipment can be determined and the operating costs be evaluated by using the other available programs for the calculation of, respectively:

A. Net heat gains and losses per building module for all relevant average and design conditions such as normal workdays and holidays during clear and cloudy weather throughout the year (Program LK012).

B. Installation and operating data for:

Two-pipe induction systems with variable primary air temperature (Program LK022).

Four-pipe induction systems (Program LK042). This program can also be used for calculating two-pipe terminal reheat systems.

Dual-duct systems (Program LK062).

C. Dimensions of duct systems made up of sheet metal ducts of circular sections and consequent fan pressures (Program LK002).

## 3. Particulars of the Programs

### 3.1 General Observations

When the work of calculating is entrusted to a computer it becomes feasible to take full consideration of such factors as non-simultaneousness of maximum cooling demands, heat storage in the building structure in connection with irregular times of insolation due to moving shadows, etc.

The potential of the computer has not been fully utilized if it is merely employed to carry out the same table references and calculations as the designer has previously performed manually. On the other hand, the program designer should not seek theoretical perfection. This could easily result in the loss of other important attributes, such as wider usefulness, since there is a limit even to the capacity of computers.

An excessively thorough treatment of certain factors in the calculation process may also frequently result in the take-off sheets being so extensive in scope as to daunt the ordinary, practical design engineer.

Moreover an unlimited refinement of the calculation methods entails a rapid increase of the CPU-time in the computer and so a sensible balance between calculation costs and scientific perfection must always be observed.

### 3.2 Meteorological Data

This philosophy is also apparent in the method we have adopted for the specification and subsequent use of the meteorological data. The calculation requires a continuous course of outdoor temperatures during an average and a design-day in every month under clear and overcast weather conditions. These temperatures are calculated by the computer on the assumption that the daily variation describes a sine curve (with alternate long and short half-periods), the maximum and minimum points of which are given in a special meteorological data form.

When a calculation is to be carried out for a specific geographic area for the first time, the requisite climatic particulars must be entered on this form. The particulars are then filed for future reference.

The varying humidity of the outdoor air is treated in a similar manner. For direct and diffuse solar radiation intensity only information as to the maximum values for a clear day in each month is required. The variation in intensity during the day is then calculated by the computer for every required aspect by assuming that direct solar heat striking an external wall varies during the day from zero to a maximum value and back to zero in conformity with the positive portion of a sine curve. The half-period is equal to the time between the instant when solar radiation first strikes the facade and that at which it ceases to strike it, regardless of whether these points in time are determined by sunrise and/or sunset, or by the sun appearing and/or disappearing round the corners of the facade.

### 3.3 Basic Calculation Method

The variation of room air and wall temperatures and of heating and cooling demands per module is calculated by determining the balance of heat stored in the building mass and transferred to the room air. This balance is computed hour by hour and includes all internal and external heat gains and losses. The heat balance at a certain point of time, called "the hour (CL+T)" is calculated with the assumption that outdoor conditions, solar radiation, as well as internal heat loads that do not vary with room temperature are constant from the preceding point of time, "the hour CL" for which they were given or previously calculated, until and not including the hour CL+T. If the room air temperature varies it is supposed to be subject to a sudden change immediately after the hour CL and then to remain constant until and including the hour CL+T. Those heat loads which depend on room air temperature (e.g. transmission) as well as the wall surface temperature and the wall core temperature are supposed to vary in the same manner.

The results of each heat balance calculation are new values of wall surface and wall core temperatures for the hour CL+T as well as a new room air temperature to be reached at that hour or, when room temperature is prescribed, a new cooling or heating demand. Thus a "backward discretization of time" is applied avoiding any instability in the calculations.

Calculations are started from 1 a.m. on the assumption that both the air in the room and the walls and floor have a temperature of 71°F then. After calculations have been completed for a full 24 hours, the room and wall temperatures obtained are usually different and these new values are taken as starting points for the next day, and so forth. Calculations are carried out in this way for four consecutive days, in general, this being adequate in most cases to achieve a stabilized temperature history i.e., the values for 12 p.m. are largely identical with those calculated for 1 a.m. the preceding night. In buildings with heavy structures, however, stabilization may not be achieved at the end of the fourth day calculated and therefore the number of consecutive days can be changed arbitrarily. In most cases, though, this number is limited to four days in view of the fact that the majority of 'heat-wave' periods (representing design weather) seldom last longer than four days, at any rate in Western Europe type climates.

### 3.4 The Influence of Sudden Load Variations, Manipulation of Blinds, etc

Using the method described, it is also possible to take account of sudden load changes, such as the switching-on or off of the lighting, the drawing or rolling-up of sunblinds, or the start and finish of office hours with the associated change in the required room temperature, occupant load, heat gain from lighting, etc. The cooling requirement in an office, for example, is greatly affected by assumptions such as that venetian blinds will be drawn all day on sunny days, or that they will be drawn only when the sun shines directly on the wall concerned during office hourse.

The take-off sheets provide for an indication of the times between which office hours may be assumed to lie, and the temperature limits to be maintained during these hours and at night or during holidays. As stated previously, the solar heat gain through the windows is calculated for each hour. The program makes the assumption that when this gain exceeds a certain level the sunblinds will be drawn, as long as the room is occupied. Otherwise it is assumed that the blinds, etc., are not drawn. Outside office hours, the client may specify that blinds will always be drawn or open.

Similarly, the light intensity due to natural lighting at a given distance from the windows is calculated for each hour of the day. It is assumed in the program that when this falls below a certain level the artificial lighting will be switched on and the heat gain from this source is then included in the heat balance equation for the hour concerned. It is taken for granted in this context that lighting is likely to be switched on only during office hours. When the intensity of natural lighting in the room is on the increase, so that it exceeds the limiting value up to which supplementary lighting is required, then it is normally assumed that artificial lighting will be switched off. It is possible, however, to provide for a degree of 'lighting negligence' by assuming that once the lighting has been switched on it will be left on to the end of office hours, regardless of the subsequent need or otherwise.

### 3.5 Effect of Shadows from External Wall Projections and Other Buildings

The reduction in the solar radiation through windows depending on shadowing by external wall features such as balconies or columns, or to the recessing of windows is taken into due account. This reduction will naturally vary according to the angle of incidence of the sun's rays, i.e. according to the time of day, and is calculated stereometrically on the basis of the sun's varying altitude and azimuth at the geographic latitude concerned.

The tables published in current A.C.-design manuals which are normally used to determine 'storage factors' are limited, for practical reasons, to external walls on which there are no shadowing features. This means that in such manual calculations the same reduction factor for shadows is employed not only for the direct solar radiation at the time considered, but also for the solar heat stored during the preceding hours. This may lead to tangible errors in certain cases, but these are completely eliminated in the computer programs, which calculate the shadow factor and stored heat hour by hour.

The programs also permit moving shadows from other neighbouring buildings or parts of the same building to be taken into account. The edge of such shadows moves constantly and, at any given time, the different parts of the facade will have been in sunlight for different lengths of time. The stored solar heat will therefore be different for these different parts and thus the resultant room temperatures, or the cooling demand will differ also.

A maximum of 12 shadowing rectangular buildings can be allowed for. Buildings with complex shapes, such as horizontally or vertically L-shaped structures, are considered to consist of two or more adjacent single rectangular buildings. The locations of the corners of the shadow-throwing structures in relation to the facade investigated are defined by means of a co-ordinate system which can be laid arbitrarily over a site plan of the building area (see fig. 2).

A division of the facade into a maximum number of 100 surface elements is then assumed by the computer. The sequence of room air temperatures or thermal loads is calculated for one module in every such surface element or group of elements. The different local successions of sunny and shaded periods on every part of the facade are thereby taken into account and also the corresponding switching on and off of electric lighting, the drawing and opening of sun blinds and the storage effect due to these factors.

## 4. Lay-out and Versatility of Take-off Sheets

For every program only two different kinds of take-off sheets are used. One of these contains all the data which are common for the entire building or describe the basic conception and central control of the A.C.-plant, whereas the other contains all the data pertaining to each individual "zone" or group of identical modules.

In addition to comprehensive instructions given in the "Program Users Manuals", the take-off sheets contain a considerable amount of guiding text to facilitate the filling-in. This is illustrated in figure 3 showing the "Common Plant Data"form for the four-pipe induction system program (LK042).

Within each A.C.-system many alternative variations can be specified such as different combinations and consecutive orders of components in the central air treating unit, free cooling through "dry" or "sprayed" recovery coil, automatic or manual terminal unit control, etc. Further, the user can specify by how many deg. F personnel in a room should be able to regulate the room temperature upwards or downwards, by how many degrees the room temperature should be allowed to glide upwards above the normally desired value in the event of exceptionally hot weather (at summer design temperature), whether 'lighting negligence' is to be taken into consideration, and so forth.

## 5. Calculation Results

### 5.1 Room Temperature Program

The results of the room temperature program LK015 comprise one printed page for every building zone and month investigated. When moving shadows occur on a facade this page gives the daily range of room temperatures obtained in the two facade elements in which the highest and the lowest top values, respectively, of the entire facade occur on a clear day. Further, the print-out gives the temperature range obtained on a cloudy day during which all modules, of course, are subject to equal loads.

When the computer, on the basis of prevailing solar intensity at any point of time, states that lighting should be switched on or that venetian blinds, jalousies or similar devices should be shut, this is indicated by the letters 'L' and 'J' respectively, after the room temperature value at the hour in question. In addition frequency tables are printed giving information on the percentage of the total time of occupancy in the zone during which different room temperatures are reached or exceeded. The varying boundary line of the moving shadows is also shown in the computer results, (fig. 4).

### 5.2 Basic Load Calculating Program

The results obtained with the program LK012 include the heating and cooling requirements per module during any specified month or all 12 months of a statistically normal year, as well as the maximum requirements occurring under outdoor design conditions. These results are based on room temperatures being prescribed either at fixed values or between given limits.

The normal procedure following this calculation is to use the data calculated and stored by the computer for designing an A.C.-installation with the aid of one of the "plant programs". However, in many cases the result of the LK012 program has a value of its own - for instance when different alternative building features such as size of windows, kind of blinds, etc., are to be compared.

### 5.3 Plant Designing Programs

An example of one page of the results produced by the plant designing programs (LK022-LK062) is shown in figure 5. This page supplies, for one zone, the information necessary for the selection of the induction units in a four-pipe system (max. required unit coil cooling and heating capacities in connection with primary air supply). Comments are also printed as to the specified comfort requirement or some other consideration that has been decisive for the capacity values stated.

In addition, a number of tables are printed showing the varying thermal output of the induction unit during the day together with room temperatures actually obtained for several critical running conditions.

In the results of the dual-duct plant program (LK062) corresponding information is given concerning the characteristics of the mixing boxes.

When moving shadows occur on any facade a special table is printed indicating on which parts of that facade it is possible to install terminal units with a lower capacity than the maximum required capacity owing to the reduced solar heat gain in these parts. After determining the capacity of the induction units or the mixing boxes, the computer calculates the total heating and cooling energy consumption in the whole building for all consecutive hours of a clear, as well as a cloudy day in every month. This is done both for normal and extreme ("design") outdoor conditions.

The highest value of the total momentary cooling demand (including all required sensible and latent cooling of outdoor air) which the computer encounters in the course of this calculation is stored and later shown in the printed calculation result. The result, apart from the maximum total cooling load, also states the daily course of unit outputs and room temperatures in every zone during the day on which this maximum total load occurs (fig. 6).

In addition to the design cooling load with the appropriate times and temperatures, the maximum total heating demand and a large number of other data required for the design of the plant, such as central cooling and heating coil characteristics, total water flow rates, etc., are also provided.

The total consumption of heating and cooling energy at every hour, calculated over the normal year referrred to above, is collected in a number of items so that the totals may be obtained at the end of each month and of the whole year. These totals are composed of the summed-up hourly consumption figures obtained in "real" (digitally simulated) over-all plant operation during clear and cloudy workdays and weekends in statistically correct proportions for every month. Apart from the yearly totals, the over-all energy consumption is stated per hour and per month.

### 5.4 Duct Calculating Program

After determining all main thermal components in the way described above, the dimensioning of the duct system and the fans remains to be calculated. This is done with the aid of the duct system calculating program (LK002).

This program yields all the necessary information for each individual pipe section of the system such as quantity of air passing, air velocity, pressure loss and, in the case where the pipe section ends in a supply or extract point, the available static pressure at these points. Any throttling devices required are indicated on the appropriate sections giving the pressure drop need. The diameters of the pipe sections and the throttling devices that may be required at the main junctions are given final values by the computer, often after repeated calculations in order that a certain maximum permissible pressure difference between the different supply points will not be exceeded. The table further indicates the total quantity of air and the fan pressure.

Further, the results of this program include a complete list of the total lengths of all ducts of different diameters used in the system, as well as the total number of bends, tees, etc.

When required, the print-out can also include a sub-division of the list in accordance with the different stages of erection of the building. Each partial list then comprises the ducts and accessories required for one floor, wing or similar part of the building and the erection engineer is thus provided with means to order the duct parts for delivery to the building site in smaller portions just when they are required.

## 6. Practical Applications and Consequences

Amongst air-conditioning engineers in Europe there is a general wish to be involved in the planning of new buildings at as early a stage as possible in order that the architectural design and the lay-out of the A.C.-plant should be co-ordinated from the very beginning, and this desirability is pointed out on every possible occasion.

However, this recommended teamwork has only too seldom been applied in actual practice. One of the main causes of this regrettable state of affairs has most probably been the difficulty for A.C.-people to give immediate and correct answers to essential questions raised by architects and principals. Such questions are, e.g., "How does the glazing percentage of facades influence costs and required space for cooling equipment?" or "What type of A.C.-system is the most advantageous in any given case?"

During the last couple of years, however, a noticeable change in this situation has taken place - made possible by the development of computer programs such as those described in this paper.

An increasing understanding by architects and building owners of the importance of air-conditioning and of the problems involved can be observed. Thus, there is now a growing practice for all the parties concerned in the planning of a building to form a team already at the financial stage, resulting in an optimal total design.

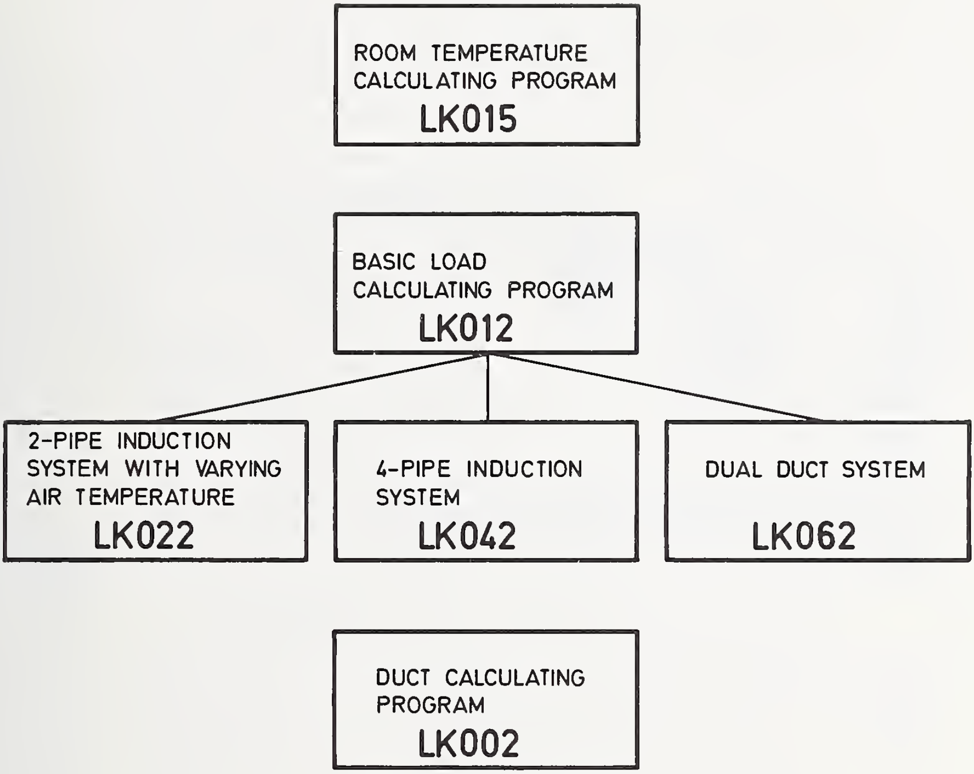


Fig. 1 Basic conception of a set of A.C.-design programs

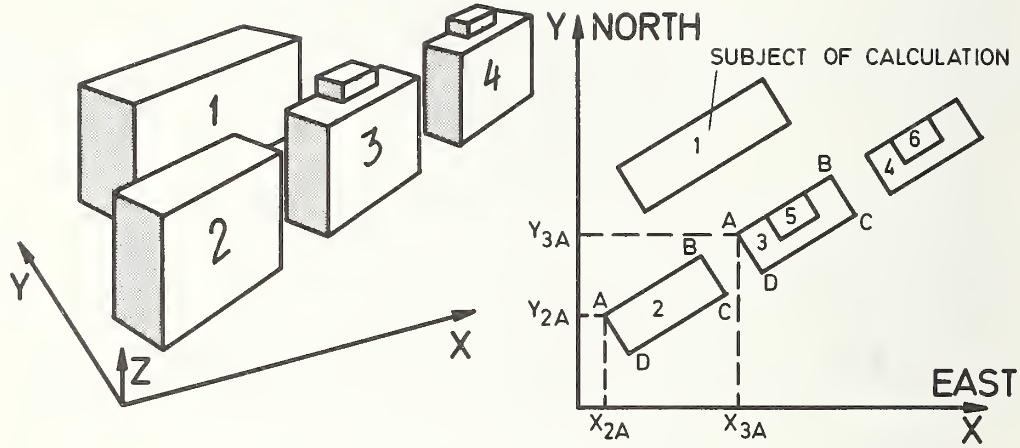
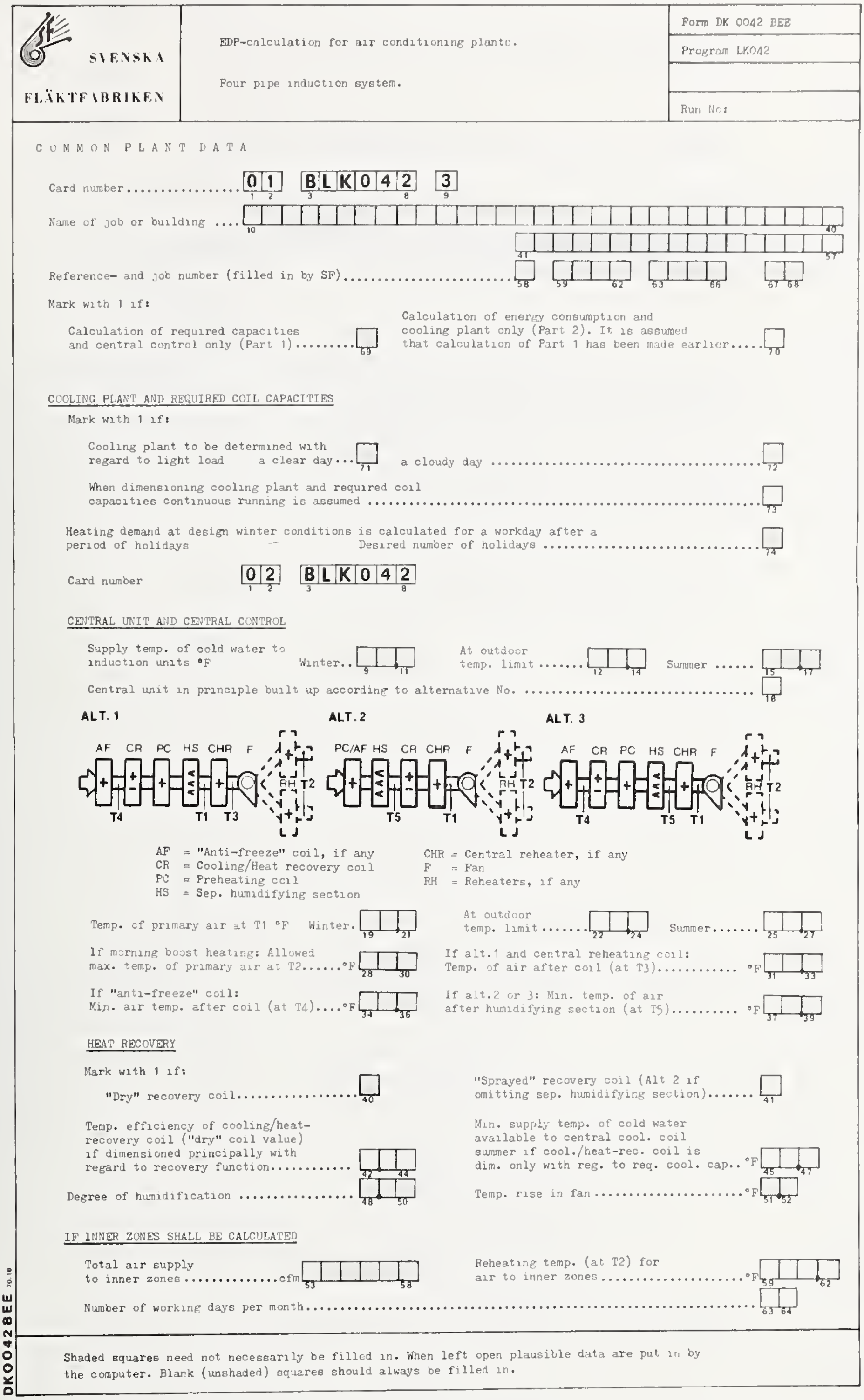


Fig. 2 Configuration of buildings with site plan



SVENSKA FLÄKTFABRIKEN

EDP-calculation for air conditioning plants.

Four pipe induction system.

Form DK 0042 BEE

Program LK042

Run No:

COMMON PLANT DATA

Card number.................. 01 (1 2) BLK042 (3 8) 3 (9)

Name of job or building .... ☐ (10–40) ☐ (41–57)

Reference- and job number (filled in by SF)........................ ☐ 58 ☐ 59–62 ☐ 63–66 ☐ 67 68

Mark with 1 if:

Calculation of required capacities and central control only (Part 1).........☐ 69

Calculation of energy consumption and cooling plant only (Part 2). It is assumed that calculation of Part 1 has been made earlier.....☐ 70

COOLING PLANT AND REQUIRED COIL CAPACITIES

Mark with 1 if:

Cooling plant to be determined with regard to light load a clear day...☐ 71 a cloudy day ......................................☐ 72

When dimensioning cooling plant and required coil capacities continuous running is assumed ...........................................................☐ 73

Heating demand at design winter conditions is calculated for a workday after a period of holidays Desired number of holidays ..............................☐ 74

Card number 02 (1 2) BLK042 (3 8)

CENTRAL UNIT AND CENTRAL CONTROL

Supply temp. of cold water to induction units °F Winter..☐ 9–11 At outdoor temp. limit.......☐ 12–14 Summer ......☐ 15–17

Central unit in principle built up according to alternative No. .................................☐ 18

AF = "Anti-freeze" coil, if any
CR = Cooling/Heat recovery coil
PC = Preheating coil
HS = Sep. humidifying section
CHR = Central reheater, if any
F = Fan
RH = Reheaters, if any

Temp. of primary air at T1 °F Winter.☐ 19–21 At outdoor temp. limit.......☐ 22–24 Summer.......☐ 25–27

If morning boost heating: Allowed max. temp. of primary air at T2......°F☐ 28–30

If alt.1 and central reheating coil: Temp. of air after coil (at T3)........... °F☐ 31–33

If "anti-freeze" coil: Min. air temp. after coil (at T4)....°F☐ 34–36

If alt.2 or 3: Min. temp. of air after humidifying section (at T5).......... °F☐ 37–39

HEAT RECOVERY

Mark with 1 if:

"Dry" recovery coil.....................☐ 40

"Sprayed" recovery coil (Alt 2 if omitting sep. humidifying section).......☐ 41

Temp. efficiency of cooling/heat-recovery coil ("dry" coil value) if dimensioned principally with regard to recovery function............☐ 42–44

Min. supply temp. of cold water available to central cool. coil summer if cool./heat-rec. coil is dim. only with reg. to req. cool. cap..°F☐ 45–47

Degree of humidification .................☐ 48–50

Temp. rise in fan.......................°F☐ 51 52

IF INNER ZONES SHALL BE CALCULATED

Total air supply to inner zones..............cfm☐ 53–58

Reheating temp. (at T2) for air to inner zones.....................°F☐ 59–62

Number of working days per month.........................................................☐ 63 64

DKOO42BEE 70.10

Shaded squares need not necessarily be filled in. When left open plausible data are put in by the computer. Blank (unshaded) squares should always be filled in.

Fig. 3 "Common Plant Data" form for the four-pipe induction system program

```
SHADOW BOUNDARIES IN AUG  ON ZONE  9  BEARING 166.0 DEGR.

- = SURFACE ELEMENT IN SHADOW OF NEIGHBOURING BUILDINGS
* = SURFACE ELEMENT IN DIRECT SUN

H= 7                        H= 8                        H= 9

* * * * * * * * - -         - - * * * * * * * *         * - - - * * * * * *
* * * * * * - - - -         - - - - - * * * * -         - - - - - - * * * *
- - * * * - - - - -         - - - - - * * * - -         - - - - - - * * * *
- - * - - - - - - -         - - - - - * * - - -         - - - - - - * * * *
- - - - - - - - - -         - - - - - * - - - -         - - - - - - * * * -
- - - - - - - - - -         - - - - - * - - - -         - - - - - - * * * -
- - - - - - - - - -         - - - - - - - - - -         - - - - - - - * * -
- - - - - - - - - -         - - - - - - - - - -         - - - - - - - * * -
- - - - - - - - - -         - - - - - - - - - -         - - - - - - - * * -
- - - - - - - - - -         - - - - - - - - - -         - - - - - - - * * -

H=10                        H=11                        H=12

* * * - - * * * * *         * * * * - - * * * *         * * * * * * * * * *
* * - - - * * * * *         * * * * - - * * * *         * * * * * - - * * *
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H=13                        H=14                        H=15

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- * * * - - - - - -         - - * * * - - - - -         - - - - - * * * * -
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- * * * - - - - - -         - - - - * - - - - -         - - - - - - * * * -
- * * * - - - - - -         - - - - * - - - - -         - - - - - - - * * -
```

Fig. 4 Calculated boundary of moving shadow

```
ZONE  2 AH   ASPECT   59.0 DEGREES   FACADE NE


DAILY COURSE OF TEMPERATURES AND THERMAL CAPACITIES PER MODULE
FOR SUMMER AND WINTER OUTDOOR DESIGN CONDITIONS

            SUMMER-WORKDAY                         SUMMER-HOLIDAY                   DESIGN WINTER DAY
                                                                                           ROOM UNOCCUPIED
      DAY WITH MAXIMUM REQUIRED      DAY WITH MAX.   DAY WITH MAX. ROOM TEMP.
      INDUCTION UNIT COIL CAPACITY    ROOM TEMP.                                  NORMAL PLANT RUNNING     CONTINUOUS RUNNING
                                                                                  TIME
      A CLEAR DAY IN MONTH  7        A CLEAR DAY     A CLEAR DAY IN MONTH  8      A CLOUDY WORKDAY
                                     IN MONTH  7                                  AFTER A PERIOD OF
                                                                                  5 CLOUDY HOLIDAYS
      OUT-  UNIT   PR.AIR  ROOM      OUT-   ROOM     OUT-  UNIT   PR.AIR  ROOM    OUT-  UNIT   PR.AIR  ROOM    UNIT   PR.AIR  ROOM
      DOOR  COIL   THERM.  AIR       DOOR   AIR      DOOR  COIL   THERM.  AIR     DOOR  COIL   THERM.  AIR     COIL   THERM.  AIR
      TEMP. OUTPUT OUTPUT  TEMP.     TEMP.  TEMP.    TEMP. OUTPUT OUTPUT  TEMP.   TEMP. OUTPUT OUTPUT  TEMP.   OUTPUT OUTPUT  TEMP.
HR    DEG.F BTU/H  BTU/H   DEG.F     DEG.F  DEG.F    DEG.F BTU/H  BTU/H   DEG.F   DEG.F BTU/H  BTU/H   DEG.F   BTU/H  BTU/H   DEG.F

 1    61.6      0      0   73.1      64.9   74.2     62.6     0      0    81.7    30.9  1040      0    59.0    1972   -460    72.0
 2    60.6      0      0   72.7      63.9   73.6     61.6     0      0    81.1    30.2  1067      0    59.0    1996   -460    72.0
 3    59.7      0      0   72.5      63.0   73.4     60.7     0      0    80.9    29.5  1087      0    59.0    2020   -460    72.0
 4    59.5      0      0   72.4      62.8   73.3     60.5     0      0    80.8    29.1  1107      0    59.0    2036   -460    72.0
 5    59.8      0      0   72.2      63.1   73.3     60.8     0      0    80.6    28.8  1119      0    59.0    2052   -460    72.0
 6    60.7    365   -460   72.2      64.0   73.3     61.7     0      0    80.6    28.4  5115   -440    59.0    2060   -460    72.0
 7    62.4   -409   -460   72.0      65.7   72.0     63.4     0      0    80.6    28.4  5099   -460    70.9    2060   -460    72.0
 8    64.7  -1482   -511   72.0 J    68.0   72.0 J   65.7     0      0    83.3    29.1  4849   -460    72.0    2036   -460    72.0
 9    67.0  -1466   -506   74.0 J    70.3   74.5 J   68.0     0      0    84.6    30.9  4579   -460    72.0    1972   -460    72.0
10    69.7  -1400   -483   73.8 J    73.0   74.1 J   70.7     0      0    85.3    33.3  4298   -460    72.0    1881   -460    72.0
11    72.1  -1214   -460   72.9      75.4   73.4     73.1     0      0    85.5    35.6  4028   -460    72.0    1790   -460    72.0
12    74.2  -1262   -460   72.0      77.5   72.1     75.2     0      0    85.1    37.4  3798   -460    72.0    1726   -460    72.0
13    75.9  -1278   -460   72.0      79.2   72.3     76.9     0      0    85.5    38.1  3623   -460    72.0    1702   -460    72.0
14    76.9  -1286   -460   72.0      80.2   72.3     77.1     0      0    85.6    38.1  3484   -460    72.0    1702   -460    72.0
15    77.3  -1270   -460   72.0      80.6   72.3     78.3     0      0    85.8    37.8  3361   -460    72.0    1710   -460    72.0
16    77.1  -1230   -460   72.0      80.4   72.3     73.1     0      0    85.8    37.4  3254   -460    72.0    1726   -460    72.0
17    76.1      0      0   72.0      79.4   72.1     77.1     0      0    85.8    37.0   679      0    72.0    1742   -460    72.0
18    75.1      0      0   74.5      78.4   75.4     76.1     0      0    85.8    36.3   694      0    65.2    1766   -460    72.0
19    73.5      0      0   74.3      76.8   75.2     74.5     0      0    85.6    35.6   718      0    65.2    1790   -460    72.0
20    71.5      0      0   73.9      74.8   75.0     72.5     0      0    85.5    34.9   742      0    65.0    1317   -460    72.0
21    69.3      0      0   74.2      72.6   74.8     70.3     0      0    85.1    34.2   766      0    64.8    1849   -460    72.0
22    67.3      0      0   73.8      70.6   74.6     68.3     0      0    84.9    33.3   794      0    64.8    1881   -460    72.0
23    65.2      0      0   73.6      68.5   74.1     66.2     0      0    84.6    32.4   817      0    64.6    1913   -460    72.0
24    63.2      0      0   73.4      66.5   73.9     64.2     0      0    84.2    31.6   845      0    64.6    1944   -460    72.0


PRIMARY AIR SUPPLY PER MODULE   29 CFM

SETTLED UNIT COIL COOLING CAPACITY PER MODULE   -1333 BTU/H
REFERRED TO 18 DEGR.F TEMP. DIFF. BETWEEN ROOM AIR AND CHILLED WATER AND DUE TO SPECIFIED SCOPE OF INDIV. TEMP. CONTROL
THIS SETTLED CAPACITY CAN ALSO BE WRITTEN:  -1482 BTU/H AT ROOM TEMP.  74.0 DEGR.F AND WATER TEMP.  54.0 DEGR.F

REQUIRED UNIT COIL COOLING CAPACITY PER MODULE FOR COVERING COOLING DEMAND ONLY  -893 BTU/H
REFERRED TO 18 DEGR.F TEMP. DIFF. BETWEEN ROOM AIR AND CHILLED WATER
OCCURS ON A DAY WITH LOWER OUTDOOR TEMPERATURES THAN ON DESIGN DAY DEPENDING ON ROOM TEMPERATURE DEMANDS

REQUIRED UNIT COIL HEATING CAPACITY

      NORMAL PLANT RUNNING TIME     5115 BTU/H
      CONTINUOUS RUNNING            2060 BTU/H
```

Fig. 5 Result of calculation of max. required unit coil cooling and heating capacities

```
C O O L I N G   P L A N T

DAILY COURSE OF TEMPERATURE AND THERMAL CAPACITIES PER MODULE FOR EACH ZONE THE DESIGN DAY FOR COOLING PLANT

IN THOSE ZONES WHERE SHADOWS FROM OTHER BUILDINGS (MSH) OCCUR THIS DAY THE DAILY COURSE IS ONLY SHOWN FOR THE MODULE
WITH THE HIGHEST TOP VALUE

A CLEAR DAY IN MONTH 7

          HR                              8       9      10      11      12      13      14      15      16     17     18     19

          OUTDOOR AIR TEMP.     DEG.F   68.0    70.3    73.0    75.4    77.5    79.2    80.2    80.6    80.4   79.4   78.4   76.8
          OUTD. AIR ENTHALPY    8TU/L8  18.5    19.7    20.5    21.6    22.5    23.0    23.6    23.8    23.6   23.0   22.9   22.1

ZONE   1 UNIT COIL OUTPUT       8TU/H   -128L   -132L   -126    -130    -134    -140    -142    -151    -159      0      0      0
 MSH     PR.AIR THERM.OUTPUT    BTU/H   -529    -543    -518    -537    554     -579    -586    -621    -656      0      0      0
         ROOM AIR TEMP.         DEG.F   72.0    73.6    74.1    73.2    73.9    74.5    75.4    75.7    77.0   78.3   81.3   81.1

ZONE   2 UNIT COIL OUTPUT       BTU/H  -1519J  -1489J  -1438J  -1341   -1356   -1356   -1356   -1356   -1341      0      0      0
 MSH     PR.AIR THERM.OUTPUT    BTU/H   -524    -514    -496    -463    -468    -468    -468    -468    -463      0      0      0
         ROOM AIR TEMP.         DEG.F   72.0    74.5    74.1    73.4    72.1    72.3    72.3    72.3    72.3   74.7   75.7   76.5

ZONE   3 UNIT COIL OUTPUT       8TU/H  -1294J  -1369J  -1420J  -1432J  -1395J  -1395JL -1306JL -1193L  -1180L     0      0      0
         PR.AIR THERM.OUTPUT    BTU/H   -551    -584    -605    -611    -594    -594    -557    -508    -503      0      0      0
         ROOM AIR TEMP.         DEG.T   72.0    74.5    75.7    76.5    76.7    76.1    76.1    74.7    72.9   72.7   81.0   79.6

ZONE   4 UNIT COIL OUTPUT       BTU/H   -925J   -976J  -1008J  -1031J  -1012JL -1058JL -1067JL  -958L   -958L      0      0      0
         PR.AIR THERM.OUTPUT    BTU/H   -548    -578    -597    -610    -599    -626    -632    -567    -567      0      0      0
         ROOM AIR TEMP.         DEG.F   72.0    74.3    75.4    76.1    76.6    76.2    77.2    76.4    75.0   75.0   82.4   81.8

TOTAL UNIT COILS COOLING
OUTPUT FOR ENTIRE PLANT         BTU/H   -197107 -198103 -184578 -181791 -181053 -183078 -177491 -184350 -183914  0      0      0

CENTRAL COOL. COIL OUTPUT       BTU/H   -114648 -139443 -165320 -190184 -212012 -229022 -239820 -243522 -240867  0      0      0

MAX. REQUIRED COOLING PLANT CAPACITY ON A CLEAR DAY HOUR 15 MONTH  7

   TOTAL UNIT COIL COOL OUTPUT FOR ENTIRE PLANT                      184350  BTU/H
   CENTRAL COOLING COIL OUTPUT                                       243522  BTU/H

                C O O L I N G   P L A N T                            427872  BTU/H
```

Fig. 6  Result of calculation of central cooling plant

# Standardized Method for Optimizing Building Construction and Heating and Ventilating Installations for Various Indoor Climate Criteria

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Heating and ventilating installations are used to create and maintain certain room temperatures under variable internal and external heat loads. If these room temperatures can be calculated for a given building, taking into account the thermal properties of the building, the calculations may be used as a tool for an optimal design of the building as a whole. It will be possible to choose between a building with a high thermal storage and relatively simple heating and ventilating installations, or a light building with more complex installations.

Such calculations are sometimes made for extreme climatic conditions. Those conditions are, however, not representative of the accumulated heat stress during a long period of time, and calculation cost or time does not allow a greater number of calculations. A method will, however, be presented whereby a few calculations make it possible to judge the resultant high room temperature over a period of one month. The method is applied to classrooms.

In a building code this method can be used to regulate the heating and ventilating installations. It is, however, then necessary to standardize most of the factors that go into the calculation and only vary those factors which have the most direct influence upon the design of the heating and ventilating system. It is also necessary to find a simple rating of the room temperatures, so that comparisons of different solutions will be feasible. This rating will be presented in addition to a few calculations showing some, typical results. This paper is based upon part of a research project, which has been published as Report 50/69 from the National Swedish Institute for Building Research [1] 1). A more detailed report, also giving results of calculations according to the method described in this paper, will be published in the near future.

Key Words: Building construction, classroom, duration of heat stress, heating and ventilation, indoor climate criteria, optimized design, performance requirements, room temperature.

## 1. Factors Affecting Room Temperature

Room temperature may be affected by many different factors. The correlations are perhaps shown most clearly by a diagram where the factors can be arranged in four different categories; i.e. internal loads, external loads, building and installations.

$$\frac{\boxed{A}}{\boxed{B}} + \boxed{C} + \boxed{D} = \boxed{E}$$

Internal loads are normally heat emission from persons, lighting, machines and equipment. The internal loads for the type of premises to be discussed here - namely, ordinary classrooms - consist mainly of heat from pupils and teachers and heat from lighting.

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Figures in brackets indicates the literature references at the end of this paper.

External loads are solar heat, transmission losses or transmission gains, influence due to the dependence of the ventilation system on wind conditions and so on.

The influence of both external and internal loads is affected by the building through, for example, its orientation, size of windows, thermal insulation etc. The heat storage properties of buildings are, however, at least as important; heat is stored, for example, in the structure of a building and in its furniture, fittings and fixtures. The volume of air stores air pollutants but also to a certain extent heat. The damping effect exerted by a building is thus in part fixed once and for all by the method of construction and the site, but can also vary, for instance, according to the use of adjustable sun shading devices and opening of windows.

However, the range of variation possible in a building is as a rule not sufficient to be able to compensate completely for the variations which occur in loads. The final compensation must therefore come from installations, in this case heating and ventilation systems.

## 2. Calculations of Room Temperature

The traditional method of calculating the requisite thermal effect of these systems is to take account of external loads, the building itself and an assumed room temperature. The heat flow is assumed to have attained an equilibrium, and it is further assumed that the values of the factors included in the calculations are not dependent upon time. This method involves great simplification of actual conditions, simplifications which can possibly be accepted for calculating the requisite heating effect, but which cannot be accepted for calculations of the necessary means of controlling temperature, or for calculation of the required cooling effects.

In the case of these calculations the internal loads, the dynamic variation in these loads and the effect of the building must be taken into account. These calculations are complicated and time-consuming. In the normal sequence of calculation it is, however, possible to make simplifications of varying extent. Different experts have suggested different simplifications, with the result that we have today a large number of different calculation methods to choose between. The simpler methods are suited to manual calculations, but the more detailed require computers. One problem is that different methods yield different values and it is hardly possible to find such simple correlations between methods that results from different methods can be converted and thus permit full-scale comparison. A method proposed by Dr Gösta Brown [2] of the Royal Institute of Technology in Stockholm was used for the calculations in this paper. This is probably one of the most comprehensive methods and provides excellent scope for computing most of the data which can be required for estimating the temperature of a room.

The way in which people react to the temperature of a room is partly conditioned by personal preferences and partly influenced by other factors, - for instance, clothing. Apart from this it is primarily three factors, air temperature, surface temperature and air velocities, which play a significant part. Speeds of air currents are, however, low and hardly possible to calculate. An estimate of the temperature of a room - the operative temperature - is therefore based on the prevailing surface and air temperatures.

In the case of the temperature levels dealt with here, heat exchange between a person and his surroundings is generally assumed to occur as much by radiation as by convection. This means that the mean between air temperature and the average surface temperature of walls, floors and ceilings can be taken as the operative temperature of the room. In calculating the average surface temperature, the temperatures of the respective surfaces are weighed in relation to their solid angle, meaning that the average surface temperature varies from point to point within the same room.

Thus the original diagram can now be modified as follows:

$$\begin{matrix}\boxed{A}\\ \\ \boxed{B}\end{matrix} + \boxed{C} + \boxed{D} = \begin{matrix}\boxed{E_1}\\ \\ \boxed{E_2}\end{matrix}$$

where $\boxed{E_1}$ = air temperature

$\boxed{E_2}$ = different surface temperatures and

$$\boxed{K} = \frac{\boxed{E_1} + \boxed{E_2}}{2}$$

where $\boxed{K}$ = the operative temperature

## 3. Relevant Point and Time

The importance of taking into account the point in the room to which the temperature estimate refers is illustrated in figure 1. The diagram shows conditions in the vicinity of windows whose surfaces usually show the greatest temperature deviation from the others. The part of the body facing a window does not experience the same operative temperature as the part facing away from it. The scale of temperatures gives the difference between these values.

The difference near to the window may be as much as 2°C, while at the wall of the room farthest away from the window it is almost non-existent. A difference in operative temperature of approximately 1°C is thus possible between these two points, which in view of the variations permitted in room temperature is quite considerable.

Despite the fact that there may be considerable differences in temperature between different points in a room it may be possible to disregard this complication when computing room temperature. In such a case the temperature of a point in the inner part of the room is calculated. Whether the value obtained will have to be adjusted for other points need only be considered when the result is to be assessed.

Even with due consideration for the points of view which have been put forward, this is not sufficient to render calculations of room temperature meaningful. We must also be careful to decide for which time calculations are relevant.

If calculations are to be used for dimensioning of air conditioning systems, they should, of course, refer to the occasions which make the greatest demands on the heat removal capacity of the system. However, the problem is as a rule somewhat different in schools. There, it is hardly a question of complete air conditioning but of trying to achieve a reasonably comfortable room temperature by other means. It is true that we may also wish to know the maximum value, i.e. the highest temperature likely to occur, but discomfort is also a question of duration. It is, for example, then possible to choose between calculating the frequency for days with different maximum temperatures or the true duration of high temperatures in hours.

The latter alternative is probably a better measure of the discomfort which occurs; i.e. in this case, deterioration in the performance of the schoolchildren.

## 4. Results of Temperature Calculations on Sunny Days

We want to show some example of results of calculations of room temperature as a function of different parameters. The calculations were made for a sunny day in May at the same latitude as Stockholm assuming a room facing due south and with an average outdoor temperature corresponding to the average for the whole month. This date was chosen in view of the fact that the school year finishes at the beginning of June and that climatic conditions in May are therefore of particular importance.

The calculations are, of course, also dependent upon many other factors but since the aim is in this case to present the method rather then the results, these factors have been omitted from this paper.

The influence of the structure is shown in figure 2. Case T represents a building with a high degree of heat accumulation; external walls and floor slabs are of concrete, internal walls of double plaster-board panels enclosing a layer of mineral wool. Case M is a building in lightweight concrete and case L a timber structure having mineral wool as thermal insulation. All the alternatives represent entirely normal constructions.

Figure 3 shows the effect of the size of windows and sun shading devices in a lightweight concrete building; window sizes were 4 $m^2$, 7 $m^2$ and 10 $m^2$ glazed area (double glazing) and as sun shading devices Venetian blinds between the panes or curtains on the inside were assumed. Compared to the solar heat gain through unshaded double-glazed windows, these arrangements admit 40 % and 60 % respectively.

The curves in this diagram are not entirely comparable if, for example, the minimum requirement is 300 lux at the desks with the poorest illumination. In some of the cases this value is not attained unless the lights are switched on. In rooms with the smallest window size fitted with Venetian blinds the lights must be left on all day which raises the corresponding curve so that it almost coincides with that for the largest window with Venetian blinds. With curtains the lights need only be on for an hour in the morning and the paradoxical result is that a less effective sun shading device gives a lower room temperature. This result is, however, not generally valid; it is a consequence of the special conditions which were assumed to prevail.

Figure 4 shows how different ventilation systems can affect room temperature. The building has a structure of lightweight concrete and windows with an area of 7 $m^2$ fitted with Venetian blinds ($F_1$ = 0.4).

The five curves represent three different rates of air flow; 7.5 $m^3/h$ per capita, 15 $m^3/h$ per capita and 30 $m^3/h$ per capita, combined with supply air of different temperatures, the lowest temperature being limited to + 20°C, or + 15°C, or outdoor temperature without a lower limit. The conditions thus illustrate a number of interesting cases. The rate of air flow of 15 $m^3$/h per capita is more or less the rate required according to current Swedish regulations. 7.5 $m^3$/h per capita is approximately the rate that can be expected in the case of many simple systems without mechanical ventilation or possibly with only an extract fan. 30 $m^3$/h per capita is the rate attained using ventilating equipment which is specially designed for classrooms. In the latter case it is possible to supply air with a minimum temperature of + 15°C without risk of draughts; with the simpler systems air is supplied without preheating and thus without minimum temperature. In addition, central air conditioning plants are to be found; these supply air to classrooms at room temperature, i.e. ca. 20°C, provided that the outdoor temperature is not higher.

The last type of system mentioned has a poor cooling effect and induces high room temperatures. The simplest system (7.5 $m^3$/h per capita) would seem to be better, but this is a consequence of the conditions regarding outdoor temperature which were assumed for the calculations. As soon as the outdoor temperature rises a degree or two the simplest systems will give the highest room temperatures.

## 5. Calculating for Longer Periods

The results published apply for school hours during just one day. These results, though limited, have of course a certain interest but in the majority of cases the prime aim is to extend the analysis to cover longer periods of time. This can also be done by determining for each day of the period the values of the calculation factors that have to be used.

Among the external factors it is primarily the temperature and the solar energy which vary. In the case of the internal factors the use of classrooms and the number of persons varies. As for the structural factors, it is conceivable that the use of the sun shading devices might vary or that windows might be opened.

However, it is not sufficient to know how each factor varies individually; the simultaneous variation must also be known. Since correlations do not always exist between the factors, it is impossible to try to find a logical pattern in the covariation of all these factors and it therefore becomes necessary to make certain simplifications.

With regard to sun shading devices, it is reasonable to assume that these are always used when the windows are sunlit during school hours, since this assumption is easy to handle mathematically and since it is impossible from the practical point of view to maintain a reasonable room temperature on any other grounds.

Opening of windows gives as a rule better ventilation and a lower room temperature. On the other hand, work in the classroom is disturbed by noise from outside and for this reason airing of rooms by opening windows cannot be relied upon as being a generally acceptable method of controlling room temperature. It is therefore reasonable in the case of the calculations we wish to make to assume that windows are closed during lessons. During breaks, however, they are assumed to be opened to reduce the room temperature.

The use of classrooms is of course governed by a time-table and this may mean that on certain days a room is used for a smaller number of school hours than on other days. The trend, however, is towards an increasingly intensive use of rooms and minor variations or displacements do not affect the room temperature to any great extent. It is therefore safe to assume that in the case of these calculations the daily schedule has not been changed. Similarly, it may be assumed that the number of persons present in the room is the same from day to day.

Thus outdoor temperature and solar energy remain, for both of which extensive meteorological statistics are available.

The mean temperatures per 24 h for four places over a period of thirty years are more or less normally distributed for each place with a standard deviation of ca. 3.5°C for months which are of interest in this context as shown in figure 5. The same would appear to be generally applicable. We have at any rate found that the monthly means of 30 places chosen at random vary considerably, but that the standard deviation lies between 3.0 and 4.0, Thus, using the monthly mean as a basis and having fixed the standard deviation at 3.5°C, it is possible to predict the average numbers of days that will have higher temperatures. A still higher degree of precision can be obtained if the prediction is based upon the temperature which represents the true standard deviation above the monthly mean temperature (Fig. 6).

As in the case of temperatures, differences exist between different times of the year and different places for solar energy on clear days. Furthermore, differences in room orientation must be taken into account. The difference for the same place and orientation are, however, not very great over a limited period of time (Fig. 7).

If we examine the simultaneous variation of outdoor temperature and solar energy, we find that the daily variation in the temperature is greater on sunny days than the average, and that high daily mean temperatures seldom coincide with maximum radiation of solar energy. When temperatures are more than one or two degrees higher than the monthly mean we can estimate the solar energy to amount to 80-85 % of the maximum value. [3]

The position is thus in brief as follows:

1. High classroom temperatures may be anticipated when a high outdoor temperature coincides with solar radiation.

2. The high outdoor temperatures have a frequency corresponding to normal distribution.

3. At these high outdoor temperatures solar radiation on clear days is approximately 80-85 % of the maximum radiation.

These correlations make it possible to obtain a good idea of the duration of the high room temperatures which can be expected by calculating the room temperature for the current outdoor temperatures and then weighting the results according to frequency. Each outdoor temperature is given a weight corresponding to normal distribution with a standard deviation of 3.5.

As many as 8-10 outdoor temperatures may be current at any one time. The total calculation volume is thus considerable. It can, however, be reduced by utilizing the fact that the correlation between outdoor temperature and the room temperature on the whole is linear. It is then sufficient to make, for example, three calculations and to interpolate and extrapolate the remaining values, figure 8.

## 6. An Aid to Construction

This method can now be used as an aid in the construction of school premises. It is possible to establish the temperatures that will occur at different orientations, the effects of the structure and the building materials and the effect of different heating and ventilation installations etc. (fig. 9). It is thus possible to make a completely objective choice of the combination which yields the best results. If the costs of the different alternatives are also included we can judge which will be the optimum solution from the point of view of building costs.

The different results obtained from the calculations show quite clearly that different structural factors can exert just as great an influence on room temperature as different heating and ventilating installations. We realize that it is futile to try to guarantee a good room climate solely by drawing up standards for installations. This point has also been proved practical in a large number of schools built in recent years. It is possible, though nowadays hardly economically motivated, to concentrate entirely on structural measures for controlling room temperature. Installations are essential and with the calculation demonstrated it is possible to adapt them according to the building in question. The method thus provides us with a means of making demands on the room temperature and from these demands derive the conditions for the installations. We have thus the chance of establishing highly functional standardized rules for performance requirements regarding heating and ventilation systems. Such a rule has been suggested, stating that "The classroom temperature, calculated according to this method, may exceed 25°C during, as a maximum, 20 % of lesson time in months of May".

## 7. Functional Standard for Heating, Ventilating and Air Conditioning Plants

Such standardization can hardly be considered without first including model values for all the factors wherever possible. Some of these values have been already touched upon, but not all of them. It goes without saying that the model values must be chosen with great care; they must be realistic for modern schools and modern work routines in schools, while at the same time deviations from the model values may not produce excessively large deviations in the final results. Nevertheless, however much care is taken deviations cannot be entirely avoided.

Here we should like to draw attention to one such deviation. As we said earlier, the statement made regarding the curve showing normal distribution for outdoor temperature is based on statistics from the years 1931-1960. This applies then for a long period of time but not necessarily for individual years. This means that in certain years the high outdoor temperatures may have longer duration, causing the high room temperatures also to last longer than intended. Conversely, it is possible to obtain shorter values. In other words, there is very little chance of checking in an existing building, if the standard is fulfilled. The standard will be purely for purposes of calculation.

It may be acceptable from the point of view of the community that a standard of this nature in the long run should give reasonable temperature conditions. For the individual pupil, however, this is unsatisfactory as he or she may be spending only one year in the classroom in question.

This weakness, if indeed it is to be regarded as such, would appear to be inevitable. It is, however, possible to prevent the most unsuitable conditions through choice of temperature limit or duration value in the standard.

Thus, in this case it is possible to use computer techniques for a standard concerning heating and ventilating installations in classrooms which is functionally constructed in that it adopts the permissible room temperature as a basis and compels us to take into account the thermal properties of the building and the purpose for which the premises are to be used. This standard is probably the first of its kind and should represent a great step forward in comparison with other regulations applied to date which have been proved largely incapable of preventing unsuitable classroom temperatures.

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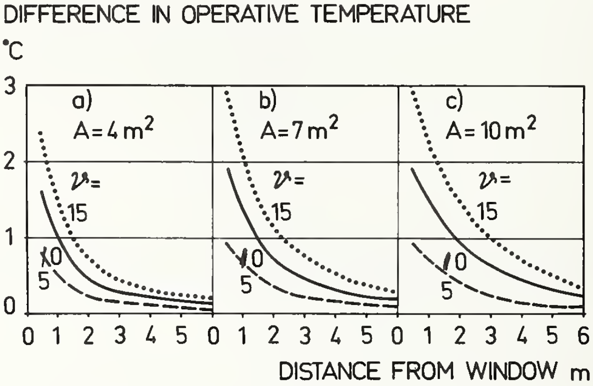

Figure 1

Difference in operative temperature (between two opposite directions) as a function of the distance from windows and glazed area. The values have been calculated on the assumption that the difference between the temperature of the glass and the mean of the room air and the temperature of the interior surfaces is 5, 10 and 15°C respectively. Cases a, b and c show the conditions for different sizes of window (A = = $4\ m^2$, $7\ m^2$ and $10\ m^2$).The point of reference lies on the centre line in relation to the window and on a level with the lower edge of the window.

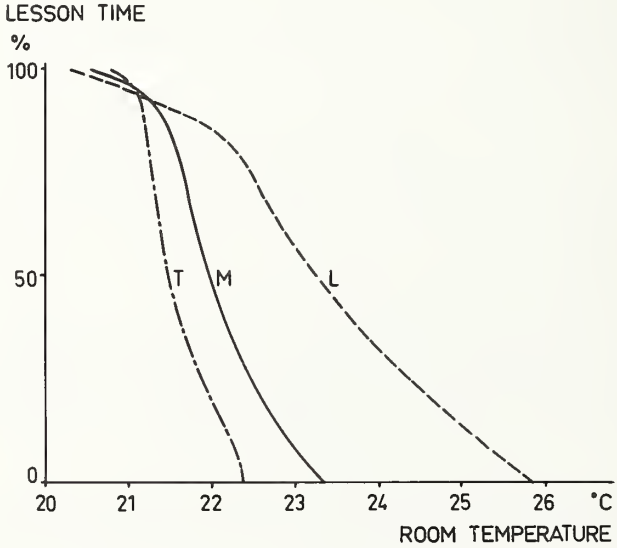

Figure 2

Duration of room temperature during lessons in classrooms in neavy (T), medium-heavy (M) and light (L) structures; the prerequisites are among others a sunny day in May, due south orientation of the room and the same latitude as Stockholm. Daily mean temperature + 11 °C.

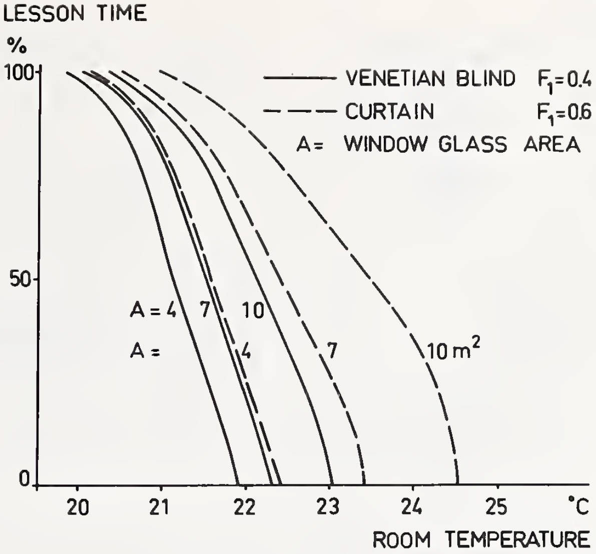

Figure 3

Duration of room temperature during lessons in classrooms with different sizes of window (A) and sun shading arrangements (F 1); the prerequisites are among others a sunny day in May, due south orientation of the building and the same latitude as Stockholm. Daily mean temperature + 11 °C.

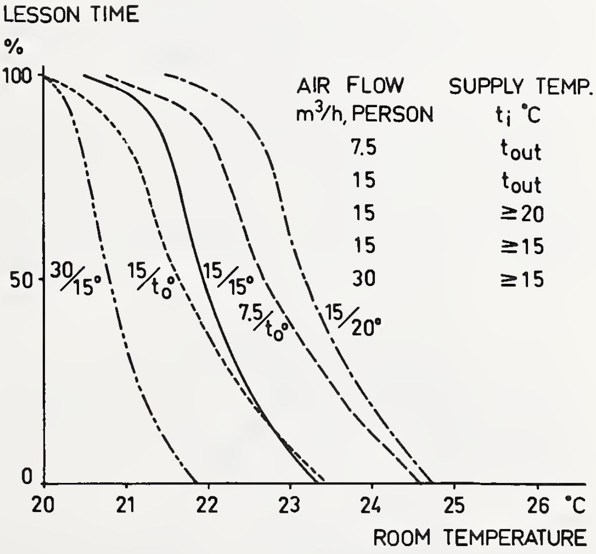

Figure 4

Duration of room temperature during lessons in classrooms with different rates of air flow and supply temperature; the prerequisites are among others a sunny day in May, due south orientation of the room and the same latitude as Stockholm. Daily mean temperature + 11 °C.

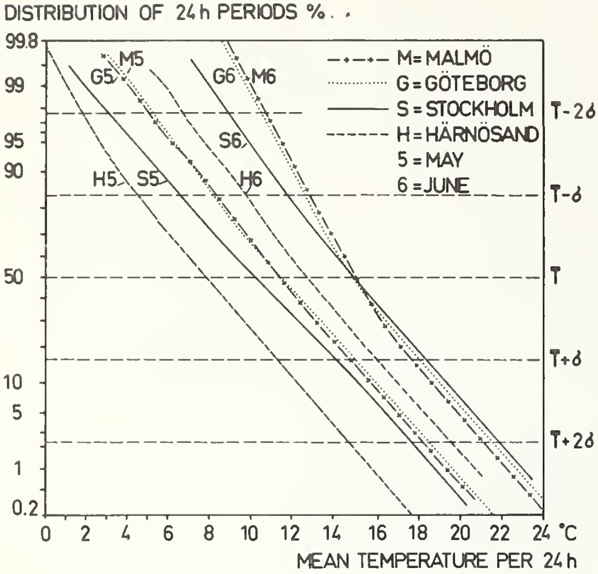

Figure 5

Frequency of different daily mean temperatures during the period 1931-1960. The values refer to four towns and the months of May (5) and June (6).

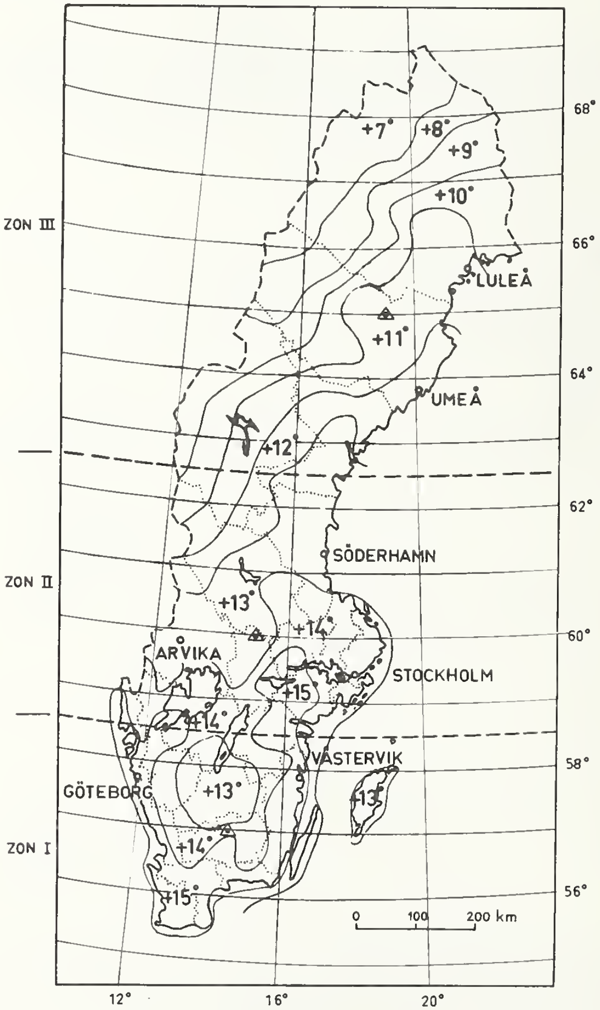

Figure 6

Isotherm map for May. The isotherms refer to the mean monthly value of the outdoor temperature + the standard deviation in the daily mean temperature.

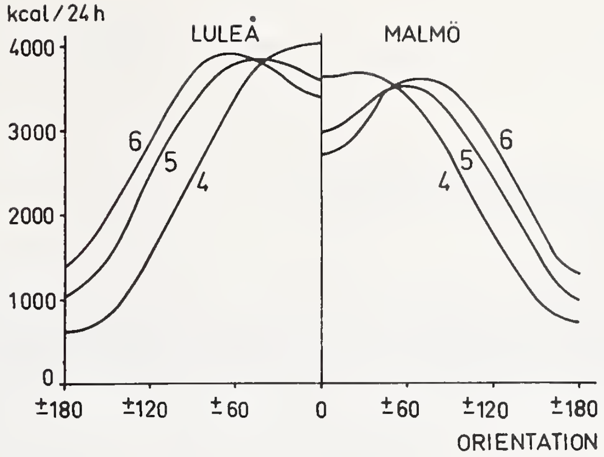

Figure 7

Heat gain from direct and indirect solar radiation through an unshaded double-glazed window. Values are given for two towns, one in southern Sweden and one in northern Sweden. The diagram is valid for the 15th day of each month, ground reflection r = 0.25.

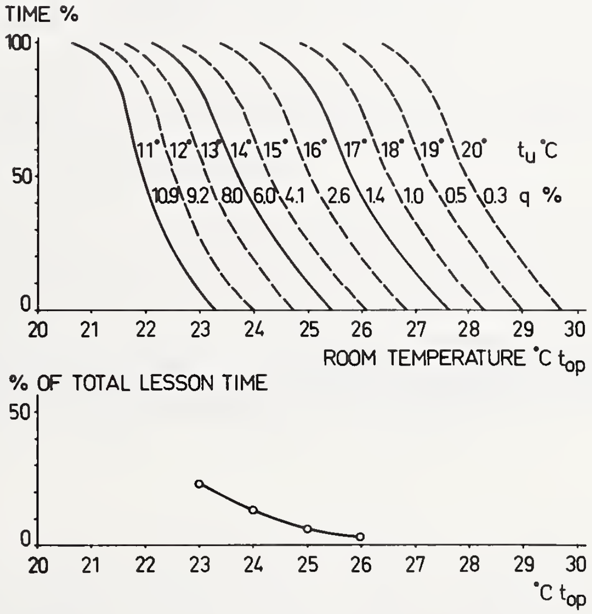

Figure 8

Duration of room temperature during lessons at different values of the daily mean of the outdoor temperature. The lower part of the figure is a sum of the curves in the upper part, account having been taken of the relative frequency q % of different outdoor temperatures. Prerequisites are among others sunny days in May, due south orientation of the room and the same latitude as Stockholm.

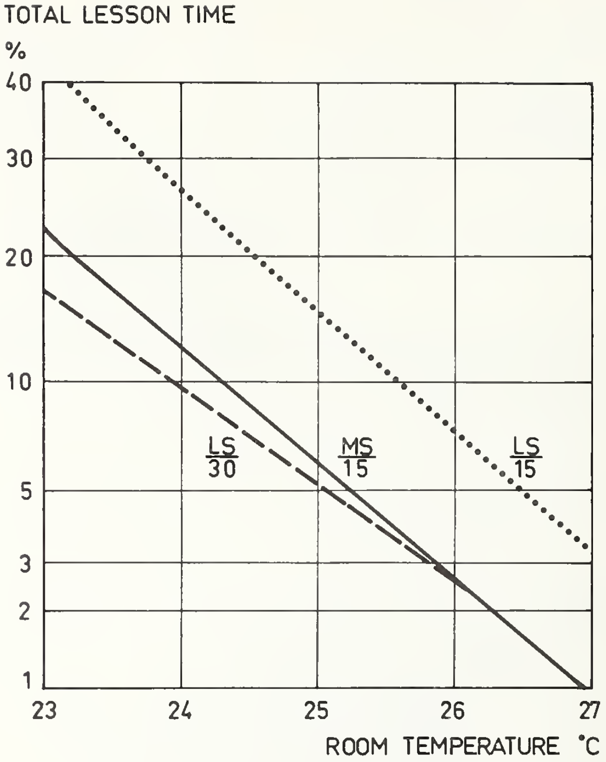

Figure 9

Duration of room temperature in classrooms with different combinations of structure and ventilation systems. More or less the same results are obtained in a lightweight building with an air flow of 30 $m^3$/h per capita as in a medium-heavy building with an air flow of 15 $m^3$/h per capita. Prerequisites are among others sunny days in May, due south orientation of the room and the same latitude as Stockholm.

# Designing Installations by Computer in Sweden

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Designing buildings and their installations has become more complicated. The building-time is shorter now than earlier because of improved building-methods. Increased demands for exterior and interior environment and more sophisticated installation equipments will increase the demands upon the designer. To make it possible for him to follow this development he has to use effective means of assistance. The computer can offer the designer this help, but first after a wide development work.

In 1968 the National Swedish Council for Building Research commissioned us to investigate the possibility for rational designing of installations by computer. The purpose of the investigation was to make an inventory of existing computer programs, to analyse and systematically compare them and give recommendations for the continuance of the work of development. The investigation showed that there are computer programs only for some routines of calculation. However they are made in a way which makes it difficult to use them practically for designing. The programs are written in different languages. There are also no standards for forms and documentations of programs. Due to this investigation the coming work will be concentrated on working out a basis for rational applications of data processing. The work will be done with grants from the National Swedish Council for Building Research according to the following principles:

- A data coordination group for the building trade, including the installation trade, has been appionted in order to investigate which program language is giving the best qualifications for a flexible use of new programs. Rules for description of programs, disposition of forms, presentation of results and so on will be drawn up.

- On the basis of the recommendations of the coordination group the calculation routines for designing installations will be programmed. This work has already begun by collecting theoretical formulas etc. We shall also investigate the possibilities how to use the computer for choosing installation components such as fans, pumps, boilers, valves and so on.

Key Words: Computer program, designing, electrical installations, environmental engineering, installation in buildings, mechanical installations, sanitary installations, Sweden.

## 1. Introduction

It is becoming more and more complicated to design buildings and the installations that go with them. Construction takes place more rapidly because of improved building methods. Future buildings will probably require even more installations, and more advanced versions of them, because of the increased demands that are being made on the indoor and outdoor environment. Equipment installed today must be capable of future modernisation or replacement, within the useful life of buildings as a whole, because of the rapid development that is taking place in the field of installation systems and products.

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The above factors make great demands on the planning and construction of installations, and will continue to do so. It is now more than ever necessary to plan well in advance, to make preliminary investigations and to carry out trial calculations to compare alternative systems with each other in terms of installation and running costs. Naturally comparison should also be made with other possible technical solutions, based on different layouts or methods of construction. The technical and economic calculations involved in planning a comprehensive and complex projekt cannot usually be carried out with the help of approximate formulae, diagrams and slide-rules alone. For these purposes computer techniques should be exploited to the full.

In 1968 the National Swedish Council for Building Research commissioned us to investigate the possibility for rational designing of installations by computer. The investigation was carried out with the above background. The aim was to make an inventory of the computer-based methods that are currently applicable to the planning of installations, and to suggest ways in which they could be developed. The results may be found in a report (1) that has been published,the contents of which are summarized in this paper. The investigation showed that there are computer programs only for some routines of calculation. However they are made in a way which makes it difficult to use them practically for designing. The programs are written in different languages. There are also no standards for forms and documentations of programs. Due to this investigation the coming work will be concentrated on working out a basis for rational applications of data processing.

## 2. The Available Choices of Machine

The choice of machine depends firstly upon which program or programs are available for the purpose. Programs owned by other users are often linked to some special type of machine, or even to a certain machine. It is usually possible to obtain the use of programs stored at other data centres.

Programs whose development has been supported by the Building Research Council are at present available without cost. The user must pay for the machine time that he uses in running the program. Certain non-recurring costs can be involved in adapting the program for use at the chosen data centre. The following choice of machines is available:

1. Rent of machine time at data centre.
2. Rental of a small machine.
3. Terminal of teleprinter type.

The various alternatives are discussed in more detail in the report. It is thought, however, that alternative 3 will prove to be the most useful, in the majority of cases.

## 3. Rationalisation using Computer Techniques

In the planning of installations calculations must be made at the various stages of construction. Shortage of time often leads to the acceptance of values based only on personal experience as the basic criteria for various judgements that have to be made. Approximate formulae and rules of thumb are often allowed to serve in place of exact calculation.

Such short cuts are often satisfactory, but they seldom allow alternative interpretations. What influence would different solar screens or differently constructed windows have? How would the air heaters and their control systems behave under different loads? What different lighting effects would be achieved by combining three different types of lighting element with four different floor coverings in a landscape office? How would the choice of ventilation system affect the heat dissipation from the light fittings? Naturally these questions can be answered by manual calculation, but the necessary time is not usually provided in the schedule. Computer techniques make it possible to weigh the different alternatives against each other, often for considerably less than it would cost to have the calculations for a single alternative made by an engineer. It is true that calculations can be made quite rapidly with the help of tables and nomograms but these are always based upon certain constants that cannot be changed without drawing up a completely new table or nomogram. Changing conditions can render such constants inapplicable, but the necessary changes are seldom made. The constants in a computer program can easily be changed, which makes data processing by this method more flexible. Experience has shown that builders and architects quickly realise that automatic data makes a more subtle analysis possible, even if they have originally been sceptical.

Few programs that are both suitable for the present purpose and intended for use via a data terminal are as yet available. Most of the available programs are very large and ambitious. Some of them are also closely linked to a certain type of machine. A number of computer manufacturers are interested in producing a "packaged program" for use via data terminals. For this to be realised, private users will have to make their programs available and further programs will have to be specially written for use via data terminals.

## 4. Existing programs

Several programs are currently available for the planning of installations. There are also programs written for the purpose of research, which could be adapted for practical use merely by inventory of programs in Scandinavia, in order to give a systematic overview of all these programs and their possible areas of use. They are described in a simple way in order to give design engineers a summary they can easily use.

The programs described in the report are listed below. The author or owner of each program is given in parantheses.

### a. Room Temperatures; Cooling and Heating Loads

Calculation of cooling and heating loads for a building and design parameters for comfort ventilation systems (AB Svenska Fläktfabriken). - The program calculates the loads for one module and one hour based on a sunny day and a cloudy day for each month of a metheorological normal-year and also for an extreme day in summer and in winter. The calculations are not bound to any special comfort system. The influence of moving shadows are also calculated.

Calculation of room temperatures (AB Svenska Fläktfabriken). - This is a similar program to the first, but no cooling of the air and also no supply air flow can be simulated.

These two programs are presented in a separate paper by W Boeke.

Calculation of room temperatures and of cooling and heating loads (G Brown, KTH). - This program calculates for one or more rooms during an arbitrary period of time, one quantity of each: cooling and heating load, room temperature, sufficient air flow, temperature of supply air. Max and min values can be given to one or two of the quantities. This is a research program and intended to be very flexible. It is presented in a separate paper by G Brown.

Calculation of room temperatures (C Allander, E Abel, KTH). - The program calculates the room-temperature in a multi-room building. It is limited to rooms with a facade wall and mechanical ventilation and is designed for the summer period in the southern and middle parts of Sweden, especially Stockholm. The program is based on a method, described in the ASHRAE Guide and Data Book 1963, p. 459-504.

Calculation of room temperatures and cooling and heating loads (Richard Nilssons Konstruktionsbyrå AB). - The program calculates the room-temperature, airflow, air inlet temperature, cooling and heating load for one or more rooms in a building. The calculation is made for each hour in any chosen month. The input data can be chosen freely. Either can the cooling load be calculated from a certain airflow, or the airflow can be calculated from the cooling load. The calculation can also be done with regard to the influence of moving shadows. The outdoor temperature is approximated to a sinus function.

Calculation of cooling and heating loads and of the energy requirements of a building (Ekono, Finland). - A series of programs have been developed by the Finnish company Ekono. One program calculates the heating load with regards to the wind pressure and unwanted ventilation. This program is described in JIHVE, March 1968, p. 357-368. Two programs calculate the cooling and heating load respectively the energy requirement. They are described in the ASHRAE Journal, Sept. 1967, p. 63-68.

### b. Temperatures in Structures

Calculation of temperatures in a structure of parallel layers (B Ludvigson Ingenjörsbyrå AB). The program is calculating the variation in temperature in a structure, consisting of parallel layers. The temperature on each side of the structure must be known, either constant or as a function of time. Maximum 15 parallel layers can be used. For each layer the temperature in the center is calculated. The program is specially written for testing different kinds of insulation.

Calculation of temperatures in a cross section of an arbitrary structure (Industridata AB). This program is calculating the temperature in a structure and the layers need not be parallel. The heat flow can be constant or suddenly changing. A heat source within the calculated zone is allowed. The border of the zone and the heat transfer coefficient may be a linear function of time. It is useful for complicated constructions.

### c. Heating Systems

Calculation of the design parameters for one and two-pipe heating systems and their distribution networks (AB Databeräkning). - The program calculates the type and size of the radiators, the pipe dimension, the friction loss, the valve dimension and the size of the distribution pump. A quantity list including all prices can also be calculated. Any make of radiators can be used. The program is useful when deciding the hot water temperature, the pipe dimension regarding energy consumption etc in order to find the lowest annual cost.

Calculation of the design parameters for single-pipe heating systems (Fellingsbro Verkstäder). The program calculates the size of radiators, valves, pump and a quantity list. The heating load can also be calculated. The program is specially designed for radiators made by the company Fellingsbro, Sweden.

### d. Water-pipe Networks

Calculation of the pressure and flow conditions in a network of water pipes (Industridata AB). The program is simulating a distribution network, e.g. a part of a town. The pressure and the flow for every connection in the network is calculated for a certain moment. Alternative consumptions of water can be simulated, as well as alternative data for pumps and reservoirs.

### e. Waste-pipe Networks

Calculation of flow conditions in a waste-pipe network (Industridata AB). - The program is designed for a rather large network. The flow is calculated as a function of time, depending on the flow on every terminal point.

### f. Stress in Pipe Systems

Stress calculations for pipe systems (Industridata AB). - The program calculates deformations, loads and stress forces in a pipe system due to temperature alterations, inside overpressure, dead weight, concentrated forces and moment, forces from flow of fluids, and forced deformations. The network system is supposed to be anchored in one or more points, with or without springs or sliding support.

Stress calculations for pipe systems (IBM). - The program calculates the pipe system based on an electrical analogical method and gives forces, moments, distorsion and stresses in different joints. No respect is taken to dead weight or the forces from flow of fluids.

### g. Ventilation Systems

Calculation of the design parameters of a ventilation system for constant static pressure (Wahlings Konstruktionsbyrå AB). - The program sizes ventilation duct systems based on the constant static pressure method. Duct types, dimensions etc are given in tables. The program calculates loss of pressure from the fan to each section and static regain. Friction loss can also be calculated only regarding to highest air speed.

Calculation of the design parameters for ventilation systems (AB Svenska Fläktfabriken). The program sizes ventilation duct systems regarding to less sheet area and most possible even pressure over air inlet or air outlet. The fan pressure can either be calculated or given as input. Necessary dampers are calculated. A quantity list can also be printed.

### h. Heat Loss from Pipes

Calculation of heat loss from pipes, flooring or ground (Hugo Theorells Ingeniörsbyrå AB). The program calculates the heat emitted from hot water pipe loops embedded in concrete, sand etc, for floors and pavements. The program can also calculate the necessary heat for melting the snow on pavements.

### i. Viscous Resistance

Calculation of viscous resistance in pipes (Hugo Theorells Ingeniörsbyrå AB). - The program calculates the flow, the speed, the dynamic pressure and the friction loss pr lenght unit for any desired diameter and fluid in a pipe. The result is given in tables.

### k. Sun, Shadow, Lighting

Irradiation from sun and sky in Sweden on clear days (G Brown, E Isfält, KTH). - The program is calculating the irradiation together with the position of the sun in the sky for each hour from sunrise to sunset on a clear day. The radiation transmitted through horizontal and vertical double-glazed windows and the irradiation onto horizontal and vertical surfaces are also calculated.

Calculation on shadows moving across building facades (G Brown, E Isfält, KTH). - The program calculates for any desired time on a sunny day the shadow on a facade, caused by narrow buildings or other objects. The facade is divided into an arbitrary number of squares, thus giving a picture in scale of the facade. The shadowed squares and the contour of the building are printed.

Calculation of the light distribution in a room (G Brown, E Isfält, KTH). - The program calculates how the irradiation in the state of diffused light is distributed in a room. The light source should be a part of a wall, e.g. windows and built-in fluorescent tubes. The irradiation is given for a small area, the position choosen freely in the room.

### l. Electrical Networks

Calculation of radial high and low voltage networks (Industridata AB). - The program can be used for continous control of existing radial high or low voltage networks or combined networks, concerning potential drop, loads, fuses, response to sudden loads, short-circuit currents and currents to earth. It is also useful as a simulation program when planning a new network.

### m. Lift Usage Patterns

Simulation of lift usage patterns (Asea-Graham). - This program can simulate lift (elevator) usage in office buildings, hospitals, hotel, department, stores and other similar buildings. The simulation is made by a simulator, special built for this purpose. Due to the number of floors, persons etc the waiting time after a call for the lift is calculated.

## 5. Suggestions for Future Development Work

### 5.1 Supplementary Research

Programs written for research purposes could often be of great practical use for engineering work. However, most of them require certain additions before they can be taken into practical use, e.g. data forms, program descriptions and adaptation to different machines. This kind of program development can most easily be done by the original author in consultation with design engineers familiar with the proposed area of use.

### 5.2 Continued Work

A review of the various routines used when planning installation projects was also included in the report. In view of this, a proposal for new programmes was made. Example of programmes: Heat exchanger system, Chimneys (dust distribution), Two-way valve system, Heat loss in pipe or duct network, Distribution of air within the duct system, Estimation of costs and quality, Optimum planning of a plant, Calculation of control circuits.

## 6. Selection of Products by Computer

### 6.1 Data Terminals

The day will probably come when each installation consultant will rent a data terminal, that is connected to a data centre via the telephone network. In this way, with only a small capital outlay, using the capacity of a large computer, one will be able to carry out very complicated calculations.

Details of the various products are needed continually during designing in order to facilitate selection of products. This information is generally obtained from catalogues and brochures. It is often very difficult to find just the right product amongst such extensive material. Some companies arrange their brochures in files; there is, however, a risk that some brochures are either missing or have become obsolete. There are so many pages for some code indexes, that it is impossible to check the complete details for each of the products.

## 6.2 Selection Methods

In order to make selection quicker and more accurate, data terminals can be used. This makes it possible to find the right product via an automatic selection system.

In order to utilize such a system, which should be based on an interchange of information between computer and designer, details of every product given by the manufacturer must be available. These facts are keys to the selection system.

One condition for developing a selection system is that the user must be able to find his product by freely specifying his requirements. He commences the dialogue with the computer by giving a general classification, for example, "pump" or "valve". There must only be a few general classifications to cover the whole range of existing products and any others that may be included later.

The computer answers by supplying a list of the various sub-group, for example, "pump for water", "pump for oil" or "manual two-way valves", "automatic controlling valve". The user then chooses a sub-group. These sub-groups should be small but still it should not be necessary to go through more than one of them in order to find the required product.

Once the sub-group has been decided upon the process of selection really begins. The computer then lists the various characteristics of the products within the group. With the help of these the user can select the features he wishes to specify. It can be details concerning size, material, media, geometrical design, sound, standard, brand etc.

It is the computer that does the selecting all the time, by giving information about the particular characteristics of the various products.

Such a system of selection has to be very flexible. It must be possible to select a product by carrying out a detailed dialogue as just explained, as well as being able to give merely the sub-group and requisite features, in order to shorten the dialogue.

Any changes in the existing products or details of new products must be easily included so that a manufacturer can immediately inform all the customers using the data terminals.

## 6.3 Handling of Information

The selection system leads up to an identification of the products fulfilling certain requirements. The selection system does not include complete information about each product. However, data techniques can even be used to help in this respect.

Three alternative methods for storing information entailing varying degrees of automation will now be described.

### a. Alternative 1

The information is stored as at present, in systematically arranged brochures and catalogues. The selection system refers you to certain pages in the files by giving the code and name of the product and the manufacturer. The appropriate pages are then picked out manually.

In this case the selection system facilitates the use of the files of brochures, besides which you are informed of products not represented in the files. In this way the files on relevant products are kept up to date.

It is only necessary in this case, to have a data terminal for calculation routines.

### b. Alternative 2

The terminal users need no brochure collections of their own. Such files are only kept in a pool, but in turn may be linked to a computer. All brochures are micro-filmed, thus enabling the whole range of products and their various details to be stored conveniently in a number of microfilm cartridges.

Every terminal user has a complete set of cartridges. He also has a so-called "reader-printer", i.e. an apparatus which enables one to get an instant reference view of any desired frame on a large screen. Prints of the relevant pictures can also be produced within a few seconds. The contents of all cartridges are reviewed and up-dated each year.

Any changes that occur during the year can be immediately put into a supplementary cartridge, which is distributed regularly to all customers.

The selection system is adapted to provide not only the name and manufacturer of the product but also to give information regarding the cartridge number of the frame in the cartridge. This alternative entails a certain amount of capital investment and some operating costs, but at the same time it cuts out the expense of producing and distributing brochures to all the terminal users.

### c. Alternative 3

The terminal user has no files at all. All information regarding products is stored centrally, on video tape, microfilm or something similar, and linked to a computer.

The system necessitates selection by dialogue with the computer via the terminal. When selection has been completed, relevant frames are projected directly onto a screen after being transferred from the data centre via the telecommunication network. Prints can also be obtained when required.

This system completely eliminates the risk of getting obsolete information, as any changes can be made quickly and are immediately available to all users.

Capital investment and operating costs will, however, be considerable. Despite this, there are a few such systems in the USA at present and there is a wide general interest in this type of system. As a result manufacturers of data and telecommunication systems are carrying out intensive research.

## 7. References

(1) Allan Weström & Teddy Rosenthal. Computer techniques for the planning of installations. National Swedish Building Research. Report R1:1970, 100 p.

# A Cost Analysis Service Helps Optimize Building Costs and Environmental Benefits

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During the past decade architectural glass performance has been improved to help achieve a comfortable indoor environment. Lower shading coefficients reduce solar heat gain 75%. Lower U-values reduce heat loss 65%. This performance results in cooler indoor glass surfaces in summer, warmer indoor glass surfaces in winter enabling easier system control and a more comfortable thermal and visual environment indoors.

A Cost Analysis Service has been developed to provide a direct cost comparison of the effect of improved fenestration on overall building costs. This service, called Building Cost Analysis, utilizes a computer program, cost estimates and rough project design criteria early in the design stage to obtain a first approximation of the effect of glass performance on construction and operating costs. This rough economic analysis obtained before glazing and mechanical system design is firm, may indicate that a more detailed professional study of glass selection best suited to the needs of the project is warranted.

The program computes initial heating and cooling equipment costs, annual heating and cooling operating costs and long term owning and operating costs for each glass under consideration.

Two case histories, a two-story office building in Madison, Wisconsin, and a 57-story office building in Columbus, Ohio, illustrate the potential savings high performance architectural glass products may have for the owner, greater design freedom for the architect and for the engineer, summer and winter insulating performance enabling more accurate control of the indoor system creating a year-round comfortable environment.

Key words: Shading coefficients, U-values, approximation, heating and cooling equipment costs, annual operating costs, present worth, owning and operating costs, potential savings, comfort.

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[1] Mechanical Engineer

# A Cost Analysis Service Helps Optimize Building Costs and Environmental Benefits

Building owners and managers who rent space to make money recognize that tenants pay premium prices for offices with large window areas. They know also that sophisticated control of temperature, air movement, humidity and radiant temperature within the comfort range is a necessity in the high rent district. Glare, condensation and drafts long have been associated with simple clear glass windows.

To meet these comfort needs new types of windows have developed and the computer has been put to work to provide an objective means for comparing glass performance and economics on specific jobs.

During the past decade architectural glass performance has been improved to help achieve a comfortable indoor environment. Lower shading coefficients reduce summer solar heat gain 75%. Lower U-values reduce winter heat loss 65%. This performance results in cooler indoor glass surfaces in summer, warmer indoor glass surfaces in winter enabling easier system control and a more comfortable thermal and visual environment indoors.

A Cost Analysis Service has been developed to provide a direct cost comparison of the effect of improved fenestration on overall building costs. This service utilizes cost estimates and rough project design criteria early in the design stage to obtain a first approximation of the effect of glass performance on construction and operating costs. This rough economic analysis obtained before glazing and mechanical systems design is firm, may indicate that a more detailed professional study of glass selection best suited to the needs of the project is warranted. Also, because it relies on a sophisticated computer program for processing and analyzing the data, it is quick and easy to use.

The Building Cost Analysis service utilizes 38 data input items. The program selects a summer and winter design day based on typical weather data for 30 geographical areas throughout the United States. Then, considering building orientation, materials, construction, energy systems and heating and cooling methods, determines the peak and annual heating and cooling loads. With this information, mechanical heating and cooling equipment size is determined and annual heating and cooling operating costs are estimated.

Also, the program uses estimated initial costs of the building, land, interest and taxes to compute the present worth and long term cost of owning and operating the building.

The Building Cost Analysis program is written in Fortran IV-G language and is run on an IBM-360-40. The memory area required for the program is 140 K. Each run takes approximately two minutes.

The program takes the input data (Chart I), accesses the design program from storage discs, calculates the latent (Design 1) and sensible (Design 2) heat loads thus determining the peak heating and cooling loads. Next, it accesses the energy program from storage discs and calculates the annual energy required for heating and cooling loads. Then, for the type of energy used - all electric, electric air conditioning - gas heating, all gas, the program estimates the annual heating and cooling operating costs. Finally, the program computes the building present worth and the annual cost of owning and operating the building.

The Building Cost Analysis program is designed to accept multiple runs for any geographical area or building variable.

Since the program has been designed to investigate the effect of various fenestration materials on buildings, glass performance is the variable we usually compare. Other building parameters for a specific product usually are fixed from run to run. The same program could be used to make other comparative analyses.

By introducing performance properties of alternate glasses, such as single clear glass vs. single tinted glass or clear insulating glass vs. reflective insulating glass, the program provides a direct comparison of the cost effect of these glasses on building cost. The most significant difference usually occurs in the initial cost of the mechanical equipment and the initial cost of the glass. Operating costs can be significant too.

Two case histories illustrate the effect architectural glass in buildings may have on initial and long term building costs.

## CASE HISTORY #1

The first case history is a two-story building in Madison, Wisconsin. This building consists of approximately 96,000 sq. ft. of rentable floor area. The proposed building will be occupied by 450 people during the hours of 8:00 a.m. to 5:00 p.m. Electric lighting will utilize 6 watts per sq. ft. Heating and cooling energy will be all-gas. The building facade will be all-glass construction - vision and spandrel area totaling approximately 15,000 total sq. ft.

The owner, architect and engineer selected three glass alternatives which were compatible with the aesthetic design. The glass alternatives selected were: 1/4-inch SOLARBRONZE Plate, 1-inch LHR SOLARBRONZE TWINDOW[2] (a fired-on reflective coating on the air space surface of the outdoor glass in an insulating unit) and 1-inch SOLARBAN (2) TWINDOW (an insulating glass unit with a metallic reflective solar control coating on the air space side of the outdoor light).

Table I illustrates the performance properties of the architectural glass alternatives and the installed cost of the glass per sq. ft. (The installed cost includes glass and installation charges).

Table II illustrates the Building Cost Analysis results. These represent building heating and cooling peak loads for each glass alternative. Note the improved performance (insulating value) of the more sophisticated glasses results directly in reduced heat gains and heat losses.

Table III is a summary of each glass alternative which includes the initial cost of the glass, the heating and cooling system costs and the cost difference between one glass and another. Initial cost comparisons reveal a potential savings of $15,000 if the more sophisticated reflective insulating glass - the SOLARBAN were used instead of the 1/4-inch SOLARBRONZE Plate. Thus, the owner and architect may elect to use the more sophisticated glass product at no increased initial construction cost.

Table IV summarizes the Building Cost Analysis program output on the building and shows that though the potential savings (initial and operating costs) are significant, the present worth and owning and operating costs remain relatively unchanged. This is typical when savings due to sophisticated architectural glass performance offset increased glass costs.

The principals used this information as incentive to investigate architectural glass alternatives more thoroughly. They elected to use the SOLARBAN product realizing that in addition to the potential savings, the glass provided added comfort due to warmer indoor glass surfaces in winter and cooler indoor glass surfaces in summer.

## CASE HISTORY #2

The second case history is a proposed 57-story office building in Columbus, Ohio. This building will consist of approximately 645,000 sq. ft. of rentable floor area; 3,000 people occupy the building from 8:00 a.m. to 6:00 p.m., electric air conditioning and gas heat are the energy requirements. The building facade is approximately 56% glass - 210,000 sq. ft.

The owner, architect and engineer selected three glass alternatives which they felt were compatible with the building aesthetic design.

The glass alternatives selected were: 1/4-inch SOLARBRONZE Plate, 1-inch SOLARBRONZE TWINDOW (insulating glass with the outdoor light tinted bronze), 1-inch SOLARBAN (2-3) TWINDOW (insulating glass with the metallic oxide solar control coating on the air space surface of both indoor and outdoor lights).

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[2] TWINDOW insulating glass construction consists of two lights of 1/4-inch clear or tinted glass separated by a 1/2-inch air space and retained in a stainless steel compression channel about the perimeter of the unit. Solar control reflective coatings normally appear on the air space side of the indoor or outdoor lights. (Figure 1)

Table V illustrates the performance properties of the architectural glass alternatives and the estimated installed cost per square foot.

Table VI illustrates the proposed building heating and cooling peak loads for each glass alternative. The improved performance properties of the sophisticated architectural glasses result in reduced heating and cooling equipment requirements.

Table VII is a summary of each glass alternative including the initial cost or the glass plus the cost of the heating and cooling equipment. The initial cost comparison reveals a potential savings of approximately $570,000 if the SOLARBAN TWINDOW were selected over the 1/4-inch SOLARBRONZE Plate.

Table VIII summarizes the Building Cost Analysis program output for the proposed building and shows, in addition to the potential initial savings, an approximate $9,500 savings per year in heating and cooling equipment operating costs. Also, present worth and owning and operating costs are shown.

The potential savings of over half a million dollars plus $14,000 per year in heating and cooling operating costs encouraged the architect and engineer to enthusiastically conduct professional studies on the glass alternatives to demonstrate to the principals' satisfaction that the potential savings through discriminate selection of architectural glasses is realistic.

Often the Building Cost Analysis program shows that a potential justification for use of more expensive sophisticated glass products leading to increased indoor thermal and visual comfort is achievable at little or no initial cost to the owner.

"Potential savings" is used throughout because the results shown represent a savings of heating and cooling equipment based on estimated "installed heating and cooling equipment" costs. Depending on the timing of a Building Cost Analysis, however, the "achievable savings" may be somewhat less. Potential savings can be realized if the architect and engineer conduct architectural glass studies early in the design process. This service provides rough cost estimates early in the design stage to encourage architects and engineers to conduct a professional design study of the economic feasibility of sophisticated, aesthetically desirable architectural glass products.

The case histories illustrate the potential savings high performance architectural glass products may provide the owner, the greater design freedom for the architect, and for the engineer, summer and winter insulating performance enabling more accurate control of the indoor systems creating a year-round comfortable environment.

The Building Cost Analysis program helps professionals relate an increased client awareness of potential savings with sophisticated high performance glasses which may lead to improved occupant comfort and satisfaction of the client's needs.

CHART 1 - BUILDING COST ANALYSIS

BLOCK DIAGRAM

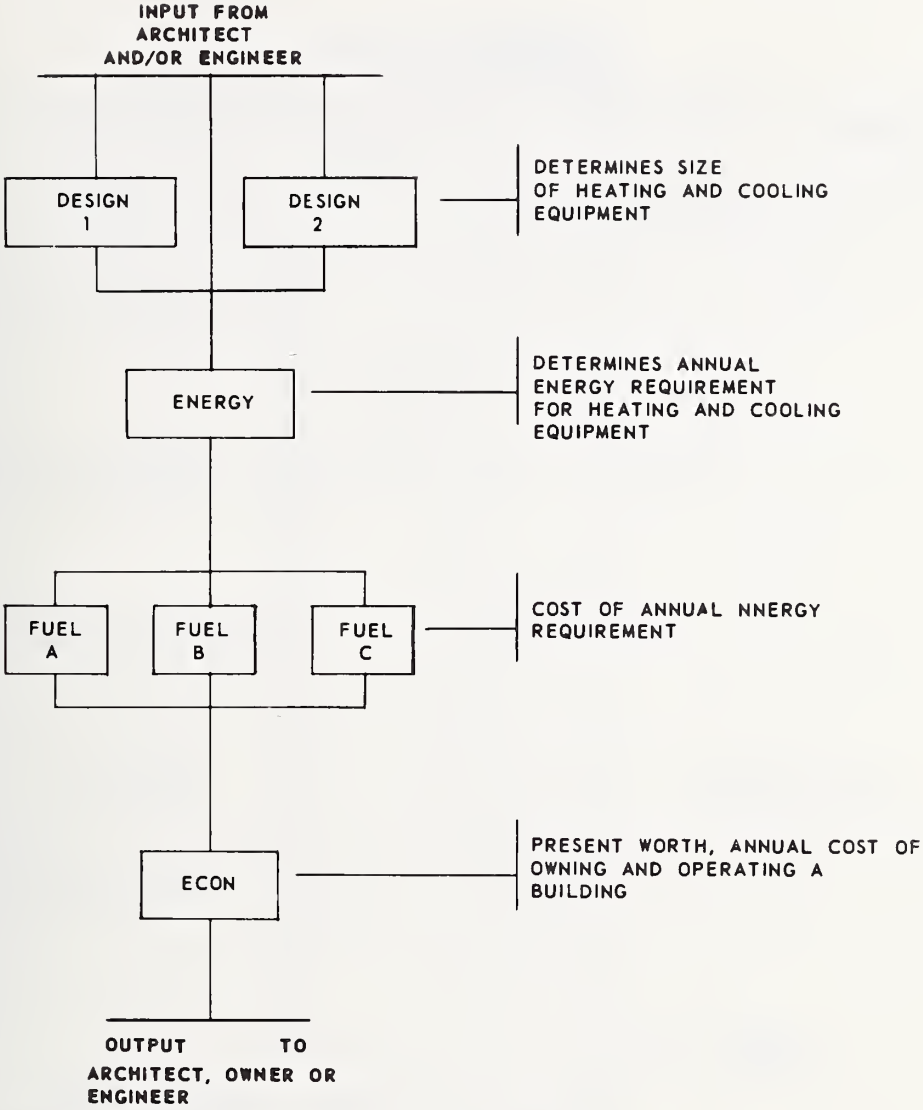
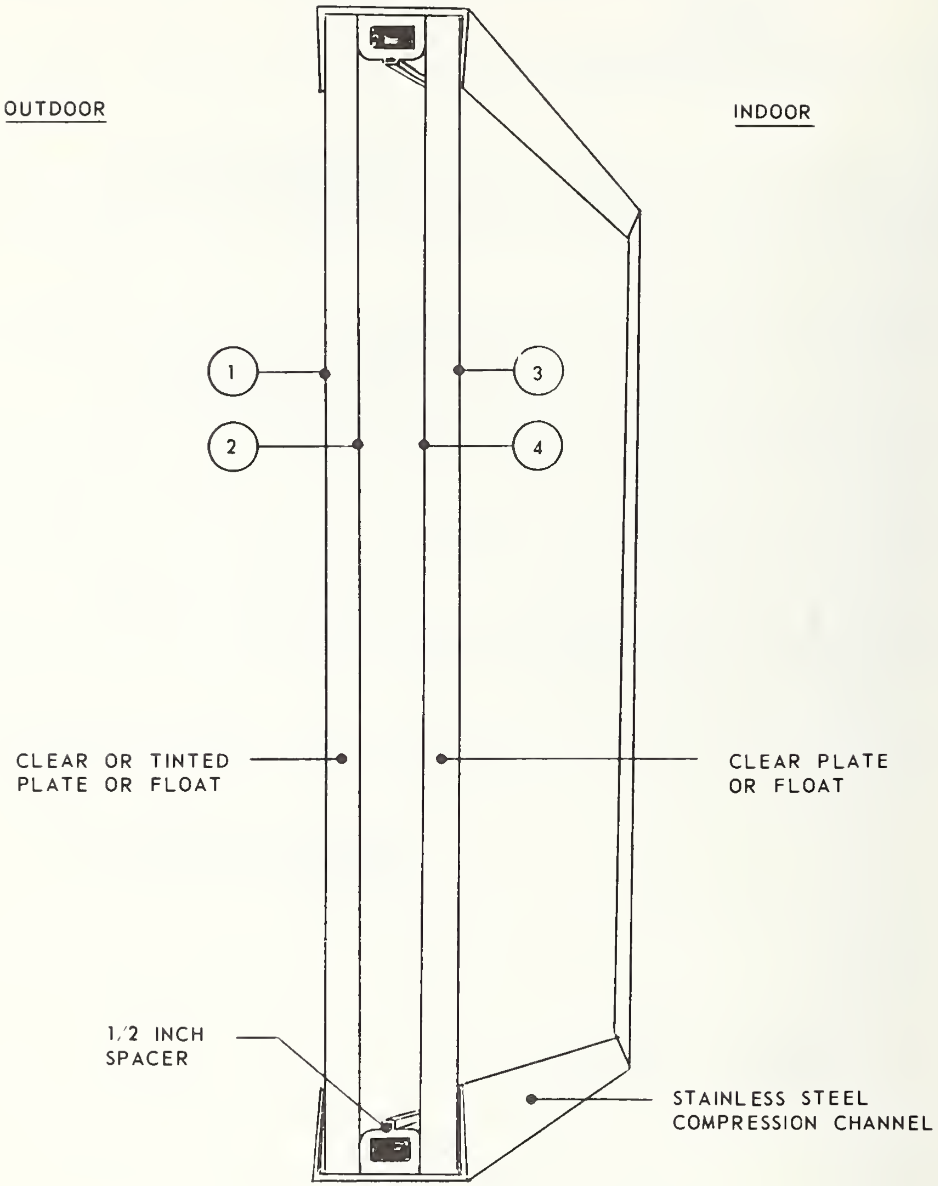


FIGURE 1 - TYPICAL TWINDOW CONSTRUCTION



## MADISON BUILDING

## TABLE I - GLASS ALTERNATIVES

| DESCRIPTION* | U-VALUE | SHADING COEFFICIENT | INSTALLED COST $ PER SQ. FT. |
|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | 0.8 | 0.54 | $ 1.25 |
| B. 1-INCH LHR SOLARBRONZE TWINDOW | 0.5 | 0.32 | 4.10 |
| C. 1-INCH SOLARBAN (2) TWINDOW | 0.30 | 0.16 | 3.60 |

*ALL WITH INDOOR SHADING

## MADISON BUILDING

## TABLE II - RESULTS

| DESCRIPTION* | COOLING | | HEATING | |
|---|---|---|---|---|
| | TONS | $ X 1000 | MIL-BTU | $ X 1000 |
| A. 1/4-INCH SOLARBRONZE PLATE | 657 | $ 657 | 11.6 | $ 198 |
| B. 1-INCH LHR SOLARBRONZE TWINDOW | 641 | 641 | 11.3 | 193 |
| C. 1-INCH SOLARBAN (2) TWINDOW | 631 | 631 | 11.1 | 190 |

*ALL WITH INDOOR SHADING

## MADISON BUILDING

## TABLE III - HEATING & COOLING SUMMARY ($ X 1000)

| DESCRIPTION* | INSTALLED GLASS COST | HEATING & COOLING EQUIPMENT COSTS | TOTAL | RELATIVE DIFFERENCE |
|---|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | 10.6 | 855 | 865.6 | - |
| B. 1-INCH LHR SOLARBRONZE TWINDOW | 34.8 | 834 | 868.8 | 3.2<br>INCREASE |
| C. 1-INCH SOLARBAN (2) TWINDOW | 30.6 | 820 | 850.6 | 15.0<br>SAVINGS |

*ALL WITH INDOOR SHADING

## MADISON BUILDING

## TABLE IV - COST ANALYSIS SUMMARY

| DESCRIPTION* | INSTALLED GLASS COST ($) | HEATING & COOLING EQUIPMENT COSTS ($) | HEATING & COOLING OPERATING COSTS ($) | PRESENT WORTH ($ per sq.ft. rentable floor area) | OWNING & OPERATING COSTS ($ per sq.ft. rentable floor area) |
|---|---|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | 10,600 | 855,000 | 18,800 | 53.04 | 3.09 |
| B. 1-INCH LHR SOLARBRONZE TWINDOW | 34,800 | 834,000 | 18,300 | 53.36 | 3.11 |
| C. 1-INCH SOLARBAN (2) TWINDOW | 30,600 | 820,000 | 18,000 | 53.01 | 3.09 |

*ALL WITH INDOOR SHADING

## COLUMBUS BUILDING

## TABLE V - GLASS ALTERNATIVES

| DESCRIPTION* | U-VALUE | SHADING COEFFICIENT | INSTALLED COST $ PER SQ. FT. |
|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | 0.8 | 0.53 | 3.30 |
| B. 1-INCH SOLARBRONZE TWINDOW | 0.5 | 0.42 | 4.50 |
| C. 1-INCH SOLARBAN (2-3) TWINDOW | 0.28 | 0.10 | 5.59 |

*ALL WITH INDOOR SHADING

## COLUMBUS BUILDING

## TABLE VI - RESULTS

| DESCRIPTION* | COOLING | | HEATING | |
|---|---|---|---|---|
| | TONS | $ X 1000 | MIL-BTU | $ X 1000 |
| A. 1/4-INCH SOLARBRONZE PLATE | 3,114 | 3,114 | 23.21 | 371.4 |
| B. 1-INCH SOLARBRONZE TWINDOW | 2,763 | 2,763 | 17.41 | 278.6 |
| C. 1-INCH SOLARBAN (2-3) TWINDOW | 2,226 | 2,226 | 13.06 | 209.0 |

*ALL WITH INDOOR SHADING

## COLUMBUS BUILDING

## TABLE VII - HEATING & COOLING SUMMARY ($ X 1000)

| DESCRIPTION* | INSTALLED GLASS COST | HEATING & COOLING EQUIPMENT COSTS | TOTAL | RELATIVE DIFFERENCE |
|---|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | $ 693 | 3,485 | 4,178 | |
| B. 1-INCH SOLARBRONZE TWINDOW | $ 945 | 3,041 | 3,986 | 192 SAVINGS |
| C. 1-INCH SOLARBAN (2-3) TWINDOW | $1,174 | 2,435 | 3,609 | 569 SAVINGS |

*ALL WITH INDOOR SHADING

## COLUMBUS BUILDING

## TABLE VIII - COST ANALYSIS SUMMARY

| DESCRIPTION* | INSTALLED GLASS COST ($) | HEATING & COOLING EQUIPMENT COSTS ($) | HEATING & COOLING OPERATING COSTS ($) | PRESENT WORTH ($ per sq.ft. rentable floor area) | OWNING & OPERATING COSTS ($ per sq.ft. rentable floor area) |
|---|---|---|---|---|---|
| A. 1/4-INCH SOLARBRONZE PLATE | 693,000 | 3,485,000 | 75,300 | 44.79 | 3.76 |
| B. 1-INCH SOLARBRONZE TWINDOW | 945,000 | 3,041,000 | 70,250 | 44.42 | 3.72 |
| C. 1-INCH SOLARBAN (2-3) TWINDOW | 1,174,000 | 2,435,000 | 60,800 | 43.64 | 3.66 |

*ALL WITH INDOOR SHADING

# Comparative Computer Analysis of the Thermal Cost Performance of Building Enclosures

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The computer programs on thermal-cost performance of building enclosure systems developed to be used in computer-aided architectural design are described as to logic, data structure and man machine interaction. The solar-climatic simulation approach using normal Weather Bureau data for predicting the thermal performance is compared to an hour by hour computer analysis of selected examples. A mathematical model of the total thermal energy system of a specific building is described and used to evaluate the relative importance of the building enclosure as well as compare computed data with that of metered energy. This study proposes that specific modeling of specific building thermal systems can be used to develop simulation packages for use in computer aided design.

Key Words: Building enclosures, computer-aided design, initial wall costs, man-machine interaction, owning cost, present worth, building mechanical equipment, thermal performance, solar-climatic simulation.

## 1. Introduction

The objective of the computer programs on thermal-cost performance is to provide the architect with information in the design development or selection of components for the building envelope, i.e., the walls and roofs. The programs can be used for a variety of comparative studies involving thermal cost trade-offs. Initial and total owning costs can be compared for specific building situations in specific locations.

The basic computer-aided design package was developed in 1967 and 1968 by Smith, Hinchman & Grylls Associates, Inc., Architects, Engineers and Planners of Detroit for use on an IBM1130. Subsequently, it was adapted to the IBM360/67 Time Sharing System of the University of Michigan. This program package and video tape instructions have been made available to the Department of Architecture for Educational purposes.

The objective of this paper is 1) to describe and evaluate the programs as a computer-aided design method 2) describe and evaluate the use of Weather Bureau data for such simulation and 3) to explore the concept of modeling of the total building energy system as a means of checking simulation approaches used in computer-aided design.

## 2. Design Development-Man and Machine

Many factors such as those related to the thermal, cost, luminous, sonic and visual performance are considered by an Architect-Engineer in the design development of the building envelope as part of the total building design. Of these, thermal performance is particularly significant since it has implications from the standpoints of comfort, direct and indirect costs and weatherability. The mechanical systems are dependent on the nature of the envelope thermal loads. Therefore, not only are decisions by the

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mechanical engineer required but effective design would indicate an optimizing approach or at least consideration of interdependent trade-offs between building components and mechanical equipment.

Since the computer program involves estimates of mechanical equipment costs, the thermal loads must necessarily be compatible with the design practice of the mechanical engineer, and as such he is logically involved in the entire process. Other facets of the problem of program development involve the need for flexibility. Combinations of advancing technology, changing building needs and variety of microclimates indicate a maximum of flexibility in any data structure. However, tempering this requirement is the need for simplicity of imput. These points together with the lack of computer background of many who might logically use the system indicates the nature of the challenge faced.

Experience to date indicates that the remote terminal is an ideal device for learning to use the programs and, in effect, to understand the system. Effort has been made to improve the man-machine interaction with these programs using conversational imput, default files for physical property and cost data along with an editing system. Once the user has learned the system he may exercise the option of using either batch or remote terminal as is appropriate in any particular case. Graphic display devices are a logical next step when such are readily available. The user should in effect be a team of a mechanical engineer and an architect or possess the insight of both for effective use of the computer programs.

The problem of computational cost must also be considered. The general objective is to obtain accurate information fast and at low cost. At one extreme is the complete design and detail evaluation of complete buildings for comparative purposes. The approach to simulation used in this system is intended to give specific effective information at a minimum cost. As an example, with the University of Michigan system the time used for the simulation-study is one fifth of that required for the detailed hour by hour analysis noted subsequently in this paper. On an IBM1130, runs will consist of from 5 to 10 minutes for several comparisons.

## 3. Data Structure

The heart of the problem from the user's standpoint is identification of the specifics of the building and finding required information for use in the system. The required data can be divided into five categories: wall type, mechanical systems, wall conditions, solar-climatic conditions and economic data.

The approach used relative to wall types is that of identifying separate components such as brick, block and glass, as shown in figure 2. Air spaces are identified as separate components. Inside shading is included with the glass as a separate component. A data assembly program TASSEM identified in figure 1, is used to select data from a master file. The data includes cost information and physical properties. As an example the user might type:

```
BRICK/EXTERIOR/4IN/
```

The computer would echo the data line with values in succession identified as: key number, thickness, density, conductivity, specific heat, initial cost, maintenance cost and maintenance interval.

```
14.1420  4.  130.  9.  .18  1.73  .12  20.  BRICK/EXTERIOR/4IN/
```

Transparent component are similar except the second, third, fourth items are, respectively, total solar energy transmission, solar heat gain factor and the "U" value.

The type of mechanical system is identified in similar manner:

```
UNITARY/AIRCOND/HWBASE/
```

From the master file, figure 2, the following cost information is identified: key number, cooling equipment costs, energy costs for cooling, heating equipment costs and energy costs for heating all in dollars per BTU. The line would be echoed as:

```
81.1000  .04  000004  .024  .0000011  UNITARY/AIRCOND/HWBASE/
```

Wall conditions, figure 3, are defined with information on relative position, area, percentage of glass, nature of ground reflecting plane and exterior shading as noted in figure 4. In the latter case the actual position of the glass needs to be identified in relation to the projecting surfaces. All of this data is sequenced and stored in a Building Specifics File, figure 2. With the three groups of data the user in effect identifies the building envelope and mechanical system.

The solar-climatic data is stored in a file and this is referenced in respect to a particular location. The data is developed in accordance with the block diagram, figure 5. The program identified as SOLCLIM in effect determines diurnal variations from a year's data of drybulb temperatures, and cloud cover and with recorded normals as input assembles the simulation data base of twelve prototypical days. The data for the July day is shown in table 1. The first line from left to right consists of latitude, elevation above sea level, outside surface film coefficient, solar radiation constant for that month, days from equinox, season, average percent of sunshine. The second line consists of projected diurnal temperature data for a day with average temperature values at 3 hour intervals corresponding to those days with an average cloud cover within the range of 31 to 69 percent. The average temperature is the normal value for the month. The third line is similar except it consists of expected highs with temperature pattern corresponding to 0 to 30 percent cloud cover. The maximum is average of normal daily maxima and extreme high as recorded. The fourth line is similar except that the cloud cover is 70 to 100 percent and lowest temperature is the average of normal daily minima and recorded extreme low.

Table 1. Solar climatic normals for a prototypical day in July.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | General | 42 | 750 | 4 | 344 | 68 | 1 | 39 | |
| | Time | 1 A.M. | 4 A.M. | 7 A.M. | 10 A.M. | 1 P.M. | 4 P.M. | 7 P.M. | 10 P.M. |
| 2 | Average | 64 | 63 | 64 | 72 | 77 | 77 | 75 | 67 |
| 3 | Highs | 71 | 69 | 69 | 80 | 85 | 85 | 82 | 73 |
| 4 | Lows | 63 | 62 | 62 | 68 | 70 | 71 | 68 | 63 |

The data noted here and subsequent information on wet bulb temperatures were keypunched directly from Local Climatological Data Summary sheets (1). The same information could be obtained from the Environmental Data Center in Asheville, N. C. and processed for the immediate locale related to the observation station.

Information on long term costs are based on the concept of present worth of future payments. Interest rate, years of anticipated use and the room temperature are entered in running the simulation programs. Increasing attention is being placed on long term or total owning costs. Although many more factors are involved in the total cost of a building, the factors included in this system do permit effective comparative studies.

## 4. Program Logic

### 4.1 Basic Approach

Three concepts, thermal neutralization, solar climatic simulation and present worth of future costs are fundamental to the logic of these programs. First, the concept that mechanical equipment and energy will be considered to counteract any loss or gain from the wall or roof; in effect, to obtain a thermally neutral surface. Involved are solar and climatic effects on both transparent and opaque surfaces. Second; fundamental to the prediction of the thermal performance is the simulation of the solar climatic environment. The system included in these computer programs is intended to be micro-oriented, that is, local temperatures, radiation intensities and ground reflections are considered. Third, in order to compare future payments for maintenance or energy with initial costs the concept of present worth of future payments is utilized. The equivalent of a dollar invested with identified interest for future payments is assumed to be comparable to the initial construction dollar.

*Note: Figures in brackets ( ) indicate literature referenced at the end of the paper.

In general, the approach has been to minimize the use of tables, in effect, to use the methods that originally were used to generate the tabular values. This distinction is a logical step from that of handbook engineering to computer-aided design. Further, it has been necessary to improvise from basic research certain projected empirical relationships to fill program "logic blocks" where such have not been advanced to the Engineering level. As an example, the interrelationship of cloud cover to direct and indirect solar radiation and related transmission through glass required programming expediency. In the latter case a second problem occurs when technical data on product performance is not available on the detail required.

The primary problem is that of using logic that accounts for changing parameters and is consistent in complexity throughout the program with intended use of the information.

## 4.2 Solar Radiation and Heat Transfer

The basic program logic used for heat transfer and solar radiation was developed in 1966 as part of research at the University of Michigan in reference to an evaluation of polyurethane roof system (2). This was prior to publication of 1967 ASHRAE handbook of Fundamentals (3) in which the first step was taken toward computer-aided design.

Direct solar radiation is computed for each of the hour-dates considered for both a clear sky and a state of cloudiness as indicated. Factors considered are: latitude, elevation above sea level, changing earth-sun distance, wall solar azimuth, wall vertical position and cloudiness. For indirect radiation, cloudiness, intensity of direct radiation (4) altitude of sun, wall position and irradiation (5) from the ground are considered. The distinction is kept between direct and indirect radiation in considering gain through transparent surfaces. The effect of direct radiation is considered only in sunlit areas as determined by exterior shading patterns.

Heat transfer through opaque walls is based on an adaptation of the exact method proposed by Mackey and Wright in 1944 (6) for determining periodic heat flow through walls and roofs. Decrement factors and lag angles for the first and second harmonics are utilized. Factors considered are absorptivity of outer surface, outside film coefficient, ambient air temperatures for a 24 hour period, direct and indirect solar radiation, the density, specific heat and conductivity of each component and the inside air temperature and inside surface film coefficient in respect to direction of heat flow and position.

The program logic now incorporated for solar effects on the glass has been adapted to use the coefficients (3) for transmittance and absorptance of glass. Total heat transfer is based on prorated square foot of wall accounting for the proportions of glass and opaque surfaces. Heat transfer from solar radiation and air to air temperature difference are considered for glass at the particular hour under consideration. Transfer through opaque is considered at a prior lag angle hour using solar radiation and ambient air temperatures of that hour.

In the research on polyurethane shell roofs (3) noted earlier limited field observations were made as a check on the logic related to solar radiation and periodic heat flow. The prototype structure consisted of seven, 14 foot hexagonal shells. A temporary enclosure and heating was provided for the test period. Iron-constantan thermocouples were installed in the roof and monitored manually at selected times over a two month period and hourly over selected days. Results for a typical day are shown in figure 6. Observations of cloud cover, wind velocity, air temperatures, surface absorptivity were intended to parallel the computer logic. On the basis of this correlation the idea of an hour by hour analysis was initially used in the development of the concepts presently used in the solar climatic simulation.

## 4.3 Thermal Loads

In the simulation program heat transfer for expected high and low temperatures are computed and stored separately in 12 by 8 arrays. As each surface in an identified zone is computed it is added to respective array. Peak heat gain and loss are determined by searching the accumulative values in the arrays. These are based on a clear sky and should, in effect, be the basis of design. Equipment costs are based on these values.

In addition, heat transfer under specified cloudiness with both highs and average temperatures are accumulated for each wall at the 96 prototypical hours for annual gross loads per se in these programs.

## 4.4 Costs

Initial costs of wall are the accumulative prorated sum of the costs of identified components. Initial cost of mechanical equipment for thermal neutralization is the product of peak loss and gain and the respective estimates of costs per BTU capacity for the particular system.

Energy costs are based on annual costs for energy for heating and cooling and are the products of accumulative loads and respective energy costs per BTU. The present worth (7) of future annual payments is used for the comparative figures. As an example for twenty equal payments the present worth equal 11.47 times the annual value.

Maintenance costs are based on the accumulated prorated sum of the present worth of future payments for maintenance on each of identified components.

Total owning costs then are the sums of initial, energy and maintenance costs.

## 5. Simulation-Thermal Cost Performance

The first example used to illustrate the application of the programs and as a test case for comparing the simulation programs with that of the detailed analytical program is the Cooley Building located on the North Campus, University of Michigan. The two story rectilinear building, figure 7 was built twenty years ago without air conditioning. The thermal performance of the building was such that an exterior shading screen was subsequently installed on the south glass wall. The summary of the output, table 2, indicates the trade-off for changes in the south wall with and without air conditioning. It should be noted that the south wall accounts for only one-quarter of the total envelope area and as such the substantial changes in heat gain from 124 to 58 BTUS per square foot for the south wall without and with the screen, is reflected in reduced values when considering the entire building surface. The total owning cost for air conditioning with the exterior screening is $7.45 per sq. ft. of envelope as compared to item 4 involving an alternate of venetian blinds.

Table 2. Thermal Cost Comparison for the Cooley Building, North Campus, University of Michigan, Ann Arbor, Michigan

| No. | Type | South Wall | Mech. Sys. | Initial Wall Cost | Mech. Equip. Cost | Maintenance Cost | Energy Cost | Total Cost | Gain/Loss BTU/Sq.Ft. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | SN | Basic | Heating Only | 2.67 | 0.75 | .39 | 1.28 | 5.08 | 28.5/-37.3 |
| 2 | SN | +Outside screen | Heating Only | 2.90 | 0.75 | .39 | 1.27 | 5.31 | 16.9/-37.3 |
| 3 | SN | Basic | Air Cond. | 2.67 | 3.60 | .39 | 1.96 | 8.61 | 28.5/ 37.3 |
| 4 | SN | +Venetian Blinds | Air Cond. | 2.78 | 2.44 | 0.42 | 1.72 | 7.63 | 19.9/-35.6 |
| 5 | S69 | +Venetian Blinds | Air Cond. | 2.78 | 2.60 | 0.42 | 1.69 | 7.50 | 19.2/ 34.1 |
| 6 | A69 | Venetian Blinds | Air Cond. | 2.78 | 2.99 | 0.42 | 1.83 | 8.03 | 22.2/-38.2 |

SM - Simulation Normals; S69 - Simulation Data Equivalent for 1969
A69 - Detailed "Year-Hour" Analysis -
All values are for a prorated average square foot for the five surfaces.
Basic south wall consists of brick and cinder block with 71% clear glass.

Items 4, 5 and 6 in table 5 are for the same building condition, in effect, the same Building Specific File is used in each. The simulation data for Ann Arbor is used in item 4 along with simulation program WTHERM, figure 8. Item 5 involves 1969 values in a simulation form, figure 5. Item 6 involves results from program WTHERMA, figure 9, a detailed analysis for 1969. The results are not exact but reasonably close for comparative purposes.

The second example the Institute of Science and Technology on North Campus, University of Michigan, involves a unique shadow box shading device. Results from the simulation study indicate an initial cost of glass of $3.10 and mechanical equipment cost of $10.20 and present worth of energy costs of $4.30 per sq. ft. of glass. The minimum shadow during period of gain for the west wall is 22 percent, figure 10. The cost trade-off for the shading device for the west wall is $2.57 per sq. ft. of glass. This was obtained by another run omitting the exterior shading.

## 6. Modeling of Building Thermal Energy Systems

It is proposed that the accuracy of the simulation approach can be evaluated by modeling the total thermal energy system for a building and comparing the prediction of energy consumption with that metered. The Undergraduate Library, figure 11, at the University of Michigan was used as a case study. The building has nearly a constant year around occupancy, further, monthly turnstile figures, recorded electrical energy consumption for the building and monthly recordings of condensate were available. The computer program system used for this building is blocked out in figure 9. Program WTHERMA referred to in the previous section was used to obtain the 1969 3-hour interval heat loss or gain of the total building envelope. Although the building appears simple in form, 12 separate surfaces were identified to cover combinations of brick and glass, stone and glass and porcelain steel panels and glass combinations. The maximum heat loss for the 63,600 sq. ft. of envelope is only 16.1 BTUS per sq. ft. and the gain of 7.1 BTUS, indicating a minimum influence of the envelope.

The program ULGUM in effect, is a model of the energy aspects of the air conditioning system. The system is a central system consisting of 6 supply fans with a total capacity of 163,000 C.F.M., a reheat system for zone control, a hot water convector system and a chilled water system using steam in Lithium Bromide absorption machine with a cooling tower. Cooling equipment was turned on April 28 and off on November 2 in 1969. On the same dates basic control of outside and return air damper were changed. Involved in this program are predictions of room temperatures and relative humidity of room air. For each of the 2920 three hour periods in the year, room conditions, percentage of outside air, as well as required wall heat, reheat and cooling are determined. Lighting and number of occupants are predicted for that specific hour. A summary for each month is included in table 3. Several runs with different cold deck temperatures and slight-room temperature variation resulted in sufficient difference in total energy to indicate that the preciseness of system control is essential for closer correlation.

Table 3. Thermal Cost Comparisons for the 1969 Operation of the Undergraduate Library, University of Michigan

| | Envelope-Energy | | Occupancy | Reheat | Cooling | Total | Metered | Total |
|---|---|---|---|---|---|---|---|---|
| Month | Gain | Loss | Gain-Energy | Energy | Energy | Energy | Energy | Cost $ |
| Jan. | 0 | 494.5 | 1,113.9 | 1,471.4 | 0 | 1,963.9 | 1,600.0 | 3,224. |
| Feb. | 0 | 395.9 | 1,180.9 | 1,338.6 | 0 | 1,734.4 | 1,250.0 | 2,844. |
| Mar. | 0.4 | 362.2 | 1,053.9 | 1,758.9 | 0 | 2,121.1 | 1,100.0 | 3,478. |
| Apr. | 4.2 | 180.8 | 1,133.0 | 1,139.9 | 0 | 1,319.7 | 1,821.0 | 2,164. |
| May | 18.7 | 103.6 | 712.6 | 1,632.5 | 577.3 | 2,313.3 | 2,120.0 | 3,793. |
| June | 36.8 | 53.5 | 832.1 | 1,251,541 | 2,574.6 | 3,878.6 | 2,011.0 | 6,360. |
| July | 72.6 | 7.1 | 705.0 | 1,340,173 | 5,200.6 | 6,547.9 | 5,103.0 | 10,737. |
| Aug. | 83.2 | 7.1 | 731.9 | 1,222.5 | 4,523.0 | 5,752.6 | 5,747.0 | 9,433. |
| Sept. | 16.6 | 66.1 | 873.6 | 1,269.0 | 2,293.3 | 3,628.4 | 6,246.0 | 5,949. |
| Oct. | 0.8 | 197.1 | 1,016.1 | 2,563.5 | 614.9 | 3,375.6 | 4,449.0 | 5,535. |
| Nov. | 0.1 | 330.3 | 1,046.5 | 1,642.6 | 0 | 1,972.9 | 1,579.0 | 3,235. |
| Dec. | 0 | 463.1 | 1,083.9 | 1,661.4 | 0 | 2,124.5 | 2,266.0 | 3,484. |
| Total | 233.3 | 2,661.1 | 11,483.3 | 18,291.1 | 15,783.6 | 36,734.8 | 35,292.0 | 60,244. |

Note: Total energy equals the sum of envelope loss, reheat and cooling.
All energy values are noted in MIL BTUS
Costs are based on a projected steam cost of $1.65 per 1000 lb.

The relative influence on costs of the envelope are shown in table 4, for the Library and a hypothetical library occupancy for the Cooley Building, Case II. The latter having a larger wall to floor area ratio and a thermally responsive glass south wall. The system cost figures used in this case have been adjusted to be consistent with those used for equipment and energy in the Library, Case I.

Table 4. Comparison of Building Envelope and Mechanical Equipment Costs in Buildings

| | Case I.<br>Undergraduate Library | Case II.<br>Cooley Bldg.<br>Hypothetical Library |
|---|---|---|
| Floor Area | 136,680 sq. ft. | 26,072 sq. ft. |
| Envelope Surface Area | 63,363 sq. ft. | 28,644 sq. ft. |
| Envelope Cost | $ 1.46 | $ 3.06 |
| Equipment Costs for Envelope Neutralization | $ 1.64 | $ 3.77 |
| Equipment Cost for Total Building | $10.08 | $10.99 |
| Energy Cost for Envelope Neutralization - Present Worth 20 Years | 0.40 | 2.30 |
| Energy Cost for Total Building - Present Worth 20 Years | 6.19 | 8.23 |

Cost Values are in $ per sq. ft. of floor area.

## 7. Conclusions

The simulation approach for identifying wall performance is an effective computer-aided design tool. Expansion of the concept to the total system is required to give the design team an accurate cost performance picture of the building.

The use of normals from accumulative Weather Bureau Data for this simulation has been compared to that obtained from a detailed year-hour analysis. The same procedure should be used for a larger number of years to provide a statistical base for correlation. Detailed checking of program logic or blocks can be accomplished by isolating the parameters for specific correlation with field measurement of temperatures.

The results of the total thermal energy study are inconclusive but do indicate the potential not only as a check for this simulation approach but as a direct benefit to the owner in optimizing the operation of his building. Further, idiosyncrasies of the building operation, such as down time for cooling equipment; and accuracy of metering must be taken into account.

## 8. References

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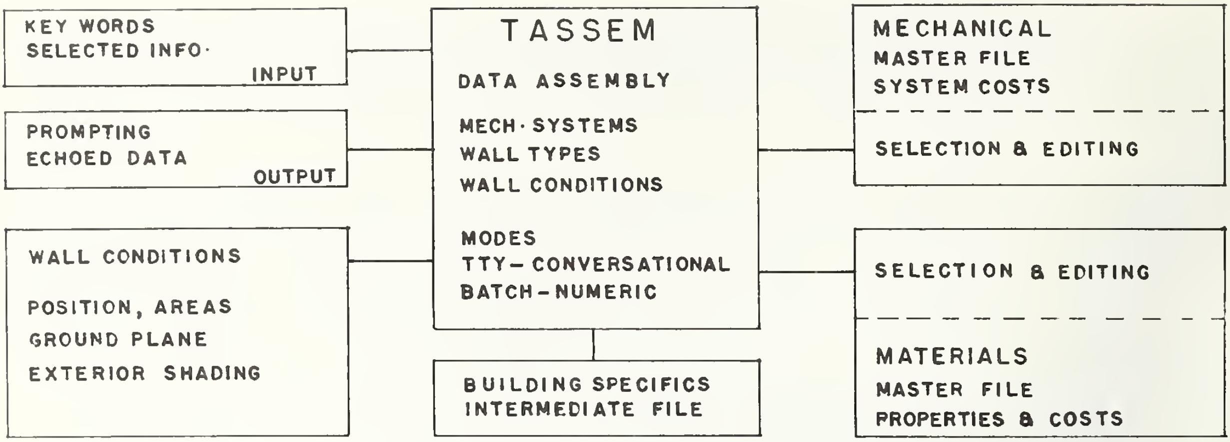


Figure 1  Block diagram - data assembly of building specifics

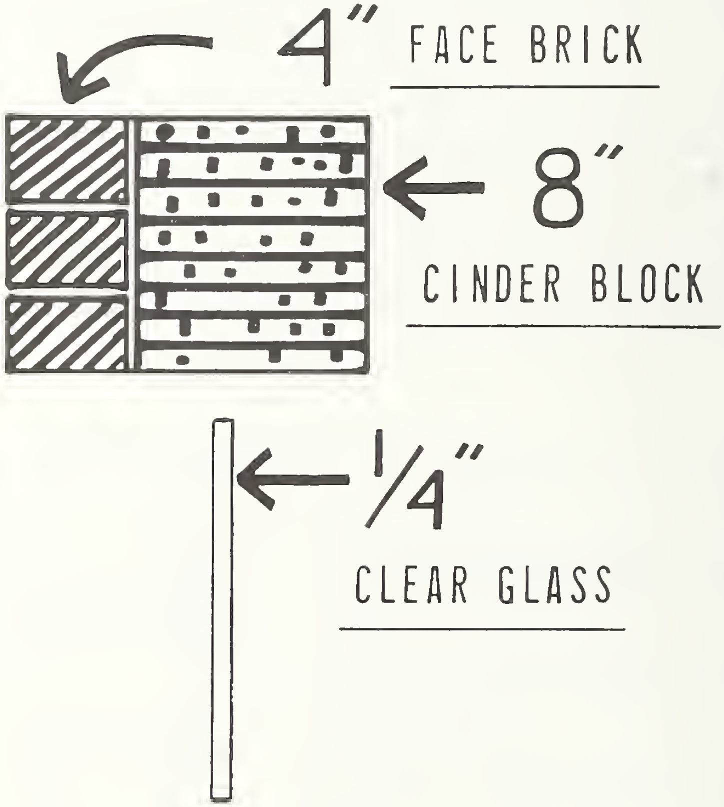


Figure 2  Wall types - Cooley Building South Wall

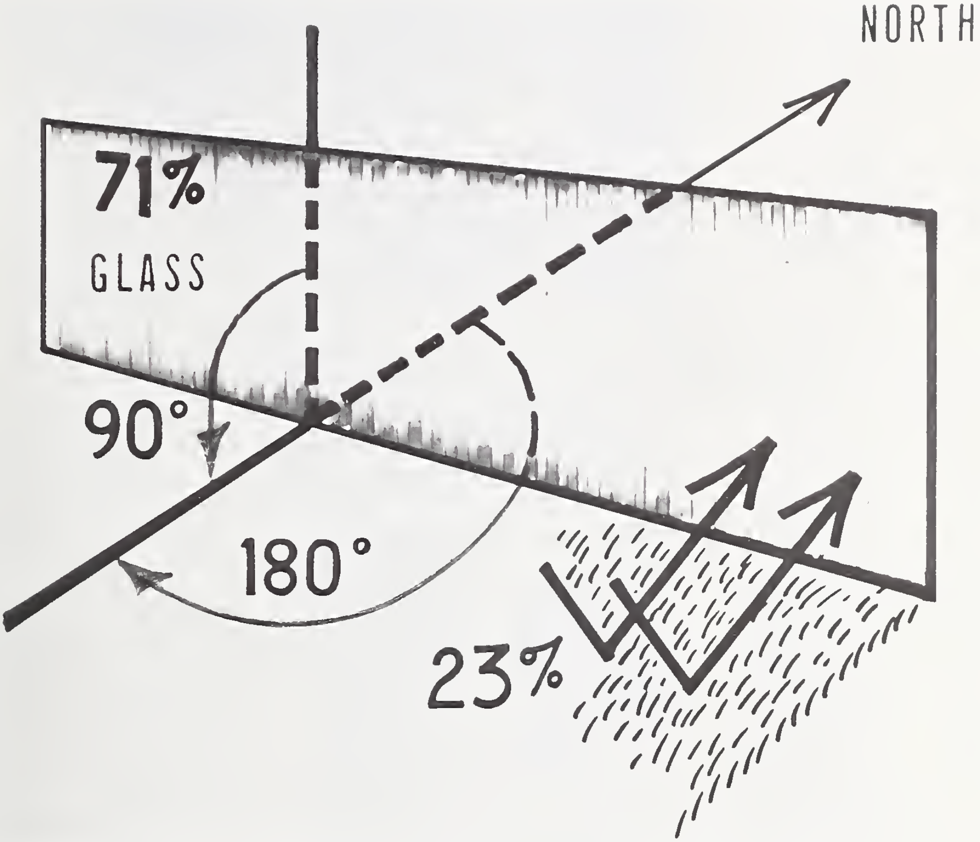
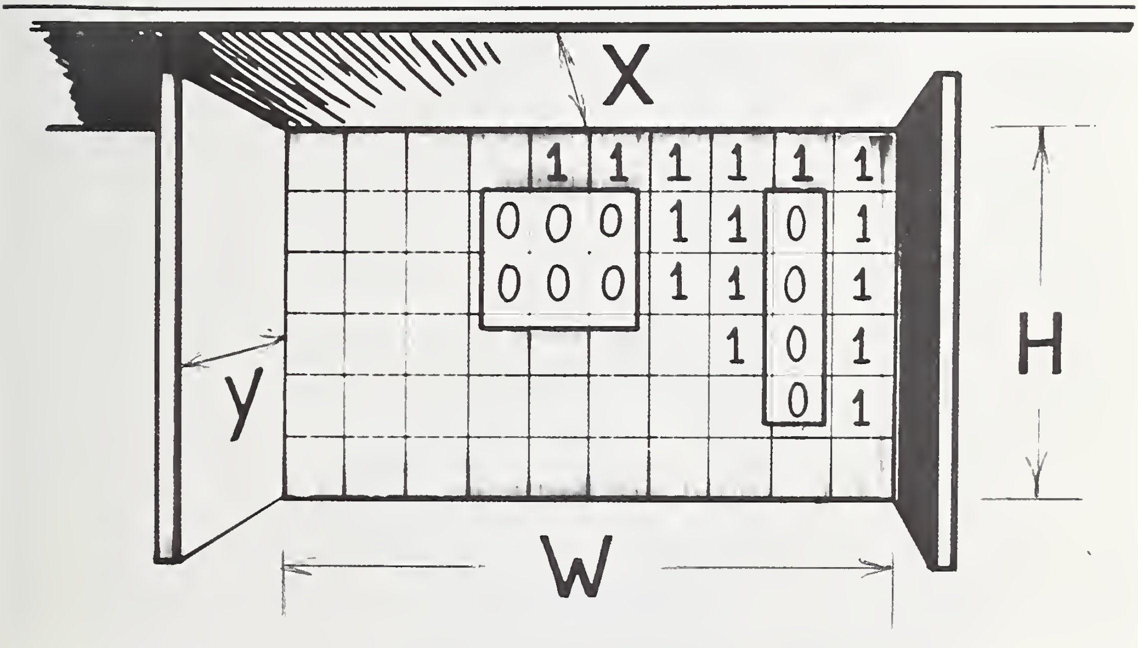


Figure 3 Wall Conditions - Cooley Building South Wall



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 1 | 1 | 1 | 1 | 1 |
| | | | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| | | | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| | | | | | | | 1 | 0 | 1 |
| | | | | | | | | 0 | 1 |
| | | | | | | | | | |

Figure 4 Exterior shading - data representation.

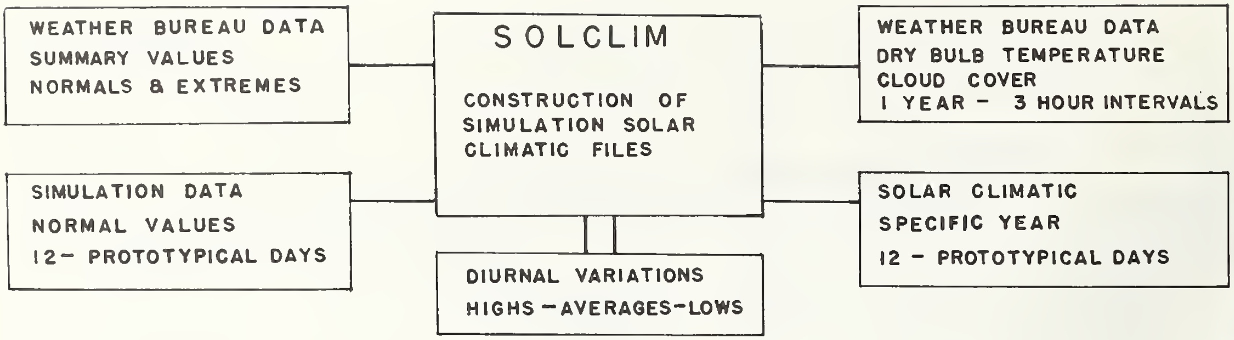
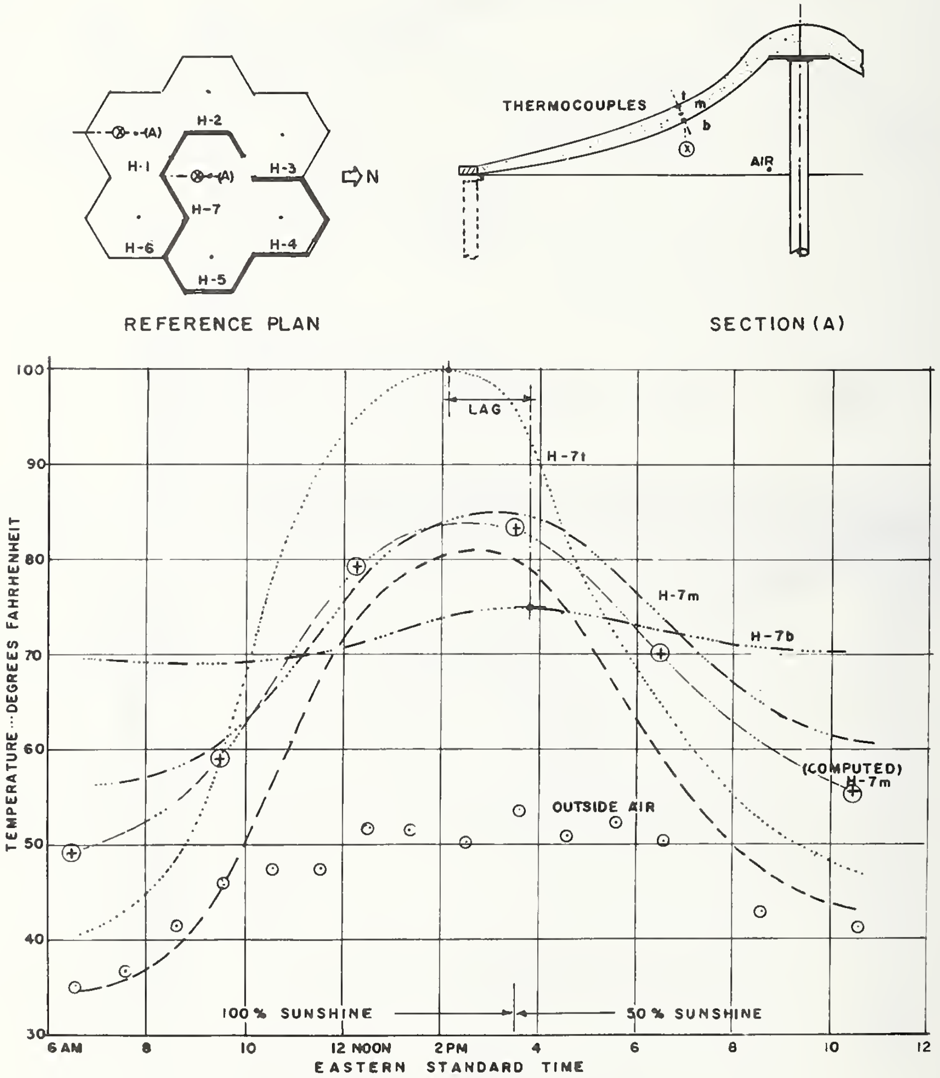


Figure 5 Block diagram - solar climatic data



NOTES CURVES ARE FOR RECORDED VALUES EXCEPT AS NOTED
DATE MAY 2, 1966
AVERAGE WIND VELOCITY = 7.9 M.P.H.

Figure 6 Correlation studies of diurnal variation of temperatures in a polyurethane shell roof.

Figure 7 Cooley Building, North Campus, University of Michigan

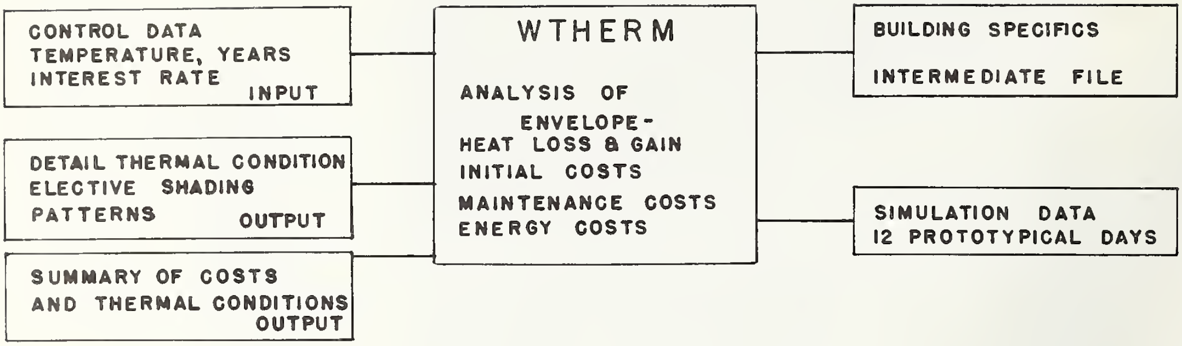


Figure 8 Block diagram of the thermal cost performance simulation program

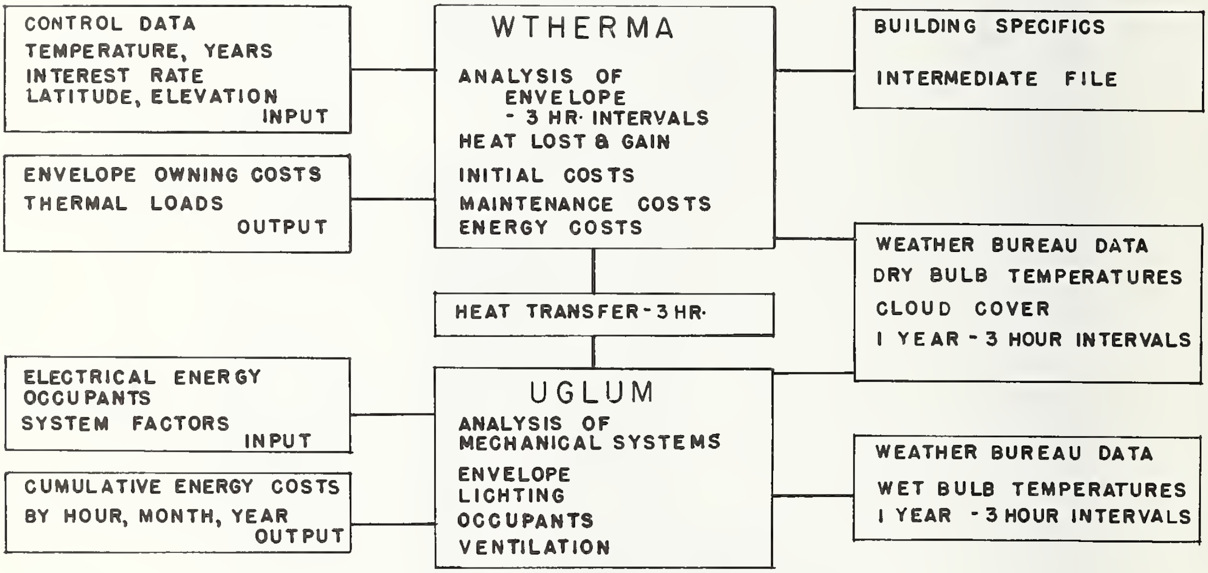


Figure 9 Block diagram of the computer programs for modeling of the thermal energy system in buildings.

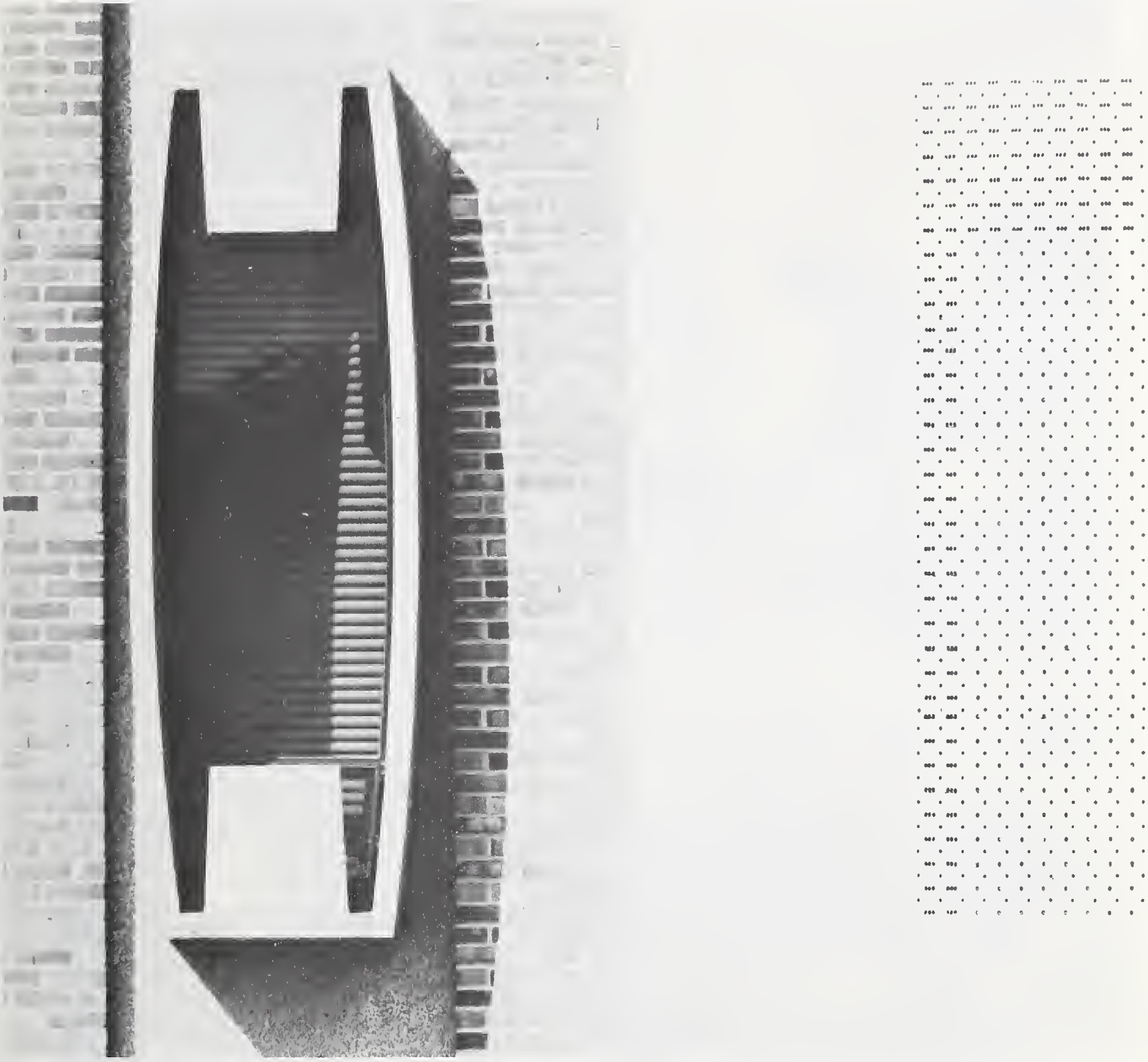


Figure 10 Shadow box window enclosure and line printer output of shadow pattern Institute of Science and Technology, North Campus, University of Michigan

Figure 11 Undergraduate Library, University of Michigan.

# A Numerical Method for Computing the Non-Linear, Time Dependent, Buoyant Circulation of Air in Rooms

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A method is described which solves the dynamic equations for air circulation at Grashof numbers that are in the range of environmental temperatures of rooms. Previous computation techniques were limited to $G \sim 10^5$, but environmental conditions require $G \sim 10^{12}$. The higher Grashof numbers are attained by improved non-linear approximation methods. Fourth order difference equations are discussed relative to linear stability properties which determine the fidelity of the convective process in the finite net of points. Non-linear instability of time-splitting methods is described and the means of overcoming this type of instability are given.

Key Words: Heat transfer, finite difference, computation, convective approximations, air circulation, time dependent solution.

## 1. Introduction

In the past decade numerous advances have been made in the use of numerical means to solve coupled non-linear partial differential equations. The methods for obtaining solutions fall roughly into two categories: 1. Expansion methods which make use of orthogonal polynomials; 2. Finite difference methods. The former of these was used by Poots[1] and the latter by Wilkes[2] for the particular problem of interest here. The success of the two methods of solution relative to the work of the cited authors was about the same. Comparable solutions were obtained for the problems of buoyant fluid flow in a two dimensional enclosure with vertical walls at a fixed temperature difference and horizontal walls either insulated or with a fixed temperature which varied linearly between the upright walls. These successes were important ones and led to the hope that the methods could readily be extended beyond the range of parameters of those studied into the practical problems of everyday engineering experience, in particular to the study of air circulation at environmental room temperatures or under fire conditions.

Both the methods indicated above do possess the potential of providing solutions to such problems. Currently, the greatest emphasis is on finite difference methods because they may be applied in a straightforward manner if sufficiently accurate algorithms are known. Unfortunately, such algorithms have been slow in realization. Here we have utilized higher order approximations that depend upon flow direction. This is essential in order that detail may be maintained without introduction of noise components which are of numerical origin. The nature of the flow in the rectangular enclosure is such that the common weaknesses of non-linear difference approximations are very pronounced. Thus while the geometrical situation is fairly simple it provides a crucial test of the methods.

Our primary objectives here are to outline the numerical methods and to give details of the algorithms that are required. The general procedure of finite difference computation using the vorticity and streamfunction as field variables is fairly well established[3,4,5]. Also, the evolution of thought on requirements relative to reduction of phase error, which is the primary cause of computational noise, has been documented[6,7]. The additional modifications of the basic algorithms that are pertinent for the success of high Grashof number calculations are emphasized here.

As a result of these efforts toward minimizing numerical noise and numerical damping, the two-dimensional computation can reasonably be carried out up to Grashof numbers of $10^{12}$. At these high values there are uncertainties about the magnitude of viscosity and heat transfer coefficients relative to truncation errors. Further, very small time increments are required for computation. Hence, the limiting calculation of infinite Grashof number would not be a reasonable one.

Because turbulence is surely a factor in the real world, the present calculations using molecular coefficients should be regarded as idealizations. This does not mean that they are of little value. On the contrary, we can analyze the flow behavior, explaining the flow features that occur and why they occur. Further, gross properties of the flows can be given within the framework of the idealization. With this information it should be possible, with the help of experiments, to determine where discrepancies occur between observation and computation. Information so obtained is pertinent to modeling efforts where turbulent diffusion and heat transport must be parameterized.

## 2. Governing Equations and Problem Description

The conservation equations to be solved in a two dimensional rectangular region are:

$$\text{Mass:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad , \tag{1}$$

$$\text{Momentum:} \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_o}\frac{\partial P}{\partial x} + \nu\nabla^2 u \quad , \tag{2}$$

$$\frac{\partial \nu}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_o}\frac{\partial P}{\partial y} - g\left(\frac{T-T_o}{T_o}\right) + \nu\nabla^2 v \quad , \tag{3}$$

$$\text{Energy:} \quad \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial X} + v\frac{\partial T}{\partial y} = \kappa\nabla^2 T \quad . \tag{4}$$

Here $u$ and $v$ are velocity components in the $x$ and $y$ directions respectively. The thermodynamic variables $\rho$, $P$, and $T$ are the density, pressure and temperature respectively. The subscript $o$ indicates constant reference values to be defined. $\nu$, $\kappa$, and $g$ are the kinematic viscosity, thermometric conductivity and gravitational constant respectively.

Numerical computation is carried out using the vorticity and streamfunction. We define

$$\omega = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \tag{5}$$

and

$$u = \frac{\partial \psi}{\partial y} \; , \; v = -\frac{\partial \psi}{\partial x} \; . \tag{6}$$

Thus (1) is satisfied identically by (6), (5) becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad , \tag{7}$$

and $P$ can be eliminated from (2) and (3) yielding the vorticity equation

$$\frac{\partial \omega}{\partial t} + \frac{\partial u\omega}{\partial x} + \frac{\partial v\omega}{\partial y} = -\frac{g}{T_o}\frac{\partial T}{\partial x} + \nu\nabla^2\omega \quad . \tag{8}$$

Our system of computation equations are then (7), (8), (4) and the definitions (6).

We take $x$ as the horizontal coordinate and impose the boundary conditions for a square enclosure as

$$u=v=o, \; T=T_1 \quad \text{at} \quad x=o \text{ for } o \le y \le d$$

$$u=v=o, \; T=T_o \quad \text{at} \quad x=d \text{ for } o \le y \le d$$

$$u=v=o \quad \text{at} \quad y=0,d \quad \text{for } o \le x \le d$$

$$T=T_1-x(T_1-T_o)/d \quad \text{at} \quad y=o,d \quad \text{for } o \le x \le d \tag{9}$$

It is found more convenient to deal numerically in terms of the dimensional variables because of physical interpretation and number sizes encountered in numerical solution. Following Poots we use the nomenclature

$$\sigma = \nu/\kappa$$

as the Prandtl number and

$$A = \sigma\ G = (T_1 - T_o)\ gd^3/T_o \kappa\nu$$

for the Rayleigh and Grashof numbers respectively.

While numerous gross properties could be evaluated to lend understanding to the flows, we here consider the dimensionless heat transfer for comparison with results by Poots and Wilkes. Heat transmission through the cold vertical boundary is not the total heat transmission but is the value given by Poots. Numerically, one must also include the convection flux because the discrete description requires it. For the square region considered we define the Nusselt number

$$N = \frac{\kappa \int_o^d \frac{\partial T}{\partial x}\, dy + \int_o^d uT dy}{\kappa . (T_1 - T_o)} \quad .$$

The total heat transmission can be evaluated only by having the end points of the integration correspond to $x=d/2$ and $y=o,d$. Nevertheless, for comparison purposes we also measure the transmission at $x=o$ with integration limits $y=o,d$.

## 3. The Numerical Approximations

We now describe the non-linear approximation method that was required for the success of the given calculations. For completeness we shall outline the numerical procedure, giving the numerical approximations but postponing the convective flux methods for later discussion.

For small values of $A$ or $G$ it may be advisable to initialize calculation with no flow and an initially prescribed linear horizontal temperature variation inside the fluid region. If $A$ is small, steady state results are of primary interest and computation time may be preserved by relay type calculations, always using steady state results from lower values of $A$ to proceed to higher values. However, beyond $A=10^6$ the solutions are unsteady, in fact $A=10^6$, $\sigma=0.1$ gives an oscillating solution. While it is always useful to begin with an approximate solution to the expected solution at late time, it is more reasonable to initialize calculation at high values of $A$ with internal temperatures specified to be everywhere the mean of the given wall temperatures. If the initial interior temperature is specified to be that of the low temperature wall, much computation would be required to approach true late time conditions. In all cases reported here, we initialized the temperature field to have the mean wall temperature value. The streamfunction and vorticity were perscribed to be everywhere zero initially.

With all field values given at the time $t=o$ we proceed by updating the temperature field toward ultimate prescription of field values $T^n_{i,j} = T(i\Delta x, j\Delta y, n\Delta t)$ for $n=1$. Through convective flux computation we first obtain everywhere

$$\tilde{T}_{i,j} = T^o_{i,j} + \frac{\Delta t}{\Delta x}\left(F^o_{i-1/2,j} - F^o_{i+1/2,j}\right) + \frac{\Delta t}{\Delta y}\left(F^o_{i,j-1/2} - F^o_{i,j+1/2}\right) \quad , \tag{10}$$

where $F$ is the flux at half cell distances from the point $(i,j)$. Using everywhere the tentative values (except for fixed values at the boundaries) we add the conduction contribution to complete the update of $T$. Thus

$$\begin{aligned} T^1_{i,j} = \tilde{T}_{i,j} &+ \frac{\kappa\Delta t}{(\Delta x)^2}\left(\tilde{T}_{i+1,j} - 2\tilde{T}_{i,j} + \tilde{T}_{i-1,j}\right) \\ &+ \frac{\kappa\Delta t}{(\Delta y)^2}\left(\tilde{T}_{i,j+1} - 2\tilde{T}_{i,j} + \tilde{T}_{i,j-1}\right) \quad . \end{aligned} \tag{11}$$

The use of tentative updated values in the conduction calculation provides for less restrictive conditional stability of this explicit difference equation. This has been suggested in the work of Marchuk[8] and has been observed empirically in the present calculations. Through this procedure the stability condition

$$\kappa\Delta t/\ \min[(\Delta x)^2,(\Delta y)^2] < 1/4 \tag{12}$$

is strictly applicable. For small A a more stringent condition would otherwise be necessary.

At this stage one proceeds similarly with the vorticity equation, first obtaining

$$\tilde{\omega}_{i,j} = \omega^{o}_{i,j} + \frac{\Delta t}{\Delta x}\left(H_{i-1/2,j} - H_{i+1/2,j}\right) + \frac{\Delta t}{\Delta y}\left(H^{o}_{i,j-1/2} - H^{o}_{i,j+1/2}\right) , \tag{13}$$

where H is the vorticity flux analogous to F of (10). Here two tentative updates are necessary because of the buoyancy term. For the diffusion update we write

$$\overset{*}{\omega}_{i,j} = \tilde{\omega}_{i,j} + \frac{\nu\Delta t}{(\Delta x)^2}\left(\tilde{\omega}_{i+1,j} - 2\tilde{\omega}_{i,j} + \tilde{\omega}_{i-1,j}\right) + \frac{\nu\Delta t}{(\Delta y)^2}\left(\tilde{\omega}_{i,j+1} - 2\tilde{\omega}_{i,j} + \tilde{\omega}_{i,j-1}\right) . \tag{14}$$

Finally the buoyancy contribution gives the complete update of $\omega$ thus:

$$\omega^{1}_{i,j} = \overset{*}{\omega}_{i,j} + \frac{g\Delta t}{2T_o\Delta x}\left(T^{1}_{i+1,j} - T^{1}_{i-1,j}\right) \tag{15}$$

Note that the values obtained in (11) are necessary in (15). If in (15) the old time values were used instability would result.

With T and $\omega$ both obtained for new times we next consider the streamfunction field. Simultaneous solution of all net points is required to satisfy

$$\frac{\psi^{1}_{i+1,j} - 2\psi^{1}_{i,j} + \psi^{1}_{i-1,j}}{(\Delta x)^2} + \frac{\psi^{1}_{i,j+1} - 2\psi^{1}_{i,j} + \omega^{1}_{i,j-1}}{\Delta y^2} = -\omega^{1} . \tag{16}$$

Because of the simplicity of the boundary conditions on $\psi$ a direct method may readily provide solution. We have here made use of a program developed by Buneman[9]. Buneman's method involves cyclic matrix reduction in two directions. It is applicable to an enclosed rectangular region with zero value specified for $\psi$ at the boundaries. The program is fast, accurate to computer round off, and compact. It is estimated that an iterative method would have taken more than ten times as much computation for equal accuracy.

The final step in computation of the field variables, to have a complete solution at the advanced time step, is to apply no-slip conditions at the walls, i.e., obtain boundary vorticity values. The approximations here used are

$$\begin{aligned}
\omega^{1}_{o,j} &= -2\left(\psi^{1}_{1,j} - \psi^{1}_{o,j}\right)/\Delta x^2 \\
\omega^{1}_{I,j} &= 2\left(\psi^{1}_{I,j} - \psi^{1}_{I-1,j}\right)/\Delta x^2 \\
\omega^{1}_{i,o} &= -2\left(\psi^{1}_{i,1} - \psi^{1}_{i,0}\right)/\Delta y^2 \\
\omega^{1}_{i,J} &= 2\left(\psi^{1}_{i,J} - \psi^{1}_{i,J-1}\right)/\Delta y^2 ,
\end{aligned} \tag{17}$$

where o refers to left or lower boundary and I and J refer to right and upper boundaries respectively. The equations (17) may readily be derived from (16). Corner values of $\omega$ are always zero as implied by successive use of appropriate parts of (17).

Stability of the explicit form of solution is tested at all time steps and provision is made to double or halve the time step. If the larger of $u\Delta t/\Delta x$ and $v\Delta t/\Delta y$ is less than 0.4 the time step is doubled. If the larger of $u\Delta t/\Delta x$ and $v\Delta t/\Delta y$ is greater than 1.0 the time step is cut in half. Applying these conditions along with (12) maintains stability throughout the forward marching problem. While an initial estimate of $\Delta t$ is not required by this procedure some early time rapid adjustment of $\Delta t$ does occur for large A. In these cases a maximum early time $\Delta t$ should be used for the first several time steps because a very large $\Delta t$ is implied by the stability conditions. Eq. (15) is overstable and no test is necessary for this part of the procedure.

With a view of the overall procedure we now give further consideration to F of Eq. (10) or H of Eq. (13). Both F and H are, in computation, handled by the same program. Prior experience led to fourth order one dimensional approximations for F on the assumption that "time splitting" methods would be used to provide isotropic behavior relative to the finite lattice. "Time splitting" is a process which involves computation of tentative values through the addition of horizontal flux contributions followed by additions of vertical flux contributions. Here the latter computation makes use of the tentative values derived from the horizontal additions. In a linear sense "time splitting" results in the inclusion of diagonal values of the field variable such that truly two dimensional flow is treated. It is an efficient way to obtain isotropic behavior since without "time splitting" or some equivalent procedure one may, in the extreme case, experience instability in the flow direction simultaneous to numerical diffusion transverse to the flow direction. Often these two effects can hold the numerical computation in check but the results may be badly in error since distortion occurs here even if the flow is uniform but not along the coordinate axes. In two-dimensions then we must in some sense bring in mesh values of the field variables that are diagonally distributed relative to the coordinate directions at (i,j).

Unfortunately, while "time splitting" is an efficient procedure to use it has been found that a new type of instability can occur. This instability is slow and occurs only in non-linear cases. In the given problem a vortex originating from buoyant effects may grow in intensity in a non-physical manner. This difficulty is not very different from that of using non-conservative difference methods. It is perhaps less severe. Its degree of severity depends upon the non-linearity of the problem and the amplitude characteristics of the approximation. The difficulty has its origin in an inconsistency that occurs in the velocity values that are effectively present in the implied cross differences. Since the cross derivatives are implied through "time splitting", this shortcoming goes unnoticed until the slow growing instability occurs.

Here the difficulty is circumvented by explicitly programming cross derivatives as required to provide the linear equivalent of "time splitting" procedures. By so doing the linear stability conditions are maintained and the inconsistencies in velocity values are avoided. If one expands the "time splitting" equations into a single step the cross differences will appear. Using these terms as a guide, final layout of conservative expressions are developed from a Taylor's series expansion*. Consider

$$T^{n+1} = T^n + \Delta t \left(\frac{\partial T}{\partial t}\right) + \frac{\Delta t}{2}\left(\frac{\partial^2 T}{\partial t^2}\right) + \ldots \quad . \tag{18}$$

Because we are numerically separating convection and conduction we take

$$\frac{\partial T}{\partial t} = -\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} \quad . \tag{19}$$

If we make use of (19) in (18) and drop velocity time derivatives we obtain

$$\frac{\partial^2 T}{\partial t^2} = \frac{\partial}{\partial x}\left(u^2 \frac{\partial^2 T}{\partial x^2} + uv\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(uv\frac{\partial T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2}\right) \tag{20}$$

Here the (uv) terms are the first cross terms that must be explicitly programmed rather than implied as in "time splitting". The (uv) term has two parts, one part is regarded as a skew flow correction to the flux in the x direction, the other part as a skew flow correction to the flux in the y direction. With similar expansion of (18) to eighth order all cross differences for fourth order approximation may be developed. Proper combination of these terms can lead to simple expressions and reasonably fast computation. The fourth order forms are given in the Appendix.

One further consideration in the convective approximation is phase distortion. It is particularly

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* This technique was first employed by S. K. Jordan in his Doctoral thesis "Numerical Solutions for the Time-Dependent, Viscous, Incompressible Flow Past a Circle," Department of Aeronautics and Astronautics, Stanford University, 1970.

pertinent to the problem at hand since the usual difference methods would quickly lead to meaningless results for $A>10^7$. It is essential to success of such calculations to have leading phase errors and these should be small. Upstream approximations may be used to achieve this and if necessary several different approximations may be linked through different magnetudes of $\alpha = u\Delta t/\Delta x$. Here we have used a combined fourth order upstream and central approximation (flow direction must here also be tested) for $\alpha \geq 0.5$. For $0.90 \leq \alpha \leq 0.5$ the fourth order upstream approximation alone is used. If $\alpha < 0.09$ a second order upstream approximation is used. Central approximations are used only for flow away from solid boundaries if data points for upstream calculation are not available. In these latter instances lagging phase errors do not lead to noise. The use of second order upstream approximations for $\alpha < 0.09$ has to do with an anomoly in fourth order characteristics that is not present in second order. The above procedure is not the only one that may be used. In the given case it was the most expedient on the basis of available data on phase properties of the methods. The choice made also included consideration of amplitude properties to limit numerical damping.

## 4. Flow Behavior

At the time of this writing we have computed several cases to give an overall view of variation of flows with different Rayleigh number and Prandtl number. Computations have been made for $A = 10^4$ through $A = 10^{12}$ at intervals of a factor of 10 and with $\sigma=1.0$ For even powers of A, runs were also made for $\sigma = 10.0$ and $\sigma = 0.1$. From these computations the flows can be classed roughly into three groups. 1. Steady state flows $(G < 10^7)$; 2. Transition flows, unsteady and highly variable in behavior $(10^7 < G < 10^9)$; and 3. Single circulation, high Grashof number flows $(10^9 \leq G < 10^{13})$.

The steady state flows are also single circulation flows characterized by the presence of a vertical temperature gradient in the central region of the enclosure but little horizontal temperature gradient. The temperature gradients increase strongly near the walls. This type of behavior is illustrated in the steady solution of $A = 10^5$, $\sigma = 1.0$ of Fig. 1.

In the transition flows the variable behavior includes sinusoidal behavior at the low values of G. Mid-transition flows deviate from sinusoidal to include random occurances of buoyant plumes along the horizontal boundaries. In the upper transition flows randomness is increased and boundary layer entrainment leads to counter circulations mixed with buoyant circulations. Figure 2 is an example of upper transition range flow.

Members of the last group are classed together because visually they exhibit little difference in behavior. A single circulation (in the square region) is the main feature and is common to the whole group. The interior fluid is essentially isothermal at the mean temperature and vorticity is almost completely absent there. One can describe these flows as having an inner core of ideal fluid flow and a highly variable boundary layer behavior. The thermal and velocity boundary layers are thin but corner regions are involved in the flow related to these layers. In Fig. 3 we include an example from this group.

Returning now to Fig. 1 we consider this group in more detail. In Fig. 1A the streamlines of the flow are given. The highest speeds,as expected, are of flow near the vertical boundaries while the flow near the center is slow. The separated centers of circulation are in the same direction of rotation. They occur because of two regions of buoyant vorticity associated with the heated and cooled vertical walls. They are separated by a reverse flow tendency which has its origin in a slight reversal in the horizontal temperature gradient. In Fig. 1B one notes that the highest values of vorticity occur at the boundaries. The sign is opposite to the two buoyant vorticity extremals occuring near the vertical walls. This is the boundary layer vorticity which follows from our no-slip condition. The central extremal of vorticity is also of opposite sign to the main buoyant contributions. It also has its origin in buoyancy but here in reverse to the main circulation. This reversed vorticity contributes to the attainment of steady flow in this range, along with the dissipation or frictional restraining forces. The latter mechanism is of coarse strongest at the boundaries. In the isotherms of Fig. 1C one clearly sees the slight reversal in horizontal temperature gradient at the center (the slant of the isotherms is downward to the right). Increased heat transfer relative to a non-fluid material is obvious from Fig. 1C in that the flow leads to high gradients in temperature at the vertical walls. Near the walls conduction is inhanced by the high gradients and in turn the flow in the center leads to exchange of warm and cold fluid through the convective process.

The results of Fig. 1 are not new, similar results have been obtained by Wilkes and Poots. To establish a point of departure from this earlier work we have compared a calculation of $A = 10^4$, $\sigma = .73$ with the results by Poots. The results are the same in all essentials in the visual sense of the solution. To further compare results we have numerically obtained the Fourier coefficients of our computed result for the disturbance temperature field (the linear gradient with $T_1-T_o=1.0$ must be added to obtain the equivalent of Fig. 1C). In Table I we compare the numerical coefficients with coefficients given by Poots. Differences in the results cannot be construed as error in the numerical results because the analytic solution is also approximate. The differences in the second group of coefficients of Table I is puzzling and may be an error by Poots in recording the numbers. Consistencies for even smaller coefficients is good. Measured heat transfer at the cold wall as obtained by Poots is $N = 1.706$. We obtained $N = 1.753$ numerically.

Table I. Fourier coefficients of disturbance temperature distribution for $A = 10^4$, $\sigma = .73$

| Term (kx,ky) | Poot's | Finite Difference |
|---|---|---|
| 1,2 | -0.1875 | -0.1868 |
| 2,1 | -0.1874 | -0.1840 |
| 1,4 | 0.0019 | -0.0082 |
| 2,3 | 0.0063 | -0.0166 |
| 3,2 | -0.0107 | 0.0034 |
| 4,1 | -0.0314 | -0.0359 |
| 1,6 | 0.0017 | 0.0024 |
| 2,5 | 0.0026 | -0.0000 |
| 3,4 | -0.0004 | -0.0014 |
| 4,3 | 0.0016 | 0.0014 |
| 5,2 | 0.0017 | 0.0024 |
| 6,1 | -0.0071 | -0.0067 |
| 1,8 | 0.0009 | 0.0010 |
| 2,7 | 0.0018 | 0.0011 |
| 3,6 | 0.0008 | -0.0001 |
| 4,5 | 0.0007 | 0.0006 |
| 5,4 | -0.0003 | -0.0003 |
| 6,3 | 0.0006 | 0.0002 |
| 7,2 | 0.0008 | 0.0005 |
| 8,1 | -0.0018 | -0.0015 |
| 1,10 | 0.0001 | 0.0003 |
| 2,9 | 0.0004 | 0.0004 |
| 3,8 | 0.0003 | 0.0001 |
| 4,7 | 0.0003 | 0.0001 |
| 5,6 | -0.0002 | -0.0001 |
| 6,5 | 0.0001 | 0.0002 |
| 7,4 | 0.0001 | -0.0001 |
| 8,3 | 0.0003 | 0.0001 |
| 9,2 | 0.0003 | -0.0001 |
| 10,1 | -0.0006 | -0.0005 |
| 1,12 |  | 0.0001 |
| 2,11 |  | 0.0001 |
| 11,2 |  | 0.0000 |
| 12,1 |  | -0.0002 |

Proceeding to Fig. 2 we include results for $A = 10^8$, $\sigma = 1$. In this range we have passed considerably beyond the point where solution was previously possible. The results portrayed in Fig. 2 are a single still portrait of a rapidly varying flow. Standing alone these photos do not yield much understanding of detailed behavior, nor do we necessarily need detail of the origin of every small vortex. Animations of these solutions have proved to be valuable. With motion picture films of various representations new insights into the behavior become possible. The most important mechanisms beyond those evident in Fig. 1 is that boundary layer separation occurs and counter circulations to the main buoyant tendency can arise. When these do arise they may influence the temperature field sufficiently to be enhanced by buoyant effects. Reinforcement of counter circulations by buoyant forces is present in the case of Fig. 2. The instabilities that occur because of these successive effects are probably only part of the influences that prevent the emergence of a steady state. In the transition range, steady state is probably impossible although resonant effects might possibly be achieved through a highly critical choice of rectangular dimensions of the enclosure.

In Fig. 2A the separation streamline is given and counter circulations may be identified by tracing this streamline. Those circulations falling in regions toward the boundaries (exterior to the separation streamline are counter circulations). The remaining significant centers of rotation are in the direction of the basic buoyant tendency. In the vorticity plots of Fig. 2B we note that the buoyant contribution from the vertical walls is confined to a very narrow vertical strip and this strip may be broken up into small concentrations. These concentrations move upward along the heated wall much like air bubbles in water and give a wave character to the streamlines. Dominant characteristics in this range of A, even in the presence of the rapid time variation, are the strong circulations in the upper left and lower right corners. These circulations are inhanced by the rising and falling concentrations of bouyant vorticity from the heated and cooled walls. Boundary layer separation occurs immediately downstream from these dominant circulations. The counter circulations so developed usually migrate along the horizontal boundaries. Buoyant inhancement of the counter circulation is most likely in the vicinity of the lower left and upper right corners where thermal boundary layers become stretched into plumes. In Fig. 2C such a plume is evident near the left side of the lower boundary.

Measurement of the heat transfer in the transitional and upper range is difficult because the unsteadyness can lead to wide extremes (numerically we cannot measure transfer right at the wall). Long term averages must be taken. Currently we are still in the process of dealing with these measurements along with necessary work that must be done on overall data reduction. For the calculation of Fig. 2A ($A = 10^8$, $\sigma = 1$) we estimate $N \sim 32.0$. Numerical programs to give late time average heat transfer values have not been employed at this writing.

Finally we consider the last group (Fig. 3) in greater detail. The single circulation behavior of the high Rayleigh number flows is somewhat of a surprise because here the character of the flow is almost the same for a very wide range of values of the parameters. Quantitatively the flows remain different particularly in the speed of circulation at late times or the rate at which the circulation speed increases from the initial no flow state. In this range the viscous and conduction effects are essentially absent at the central region but intense at the boundaries. The results show dramatically why boundary layer theory has been so successful in many theoretical studies of fluid flow. While the unsteadyness far exceeds that of the transition range it is almost entirely confined to a thin layer at the boundaries and to the corner regions. There is eccentricity in the flows that does not appear to diminish over the range of time covered by the calculations. An almost perfectly circular flow may be disrupted by migrating corner activity. Such a disruption may lead to increasingly eccentric flow but never a return to the type of flow of the transitional range.

In these flows it is not uncommon for plume roll up to occur so that a hot or cold spot is carried out into the main circulation. Once caught up in the main circulation it may undergo several revolutions before intermingling again with the thermal boundary layer or getting trapped in a corner of the flow region.

The single circulation behavior must be a consequence of the very low dissipation and slow conduction process in the interior flow region. In the absence of some roughness or protrusion from the boundary to inhance mixing the interior region remains isothermal. It is also free of vorticity because boundary generated vorticity does not penetrate to the central region and the isothermal fluid does not contribute buoyant vorticity.

Early transitional behavior from the no flow initial condition does involve similar behavior to that of the transitional range flows. It may however be of value to investigate the effects of the initial state. Presumably late time solutions for an initial state of the interior fluid at the cold temperature would ultimately be the same as the given solutions of Fig. 3, but no calculations of this type were made.

In Fig. 3A we note that counter circulations to the main circulation are very limited. The apparent smooth appearance of this still shot is not indicative of the actual behavior. Small extremals of vorticity (Fig. 3B) or hot spots of temperature (Fig. 3C) that occur on occasion in the central part of the flow have been timed to estimate a mean speed of the main circulation. In an 8 ft. square enclosure the circulation time is roughly 5 seconds. This is not unreasonable but much work still needs to be done in comparing our results with experiment. It is uncertain to what extent the numerical result provides for the mixing that would occur in the turbulent flows as observed in the laboratory.

## 5. Acknowledgements

The author wishes to acknowledge the assistance of Donald E. Schreiber[10] whose graphics programs for the IBM 2250 made it possible to reduce many numbers to very meaningful pictures. Acknowledgement is also due William E. Langlois for his review and recommendations of the work.

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# Appendix

In this appendix we describe the form of the convective approximation used in the calculations. We shall give the form of the flux term in the u direction only, but for the various ranges of $\alpha = u\Delta t/\Delta x$ as indicated earlier. It must be remembered that the flux as given is only half of the required computation for a given mesh point. If the flux is added to the point on the right and subtracted from the point on the left the speed of computation can be reduced by half. Also the same convective program can be used for both vorticity and temperature if these variables are defined the same relative to the grid. The fluxes in the vertical direction may be inferred from those given here for the horizontal.

We define

$$\alpha_{i-1/2,j} = \frac{u_{i-1/2,j}\ \Delta t}{\Delta x} \quad \text{and} \quad \beta_{i-1/2,j} = \frac{v_{i-1/2,j}\ \Delta t}{\Delta y}$$

where

$$u_{i-1/2,j} = (\psi_{i-1,j+1} + \psi_{i,j+1} - \psi_{i-1,j-1} - \psi_{i,j-1}) / 4\Delta y$$

$$v_{i-1/2,j} = (\psi_{i-1,j} - \psi_{i,j}) / \Delta x$$

If $|\alpha|<.09$ we use an upstream second order approximation. We here write the central approximation terms with the understanding that if $\alpha>0$, $\alpha$ is replaced by $(\alpha-1)$ and i indices are reduced by 1. Further if $\alpha<0$, $\alpha$ is replaced by $(\alpha+1)$ and i indices are increased by 1. To make addition and subtraction of flux contributions possible as previously mentioned the powers of $\alpha$ must further be modified. For example $\alpha^2$ must be replaced by $[(\alpha-1)^2-1] = \alpha^2-2\alpha$ so that Eq. (11) does indeed apply without a shift of index on the leading term of this equation. Cancellation of the appropriate linear terms (terms that do not contain $\alpha$) can readily be varified for the difference forms here given. This however requires writing out the complete expression of both the flux into and out of the cell of interest.

Define

$$S_{3,0} = T_{i-1,j} + T_{i,j} \ , \quad D_{3,0} = T_{i-1,j} - T_{i,j} \ ,$$

$$S_{6,2} = T_{i-1,j+1} + T_{i,j+1} \ , \quad D_{6,2} = T_{i-1,j+1} - T_{i,j+1} \ ,$$

and

$$S_{7,4} = T_{i-1,j-1} + T_{i,j-1} \ , \quad D_{7,4} = T_{i-1,j-1} - T_{i,j-1} \ .$$

Here the subscript numbers follow a procedure of successively numbering points in a counter clockwise direction starting with the point just to the right of a point of interest. The symbol (i,j) and the symbol 0 are synonymous.

Now with index i-1/2 implied for $\alpha$, $\beta$ and F we write

$$\begin{aligned}\frac{\Delta t}{\Delta x}\ F = {} & 1/2\ \alpha\ S_{3,0} + 1/8\ \alpha\beta(S_{7,4} - S_{6,2}) \\ & + 1/2\ \alpha^2\ D_{3,0} + 1/12\ \alpha\beta^2(S_{7,4} - 2\ S_{3,0} + S_{6,2}) \\ & + 1/6\ \alpha^2\beta(D_{7,4} - D_{6,2}) + 1/8\ \alpha^2\beta^2(D_{7,4} - 2\ D_{3,0} + D_{6,2})\end{aligned}$$

For $0.9 \leq |\alpha| \leq .5$ we use a fourth order upstream expression and for $.5 < |\alpha| \leq 1$ we use the average of the upstream and central expressions. Here again we give only the central difference expression. The same rules apply on replacment of $\alpha$ appropriately with $(\alpha-1)$ or $(\alpha+1)$ and shifting indices as previously indicated.

Define in addition to the above sums and differences

$$S_{15,10} = T_{i-1,j+2} + T_{i,j+2} \quad , \quad D_{15,10} = T_{i-1,j+2} - T_{i,j+2}$$

$$S_{18,12} = T_{i-1,j-2} + T_{i,j-2} \quad , \quad D_{18,12} = T_{i-1,j-2} - T_{i,j-2}$$

$$S_{11,1} = T_{i-2,j} + T_{i+1,j} \quad , \quad D_{11,1} = T_{i-2,j} - T_{i+1,j}$$

$$S_{16,5} = T_{i-2,j+1} + T_{i+1,j+1} \quad , \quad D_{16,5} = T_{i-2,j+1} - T_{i+1,j+1}$$

$$S_{17,8} = T_{i-2,j-1} + T_{i+1,j-1} \quad , \quad D_{17,8} = T_{i-2,j-1} - T_{i+1,j-1}$$

$$S_{22,14} = T_{i-2,j+2} + T_{i+1,j+2} \quad , \quad D_{23,14} = T_{i-2,j+2} - T_{i+1,j+2}$$

and

$$S_{23,19} = T_{i-2,j-2} + T_{i+1,j-2} \quad , \quad D_{23,19} = T_{i-2,j-2} - T_{i+1,j-2}$$

We may now write, with index $i-1/2$ implied for $\alpha, \beta$ and $F$,

$$\frac{\Delta t}{\Delta x} F = \sum_{\ell=1}^{42} B_\ell X_\ell \quad .$$

Where the B's and X's are given in Table II (A and B), and the numerical coefficients associated with the B's are given in Table III.

Table II-A. Leading terms and differences of fourth order approximation.

| B | | X |
|---|---|---|
| 1 | $A_{11}\ \alpha$ | $S_{3,0}$ |
| 2 | $A_{21}\ \alpha$ | $S_{11,1}$ |
| 3 | $A_{11}A_{11}\ \alpha\beta/2$ | $S_{7,4} - S_{6,2}$ |
| 4 | $A_{11}A_{21}\ \alpha\beta/2$ | $S_{18,12} - S_{6,2} - S_{15,10}$ |
| 5 | $A_{21}A_{11}\ \alpha\beta/2$ | $S_{17,8} - S_{16,5}$ |
| 6 | $A_{21}A_{21}\ \alpha\beta/2$ | $S_{23,19} - S_{17,8} + S_{16,5} - S_{12,14}$ |
| 7 | $A_{12}\ \alpha^2$ | $D_{3,0}$ |
| 8 | $A_{22}\ \alpha^2$ | $D_{11,1}$ |
| 9 | $A_{11}A_{12}\ \alpha\beta^2/3$ | $S_{7,4} - 2S_{3,0} + S_{6,2}$ |
| 10 | $A_{11}A_{22}\ \alpha\beta^2/3$ | $S_{18,12} - S_{7,4} - S_{6,2} + S_{15,10}$ |
| 11 | $A_{21}A_{12}\ \alpha\beta^2/3$ | $S_{17,8} - 2S_{11,1} + S_{16,5}$ |
| 12 | $A_{21}A_{22}\ \alpha\beta^2/3$ | $S_{23,19} - S_{17,8} - S_{16,5} + S_{22,14}$ |
| 13 | $2A_{12}A_{11}\ \alpha^2\beta/3$ | $D_{7,4} - D_{6,2}$ |
| 14 | $2A_{12}A_{21}\ \alpha^2\beta/3$ | $D_{18,12} - D_{7,4} + D_{6,2} - D_{15,10}$ |
| 15 | $2A_{22}A_{11}\ \alpha^2\beta/3$ | $D_{17,8} - D_{16,5}$ |
| 16 | $2A_{22}A_{21}\ \alpha^2\beta/3$ | $D_{23,19} - D_{17,8} + D_{16,5} - D_{22,14}$ |
| 17 | $A_{23}\ \alpha^3$ | $X_2 - X_1$ |

Table II-A. continued.

| | B | X |
|---|---|---|
| 18<br>19 | $A_{11}A_{23}\ \alpha\beta^3/4$<br>$A_{21}A_{23}\ \alpha\beta^3/4$ | $X_4 - X_3$<br>$X_6 - X_5$ |
| 20<br>21<br>22<br>23 | $A_{12}A_{12}\ \alpha^2\beta^2/2$<br>$A_{12}A_{22}\ \alpha^2\beta^2/2$<br>$A_{22}A_{12}\ \alpha^2\beta^2/2$<br>$A_{22}A_{22}\ \alpha^2\beta^2/2$ | $D_{7,4} - 2D_{3,0} + D_{6,2}$<br>$D_{18,12} - D_{7,4} - D_{6,2} + D_{15,10}$<br>$D_{17,8} - 2D_{11,1} + D_{16,5}$<br>$D_{23,19} - D_{17,8} - D_{16,5} + D_{22,14}$ |

Table II-B. Leading terms and differences of fourth order approximation. (continued)

| | B | X |
|---|---|---|
| 24<br>25 | $3A_{23}A_{11}\ \alpha^3\beta/4$<br>$3A_{23}A_{21}\ \alpha^3\beta/4$ | $X_5 - X_3$<br>$X_6 - X_4$ |
| 26 | $A_{24}\ \alpha^4$ | $X_8 - 3X_7$ |
| 27<br>28 | $A_{11}A_{24}\ \alpha\beta^4/5$<br>$A_{21}A_{24}\ \alpha\beta^4/5$ | $X_{10} - 3X_9$<br>$X_{12} - 3X_{11}$ |
| 29<br>30 | $2A_{12}A_{23}\ \alpha^2\beta^3/5$<br>$2A_{22}A_{23}\ \alpha^2\beta^3/5$ | $X_{14} - X_{13}$<br>$X_{16} - X_{15}$ |
| 31<br>32 | $3A_{23}A_{12}\ \alpha^3\beta/5$<br>$3A_{23}A_{22}\ \alpha^3\beta/5$ | $X_{11} - X_9$<br>$X_{12} - X_{10}$ |
| 33<br>34 | $4A_{24}A_{11}\ \alpha^4\beta/5$<br>$4A_{24}A_{21}\ \alpha^4\beta/5$ | $X_{15} - 3X_{13}$<br>$X_{16} - 3X_{14}$ |
| 35<br>36 | $A_{12}A_{24}\ \alpha^2\beta^4/3$<br>$A_{22}A_{24}\ \alpha^2\beta^4/3$ | $X_{21} - 3X_{20}$<br>$X_{23} - 3X_{22}$ |
| 37 | $A_{23}A_{23}\ \alpha^3\beta^3/2$ | $X_{19} - X_{18}$ |
| 38<br>39 | $2A_{24}A_{12}\ \alpha^4\beta^2/3$<br>$2A_{24}A_{22}\ \alpha^4\beta^2/3$ | $X_{22} - 3X_{20}$<br>$X_{23} - 3X_{21}$ |
| 40 | $3A_{23}A_{24}\ \alpha^3\beta^4/7$ | $X_{28} - X_{27}$ |
| 41 | $4A_{24}A_{23}\ \alpha^4\beta^3/7$ | $X_{30} - 3X_{29}$ |
| 42 | $A_{24}A_{24}\ \alpha^4\beta^4/2$ | $X_{36} - 3X_{35}$ |

Table III. Coefficients of fourth order approximation.

| | |
|---|---|
| $A_{11}$ | 7/12 |
| $A_{12}$ | 15/24 |
| $A_{13}$ | - 1/12 |
| $A_{14}$ | - 3/24 |
| $A_{21}$ | - 1/12 |
| $A_{22}$ | - 1/24 |
| $A_{23}$ | 1/12 |
| $A_{24}$ | 1/24 |

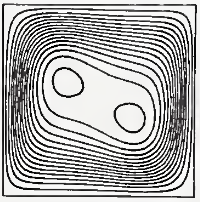

Figure 1A. Steady streamline solution for $A = 10^5$, $\sigma = 1.0$. The plot increment $\Delta\psi = 0.05\ \text{ft}^2/\text{sec}$. $\psi = 0$ at the boundary and 0.025 at the first contour from the boundary.

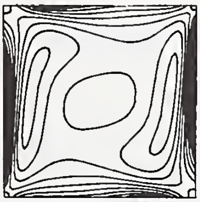

Figure 1B. Contours of constant vorticity for $A = 10^5$, $\sigma = 1.0$. The plot increment $\Delta\omega = 0.2/\text{sec}$. The minimum displayed contour value is -0.5 and is the inner contour of the two symetric extremals. $\omega$ maximum at the wall is 2.47.

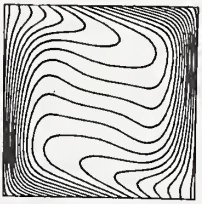

Figure 1C. Isotherms for $A = 10^5$, $\sigma = 1.0$. The plot increment is $\Delta T = 0.5°F$ for the overall wall temperature difference $T_1 - T_0 = 10.0$.

Figure 2A. Late time streamline solution for $A = 10^8$, $\sigma = 1.0$. The plot increment $\Delta\psi = 0.02$ ft $^2$/sec. The separation streamline $\psi = 0$ is given and is the reference contour.

Figure 2B. Vorticity contours corresponding to Figure 2A. $\Delta\omega = 0.75$/sec. $\omega$ maximum at the wall is 12.9.

Figure 2C. Isotherms corresponding to Figure 2A. $\Delta T = 0.5$°F.

Figure 3A. Late time streamline solution for $A = 10^{12}$, $\sigma = 1.0$. The plot increment $\Delta\psi = 1.0$ ft $^2$/sec. The separation streamline $\psi = 0$ is given and is the reference contour.

Figure 3B. Vorticity contours corresponding to Figure 2A. $\Delta\omega = 10.0$/sec. $\omega$ maximum at the wall is 145.4.

Figure 3C. Isotherms corresponding to Figure 2A. $\Delta T = 0.5$°F.

# Fortran IV Program to Calculate Absorption and Transmission of Thermal Radiation by Single and Double-glazed Windows

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In the calculations of the heating or cooling load for a room, it is necessary to know the fraction of the solar radiation incident on the outside of the window that is absorbed by the glass and the fraction that is transmitted to the interior of the room. This program calculates the absorptivity and transmissivity of windows made of common glass. In addition the coefficients for a 5th degree polynomial are calculated to allow rapid (although less accurate) determination of these factors for a given window and given incident angle of solar beam.

The calculations by this program are based on:

(a) Fresnel's formulae (relation between incident angle, refraction angle and reflection of parallel and perpendicular polarization components of radiation).

(b) Snell's law (relation between refraction and incident angles).

(c) Exponential extinction law (relation between glass sheet thickness, extinction coefficient for glass and absorption of radiation in a single pass).

The calculated factors for single and double glazed windows account for the multiple reflections and absorptions that occur when radiation passes through more than one air-glass interface.

The 5th order polynomials, that relate the factors and the cosine of incident angle, are fitted by the least-squares method using the calculated values for incident angle 0° to 90° in one degree steps. The coefficients of these polynomials are used to calculate diffuse radiation factors assuming that the diffuse radiation from the sky and the ground has equal intensity at all incident angles.

Key Words: Absorbtion, glass, radiation, solar transmission, window.

---

[1] Research Officer and Computer Systems Programmer, respectively.

### 1. Expressions for Absorptivity and Transmissivity of a Window

The single air-glass interface reflectivity is given by Fresnel's formulae

$$r_{//} = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} \tag{1}$$

$$r_{\perp} = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \tag{2}$$

where $r_{//}$ and $r_{\perp}$ = the reflectivity for radiation that is polarized with the electric vector parallel and perpendicular, respectively, to the plane that contains the incident beam and a normal to the interface.

$\theta_1$ = incident angle

$\theta_2$ = refraction angle

The refraction angle is related to the incident angle by Snell's law.

$$\mathrm{Sin}\ \theta_1 = n\ \mathrm{Sin}\ \theta_2 \tag{3}$$

where $n$ = index of refraction for the glass.

For normal incidence $\theta_1 = \theta_2 = 0$

and $r_{//} = r_{\perp} = \left(\frac{n-1}{n+1}\right)^2$ (4)

The fraction of the radiation that is absorbed in a single pass through a glass sheet of thickness L is given by

$$a = 1 - \exp(-KL/\mathrm{Cos}\ \theta_2) \tag{5}$$

where $K$ = extinction coefficient for glass.

The absorbtivity, A, transmissivity, T, and reflectivity, R, of a sheet of glass and double-glazed window are calculated for parallel and perpendicular polarization separately. The average values of $A_{//}$ and $A_{\perp}$, $T_{//}$ and $T_{\perp}$, or $R_{//}$ and $R_{\perp}$ are applicable for non-polarized incident beam.

The factors A, T, and R of a single sheet of glass (taking account of multiple reflections of both surfaces and multiple absorbtions) are given by

$$A = \frac{a(1-r)\,[1 + r(1-a)]}{1 - r^2(1-a)^2} \tag{6}$$

$$T = \frac{(1 - r)^2 (1 - a)}{1 - r^2 (1 - a)^2} \quad (7)$$

and

$$R = r + \frac{r(1 - r)^2 (1 - a)^2}{1 - r^2 (1 - a)^2} \quad (8)$$

where $r$ and $a$ are the single pass factors. The factors for parallel and perpendicular polarization components are calculated when $r = r_{//}$ and $r = r_{\perp}$ respectively.

The double-glazed window absorptivity, $A_{1D}$ and $A_{2D}$, and transmissivity, $T_D$, are given by

$$A_{2D} = A_2 \left\{ 1 + R_1 T_2 \left( \frac{1}{1 - R_1 R_2} \right) \right\} \quad (9)$$

$$A_{1D} = \frac{A_1 T_2}{1 - R_1 R_2} \quad (10)$$

and

$$T_D = \frac{T_1 T_2}{1 - R_1 R_2} \quad (11)$$

where the quantities with subscript D are for double-glazed windows and those without subscript D are for a single sheet of glass. The subscript 2 denotes the factors for the outer pane and subscript 1 refers to the inner one.

## 2. Polynomial Coefficients

The 5th order polynomials, that relate the factors and the cosine of incident angle, are fitted by the least-squares method. For example, the polynomial that relates A and $\cos \theta_1$ is

$$A = \sum_{i = o}^{5} C_{A,i} (\cos \theta_1)^i \quad (12)$$

where $C_{A,i}$ = calculated polynomial coefficients for an absorbtivity A.

Polynomials are fitted only to the non-polarized beam factors using the calculated values for $\theta_1 = 0°$ to $90°$ in one degree steps.

## 3. Diffuse Radiation

For heat-gain calculations through a window it is usually assumed that the diffuse radiation from the sky or the ground has equal intensity at all incident angles. The factors for diffuse radiation of this nature are given by

$$F_{diffuse} = \int_{o}^{\pi/2} F(\theta) \sin 2\theta \, d\theta \quad (13)$$

where F(θ) is the factor for direct solar beam and is a function of the incident angle θ. The substitution of the polynomial expression for F(θ) and integration gives

$$F_{diffuse} = 2 \sum_{i = o}^{5} \frac{Ci}{i + 2} \tag{14}$$

where $Ci$ = 5th order polynomial coefficients that relates the factor F and $\cos \theta_1$.

## 4. General Description of the Program

This Fortran IV program is for an IBM-System/360 operating system.

The coding sheets, a sample of output and the flow diagram (fig. A1) are given on pages A-1 to A-7 of Appendix A.

Input:

Card 1 - columns 1-10 n, index of refraction

Card 2 - columns 1-10 KL inside
11-20 KL outside.

Format: Floating point, 10 columns.

This paper is a contribution of the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

# APPENDIX A

```
KL INSIDE=0.12    KL OUTSIDE=0.60    N=1.52

INCIDENT  COSINE       PERPENDICULAR                 PARALLEL                     AVERAGE
 ANGLE     ANGLE     A1      A2      T          A1      A2      T          A1      A2      T

    0     1.0000   0.0569  0.4587  0.4114     0.0569  0.4587  0.4114     0.0569  0.4587  0.4114
   10     0.9848   0.0568  0.4610  0.4067     0.0572  0.4606  0.4122     0.0570  0.4608  0.4095
   20     0.9397   0.0565  0.4677  0.3922     0.0581  0.4661  0.4147     0.0573  0.4669  0.4034
   30     0.8660   0.0558  0.4784  0.3667     0.0597  0.4751  0.4190     0.0578  0.4768  0.3929
   40     0.7660   0.0542  0.4919  0.3284     0.0619  0.4871  0.4247     0.0581  0.4895  0.3765

   45     0.7071   0.0529  0.4987  0.3036     0.0631  0.4940  0.4271     0.0580  0.4964  0.3653
   50     0.6428   0.0511  0.5047  0.2746     0.0642  0.5015  0.4279     0.0577  0.5031  0.3513
   55     0.5736   0.0486  0.5086  0.2411     0.0651  0.5095  0.4249     0.0569  0.5091  0.3330
   60     0.5000   0.0454  0.5084  0.2033     0.0653  0.5184  0.4139     0.0553  0.5134  0.3086
   65     0.4226   0.0410  0.5007  0.1617     0.0641  0.5281  0.3883     0.0526  0.5144  0.2750

   70     0.3420   0.0355  0.4803  0.1178     0.0605  0.5375  0.3394     0.0480  0.5089  0.2286
   72     0.3090   0.0329  0.4668  0.1002     0.0581  0.5400  0.3115     0.0455  0.5034  0.2058
   74     0.2756   0.0300  0.4494  0.0828     0.0551  0.5407  0.2784     0.0425  0.4951  0.1806
   76     0.2419   0.0269  0.4270  0.0660     0.0513  0.5382  0.2404     0.0391  0.4826  0.1532
   78     0.2079   0.0234  0.3987  0.0502     0.0467  0.5303  0.1983     0.0351  0.4645  0.1243

   80     0.1736   0.0197  0.3631  0.0357     0.0412  0.5136  0.1537     0.0305  0.4384  0.0947
   82     0.1392   0.0156  0.3187  0.0230     0.0348  0.4836  0.1088     0.0252  0.4011  0.0659
   84     0.1045   0.0113  0.2633  0.0126     0.0272  0.4329  0.0666     0.0193  0.3481  0.0396
   86     0.0698   0.0068  0.1943  0.0051     0.0184  0.3506  0.0310     0.0126  0.2725  0.0181
   88     0.0349   0.0025  0.1083  0.0009     0.0082  0.2179  0.0071     0.0054  0.1631  0.0040

POLYNOMIAL COEFFICIENTS FOR AVERAGE ABSORPTION, TRANSMISSION, AND REFLECTION.

                      A1          A2          T           R
    C0            -0.002607    0.035383   -0.006516    0.973741
    C1             0.241959    4.179491    0.137991   -4.559441
    C2            -0.316419  -14.234123    4.068337   10.482203
    C3             0.091218   23.624634  -10.116615  -13.599236
    C4             0.107813  -19.170715    9.522376    9.540527
    C5            -0.065152    6.026407   -3.195348   -2.765906

DIFFUSE VALUES     0.054299    0.486093    0.334166    0.125438

LEGEND
A1=ABSORPTION INSIDE PANE
A2=ABSORPTION OUTSIDE PANE
T=TRANSMISSION
R=REFLECTION
IF KL OUTSIDE=0.00 THEN THE VALUES ARE FOR A SINGLE GLAZED WINDOW
KL=PRODUCT OF GLASS ABSORPTION COEFFICIENT AND GLASS THICKNESS
N=INDEX OF REFRACTION OF THE GLASS
```

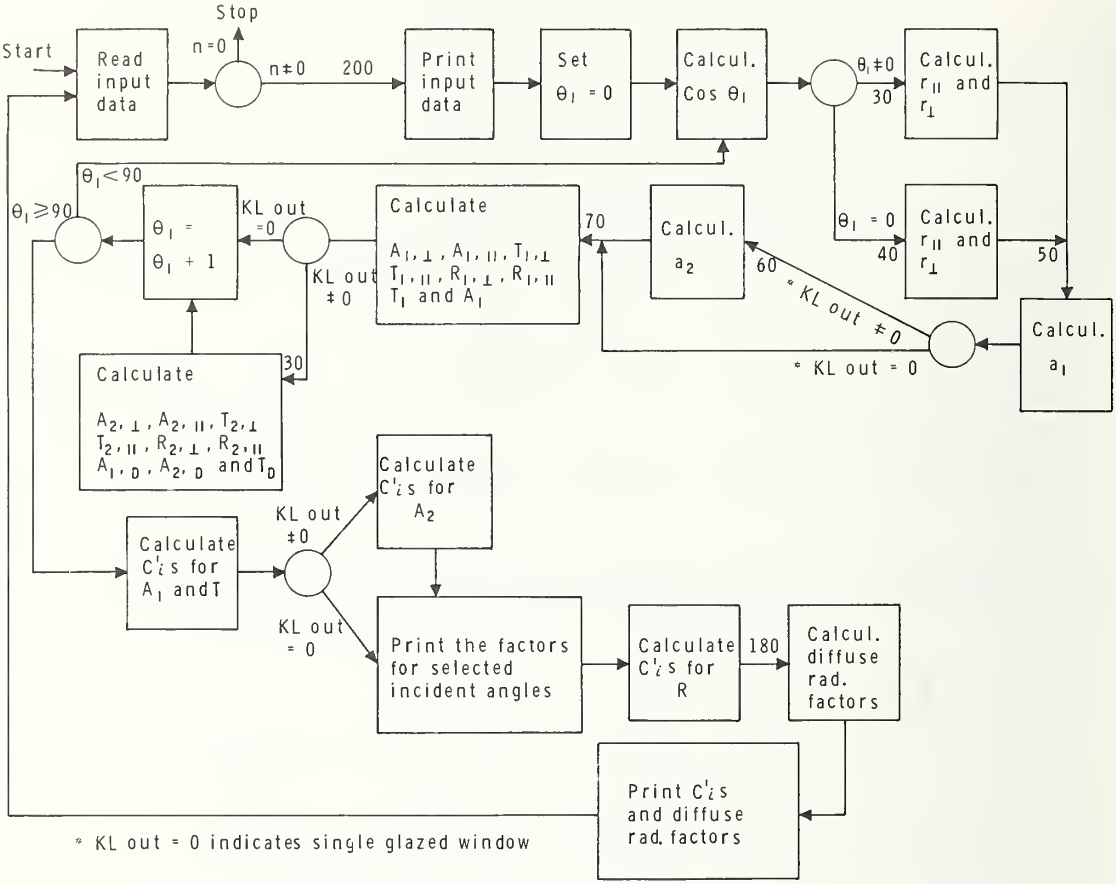

Figure A1 - Flow Diagram for Absorption and Transmission Factor Calculation Program.

```
C      PROGRAM BR-81.
C      ABSORPTION AND TRANSMISSION OF THERMAL RADIATION
C      BY SINGLE AND DOUBLE GLAZED WINDOWS.
       DIMENSION A(100),B(100),C(100),D(100),E(100),F(100),X(100),
      9Y(100),Z(100),G(100),STORE(16,15),W(100),CC(4,7)
       DOUBLE PRECISION PI,X,Y,Z,G,STORE,W,TOL
       INTEGER CARD,PRINT
       REAL N,KLI,KLO
       CARD=1
       IOUT=2
       PRINT=3
       PI=3.14159265
       TOL=0.0000001
  2000 READ(CARD,1) N
     1 FORMAT(2F10.4)
       IF(N)10,1000,10
    10 READ(CARD,1)KLI,KLO
       WRITE(PRINT,5) KLI,KLO,N
     5 FORMAT(1H1,'KL INSIDE=',F4.2,4X,'KL OUTSIDE=',F4.2,4X,'N=',F4.2)
       WRITE(PRINT,1)
       DO 20 I=1,90
       THETA=(I-1)*PI/180.0
       G(I)=COS(THETA)
       IF(THETA)30,40,30
    40 RPA=(N-1.0)/(N+1.0)
       RPA=RPA*RPA
       RPE=RPA
       GO TO 50
    30 TETA=SIN(THETA)/N
       TEMP=SQRT(1.0-TETA*TETA)
       TETA=ATAN(TETA/TEMP)
       RPE=SIN(THETA-TETA)/SIN(THETA+TETA)
       RPA=RPE*COS(THETA+TETA)/COS(THETA-TETA)
       RPA=RPA*RPA
       RPE=RPE*RPE
    50 TEMP=1.0-EXP(-N*KLI/SQRT(N*N-SIN(THETA)*SIN(THETA)))
       APA=TEMP
       APE=TEMP
       IF(KLO)60,70,60
    60 TEMP=1.0 -EXP(-N*KLO/SQRT(N*N-SIN(THETA)*SIN(THETA)))
       APAO=TEMP
       APEO=TEMP
    70 TEMP=1.0 -RPA*RPA*(1.0-APA)*(1.0-APA)
       AIPA=APA*(1.0-RPA)*(1.0+RPA*(1.0-APA))/TEMP
       TIPA=(1.0-RPA)*(1.0-RPA)*(1.0-APA)/TEMP
       TEMP=1.0-RPE*RPE*(1.0-APE)*(1.0-APE)
       AIPE=APE*(1.0-RPE)*(1.0+RPE*(1.0-APE))/TEMP
       TIPE=(1.0-RPE)*(1.0-RPE)*(1.0-APE)/TEMP
       RIPE=1.0-AIPE-TIPE
       RIPA=1.0-AIPA-TIPA
       TI=(TIPE+TIPA)/2.0
       AI=(AIPE+AIPA)/2.0
       A(I)=AIPE
```

```
      B(I)=0.0
       C(I)=TIPE
      D(I)=AIPA
      E(I)=0.0
      F(I)=TIPA
      X(I)=AI
      Y(I)=0.0
      Z(I)=TI
      IF(KLO)80,20,80
   80 TEMP=1.0-RPA*RPA*(1.0-APAO)*(1.0-APAO)
      AOPA=APAO*(1.0-RPA)*(1.0+RPA*(1.0-APAO))/TEMP
      TOPA=(1.0-RPA)*(1.0-RPA)*(1.0-APAO)/TEMP
      TEMP=1.0-RPE*RPE*(1.0-APEO)*(1.0-APEO)
      AOPE=APEO*(1.0-RPE)*(1.0+RPE*(1.0-APEO))/TEMP
      TOPE=(1.0-RPE)*(1.0-RPE)*(1.0-APEO)/TEMP
      ROPE=1.0-AOPE-TOPE
      ROPA=1.0-AOPA-TOPA
      AO=AOPE*(1.0+RIPE*TOPE/(1.0-RIPE*ROPE))
      B(I)=AO
      AO=AO+AOPA*(1.0+RIPA*TOPA/(1.0-RIPA*ROPA))
      E(I)=AO-B(I)
      Y(I)=AO/2.0
      AI=AIPE*TOPE/(1.0-ROPE*RIPE)
      A(I)=AI
      AI=AI+AIPA*TOPA/(1.0-ROPA*RIPA)
      D(I)=AI-A(I)
      X(I)=AI/2.0
      TI=TOPE*TIPE/(1.0-RIPE*ROPE)
      C(I)=TI
      TI=TI+TOPA*TIPA/(1.0-RIPA*ROPA)
      F(I)=TI-C(I)
      Z(I)=TI/2.0
   20 CONTINUE
      CALL POLY(G,X,W,STORE,90,2,2,2,TOL,5)
      DO 90 I=1,6
      CC(2,I)=0.0
   90 CC(1,I)=STORE(I,5)
      CALL POLY(G,Z,W,STORE,90,2,2,2,TOL,5)
      DO 100 I=1,6
  100 CC(3,I)=STORE(I,5)
      IF(KLO)110,120,110
  110 CALL POLY(G,Y,W,STORE,90,2,2,2,TOL,5)
      DO 130 I=1,6
  130 CC(2,I)=STORE(I,5)
  120 WRITE(PRINT,2)
    2 FORMAT(1X,'INCIDENT',2X,'COSINE',7X,'PEPPENDICULAR',16X,
     9'PARALLEL',19X'AVERAGE')
      WRITE(PRINT,3)
    3 FORMAT(2X,'ANGLE',5X,'ANGLE',5X,'A1',6X,'A2',7X,'T',
     98X,'A1',6X,'A2',7X,'T',8X,'A1',6X,'A2',7X,'T')
      WRITE(PRINT,1)
      DO 140 I=1,41,10
      IT=I-1
```

```
  140 WRITE(PRINT,4) IT,G(I),A(I),B(I),C(I),D(I),E(I),F(I),X(I),
     9Y(I),Z(I)
      WRITE(PRINT,1)
    4 FORMAT(1X,I5,F11.4,F9.4,2F8.4,F10.4,2F8.4,F10.4,2F8.4)
      DO 150 I=46,66,5
      IT=I-1
  150 WRITE(PRINT,4) IT,G(I),A(I),B(I),C(I),D(I),E(I),F(I),X(I),
     9Y(I),Z(I)
      WRITE(PRINT,1)
      DO 160 I=71,79,2
      IT=I-1
  160 WRITE(PRINT,4) IT,G(I),A(I),B(I),C(I),D(I),E(I),F(I),X(I),
     9Y(I),Z(I)
      WRITE(PRINT,1)
      DO 165 I=81,89,2
      IT=I-1
  165 WRITE(PRINT,4) IT,G(I),A(I),B(I),C(I),D(I),E(I),F(I),X(I),
     9Y(I),Z(I)
      DO 170 I=1,100
  170 X(I)=1.0 -X(I)-Y(I)-Z(I)
      CALL POLY(G,X,W,STORE,90,2,2,2,TOL,5)
      DO 180 I=1,6
  180 CC(4,I)=STORE(I,5)
      DO 190 I=1,4
  190 CC(I,7)=0.0
      DO 200 I=1,6
      II=I+1
      DO 200 J=1,4
  200 CC(J,7)=CC(J,7)+CC(J,I)/II*2.0
      WRITE(PRINT,6)
    6 FORMAT(1H0,'POLYNOMIAL COEFFICIENTS FOR AVERAGE ABSORPTION, TRANSM
     9ISSION, AND REFLECTION.')
      WRITE(PRINT,1)
      WRITE(PRINT,8)
    8 FORMAT(25X,'A1',9X,'A2',10X,'T',10X,'R')
      WRITE(PRINT,7) ((CC(I,J),I=1,4),J=1,7)
    7 FORMAT(6X,'C0',11X,4F11.6/6X,'C1',11X,4F11.6/6X,'C2',11X,4F11.6/
     96X,'C3',11X,4F11.6/6X,'C4',11X,4F11.6/6X,'C5',11X,4F11.6/
     9          /1X,'DIFFUSE VALUES',4X,4F11.6)
      WRITE(IOUT,11) ((CC(I,J),J=1,7),I=1,4)
   11 FORMAT(7F11.6)
      WRITE(PRINT,1)
      WRITE(PRINT,9)
    9 FORMAT(1X,'LEGEND'/1X,'A1=ABSORPTION INSIDE PANE'/
     91X,'A2=ABSORPTION OUTSIDE PANE'/
     91X,'T=TRANSMISSION'/
     91X,'R=REFLECTION'/
     91X,'IF KL OUTSIDE=0.00 THEN THE VALUES ARE FOR A SINGLE GLAZED WIN
     9DOW'/1X,'KL=PRODUCT OF GLASS ABSORPTION COEFFICIENT AND GLASS THIC
     9KNESS'/1X,'N=INDEX OF REFRACTION OF THE GLASS')
      GO TO 2000
 1000 CALL EXIT
      END
```

```
FORTRAN IV G LEVEL  18                 MAIN               DATE = 70126          14/00/52              PAGE 0001

                    C  MODIFICAT+ON TO GRAVES POLYNOMIAL CURVE FITTING TO FORTRAN  IV        GA  0046
                    C     N IS THE NUMBER OF OBSERVATIONS                                    GA  0048
                    C    TOL IS THE ALLOWABLE VALUE OF THE STANDARD ERROR                    GA  0050
                    C  LAST IS THE VALUE OF THE HIGHEST ORDER OF THE POLYNOMIAL TO BE FITTEDGA  0052
                    C  III=1 FOR WEIGHTED INPUT                                              GA  0054
                    C   III=2 FOR UNWEIGHTED INPUT                                           GA  0056
                    C  ISS2=1 FOR INTERMEDIATE OUTPUT                                        GA  0058
                    C  ISS2=2 FOR FINAL OUTPUT ONLY                                          GA  0060
                    C  ISS3=1 FOR OUTPUT OF OBSERVED AND CALCULATED VALUES                   GA  0062
                    C  ISS3=2 FOR NO OUTPUT                                                  GA  0064
                    C    ISS3 ONLY USED IF ISS2=1                                            GA  0066
 0001                     SUBROUTINE POLY(X,Y,W,STORE,N,III,ISS2,ISS3,TOL,LAST)              GA  0068
 0002                     DOUBLE PRECISION X(100),Y(100),A(16,31),SUMX(31),SUMY(15),W(100),CGA  0070
                         1,S1,S2,S3,B,TOL,E(16),S4,STORE(16,15)                              GA  0072
 0003                     IN=1                                                               GA  0074
 0004                     IOUT=3                                                             GA  0076
 0005                     DO 1 I=1,16                                                        GA  0078
 0006                     DO 1 J=1,15                                                        GA  0080
 0007                   1 STORE(I,J)=0.0                                                     GA  0082
 0008                     GO TO (70,50) , III                                                GA  0084
 0009                  50 DO 60 I=1,N                                                        GA  0086
 0010                  60 W(I)=1.                                                            GA  0088
 0011                  70 SUMX(1)=0.                                                         GA  0090
 0012                     SUMX(2)=0.                                                         GA  0092
 0013                     SUMX(3)=0.                                                         GA  0094
 0014                     SUMY(1)=0.                                                         GA  0096
 0015                     SUMY(2)=0.                                                         GA  0098
 0016                     DO 90 I=1,N                                                        GA  0100
 0017                     SUMX(1)=SUMX(1)+W(I)                                               GA  0102
 0018                     SUMX(2)=SUMX(2)+W(I)*X(I)                                          GA  0104
 0019                     SUMX(3)=SUMX(3)+W(I)*X(I)*X(I)                                     GA  0106
 0020                     SUMY(1)=SUMY(1)+W(I)*Y(I)                                          GA  0108
 0021                  90 SUMY(2)=SUMY(2)+W(I)*X(I)*Y(I)                                     GA  0110
 0022                     NORD=1                                                             GA  0112
 0023                  91 L=NORD+1                                                           GA  0114
 0024                     KK=2*L+1                                                           GA  0116
 0025                     KA=L+1                                                             GA  0118
 0026                     DO 101 I=1,L                                                       GA  0120
 0027                     DO 100 J=1,L                                                       GA  0122
 0028                     IK=J-1+I                                                           GA  0124
 0029                     JL=J+L+1                                                           GA  0126
 0030                     A(I,JL)=0.                                                         GA  0128
 0031               100   A(I,J)=SUMX(IK)                                                    GA  0130
 0032                     JL=I+L+1                                                           GA  0132
 0033                     A(I,JL)=1.0                                                        GA  0134
 0034               101   A(I,KA)=SUMY(I)                                                    GA  0136
 0035                     DO 140 I=1,L                                                       GA  0138
 0036                     A(KA,I)=-1                                                         GA  0140
 0037                     KKK=I+1                                                            GA  0142
 0038                     DO 110 J=KKK,KK                                                    GA  0144
 0039               110   A(KA,J)=0.                                                         GA  0146
 0040                     C=1./A(1,I)                                                        GA  0148
 0041                     DO 120 II=2,KA                                                     GA  0150
```

```
      DO 120 J=KKK,KK                                                   GA  0152
  120 A(II,J)=A(II,J)-A(1,J)*A(II,I)*C                                  GA  0154
      DO 140 II=1,L                                                     GA  0156
      DO 140 J=KKK,KK                                                   GA  0158
  140 A(II,J)=A(II+1,J)                                                 GA  0160
      DO 141 II=1,L                                                     GA  0162
      J=NORD                                                            GA  0164
  141 STORE(II,J)=A(II,KA)                                              GA  0166
C TO CALCULATE THE ERROR IN THE COEFFICIENTS YOU NOW NEED               GA  0168
      S4=0.                                                             GA  0170
      DO 160 J=1,N                                                      GA  0172
      S1=A(1,KA)                                                        GA  0174
      DO 150 I=1,NORD                                                   GA  0176
  150 S1=S1+A(I+1,KA)*X(J)**I                                           GA  0178
  160 S4=S4+W(J)*(S1-Y(J))*(S1-Y(J))                                    GA  0180
      B=N-L                                                             GA  0182
      S4=(S4/B)**.5                                                     GA  0184
      S2=S4                                                             GA  0186
      DO 1000 I=1,L                                                     GA  0188
      J=I+L                                                             GA  0190
 1000 E(I)=(A(I,J+1   ))**.5*S4                                         GA  0192
      GO TO (163,161),ISS2                                              GA  0194
  161 IF(NORD-LAST)162,173,162
  162 IF(S2-TOL)163,163,171                                             GA  0198
  163 WRITE (IOUT,10)                                                   GA  0200
   10 FORMAT (1H0,5HORDER,5X9HTOLERANCE,15X14HSTANDARD ERROR,6X3HOBS)   GA  0202
      WRITE (IOUT,8) NORD,TOL,S2,N                                      GA  0204
    8 FORMAT (1X  I3,2D24.16,I3//)                                      GA  0206
  210 DO 164 I=1,L                                                      GA  0208
      J=I-1                                                             GA  0210
      IF (I-1) 164,11,164                                               GA  0212
   11 WRITE (IOUT,12)                                                   GA  0214
   12 FORMAT (8X12HCOEFFICIENTS,14X10HSTD. ERROR,/)                     GA  0216
  164 WRITE (IOUT,6) J,A(I,KA),E(I)                                     GA  0218
    6 FORMAT (1XI3,D24.16,D24.16)                                       GA  0220
      GO TO (167,165),ISS3                                              GA  0222
  165 IF(NORD-LAST)166,167,166                                          GA  0224
  166 IF(S2-TOL)167,167,171                                             GA  0226
  167 DO 169 I=1,N                                                      GA  0228
      S1=A(1,KA)                                                        GA  0230
      DO 168 J=1,NORD                                                   GA  0232
  168 S1=S1+A(J+1,KA)*X(I)**J                                           GA  0234
      S3=Y(I)-S1                                                        GA  0236
      IF (I-1) 169,14,169                                               GA  0238
   14 WRITE(IOUT,15)                                                    GA  0240
   15 FORMAT (1H0,9X6HOBS(X),17X6HOBS(Y),17X6HCAL(Y),17X10HDIFFERENCE,/)GA  0242
  169 WRITE(IOUT,7) X(I),Y(I),S1,S3                                     GA  0244
    7 FORMAT (1X4D24.16)                                                GA  0246
      IF(NORD-LAST)170,173,173                                          GA  0248
  170 IF(S2-TOL)173,173,171                                             GA  0250
  171 NORD=NORD+1                                                       GA  0252
      J=2*NORD                                                          GA  0254
      SUMX(J)=0.                                                        GA  0256
```

```
 0094               SUMX(J+1)=0.                                                    GA  0258
 0095               SUMY(NORD+1)=0.                                                 GA  0260
 0096               DO 172  I=1,N                                                   GA  0262
 0097               SUMX(J)=SUMX(J)+X(I)**(J-1)*W(I)                                GA  0264
 0098               SUMX(J+1)=SUMX(J+1)+X(I)**J*W(I)                                GA  0266
 0099           172 SUMY(NORD+1)=SUMY(NORD+1)+Y(I)*X(I)**NORD*W(I)                  GA  0268
 0100               GO TO 91                                                        GA  0270
 0101           173 CONTINUE                                                        GA  0272
 0102           200 RETURN                                                          GA  0274
 0103               END                                                             GA  0276
```

# A Computer Analysis of Window Shading Coefficients by Calculating Optical and Thermal Transmission

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Measurements of solar heat gain through windows with different shading devices are rather complicated and need a calorimeter or some other kind of expensive equipment. The optical properties of a single layer can be found quite easily by spectrometer measurements in a laboratory. The paper describes a method to calculate the solar heat gain through combinations of layers outgoing from the optical data for the single layers. The ALGOL-program developed to solve this problem allows an arbitrary number of layers, anyone could be diffusing. The direct and total transmission through the combination, the reflexion, and the shading coefficients are calculated. The results of a theoretical investigation of different window combinations are presented.

Key Words: Artificial lighting, daylighting, daylight distribution, fenestration, light transmission, polarization, shading coefficient, shading devices, solar heat transmission, solar optical properties, solar radiation, window shading coefficients.

## 1. Introduction

For a correct choice of shading devices many different aspects should be taken into consideration. Apart from the solar heat reduction, illumination demands, glare, operating possibilities, dirtying, sound, aesthetical demands etc. should be paid regard to. In most cases these points of view should be taken into account at the lowest possible cost. Information on shading devices is often restricted to values of the solar heat reduction. Below it is indicated how calculations of the transmission of sunlight as well as solar heat can be made starting from optical data for the different layers of a window. Thereby it is possible to determine the heat flow into a room with regard to solar heat and artificial illumination.

## 2. Concepts and Definitions

The optical properties of a translucent layer is indicated by its transmission, T, reflexion,R, and absorption, A, of incident radiation. For these quantities the following is always applicable

$$T + R + A = 1$$

The optical properties are due to the wave-length of the radiation. Solar radiation is within a range of wave-length with a certain distribution within this range.

On calculations of this kind it is presupposed that the optical properties are average values for solar radiation, i.e. they are valid with regard to the distribution of wave-length of the solar radiation and the properties of the layers (Solar-Optical Properties).

The part of the incident radiation which is absorbed, part A, causes a heating of the layer. Hereby part of A will be transmitted into the room by long-wave radiation and convection, the secondary transmission.

Thus the solar heat transfer through a window consists of two parts, the direct and the secondary transmission.

The direct and total transmission is due to the angle of incidence of the radiation. For most window combinations curves of almost the same appearance are applicable. The curves diverge approximately by a constant factor. This implies that the transmission of a certain window can be related to

For continuity reasons this is also achieved

$$q_2 = q_1 + A_{1(2)}$$

$$q_{sec} = q_2 + A_{(1)2}$$

Finally this definition is made

$$m_{tot} = m_y + m_f + m_i$$

From these equations $q_1$, $q_2$, $t_1$ and $t_2$ are eliminated and the secondary transmission is obtained from the expression

$$q_{sec} = \frac{t_l - t_i}{m_{tot}} + A_{1(2)} \cdot \frac{m_y}{m_{tot}} + A_{(1)2} \cdot \frac{m_y + m_f}{m_{tot}}$$

Here the first term constitutes the heat transmission owing to the difference of temperature between the outside and the inside, the other terms are contributions from absorbed solar radiation.

Similar derivations can be made for several layers. It can be observed that the inward-flowing fraction of the absorbed radiation in a layer is always the ratio of the thermal resistance from the layer to the outside air to the total thermal resistance of the window.

## 4. Arbitrary number of layers. Computer Program

The extensiveness of the calculations increases rapidly by the number of layers. The equations for the course of radiation in the layer combination vary owing to where a possible diffusing layer may have its position. Instead of using different equations for different cases when calculating with a computer the radiation components are followed and added successively until all contributions are exceedingly small.

The core of the ALGOL-program which has been drawn up for these calculations consists of a procedure which divides the radiation components at each layer into one transmitted, one reflected and one absorbed part. The radiation components are stored in a two-dimensioned array

IN [FROM, TO]

The layers are numbered 1, 2, 3 etc... NUMBER from the outside. (Capital letters for identifiers). The calculations proceed by steps forwards and backwards through the window. At each step one element is divided up according to the pattern below

$$\begin{tikzcd}
 & \text{ATOT}[1] & & \text{ATOT}[2] & & \text{ATOT}[3] & \\
\text{etc.} & \text{IN}[2,1] \arrow[l] \arrow[u] \arrow[dr] & & \text{IN}[1,2] \arrow[ll] \arrow[rr] \arrow[u] & & \text{IN}[2,3] \arrow[r] \arrow[u] \arrow[dl] & \text{etc.} \\
 & & \text{etc.} & & \text{etc.} & &
\end{tikzcd}$$

When the radiation has passed through or has been reflected against a diffusing layer the optical properties of the different layers which are stored in the arrays T NR , R NR and A NR are exchanged for the values applicable to diffuse radiation.

The direct transmission constitutes the sum of all elements IN [NUMBER, NUMBER + 1] and the reflexion the sum of all elements IN [1, 0] .

When the calculation has proceeded so far that all contributions are smaller than a given tolerance, the direct transmission, the reflexion and the absorption in each layer are summed up. This sum should be = 1. Finally the secondary transmission is determined with the aid of the thermal resistances which were given as data and the calculated values of the absorption in the different layers.

the transmission through a reference window and thus be characterized with the aid of one single factor which is independent of the angle of incidence. This factor is called the Shading Coefficient.

If the solar radiation values are determined once for all with regard to the reduction in the reference window the angles of incidence of the solar radiation need not be determined any more. In Sweden an unshaded two-glass window of ordinary window-glass is used as a reference. (The total transmission when the radiation is falling in perpendicularly = 0,79).

The directly transmitted short-wave radiation and the secondarily transmitted solar heat are distributed in different ways in a room. The short-wave radiation is reflected between the room-surfaces, whereas the long-wave radiation broadly speaking interprets the room-surfaces as being black. When calculating with our main program [2] this difference is taken into consideration. For this reason two shading coefficients $F_1$ and $F_2$ have been introduced: $F_1$ multiplied by the incident solar radiation through the reference window gives the totally transmitted solar heat, $F_2$ similarly the directly transmitted part. If the spectral changes of the radiation are small at the passage through the window, $F_2$ is a value of the light transmission.

## 3. Double panes

### 3.1 Short-wave radiation

When several layers are combined they affect each other by reflexions between themselves. The course of radiation when there are two translucent layers can be seen in Fig. 1, in which the denominations used below are also given. The repeated reflexions give rise to geometrical series, the sums of which are included in the following expressions:

The direct transmission is achieved from the expression

$$T_{12} = \frac{T_1 \cdot T_2}{1 - R_1 \cdot R_2}$$

The reflexion

$$R_{12} = R_1 + \frac{T_1^2 \cdot R_2}{1 - R_1 \cdot R_2}$$

The absorption in layer No. 1 with contribution of layer No. 2

$$A_{1(2)} = A_1 \left(1 + \frac{T_1 \cdot R_2}{1 - R_1 \cdot R_2}\right)$$

The absorption in layer No. 2

$$A_{(1)2} = \frac{T_1 \cdot A_2}{1 - R_1 \cdot R_2}$$

### 3.2 Long-wave radiation and convection

The secondary transmission can be derived from Fig. 2.

For the denominations introduced in this figure the following is applicable

t = temperature
m = thermal resistance
q = heat flow

The heat flows are given by

$$q_1 = \frac{1}{m_y} (t_l - t_1)$$

$$q_2 = \frac{1}{m_f} (t_1 - t_2)$$

$$q_{sec} = \frac{1}{m_i} (t_2 - t_i)$$

# 5. Applications

## 5.1 Influence of polarization

As to calculations including several panes the effect of the polarization is often taken into account. This is not the case in this program. Fig. 3 shows the direct transmission through a two-glass window determined with as well as without regard to the polarization. It is evident that the imperfection is not noticeable until the angles of incidence are larger than about $40^{o}$. If the determination of $F_1$ and $F_2$ is made with layer data valid at an angle of incidence of for instance $30^{o}$ the imperfection is negligible.

## 5.2 Different numbers of panes

The availability of the program is first illustrated by the following example:

Calculations with the number of ordinary window-glasses (T = 0.86, R = 0.079) increasing from 2 to 6 have been made. Thermal resistances: outside 0.06, inside 0.11, between glasses 0.17 $m^2\ {}^{o}C\ W^{-1}$. Fig. 4 shows the result. The secondary transmission increases by the number of glasses. The direct transmission, however, decreases more so that the total transmission decreases by an increasing number of panes.

## 5.3 Windows with drapes

The optical properties of a drape fabric is variable within wide limits owing to the fact that the closeness of the texture as well as the color of the fabric can be varied. Calculations have been made for varying types of drapes in combination with two panes of ordinary window-glass ($T_N$ = 0.86) placed on the outside, between the panes and on the inside. The direct transmission indicated by $F_2$, is supposed to be the same for visible light as for radiation in the whole solar spectrum. Since the primary purpose of the window is to let in the daylight it is natural to start from $F_2$ as an independent variable, which is the case in the figures 5 - 7. The total transmission indicated through $F_1$ is a dependent variable and T and R of the drape fabric are parameters.

The following values of heat resistance in $m^2\ {}^{o}C\ W^{-1}$ have been used:

| | |
|---|---|
| outside 0.06 | |
| inside 0.11 | |
| glass to glass | 0.17 |
| glass to drape | 0.13 when drape outside, else 0.15 |

# 6. Utilization of calculated data

The following example is intended to illustrate how the values which are achieved from the diagrams can be used in a chain of calculations concerning the heat flow into a room due to sun and illumination.

## 6.1 Assumptions

The example refers to a room with the dimensions

breadth = 2.5 m
depth = 4.0 "
height = 2.9 "
window area = 3.24 $m^2$

Other pre-requisites:

| | |
|---|---|
| latitude | $60^{o}$N |
| orientation | South |
| climate | August, clear day |

Window: drape between 2 ordinary window-panes

The shading coefficient $F_1$ = 40%. This value is achieved according to Fig. 6 for instance with combinations according to the table below.

| Fabric Transmittance % | Fabric Reflectance % | $F_2$ % |
|---|---|---|
| 11 | 38 | 10 |
| 21 | 50 | 20 |
| 29 | 60 | 30 |

The value of $F_2$ is decisive for the daylight in the room. Owing to the demands of illumination $F_2$ therefore is determining for the heat energy from artificial illumination.

### 6.2 Solar radiation

The incident solar radiation, in $Wm^{-2}$, through the reference glass (unshaded double glazing) is determined by a program from information about latitude, longitude, date and orientation.

### 6.3 Visible radiation

The sunlight is supposed to be in keeping with the incident solar radiation and is indicated by a luminous efficiency factor.

### 6.3.2 Distribution of daylight

The distribution of daylight in the room is achieved with the aid of a program which determines the absorption factors for radiation from the window against horizontal surface elements placed arbitrarily in the room. The calculations thus presuppose a diffuse distribution of the light. In the result an infinite amount of reflexions between the room surfaces is included.

For the room in question the program gave the following values of the ratio of the lumance of the window surface to the daylight illumination along the central line of the room at a height of 0.8 m:

| Distance from window, m | ratio | |
|---|---|---|
| 1 | 0.212 | |
| 2 | 0.108 | (middle of room) |
| 3 | 0.068 | |

### 6.4 Artificial lighting

The calculations made so far have given the incident solar radiation and the daylight hour by hour in the room. Fig. 8 shows the incident solar radiation in $W m^{-2}$ window area and the illumination in lux in the centre of the room for $F_2$ = 10, 20 and 30%.

The consequence of different demands of illumination in the centre of the room can now be examined.

The calculations concern the time between 8 and 16. When the daylight is insufficient it is presupposed that the illumination is switched on. With different demands of illumination Fig. 8 gives the following illumination periods for different values of $F_2$

Artificial lighting, hours

| | | Lux | |
|---|---|---|---|
| $F_2$ | 500 | 1000 | 1500 |
| 10 | 3.5 | 8.0 | 8.0 |
| 20 | 0.5 | 3.5 | 8.0 |
| 30 | 0 | 1.4 | 3.5 |

### 6.5 Total load

Provided that the armature gives 20 lumens per watt the energy with which the room is supplied from sun and illumination during the day gets the following values:

Energy from sun and artificial lighting, W h

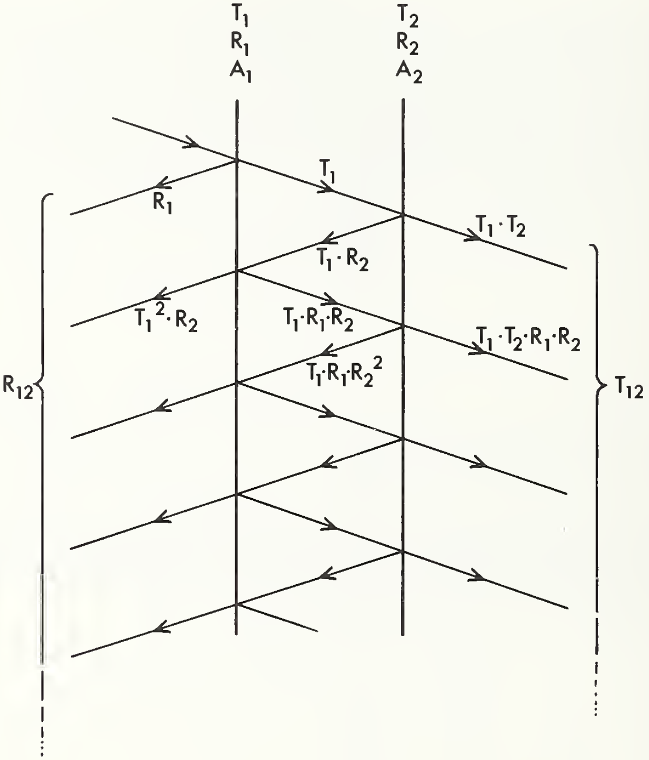
| | | Lux | |
|---|---|---|---|
| $F_2$ | 500 | 1000 | 1500 |
| 10 | 6100 | 9200 | 11200 |
| 20 | 5300 | 6950 | 11200 |
| 30 | 5200 | 5900 | 7800 |

On the basis of these values Fig. 9 has been drawn. The figure shows the great importance of considering the light as well as the solar heat transmission when choosing shading devices. The heat load as a result of sun and illumination at a demand of illumination of 1000 lux is about 50% larger for a drape with $F_2$ = 15% than for one with $F_2$ = 30%. The incident solar heat is the same in both cases ($F_1$ = 40%).

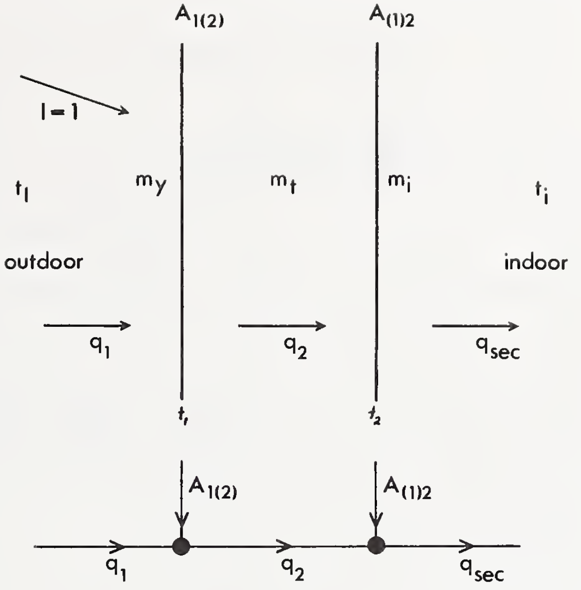
## 7. References

1 ASHRAE Handbook of Fundamentals, Ch. XXVIII pp 467-512 (1967)

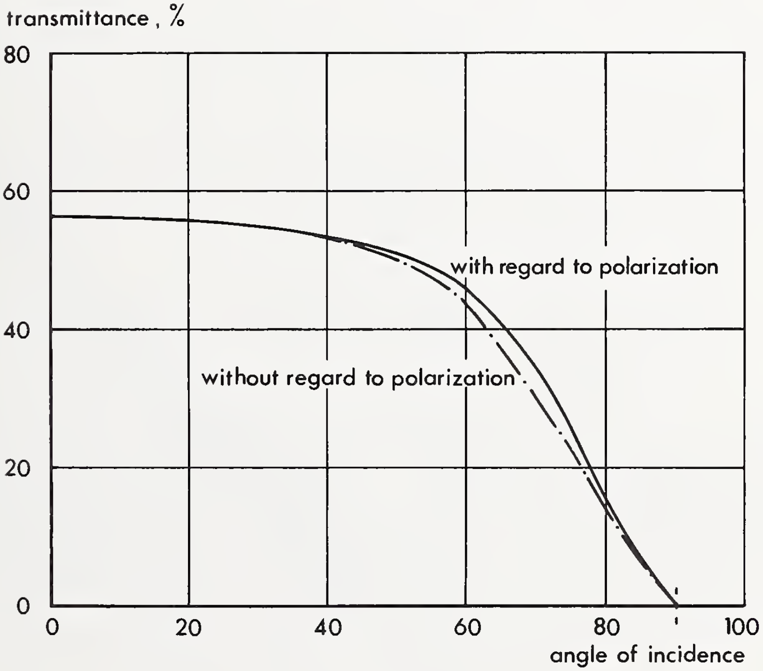
2 Brown, G., Simulation by digital computer program of the temperature variation in a room. Contribution to this symposium.



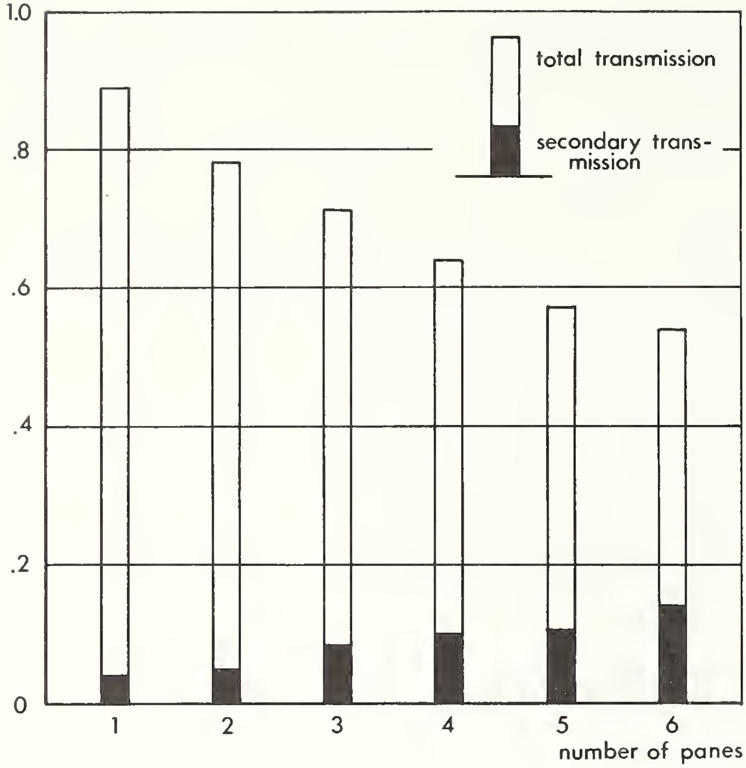
1. The course of radiation in double glazing. Denominations $T_1$, $R_1$ ... etc. are applicable for the layer alone.



2. Denominations used to derive the secondary transmission through a two-glass window.

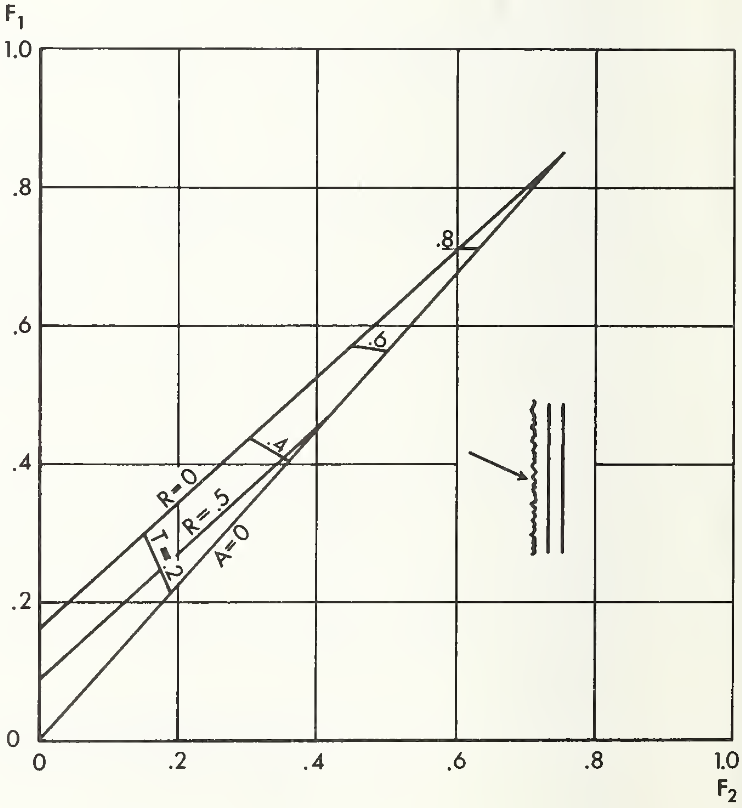


3. Direct transmission through a two-glass window determined with as well as without regard to the polarization.



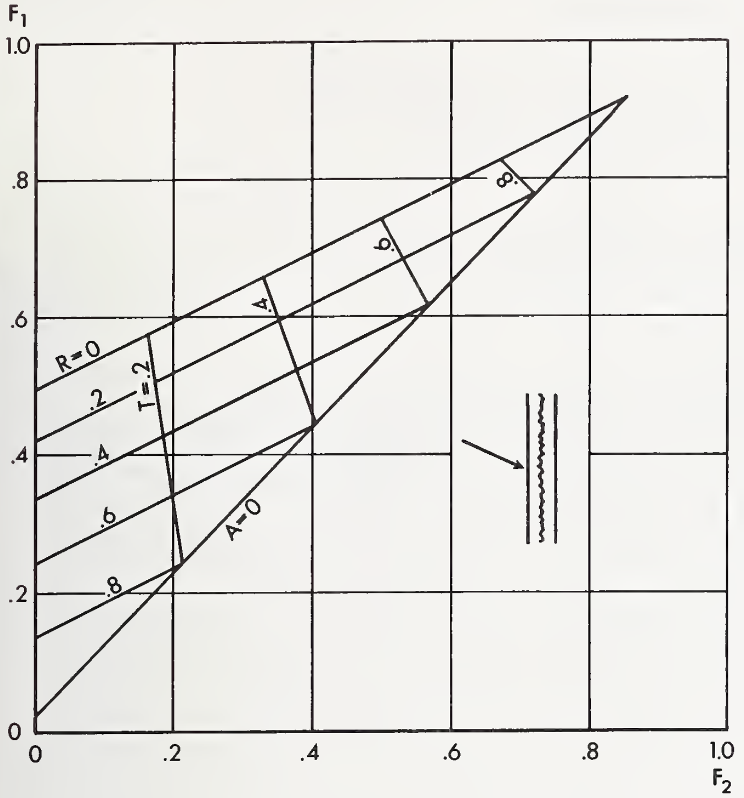
4.

Total and secondary transmission through different numbers of panes.



5

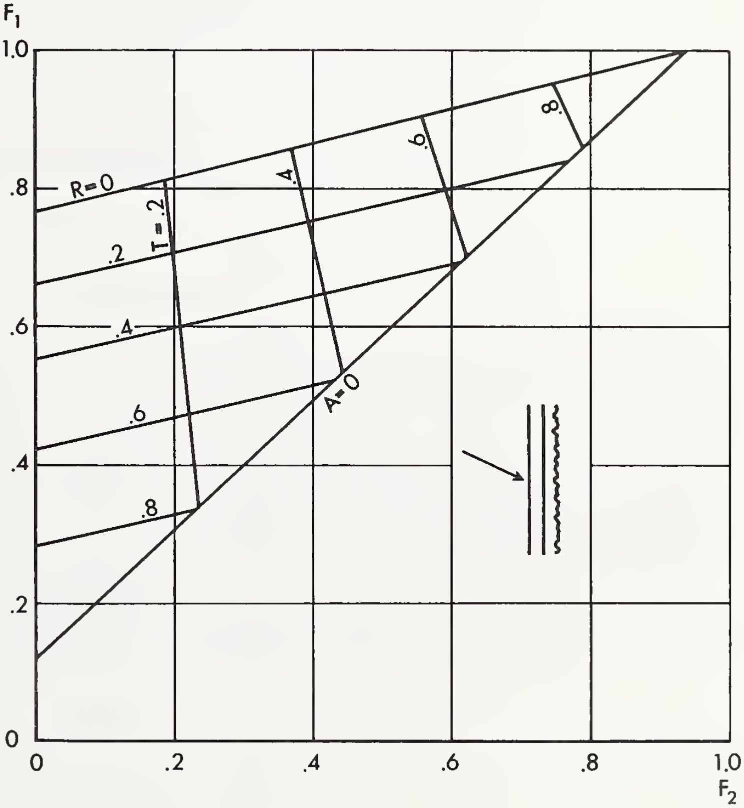
Shading coefficient $F_1$ (for total transmission) as function of coefficient $F_2$ (for direct transmission) for double glazing with drapes. Drape solar optical properties are parameters.

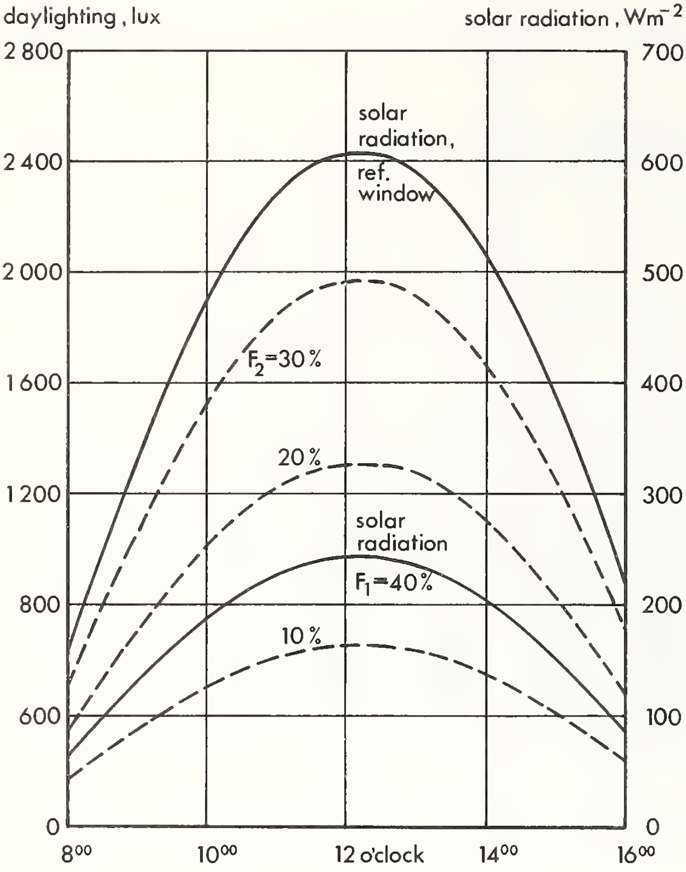


6
Shading coefficient $F_1$ (for total transmission) as function of coefficient $F_2$ (for direct transmission) for double glazing with drapes. Drape solar optical properties are parameters.

7

Shading coefficient $F_1$ (for total transmission) as function of coefficient $F_2$ (for direct transmission) for double glazing with drapes. Drape solar optical properties are parameters.



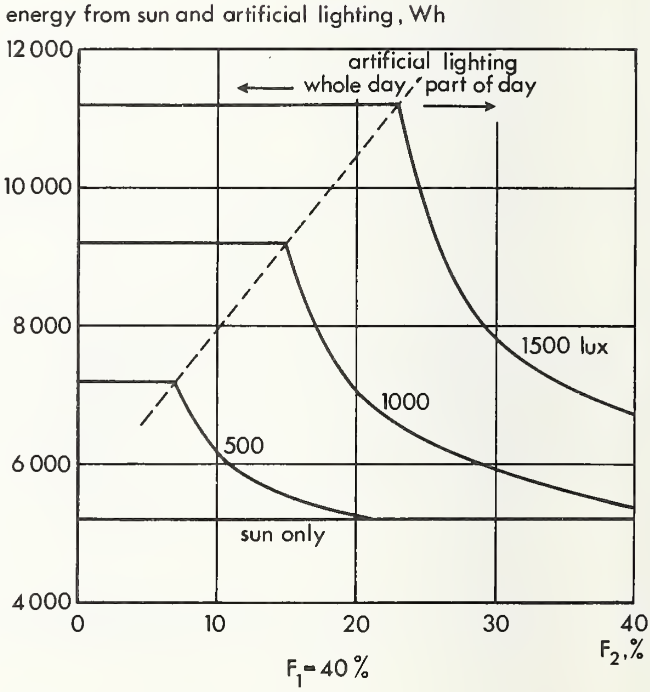


8.

Solar radiation through the window and daylighting (dotted lines) hour by hour in the middle of a room.

9.

Energy from sun and artificial lighting as function of coefficient $F_2$ at different demands of illumination.
Shading coefficient $F_1$ = 40%.



# Optimum Shape of External Shade for the Window to Minimize Annual Solar Heat Gain and to Maximize View Factor

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One of the most fluctuating components of space cooling load is associated with solar heat gain from windows. Overhangs, side fins or screen type of sun shades are widely used to avoid unwanted heat from the sun. On the other hand the window is expected to provide sunlight into the occupied space and allow people to look outside through it. Considering that the design of shading devices should be coordinated to both of these factors in conformity with aethetics, the author developed a procedure to determine the optimumshape of external shade for a rectangular window of a given proportionfacing to a specified orientation so that it could intercept direct radiation as much as possible and be fabricated with a minimum amount of material, providing a maximum possible view for the occupants. Iteration process is required to obtain the geometry of the sun shade composed of two pairs of horizontal and vertical flat plates with optimum width and tilt angle. This is followed by the estimation of the annual total solar heat gain from the window with the brise-soleil thus designed, taking account of reflection and re-radiation from sunlit surfaces of sun shade as well as direct and diffuse solar radiation transmitted through the shade. The heat exchange problem around the shade assembly is approximately solved by the method similar to that to calculate the weighting factors relating heat gain to cooling load. The routines developed are intended to be incorporated with graphical display system or plotter drawing system so as to be an effective aid for the architectural design.

Key Words: Computer, optimization, brise-soleil, window, solar radiation, shadow, overhang, side fin, view factor, heat gain, design.

## 1. Introduction

Design of sun shades for the building façade is one of the most interesting phases of architectural design associated with environmental engineering. In determining the shape of sun shades the designer wants to know the actual effects of shading devices in terms ofannual energy savings owing to the reduction of solar heat gain as well as the cost to fabricate it. It is considered necessary, therefore, to offer the designer the optimum range of key elements defining the geometry of sun shade so that he can determine the shape of it with his idea and aethetical judgement, eliminating such unnecessary steps that he has to draw many figures of obviously ineffective sun shades. Determining the shape, the computer can calculate the cooling load for air conditioning engineers as well as the manufacturing cost to justify the installation of sun shades.

Conventional architectural design involves various phases of design process such as working on drawings, refering to informations, making discussions and so on. In order to use the computer as a design partner the design procedure must be systematized and the design operations be performed in a logical flow without losing artistic inputs. This

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[1] Associate Professor, Department of Architecture.

is the general concept of computer aided design and the attempts made on the sun shade design is based on it.

## 2. Basic Concept

### 2.1 Design Process and Use of Computer

The process of architectural design is too complexed to be thoroughly systematized, but efforts are being made by computerists to seek the possibilities how far the design process can be computerized taking various human factors into account. This approach is being attempted by Negroponte (1)[1] and his colleagues in their program development for urban design. For simplicity in this paper it is presupposed that the design process is composed of the combination of various minor processes and they may be either inductive or deductive depending on situations.

It is considered that the conventional process of architectural design consists of multiple repetitions of making decisions based on various requirements and designer's idea and judgement through the routines of deriving effects caused by the decision and checking conflicts among other decisions in reference to the design criteria. This is shown on the left side of Fig. 1. Since it is always advantageous to use computer when calculation involves a number of repetitive routines, the design process with computer should be so arranged as to include repetitive routines. The right side of Fig. 1 shows a flow of the deductive design process where the computer gives a number of answers that meet the requirements and suffice the criteria and the designer only has to select one out of them based on his idea and judgement. If the computer gave only one answer, which might be the most optimum, there would be no room for selection. It is desirable, therefore, that the optimum range of key elements defining the object should be provided, because the most optimum one may be much inferior to another when refered to the designer's idea.

On the other hand conversational or on-line type of design process with computer, which might be called inductive design process, can go through the same routine as the conventional one as shown on the left side of Fig. 1 again. When the graphical display system were used, for example, the calculation of checking conflicts and deriving effects can simultaneously be made by the main processor and the designer can change as many decisions as he likes while sitting down at the C.R.T. console.

### 2.2 Basic Approach to Design of Sun Shades

The object problem for sun shade design presented in this paper is to determine the optimum shape and to calculate heat gain from windows excluding the cost estimation routine. The overall phase of it is shown in Table 1 in the list up form of input and output items.

Table 1. Input and output of object problem

| I/O | Description | Symbol | Item | Unit |
|---|---|---|---|---|
| Input | Location of building | LAT<br>LONG | Latitude<br>Longitude | degrees<br>degrees |
| | Wall orientation | WO | Normal direction to wall surface | degrees from south |
| | Shape of window to which sun shade is to be attached | B | Proportion of width to height of window | --- |
| | Design criteria | PSA | Upper limit of sunlit area per window area | --- |
| | Optimum Shape of sun shade | DL | Projection of overhang per window height | --- |
| | | FL | Projection of side fin per window height | --- |

---

[1] Figures in brackets indicate the literature references at the end of this paper.

| | | | | |
|---|---|---|---|---|
| | | AA | Tilt angle of overhang | degrees |
| | | AB | Tilt angle of side fin | degrees |
| | | | Drawings | |
| Output | Reference data | AM | Area of material per window area | --- |
| | | AS | Sunlit area per window area | --- |
| | | VF | View factor | --- |
| | | HG | Heat gain through fenestration | watt/m$^2$ or kcal/m$^2$h |

Table 2 shows the overall flow of the object problem subdivided into three phases for the conversational type of design process in the sequence form of execution routines, where Enter means to enter the input data at the keyboard and Xeq. means to execute the calculation routine marked after that. These three phases also can be applied to the deductive design process which does not necessarily require C.R.T. console or terminal units.

Table 2. Sequence of execution routines

| Phase | Operation | | Output | |
|---|---|---|---|---|
| I | Enter | LAT,LONG,WO | | |
| | Xeq. | MAXSOL | IDV | Maximum direct radiation upon window |
| | | | TIMAX | Time when IDV occurs |
| | | | TPM | Tangent of profile angle at TIMAX |
| | | | TGM | Tangent of wall solar azimuth at TIMAX |
| II | Enter | B, PSA | | |
| | Xeq. | SHAPE | DL, FL, AA, AB<br>Drawings | |
| | Xeq. | REFER | AS, AM, VF | |
| III | Enter | TIME | | |
| | Xeq. | DSOLT | IDVT | Direct solar radiation transmitted through shade throughout the year |
| | Enter | WEATA (weather data) | | |
| | Xeq. | HGAIN | RDT | Direct radiation transmitted through shade and glass |
| | | | RFT | Diffuse radiation transmitted through shade and glass |
| | | | HTTR | Overall heat transfer at inner surface of fenestration |

*MAXSOL, SHAPE, REFER, DSOLT, HGAIN are the back-up routines the programs of which are loaded in the main processor.

The basic approach to the problem to find the optimum geometry of sun shade is, first of all, that the sun shade should be designed so as to intercept the direct solar radiation as much as possible when it amounts to its maximum for the orientation to which the window is faced. It would not be probable that the total solar radiation transmitted through the window plus the heat gain associated with outside air temperature at other times of the year would exceed the rate when the maximum radiation occurs, but this must be checked before the decision is made.

## 3. Description of Routines with Example Results

### 3.1 Phase I

According to the above basic approach, the objective of Phase I is to find the maximum rate of direct solar radiation upon the surface for the window orientation and the sun's position relative to the window surface when it occurs. Useful data on the sun's position for the Phase II calculation are the tangent of profile angle as defined by Parmelee (2) and the tangent of wall solar azimuth.

Example calculation results are shown in Table 3 for the window faced with south-west and for the building located in Tokyo (35°41'N, 139°46'E). This shows the maximum radiation occurs in January. Considering that the total cooling load usually amounts to be higher in summer, the data for August were selected for the determination of optimum shape of sun shade for Phase II calculation.

Table 3. Output data example of Phase I

| Month | Sun time | Direct solar radiation on vertical surface (kcal $m^{-2}h^{-1}$) | Tangent of Profile angle | Tangent of wall solar azimuth |
|---|---|---|---|---|
| January | 14 | 677.9 | 0.52885 | 0.23383 |
| February | 15 | 675.0 | 0.49787 | 0.09798 |
| March | 15 | 595.1 | 0.72571 | 0.26333 |
| April | 15 | 522.5 | 1.03434 | 0.48731 |
| May | 15 | 481.2 | 1.36358 | 0.72624 |
| June | 15 | 342.5 | 1.52520 | 0.84353 |
| July | 15 | 385.4 | 1.36358 | 0.72624 |
| August | 15 | 455.3 | TPM=1.03434 | TGM=0.48731 |
| September | 15 | 581.5 | 0.72571 | 0.26333 |
| October | 14 | 610.8 | 0.70200 | 0.14790 |
| November | 14 | 610.9 | 0.52885 | 0.23383 |
| December | 14 | 605.7 | 0.46956 | 0.26325 |

### 3.2 Phase II

The objective of Phase II calculation is to determine the optimum shape of sun shade for a given rectangular window whose height is unity and breadth is B based on the output information of Phase I. Studied in this paper is with an egg-crate type of sun shade that consists of horizontal overhang and vertical side fins, both of which are made of rectangular flat plates and may be tilted if desirable as shown in Fig. 2, where the general shadow patterns are also illustrated. The design criteria adopted is the upper limit of sunlit area of window when direct solar radiation reaches its maximum in summer. The problem then is to find the optimum projection and optimum tilt angle of both overhang and side fins.

Fig. 3 is the flow chart showing the actual routines of Phase II calculation and the followings are the step by step descriptions.

(1) Input data:
B = 0.5 ( width of window relative to height of window )

TPM = 1.03434 ( tangent of profile angle at 3 pm in August, cf. Table 3 )
TGM = 0.48731 ( tangent of wall solar azimuth at 3 pm in August, cf. Table 3 )
PSA = 0.1 ( upper limit of sunlit area per window area as design criteria )

(2) Initial set values of tilt angle of overhang and side fin:
AA = 0
AB = 0

(3) Calculate the tentative values of overhang projection DL and side fin projection FL to have the sunlit area proportion AS made about 0.1 (PSA) with the following formulas:

$$DL = 0.7 / ( \tan AA + TPM ) \qquad (1)$$

$$FL = 0.7 / ( \tan AB + TGM ) \qquad (2)$$

(4) Calculate sunlit area AS including the triangle area as shown in Fig. 2.

(5) Calculate area of material AM required per window area as the sum of overhang area and side fin area.

(6) Calculate view factor VF.

(7) Output data:
DL, FL, AA, AB, AS, AM, VF.

(8) Repeat calculation with the shape of overhang and side fin modified.

(a) Modify the tilt angle of overhang or side fin whichever gives larger shadow area. Go to (3).

(b) Modify the projection of overhang or side fin whichever gives larger shadow area. Go to (4).

Example calculation results for the window whose relative breadth to height B is 0.5 are shown in Table 4.

It can be seen that the area of material requires gets smaller as the tilt angles of overhang and side fin increase , whereas view factor hardly varies regardless of the shape as long as the sunlit area stays the same around 0.1 which is the specified values as design criteria.

All of these results shown in Table 4 from the serial number I = 0 to I = 30 are regarded optimum as far as the design criteria adopted is concerned.

Fig. 4 shows the plotter drawings of section and plan of the four selected types of brise-soleil with the shadow pattern casted upon the window, assuming that the designer made his selection based on his idea and judgement to see the actual shape of them. In Fig. 4, (a) shows the figure of the type I = 0 in Table 4, (b) I = 9, (c) I = 19 and (d) I = 29. It would be more desirable to show the perspective drawings.

### 3.3 Phase III

Calculation of heat gain from the glass window combined with the sun shade the shape of which is determined in Phase II is to be made in Phase III. There are three basic components of heat gain that appear inside of glass: direct solar radiation transmitted through shade and glass, diffuse solar radiation transmitted through shade and glass including the reflected component from the sunlit surfaces of the shade and the heat transfer from outside across glass including the long wave length re-radiation from the shade surfaces the temperature of which could get considerably higher than the air temperature because of the absorption of solar radiation. Fig. 5 shows the overall phase of direct and diffuse solar radiation that turn out heat gain through the combination of shade assembly and glass.

The direct transmission component can be calculated using SHADOW routines (3),(4). The diffuse component including the reflected radiation at the shadow surfaces can be calculated with the view factor formulas. As the rigorous calculation of re-radiation component is very complicated and attempts were made to simplify the situation and estimate it in the form of equivalent temperature rise based on sol-air temperature

Table 4. Output data example of Phase II

```
       B =        0.50

       I          DL         FL         AA         AB         AS         AM         VF

       0        0.68       0.72       0.00       0.00       0.09       2.11       0.19
       1        0.68       0.61       0.00       5.00       0.09       1.90       0.20
       2        0.74       0.61       0.00       5.00       0.08       1.97       0.18
       3        0.62       0.61       5.00       5.00       0.09       1.85       0.21
       4        0.69       0.61       5.00       5.00       0.07       1.91       0.19
       5        0.62       0.53       5.00      10.00       0.09       1.70       0.21
       6        0.69       0.53       5.00      10.00       0.08       1.76       0.18
       7        0.58       0.53      10.00      10.00       0.09       1.66       0.22
       8        0.64       0.53      10.00      10.00       0.07       1.72       0.19
       9        0.58       0.46      10.00      15.00       0.10       1.55       0.21
      10        0.64       0.46      10.00      15.00       0.08       1.61       0.19
      11        0.54       0.46      15.00      15.00       0.09       1.52       0.22
      12        0.59       0.46      15.00      15.00       0.08       1.57       0.20
      13        0.54       0.41      15.00      20.00       0.10       1.43       0.22
      14        0.59       0.41      15.00      20.00       0.09       1.49       0.19
      15        0.50       0.41      20.00      20.00       0.09       1.41       0.22
      16        0.55       0.41      20.00      20.00       0.08       1.46       0.20
      17        0.50       0.37      20.00      25.00       0.10       1.34       0.22
      18        0.55       0.37      20.00      25.00       0.08       1.40       0.20
      19        0.47       0.37      25.00      25.00       0.09       1.32       0.23
      20        0.51       0.37      25.00      25.00       0.08       1.38       0.20
      21        0.47       0.33      25.00      30.00       0.10       1.27       0.22
      22        0.51       0.33      25.00      30.00       0.08       1.33       0.20
      23        0.43       0.33      30.00      30.00       0.10       1.26       0.23
      24        0.48       0.33      30.00      30.00       0.08       1.31       0.20
      25        0.43       0.29      30.00      35.00       0.10       1.22       0.22
      26        0.48       0.29      30.00      35.00       0.08       1.27       0.20
      27        0.40       0.29      35.00      35.00       0.10       1.21       0.23
      28        0.44       0.29      35.00      35.00       0.08       1.26       0.21
      29        0.40       0.26      35.00      40.00       0.10       1.18       0.22
      30        0.44       0.26      35.00      40.00       0.07       1.23       0.20
```

Table 5. Output data example of Phase III

| Time | Weather data | | | Output information | | | Equivalent temperature rise | | |
|---|---|---|---|---|---|---|---|---|---|
| | Direct solar radiatn | Diffuse solar radiatn | Outside air temp. | Direct radiatn transmtd | Diffuse radiatn transmtd | Heat transfrd across | Radiatn absorbed by glass | Re-radtn from shade | Atmosphrc radiatn |
| | --(kcal m$^{-2}$h$^{-1}$)- | | degC | -----(kcal m$^{-2}$h$^{-1}$)------ | | | degC | degC | degC |
| 9 | 2 | 34 | 28.6 | 0 | 11 | 7 | 0.16 | 0.19 | -1.28 |
| 10 | 3 | 49 | 29.6 | 0 | 21 | 16 | 0.28 | 0.49 | -1.30 |
| 11 | 71 | 56 | 30.3 | 0 | 29 | 23 | 0.39 | 1.05 | -1.31 |
| 12 | 215 | 57 | 30.7 | 0 | 34 | 31 | 0.46 | 2.02 | -1.32 |
| 13 | 340 | 56 | 31.1 | 0 | 37 | 39 | 0.51 | 3.19 | -1.33 |
| 14 | 425 | 54 | 31.0 | 23 | 37 | 47 | 1.27 | 4.13 | -1.32 |
| 15 | 455 | 51 | 30.7 | 29 | 35 | 50 | 1.42 | 4.74 | -1.32 |
| 16 | 416 | 47 | 30.5 | 16 | 30 | 47 | 0.91 | 4.91 | -1.31 |
| 17 | 288 | 39 | 29.6 | 0 | 21 | 37 | 0.29 | 4.41 | -1.30 |
| 18 | 54 | 22 | 28.4 | 0 | 7 | 22 | 0.10 | 2.94 | -1.28 |

concept against the glass surface (5) using the weighting factor technique (6) as used for space cooling load calculation. Detail of this approximation process is described in Appendix.

Results of example calculation with the brise-soleil whose type is I = 29 in Table 4 are shown in Table 5. It can be seen that the re-radiation effect can be quite large and the equivalent temperature rise amounts to over 7 degC especially when the shade intercepts a considerable amount of incident solar radiation. The author's experiments previously made (5) showed that the equivalent temperature rise was 3 - 4 degC at maximum in summer.

In the process of heat gain calculation it is desirable to check whether or not the shape of sun shade determined in Phase II could effectively reduce heat gain at other times of the year as well as other hours of the month. It sometimes happens in winter that the transmitted solar radiation amounts so high that one cannot control the space temperature. Two methods to check the possibility of this situation throughout the year are conceivable: one is to calculate only the direct solar radiation transmitted through the sun shade because it is a predominant factor and the other is to calculate heat gain or cooling load including other excitation components to make an overall judgement. If the heat gain for any month turned out too much, the alternative shape should be selected and the Phase III calculation repeated until the conditions regarding both aethetical and thermal effects are satisfied. If it is still unsatisfactory, Phase II routine must be repeated with input data revised from the results of Phase I calculation.

## 4. Conclusion

(1) As an example of optimization problem with computer the procedure to determine the optimum shape of sun shade is presented. This is based on the realistical design process so that an architectural designer could use his aethetical judgement in the course of the computer calculation which could be combined with graphical display system or plotter drawing system.

(2) The basic routines developed can be used both for the deductive and inductive design processes by a simple application technique so that they could be an effective aid in the architectural design of building façade.

(3) The procedure to calculate the heat gain from the window with brise-soleil taking account of reflection and re-radiation from sunlit surfaces is also developed so that the results obtained could directly be used for air conditioning load calculation.

## 5. References

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## 6. Appendix

### 6.1 Weighting Factors to Estimate the Heat which is Discharged from Sunlit Shade Assembly

Part of incident solar radiation transmits through the shade as shown in figure 5 and the remainder naturally falls upon the surfaces of the shade. Part of the radiation absorbed in the shade material is once stored but eventually discharged to the air by convection and to the glass surface by radiation, the remainder being emmitted to the sky. The radiation component from sunlit shade surface to the glass is defined as re-radiation and the effect of it can approximately be estimated using weighting factor technique.

Refering to figure 6 the weighting factors $W_j$ relating the incident solar radiation into the shade assembly to the heat discharged from all the surfaces of the shade can be expressed by the following equations using the response factors of shade material $X_j$ and

$Y_j$. Namely,

$$W_0 = 1 - ( X_0 - Y_0)*a_s/\alpha_{so} \qquad (A\text{-}1)$$

for j = 1 $$W_j = - ( X_j - Y_j)*a_s/\alpha_{so}, \qquad (A\text{-}2)$$

where $a_s$ is the absorptivity of shade assembly and $\alpha_{so}$ is film coefficient along the shade surface.

Then $HS_n$ the heat discharged from the shade surfaces per unit window area at time $t = n\,\Delta t$, where $\Delta t$ = time interval, can be obtained by

$$HS_n = \sum_{j=o}^{\infty} W_j * I_{n-j} , \qquad (A\text{-}3)$$

where $I_n$ is the incident solar radiation into the shade assembly including the reflection from glass as shown in figure 5 per unit window area.

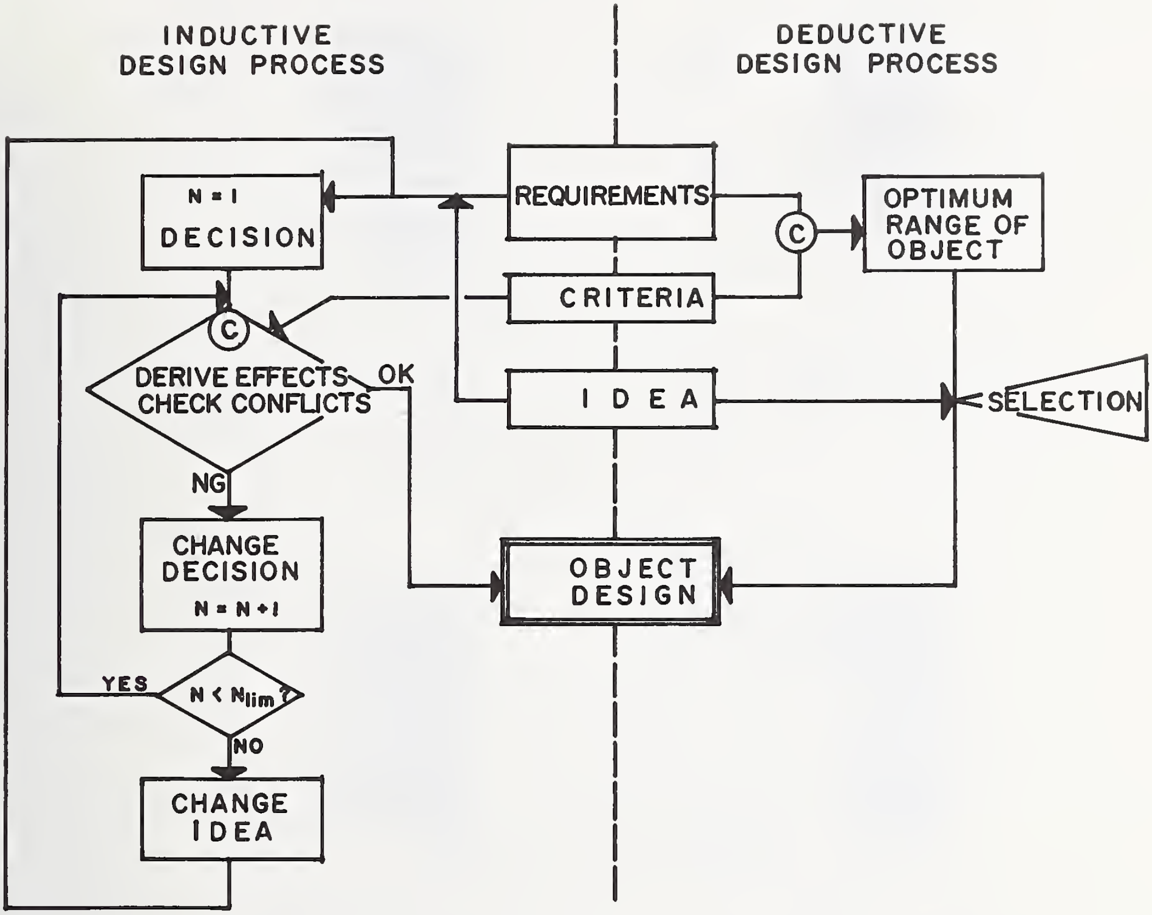

Figure 1 Two basic types of design process

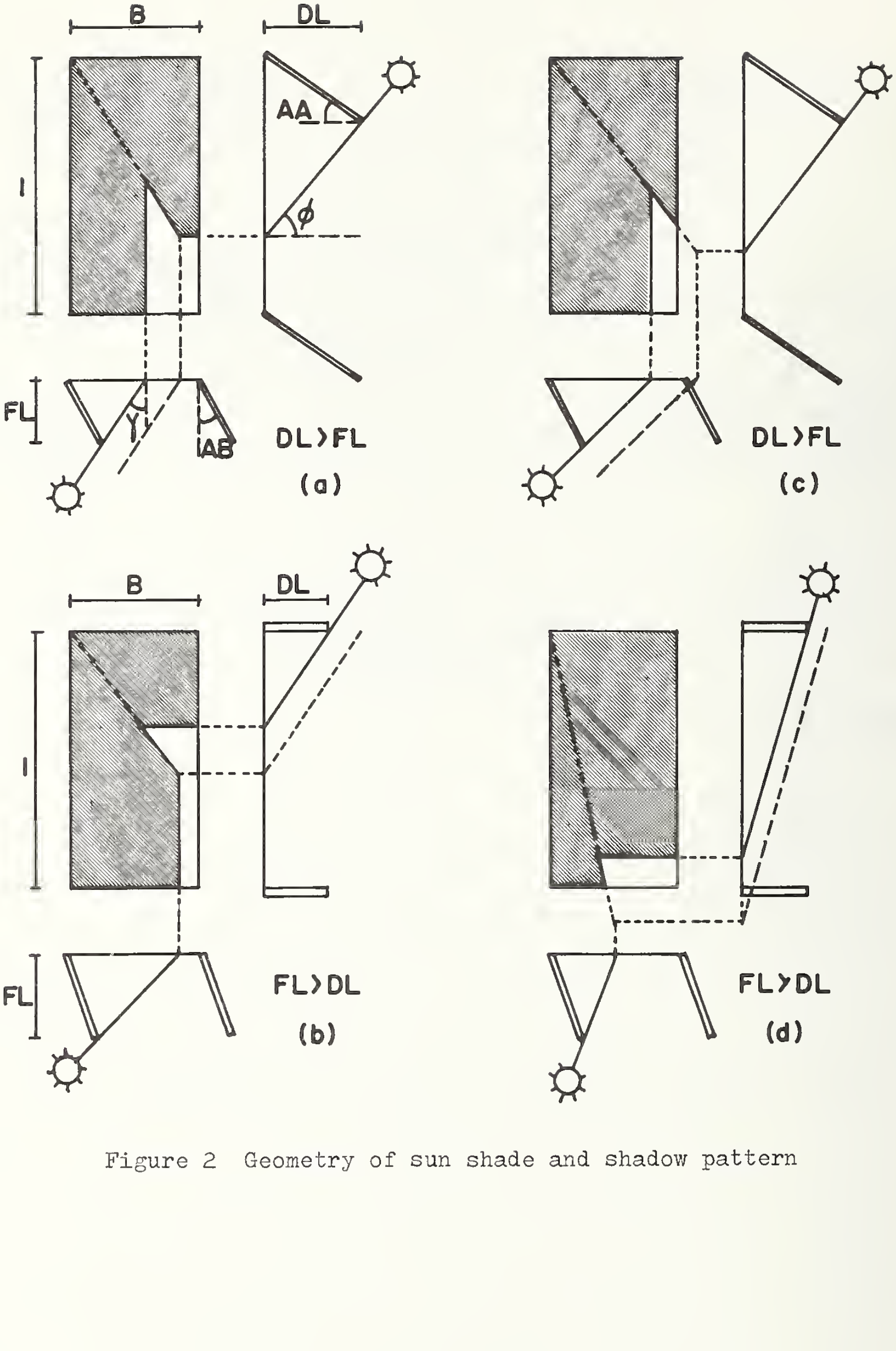

Figure 2 Geometry of sun shade and shadow pattern

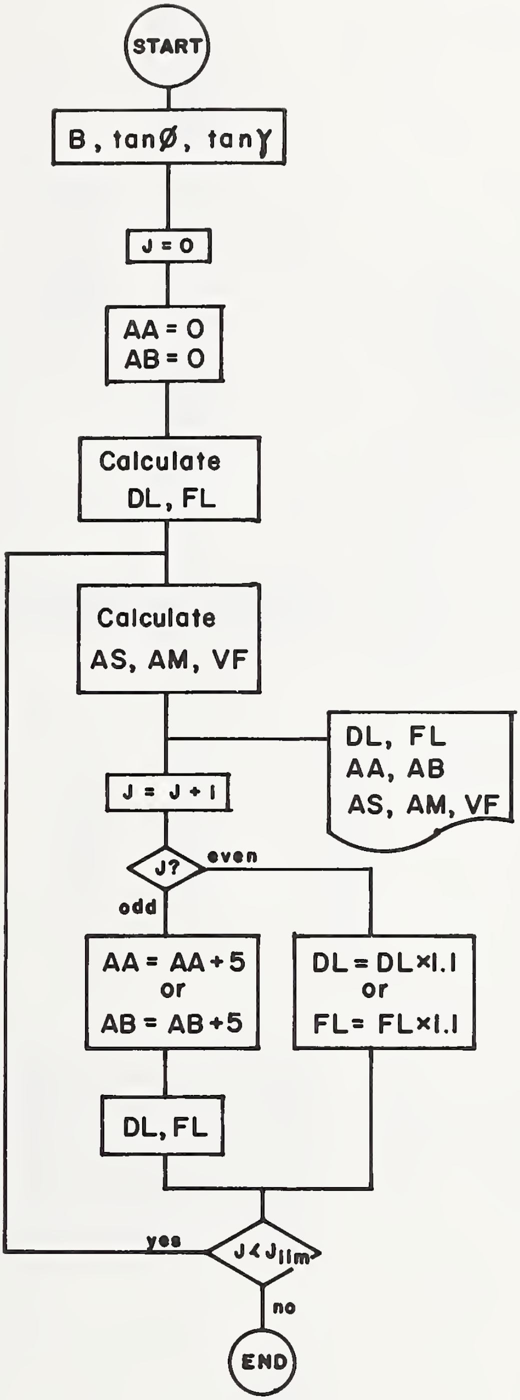

Figure 3 Flow chart of Phase II calculation

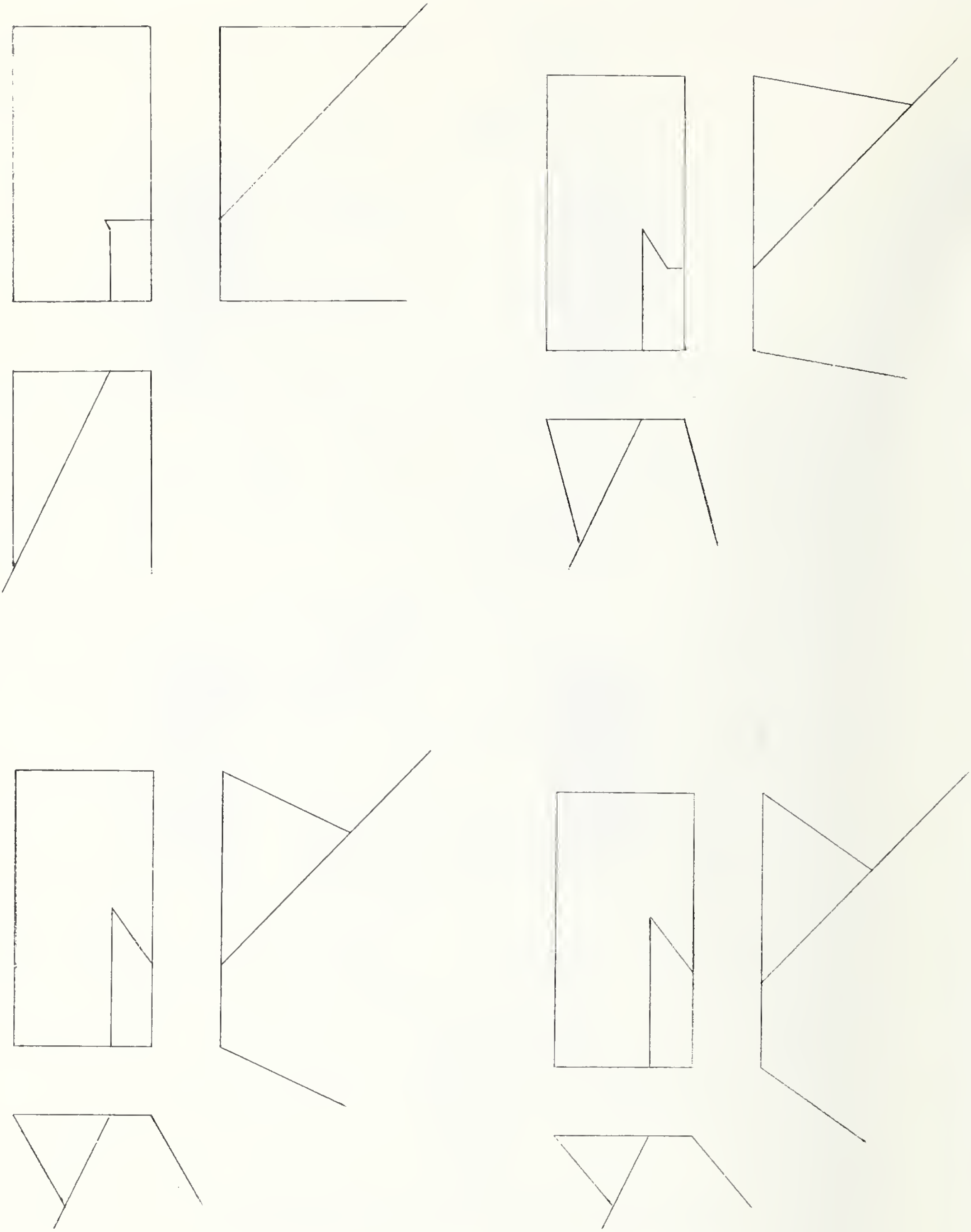

Figure 4 Plotter drawings of example shapes of sun shade

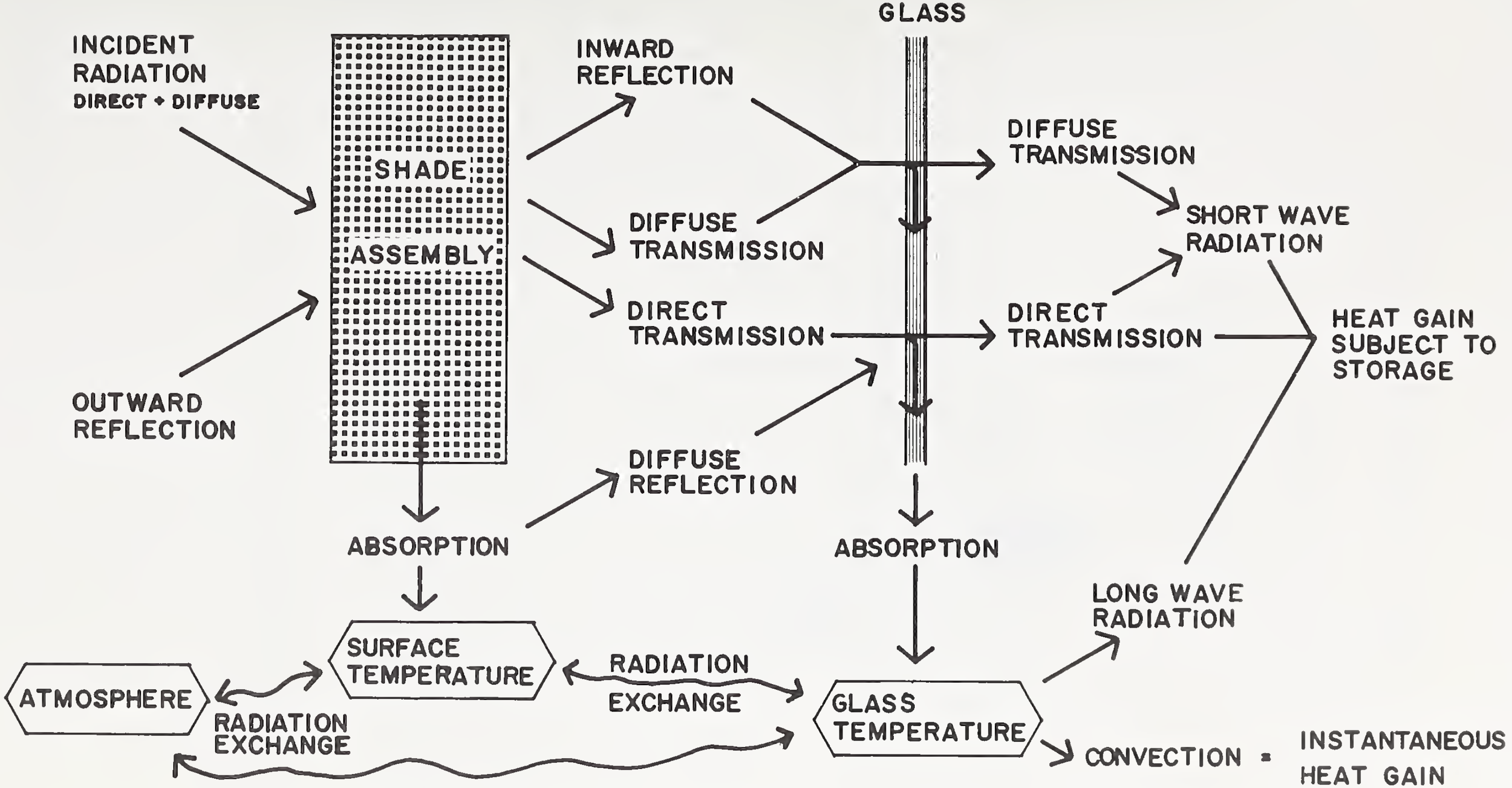

Figure 5 Route of the incident solar radiation converted into heat gain through the shade assembly and the glass

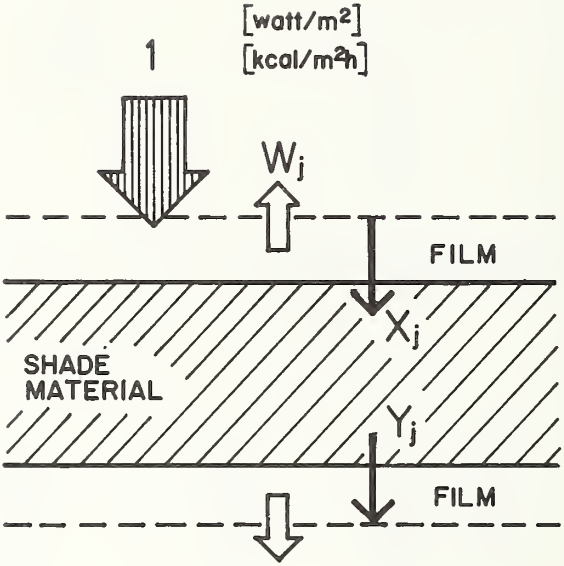

Figure 6 Weighting factors relating the incident solar radiation into the shade assembly to the heat discharged from the shade surfaces

# Calculation of Smoke Movement in Buildings

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For the purpose of personal safety from the hazard of smoke in building fires, it is necessary to keep escape routes clear of smoke or to keep the smoke concentration on the routes thinner than allowable one until the evacuation of the occupants is completed safely. For this, it is naturally required to control smoke movement by some mean. There should be a need for some design system, so called "smoke protection design system", to plan the reasonable and effective system suitable for every building to secure occupants from the hazard of smoke. Some calculation to estimate smoke movement or to evaluate smoke control methods will be indispensable to the design system.

Smoke movement is, needless to say, substantially caused by stack and wind actions the same way as air flow in a building. The movement is, therefore, practically under the influence of such conditions and factors as burning severity in fire compartment, composition of vertical shafts and other various flow paths and openings, temperatures on each part of a building and the outside air, the air handling systems involving the smoke tower or the pressurization method, and the outside wind. If these data are given, calculation of smoke movement can be made using a digital computer.

This paper presents a fundamental concept of the smoke protection design system, computer techniques for calculating smoke movement and computed results of sample buildings. A computer program has been formulated to calculate mass rate of smoke or air flowing through each path or opening and concentration of smoke and to evaluate methods of controlling smoke movement under various conditions for various buildings.

Key Words: Air, building, computer, concentration, control, design, density, escape, evacuation, fire, flow, movement, path, pressure, rate, shaft, smoke, stack action, wind action.

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## 1. Introduction

The increase in the number of high rise buildings and the tragic results of fires continuously happened in Japan in recent years have attracted much attentions to the hazard of smoke and toxic gases produced by fires in the buildings. As a result, various suggestions have been offered as counter-measures against smoke hazars, such as the restriction on the use of combustible linings, the effectivity of balconies on outside wall for escape, the necessity of smoke tower or the pressurization of escape routes as measures to control smoke movement and so on. These are surely available for personal safety from smoke in its own ways, however it should not be easy in practice to apply all of them in a building. To evaluate these measures, and to solve the problem how rationally and effectively to protect occupants against smoke, it will be the most important and necessary approach to establish a design system enable to evaluate various measures mentioned above, by means of estimating smoke movement or concentrations in escape routes and the time required to evacuate the occupants. Since the calculation to estimate the time for the evacuation is now enabled to apply practically (1), the only remained difficult problem to be solved is how to assume the conditions and to calculate smoke movement under the conditions.

## 2. Smoke Protection Design System

If a building has a perfect security to prevent an outbreak of fire, with all incombustible linings and furnitures or some complete set of fire extinguishers as sprinklers, the building should evidently need no further measure against fire or smoke. However, actually it is almost impossible to expect to be that way. Therefore, the most important problem is how to make occupants escape safely in fire by some measure, and then how reasonably and effectively to provide and plan the measures or the facilities. The system to solve this problem is, so to speak, the smoke protection design system.

Fig. 1 shows a flow chart illustrating a concept of the design system. When the conditions and factors of a building are given or determined, one can estimate whether the occupants will be safe or not in fire, and one can take measures to meet the situation or consider how to plan facilities against smoke per the procedure shown in Fig. 1. The smoke concentration "Cs ( t )" can be estimated for each part of the building as a function of the time "t" after outbreak of fire by the calculation of smoke concentration or mass rates of smoke and air flowing through each opening. The smoke load to each compartment is given as the product of the mass rate by the concentration of smoke flowing into the compertment. The original concentration of smoke in a compartment in fire (3) can be assumed from the combustibles and the temperature in the compartment (4). The evacuation time for the design "Td" is given as a product of the minimum time required to evacuate the occupants "Tm" by a safety coefficient "$\varepsilon$" as $Td = \varepsilon Tm$. Tm can be calculated for each escape route based on width and length of escape routes and its number in quantity, type and quantity of fire alarm system, lighting and guiding sign, constitution of the occupants, and human psychology in emergency. The allowable smoke concentration "$Cs_A$" may be given from the visible distance required for the occupants to escape (5). According to the procedure shown in the Fig. 1, we can predict from the values of Cs(t), Td and $Cs_A$, whether the occupants will be safe or not in fire, or whether $Cs(Td) \leq Cs_A$ or $Cs(Td) > Cs_A$. When $Cs(Td) > Cs_A$, it indicates need for some further consideration to reduce Cs(Td).

## 3. Analysis of Air Flow in Building

Smoke is probably not so much different from air in the nature such as the density (10), the viscosity and so on. To understand smoke movement in buildings, therefore, it is necessary to solve the problem of how much air flows through each component and how much smoke is contained in the air.

### 3.1 Basic Pattern of Flow through a Path

In general, the mass rate of air through a path Q can be represented as,

$$Q = \pm\sqrt{|\triangle P| / R}, \quad \text{or} \quad \triangle P = R \cdot Q^2$$

where R: flow resistance of a path,

P: pressure difference across a path.

When two spaces adjoining each another with different temperatures, there is pressure distribution on the separation attributed to the density difference of the air. In the case of Fig. 2(a), where the neutral plane exists between the levels of the top and the bottom of the opening, each mass rate $Q_1$ and $Q_2$ of air flows through the upper or the lower part of the opening can be thought as,

$$Q_1 = \pm\sqrt{|\triangle Pa_1| / R_1}, \quad \text{or} \quad Q_2 = \pm\sqrt{|\triangle Pa_2| / R_2}$$

where $\triangle Pa_1 = 4\triangle P_1/9$, $\triangle Pa_2 = 4\triangle P_2/9$, $R_1 = 1/(2g\gamma_1(\alpha A_1)^2)$, $R_2 = 1/(2g\gamma_2(\alpha A_2)^2)$

g: acceleration of gravity, $\gamma$ : density of air, $\alpha A$: effective area of flow path,
$\triangle P_1$, $\triangle P_2$ : pressure difference at the level of the top or the bottom of the opening.

In the case of Fig. 2(b), where the air flow is one-way, the rate Q can be given as,

$$Q = \pm\sqrt{|\triangle Pa| / R} \quad \text{where} \quad R = 1/(2g\gamma_2 K(\alpha A)^2),$$

$$K = \frac{8}{9}\left(1 + \frac{n}{(1+n)(1+n+2\sqrt{n})}\right), \quad n = h_1/h_2$$

$\triangle Pa$: pressure difference between two spaces at the center of the opening.

## 3.2 Wind Action

Wind pressure on a outside wall Pw is determined by wind velocity v and wind pressure coefficient C which depends on directions of wind and a wall face, that is $Pw = C\gamma v^2/2g$. If the velocity profile, surrounding conditions and the nature of wind are neglected, the wind pressure coefficient C is given by an angle Aww between wind direction and the normal to a wall face as follows (9),
$C = 0.75$ when $0° \leq |Aww| \leq 30°$, $C = -0.021Aww + 1.38$ when $30° < |Aww| < 90°$, $C = -0.5$ when $|Aww| \geq 90°$.

## 3.3 Computer Technique for Flow Analysis

### a. Basic Model and Computer Technique

Pressure in each part of a building and air flow rate through each opening or path can be obtained by means of solving an air flow network composed of flow resistances of openings and such motive forces as wind pressures and air gravities. There are two methods, so to speak, "loop method" and "knot method", as computer techniques to solve these kinds of networks. Two basic patterns of the network simulating the electric circuit are illustrated in Fig. 3 and 4, relating to the both methods.

The loop method is to solve, based on the 1st and 2nd lows of Kirchhoff, the equation of the through quantity on the knot $\Sigma Q = 0$ and the equation of the across quantity on the loop of the network $\Sigma\triangle P = 0$. This method has already been described in (2), but the simplest model is shown in Fig. 3.

The knot method is to solve the network with the mass rate balance for each knot or the equation of the through quantity $\Sigma Q = 0$, as shown in Fig. 4 where a basic model of the network is shown for this method. In this method, pressures on all knots are assumed at first, then they are corrected iteratively until mass rate balances of all knots are satisfied. In Fig. 4, when static pressures $p_1 \sim p_4$ are given, the pressure P to be solve is obtained as a root of the following equation;

$$\Sigma Q = \Sigma \pm\sqrt{|P_i - E_i - P| / R_i} = 0$$

The flow chart shown in Fig. 5 illustrates an iteration procedure for correcting unbalanced mass rates in the above equation.

### b. Network for Practical Building

Fig.6 shows practical patterns of the network of air flow in buildings. A computer program was formulated using the iterative technique shown in Fig.5, to solve the network composed of the patterns shown in Fig.6, by which it is enabled to calculate air and smoke flows under various conditions on various building. Fig.7 shows a network of an actual building in Tokyo, as a simple example. The results for this building, calculated by the loop method, has already been published (2), but a part of them is shown afterwards.

## 4. Smoke Spread and Smoke Concentration

To estimate the safety of occupants in escape routes, one must calculate the rising rate of smoke concentration in each escape route. So it is necessary to obtain the time in which smoke appears at each escape route and to calculate smoke concentration after smoke reaches there. As a whole, it is very difficult to grasp the smoke movement in inicial stage of a fire, because temperature of smoke is transient and under unsteady state. So, calculations was made based on the following assumptions;

(1) temperature in each part is constant during fire,

(2) smoke diffuses uniformly itself all over the space instantaneously except in vertical shafts and the corridor of the floor on fire.

### 4.1 Smoke Movement in Corridor of Floor on Fire

The nature of smoke, particularly its temperature, in the corridor of the floor on fire is very important for calculations of spread and diffusion of smoke to other floors. Layers of smoke and air are generally formed in the corridor, as shown in Fig.8, the one of smoke flowing on upper part of the corridor from burning compartment, and the other of air flowing on the lower part to the compartment.

The temperature of the smoke layer varies according mainly to the distance from the opening of burning compartment and the temperature of burning compartment which varies not so much after the flash-over. Since the temperature as a function of the distance and the time after breaking out of fire is difficult to solve and not so much usable for the purpose of these calculations, one may assume it as constant for the time, and then regard it as a function of the distance. Fig.8 shows a cross section of a smoke layer of thickness H and width W (equal to the width of corridor). Assuming that heat transfer coefficient to the surroundings h, the specific heat of smoke at constant pressure Cp and the temperature of the air $\theta_0$ are constant, a heat balance is shown by the following equation (1) on the crosshatched finite slice of thickness dx and distance x from the opening of the burning compartment.

$$-Q \cdot Cp \cdot \left(\frac{\partial \theta}{\partial X}\right) \cdot dx \cdot dt - 2h(H+W)(\theta-\theta_0)(1-\beta)\,dx\,dt = HW\gamma Cpdx \triangle\theta \quad \cdots\cdots\cdots\cdots\cdots(1)$$

where Q is mass rate of smoke in its layer, $\theta$ is the temperature of the slice or smoke x distant from the opening, $\triangle\theta$ is the varied temperature of the slice after the elapse of a finite time increment dt, and $\beta=(\theta_{SX}-\theta_0)/(\theta-\theta_0)$ where $\theta$ sx is the temperature on the surfaces of the surroundings at the position x from the opening.

Assuming the temperature to be constant or $\triangle\theta = 0$, Eq. (1) becomes

$$\frac{d\theta}{dx} = -\frac{2h(H+W)(1-\beta)}{Q\ Cp}(\theta-\theta_0)$$

Therefore, $\theta$ is given as

$$\theta = \theta_0 + ( \theta_F - \theta_0 ) \ \exp \ ( - \alpha \Phi x ) \qquad \cdots\cdots (2)$$

where $\alpha = \dfrac{2 ( H + W )}{Q \ Cp}$

$\Phi = h \cdot \exp(h^2 t/\lambda c\rho) \ \text{erfc} \ h \sqrt{t/\lambda c\rho}$

$\theta_F$ : temperature of smoke in burning compartment

t : the time representing the elapse of fire

$\lambda . c . \rho$: heat conductivity, specific heat and density of the materials of surrounding walls of the corridor respectively

In Eq. (2), $\Phi$ can be obtained by assuming that the surroundings of the corridor are made of semi-infinite solid. Fig. 9 shows a graph of $\Phi$ vs. $t/\lambda c\rho$ for the values 40, ..., 120 of h(Kcal/m$^2$hr °C). One may use $\Phi$ at an average time like the half of the evacuation time based on the assumption of steady state.

Assuming that t = 0 or $\Phi$ = h for initial smoke flow, to calculate the time Tx in which smoke arrives at a position x distant from an opening of the fire, the velocity Vx of the smoke head at x is thought as:

$$Vx = \frac{Q ( \theta_0' + ( \theta_F - \theta_0 ) \exp ( - \alpha hx ))}{\theta_0' \gamma_0 \ HW} \qquad \cdots\cdots (3)$$

where $\gamma_0$ is density of air at temperature $\theta_0$ , $\theta_0' = 273 + \theta_0$ , $\theta_F' = 273 + \theta_F$

$$Tx = \int_0^x \frac{dx}{Vx} = \frac{\gamma_0 HW}{\alpha h Q} ( \alpha hx + \ln( \frac{\theta_0' + ( \theta_F - \theta_0 ) \ \exp \ (- \alpha hx)}{\theta_F'} )) \qquad \cdots\cdots (4)$$

Therefore the time Tc in which smoke head arrives at a shaft opening in the floor on fire can be obtained by inserting the distance from the opening of burning compartment to the shaft into Eq. (4).

### 4.2 Smoke Movement in Vertical Shaft

To obtain the smoke load and the time for smoke to reach other floors, it is necessary to analyse the flow in vertical shafts such as a staircase, an elevator shaft or a vertical duct for the HVAC. Let's assume a simple condition or layout as illustrated in Fig. 10. The time Ti, in which smoke flows through a duct from the burning floor "f" to any floor "i", and the concentration of smoke Ci in a shaft on the "i"-th floor level are:

$$T_i = \sum_{\mu = i}^{f-1} \frac{\gamma HA}{\sum_{k=1}^{\mu} Q_k} \quad \text{when} \quad 1 \leq i < f , \qquad \text{or} \qquad T_i = \sum_{\mu}^{i=1} \frac{\gamma HA}{\sum_{k=1}^{\mu} Q_k} \quad \text{when} \quad i > f$$

$$C_i = \frac{\sum_{k=1}^{i} C_k \ Q_k}{\sum_{k=1}^{i} Q_k}$$

where Q is (+) for flow out of a shaft,
or (-) for flow into a shaft

where H is floor height, A is effective cross section area of a shaft.
When $\Sigma(\gamma HA/Q) \leq 0$ or $T_i < 0$ in the i-th floor, no smoke will consequently flow into the floor through the shaft.

### 4.3 Smoke Concentration in General Floors

If smoke enters into a compartment "j" in the i-th floor through openings n in number, the smoke concentration will be:

$$C_{j}i(t) = \sum_{k=1}^{n} \frac{2C_k \cdot Q_k}{\Sigma|Q_j|} (1 - \exp(-\frac{\Sigma|Q_j|}{2\gamma V}(t - Tk)))$$

where Cji (t) : smoke concentration in the compartment "j" in i-th floor for the time t after elapse of fire,

$C_k$ : concentration of smoke flowing into the compartment through the "k" opening,

$Q_k$ : mass flow of the smoke,

Tk : the time in which smoke appears to the compartment through the "k" opening, Tk = Tc + Ti,

Qj : total mass flow of air and smoke flowing in and out of the compartment,

V : volume of the compartment,

$\gamma$ : density of air in the compartment.

Transient smoke concentration of the equation assumes instantaneous mixing of smoke with air in the compartment.

## 5. Calculation of Example Buildings

Calculations were made of smoke movement on two buildings, using the techniques as mentioned above. These calculations were assumed temperatures at steady state condition.

### 5.1 Example - 1

Assumptions: The example is of 9 story building actually existing in Tokyo, and a plan and openings of the building are shown in Fig. 11. The building has two duct systems for HVAC which are independent of each other. The one has air diffusers through which fresh air can be supplied into the center core in fire. The other has 8 air diffusers per an office room in each floor - the total leakage area is 0.072 sq m -, but fresh air does not be supplied through this system during fire. Fig. 7 shows a simplified cross section and a simulated network of the building. The optical concentration of smoke in fire compartment is Cs = 10 (1/m). The calculation is performed under 32 conditions composed of the followings;

(1) floor on burning ............................................2nd or 5th (floor)
(2) size of opening window of burning room (height x width) ................2 x 1 or 2 x 5 (m)
(3) number of opening door leading to the outside on 1st floor ..............0 or 2
(4) season of fire ............................................summer or winter
(5) wind velocity (wind direction toward the opened window of burning room) ..0 or 10 (m/s)

Temperature (°C)

| season | the outside | rooms and core | duct | burning room |
|---|---|---|---|---|
| summer | 30 | 24 | 27 | 727 |
| winter | 0 | 17 | 17 | 727 |

Calculated results: The results are shown in Table 1 and Fig. 12. Table 1 shows mass rates of air or smoke flowing through each opening or path, and the safe time during that occupants are sequred against smoke in each part of the building. These results show clearly that the mass rates and the safe time are so much influenced by wind and stack actions. Fig. 12 shows the relationship between volume of supplied air into the core and the safe time, under only no wind. Under every condition, as the supplied air increases more, the powerful effect appears more as shown in the figure. The tendency is particularly appeared when the doors are closed. So it may be said generally that the effect of pressurization is more powerful to thin smoke concentration than one of ventilation. The highest pressure on a door under pressurized condition is calculated to be about 3kg/sq m or 10kg/leaf, which suggests no difficulty for occupants to open the doors.

## 5.2 Example - 2

This example has a purpose to calculate smoke movement under various conditions for a building, using the computer program which is formulated on the pattern of network shown in Fig. 6, based on the techniques mentioned above.

Assumptions: The example building is 10 story, and its plan and sizes of openings are shown in Fig. 13. This building is consisted of five rooms ① - ⑤ , a corridor and a core on each floor. The core is consisted of two symmetrical parts, and each of them has a staircase, a lobby, an elevator shaft, a smoke tower, a HVAC duct, a return duct and a duct for air supply into a lobby or a staircase. During fire the return duct can be used for air supply only to a corridor of the floor on fire. Each smoke tower has two openings which are possible to open and shut to the lobby and the corridor of the floor on fire. The elevator shafts can be regarded as one shaft for the calculation by combining their flow resistances. The smoke concentration in burning room is $Cs = 3$ (1/m). Room ① of 2nd floor is on fire. Temperatures of the corridor of the floor on fire smoke towers are calculated by Eq.(2). Other temperatures are 0°C outside, 20°C in general part of the building, and 800°C in the room burning. The calculation was performed under eight conditions as shown in Table 2. Smoke control measures as smoke tower and pressurization on escape routes are also included in the assumptions. The efficiencies can be evaluated by these calculations.

Calculated results: The details of calculated results for condition No. 1 are shown in Table 3 and Fig. 14 - 17. Table 3 shows mass rates of air and smoke flowing through each opening and pressure on corridor of each floor. Fig. 14 - 16 show respectively the smoke and air flows through the openings of burning room, smoke movement on the corridor of burning floor and vertical shafts - elevator shafts and staircases -. Fig. 17 shows smoke concentration on each corridor. Calculated results for every condition are shown in Table 4 and 5. Table 4 shows the time in which smoke concentration on each corridor reaches to allowable one $Cs_A = 0.2$ (1/m), when visible distance is 10 - 15m. Table 5 shows the smoke concentrations in staircases Ⓘ and Ⓘⓘ . From these tables, one can easily estimate whether occupants are safe or not in fire and evaluate how to control smoke.

## 6. Discussion

This paper described computer techniques to calculate smoke movement in building fires, are an important and necessary approach to establish a smoke protection design system. For the background of this establishment, there have been done many valuable works on the smoke generation from various materials in fire (3), the temperature in burning compartment (4), the allowable smoke concentration for occupants to escape (5) and the time required for evacuation (1). These research works have already reached to the stage of practical apprication.

Therefore the only remained difficult problem was how to grasp the movement of smoke in fire, though it has been possible to calculate air and smoke flows when the conditions are assumed (2), (6), (7). The difficulty in this problem is however how to assume the conditions for such calculations. In practice, therefore, it should be required to simplify the conditions of the calculation particularly the compositions of flow paths and to calculate only significant flows under various conditions such as altering location of burning compartment, temperatures of the outside and the inside of a building, the outside wind and so on (8).

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(*): Report in Japanese

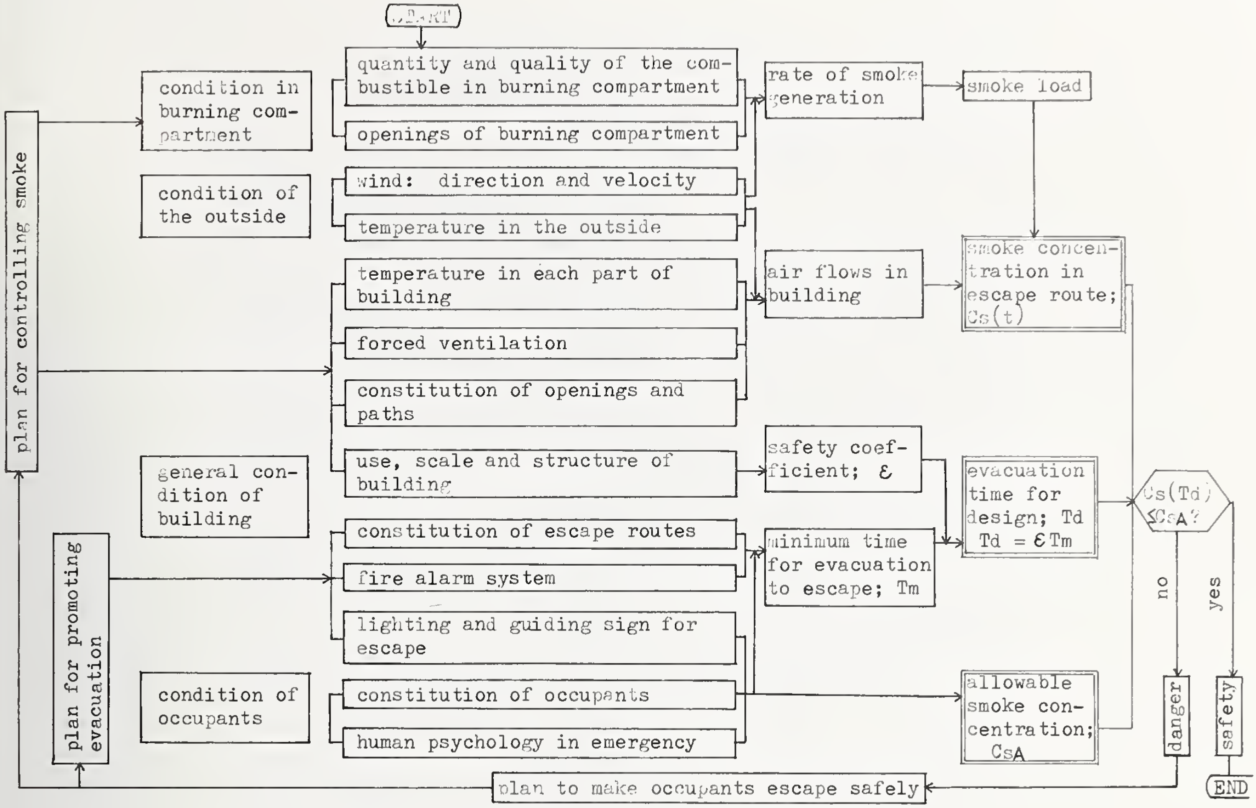


Fig.1 Flow Chart of Smoke Protection Design System

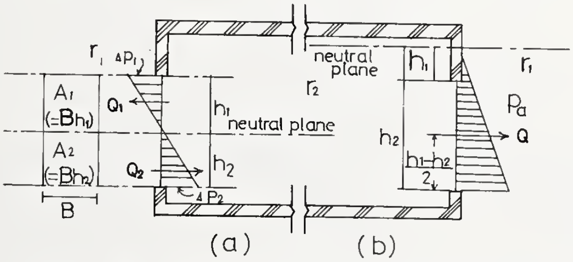


Fig.2 Basic Pattern of Flow Through Opening

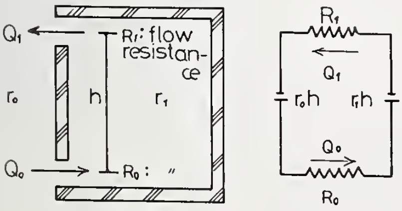


Fig.3 Basic Pattern of Loop Method

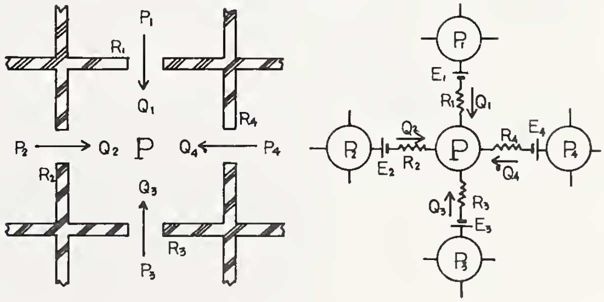


Fig.4 Basic Pattern of Knot Method

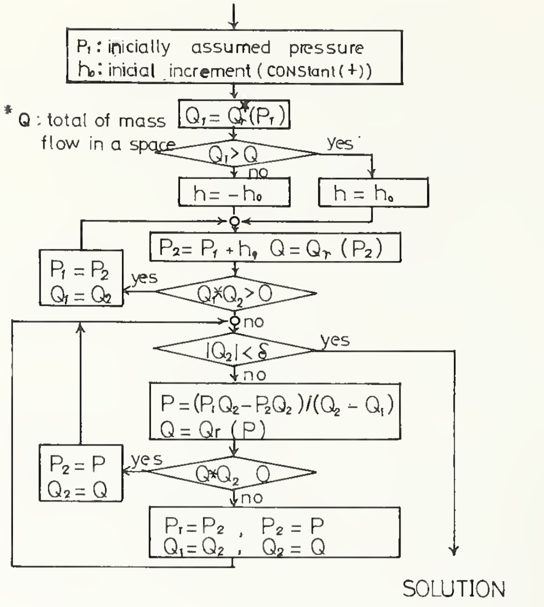
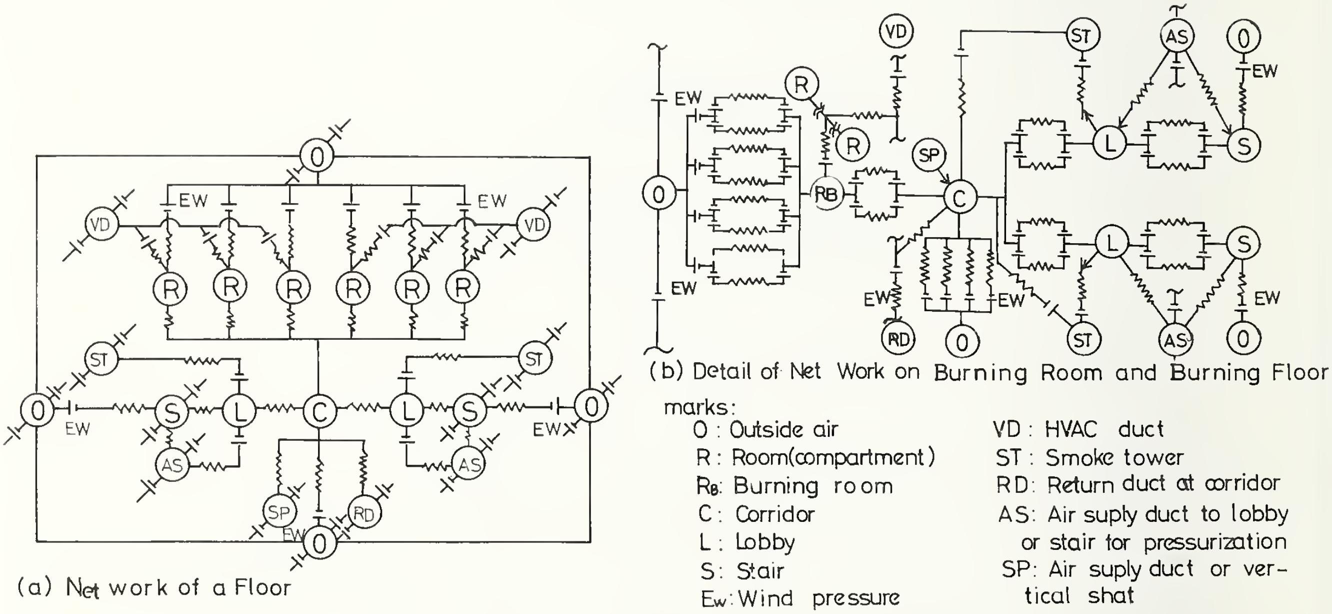


Fig.5 Flow Chart of Interation Procedure for Correcting Unbalanced Mass Flow in a Space



(a) Net work of a Floor

(b) Detail of Net Work on Burning Room and Burning Floor

marks:

O : Outside air
R : Room(compartment)
RB: Burning room
C : Corridor
L : Lobby
S : Stair
Ew: Wind pressure
VD : HVAC duct
ST : Smoke tower
RD: Return duct at corridor
AS: Air suply duct to lobby or stair for pressurization
SP: Air suply duct or vertical shat

Fig.6 Net Work Shows a Pattern of Air Flow Paths under Wind and Stack Action

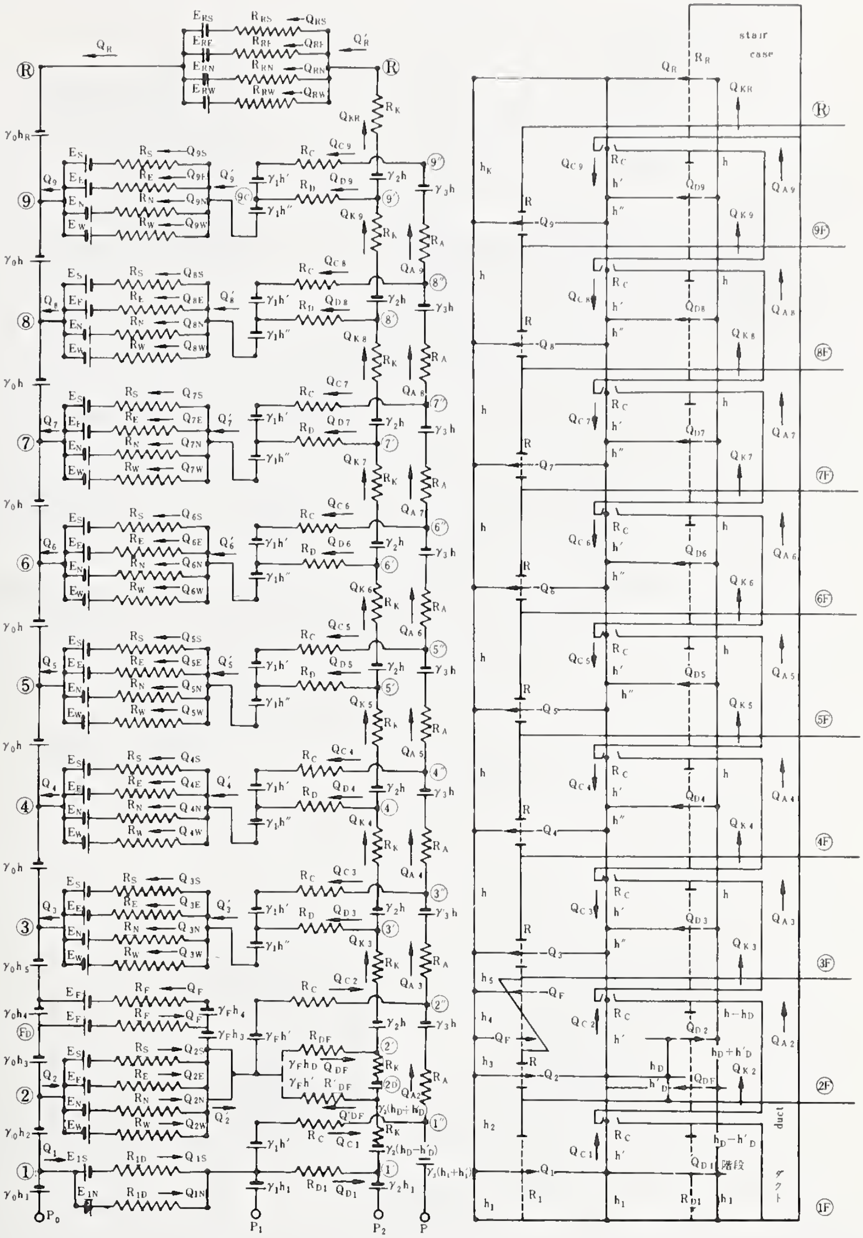


Fig.7  Simulated Net Work and Simplified Section of Example Building (Example - I)

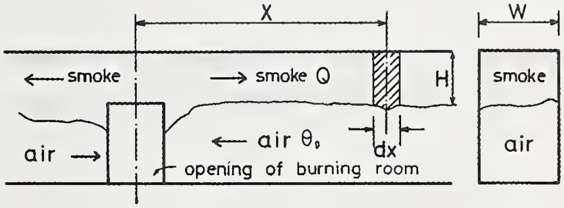


Fig.8  Smoke on Corridor of Fire Floor

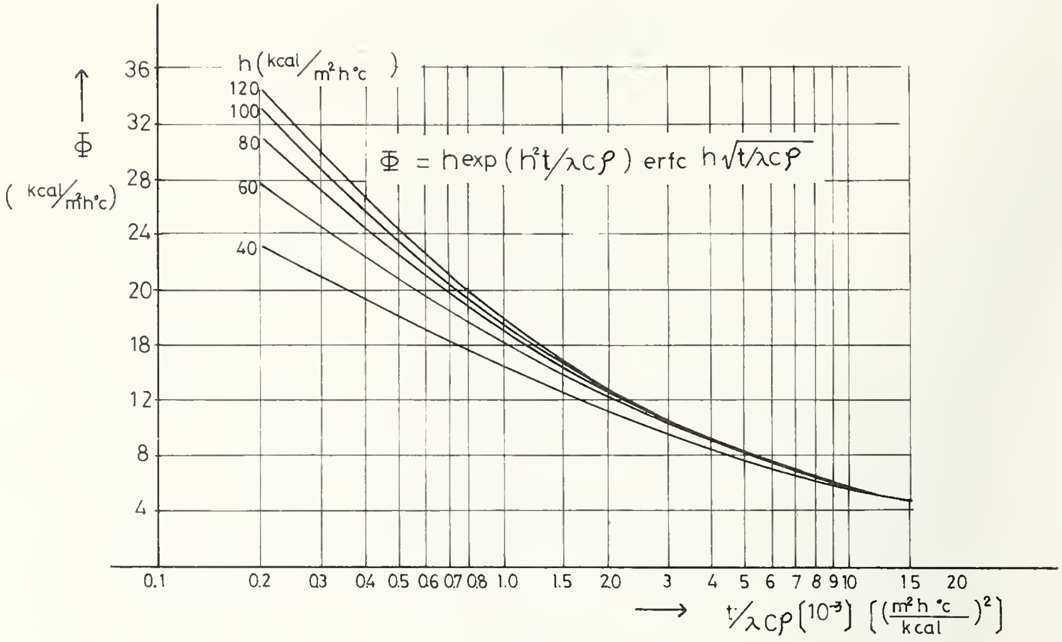
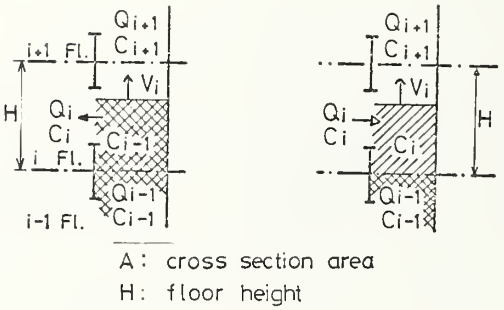


Fig.9 Heat Loss Factor on Surrounding Surface



(a) $Q_i > 0$

$$V_i = \frac{(Q_{i-1} - Q_i)}{\gamma A}$$

$$C_i = C_{i-1}$$

(b) $Q_i \leq 0$

$$V_i = \frac{(Q_{i-1} + Q_i)}{\gamma A}$$

$$C_i = \frac{C_{i-1} Q_{i-1} + C'_i Q_i}{Q_{i-1} + Q_i}$$

Fig.10 Smoke Flow in Vertical Shaft

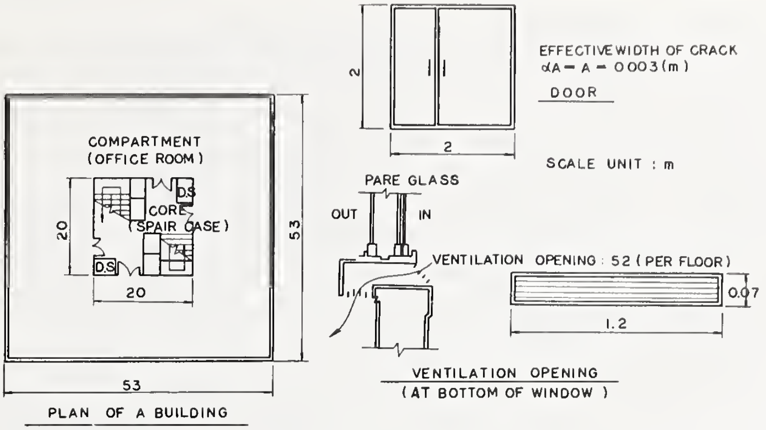


Fig.11 Plan and Opening of Example Building (Example-1)

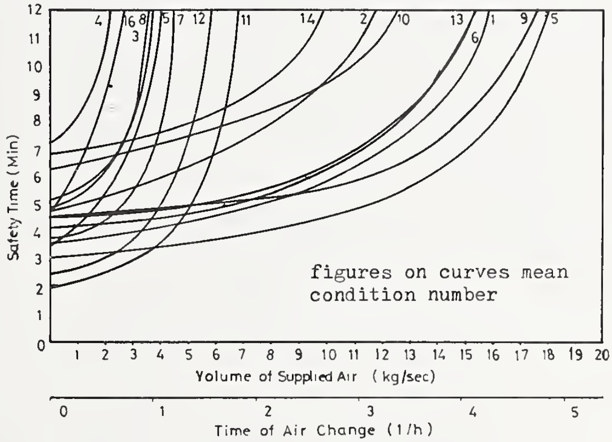


Fig.12 Relationship between safety time for escape and volume of supplied air or times of air changes in staircase

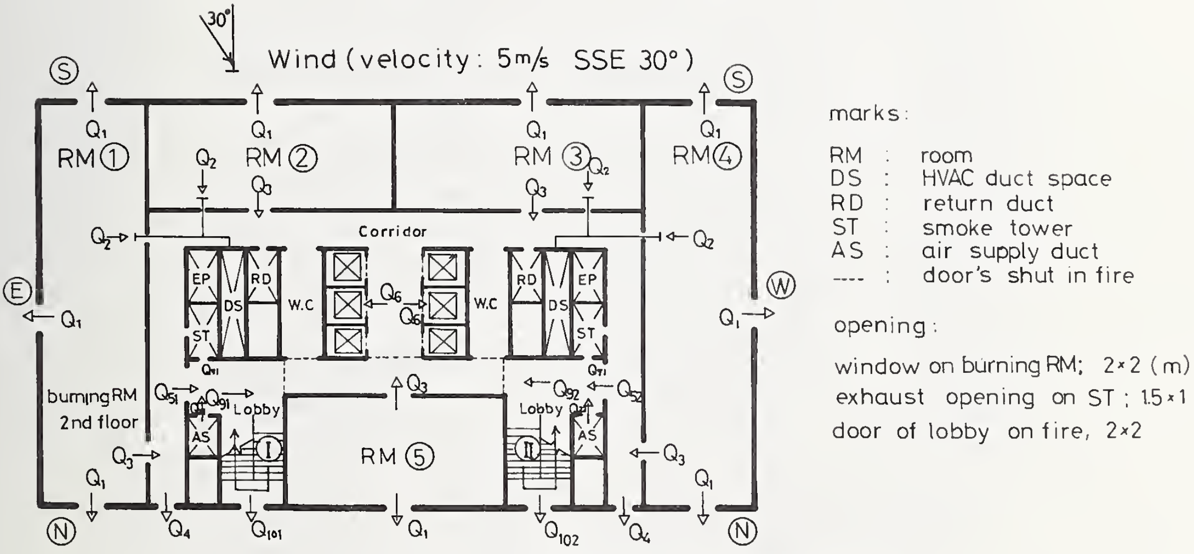


Fig.13 Plan of Example Building (Example - 1)

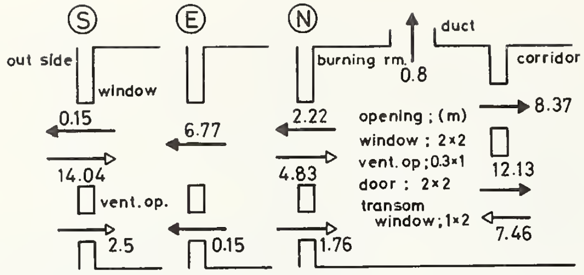


Fig.14 Mass Rates (kg/s) of Smoke and Air of Burning Room

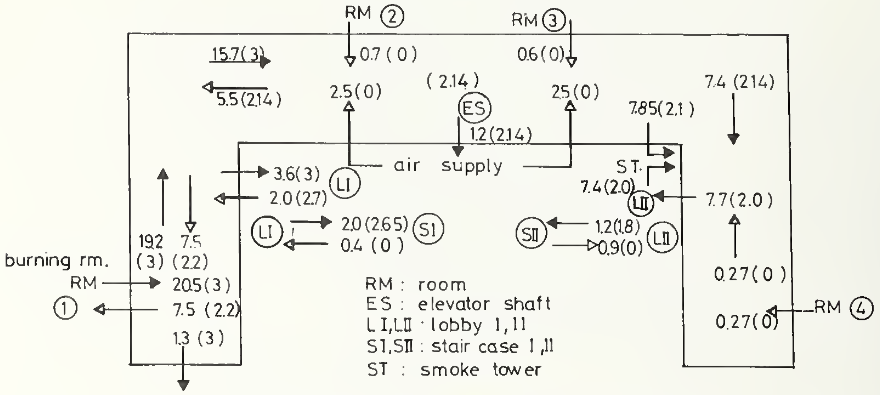
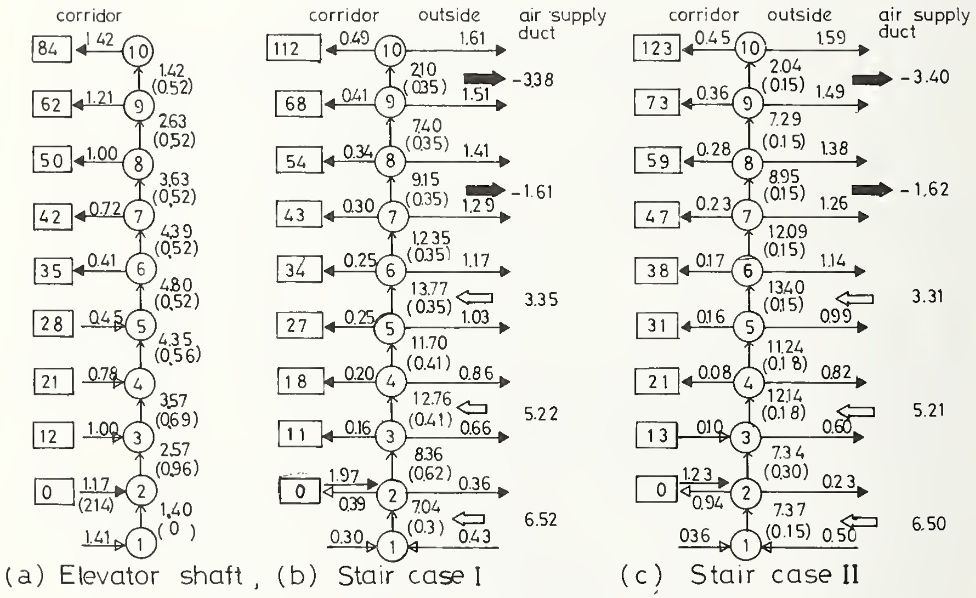


Fig.15 Smoke Flow on Corridor of Fire Floor



(a) Elevator shaft, (b) Stair case I (c) Stair case II

☐ : time in which smoke head arrives at each floor level, (sec.)

( ) : smoke concentration, (optical), (1/m)

other figures : mass rate of smoke or air (kg/sec)

Fig.16 Smoke Flow in Vertical Shaft

| Season | | Summer | | | | | | | | | | | | | | | | Winter | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Fire Floor | | 2 | | | | | | | | 5 | | | | | | | | 2 | | | | | | | | 5 | | | | | | | |
| Opening at two door at Ist FL | | OPEN | | | | CLOSE | | | | OPEN | | | | CLOSE | | | | OPEN | | | | CLOSE | | | | OPEN | | | | CLOSE | | | |
| Width of opened window on burning compartment (hight: 2m) | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | | 1 | | 5 | |
| Wind velocity(m/s)(souther) | | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 | 0 | 10 |
| Floor NO. | Condition NO. | 1 | 1' | 2 | 2' | 3 | 3' | 4 | 4' | 5 | 5' | 6 | 6' | 7 | 7' | 8 | 8' | 9 | 9' | 10 | 10' | 11 | 11' | 12 | 12' | 13 | 13' | 14 | 14' | 15 | 15' | 16 | 16' |
| Flow rate [Qsi (kg/s)] Stairway (core) (+) ⇄ (−) Room | 1 | 2.0 | -8.9 | 1.9 | -8.8 | 0.3 | -0.9 | 0.3 | -0.9 | 1.3 | 2.9 | 1.2 | 2.9 | 0.3 | -0.6 | 0.3 | -0.6 | -4.1 | -11.1 | -4.1 | -11.0 | -0.6 | -1.2 | -0.6 | -1.2 | -1.0 | -9.3 | -1.2 | -9.2 | -0.6 | -1.0 | -0.6 | -1.0 |
| | 2 | -0.1 | 1.0 | 0.1 | 0.9 | 0.1 | 0.0 | 0.2 | -0.3 | 0.1 | -1.0 | 0.1 | -0.3 | 0.3 | -0.7 | 0.3 | -0.6 | 0.1 | 1.1 | 0.2 | 1.0 | -0.3 | -0.3 | -0.2 | -0.5 | -0.4 | -6.0 | -0.4 | -5.9 | -0.6 | -0.9 | -0.6 | -0.9 |
| | 3 | -0.2 | 1.2 | -0.2 | 1.2 | 0.2 | 0.3 | 0.2 | 0.3 | -0.1 | -2.9 | -0.1 | -2.9 | 0.2 | -0.7 | 0.2 | -0.6 | 0.4 | 1.4 | 0.4 | 1.4 | -0.5 | -0.3 | -0.5 | -0.3 | -0.3 | 1.4 | -0.3 | -1.5 | -0.5 | -0.9 | -0.5 | -0.9 |
| | 4 | -0.3 | 1.2 | -0.3 | 1.2 | 0.2 | 0.3 | 0.2 | 0.3 | -0.2 | -4.6 | -0.2 | -4.6 | 0.2 | -0.7 | 0.2 | -0.7 | 0.5 | 1.4 | 0.5 | 1.4 | -0.4 | -0.2 | -0.2 | -0.4 | -0.0 | -0.1 | -6.6 | -0.1 | -6.7 | -0.4 | -0.4 | -0.8 |
| | 5 | -0.3 | 1.1 | -0.3 | 1.1 | 0.1 | 0.2 | 0.1 | 0.3 | -0.1 | 1.0 | -0.1 | 0.9 | -0.1 | 0.4 | 0.1 | 0.1 | 0.6 | 1.4 | 0.6 | 1.4 | -0.2 | 0.2 | -0.2 | 0.2 | 0.1 | 1.2 | 0.2 | 1.1 | 0.1 | 0.4 | 0.1 | 0.2 |
| | 6 | -0.3 | 1.1 | -0.3 | 1.1 | -0.1 | 0.2 | -0.1 | 0.2 | -0.3 | 1.2 | -0.3 | 1.2 | -0.1 | 0.6 | -0.1 | 0.6 | 0.6 | 1.4 | 0.6 | 1.4 | 0.2 | 0.3 | 0.2 | 0.3 | 0.4 | 1.4 | 0.4 | 1.4 | 0.2 | 0.7 | 0.2 | 0.8 |
| | 7 | -0.4 | 1.1 | -0.4 | 1.1 | -0.2 | 0.1 | -0.2 | 0.2 | -0.3 | 1.2 | -0.3 | 1.2 | -0.2 | 0.6 | -0.2 | 0.6 | 0.7 | 1.5 | 0.7 | 1.5 | 0.4 | 0.4 | 0.4 | 0.4 | 0.5 | 1.5 | 0.5 | 1.5 | 0.4 | 0.8 | 0.4 | 0.8 |
| | 8 | -0.4 | 1.1 | -0.4 | 1.1 | -0.3 | -0.1 | -0.3 | 0.0 | -0.4 | 1.2 | -0.4 | 1.2 | -0.3 | 0.6 | -0.3 | 0.6 | 0.8 | 1.5 | 0.8 | 1.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.6 | 1.5 | 0.6 | 1.5 | 0.5 | 0.8 | 0.5 | 0.9 |
| | 9 | -0.4 | 1.1 | -0.4 | 1.1 | -0.3 | -0.2 | -0.3 | -0.1 | -0.4 | 1.2 | -0.4 | 1.2 | -0.3 | 0.5 | -0.3 | 0.6 | 0.8 | 1.5 | 0.8 | 1.5 | 0.6 | 0.6 | 0.6 | 0.6 | 0.7 | 1.5 | 0.7 | 1.5 | 0.6 | 0.9 | 0.6 | 0.9 |
| Flow rate [Qci (kg/s)] Duct (+) ⇄ (−) Room | 1 | 0.9 | -4.0 | 0.8 | -0.4 | 0.9 | -3.1 | 0.9 | -3.1 | 0.6 | -2.9 | 0.6 | -2.8 | 0.9 | -2.0 | 0.9 | -2.0 | -2.0 | -4.9 | -2.0 | -5.0 | -2.1 | -4.0 | -2.1 | -4.0 | -1.0 | -3.8 | -1.0 | -3.7 | -2.0 | -3.1 | -2.0 | -3.0 |
| | 2 | -1.2 | -1.3 | -1.1 | -1.6 | -1.2 | -1.3 | -1.1 | -1.6 | 0.8 | -2.9 | 0.8 | -2.8 | 0.8 | -2.1 | 0.8 | -2.0 | -1.6 | -1.6 | -1.5 | -1.9 | -1.6 | -1.6 | -1.5 | -1.9 | -1.8 | -3.7 | -1.8 | -3.6 | -1.7 | -3.0 | -1.7 | -2.9 |
| | 3 | 0.7 | 1.0 | 0.7 | 1.0 | 0.7 | 0.8 | 0.7 | 0.9 | 0.7 | -2.9 | 0.7 | -2.8 | 0.7 | -2.1 | 0.7 | -2.0 | -1.5 | -0.9 | -1.5 | -0.9 | -1.5 | -1.0 | -1.5 | -0.9 | -1.5 | -3.6 | -1.5 | -3.5 | -1.4 | -2.7 | -1.4 | -2.7 |
| | 4 | 0.6 | 0.9 | 0.6 | 1.0 | 0.6 | 0.8 | 0.6 | 0.8 | 0.6 | -2.9 | 0.6 | -2.8 | 0.6 | -2.1 | 0.6 | -2.0 | -1.1 | -0.2 | -1.1 | -0.1 | -1.1 | -0.4 | -1.1 | -0.3 | -1.1 | -3.5 | -1.1 | -3.4 | -0.9 | -2.6 | -0.9 | -2.5 |
| | 5 | 0.4 | 0.8 | 0.4 | 0.9 | 0.4 | 0.7 | 0.4 | 0.7 | -1.3 | 2.8 | -1.1 | -0.2 | -1.3 | -0.6 | -1.2 | -1.0 | -0.3 | 0.8 | -0.4 | 0.1 | -0.3 | 0.6 | -0.4 | 0.7 | -1.3 | 0.7 | -1.2 | 0.1 | -1.3 | -0.4 | -1.1 | -1.0 |
| | 6 | 0.2 | 0.8 | 0.1 | 0.8 | 0.1 | 0.6 | 0.1 | 0.7 | 0.3 | 2.8 | 0.2 | 2.9 | 0.1 | 2.3 | 0.1 | 2.3 | 1.0 | 1.3 | 1.0 | 1.3 | 1.0 | 1.1 | 1.0 | 1.2 | 1.0 | 3.2 | 1.0 | 3.3 | 1.2 | 2.7 | 1.2 | 2.7 |
| | 7 | -0.4 | 0.7 | -0.4 | 0.7 | -0.4 | 0.6 | -0.4 | 0.6 | -0.3 | 2.8 | -0.3 | 2.9 | -0.4 | 2.3 | -0.4 | 2.3 | 1.4 | 1.6 | 1.4 | 1.6 | 1.4 | 1.5 | 1.4 | 1.5 | 1.5 | 3.4 | 1.4 | 3.5 | 1.6 | 2.9 | 1.6 | 2.9 |
| | 8 | -0.5 | 0.6 | -0.5 | 0.7 | -0.5 | 0.5 | -0.5 | 0.5 | -0.5 | 2.8 | -0.5 | 2.9 | -0.5 | 2.2 | -0.5 | 2.3 | 1.8 | 1.9 | 1.8 | 1.9 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 | 3.5 | 1.8 | 3.6 | 1.9 | 3.0 | 1.9 | 3.1 |
| | 9 | -0.7 | 0.5 | -0.7 | 0.6 | -0.7 | 0.4 | -0.7 | 0.5 | -0.6 | 2.7 | -0.6 | 2.8 | -0.7 | 2.2 | -0.7 | 2.3 | 2.1 | 2.1 | 2.1 | 2.1 | 2.1 | 2.0 | 2.1 | 2.0 | 2.1 | 3.7 | 2.1 | 3.7 | 2.2 | 3.2 | 2.2 | 3.2 |
| Smoke rate to Stair way | | 0.16 | 0 | 0.12 | 0 | 0.12 | 0.14 | 0.08 | 0.26 | 0.18 | 0 | 0.14 | 0 | 0.15 | 0.03 | 0.11 | 0.10 | 0.13 | 0 | 0.09 | 0 | 0.27 | 0.28 | 0.23 | 0.36 | 0.12 | 0 | 0.08 | 0 | 0.16 | 0.02 | 0.12 | 0.10 |
| Safe time for occupants (min) S: safe more than 30 min | 1 | 4.3 | S | 4.5 | | 4.5 | | 4.6 | | 7.1 | S | 7.2 | S | 5.1 | S | 5.2 | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S | S |
| | 2 | — | — | — | — | — | — | — | — | 4.9 | 〃 | 5.1 | 〃 | 4.9 | 〃 | 5.1 | 〃 | — | — | — | — | — | — | — | — | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 |
| | 3 | S | 7.0 | S | 6.2 | S | 6.4 | S | 5.5 | 5.2 | 〃 | 5.3 | 〃 | 5.1 | 〃 | 5.3 | 〃 | S | S | S | S | S | S | S | S | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 | 〃 |
| | 4 | 〃 | 7.6 | 〃 | 6.7 | 〃 | 7.0 | 〃 | 6.2 | 5.8 | 〃 | 6.0 | S | 6.0 | S | 6.2 | S | S | S | S | 5 | S | S | S | S | S | S | S | S | S | S | S | S |
| | 5 | 〃 | 8.4 | 〃 | 7.4 | 〃 | 8.1 | 〃 | 6.9 | — | — | — | — | — | — | — | — | S | 9.5 | S | 8.3 | S | 9.9 | S | 8.4 | — | — | — | — | — | — | — | — |
| | 6 | 〃 | 9.4 | 〃 | 8.2 | 〃 | 9.3 | 〃 | 8.0 | S | 〃 | S | 27.5 | S | 7.6 | S | 4.5 | 3.9 | 6.4 | 3.9 | 5.6 | 3.9 | 5.8 | 3.9 | 5.2 | 3.5 | S | 3.7 | 27.5 | 3.0 | 11.9 | 3.1 | 5.0 |
| | 7 | 〃 | 10.6 | 〃 | 9.3 | 〃 | 11.1 | 〃 | 9.4 | 〃 | 〃 | 〃 | 27.9 | 〃 | 7.8 | 〃 | 4.6 | 3.0 | 5.2 | 3.0 | 4.6 | 2.9 | 4.7 | 3.0 | 4.1 | 2.6 | 〃 | 2.8 | 27.9 | 2.4 | 11.2 | 2.4 | 4.9 |
| | 8 | 〃 | 12.5 | 〃 | 10.8 | 〃 | 14.8 | 〃 | 11.8 | 〃 | 〃 | 〃 | 28.1 | 〃 | 8.0 | 〃 | 4.9 | 2.6 | 4.6 | 2.7 | 4.2 | 2.6 | 4.0 | 2.6 | 3.8 | 2.4 | 〃 | 2.4 | 28.1 | 2.2 | 10.8 | 2.3 | 4.6 |
| | 9 | S | 15.9 | S | 13.6 | S | 19.2 | S | 15.9 | S | S | S | 28.5 | S | 8.4 | 5 | 5.2 | 2.6 | 4.3 | 2.6 | 4.1 | 2.6 | 4.0 | 2.6 | 3.8 | 2.2 | S | 2.4 | 28.5 | 2.1 | 10.4 | 2.3 | 4.6 |
| | Stair way | 14.9 | ∞ | 20.9 | ∞ | 19.9 | 16.6 | 31.1 | 8.8 | 12.6 | ∞ | 16.8 | ∞ | 15.2 | ∞ | 21.1 | 25.7 | 20.8 | ∞ | 34.8 | ∞ | 8.6 | 8.5 | 10.3 | 6.5 | 20.4 | ∞ | 33.4 | ∞ | 14.8 | ∞ | 21.1 | 32.2 |

Table 1. Mass Rate of Air or Smoke and Safe Time for Occupants ( Example - 1 )

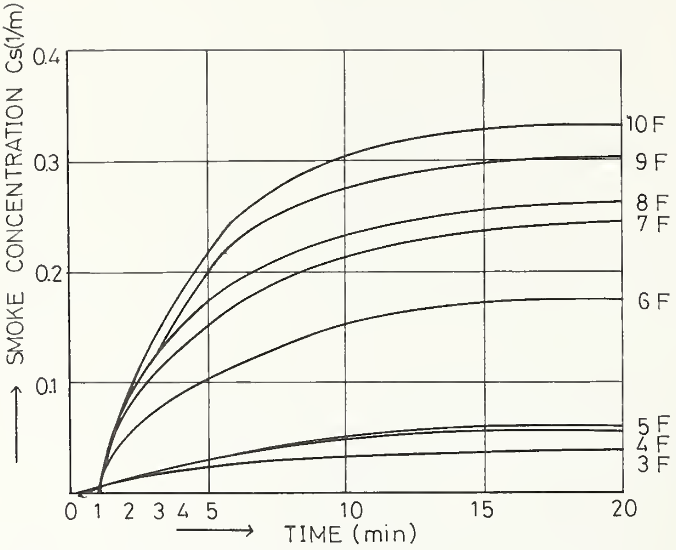


Fig.17 Smoke Concentration in Corridors on Each Floor

Table 2 Conditions of Calculation (Example - 2)

| | | Condition No. | | | | | | | | Marks |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Outside wind, | velocity(m/s) | 5 | - | - | - | - | - | - | - | -: no wind |
| | direction | SSE 30° | | | | | | | | |
| Window | on staircases | OP | OP | OP | CL | OP | OP | CL | CL | OP: opened |
| | on corridors | OP | OP | OP | CL | OP | OP | CL | CL | CL: closed |
| Mass rate of supply air (kg/s) | to staircases | 10 | 0 | 10 | 10 | 10 | 10 | 5 | 10 | *: on fire floor |
| | to corridors* | 5 | 0 | 0 | 0 | 0 | 0 | 10 | 20 | |
| Opening of smoke tower | smoketower (I) | - | - | - | - | L | C | - | - | L: opened to lobby |
| | " (II) | L.C. | - | - | - | L | C | - | - | C: opened to corridor |

Table 3 Mass Rate of Air and Smoke and Pressure on Corridor (Example - 2)

| Floor NO, | $Q_1$ | | | | | | | | $Q_2$ | | | | $Q_3$ | | | | $Q_4$ | $Q_5$ | | $Q_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ① | | | ② | ③ | ④ | | | ① | ② | ③ | ④ | ① | ② | ③ | ④ | | $Q_{51}$ | $Q_{52}$ | |
| | S | N | E | S | S | S | W | N | | | | | | | | | | | | |
| 1 | 1.28 | -0.65 | 0.78 | 1.30 | 1.33 | 1.41 | -0.27 | -0.27 | -0.75 | -0.70 | -0.78 | -0.61 | -0.65 | -0.60 | -0.55 | -0.27 | -0.01 | -0.30 | -0.36 | 1.41 |
| 2 | 0 | 0 | 0 | 1.25 | 1.27 | 1.39 | -0.37 | -0.37 | -0.80 | -0.58 | -0.66 | -0.37 | -13.03 | -0.67 | -0.61 | -0.27 | -1.28 | -1.58 | -7.70 | 1.17 |
| 3 | 1.18 | -0.82 | 0.60 | 1.14 | 1.17 | 1.30 | -0.61 | -0.61 | -0.43 | -0.53 | -0.61 | -0.23 | -0.53 | -0.61 | -0.56 | 0.14 | -0.62 | 0.16 | -0.10 | 1.00 |
| 4 | 1.11 | -0.91 | 0.44 | 1.03 | 1.07 | 1.21 | -0.78 | -0.78 | -0.21 | -0.45 | -0.55 | 0.13 | -0.42 | -0.58 | -0.52 | 0.22 | -0.81 | 0.21 | 0.09 | 0.78 |
| 5 | 1.02 | -1.01 | 0.05 | 0.91 | 0.95 | 1.12 | -0.90 | -0.90 | 0.27 | -0.36 | -0.47 | 0.35 | -0.32 | -0.55 | -0.48 | 0.34 | -0.96 | 0.25 | 0.16 | 0.45 |
| 6 | 0.92 | -1.10 | -0.42 | 0.76 | 0.81 | 1.03 | -1.00 | -1.00 | 0.45 | -0.27 | -0.40 | 0.49 | 0.15 | -0.50 | -0.41 | 0.48 | -1.11 | 0.25 | 0.17 | -0.41 |
| 7 | 0.83 | -1.17 | -0.59 | 0.58 | 0.66 | 0.95 | -1.08 | -1.08 | 0.59 | -0.10 | -0.28 | 0.63 | 0.35 | -0.48 | -0.37 | 0.58 | -1.22 | 0.30 | 0.23 | -0.76 |
| 8 | 0.74 | -1.23 | -0.70 | 0.28 | 0.44 | 0.87 | -1.14 | -1.14 | 0.71 | 0.21 | -0.09 | 0.75 | 0.48 | -0.49 | -0.35 | 0.67 | -1.32 | 0.34 | 0.28 | -1.00 |
| 9 | 0.65 | -1.28 | -0.78 | -0.14 | 0.01 | 0.80 | -1.19 | -1.19 | 0.84 | 0.50 | 0.33 | 0.86 | 0.56 | -0.36 | -0.33 | 0.73 | -1.40 | 0.41 | 0.36 | -1.22 |
| 10 | 0.57 | -1.32 | -0.84 | -0.38 | -0.33 | 0.73 | -1.24 | -1.24 | 0.96 | 0.66 | 0.55 | 0.97 | 0.62 | -0.29 | -0.22 | 0.77 | -1.46 | 0.49 | 0.45 | -1.41 |

| Floor NO. | $Q_9$ | | $Q_{10}$ | | $Q_{100}$ | | pressure on corridor |
|---|---|---|---|---|---|---|---|
| | $Q_{91}$ | $Q_{92}$ | $Q_{11}$ | $Q_{12}$ | $Q_{101}$ | $Q_{102}$ | |
| 1 | -0.30 | -0.36 | 6.52 | 6.53 | 0.43 | 0.50 | -0.82 |
| 2 | -0.58 | -0.29 | — | — | -0.36 | -0.23 | -0.76 |
| 3 | 0.16 | -0.10 | 5.22 | 5.21 | -0.66 | -0.60 | -0.44 |
| 4 | 0.21 | 0.09 | — | — | -0.86 | -0.82 | -0.17 |
| 5 | 0.25 | 0.16 | 3.35 | 3.31 | -1.03 | -0.99 | 0.10 |
| 6 | 0.25 | 0.17 | — | — | -1.17 | -1.14 | 0.41 |
| 7 | 0.30 | 0.23 | -1.61 | -1.65 | -1.29 | -1.26 | 0.67 |
| 8 | 0.34 | 0.28 | — | — | -1.41 | -1.38 | 0.92 |
| 9 | 0.41 | -0.36 | -3.38 | -3.40 | -1.51 | -1.49 | 1.13 |
| 10 | 0.49 | 0.45 | — | — | -1.61 | -1.59 | 1.30 |

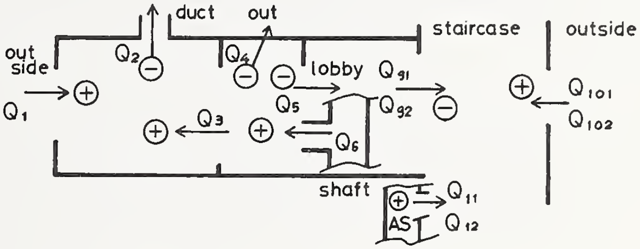


Table 4 Safe Time in Corridor (min.)

| Floor No. \ Condition No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | S* | S | S | S | S | S | S | S |
| 2 | - | - | - | - | - | - | - | S |
| 3 | S | S | S | S | S | S | 13.0 | S |
| 4 | S | S | S | S | S | S | 13.0 | S |
| 5 | S | 5.0 | 9.0 | S | S | S | 8.0 | S |
| 6 | S | 2.5 | 4.5 | 13.0 | 7.5 | 13.0 | 4.5 | S |
| 7 | 8.0 | 2.5 | 3.5 | 5.5 | 4.5 | 6.0 | 3.0 | S |
| 8 | 6.0 | 2.5 | 3.0 | 4.5 | 3.5 | 5.0 | 3.0 | S |
| 9 | 5.0 | 2.5 | 3.0 | 4.0 | 3.5 | 4.5 | 3.0 | S |
| 10 | 4.5 | 2.5 | 3.0 | 4.0 | 3.5 | 4.5 | 3.5 | S |

*S : safe constantly; $C_{SA} \leq 0.2$, $0 \leq t < \infty$

Table 5 Smoke Concentration in Staircase (1/m)

| condition NO.<br>stair cose NO.<br>floor NO. | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (I) | (II) | (I) | (II) | (I) | (II) | (I) | (II) | (I) | (II) | (I) | (II) | (I) | (II) | (I) | (II) |
| 1 | 0.12 | 0.05 | 0 | 0 | 0.09 | 0.08 | 0.01 | 0.01 | 0.06 | 0.03 | 0.03 | 0 | 0.19 | 0.07 | 0 | 0 |
| 2 | 0.62 | 0.30 | 1.7 | 1.2 | 0.48 | 0.45 | 0.06 | 0.06 | 0.44 | 0.19 | 0.17 | 0 | 0.58 | 0.22 | 0 | 0 |
| 3 | 0.41 | 0.18 | 1.4 | 1.1 | 0.31 | 0.29 | 0.02 | 0.02 | 0.21 | 0.10 | 0.09 | 0 | 0.37 | 0.14 | 0 | 0 |
| 4 | 0.41 | 0.18 | 1.3 | 0.98 | 0.31 | 0.29 | 0.02 | 0.02 | 0.21 | 0.10 | 0.09 | 0 | 0.37 | 0.14 | 0 | 0 |
| 5 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |
| 6 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |
| 7 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |
| 8 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |
| 9 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |
| 10 | 0.35 | 0.15 | 1.3 | 0.98 | 0.27 | 0.25 | 0.02 | 0.02 | 0.18 | 0.08 | 0.08 | 0 | 0.37 | 0.13 | 0 | 0 |

# Use of Actual Observed Solar Radiation Values in the Determination of Building Energy Requirements

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The effect of solar radiation is one of the many items that must be considered to properly determine a building's energy requirements for heating and cooling. The intensity of direct and diffuse solar radiation impinging upon a building's surfaces depends upon many factors; the season, locality, time of day and various types of sky contamination.

Numerous computer programs to determine the total annual energy consumption of buildings for heating, cooling and other uses have been written by various companies and organizations. Different approaches are used in these programs to estimate the effect of solar radiation on a building's heat loss or heat gain which in turn affect the energy requirements of the heating and cooling systems being evaluated.

This paper describes a practical, rather than theoretical, approach for estimating the effect of solar radiation on a building's heat loss or heat gain. The approach described is incorporated in a computer program that calculates a building's total annual energy requirements on an hourly basis. Basically, available solar radiation data as observed on a horizontal plate at several locations in the Southeastern United States over a period of years are converted to solar intensity values normal to the sun's rays for each hour of a year. The advantages and disadvantages over the theoretical approach are discussed.

Key Words: Solar radiation, actual radiation, observed radiation, actual observed radiation, building energy requirements, heating and cooling requirements.

## 1. Introduction

Consulting engineers and utility personnel often are required to estimate the annual heating, cooling and electrical energy requirements and energy cost of different buildings. Such estimates can be much more difficult and time consuming to calculate than the design point heat loss or heat gain for sizing the building's heating and cooling equipment. Calculation of the annual energy requirements involves a summation of the combined effect of many factors over a lengthy time period, and these factors may vary considerably with time.[1][2] Though the electrical requirements of a building are affected only by the building's mode of operation, the heating and cooling requirements are affected by five factors: (1) building location, (2) building construction, (3) building mode of operation, (4) building thermal load and (5) weather at the location.

## 2. Complexity of Problem

Because of the complex and interrelated nature of the factors involved, no simple and quick, non-computerized method has to my knowledge yet been developed for very accurately estimating the energy requirements of various types of buildings; however, there are many different analysis methods in use today that provide reasonably accurate estimates of this nature. Use of past operating records on a particular building, when these are available, may provide a reasonably accurate prediction of future energy requirements for a building being designed - provided that the factors affecting its energy requirements are almost identical in every respect. Unfortunately, many new buildings will differ from those existing in not just one, but in many ways. The use of averages of operating records available from similar, yet

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[2] "Figures in brackets indicate the literature reference at the end of this paper."

not identical, buildings can be misleading. There is no "average" building.

## 3. Digital Computer Offers Means

The most direct and reliable estimating procedure for determining a building's annual energy requirements to provide its heating, cooling and electrical needs is an hourly integration of the simultaneous calculated loads of each item involved as a function of the weather and use schedules. The modern digital computer offers the means for handling the vast amounts of data and calculations associated with preparing such an estimate. Development of computer techniques together with improvements in the availability of the basic data needed for the calculations are making it possible to determine the annual energy requirements with a high degree of accuracy. Numerous computer programs and methodologies have been written in recent years by various companies and organizations to perform this type of work. Different approaches are used in the programs to estimate the effect of solar radiation.

## 4. Solar Radiation a Major Factor

The amount of solar radiation impinging on the exterior surfaces of a building has a significant effect on its heating and cooling requirements. In many buildings as a result of a particular orientation and construction, solar radiation can have a very decided effect on the heating and cooling requirements. Due to the numerous factors that affect the total amount of solar radiation finally reaching the earth's surface, the exact magnitude impinging on the exterior surfaces of the building can be difficult to predict.

The intensity of direct solar radiation on a surface normal to the sun's rays above the earth's atmosphere is generally assumed to be approximately 445 btu/hour/square foot. In passing through the earth's atmosphere, the sun's radiation becomes scattered and absorbed to varying degrees by water vapor, dust, ozone and gas molecules that are present. Thus, what was only direct radiation now becomes both direct and diffuse. The amount of total solar radiation reaching a particular surface is the sum of the direct solar radiation, the diffuse sky radiation and the solar radiation reflected from surrounding surfaces. Its magnitude at any given time is determined by the moisture, the amount of smoke and dust in the air, the type and thickness of cloud cover, the locality, time of year and time of day [2].

## 5. One Way to Calculate Solar Radiation

It is recognized that some of the approaches used in existing computer programs and methodologies are a result of insufficient solar radiation data being available for the entire United States at the time the program was written. In other instances, the approaches used were probably thought to be, and may be, reasonably accurate. One of the more prevalent approaches has been to calculate assumed hourly solar radiation values for cloudless days and to apply available percentage cloud cover hourly data in an attempt to compensate for the effect of cloud cover on radiation. With this procedure, values of solar azimuth and solar altitude angles are input in table form on a one set of hourly values per month basis. Values for cloudless days of direct solar radiation received at normal incidence at the earth's surface and values of diffuse or sky solar radiation assumed to be received by variously oriented surfaces for clear or industrial atmospheres are also input for the varying solar altitudes. The program then interpolates the data to determine the hourly location of the sun and to estimate the hourly values of radiation impinging on the various building surfaces.

## 6. Use of Observed Solar Radiation Values

Another approach that can be utilized to calculate the hourly effect of solar radiation on a building's heating and cooling energy requirements is to use actual observed radiation data on a horizontal plate as an integral part of the program. This is the approach that was utilized in a computer program developed several years ago for the group of electric utility companies in the Southeast with which I am affiliated. The program was developed by the Mechanical Engineering Department of the University of Florida on a research contact with Southern Services, Inc. It considers the various factors mentioned earlier and performs an hour-by-hour synthesis of a building's heating, cooling and electrical requirements on a multi-zone basis, up to a maximum of 24-zones. After the hourly requirements are calculated, the program simulates the performance of specific equipment to meet these requirements and determines the total monthly and annual energy consumptions and demands. From these values, the monthly electric, fossil fuel and/or steam bills can be calculated.

This computer program was developed on the basis of having the capability to be used on any type of building. Its development was predicated on being able to obtain the most accurate answer that present scientific knowledge and available information would permit, using basic equations. In keeping with this basis, it was concluded that the use of actual observed hourly solar radiation data on a horizontal plate for energy calculation purposes would be preferable to calculating assumed hourly values and adjusting them by the application of cloud cover factors. The actual observed horizontal plate values would already include the effect of the type and thickness of cloud cover and the moisture, smoke and dust content of the air. Use of these values would also eliminate any error due to an improper cloud cover percentage being estimated by the observer. It was recognized that the effect of an industrial atmosphere would still need to be considered.

## 7. Details on Use of Observed Radiation Values

Research efforts located eleven year averages of actual observed hourly solar radiation data on a horizontal plate for various cities. This data was developed through a joint effort by the Boeing Company and the U. S. Weather Bureau [3]. Data was extracted from the report for the southeastern cities of Apalachicola, Florida, Lake Charles, Louisiana, and Charleston, South Carolina. (See figure number one for a comparison of the average values of solar radiation on which this program is based and the extremes which occurred over a 14-year period.) From the mean hourly solar radiation data in the report for these three cities, average values in Langleys were calculated for each sunlit hour of each month. One Langley is equal to one gram calorie per square centimeter and one gram calorie is equal to .003968 btus. These values were converted to btu/hour/square foot of horizontal surface. (See figure number two for curves depicting typical hours.) Then, these values were converted to intensity normal to the sun's rays, still in the same units. A review of these values indicated that afternoon values were somewhat excessive due to the higher level of diffuse radiation energy existing in the afternoon hours. To obtain more realistic values of direct solar radiation normal to the sun's rays in the afternoon, the morning horizontal plate readings were subtracted from the equivalent afternoon hour's readings, with the difference considered as diffuse radiation energy. New afternoon values were then created by adding this difference to the morning values at the same solar altitude. Equations were developed to calculate these values as well as the sun's location for each hour of the year. (See figure number three for typical curves of the solar radiation normal to the sun's rays as used in the program.)

## 8. Other Considerations

The effect of atmospheric clearness is considered by the program and the effect of an industrial atmosphere can be considered. An atmospheric clearness number of 0.95 is used in the equations rather than a value of 1.00 because according to Threlkeld and Jordan [4] in the southeastern states there are relatively high concentrations of atmospheric water vapor. The effect of an industrial atmosphere on direct normal solar radiation is considered as a function of the solar altitude angle and can be varied from 0 to 100 percent. Moon's data for 40 degrees North latitude on about August 1 relating the direct normal radiation and diffuse radiation of clear and industrial atmospheres to solar altitude is used as a basis [5].

## 9. Concluding Remarks

The above comments reflect the treatment of solar radiation for hourly energy calculation purposes. I have not discussed the procedure used for design condition energy calculation purposes. The procedure used for these purposes is somewhat different. One may question the accuracy attained by use of eleven year averages of hourly solar radiation data for three locations as opposed to other analysis methods. It was concluded that an analysis procedure predicated on being able to use long-term averages of actual observed hourly data for three locations covering The Southern Company service area was more factual and inherently more accurate than other possible methods.

## 10. References

[1] ASHRAE Guide and Data Book, Applications, Chapter 54, p. 645-656 (1968).

[2] ASHRAE Handbook of Fundamentals, Chapter 28, p. 469-475 (1967).

[3] "Summary of Solar Radiation Observations", U.S. Weather Bureau Report Number D2-90577-1) December 1964.

[4] Threlkeld, J. L., and Jordan, R. C., "Direct Solar Radiation Available on Clear Days", ASHRAE Transaction Number 1622.

[5] ASHRAE Guide, Chapter 13, p. 298, Table 4 (1958).

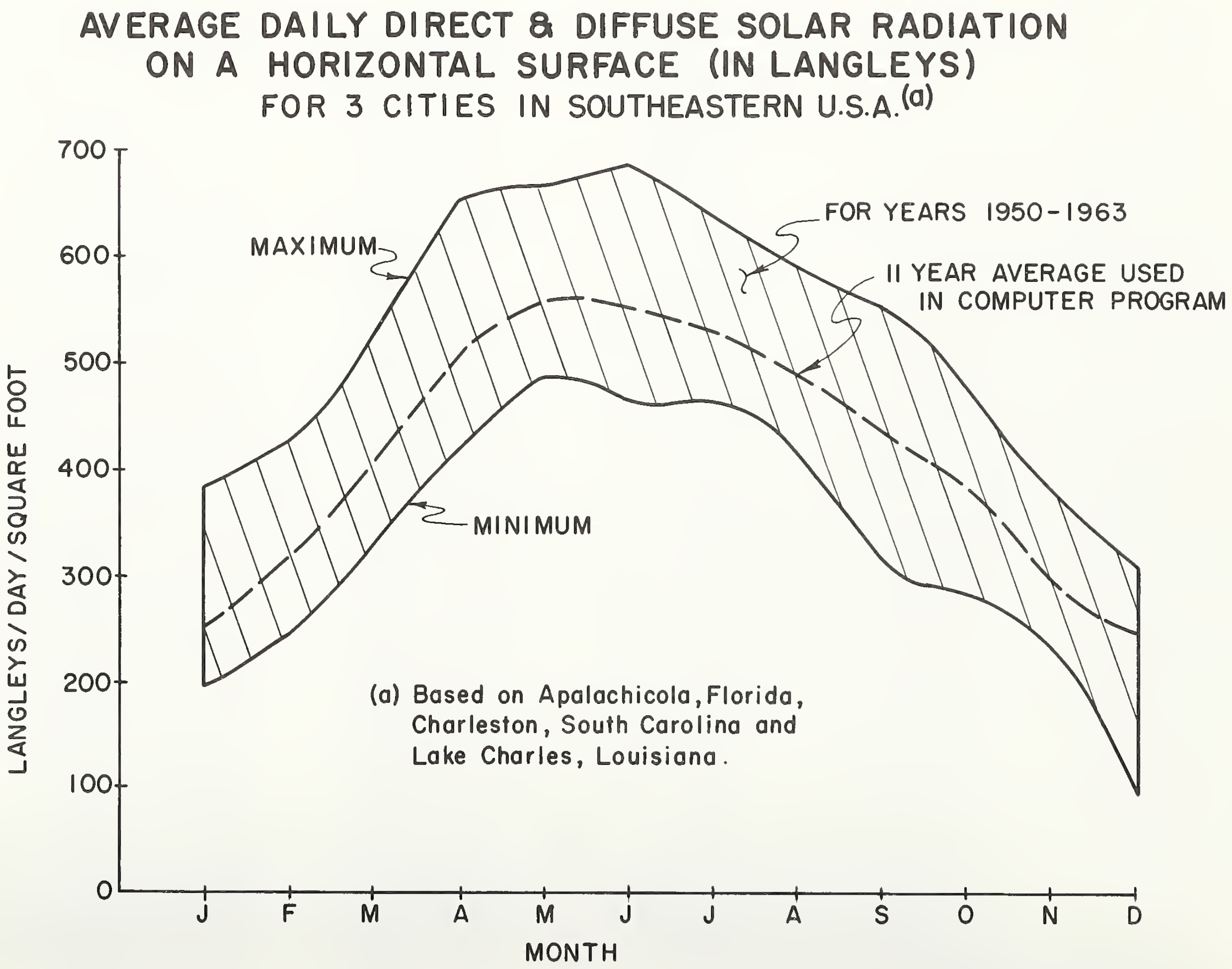


Figure number one: Average Daily Direct and Diffuse Solar Radiation on a Horizontal Surface (In Langleys) For 3-Cities in Southeastern U.S.A.

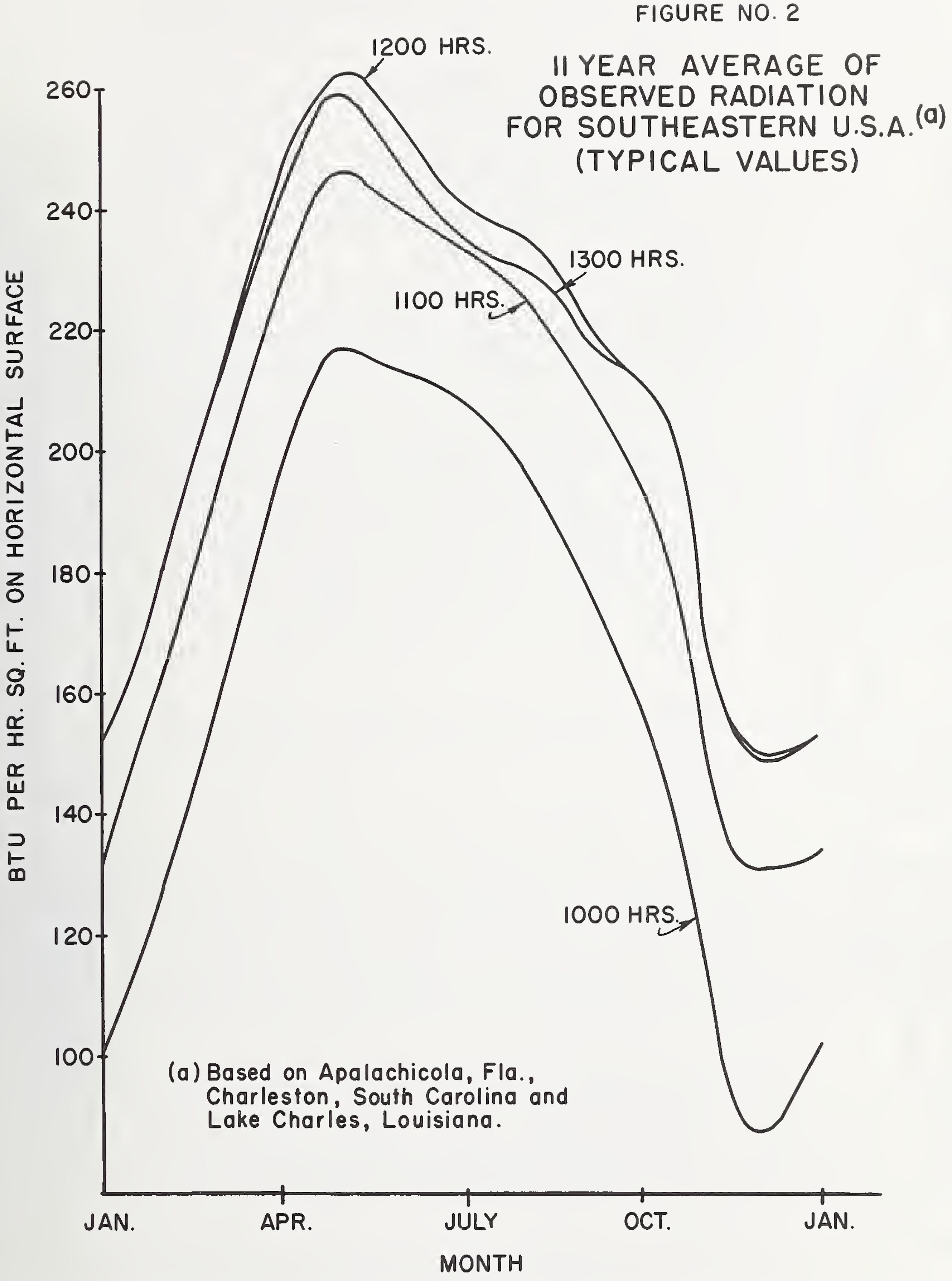
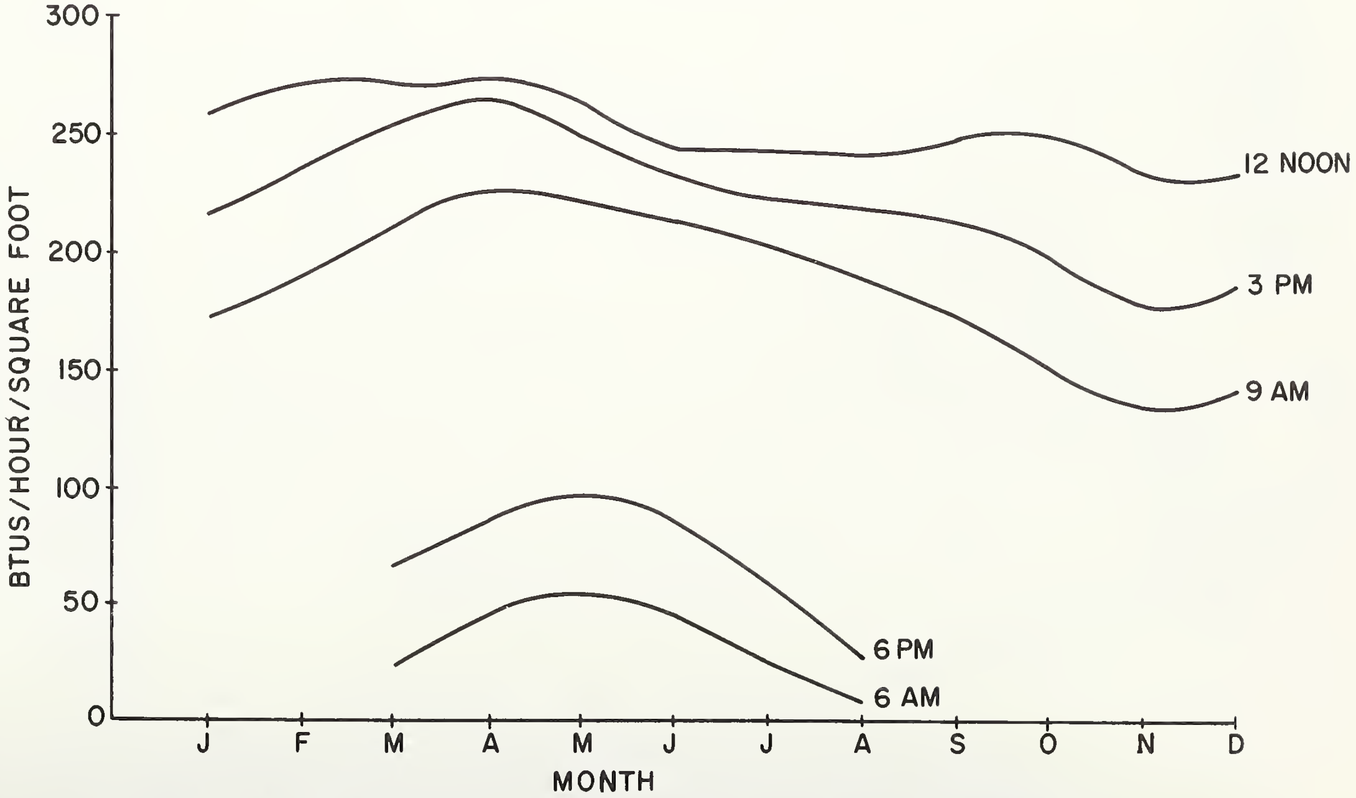


Figure number two: 11-Year Average of Observed Radiation For Southeastern U.S.A. (Typical Values)



CALCULATED SOLAR RADIATION NORMAL TO SUN'S RAYS CONSIDERING EFFECT OF CLOUD COVER, ETC. AS USED IN COMPUTER PROGRAM[(a)]
(TYPICAL VALUES)

(a) Values are shown for 19th of each month, based on an atmospheric clearness number of 0.95.

Figure number three: Calculated Solar Radiation Normal to Sun's Rays Considering Effect of Cloud Cover, Etc. As Used in Computer Program (Typical Values)

# Design of Direct-Expansion Evaporator Coils by Digital Computer

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Heat and water vapor transfer normally occur together in the use of an evaporator coil for air conditioning, making the process diverse and complex. As knowledge of this process has developed, together with the ability to describe it analytically, so has the need evolved for more exact procedures in practical design. This paper describes how the digital computer may be used for this purpose.

Beginning with a discussion of overall program objectives with respect to input and output, the development of a program logic for evaporator design is described. Decision points within the program include a logical procedure for selection of coil dimensions and refrigerant-side circuiting. Other decision points of particular significance are those involving iterative procedures for arriving at designs which match coil load conditions. Problems of convergence of these procedures are discussed. Basic equations used for performing the calculations are outlined, including methods of determining air-side, refrigerant-side and metal thermal resistances. The effect of variations in these resistances on overall performance is considered as related to methods of circuiting and operating conditions.

Sample solutions illustrating use of the program and output format are given.

Key Words: Air conditioning coils, coils (plate-fin), cooling and dehumidifying, computer design (digital), evaporators, heat exchangers, refrigerant heat transfer.

## 1. Introduction and Objectives

The large volume production of finned-tube heat exchangers for the air conditioning and refrigeration industry provides strong incentive for improving their performance and lowering their cost. The computer program described here, as an aid to the design of direct-expansion coils, was developed to assist design engineers in meeting these cost and performance objectives. The program should be viewed as providing a tool for "computer-aided" design, rather than design by computer. Although many design decisions are made automatically, key decisions, and a final choice among several alternative "good" designs, are still required by the design engineer. Introduction of the program into actual use at the Carrier Corporation has shown it to be of considerable assistance as a time and cost saver in identifying that "best" design. And its acceptance, even by experienced designers, has been excellent.

Specific objectives for the development of the computer program were as follows:

1. To permit more effective use of the accumulating data on heat transfer and pressure drop for refrigerants evaporating inside tubes and air flow over coil surfaces.
2. To study the differences which would result from computer-aided design, using more detailed and fundamental heat transfer relationships, as compared with existing design procedures.
3. To provide a tool which would permit the designer to arrive nearer the optimum coil configuration for a given task.
4. To identify critical areas in which present information is not sufficiently precise to satisfactorily determine an optimum design, and to plan research efforts to overcome them.

## 2. Overall Program Logic

The design of an evaporator coil, like most design procedures, is not a straightforward process. It is necessary to make a number of initial assumptions regarding coil configuration, air flow rates, and refrigerant conditions. The more experienced designer will have acquired certain rules and judgments which make it possible for him to choose initial values that produce a near-optimum solution. Nevertheless, even he will have to make a few trials and adjustments to match satisfactorily the particular constraints for a given system.

Our most challenging task in developing a computer-aided design procedure for evaporators was to bring into its logic relationships which express quantitatively the judgment of experienced designers. In addition there was the overriding goal to make the program a flexible tool in the hands of the designer. While key assumptions and decisions could be made by the program, they must not be hidden from the user. In fact it would be desirable for the designer to override key program logic, and to specify particular constraints on certain variables where he desires. In general he should be able to guide the computer calculations, when desired, to conform with his experience and judgment in obtaining a satisfactory design.

### 2.1 What it does - general

The program determines the physical dimensions, leaving air conditions, heat transfer characteristics, and manufacturing cost of a direct-expansion air cooling and dehumidifying coil. As input it requires geometric data defining the type of plate-fin heat transfer surface, constants and exponents to calculate air-side pressure drop and heat transfer performance, required cooling load, system operating conditions, and cost data.

The procedure is to make an initial assumption with regard to air quantity (about 400 cfm/ton for air conditioning applications), and air face velocity (usually between 400 to 500 fpm) and calculate the coil dimensions. An iterative calculation then matches the heat transfer rate to the required cooling load. The iterative procedure either adjusts the air face velocity or the coil size to match the loads to within one-half of one percent. Finally, the program calculates the leaving air conditions, the air pressure drop, the sensible heat factor, and the coil cost. The program also has features to automatically index to another air velocity or coil size and repeat the above calculations. The designer is thereby provided with several alternate designs to assist him in either making a final choice, or respecifying key conditions in a continued search for an "optimum" coil design.

### 2.2 What it requires

The user of the program must supply data on the plate-fin type heat exchanger surface, the system operating conditions, and the manufacturing costs. Figure 1 is a typical input data sheet. The quantities appearing thereon are as follows:

#### a. Coil data

(1) Description, Refrigerant (12 or 22), and Identification Number.

(2) Tube outside diameter and wall thickness - $D_o$ and $t_w$ (inches).

(3) Hairpins or return bends, and return bend inside diameter - $D_{ib}$ (inches).

(4) Tube spacing (transverse and longitudinal) - $P_t$ and $P_r$ (inches).

(5) Fin spacing and thickness - $N_f$ (fins/inch) and $t_f$ (inches).

(6) Surface heat transfer constants - $C_1$ and $n_1$ (for the determination of the air-side thermal resistance, as explained in Section 3.2).

(7) Surface pressure drop constants - $C_2$ and $n_2$ (for the determination of the air-side frictional pressure drop, as explained in Section 3.2).

By storing the heat transfer and pressure drop characteristics of a given type coil in a disc file, items (6) and (7) can be replaced by an identification number which is related to the location on the disc where the information is stored. This number is 5 in the example given in Figure 1.

#### b. System operating conditions

(1) Cooling load - $q$(Btu/hr)

(2) Saturation temperature at suction - $t_{ro}$(°F)

(3) Subcooled liquid temperature at inlet - $t_{sub}(^\circ F)$

(4) Suction superheat - $\Delta t_{sup}(^\circ F)$

(5) Entering air dry-bulb temperature - $t_{a_1(db)}(^\circ F)$

(6) Entering air wet-bulb temperature - $t_{a_1(wb)}(^\circ F)$

(7) Atmospheric pressure - $Patm^{(in.\ Hg)}$

(8) Safety factor (see Eq. 1) - $F_s$

(9) Minimum air face velocity - $V_{70(min)}(ft/min)$

(10) Air face velocity increment - $\Delta V_{70}(ft/min)$

(11) Maximum air face velocity - $V_{70(max)}(ft/min)$

### c. Cost data

The cost of materials for construction of the coil core can be expressed as the sum of the costs for the tubing, the fin material, and the return bends. The tube and fin costs will depend on the weight or volume of material used. The total material cost is then written as

$$C_m = C_t V_t + C_f V_f + C_b N_b$$

The input data then requires

(1) Cost of tube material - $C_t$ (\$/cu in)

(2) Cost of fin material - $C_f$ (\$/cu in)

(3) Cost of return bends - $C_b$ (\$/bend)

More detailed cost calculations would bring into consideration costs of the refrigerant distributor and feeder tubes, solder, labor, etc.

## 2.3 Decision Points

A first and major decision is where to start the design calculation. Other key decisions involve the refrigerant circuiting of the coil, and how to direct the search to insure a final design choice which is a near optimum one. The program logic which makes these decisions originally evolved from the judgment of experienced designers, and underwent refinements as it was put to use on a variety of evaporator designs.

### a. Initializing coil dimensions

Three choices are possible as follows:

(1) Case 1 - Aspect ratio

Under this case, the ratio of coil length between tube sheets to coil height (aspect ratio) is specified by the designer. In iterating to arrive at the design cooling load the length between tube sheets is varied, hence the final design does not correspond to the exact value of aspect ratio.

(2) Case 2 - Length between tube sheets

With this selection the program iterates on air face velocity so that the final design does conform to the specified length dimension.

(3) Case 3 - Number of tubes in the coil face

This is equivalent to the selection of the face height of the coil. With this design constraint the program holds the given height and iterates on the length between tube sheets to match the design load.

If the user does not choose to select any of the above, the program sets the aspect ratio at a value of 2, and proceeds as in Case 1.

### b. Air velocities and flow rates

Air face velocities for DX evaporator coils have a restricted range. Low velocities require large coil size which may not be economically competitive. High velocities may cause entrainment of condensed water on the coil surface, and carry over into the discharge air stream. As listed above for the system operating conditions - input data, the user must specify a minimum, maximum, and incremental air face velocity. Reasonable limits are 300 fpm and 600 fpm for the minimum and maximum velocities, although velocities outside this range may be used in special cases. The incremental air velocity, $\Delta V_{70}$, directs the program to index upwards from $V_{70(min)}$ to $V_{70\ (max)}$ at that increment. A good initial selection would be 100 fpm. In the final search for a best design the increment may be made smaller.

An air flow rate of 400 cfm per ton has proven to be a good choice for starting the design in air conditioning applications. This choice usually leads to an operating sensible heat factor of between 0.7 and 0.8 for a 3 row deep coil at 400 to 500 fpm air face velocity. In iterating to arrive at the design cooling load, and in automatically indexing to new design configurations, the cfm/ton changes.

### c. Refrigerant circuiting

If the number of circuits is not given with the input data a subroutine is called which determines circuiting automatically. In this subroutine the number of circuits is selected so that the refrigerant flow rate is within the range of good design practice. In addition, circuiting selections are limited to those which will provide balanced refrigerant distribution, a factor of considerable importance to good performance.

For an air conditioning coil with 1/2 inch tubes, experience has shown that a good starting point for determining the number of circuits is to assume a circuit load of 16,000 Btu/hr. At smaller or larger tube diameters the loading factor of 16,000 must be decreased or increased correspondingly. Changes in the pressure level also affect the loading factor because of density changes of the fluid. Relationships to account for these effects are incorporated into the program logic to calculate an initial value for the number of circuits. The program next requires that both $N_c$ and the ratio $N_t/N_c$ be integers. For example, if $N_t = 24$ tubes in the coil face, then the number of circuits, $N_c$, permitted is 1, 2, 3, 4, 6, 8, 12, or 24. The selection will be the smallest of these numbers which does not exceed by more than 30 per cent the initial value.

The above procedure insures that a balanced refrigerant circuiting arrangement will result. The experienced designer may recognize a satisfactory circuiting arrangement not permitted by the simple program logic. In this case he may specify $N_c$ in the input data, and override the computer selection procedure.

### d. Indexing to new designs

Depending upon the designer's choice of Case 1, 2, or 3 in Section (a) above, the computer is directed to iterate on either air face velocity or face length to match the design load. After the first design has been found, provision has been made for automatically seeking alternate designs using the same method of iteration. The first method of indexing to a new design is to change the number of tubes in the coil face by first two greater and then two less than that found in the initial design.

When the design is proceeding on the basis of aspect ratio (Case 1 or no designer specification), or on tubes in the face (Case 3), the design starts with the minimum air face velocity of $V_{70(min)}$. After the initial design, and those at ± 2 additional tubes in the face are completed, the computer indexes to a higher face velocity by the velocity increment $\Delta V_{70}$. Again at the completion of an initial design for this velocity the program calls for designs at ± 2 additional tubes in the face. The indexing to increasing air velocities continues according to the equation

$$V_{70} = V_{70} + I\ (\Delta V)_{70}$$

with I = 1, 2, 3, ---, until $V_{70}$ exceeds $V_{70(max)}$.

In Case 2 where the length between tube sheets is specified, the procedure for indexing to ± 2 tubes in face is used, but the incremental air velocity is not. If the number of refrigerant circuits is specified, or if the face area of the coil is fixed by giving both $L_p$ and $N_t$ with the input data, then no automatic indexing occurs.

## 2.4 Flow Diagram for the Main Program

A condensed version of the main program flow diagram, labeled DX - 12/22, is presented as figure 2. As used on an IBM 1130 computer it calls on nine subroutine programs, designated: R, S, T, A, B, C, D, E, and H. Referring to circled numbers on this figure, the explanatory phrases below will assist in follow- the program logic.

(1) Subroutine program 'A' reads the input data of figure 1, and supplies index numbers to carry out the automatic indexing to new designs, described in (d) above.

(2) Subroutine 'R' calculates refrigerant properties for either R-12 or R-22.

(3) Subroutine 'S' calculates air-side thermal resistance and pressure drop at standard air conditions for the specified plate-fin surface.

(4) Subroutine 'T' calculates the metal resistance of the finned coil by the sector method.

(5) Subroutine 'H' calculates the enthalpy of moist air at the coil inlet. The other calculations here are to make a temperature correction to the air-side thermal resistance, and set an initial value of air flow by the rule of 400 cfm/ton.

(6) Subroutine 'B' calculates the coil geometry, including the number of refrigerant circuits.

(7) Subroutine 'C' calculates refrigerant-side heat transfer coefficients and pressure drops, the mean temperature difference in the superheat section and the superheat length.

(8) Subroutine 'D' calculates the mean temperature difference in the evaporating section of the coil.

(9) The load calculation is made by determination of heat exchange in both the evaporating and superheating portions of the coil.

(10) The convergence routine to match calculated to design load. (See Section 4.2).

(11) Subroutine 'E' calculates the leaving air conditions, the sensible heat factor, coil apparatus dew point, and materials or manufacturing costs.

(12) This is the automatic indexing sequence to set up for additional designs at ± 2 additional tubes in the face, and at incrementally higher air velocities.

## 3. Basic Relations

The flow and heat-transfer processes occurring in a direct expansion evaporator are exceedingly complex. On the air side heat and water vapor transfer simultaneously between the air and the fin surfaces. On the refrigerant side a series of changing two-phase gas-liquid flow patterns occur due to the increasing ratio of vapor to liquid flow rate. These changing flow patterns and velocities profoundly affect the heat-transfer coefficient and pressure gradient.

Figure 3 illustrates, in an approximate way, the heat-transfer coefficient and temperature behavior of a refrigerant as it moves through an evaporator tube. Two sets of curves are shown, one for high loading and the other for low loading. As shown in this figure, the heat transfer coefficient increases to a maximum and then decreases sharply, eventually reaching a value corresponding to that for pure gas flow in the superheat section. The increase in coefficient is due to increasing shear at the vapor-liquid interface. The decrease is due to the development of dry areas on the wall, a condition which occurs at vapor qualities on the order of 0.8 in typical evaporators. Experimental determination of these variations in coefficient have been reported by Anderson, Rich and Geary (1)[1]

It is clear from figure 3 that an accurate calculation of the heat transfer in an evaporator cannot be made using a single average value of the refrigerant coefficient. Ideally, a step-by-step calculation could be performed such that local variations in coefficient, refrigerant temperature, air enthalpy, and surface temperature are accounted for. Such a procedure, however, would be complex and time consuming. As a first approximation, therefore, the present program divides the coil into two sections - an evaporation section and a superheat section - with average coefficients determined for each. Coil capacity is then calculated as follows:

$$q = \left.\frac{F_s \, S_o \Delta t_m}{R_r + R_m}\right)_{evap} + \left.\frac{F_s \, S_o \Delta t_m}{R_r + R_m}\right)_{sup} \qquad (1)$$

where $S_o$ = external surface area (prime plus finned)

$\Delta t_m$ = mean temperature difference between the refrigerant and the wetted coil surface

$R_r$ = thermal resistance to heat transfer between the refrigerant and the inside surface of the tube wall per unit of external surface area

$R_m$ = thermal resistance of the fin and tube material per unit of external surface area

$F_s$ = a safety factor specified by the designer

Procedures used for calculating $\Delta t_m$, $R_r$ and $R_m$ are described in the following sections.

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[1] Figures in brackets indicate the literature references at the end of this paper.

## 3.1 Mean Temperature Differences - Surface Temperatures

Figure 4 presents two examples of circuiting arrangements which might be used in direct-expansion evaporator coils. In 4A (labeled thermal parallel flow) the refrigerant enters at the coil leaving air face, flows through the coil generally counter to the air flow, and discharges at the entering air face. In 4B (labeled thermal counterflow) the refrigerant enters at the entering air face, flows through the coil generally parallel to the air flow, but returns to the face for the final pass (tube marked S). Both of these arrangements have the advantage of providing superheating in the region where the air temperature is maximum. The second arrangement has the additional advantage of providing an approximate thermal counterflow relationship between the air and refrigerant. Although other arrangements may be used in practice, depending upon economic considerations related to the application, those shown in figure 2 are sufficiently typical to be useful as models.

Figure 5 is a simplified thermal diagram for an evaporator depicting the evaporation and superheat sections and defining the various terminal temperatures and enthalpies used to calculate the mean temperature differences. Circuiting is the thermal counterflow type as shown in figure 4B, with superheating assumed to occur in tubes at the coil face. In figure 5 this results in a discontinuity for the plot of air enthalpy ($h_a$) and coil surface temperature ($t_s$) vs circuit length. As illustrated the superheat tube section is in contact with entering air ($h_a = h_{a1}$).

Using temperature symbols as identified in figure 5, the following equations can be written for the mean temperature differences:

Evaporation Section

$$\Delta t_m = \frac{(t_{s1} - t_{r1}) - (t_{s2} - t_{r2})}{\log\left(\frac{t_{s1} - t_{r1}}{t_{s2} - t_{r2}}\right)} \qquad (2)$$

thermal parallel flow

$$\text{or} \quad \Delta t_m = \frac{(t_{s1} - t_{r2}) - (t_{s2} - t_{r1})}{\log\left(\frac{t_{s1} - t_{r2}}{t_{s2} - t_{r1}}\right)} \qquad (3)$$

thermal counter flow

Superheat Section

$$\Delta t_m = \frac{(t_{s3} - t_{r2}) - (t_{s4} - t_{r3})}{\log\left(\frac{t_{s3} - t_{r2}}{t_{s4} - t_{r3}}\right)} \qquad (4)$$

Referring to figure 5, note that $t_{ro}$ is the temperature at saturation corresponding to the refrigerant pressure at the coil outlet. This value is an input to the program, as is the superheat, $t_{r3} - t_{ro}$. The other refrigerant temperatures are direct functions of the tube-side pressure drop; $t_{r2} - t_{ro}$ is the temperature drop due to pressure drop in the superheat section and $t_{r1} - t_{r2}$ is the temperature drop due to pressure drop in the evaporation section.

The coil surface temperatures will depend upon the air flow conditions, the refrigerant conditions and whether the surface is wet or dry. The computer program presented here assumes that the coil surface will be completely wetted. This is not a serious limitation for a design program since evaporator coils normally operate fully wetted at normal design conditions. With this assumption, each of the surface temperatures can be related to a coil characteristic, C, by the equation(2)

$$\frac{t_s - t_r}{h_a - h_s} = \frac{R_m + R_r}{c_p R_a} = C \qquad (5)$$

where $t_s$ = surface temperature

$t_r$ = refrigerant temperature

$h_a$ = air enthalpy

$h_s$ = air enthalpy at saturation at $t_s$

$c_p$ = humid specific heat ($\approx$ 0.243 Btu/$lb_{da}$ °F)

$R_a$ = thermal resistance between the air and surface per unit of external surface area

For the evaporation section of the coil circuit a single value of the coil characteristic is used, based on an average value of the refrigerant-side thermal resistance. Values of $t_{s1}$ and $t_{s2}$ are then determined from eq (5) using $h_{a1}$ - and $h_{a2}$ and, respectively, $t_{r1}$ and $t_{r2}$ (or $t_{r2}$ and $t_{r1}$ for thermal parallel flow).

For the superheat section of the coil circuit two values of the characteristic are used. At the entrance the surface temperature ($t_{s3}$) is based on the evaporating coefficient and $t_{r2}$; at the exit the surface temperature ($t_{s4}$) is based on the superheat coefficient and $t_{r3}$. The entering air enthalpy, $h_{a1}$, is used in both cases reflecting the assumption that the superheating tube passes are located on the entering-air side of the coil.

A computer procedure for solving eq (5) is given in Section 4.1.

### 3.2 Air-Side Thermal Resistance

Because of the many geometric variables involved, the air-side thermal resistance for plate-fin coils is determined by experiment. Fin spacing and thickness, tube spacing and diameter, number of tube rows and fin surface configuration are variables upon which the coefficient depends. Generalized relationships which can predict the effect of these variables are lacking. Detailed procedures for experimentally determining the air-side thermal resistance of cooling and dehumidifying coils are given in ASHRAE Standard 33-64 (3).

For the small temperature differences and limited velocity range encountered in normal air conditioning and refrigeration applications, convective heat transfer data can be represented accurately by the Nusselt equation.

$$\frac{hD}{k} = K_1 \left(\frac{GD}{\mu}\right)^{n_1} P_r^{\ m} \qquad (6)$$

Setting m = 0.4, eq (6) can be solved for $R_a$ (= 1/h), to give

$$R_a = \left(\frac{D^{1-n_1}}{K_1}\right) \left(\frac{\mu^{n_1}}{k\ P_r^{\ .4}}\right) G^{-n_1}$$

$$\text{or } R_a = C_1\ \varphi_1\ V_{70}^{-n_1} \qquad (7)$$

where $\varphi_1$ is a properties function to account for variations from standard air conditions.

An approximate equation for $\varphi_1$, normalized to standard air conditions of 70°F and 1 atmosphere, is

$$\varphi_1 = \frac{1}{1 + 0.00143\ (1.109 + n_1)\ (t - 70)} \qquad (8)$$

Equation (8) is accurate within ± 1 per cent from -25°F to 175°F.

The frictional pressure drop can be calculated in a similar manner. Thus

$$\Delta p = \frac{K_2}{R_e^{\ m}}\ \frac{G^2}{2\rho\ g_c}\ \frac{L}{D} = \left(\frac{K_2}{2\ g_c\ D^{1\ +\ m}}\right)\left(\frac{\mu^m}{\rho}\right) G^{2-m}$$

$$\text{or } \Delta p = C_2\ \varphi_2\ V_{70}^{\ n_2} \qquad (9)$$

where $\varphi_2$ accounts for variations from standard air conditions.

$C_1$, $n_1$, $C_2$ and $n_2$ are empirical constants fitted to the experimental data for a particular geometric coil configuration. A file of these constants for a variety of coil types can be stored for automatic access by the computer. Alternately, the designer can supply values to be read as part of the input data.

### 3.3 Metal Thermal Resistance

The metal thermal resistance is the sum of the resistance of the fins and the resistance of the tube wall. The following equation relating the metal resistance, $R_m$, to the fin efficiency $\phi$, is taken from Reference 2.

$$R_m = \left[ \frac{1 - \phi}{\phi + S_p/S_f} \right] \frac{R_a c_p}{h_s'} + \frac{D_i \; S_o/S_i}{2 k_t} \log\left( \frac{D_o}{D_i} \right) \qquad (10)$$

Herein, $h_s' = dh_s/dt_s$; $k_t$ is the tube conductivity; $D_o$ and $D_i$ are the tube OD and ID; and $S_p$, $S_f$, $S_o$ and $S_i$ are the external prime, external finned, total external, and internal surface areas respectively.

Various methods have been proposed for calculating the fin efficiency. Where the fin shape associated with each tube is nearly square the efficiency of an annular fin of equal area can be used (4). Otherwise, more accurate results can be obtained using the sector method (4) (5). The present program utilizes a subprogram based on the sector method.

### 3.4 Refrigerant-side Thermal Resistance and Pressure Drop

Experimental measurements of evaporating heat transfer coefficients in tubes have been reported in the literature, and several methods of correlating these data have been proposed. Reference 6 contains a summary of experimental results and correlating equations which are applicable to forced convection evaporation of refrigerants. None are completely satisfactory for the complete range of conditions encountered in evaporators. As stated in Reference 6, "the single equation which can be recommended for broadest application to refrigerant evaporation in tubes is that of Pierre (7). Altman, Norris and Staub (8), and Chaddock (9), have fitted it to a wide range of Refrigerant 12 and 22 data."

Pierre's equation for exit qualities $\leq$ 90 per cent is

$$Nu = 0.0009 \left(R_e^2 K_f\right)^{0.5}$$

$$\text{or} \quad h_r = 0.0009 \left( \frac{k_\ell}{D_i} \right) \left[ \left( \frac{G \; D_i}{\mu_\ell} \right)^2 \left( \frac{J \; \Delta x \; h_{fg}}{L} \right) \right]^{0.5} \qquad (11)$$

where $\Delta x$ = change in vapor fraction

$L$ = tube length

$D_i$ = tube inside diameter

$k_\ell$ = liquid thermal conductivity

$\mu_\ell$ = liquid viscosity

$h_{fg}$ = latent heat

$J$ = mechanical equivalent of heat (778 ft-lb/Btu)

In the present program a modified form of Eq. (11) was used in order to cover vapor fractions up to unity.

The heat-transfer coefficient for the superheat section can be calculated by the well-known methods for single-phase forced convection in tubes. McAdams (10), after critically evaluating a large amount of data covering a wide range of conditions, gives three equations. One of these is called the Dittus-Boelter equation, and is expressed as:

$$\frac{h\ D_i}{k} = .023 \left(\frac{D_i\ G}{\mu}\right)^{0.8} \ Pr^{0.4}$$

$$\text{or} \quad h = \varphi_3 \ \frac{G^{0.8}}{D_i^{\ 0.2}} \tag{12}$$

where $\varphi_3$ is a function of the fluid properties.

Reference 6 tabulates values of $\varphi_3$ as a function of temperature for various refrigerants. The following simple polynomial expressions can be used to calculate $\varphi_3$ with a maximum error of 0.7 per cent over the temperature range 0 to 160°F.

R-12 vapor $\quad \varphi_3 = 1.816 \times 10^{-3} + 3.214 \times 10^{-6}\ t + 1.786 \times 10^{-9}\ t^2$

R-22 vapor $\quad \varphi_3 = 1.974 \times 10^{-3} + 3.039 \times 10^{-6}\ t + 1.116 \times 10^{-8}\ t^2$

The pressure drop in a direct-expansion evaporator has several components. These include pressure drop across the bends, pressure drop due to friction in the tubes, pressure drop within the suction header, inlet and exit losses, and a pressure drop due to acceleration of the refrigerant as it flows through the coil. In the present program header losses are not included (pressure at the exit of the tubes has been chosen as a program input), and inlet and exit losses have been neglected. Of the remaining losses (bends, friction and acceleration) the acceleration loss is generally smallest and friction largest, although bend losses can predominate in very narrow coils.

Several methods have been proposed for calculating friction pressure drop for two-phase flow inside tubes. The one proposed by Martinelli and coworkers (11), (12) has received widest acceptance because of its general applicability. A somewhat simpler method is that based on the friction-factor equation. It has proven to be satisfactory in correlating data for restricted ranges of conditions.

The basic form of the friction-factor equation is:

$$\Delta p_f = \frac{G^2}{2\rho}\ f\ \frac{L}{D_i} \tag{13}$$

where L is the total circuit length and G is the refrigerant mass velocity based on the total flow rate (liquid plus vapor). Various definitions of the two-phase friction factor, f, and two-phase density, $\rho$, have been proposed. Pierre (13) uses an average between the liquid and vapor densities, and defines his friction factor in terms of $K_f/R_e$ (see eq 11). A similar approach was taken in the present program except the vapor density was used and the effects of inlet vapor mass fraction were incorporated in the friction factor.

In addition to the pressure drop due to friction there is also a loss in pressure to accommodate the acceleration of the refrigerant as it changes from a liquid to a vapor. The acceleration pressure drop can be calculated by the relation proposed by Martinelli and Nelson (12). For complete evaporation this relation is

$$\Delta p_a = \frac{G^2}{g_c} \left\{ \frac{1}{\rho_g} - \frac{(1 - x_1)^2}{\rho_\ell\ R_L} + \frac{x_1^{\ 2}}{\rho_g\ (1 - R_L)} \right\} \tag{14}$$

where $R_L$ is the fraction of the tube filled with liquid at the inlet.

The following polynomial expression, based on the curve given in Reference (11), can be used to calculate $R_L$.

$$\log_{10} R_L = -\ 0.64413 + 0.48554y - 0.1308\ y^2 + 0.01929\ y^3$$

$$\text{where} \quad y = \log_{10} \left\{ \left(\frac{1 - x}{x}\right)^{.9} \left(\frac{\rho_g}{\rho_\ell}\right)^{.5} \left(\frac{\mu_\ell}{\mu_g}\right)^{.1} \right\}$$

Virtually no data is available in the literature for pressure drop of refrigerants in bends. As an approximation, therefore, the pressure drop for the bends is calculated in the same manner as a straight tube, using a length equal to 75 diameters. This practice is commonly used for single-phase flow (10).

### 3.5 Leaving Dry Bulb Temperature, Sensible Heat Factor and Coil Costs

After the program calculations have converged to a coil design whose capacity matches the given load, a final subroutine is called which calculates the leaving dry-bulb temperature, the sensible heat factor, and an estimated cost.

Using the relationships recommended in Reference 2, the leaving dry bulb temperature is calculated by first determining the saturated air enthalpy, $\bar{h}_s$, at the coil surface by

$$\bar{h}_s = h_{a1} - \frac{(h_{a1} - h_{a2})}{1 - e^{-\lambda}}$$

$$\text{where } \lambda = -S_o/w\ c_p\ R_a$$

The surface temperature $\bar{t}_s$ corresponding to $\bar{h}_s$ is readily found, and the leaving dry bulb temperature and sensible heat factor follow directly from the relations

$$t_{a2} = \bar{t}_s + (t_{a1} - \bar{t}_s)\ e^{-\lambda} \qquad (15)$$

$$SHF = c_p\ (t_{a1} - t_{a2})/(h_{a1} - h_{a2}) \qquad (16)$$

Once the coil size has been determined, fin and tube costs can be calculated based on unit material costs furnished as input. Labor costs may also be included provided information is available which relates the time required for coil fabrication to the various geometric parameters. Obviously, labor costs will vary greatly depending upon manufacturing methods, plant layout, production volumes and other factors and generalized relationships cannot be given.

## 4. Iterative Procedures

An important consideration in the development of computer programs for designing heat exchangers is the selection of iterative procedures which converge quickly and consistently to a correct solution. In the present program iterative procedures are involved in solving eq (5) for surface temperatures, and also to find coil size or cfm to match the specified load. A discussion of these procedures is given below.

### 4.1 Surface Temperatures

"When the derivative of f(x) is a simple expression and easily found, the real roots of f(x) = 0 can be computed rapidly by a process called the Newton-Raphson Method" (14). According to this method, successive approximations to the root can be formed by the following steps.

$$a_1 = a_o - f\ (a_o)/f'\ (a_o)$$

$$a_2 = a_1 - f\ (a_1)/f'\ (a_1)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$a_n = a_{n-1} - f\ (a_{n-1})/f'\ (a_{n-1})$$

where $a_o$ is the first approximation to the root. Applying this procedure to eq (5) we obtain

$$f(t_s) = t_s - t_r - C\ (h_a - h_s) = 0$$

$$f'(t_s) = 1 + C h_s'$$

Therefore

$$t_s(\text{new}) = t_s - \left\{ \frac{t_s - t_r - C(h_a - h_s)}{1 + C h_s'} \right\} \tag{17}$$

Since $f'(t_s)$ is always greater than unity, a routine based on eq (17) will always converge rapidly.

Following are a series of FORTRAN statements based on eq (17) which can be used to calculate surface temperature to within E degrees of its correct value.

```
  C  =  (RR + RM)/(.243*RA)
  TS  =  TR
1 CALL HAIR (TS, P, HS, HSP)
  DT  =  (TR-TS + C* (HA - HS) )/(1. + C * HSP)
  TS  =  TS + DT
  IF(ABS (DT - E) ) 2, 2, 1
2 CONTINUE
```

In the above routine HAIR is a subroutine which calculates air enthalpy at saturation (HS) and its derivative (HSP) as a function of surface temperature (TS) and air pressure (P).

### 4.2 Air Flow Rate

As previously described, the program may, in some cases, be called upon to match the load by varying the coil length between tube sheets, with the air face velocity held constant. In other cases the air flow rate is varied to match the load while holding the coil size constant.

Let us first consider the case where the coil size is fixed. Figure 6A is a graph showing for a typical case how the coil capacity q as calculated by the program will vary with the assumed flow rate w. Note that as flow rate increases the capacity asymptotically approaches a maximum value. This is the value corresponding to negligible air-side thermal resistance; it must be greater than the given load if a solution is to be found.

As the air quantity decreases, the capacity approaches zero at a finite value of air flow equal to $w_o$. This is due to a decreasing enthalpy difference (approach) at the leaving air side of the coil. This zero approach condition is possible because the leaving air enthalpy, $h_{a_2}$, is calculated as a function of the given load, $q_o$, rather than as a function of the calculated capacity, q, since the latter is not known at the beginning of the computation.

Figure 6B is a similar plot to figure 6A for the case where the length between tube sheets is varied to match the load. In this case the calculated capacity can reach a maximum and then decrease with increasing length and flow rate. The decrease occurs when the effect of refrigerant-side pressure drop becomes dominant. Obviously, the given load must be below this maximum value if a solution is to be found.

Depicted graphically on figure 6B is an iterative procedure which can be used in both of the above cases to find the value of w for which $q = q_o$. The procedure calculates successive approximations to w by interpolating along a straight line between the points $(0, w_o)$ and $(q_n, w_n)$. A general equation relating the new value of flow rate, $w_{n+1}$, to the previous value, $w_n$, follows directly.

$$w_{n+1} = w_o + (w_n - w_o) \frac{q_o}{q_n}$$

Although convergence is not particularly rapid with this procedure, it offers the important advantage of insuring convergence for all values of $w_1$ less than $w_c$.

## 5. Sample Output

Figure 7 gives the output corresponding to the input provided in figure 1. In addition to the calculated quantities for the dimensions and performance of the coil, most of the input data values are recorded also. This is to assist the user in identifying the coil design and in making comparisons and selections. Comparisons are also facilitated by the columnar format used for output.

In the example given the program was asked for solutions at only one air velocity, 450 ft/min. Three solutions were given. As a first choice the program arrived at a design with 24 tubes in the face and 12 circuits. It then increased the tubes in the face to 26 and selected 13 circuits. Finally, the tubes in the face were reduced to 22 and a solution given for 22 circuits. In this last case 11 circuits was rejected by the program as being beyond good design practice; the search for a solution at a greater number of circuits resulted in 22 as the only number which would meet the requirements that $N_t/N_c$ be an integer.

The last case illustrates a situation where designer selection of the number of circuits may result in an improved design. If, for example, space limitations dictate a maximum of 22 tubes in the face the designer may wish to override the program by specifying the number of circuits at some value between 11 and 22. While this may present a more difficult circuit balancing problem, it could result in a more economical design.

An important constraint which frequently is imposed is the cfm/ton. As presently written the program starts with a value of cfm/ton dependent upon the refrigerant temperature and the entering air enthalpy. The final value will show variations from this initial selection, depending on the ratio of the design load to the first trial value of the calculated load. To arrive at or near a specific cfm/ton, therefore, some trial and error computations may be necessary involving face velocity, fins per inch, number of tube rows or face area. Here again the designer's experience and judgment plays an important role in arriving at a good final choice.

From the foregoing it is clear that the program described here is a powerful design tool. It has made practical the application of improved fundamental relationships, and more accurate design procedures. As our understanding of the fluid flow and heat transfer processes is increased through new research results, modifications can be made to reflect that improved knowledge. In this way the program plays an important role in accelerating the process of bridging the gap between research results and engineering application.

## Acknowledgement

The authors express their gratitude to Mrs. Anna D. Mathill of the Mathematics Section, Research Division, Carrier Corporation, for her major contributions to the development of this program.

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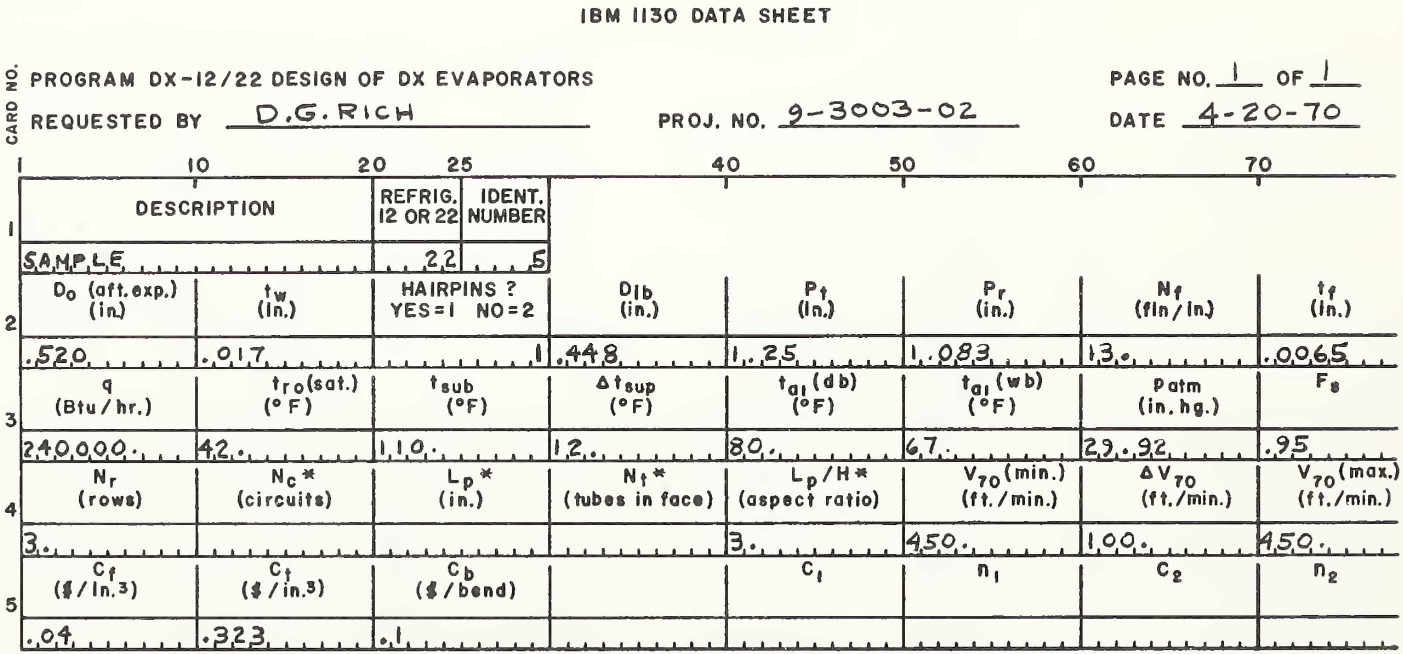


IBM 1130 DATA SHEET

PROGRAM DX-12/22 DESIGN OF DX EVAPORATORS PAGE NO. 1 OF 1

REQUESTED BY D.G.RICH PROJ. NO. 9-3003-02 DATE 4-20-70

| CARD NO. | 1 | 10 | 20 | 25 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|---|
| 1 | DESCRIPTION | | REFRIG. 12 OR 22 | IDENT. NUMBER | | | | |
| | SAMPLE | | 22 | 5 | | | | |
| 2 | $D_o$ (aft.exp.) (in.) | $t_w$ (in.) | HAIRPINS ? YES=1 NO=2 | $D_{lb}$ (in.) | $P_t$ (in.) | $P_r$ (in.) | $N_f$ (fin/in.) | $t_f$ (in.) |
| | .520 | .017 | 1 | .448 | 1.25 | 1.083 | 13. | .0065 |
| 3 | q (Btu/hr.) | $t_{ro}$(sat.) (°F) | $t_{sub}$ (°F) | $\Delta t_{sup}$ (°F) | $t_{al}$ (db) (°F) | $t_{al}$ (wb) (°F) | $P_{atm}$ (in. hg.) | $F_s$ |
| | 240000. | 42. | 110. | 12. | 80. | 67. | 29.92 | .95 |
| 4 | $N_r$ (rows) | $N_c$* (circuits) | $L_p$* (in.) | $N_t$* (tubes in face) | $L_p/H$* (aspect ratio) | $V_{70}$(min.) (ft./min.) | $\Delta V_{70}$ (ft./min.) | $V_{70}$(max.) (ft./min.) |
| | 3. | | | | 3. | 450. | 100. | 450. |
| 5 | $C_f$ (\$/in.$^3$) | $C_t$ (\$/in.$^3$) | $C_b$ (\$/bend) | | $C_1$ | $n_1$ | $C_2$ | $n_2$ |
| | .04 | .323 | .1 | | | | | |

A BLANK CARD MUST FOLLOW LAST DATA SET

Figure 1 Typical Input Data Sheet

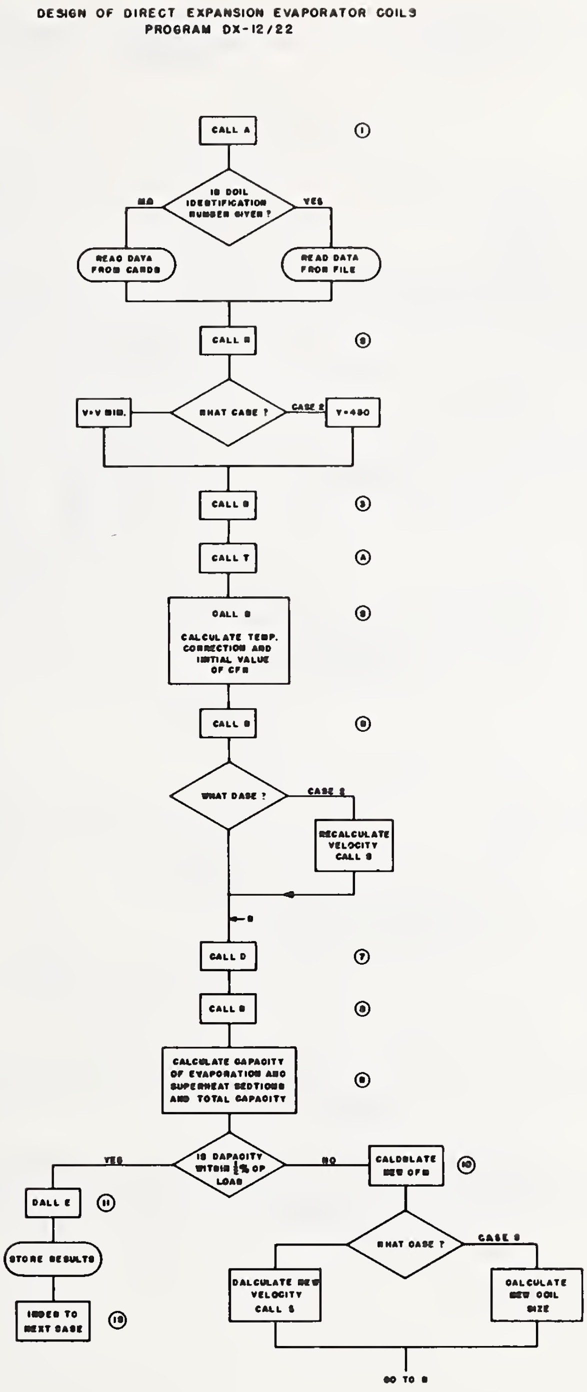


Figure 2 Main Program Flow Diagram

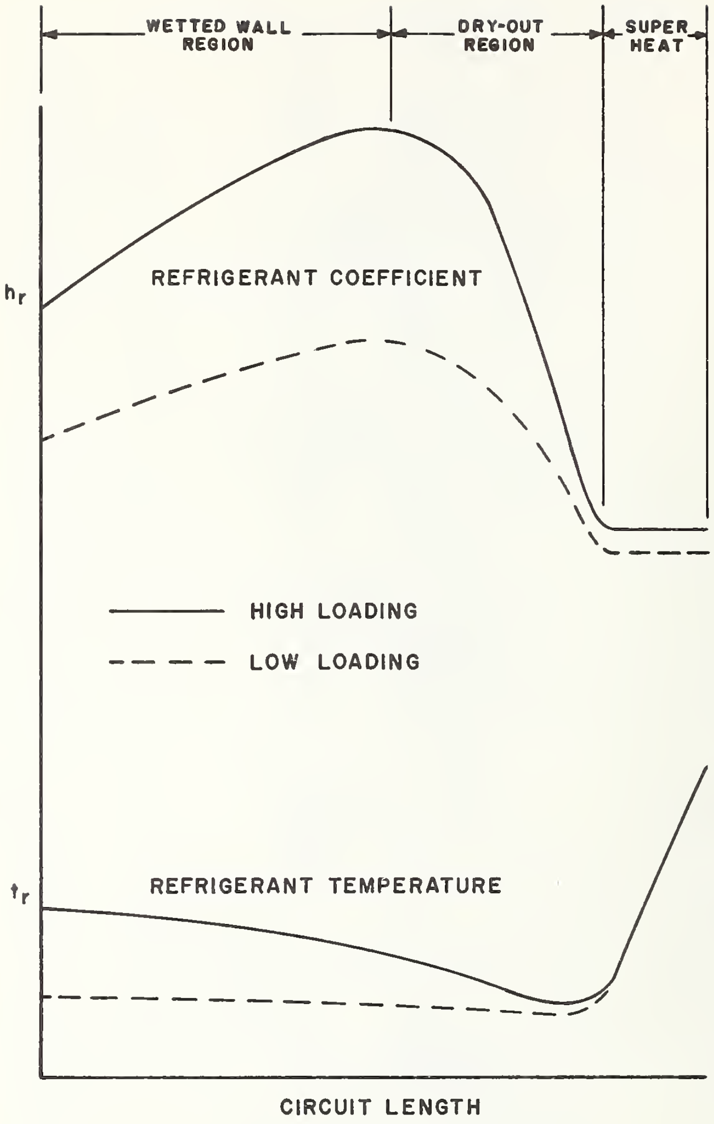


Figure 3 Heat Transfer Coefficient and Temperature Versus Circuit Length for an Evaporating Refrigerant in a Tube

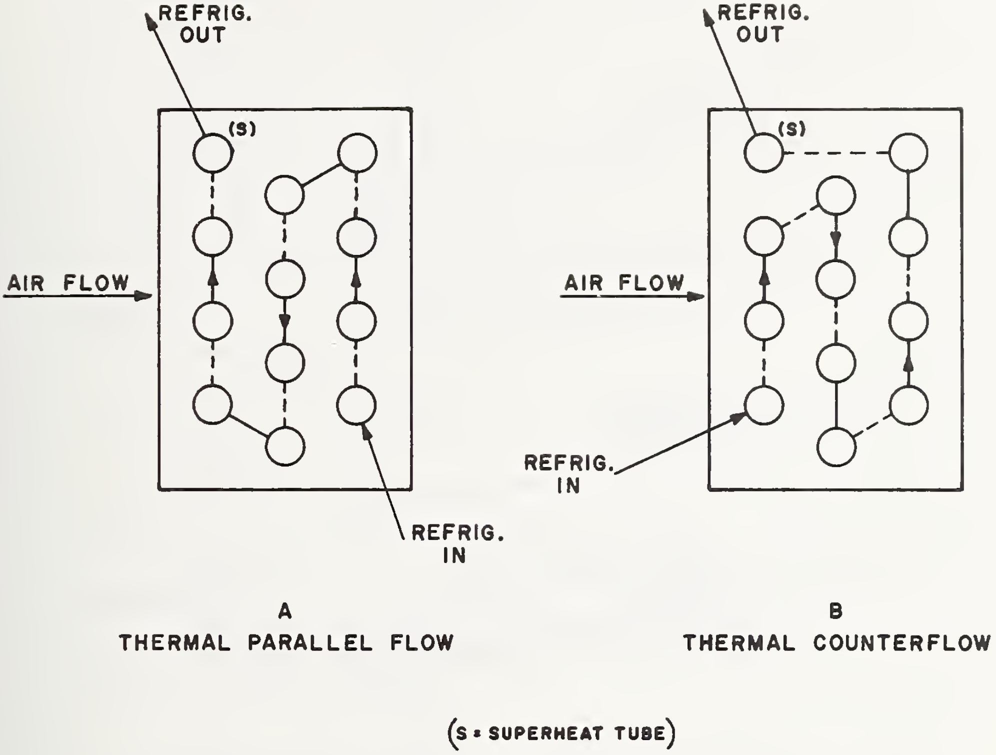


Figure 4 Two Evaporator Circuiting Arrangements

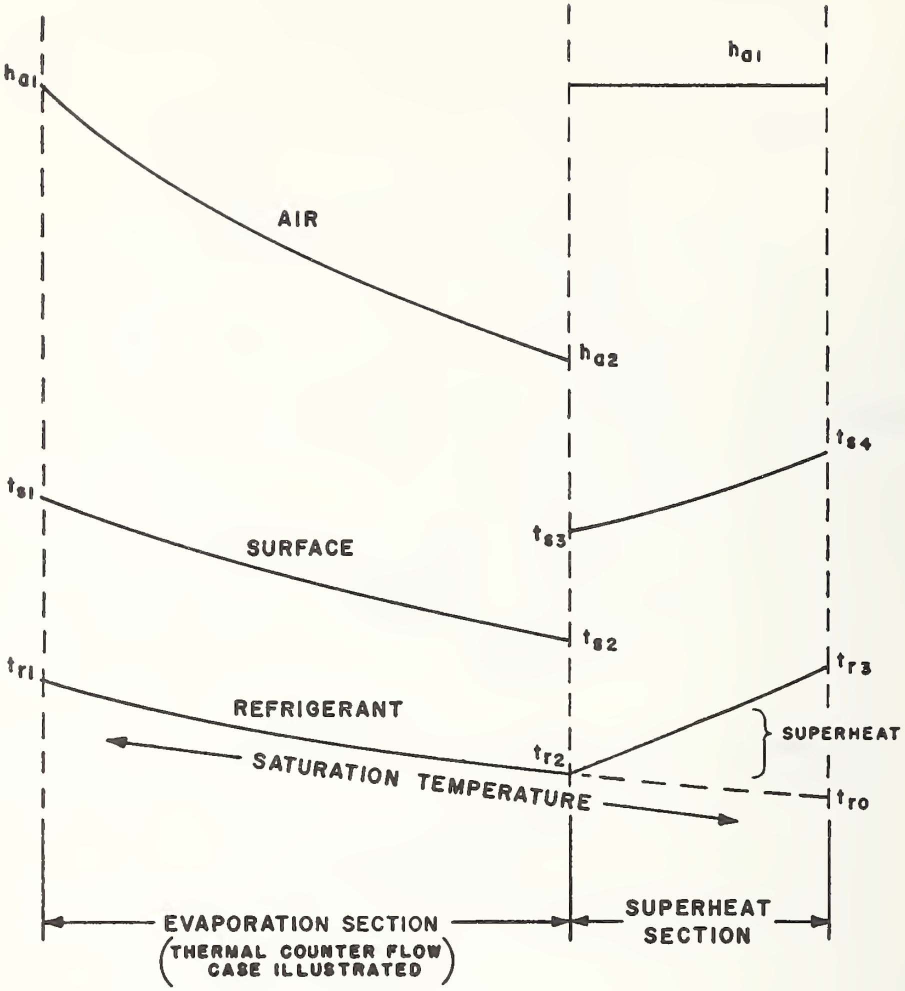
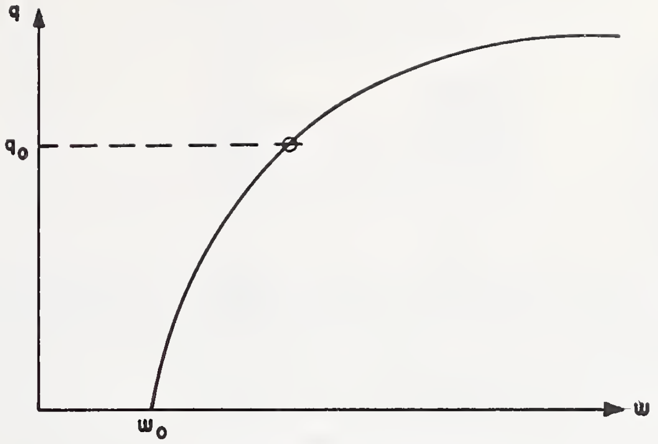
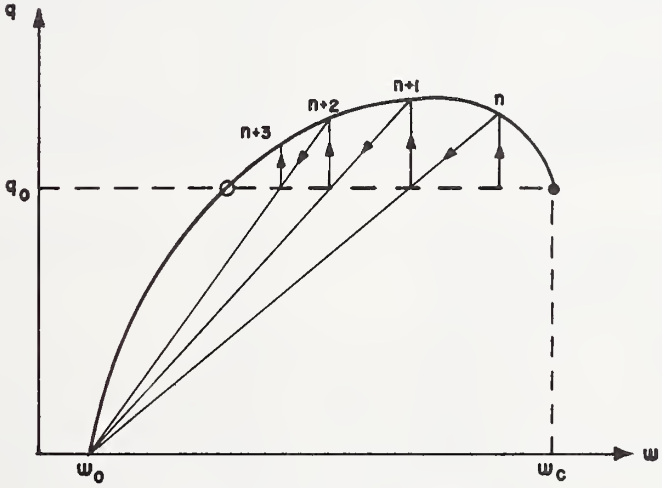


Figure 5 Thermal Diagram for an Evaporator Circuit



A FACE AREA CONSTANT



B FACE VELOCITY CONSTANT

Figure 6 Calculated Capacity Versus Assumed Air Flow Rate

```
                              COOLING AND DEHUMIDIFYING COIL DESIGN DX-12/22

DESCRIPTION                                        SAMPLE           SAMPLE           SAMPLE

CURVE NUMBER                                            5                5                5
REFRIGERANT                                            22               22               22
FINS PER INCH                                        13.0             13.0             13.0
NUMBER OF ROWS                                          3                3                3
TUBE FACE SPACING           INCHES                  1.250            1.250            1.250
TUBE ROW SPACING            INCHES                  1.083            1.083            1.083
TUBE O.D.                   INCHES                   .520             .520             .520
TUBE WALL THICKNESS         INCHES                   .017             .017             .017
FIN THICKNESS               INCHES                  .0065            .0065            .0065
SURFACE EFFICIENCY                                    .95              .95              .95

AIR VELOCITY(STD)              FPM                    450              450              450
CFM(STD)                       CFM                   7932             7883             8236
AIR PRESSURE               IN. HG                   29.92            29.92            29.92
NUMBER OF TUBES IN FACE                                24               26               22
NUMBER OF CIRCUITS                                     12               13               22
NUMBER OF RETURN BENDS                                 24               26               22
COIL HEIGHT                 INCHES                  30.00            32.50            27.50
LENGTH BETWEEN TUBE SHEETS  INCHES                  84.61            77.61            95.84
SURFACE RATIO                                        20.3             20.3             20.3
SURFACE/FACE/ROW                                     24.7             24.7             24.7

LOAD                        BTU/HR                 240000           240000           240000

SUCTION TEMPERATURE           DEGF                   42.0             42.0             42.0
COMD. LIQ. TEMPERATURE        DEGF                  110.0            110.0            110.0
SUPERHEAT                     DEGF                  12.00            12.00            12.00
SURF/REFR. MTD (EVAP)         DEGF                  10.23            10.81            13.36
SURF/REFR. MTD (SUPHT)        DEGF                  13.54            13.68            14.63
ENT. DRY BULB TEMPERATURE     DEGF                  80.00            80.00            80.00
ENT. WET BULB TEMPERATURE     DEGF                  67.00            67.00            67.00
LVG. DRY BULB TEMPERATURE     DEGF                  59.21            59.14            59.60
LVG. WET BULB TEMPERATURE     DEGF                  57.51            57.45            57.89
TADP                          DEGF                  55.74            55.66            56.20

AIR TH.RES.(STD) HR-SQFT-DEGF/ BTU                  .0777            .0777            .0777
AIR PRESSURE DROP(ACT)   IN. WATER                   .268             .268             .268
METAL RESISTANCE  HR-SQFT-DEGF/BTU                  .0172            .0172            .0172
REFR.COEFF (EVAP)BTU/HR-SQFT - DEGF                   655              614              451
REFR.COEFF(SUPHT)BTU/HR-SQFT - DEGF                    82               77               50
REFR. MASS VELOCITY    LBS/HR/SQFT                 228311           210749           124533
REFR. TEMP. DROP              DEGF                   6.08             4.87             1.08

FACE AREA                     SQFT                  17.63            17.52            18.30
SUPERHEAT LENGTH              FEET                   5.01             4.86             4.00
SENSIBLE HEAT FACTOR                                 .752             .749             .766
CFM PER TON                                           397              394              412

FIN MATERIAL COST          DOLLARS                  27.87            27.70            28.94
HAIRPIN TUBE COST          DOLLARS                  55.62            55.38            57.61
TOTAL MATERIAL COST        DOLLARS                  85.89            85.68            88.75
```

FIGURE 7    SAMPLE OUTPUT

# Simulation of a Multicylinder Reciprocating Refrigeration System with Chilled Water Coil and Evaporative Condenser

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Newark, New Jersey 07102

A refrigeration system consisting of a packaged chiller, chilled water coil, and evaporative condenser was simulated mathematically in an effort to determine it's performance under varying load conditions. While the simulation was done for the above system only, the method used is sufficiently general to be applied to refrigeration systems with other components. Components were selected from manufacturers catalogues to meet design load conditions for an assumed building whose load is time dependent. The response of the refrigeration system to these changing building loads is found by simulating the performance of each component by a polynomial function of two independent variables and solving for the balance point by matching the polynomials.

The water coil's performance is expressed by finding its effectiveness as a function of air and mass flow rates. The chiller's performance is given in terms of the leaving chilled water temperature and condensing temperature. The evaporative condenser's performance is a function of the condensing temperature and outdoor wet bulb temperature.

Manufacturer's catalogue data were used with a computer program to determine the nine constants needed in the polynomial expressions for each system component and the system balance was then established using a computer program for each load on the system.

Key Words: Simulation, system simulation, refrigeration system simulation, component simulation.

## 1. Introduction

The calculation of energy requirements of a building and the use of a computer to control the starting and stopping of components of the building's air conditioning system require the simulation of the performance of both the air distribution system and the refrigeration system. Methods for calculating building cooling and heating loads via computer have been developed by the sub-committee on cooling and heating loads of ASHRAE's Task Group on Energy Requirements for Heating and Cooling and presented in the publication of this sub-committee [1][2]. Suggested methods of component simulation and forms of the equations to be used were presented in the publication of the Task Group's sub-committee on System Simulation [2]. An example of the simulation of a dual duct system is also given in that publication.

This paper uses the methods suggested for component simulation to simulate the performance of a multicylinder reciprocating compressor water chiller, evaporative condenser and chilled water coil. The output of the calculations give air flow over the cooling coil, coil bypass factor, coil effectiveness as well as power input to the compressor, total heat rejection, condensing temperature as well as percent of the time that the compressor is running.

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[1] Professor of Mechanical Engineering and Graduate student, currently Desalting Engineer, Mekorath Water Co., Israel, respectively.

[2] Figures in brackets indicate the literature references at the end of this paper.

## 2. Cooling Load

### 2.1 Building

It is necessary to have cooling load data in order to select equipment to meet the design load as well as to determine how the equipment will perform under part load conditions. The fictitious building shown in figure 1 was used to calculate the cooling loads. The building was located in New York City and is three stories high with wall, roof and glass areas and construction indicated in the figure.

### 2.2 Cooling Load Calculation

The cooling load was calculated by hand at two hour intervals from 7 A.M. to 7 P.M. for two typical days, July 21 and August 21. The calculations were based upon recommended procedures in the 1967 ASHRAE Handbook of Fundamentals [3]. A typical outdoor temperature variation was taken from figure 15.16 in Threlkeld [4]. The number of people assumed to work in the building is 375, starting between 7 and 9 A.M. and leaving between 5 and 7 P.M. The lighting load was assumed constant at 4.75 watts ft.$^{-2}$. Outdoor design conditions are 94F dry bulb and 77F wet bulb. To allow for vacations 80% occupancy was assumed for the July 21 calculation. Outside air is introduced at the rate of 15 cfm per person and at the outdoor weather conditions.

Results of the load calculations are in Tables 1 and 2. These results were used in selecting the components and were used in determining the necessary operating conditions for each component under part load.

Table 1

Summary of Cooling Load for August 21

| 1. Sensible Load | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
|---|---|---|---|---|---|---|---|
| a. Transmission | | | | | | | |
| Roof | -18,202 | -17,971 | - 2,304 | 21,888 | 61,517 | 76,262 | 71,885 |
| North Wall | - 5,400 | - 4,140 | 360 | 4,860 | 8,640 | 11,520 | 10,440 |
| West Wall | 840 | 2,040 | 4,200 | 6,360 | 8,280 | 9,720 | 11,760 |
| East Wall | - 9,083 | - 6,896 | 841 | 8,746 | 14,968 | 17,491 | 15,809 |
| South Wall | -12,334 | -10,765 | - 9,157 | 2,018 | 10,316 | 17,268 | 19,062 |
| b. Transmitted Solar Radiation & Transmission | | | | | | | |
| East Windows | 28,200 | 62,150 | 59,506 | 40,727 | 47,025 | 70,548 | 56,272 |
| South Windows | 15,635 | 16,592 | 40,743 | 67,864 | 71,682 | 50,330 | 27,709 |
| South Door | 10,992 | 13,552 | 33,440 | 59,904 | 57,320 | 39,136 | 20,304 |
| c. Internal Load | | | | | | | |
| Infiltration | 781 | 540 | 1,404 | 1,836 | 2,052 | 1,944 | 3,384 |
| Lights | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 |
| Fan Motor | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 |
| People | - | 93,750 | 93,750 | 93,750 | 93,750 | 93,750 | - |
| Total Sensible Load | 513,357 | 650,834 | 724,711 | 809,881 | 877,478 | 889,897 | 738,553 |
| 2. Latent Load | | | | | | | |
| Infiltration | 12,481 | 4,889 | 4,017 | 3,533 | 3,340 | 3,436 | 8,515 |
| People | - | 75,000 | 75,000 | 75,000 | 75,000 | 75,000 | - |
| Total Latent Load | 12,481 | 79,889 | 79,017 | 78,533 | 78,340 | 78,436 | 8,515 |
| 3. Ventilation Load | 316,406 | 315,141 | 313,875 | 312,609 | 311,344 | 311,344 | 313,875 |
| 4. Total Cooling Load | 842,244 | 1,045,864 | 1,117,603 | 1,201,023 | 1,267,162 | 1,279,667 | 1,060,943 |

Table 2

Summary of Cooling Load for July 21

| | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
|---|---|---|---|---|---|---|---|
| 1. Sensible Load | | | | | | | |
| a. Transmission | | | | | | | |
| Roof | -28,570 | -28,339 | -14,285 | 8,295 | 46,541 | 61,287 | 57,830 |
| North Wall | -13,500 | -11,700 | - 9,000 | - 5,760 | - 2,500 | -180 | 0 |
| West Wall | - 4,560 | - 3,720 | - 1,680 | -720 | 480 | 1,920 | 4,800 |
| East Wall | -16,651 | -14,464 | - 7,905 | - 1,177 | 4,542 | 6,560 | 6,063 |
| South Wall | -22,426 | -20,855 | -16,819 | -11,213 | - 3,589 | 2,691 | 5,382 |
| b. Solar Radiation and Transmission | | | | | | | |
| East Window | 26,341 | 52,877 | 59,383 | 35,821 | 61,482 | 57,601 | 30,540 |
| South Windows | - 2,074 | 5,599 | 27,139 | 43,359 | 29,939 | 11,897 | 3,525 |
| South Door | -588 | 5,536 | 22,680 | 36,752 | 24,376 | 9,352 | 2,804 |
| c. Internal Load | | | | | | | |
| Infiltration | - 1,458 | -540 | 108 | 324 | 540 | 432 | 208 |
| Lights | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 | 471,400 |
| Fan Motor | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 | 30,528 |
| People | - | 75,000 | 75,000 | 75,000 | 75,000 | 75,000 | - |
| Total Sensible Load | 438,442 | 561,322 | 636,489 | 682,609 | 738,719 | 728,488 | 613,080 |
| 2. Latent Load | | | | | | | |
| Infiltration | 4,386 | 2,081 | 1,307 | 920 | 726 | 823 | 2,606 |
| People | - | 60,000 | 60,000 | 60,000 | 60,000 | 60,000 | - |
| Total Latent Load | 4,386 | 62,081 | 61,307 | 60,920 | 60,726 | 60,828 | 2,606 |
| 3. Ventilation Load | 91,125 | 91,125 | 89,962 | 88,596 | 88,596 | 88,596 | 89,962 |
| 4. Total Cooling Load | 533,953 | 714,528 | 787,758 | 832,125 | 884,041 | 877,912 | 705,648 |

## 3. Component Selection

### 3.1 Cooling Coil

The peak load occurs at 1700 hours (5 P.M. EDST) on August 21. The load is 1,279,667 BTU-hr$^{-1}$ or 106.64 tons of refrigeration. Of this total the space sensible load is 889,367 BTU-hr$^{-1}$ and the space latent load is 78,436 BTU-hr$^{-1}$. To meet this load the water flow through the coil is given by eq (1)

$$Q = WC_p \Delta T \qquad (1)$$

where Q = total load (BTU-hr$^{-1}$), $C_p$ = 1.0 BTU-lb$^{-1}$- $^{o}F^{-1}$

$\Delta T$ = water temperature rise through the coil, $^{o}$F, taken as 10$^{o}$ for the selection made

W = water flow rate (lb-hr$^{-1}$). From eq (1)

$$W = \frac{1,279,667}{10} = 127,967 \text{ lb-hr}^{-1} = 255.9 \text{ gal-min}^{-1}$$

The air flow across the coil (cfm) is given by eq (2)

$$G_1 = \frac{Q_{sens.}}{1.08(T_{ent.} - T_{lvg.})} \qquad (2)$$

$T_{ent.}$ and $T_{lvg.}$ are the dry bulb temperatures before and after the coil

$$G_1 = \frac{889897}{1.08(75-56)} = 43270 \text{ cfm}$$

The coil face area required is $FA = \frac{43270}{500} = 86 \text{ ft}^2$ where a face velocity of 500 $\text{ft-min}^{-1}$ is used. 12 coils with a length of 36" and height of 30" and a total face area of 90 $\text{ft}^2$ and face velocity of 480 $\text{ft-min}^{-1}$ were selected. From a manufacturer's catalogue to meet the required $\frac{106.6}{90} = 1.18 \text{ tons-ft}^2$ a set of 12, 6 row, 80 fins per inch coils 36" x 30" were selected.

### 3.2 Chiller

With the required tonnage of 106.64, leaving water temperature 45°F, entering water temperature of 55°F, two chillers were selected as possible alternatives. The simulation was run with both possibilities to show how the simulation is helpful in selecting the optimum chiller to meet the building load characteristics. The final selection is discussed under results. The two chillers selected had as full load characteristics,for chiller A; saturated discharge temperature 103.7°F, compressor power input 93.4 KW, total heat rejection 133.4 tons; while for chiller B the corresponding values are 114.3°F, 110.2 KW and 138 tons.

Each compressor is equipped with cylinder unloaders to operate at 100%, 75%, 50% and 25% of full load capacity.

### 3.3 Evaporative Condenser

The evaporative condenser chosen with a saturation temperature of 103.7°F, outdoor wet bulb of 77°F would have a capacity of 133.2 tons with 20° sub-cooling. This was chosen to meet the conditions of chiller A. This condenser has a capacity greater than the required 138 ton heat rejection needed for chiller B at it's higher condensing temperature of 114.3°F.

## 4. Component Simulation

Each component's performance is to be expressed as a function of two independent variables [2] in the form:

$$Z = a_1 + a_2x + a_3x^2 + a_4y + a_5y^2$$

$$+ a_6xy + a_7x^2y + a_8xy^2 + a_9x^2y^2 \qquad (3)$$

The coefficients $a_1$ through $a_9$ should be chosen so as to predict the performance of the individual component over a wide range of load conditions. It is hoped that the manufacturers will present the performance of their equipment in the form of eq (3). Currently this data is not generally available and the coefficients used in simulating each component in this paper were computed from tabulated manufacturer's values (which are generally at or near full load). The equations so derived were used to predict the part load performance. Clearly the results will be more reliable when more reliable part load data becomes available.

### 4.1 Simulation of Chilled Water Coil

In accordance with the suggestions of Stoecker [2], the coil's performance should be given by the effectiveness expressed as a function of the air flow (G) and water flow (W) rates.

The effectiveness is defined as

$$E = \frac{h_{in} - h_{out}}{h_{in} - h_w} = \frac{\Delta h}{\Delta h_w} \qquad (4)$$

where $h_{in}$ and $h_{out}$ are the enthalpies of the air before and after the coil and $h_w$ is the enthalpy of saturated air at the entering water temperature, so that $\Delta h_w$ is an enthalpy potential.

The maximum possible effectiveness $E_{max} = h_{in} - \bar{h}_w / h_{in} - h_w$ where $\bar{h}_w$ = enthalpy of saturated air at the average water temperature.
The effectiveness can be expressed in terms of G and W because

$$Q = G\Delta h = GE\Delta h_w \tag{5}$$

$$\text{and} \quad Q = W\ (T_{in} - T_{out}) = W\Delta T \tag{6}$$

where $\Delta T$, the cooling range is the water temperature difference across the coil.

In terms of the coil face velocity and face area

$$G = \rho VA \tag{7}$$

For this example the coil was used as a "wild" coil, i.e. the temperature of the water entering the coil and the mass flow of water across the coil is constant. Since the load on the coil is the building load, and the outdoor conditions are known, and the water inlet temperature is constant, $h_{in}$ is known and $h_w$ is constant; the coil effectiveness can be computed from manufacturer's catalogue data. Therefore, for a given coil with part load data given, the constants in:

$$\begin{aligned} E = {} & a_1 + a_2 G + a_3 G^2 + a_4 W + a_5 W^2 \\ & + a_6 GW + a_7 G^2 W + a_8 GW^2 + a_9 G^2 W^2 \end{aligned} \tag{8}$$

can be evaluated by solving nine simultaneous equations using manufacturer's data for G, W and computed values of effectiveness based upon manufacturer's data.

## 4.2 Simulation of the Chiller

From manufacturer's data at full and part loads the chiller capacity, $Q_{ch}$ and power input, $P_{ch}$ may be expressed in terms of $T_{ch}$ and $T_{cd}$, the temperature of water leaving the chiller, and the condensing temperature of the refrigerant.

$$\begin{aligned} Q_{ch} = {} & b_1 + b_2\ T_{ch} + b_3\ T_{ch}^{2} + b_4\ T_{cd} + b_5\ T_{cd}^{2} \\ & + b_6\ T_{ch}\ T_{cd} + b_7\ T_{ch}^{2}\ T_{cd} + b_8\ T_{ch}\ T_{cd}^{2} + b_9\ T_{ch}^{2}\ T_{cd}^{2} \end{aligned} \tag{9}$$

$$\begin{aligned} P_{ch} = {} & c_1 + c_2\ T_{ch} + c_3\ T_{ch}^{2} + c_4\ T_{cd} + c_5\ T_{cd}^{2} + c_6\ T_{ch}\ T_{cd} \\ & + c_7\ T_{ch}^{2}\ T_{cd} + c_8\ T_{ch}\ T_{cd}^{2} + c_9\ T_{ch}^{2}\ T_{cd}^{2} \end{aligned} \tag{10}$$

Again, standard programs for solution of simultaneous equations evaluate the constants if nine data points are given.

The performance of the chiller is complicated by the fact that the compressor is equipped with unloaders to lower the power requirements in steps to 75%, 50% and 25% of full load requirements. However, to deliver the exact cooling load the compressor must cycle and operate only part of the time.

The starting current is higher than normal running current so that the effect of cycling is to increase equivalent power consumption; e.g. when the compressor operates say 80% of the time, the power requirement is greater than 80% of the full time operating power. Manufacturers suggest that the ratio of part time power required to full time power (RP) is expressible in terms of the percent time operating (RL) by

$$RP = R + .8\ RL \tag{11}$$

so that the heat rejected in the evaporative condenser is given by

$$Q_{REJ} = Q + RP\ (P_{ch}) \tag{12}$$

### 4.3 Simulation of the Evaporative Condenser

The evaporative condenser heat rejection is given by:

$$\begin{aligned} Q_{REJ} &= d_1 + d_2(WBT) + d_3(WBT)^2 + d_4(T_{cD}) \\ &+ d_5\ T_{cd}^{\ 2} + d_6\ (WBT)\ T_{cD} + d_7\ (WBT)^2\ T_{cD} \\ &+ d_8\ (WBT)\ T_{cd}^{\ 2} + d_9\ (WBT)^2\ (T_{cd})^2 \end{aligned} \tag{13}$$

It is seen from eq.'s 10 through 13 that the power requirement of the chiller and heat rejected by the condenser are functions of the condensing temperature, so that $T_{cD}$ will be determined by the relative capacity of both components.

## 5. System Simulation

### 5.1 Control System

The coil was selected to operate without a control valve so that the water flow rate and entering water temperature are constant. To control the output some of the air is bypassed around the coil and then at low loads the amount of coil surface exposed to the air is decreased.

The blocking of part of the coil surface is necessary because at low air flow rates the leaving air temperature becomes uncomfortably low.

In the mathematical model the requirement of blocking part of the coil may be sensed by values of E that are too high, approaching $E_{max}$ (the DBT of the leaving air approaches the average water temperature).

### 5.2 Mathematical Model

At a given load eq.'s (8) and (5) must be satisfied simultaneously. Since W is constant, eq. (8) may be rewritten as:

$$E = B' + F'G + D'\ G^2 \tag{14}$$

where

$$B' = a_1 + a_4W + a_5W^2$$

$$F' = a_2 + a_6W + a_8W^2$$

$$D' = a_3 + a_7W + a_9W^2$$

Using the identity $G = \frac{GE}{E}$ in eq (14) we get:

$$E^3 - B'E^2 - (GE)F'E - (GE^2)D' = 0$$

This equation is solved for E with constant G such that

$$0 < E < E_{max}$$

If no such root exists, G is reduced (part of the coil is blocked) until such a root is found.

The operation of the packaged chiller and evaporative condenser is dependent upon the cooling load and the outdoor wet bulb temperature only. If the chiller leaving water temperature is fixed, the condensing temperature $T_{cD}$ is the only variable.

The capacity of the chiller is: $Q_{chil} = f(T_{ch}, T_{cd})$, if $T_{ch}$ is fixed at $45^o$. Based on the actual load Q the number of cylinders required is found.

For example if:

$$RL = Q/Q_{chill} = 0.6$$

so $.50 < RL < .75$, 75% of the cylinders are required and the percent operating time is:

$$RLI = \frac{.60}{.75} (100) = 80\%$$

The power ratio is then $RP = .2 + .8RLI = .84$ and the actual power required

$$PWI = P_{ch} \text{ x } .75 \text{ x } .84 = .63\ P_{ch}$$

$$P_{ch} = f\ (T_{cw}, T_{cD}) \text{ and } Q_{REJ} = Q + PWI$$

The capacity of the evaporative condenser is evaluated at $T_{cD}$

$Q_{evcon} = f(WBT, T_{cD})$ and the two values are compared.

If $Q_{evcon} > Q_{rej}$ then $T_{cD}$ is decreased and if $Q_{evcon} < Q_{rej}$, $T_{cD}$ is increased.

A flow chart of the computer program is shown in figure 3.

## 6. Results

Typical results are shown in Tables 3 and 4 for the full load case on August 21 and 5 P.M. The two alternate chillers chosen result in an input of 94.3KW with all cylinders in operation 100% of the time, and 96.4KW with all the cylinders in operation 91.9% of the time. The result for the full load period are typical throughout. One chiller chosen generally has a smaller power input and cycles less often. Thus, the simulation is helpful in equipment selection as well as in energy computation.

An exception to the first alternate giving better results is shown in Tables 5 and 6 where the outputs at 9 A.M. July 21 are presented. Here alternate 2 has a lower power input and will cycle less. The importance of examining the entire range of outputs is evident.

Table 3

| | |
|---|---|
| Date - August 21 | |
| Time - 17.00 | |
| Cooling Load, BTU/HR | 1279667. |
| Outdoor Dry Bulb Temp.,F | 93.0 |
| Outdoor Wet Bulb Temp.,F | 77.0 |
| Water Coil Data | |
| Percent of Face Area Used | 100.0 |
| Water Temperature (Entering Cooling Coil),F | 45.0 |
| Water Flow In Cooling Coil,GPM | 255.9 |
| Air Temp. (Entering Cooling Coil),F | 77.0 |
| Air Flow Over Cooling Coil,CFM | 42076. |
| Coil Effectiveness Is | 0.571 |
| Bypass Factor Is, Percent | 15.8 |
| Water Chiller Data | |
| Water Temp. Living Chiller, F | 45.0 |
| Condensing Temp. of Chiller,F | 103.7 |
| 100 Percent of Cylinders are in Operation Operating 100.0 Percent of the Time | |
| Actual Power Requirements,KW | 94.3 |
| Total Heat Rejection,Tons | 133.4 |

Table 4

| | |
|---|---|
| Date - August 21 | |
| Time - 17.00 | |
| Cooling Load, BTU/Hr | 1279667. |
| Outdoor Dry Bulb Temp.,F | 93.0 |
| Outdoor Wet Bulb Temp.,F | 77.0 |
| Water Coil Data | |
| Percent of Face Area Used | 100.0 |
| Water Temperature (Entering Cooling Coil),F | 45.0 |
| Water Flow in Cooling Coil,GPM | 255.9 |
| Air Temp. (Entering Cooling Coil),F | 77.0 |
| Air Flow Over Cooling Coil,CFM | 42076. |
| Coil Effectiveness Is | 0.571 |
| Bypass Factor Is, Percent | 15.8 |
| Water Chiller Data | |
| Water Temp. Living Chiller, F | 45.0 |
| Condensing Temp. of Chiller, F | 103.7 |
| 100 Percent of Cylinders are in Operation Operating 91.9 Percent of the Time | |
| Actual Power Requirements,KW | 96.4 |
| Total Heat Rejection,Tons | 133.4 |

Table 5

| | |
|---|---|
| Date - July 21 | |
| Time - 9.00 | |
| Cooling Load, BTU/HR | 714528. |
| Outdoor Dry Bulb Temp.,F | 70.0 |
| Outdoor Wet Bulb Temp.,F | 67.0 |
| Water Coil Data | |
| Percent of Face Area Used | 75.0 |
| Water Temperature (Entering Cooling Coil),F | 45.0 |
| Water Flow in Cooling Coil,GPM | 255.9 |
| Air Temp. (Entering Cooling Coil),F | 74.4 |
| Air Flow Over Cooling Coil,CFM | 20207. |
| Coil Effectiveness Is | 0.724 |
| Bypass Factor Is, Percent | 59.6 |
| Water Chiller Data | |
| Water Temp. Living Chiller, F | 45.0 |
| Condensing Temp. of Chiller, F | 91.9 |
| 75 Percent of Cylinders are in Operation Operating 68.1 Percent of the Time | |
| Actual Power Requirements,KW | 48.9 |
| Total Heat Rejection,Tons | 73.4 |

Table 6

| | |
|---|---|
| Date - July 21 | |
| Time - 9.00 | |
| Cooling Load, BTU/HR | 714528. |
| Outdoor Dry Bulb Temp.,F | 70.0 |
| Outdoor Wet Bulb Temp.,F | 67.0 |
| Water Coil Data | |
| Percent of Face Area Used | 75.0 |
| Water Temperature (Entering Cooling Coil),F | 45.0 |
| Water Flow in Cooling Coil,GPM | 255.9 |
| Air Temp. (Entering Cooling Coil),F | 74.4 |
| Air Flow Over Cooling Coil,CFM | 20207. |
| Coil Effectiveness Is | 0.724 |
| Bypass Factor Is, Percent | 59.6 |
| Water Chiller Data | |
| Water Temp. Living Chiller, F | 45.0 |
| Condensing Temp. of Chiller, F | 91.7 |
| 50 Percent of Cylinders are in Operation Operating 94.1 Percent of the Time | |
| Actual Power Requirements,KW | 45.4 |
| Total Heat Rejection,Tons | 71.9 |

## 7. References

[1] Proposed Procedure for Determining Heating and Cooling Loads for Energy Calculations. Task Group on Energy Requirements for Heating and Cooling - ASHRAE, edited by M. Lokmanhekim.

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[3] Handbook of Fundamentals, 1967, ASHRAE.

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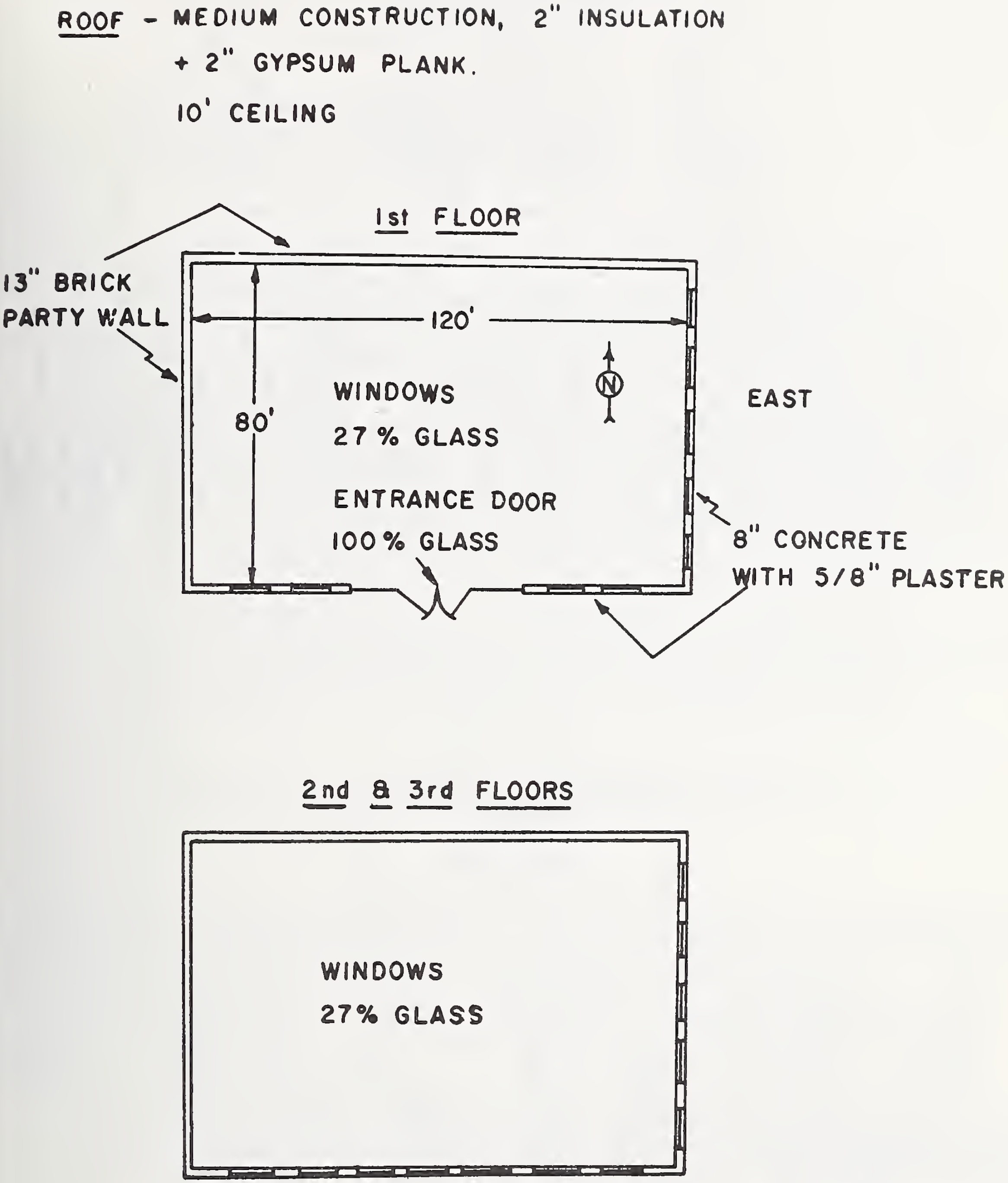

Figure 1. Building Orientation and Structure

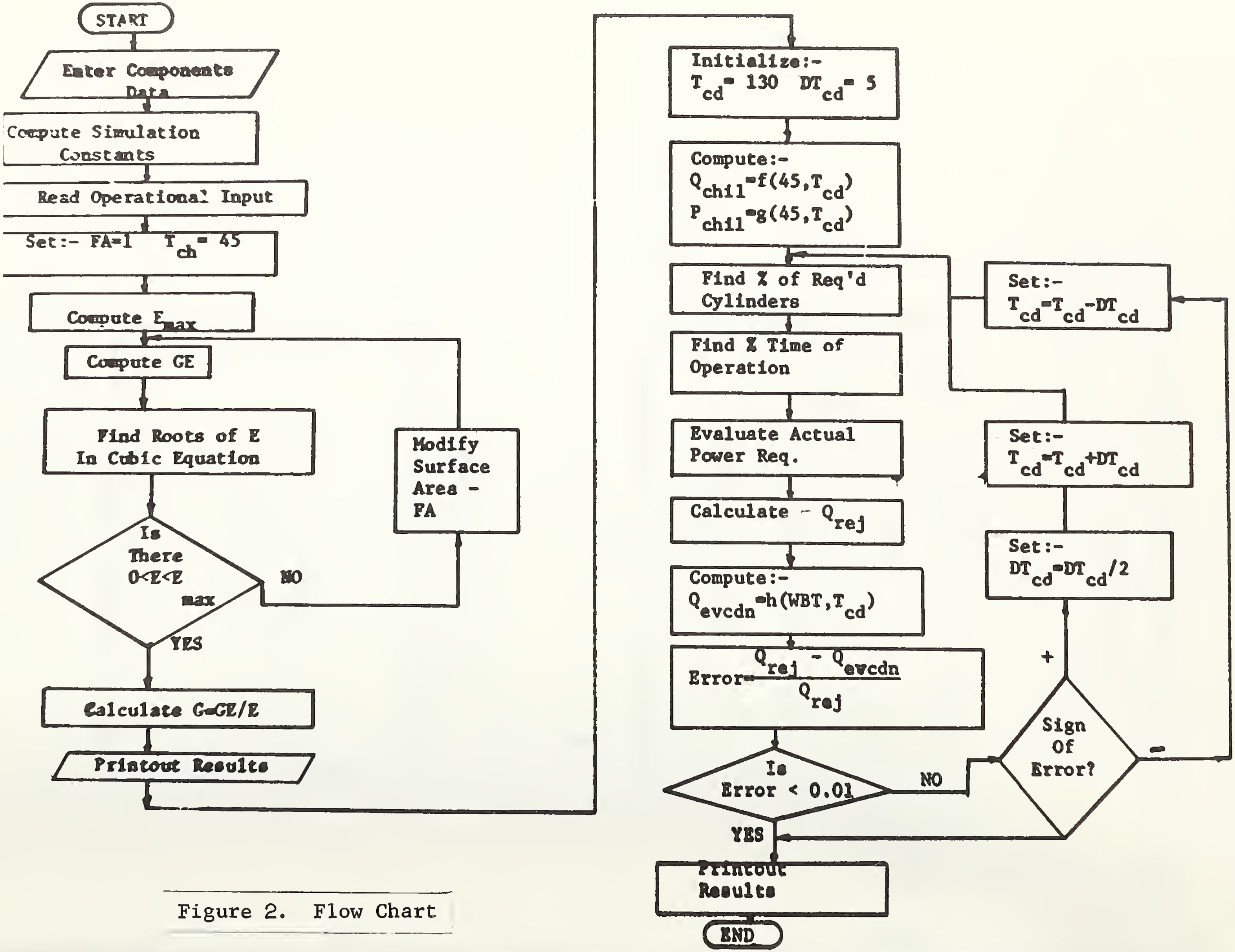

Figure 2. Flow Chart

# Use of Digital Computers For the Heat and Mass Transfer Analyses of Controlled Environment Greenhouses

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Heat and mass balance equations for the controlled-environment Greenhouses yielded simultaneous non-linear differential equations that were modified to finite difference equations for computer solution. These equations were linearized before attempts were made to solve them, using a CDC 6400 Computer. The unsteady state analysis was carried out to determine the variation of temperatures due to storage effects, especially that of soil. The plastic cover, plant and air temperatures responded to any change of radiation or external disturbance fast enough to justify a quasi-steady state analysis in which the radiation level was modified step-by-step in hourly intervals. Various programs were written to solve the problem at different phases; namely, the analysis without any moisture effects, studies with transpiration taken into consideration and, ultimately, the model with evaporation from the soil, transpiration from the plants and condensation over the plastic cover whenever its temperature drops below dew point of the air stream. Since the final phase covers all phenomena involved, only its formulation and computer programming are emphasized. The computer programs developed may be used in predicting the controlled-environment greenhouse performance under continuous operation conditions for any locality, if weather data are supplied. This information also will enable the designer to determine the capacity of the packed bed humidifiers, circulation fans and water requirements of humidifiers and irrigation purposes.

Key Words: Controlled environments, greenhouse energy budget program, greenhouse heat and mass balances.

## 1. Introduction

The controlled-environment greenhouse is the vital component of the POWER-WATER-FOOD complex developed by the University of Arizona. The principle of operation and features of a large-scale scheme which is being applied in Abu Dhabi on the Arabian Gulf already has been reported. [1], [2], [3][2]

The present paper outlines the computation methods and details of the computer programming of an extensive study of the heat and mass balances of the controlled environment greenhouses [4].

[1] Author is an Assistant Professor at the Mechanical Engineering Department Middle East Technical University of Ankara, Turkey. This paper is based upon a research carried out while he was on leave at the Environmental Research Laboratory of the University of Arizona.

[2] Numerals in brackets refer to references at the end of this paper.

## 2. Mathematical Models

The formulation of the heat and mass transfer in the inflated plastic controlled-environment greenhouse was carried out at a number of stages, from simple to elaborate. The purpose has been to obtain approximate values of unknowns for use in the iterative process to compute temperatures and moistures accurately.

Fig. 1) is a sketch of the greenhouse, its dimensions and definition of unknowns investigated as well as initial and boundary conditions.

The ambient db and wb air temperatures, outside wind velocity, inside air velocity, solar radiation intensity and initial db and wb air temperatures are specified as input data for the computer program. The present analyses enable one to compute temperatures of the plastic cover, air stream plant leaf surface temperatures, soil temperature at the surface and various levels below the surface, at selected space intervals for a 24-hour operation period.

The following steps were taken in formulating the problem:

(a) The analysis of the system neglecting the moisture effects, steady state,

(b) The analysis of the system with plant transpiration only, steady state,

(c) The analysis of the system neglecting the moisture effects, unsteady state,

(d) The analysis of the system with plant transpiration only unsteady state,

(e) The analysis of the system with the effects of soil moisture evaporation, plant transpiration and condensation on the cover, unsteady state,

(f) An analysis to predict the greenhouse performance for year-round operation based upon the formulation (e).

Since the detailed description of these formulations would be too lengthy, only the study which is covered in formulation (e) will be outlined.

It should be noted that formulations (a) through (d) are special cases of formulation (e) in which some of the parameters have been neglected and analysis is applied to the steady state operation.

### 2.1 The Analysis of the System with Soil Water Evaporation, Plant Transpiration and Condensation on the Cover

The model employed in this analysis is outlined in Fig. 2). The computer program is labeled KUD 81.

#### 2.1.1 Heat Balance on the Cover

Neglecting the heat capacity of plastic cover and employing quasi-state analysis:

$$hc_i P\Delta x(T_l - \theta_c) + K_c P\Delta x h_{fg}(\omega_a - \omega_c) + hr_{p,c}\varepsilon_{p,Lw}\varepsilon_{c,Lw}A_{pr}(\theta_p - \theta_c)F_{p,c}$$

$$+ I\alpha_{c,sw}P\Delta x + hr_{s,c}\varepsilon_{s,Lw}\varepsilon_{c,Lw}F_{s,c}P\Delta x(\theta_s - \theta_c) = hc_w P\Delta x(\theta_c - T_a)$$

$$+ hr_{c,sk}P\Delta x(\theta_c - T_{sky})\varepsilon_{c,Lw} \qquad (1)$$

where $K_c$ is the condensation rate in (lb $hr^{-1}ft^{-2}$)

Taking $F_{p,c}$, $F_{s,c} = 1$, and from $F \cdot \varepsilon = \mathcal{F}$.

Rearranging for the unknowns of $\theta_c, T_l, \theta_p$ and $\theta_s$ and dividing by $\Delta x$:

$$[-hc_i P - hr_{p,c}\mathcal{F}_{p,c}A_{pr}/\Delta x - hr_{s,c}\mathcal{F}_{s,c}P - hc_w P - hr_{c,sk}p\varepsilon_{c,Lw}]\theta_c$$

$$+ [hc_i P]T_l + [hr_{p,c}\mathcal{F}_{p,c}A_{pr}/\Delta x]\theta_p + [hr_{s,c}\mathcal{F}_{s,c}P]\theta_s = - K_c P h_{fg}(\omega_a - \omega_c)$$

$$- I\alpha_{c,sw}P - hc_w PT_a - hr_{c,sk}P\varepsilon_{c,Lw}T_{sky} \qquad (2)$$

Mass Balance of Airstream, enables one to obtain $\omega_a$ in terms of $\omega_s$, $\omega_p$, $\omega_c$ and the initial humidity ratio of the airstream ($\omega_{a,in}$).

### 2.1.2 Mass Balance on Airstream

Change of water vapor content of the airstream is:

$$m_a(\omega_a - \omega_{a,in}) = K_s(\omega_s - \omega_a)A_g + K_pA_p(\omega_p - \omega_a) - K_c(\omega_a - \omega_c)P\Delta x \tag{3}$$

The transpiration rate is proportional to:

$$K_p = \frac{1}{\frac{1}{(hc_p/C_{pm}} + R_{stom}} \tag{4}$$

and $R_{stom}$ is stomatal resistance in (hr ft$^2$ lb$^{-1}$)

Rearranging (3) and solving for ($\omega_a$):

$$\omega_a = (m_a\omega_{a,in} + K_s\omega_sA_g + K_pA_p\omega_p + K_c\omega_cP\Delta x)/(m_a + K_sA_g + K_pA_p + K_cP\Delta x) \tag{5}$$

from the Psychrometric Relations, assuming saturation conditions at the cover, leaf and soil surfaces:

$$\omega_c = [0.622\ P_{v(\theta c)}]/[P_m - P_{v(\theta c)}] \tag{6}$$

$$\omega_p = [0.622\ P_{v(\theta p)}]/[P_m - P_{v(\theta p)}] \tag{7}$$

$$\omega_s = [0.622\ P_{v(\theta_s)}]/[P_m - P_{v(\theta s)}] \tag{8}$$

Equations (5), (6), (7) and (8), together with 4 heat balance equations corresponding to cover, airstream, plant and soil are 8 non-linear simultaneous equations which should allow one to solve the unknowns $\omega_p$, $\omega_c$, $\omega_s$, $\omega_a$, $\theta_c$, $T_1$, $\theta_p$ and $\theta_s$. To ease the solution, $\omega_c$, $\omega_p$ and $\omega_s$ will be expressed as linear functions of temperature in the form of $\omega = X\theta + Y$. Rather than using relations (6), (7) and (8), the constants X and Y are to be individually determined within the temperature region of interest.

For the cover at saturation conditions: $\omega_c = X_c\theta_c + Y_c$ (9)

For the plant at saturation conditions: $\omega_p = X_p\theta_p + Y_p$ (10)

For the soil at saturation conditions: $\omega_s = X_s\theta_s + Y_s$ (11)

$\theta_c$, $\theta_p$ and $\theta_s$ were approximately determined employing the analysis with plant transpiration only. Details of this analysis are described in another publication by the author [5]. The coefficients $X_c$, $X_p$, $X_s$, $Y_c$, $Y_p$, and $Y_s$ can be closely estimated from the saturation curve for air-water vapor mixture once $\theta_c$, $\theta_p$ and $\theta_s$ are known.

Since the mass flow rate is:

$$m_a = \rho_m vA_s \ : \ \text{lb/hr}$$

The mass balance equation (5) for the airstream then could be written in terms of temperatures alone:

$$\omega_a = [(\rho_m vA_s/\Delta x)\omega_{a,in} + (K_sA_g/\Delta x)\ (X_s\theta_s + Y_s) + (K_pA_p/\Delta x)\ (X_p\theta_p + Y_p) + K_cP(X_c\theta_c + Y_c)]/[\rho_m vA_s/\Delta x + K_sA_g/\Delta x + K_pA_p/\Delta x + K_cP] \tag{12}$$

separating $\theta_c$, $\theta_p$ and $\theta_s$,

$$\omega_a = \frac{1}{(\rho_m vA_s/\Delta x + K_s A_g/\Delta x + K_p A_p/\Delta x + K_c P)} [X_c K_c P\theta_c + (X_p K_p A_p/\Delta x)\theta_p + (X_s K_x A_g/\Delta x)\theta_s + (\rho_m vA_s/\Delta x)\,\omega_{a,in} + Y_c K_c P + Y_p K_p A_p/\Delta x + Y_s K_s A_g/\Delta X] \qquad (13)$$

Substituting $\omega_a$ from equation (13) and $\omega_c$ from equation (9), into equation (2):

$$[-hc_i P - hr_{p,c}\mathcal{F}_{p,c}A_{pr}/\Delta x - hr_{s,c}\mathcal{F}_{s,c}P - hc_w P - hr_{c,sk}P\varepsilon_{c,lw}]\theta_c + [hc_i P]T_i + [hr_{p,c}\mathcal{F}_{p,c}A_{pr}/\Delta x]\theta_p + [hr_{s,c}\mathcal{F}_{s,c}P]\theta_s = -K_c Ph_{fg}[D\{X_c K_c P\,\theta_c + (X_p K_p A_p/\Delta x)\theta_p + (X_s K_s A_g/\Delta x)\theta_s + (\rho_m vA_s/\Delta x)\omega_{a,in} + Y_c K_c P + Y_p K_p A_p/\Delta x + Y_s K_s A_g/\Delta x\} - X_c\theta_c - Y_c] - I\alpha_{c,sw}P - hc_w PT_a - hr_{c,sk}P\varepsilon_{c,Lw}T_{sky} \qquad (14)$$

Where $D = \dfrac{1}{\rho_m vA_s/\Delta x + K_s A_g/\Delta x + K_p A_p/\Delta x + K_c P}$

### 2.1.3 Heat Balance on Airstream

For a system of $\Delta x$ length:

$$m_a C_{pm}(T_1 - T_{1,in}) = hc_s A_g(\theta_s - T_1) + hc_p A_p(\theta_p - T_1) - hc_i P\Delta x(T_1 - \theta_c)$$

For unit length:

$$(\rho_m v/\Delta x)A_s C_{pm}(T_1 - T_{1,in}) = hc_s A_g/\Delta x(\theta_s - T_1) + hc_p(A_p/\Delta x)\ (\theta_p - T_1) - hc_i P(T_1 - \theta_c) \qquad (15)$$

$(A_p/\Delta x)$: The effective plant surface per unit length of the greenhouse has to be worked out separately for the plant under study and at various stages of growth.

### 2.1.4 Heat Balance Over the Plant Canopy

The plant canopy does not have a definite geometrical shape, nor uniform heat and mass transfer coefficients. The velocity field around and across the canopy is not uniform. Therefore, those coefficients, areas, temperatures and velocities have been bulked in this analysis. For the unit length of greenhouse, $\Delta x = 1$.

$$I_D\tau_{c,sw,D}\alpha_{p,sw,D}A_{D,p} + I_d\tau_{c,sw,d}\alpha_{p,sw,d}A_{d,p} = hc_p A_p(\theta_p - T_1) + h_{fg}K_p A_p(\omega_p - \omega_a) + hr_{s,p}\mathcal{F}_{s,p}A_p(\theta_s - \theta_p) + hr_{p,c}\mathcal{F}_{p,c}A_{pr}(\theta_p - \theta_c) + hr_{p,sk}\tau_{c,Lw}\varepsilon_{p,Lw}A_{pr}(\theta_p - T_{sky}) \qquad (16)$$

where the subscript D refers to Direct, and d refers to diffuse.

$(hc_p A_p)$, $(K_p A_p)$, $(\mathcal{F}_{s,p}A_p)$ and $(\mathcal{F}_{p,c}A_{pr})$ are bulked as indicated above.

Substituting $\omega_p$ from equation (10) and $\omega_a$ from equation (13):

$$
\begin{aligned}
I_D\tau_{c,sw,D}\alpha_{p,sw,D}A_{D,p} + I_d\tau_{c,sw,d}\alpha_{p,sw,d}A_{d,p} &= hc_pA_p(\theta_p - T_1) \\
&+ h_{fg}K_pA_p[X_p\theta_p + Y_p - D\{(X_cK_cP)\theta_c + (X_pK_pA_p/\Delta x)\theta_p + (X_sK_sA_g/\Delta x)\theta_s \\
&+ (\rho_m vA_s/\Delta x)\omega_{a,in} + Y_cK_cP + Y_pK_pA_p/\Delta x + Y_sK_sA_g/\Delta x)] \\
&+ hr_{s,p}\mathcal{F}_{s,p}A_p(\theta_s - \theta_p) + hr_{p,c}\mathcal{F}_{p,c}A_{pr}(\theta_p - \theta_c) \\
&\qquad + hr_{p,sk}\tau_{c,Lw}\varepsilon_{p,Lw}A_{pr}(\theta_p - T_{sky})
\end{aligned}
\tag{17}
$$

### 2.1.5 Heat Balance on Moist Soil

The soil is assumed to be saturated to moisture. Evaporation from the soil surface results in reduction of the soil surface temperature and affects the humidity ratio of the airstream. The area of the exposed soil surface is the same for conduction convection, radiation H.T. and evaporation.

In case of one-dimensional heat flow into soil, for unit soil surface dividing by $A_g$

$$
\begin{aligned}
I\tau_{c,sw}\alpha_{s,sw} &= h_{fg}K_s(\omega_s - \omega_a) + hc_s(\theta_s - T_1) + hr_{s,sk}\varepsilon_{s,Lw}\tau_{c,Lw}(\theta_s - T_{sky}) \\
&+ hr_{s,c}\varepsilon_{c,Lw}\varepsilon_{s,Lw}F_{s,c}(\theta_s - \theta_c) + (hr_{s,p}\quad_{s,p}A_p/A_g)(\theta_s - \theta_p) \\
&\qquad + (k_s/\delta_s)(\theta_s - \theta_{s1}) + \rho_s\delta_sC_s(\theta_s - \theta_{s,in})/\Delta t
\end{aligned}
\tag{18}
$$

Substituting $\omega_s$ from equation (8) and $\omega_a$ from equation (13):

$$
\begin{aligned}
&-h_{fg}K_s\{X_s\theta_s + Y_s - D[X_cK_cP\theta_c + (X_pK_pA_p/\Delta x)\,\theta_p + (X_sK_sA_g/\Delta x)\theta_s \\
&\quad + (\rho_m vA_s/\Delta x)\omega_{a,in} + Y_cK_cP + Y_pK_pA_p/\Delta x + Y_sK_sA_g/\Delta x]\} - hc_s(\theta_s - T_1) \\
&\quad - hr_{s,sk}\varepsilon_{s,Lw}\tau_{c,Lw}(\theta_s - T_{sky}) - hr_{s,c}\quad_{s,c}(\theta_s - \theta_c) \\
&\quad - hr_{s,p}\mathcal{F}_{s,p}A_p/A_g(\theta_s - \theta_p) - K_s(\theta_s - \theta_{s1})/\delta_s - \rho_s\delta_sC_s(\theta_s - \theta_{s,in})/\Delta t \\
&\qquad = -\, I\tau_{c,sw}\alpha_{s,sw}
\end{aligned}
\tag{19}
$$

### 2.1.6 Heat Balance on the First Soil Slab

The first soil slab temperature after the time interval of ($\Delta t$) ($\theta_{s1}$), can be obtained from:

$$(\theta_{s,in} + \theta_{s2,in} - 2\theta_{s1,in}) = \frac{\delta_s^2}{a\Delta t}(\theta_{s1} - \theta_{s1,in}) \tag{20}$$

or

$$\theta_{s1} = \frac{2a\Delta t}{\delta_s^2}[(\theta_{s,in} + \theta_{s2,in})/2 - \theta_{s1,in}] + \theta_{s1,in} \tag{20-1}$$

Similarly for the second soil slab:

$$\theta_{s2} = \frac{2a\Delta t}{\delta_s^2}[(\theta_{s1,in} + \theta_{s3,in}(/2 - \theta_{s2,in}] + \theta_{s2,in} \tag{20-2}$$

For the kth soil slab:

$$\theta_{sk} = \frac{2a\Delta t}{\delta_s^2}[(\theta_{s(k-1),in} + \theta_{s(k+1),in})/2 - \theta_{sk,in}] + \theta_{sk,in} \tag{20-k}$$

where (k=m-4), and m is the number of unknowns. 4 equations refer to the cover, stream of air, plant and the soil slab at the surface.

Those equations could be further simplified taking

$$\delta_s = \sqrt{2a\Delta t}. \qquad (21)$$

For 20 slabs below soil surface:

$$\theta_{s1} = (\theta_{s,in} + \theta_{s2,in})/2 \qquad (22\text{-}1)$$

$$\theta_{s2} = (\theta_{s1,in} + \theta_{s3,in})/2 \qquad (22\text{-}2)$$

$$\theta_{sk} = (\theta_{s,k-1,in} + \theta_{s,k+1,in})/2 \qquad (22\text{-}k)$$

$$\theta_{s20} = (\theta_{s19,in} + \theta_{s21,in})/2 \qquad (22\text{-}20)$$

The initial and boundary conditions are:

$T_a$ : The ambient air temperature

$T_{sky}$ : Effective sky temperature

$T_{1,in}$ : Initial air temperature

$\theta_{s,in}$, $\theta_{s1,in}$, $\theta_{s2,in} \cdots \theta_{s21,in}$ : Initial soil temperature profile

$\omega_{a,in}$ : Initial humidity ratio of the airstream

Equation (14), (15), (17), (19) and (22-1) to (22-20) are 24 simultaneous equations in which $\theta_c$, $T_1$, $\theta_p$, $\theta_s$, $\theta_{s1}$, $\theta_{s2}, \ldots\; \theta_{s20}$ are 24 unknowns to be solved.

The solution techniques and features of various programs follow.

## 3. Computer Programming

Since the formulation of the problem was carried out at various stages, the solution also was obtained for the corresponding steps as listed in Table I.

Table I - List of Computer Programs

| | |
|---|---|
| 00 Series | |
| KUD 03 | No-moisture Analysis, Steady State Printout + Variation of ($\theta_c$, $T_1$, $\theta_p$, $\theta_s$) along the Greenhouse, Computer Plotted |
| 10 Series | |
| KUD 12 | Analysis with Transpiration Only, Steady State Linear Interpolation for the Plant Temperature Correction |
| KUD 15 | Same as Above - Newton-Rhapson Method Used in Approximation |
| 20 Series | |
| KUD 21 | Same as KUD 15, Greenhouse End Effects Included |
| 50 Series | |
| KUD 53 | Unsteady State Analysis, No Moisture Effects 4 Equations + Schmidt-Binder Technique |
| KUD 55 | Same as KUD 53, but 4 Original Unknowns + 20 Soil Temperatures Solved by 24 Equations |
| KUD 57 | Same as KUD 53, but Design Modified for E.R.L. Tucson Greenhouses |
| 60 Series | |
| KUD 61 | Unsteady State with Transpiration only 4 Equations + Schmidt-Binder Technique |

Table I - List of Computer Programs (Continued)

| | |
|---|---|
| 80 Series | |
| KUD 81 | Unsteady State, with Soil Moisture Evaporation, Plant Transpiration and Condensation on the Cover 4 Equations + Schmidt-Binder Technique |
| KUD 82 | Same as Above, Effect of Time Interval |
| KUD 84 | Same as KUD 81, Effect of Stomatal Resistance |
| 90 Series | |
| KUD 91 | 12 Month Performance Prediction from KUD 81 Printout |
| KUD 92 | Same as KUD 91, Result Computer Plotted |
| KUD 93 | Same as KUD 92, Effect of Shading |

### 3.1 The Analysis with no Moisture Effects

The formulation of the problem which is described elsewhere [4] yielded four non-linear, simultaneous equations, due to the radiation terms of $(\theta_c + 460)^4$ etc. These were linearized using the equivalent radiation heat transfer coefficients, $hr_{c,sk}$....etc. The program is labeled KUD 03. The solution of this and other models was obtained using CDC Model 6400 computer, of the University of Arizona's Computer Center.

(4 x 4) matrix A and 4 element column vector C were generated and the solution of linear simultaneous equations was obtained using the Gauss-Jordan elimination technique.

The computer program of the final model, however, includes the same matrix A and vector C which is used to predict the temperatures of $\theta_c$, $\theta_p$ and $\theta_s$.

### 3.2 Solution of the Model with Transpiration Effects Only

The formulation with transpiration effects which allows prediction of plant temperatures more accurately has been described in Reference [5].

Starting with an estimated transpiration rate based upon the enthalpy potential between the plant interface at approximate temperature and the airstream, both transpiration rate and the final leaf temperature were iterated until the assumed and calculated leaf temperature agreed with one another. The convergence of the iteration was accelerated utilizing the Newton-Rhapson method as shown in Fig. 3). This process which utilizes the matrix E (4,4) and vector G(4) is included in the second part of the computer program KUD 81.

### 3.3 Solution of the Model with Condensation on the Cover, Transpiration from the Plant and Evaporation from the Soil

24 simultaneous equations previously derived were utilized. Those equations were arranged for the 24 unknowns mentioned, and the coefficient matrix Q(24,24) and vector Z(24) which are presented in Table II, were obtained. $q_{mn}$ corresponds to the matrix element of Q(M,N) and $z_m$ to Z(M).

Table II - Coefficient Matrix Q(24,24) and Vector Z(24) for the Simultaneous Equations, Program KUD 81

| Q(24,24) | | | | | | | | | | | Z(24) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $q_{11}$ | $q_{12}$ | $q_{13}$ | $q_{14}$ | 0 | 0 | 0 | 0 | 0 | 0 | . . . . | $= z_1$ |
| $q_{21}$ | $q_{22}$ | $q_{23}$ | $q_{24}$ | 0 | 0 | 0 | 0 | 0 | 0 | . . . . | $= z_2$ |
| $q_{31}$ | $q_{32}$ | $q_{33}$ | $q_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | . . . . | $= z_3$ |
| $q_{41}$ | $q_{42}$ | $q_{43}$ | $q_{44}$ | 0 | 0 | 0 | 0 | 0 | 0 | . . . . | $= z_4$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | . . . . | $= (\theta_{s,in} + \theta_{s2,in})/2$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | . . . . | $= (\theta_{s1,in} + \theta_{s3,in})/2$ |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | . . . . | $= (\theta_{s,k-1,in} + \theta_{s,k+1,in})/2$ |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| ' | ' | ' | ' | ' | ' | ' | ' | ' | ' | . . . . | ' |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 0 0 1 | $= (\theta_{s19,in} + \theta_{s21,in})/2$ |

The coefficient matrix Q and vector Z yield the final solution by means of Gauss-Jordan elimination technique.

The other alternative is to solve the original 4 equations written for the cover, airstream, plant and the soil surface slab, then apply the Schmidt-Binder method for an hour's interval. The solar radiation intensity during an hour's period is assumed to be constant. The soil surface temperature is determined starting with an initial temperature profile following the method outlined in Reference [6].

Then, temperature profiles, at succeeding hours, within the soil are obtained at intervals defined by $\Delta t = \delta_s^2/2a$.

The logic diagram of the computer solution is presented in Fig. 4).

The ambient air temperature, solar radiation intensity, and the sky temperature were supplied as boundary conditions.

The air outlet temperature from the control volume $T_1$, was set equal to the initial air temperature of the following control volume $(T_{1,in})$. Similarly, the solution of the present analysis $\omega_a$ was introduced as the initial humidity ratio for the adjacent control volume. This process was repeated until the length of the greenhouse was traversed.

Comparing the results obtained using these two approaches suggested for the soil temperature profiles, it can be concluded that the accuracy attained employing (n + 4) equations is not worth the excess computer time required.

Typical soil surface temperatures obtained using 24 equations with 24 unknowns, which are 60.9°F, 95.8°F, 128.0°F, 136.8°F, were modified as 61.2°F, 95.4°F, 126.1°F, 134.9°F, respectively, using the simplified method based upon Schmidt-Binder Technique. On the other hand, the variation of computed air temperatures using either method has been less significant.

Typical air outlet temperatures obtained by 24 equations have been 58.69°F, 69.88°F, 85.75°F, and 93.76°F; whereas, using 4 equations and Schmidt-Binder Technique, respective temperatures have been 58.81°F, 69.82°F, 85.46°F, and 93.54°F.

## 4. Results of the Analysis and Experimental Verification

24-hour tests were run to verify the theories developed. The same experimental data was used for a variety of computer programs and computed values were compared with measurements as listed in Table III.

Table III - Comparison of Computed Values for Various Programs and Measurements at Puerto Peñasco on November 21, 1969, 12 Noon

| | | COMPUTER PROGRAM NUMBER | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ITEM | UNIT | KUD03 | KUD15 | KUD21 | KUD53 | KUD61 | KUD81 | MEASURED |
| Ambient Temp., DB | °F | | | | | | | 70.8 |
| Ambient Temp., WB | °F | | | | | | | 63.2 |
| Air Inlet, DB | °F | | | | | | | 85.2 |
| Air Inlet, WB | °F | | | | | | | 84.2 |
| Air Outlet, DB (200 ft) | °F | 95. | 90.16 | 91.2 | 89.7 | 88.2 | 89.6 | 89.0 |
| Air Outlet, WB (200 ft) | °F | | 87. | 86.8 | | 86.8 | 88. | 86.5 |
| Wind Velocity | mph | | | | | | | 4.5 |
| Air Velocity | ft $hr^{-1}$ | | | | | | | 6870. |
| Solar Radiation | Btu $hr^{-1} ft^{-2}$ | | | | | | | 208. |
| Plastic Cover Temp. at 50' | °F | 84.1 | 90.16 | 87.09 | 81.5 | 83.8 | 83.7 | |
| Plant Temp. at 50' | °F | 93.6 | 85.9 | 86.03 | 89.8 | 86.1 | 86.8 | 87.4 |
| Soil Temp. Surface | °F | 111.9 | 102.2 | 107.8 | 92.4 | 100.3 | 95.1 | 97.5 |
| Soil Temp., 2" | °F | | | | 84. | 91.4 | 88.5 | 89.9 |
| Soil Temp., 4" | °F | | | | 79.5 | 84.5 | 83.9 | 83.4 |
| Soil Temp., 7" | °F | | | | 78.9 | 78.9 | 79.4 | 79.4 |
| Condensation on the Cover | lb $hr^{-1}$ | | | | | | 25.9 | 53.7 |
| $\omega_{a,in}$ | $lb_v/lb_{da}$ | | 0.0263 | 0.0263 | | 0.0263 | 0.0263 | 0.0255 |
| $\omega_{a,in}$ (200 ft) | $lb_v/lb_{da}$ | | 0.0274 | 0.0273 | | 0.0276 | 0.0291 | 0.0271 |

The analysis with transpiration effects improves the computed leaf temperature whereas considering evaporation from the soil allows to predict the soil surface temperature more closely. The condensation rates could only be computed using the program KUD81.

It should be noted that the analyses with no moisture effects and end effects were developed for steady state operation; therefore, the typical radiation and ambient conditions at 1200 noon were used to compare the computed temperatures. Soil temperatures below the surface are not available in the programs labeled KUD03, KUD15 and KUD21.

The choice of program to be used depends upon the degree of accuracy desired, as well as the cost of computation time. Typical computation times have been 8.954 seconds for KUD03, 13.968 seconds for KUD15, 16.816 seconds for KUD21, 26.599 seconds for KUD53, 71.660 seconds for KUD61, and 51.058 seconds for KUD81, on a CDC 6400 computer.

The use of the advanced version of KUD 61 and 81 may be a drawback, especially in case of using a low-speed computer.

## 5. Discussions and Conclusions

The prediction of temperatures and humidities in a controlled-environment greenhouse enables the design engineer to maintain the temperature of air and leaf within the safe and productive margins given by the horticulturalist.

Determination of the fan capacity, cooling or heating load where required, should also be possible. The present mathematical formulation of the problem has proven to represent the actual performance accurately enough for any engineering application, since test results have agreed with the computer program.

The physical phenomena of transpiration, condensation effects on the plastic cover, turbulence over the plant canopy, flow field through and around the canopies, penetration of solar radiation through plant communities have been separately treated by investigators in various disciplines [7], [8], [9], [10]. However, the results of those surveys are not directly applicable to the specific system under study, since they mostly consider each component separately. Physical properties of plastic cover, plant and soil are not exactly the same as the present system. Besides, heat and mass transfer coefficients must be determined for the surfaces under consideration. Those studies would require sophisticated instruments and tedious experimentation techniques while the present analysis aims to predict the overall performance of the system rather than the detailed analysis of each component such as determination of stomatal and boundary layer resistance of a single leaf.

The computer program developed however, is versatile enough to consider these modifications on the analysis and in its present form, supplies the essential design data on temperatures, humidities, and heating or cooling load.

Year-round performance prediction for the controlled-environment greenhouse has been possible through the program KUD 91 which is virtually the same as KUD 81 except the repeated number of runs and computer plotted curves.

A typical set of performance curves for the month of July at Puerto Peñasco and year-round variation of temperatures, humidities and condensation at 11:00 AM and 5:00 AM are sampled in Fig. 5) and Fig. 6), respectively.

An extensive program is being launched by the Environmental Research Laboratory of the University of Arizona, to investigate all those interrelated parameters in a coordinated research effort. The findings of this survey should provide the most accurate data for the prediction of the performance of an inflated plastic controlled environment greenhouse.

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## Nomenclature

| | | UNIT | FORTRAN NOTATION |
|---|---|---|---|
| $A_{D,p}$ | Area of plant receiving direct radiation | ft$^2$ | ADIRP |
| $A_{d,p}$ | Area of plant receiving diffuse radiation | ft$^2$ | ADIFP |
| $A_g$ | Ground area (Exposed to Solar Radiation) | ft$^2$ | AGROU |
| $A_p$ | Area of plant (convection H.T.) | ft$^2$ | AP |
| $A_{pr}$ | Area of plant (Radiation H.T.) | ft$^2$ | APR |
| $A_s$ | Area of the cross section | ft$^2$ | CROSEC |
| a | Thermal diffusivity (soil) | ft$^2$ hr$^{-1}$ | THDIF |
| $C_p$ | Specific heat of dry air | Btu lb$^{-1}$ °F$^{-1}$ | |
| $C_{pm}$ | Specific heat of moist air | Btu lb$^{-1}$ °F$^{-1}$ | CEAIR |
| $C_s$ | Specific heat of soil | Btu lb$^{-1}$ °F$^{-1}$ | CESO |
| F | Radiation Heat Transfer Shape factor | Dimensionless | |
| $\mathcal{F}$ | $\mathcal{F} = F \cdot \varepsilon$ Radiation H.T. Shape Emissivity Factor | Dimensionless | |

| | | UNIT | FORTRAN NOTATION |
|---|---|---|---|
| $\mathcal{F}_{p,c}$ | $(F_{p,c} \cdot \varepsilon_{p,Lw} \cdot \varepsilon_{c,Lw})$ Radiation heat transfer between plant and cover | Dimensionless | FRPC |
| $\mathcal{F}_{s,c}$ | $(F_{s,c} \cdot \varepsilon_{s,Lw} \cdot \varepsilon_{c,Lw})$ Radiation heat transfer between soil and cover | Dimensionless | FRSC |
| $\mathcal{F}_{s,p}$ | $(F_{s,p} \cdot \varepsilon_{s,Lw} \cdot \varepsilon_{p,Lw})$ Radiation heat transfer between soil and plant | Dimensionless | FRSP |
| $h_a$ | Enthalpy of dry air | Btu $lb^{-1}$ | HDA |
| $h_f$ | Enthalpy of saturated liquid | Btu $lb^{-1}$ | HF |
| $h_{fg}$ | Enthalpy of phase change, evaporation | Btu $lb^{-1}$ | HFG |
| $h_g$ | Enthalpy of saturated vapor | Btu $lb^{-1}$ | HG |
| $h_{interf}$ | Enthalpy of air stream at the plant interface | Btu $lb^{-1}$ | |
| $hc_i$ | Convective H.T. inside | Btu $hr^{-1}$ $ft^{-2}$ $^{\circ}F^{-1}$ | HCIN |
| $hc_p$ | Convective H.T. over plants | " | HCPL |
| $hc_s$ | Convective H.T. over soil | " | HCSP |
| $hc_w$ | Convective H.T. due to wind | " | HWIN |
| $hr_{c,sk}$ | ERHTC - cover to sky | " | HRCOSK |
| $hr_{p,sk}$ | ERHTC - plant to sky | " | HRPLSK |
| $hr_{s,sk}$ | ERHTC - soil to sky | " | HRSOSK |
| $hr_{p,c}$ | ERHTC - plant to cover | " | HRPC |
| $hr_{s,c}$ | ERHTC - soil to cover | " | HRSC |
| $hr_{s,p}$ | ERHTC - soil to plane | " | HRSP |
| I | Solar radiation intensity | Btu $hr^{-1}$ $ft^{-2}$ | RADI |
| $I_D$ | Direct solar radiation intensity | " | RADIR |
| $I_d$ | Diffuse solar radiation intensity | " | RADIF |
| $K_D$ | Mass Diffusion coefficient | lb $hr^{-1}$ $ft^{-2}$ | |
| $K_C$ | Mass Diffusion coefficient at cover (Condensation Rate) | " | OKC |
| $K_p$ | Mass Diffusion coefficient at plant (Transpiration) | " | OKP |

|  |  | UNIT | FORTRAN NOTATION |
|---|---|---|---|
| $K_s$ | Mass Diffusion coefficient over soil (Evaporation) | $lb\ hr^{-1}\ ft^{-2}$ | OKS |
| $k$ | Thermal Conductivity | $Btu\ hr^{-1}\ ft^{-1}\ {}^{\circ}F^{-1}$ |  |
| $k_s$ | Soil Thermal Conductivity | " | CONSO |
| $m_a$ | Mass Flow Rate of air | $lb\ hr^{-1}$ |  |
| $P$ | Perimeter : $\Pi \cdot R$ | ft | PERIM |
| $P_a$ | Partial pressure of air | psia |  |
| $P_{bar}$ | Barometric pressure | psia | PBAR |
| $P_m$ | Total (mixture) pressure | psia | PMIX |
| $P_v$ | Partial pressure of water vapor | psia |  |
| $P_{v,a}$ | Partial pressure of water vapor at ambient air | psia | PVAM |
| $P_{v,in}$ | Partial pressure of water vapor initially | psia | PVI |
| $Q$ | Total heat transferred | $Btu\ hr^{-1}$ |  |
| $q$ | Heat transfer rate | $Btu\ hr^{-1}\ ft^{-2}$ |  |
| $R$ | Greenhouse cross section radius | ft | RADIUS |
| $R_{stom}$ | Stomatal resistance to mass diffusion | $hr\ ft^{-2}\ lb^{-1}$ | RSTOM |
| $t$ | Time | hrs | TIME |
| $T$ | Temperature |  |  |
| $T_i$ | Temperature of air inside the greenhouse | °F | TEAIR |
| $T_a$ | Ambient Air temperature | °F | TAM |
| $T_{db}$ | Dry bulb temperature | °F |  |
| $T_{wb}$ | Wet bulb temperature | °F |  |
| $T_{sky}$ | Equivalent sky temperature | °F | TSKY |
| $T_{1,in}$ | Initial temperature (Dry Bulb) | °F | TINIDB |
| $T_{1,in}$ | " " (Wet Bulb) | °F | TINIWB |
| $V$ | Volume flow rate | $ft^{3}\ ft^{-1}$ |  |
| $v$ | Velocity | $ft\ hr^{-1}$ | VEL |
| $W$ | Wind velocity | mph | WIND |

## Nomenclature

### Greek Symbols

| | | UNIT | FORTRAN NOTATION |
|---|---|---|---|
| $\alpha_{c,sw}$ | Absorptivity, cover shortwave | Dimensionless | ALFCSW |
| $\alpha_{c,Lw}$ | Absorptivity, cover longwave | Dimensionless | ALFCLW |
| $\alpha_{p}$ | Absorptivity, plant | Dimensionless | |
| $\alpha_{p,sw}$ | Absorptivity, plant shortwave | Dimensionless | ALFPSW |
| $\alpha_{p,Lw}$ | Absorptivity, plant longwave | Dimensionless | ALFPLW |
| $\alpha_{s,sw}$ | Absorptivity, soil shortwave | Dimensionless | ALFSSW |
| $\alpha_{s,Lw}$ | Absorptivity, soil longwave | Dimensionless | ALFSLW |
| $\alpha_{p,sw,D}$ | Absorptivity, plant shortwave, direct radiation | Dimensionless | AFPSWR |
| $\alpha_{p,sw,d}$ | Absorptivity, plant shortwave, diffuse radiation | Dimensionless | AFPSWF |
| $\varepsilon_{c,Lw}$ | Emissivity, cover longwave | Dimensionless | EMCLW |
| $\varepsilon_{p,Lw}$ | Emissivity, plant longwave | Dimensionless | EMPLLW |
| $\varepsilon_{s,Lw}$ | Emissivity, soil longwave | Dimensionless | EMSOLW |
| $\tau_{c}$ | Cover, transmissivity | Dimensionless | TRAN |
| $\tau_{c,Lw}$ | Cover, transmissivity longwave | Dimensionless | TAUCLW |
| $\tau_{c,sw}$ | Cover, transmissivity shortwave | Dimensionless | TAUCSW |
| $\delta_{s}$ | Thickness of the soil slab | ft | DELSO |
| $\Delta x$ | Space interval (Length of the System) | ft | DELX |
| $\Delta t$ | Time Interval | hr | DELTIM |
| $\omega$ | Humidity ratio | $lb_{v}/lb_{da}$ | |
| $\omega_{a}$ | Humidity ratio of air | " | OMEGAI |
| $\omega_{a,in}$ | Initial humidity ratio | " | OMEGIN |
| $\omega_{c}$ | Humidity ratio of saturated air, at the cover temp. | " | OMEGCO |
| $\omega_{p}$ | Humidity ratio of saturated air at plant temp. | " | OMEGPL |
| $\omega_{s}$ | Humidity ratio of saturated air, at soil surface temp. | " | OMEGSO |
| $\Theta_{init}$ | Relative humidity of air initially | Dimensionless | PHIN |

| | | UNIT | FORTRAN NOTATION |
|---|---|---|---|
| $\theta_c$ | Transparent cover temp. | °F | TECOV |
| $\theta_p$ | Plant temperature | °F | TEPLA |
| $\theta_s$ | Soil surface temperature | °F | TESOI |
| $\theta_{s1}$ | Soil temperature 1st slab | °F | TS1 |
| $\theta_{s2}$ | Soil temperature 2nd slab | °F | TS2 |
| $\theta_{s3}$ | Soil temperature 3rd slab | °F | TS3 |
| $\theta_{sk}$ | Soil temperature kth slab | °F | |
| $\rho_a$ | Density of dry air | $lb_m\ ft^{-3}$ | ROAIR |
| $\rho_m$ | Density of moist air | " | ROMAIR |
| $\rho_s$ | Density of the soil | " | ROSO |

Subscripts

| | |
|---|---|
| a | air |
| am | ambient |
| c | cover |
| D | Diffusion, Direct for solar radiation |
| d | diffuse, solar radiation |
| da | dry air |
| db | dry bulb |
| i | inlet, inside |
| init.,in | initial |
| interf. | interface |
| Lw | longwave |
| m | moist air |
| o | outlet, outside |
| p | plant |
| s | soil |
| sat | saturation |
| sk | sky |
| sw | shortwave |
| V | Vapor |
| w | water |
| wb | wet bulb |

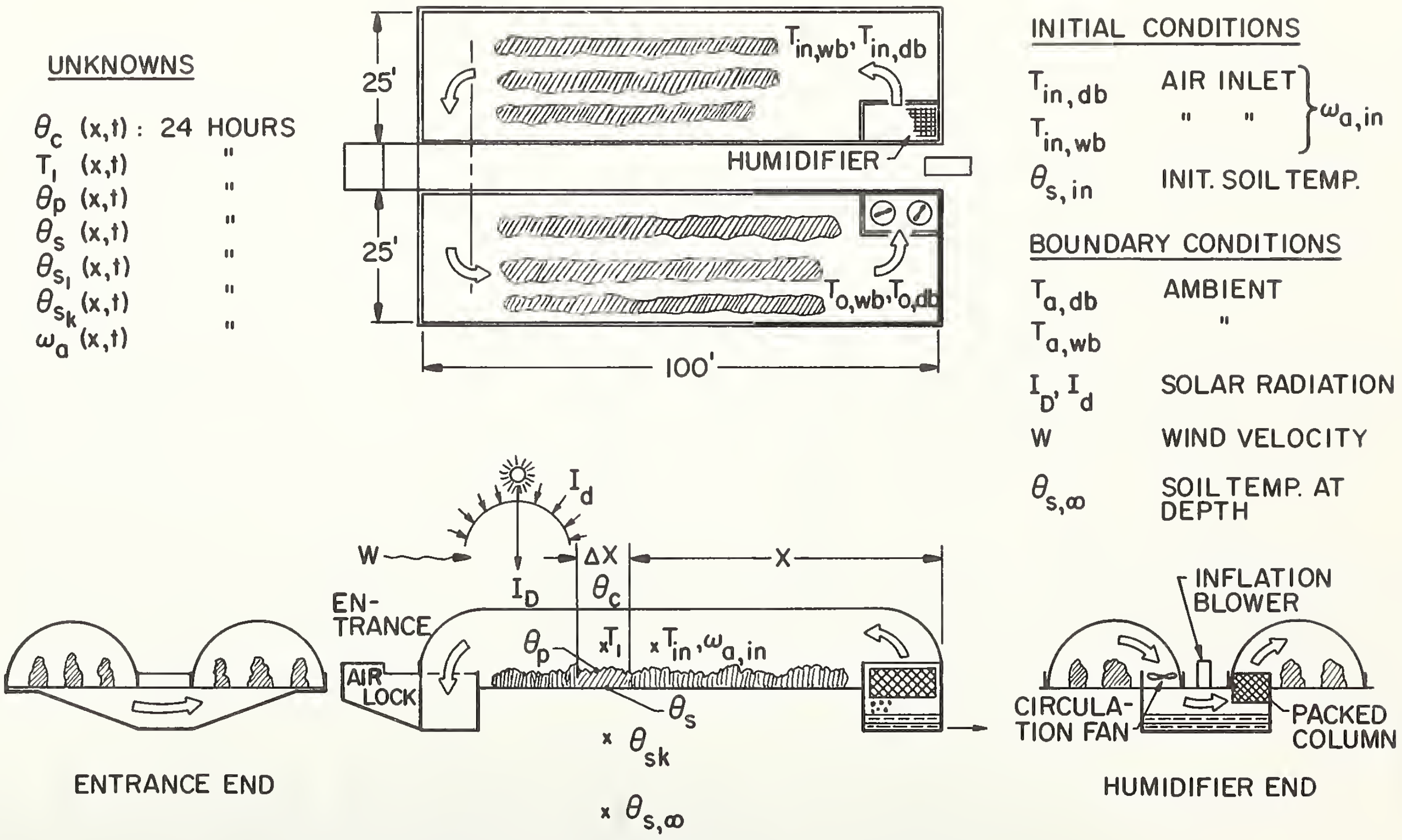


Fig. 1) Controlled Environment Greenhouse, Dimensions, Boundary Conditions and Unknowns for the Performance Analysis

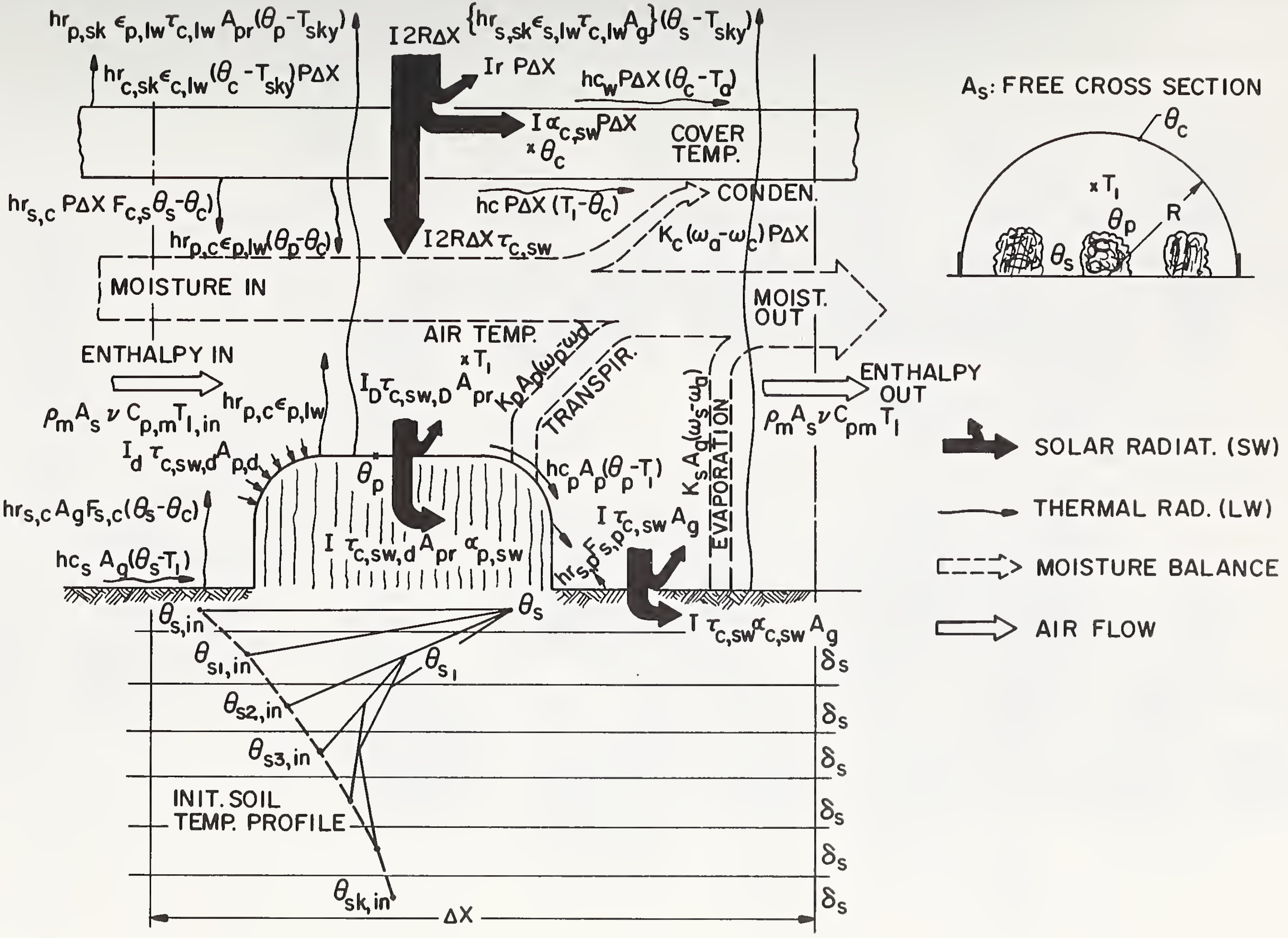


Fig. 2) Mathematical Model for the Heat and Mass Transfer

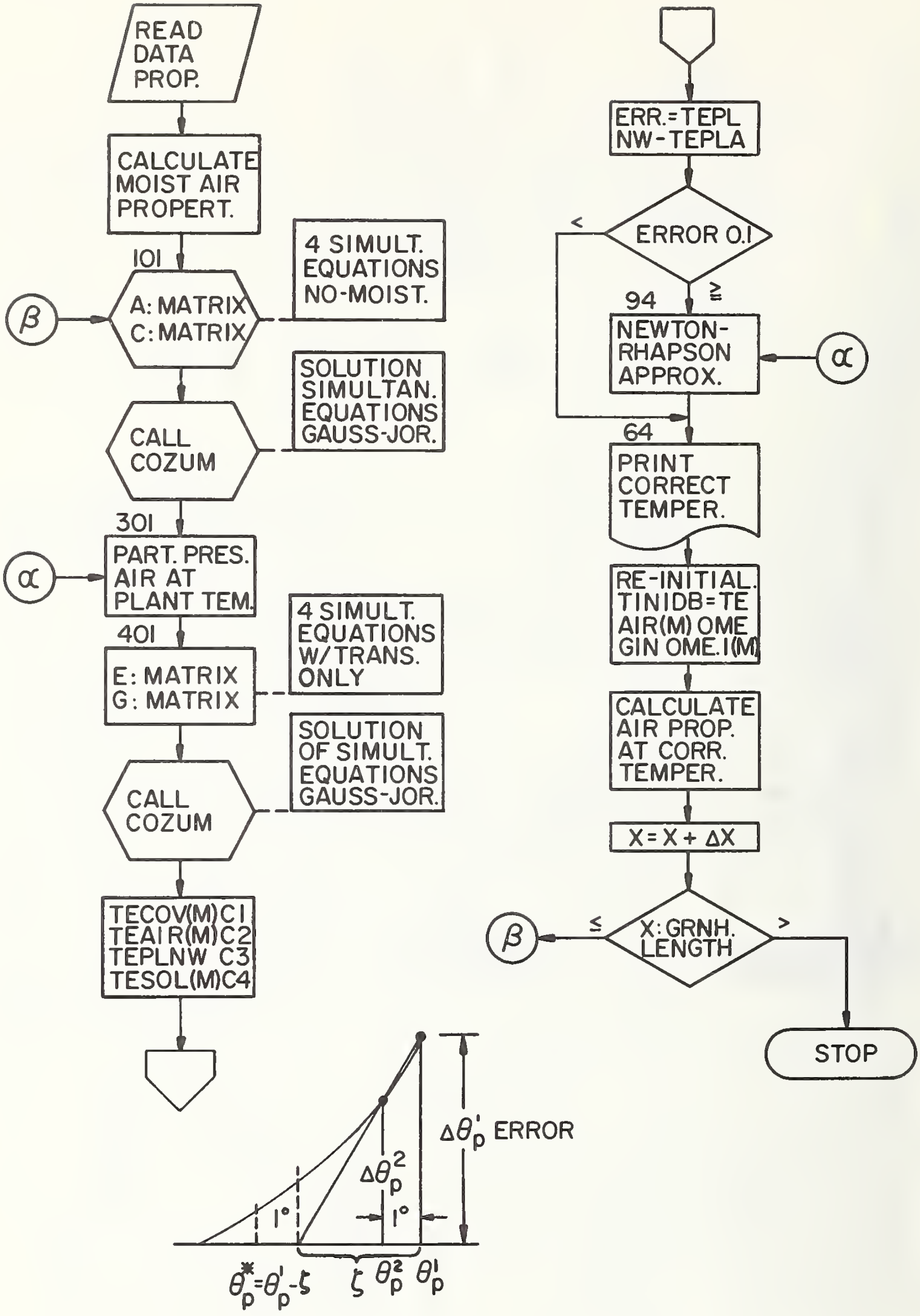


Fig. 3) Flow Chart for the Analysis with Transpiration Effects Only, Program KUD 15

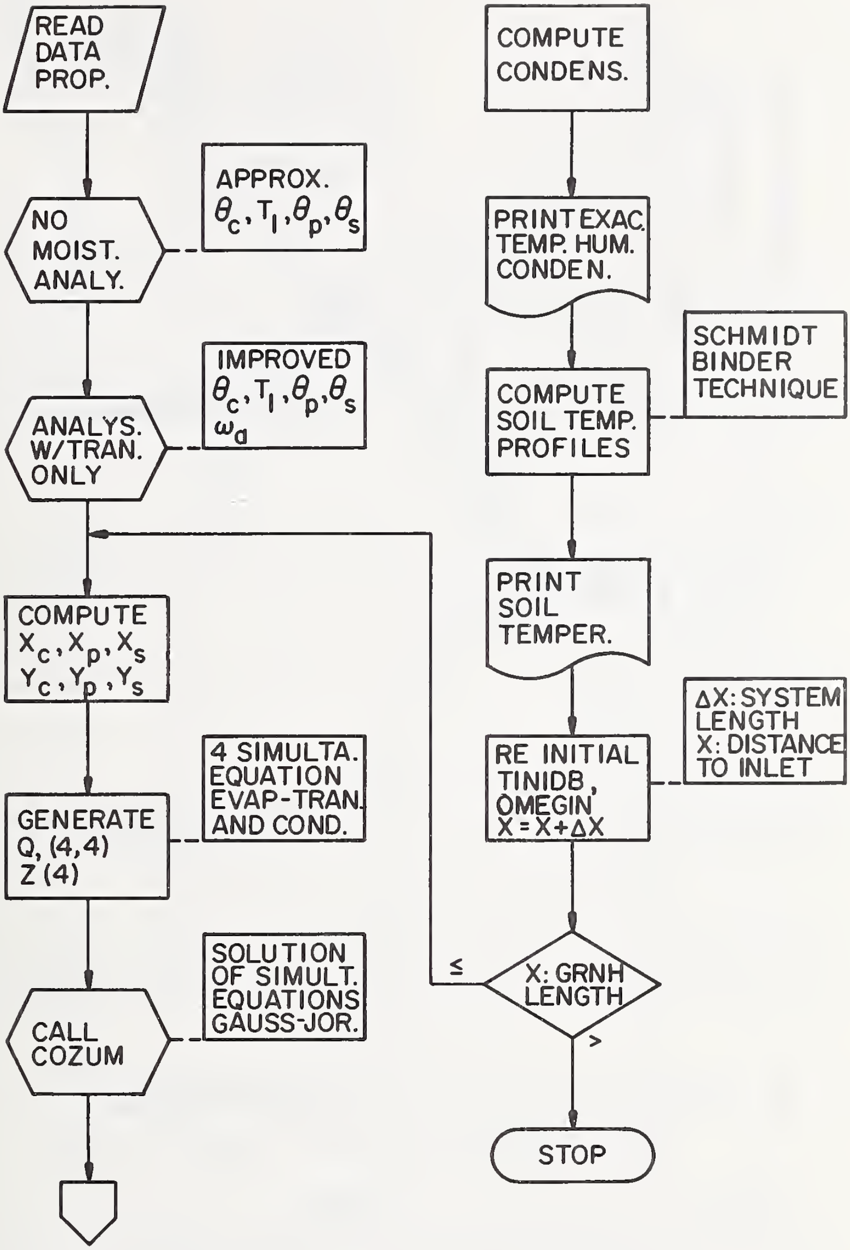


Fig. 4) Flow Chart for the Analysis with Evaporation-Transpiration and Condensation Effects

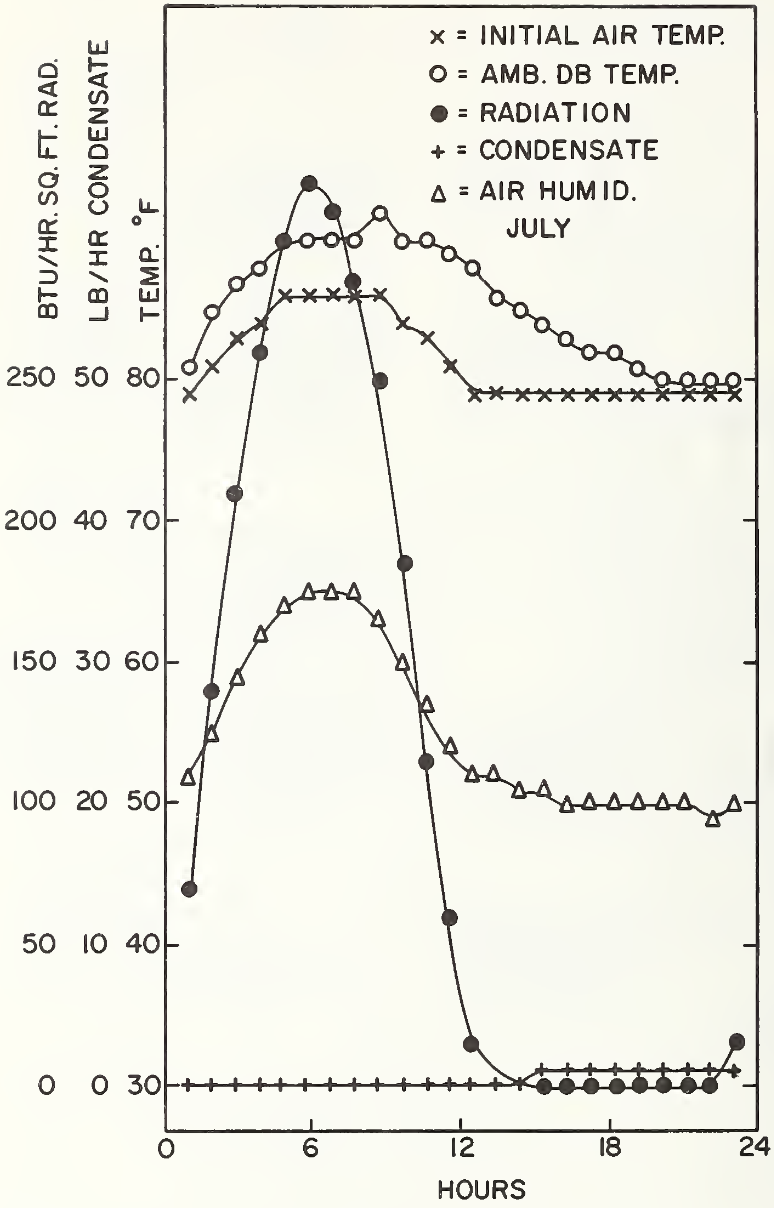


Fig. 5a. Computer Plotted Performance Curves for the Month of July at Puerto Peñasco, Sonora, Mexico.

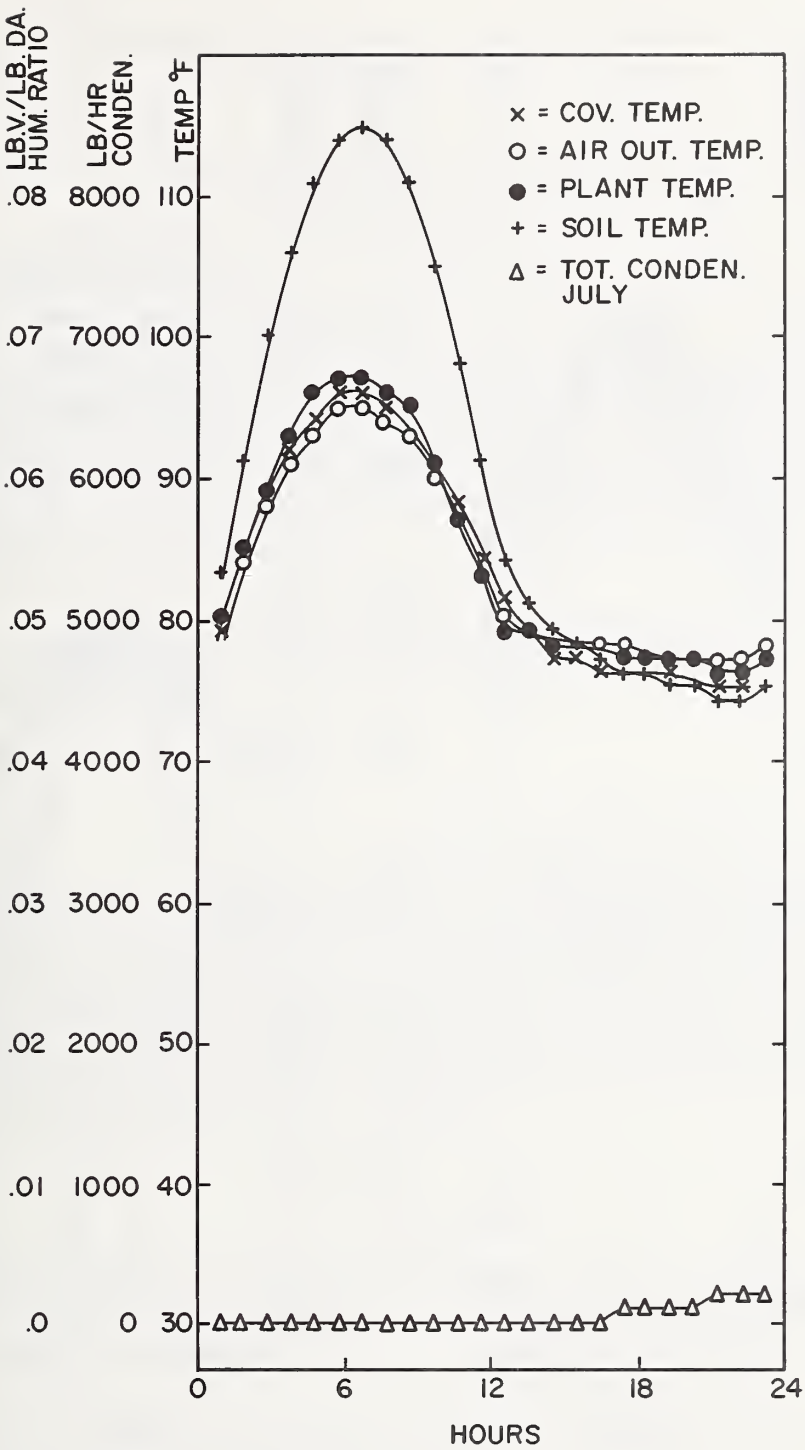
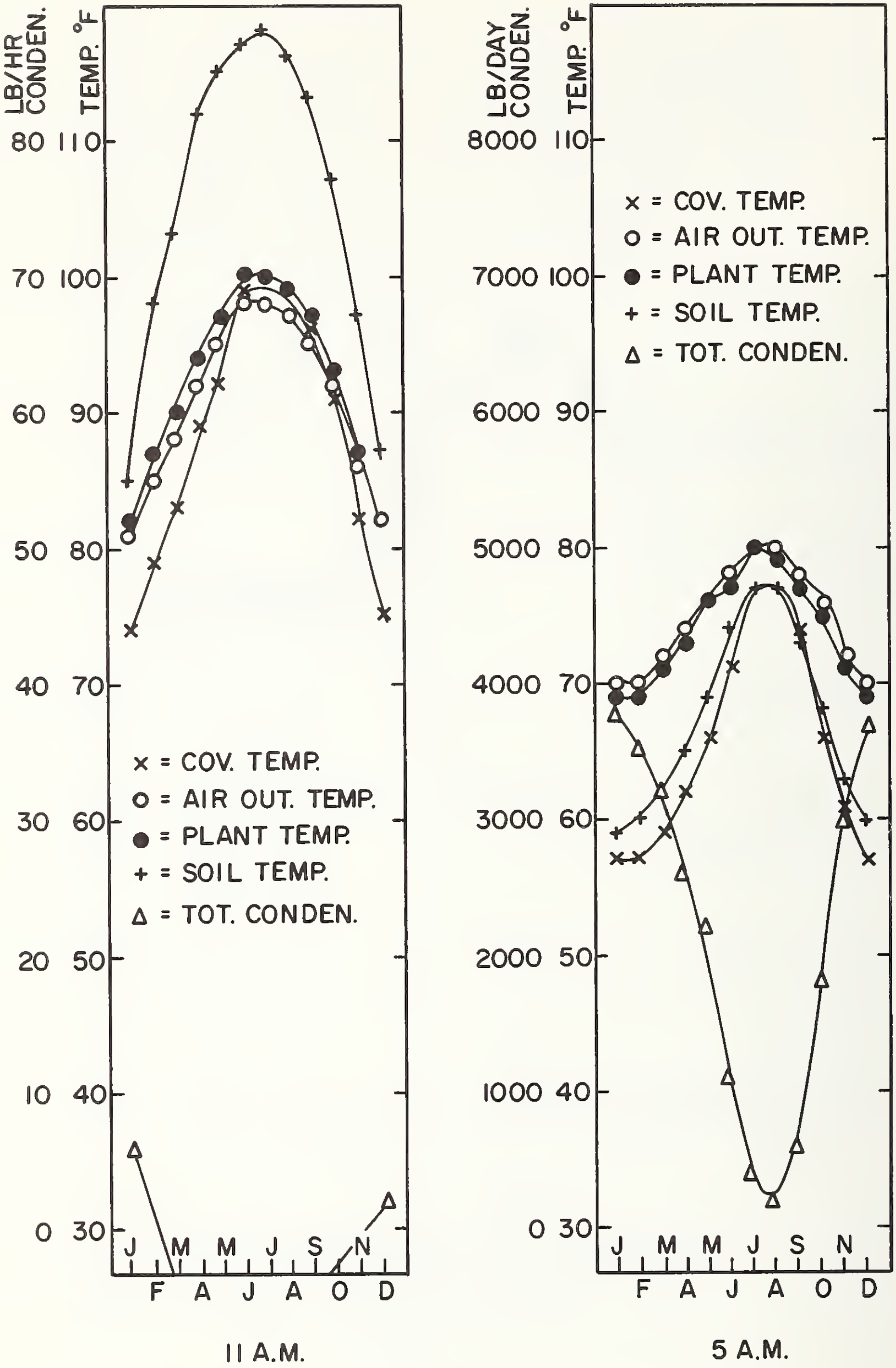


Fig. 5b. Computer Plotted Performance Curves for the Month of July at Puerto Peñasco, Sonora, Mexico.



11 A.M.

5 A.M.

Fig. 6) Computer Plotted Performance Curves at Noon and Midnight for 12 Months at Puerto Peñasco, Sonora, Mexico

# Automated Design Program for Air-Handling Apparatus

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This paper describes a computer program that can be used to calculate the heating and cooling load, the supply air volume, the psychrometric data, and the specifications for an air-handling apparatus. The program is based on equations that express the change in the state of moist air. The program can be used to select the appropriate apparatus and calculate the required air volume for either a non-reheat system or a reheat system. It also can be used to quickly optimize the design by performing alternative calculations for various hypothetical conditions.

Key Words: Air-handling apparatus, apparatus dew point, automated design, cooling and dehumidifying coil, cooling load, psychrometric chart, reheat system, single-duct air system, supply air volume.

## 1. Introduction

The process of designing single-duct air systems requires the calculation of cooling-heating load, a psychrometric analysis, the calculation of supply air volume, and the selection of the apparatus. The calculation of the cooling and heating loads by computers has been widely studied and many programs have been developed. Programs that consistently treat the entire design process have not been formulated. Because it is difficult to treat the psychrometric analysis by computer and to establish and systematize a standard for engineering judgement during the design process. This paper describes a program that consistently treats the design process by setting up approximate formulas to express the state of moist air, simplifying the phenomena under several hypothetical conditions, and establishing the standard for engineering judgment.

## 2. Program Outline

This program is to be used after the preliminary design of the air-conditioning facilities is completed and the types of systems, the zones, and the kind and conditions of the energy source are determined.

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The input data are as follows:

a. Data for load calculation: outdoor design conditions, room design conditions, thermal properties and quantities of the materials constituting the room, internal heat load, etc.

b. Data for system-design: type of system, chilled-water temperature, steam pressure, type of humidifier, type of filter, etc.

The program consists of three blocks. The first calculates cooling and heating loads, based on the theory of periodic heat conduction, which assumes the room temperature to be constant. From the input data of the load calculation, summertime cooling load and wintertime heating load for every room are calculated by hour and summed up by zones. The second block takes the results of load calculation and calculates the states and the volume of supply air by the psychrometric analysis of system-design data. If no solution can be obtained by that process, the room conditions are automatically changed and the load calculations are repeated. The third block determines the specifications of air-handling apparatus, calculating the apparatus load as well as the energy source capacity from the system-design data and psychrometric data, and selects the equipment. The data showing the apparatus efficiency required for the apparatus selection are included in the program with the performance value stated in the manufacturer's catalogue converted into formulas.

The data to be printed out are as follows:

- The peak load of each room.
- The supply air volume of the room.
- The hourly load of each zone.
- The specifications of air-handling apparatus, the psychrometric data, and the capacity and conditions of the energy source.

## 3. Algorithms of Psychrometrics

A psychrometric chart is generally used to obtain and analyze the data concerning the state of moist air. In order to do the same with a computer, all the characteristics of moist air have to be expressed by mathematical formulas. The general psychrometric chart has enthalpy and humidity ratio drawn as oblique coordinates, and dry-bulb temperature, wet-bulb temperature, and relative humidity as constant lines. The data of moist air necessary for the calculations of supply air volume and the selection of the coil can be expressed by dry-bulb temperature, humidity ratio, and enthalpy. To simplify the numerical analysis, approximate formulas may be used, so long as no practical problem arises. In this program, therefore, the state of moist air and its changes are expressed by using the enthalpy and the humidity ratio. The dry-bulb temperature of moist air employed in the following formulas ranges from 0°C to 40°C when the atmospheric pressure is 760 mmHg.

Equations for obtaining humidity ratio with dry-bulb temperature and relative humidity, given:

$$\begin{aligned}\mathrm{PWS\,(T)} = {} & 735.557 \ast \mathrm{EXP}\,(-7.90298 \ast (\mathrm{TS/T}-1.0) + 5.02808 \ast \mathrm{ALOG}\,(\mathrm{TS/T}) \\ & -1.3816\mathrm{E}{-7} \ast (10.0 \ast\ast (11.349 \ast (1.0-\mathrm{T/TS}))-1.0) \\ & +8.1328\mathrm{E}{-3} \ast (10.0 \ast\ast (-3.49149 \ast (\mathrm{TS/T}-1.0))-1.0))\end{aligned} \tag{1}$$

$$\mathrm{W} = 0.0062 \ast \mathrm{RH} \ast \mathrm{PWS}\,(\mathrm{DB}+273.16) \,/\, (760.0 - 0.01 \ast \mathrm{RH} \ast \mathrm{PWS}\,(\mathrm{DB}+273.16)) \tag{2}$$

Equation for obtaining enthalpy with dry-bulb temperature and humidity ratio, given:

$$\mathrm{H} = 0.240 \ast \mathrm{DB} + (597.3 + 0.441 \ast \mathrm{DB}) \ast \mathrm{W} \tag{3}$$

Equations for obtaining humidity ratio and relative humidity with dry-bulb temperature and wet-bulb temperature, given:

$$\mathrm{PW} = \mathrm{PWS}\,(\mathrm{WB}+273.16) - 0.5 \ast (\mathrm{DB} - \mathrm{WB}) \ast 760.0/755.0 \tag{4}$$

$$\mathrm{W} = 0.622 \ast \mathrm{PW} \,/\, (760.0 - \mathrm{PWS}\,(\mathrm{DB}+273.16)) \tag{5}$$

$$RH = 100.0*PW / PWS (DB+273.16) \quad (6)$$

Equation for obtaining humidity ratio $W_2$, when the air with dry-bulb temperature $DB_1$ and humidity ratio $W_1$ changes with the given Δ H/ Δ W constant and dry-bulb temperature becomes $DB_2$:

$$W_2 = \frac{C+597.3+0.441*DB_1}{C-597.3-0.441*DB_2} * W_1 + \frac{0.24*(DB_2+DB_1)}{C-597.3-0.441*DB_2} \quad (7)$$

Equations for the approximate relation of enthalpy and humidity ratio when relative humidity is constant:

(Within dry-bulb temperature of 0° - 30°C, the error is limited to ± 0.4%.)

$$HS(W) = -3.32635 + 1673.25*W - 49827.4*W**2 + 1005561.0*W**3 \quad (8)$$

$$H95(W) = -3.13977 + 1669.23*W - 49134.4*W**2 + 983042.0*W**3 \quad (9)$$

$$H85(W) = -2.93914 + 1726.00*W - 54017.6*W**2 + 1128573.0*W**3 \quad (10)$$

To obtain enthalpy $H_2$ and humidity ratio $W_2$ when the air with enthalpy $H_1$ and humidity ratio $W_1$ changes with the given constant Δ H/ Δ W, and relative humidity becomes 95%, solve the following equation:

$$H95(W_2) = C*(W_2 - W_1) + H_1 \quad (11)$$

Equation (11) can be solved by applying the Newton-Raphson Method.

To obtain dew-point temperature of air with humidity ratio $W_1$, first obtain dew-point enthalpy by eq(8) and then use eq(3).

## 4. Calculations of Supply Air Volume

### 4.1 Fundamental

If the state of supply air and its volume can be set to satisfy eqs(12) and (13), the room temperature and humidity can be maintained under conditions similar to those of the design.

$$QT = 1.2*AQ*(H_1 - H_6) \quad (12)$$

$$\frac{H_1 - H_6}{W_1 - W_6} = \frac{597.3}{1.0 - SHF} \quad (13)$$

In determining the state of supply air and its volume, however, there are the following limitations:

- There is a lower limit of the supply air temperature which prevents vapor condensation at the outlet.
- There are limits of the cooling and dehumidifying ability of the chilled-water coil.
- There are upper and lower limits of air volume for the air distribution within a room and for economic and health reasons.

When the air volume within the range of the above limitations is minimized, the equipment will be most economical. If such limitations are established and standaridized, it will not be hard to use the computer for psychrometric analysis. The conditions set up in this program for such limitations and the procedure for air-volume calculations are given in Figure 1. As to the change of the state of air passing through the cooling-dehumidifying coil, Δ H/ Δ W is assumed to be a constant. It is also assumed that the change of the state of air makes no change in the air volume.

The state of supply air should be above the dew point temperature of the room, to prevent vapor condensation at the outlet, and below 85% relative humidity, to prevent the state of air leaving the coil from being too close to the saturation line.

The relative humidity of the air leaving the cooling-dehumidifying coil is to be below 95% and the apparatus dew point temperature is to be above its specified dew point temperature. Considering the coil performance, the variation of chilled-water temperature, and the value of $\Delta H / \Delta W$ of the air passing through the coil, the designer must set the specified apparatus dew point temperature to be higher than the chilled-water temperature by 3°C or more.

As to the minimum volume of supply air, choose the larger one from either between outdoor air volume or exhaust air volume, as is required by the design. The maximum will be, as a rule, below 15 times of air changes per hour, but this can be ignored if it interferes with other factors.

The heat gain caused by the fan can be determined by the fan efficiency and its total pressure. Their exact value cannot be estimated until the duct system is designed. So they most be assumed. The centrifugal type is one of the most common types of fan used in the air-handling apparatus ; its efficiency is 0. 5-0. 65. Thus, if the efficiency is assumed to be 0. 5, the relation between the total pressure of the fan and the rise of temperature caused by heat gain is as follows:

$$FTD = 0.0162 * PT \tag{14}$$

The total pressure of fan will be assumed by the designer according to the layout of the duct system and the type of filter. Since the actual temperature rise is 1. 0-1. 8°C, it is not necessary to maintain complete accuracy. It is assumed that the fan is located in the discharge-side of the air-handling apparatus.

The supply duct heat gain is ignored. It can, however, be estimated to be about 5 percent of the sensible heat load of the room and added to the apparatus load.

### 4. 2 Non-Reheat System

If the total heat load of the room, the sensible heat factor, the required outdoor air volume, and the room and outdoor conditions are given, the psychrometric data and the supply air volume are calculated by eqs (7), (8), (10), (12), (13), and (14). In case the solution cannot be obtained or the apparatus dew point falls below the specified apparatus dew point, the room conditions are automatically modified and the load calculation is repeated. The dotted line in Figure 2 shows such a case.

As in the basement zone, where the cooling load is small, the supply air volume calculated in the above way may turn out to be too small. In this case, the temperature difference between supply air at the outlet and the room air will be reduced so that the supply air volume can be equal to the required minimum volume. In the example in Figure 3, the state of supply air changes from 6' to 6.

In modifing the room design conditions, the dry-bulb temperature and the humidity of the room air will be modified to keep the effective temperature of the room constant. In this program the dry-bulb temperature is reduced by 0. 5°C and the relative humidity is increased by 5%. In the case of Figure 2, 1' is moved to 1. When the room conditions are modified, then the load calculation is repeated under the new conditions. If the apparatus dew point is too low even when the conditions are modified, reheating calculations should be performed.

### 4. 3 Reheat System

In the case of a reheat system, the specified apparatus dew point is initially set to be the apparatus dew point, and the relative humidity of the air leaving the coil should be 95% to maximize the temperature difference between the supply air at the outlet and the room air, and thus to minimize the supply air volume. In the case of Figure 4, if the supply air is assigned the room dew point 6', the apparatus dew point cannot be obtained, as indicated by the dotted line, because of the inadequate selection of the state of supply air. If the total heat load, the sensible heat factor, the required outdoor air volume, and the room and outdoor conditions are given, the appropriate state of supply air can be obtained by solving the following simultaneous equations:

$$\frac{1.2 * (H_1 - H_6) * OAQ}{QT} = \frac{W_3 - W_1}{W_2 - W_1} \tag{15}$$

$$W_4 = W_6 \tag{16}$$

$$H_4 = H95(W_4) \tag{17}$$

$$\frac{H_3 - H_7}{W_3 - W_7} = \frac{H_4 - H_7}{W_4 - W_7} \tag{18}$$

$$\frac{W_3 - W_1}{W_2 - W_1} = \frac{H_3 - H_1}{H_2 - H_1} \tag{19}$$

$$\frac{H_6 - H_1}{W_6 - W_1} = \frac{597.3}{1.0 - SHF} \tag{20}$$

If the state of supply air is known, the state of air entering the fan or leaving the reheater can be obtained by eq(14). When the volume of outdoor air and the latent heat load of the room are large, however, the air temperature at the outlet approaches the room air temperature; consequently, the air volume may increase and becomes greater than the specified air volume. In this case, the room conditions must be modified and the calculations performed as in the non-reheat system.

## 5. Example of Computation

This program has been applied to the design of the equipment for a hotel, to be built in Tokyo. The hotel is 47 stories high, has 3 stories underground, and has a gross floor area of 114,600m$^2$. It has 44 cooling zones and about 330 types of rooms. Thirty of the cooling zones are to be serviced by all-air systems, of which 18 will employ the reheat system. The other 14 zones will be serviced by the primary air fan-coil system.

The input data to the program consist of about 2,500 punch cards, mostly for load calculations; the data cards for the computation of designing the system number about 50. The computer calculations take approximately 15 minutes.

The specifications of the air-handling apparatus and the psychrometric data are shown in Figure 5 as an example of the output. As can be seen from this example, the computation of psychrometric data was sufficiently accurate as compared to the manual calculations using the psychrometric chart. The manpower required to prepare the input data was approximately 50 man-hours, or about the same manpower required for the load calculations.

## 6. Conclusion

The formulas and the procedure for supply air volume computation and the psychrometric analysis can be applied to the computer design of not only a single-duct system but also a dual-duct system but also a dual-duct system or a primary air-handling apparatus of the water system.

The advantages of using the Automated Design Program are as follows:

- Great amounts of design labor and time can be saved, and engineers can promptly follow up on any change in the architectural design.
- The discrepancies arising from the designers' experience and personality can be eliminated, and even inexperienced designers can obtain the same results with equal safety and accuracy.
- Comparative studies of computations with alternative design factors are easy, thus an optimum design can be achieved.

We express our deepest gratitude to the staffs of the Kajima Corporation Designing Department and the Kajima Institute of Construction Technology for their assistance in developing this program.

## 7. References

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(2) ASHRAE, Handbook of Fundamentals (1967).

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(4) Carrier Air Conditioning Company, Handbook of Air Conditioning System Design, McGraw-Hill (1965).

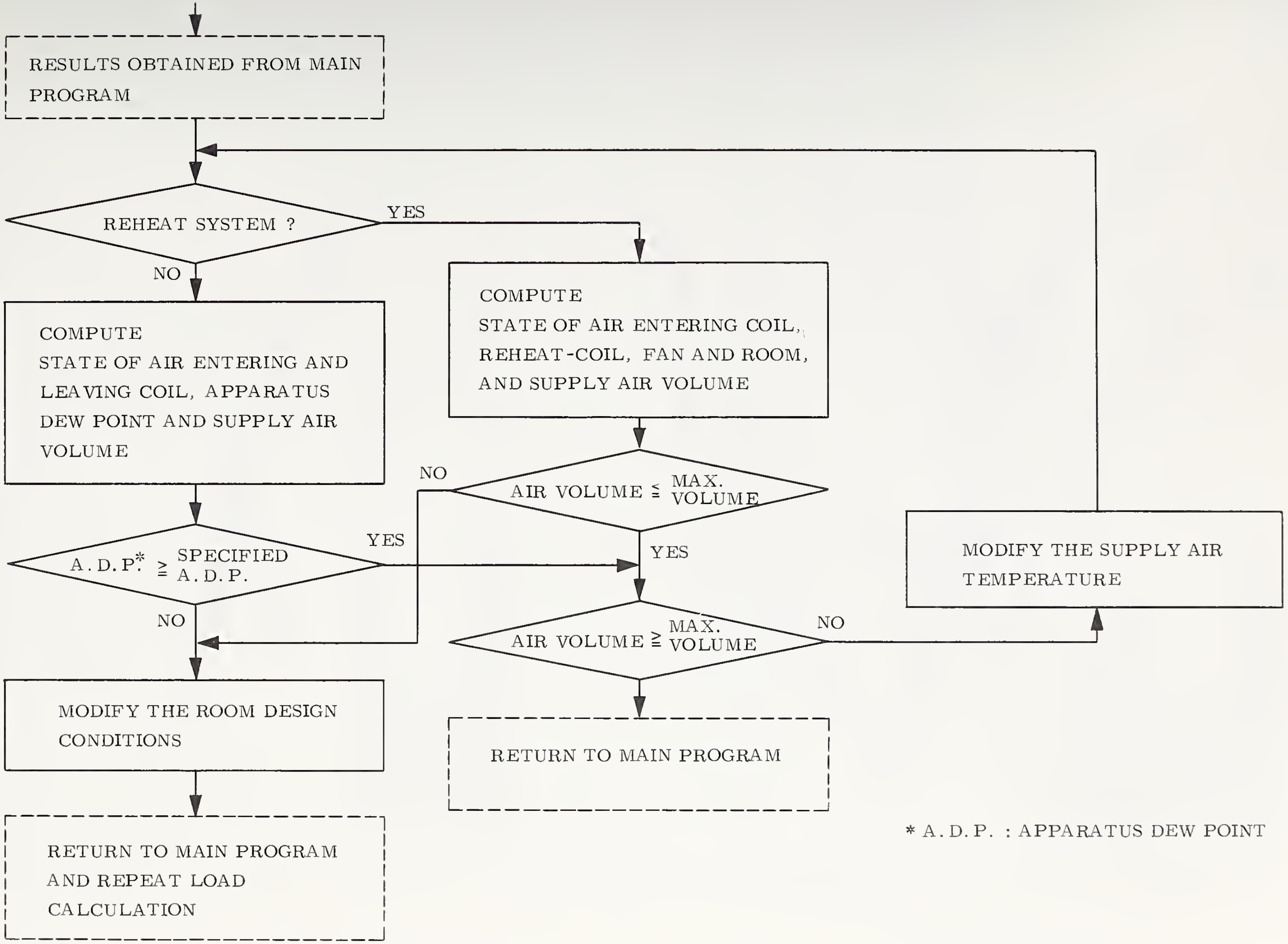
## 8. Notation

Letter symbols used in this paper are defined as follows:

AQ = Supply air volume ($m^3$/hr)
C = Enthalpy-humidity difference ratio, $\Delta H / \Delta W$ (kcal/kg)
DB = Dry-bulb temperature (°C)
DP = Dew-point temperature (°C)
FTD = Temperature rise caused by fan load (°C)
H = Enthalpy of moist air (kcal/kg)
HS = Enthalpy of moisture saturated air (kcal/kg)
H95 = Enthalpy of 95% relative humidity air (kcal/kg)
H85 = Enthalpy of 85% relative humidity air (kcal/kg)
OAQ = Outdoor air volume ($m^3$/hr)
PT = Fan total pressure (mmAq)
PW = Partial pressure of water vaper in moist air (mmHg)
PWS = Partial pressure of water vaper in moisture saturated air (mmHg)
QT = Total heat load of room (kcal/hr)
RH = Relative humidity (%)
SHF = Sensible heat factor (non-dimension)
T = Absolute temperature (°K)
TS = Absolute temperature, 373.16 (°K)
W = Humidity ratio of moist air (kg/kg of dry air)
WB = Wet-bulb temperature (°C)

Subscript symbols are used as follows:

1. refers to room design condition
2. refers to outdoor design condition
3. refers to condition of air entering coil
4. refers to condition of air entering reheater
5. refers to condition of air entering fan
6. refers to condition of supply air
7. refers to apparatus dewpoint



* A.D.P. : APPARATUS DEW POINT

Fig. 1 Generalized Flow Chart of Supply Air Volume Calculation Subprogram.

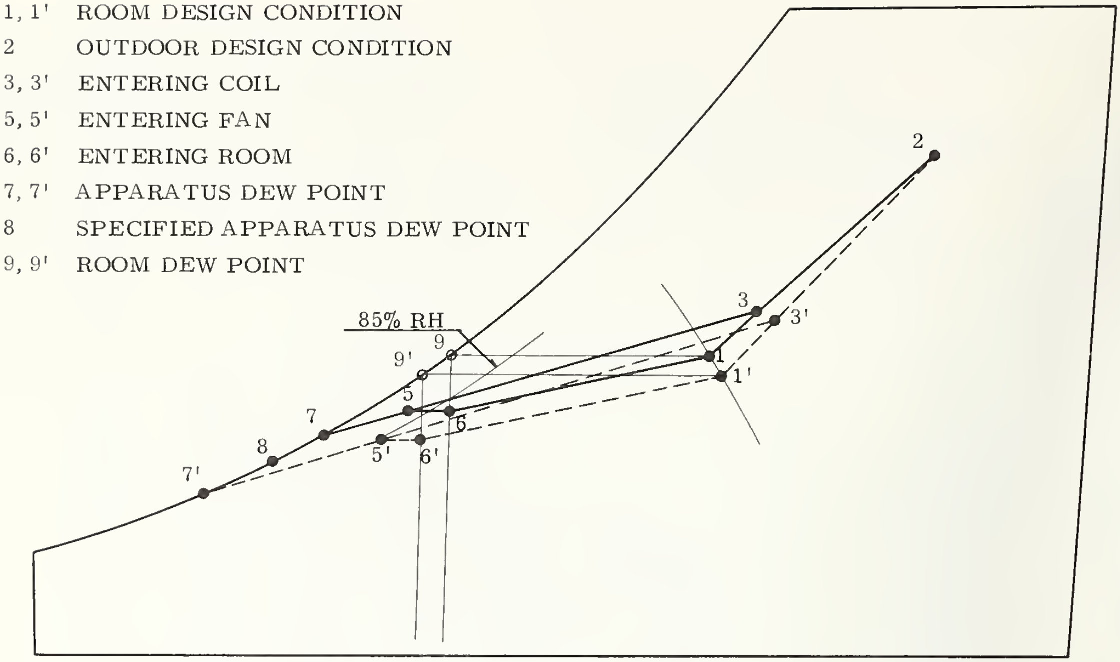


Fig. 2 Psychrometrics of Non-Reheat System.

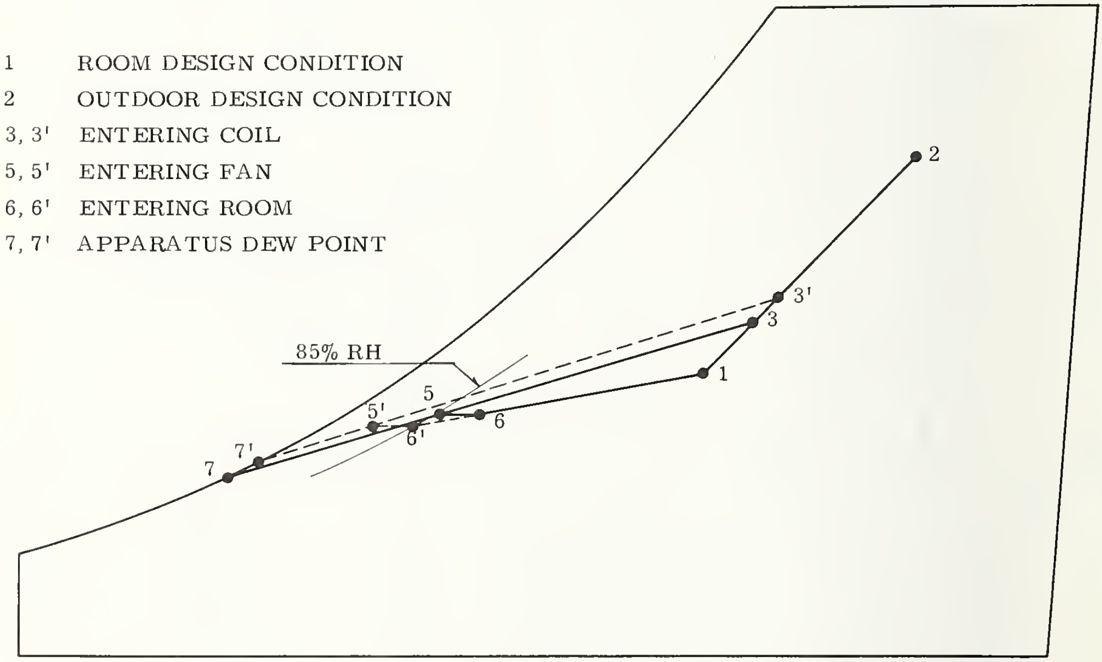


Fig. 3 The Case where the State of Supply Air is Determined under the Limited Condition of Minimum Air Volume.

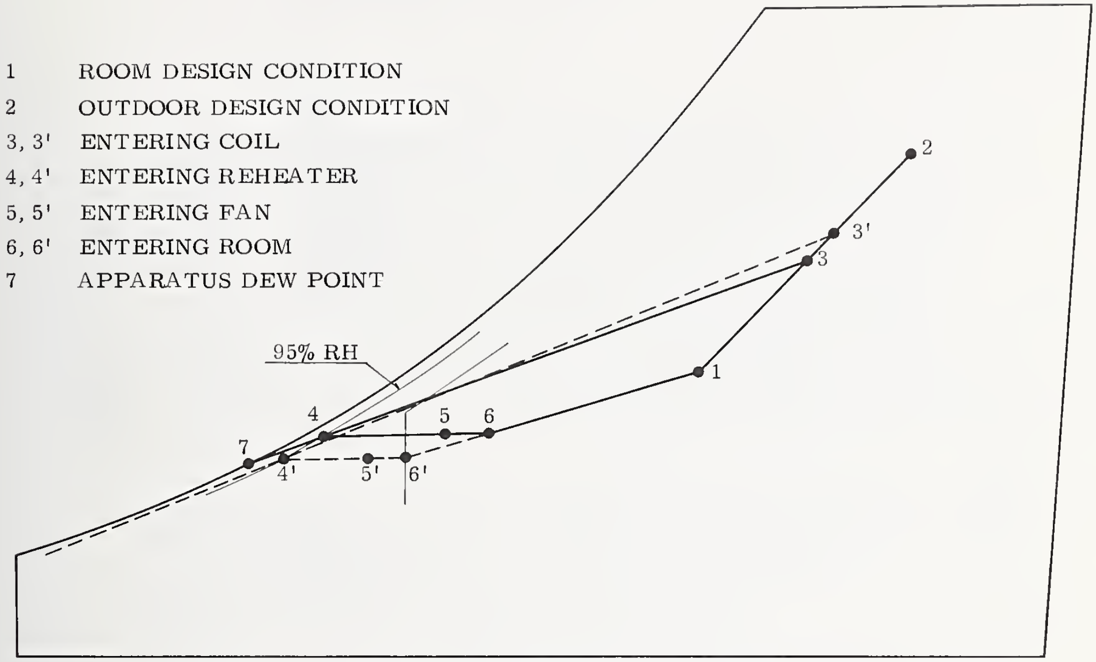


Fig. 4 Psychrometrics of Reheat System.

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** SPECIFICATION OF AIR HANDLING UNIT

   ZONE NO.  2  CONTROL CENTER            ***** SINGLE DUCT SYSTEM


  *  UNIT SIZE
                NO.   9

  *  FAN
                NO.         *   7000. CM/H    *   80. MMAQ  *     3.75KW   *   1

  *  COOLING COIL
                F.P. 3.5 MM  *  20 TUBES/ROW  *  1150. MM    *   6.ROWS(S) *   1

  *  HEATING COIL
                F.P. 3.0 MM  *  20 TUBES/ROW  *  1150. MM    *   1.ROWS    *   1

  *  HUMIDIFIER
                STEAM GRID   *    29. KG/H

  *  AIR FILTER
                ROLL         *   0.9 SM       *    0.2 KW


** SYSTEM DESIGN DATA

  * PSYCHROMETRIC DATA             ( COOLING )                                                  ( HEATING )
                              DB       HR       ENTH      DP                                DB       HR        ENTH

      1 ROOM DESIGN          26.0    0.01049   12.63                1 ROOM DESIGN          24.0    0.00740    10.26
      2 OUTDOOR              33.6    0.01840   19.33                2 OUTDOOR              -2.9    0.00160     0.26
      3 ENTERING COIL        30.6    0.01529   16.69     20.5       3 ENTERING COIL         7.7    0.00389     4.18
      4 ENTERING FAN         14.5    0.00953    9.23                4 ENT. HUMIDIFIER      39.6    0.00389    11.90
      5 OUTLET               15.9    0.00953    9.58                5 OUTLET               40.0    0.00740    14.16

  * HEAT SOURCE DATA
                         CAPACITY   WATER VOL.   WATER TEMP.    STEAM Q.   STEAM PRESS.
                         (KCAL/H)    (L/MIN)    INLET  OUTLET    (KG/H)     (KG/SCM)

       COOLING COIL       62664.      200.        6.3    11.5
       HEATING COIL       64829.                                 125.3        2.00
       HUMIDIFIER         18934.                                  29.1        0.35
```

Fig. 5   Example of the Output.

# Computer-aided System
# for
# Preliminary Air Conditioning Design

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In preliminary designing stage of air conditioning system, there is a time when the information necessary for deciding the zoning, type of system, general layout, types of equipment, cost, etc. must be obtained by preparing a schematic design of the proposed system to see if it will meet the basic design requirements. The information thus obtained may be fed back to those concerned so that they can make necessary project coordination. In order to make it possible to deal with the work required at such stage effectively by utilizing a computer, the authors developed a system titled COSMA which was intended for continuous processing of heat load computation, equipment selection, pipe and duct designing, etc. Based on the experienced gained through its trial use, COSMA is now being reorganized into SPACE (Subprogram and Phrase for Air Conditioning Engineers) which is featured by the following characteristics:

1. A variety of subprograms and files is available so as to enable the selection of various equipment, basic design of ductwork and piping work, cost estimating of principal elements, etc. on the basis of heat loads.
2. The system is flexible and allows free combinations of phrases in accordance with thinking sequence of the designers.
3. Simplified input data are another feature of this system.
4. The system is composed of subprograms which are minutely divided into functional units by well documented and classified files. These subprograms and files are designed to be easily expanded, modified or adapted to cope with varied conditions. Moreover, considerations have been given so that they can, in future, be coupled with the computer-aided systems for other engineering areas such as electrical, plumbing, architectural and structural.

This system enables the designers to obtain the basic data which are more concrete and objective in a shorter time, and thus makes it possible to study more alternatives within the given time.

Key Words: Preliminary air conditioning design, computer-aided system, equipment selection program, duct design program, pipe design program, AC system design program.

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[1] Mechanical Engineers

# 1. Need of Developing a Computer Program for Preliminary Design

At an early stage of architectural planning, an air conditioning engineer must propose a preliminary air conditioning design which will satisfy the given conditions. At this stage, he must make necessary investigations and propose a creative design based on his professional intuition and experience. Then, he must check to what extent the aforesaid proposal will satisfy the given conditions, decide whether the proposal should be adopted, and, if it is to be adopted, determine the optimum scope of the system. If not, he will have to make some alternative proposal, or request the architect to restudy and modify the architectural design, or in some cases, he will have to ask to modify the basic design conditions and requirements in conformity to which he has prepared the original proposal.

The data required at this stage will include: (1) the initial instllation cost; (2) the operating cost; (3) the physical size of each element of the system to give the basis for coordinating the relation of architectural and structural designs with the air conditioning design; (4) the data indicating the anticipated ambient conditions (including the values, changes and distribution of temperature, humidity, air flow, etc. in each room and parts thereof) which will be created by the proposed system. The data is important because they will enable the mechanical engineer to foresee if these conditions meet the design requirements.

The data described above need be prepared and evaluated by two methods: in the method (A), the evaluation of the data (1), (2) and (3) will be made through a statistic approach, namely on the basis of analyses of assembled data, while the data (4) will be assessed on the basis of professional experience; and in the method (B), the data (1), (2) and (3) will be obtained by the schematic design studies of the proposed system and the data (4), by means of simulation. (It should be noted that the methods (A) and (B) are mutually supplementary, and one is not enough without the other to insure an accurate and proper determination.)

It, however, may be generalized that the method (B) is more direct and convenient in seeing quantitative differences beween a number of proposed systems with respect to the data (1), (2) and (3).

Since a considerable part of the work required at this stage can be systematically structured, a utilization of a computer at this stage should enable a mechanical designer to evaluate a larger number of alternative proposals in a comparatively short time, and this should make it possible to make feed-backs to the architect and the owner in a more effective manner. For this reason, the development of a computer-aided system for the aforesaid schematic design studies is highly desirable.

The firm of Nikken Sekkei took to the development of such a computered system in 1967, and completed the first edition of the program in 1969. Based on the experience in using this program, the firm began to revise the program at the end of 1969. In the following paragraphs, the authors intend to describe briefly the first edition of the program, the advantages derived from it, the reasons why the first edition had to be revised, and the concept and present status of the new system now in the course of development.

# 2. Program Group for Schematic Designs COSMA - First Edition

## 2. 1. General Description

The scope of COSMA First Edition is as shown in figures 1 and 2 and the program has been in use at Nikken on some of the actual project on a trial basis.

Basically, COSMA is composed of the following three groups of programs:

(1) Programs for a continuous data processing from heat load computation to selection of various equipment.

(2) Programs for designing duct and piping systems

(3) Programs which, for some typical perimeter zone and interior zone systems often adopted in practice, enable a continuous data processing, by means of simplified input data, for all design stages such as heat load computation, selection of terminal units, pipe and duct design and cost estimation.

### (1) Outline and features of program for heat load computations and equipment selections

The first step in a series of processes described in this paragraph is to define space units and zones in the building for the purpose of planning. The word "space-unit" may be defined as a minimum unit of space which is used as a basis for the load computation. The term "zone" as used here denotes a building space which is regarded as a minimum independent area in air conditioning planning, and zones are classified depending on their load characteristics and the intended purposes. Each zone is defined as a group of space units. The program to be discribed later in this paragraph use the aforesaid unit space and zone as a basis. The program is capable of dealing with up to 36 types of space units and 60 zones for one building.

The second step is to make heat load computations to give a basic data for the entire air conditioning system design which includes the selection of equipment and the designing of ducts and piping. The program introduced here is so contrived as to enable numerous detailed computations required for large buildings with well-organized simple input data. The computation according to this program follow the steps mentioned below: (1) data for calculation for the various types of interior are fed to the computer, each input data card representing one type; (2) next, various combinations of the perimeter and the interior types and requirements for fresh air intake are fed for each space unit with two cards per space unit. Based on the foregoing input data, the computer calculates the heat loads for all space units and then sums them up to give the heat load for each zone. The zonal heat loads are further summed up to give the heat load for the whole building. Then, as the last step in heat load computation, flow rates of air, chilled and hot water, steam and other media are computed for each zone.

The third step in this program is for the user to assign a group of chillers, boilers, pumps, air conditioners, etc. to each zone according to the air conditioning system under consideration. Each group of chillers etc. is given an identification number; and the zone or zones to be served by the respective group are indicated by the aforesaid identification number. Also, the number of equipment needed for each group is determined.

The input data are given in the form of cards, one card being provided for each zone. The number given at the equipment type column on each card indicates the identification number of the set of equipment which takes care of the zone the card represents. The number of equipment for each group is indicated on the same card. A pump may be designated to handle chilled water, hot water or condenser water to meet seasonal requirement. Thus, if the chilled water pump column and hot water pump column of the card are given the same number, it means that a single pump is to handle both chilled water and hot water.

The fourth step is to select equipment on the basis of the heat loads, flow rates and system designations obtained in the previous step. At this stage, such requirements as types of apparatus are also specified on the input data. The data necessary for specifying such requirements can be taken from the information supplied by the manufacturers, arranged and stored on disks.

The output data give the information on the type, performance, size, weight, cost, etc. of each equipment. For some kinds of equipment, this information is given in the form of a comparative table showing performance etc. of various makes and models.

Sample input data showing zone-equipment-system relations (Simplified for explanatory purposes)

| Zone | Chiller | Boiler | Pump | | | | Air Conditioner |
|---|---|---|---|---|---|---|---|
| | | | Condenser Water | Primary Chilled Water | Secondary Chilled Water | Hot Water | |
| 1 | 1 | 1 | 1 | 4 | 7 | 1 | 1 |
| 2 | 1 | 1 | 1 | 4 | 8 | 1 | 2 |
| 3 | 1 | 1 | 1 | 4 | 9 | 1 | 3 |
| 4 | 2 | 1 | 2 | 5 | 5 | 10 | 4 |
| 5 | 2 | 1 | 2 | 5 | 5 | 10 | 5 |
| 6 | 2 | 1 | 2 | 5 | 5 | 10 | 6 |
| 7 | 3 | 1 | 3 | 6 | 6 | 6 | 7 |

The meaning of the above input data is:
No. 1 chiller is for Zones No. 1, No. 2 and No. 3.
No. 1 boiler is for all zones.
No. 1 pump is for condenser water and hot water for Zones No. 1, No. 2 and No. 3.
No. 2 pump is for condenser water for Zones No. 4, No. 5 and No. 6.
No. 6 pump is for primary-secondary chilled water and hot water for Zone No. 7.

## (2) Outline and features of piping and duct programs

In the piping program, the locations of equipment, the required flow rates of water, the desired flow resistance, etc. are fed as input data, according to which the computer calculates the appropriate pipe diameters at respective sections of pipeline, and gives the pipe sizes, flow resistance, lengths which are useful for cost estimation, etc. Pipe sizes are determined to equalize the resistances of all branches. Input data are prepared by nodal description method. In the duct program, the design, cost estimate, outlet and mixing box selection, etc. are processed on the basis of input data which are prepared by nodal description method.

## (3) Outline and features of the programs for AC system

The flow chart in figure 2 indicates what can be processed by this set of programs. In computerizing the process of obtaining necessary information through schematic designs of a fan-coil unit, an induction unit, a dual-duct or a single duct system, the greatest difficulty is usually encountered in how to computerize the designing of duct and piping systems. This difficulty results from the fact that the input data become, more often than not, highly complicated in proportion with the importance of the information. To avoid this difficulty, the nodal description method is dispensed with in preparing input data for duct and piping design as shown in figures 4 and 5.

Instead, a set of input sheets into which an architectural modular grid is entered in the form of matrices based on the elevations and plans is used. On these input sheets, the designer writes either in symbols or in numerals, the designations as to the following: (1) the types of systems to be studied (the name of systems and designation of the number of pipes, i. e. , 2-pipe, 4-pipe, etc. ); (2) the type designation of duct and piping system; (3) the type of piping (reverse-return etc. ); the data necessary for computations (computation method, friction loss per unit length, etc. ). The program is formulated in such a way that heat load computation, selection of terminal units, and further duct and pipe computations and cost estimating can be performed by a computer. As shown in figure 3 , these program requires only two input data sheets to study up to three different perimeter systems.

(4) Scope of the program

| | No. of Main Programs | No. of Sub-Programs | Steps | Fixed Data (Sectors) |
|---|---|---|---|---|
| Psychrometirc Chart | 0 | 18 | 300 | |
| Heat Calculation and Flow Rate | 8 | | 1,000 | 35 |
| Equipment Selection | 38 | 9 | 5,000 | 715 |
| Perimeter System | 1 | 8 | 3,500 | 23 |
| Interior System | 1 | 9 | 3,000 | 28 |
| Piping | 1 | 1 | 500 | 1 |
| Ductwork | 2 | 7 | 2,000 | 10 |

Altogether, these amount to one and one half disks in area.

## 2. 2. COSMA promoted design efficiency

The authors utilized COSMA when they designed the air conditioning system for a 20-storied building having a floor area of about 10,000 sq. m or 106,500 sq ft. In this case, the computer (IBM 1130) was used to process those data which are shown above the dotted line in figure 1, or in other words, the data concerned with the steps from the heat load computation to the equipment selection and study. The relevant information desired by the designers was obtained in less than five hours, the designers spending two to three hours for input data preparation and the computer personnel spending about two hours for punching, running, etc.

Also, COSMA was utilized in designing the air conditioning system for another 20-storied building with a floor area of some 30,000 sq. m (318,000 sq ft). In this case, it was assumed that the whole system was composed of 8 perimeter systems and 12 interior systems, and three alternative system types (induction unit system, fan coil system and dual duct system) for the perimeter and also three system types (single duct system, dual duct system and individual zone system) for the interior were considered. It took about 8 hours for input data preparation, 3 hours and 15 minutes for computer processing, and another 8 hours for manual assorting and editing of the output data.

From these experiences, it was learned that all the data processing for COSMA System could be done in one day if the output data were limited to the point of equipment studies, and in about three or four days if the output data were to include those concerning duct and piping designs and comparison of alternatives. With the efficiency thus achieved, the system was considered capable of serving the intended purpose satisfactorily.

## 2. 3. Some problems with COSMA

After having been used for the past year on a trial basis by some machinery engineers of our firm, it is considered that COSMA should be improved with respect to the following:

a. COSMA consists of a number of subsystems; however, it is rather strongly characterised by its being one inseparable system. This character of the system makes it difficult to pick out and combine desired programs freely as the occasion demands.

b. The need is felt that the system should allow for a greater freedom in setting up the most desirable flow of data processing for each individual project.

c. COSMA is capable of dealing with the air conditioning systems generally used at present. The improvement of the system to make it easily adaptable to new design situations, such as the development of new equipment or unique air conditioning system combinations, is considered necessary.

d. It is desired that COSMA have a means of output control so that the computer can give the specific information required at any given phase of the data processing.

e. It is desired that COSMA be made capable of aiding the design work from the earlier stage of design than is possible with the present program.

## 3. Development an Assembly of Phrases for Schematic Designs - Development of SPACE (Subprograms & Phrases for Air Conditioning Engineers)

### 3. 1. Two approaches to the development

There appear to be two approaches to arrive at the solutions for the problems described in the paragraph 2. 4. , with the original system being used as basis of the further development.

#### (1) Developing a group of approximation formulas

In one way of approaching the solutions, the whole group of programs is regarded as a data generating system, with the conditions given to COSMA being changed systematically. The resulting output data can be handled and organized as in the case of statistic data. Then, a variety of approximation formulas can be obtained for these output data, and this group of formulas can be converted into programs to obtain important information.

#### (2) Subprograms and phrases

Another approach to the improvement of original COSMA is to decompose it into as many minute mono-functional subprograms as possible and, at the same time, to standardize all the files.

These subprograms and files can then be organized as required by application of IBM-plan technique (1)[2] into a set of phrases for air conditioning designs. It is expected that this process can eliminate the drawbacks of original COSMA to a large extent without losing its features.

The air conditioning designers who use this system (SPACE) are not supposed to look at the input data sheets and make entries for the necessary items disregarding the inapplicable optionals, all according to the designated processing sequence. Instead, the designers are requested to arrange the phrases and data as necessary for the designs, so that they can use the program in a more flexible and versatile manner. The system as has been developed to date is as outlined in the following paragraphs.

### 3. 2. General setup of SPACE

---

[2] Figure in parenthesis indicates the literature referenced at the end of this paper.

### (1) Outline of the system

Subprograms:

At present, our primary efforts are being directed to dividing COSMA subroutines into as minutely classified subprograms as possible and setting up appropriate output controls which precede output instructions. The number and scope of such programs, therefore, can be surmised by reference to the scope of COSMA described in the section 2 of this paper.

File group:

The files are generally organized as follows:

(i) Common data files - These files contain the data applicable to all kinds of projects.

(ii) Project files - For these files, only the framework of data storage system (the name of file, the sequence of data filing, etc. ) is predetermined, so the designer must compile a file for each individual project by using the applicable phrases.

The major files of each category will be described in Appendix A.

Phrases:

Presently, the COSMA subprograms which have been decomposed are being reorganized into a set of new phrases. The composed phrases will serve the purpose of primary operations. As the next step, the need is felt to develop the phrases which can be used for secondary operations. These phrases for secondary operations should be capable of modifying a part, not all, of the input data given out in primary operations, and of reprocessing the phrases which have been modified. For instance, such a phrase may read "SELECT CHILLERS FOR CHANGED CONDITIONS."

### (2) Relationship between subprograms and phrases

Since it is apparently impossible to describe the computered design process for an entire air conditioning system within the space allowable for this paper, the functions of subprograms and the subprogram-phrase relationships will be described for some of the phrases used in perimeter system designs. The function of individual subprograms will be further described in Appendix B.

| | |
|---|---|
| ALC-33 | Instructions to read the project titles etc. |
| ALC-34 | Instructions to read the design climatic conditions |
| LAC-44 | Instructions to compute heat loads |
| AAV-44 | Air volume computation |
| AOAV 1 | Designation of primary air |
| AFCO 2 | Selection of fan coils |
| AZON 3 | Subprogram for defining the zoning by matrix |
| APMO 1 | Piping design for unit space expressed by matrix |
| ADMO 1 | Duct design for unit spaces expressed by matrix |

#### (a) Some examples of phrase usage

The design can be worked out by a variety of methods by utilizing the foregoing subprograms as shown by the following examples. (The subprograms listed in parentheses are shown for explanation only, and are necessary only for defining the phrases.) In actual use for a specific project, phrases and data only need be stated.

On the outset of a project, the heat computation is conducted:

LOAD CALC BY ALC : (ALC-33, ALC-34 & ALC-44)

At the next step, the phrases such as the following may ensue:

(1) SELECT FAN COIL (AFCO 2):
The instructions that proper fan coil units be selected for each space unit.

(2) SELECT FAN COIL WITH PA (AOAV 1 and AFCO 2):
The instructions that proper fan coil units be selected against the load differencial computed by deducting the cooling capacity of the primary air supplied to each unit space according to the, applicable criterion from the loads on space units.

(3) ZONING BY MATRIX; ZONE _____ (AZON 5)
DESIGN FAN COIL SYSTEM (AFCO 2 APMO 1)
By these phrases, a computer is instructed to select fan coils and execute piping design and cost estimating on the basis of the given data indicating the proposed space unit layout and piping system.

(4) ZONING BY MATRIX; ZONE ______(AZON 5)
DESIGN FAN COIL SYSTEM WITH PA (AOAV 1, AFCO 2, APMO 1 and ADMO 1)
After the primary air supply data have been fed and fan coil units have been selected as described in (2), the piping for fan coil units and the ductwork for primary air are designed and estimated by the instructions phrased as above. In this example, space unit layout only has been instruted by ZONING phrase, and separate instructions as to how the pipes and ducts should be laid out must be given by the imput data included in APMO 1 and ADMO 1. Therefore, no trouble will ensue even if the duct layout differs from the piping layout.

For single duct and other systems, the phrases may be combined generally in the same manner as described above.

## 4. Conclusion - Versatility of SPACE System

The authors wish to conclude this paper with a brief statement on the development potentiality of this system.

i. The input data for this system can be further simplified by unifying them with the files for other engineering disciplines. If the physical data on space units are included in the files prepared by architects, structural engineers or cost estimators, the physical data necessary for heat load computations can be taken from such files.

ii. The system is adaptable to the future development of equipment. For instance, if it is assumed that a new type of equipment is developed for air and water system or for water system, and that the equipment is named XUNIT and a new program named XPROG is developed for selecting this equipment (such a program should be developed without difficulty because it is to serve a simple purpose of selecting a type of equipment and computing water and air flow rates), the phrases to be used for the design of the system wherein such new equipment is used can be completed simply by replacing the word FANCOIL in the phrases (1) - (4) with the words XUNIT and by using the word XPROG in lieu of AFCO 2 in the program list.

iii The system has high adaptability to a variety of system combinations.

iv The system has an advantage in that,in developing programs, subprograms may be developed by an engineer or a programmer, and the phrases can be developed by someone else who systematizes that subprogram.

## 5. References

(1) IBM, Program language analyzer (Plan), Program description manual, January 1969

# 6. Appendix

## Appendix A Structure of files

(1) Common Data Files
- a. Foundamental data file consisting of basic data on climatic conditions, physical properties, etc.
- b. Catalogue data file consisting of the data on equipment performance, sizes, cost, etc. which have been taken for storage from trade literature.
- c. Statistic data file (Data to be stored in future)
- d. Standard data file (Data to be stored in future)

(2) Project Files
- a. Physical data files concerning space units:
  The data in these files shows the shape, dimensions (length, width and height), and other physical data on space unit as a shell.
- b. Air conditioning system files:
  These files contains the information relating to the zoning and system designations.
- c. Equipment data files:
- d. Heat load files:
  The results of heat load computation and the loads as determined from the required fresh air volume are filed.
- e. Air data files:
  These contain the data relating to the air volume, primary air volume, pressure loss, etc. for each space unit.
- f. Water data files:
  Water flow rate, head loss, etc. for each space unit are filed.
- g. Files of data on selected equipment:
  The framework to be determined in future.

## Appendix B Description of subprograms

| | |
|---|---|
| ALC-33 | This subprogram instructs a computer to read the title of project, the names of designers, the data the design was executed, etc. |
| ALC-34 | This instructs a computer to read such design conditions as the temperature and humidity of outside and room air, and the shadow factors. ( → Design conditions and → shadow factors files) |
| ALC-44 | This instructs a computer to read the data on each space unit ( → Space unit file), to compute the area, volume, etc. ( → Physical data file for space unit), and to compute heat loads ( → Heat data files). Where the zoning is defined at this stage, the zoning information is also stored in the appropriate file ( → System file). |
| AAV-44 | The name of zone for which computation is to be made; the computation method(① by ADP, ② by the predetermined temperature difference of supply air, ③ by the number of air changes, ④ by the air volume per unit area, or ⑤ by the predetermined air volume per person in the unit space), and the criteria for fresh air volume ( ① per area, ② per person, ③ per air change, or ④ by percentage of the total air supply) are to be instructed. On the basis of these instructions, a computer will, by reference to Heat data files, calculate air volume ( → Air data files), fresh air volume ( → Fresh air data files), the conditions at coil inlets and outlets ( → Coil data files) for both summer and winter and will store all these data in the appropriate files. |
| AOAV 1 | In accordance with the supply requirements (per area, per person or on the basis of designated value) provided for each unit space within a designated zone, this subprogram |

instructs a computer; to calculate, making reference to the data on area, number of persons, etc. in architectural file, the primary supply air volume ( → Primary air files); to compute the cooling or heating capacity of primary air under the given supply conditions ( → Coil files, with the design outside air data being filed also in Coil files as the coil inlet conditions); and to compute the coil loads ( → Coil files).

| | |
|---|---|
| AFCO 2 | This subprogram instructs a computer to select appropriate fan coil units for each unit space within the zone under consideration, making reference to Heat files, and also to Cooling and Heating capacity files if so instructed (Type, number, water flow rate, pressure loss of the units to be filed in Water files). |
| AZON 3 | This subprogram instructs a computer to transform the simplified space unit arrangement given in the form of a matrix into a perfect arrangement and produce it as output. ( → Zoning files) |
| APMO 1 | Where unit space arrangements are shown by matrices, this subprogram instructs a computer: to determine the flow rate, diameter and flow velocity for each branch line according to such input data as the piping layout pattern numbers, the designation as to the loop-reverse, design friction head loss, etc. ; and further to obtain the total head loss ( → Water files) and the pipe lengths by the diameter. ( → Material files) |
| ADMO 1 | This subprogram is basically same as APMO 1. |

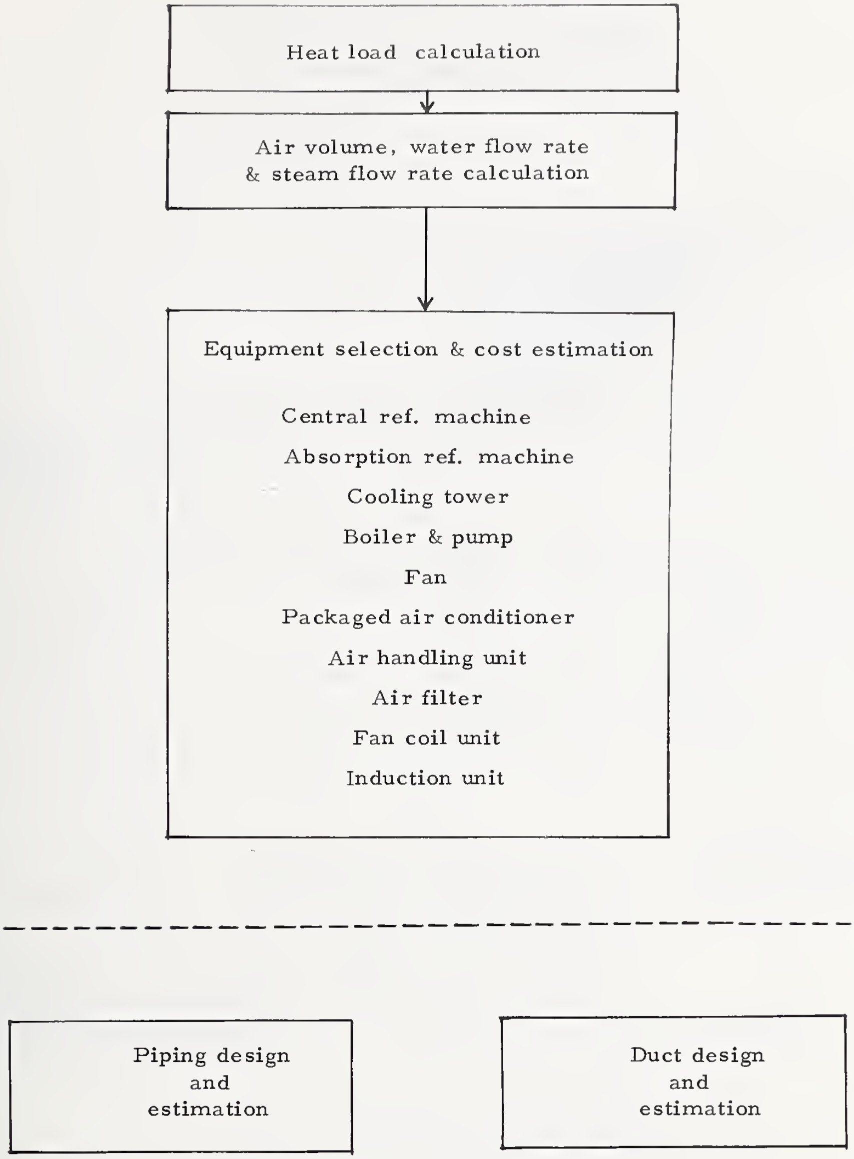


Fig. 1 The scope of COSMA

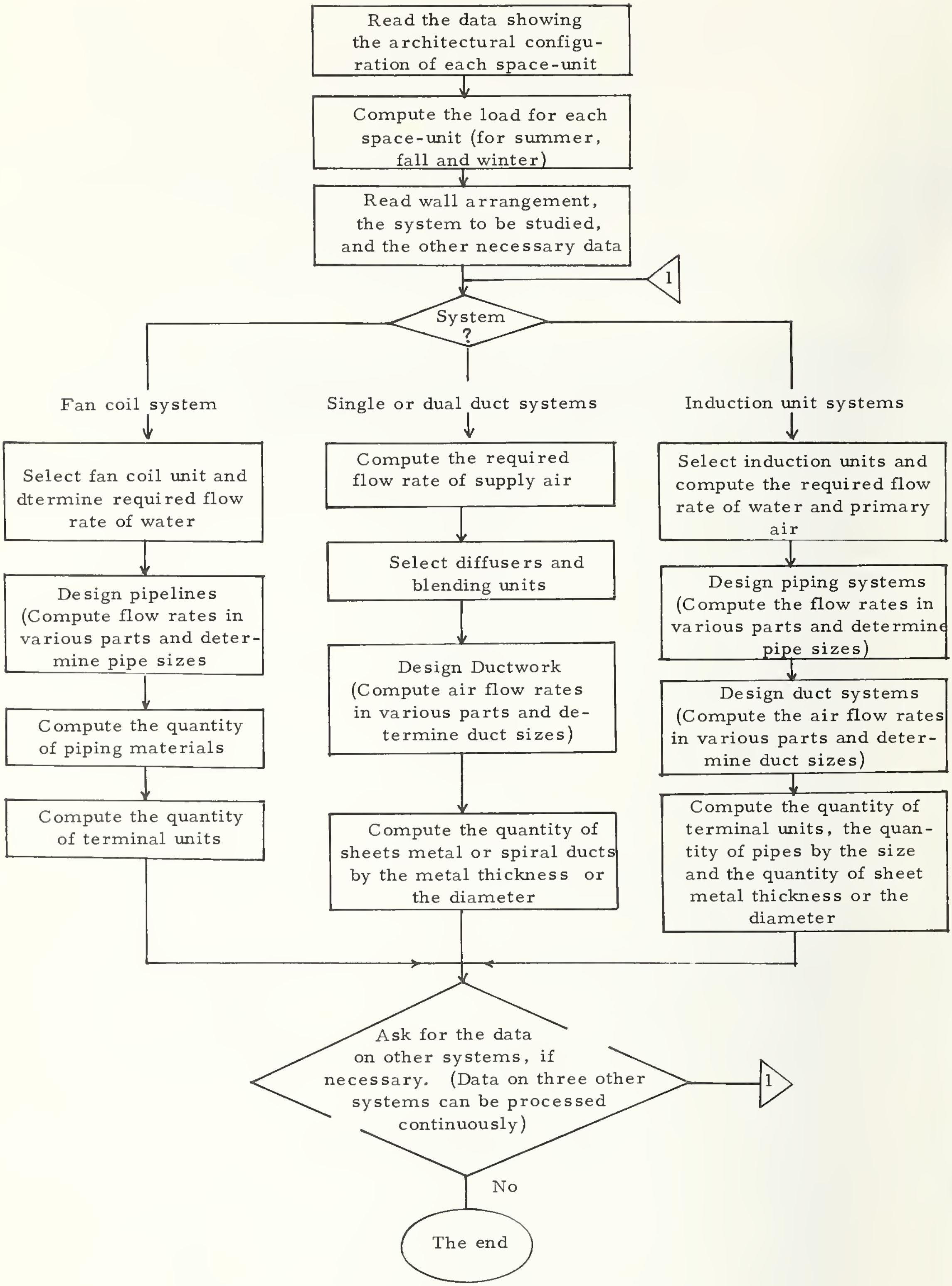
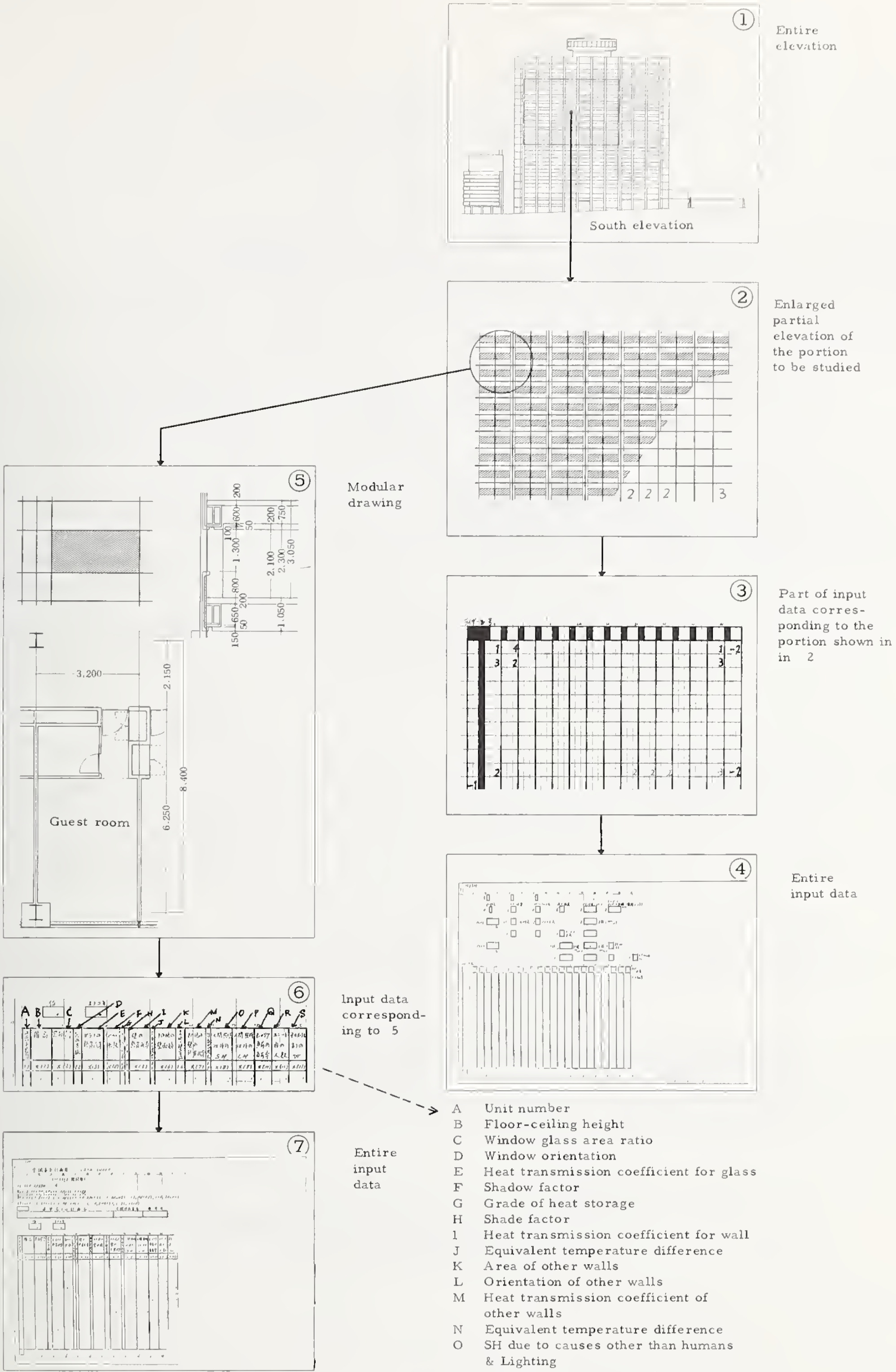


Fig. 2 Flow chart for perimeter system program



| | |
|---|---|
| A | Unit number |
| B | Floor-ceiling height |
| C | Window glass area ratio |
| D | Window orientation |
| E | Heat transmission coefficient for glass |
| F | Shadow factor |
| G | Grade of heat storage |
| H | Shade factor |
| I | Heat transmission coefficient for wall |
| J | Equivalent temperature difference |
| K | Area of other walls |
| L | Orientation of other walls |
| M | Heat transmission coefficient of other walls |
| N | Equivalent temperature difference |
| O | SH due to causes other than humans & Lighting |
| P | LH due to causes other than humans & Lighting |
| Q | Load factor for interior loads |
| R | No. of occupants in a unit |
| S | W per sq. m |

Fig. 3 Input data arrangement (for perimeter system study program)

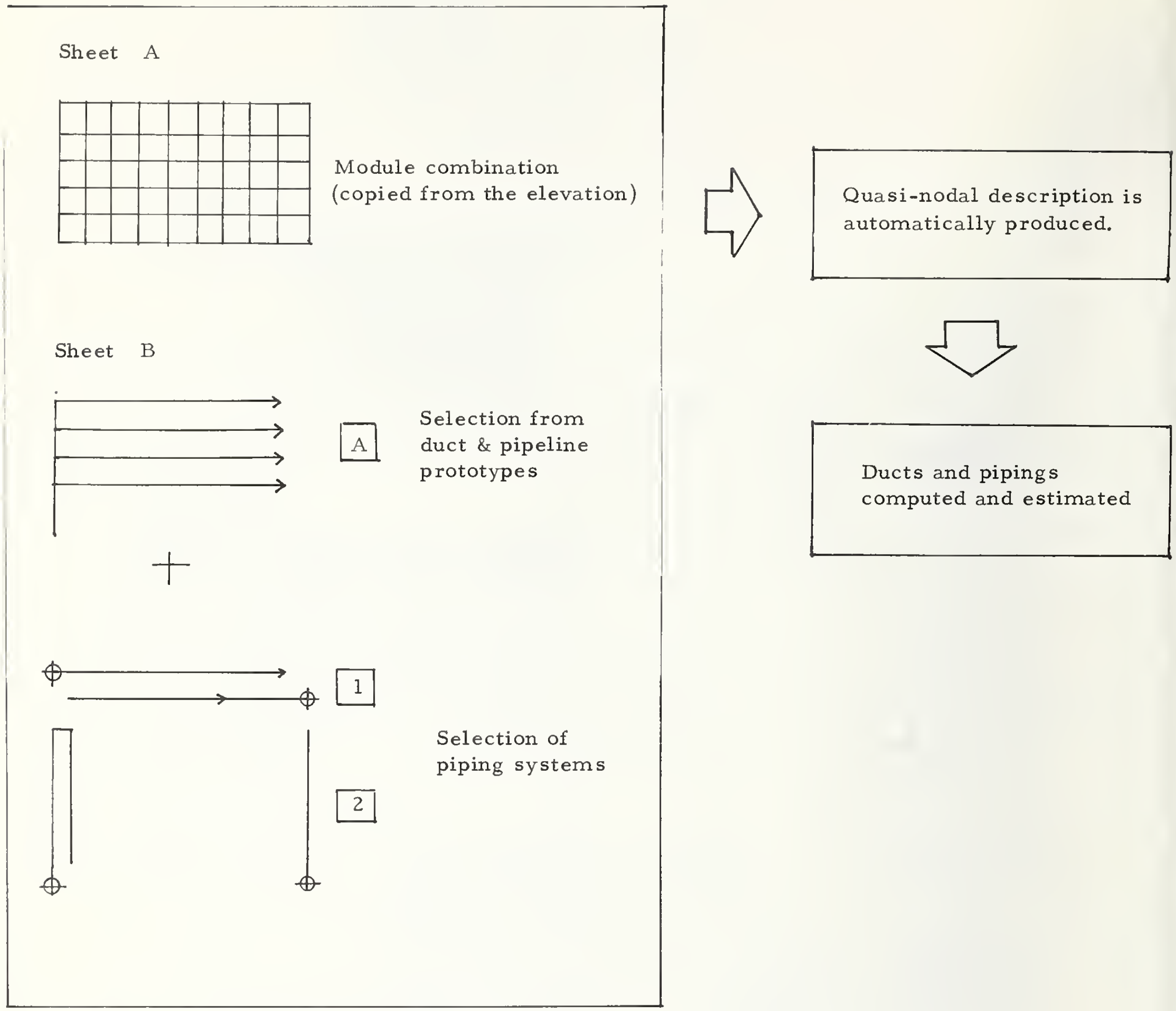


Fig. 4 Schematic chart showing the duct and piping data processing procedures (for perimeter system program)

# Computer selection and evaluation of Design Weather Data

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The need for hourly design weather data, particularly in climatic situations typified by large diurnal variations in the individual elements, is stressed.

For the purpose of determining design data, typically hot days are first selected on the basis of either computed daily maximum sol-air temperatures or daily maximum dry-bulb temperature occurring on 10 per cent, 5 per cent and 2.5 per cent of the occasions for the period under consideration. Typically cold days, on the other hand are selected on the basis of daily minimum temperatures occurring for the same percentages of the time.

The design data are evaluated from coincident values of the four elements, viz. dry-bulb temperature, humidity, wind and solar radiation, as they actually occur in practice.

The foregoing is accomplished by the following Fortran IV computer programmes:

SELECT - This programme computes daily maximum sol-air temperature, threshold daily maximum sol-air temperatures. It lists the year and day of year on which the daily maximum sol-air temperature equals or exceeds the threshold values. In addition it computes hourly values of Linke turbidity factor and precipitable water vapour content of the atmosphere.

SOLAR - This programme computes hourly values of direct solar radiation, diffuse solar radiation and atmospheric transmission coefficient for clear sky conditions for places for which such information is not available, from estimated hourly values of turbidity coefficient and solar constant.

DWDATA - This programme computes design weather data selected by programme SELECT, OR, from hot days selected on the basis of daily maximum dry-bulb temperature, OR, from cold days selected on the basis of daily minimum dry-bulb temperature. The solar radiation information is computed for horizontal as well as vertical surfaces.

Key Words: Weather data, design, computer, computer programmes, FORTRAN, thermal performance, buildings, Sol-air temperature, Linke turbidity factor.

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## 1. Introduction

The climate over the major portion of Southern Africa can generally be described as warm and dry. Such climates are usually typified by large diurnal variations in air temperature, high solar radiation intensities and an abundance of sunshine. For instance, the mean annual duration of sunshine for the western interior of South Africa is more than 80 per cent of the maximum possible duration, whilst over the coastal areas it rarely falls below 60 per cent except along the west coast where 50 per cent is a more likely value.(1) Under such circumstances the outside surface of the exposed elements of a building are alternately heated strongly during the day and cooled strongly at night so that there is a marked periodic flow of heat in and out of such elements.

## 2. Selection of Weather Data

Design weather data for evaluating the thermal performance of buildings subjected to periodically fluctuating heat flow conditions must be given on at least an hourly basis. Furthermore any method of selecting weather data from which design information is to be evaluated should be aimed at describing the weather on typically hot days in summer and typically cold days in winter. Defining a hot or cold day with respect to the thermal performance of buildings, however, is still a matter of speculation, although computed daily maximum sol-air temperature seems to offer the best possibility.

Winter design data should obviously be based on weather conditions that induce the maximum cooling of a structure. Under South African conditions, days with the lowest minimum temperature would represent such conditions in the interior, since, generally speaking, these low temperatures are due to high rates of radiative cooling during the night. However, in certain areas, such as the South-Western Cape, maximum cooling rates are usually experienced during the passage of a cold front with accompanying high winds and low temperatures.

More specifically, the selection of weather data for design purposes is more usefully made in accordance with the probability that similar or worse weather conditions will occur on as relatively few occasions during the summer and winter as will justify the specific design criteria; the data should therefore be selected from coincident values of the pertinent weather elements as they actually occur in practice and should be expressed in terms of the diurnal variation of the elements. Runs of successive days of hot or cold weather of different duration must also be evaluated.(2,3,4)

Five years of hourly air temperature, relative humidity, wind and solar radiation data for 15 places in South Africa, South West Africa and Botswana, have been selected for analysis in this way. It is envisaged that the number of stations for inclusion in this study will be significantly increased shortly.

The principles outlined above have been used in the preparation of computer programmes for selecting weather data and evaluating design weather conditions for the thermal performance assessment of buildings. The programmes are written in FORTRAN IV for implementation on an IBM S/360 model 65 IH operating under Operating System Release 18 MVT (Multiprogramming with Variable number of Tasks). The package consists of three main programmes, viz SELECT, SOLAR and DWDATA and sixteen subroutine sub-programmes.

## 3. Description of Computer Programmes

### 3.1 SELECT

The thermal interaction between a building surface and its immediate micrometeorological environment is a function of the integration, in some way, of the relevant meteorological parameters.(5) The concept of sol-air temperature, introduced by Mackey and Wright (6) provides such an integrated meteorological parameter. It therefore seems reasonable to select summer design weather data on the basis of sol-air temperature. For purposes of comparison, however, days are also selected purely on the basis of maximum dry bulb temperature.

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(1) Figures in brackets indicate the literature references at the end of this paper.

Sol-air temperature may be expressed as follows: (7)

$$\theta_{sa} = \theta_{oa} + \frac{\alpha I_s - I_{lt}}{h_{oc}} \qquad (1)$$

where $\theta_{sa}$ is the sol-air temperature, $\theta_{oa}$ the dry-bulb temperature, $\alpha$ the absorptivity of surface for solar radiation, $I_s$ the Intensity of solar radiation, $I_{lt}$ the long-wave radiation balance of the surface and $h_{oc}$ the coefficient of convective heat transfer between the surface and the air.

The outside convection coefficient and low-temperature radiation exchange of the surface and its surroundings are not generally known and the values for both these factors are therefore derived from, amongst other things, a knowledge of the outside surface temperature of the surface under consideration. Since this temperature is mostly not known, it has been suggested (7) that eq (1) be written as

$$\theta_{sa} = \theta_{oa} + \frac{\alpha I_s}{h_o} \qquad (2)$$

where the values of $I_{lt}$ and $h_{oc}$ are replaced by a so-called combined outside surface coefficient given by

$$\begin{aligned} h_o &= h_{oc} + \frac{\epsilon\sigma\, T_a^4 - T_s^4}{\theta_a - \theta_s} \\ &\approx 4\epsilon\sigma T_F^3 \qquad (3) \end{aligned}$$

where $T_F$ is the film temperature, $\frac{T_a + T_s}{2}$, $\theta_a$ the air temperature °C, $\theta_s$ the surface temperature °C, $T_a$ the air temperature °K, $T_s$ the surface temperature °K, $\epsilon$ the emissivity of the surface and $\sigma$ the Stefan-Boltzman constant.

Roux (7) found that, for South African conditions the combined outside coefficient was, for practical purposes, constant with a value of 19.873 $W/m^2$ degC (3.5 $Btu/ft^2$ h degF). Assuming a surface absorptivity for solar radiation of 0.7, sol-air temperature can be computed from the relationship.

$$\theta_{sa} = \theta_{oa} + .0352 \; I_s \qquad (4)$$

where $I_s$ is the solar radiation in $W/m^2$

In practice in South Africa solar radiation data are summed over hours L.A.T., and all other data, with the exception of sunshine, are recorded on the hour, clock time. Therefore solar radiation data has to be corrected for time, if sol-air temperature is to be computed.

For computer application the equation of time, employed in determining the true correction, E, is represented in fourier form as follows:-

$$\begin{aligned} E = {} & .005 + .016\cos\left(\frac{2\pi n}{365}\right) - .133\sin\left(\frac{2\pi n}{365}\right) - .059\cos \\ & \left(\frac{4\pi n}{365}\right) - .169\sin\left(\frac{4\pi n}{365}\right) - .014\cos\left(\frac{6\pi n}{365}\right) - .010\sin\left(\frac{6\pi n}{365}\right) \quad (5) \end{aligned}$$

where n is the day of year

Programme SELECT computes hourly sol-air temperatures from 08:00 to 16:00 hours for every day from 1 September through 30 April as well as the daily maximum sol-air temperature. This is followed by a frequency analysis of daily maximum sol-air temperature. From a cumulative frequency table set up in the computer, the days on which the daily maximum sol-air temperatures are equal to or higher than the value that is equalled or exceeded on 10 per cent of the occasions are listed in a table. This table is then sorted into ascending order of sequence on sol-air temperature. The sorted table is then printed along with the year and day of the year on which each individual value occurred. The 5 per cent and 2.5 per cent threshold values of daily maximum sol-air temperature are also determined. The associated sequences of days are, of course, contained in the 10 per cent tabulation.

Unfortunately solar radiation is only measured at 9 of the 15 stations available for analysis, so that sol-air temperature cannot be directly computed in all cases. Hourly solar radiation data can, however, be computed theoretically for clear sky conditions. If it can be assumed that hot days are free of clouds, sol-air temperature can then be computed and hot days selected as before. This assumption is not generally valid, since clouds appear on most afternoons in summer over most of the interior. However, at the time of occurrence of maximum sol-air temperature, around noon, analysis of Pretoria and Maun data indicates that it can be assumed that on the hottest days the skies are relatively cloudless. Consequently a programme called SOLAR has been written to compute hourly solar radiation totals (both global and diffuse radiation) for clear sky conditions for stations for which recorded information is not available. This programme is discussed in more detail in subsequent paragraphs. At this stage, it will suffice to state that the radiation data are computed from estimated mean hourly values of Linke turbidity factor and extra-terrestrial solar radiation fluxes. One of the functions of programme SELECT is also to compute hourly values of Linke turbidity factor between 08:00 and 16:00 hrs for the same period of the year as above. This is done for the nine stations for which solar radiation data are available. From these data some idea of the mean hourly turbidity climate of the country can be derived, to make a more reliable estimate of the mean hourly Linke turbidity factor for other places.

The Linke turbidity factor was selected since it is a simple measure of the haze and water-vapour content of the atmosphere. For total radiation, it gives the number of clear dry atmospheres that would be necessary to produce the attenuation of the extra-terrestrial radiation that is produced by the actual atmosphere containing water vapour and haze. In fact, this type of turbidity factor was recommended for meteorological use by the International Radiation Conference of Davos 1956 in cases where only the total solar radiation is measured, i.e., no filter measurements.(8)

The extinction by haze and water vapour differs from the extinction by pure air (molecules), and the deviation of the dependence on wave-length of both these extinctions gives rise to a diurnal variation in the turbidity factor even when the water vapour and haze content are constant throughout the day. This variation is generally referred to as a 'virtual variation' and is different for different water vapour and haze contents. It is for this reason that hourly values of turbidity factor are computed.

The Linke turbidity factor can be computed from the following relationship:-

$$I = SI_o \, e^{-(T.\bar{a}_R \, (m) \, . \, m)} \qquad (6)$$

where I is the solar radiation intensity at the earth's surface (measured),

$I_o = \int_0^\infty I_{o\lambda} \, d\lambda$ , representing the extra-terrestrial solar radiation intensity,

T the Linke turbidity factor, $\bar{a}_R(m)$ the mean value over all wavelengths of the extinction coefficient in a clean dry atmosphere (Rayleigh atmosphere), weighted according to the distribution of the transmitted energy (this complex extinction coefficient depends on the air mass m because of the shifting of the optical centre of gravity of the radiation as m changes, $S = \frac{1}{R^2}$ where R is the radius vector of the earth and m the absolute optical air mass.

For computer application the value of S can be approximated by a Fourier series, such as

$$S = 0.981 + .037\cos\left(\frac{2\pi n}{365}\right) \qquad (7)$$

where n is the day of year.

(If more accuracy is required additional terms can be added.) The relative air mass is computed from the relationship

$$m_r = \left[\left(\frac{R}{H}\cos z\right)^2 + \frac{2R}{H} + 1\right]^{\frac{1}{2}} - \frac{R}{H}\cos z \qquad (8)$$

where $m_r$ is the relative air mass, R the radius of the earth = 6371Km, H the height of a homogeneous atmosphere given by $\frac{P_o}{g\rho_o}$, where $P_o$ is the atmospheric pressure at sea level = 1.014 x $10^5$ Kgm/sec$^2$ m$^2$, g the acceleration due to gravity = 9.80629 m/sec$^2$, $\rho_o$ the atmospheric density at sea-level = 1.2923 Kg/m$^3$

Thus H = 8.001 Km

$z$ = Zenith angle given by $\cos z = \sin\phi \sin\delta + \cos\phi \cos\delta$ cost where $\phi$ is the latitude angle (negative in Southern hemisphere), $\delta$ the apparent solar declination, and t the hour angle of the sun given by $\frac{\pi T}{12}$, where T is time of day.

The apparent declination of the sun can be approximated by a Fourier series, such as

$$\begin{aligned}\delta = {} & .009 - .401\cos\left(\frac{2\pi n}{365}\right) + .066\sin\left(\frac{2\pi n}{365}\right) \\ & - .007\cos\left(\frac{4\pi n}{365}\right) + .001\sin\left(\frac{4\pi n}{365}\right) \\ & - .003\cos\left(\frac{6\pi n}{365}\right) + .001\sin\left(\frac{6\pi n}{365}\right) \qquad (9)\end{aligned}$$

where n is the day of year.

Traditionally the hour angle is measured from solar noon. For computer application it is more convenient to increment time from midnight. Thus the equation for zenith angle becomes

$$\cos z = \sin\phi \sin\delta - \cos\phi \cos\delta \cos t \qquad (10)$$

FOR $\bar{a}_R$ (m), Fuessner and du Bois (9) give an empirical relation:-

$$e^{-\bar{a}_R(m)} = 0.907m^{0.018} \qquad (11)$$

$$-\bar{a}_R(m) = -(\ln 0.907 + 0.018 \ln m) \qquad (12)$$

Equation (6) can also be written in the form:

$$\ln I = \ln S + \ln I_o - T\bar{a}_R(m)\, m \ln e$$

hence $$T = P(m)\ (\ln I_o + \ln S - \ln I) \qquad (13)$$

where $$P_{(m)} = \left[\bar{a}_R(m)\, m\right]^{-1}$$

From the above relations it is now possible to compute turbidity factor.

For future reference and in case some alternative method for computing solar radiation should be attempted, precipitable water vapour content of the atmosphere is also computed for the same hours and days as turbidity. The Hann formula (10) was used for computing precipitable water vapour content from surface vapour pressure, <u>viz</u>:

$$w = c\ e_W\ (t) \qquad (14)$$

where w is the precipitable water vapour content in cm, c the proportionality constant and $e_W$ (t) the vapour pressure at temperature, t.

Hann gives a value of 0.25 for e. However, the following values were found for different centres in South Africa.

Table 1. Mean values of the proportionality constant C for January and July

| Station | Mean value of constant, C | |
|---|---|---|
| | January | June |
| Pretoria (morning) | 0.17 | - |
| Pretoria (afternoon) | 0.17 | 0.13 |
| Durban | 0.18 | 0.12 |
| Windhoek | 0.16 | 0.13 |
| Bloemfontein | 0.18 | 0.12 |
| Port Elizabeth | 0.14 | 0.11 |
| Cape Town | 0.13 | 0.12 |

The surface vapour pressures are computed using the well-known Goff-Gratch formula.

### 3.2 SOLAR

Programme SOLAR computes hourly global and diffuse solar radiation fluxes on a horizontal surface.

Under clear sky conditions for any hour for any place the direct component of global solar radiation is computed by means of eq (6):

$$I = SI_o e^{-(T.\bar{a}_R\ (m)\ m)} \cos z \qquad (15)$$

The diffuse component of radiation is computed from a relationship given by Liu and Jordan (11), viz:

$$\tau_d = 0.2710 - 0.2939 \tau_D \qquad (16)$$

where $\tau_d$ = $I_{dh}/I_{oh}$ and $\tau_D$ = $I_{Dh}/I_{oh}$ and $I_{oh}$ = extra-terrestrial intensity of solar radiation incident on a horizontal surface, = $SI_o$ cos , $I_{dh}$ the intensity of direct radiation on a horizontal surface, and $I_{Dh}$ the intensity of direct radiation incident on a horizontal surface.

Alternatively

$$D_h = 60\ (.2710\ I_{oh} - .2939\ I \cos z) \qquad (17)$$

where $D_h$ is the hourly diffuse radiation on a horizontal surface.

Spencer (12) has found that the coefficients in eq (16) require some adjustment for Melbourne. However, Liu and Jordan's coefficients will be used until it has been possible to verify them for South African conditions.

## 3.3 DWDATA

It was stated above that SELECT produces a list of days in ascending order of daily maximum sol-air temperatures that are equal to or higher than the 90th percentile of daily maximum sol-air temperature. The punched cards for the various elements, i.e. hourly drybulb temperatures, hourly relative humidity, hourly wind speeds and hourly global and diffuse solar radiation values are then selected according to this list and form the input for DWDATA.

The programme DWDATA is flexible in the sense that, with the aid of a control parameter, any one of four methods of evaluating design weather data can be selected; and with the aid of a further control parameter, hourly solar radiation data can either be read from cards or computed.

Briefly, the four different methods of evaluating design weather data are:-

### a. Method 1

According to this method summer design data, selected on the basis of daily maximum dry-bulb temperature, are evaluated. The first step in this analysis is to determine the mean dry-bulb temperature for each hour of the day, its standard deviation and maximum and minimum values for all days selected, as described before. The next step is to select all the occasions when the hourly temperature of each hour of the day taken in succession $t_{i,j}$ is such that $(\bar{t}_j - 0.5) \leqslant t_{i,j} \leqslant (\bar{t}_j + 0.5)$ where i refers to the day and j to the hour under consideration and $\bar{t}_j$ is the appropriate mean hourly temperature for the days selected for each of the probability levels taken separately. The other weather elements for these occasions are then selected and their means, standard deviations and maximum and minimum values computed. A tabulation of the mean dry-bulb temperature and the associated relative humidity, wind speed and solar radiation values for the various probability levels, *viz* maximum, 2.5, 5 and 10 per cent levels, represents the required hot weather design data. The programme furthermore computes the hourly solar radiation fluxes on vertical surfaces facing north, south, east and west, and the following humidity parameters based on the algorithms published by Kusuda (13): wet-bulb temperature, dew-point temperature, vapour pressure, humidity mixing-ratio, enthalpy of moist air, entropy of dry air and specific volume of moist air for each hour for each probability level. This information is tabulated together with the other hourly information, referred to above.

The method of computing solar radiation fluxes on vertical surfaces consists of computing the direct component, diffuse component from an unobstructed sky and diffuse component reflected from a flat, unobstructed surrounding terrain.

The direct component is simply given by:

$$I_v = \frac{G_h - D_h}{\cos z} \cos i \qquad (18)$$

where, $I_v$ is the direct component of solar radiation normal to given vertical surface and i the angle of incidence of direct solar beam on given vertical surface.

For computing the diffuse sky component, clear and cloudy skies have to be differentiated. Since no hourly cloudiness data have been recorded on punched cards it was decided to compute the Linke turbidity factor and compare this with some threshold value of turbidity factor, the latter being for cloudy conditions, defined as a sky having approximately one-quarter cloud cover. The turbidity factor is computed as follows:-

$$T = \frac{\log\left(\frac{1381 \text{ S} \cos z}{G_h - D_h}\right).60}{0.434m\ (\ln 0.907 - 0.018 \ln m)} \qquad (19)$$

where T is the Linke turbidity factor, $G_h$ the hourly global solar radiation on a horizontal surface, w/m$^2$, and $D_h$ the hourly diffuse radiation on a horizontal surface w/m$^2$

Based on an analysis of the data for Pretoria and Maun, a threshold turbidity of 5.0 seems to be about right.

For computed turbidity $<$ 5.0, the following relationships for computing the diffuse sky radiation component on a vertical surface have been found for Pretoria (14):-

for cos i $<$ 0.2:

$$D_V = D_H \, (0.31 - 0.1 \cdot \cos i) \tag{20}$$

for cos i $\geqslant$ 0.2:

$$D_V = D_H e^{(1.18 \cos i - .8)} \tag{21}$$

For computed turbidity $\geqslant$ 5.0, the simple relationship (14)

$$D_V = 0.5 \, D_H \tag{22}$$

has been found to be valid for Pretoria for average cloudy conditions.

As for the ground reflected solar radiation component, it has been shown (15) that, for practical purposes, for an unobstructed horizon for Pretoria, this component can be approximated by

$$R_V = \tfrac{1}{2} \rho' G_H \tag{23}$$

where $\rho'$ = reflectivity of the ground.

### b. Method 2

According to this method summer design data, are evaluated in exactly the same way as a above, except that they are now being determined on the basis of daily maximum sol-air temperature in stead of on maximum dry bulb temperature.

### c. Method 3

In this case hot day design data, selected on the same basis as in b above, are evaluated. However, means, standard deviations, maximum and minimum values are determined for each of the basic and derived elements in the way described for dry-bulb temperature in a above.

### d. Method 4

When the parameter, METHOD, is equal to 4, cold day design data are evaluated. The basic data are selected on the basis of those daily minimum dry-bulb temperatures that are equal to or less than those daily minimum dry-bulb temperatures occurring on 2.5, 5 and 10 per cent of the occasions for the period under consideration.

The programme DWDATA also arranges the selected information in ascending order of sequence on year and day of the year. Runs of days when the daily maximum dry-bulb temperature or daily maximum sol-air temperature, depending on which of these elements was employed in defining hot days are equalled or exceeded, can then be readily determined. In a similar way, runs of cold days can be assessed.

In the case of light-weight elements the effects of heat storage capacity is relatively unimportant. Therefore, there is no significant cumulative effect due to a succession of hot or cold days. However, in the case of heavy-weight elements, the cumulative effect is very important. This cumulative effect of sequences of hot days of varying duration can be computed for such elements and related to the probability of occurrence of sequences of different duration as they actually occur in practice.

## 4. Conclusions

In the field of computer application to environmental engineering problems at the National Building Research Institute in Pretoria, the stress, up to the present time, has been on producing design weather data for the thermal performance design of buildings rather than on the development of sophisticated procedures for computing thermal performance and heating and cooling loads. The philosophy has been that solutions of such problems can be no better than the physical data on which they are based, no matter how sophisticated they are. It is believed that the design data evaluated according to the methods outlined in this paper will, with perhaps small refinements, provide the basic information required for predicting thermal performance and evaluating heating and cooling loads with acceptable accuracy.

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# Quality Rules for Thermal Performance of Low Cost Dwellings
## (Building Climatology for Argentine)

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Climatic data from the hourly records of the National Meteorological Service were collected from approximately 22 locations throughout Argentina. These data were then analyzed on the basis of (1) monthly, seasonal and annual averages, (2) typical hot and cold days, (3) probability of occurrence of solar radiation, humidity, and wind for typical hot and cold days and (4) probability of N successive days "equal" to one intitial typical day. Described in this paper are computer programs for the analysis described above and charts and tables developed for the practical use.

Key Words: Climatic data, probability of occurrence of values, probability of runs of N equal days, typical days.

## 1. Preliminary

Since 1967 the INTI (National Institute of Industrial Technology) and Bouwcentrum Argentina (Center for Building and Housing Research and Information of INTI's System of Centers) are at work in the complex problem of searching for the minimal higro-thermal requirements to be established for the housing programs of this country. Up to that year only sporadic efforts of isolated researchers were recorded in the Argentine Republic. This is then the first research set up with the necessary means and with the long-range goal of writing down those requirements for the different climatic regions and socio-economic strata of the country. (Ref. No. 1).

## 2. Steps of the research

The steps or successive goals of this task were laid down as follows:

2.1. To know the climatic zones of the Argentine Republic, for building purposes.

2.2. To define the convenient levels of thermal comfort for different geographical and socio-economic circumstances.

2.3. To translate those levels into "technical requirements" to be fulfilled by the elements and/or the whole of the buildings or the urban design.

2.4. To choose experimental controls for the task.

2.5. To put up a theory of thermal comfort and thermal performance of buildings, useful for the Argentine problems in this area, and based in computer's capibility.

2.6. To translate the findings into the pertinent legal instruments such as codes, norms, etc.

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## 3. Chronological developments

This kind of research is obviously a long-range and long-duration task, with many feed-backs and probably several re-thinking phases about the whole affair. In this moment we deem it possible to make fair advances in the three years 1969-1970-1971. The work's progress is now following the guilding lines established at the beginning of the period.

Those guide-lines are the steps before mentioned in paragraph 2. In this paper the first step of the pont 2.1., i.e., deals with the knowledge of the climate of the capital city for building purposes. This task will be eventually expanded to cover the whole of the country.

## 4. Source of raw data and processing equipment

Source of raw data is the National Meteorological Service. We have an arrangement made with this official agency for the access and use of the climatic data recorded in punched IBM cards.

The National Meteorological Service, founded in 1872 (by suggestion of the North-American astronomer Gould) by President D. F. Sarmiento, has now up to 8 million IBM cards with data from many parts of the country. This data covers the last 15 years.

The cards are the international type used in 700 stations over the world (350 in the U. S.) (Fig. 1).

The type "A" card is prepared for hourly data. In practice it is used with 24 observations/day and also with 8 to 3 observations/day.

Unfortunately such different type of records are made often at the same place in different periods or runs of years.

The type "B" card is prepared for daily data or parameters and as such as it records only one or two values in each 24 hour period.

The type "77" card is used with monthly parameters.

In this research we used only the "A" and "B" types of cards.

For instance, in the first city studied (Buenos Aires) we used a record of 5 years with hourly data and another, also of 5 years, with data at the hours 2 and 8 a.m., and 2 and 8 p.m. These data come from the station "Central observatory" placed near the geographical center of the city. We have only two other stations in the city and it outskirts with data in punched-cards. One is by the River Plate and the other about 25 km inland. We hope to check those stations' data with the former, in a later phase of our task.

The IBM 1130 machine works with disks (each capable of 248.000 words).

This computer is in the CITMADE (INTI's computer center) and our research has a part-time arrangement for its use.

## Climatic parameters for building purposes in typical hot or cold days

### 1. Goals of the Work

This research aims at the statistic specification of all the relevatn climatic parameters of a "typical hot day" and a "typical cold day".

Those days are to be selected following statistical criteria with 2 different levels of complexity.

We deem this work suitable for calculations and specifications in the following fields.

(a) -Design of buildings and/or its parts for suitable thermal insulation, good natural ventilation, efficient vapour barriers, convenient sun exposures, etc.

(b) -Urban design of new communities and housing schemes, etc.

(c) -Design of heating, ventilating and air conditioning installations or appliances, from the following points of view:

i. Outdoor design basis for the project.

ii. Outdoor conditions for testing purposes.

## 2. Summary of the first task

The first task was then to get acquainted with the climatic parameters of different zones and/or cities of this country.

As the climatic parameters as processed by the standard methods of our National Meteorlogical Services for other ends were inadequate for building purposes, it was decided, then, to process the raw data (in the form of IBM cards provided by the NMS) in the CITMADE's computer, following several new programs written down especially for this goal. The plan is to record the data in up to a million cards of the NMS in about 12 disks, according to those programs. These data are from about 20 cities or regions of the country. It is expected to develop such work in a span of about 3 years.

The hypothesis laid down for the task were these:

(h-i) The design conditions must take into account all the pertinent climatic parameters:

(a) air temperature

(b) humidity (by one or more of the several variable used for measuring it)

(c) speed and direction of the mind

(d) sun radiation

(h-ii) The study of the parameters one by one was deemed not satisfactory in this problem, so we stated that the statistical probability of concurrence of typical values of those variables had to be worked out.

(h-iii) A "typical hot day" and a "typical cold day" had in turn to be analyzed hour by hour and the results submitted to a statistical probability analysis in order to fulfill the item (h-ii).

(h-iiii) The probability of occurrence of successive runs of 2, 3, N typical hot-days and typical cold-days had to be worked out.

## 3. Main features of the Argentine climate

This country stretches from 22° to 55° south latitude, and besides has several territories further to the South, on islands and a segment of the polar continent. On the mainland we have the following broad climatic zones: (each zone is subdivided in two, as shown in the map , fig. 2).

i. Hot: 1 - humid; 2 - dry

ii. Temperate: 1 - humid; 2 - dry

iii. Dry: 1 - semi-arid; 2 - "patagonic"

iiii. Cold: 1 - dry; 2 - "humid"

Nevertheless, the geographic and seasonal climatic variations are not so marked as in the N. W. or central U. S. or central and east Europe. This is because South America is really a tongue of land between two huge masses of water, the South Atlantic and the South Pacific Oceans; and the Polar Cape is more than 1.000 km. South of Cape Horn, the tip of the Continental Block and adjacent islands. But the long Cordillera de los Andes forming the West limit of the country (about 2.500 km) is really a great "climatic dam" and a decisive factor of our climatic patterns; it produces marked alterations in the overall atmospheric circulation, and superimposes an West-East influence over the normal North-South axis of climatic levels.

The most remarkable climatic factors are: hot summers in NE, NW and W zones; mild conditions in both seasons in the Center; cool and very windy in the lower half (Patagonia). Snow is a problem only in high mountain and plateaux zones. The East half of the country has abundant rains, but the rest is under the level of 500 mm (20 inches) in the average year and so, under the minimum rain level for agricultural purposes (except zone in the N.W. and other in the southern cordillera and the island of Tierra del Fuego, both cold and rainy).

The first region occupies about 1.700.000 $km^2$ and the second 1.000.000 $km^2$. More than 3/4 of the total population of 22 million inhabitants live in the humid region.

## 4. Fundamental criteria for choosing and describing the typical or design day (hot or cold)

One climatic parameter was to be chosen as the fundamental, and the probability of concurrence of values of the other three (excepting precipitation) to be studied.

The first one had to be the most important for the higro-thermal state. A review of the climatic zones of our country showed that not always the temperature of the air - the best known and more often used parameter - seems to be the best choice. Maybe the solar radiation in the hot zones or the wind in the Patagonia would be better. But for the sake of uniformity and also because the temperature of air has always the best record in each locality, this was our choice for the role of "leading parameter". As mentioned before, the work was begun with Buenos Aires and we used type "A" (hourly) and type "B" (daily) IBM cards. For the 5 years 1959 to 1963 we had hourly data and for the next 5 years 1964 to 1968 we had only data from 2 and 8 a.m. and 2 and 8 p.m. In both runs we had also the daily card of the "B" type. (Fig. 2 shows the two cards and the relevant items of the punching-code used by the NMS).

The first computing task was to get the usual average, maximum and minimum values of the relevant variables. This was checked with other period's results, giving in general a good agreement (of course, up to this point the work was rather of the usual climatic type). (Fig. 3).

The second task was different and implied an entirely new approach - at least for our country - aimed at the determination of the design days, for building purposes.

This was done as follows:

4-1 The ogive of the average daily temperatures of the 3650 days (10 years, without the 29th of February for the sake of simplicity) was worked out in absolute and relative frequencies by the computer. Let's call "p" the cumulated relative frequencies.

4-2 For each of the levels of p = 2,5%, 5%, 10%, 90%, 95%, 97,5%, and 99%, we worked out the steps that follow.

Here we give the general explanation for the p = 2,5% level, because we take p = 2,5% for the typical cold day, but in the graphs and comments we give also the results for the p = 97,5% level for the typcial hot day.

4-3 The machine was ordered:

(a) to find the p = 2,5% day and pick up its card;

(b) to pick up the cards of the "next" ten days over that day; and the "next" ten days below that day. "next" means here the days whose cards showed average daily temperature numerically (not chronologically) in the neighborhood of the p = 2,5% day.

4-4 For this set of similar 21 days the machine then did the following:

(a) worked out the relative frequencies of average values of temperature, humidity and insulation (Fig. 5)

(b) worked out the average hourly temperature, i.e.,the average 1 h, 2 h, 3 h, ...........24 h temperature; with those values we traced the curve (Fig. 4)

(c) the same calculations were made for the relative humidity, hour by hour and also the curve was traced (Fig. 4)

(d) worked out the frequency of the wind in 8 directions and one "no-wind" state, also hour by hour. The machine also got the average speed for each direction. (Fig. 5 bottom)

(e) in the drawing (Fig. 5, lower) for the solar radiation we used "heliophany" or relative insulation parameter (the ratio of the real sunny time of a day to the astronomical one) and with such basis we made the calculation of the average direct sun radiation on 8 vertical planes and the horizontal plane. (Fig. 6).

For the typical hot days see Figs. 7, 8 and 9.

The radiation values were worked out using the tabulated results of a sun thermal radiation program prepared by Architect Victor Olgyay, and generously given to us during his 1969 stay in Bouwcentrum. (Ref. 14)

The program was processed in the IBM machine and gave for each degree of latitude and for the mid-month day the before-mentioned radiation values, for a theoretically cloud free and clean air day. Those values are then the maximum possible ones. For Buenos Aires we used the latitude 34° 1/2 South.

In order to get the real magnitudes, these theoretical values were affected by a "F" factor, less than the unit established for Montevideo (a city only 200 km from Buenos Aires and almost in the same latitude) in a statistical calculation summarized in appendix "A".

As the two cities have very similar climates, we felt that we could use values of the insulation factor "F" of Montevideo for our work about Buenos Aires (Ref. No. 2).

4-5 Another useful statistical criterium was used: the machine worked out the probabilities of chronological runs of 2, 3 ..............N successive typical cold days, following this sequence (Figs. 10 and 11):

(i) we had established before the average temperature of the "first" cold day and chosen the respective card (paragraph 3, item (a).

(ii) the machine then worked out the probability of having the next day with an average temperature equal or less than the value of this parameter in the "first" cold day (item (i); and also the probability of having 3, 4........10 successive "cold days" so defined. Of course, in the case of the typical hot day, in the item (ii) we read "equal or more" instead of "equal or less".

(iii) then it did the same with an allowance of + 1°C and + 2°C for the typical cold day's average; and 1°C and 2°C for the typical hot day's average.

## 5. Comments and remarks

Several comments and remarks could be made on the method just explained. We state here a few of them:

5-1 of course, the "desing days" obtained with such processes are a kind of statistically representative or meaningful days, for the hot or cold condition studied.

The various parameters will evolve, during such a day, in a typical manner, represented by our tables and graphs.

5-2 In the typical cold day it is observed that:

(a) the lowest temperature is above the freezing point (+ 2.7°C)

(b) the wind blows from the W or SW at the coldest hours

(c) the sun shines bright, it is a clear day

(d) the daily span of the temperature is significant (9.3°C)

(e) the humidity has also a marked oscillation being very high at the coldest hours, and rather low at the hottest hours

(f) the probability of chronological runs of such days goes down very fast and is negligible after a few days

5-3 In the typical hot day it is observed that:

(a) the maximum temperature is moderately high (+ 32°C)

(b) the wind blows from the N, NE and E in the hottest hours

(c) the sun shines brightly - it is a clear day

(d) the daily span of temperature is significant (9°C)

(e) the relative humidity has a marked oscillation, being low at the hottest hours (45%)

(f) the probability of chronological runs of such days goes down very fast and is negligible after a few days

5-4 The findings are of interest from several points of view:

(a) from the standpoint of the thermal lag of buildings

(b) from the standpoint of the heating or conditioning system, their thermal lag and the control systems to be used

(c) from the standpoint of the possible use of analogue computers for simulating the conditions (electronic or hydraulic)

(d) from the standpoint of the possible use of digital computers for the same end (i.e., for resolving the electrical circuit)

(e) from the standpoint of the different levels of probability likely to be used in different problems

On the points (a) and (b) it can be said that in this city it is important to have a heating or winter conditioning system with great "time flexibility" and with zonification according to the rose, i.e., "spatial flexibility".

This is because of the important daily oscillations of temperature and the high solar radiation at the NE; N and NW exposures.

The summer service of air conditioning in buildings with great exposure requires also, both flexibilities, as the typical hot day shows.

The prevailing winds blowing from the N, NE and E in those hot days can be used for natural ventilation of dwellings, schools, industrial buildings, etc., not equipped with air conditioning.

It is also possible to see that light buildings can be brought in "resonance" with the exterior conditions (transient period 1 to 3 days, for instance), but heavy structures cannot attain such condition (transient periods from 4 days up). This problem is very important for low-cost dwellings in summer, because it is not economically possible to furnish such houses with air-conditioning appliances and therefore, it is necessary to study the spontaneous evolution of the internal conditions as a function of the external ones, (a very difficult problem from a physico-mathematical point of view).

About the points (c) and (d), it is obviously necessary to know the chronological recurrences of such days in order to simulate the real situation in the computers.

(f) This study shows only the results for the "2,5% day" as a typical cold day and the "97,5% day" as the typical hot day. However, we worked out, as stated before, the levels Ps 1%, 5%, 10%, 90%, 95%, and 99% because we felt that for idfferent problems it may be necessary to use such different levels.

Of course, with the levels 5%, 10%, 90%, and 95% the "character" of the typical cold or hot day is not so well marked and it is a matter open to discussion as to the meaning of such days.

We want to state here our thanks to the authorities of the following instiuttions for their decisive support to this research:

INTI (National Institute of Industrial Technology);
CIBA (Bouwcentrum Argentina, Center for the Research and Documentation in the Building Industry);
CITMADE (Center of Mathematical and Computer's Techniques);
SMN (National Meteorological Service).

## Appendix "A:

Factor (f) of conversion of the sun thermal directo radiation for clear sky in the value for mean sky (R2) taking into account the insulation and the nubosity (Ref. No. 2).

These auxiliary symbols are used: R1 direct radiation for clear sky.

<u>Insolation: (%)</u> I = Real time with bright sun/Astronomical length of the day

NOTE: Insolation is called "heliophany" in the figures.

N = Geometrical area of the sky

<u>Clarity: (%)</u> C = 100 - N

The paper (Ref. No. 2) shows that with an error not in excess of 4% for Montevideo it is correct to write:

$$F\ (\%) = 1.93\ \frac{C \times I}{C \times I}\ :\ R_2 = F \times R_1$$

This factor is to be applied to any hour in the day if that day has insolation I (%) and nubosity N (%) = 100 - C (%), and gives $R_2$ for those conditions.

The day in question can be a single one or a statistical average day.

Further, the paper shows that the coefficient M = I/C has in Montevideo a maximum value of 1.461 and a minimum of 1.327. This means that it exists in fact a correlation between the values of I and C as can be expected, and this justifies the use of F.

## Appendix "B"

## Prehistory of this paper

The choice of the statistical criteria for this study was the result of a rather long process. Maybe it is interesting to outline its principal steps. It must be remembered that we started with a totally new approach in our country, and we had only foreign bibliography on the subject; then we made the following remarks in the course of the studies:

First: We were at a loss when we tried to understand clearly the reasons that several researchers had in mind when they stipulated their climatic parameters for building purposes. Such reasons were not explained, or maybe were not good between-lines readers (Refs. 3, 4, 5, 6 and 7).

Second: We felt that in order to adapt, modify or reject some of this pre-existent work, it was necessary to gain knowledge about such a background.

Third: We deemed necessary to find a way to state not only the parameters as if its influences over the building and the human comfort were independent of the others, but also in their mixed or complex effects. As an example, the Sol-Air theory (Ref. 8) showed a good advance towards such an "upper level" of complexity, fusing the sun radiation and the heat of the air in one parameter.

Fourth: We saw that, for obvious reasons, in each country the parameters and criteria related to the outstanding characteristics of their climate were those more studied. This showed that for our purposes it would be necessary to study with special dedication the technical documentation produced by countries with climates not very dissimilar from ours. And between these, the papers about the problems in temperate, hot-arid and hot-humid regions were the most useful because the thermal comfort is more difficult to attain in those circumstances than in the moderate cold prevailing in our winters (Ref. 9, 10, and 11).

All this boiled down to the following "obvious" remark: The building climatology is a science "in the making" and as such shows plenty of disagreement between authorities, not only about the answers, but also - and maybe in the first place - about which are the problems.

An illunimating paper came then in our hands (Ref. 12). Here the two authors stated clearly what we had been searching for: the basis for new criteria in the "upper level" in question.

It is an act of justice to mention here that one of us was in England during 6 months of 1968, working with Professor A. Pratt (Ref. 13), and that during three months of 1969, we had the technical assistance of Professor Victor Olgyay (Ref. 14). Working and discussing with those authorities was a very useful experience for our background.

We decided then, to try our hand at the problem, reduced for the first effort, to the definition of the typical hot and typical cold days working in such an "upper level" of complexity, and using Buenos Aires climatic data.

For this we made, as a first step, a classification of all the parameters used for the others researchers following several criteria:

i. extensive, specific and intensive variables

ii. types of periodic functions approached by these

iii. possible useful combinations of variables in the "upper level" of complexity

iiii. miscellaneous.

And finally, with all this background, we asked ourselves: what is a hot day in Buenos Aires like? - A cold day? How would such a day affect the thermal comfort of a person? And the thermal performance of buildings? - And with those "simple" questions in mind we write down the criteria to be used by the programmer of the IBM machine and we passed the question and the answers over and over again through this filter. Up to now, the results are described in the present paper.... and this is the little story about the making of such research.

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(12) E.N. van Deventer and J. F. van Straaten, "A rational basis for assessing Climatic Data for use in Building design", National Building Research Institute. Pretoria (South Africa), 1967.

(13) Prof. A. W. Pratt, Head of Department of Building, Aston University in Birmingham; directs research in thermal problems of building.

(14) Prof. Victor Olgyay (1910-1970). Worked with his brother Aladar Olygyay (1910-1960) at the Princeton School of Architecture, in the field of architecture and climate.

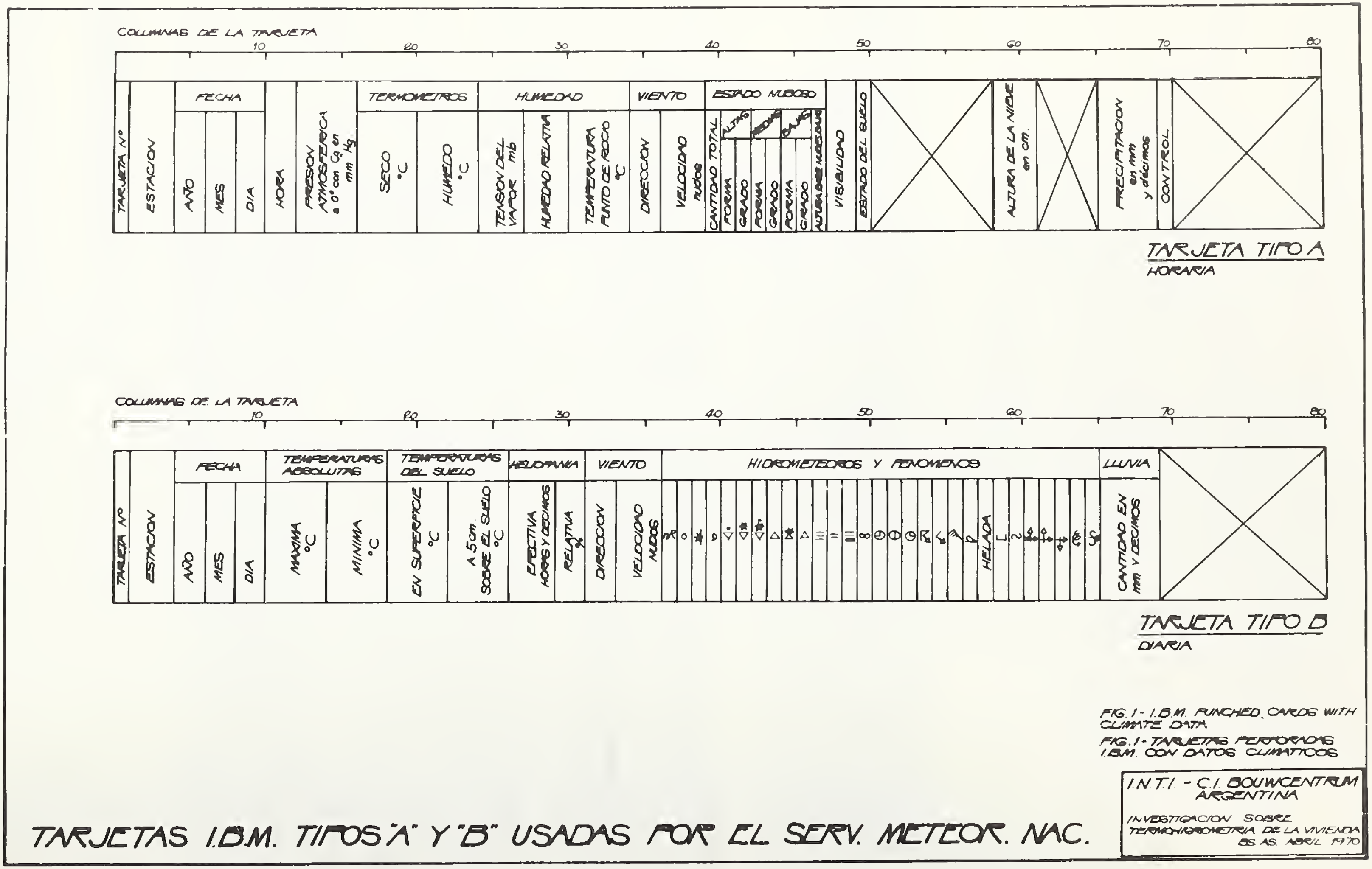

Figure 1. IBM card format for weather data

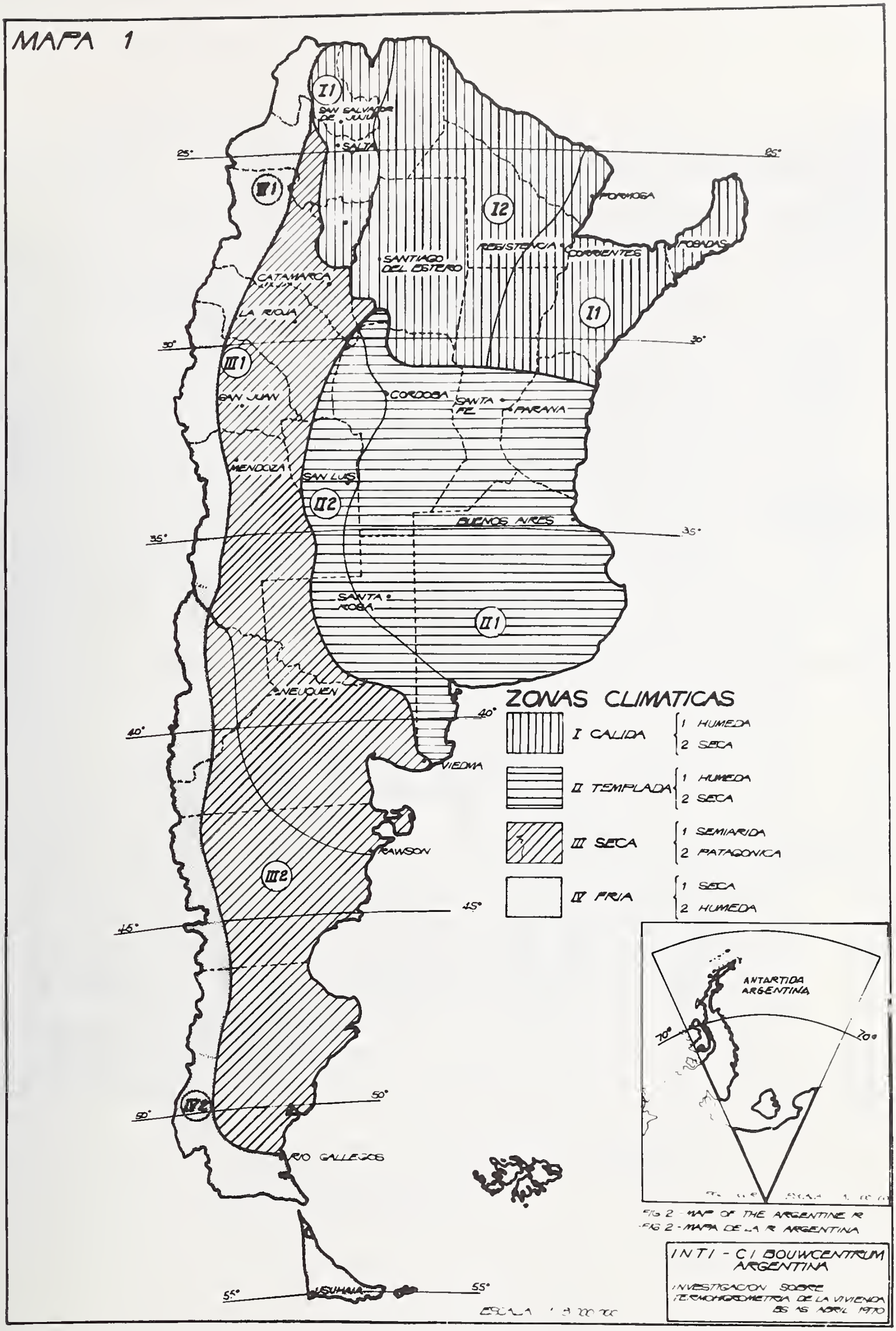

Figure 2. Map of the Argentine

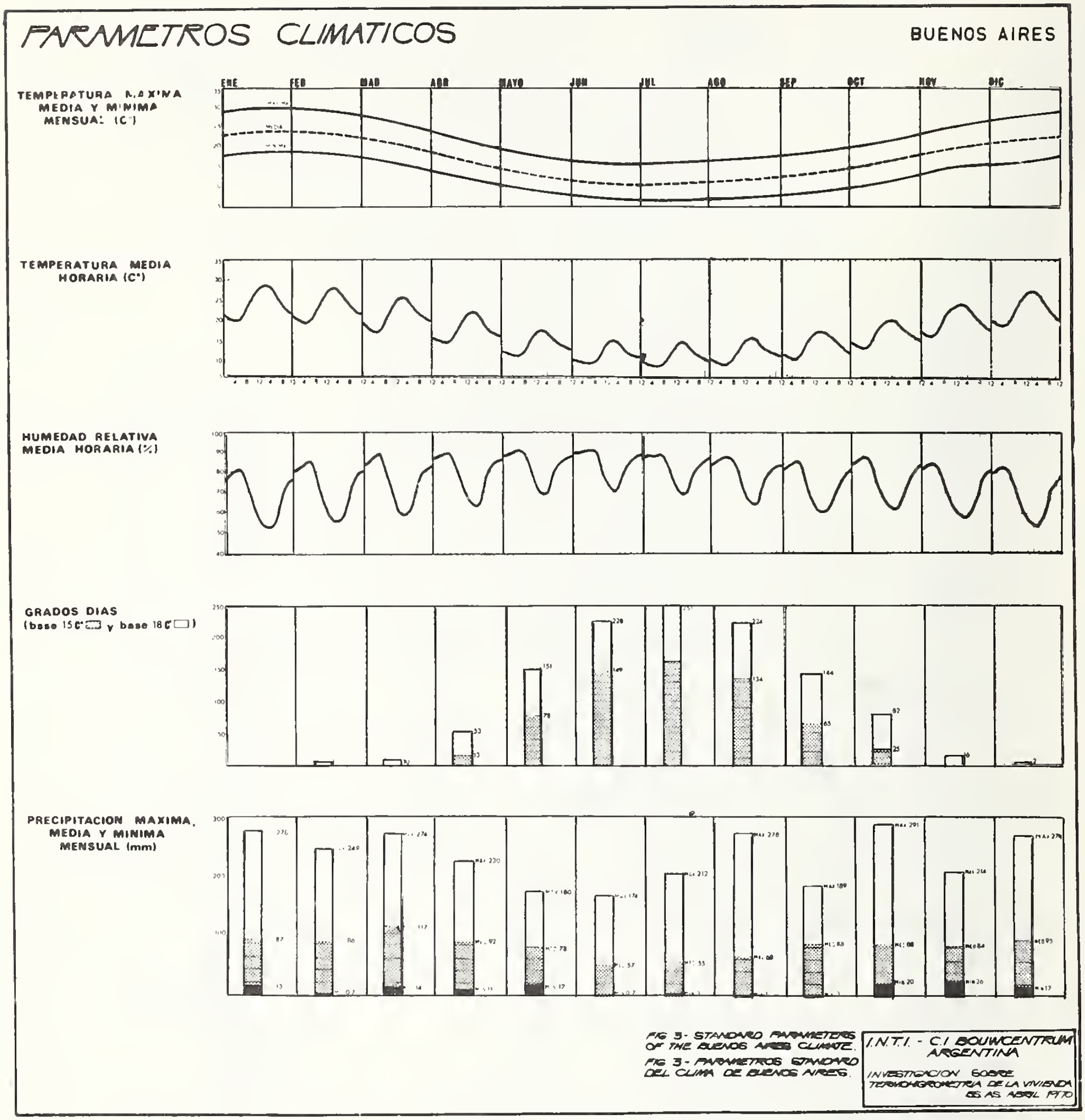

Figure 3. Standard climatic data of Buenos Aires

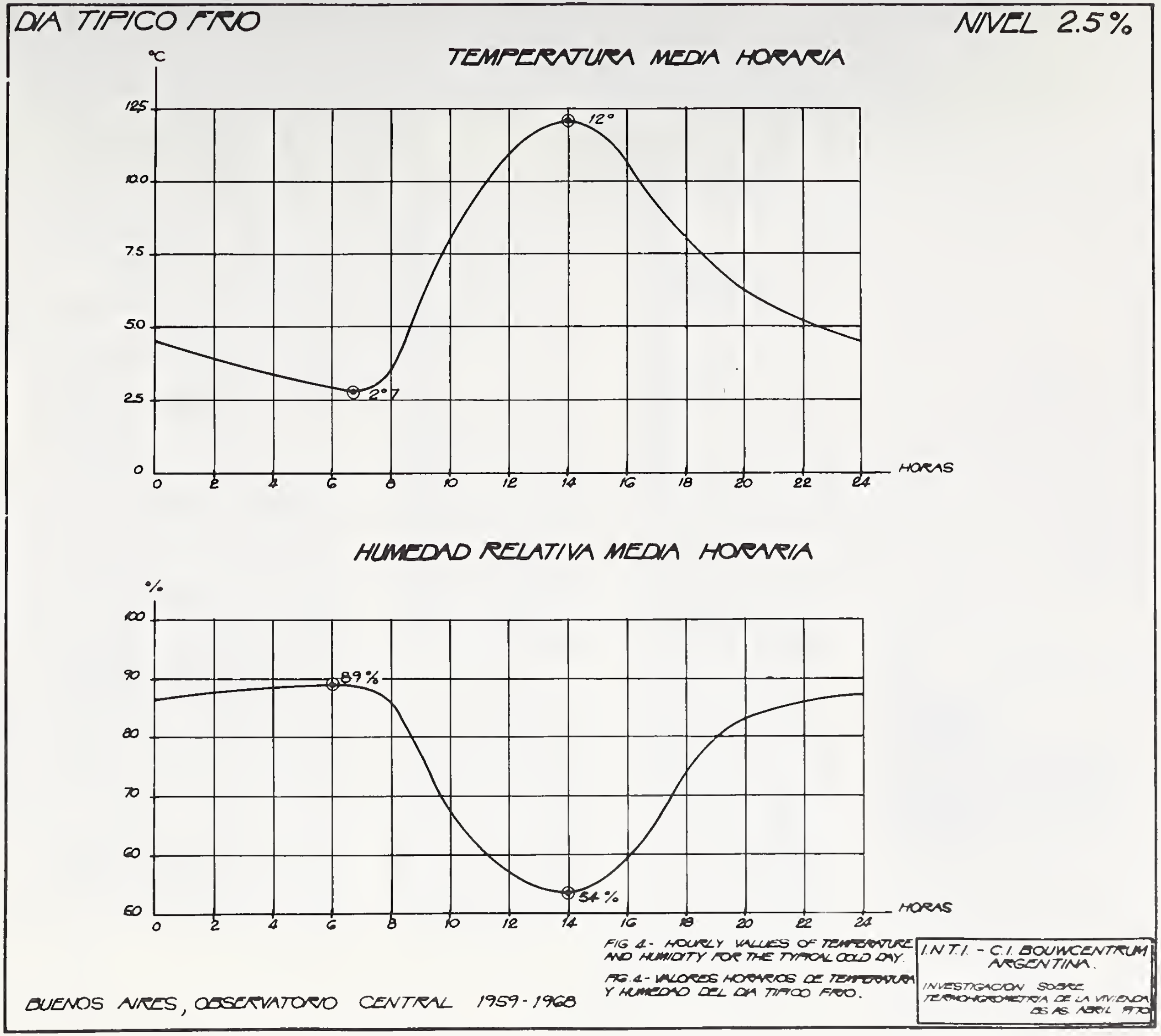

Figure 4. Hourly profiles of temperature and relative humidity during a typical cold day

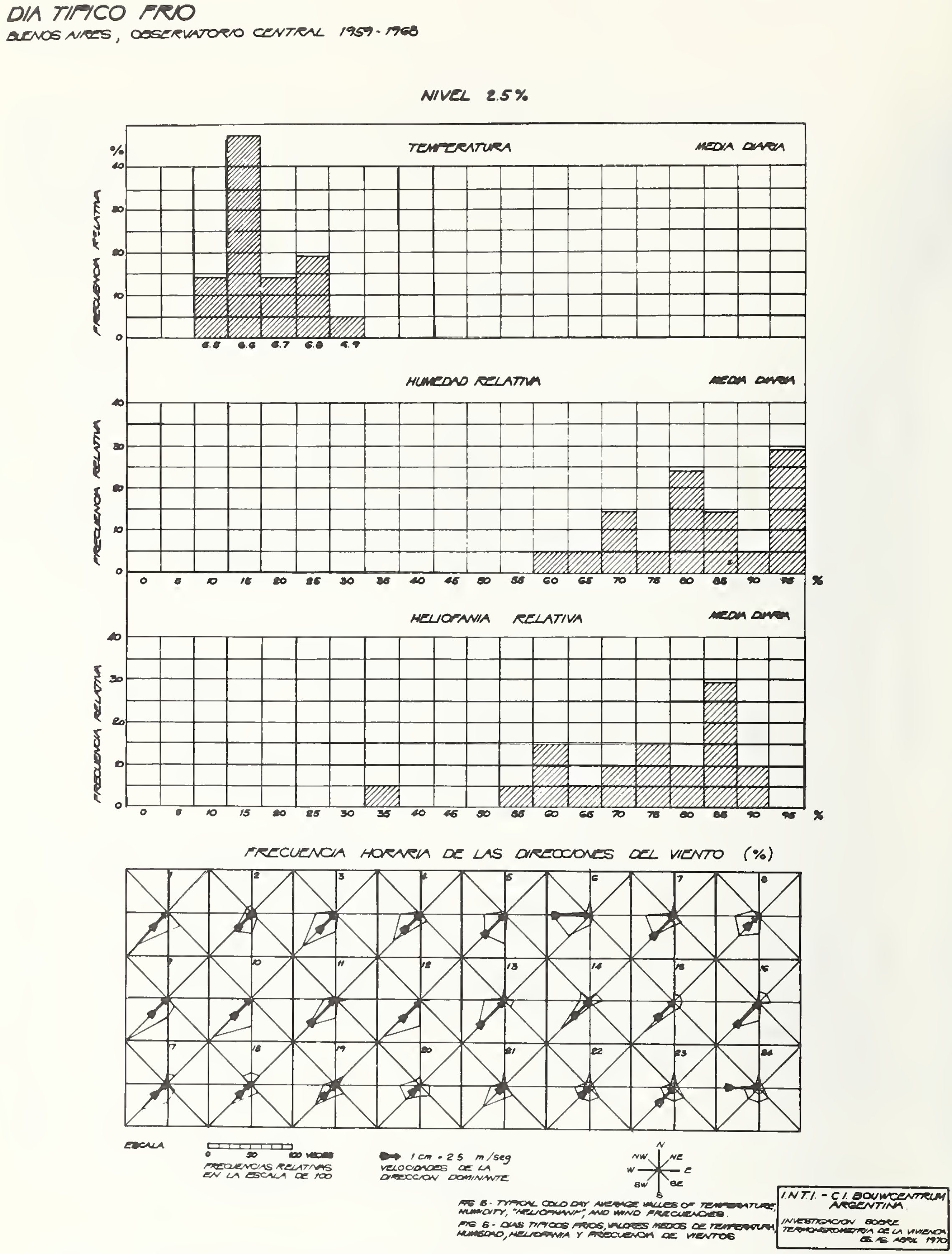

Figure 5. Average values of temperature, humidity and wind frequency for typical cold days

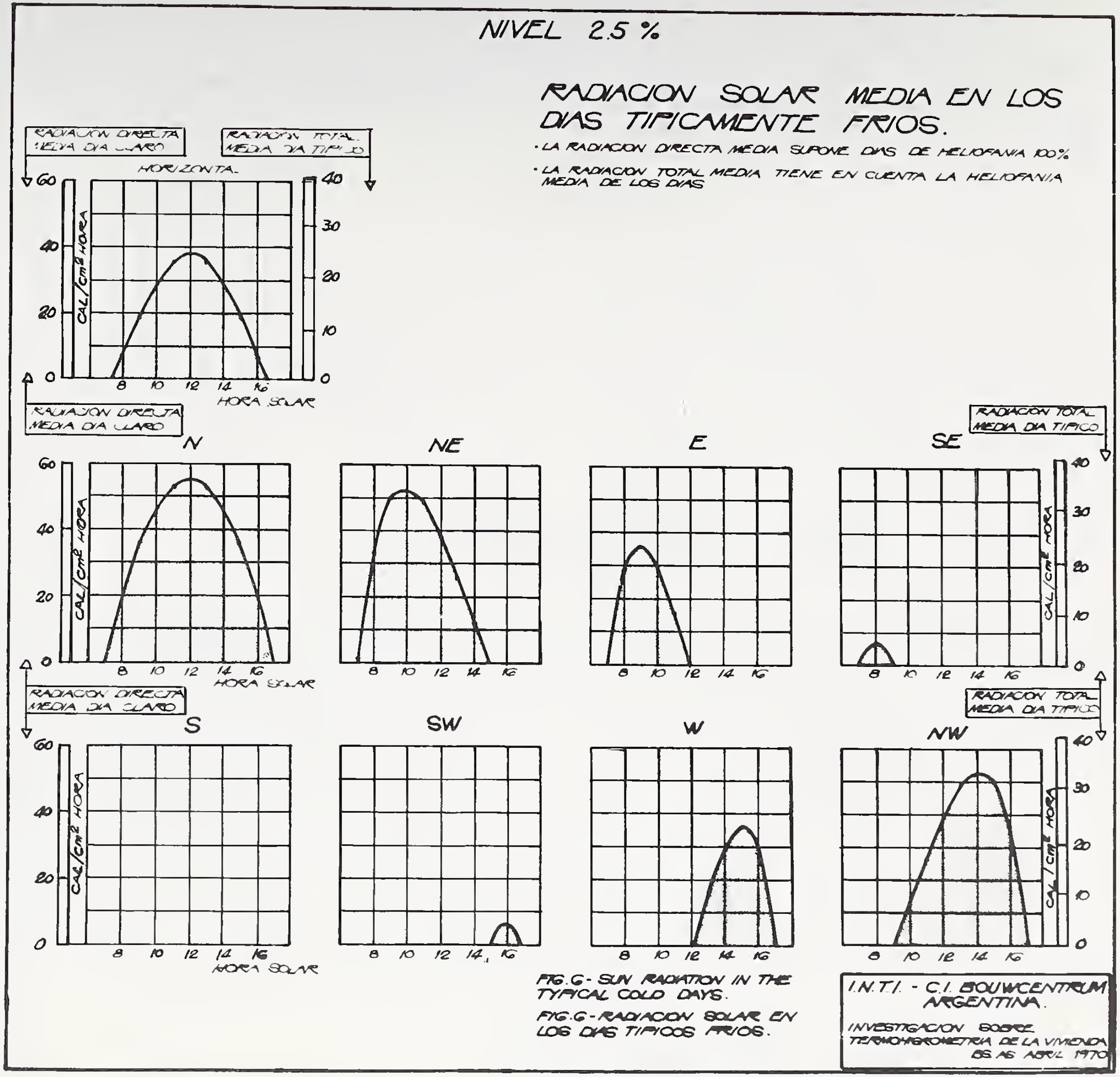

Figure 6. Solar radiation for typical cold days

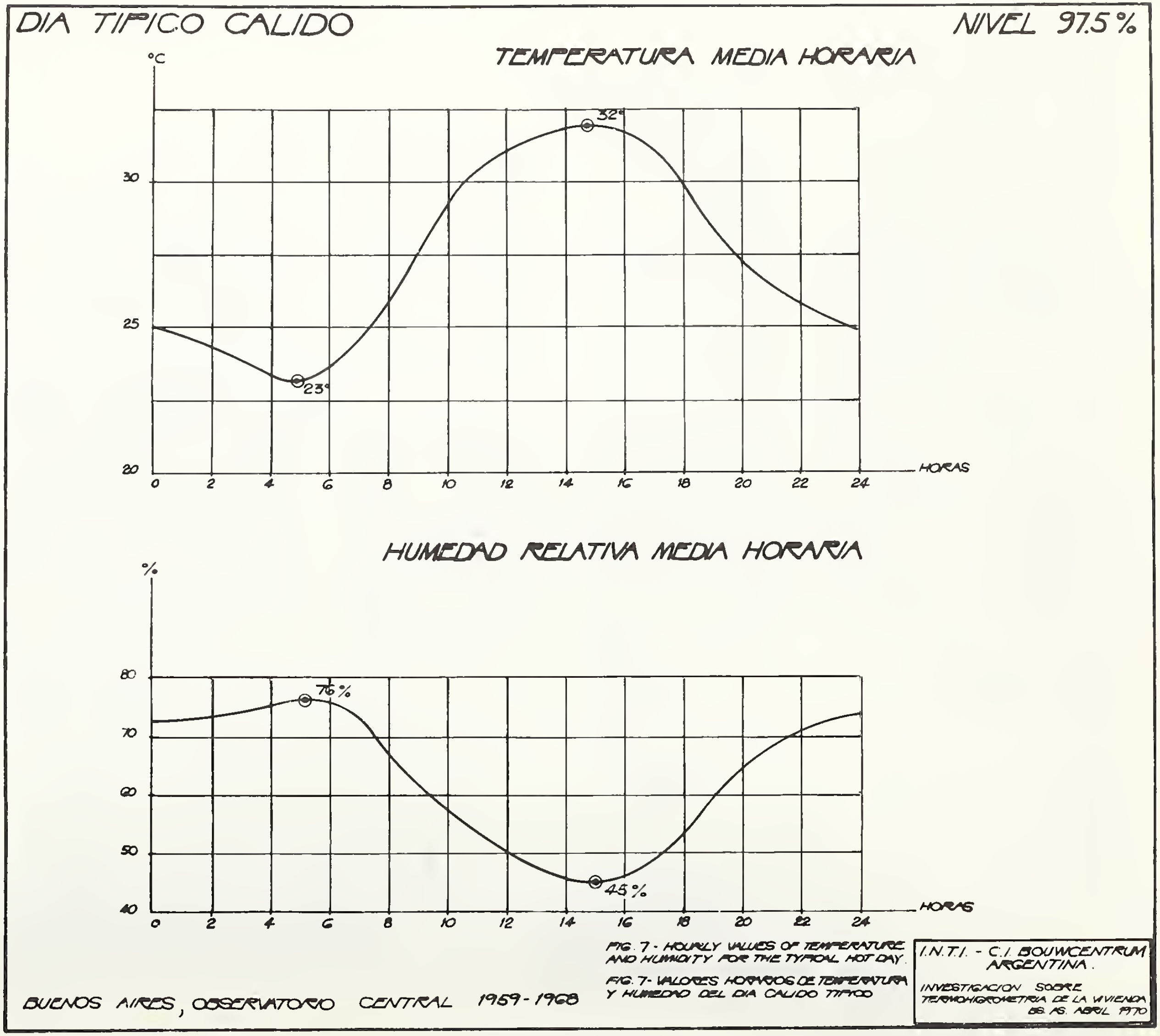

Figure 7. Hourly profiles of temperature and relative humidity during a typical hot day

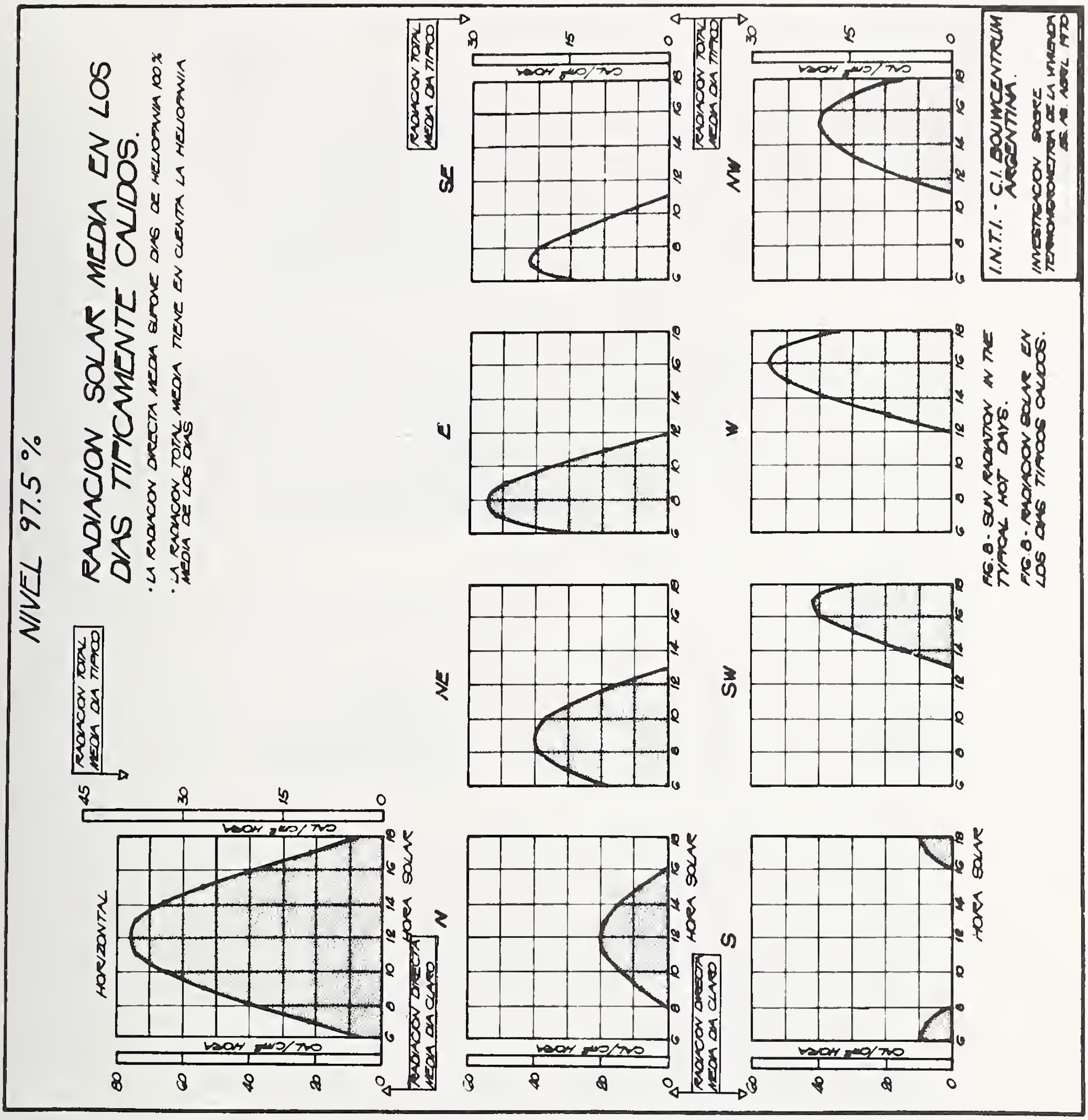

Figure 8. Solar radiation of typical hot days

DIA TIPICO CALIDO

BUENOS AIRES, OBSERVATORIO CENTRAL 1959-1968

NIVEL 97.5 %

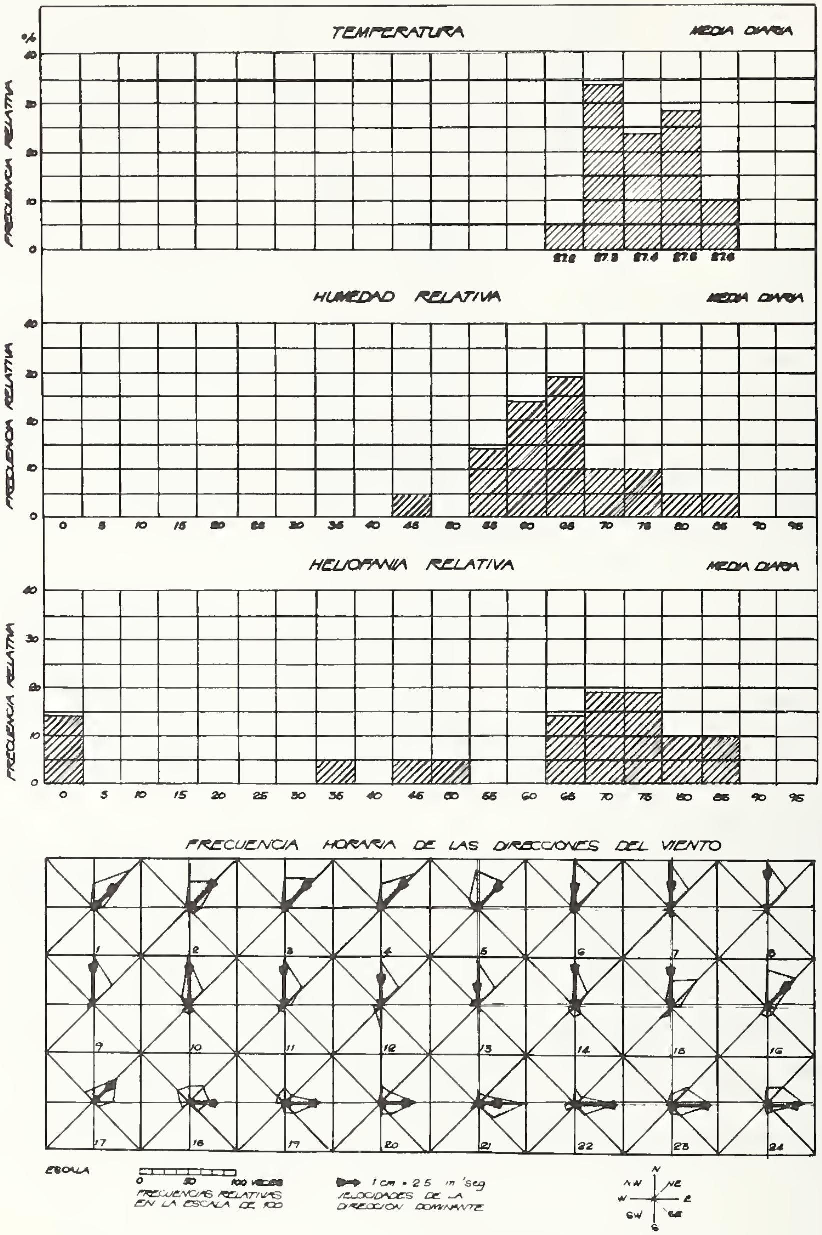

FIG 9 - TYPICAL HOT DAY AVERAGE VALUES OF TEMPERATURE, HUMIDITY, "HELIOPHANY" AND WIND FRECUENCIES

FIG 9 - DIAS CALIDOS TIPICOS, VALORES MEDIOS DE TEMPERATURA, HUMEDAD, HELIOFANIA Y FRECUENCIA DE VIENTOS

INTI - CI BOUWCENTRUM ARGENTINA

INVESTIGACION SOBRE TERMOHIGROMETRIA DE LA VIVIENDA

BS AS. ABRIL 1970

Figure 9. Average values of temperature, humidity and wind frequency for typical hot days

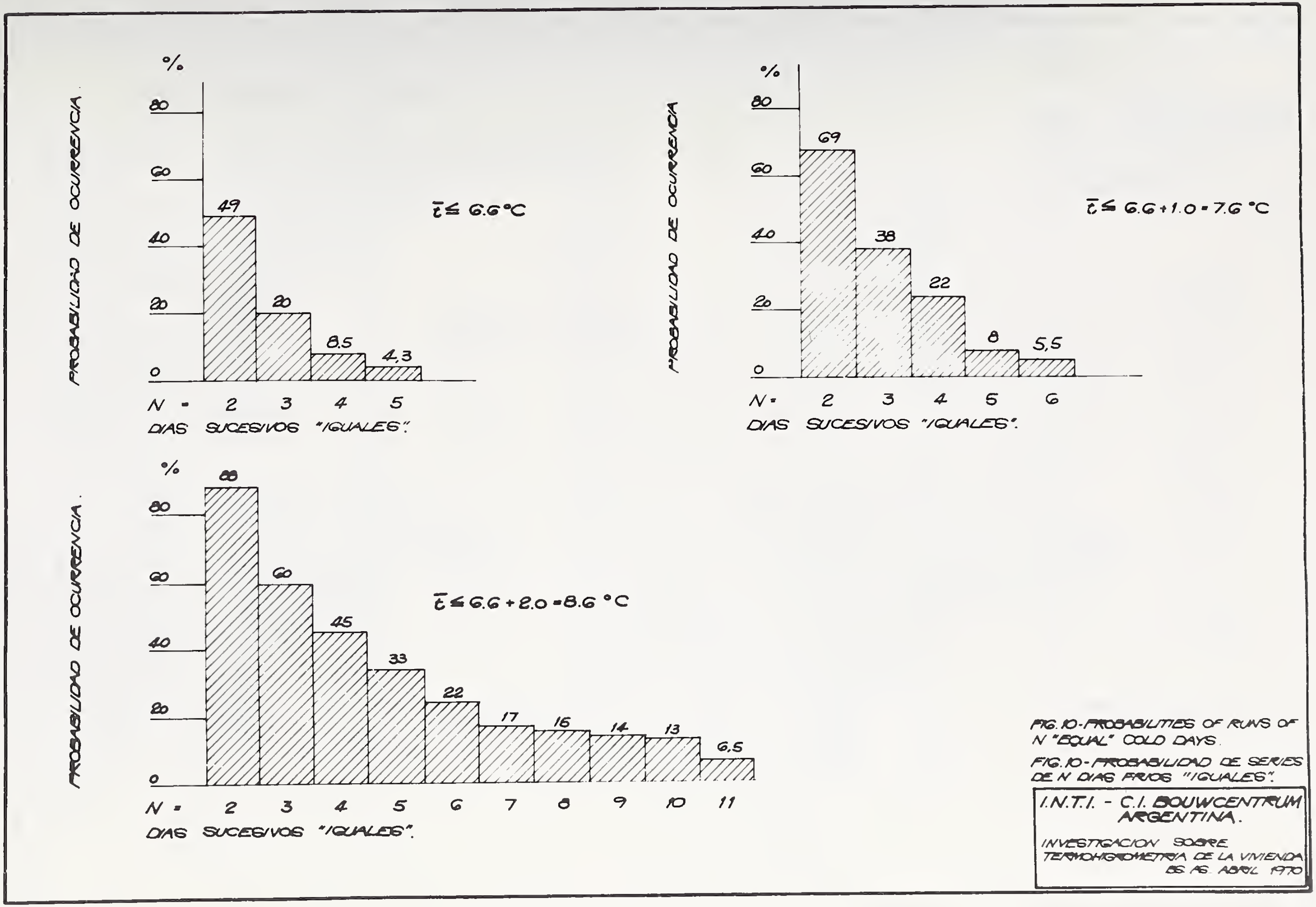

Figure 10. Probability of N equal cold day occurrence

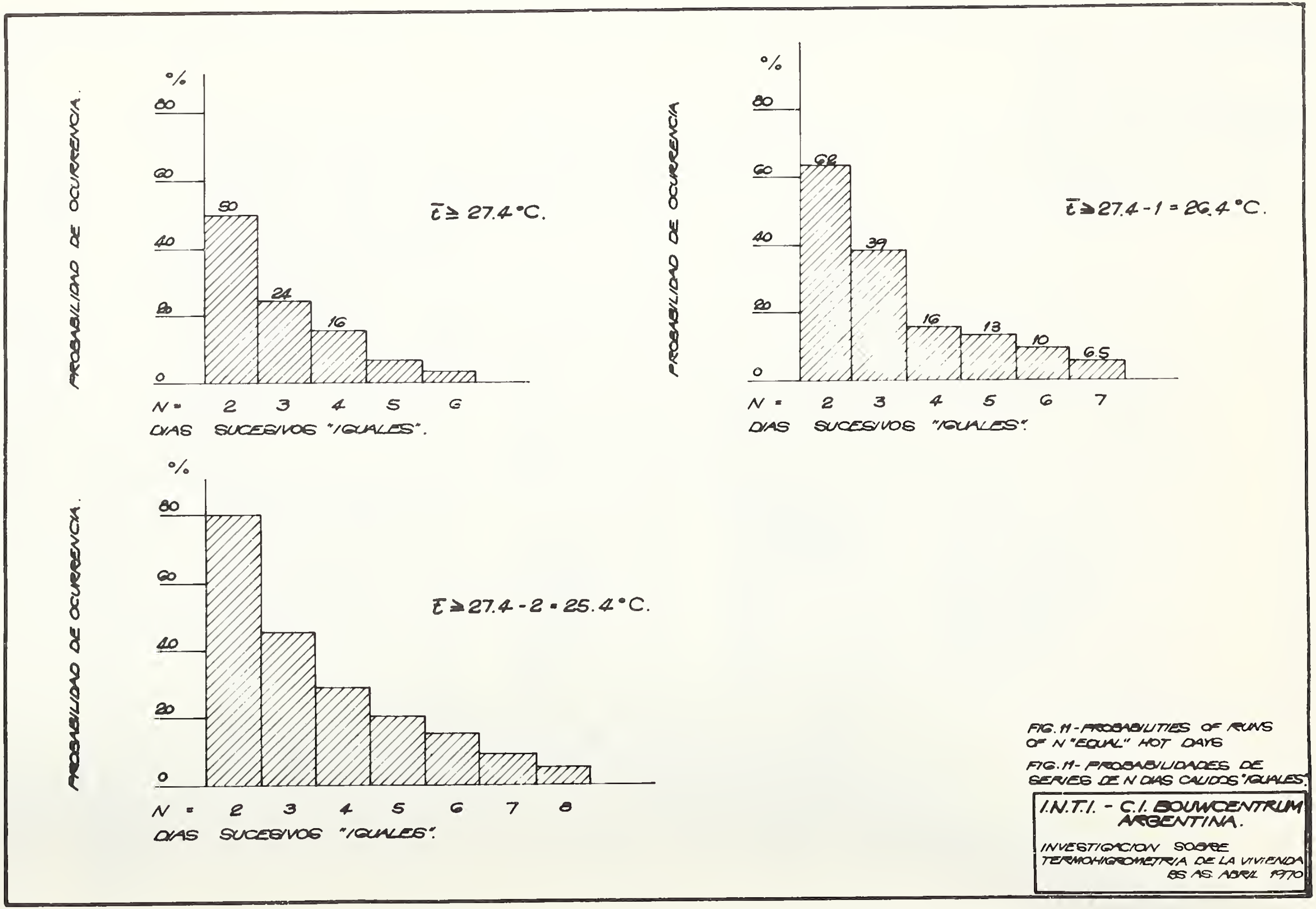

Figure 11. Probability of N equal hot day occurrence

# Fortran IV Program to Calculate z-Transfer Functions for the Calculation of Transient Heat Transfer through Walls and Roofs

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Division of Building Research
National Research Council of Canada
Ottawa

The heat transmission matrix for a wall or roof has elements A, B, C and D; i.e.,

$$\begin{vmatrix} \theta_o \\ Q_o \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \cdot \begin{vmatrix} \theta_i \\ Q_i \end{vmatrix}$$

where $\theta$ = Laplace transform of surface temperature, and

$Q$ = Laplace transform of surface flux

The elements A, B, C and D are functions of the thermal properties, thickness and position of materials in the wall. When the boundary conditions are of the first kind (i.e. when $\theta_o$ and $\theta_i$ are given), the fluxes are given by

$$\begin{vmatrix} Q_o \\ Q_i \end{vmatrix} = \frac{1}{B} \begin{vmatrix} D\ , & -1 \\ 1\ , & -A \end{vmatrix} \cdot \begin{vmatrix} \theta_o \\ \theta_i \end{vmatrix} \quad ;$$

and when boundary conditions are of the second kind, the equations invert to

$$\begin{vmatrix} \theta_o \\ \theta_i \end{vmatrix} = \frac{1}{C} \begin{vmatrix} A\ , & -1 \\ 1\ , & -D \end{vmatrix} \cdot \begin{vmatrix} Q_o \\ Q_i \end{vmatrix}$$

The program presented in this paper evaluates the coefficients of a set of z-transfer functions that are equivalent to the Laplace transfer functions D/B, 1/B, A/B, A/C, 1/C and D/C. These z-transfer functions relate to the z-transforms of the surface temperatures and heat fluxes in the same way as their counterpart Laplace transfer functions relate to the expressions above.

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[1] Research Officer and Computer Systems Programmer, respectively.

The program will evaluate z-transfer functions that are exact for either a unit step input, a ramp type input or a periodic input with specified harmonic components. The user can choose, therefore, the option that best suits a particular problem.

Key Words: Frequency response, roofs, transient heat conduction, walls, z-transforms.

The heat transmission matrix for a wall or roof has elements A, B, C and D, i.e.,

$$\begin{vmatrix} \theta_o \\ Q_o \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \cdot \begin{vmatrix} \theta_i \\ Q_i \end{vmatrix}$$

where $\theta$ = Laplace transform of surface temperature

$Q$ = Laplace transform of surface flux

The elements A, B, C and D are functions of the Laplace parameter, s, and of the thermal properties, thickness and position of materials in the wall. When the boundary conditions are of the first kind (i.e. when $\theta_o$ and $\theta_i$ are given), the fluxes are given by

$$\begin{vmatrix} Q_o \\ Q_i \end{vmatrix} = \frac{1}{B} \begin{vmatrix} D \, , & -1 \\ 1 \, , & -A \end{vmatrix} \cdot \begin{vmatrix} \theta_o \\ \theta_i \end{vmatrix} ;$$

and when boundary conditions are of the second kind, the equations invert to

$$\begin{vmatrix} \theta_o \\ \theta_i \end{vmatrix} = \frac{1}{C} \begin{vmatrix} A \, , & -1 \\ 1 \, , & -D \end{vmatrix} \cdot \begin{vmatrix} Q_o \\ Q_i \end{vmatrix}$$

The program presented in this paper evaluates the coefficients of a set of z-transfer functions that are equivalent to the Laplace transfer functions D/B, 1/B, A/B, A/C, 1/C and D/C. These z-transfer functions relate to the z-transforms of the surface temperatures and heat fluxes in the same way as their counterpart Laplace transfer functions relate to the expressions above.

The program will evaluate z-transfer functions that are exact for either a unit step input, a ramp type input or a periodic input with specified harmonic components. The user can choose, therefore, the option that best suits a particular problem.

## 1. Calculations of z-Transfer Functions [Ref. 1] [2]

The z-transfer functions for a multilayer wall can be calculated in two ways:

> Method 1 consists of choosing either a step or a ramp input function, I(z), and evaluating the output, 0(z), that corresponds to $1/s$ or $1/s^2$ times one of the Laplace transfer functions. The related z-transfer function is determined from 0(z)/I(z).

---

[2] The literature reference is at the end of the main text of this paper.

Method 2 involves solving a set of simultaneous linear algebraic equations to obtain the coefficients of a z-transfer function whose frequency response matches the exact frequency response of the multilayer slab at certain selected frequencies.

The z-transfer function corresponding to any one of the Laplace transfer functions can be expressed as the ratio of two finite polynomials, N(z)/D(z). The denominator, D(z), is the same for all the transfer functions that have a common denominator in their Laplace transfer function equivalents, and is the same for Method 1 and Method 2. The procedure for finding the coefficients of the denominator polynomial involves two steps.

(1) Determination of the poles of the associated Laplace transfer function:.

i.e., find $\beta_n$, the roots of B = 0;

or $\gamma_n$, the roots of C = 0;

The elements of the transmission matrix for a wall have an infinite set of roots, which lie along the negative real axis in the s-plane. The position of the roots depends on the dimensions and thermal properties of all the layers, and cannot be expressed in any simple way. The necessary poles can be found numerically, however, by evaluating the functions B or C for a sequence of negative real values of s. This program evaluates the roots of B between zero and $-30/\Delta$, and the roots of C between zero and $-450/\Delta$, where $\Delta$ is the specified sampling interval of the z-transform.

(2) The evaluation of the product:

$$D(z) = \prod (1 - e^{-\beta_n \Delta} z^{-1})$$

when the parent Laplace transfer function has the element B in the denominator, or

$$D(z) = \prod (1 - e^{-\gamma_n \Delta} z^{-1})$$

when Laplace transfer function has C in the denominator.

Methods 1 and 2 differ only in the way the numerator polynomial is determined. Method 1 requires the evaluation of the time function that corresponds to $1/s$ (step input) or to $1/s^2$ (ramp input) times the appropriate Laplace transfer function, for $t = \Delta, 2\Delta, 3\Delta, \ldots\ldots$ The coefficients of 0(z) are evaluated by finding the residues of the Laplace transfer function at the previously determined poles.

The numerator (N(z) is then evaluated using the expression

$$N(z) = \frac{D(z) \quad . \quad 0(z)}{I(z)}$$

where $$I(z) = \frac{1}{1 - z^{-1}} \quad \text{for a step input}$$

or $$I(z) = \frac{\Delta}{z(1 - z^{-1})^2} \quad \text{for a ramp input.}$$

Method 2 requires the evaluation of the Laplace transfer function of the wall at $s = i\omega_n$, and the calculation of the denominator $D(z)$ at $z = e^{i\omega_n \Delta}$, where $\omega_n$ is the angular velocity at which the z-transfer function is to match the exact frequency response. This gives a pair of equations for each value of $\omega_n$ (i.e. real and imaginary parts are equated separately) except at $\omega_n = 0.0$ (i.e. steady state) where only the real part of the equation exists. The resulting set of equations for a series of values of $\omega_n$ can be expressed in matrix form, viz.:

$$\begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & \text{Cos}\,\omega_1\Delta & \text{Cos}\,2\omega_1\Delta & \cdots & \text{Cos}\,J\omega_1\Delta \\ 0 & \text{Sin}\,\omega_1\Delta & \text{Sin}\,2\omega_1\Delta & \cdots & \text{Sin}\,J\omega_1\Delta \\ 1 & \text{Cos}\,\omega_2\Delta & \text{Cos}\,2\omega_2\Delta & \cdots & \text{Cos}\,J\omega_2\Delta \\ 0 & \text{Sin}\,\omega_2\Delta & \text{Sin}\,2\omega_2\Delta & \cdots & \text{Sin}\,J\omega_2\Delta \\ & & & & \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ & & & & \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ \\ \cdot \\ \cdot \\ \cdot \\ a_J \end{bmatrix} = \begin{bmatrix} X(0) \\ X(\omega_1) \\ Y(\omega_1) \\ X(\omega_2) \\ Y(\omega_2) \\ \\ \cdot \\ \cdot \\ \cdot \\ \\ \end{bmatrix}$$

where the a's are the unknown coefficients of the N(z) polynomial and $X(\omega_n)$ and $Y(\omega_n)$ are real and imaginary parts of the product of the Laplace transfer function and denominator D(z) evaluated at $s = i\omega_n$ and $z = e^{i\omega_n \Delta}$ respectively. The solution of this matrix equation gives the unknown coefficients. It should be noted that in setting up this matrix, $\omega_n > \frac{2\pi}{3\Delta}$ should not be used since higher frequencies than this tend to give poorer results.

## 2. General Description of the Program

This Fortran IV program is designed for an IBM-360 computer with line printer. Appendix A consists of the coding sheets (A-1 to A-20), a sample of output (A-21 to A-25), and the flow diagrams (A-26 to A-31) for this program.

The program can handle slabs that are comprised of no more than 20 layers of homogeneous material and no more than 100 significant poles. The poles are evaluated to $10^{-14}$ precision, and the limit for the numerator and denominator series of the z-transform is set at $10^{-7}$. At least one of the layers of the composite slab must have significant heat-storage capacity.

The program is designed to operate continuously; i.e., after the z-transforms for one wall have been completed, the program automatically reads the data for the following calculation. The program terminates when $\Delta = 0.0$ is read.

### 2.1 Input

| | |
|---|---|
| Card 1 | Sampling time interval Δ<br>Format: (F 10.3) |
| Card 2 and 3 | Description of the slab for title purpose only.<br>Format: (80 A1) |
| Cards 4 to I | Groups of cards giving thermal properties, thickness, and description of the layers. Whenever applicable, the first card of the group contains values of<br>thickness of layer,<br>thermal conductivity,<br>density, |

specific heat, and
resistance of radiation path.

Otherwise, the first card contains the thermal resistance of a layer that has negligible heat storage capacity.
Format: (5F 10.4)

The second, third ... or more cards of the group can be used for the description of the layer if an integer is inserted in Column One.
Format: (30 A1)

Card I + 1 Blank card to terminate the above input of thermal properties and their descriptions.

Card I + 2 Code number, ICASE, and the number of frequencies, NW, to be fitted when Method 2 is to be used (see Table 1).
Format: (I1, I1)

Card I + 3 This card is read only when ICASE = 2 or 5. It specifies the periods of the harmonics to be used in frequency response calculations.

Table 1. Code Number ICASE

| Input Function / Boundary Condition | Method 1 | | Method 2 |
|---|---|---|---|
| | Square Pulse | Triangle Pulse | Group of Harmonics |
| First Kind | Invalid Combination | 1 | 2 |
| Second Kind | 3 | 4 | 5 |

## 3. Reference

[1] Stephenson, D.G. and Mitalas, G.P., Calculation of heat conduction transfer functions for multilayer slabs. Submitted to ASHRAE for presentation January 1971.

## 4. Acknowledgement

The authors gratefully acknowledge the many helpful suggestions made by Dr. D.G. Stephenson.

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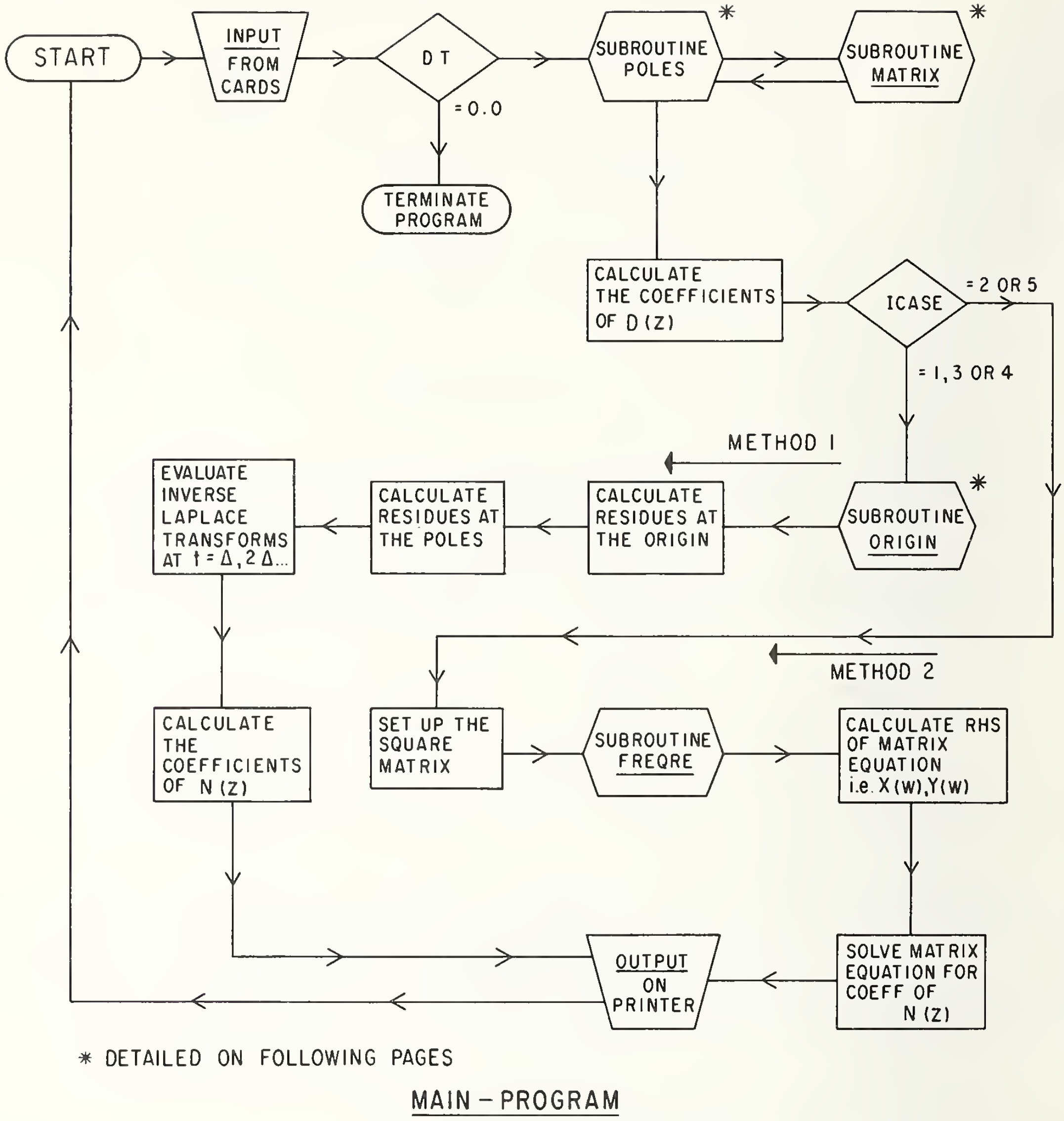
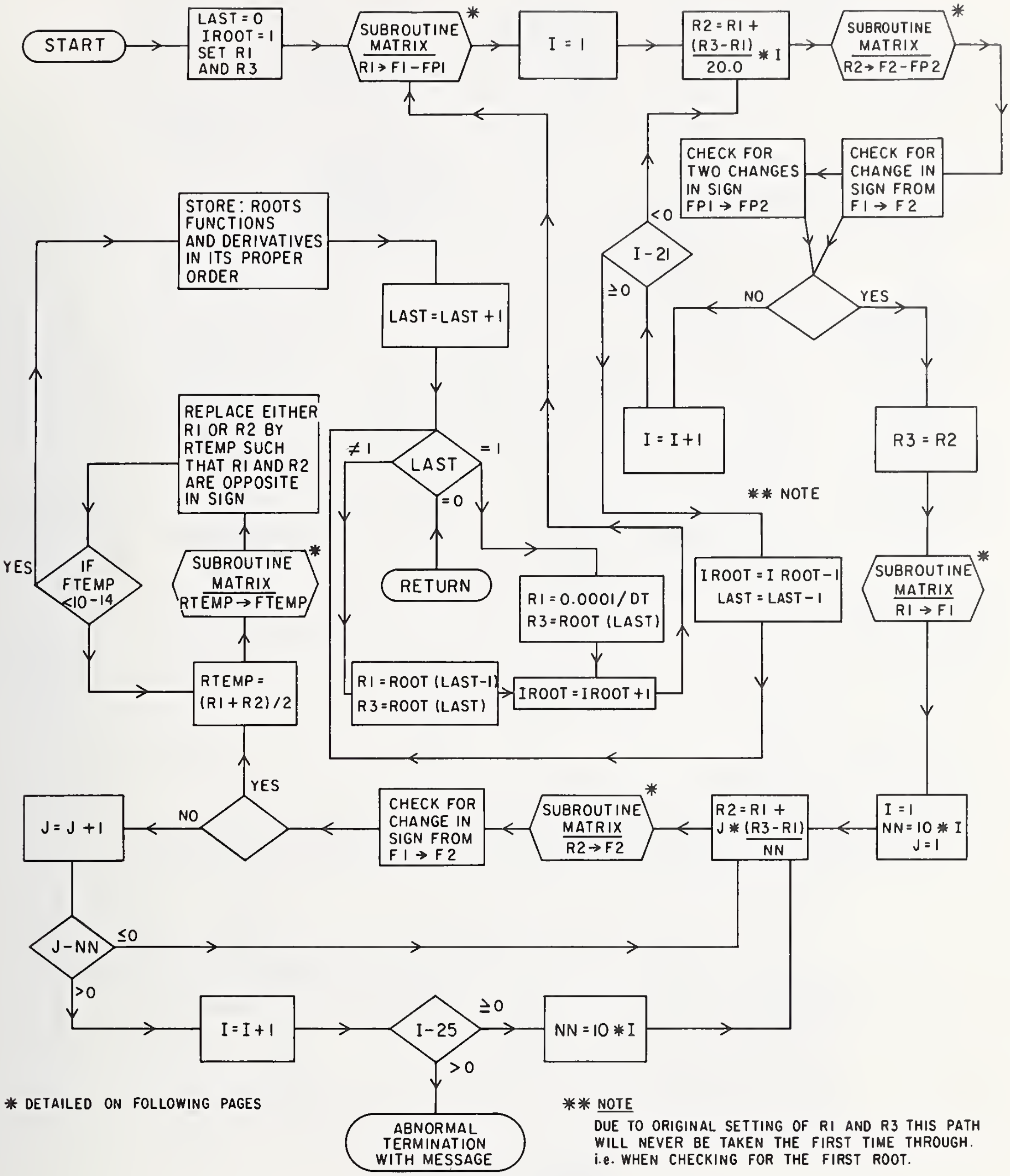


Figure A-1. Flow diagram for z-transfer function calculation program.



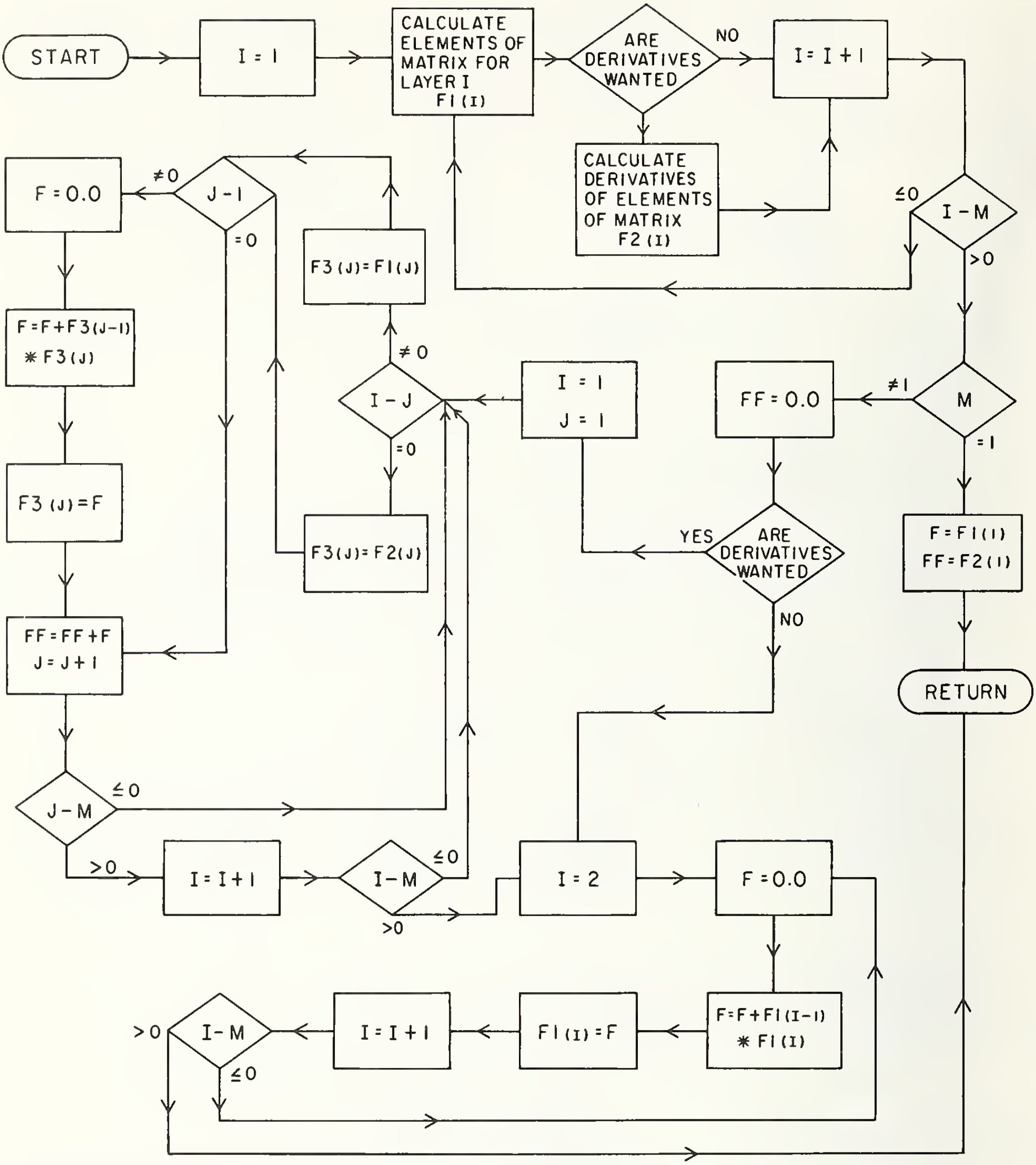
* DETAILED ON FOLLOWING PAGES

** NOTE

DUE TO ORIGINAL SETTING OF RI AND R3 THIS PATH WILL NEVER BE TAKEN THE FIRST TIME THROUGH. i.e. WHEN CHECKING FOR THE FIRST ROOT.

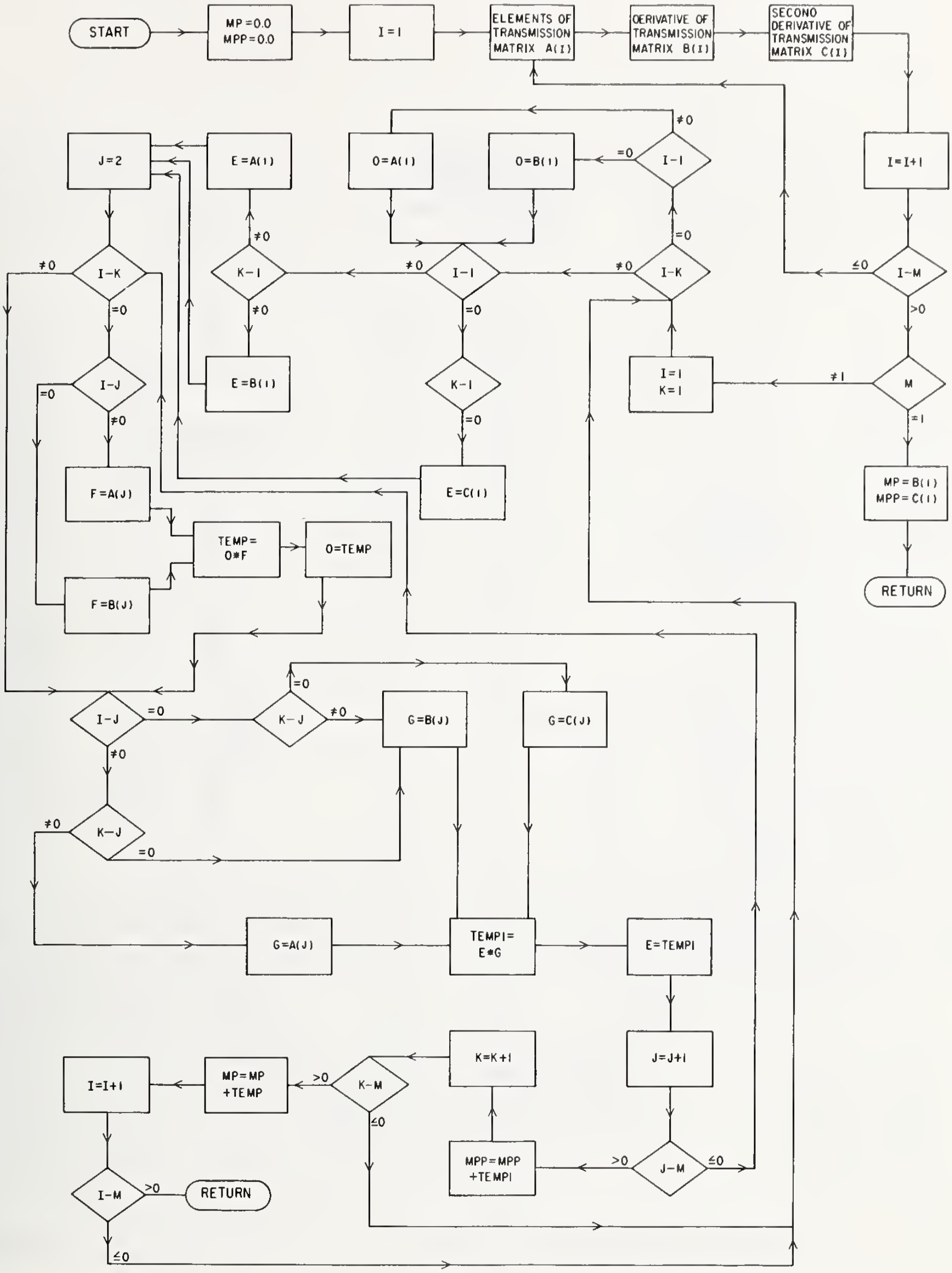
SUBROUTINE — POLES

Figure A-2. Flow diagram for determination of poles of Laplace transfer function and calculation of the values of A, B, C, D, and their derivatives at these poles.



SUBROUTINE - MATRIX

Figure A-3. Flow diagram for the evaluation of A, B, C, D, and their derivatives for any real negative value of s.



SUBRROUTINE – ORIGIN

Figure A-4. Flow diagram for the evaluation of first and second derivative of A, B, C and D at s = o.

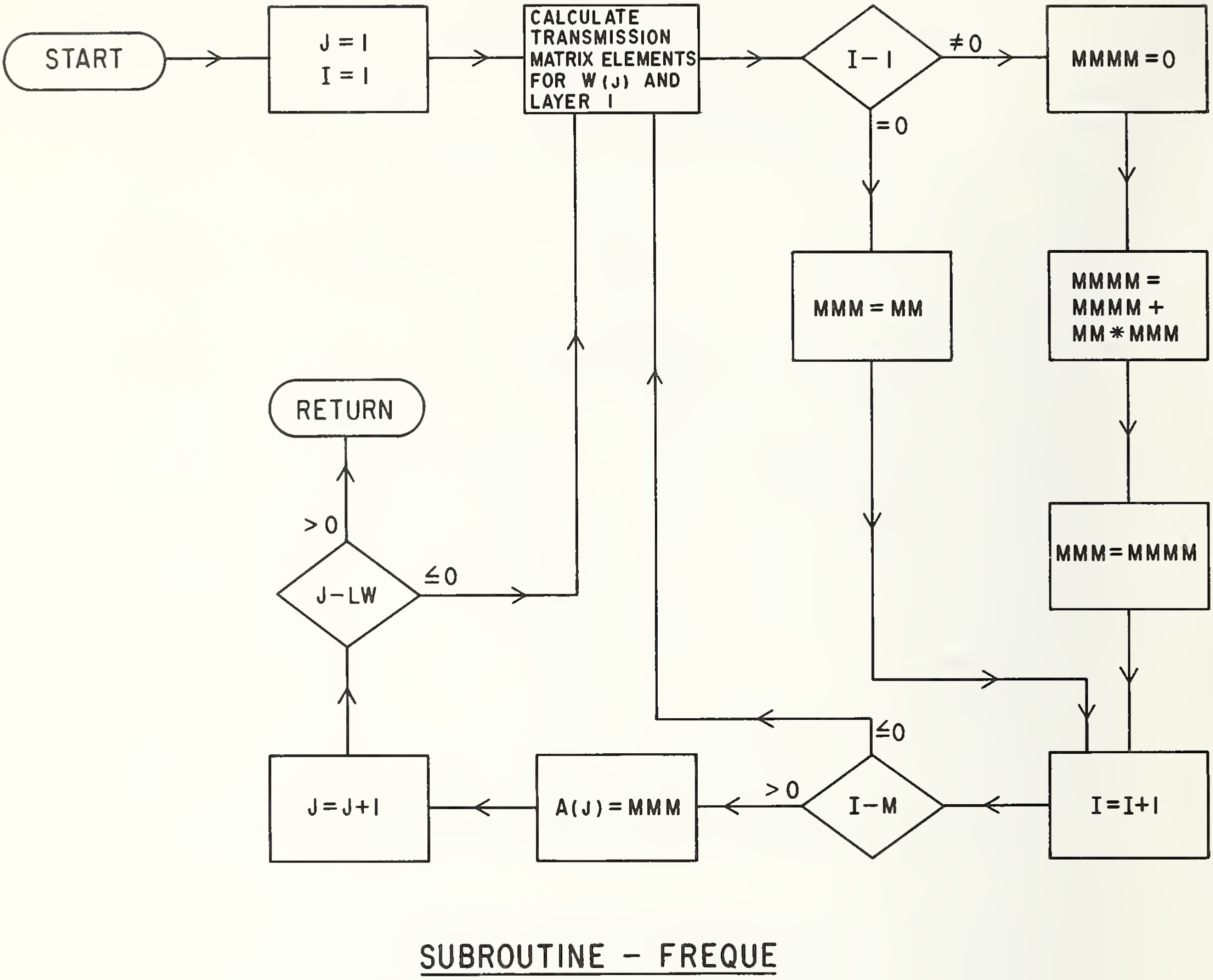


Figure A-5. Flow diagram for the evaluation of A, B, C and D for pure imaginary arguments.

# APPENDIX A

```
FORTRAN IV G LEVEL  18              MAIN              DATE = 70132          14/14/56             PAGE 0001
            C           FORTRAN PROGRAM TO EVALUATE Z-TRANSFORMS FOR CALCULATION OF
            C     TRANSIENT HEAT TRANSFERS THROUGH WALLS AND ROOFS.
            C
            C         THIS PROGRAM WILL DERIVE THE Z-TRANSFER FUNCTIONS FOR TWO
            C     TYPES OF BOUNDARY CONDITIONS AND THE FORM OF BOUNDARY PARAMETERS
            C     MUST BE SPECIFIED.
            C
            C     BOUNDARY CONDITIONS:
            C         OF THE FIRST KIND; (TEMPERATURE GIVEN FOR BOTH SURFACES).
            C     A) RAMP INPUT                ICASE=1
            C     B) FREQUENCY RESPONSE        ICASE=2
            C
            C         OF THE SECOND KIND; (FLUX GIVEN FOR BOTH SURFACES).
            C     A) STEP INPUT                ICASE=3
            C     B) RAMP INPUT                ICASE=4
            C     C) FREQUENCY RESPONSE        ICASE=5
            C
            C         INPUT TO PROGRAM:
            C     CARD(1)       DT (F10.3)     DT=SAMPLING TIME INTERVAL.
            C     CARD(2)*
            C            * DESCRIPTION OF THE SLAB FOR TITLE PURPOSE ONLY (80A1).
            C     CARD(3)*
            C
            C     CARD(4)   *
            C       "       *
            C     CARD(I+3)* XL(I),XK(I),D(I),SH(I),RES(I),(TEXT(I,J),J=1,30) WHERE
            C       "       * I INDICATES THE I'TH LAYER OF THE SLAB (5F10.4,30A1).
            C       "       *
            C     CARD(M+3)*
            C         WHERE XL=THICKNESS OF LAYER.
            C               XK=THERMAL CONDUCTIVITY.
            C                D=DENSITY.
            C               SH=SPECIFIC HEAT.
            C              RES=RESISTANCE OF RADIATION PATH WHENEVER APPLICABLE
            C                  OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
            C                  NEGLIGIBLE HEAT STORAGE.
            C             TEXT=DESCRIPTION OF LAYER, A SECOND CARD AND SO ON CAN BE
            C                  USED BY INSERTING ANY INTEGER IN COLUMN ONE(1).
            C                M=NUMBER OF LAYERS THE SLAB IS COMPOSED OF.
            C     CARD(M+4)    BLANK CARD TO STOP ABOVE INPUT.
            C     CARD(M+5)    ICASE,NW (I1,I1)
            C                  WHERE:  NW=NUMBER OF FREQUENCIES TO BE USED WHEN
            C                        FREQUENCY RESPONSE IS INVOLVED.
            C     CARD(M+6)     W(2),W(3)..............W(NW+1) (8F10.4)
            C                   ABOVE CARD ONLY READ WHEN FREQUENCY RESPONSE IS
            C                   INVOLVED. (ICASE=2 OR ICASE=5) W(1) IS SET TO 0.0
            C                   W(I)'S ARE THE PERIODS.
            C
            C     NOMENCLATURE:
            C         RR=THICKNESS/THERMAL CONDUCTIVITY (XL/XK)
            C            OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
            C            NEGLIGIBLE HEAT STORAGE. THEN RES=0.0
            C         BETA*BETA=XL*XL*D*SH/XK
```

```
C         CO=THERMAL CONDUCTANCE AND USED AS 1/C'AT THE POLE FOR ICASE=4.
C         AFTER A RUN IS COMPLETED CONTROL RETURNS TO READING CARD(1)
C         THEREFORE A BLANK CARD IN THIS LOCATION TERMINATES THE PROGRAM.
C
C
      DOUBLE PRECISION RR(20),BETA(20),XL(20),XK(20),D(20),SH(20),
     9RES(20),ROOT(100),FUNC(100,4),DER(100,4),DN(100,3),MP(2,2),
     8MPP(2,2),POL1(100),POL2(100),POL3(100),POL4(100),POL5(100),
     7POL6(100),MX(11,11),X(11),Y(11),Z(11),TEM1(2),TEM2(2),TEM3(2),
     6DT,CO,C1X,C1Y,C1Z,C2X,C2Y,C2Z,T,ARG1,ARG2,PREC,DET,TEST,W(6)
      COMPLEX*16 A(6,2,2),TEMP,TEMP1,TEMP2,TEMP3
      INTEGER CARD,PRINT,TEXT(20,30),TEXT1(2,80)
      EQUIVALENCE (TEMP1,TEM1(1)),(TEMP2,TEM2(1)),(TEMP3,TEM3(1))
      CARD=1
      PRINT=3
      PREC=0.0
      W(1)=0.0
 1000 READ(CARD,1) DT
    1 FORMAT(F10.3)
      IF(DT.EQ.0.0) GO TO 2000
      READ(CARD,2) ((TEXT1(I,J),J=1,80),I=1,2)
    2 FORMAT(80A1)
      WRITE(PRINT,3) ((TEXT1(I,J),J=1,80),I=1,2)
    3 FORMAT(1H1,26X,80A1/27X,80A1)
      WRITE(PRINT,4)
    4 FORMAT(1HO,'LAYER',1X,'THICKNESS',4X,'CONDUCTIVITY',8X,'DENSITY',
     18X,'SP HEAT',11X,'RESISTANCE')
      WRITE(PRINT,5)
    5 FORMAT(2X,'   ',5X,'  ',4X,'                          ',3X,'         '
     1,2X,'           ',2X,'                           ',4X,'DESCRIPTION
     2OF LAYER'/)
      CO=0.0
      DO 10 I=1,20
      M=I
      READ(CARD,6) XL(I),XK(I),D(I),SH(I),RES(I),(TEXT(I,J),J=1,30)
    6 FORMAT(5F10.4,30A1)
      IF(RES(I).EQ.0.0.AND.SH(I).EQ.0.0.AND.XL(I).NE.0.0) GO TO 20
      IF(RES(I).EQ.0.0.AND.XL(I).EQ.0.0) GO TO 30
      IF(XL(I).NE.0.0) GO TO 40
      RR(I)=RES(I)
      BETA(I)=0.0
      GO TO 50
   40 RR(I)=XL(I)/XK(I)
      BETA(I)=XL(I)*DSQRT(D(I)*SH(I)/XK(I))
      CO=CO+RES(I)
   50 CO=CO+RR(I)
      WRITE(PRINT,7) I,XL(I),XK(I),D(I),SH(I),RES(I),(TEXT(I,J),J=1,30)
    7 FORMAT(1X,I3,F10.4,F15.4,F18.4,F15.4,F20.4,13X,30A1)
      GO TO 10
   20 WRITE(PRINT,8) (TEXT(I,J),J=1,30)
    8 FORMAT(95X,30A1)
      I=I-1
   10 CONTINUE
```

```
      M=21
   30 M=M-1
      DO 60 I=1,M
      IF(XL(I).NE.0.0) GO TO 60
      RES(I)=0.0
   60 CONTINUE
      CO=1.0/CO
      WRITE(PRINT,1)
      WRITE(PRINT,1)
      WRITE(PRINT,9) CO
    9 FORMAT(30X,'THERMAL CONDUCTANCE,    U=',F6.3,2X,'
     1    ')
      WRITE(PRINT,1)
      WRITE(PRINT,11) DT
   11 FORMAT(39X,'SAMPLING TIME INTERVAL,  DT=',F9.3,'    ')
      WRITE(PRINT,1)
      READ(CARD,12) ICASE,NW
   12 FORMAT(I1,I1)
      NW=NW+1
      MW=NW*2-1
      IF(ICASE.NE.2.AND.ICASE.NE.5) GO TO 65
      READ(CARD,13) (W(I),I=2,NW)
   13 FORMAT(8F10.4)
      DO 61 I=2,NW
   61 W(I)=2.0*3.14159265/W(I)
   65 IF(ICASE.LT.3) GO TO 70
      IX=2
      JX=1
      GO TO 80
   70 IX=1
      JX=2
   80 CALL POLES(RR,BETA,RES,ROOT,DER,FUNC,M,IX,JX,DT,IROOT,ICASE)
      DO 90 I=1,100
      POL1(I)=0.0
      POL2(I)=0.0
      POL3(I)=0.0
      POL4(I)=0.0
      POL5(I)=0.0
   90 POL6(I)=0.0
      MMM=1
      NNN=0
      POL1(1)=1.0
      DO 100 I=1,IROOT
      POL2(1)=1.0
      POL2(2)=-DEXP(-ROOT(I)*DT)
      IF(DABS(POL2(2)).LT.1.0D-16)GO TO 110
      CALL POLYM(POL1,POL2,NNN,MMM)
  100 CONTINUE
  110 IF(ICASE.LT.3) GO TO 120
      POL2(1)=1.0
      POL2(2)=-1.0
      CALL POLYM(POL1,POL2,NNN,MMM)
  120 ID=NNN+1
```

```
      IF(ICASE.EQ.2.OR.ICASE.EQ.5) GO TO 130
      CALL ORIGIN(RR,BETA,RES,M,MP,MPP)
      GO TO(140,140,150,150,150),ICASE
  140 C1Y=-CO*CO*MP(1,2)
      C1X=C1Y+MP(2,2)*CO
      C1Z=C1Y+MP(1,1)*CO
      GO TO 160
  150 CO=1.0/MP(2,1)
      C1Y=-CO*CO*MPP(2,1)/2.0
      C1X=MP(2,2)*CO+C1Y
      C1Z=MP(1,1)*CO+C1Y
  160 C2X=0.0
      C2Y=0.0
      C2Z=0.0
      DO 170 I=1,IROOT
      IF(ICASE.GT.1) GO TO 180
      DN(I,2)=1.0/ROOT(I)/ROOT(I)/DER(I,2)
      GO TO 200
  180 IF(ICASE.EQ.4) GO TO 190
      DN(I,2)=-1.0/ROOT(I)/DER(I,3)
      GO TO 200
  190 DN(I,2)=1.0/ROOT(I)/ROOT(I)/DER(I,3)
  200 DN(I,1)=DN(I,2)*FUNC(I,4)
      DN(I,3)=DN(I,2)*FUNC(I,1)
      IF(ICASE.NE.4) GO TO 170
      C2X=C2X-DN(I,1)
      C2Y=C2Y-DN(I,2)
      C2Z=C2Z-DN(I,3)
  170 CONTINUE
      DO 210 I=1,ID
  210 POL2(I)=POL1(I)
      IF(ICASE.EQ.3) GO TO 220
      POL3(1)=1.0/DT
      POL3(2)=-2.0/DT
      POL3(3)=1.0/DT
      MMM=2
      GO TO 235
  220 POL3(1)=1.0
      POL3(2)=-1.0
      MMM=1
  235 CALL POLYM(POL2,POL3,NNN,MMM)
      POL3(1)=0.0
      POL3(2)=0.0
      POL3(3)=0.0
      DO 230 I=1,100
      II=I
      T=I*DT
      DO 240 J=1,IROOT
      IF(ROOT(J)*T.GE.40.0) GO TO 250
      POL3(I)=POL3(I)+DEXP(-ROOT(J)*T)*DN(J,1)
      POL4(I)=POL4(I)+DEXP(-ROOT(J)*T)*DN(J,2)
      POL5(I)=POL5(I)+DEXP(-ROOT(J)*T)*DN(J,3)
      IF(J.LE.10) GO TO 240
```

```
 0147                IF(DABS(DEXP(-ROOT(J)*T)*DN(J,2)).LT.1.0D-16) GO TO 250
 0148            240 CONTINUE
 0149            250 IF(ICASE.EQ.4) GO TO 260
 0150                POL3(I)=POL3(I)+CO*T+C1X
 0151                POL4(I)=POL4(I)+CO*T+C1Y
 0152                POL5(I)=POL5(I)+CO*T+C1Z
 0153                GO TO 270
 0154            260 POL3(I)=POL3(I)+CO*T*T /2.0+C1X*T+C2X
 0155                POL4(I)=POL4(I)+CO*T*T /2.0+C1Y*T+C2Y
 0156                POL5(I)=POL5(I)+CO*T*T /2.0+C1Z*T+C2Z
 0157            270 IF(I.LE.10) GO TO 230
 0158                IF(DABS(POL4(I)).LT.1.0D-16) GO TO 280
 0159            230 CONTINUE
 0160            280 MMM=II-1
 0161                NN=NNN+1
 0162                DO 290 I=1,NN
 0163            290 POL6(I)=POL2(I)
 0164                CALL POLYM(POL6,POL3,NNN,MMM)
 0165                NN1=NNN+1
 0166                DO 300 I=1,NN1
 0167            300 POL3(I)=POL6(I)
 0168                DO 310 I=1,NN
 0169            310 POL6(I)=POL2(I)
 0170                NNN=NN-1
 0171                CALL POLYM(POL6,POL4,NNN,MMM)
 0172                NN2=NNN+1
 0173                DO 320 I=1,NN2
 0174            320 POL4(I)=POL6(I)
 0175                DO 330 I=1,NN
 0176            330 POL6(I)=POL2(I)
 0177                NNN=NN-1
 0178                CALL POLYM(POL6,POL5,NNN,MMM)
 0179                NN3=NNN+1
 0180                DO 340 I=1,NN3
 0181            340 POL5(I)=POL6(I)
 0182                GO TO 350
 0183            130 DO 360 I=1,MW,2
 0184                DO 360 J=1,MW
 0185                IF(I.EQ.1) GO TO 370
 0186                K=(I+1)/2
 0187                MX(I,J)=DSIN((J-1)*DT*W(K))
 0188                GO TO 360
 0189            370 MX(I,J)=1.0
 0190            360 CONTINUE
 0191                LW=MW-1
 0192                DO 380 I=2,LW,2
 0193                DO 380 J=1,MW
 0194                K=I/2+1
 0195            380 MX(I,J)=DCOS((J-1)*DT*W(K))
 0196                IF(ICASE.EQ.2) GO TO 381
 0197                MW=MW-1
 0198                DO 382 I=1,MW
 0199                DO 382 J=1,MW
```

```
  382 MX(I,J)=MX(I+1,J)
  381 CALL SOLVD(MX,11,MW,MW,PREC,DET,TEST)
      IF(ICASE.EQ.5) MW=MW+1
      CALL FREQRE (RR,BETA,RES,XL,XK,D,SH,M,W,A,NW)
      DO 390 I=1,MW,2
      IF(I.EQ.1) GO TO 390
      K=(I+1)/2
      ARG1=0.0
      ARG2=0.0
      DO 410 J=1,ID
      ARG1=ARG1+POL1(J)*DCOS((J-1)*DT*W(K))
  410 ARG2=ARG2-POL1(J)*DSIN((J-1)*DT*W(K))
      TEMP=DCMPLX(ARG1,ARG2)
      IF(ICASE.EQ.2) GO TO 420
      TEMP1=TEMP*A(K,2,2)/A(K,2,1)
      TEMP2=TEMP/A(K,2,1)
      TEMP3=TEMP*A(K,1,1)/A(K,2,1)
      GO TO 430
  420 TEMP1=TEMP*A(K,2,2)/A(K,1,2)
      TEMP2=TEMP/A(K,1,2)
      TEMP3=TEMP*A(K,1,1)/A(K,1,2)
  430 X(I-1)=TEM1(1)
      X(I)=-TEM1(2)
      Y(I-1)=TEM2(1)
      Y(I)=-TEM2(2)
      Z(I-1)=TEM3(1)
      Z(I)=-TEM3(2)
  390 CONTINUE
      IF(ICASE.EQ.2) GO TO 391
      MW=MW-1
      DO 392 I=1,MW
      X(I)=X(I+1)
      Y(I)=Y(I+1)
  392 Z(I)=Z(I+1)
      GO TO 393
  391 ARG1=0.0
      DO 440 J=1,ID
  440 ARG1=ARG1+POL1(J)
      TEMP=DCMPLX(ARG1,0.0D+01)
      IF(ICASE.EQ.2) GO TO 450
      TEMP1=TEMP*A(1,2,2)/A(1,2,1)
      TEMP2=TEMP/A(1,2,1)
      TEMP3=TEMP*A(1,1,1)/A(1,2,1)
      GO TO 460
  450 TEMP1=TEMP*A(1,2,2)/A(1,1,2)
      TEMP2=TEMP/A(1,1,2)
      TEMP3=TEMP*A(1,1,1)/A(1,1,2)
  460 X(1)=TEM1(1)
      Y(1)=TEM2(1)
      Z(1)=TEM3(1)
  393 DO 470 I=1,MW
      POL3(I)=0.0
      POL4(I)=0.0
```

```
  470 POL5(I)=0.0
      DO 480 I=1,MW
      DO 480 J=1,MW
      POL3(I)=POL3(I)+MX(I,J)*X(J)
      POL4(I)=POL4(I)+MX(I,J)*Y(J)
  480 POL5(I)=POL5(I)+MX(I,J)*Z(J)
      NN1=MW
      NN2=MW
      NN3=MW
  350 IF(ICASE.EQ.3) GO TO 490
      IF(ICASE.EQ.2.OR.ICASE.EQ.5) GO TO 500
      WRITE(PRINT,14)
   14 FORMAT(44X,'COEFFICIENTS FOR RAMP INPUT')
      GO TO 510
  490 WRITE(PRINT,15)
   15 FORMAT(44X,'COEFFICIENTS FOR STEP INPUT')
      GO TO 510
  500 WRITE(PRINT,16)
   16 FORMAT(44X,'COEFFICIENTS BY FREQUENCY RESPONSE')
      DO 501 I=2,NW
  501 W(I)=2.0*3.14159265/W(I)
      WRITE(PRINT,22) (W(I),I=2,NW)
   22 FORMAT('0',20X,'PERIODS',8F10.1)
  510 WRITE(PRINT,1)
      IF(ICASE.GT.2) GO TO 520
      WRITE(PRINT,17)
   17 FORMAT(14X,'J',18X,'D/B',19X,'1/B',19X,'A/B',17X,'D(Z)')
      GO TO 530
  520 WRITE(PRINT,18)
   18 FORMAT(14X,'J',18X,'D/C',19X,'1/C',19X,'A/C',17X,'D(Z)')
  530 NN=MIN0(NN1,NN2,NN3)
      N=MAX0(NN,ID)
      DO 540 I=1,N
      J=I-1
      IF(I.LE.ID.AND.I.LE.NN) GO TO 550
      IF(NN.LT.ID) GO TO 570
      GO TO 560
  570 WRITE(PRINT,19) J,POL1(I)
   19 FORMAT(9X,I6,68X,F20.6)
      GO TO 540
  560 WRITE(PRINT,21) J,POL3(I),POL4(I),POL5(I)
   21 FORMAT(9X,I6,F24.6,F22.6,F22.6,F20.6)
      GO TO 540
  550 WRITE(PRINT,21) J,POL3(I),POL4(I),POL5(I),POL1(I)
  540 CONTINUE
      GO TO 1000
 2000 WRITE(PRINT,3)
      CALL EXIT
      END
```

```
C     SUBROUTINE POLES(RR,BETA,RES,ROOT,DER,FUNC,M,II,IJ,DT,IROOT)
C
C         SUBROUTINE TO CALCULATE THE ROOTS OF THE HEAT TRANSFER MATRIX
C     AND WILL STORE THE VALUE OF THE FUNCTIONS AND THE FIRST DERIVATIVE
C     AT THE ROOTS.
C
C         THE MAXIMUM NUMBER OF ROOTS THAT CAN BE OBTAINED IS SET
C     AT ONE HUNDRED(100)
C
C         THIS METHOD WILL FIRST FIND A ROOT BETWEEN 30.0/DT AND
C     100.0/DT, BEING ASSUMED THAT A ROOT EXIST IN THIS INTERVAL. THIS
C     ROOT IS ALSO LARGE ENOUGH TO GIVE SUFFICIENT ACCURACY TO EVALUATE
C     THE RESPONSE FACTORS.
C         THE METHOD CHECKS THE INTERVAL BETWEEN THE ORIGIN AND THIS
C     FIRST ROOT AND WHEN ANOTHER ROOT IS FOUND THE INTERVAL NEXT TO BE
C     CHECKED BECOMES THE INTERVAL BETWEEN THIS NEW ROOT AND THE NEXT
C     LARGEST ROOT AND SO ON. WHEN NO ROOT EXIST IN AN INTERVAL THE NEXT
C     SMALLEST INTERVAL IS SELECTED AND SO ON WORKING TOWARDS THE ORIGIN
C     UNTIL ALL ROOTS ARE FOUND.
C         TO CHECK FOR A ROOT THE METHOD SUBDIVIDES THE INTERVALS IN
C     RELATIVELY LARGE SEGMENTS AND CHECKS FOR BOTH A CHANGE IN SIGN OF
C     THE FUNCTION AND FOR TWO CHANGES IN DIRECTION OF THE SLOPE OF THE
C     FUNCTION. IF A ROOT EXIST, BY MAKING THESE TWO CHECKS, IT IS
C     INDICATED SO IN A RELATIVELY SHORT TIME. ONCE IT IS INDICATED THAT
C     A ROOT DOES EXIST IN A CERTAIN SEGMENT OF AN INTERVAL, THIS
C     SEGMENT IS FURTHER SUBDIVIDED AND USING A SIMILAR ROUTINE AS ABOVE
C     EXCEPT CHECKING FOR A CHANGE IN SIGN OF THE FUNCTION ONLY. IF ON
C     THE FIRST PASS A CHANGE IN SIGN IS NOT FOUND THE SEGMENT IS FURTHER
C     SUBDIVIDED INTO EVEN SMALLER PARTS UNTIL A CHANGE IN SIGN DOES
C     OCCUR. ONCE A CHANGE IN SIGN OCCURS THE ROOT IS ARRIVED AT BY
C     SPLITTING THIS INTERVAL SUCCESSIVELY IN HALF USING THE NEW SEGMENT
C     WITH FUNCTION VALUE OF OPPOSITE SIGN UNTIL A ROOT IS REACHED
C     WITHIN AN ACCURACY OF 10-14.
C         THE SPLITTING OF THE SEGMENTS TO ARRIVE AT A ROOT IS USED
C     BECAUSE A RELATIVELY CONSTANT NUMBER OF ITERATIONS ARE REQUIRED
C     TO OBTAIN THE ACCURACY WANTED. IN THE CASE OF THE REGULA FALSI
C     METHOD IT WAS FOUND THAT THE NUMBER OF ITERATIONS VARIED FROM AS
C     LOW AS FIVE (5) TO MORE THAN THREE HUNDRED (300) ITERATIONS. IN THE
C     LONG RUN IT WAS FOUND THAT THE SPLITTING OF THE POINTS REQUIRED
C     LESS RUNNING TIME.
C
C     NOMENCLATURE:
C         RR=THICKNESS/THERMAL CONDUCTIVITY (XL/XK)
C            OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
C            NEGLIGIBLE HEAT STORAGE
C         BETA*BETA=XL*XL*D*SH/XK
C                   WHERE      D=DENSITY.
C                              SH=SPECIFIC HEAT.
C         RES=RESISTANCE OF RADIATION PATH WHENEVER APPLICABLE.
C
C         ROOT=CONTAINS THE ROOTS OF THE HEAT TRANSFER FUNCTIONS
C              ON RETURN.
C         DER=CONTAINS THE DERIVATIVE OF THE HEAT TRANSFER FUNCTIONS
```

```
                C          AT THE ROOTS ON RETURN.
                C       FUNC=CONTAINS THE VALUE OF THE HEAT TRANSFER FUNCTIONS
                C            AT THE ROOTS ON RETURN.
                C
                C       M=NUMBER OF LAYERS THE SLAB IS COMPOSED OF.
                C
                C       II AND IJ ARE THE ROW AND COLUMN SUBSCRIPTS OF THE ELEMENT OF
                C       THE MATRIX FOR WHICH THE ROOT IS FOUND.
                C       II=1*
                C           * BOUNDARY CONDITION OF THE FIRST KIND.
                C       IJ=2*
                C
                C       II=2*
                C           * BOUNDARY CONDITION OF THE SECOND KIND.
                C       IJ=1*
                C
                C       DT=TIME INTERVAL OF SAMPLING
                C
 0001                 SUBROUTINE POLES(RR,BETA,RES,ROOT,DER,FUNC,M,II,IJ,DT,IROOT,ICASE)
 0002                 DOUBLE PRECISION RR(20),BETA(20),RES(20),ROOT(100),DER(100,4),
                     1FUNC(100,4),F(2,2),FF(2,2),R1,R2,R3,F1,F2,F3,FP1,FP2,FP3,RTEMP,
                     2FTEMP,DT
 0003                 DO 10 I=1,100
 0004                 DO 20 J=1,4
 0005                 FUNC(I,J)=0.0
 0006              20 DER(I,J)=0.0
 0007              10 ROOT(I)=0.0
 0008                 LAST=0
 0009                 R1=30.0/DT
 0010                 R3=100.0/DT
 0011                 IF(ICASE.EQ.4) R1=450.0/DT
 0012                 IF(ICASE.EQ.4) R3=700.0/DT
 0013                 DO 30 IROOT=1,100
 0014                 IF(IROOT.EQ.1) GO TO 40
                C     FOLLOWING IS THE ROUTINE CHECKING FOR A CHANGE IN THE SIGN OF THE
                C     FUNCTION AND ALSO FOR TWO(2) CHANGES IN DIRECTION OF THE SLOPE, TO
                C     FIND WHETHER A ROOT EXIST OR NOT.
 0015             300 CALL MATRIX(RR,BETA,RES,R1,M,F,FF,2)
 0016                 F1=F(II,IJ)
 0017                 FP1=FF(II,IJ)
 0018                 IC=0
 0019                 DO 50 I=1,20
 0020                 R2=R1+(R3-R1)/20.0*I
 0021                 CALL MATRIX(RR,BETA,RES,R2,M,F,FF,2)
 0022                 F2=F(II,IJ)
 0023                 FP2=FF(II,IJ)
 0024                 IF(F1.GT.0.0) GO TO 60
 0025                 IF(F2.LE.0.0) GO TO 70
 0026                 GO TO 80
 0027              60 IF(F2.LE.0.0) GO TO 80
 0028              70 IF(FP1.GT.0.0) GO TO 90
 0029                 IF(FP2.GT.0.0) GO TO 100
 0030                 GO TO 110
```

```
 90 IF(FP2.GT.0.0) GO TO 110
100 IC=IC+1
    IF(IC.EQ.2) GO TO 80
110 F1=F2
    FP1=FP2
 50 CONTINUE
    IROOT=IROOT-1
    LAST=LAST-1
120 IF(LAST.EQ.0) GO TO 130
    IF(LAST.NE.1) GO TO 125
    R1=0.0001/DT
    GO TO 140
125 R1=ROOT(LAST-1)
140 R3=ROOT(LAST)
    R1=R1+0.00001/DT
    R3=R3-0.00001/DT
    GO TO 30
 80 R3=R2
 40 N=1
    CALL MATRIX(RR,BETA,RES,R1,M,F,FF,1)
    F1=F(II,IJ)
    DO 150 I=N,25
    NN=10*I
    DO 160 J=1,NN
    R2=R1+J*(R3-R1)/NN
    CALL MATRIX(RR,BETA,RES,R2,M,F,FF,1)
    F2= F(II,IJ)
    IF(F1.GT.0.0) GO TO 170
    IF(F2.LE.0.0) GO TO 160
    GO TO 190
170 IF(F2.LE.0.0) GO TO 190
    GO TO 160
190 RTEMP=(R1+R2)/2.0
    CALL MATRIX(RR,BETA,RES,RTEMP,M,F,FF,1)
    FTEMP=F(II,IJ)
    IF(FTEMP.EQ.0.0) GO TO 200
    IF(FTEMP.GT.0.0) GO TO 210
    IF(F1.GT.0.0) GO TO 220
    F1=FTEMP
    R1=RTEMP
    GO TO 230
220 F2=FTEMP
    R2=RTEMP
    GO TO 230
210 IF(F1.GT.0.0) GO TO 215
    F2=FTEMP
    R2=RTEMP
    GO TO 230
215 F1=FTEMP
    R1=RTEMP
230 IF( DABS((R1-R2)/R1)-1.0D-14 .GT.0.0) GO TO 190
200 CALL MATRIX(RR,BETA,RES,R2,M,F,FF,2)
    GO TO 240
```

```
      160 CONTINUE
      150 CONTINUE
          WRITE(3,1)
        1 FORMAT('OUNABLE TO FIND A ROOT AFTER INDICATION THAT A ROOT EXISTE
         1D')
          CALL EXIT
      240 DO 250 I=1,IROOT
          J=I+1
          IF(R2.GT.ROOT(I)) GO TO 250
          J=I+1
          GO TO 260
      250 CONTINUE
      260 LAST=LAST+1
          IF(IROOT.EQ.1) GO TO 270
          JJ=IROOT+1
          DO 280 I=J,IROOT
          JJ=JJ-1
          ROOT(JJ)=ROOT(JJ-1)
          DO 280 K=1,4
          DER(JJ,K)=DER(JJ-1,K)
      280 FUNC(JJ,K)=FUNC(JJ-1,K)
      270 J=J-1
          ROOT(J)=R2
          DO 290 K=1,4
          KX=(K+1)/2
          KY=K/KX
          DER(J,K)=FF(KX,KY)
      290 FUNC(J,K)=F(KX,KY)
          GO TO 120
       30 CONTINUE
      130 RETURN
          END
```

```
FORTRAN IV G LEVEL  18                 MATRIX            DATE = 70132          14/14/56              PAGE 0001

0001                   SUBROUTINE MATRIX(RR,BETA,RES,W,M,F,FF,ICONT)
                C
                C          SUBROUTINE TO CALCULATE THE HEAT TRANSFER MATRIX FOR A SLAB,
                C      AND THE DERIVATIVE OF THIS MATRIX.
                C
                C      IF ICONT=1 THE ROUTINE CALCULATES HEAT TRANSFER MATRIX ONLY.
                C      IF ICONT=2 THE ROUTINE CALCULATES HEAT TRANSFER MATRIX AND ITS
                C                 DERIVATIVE.
                C
                C      NOMENCLATURE:
                C          RR=THICKNESS/THERMAL CONDUCTIVITY (XL/XK)
                C             OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
                C             NEGLIGIBLE HEAT STORAGE.
                C          BETA*BETA=XL*XL*D*SH/K.
                C                    WHERE D=DENSITY.
                C                          SH=SPECIFIC HEAT.
                C          RES=RESISTANCE OF RADIATION PATH WHENEVER APPLICABLE.
                C
                C          W=VALUES ALONG THE AXIS FOR WHICH THE MATRIX OR THE MATRIX
                C            AND DERIVATIVES ARE FOUND.
                C          M=NUMBER OF LAYERS THE SLAB IS COMPOSED OF.
                C          F=CONTAINS THE VALUE OF THE HEAT TRANSFER MATRIX ON RETURN
                C          FF=CONTAINS THE VALUE OF THE DERIVATIVE ON RETURN.
                C
0002                   DOUBLE PRECISION RR(20),BETA(20),F1(20,2,2),F2(20,2,2),F3(20,2,2),
                      1F(2,2),FF(2,2),RES(20),P,R,ALPHA,SQ,W,TEMP,TEMP1
0003                   DO 10 I=1,M
0004                   P=DSQRT(W)*BETA(I)
0005                   R=RR(I)
0006                   ALPHA=BETA(I)
0007                   SQ=DSQRT(W)
0008                   IF(P.NE.0.0) GO TO 20
                C      ELEMENTS OF THE MATRIX FOR LAYER I WHERE THERE IS NEGLIGIBLE
                C      HEAT STORAGE.
0009                   F1(I,1,1)=1.0
0010                   F1(I,1,2)=R
0011                   F1(I,2,1)=0.0
0012                   F1(I,2,2)=1.0
0013                   IF(ICONT.EQ.1) GO TO 10
                C      DERIVATIVES OF THE ELEMENTS OF THE MATRIX FOR LAYER I WHERE THERE
                C      IS NEGLIGIBLE HEAT STORAGE.
0014                   F2(I,1,1)=0.0
0015                   F2(I,1,2)=0.0
0016                   F2(I,2,1)=0.0
0017                   F2(I,2,2)=0.0
0018                   GO TO 10
                C      ELEMENTS OF THE MATRIX FOR LAYER I FOR HEAT TRANSFER BY CONDUCTION
                C      ONLY.
0019                20 F1(I,1,1)=DCOS(P)
0020                   F1(I,1,2)=R/P*DSIN(P)
0021                   F1(I,2,1)=-P/R*DSIN(P)
0022                   F1(I,2,2)=F1(I,1,1)
0023                   IF(ICONT.EQ.1) GO TO 30
```

```
FORTRAN IV G LEVEL  18              MATRIX           DATE = 70132          14/14/56              PAGE 0002

C       DERIVATIVES OF THE ELEMENTS OF THE MATRIX FOR LAYER I FOR HEAT
C       TRANSFER BY CONDUCTION ONLY.
        F2(I,1,1)=ALPHA*DSIN(ALPHA*SQ)/2.0/SQ
        F2(I,1,2)=-R*DCOS(ALPHA*SQ)/2.0/W+R*DSIN(ALPHA*SQ)/ALPHA/2.0/SQ/SQ
       1/SQ
        F2(I,2,1)=ALPHA*ALPHA*DCOS(ALPHA*SQ)/2.0/R+DSIN(ALPHA*SQ)/2.0/SQ*
       1ALPHA/R
        F2(I,2,2)=F2(I,1,1)
     30 IF(RES(I).EQ.0.0) GO TO 10
C       ELEMENTS OF THE MATRIX FOR LAYER I WHERE THERE IS HEAT TRANSFER BY
C       CONDUCTION AND THERMAL RADIATION USING CONDUCTION PART FROM ABOVE.
        TEMP=1.0/(F1(I,1,2)+RES(I))
        F1(I,2,1)=(F1(I,2,1)*RES(I)+2.0*F1(I,1,1)-2.0)*TEMP
        F1(I,1,1)=(F1(I,1,1)*RES(I)+F1(I,1,2))*TEMP
        F1(I,2,2)=F1(I,1,1)
        F1(I,1,2)=F1(I,1,2)*RES(I)*TEMP
        IF(ICONT.EQ.1) GO TO 10
C       DERIVATIVES OF THE ELEMENTS OF THE MATRIX FOR LAYER I WHERE THERE
C       IS HEAT TRANSFER BY CONDUCTION AND THERMAL RADIATION USING
C       CONDUCTION PART FROM ABOVE
        TEMP1=F2(I,1,2)*TEMP
        F2(I,2,1)=(F2(I,2,1)*RES(I)+2.0*F2(I,1,1))*TEMP-F1(I,2,1)*TEMP1
        F2(I,1,1)=(F2(I,1,1)*RES(I)+F2(I,1,2))*TEMP-F1(I,1,1)*TEMP1
        F2(I,2,2)=F2(I,1,1)
        F2(I,1,2)=F2(I,1,2)*RES(I)*TEMP-F1(I,1,2)*TEMP1
     10 CONTINUE
C       RETURN IF ONLY ONE LAYER INVOLVED.
        IF((M-1).NE.0) GO TO 50
        DO 40 K=1,2
        DO 40 L=1,2
        IF(ICONT.EQ.1) GO TO 40
        FF(K,L)=F2(1,K,L)
     40 F(K,L)=F1(1,K,L)
        RETURN
     50 DO 60 K=1,2
        DO 60 L=1,2
     60 FF(K,L)=0.0
        IF(ICONT.EQ.1) GO TO 150
C       FOLLOWING IS THE ROUTINE TO COMBINE INDIVIDUAL DERIVATIVES OF THE
C       HEAT TRANSFER MATRICES TO GET THE OVERALL DERIVATIVE.
        DO 140 I=1,M
        DO 120 J=1,M
        DO 80 K=1,2
        DO 80 L=1,2
        IF((I-J).EQ.0) GO TO 70
        F3(J,K,L)=F1(J,K,L)
        GO TO 80
     70 F3(J,K,L)=F2(J,K,L)
     80 CONTINUE
        IF((J-1).EQ.0) GO TO 120
        DO 90 K=1,2
        DO 90 L=1,2
     90 F(K,L)=0.0
```

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FORTRAN IV G LEVEL  18                MATRIX            DATE = 70132         14/14/56            PAGE 0003

 0065              DO 100 K=1,2
 0066              DO 100 L=1,2
 0067              DO 100 N=1,2
 0068          100 F(K,L)=F(K,L)+F3(J-1,K,N)*F3(J,N,L)
 0069              DO 110 L=1,2
 0070              DO 110 K=1,2
 0071          110 F3(J,K,L)=F(K,L)
 0072          120 CONTINUE
 0073              DO 130 K=1,2
 0074              DO 130 L=1,2
 0075          130 FF(K,L)=FF(K,L)+F(K,L)
 0076          140 CONTINUE
             C     FOLLOWING IS THE ROUTINE TO COMBINE INDIVIDUAL HEAT TRANSFER
             C     MATRIX TO GET THE OVERALL HEAT TRANSFER MATRIX.
 0077          150 DO 190 I=2,M
 0078              DO 160 K=1,2
 0079              DO 160 L=1,2
 0080          160 F(K,L)=0.0
 0081              DO 170 K=1,2
 0082              DO 170 L=1,2
 0083              DO 170 N=1,2
 0084          170 F(K,L)=F(K,L)+F1(I-1,K,N)*F1(I,N,L)
 0085              DO 180 K=1,2
 0086              DO 180 L=1,2
 0087          180 F1(I,K,L)=F(K,L)
 0088          190 CONTINUE
 0089              RETURN
 0090              END
```

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FORTRAN IV G LEVEL  18                    ORIGIN            DATE = 70132          14/14/56              PAGE 0001

0001                   SU8ROUTINE ORIGIN(RR,BETA,RES,M,MP,MPP)
                 C
                 C     SUBROUTINE TO CALCULATE THE RESIDUES AT THE POLES OF THE
                 C     Z-TRANSFER FUNCTIONS. (FIRST AND SECOND DERIVATIVES)
                 C
                 C     NOMENCLATURE:
                 C         RR=THICKNESS/THERMAL CONDUCTIVITY (XL/XK).
                 C            OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
                 C            NEGLIGIBLE HEAT STORAGE.
                 C         BETA*BETA=XL*XL*D*SH/XK.
                 C                  WHERE D=DENSITY.
                 C                        SH=SPECIFIC HEAT.
                 C         RES=RESISTANCE OF RADIATION PATH WHENEVER APPLICABLE.
                 C
                 C         M=NUMBER OF LAYER THE SLAB IS COMPOSED OF.
                 C
                 C         MP=CONTAINS THE VALUE OF THE FIRST DERIVATIVE AT THE
                 C            POLES ON RETURN.
                 C         MPP=CONTAINS THE VALUE OF THE SECOND DERIVATIVE AT THE POLES
                 C             ON RETURN
                 C
0002                   DOUBLE PRECISION RR(20),BETA(20),RES(20),MP(2,2),MPP(2,2),
                      1A(20,2,2),B(20,2,2),C(20,2,2),D(2,2),E(2,2),F(2,2),G(2,2),
                      2TEMP(2,2),TEMP1(2,2),P,R
0003                   DO 10 I=1,2
0004                   DO 10 J=1,2
0005                   MP(I,J)= 0.0
0006                10 MPP(I,J)=0.0
0007                   DO 40 I=1,M
0008                   P=BETA(I)*BETA(I)
0009                   R=RR(I)
                 C     ELEMENTS OF THE MATRIX AT THE POLE FOR LAYER I, FOR CONDUCTION
                 C     OR NEGLIGIBLE HEAT TRANSFER.
0010                   A(I,1,1)=1.0
0011                   A(I,1,2)=R
0012                   A(I,2,1)=0.0
0013                   A(I,2,2)=1.0
0014                   IF(RES(I).EQ.0.0) GO TO 20
                 C     ELEMENTS OF THE MATRIX AT THE POLE FOR LAYER I, WHERE THERE IS
                 C     HEAT TRANSFER BY CONDUCTION AND THERMAL RADIATION.
0015                   A(I,1,2)=R*RES(I)/(R+RES(I))
                 C     FIRST DERIVATIVE OF THE ELEMENTS OF THE MATRIX AT THE POLE
                 C     FOR LAYER I, FOR CONDUCTION OR NEGLIGIBLE HEAT STORAGE.
0016                20 B(I,1,1)=P/2.0
0017                   B(I,1,2)=R*P/6.0
0018                   B(I,2,1)=P/R
0019                   B(I,2,2)=P/2.0
0020                   IF(RES(I).EQ.0.0) GO TO 30
                 C     FIRST DERIVATIVE OF THE ELEMENTS OF THE MATRIX AT THE POLE
                 C     FOR LAYER I, WHERE THERE IS HEAT TRANSFER BY CONDUCTION AND
                 C     THERMAL RADIATION.
0021                   B(I,1,1)=RES(I)*P/2.0/(R+RES(I))
0022                   B(I,1,2)=(1.0-R/(R+RES(I)))*RES(I)*R*P/6.0/(R+RES(I))
```

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FORTRAN IV G LEVEL  18                 ORIGIN              DATE = 70132           14/14/56              PAGE 0002

                  B(I,2,1)=(RES(I)*P/R+P)/(R+RES(I))
                  B(I,2,2)=B(I,1,1)
            C     SECOND DERIVATIVE OF THE ELEMENTS OF THE MATRIX AT THE POLE FOR
            C     LAYER I, FOR CONDUCTION OR NEGLIGIBLE HEAT STORAGE.
               30 C(I,1,1)=P*P/12.0
                  C(I,1,2)=P*P*R/60.0
                  C(I,2,1)=P*P/3.0/R
                  C(I,2,2)=C(I,1,1)
               40 CONTINUE
            C     RETURN IF ONLY ONE LAYER INVOLVED.
                  IF((M-1).NE.0) GO TO 60
                  DO 50 I=1,2
                  DO 50 J=1,2
                  MP(I,J)=B(1,I,J)
               50 MPP(I,J)=C(1,I,J)
                  RETURN
            C     FOLLOWING IS THE ROUTINE TO CALCULATE THE FIRST AND SECOND
            C     DERIVATIVE OF THE HEAT TRANSFER MATRIX AT THE POLES FOR A
            C     MULTILAYER SLAB.
               60 DO 280 I=1,M
                  DO 260 K=1,M
                  IF(I.NE.K) GO TO 80
                  IF(I.NE.1) GO TO 70
                  D(1,1)=B(1,1,1)
                  D(1,2)=B(1,1,2)
                  D(2,1)=B(1,2,1)
                  D(2,2)=B(1,2,2)
                  GO TO 80
               70 D(1,1)=A(1,1,1)
                  D(1,2)=A(1,1,2)
                  D(2,1)=A(1,2,1)
                  D(2,2)=A(1,2,2)
               80 IF(I.NE.1) GO TO 90
                  IF(K.NE.1) GO TO 100
                  E(1,1)=C(1,1,1)
                  E(1,2)=C(1,1,2)
                  E(2,1)=C(1,2,1)
                  E(2,2)=C(1,2,2)
                  GO TO 120
               90 IF(K.NE.1) GO TO 110
              100 E(1,1)=B(1,1,1)
                  E(1,2)=B(1,1,2)
                  E(2,1)=B(1,2,1)
                  E(2,2)=B(1,2,2)
                  GO TO 120
              110 E(1,1)=A(1,1,1)
                  E(1,2)=A(1,1,2)
                  E(2,1)=A(1,2,1)
                  E(2,2)=A(1,2,2)
              120 DO 240 J=2,M
                  IF(I.NE.K) GO TO 170
                  IF(I.EQ.J) GO TO 130
                  F(1,1)=A(J,1,1)
```

```
      F(1,2)=A(J,1,2)
      F(2,1)=A(J,2,1)
      F(2,2)=A(J,2,2)
      GO TO 140
  130 F(1,1)=B(J,1,1)
      F(1,2)=B(J,1,2)
      F(2,1)=B(J,2,1)
      F(2,2)=B(J,2,2)
  140 DO 150 L=1,2
      DO 150 LL=1,2
      TEMP(L,LL)=0.0
      DO 150 LLL=1,2
  150 TEMP(L,LL)=TEMP(L,LL)+D(L,LLL)*F(LLL,LL)
      DO 160 L=1,2
      DO 160 LL=1,2
  160 D(L,LL)=TEMP(L,LL)
  170 IF(I.EQ.J) GO TO 180
      IF(K.EQ.J) GO TO 190
      G(1,1)=A(J,1,1)
      G(1,2)=A(J,1,2)
      G(2,1)=A(J,2,1)
      G(2,2)=A(J,2,2)
      GO TO 210
  180 IF(K.EQ.J) GO TO 200
  190 G(1,1)=B(J,1,1)
      G(1,2)=B(J,1,2)
      G(2,1)=B(J,2,1)
      G(2,2)=B(J,2,2)
      GO TO 210
  200 G(1,1)=C(J,1,1)
      G(1,2)=C(J,1,2)
      G(2,1)=C(J,2,1)
      G(2,2)=C(J,2,2)
  210 DO 220 L=1,2
      DO 220 LL=1,2
      TEMP1(L,LL)=0.0
      DO 220 LLL=1,2
  220 TEMP1(L,LL)=TEMP1(L,LL)+E(L,LLL)*G(LLL,LL)
      DO 230 L=1,2
      DO 230 LL=1,2
  230 E(L,LL)=TEMP1(L,LL)
  240 CONTINUE
      DO 250 L=1,2
      DO 250 LL=1,2
  250 MPP(L,LL)=MPP(L,LL)+TEMP1(L,LL)
  260 CONTINUE
      DO 270 L=1,2
      DO 270 LL=1,2
  270 MP(L,LL)=MP(L,LL)+TEMP(L,LL)
  280 CONTINUE
      RETURN
      END
```

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FORTRAN IV G LEVEL  18                    FREQRE            DATE = 70132           14/14/56              PAGE 0001

 0001                SUBROUTINE FREQRE (RR,BETA,RES,XL,XK,D,SH,M,W,A,LW)
               C
               C         THIS SUBROUTINE CALCULATES THE FUNCTIONS OF THE HEAT
               C     TRANSFER MATRIX WHEN S=IW  WHERE I=SQRT(-1.0)
               C
               C     NOMENCLATURE:
               C         RR=THICKNESS/THERMAL CONDUCTIVITY (XL/XK)
               C            OR THERMAL RESISTANCE OF LAYER WHEN THERE IS
               C            NEGLIGIBLE HEAT STORAGE.
               C         BETA*BETA=XL*XL*D*SH/XK
               C                    WHERE D=DENSITY
               C                          SH=SPECIFIC HEAT
               C         RES=RESISTANCE OF RADIATION PATH WHENEVER APPLICABLE.
               C
               C         M=NUMBER OF LAYER THE SLAB IS COMPOSED
               C
               C         W=ARRAY CONTAINING THE FREQUENCIES AT WHICH THE FUNCTIONS
               C           ARE EVALUATED
               C
               C         A=CONTAINS THE VALUES OF THE FUNCTIONS AT S=IW FOR THE
               C           VARIOUS FREQUENCIES ON RETURN FROM THE SUBROUTINE.
               C
 0002                DOUBLE PRECISION RR(20),BETA(20),RES(20),XL(20),XK(20),D(20),
                    1SH(20),AA,BB,CC,DD,EE,FF,P,R,ALPHA,PHI,PI,ARG1,ARG2,TEMP,W(6)
 0003                COMPLEX*16 A(6,2,2),MM(2,2),MMM(2,2),MMMM(2,2)
 0004                PI=3.14159265
 0005                DO 60 J=1,LW
 0006                DO 10 I=1,M
 0007                R=RR(I)
 0008                IF(W(J).NE.0.0.AND.XL(I).NE.0.0) GO TO 5
 0009                MM(1,1)=(1.0D0,0.0D0)
 0010                ARG2=0.0
 0011                MM(1,2)=DCMPLX(R,ARG2)
 0012                MM(2,1)=(0.0D0,0.0D0)
 0013                GO TO 6
 0014              5 P=2.0*PI/W(J)
 0015                ALPHA=XK(I)/D(I)/SH(I)
 0016                PHI=DSQRT(PI*XL(I)*XL(I)/ALPHA/P)
 0017                AA=DSIN(PHI)
 0018                BB=DCOS(PHI)
 0019                CC=DEXP(PHI)
 0020                DD=DEXP(-PHI)
 0021                EE=(CC-DD)/2.0
 0022                FF=(CC+DD)/2.0
 0023                ARG1=FF*BB
 0024                ARG2=EE*AA
 0025                MM(1,1)=DCMPLX(ARG1,ARG2)
 0026                ARG1=RR(I)*(FF*AA+EE*BB)/2.0/PHI
 0027                ARG2=RR(I)*(FF*AA-EE*BB)/2.0/PHI
 0028                MM(1,2)=DCMPLX(ARG1,ARG2)
 0029                TEMP=2.0*ARG1*PHI*PHI/RR(I)/RR(I)
 0030                ARG1=-ARG2*2.0*PHI*PHI/RR(I)/RR(I)
 0031                ARG2=TEMP
```

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FORTRAN IV G LEVEL  18                FREQRE              DATE = 70132          14/14/56            PAGE 0002

      MM(2,1)=DCMPLX(ARG1,ARG2)
    6 MM(2,2)=MM(1,1)
      IF(RES(I).EQ.0.0) GO TO 7
      MM(2,1)=(MM(2,1)*RES(I)+2.0*MM(1,1)-2.0)/(MM(1,2)+RES(I))
      MM(1,1)=(MM(1,1)*RES(I)+MM(1,2))/(MM(1,2)+RES(I))
      MM(1,2)=(MM(1,2)*RES(I))/(MM(1,2)+RES(I))
      MM(2,2)=MM(1,1)
    7 IF(I.EQ.1) GO TO 20
      DO 30 K=1,2
      DO 30 L=1,2
      MMMM(K,L)=(0.0,0.0)
      DO 30 N=1,2
   30 MMMM(K,L)=MMMM(K,L)+MM(N,L)*MMM(K,N)
      DO 40 K=1,2
      DO 40 L=1,2
   40 MMM(K,L)=MMMM(K,L)
      GO TO 10
   20 DO 50 K=1,2
      DO 50 L=1,2
   50 MMM(K,L)=MM(K,L)
   10 CONTINUE
      DO 70 K=1,2
      DO 70 L=1,2
   70 A(J,K,L)=MMM(K,L)
   60 CONTINUE
      RETURN
      END
```

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FORTRAN IV G LEVEL  18                 POLYM              DATE = 70132          14/14/56                PAGE 0001

 0001                SUBROUTINE POLYM(A,B,N,M)
             C
             C       THIS SUBROUTINE FINDS THE PRODUCT OF TWO POLYNOMIALS OF ORDER
             C       N AND M RESPECTIVELY.(N=ORDER OF NEW POLYNOMIAL A ON RETURN).
             C
 0002                DOUBLE PRECISION A(100),B(100),C(100)
 0003                K=N+M
 0004                K=MINO(K,99)
 0005                KK=K+1
 0006                DO 10  I=1,100
 0007             10 C(I)=0.0
 0008                DO 25  I=1,KK
 0009                KKK=I
 0010                L=N+1
 0011                L=MINO(I,L)
 0012                J=I-M
 0013                J=MAXO(1,J)
 0014                DO 20  NN=J,L
 0015                INN=I-NN+1
 0016             20 C(I)=C(I)+A(NN)*B(INN)
 0017                IF(I.LT.5)GO TO 25
 0018                IF(DABS(C(I)).LT.0.0000001) GO TO 35
 0019             25 CONTINUE
 0020             35 KK=KKK
 0021                DO 30  I=1,100
 0022             30 A(I)=C(I)
 0023                N=KK-1
 0024                RETURN
 0025                END
```

# Samples of Input and Output

Note: Input and output are in consistent units. These examples are in the International System of Units (SI).

```
                                  NUMERICAL DATA FOR EXAMPLE WALL.
                                  SLAB COMPONENTS.

LAYER THICKNESS     CONDUCTIVITY        DENSITY        SP HEAT            RESISTANCE
                                                                                            DESCRIPTION OF LAYER

  1     0.0              0.0               0.0            0.0                0.1500         AIR  INSIDE SURFACE
  2     0.1000           0.7300         1600.0000       920.0000             0.0            BRICK
  3     0.1000           1.3300         2000.0000       920.0000             0.0            CONCRETE  MEDIUM WEIGHT
  4     0.0              0.0               0.0            0.0                0.0600         AIR  OUTSIDE SURFACE


                           THERMAL CONDUCTANCE,   U= 2.369

                                  SAMPLING TIME INTERVAL,  DT= 3600.000

                                       COEFFICIENTS FOR RAMP INPUT

            J                D/B                  1/B                  A/B                 D(Z)
            0             5.133114             0.000852            11.093835             1.000000
            1            -7.781125             0.049375           -17.670871            -1.337286
            2             3.105002             0.118010             7.503862             0.451402
            3            -0.255996             0.034871            -0.733343            -0.028029
            4             0.003320             0.001198             0.010846             0.000168
            5            -0.000005             0.000003            -0.000020            -0.000000
            6             0.000000             0.000000             0.000000
```

```
                              NUMERICAL DATA FOR EXAMPLE WALL.
                              SLAB COMPONENTS.

LAYER THICKNESS    CONDUCTIVITY       DENSITY      SP HEAT        RESISTANCE
                                                                                DESCRIPTION OF LAYER

  1    0.0           0.0                0.0          0.0            0.1500      AIR  INSIDE SURFACE
  2    0.1000        0.7300          1600.0000     920.0000         0.0         BRICK
  3    0.1000        1.3300          2000.0000     920.0000         0.0         CONCRETE  MEDIUM WEIGHT
  4    0.0           0.0                0.0          0.0            0.0600      AIR  OUTSIDE SURFACE

                         THERMAL CONDUCTANCE,   U= 2.369

                              SAMPLING TIME INTERVAL,  DT= 3600.000

                                 COEFFICIENTS BY FREQUENCY RESPONSE

                 PERIODS   28800.0   17280.0   12240.0   86400.0

             J               D/B                 1/B                 A/B                D(Z)
             0             5.342817           -0.001245           11.780631           1.000000
             1            -8.615756            0.038669          -20.437882          -1.337286
             2             4.556544            0.146869           12.380368           0.451402
             3            -1.843089            0.015254           -6.121494          -0.028029
             4             1.353690            0.008909            4.611540           0.000168
             5            -0.965779           -0.006880           -3.288532          -0.000000
             6             0.553785            0.004025            1.885314
             7            -0.226359           -0.001641           -0.770602
             8             0.048455            0.000348            0.164965
```

```
                              NUMERICAL DATA FOR EXAMPLE WALL.
                              SLAB COMPONENTS.

LAYER THICKNESS    CONDUCTIVITY      DENSITY       SP HEAT         RESISTANCE
                                                                                  DESCRIPTION OF LAYER

  1    0.0          0.0              0.0          0.0              0.1500         AIR  INSIDE SURFACE
  2    0.1000       0.7300        1600.0000     920.0000           0.0            BRICK
  3    0.1000       1.3300        2000.0000     920.0000           0.0            CONCRETE  MEDIUM WEIGHT
  4    0.0          0.0              0.0          0.0              0.0600         AIR  OUTSIDE SURFACE


                         THERMAL CONDUCTANCE,   U= 2.369

                              SAMPLING TIME INTERVAL,  DT= 3600.000

                                 COEFFICIENTS FOR STEP INPUT

             J              D/C               1/C               A/C               D(Z)
             0            0.103315          0.000081          0.215305          1.000000
             1           -0.159683          0.001780         -0.344502         -1.724442
             2            0.065686          0.001778          0.145840          0.799418
             3           -0.005565          0.000162         -0.012955         -0.075676
             4            0.000049          0.000001          0.000115          0.000700
             5           -0.000000          0.000000         -0.000000         -0.000000
             6                                                                  0.000000
```

```
                         NUMERICAL DATA FOR EXAMPLE WALL.
                         SLAB COMPONENTS.

LAYER THICKNESS   CONDUCTIVITY      DENSITY     SP HEAT        RESISTANCE
                                                                            DESCRIPTION OF LAYER

  1    0.0          0.0              0.0          0.0             0.1500     AIR  INSIDE SURFACE
  2    0.1000       0.7300        1600.0000     920.0000          0.0        BRICK
  3    0.1000       1.3300        2000.0000     920.0000          0.0        CONCRETE  MEDIUM WEIGHT
  4    0.0          0.0              0.0          0.0             0.0600     AIR  OUTSIDE SURFACE

                    THERMAL CONDUCTANCE,   U= 2.369

                         SAMPLING TIME INTERVAL,  DT= 3600.000

                            COEFFICIENTS BY FREQUENCY RESPONSE

               PERIODS   43200.0   21600.0   14400.0   10800.0   32400.0

            J              D/C                   1/C                   A/C               D(Z)
            0           0.085334             -0.000021              0.188244          1.000000
            1          -0.112872              0.000558             -0.273221         -1.724442
            2           0.015874              0.002638              0.070122          0.799418
            3           0.032665              0.000593              0.044606         -0.075676
            4          -0.031011              0.000076             -0.046828          0.000700
            5           0.025175             -0.000082              0.038057         -0.000000
            6          -0.018021              0.000066             -0.027246          0.000000
            7           0.011000             -0.000043              0.016631
            8          -0.004926              0.000020             -0.007447
            9           0.001351             -0.000005              0.002042
```

```
                        NUMERICAL DATA FOR EXAMPLE WALL.
                        SLAB COMPONENTS.

LAYER THICKNESS    CONDUCTIVITY       DENSITY       SP HEAT          RESISTANCE
                                                                                      DESCRIPTION OF LAYER

  1     0.0            0.0               0.0           0.0              0.1500        AIR  INSIDE SURFACE
  2     0.1000         0.7300         1600.0000      920.0000           0.0           BRICK
  3     0.1000         1.3300         2000.0000      920.0000           0.0           CONCRETE  MEDIUM WEIGHT
  4     0.0            0.0               0.0           0.0              0.0600        AIR  OUTSIDE SURFACE

                    THERMAL CONDUCTANCE,    U= 2.369

                         SAMPLING TIME INTERVAL,  DT= 3600.000

                            COEFFICIENTS FOR RAMP INPUT

            J               D/C              1/C              A/C             D(Z)
            0            0.088859         0.000011         0.193541        1.000000
            1           -0.129085         0.000749        -0.297781       -1.724442
            2            0.045849         0.002187         0.115608        0.799418
            3           -0.001759         0.000818        -0.007503       -0.075676
            4           -0.000061         0.000038        -0.000064        0.000700
            5            0.000000         0.000000         0.000000       -0.000000
            6           -0.000000         0.000000        -0.000000        0.000000
```

# Application of Multilayer Periodic Heat Flow Theory To the Design and Optimization of Roofing Systems

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Techniques for studying the periodic heat fluxes through multilayered roof systems subject to equivalent sol air input driving functions have been coupled with a multi-functional optimization procedure for the selection of material components, which, when combined into a roof section, will satisfy certain object functions. The object functions can be any or all of the following type: maximum thermal lag, minimum total integrated flux, minimum peak flux, minimum temperature variation of the most temperature susceptible material, minimum cost, minimum weight, and others related to purely structural consideration such as deflection, stresses, etc.

A computer program has been designed such that a search is made through a directory of roofing materials in order to select the right combination of materials, both from a dimensional and property standpoint as well as actual position within the roofing section. The basis of the optimization procedure is a generalized multivariable optimization criteria that essentially compromises the design objectives and is capable of handling any number of additional inequality constraints.

The multi-layer boundary value problem for temperature and flux at any position within the layers was solved by use of the Lapace transform which converts the partial differential equation of linear heat flow into an ordinary differential equation which is then solved in matrix form. When incorporated into the optimal design procedure the multi-layer heat flow theory gives the designer or analyst a powerful tool for studying roofing design.

Specific design examples are presented to show the importance of material selection and sequence of material layers on the most critical components of the roofing system. The multitude of choices possible in terms of combinations of materials, thicknesses and sequence of layers precludes solution other than by computer.

Key Words: Built-up roofing, optimization, periodic heat flow theory, roof design, sol-air temperature.

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[2] Associate Professor.

## 1. Introduction

Over the last twenty years, there has developed an awareness of, and a concern for premature failures of built-up roofing systems. Various sources have estimated that annually ten to fifteen percent of the installed new roof systems will ultimately fail prematurely; i.e., within one to five years after initial installation [1].[3]

There are a large number of different types of failures and associated factors causing the failures[1]. Many failures can be directly attributed to poor design, poor workmanship, poor installation, or simply attributed to the hostile environment in which the built-up roofing must function. Cullen [2] [3], Handegord [4] and Joy [5] have singled out thermal cycling and solar radiation exposure as two of the most important factors influencing roofing membrane durability and integrity. Specifically thermal cycling can be related to thermal shock and thermal expansion-contraction movements, while the solar radiation can be related to photo-oxidation of the asphaltic materials.

Some knowledgeable researchers [1] in the field of roofing design have suggested that roofs should be designed so as to provide a more favorable environment for the waterproof membrane. To quote Baker, "Either improve membranes to withstand the harsh real world environment, or protect the membranes from the environment." The economics of conventional built-up roofing being what they are, it is unlikely that they will be replaced in the near future. Therefore, the protected membrane system is the more likely to offer immediate solutions to the problem.

In order to protect the membranes, it is necessary to determine the thermal fluctuations of the felts. In order to do this, it is necessary to have an analytical technique for handling multilayer periodic heat transfer. However, the choice, location and thickness of the insulating material to protect the felts is governed by certain performance characteristics for the roofing system as a whole. The former aspect of this problem requires a thermal model for multilayer heat transfer while the later aspect of the model requires a multi-object function optimization scheme. The purpose of this paper is to present a combination of an analytical model for heat transfer and an optimization scheme which will be used to design a roofing system to protect the roofing felts from thermal fluctuations but at the same time provide a suitable thermal barrier at a reasonable cost.

## 2. Roof Model

For purposes of illustration and comparison, a relatively simple roofing model was chosen, see figure 1. The three major components of the roof system are the structural deck, in this case a concrete slab, the insulation, arbitrary, and the built-up membranes, i.e., four ply hot asphalt-asphalt saturated organic felts. The roof system ideally must perform two main functions: (1) insulate the building; (2) provide a weather barrier. The roof model is bounded by an internal space of constant air temperature and an external exposed surface upon which is imposed a sol-air temperature input.

## 3. Sol-Air Temperature Equivalent

The heat flow into a roof is due to the outdoor temperature $\theta_o(t)$ and the incident solar flux $I(t)$. The rate of heat transfer $q_o$ will be given by

$$q_o(t) = h_o(\theta_o(t) - \theta(0,t)) + \alpha I(t) \quad (1)$$

where $h_o$ is the surface conductance of the roof surface, $\theta(0,t)$ is the surface temperature at distance of $x = 0$ and time t, and $\alpha$ is the solar absorptivity of the roof surface. This may be also expressed as

$$q_o(t) = h_o(\theta_E(t) - \theta(0,t)) \quad (2)$$

where $\theta_E$ is a fictitious temperature, the sol-air temperature and may be written as

$$\theta_E(t) = \theta_o(t) + \alpha I(t)/h_o \quad (3)$$

Since the surface absorptivity and outside surface heat transfer coefficient is contained in this expression, the relation is only good for a particular surface. A rather complex computer subroutine was developed to automatically determine the sol-air temperature for any specified location, time of the year, and for any given surface condition and orientation.

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[3] Figures in brackets indicate the literature references at the end of this paper.

## 4. Multilayer Periodic Heat Flow

Periodic heat flow through composite roof systems have been studied by a number of investigators, Mackey and Wright [7] and more recently, Hoglund, Mitalas and Stephenson [6]. The present study employs a computer program especially developed for determining temperatures and fluxes at any position within a layered system and is a modification of an earlier program developed for making thermal diffusivity measurements [8]. A Laplace transform was employed to convert the partial differential equation of linear heat flow into an ordinary differential equation which was then solved in matrix notation form. The input driving function at the exposed outer roof surface boundary consisted of the sol-air temperature expressed as a Fourier Series.

The temperature distribution, $\theta_m(x,t)$ within a given layer m of a multi-layer infinite slab of total thickness $L = \sum_1^M L_m$ composed of M layers of thickness $L_m$, is given by the solution of the one dimensional equation of linear heat flow with specified boundary condition,

$$a_m \theta_{m\delta\delta}(x,t) = \theta_{mt}(x,t). \qquad (4)$$

The distance x is measured from the input face of the first layer. The thermal diffusivity, $a_m$ of each layer is defined as $a_m = \lambda_m / d_m C_m$ with $\lambda_m$ being the thermal conductivity; $d_m$, the density; and $C_m$, the specific heat. It is assumed that each diffusivity is independent of position, time, and temperature. The subscripts, δ and t, denote differentiation with respect to distance and time.

The boundary condition at the input face, x = 0 will be the sol-air temperature, $\theta_E(t)$ expressed as a Fourier Series,

$$\theta_E(t) = A_o + \sum_1^\infty A_n \sin(2\pi nt + \delta_n) \qquad (5)$$

and

$$\theta_m(x,t) = 0 \quad t \leq 0 \qquad (6)$$

for all M layers. The temperature at the output face will be maintained constant at $T_o$ for all time,

$$\theta(L,t) = T_o \quad t > 0 \qquad (7)$$

In a multi-layer slab of M layers, an additional pair of boundary conditions is required at each interface; namely, the flux and temperature must be continuous. At the interface between the m'th and m'th + 1 layer, the boundary condition may be expressed as

$$\theta_m\left(\sum_1^m L_m, t\right) = \theta_{m+1}\left(\sum_1^m L_m, t\right) \qquad (8)$$

$$\lambda_m \theta_{m\delta}\left(\sum_1^m L_m, t\right) = \lambda_{m+1} \theta_{m+1\delta}\left(\sum_1^m L_m, t\right) \qquad (9)$$

This multi-layer boundary value problem can be solved by the use of the Laplace transformation which converts the partial differential equation in θ(x,t) to the ordinary differential equation in u(x,p). The transformed equation for the m'th layer becomes

$$u_{m\delta\delta}(x,p) - q_m^2 u_m(x,p) = 0 \quad \sum_1^{m-1} L_m < x < \sum_1^m L_m \qquad (4')$$

$$q_m^2 = p/a_m$$

The interface boundary conditions at the input face of the m'th layer transform to the form

$$u_{m-1}\left(\sum_1^{m-1} L_m, p\right) = u_m\left(\sum_1^{m-1} L_m, p\right) \qquad (8')$$

$$\lambda_{m-1} u_{m-1\delta}\left(\sum_1^{m-1} L_m, p\right) = \lambda_m u_{m\delta}\left(\sum_1^{m-1} L_m, p\right) \qquad (9')$$

or

$$f_{m-1}\left(\sum_1^{m-1} L_m, p\right) = f_m\left(\sum_1^{m-1} L_m, p\right).$$

where f is the transformed flux.

The boundary condition at the final output face will be

$$u_m(L,p) = 0 \qquad (7')$$

while the transform of the input boundary condition at $x = 0$ will depend upon the time dependence of the input function.

The general solution for the m'th layer can be written in matrix form [8]

$$\begin{vmatrix} u_{mo} \\ f_{mo} \end{vmatrix} = \begin{vmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{vmatrix} \cdot \begin{vmatrix} u_{mi} \\ f_{mi} \end{vmatrix} = \begin{vmatrix} T_m \end{vmatrix} \cdot \begin{vmatrix} u_{mi} \\ f_{mi} \end{vmatrix} \qquad (10)$$

where the subscripts o and i designate the value of the transformed temperature or flux in the plane of the output or input face of the m'th layer. The terms in the square matrix are given by

$$T_{11} = T_{22} = \cosh(q_m L_m)$$

$$T_{12} = - \frac{\sinh(q_m L_m)}{\lambda_m q_m}$$

$$T_{21} = - \lambda_m q_m \sinh(q_m L_m) \qquad (11)$$

and $T_m$ is the matrix for the m'th layer.

If layers from 1 to M are placed in series so that the output of the m'th layer becomes the input of the m'th + 1 layer, the matrix equation becomes

$$\begin{vmatrix} u_{mo} \\ f_{mo} \end{vmatrix} = \begin{vmatrix} T_M \end{vmatrix} \cdots \begin{vmatrix} T_m \end{vmatrix} \cdots \begin{vmatrix} T_1 \end{vmatrix} \cdot \begin{vmatrix} u_{1i} \\ f_{1i} \end{vmatrix}$$

$$= \overset{m=1}{\underset{m=M}{\pi}} T_m \cdot \begin{matrix} u_{1i} \\ f_{1i} \end{matrix} \qquad (12)$$

For a general position, $\chi$ , measured from the input face of the m'th layer, the matrix equation is

$$\begin{vmatrix} u_{m\chi} \\ f_{m\chi} \end{vmatrix} = \begin{vmatrix} T_{m\chi} \end{vmatrix} \cdot \begin{vmatrix} u_{mi} \\ f_{mi} \end{vmatrix} \qquad (13)$$

where the arguments of the hyperbolic functions in the matrix $T_{m\chi}$ will be $q_m x_m$ or $q_m(x - \sum_1^{m-1} L_m)$ as x is measured from the input face of the first layer. The values of the input functions of the m'th layer are given in terms of the output functions by

$$\begin{vmatrix} u_{mi} \\ f_{mi} \end{vmatrix} = \begin{vmatrix} T_m^{-1} \end{vmatrix} \cdot \begin{vmatrix} u_{mo} \\ f_{mo} \end{vmatrix} \qquad (14)$$

where $T_m^{-1}$ is the inverse of $T_m$.

Thus, the value of the function at $\chi$ in the m'th layer is given in terms of the output

$$\begin{vmatrix} u_{m\chi} \\ f_{m\chi} \end{vmatrix} = \begin{vmatrix} T_{m\chi} \end{vmatrix} \cdot \begin{vmatrix} T_m^{-1} \end{vmatrix} \cdots \begin{vmatrix} T_m^{-1} \end{vmatrix} \cdot \begin{vmatrix} u_{Mo} \\ f_{Mo} \end{vmatrix} \qquad (15)$$

or in shortened form

$$\begin{matrix} u_{m\chi} \\ f_{m\chi} \end{matrix} = \begin{vmatrix} T_{m\chi} \end{vmatrix} \cdot \overset{m=M}{\underset{m=m}{\pi}} \begin{vmatrix} T_m^{-1} \end{vmatrix} \cdot \begin{vmatrix} u_{Mo} \\ f_{Mo} \end{vmatrix}$$

$$= \begin{vmatrix} N \end{vmatrix} \cdot \begin{vmatrix} u_{Mo} \\ f_{Mo} \end{vmatrix} \qquad (16)$$

Solving equation 12 for the input functions gives

$$\begin{vmatrix} u_{1i} \\ f_{1i} \end{vmatrix} = \overset{m=M}{\underset{m=1}{\pi}} \begin{vmatrix} T_m^{-1} \end{vmatrix} \quad \begin{vmatrix} u_{Mo} \\ f_{Mo} \end{vmatrix}$$

$$= | \; D \; | \cdot \begin{vmatrix} u_{Mo} \\ f_{Mo} \end{vmatrix} \tag{17}$$

The ratio of the temperature at x to the temperature at the input face, x = 0, for a constant temperature at the output face, x = L is given by solving eq 16 for $u_{mx}$ and eq 17 for $u_{1i}$ and dividing to obtain the desired ratio; i.e.,

$$\frac{u_{mx}}{u_{1i}} = \frac{N_{12}}{D_{12}} \cdot \frac{f_{Mo}}{f_{Mo}}$$

or

$$u(x,p)/u(0,p) = N_{12}/D_{12} = Z \tag{18}$$

The value of Z, the ratio of the transformed temperature at x to the transformed temperature at x = 0, $N_{12}/D_{12}$, is a complex quantity dependent upon q, the thickness of the layers and their physical properties. The expressions for $\theta(x,t)$ and $\theta(0,t)$ are obtained from u(x,p) and u(0,t) by use of the Inversion Theorem by which the residues at the poles of the integrand are evaluated. For the case of the steady periodic state, only the residues at the poles which occur along the imaginary axis need to be evaluated. For an input function containing a single frequency, $\omega$, these poles will be at $\pm i\omega$. Thus, for a sine wave input function, Z is obtained by evaluating $N_{12}/D_{12}$ with the factor q in each argument given by $q_m = (i\omega/a_m)^{1/2}$. The vector Z gives the attenuation and phase lag of an input sine wave of temperature when measured as a temperature at a position x in a multi-layer infinite slab.

For periodic input functions expressed as a Fourier Series, the application of the Inversion Theorem on the transform of the temperature requires the evaluation of the residues at pairs of poles at $\pm in\omega$ where the values of n depends upon the harmonic content of the input function. The residues will be given by $A_nZ_n$ where $Z_n$ is the value of Z with the factor of q of each argument given by $q_m = (in\omega/a_m)^{1/2}$; and $A_n$ is the coefficient of the n'th harmonic of the Fourier Series expansion of the input function $\theta(0,t)$.

The matrices N and D become unwieldy when used to obtain an explicit expression for Z of a slab containing more than two or three layers. The matrices can, however, be evaluated by purely numerical methods using complex-mode computer programs. When this method of calculation is used, each term in the matrices N and D is determined with equal ease. It is thus possible to find a vector Z giving the amplitude and phase lag of the temperature or flux at a point x with respect to a temperature or flux wave incident on the input face of a multi-layer slab with the output face held in an isothermal or adiabatic condition.

## 5. Design Objectives

As indicated earlier, a roof is to perform the major objectives, i.e., provide thermal insulation for the control of the internal environment and act as a waterproof weather barrier. It should accomplish these objectives within certain economic design limitations. The total roofing system also has to carry its own dead weight plus any live loading from wind, snow, rain or normal maintenance traffic. The latter objective is generally the sole responsibility of the structural deck system. Certain types of application may also require a fire rating for the roof system. In total, there may be as many as five or six functions to be performed by a given roof system, and in most cases, there will be conflicts between certain design objectives which will require some form of compromise in terms of the final design. Determining the amounts and type of insulation to be employed within the confines of a particular system can involve a number of considerations ranging from economic to thermal performance. A procedure for performing multivariable optimization has been developed previously and is described in the following section.

## 6. Basis of Optimization

The basis of the optimizing procedure is a generalized multivariable optimization criteria that attempts to compromise design objectives and any inequality constraints. The procedure was originally presented as a structural design aid by Gall and Krokosky [9]. Since every design is a compromise between conflicting goals, each design reflects the designer's ability to perform some trade-off among various goals. The present design program still requires the judgment and selectivity of the designer to determine realistic performance indices. The determination of performance indices is by far the most difficult part of the entire procedure. Variations of the procedure are limited only by the imagination and resourcefulness of the designer. However, for the untrained designer, the determination of performance indices can be developed heuristically.

## 7. Present Optimization Scheme

The current program specifically designs a three-layered composite roof system by selecting the materials from a data bank of discrete material properties. The final design represents a compromise between thermal performance variables and cost. The importance of each design objective is determined by the designer in setting up the various performance indices into a so-called ranking matrix consisting of levels of desirability.

## 8. Ranking Matrix

The heart of the present procedure utilizes a ranking matrix of system functions which reflects the designer's ability to recognize and to assess the relevance and desirability of each design function. Table I shows such an array for a typical roof design considered herein.

TABLE I

Design Ranking Matrix for Roof Section

| Cost of Insulation ($\$/ft^2$) | Total Integrated Flux ($BTU/ft^2$24-hr.period | Maximum Peak Thermal Flux ($BTU/hr/ft^2$) | Maximum Felt Temperature Differential (°F) |
|---|---|---|---|
| 0.00 | 0.00 | 0.00 | 0.00 |
| 0.25 | 50.00 | 10.00 | 25.00 |
| 0.35 | 75.00 | 10.00 | 25.00 |
| 0.45 | 100.00 | 20.00 | 25.00 |
| 0.55 | 125.00 | 30.00 | 25.00 |
| 50.00 | 150.00 | 300.00 | 300.00 |

Limits:

Thickness (ft) $0.0833 \leq t_1 \leq 0.500$; $t_2 = 0.0208$; $t_3 = 0.333$

Density ($\#/ft^3$) $1.9 \leq \rho_1 \leq 27.0$; $\rho_2 = 70.0$; $\rho_3 = 140.0$

The matrix ranks system attributes against an absolute scale of performance. The first row, J = 1, represents the most optimistic desires for the performance of the system as well as the upper bounds for each system attribute. Row two, J = 2, gives the designer's appraisal of excellent performance and so on down the following rows with each succeeding row having less desirable characteristics than the previous row. The designer can use as many rows as needed. In each case the last row is known as the funnel row and its function is to funnel the performance characteristic within ranking considerations. The only restrictions on the construction of such an array are that each row must contain values of system attributes of equal desirability and each column in the array must be montonically increasing or decreasing.

## 9. Search Procedure

The optimization search procedure relies on a pseudorandom or adaptive search techniques in which the search probability is not fixed but shifts around as the search progresses. A purely random search procedure would use a probability density that is uniform over the whole search procedure. The search relies on a probability density that is a maximum at the current best value of the design parameters. In the present program, these parameters are the density of the insulation, conductivity and the corresponding thicknesses of the insulation.

On each side of the current best value the probability density decays with some exponential value of [odd integer - 1)/odd integer]. A new choice of parameters is determined by

$$KO[I] = KB[I] + [ \left| UB[I] - LB[I] \right| R^9 ] \qquad (19)$$

in which KO = the newly chosen search parameter; KB = current best value; LB and UB = the upper and lower search parameter bounds; R = a uniform probability density between -1 and 1; and 9 is the odd integer describing the probability density decay. Every time a better point is found that results in a lower value of $J_m$, its corresponding variables are stored in KB. The maximum value of all the desirability ratings is given by $J_m$, i.e.

$$J_m = \text{maximum } [RA[N]] \qquad (20)$$

in which N = the number of system variables and RA = the desirability of each system variable which is obtained by linear interpolation of the calculated value and the design values from the ranking matrix. Thus, figure 2 shows the flow chart for the ranking optimization program. In this particular variation of the program, the density variable is used to select the materials for insulation. The random value of the density is used in conjunction with the discrete material file. An interpolation is carried out and the discrete material having the density closest to the pseudorandomly generated value is then chosen along with its properties for a given ranking. The program can be run for any given period of time until the desirability of each system variable is approximately equal or until there is no appreciable change in the $J_m$ value.

## 10. Inequality Constraints

The ranking array can be set up to act as an inequality constraint so that underdesigned quantities are the ones that are immediately reduced, see Table II. Underdesigned quantities are fixed so that they have a high $J_m$, therefore a low desirability, while the overdesigned have a low index and are not usually reduced further.

Table II

Typical Ranking Array Showing Implications of Underdesign and Overdesign

| Desirability Index | Maximum Temperature Gradient Across Felts, °F | Desirability Index | Equivalent To |
|---|---|---|---|
| J[1] Most desirable | 0 | J[1] | Overdesigned |
| J[2] | 25 | J[2] | |
| J[3] | 25 | J[3] | Allowable |
| J[4] | 25 | J[4] | |
| J[5] Funnel Row | 300 | J[5] | Underdesigned |

## 11. Illustrated Example Application

Possibly the best method of showing the usefulness of the previously described analytical techniques is to consider a specific example. The thesis previously expressed was that conventional roofing design employing the built-up roofing waterproof membranes exposed to the weather was, in fact, somehow poor design.

The aforementioned thermal analysis and optimization procedure were applied to this particular problem in an effort to search out more desirable designs. Specifically the present design considers the implications of placing the built-up membranes under the insulation.

Initially only four major parameters were considered in the ranking matrix as shown in Table I. These were the cost, total integrated flux, maximum temperature differential of the membrane layer, and maximum thermal flux transmitted through the roof section. The inclusion of cost as a comparative parameter is obvious, whereas the other parameters may not be as obvious. The temperature differential of the felt membrane layer was considered for protection against thermal shock and excessive thermal cycling. Finally, the peak thermal flux was considered to give some indication of the overall thermal design of the roof as far as the interior environment was concerned.

Initially the search was also limited to six specific insulations as shown in Table III. Other insulations or variations of insulation properties with density could have been included as well, but the present study was purposely kept simple so that the overall approach could be emphasized.

The object of the present search was to select the most economical insulation, and thickness of insulation, from the possible insulations listed in Table III which would also satisfy the criteria of the ranking matrix, Table I. Allowable insulation thicknesses are also shown in Table I.

Table III

Material Properties

| Material | Density $\#/ft^3$ | Thermal Conductivity $BTU \cdot ft/hr. ^\circ F \cdot ft^2$ | Specific Heat $BTU/lb/^\circ F$ | Cost $/Board ft. |
|---|---|---|---|---|
| Urethane | 1.9 | 0.012 | 0.240 | 0.17 |
| FOAMGLAS | 9.0 | 0.033 | 0.188 | 0.15 |
| Asphalt-coated lightweight aggregate | 15.0 | 0.038 | 0.200 | 0.08 |
| Fiberboard | 15.1 | 0.030 | 0.500 | 0.07 |
| Lightweight aggregate insulating concrete | 25.0 | 0.058 | 0.169 | 0.08 |
| Concrete-coated lightweight aggregate | 27.0 | 0.065 | 0.180 | 0.10 |
| Built-Up Roof | 70.0 | 0.093 | 0.370 | 0.22 |
| Concrete | 140.0 | 1.000 | 0.210 | 0.05 |

The specific climatic conditions considered in this paper correspond to a site just east of Pittsburgh, Pennsylvania*. Both a representative summer and winter day was evaluated. Specific information concerning dry bulb air temperatures, cloud cover, wind velocity, etc. was chosen to be representative. In the present program, any region of the United States could just as easily have been considered.

## II. Results

Summer, Case I - (Concrete structural deck) + 4 ply membrane + Insulation. The first example consists of conventional 4 ply built-up roofing membranes placed over four inches of structural concrete deck and subjected to a typical summer sol-air temperature driving function as shown in figure 3. To begin the search, a guess is made as to the yet unknown insulation and its thickness. Initial values of system object functions are then calculated and compared to the allowable limits. If the values are all within the limits, each function is then ranked according to the ranking matrix shown previously. The next cycle of the search then attempts to improve the rank of the worst ranked parameter by either a change of insulation thickness or insulation material.

Shown in Table IV-A are actual ranking output generated by the program for which several improved ranks were found. It can be quickly seen that the initial estimate was unsatisfactory in terms of the total heat gain, (i.e., rank 4·234). It should be remembered that the more desirable the function, the lower the rank value will be. After only slight improvement was achieved by an increase in thickness, the search procedure shifted to a lighter and more efficient insulation. In so doing, the cost then became the critical parameter and further searching ultimately produced a design where the cost and total flux were of approximately equal rank and the search was terminated.

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*Location of Pittsburgh Corning Research and Engineering Laboratory.

Table IV-A

Optimization Search Output

A. Summer Condition - Concrete Deck.

| Current Insulation Choice | | Design Parameters and Rank | | | |
|---|---|---|---|---|---|
| Density<br>#/ft$^3$ | Thickness<br>Ft. | Cost<br>$\$/ft^2$ | Total Flux<br>BTU/ft$^2$/24 hr. | Peak Flux<br>BTU/ft$^2$/hr. | Maximum Felt Temp. Differential<br>°F |
| 25.0 | 0.210 | 0.202 | 105.85 | 7.751 | 8.888 |
| | | (1.806) | [ 4.234] | (1.775) | (1.355) |
| 25.0 | 0.217 | 0.208 | 103.61 | 7.559 | 8.649 |
| | | (1.833) | [ 4.144] | (1.756) | (1.346) |
| 9.0 | 0.217 | 0.390 | 68.30 | 4.859 | 5.397 |
| | | [3.402] | ( 2.732) | (1.486) | (1.216) |
| 9.0 | 0.1970 | 0.355 | 73.58 | 5.266 | 5.899 |
| | | [3.046] | ( 2.943) | (1.527) | (1.236) |

Table IV-B

Optimization Search Output

B. Winter Condition - Concrete Deck.

| Current Insulation Choice | | Design Parameters and Rank | | | |
|---|---|---|---|---|---|
| Density<br>#/ft$^3$ | Thickness<br>Ft. | Cost<br>$\$/ft^2$ | Total Flux<br>BTU/ft$^2$/24 hr. | Peak Flux<br>BTU/ft$^2$/hr. | Maximum Felt Temp. Differential<br>°F |
| 9.0 | 0.210 | 0.378 | 145.37 | 7.863 | 5.560 |
| | | (3.28) | [ 5.815] | (1.786) | (1.222) |
| 1.92 | 0.172 | 0.3509 | 102.29 | 4.578 | 1.21 |
| | | (3.009) | [ 4.092] | (1.458) | (1.049) |
| 1.92 | 0.179 | 0.3647 | 98.82 | 4.421 | 1.169 |
| | | (3.147) | [ 3.953] | (1.442) | (1.047) |
| 1.92 | 0.213 | 0.4347 | 84.36 | 3.772 | 0.983 |
| | | [3.846] | ( 3.374) | (1.377) | (1.039) |

All system attributes were not equally ranked because the original ranking matrix tended to overdesign the peak flux and maximum temperature differential for the present input function.

Winter, Case I - (Concrete structural deck) + 4 ply membrane + Insulation. Since most of the northern latitudes of the United States experience a great difference in ambient outdoor temperature from summer to winter seasons and any insulation system must function year round; real designs would have to consider more than just one input sol-air condition. Although more elaborate design criteria would undoubtedly be considered in an actual design, for the sake of demonstration and comparison, a typical winter sol-air temperature was developed for the same roof and location as in the summer case just discussed. This sol-air temperature shown in figure 4 was used as input to the optimization program and a new search was made for the optimal insulation to function under winter conditions for the same design criteria as given in the ranking matrix of Table I.

Table IV-B shows the actual ranked designs for the winter input. It is seen immediately that what was the best choice of insulations for the summer sol-air conditions no longer was acceptable because of the large heat loss shown as total flux, i.e. Rank, (5.815). The next ranked design shows the effect of a lighter and more efficient insulation, but again a more costly insulation. This is reflected in a shift of the critical parameter from total flux to cost. Further searching produced a slight reduction in the cost rank by reducing the thickness.

It is interesting to note that the optimized designs for summer and winter conditions do not suggest the same insulation in each case. The summer design uses 2.36 inches of 9.0 pcf density insulation and the winter case selects 2.56 inches of 1.92 pcf density insulation. In a real case an additional criteria such as the cost of heating and cooling could be included to aid in the decision. The addition of such a criteria in the present program would present no serious problems.

Figures 3 and 4 show computer-generated data for both the sol-air equivalent driving functions and the responses of both the protected and conventional membrane roofing systems when insulated with the designed winter insulation. It is at once obvious that the protected membrane system effectively damps out the large thermal fluctuations of the driving input sol-air temperature and provides a relatively stable thermal environment. The conventional designed membrane on the other hand very closely tracks the input sol-air temperature. The conventional membrane curves and sol-air temperature are shown as a single line because the membrane temperature is so close to the input.

Since the initial example consisted of a rather massive structural deck, i.e., 4.0 inches of concrete which provides the potential for a large amount of heat storage, it was decided to test a second example with a vastly different deck construction. The second case consists of a corrugated metal deck and a 0.5 inch layer of fiberboard adhered there unto as a base for the built-up roofing membranes. Once again the task was to determine the type and thickness of insulation to protect the membranes and to provide the major insulation for the building for both the summer and winter conditions.

Summer, Case II - (Corrugated metal and fiberboard deck) + 4 ply membrane + Insulation. Table V-A shows the resulting ranks for both the initial estimate, i.e., 2.52 inches of 25.0 pcf density insulation, and succeeding generated values.

Table V-A

Optimization Search Output

A. Summer Condition - Steel Deck + Fiberboard.

| Current Insulation Choice | | Design Parameters and Rank | | | |
|---|---|---|---|---|---|
| Density #/ft$^3$ | Thickness Ft. | Cost \$/ft$^2$ | Total Flux BTU/ft$^2$/24 hr. | Peak Flux BTU/ft$^2$/hr. | Maximum Felt Temp. Differential °F |
| 25.0 | 0.210 | 0.202 | 84.51 | 11.273 | 29.711 |
| | | (1.806) | ( 3.380) | ( 3.127) | [ 5.017] |
| 9.0 | 0.217 | 0.390 | 57.83 | 7.806 | 20.587 |
| | | [3.402] | ( 2.313) | ( 1.781) | ( 1.824) |
| 9.0 | 0.197 | 0.355 | 61.73 | 8.376 | 22.098 |
| | | [3.046] | ( 2.469) | ( 1.838) | ( 1.884) |
| 15.0 | 0.197 | 0.189 | 67.52 | 9.032 | 23.809 |
| | | (1.756) | [ 2.701] | ( 1.903) | ( 1.952) |

Initially, it is seen that the membrane temperature differential was the critical, or worst, ranked parameter. Attempting to correct this, the cost then became the decisive parameter. Finally the search procedure settled on a compromise insulation where total flux transmission was the critical variable. At this point time terminated the search procedure and it can be seen that three of the four ranked parameters are approximately of equal ranked value. Had time not terminated the run, it is most likely that the thickness of insulation would have been increased, increasing cost slightly, while lowering total flux, peak flux and maximum felt temperature differential. The cost and total flux would then have been of approximately equal ranking magnitude. It is somewhat interesting to note that with this lightweight metal deck system there is an increase in the maximum felt temperature differential over over that observed with the more massive concrete deck system. The mass of the concrete tends to provide inertia which resists the rapid input fluctuations.

Winter, Case II - (Corrugated metal and fiberboard deck) + 4 ply membrane + Insulation. Once again the second part of the procedure was to evaluate the search for the winter sol-air temperature input. Table V-B shows the results from the optimization search. Initially the total flux, i.e., heat loss, was the critical parameter with a rank of [5.261]. The search procedure improved on this rank by switching to a more efficient insulation which also increased the cost. Further searching reduced the cost somewhat but not enough to remove the cost as the critical parameter. As in the former case, a different optimum design is suggested for the winter case as compared to the summer condition.

The input sol-air temperature and system thermal response are shown for both summer and winter conditions in Figure 5 and 6. As with the previous example, the insulated or protected membrane remains essentially isolated from the large input temperature fluctuations although as noted earlier, the present case does show somewhat larger fluctuations than the concrete deck system.

Table V-B

Optimization Search Output

Winter Condition - Steel Deck + Fiberboard.

| Current Insulation Choice | | Design Parameters and Rank | | | |
|---|---|---|---|---|---|
| Density<br>#/ft$^3$ | Thickness<br>Ft. | Cost<br>$\$/ft^2$ | Total Flux<br>BTU/ft$^2$/24 hr. | Peak Flux<br>BTU/ft$^2$/hr. | Maximum Felt Temp. Differential<br>°F |
| 9.0 | 0.210 | 0.378 | 131.52 | 8.739 | 21.085 |
| | | (3.28) | [ 5.261] | (1.874) | ( 1.843) |
| 1.92 | 0.210 | 0.4281 | 81.47 | 4.033 | 4.695 |
| | | [3.781] | ( 3.259) | (1.403) | ( 1.1878) |
| 1.92 | 0.207 | 0.4225 | 82.39 | 4.078 | 4.749 |
| | | [3.725] | ( 3.296) | (1.408) | ( 1.190) |

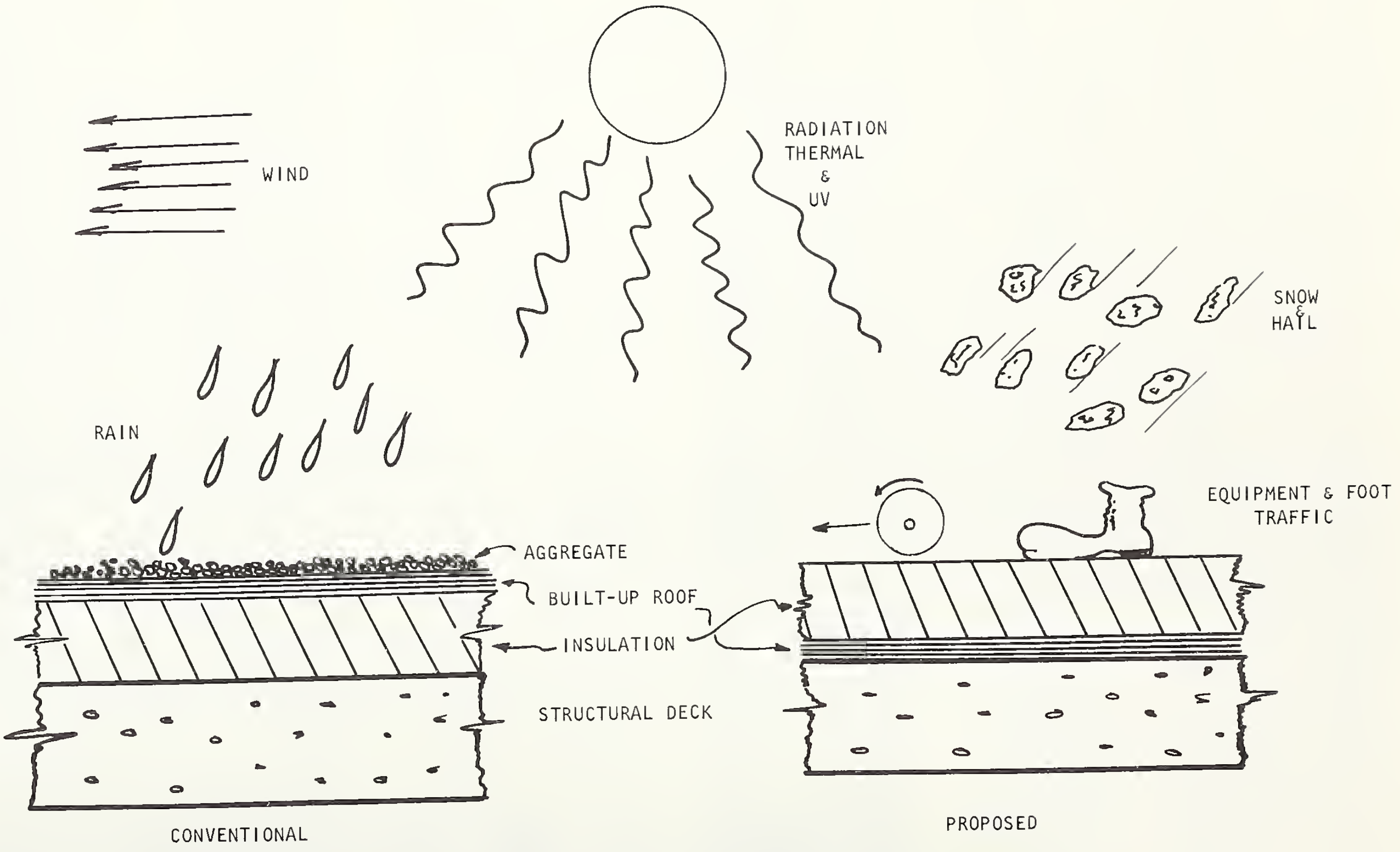
## 12. Conclusions

Although we have not had enough experience with the present program to be able to state generalizations regarding the design of protected membrane roofing systems, we feel the present approach will be extremely useful toward that eventual end. We believe it is significant that seasonal demands caused by different sol-air temperature driving functions dictate different insulation requirements for summer as compared to winter conditions. The present example has shown the benefits to be derived in terms of a stabilized thermal environment for the membrane in a protected membrane roof insulation system independent of seasonal conditions.

The present multifunctional optimization program offers the design engineer an efficient means of assessing the effects of variations in specific design parameters on the overall design of a building material system.

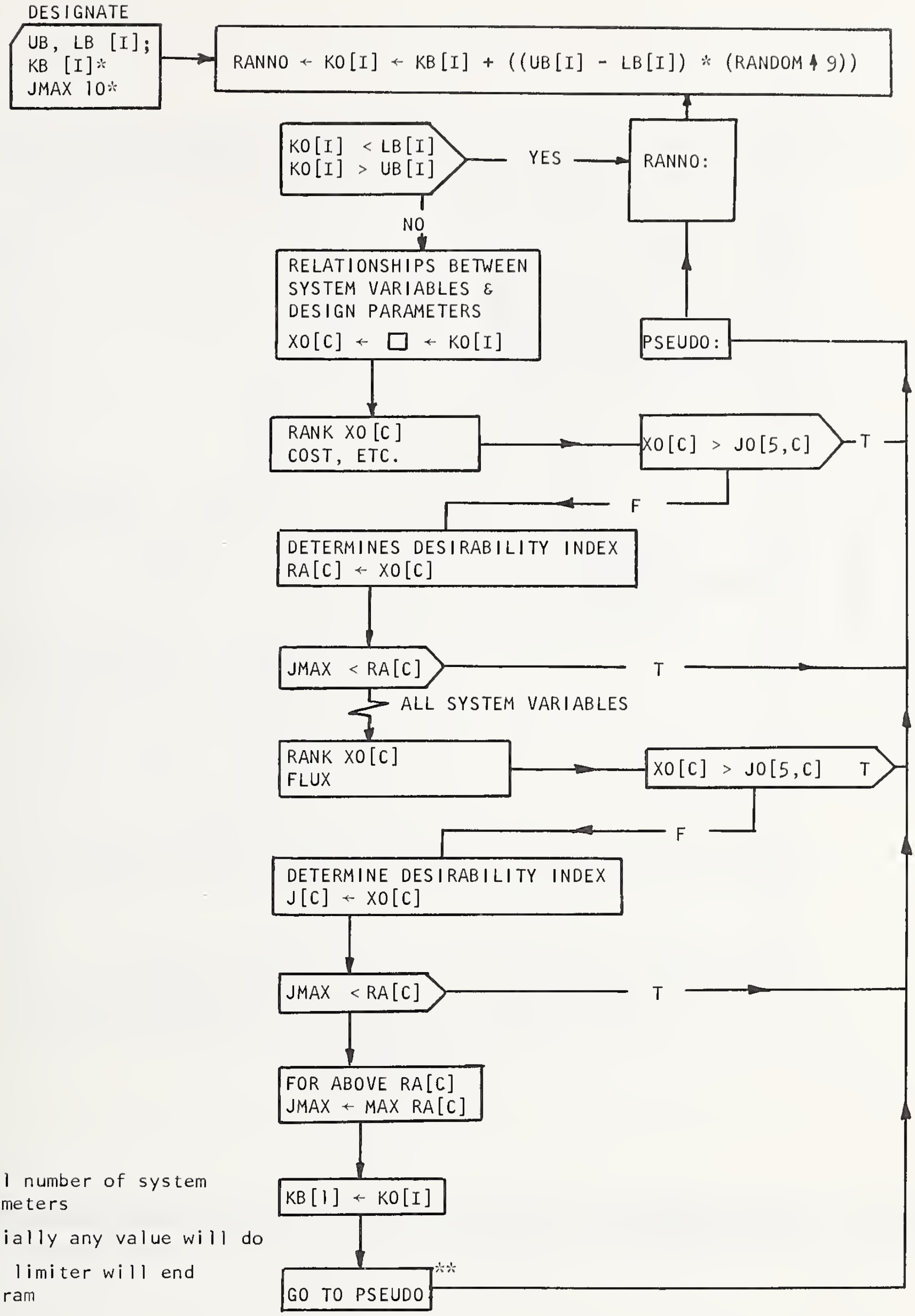
We feel that it is especially useful in discovering and resolving the conflicts between the various design criteria which do arise in multifunctional systems. The designer can then easily determine the degree of interplay and the limitations of design variables. The need for rational compromise is then made more apparent.

Since the most difficult part of the design process appears to be the specification of equivalent desirable performance criteria for the many variables involved, it is also possible to use the present program as a heuristic design learning process for the actual establishment of said performance criteria.



TYPICAL ROOF SECTIONS

Figure 1. Environment of a Roof



[I] equal number of system parameters

* initially any value will do

** time limiter will end program

Figure 2. Flow Diagram for Optimization Program

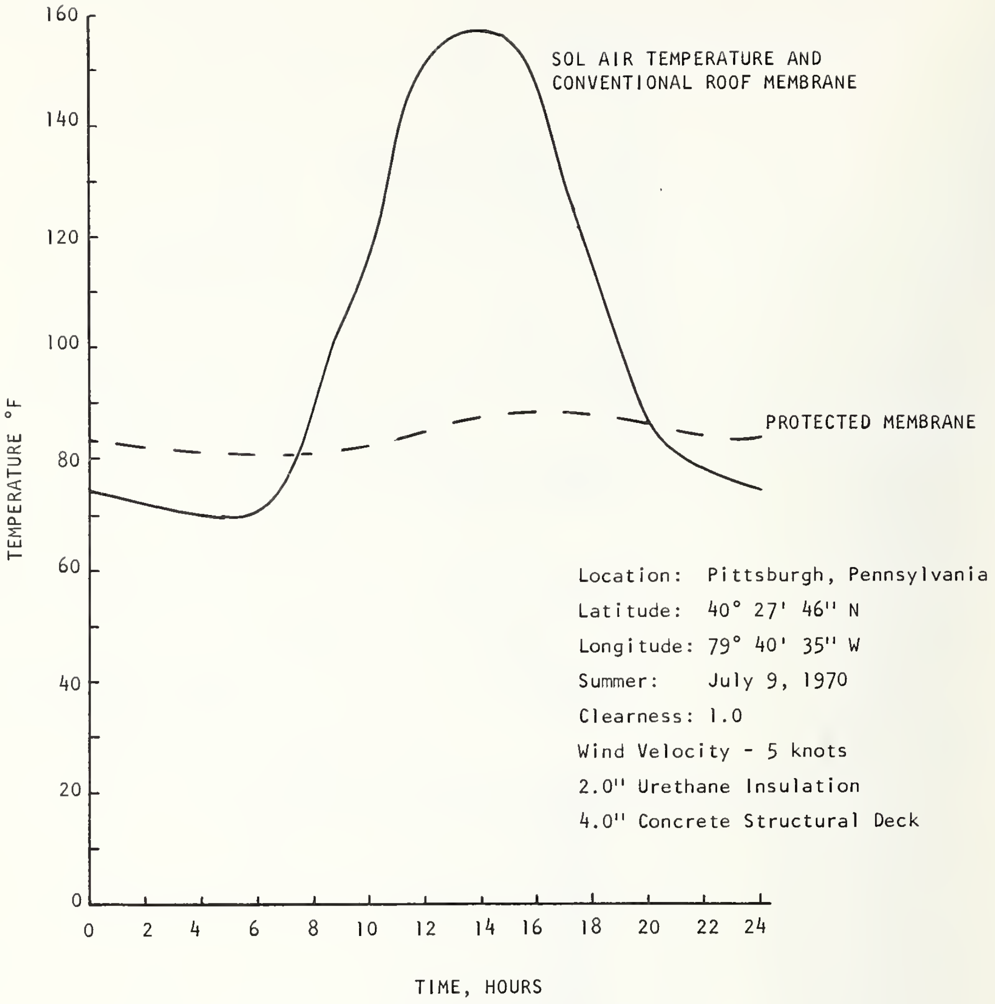


Figure 3. Periodic Temperature Variations for Conventional and Insulated Membrane Roof Systems - Summer Conditions

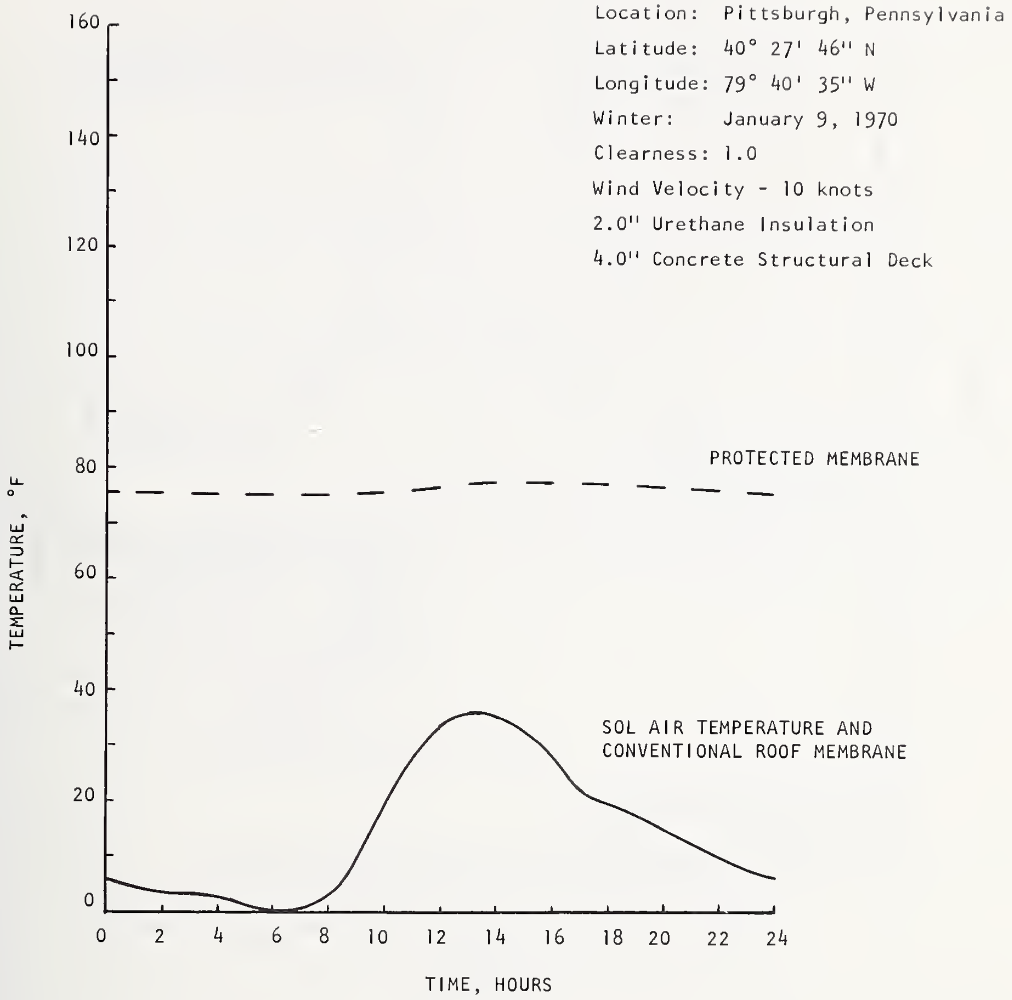


Figure 4. Periodic Temperature Variations for Conventional and Insulated Membrane Roof Systems - Winter Conditions

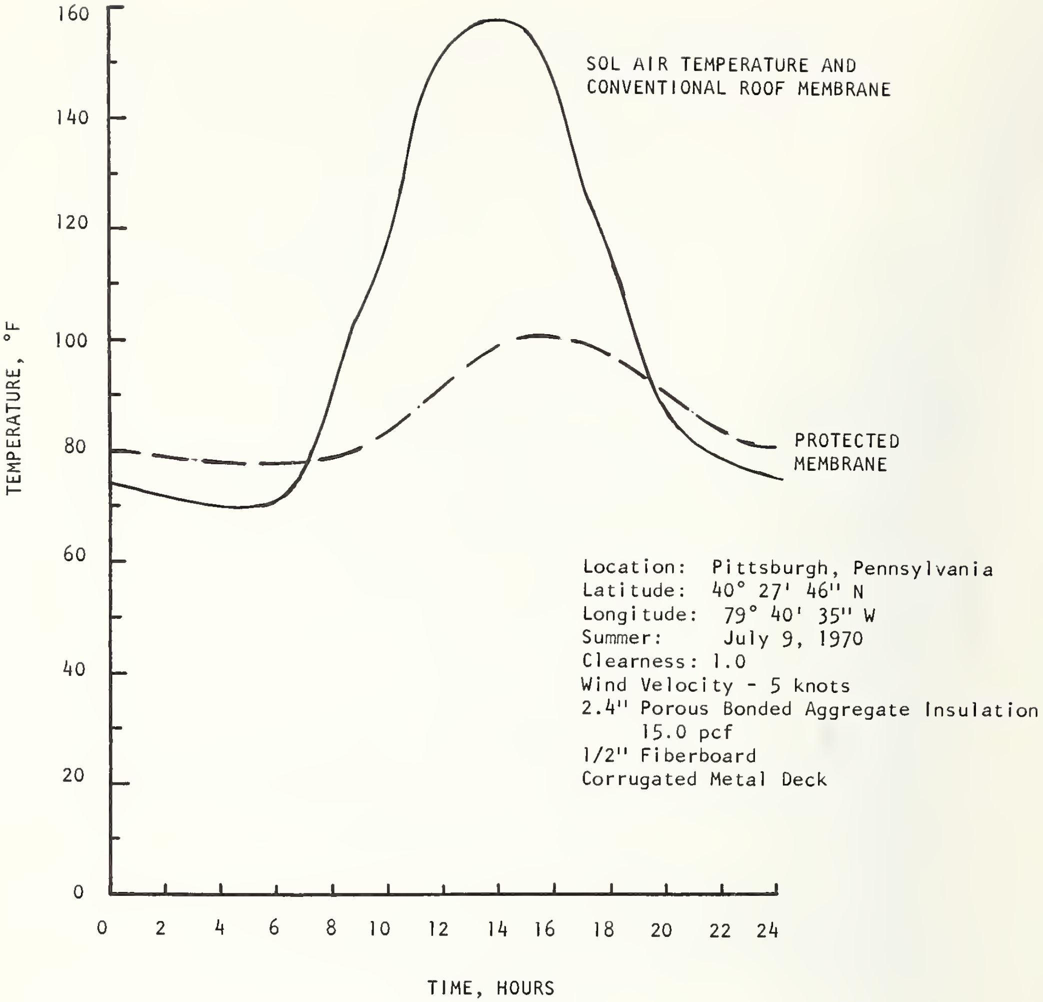


Figure 5. Periodic Temperature Variations for Conventional and Insulated Membrane Roof Systems - Summer Conditions

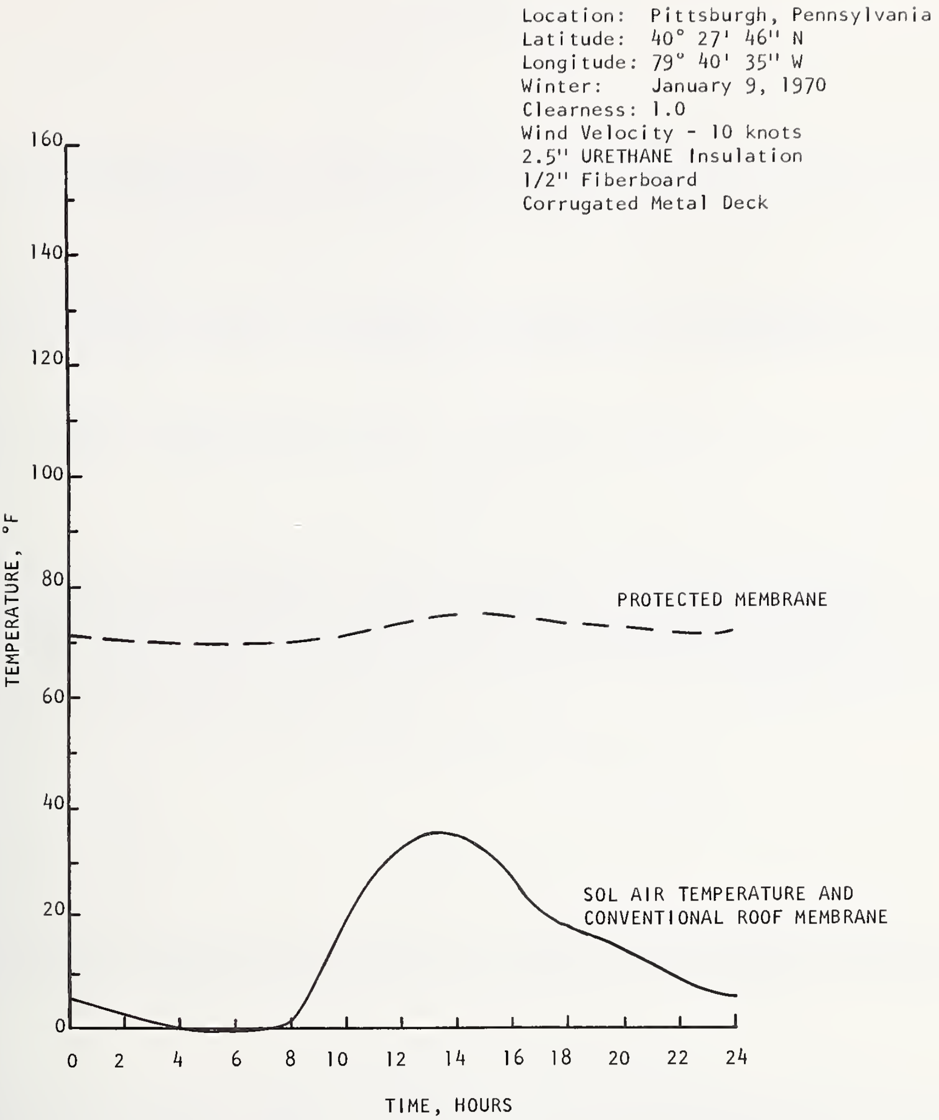


Figure 6. Periodic Temperature Variations for Conventional and Insulated Membrane Roof Systems - Winter Conditions

# Pulse Transfer Function & Its Application Related to Buildings

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## Abstract

For the digital calculation of a linear system, it is more convenient to use the Z-transform than the S-transform (Laplace transform). The transformation of the product of the system input and transfer function on the S-domain into the Z-domain is very complicated.

This difficulty is overcome, however, if we assume that the output varies very slowly in response to a sudden input change.

Discussed in this paper is the general technique of obtaining the Z-transform transfer function and the application of the technique to calculate room temperature of air-conditioned rooms under an automatic control system.

Keywords: Linear systems, transfer function, Z-transform

## 1. Introduction

Need exists to calculate dynamic changes of room temperature and heating/cooling loads. The method of these calculations are either frequency response method or the pulse response method. The frequency response method requires the solution of simultaneous equations with complex coefficients. With a careful plan the computer time for the complicated system can be decreased considerably. On the other hand, the pulse transfer function method requires polynominal expression of time series data in power of $Z^{-1}$. Although the pulse transfer method is convenient for dealing with sampled data, there is no superiority or inferiority over the frequency response method.

It is possible to obtain the pulse transfer function in three ways:

1. transfer function

2. frequency response

3. Weighting function

Reference (1) describes the method to derive the pulse transfer function from the frequency response. This paper is based on a method of obtaining approximate pulse transfer function from transfer function.

## 2. Input, Output and Components

The inputs may be the time changes of solar radiation, outdoor air temperature, internal heat generation in the room heat due to lighting and equipment or due to the occupants. The outputs may be the room temperature and heating/cooling load. These inputs and outputs are related by the thermal system components, which may be the structural characteristics of walls.

## 3. Principles of the Method

First, a method of obtaining the approximate pulse transfer function from the transfer function in a continuous system will be described. It is assumed that:

1. The system considered herein is linear

2. The change of the output is gradual even when the input is abrupt

These assumptions are not too far off in the real thermal systems. Denoting the input by F(S), the output by H(S) and the transfer function by G(S), it can be shown that

$$H(S) = G(S) \cdot F(S) \tag{1}$$

With the assumptions given above, this equation can be modified to

$$H^*(S) \cdot Gh(S) = G(S) \cdot F(S) \quad (2),$$

where $H^*(S)$ is the modified output to suit the sampling width. If Gh(S) is to represent the triangular pulse function

$$Gh(S) = (1-e^{-sT})^2 \cdot e^{sT}/(TS^2) \quad (3)$$

Equation (2) now may be represented in the Z-domain by

$$H(Z) = Ga(Z) \cdot F(Z) \quad (4)$$

Here Ga(Z) is the pulse transfer function and

$$Ga(Z) = 1/\zeta(Gh(S)/G(S)) \quad (5)$$

Thus it can be shown that the product of the linear system input and the transfer function can be obtained by multiplying the sampled input by the pulse transfer function.

## 4. Pulse Transfer Function

### 4.1 Heat Flow, Imaginary Function of a Slab

In a linear heat conduction system, for a slab of constant thermal resistance r, diffusivity $\alpha$, and thickness $\ell$, when subjected to the unit surface temperature excitation at one side and zero at another, the Laplace transform of the heat flux at the excitation side is

$$X = \sqrt{\frac{s}{\alpha}}\,\ell \ \cot h \sqrt{\frac{s}{\alpha}}\,\ell/r\ell \quad (6)$$

The heat flux at the zero temperature side is

$$Y = \sqrt{\frac{s}{\alpha}}\,\ell \ \mathrm{cosec} \sqrt{\frac{s}{\alpha}}\,\ell/r\ell \quad (7)$$

when s is the complex parameter. The solution for the multi-layer wall will be treated in the next section as a function of X and Y.

### 4.2 Surface Temperature Transfer Function For Multi-layer Wall as Temperature Input

Multi-layer wall temperature $W_o(S)$ responding to the unit surface temperature and zero at another will be (figure (2)),

$${}_iW_o(S) = Bw/Aw \quad (8)$$

where i is an index for the side of input and o is for the output, and Aw is expressed as follows,

$$Aw = a_1^* \cdot a_2^* \cdot a_3^* \ \ldots\ldots \ a_{n-1}^* \quad (9)$$

$$a_k = x_k + x_{k+1} \quad (k = 1, 2 \ \ldots \ , n - 1$$

$$a_1^* = a_1$$

$$a_k^* = a_k - Y_k^2/a^*_{k-1}$$

and if we let input i as 01 and output o as 12, Bw is

$$Bw = b_1^* \cdot b_2^* \cdot b_3^* \cdots\cdots b_{n-1}^* \qquad (10)$$

$$b_k = x_k + x_{k+1} \quad (k = 2, 3, \ldots , \ n-1)$$

$$b_1^* = Y_1$$

$$b_k^* = b_k - Y_k^2/b_{k-1}^*$$

and if we let input i as nn+1 and output o as 12, Bw is

$$B_w = b_1^* \cdot b_2^* \cdot b_3^* \cdots\cdots b_{n-1}^* \qquad (11)$$

$$b_k^* = Y_{k+1}$$

and if i is 01 and o is n-1n, Bw is

$$Bw = b_1^* \cdot b_2^* \cdot b_3^* \cdots\cdots b_{n-1}^* \qquad (12)$$

$$b_k^* = Y_k$$

### 4.3 Transfer Function of Single Room Temperature as Heat Supply

Further if we make some assumptions as

(3) Heat supplied into the room diffuses instantly into the room air

(4) Number of air changes in the room is constant

(5) Heat radiation is considered as a part of the overall heat convection coefficient

The transfer function of the single room temperature for the supply heat Rh(S) is

$$Rh(S) = 1/(Qa \cdot S{+}Qa \cdot N + \Sigma\ (A_j/Ri_j) \cdot (1-{}_{01}W_{12,j}(S)) \qquad (13)$$

where Qa is air capacity of the room, N is number of air changes, $A_j$ is area of the wall of number j, 1/Ri is overall thermal coefficient corresponding to wall j and ${}_{01}W_{12,j}(S)$ is transfer function of wall surface temperature on the side of excitation (outside surface) and S is complex parameter. In this case, we let $X_1$ and $Y_1$ as 1/Ri, and $X_n$ and $Y_n$ as $1/R_o$.

Now from eq. (13), room temperature θr(S) as supply heat Ht(S) will be in linear system,

$$\theta r(S) = Rh(S) \cdot Ht(S) \qquad (14)$$

Ht(S) includes heat exchange with outdoor air heat conduction through wall while the room temperature assumed to be 0. If we want to obtain frequency response of room temperature as heat supply, i.w would replace S in eq. (13). Where i is an imaginary number index and w is an angular frequency of heat supply.

In this system θr(S) is output, therefore we can transform room temperature θr(S) in accordance with assumption (2) and eq. (2) as follows,

$$\theta r(S) = \theta r^*(S) \cdot Gh(S) \qquad (15)$$

where $\theta r^*(S)$ is the sampled temperature of the room. Equation (14) is now modified to

$$\theta r^*(S) \;.\; Gh(S)/G(S) = Ht(S) \tag{16}$$

Equation (16) is then transformed to Z-domain as follows,

$$\theta r(Z) \;.\; \zeta(Gh(S)/G(S)) = Ht(Z) \tag{17}$$

In this case Ht(Z) corresponds to input in eq. (4). And if we consider eq. (17) as pulse transfer function of single room for heat supply,

$$Rh(Z) = 1/\zeta(Gh(S)/G(S)) \tag{18}$$

where Rh(Z) corresponds to Ra(Z) in eq. (5), then eq. (17) becomes,

$$\theta r(Z) = Rh(Z) \;.\; Ht(Z) \tag{19}$$

Now, from eq. (3), eq. (13) and eq. (18), we can get Rh(Z) in the following,

$$Rh(Z) = 1/(Qa \;.\; (Z-1)/T+Qa \;.\; N+ \Sigma\ (1-{}_{01}W_{12,j}(Z) \;.\; A_j/Ri_j)) \tag{20}$$

where ${}_{01}W_{12,j}(Z)$ is calculated by assumptions (2), (3), (4), and (5) and eqs. (3), (4), (5), (6), (7), (8), (9), and (10), as follows

$${}_{01}W_{12,j}(Z) = Bw/Aw \tag{21}$$

when Aw and Bw are given in eq. (9) and eq. (10), respectively. However, we denote X as x(Z) and Y as y(Z) in this case to represent

$$x(Z) = (1+(2/t) \;.\; \Sigma\ (Z/(Z-1) + Z/(Z-d_k))/ck)r1 \tag{22}$$

$$y(Z) = (1+(2/t) \;.\; \Sigma\ (-1)^k \;.\; (Z/(Z-1) + Z/(Z-d_k))/c_k)/r1$$

$$c_k = k^2 \;.\; \pi^2/(1^2/a^2)$$

$$d_k = EXP(-c_k \;.\; T)$$

T: sampled width

In these equations, slab number of multi-layer wall are omitted. x(Z) and y(Z) may also be expressed as

$$\begin{aligned} x(Z) &= (1+(2/T)\ (\sum_{k=1} (1-d_k)/c_k + \sum_{M=1} (\sum_{K=1} (1-d_k)^2 \;.\; d_k^{m-1})Z/c_k{}^{-m})r1 \\ y(Z) &= (1+(2/T)\ (\Sigma\ (-1)^k(1-d_k)/c_k + \Sigma\ (-1)^k(1-d_k)^2 d_k^{m-1})Z/c_k{}^{-m})r1 \end{aligned} \tag{23}$$

## 5. Pulse Transfer Function of Multi-connected Rooms Temperature

Now, if we let the input as heat supply $H_i(S)$, in which i is the room number and let output as room temperature $\theta_i(S)$, and if the transfer function of this room is $Rh_i(S)$

$$\theta_i(S) = Rh_i(S) \;.\; H_i(S) \tag{24}$$

and if we let the transfer function of a multiply-connected rooms as $Rh_{ji}(S)$, where j is the room number of input and i is the room number of output, $\theta_i(S)$ becomes

$$\theta_i(S) = \Sigma\, Rh_{ji}(S) \,.\, H_j(S) \tag{25}$$

where $Rh_{ji}(S)$ equals $Dr_{ji}/Dr$. Dr and $Dr_{ji}$ are as follows,

$$Dr = \begin{array}{ccccc} g_{11} & \cdot & \cdots & \cdot & g_{1n} \\ g_{21} & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ g_{n1} & \cdot & \cdots & \cdot & g_{nn} \end{array}$$

$$Dr_{ji} = \begin{array}{ccccc} g_{11} & \cdot\ \cdot\ \cdot & 0 & \cdot\ \cdot\ \cdot & g_{1n} \\ g_{21} & & \cdot & & \cdot \\ \cdot & & 0 & & \cdot \\ \cdot & & 0 & & \cdot \\ g_{j1} & \cdot\ \cdot\ \cdot & Rh_{ji} & \cdot\ \cdot\ \cdot & g_{jn} \\ \cdot & & 0 & & \cdot \\ \cdot & & \cdot & & \cdot \\ g_{n1} & \cdot\ \cdot\ \cdot & 0 & \cdot\ \cdot\ \cdot & g_{nn} \end{array}$$

$g_{ij} = -(A/R_i) \,.\, {}_jW_i(S) \,.\, Rh_i(S)$

(room of number i adjacent to room of number j)

$= 0$ (non-adjacent)

$= 1$ $(i = j)$

Therefore, pulse transfer function of this system $Rh_{ji}(Z)$ will be,

$$Rh_{ji}(Z) = 1/\zeta(Gh(S)/\Sigma\, Rh_{ji}(S)) \tag{27}$$

We can obtain the solution of eq. (27) by using assumption (2).

## 6. Temperature of an Automatically Air-conditioned Rooms

It is assumed that an office room is automatically air-conditioned during the office hours by a feed back controlled system. Temperature of the rooms during the off office hours will vary naturally in accordance with eq. (14).

Now, the heat supplied into the room during the air-conditioned period is $Hm(S)$. Heat supplied during the off-hours is $Ho(S)$. The programmed temperature profile is $Op(S)$, then equations will be,

$$\theta r(S) = Rh(S) \,.\, Ho(S)/(1+Rh(S) \,.\, Tf(S)) + Rh(S) \,.\, Tf(S) \,.\, \theta p(S)/(1+Rh(S) \,.\, Tf(S)) \tag{28}$$

$$Hm(S) = Tf(S) \,.\, (\theta p(S) - \theta r(S)) \tag{29}$$

and if we want to express eq. (28) by the Z-expression,

$$\theta r(Z) = Rh(Z) \;.\; Ho(Z)/(1+Rh(Z) \;.\; Tf(Z)) + Rh(Z) \;.\; Tf(Z) \;.\; \theta p(Z)/(1+Rh(Z) \;.\; Tf(Z)) \qquad (30)$$

when Rh(Z) is shown in eq. (18) and Tf(Z) is 1/(Gh(S)/Tf(S)). If we let Tf(S) as p.(1+I/S), Gh(S) as eq. (3), then

$$Tf(Z) = (1-q- \Sigma\, (1-q)^2 q^{j-1} Z^{-j})/(p.I.T) \qquad (31)$$

$$q = EXP(-I.T)$$

And the programmed temperature will be expressed in Z-expression as follows,

$$\theta p(Z) = \Sigma\, \theta r_j Z^{-j} + \sum_{b}^{e} \theta p_j Z^{-j} + \Sigma\, \theta r_j Z^{-j} \qquad (32)$$

then, if $\theta p_j$ and T are given, $\theta r_j$ can be solved from eqs. (30), (31) and (32).

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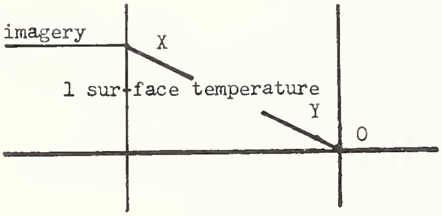
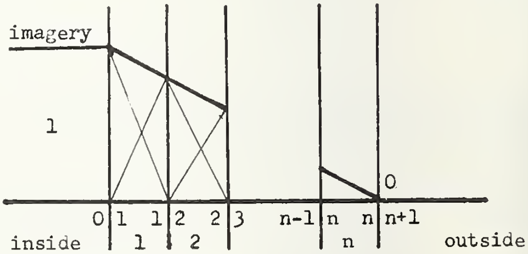


Fig(1) Heat flow imagery function of a slab



Fig(2) System of transfer function of multi-layer wall

# A Calculating Method for Heating Loads of Buildings,

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A method of calculation for heating loads of buildings, which are heated ( or cooled ) intermittently, is presented.
This method depends on the Fourier analysis. Then the thermal properties of the walls are described as the frequency transfer function, and the outdoor temperatures and the room temperatures are approximately expressed by the Fourier series.
To calculate the room air temperature is not so difficult when the supplied heat quantity is known, but it is comparatively difficult to determine the heating loads of the building which is heated intermittently.
This method is applicable for the above two cases and also for the next cases.

1. A building is consist of both heated and unheated rooms.
2. Ventilation rates in the rooms vary periodically.

This method is expressed by matrix equations. Namely, rooms of a building are divided into some blocks and the system is expressed by the tri-diagonal matrices whose elements are consist of the minor matrices.
If the rooms are laid in the grid type, the system is expressed by the transition matrices.
Since matrix calculation is used in this method, it is suitable for computer programming.
Finally, by using this method the heating loads H, which are to be supplied in the rooms to form the appointed temperatures $\overline{\theta}$ , are determined, then the room temperatures $\theta$ are determined for the obtained H. By comparing $\overline{\theta}$ and $\theta$ ,the usefulness of this method is considered.

Key Words: Heating loads, intermittent heating, temperature, Fourier analysis, frequency transfer function, Fourier series, ventilation, matrix, grid plan, tri-diagonal matrix, transition matrix.

## 1. Introduction

The calculation methods for the thermal environment of buildings can be classified roughly into the following two kinds of methods. The one is for the problems that the room temperatures are to be determined under the known supplied heats. The other is for the contrary cases in which the room temperatures are known and the heat to be supplied are unknown during some time.

Phenomena which we must calculate are generally unsteady. For the phenomena we have some calculation methods of the weighting function ( or inditial response ) which is gained on each wall. [1],[2],[3],[4],[5],[6],[7],[8] For the periodic states we have also some calculation methods of the frequency transfer functions gained on each wall. [9], [10], [11], [12][1]

As mentioned before, strictly speaking, the thermal phenomena surrounding the buildings are unsteady, therefore the periodic changes are only approximations of the phenomena. However, if we want the data for the purpose of the thermal planning of the building or the decision of the "zoning", it seems that the results calculated by the approximated periodic state meet our needs sufficiently. Moreover, in comparison with the weighting function and the frequency transfer function it is more diffcult to obtain the weighting function for multi-layer wall than the other function. Further the latter has a big advantage such as we can obtain it exactly.

This paper presents calculation methods for the periodic states in the multi-room building. The calculation methods are carried in terms of matrices as much as possible.

Namely, at first, the properties of the each layer of the composite wall are expressed in the "four terminal matrix". Then the four terminal matrix for the multi-layer wall is obtained by connecting the matrices of the all layers of the multi-layer wall, and from this matrix the "admittance matrix" is obtained.

Next the system is expressed in the three kinds of the matrix expressions by using the elements of these four terminal matrices and admittance matrices of the walls.

The first expression is shown in general matrix form of the simultaneous equation whose coefficients are complex numbers. Secondly, the system is divided into some blocks and it is expressed in the tri-diagonal matrix whose elements are consist of the minor matrices. The third, the system is expressed in the transition matrices.

The second expression needs less amounts of memory of computer than the general matrix expression. The third expression by the transition matrix is especially applicable for such problem that we want to know the relations between some two blocks in a building.

Now a method of analysis for intermittent heating ( or cooling ) is proposed. This method depends on the least squares method. Namely, the time during which the heat h is to be supplied is divided into n, then let $h'_k$ (k=1,.....,n+1) be the heat quantities at the time k. At first the unknown heat h is approximately expressed in arbitrary function by using those unknown $h'_k$, and then the function is expanded in the Fourier series as following.

$$h=\sum_{\ell}(\sum_{k}a_{\ell\sim k}\,h'_k)\cos\omega_\ell t+\sum_{\ell}(\sum_{k}b_{\ell\sim k}h'_k)\sin\omega_\ell t$$

Then the unknown $h'_k$ (k=1,...,n+1) are obtained by solving the following simultaneous equation.

$$\frac{\partial}{\partial h'_k}\int_{t_1}^{t_2}(\bar{\theta}-\theta_{h'_k})^2\,dt = 0 ,$$

where $\bar{\theta}$ is the given room temperature from time $t_1$ to $t_2$, and $\theta_{h'_k}$ is the room temperature which depends on the unknown heat $h'_k$ .

However, the number of unknowns in the simultaneous equation which is expressed for a multi-room building generally becomes too large, then it is not advisable to calculate the equation directly. Then an iterative method is presented as a practical procedure.

Finally the usefulness of the calculation method for intermittent heating is considered by using the numerical results of an example.

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[1] Figures in brackets indicates the literature references at the end of this paper.

## 2. Notation

| | |
|---|---|
| $j=\sqrt{-1}$ | symbol of imaginary number |
| $\omega$ | angular frequency ($hr^{-1}$) |
| $\lambda$ | thermal conductivity ( $W\ deg^{-1} m^{-1}$ ) |
| $c$ | specific heat ( $W\ deg^{-1} Kg^{-1}$ ) |
| $\gamma$ | specific gravity ($Kg\ m^{-3}$) |
| $a=\lambda/c\gamma$ | thermal diffusivity ($m^2 hr^{-1}$) |
| $l$ | depth of wall ( m ) |
| $\alpha$ | transfer coefficient ( $W\ deg^{-1} m^{-2}$ ) |
| $r$ | transfer resistance ( $deg\ m^2 W^{-1}$ ) |
| $t$ | time ( hr ) |
| $\theta$ | temperature ( deg ) |
| $q$, $h$ | heat quantity ( W ) |
| $C_a$ , | heat capacity ( $W\ hr\ deg^{-1}$ ) |
| ${}_c\Theta = \Theta \exp(j\omega t)$ | complex temperature ( deg ) |
| ${}_cQ = Q \exp(j\omega t)$ | complex heat quantity ( W ) |

## 3. Thermal Properties of Wall

### 3.1. Multi-layer Wall

Relation between complex number heat flows ${}_cQ_0$, ${}_cQ_1$ and complex number temperatures ${}_c\Theta_0$, ${}_c\Theta_1$ at the surfaces of a wall is expressed as following. [13]

$$\begin{bmatrix} {}_c\Theta_1 \\ {}_cQ_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0 \end{bmatrix} \tag{1}$$

where the matrix is named four terminal matrix ( or foundamental matrix), and its elements are as followings.

$$\left.\begin{aligned}
A &= \cosh\sqrt{\frac{\omega}{2a}}l \cos\sqrt{\frac{\omega}{2a}}l + j\sinh\sqrt{\frac{\omega}{2a}}l \sin\sqrt{\frac{\omega}{2a}}l \\
B &= \frac{1}{\sqrt{2\lambda c\gamma}\sqrt{\omega}}\left\{\left(\sinh\sqrt{\frac{\omega}{2a}}l \cos\sqrt{\frac{\omega}{2a}}l + \cosh\sqrt{\frac{\omega}{2a}}l \sin\sqrt{\frac{\omega}{2a}}l\right) + j\left(-\sinh\sqrt{\frac{\omega}{2a}}l \cos\sqrt{\frac{\omega}{2a}}l + \cosh\sqrt{\frac{\omega}{2a}}l \sin\sqrt{\frac{\omega}{2a}}l\right)\right\} \\
C &= \lambda\sqrt{\frac{\omega}{2a}}\left\{\left(\sinh\sqrt{\frac{\omega}{2a}}l \cos\sqrt{\frac{\omega}{2a}}l - \cosh\sqrt{\frac{\omega}{2a}}l \sin\sqrt{\frac{\omega}{2a}}l\right) + j\left(\sinh\sqrt{\frac{\omega}{2a}}l \cos\sqrt{\frac{\omega}{2a}}l + \cosh\sqrt{\frac{\omega}{2a}}l \sin\sqrt{\frac{\omega}{2a}}l\right)\right\} \\
D &= A
\end{aligned}\right\} \tag{2}$$

If there is contact resistance r between the walls or at the surface, they are expressed as following. [13]

$$A=1\ ,\ B=r\ ,\ C=0\ ,\ D=1 \tag{3}$$

For multi-layer wall, we have

$$\begin{bmatrix} {}_c\Theta_n \\ {}_cQ_n \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \cdots\cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0 \end{bmatrix} \tag{4}$$

Equation (4) can be written in the following form.

$$\begin{bmatrix} {}_c\Theta_n \\ {}_cQ_n \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0 \end{bmatrix} . \tag{5}$$

where

$$\begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \cdots\cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} . \tag{6}$$

If there are heat sources ${}_cH_k=H_k\exp(j\omega t)$ $(k=2,\ldots n)$ at the each boundary, eq (5) becomes

$$\begin{bmatrix} {}_c\Theta_n \\ {}_cQ_n \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0 \end{bmatrix} + \sum_{k=2}^{n} \left( \prod_{j=k+1}^{n} \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} \right) \begin{bmatrix} B_k \\ D_k \end{bmatrix} {}_cH_k , \tag{7}$$

in which

$$\left. \begin{aligned} \prod_{k=n}^{n} \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} &= \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} , \\ \prod_{k=n+1}^{n} \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \end{aligned} \right\} \tag{8}$$

However, I will explain for the case, H=0. Each four terminal matrix of wall has the next property. [13]

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD-CD =1. \tag{9}$$

From the property on the product of determinants,

$$\left| \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \cdots\cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \right| = \begin{vmatrix} A_n & B_n \\ C_n & D_n \end{vmatrix} \cdots\cdots \begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix} . \tag{10}$$

Substituting eq(9) into eq (10), we have also the following property for the multi-layer wall.

$$\begin{vmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{vmatrix} = \left| \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \cdots\cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \right| =1 \tag{11}$$

As an another form of the matrix, we can obtain the following equation by using eq (5).

$$\begin{pmatrix} {}_cQ_n \\ {}_cQ_0 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} {}_c\Theta_n \\ {}_c\Theta_0 \end{pmatrix} , \tag{12}$$

where

$$\left.\begin{aligned} Y_{11} &= \frac{\overline{D}}{\overline{B}} , \\ Y_{12} &= \frac{\overline{A}\overline{D} - \overline{C}\overline{B}}{\overline{B}} = \frac{1}{\overline{B}} , \\ Y_{21} &= \frac{1}{\overline{B}} = Y_{12} , \\ Y_{22} &= \frac{\overline{A}}{\overline{B}} , \end{aligned}\right\} \tag{13}$$

and the matrix in eq (12) is named admittance matrix in electric engineering.

It must be noticed that the direction of the heat flow which flows from the surface n ( or the surface 1 ) to the surface 0 is determined as the plus direction for eqs (1),(4),(5) and(7). On the other hand for the eq.(12) direction of heat flow which flows into the inside of the wall from the surfaces is determined as the plus.

### 3.2 Multi-layer Wall + Semi-infinite Wall

Relation between ${}_c\Theta_s$ and ${}_cQ_s$ which are the temperature and the heat flow at the surface of the semi-infinite wall is expressed as

$${}_cQ_s = \frac{1}{Z} {}_c\Theta_s . \tag{14}$$

Where,

$$Z = \frac{1}{\sqrt{2\lambda c \gamma}\sqrt{\omega}}(1-j) . \tag{15}$$

When a multi-layer wall, whose elements of the four terminal matrix are A, B, C and D, is adjacent to a semi-infinite wall, the relation between ${}_c\Theta_n$, ${}_c\Theta_s$ and ${}_cQ_n$ , ${}_cQ_s$ , which are the complex temperatures and heat flows of the surfaces of the multi-layer wall, are expressed as following by using eq (6).

$$\begin{pmatrix} {}_c\Theta_n \\ {}_cQ_n \end{pmatrix} = \begin{pmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{pmatrix} \begin{pmatrix} {}_c\Theta_s \\ {}_cQ_s \end{pmatrix} \tag{16}$$

Substituting eq (14) into eq (16), we obtain

$$\begin{aligned} {}_c\Theta_n &= \overline{A}\, {}_c\Theta_s + \overline{B}\, {}_cQ_s \\ &= \overline{A}\, {}_c\Theta_s + \frac{\overline{B}}{Z} {}_c\Theta_s \\ &= ( \overline{A} + \frac{\overline{B}}{Z} )\, {}_c\Theta_s , \end{aligned} \tag{17}$$

so that

$$ {}_c\Theta_s = \frac{1}{\overline{A} + \frac{\overline{B}}{Z}} \, {}_c\Theta_n $$

$$ = \frac{Z}{Z\,\overline{A} + \overline{B}} \, {}_c\Theta_n \quad . \qquad (18) $$

From eq (16), we have

$$ {}_cQ_n = \overline{C} \, {}_c\Theta_s + \overline{D} \, {}_cQ_s $$

$$ = ( \overline{C} + \frac{D}{Z} ) \, {}_c\Theta_s \; . \qquad (19) $$

Substituting eq (18) into eq (19), we have

$$ {}_cQ_n = \frac{Z}{Z\overline{A} + \overline{B}} ( \overline{C} + \frac{\overline{D}}{Z} ) \, {}_c\Theta_n $$

$$ = \frac{Z\overline{C} + \overline{D}}{Z\overline{A} + \overline{B}} \, {}_c\Theta_n $$

$$ = Y_{11} \, {}_c\Theta_n \, , \qquad (20) $$

where

$$ Y_{11} = \frac{Z\overline{C} + \overline{D}}{Z\overline{A} + \overline{B}} \; . \qquad (21) $$

## 4. Heat Balance Equations for A Multi-room Building

Now we obtained the thermal properties of the wall in the four terminal matrix and the admittance matrix. By using these matrices the system is expressed also in the matrix equations.

### 4.1. Heat Balance Equations for Each Room

Let $k_i$ be the room which is adjacent to the room i. When we express the elements of the admittance matrix of the wall, which exists between the room i and the room $k_i$, by $Y_{k_i \sim 11}$, $Y_{k_i \sim 12}$, $Y_{k_i \sim 21}$ and $Y_{k_i \sim 22}$, the heat flow ${}_cQ_{in \sim k_i}$, which flows into the room i through the wall when the temperature of the room i, ${}_c\Theta_i$, is zero and the temperature of the room $k_i$ is ${}_c\Theta_{k_i}$, is expressed as following.

$$ {}_cQ_{in \sim k_i} = - Y_{k_i \sim 12} \, {}_c\Theta_i \qquad (22) $$

On the other hand, when ${}_c\Theta_{k_i}$ equals zero and ${}_c\Theta_i$ is not zero, the heat flow which flows out from the room i is obtained by

$$ {}_cQ_{out \sim k_i} = Y_{k_i \sim 11} \, {}_c\Theta_i \quad . \qquad (23) $$

And the heat quantity ${}_cQ_{c\sim i}$ which is stored in the heat capacity of the room i, $C_{a\sim i}$ ,depending on the temperature fluctuation is expressed as follows.

$$\begin{aligned} {}_cQ_{c\sim i} &= C_{a\sim i}\frac{d}{dt}\,{}_c\theta_i \\ &= j\omega C_{a\sim i}\,{}_c\theta_i \end{aligned} \tag{24}$$

Expressing the heat quantity by sign ${}_cH_i$ which is supplied into the room i, and another known heat quantity in the room i by the sign ${}_cQ_i$ , then we have the heat balance equation for any $\omega$ , as following.

$${}_cH_i + {}_cQ_i = \left(\sum_{k_i=1}^{n_i} Y_{k_i\sim 11} + j\omega C_{a\sim i}\right){}_c\theta_i + \sum_{k_i=1}^{n_i} Y_{k_i\sim 12}\,{}_c\theta_{k_i} \tag{25}$$

By expanding the temperature and the heat into the Fourier series, which have m terms, eq (25) can be written as following.

$$\sum_{\ell=1}^{m}\left\{{}_cH_{i\sim\ell} + {}_cQ_{i\sim\ell}\right\} = \sum_{\ell}\left\{\left(\sum_{k_i} Y_{(k_i\sim 11)\sim\ell} + j\omega_\ell C_{a\sim i}\right){}_c\theta_{i\sim\ell} + \sum_{k_i} Y_{(k_i\sim 12)\sim\ell}\;{}_c\theta_{k_i\sim\ell}\right\} \tag{26}$$

If there are ventilations $G_{k_i}(t)$ from the room $k_i$ to the room i and $G'_{k_i}(t)$ from i to $k_i$, eq (26) becomes eq (27).

$$\sum_{\ell}\left\{{}_cH_{i\sim\ell} + {}_cQ_{i\sim\ell}\right\} = \sum_{\ell}\left\{\left(\sum_{k_i} Y_{(k_i\sim 11)\sim\ell} + \sum_{k_i} G'_{k_i}(t) + j\omega_\ell C_{a\sim i}\right){}_c\theta_{i\sim\ell} + \sum_{k_i}\left(Y_{(k_i\sim 12)\sim\ell} + G_{k_i}(t)\right){}_c\theta_{k_i\sim\ell}\right\} \tag{27}$$

## 4.2. Heat Balance Equation of Multi-room Building (1)
## ( General Matrix Expression )

For the room 1, eq (27) becomes

$$\left.\begin{aligned}
&\sum_{\ell}\left\{{}_cH_{1\sim\ell} + {}_cQ_{1\sim\ell}\right\} = \sum_{\ell}\left\{\left(\Sigma Y_{(k_1\sim 11)\sim\ell} + \Sigma G'_{k_1}(t) + j\omega_\ell C_{a\sim 1}\right){}_c\theta_{1\sim\ell} + \Sigma\left(Y_{(k_1\sim 12)\sim\ell} + G_{k_1}(t)\right){}_c\theta_{k_1\sim\ell}\right\}, \\
&\text{for the room 2,} \\
&\sum_{\ell}\left\{{}_cH_{2\sim\ell} + {}_cQ_{2\sim\ell}\right\} = \sum_{\ell}\left\{\left(\Sigma Y_{(k_2\sim 11)\sim\ell} + \Sigma G'_{k_2}(t) + j\omega_\ell C_{a\sim 2}\right){}_c\theta_{2\sim\ell} + \Sigma\left(Y_{(k_2\sim 12)\sim\ell} + G_{k_2}(t)\right){}_c\theta_{k_2\sim\ell}\right\}, \\
&\text{for the room n,} \\
&\sum_{\ell}\left\{{}_cH_{n\sim\ell} + {}_cQ_{n\sim\ell}\right\} = \sum_{\ell}\left\{\left(\Sigma Y_{(k_n\sim 11)\sim\ell} + \Sigma G'_{k_n}(t) + j\omega_\ell C_{a\sim n}\right){}_c\theta_{n\sim\ell} + \Sigma\left(Y_{(k_n\sim 12)\sim\ell} + G_{k_n}(t)\right){}_c\theta_{k_n\sim\ell}\right\}.
\end{aligned}\right\} \tag{28}$$

We can write eq (28) as following matrix form.

$$\sum_{\ell}\left\{[A_{\ell}] + [G(t)]\right\}[{}_c\Theta_{\ell}] = \sum_{\ell}\left\{[{}_cH_{\ell}] + [{}_cQ_{\ell}]\right\} \quad , \tag{29}$$

where $A_{\ell}$ is the complex number matrix which is determined by admittances, heat capacity of the room air, and where G(t) is time variant matrix determined by the ventilations, and ${}_cH_{\ell}$ , ${}_cQ_{\ell}$ are the supplied heat and known heat respectively.

### 4.3. Heat Balance Equation of Multi-room Building (2)

### ( Tri-diagonal Matrix Expression )

When we divide the system into three blocks as shown in figure 1, for the block 1 the heat balance equations are expressed as follows.

$$\left.\begin{array}{l} {}_1A_{11}\,{}_c\Theta_{11} + {}_1A_{12}\,{}_c\Theta_{12} + {}_1R_{11}\,{}_c\Theta_{21} = {}_cH_{11} + {}_cQ_{11} \\ {}_1A_{21}\,{}_c\Theta_{11} + {}_1A_{22}\,{}_c\Theta_{12} + {}_1A_{23}\,{}_c\Theta_{13} + {}_1R_{21}\,{}_c\Theta_{21} + {}_1R_{22}\,{}_c\Theta_{22} = {}_cH_{12} + {}_cQ_{12} \\ {}_1A_{32}\,{}_c\Theta_{12} + {}_1A_{33}\,{}_c\Theta_{13} + {}_1R_{32}\,{}_c\Theta_{22} = {}_cH_{13} + {}_cQ_{13} \end{array}\right\} \tag{30}$$

Equation (30) is expressed in the following matrix equation.

$$\begin{bmatrix} {}_1A_{11} & {}_1A_{12} & 0 \\ {}_1A_{21} & {}_1A_{22} & {}_1A_{23} \\ 0 & {}_1A_{32} & {}_1A_{33} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{11} \\ {}_c\Theta_{12} \\ {}_c\Theta_{13} \end{bmatrix} + \begin{bmatrix} {}_1R_{11} & 0 \\ {}_1R_{21} & {}_1R_{22} \\ 0 & {}_1R_{32} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{21} \\ {}_c\Theta_{22} \end{bmatrix} = \begin{bmatrix} {}_cH_{11} \\ {}_cH_{12} \\ {}_cH_{13} \end{bmatrix} + \begin{bmatrix} {}_cQ_{11} \\ {}_cQ_{12} \\ {}_cQ_{13} \end{bmatrix} \tag{31}$$

In the same way, for the block 2 the matrix equation is

$$\begin{bmatrix} {}_2L_{11} & {}_2L_{12} & 0 \\ 0 & {}_2L_{22} & {}_2L_{23} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{11} \\ {}_c\Theta_{12} \\ {}_c\Theta_{13} \end{bmatrix} + \begin{bmatrix} {}_2A_{11} & {}_2A_{12} \\ {}_2A_{21} & {}_2A_{22} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{21} \\ {}_c\Theta_{22} \end{bmatrix} + \begin{bmatrix} {}_2R_{11} & {}_2R_{12} & 0 \\ 0 & {}_2R_{22} & {}_2R_{23} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{31} \\ {}_c\Theta_{32} \\ {}_c\Theta_{33} \end{bmatrix} = \begin{bmatrix} {}_cH_{21} \\ {}_cH_{22} \end{bmatrix} + \begin{bmatrix} {}_cQ_{21} \\ {}_cQ_{22} \end{bmatrix} , \tag{32}$$

and for the block 3,

$$\begin{bmatrix} {}_3L_{11} & 0 \\ {}_3L_{21} & {}_3L_{22} \\ 0 & {}_3L_{32} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{21} \\ {}_c\Theta_{22} \end{bmatrix} + \begin{bmatrix} {}_3A_{11} & {}_3A_{12} & 0 \\ {}_3A_{21} & {}_3A_{22} & {}_3A_{23} \\ 0 & {}_3A_{32} & {}_3A_{33} \end{bmatrix}\begin{bmatrix} {}_c\Theta_{31} \\ {}_c\Theta_{32} \\ {}_c\Theta_{33} \end{bmatrix} = \begin{bmatrix} {}_cH_{31} \\ {}_cH_{32} \\ {}_cH_{33} \end{bmatrix} + \begin{bmatrix} {}_cQ_{31} \\ {}_cQ_{32} \\ {}_cQ_{33} \end{bmatrix} \tag{33}$$

Getting together these three equations, we obtain the following matrix equation.

$$\begin{Bmatrix} \begin{bmatrix} {}_1A_{11} & {}_1A_{12} & 0 \\ {}_1A_{21} & {}_1A_{22} & {}_1A_{23} \\ 0 & {}_1A_{32} & {}_1A_{33} \end{bmatrix} & \begin{bmatrix} {}_1R_{11} & 0 \\ {}_1R_{21} & {}_1R_{22} \\ 0 & {}_1R_{32} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} {}_2L_{11} & {}_2L_{12} & {}_2L_{13} \\ {}_2L_{21} & {}_2L_{22} & {}_2L_{23} \end{bmatrix} & \begin{bmatrix} {}_2A_{11} & {}_2A_{12} \\ {}_2A_{21} & {}_2A_{22} \end{bmatrix} & \begin{bmatrix} {}_2R_{11} & {}_2R_{12} & 0 \\ 0 & {}_2R_{22} & {}_2R_{23} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} {}_3L_{11} & 0 \\ {}_3L_{21} & {}_3L_{22} \\ 0 & {}_3L_{32} \end{bmatrix} & \begin{bmatrix} {}_3A_{11} & {}_3A_{12} & 0 \\ {}_3A_{21} & {}_3A_{22} & {}_2A_{23} \\ 0 & {}_3A_{32} & {}_3A_{33} \end{bmatrix} \end{Bmatrix} \begin{Bmatrix} \begin{bmatrix} {}_c\Theta_{11} \\ {}_c\Theta_{12} \\ {}_c\Theta_{13} \end{bmatrix} \\ \begin{bmatrix} {}_c\Theta_{21} \\ {}_c\Theta_{22} \end{bmatrix} \\ \begin{bmatrix} {}_c\Theta_{31} \\ {}_c\Theta_{32} \\ {}_c\Theta_{33} \end{bmatrix} \end{Bmatrix} = \begin{Bmatrix} \begin{bmatrix} {}_cH_{11} \\ {}_cH_{12} \\ {}_cH_{13} \end{bmatrix} \\ \begin{bmatrix} {}_cH_{21} \\ {}_cH_{22} \end{bmatrix} \\ \begin{bmatrix} {}_cH_{31} \\ {}_cH_{32} \\ {}_cH_{33} \end{bmatrix} \end{Bmatrix} + \begin{Bmatrix} \begin{bmatrix} {}_cQ_{11} \\ {}_cQ_{12} \\ {}_cQ_{13} \end{bmatrix} \\ \begin{bmatrix} {}_cQ_{21} \\ {}_cQ_{22} \end{bmatrix} \\ \begin{bmatrix} {}_cQ_{31} \\ {}_cQ_{32} \\ {}_cQ_{33} \end{bmatrix} \end{Bmatrix} , \qquad (34)$$

where L, A, R are complex coefficients which are determined by the admittances of the walls and by the heat capacities of all rooms, and ${}_c\Theta_{ij}$ is the temperature of the room (i,j), ${}_cH_{ij}$ is the supplied heat, and ${}_cQ_{ij}$ is the known heat in the room (i,j).

By using the minor matrices A, L, R, 0 and the vector expression, eq (34) can be described as following.

$$\begin{bmatrix} A_1 & R_1 & 0 \\ L_2 & A_2 & R_2 \\ 0 & L_3 & A_3 \end{bmatrix} \begin{bmatrix} {}_c\Theta_1 \\ {}_c\Theta_2 \\ {}_c\Theta_3 \end{bmatrix} = \begin{bmatrix} {}_cH_1 \\ {}_cH_2 \\ {}_cH_3 \end{bmatrix} + \begin{bmatrix} {}_cQ_1 \\ {}_cQ_2 \\ {}_cQ_3 \end{bmatrix} \qquad (35)$$

If there are air flows between rooms, eq (35) becomes eq (36).

$$\left\{ \begin{bmatrix} A_1 & R_1 & 0 \\ L_2 & A_2 & R_2 \\ 0 & L_3 & A_3 \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & G_{23} \\ 0 & G_{32} & G_{33} \end{bmatrix} \right\} \begin{bmatrix} {}_c\Theta_1 \\ {}_c\Theta_2 \\ {}_c\Theta_3 \end{bmatrix} = \begin{bmatrix} {}_cH_1 \\ {}_cH_2 \\ {}_cH_3 \end{bmatrix} + \begin{bmatrix} {}_cQ_1 \\ {}_cQ_2 \\ {}_cQ_3 \end{bmatrix} \qquad (36)$$

Equation (35) and eq (36) are tri-diagonal matrix equations. Same expression can be obtained for the general multi-room building.

### 4.4. Heat Balance Equation of Multi-room Building (3)
### ( Transition Matrix Expression )

Though it is possible for the general multi-room building to obtain the transition matrix expression by using eq (36), here I will explain the case of the one-story house which is built in the grid type. Assume that there are j rooms ($j=1,..,n_x$) in the direction x and i rooms ($i=1,...,n_y$) in the direction y in the house.

The following signs are adopted here.

| | |
|---|---|
| ${}_ix_j$ | wall between a room(i,j-1) and a room(i,j) |
| ${}_jy_i$ | wall between a room(i-1,j) and a room(i,j) |
| ${}_dz_{ij}$ | floor of a room(i,j) |
| ${}_uz_{ij}$ | roof of a room(i,j) |
| ${}_c\Theta_{ij}$ | room temperature of a room(i,j) |
| $C_{a\sim ij}$ | thermal capacity of the air in a room(i,j) |
| ${}_cH_{ij}$ | supplied heat in a room(i,j) |
| ${}_cQ_{ij}$ | known heat in a room(i,j) |
| ${}_cH^k_{ij}$ | heat flow into a room(i,j) from another room(k,j) when ${}_c\Theta_{ij}=0$ and ${}_c\Theta_{kj}\neq 0$ |
| ${}_cQ^i_{kj}$ | heat flow from a room(i,j) towards another room(k,j) when ${}_c\Theta_{kj}=0$ and ${}_c\Theta_{ij}\neq 0$ |
| ${}_cH^!_{ij}$ | heat flow into a room(i,j) through wall ${}_jx_{i+1}$ depending on the room temperatures, ${}_c\Theta_{ij}$ and ${}_c\Theta_{i\,j+1}$ |
| ${}_cQ^!_{ij}$ | heat flow from a room(i,j) through wall ${}_jx_i$ depending on ${}_c\Theta_{ij}$ and ${}_c\Theta_{i\,j-1}$ |
| $Y_{ij\sim11}, Y_{ij\sim12}, Y_{ij\sim21}, Y_{ij\sim22}$ | elements of the admittance matrix of wall |

### (1) Heat Balance Equation of the Wall x

For each wall ${}_i x_j$, the following equation exists.

$$\begin{bmatrix} {}_c\Theta_{ij} \\ {}_cQ'_{ij} \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} {}_c\Theta_{ij-1} \\ {}_cH'_{ij-1} \end{bmatrix} \qquad (i=1,\ldots,n_y) \tag{37}$$

Expressing as follows,

$$\left.\begin{aligned} &{}_c\Theta_j = \begin{bmatrix} {}_c\Theta_{1j} \\ \vdots \\ {}_c\Theta_{n_yj} \end{bmatrix}, \quad {}_cQ'_j = \begin{bmatrix} {}_cQ'_{1j} \\ \vdots \\ {}_cQ'_{n_yj} \end{bmatrix}, \quad {}_cH'_j = \begin{bmatrix} {}_cH'_{1j} \\ \vdots \\ {}_cH'_{n_yj} \end{bmatrix}, \\ &A_j = \begin{bmatrix} A_{1j} & & 0 \\ & \ddots & \\ 0 & & A_{n_yj} \end{bmatrix}, \quad B_j = \begin{bmatrix} B_{1j} & & 0 \\ & \ddots & \\ 0 & & B_{n_yj} \end{bmatrix}, \\ &C_j = \begin{bmatrix} C_{1j} & & 0 \\ & \ddots & \\ 0 & & C_{n_yj} \end{bmatrix}, \quad D_j = \begin{bmatrix} D_{1j} & & 0 \\ & \ddots & \\ 0 & & D_{n_yj} \end{bmatrix}. \end{aligned}\right\} \tag{38}$$

By using eq (38), eq (37) becoms eq (39).

$$\begin{bmatrix} {}_c\Theta_j \\ {}_cQ'_j \end{bmatrix} = \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} \begin{bmatrix} {}_c\Theta_{j-1} \\ {}_cH'_{j-1} \end{bmatrix} \tag{39}$$

### (2) Heat Balance Equation in the Room(i,j)

The total heat flow ${}_c\overline{H}_{ij}$ which flows into the room(i,j) is given by the following equation.

$${}_c\overline{H}_{ij} = \sum_k {}_cH^k_{ij} + {}_cQ_{ij} + {}_cH_{ij} + {}_cH'_{ij} \tag{40}$$

On the room(i,j) the following heat balance equation is obtained.

$$\sum_k {}_cH^k_{ij} + {}_cQ_{ij} + {}_cH_{ij} + {}_cH'_{ij} = \sum_k {}_cQ^i_{kj} + {}_cQ'_{ij} + {}_cQ_{c\sim ij} \tag{41}$$

Rearrangeing the eq (41), we obtain the following.

$$H'_{ij} = \sum {}_cQ^i_{kj} + {}_cQ'_{ij} + {}_cQ_{c\sim ij} - {}_cQ_{ij} - {}_cH_{ij} - \sum_k {}_cH^k_{ij}\ , \tag{42}$$

in which ${}_cQ^i_{kj}$ and ${}_cH^k_{ij}$ are expressed as follows by using admittances of the wall y,

$$\left.\begin{aligned} {}_cH_{ij}^{k} &= Y_{\ell j\sim 12}\,{}_c\Theta_{kj} \quad (\ 1=i,i+1;\ k=i-1,i+1\ ) , \\ {}_cQ_{kj}^{i} &= (\ Y_{ij\sim 11} + Y_{i+1\,j\sim 22}\ )\,{}_c\Theta_{ij}\ , \end{aligned}\right\} \tag{43}$$

and

$${}_cQ_{c\sim ij} = j\omega\, C_{a\sim ij}\,{}_c\Theta_{ij}\ . \tag{44}$$

Substituting eqs (43) and (44) into eq (42), we obtain the following equations, for i=1,

$${}_cH_{1j}' = {}_cQ_{1j}' + (\ Y_{1j\sim 11} + Y_{2j\sim 22} + j\omega C_{a\sim 1j}\ )\,{}_c\Theta_{1j} - Y_{2j\sim 12}\,{}_c\Theta_{2j} - {}_cQ_{1j} - {}_cH_{1j}\ ,$$

for i=2,

$${}_cH_{2j}' = {}_cQ_{2j}' + (Y_{2j\sim 11} + Y_{3j\sim 22} + j\omega C_{a\sim 2j})\,{}_c\Theta_{2j} - Y_{3j\sim 12}\,{}_c\Theta_{3j} - Y_{2j\sim 21}\,{}_c\Theta_{1j} - {}_c\Theta_{2j} - {}_cH_{2j}, \tag{45}$$

.......

for i=$n_y$,

$${}_cH_{n_y j}' = {}_cQ_{n_y j}' + (Y_{n_y j\sim 11} + Y_{n_y+1\,j\sim 22} + j\omega C_{a\sim n_y j})\,{}_c\Theta_{n_y j} - Y_{n_y j\sim 21}\,{}_c\Theta_{n_y-1\,j} - {}_cQ_{n_y j} - {}_cH_{n_y j}\ .$$

Equation (45) can be written as the following matrix equation.

$$\begin{bmatrix} {}_cH_{1j}' \\ {}_cH_{2j}' \\ \vdots \\ {}_cH_{n_y j}' \end{bmatrix} = \begin{bmatrix} {}_cQ_{1j}' \\ {}_cQ_{2j}' \\ \vdots \\ {}_cQ_{n_y j}' \end{bmatrix} + \begin{bmatrix} T_{j\sim 11} & T_{j\sim 12} & 0 & 0 & \cdot\ \cdot \\ T_{j\sim 21} & T_{j\sim 22} & T_{j\sim 23} & 0 & \cdot\ \cdot \\ \cdot\ \cdot & \cdot & \cdot & & \\ \cdot\ \cdot & \cdot & \cdot & & T_{j\sim n_y n_y} \end{bmatrix} \begin{bmatrix} {}_c\Theta_{1j} \\ {}_c\Theta_{2j} \\ \vdots \\ {}_c\Theta_{n_y j} \end{bmatrix} - \begin{bmatrix} {}_cH_{1j} \\ {}_cH_{2j} \\ \vdots \\ {}_cH_{n_y j} \end{bmatrix} - \begin{bmatrix} {}_cQ_{1j} \\ {}_cQ_{2j} \\ \vdots \\ {}_cQ_{n_y j} \end{bmatrix}\ , \tag{46}$$

where

$$\left.\begin{aligned} T_{j\sim kk} &= Y_{kj\sim 11} + Y_{k+1\,j\sim 22} + j\omega C_{a\sim kj} && (k=2,\ldots,n_y-1), \\ T_{j\sim k+1\,k} &= -Y_{kj\sim 21} && (k=2,\ldots,n_y)\ , \\ T_{j\sim k+1\,k} &= -Y_{k+1\,j\sim 12} && (k=1,\ldots,n_y-1). \end{aligned}\right\} \tag{47}$$

Equation (46) is simplified as following.

$${}_cH_j' = {}_cQ_j' + T_j\,{}_c\Theta_j - {}_cQ_j - {}_cH_j \tag{48}$$

Where

$$T_j = \begin{bmatrix} T_{j\sim 11} & T_{j\sim 12} & T_{j\sim 13} & \cdots \\ T_{j\sim 21} & T_{j\sim 22} & T_{j\sim 23} & \cdots \\ \cdot & \cdot & \cdot & \\ T_{j\sim n_y 1} & T_{j\sim n_y 2} & T_{j\sim n_y 3} & \cdots \end{bmatrix} . \tag{49}$$

### (3) Transition Matrix Expression

Substituting eq (48) into eq (39), we obtain the followings.

$$\left.\begin{aligned}
\begin{bmatrix} {}_c\Theta_j \\ {}_cQ_j^! \end{bmatrix} &= \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} \begin{bmatrix} {}_c\Theta_{j-1} \\ {}_cH_{j-1}^! \end{bmatrix} \\
&= \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} \begin{bmatrix} {}_c\Theta_{j-1} \\ {}_cQ_{j-1}^! + T_{j-1}\,{}_c\Theta_{j-1} - {}_cH_{j-1} - {}_cQ_{j-1} \end{bmatrix} \\
&= \begin{bmatrix} (A_j + B_j T_{j-1})\,{}_c\Theta_{j-1} + B_j\,{}_cQ_{j-1}^! \\ (C_j + D_j T_{j-1})\,{}_c\Theta_{j-1} + D_j\,{}_cQ_{j-1}^! \end{bmatrix} - \begin{bmatrix} B_j\,({}_cH_{j-1} + {}_cQ_{j-1}) \\ D_j\,({}_cH_{j-1} + {}_cQ_{j-1}) \end{bmatrix} \\
&= \begin{bmatrix} A_j + B_j T_{j-1} & B_j \\ C_j + D_j T_{j-1} & D_j \end{bmatrix} \begin{bmatrix} \Theta_{j-1} \\ Q_{j-1}^! \end{bmatrix} + \begin{bmatrix} B_j \\ D_j \end{bmatrix} (-{}_cH_{j-1} - {}_cQ_{j-1})
\end{aligned}\right\} \qquad (50)$$

$$\left.\begin{aligned}
\overline{A}_j &= A_j + B_j T_{j-1} \\
\overline{C}_j &= C_j + D_j T_{j-1} \\
\overline{{}_cH_{j-1}} &= -{}_cH_{j-1} - {}_cQ_{j-1}
\end{aligned}\right\} \qquad (51)$$

By using the eq (51), eq (50) can be written as,

$$\begin{bmatrix} {}_c\Theta_j \\ {}_cQ_j^! \end{bmatrix} = \begin{bmatrix} \overline{A}_j & B_j \\ \overline{C}_j & D_j \end{bmatrix} \begin{bmatrix} {}_c\Theta_{j-1} \\ {}_cQ_{j-1}^! \end{bmatrix} + \begin{bmatrix} B_j \\ D_j \end{bmatrix} {}_c\overline{H}_{j-1} \quad . \qquad (52)$$

Connecting the eq (52), we obtain the followings.

For j=1 $$\begin{bmatrix} {}_c\Theta_1 \\ {}_cQ_1^! \end{bmatrix} = \begin{bmatrix} \overline{A}_1 & B_1 \\ \overline{C}_1 & D_1 \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0^! \end{bmatrix} , \qquad (53)$$

for j=2 $$\begin{bmatrix} {}_c\Theta_2 \\ {}_cQ_2^! \end{bmatrix} = \begin{bmatrix} \overline{A}_2 & B_2 \\ \overline{C}_2 & D_2 \end{bmatrix} \begin{bmatrix} \overline{A}_1 & B_1 \\ \overline{C}_1 & D_1 \end{bmatrix} \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0^! \end{bmatrix} + \begin{bmatrix} B_2 \\ D_2 \end{bmatrix} {}_c\overline{H}_1 , \qquad (54)$$

finally for j=n , $$\begin{bmatrix} {}_c\Theta_{n_x} \\ {}_cQ_{n_x}^! \end{bmatrix} = \left( \prod_{j=1}^{n_x} \begin{bmatrix} \overline{A}_j & B_j \\ \overline{C}_j & D_j \end{bmatrix} \right) \begin{bmatrix} {}_c\Theta_0 \\ {}_cQ_0^! \end{bmatrix} + \sum_{k=2}^{n_x-1} \left( \prod_{j=k+1}^{n_x-1} \begin{bmatrix} \overline{A}_j & B_j \\ \overline{C}_j & D_j \end{bmatrix} \right) \begin{bmatrix} B_k \\ D_k \end{bmatrix} {}_c\overline{H}_k \quad . \qquad (55)$$

In which, let

$$\prod_{j=n}^{n} \begin{bmatrix} \overline{A}_j & B_j \\ \overline{C}_j & D_j \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} , \qquad (56)$$

and

$$\prod_{m+1}^{m}\begin{bmatrix}\overline{A}_j & B_j \\ \overline{C}_j & D_j\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} . \tag{57}$$

Equation (55) is the same expression as eq (7) for the multi-layer wall which has the heat sources at all boundaries.

Transition matrix can be written as follows.

$$\begin{aligned}\begin{bmatrix}\overline{A}_j & B_j \\ \overline{C}_j & D_j\end{bmatrix} &= \begin{bmatrix}A_j + B_j T_{j-1} & B_j \\ C_j + D_j T_{j-1} & D_j\end{bmatrix} \\ &= \begin{bmatrix}A_j & B_j \\ C_j & D_j\end{bmatrix}\begin{bmatrix}1 & 0 \\ T_{j-1} & 1\end{bmatrix}\end{aligned} \tag{58}$$

If there are no ventilations between the rooms which are laid in the x direction ( direction of heat flow Q' ), the following result is obtained.

$$\begin{aligned}\begin{vmatrix}A_j & B_j \\ C_j & D_j\end{vmatrix} &= \begin{vmatrix}\begin{bmatrix}A_1 & & 0 \\ & A_2 & \\ 0 & & \ddots\end{bmatrix} & \begin{bmatrix}B_1 & & 0 \\ & B_2 & \\ 0 & & \ddots\end{bmatrix} \\ \begin{bmatrix}C_1 & & 0 \\ & C_2 & \\ 0 & & \ddots\end{bmatrix} & \begin{bmatrix}D_1 & & 0 \\ & D_2 & \\ 0 & & \ddots\end{bmatrix}\end{vmatrix} \\ &= \left| [A][D] - [C][B] \right| \\ &= \left| [A\ D] - [C\ B] \right| \\ &= \left| [A\ D - C\ B] \right| \\ &= \begin{vmatrix}A_1D_1 - C_1B_1 & & 0 \\ & A_2D_2 - C_2B_2 & \\ & & \cdots \\ 0 & & \cdots\end{vmatrix} \\ &= 1\end{aligned} \tag{59}$$

In the same way we obtain

$$\begin{vmatrix}1 & 0 \\ T_{j-1} & 1\end{vmatrix} = 1 , \tag{60}$$

so that

$$\begin{vmatrix}\overline{A}_j & B_j \\ \overline{C}_j & D_j\end{vmatrix} = \begin{vmatrix}A_j & B_j \\ C_j & D_j\end{vmatrix}\begin{vmatrix}1 & 0 \\ T_{j-1} & 1\end{vmatrix} = 1 . \tag{61}$$

The property given by eq (61) is the same of the four terminal matrix of the wall. By this property it becomes very easy to obtain the inversion of the transition matrix.

Even if there is no x wall, we can treat the system as if there is x wall by considering an imaginary wall, whose elements of the four terminal matrix is as follows.

$$A = D = 1, \quad B = C = 0$$

## 5. A Calculation Method for the Intermittent Heating

By using eq (29), which is the general expression for the multi-room building, I will explain the calculation method. Let G(t) be zero. The calculation method for $G(t) \neq 0$ is presented in the next chapter.

Since $G(t)=0$, eq (29) can be written as follows.

$$\sum_{\ell} \left[ A_{\ell} \right] \left[ {}_{c}\Theta_{\ell} \right] = \sum_{\ell} \left\{ \left[ {}_{c}H_{\ell} \right] + \left[ {}_{c}Q_{\ell} \right] \right\} \tag{62}$$

Let t the time during which the heat flow h is supplied in the room, dividing t into m and I express the heat quantity at the time k by $h'_k$ ( $k=1,\ldots,m+1$ ).

Now the unknown heat h is approximately expressed in arbitrary function by using these unknown $h'_k$ ,and then the function is expanded in the Fourier series as following.

$$h = \sum_{\ell} \left( \sum_{k=1}^{m+1} a_{\ell \sim k} \; h'_k \right) \cos \omega_{\ell} t + \sum_{\ell} \left( \sum_{k=1}^{m+1} b_{\ell \sim k} \; h'_k \right) \sin \omega_{\ell} t \tag{63}$$

From eq (63), we obtain

$${}_{c}H = \sum_{\ell} \sum_{k} (a_{\ell \sim k} - j b_{\ell \sim k}) \; h'_k \; \exp(j \omega_{\ell} t) . \tag{64}$$

For each $\omega_{\ell}$ we can write in the matrix form as follows.

$$\left[ {}_{c}H_{\ell} \right] = \sum_{k} \left[ a_{\ell \sim k} - j b_{\ell \sim k} \right] h'_k \exp(j \omega_{\ell} t) \tag{65}$$

Considering ${}_{c}Q_{\ell} = Q_{\ell} \exp(j\omega_{\ell}t)$ and substituting eq (65) into eq (62) we have eq (66), then by multiplying the eq (66) by the inverse matrix $[A_{\ell}]^{-1}$, we have eq (67).

$$\sum_{\ell} \left[ A_{\ell} \right] \left[ {}_{c}\Theta_{\ell} \right] = \sum_{\ell} \sum_{k} \left[ a_{\ell \sim k} - j b_{\ell \sim k} \right] h'_k \exp(j \omega_{\ell} t) + \sum_{\ell} \left[ Q_{\ell} \right] \exp(j \omega_{\ell} t) \tag{66}$$

$$\begin{aligned} \left[ {}_{c}\Theta_{\ell} \right] &= \sum_{\ell} \left( \sum_{k} \left[ A_{\ell} \right]^{-1} \left[ a_{\ell \sim k} - j b_{\ell \sim k} \right] h'_k \exp(j \omega_{\ell} t) + \sum_{\ell} \left[ A_{\ell} \right] \left[ Q_{\ell} \right] \exp(j \omega_{\ell} t) \\ &= \sum_{\ell} \left( \sum_{k} \left[ H'_{\ell} \right]_k \; h_k \right) \exp(j \omega_{\ell} t) + \sum_{\ell} \left[ Q'_{\ell} \right] \exp(j \omega_{\ell} t) \quad , \end{aligned} \tag{67}$$

where

$$\left.\begin{aligned}\left[H'_{\ell}\right]_{k} &= \left[A_{\ell}\right]^{-1}\left[a_{\ell\sim k} - jb_{\ell\sim k}\right], \\ \left[Q'_{\ell}\right]_{k} &= \left[A_{\ell}\right]^{-1}\left[Q_{\ell}\right].\end{aligned}\right\} \tag{68}$$

Let

$$\left[H'_{\ell}\right]_{k} = \left[{}_{h}R_{\ell\sim k} + j\,{}_{h}I_{\ell\sim k}\right], \quad \left[Q'_{\ell}\right] = \left[{}_{q}R_{\ell} + j\,{}_{q}I_{\ell}\right],$$

then, from eq (67) the real function $\theta_{\ell}$ can be written as follows.

$$\theta_{\ell} = \sum_{k}({}_{h}R_{\ell\sim k}\cos\omega_{\ell}t - {}_{h}I_{\ell\sim k}\sin\omega_{\ell}t)h'_{k} + {}_{q}R_{\ell}\cos\omega_{\ell}t - {}_{q}I_{\ell}\sin\omega_{\ell}t \tag{69}$$

Let $\overline{\theta}$ be the given temperature from time $t_1$ to $t_2$, then the unknown heat quantities $h'_{k}$ are obtained by solving the simultaneous equation which is gained by the following equation.

$$\begin{aligned}\frac{\partial}{\partial h_{k}}\int_{t_1}^{t_2}(\overline{\theta} - \sum_{\ell}\theta_{\ell})^{2}\,dt &= \frac{\partial}{\partial h_{k}}\int_{t_1}^{t_2}\Big\{\overline{\theta} - \sum_{\ell}\Big(\sum_{k'}({}_{h}R_{\ell\sim k'}\cos\omega_{\ell}t - {}_{h}I_{\ell\sim k'}\sin\omega_{\ell}t)h'_{k'} + {}_{q}R_{\ell}\cos\omega_{\ell}t - {}_{q}I_{\ell}\sin\omega_{\ell}t\Big)\Big\}^{2}dt \\ &= \int_{t_1}^{t_2}\Big[\frac{\partial}{\partial h_{k}}\Big\{\overline{\theta} - \sum_{\ell}\Big(\sum_{k'}({}_{h}R_{\ell\sim k'}\cos\omega_{\ell}t - {}_{h}I_{\ell\sim k'}\sin\omega_{\ell}t)h'_{k'} + {}_{q}R_{\ell}\cos\omega_{\ell}t - {}_{q}I_{\ell}\sin\omega_{\ell}t\Big)\Big\}^{2}\Big]dt \\ &= 2\int_{t_1}^{t_2}\Big\{\overline{\theta} - \sum_{\ell}\sum_{k'}({}_{h}R_{\ell\sim k'}\cos\omega_{\ell}t - {}_{h}I_{\ell\sim k'}\sin\omega_{\ell}t)h'_{k'} + {}_{q}R_{\ell}\cos\omega_{\ell}t - {}_{q}I_{\ell}\sin\omega_{\ell}t\Big\} \\ &\qquad \times\Big\{\sum_{\ell}({}_{h}R_{\ell\sim k}\cos\omega_{\ell}t - {}_{h}I_{\ell\sim k}\sin\omega_{\ell}t)\Big\}dt \\ &= 0 \qquad (k=1,\cdots,m+1)\end{aligned} \tag{70}$$

## 6. An Expression for an Iterative Method

The number of unknowns of eq(70) which is expressed for a multi-room building generally becomes too large, then it is not advisable to solve the equation directly. Moreover for the case $G(t)\neq 0$, it is very difficult to obtain the analytical solution for eq (70). Then, here, as a practical procedure an iterative method is explained.

We write again eq(29).

$$\sum_{\ell}\left\{\left[A_{\ell}\right] + \left[G(t)\right]\right\}\left[{}_{c}\Theta_{\ell}\right] = \sum_{\ell}\left\{\left[{}_{c}H_{\ell}\right] + \left[{}_{c}Q_{\ell}\right]\right\} \tag{71}$$

Divide the matrix $\left[A_{\ell}\right]$ into the two matrices as follows.

$$\begin{bmatrix} A_{\ell\sim11} & A_{\ell\sim12} & A_{\ell\sim13} & \cdots \\ A_{\ell\sim21} & A_{\ell\sim22} & A_{\ell\sim23} & \cdots \\ A_{\ell\sim31} & A_{\ell\sim32} & A_{\ell\sim33} & \cdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \end{bmatrix} = \begin{bmatrix} A_{\ell\sim11} & & 0 & \\ & A_{\ell\sim22} & & \\ & & A_{\ell\sim33} & \\ 0 & & & \ddots \end{bmatrix} + \begin{bmatrix} 0 & A_{\ell\sim12} & A_{\ell\sim13} & \cdots \\ A_{\ell\sim21} & 0 & A_{\ell\sim23} & \cdots \\ A_{\ell\sim31} & A_{\ell\sim32} & 0 & \cdots \\ \vdots & & & \ddots \; 0 \end{bmatrix} \qquad (72)$$

The first term at the right side of eq (72) is a diagonal matrix and the all diagonal elements of the second matrix are zeros.

Substituting eq (72) into eq (71), we obtain

$$\sum_{\ell}\left\{\begin{bmatrix} A_{\ell\sim11} & & & 0 \\ & A_{\ell\sim22} & & \\ & & \ddots & \\ 0 & & & A_{\ell\sim nn} \end{bmatrix} + \begin{bmatrix} 0 & A_{\ell\sim12} & \cdots & \\ A_{\ell\sim21} & 0 & & \\ \vdots & & \ddots & \\ A_{\ell\sim n1} & \cdots & & 0 \end{bmatrix}\right\} \begin{bmatrix} {}_c\Theta_{\ell\sim1} \\ {}_c\Theta_{\ell\sim2} \\ \vdots \\ {}_c\Theta_{\ell\sim n} \end{bmatrix} + \sum_{\ell} \begin{bmatrix} G_{11}(t) & \cdots & & \cdot \\ \vdots & \ddots & & \vdots \\ \vdots & \cdots & & G_{nn}(t) \end{bmatrix} \begin{bmatrix} {}_c\Theta_{\ell\sim1} \\ \vdots \\ {}_c\Theta_{\ell\sim n} \end{bmatrix}$$

$$= \sum_{\ell}\left\{\begin{bmatrix} {}_cH_{\ell\sim1} \\ \vdots \\ {}_cH_{\ell\sim n} \end{bmatrix} + \begin{bmatrix} {}_cQ_{\ell\sim1} \\ \vdots \\ {}_cQ_{\ell\sim n} \end{bmatrix}\right\} . \qquad (73)$$

The iterative procedures are carried as follows by using eq (73).

Procedure 1. Let the mean ventilation rates be the approximate rates, and then assume that the room temperature to be determined is equal to the adjacent rooms.

By these approximations eq(73) becomes the following.

$$\sum_{\ell}\begin{bmatrix} \overline{A}_{\ell\sim1}+\overline{G}_1 & & 0 \\ & \ddots & \\ 0 & & \overline{A}_{\ell\sim n}+\overline{G}_n \end{bmatrix} \begin{bmatrix} {}_c\Theta_{\ell\sim1} \\ \vdots \\ {}_c\Theta_{\ell\sim n} \end{bmatrix} = \sum_{\ell}\begin{bmatrix} {}_cH_{\ell\sim1} \\ \vdots \\ {}_cH_{\ell\sim n} \end{bmatrix} + \sum_{\ell}\begin{bmatrix} {}_cQ_{\ell\sim1} \\ \vdots \\ {}_cQ_{\ell\sim n} \end{bmatrix} , \qquad (74)$$

in which

$$\overline{A}_{\ell\sim i} = \sum_{j=1}^{n} A_{\ell\sim ij} \quad (i=1,\ldots,n) \quad , \qquad (75)$$

$$\overline{G}_i = \sum_{j=1}^{n}\left\{\frac{1}{T}\int_T G_{ij}(t)\, dt\right\} \quad (i=1,\ldots,n) \quad , \qquad (76)$$

and T is the time of one period.

Thus the coefficient matrix of eq(71) results in the diagonal matrix, then we can obtain the each room temperature and heat to be surpplied in each room independently by eq(74).

After the procedure 1, the following procedure is repeated untill we obtain the satisfactory results.

Procedure 2. Let the temperatures $\Theta'$ obtained already be the temperatures of the adjacent rooms and assume that the matrix G(t) is multiplied by the temperatures $\Theta'$.

By this procedure eq (73) becomes eq (77).

$$\sum_{\ell}\begin{bmatrix} A_{\ell\sim11} & & 0 \\ & \ddots & \\ 0 & & A_{\ell\sim nn} \end{bmatrix} \begin{bmatrix} {}_c\Theta_{\ell\sim1} \\ \vdots \\ {}_c\Theta_{\ell\sim n} \end{bmatrix} = \sum_{\ell}\left\{\begin{bmatrix} {}_cH_{\ell\sim1} \\ \vdots \\ {}_cH_{\ell\sim n} \end{bmatrix} + \begin{bmatrix} {}_cQ_{\ell\sim1} \\ \vdots \\ {}_cQ_{\ell\sim n} \end{bmatrix} + \begin{bmatrix} 0 & -A_{\ell\sim12} & \cdots & \vdots \\ -A_{\ell\sim21} & 0 & & \\ \vdots & \vdots & \ddots & 0 \end{bmatrix} \begin{bmatrix} {}_c\Theta'_{\ell\sim1} \\ \vdots \\ {}_c\Theta'_{\ell\sim n} \end{bmatrix} + \Big[ -G(t) \Big] \begin{bmatrix} {}_c\Theta'_{\ell\sim1} \\ \vdots \\ {}_c\Theta'_{\ell\sim n} \end{bmatrix}\right\} \qquad (77)$$

In eq (77), the vector ${}_c\Theta'$ is known, then we obtain the following.

$$\sum_{\ell}\begin{bmatrix} A_{\ell\sim11} & & 0\\ & \ddots & \\ 0 & & A_{\ell\sim nn}\end{bmatrix}\begin{bmatrix}{}_c\Theta_{\ell\sim1}\\ \vdots\\ {}_c\Theta_{\ell\sim n}\end{bmatrix} = \sum_{\ell}\left\{\begin{bmatrix}{}_cH_{\ell\sim1}\\ \vdots\\ {}_cH_{\ell\sim n}\end{bmatrix} + \begin{bmatrix}{}_cQ_{\ell\sim1}\\ \vdots\\ {}_cQ_{\ell\sim n}\end{bmatrix} + \begin{bmatrix}{}_cQ'_{\ell\sim1}\\ \vdots\\ QQ\end{bmatrix} + \begin{bmatrix}{}_cQ''_{\ell\sim1}\\ \vdots\\ {}_cQ''_{\ell\sim n}\end{bmatrix}\right\}$$

$$= \sum_{\ell}\left\{\begin{bmatrix}{}_cH_{\ell\sim1}\\ \vdots\\ {}_cH_{\ell\sim n}\end{bmatrix} + \begin{bmatrix}{}_c\overline{Q}_{\ell\sim1}\\ \vdots\\ {}_cQ_{\ell\sim n}\end{bmatrix}\right\}, \qquad (78)$$

where

$$\begin{bmatrix}{}_cQ'_{\ell\sim1}\\ \vdots\\ {}_cQ'_{\ell\sim n}\end{bmatrix} = \begin{bmatrix} 0 & -A_{\ell\sim12} & \cdots\\ -A_{\ell\sim21} & \ddots & \vdots\\ \vdots & \cdots & 0\end{bmatrix}\begin{bmatrix}{}_c\Theta'_{\ell\sim1}\\ \vdots\\ {}_c\Theta'_{\ell\sim n}\end{bmatrix},$$

$$\begin{bmatrix}{}_cQ''_{\ell\sim1}\\ \vdots\\ {}_cQ''_{\ell\sim n}\end{bmatrix} = \begin{bmatrix} -G(t)\end{bmatrix}\begin{bmatrix}{}_c\Theta'_{\ell\sim1}\\ \vdots\\ {}_c\Theta'_{\ell\sim n}\end{bmatrix}, \qquad (79)$$

and

$${}_c\overline{Q}_{\ell\sim i} = {}_cQ_{\ell\sim i} + {}_cQ'_{\ell\sim i} + {}_cQ''_{\ell\sim i} \qquad (i=1,\ldots,n). \qquad (80)$$

Equation (78) is also the diagonal matrix, so that we can determine the unknown heats to be supplied and the room temperaturs of the each room independently. We repeat the procedure 2 untill $\Theta'$ becomes nearly equal to $\Theta$.

## 7. An Example

An one-story concrete house was taken as an example ( figure 2), and the following quantities were calculated.

1. The room temperatures $\theta_n$,which are formed when the outdoor temperature $\theta_{s\text{-}i}$ (i=1, ...,6) are those as shown in figure 3 and there are no supplied heats in the house.
2. The heat quantities H,which are to be taken off from the rooms to form the room temperatures excepting corridor in 25°C from 9 am.to 5 pm..
3. The room temperatures $\theta_c$,which are formed when the heat quantities H are taken off from the rooms excepting corridor.

For numerical calculation temperatures and heat quantities were expanded in the fifteen terms Fourier series ($\omega_1, \omega_2, \omega_3, \ldots, \omega_{15}$).

Unknown heat quantities H were divided into five parts and approximated as the broken lines respectively.

The sol-air temperatures $\theta_{s\text{-}i}$(i=1,...,6) in the figure 3 act against the east, the west, the south, the north walls, the roof and the floors respectively.

The results of calculation are shown in figure 4.

It needed four times calculations for each room to obtain the satisfactory result.

## 8. Conclusion

By using the frequency transfer function, three kinds of matrix equations whose elements are complex were expressed for the multi-room building.

The first one is expressed in the general matrix form of the simultaneous equation, the second is the tri-diagonal matrix equation whose elements are consist of the minor matrices, and the third is the transition matrix equation.

The tri-diagonal matrix expression reqires the less amounts of the memory of the computer than the general matrix expression. The transition matrix expression for the building is similar to the four terminal matrix expression for the wall. Therefore we can say that the four terminal matrix is only a special form in the transition matrices. This expression is especially usefull when we want to know the relations between some two blocks in a building.

It was introduced that the determinant of the transition matrix is unity as same as the determinant of four terminal matrix. By this property that the determinant is unity it becomes very easy to obtain the inversion of the transition matrix.

A calculation method for the intermittent heating, depending on the Fourier analysis, was presented.

As the practical procedure, an iterative method was proposed. For an example at first the heat quantities, which are to be taken off from the each room to form the predetermined room temperatures, were determined, and then the room temperatures were calculated for the obtained heat quantities.

As shown in figure 4 arround 9 and 17 o'clock there are comparatively large differences between the predetermined and the determined temperatures, but at the most of the time they are in good agreement.

Of course more detailed research is necessary for the method proposed in this paper, but from these results it seems to be quite all right to consider that this method is useful for the some problems such as the thermal planning or dicision of zoning.

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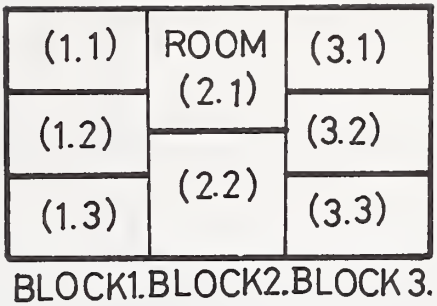

Fig 1 Block Plan of a Building

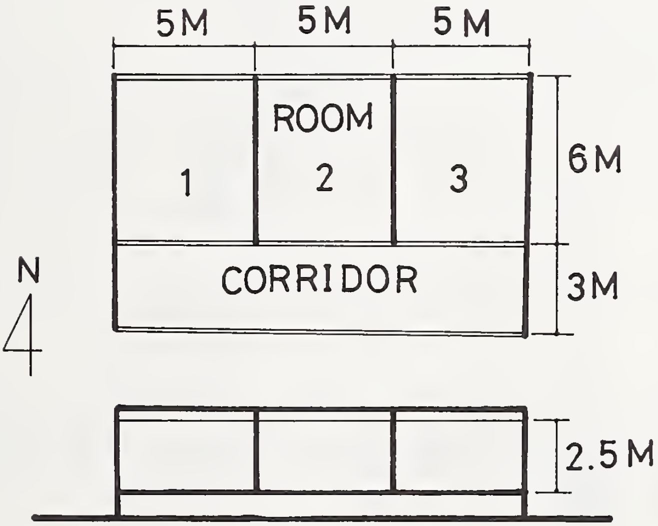

Fig 2 Plan and Section of a Model Building

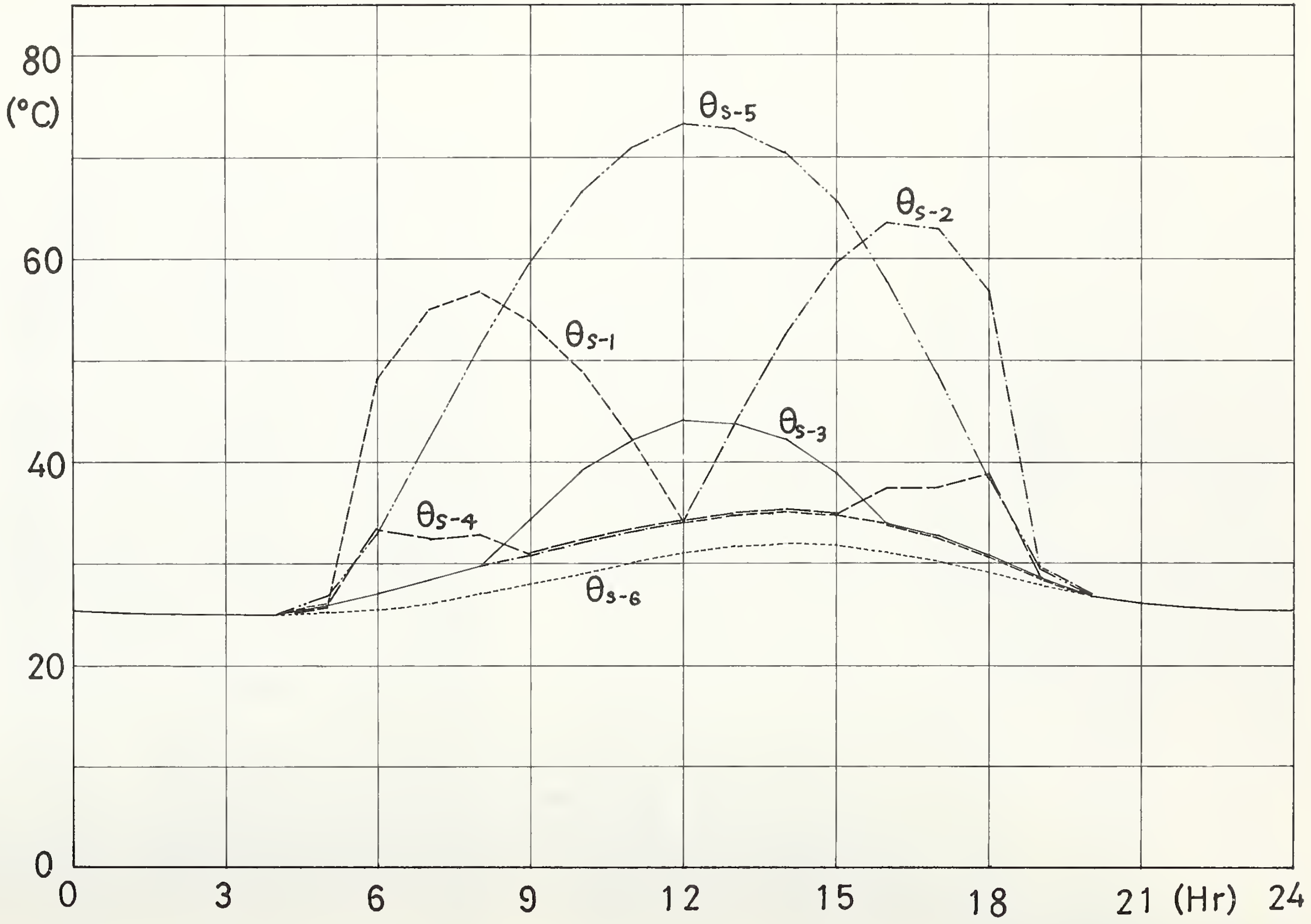

Fig 3  Sol-air Temperatures

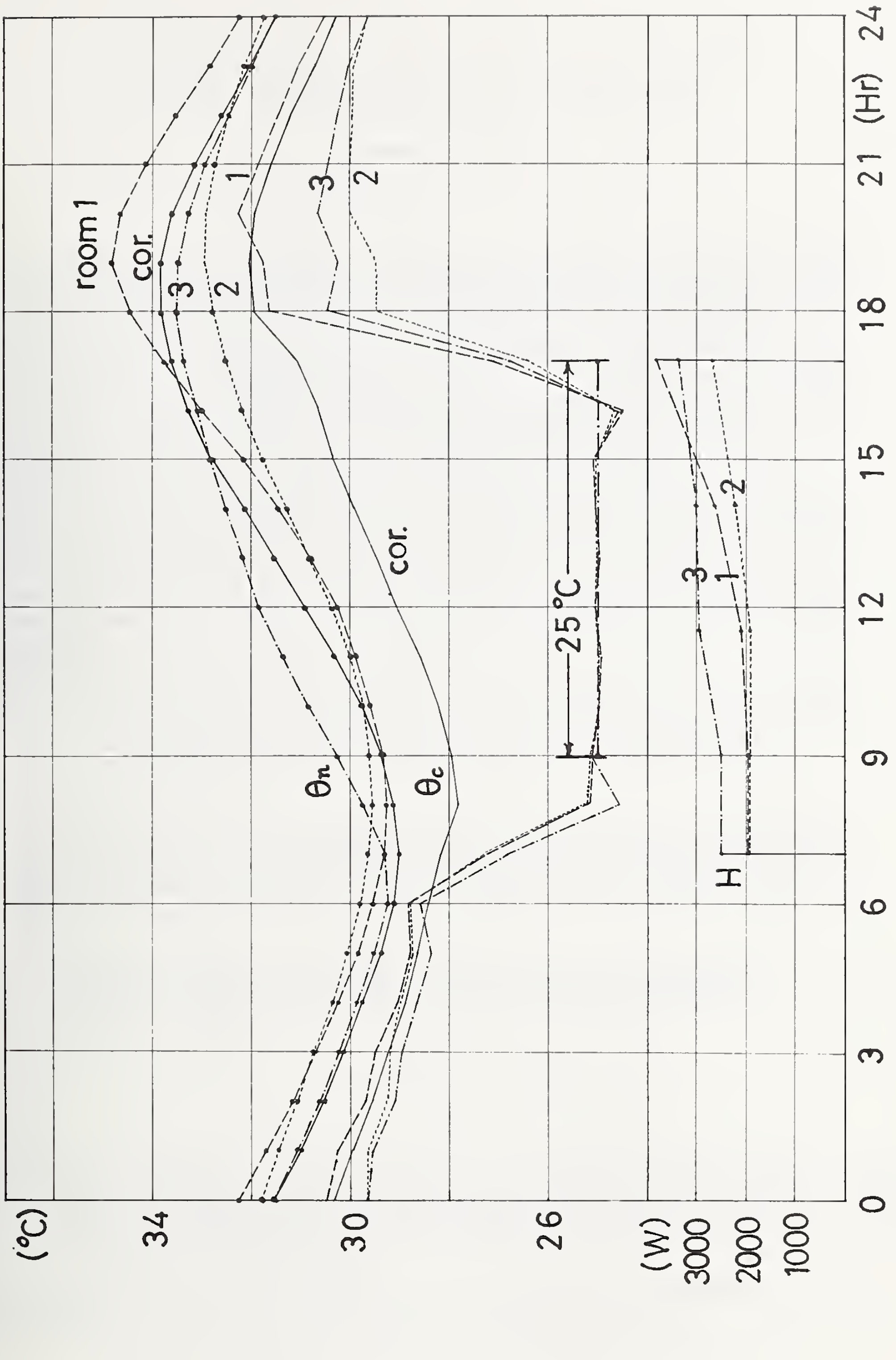

Fig 4 Room Temperatures and Cooling Loads

# An Example of Heating and Cooling Load Calculation Method for Air-Conditioning of Building by Digital Computer

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Today, many air conditioning engineers recognize that the actual load for air conditioning system can hardly be achieved by the conventional heating and cooling load calculation method. In 1966, the author worked out a load calculation system, in which the numerical calculation method was adopted to solve the problem on heat transmission, and has finally perfected this calculation system for actual use quite recently. The characteristics of this calculation method can be summarized as follows: The air conditioning system in Japan is in majority operated intermittently in a day and quite particular phenomena appear during the early hours after the commencement of operation and right after the termination of operation. These phenomena are caused by the transition of heat retained by the building and air conditioning system as heat capacity. The calculation method worked out has advantages over the others in representing the phenomina mentioned. To be more specific, while any physical system doesn't particularly need to be conditioned linear or invariable for calculating heat transmission, the storage heat load can be accurately calculated by use of this calculation method. Moreover, this calculation system is marked by its reliability in securing an accuracy within the allowable degree for load calculation. It also permits easy understanding for the practice engineers in general, hence it will be widely useful for the professional engineers. Improvement and modification to this calculation system can be also made without much difficulties. This calculation system is mainly composed of two parts, namely,

1. the calculation for obtaining the outputs of the heat source equipment of air conditioning system, and the heat transfer rate of thermal medium transportation system and the changes of room temperatures, and
2. the calculation for the heating and cooling load changes in the air-conditioned rooms.

In the text of this report, the equations adopted for this calculation are firstly illustrated, then the formation of the calculation system is explained in detail and lastly the results of the calculation made for the model building in existence are checked and assured by carrying out the actual measurements and by comparing the measured values with the values obtained by the calculation system.

Key Words: Computer, air conditioning, heating load, cooling load, numerical method, room temperature, medium temperature, simulation, radiation, measurement, intermittent operation, evaluation.

## 1. Introduction

The heating and cooling load of air conditioning is a heat that changes temperature and humidity in the room being air-conditioned, other than produced by an air-conditioning system. It occurs in the form of an external heat invading the room or an internal heat produced in the room. The heat from the air-conditioner absorbs this heating and cooling load, thereby maintaining the temperature and humidity of the air in the room at a required level.

The calculation system that will be referred to in this paper is composed of the following two principal parts:

---

[1] Mechanical Engineer

(1) Calculation of heat transfer and change in room air temperature relative to heat source equipment output and heat medium transportation system.

(2) Calculation of change in the heating and cooling load in the room being air-conditioned.

This paper will deal with the subject, starting will the details of the calculation system. Then, the result obtained from the calculation upon application to an existing building will be compared with the actual measurement data to confirm the exactness of the calculation. First of all, the features of the calculation system will be explained:

(1) The system gives a relation between cooling and heating load $Q_{(t)}$ and output of air-conditioning heat source $Q_{RB(t)}$, therewith enabling calculation of the room air temperature $\theta_{i(t)}$ under an established condition.

Despite the fact that air-conditioning systems are operated intermittently in many cases in Japan, thermal changes at the early stage of operation and after the operation have not been much regarded. By the said calculation system, such changes can be clarified.

(2) A relation between the time $t_4$ required for room air temperature $\theta_i$ to reach a predetermined level after the air-conditioning system starts operation and the capacity of the air-conditioning heat source $Q_{RBm}$ can be obtained, thereby permitting reasonable design of heat medium transportation systems.

(3) The calculation system deals with heat transfer by a numerical method. (1,2)[1] This eliminates the need for any linearity or invariability of the thermal system and permits calculation independent of such changes in heat flow and temperature. Also, the calculation system is easy for practical designers to understand, and can be applied and improved without difficulty, thus offering wide use by professional designers.

(4) In making up the calculation system, approximation methods of new concept have been introduced; (a) simulation of the heat medium transportation system in the air-conditioning system, (b) simulation of radiant heat transportation in the room, and (c) simulation of heat exchanging in the air-conditioning system.

The heating and cooling load in the room being air-conditioned is defined as a heat in a give-and-take relation with the room air, where the heat causing the room air temperature to increase is expressed as "positive" and that responsible for the decrease as "negative". Other definitions necessary in the calculation will be given as required.

## 2. Principal Equations Adopted in Calculation System

### 2.1 Excitations

Excitations are used as an input factor to achieve the calculation, which can be obtained by means of necessary relative formulae available from the design condition data. These formulae are prepared as subrutine. While the design condition involves solar radiation, sky radiation, other effective radiations, atmosphere, humanbody, animals, lights and heaters, their relation with the excitations directly related thereto has been well known.

### 2.2 Numerical method of heat transfer

(1) Fundamental equation of steady state heat transfer (2-dimensional)

$$Q_{ij} = K_{ij} (\theta_j - \theta_i) \qquad \ldots(1)$$

$$K_{ij} = \frac{\lambda . d}{d} = \lambda \qquad \ldots(1)'$$

where, $Q_{ij}$: Rate of heat flow from point j to adjacent point i, d: Divided dimensions (see figure 1), $\theta_i$, $\theta_j$: Temperatures at points i and j, $\lambda$: Thermal conductivity. Assuming $\theta_1$, $\theta_2$,... to be the temperatures at points 1, 2,... and $Q_1$ to be the rate of a heat flow to point 1, we have,

---

[1] Figures in brackets indicate the literature reference at the end of this paper.

$$Q_1/d = \theta_2 + \theta_3 + \theta_4 + \theta_5 - 4\theta_1 \qquad ...(2)$$

For the steady-state heat transfer, the left member of the eq (2) is zero.

### (2) Fundamental equation of unsteady state heat transfer (2-dimensional)

$$C = c.\gamma.d^2 \qquad ...(3)$$

$$\Delta t.Q_1 = C(\theta_{1.\Delta t} - \theta_1) \qquad ...(4)$$

$$\theta_{1.\Delta t} = P\{\theta_2 + \theta_3 + \theta_4 + \theta_5 + (\frac{1}{P} - 4)\theta_1\} \qquad ...(5)$$

$$P = \frac{a.\Delta t}{d^2} \qquad ...(6)$$

$$N = \frac{d.\alpha}{\lambda} \qquad ...(7)$$

where $\theta_{1.\Delta t}$: Temperature at point 1 after time interval of $\Delta t$, a: Thermal diffusivity, C: Heat capacity of element volume, c: Specific heat, $\gamma$: Specific weight, $\Delta t$: Time interval, $\alpha$: Heat conductivity.

The various equations of heat transfer to be produced by the above fundamental equation and the traditional equations will be illustrated in appendix.

### (3) Temperature variation in thermal medium transportation system ( 5)

With a tube as shown in a figure 2 (1) considered to be a model of the thermal medium transportation tube, it is assumed that the surface of the tube is insulated, and the temperatures of the tube and thermal medium are equal at any contacting point and time. Next, changes in temperature with time at respective points as shown in figure 2 (2) where the medium temperature is changed in step order as much as $\Delta\theta_1$ in the transportation system maintained at a constant temperature are replaced by linear changes as shown in figure 2 (3), and it is assumed that $\theta_2$ changes at a uniform rate during the period of $2m1.\Delta t$ to reach a final level, $\theta_2$ corresponding to $\theta_1$ that freely changes can be obtained by the following equation.

$$\theta_{2(n.\Delta t)} = \theta_{1(n.\Delta t-2m1.\Delta t)} + \sum_{m=1}^{2m1} \frac{2m1-m+1}{2m1} \Delta\theta_{1\{(n-2m1+m-1)\Delta t\}} \qquad ...(8)$$

$$m1 = \frac{f}{c.\gamma.v.\Delta t} \qquad ...(9)$$

where, $\theta_{1(n.\Delta t)}$: Temperature of medium fluid at inlet of system at time $n.\Delta t$, $\theta_{2(n.\Delta t)}$: Temperature of medium fluid at outlet of system at time $n.\Delta t$, $\Delta\theta_{1(n.\Delta t)}$: Temperature difference at inlet during $\Delta t$ at time $n.\Delta t$, n, m: Number showing time by multiple of $\Delta t$, f: Total heat capacity of transportation system, p, v: Flow rate of medium fluid, c: Specific heat of medium fluid, $\gamma$: Specific weight of medium liquid.

When m1 is under a relation of $0.5 \leq m1 \leq 1$, it is regarded as 1, and if under a relation of $m1 \geq 1$, an integer closet to the actual value is employed. Especially when $m1 < 0.5$, $\theta_1$ is always considered equal to $\theta_2$. This means that the effect of the heat capacity of the system is to be taken into account when it is large, and such effect may be ignored when small.

## 3. Change in Room Air Temperature and Heat Medium Transportation System

The calculation system dealt with in this Section takes the heating and cooling load Q as the input data, thereby computing the output of heat source equipment $Q_{RB}$ and the change of room air temperature $\theta_i$. Calculation system concerning Q will be explained in Section 4. Between the two calculation systems for $\theta_i$ and Q, input and output must be exchanged to carry out the calculation. The heat produced by the heat source equipment is carried into the air-conditioning room by the transportation system, and the heating or cooling load is carried into the heat source equipment by the same system. These heat transportation phenomena are under the influence of: (1) Heat capacity of the transportation system plus heat medium, (2) Thermal load caused by transportation equipment power, and (3) Heat from outside of the transportation system, whereby the thermal characteristics of the transportation system will be formed. In addition, the air-conditioning system is provided with automatic control unit. With these factors, an air-conditioning process system will be made up under restrictions by the design requirements.

In this Section, assuming a simple basic model as shown in figure 3 further explanation will be made.

### 3.1 Conditions for design calculation

Principal control values for the air-conditioning system include (a) room temperature $\theta_{im}$, (b) chilled or hot water temperature $\theta_{wlm}$ and (c) conditioning air temperature $\theta_{f1m}$. As restrictive conditions, there are (a) capacity of heat source $Q_{RBm}$, (b) flow rate of chillied or hot water p, (c) flow rate of air-conditioning air $v_1$, (d) volume of outdoor air induced $v_o$, (e) time required for room air temperature to reach a stable level at early stage of operation $t_4$, (f) time required for heat source output to reach a stable level at early stage of operation $t_1$ and (g) time required till complete elimination of heat source output after operation stop $t_5$.

Figure 4 shows the changes in $Q_{RB}$, Q and $\theta_i$ at early stage of operation, and figure 5 indicates their trend of subsquent changes after operation is over. The control values and restrictive conditions are applied as design requirements together with outdoor and indoor conditions.

The changes in $Q_{RM}$ at early stage of operation and after the operation is over which are given as the characteristic of the heat source equipment, can be regarded as linear changes in the case of ordinary refrigerators and boilers, and thus are treated as a certain restrictive factor. Assuming $n_1 = t_1/\Delta t$, and $Q_{RBm}$ = maximum heat source output, we have $\Delta Q_{RB} = Q_{RBm}/n_1$ = constant at early stage of operation. If n is between 0 and $n_1$, this equation can be rewritten as

$$Q_{RB(n.\Delta t)} = \frac{n.Q_{RBm}}{n_1} \qquad \text{...(10)}$$

Also, assuming that the time to stop operation is t = 0, and $n_5 = t_5/\Delta t$, the following relation can be obtained if n is between 0 and $n_5$

$$Q_{RB(n.\Delta t)} = \frac{n.Q_{RB(0)}}{n_5} \qquad \text{...(11)}$$

In addition, variables necessary for the calculation, such as; $W_1$: Total heat capacity of supply system of cooling or heating water, $W_2$: Total heat capacity of return system of cooling or heating water, $f_1$: Total heat capacity of supply air duct system, V: Volume of room, $c.\gamma.\ V(1+\beta)$: Total effective heat capacity of room air, $\beta$: Additional coefficient of heat capacity of room air must be assumed for calculation.

Initial conditions are related to the thermal state of the air-conditioning system at the time of commencement of the calculation, and may be assumed through engineering judgements guided by location of building, data, time, thermal characteristics of building and system, and statistical data.

Now, assuming that $\theta_o$ is outdoor air temperature, $\theta''_i$ is return air temperature at the inlet of heat exchanger, and $\theta_{f2}$ is mixed air temperature on the side of the heat exchanger, $\theta_{f2}$ can be obtained from the following equation.

$$\theta_{f2} = \frac{(V_1 - V_o)\ \theta''_i + V_o\theta_o}{V_1} \qquad \text{...(12)}$$

Slight errors in assumed values for the initial condition are negligible as long as, subsequent input data are correct, because the calculation system itself has convergence and if the time to start calculation is advance.

### 3.2 Variation in room temperature

Variation in room temperature is obtainable by the following equation.

$$\theta_{i(n.\Delta t+\Delta t)} = \theta_{i(n.\Delta t)} + \Delta\theta_{i(n.\Delta t)} = \theta_{i(n.\Delta t)} + \frac{\left[Q'''_{RB(n.\Delta t)} + Q_{(n.\Delta t)}\right]\Delta t}{c.\gamma.V(1+\beta)} \qquad ...(13)$$

where, $Q_{RB(n.\Delta t)}$: Rate of heat transported from air conditioning system to room at time $n.\Delta t$, $Q_{(n.\Delta t)}$: Heating or cooling load in room at time $n.\Delta t$, $\Delta\theta_{i(n.\Delta t)}$: Difference in room temperature between $\Delta t$ at time $n.\Delta t$. And if the supply air temperature at diffuser at time $n.\Delta t$ is $\theta''_{fl(n.\Delta t)}$,

$$Q_{RB(n.\Delta t)} = c.\gamma.v_1\left[\theta''_{fl(n.\Delta t)} - \theta_{i(n.\Delta t)}\right] \qquad ...(14)$$

According to the equation (14), it is noted that the room temperature at time $(n.\Delta t + \Delta t)$ is obtainable from respective values at time $n.\Delta t$. This fact is of great significance for the subsequent calculation. Apparent variation in retained heat of room air $Q_v$ relative to changes in room temperature at the early period operation from 0 to $t_4$ is given as,

$$Q_v = c.\gamma.V(1+\beta)\left[\theta_{i(n_4.\Delta t)} - \theta_{i(0)}\right] \qquad ...(15)$$

where $n_4 = t_4/\Delta t$. The decisive factor for the change in room temperature where the absolute value of $Q_v$ is smaller than that of $\int_0^{t_4} Q_{RB} dt$ is a balancing relation between Q and $Q'''_{RB}$. With relative increase in $Q_v$, however, its effect on $t_4$ can no longer be ignored, and must be given due consideration in selecting the value of $Q_{RBm}$. At the early stage of usual intermittent air-conditioning operation, the maximum output of the heat source equipment is required, and $Q_{RBm}$ is often determined by setting $t_4$ at a required value. The relationship between $t_4$ and $Q_{RBm}$ can be obtained. Today, there is no better way than a trial and error method to find the relationship between $t_4$ and $Q_{RBm}$.

### 3.3 Heat exchanger

Air must be used as final thermal medium when heat produced by the air-conditioning system is transmitted into the room. Heat exchangers of the air-conditioning system in general use today employ water as primary thermal medium and air as secondary thermal medium, excepting heat exchanger for heat source equipment. The capacity and service condition of the heat exchanger are subject to the related design specifications. In order to quantitatively find the characteristics of the heat exchanger under other service conditions, it is necessary to incorporate complicated characteristic relationship as it is into the calculation system. This, however, is so intricate to deal with that a practical simple method will be employed here.

General usage and performance of the heat exchanger most used recently are as follows, with the flow rate of air $v_1$ passing through the heat exchanger considered invariable, chilled or hot water at an almost constant inlet temperature $\theta'_{w1m}$ is supplied by controlling its flow rate p by means of a two-way or three-way valve. Assuming the return water temperature to be $\theta'_{w2}$, the rate of exchanged heat is expressed as $p\ (\theta'_{w1} - \theta'_{w2})$. This formula is equal, whether two-way or three-way valve is involved.

If the air temperatures at the inlet and outlet of the heat exchanger are $\theta_{f2}$ and $\theta_{f1}$, respectively, the corresponding enthalpies are $i_2$ and $i_1$, and the latent heat load to be eliminated is $Q_L$, the rate of exchanged heat is had as follows,

$$Q'_{RB} = p(\theta'_{w1} - \theta'_{w2}) = c.\gamma.v_1(\theta_{f1} - \theta_{f2}).-Q_L = \gamma.v_1(i_1 - i_2) \qquad ...(16)$$

While the values of $\theta'_{w2}$ and $\theta_{f2}$ are established spontaneously during the air-conditioning operation, $\theta'_{w1m}$ and $\theta_{f1m}$ serve as control values. Design values p and $v_1$, therefore, must be adjusted so that these control values may not be exceeded even under the maximum air-conditioning load.

Taking for example a cooling operation to simulate the action of this heat exchanger, the mutual relation with temperatures is shown in figure 6. In figure 6 (1), a relationship under maximum load is given, where the value of c/a is assumed to be invariable under medium load shown in figure 6 (2). Under this relation, $a = \theta_{f2} - \theta'_{w1}$, $b = \theta'_{w2} - \theta'_{w1}$, $b' = [\theta'_{w2}] - \theta'_{w1}$ and $c = \theta_{f2} - \theta_{f1}$. The difference between $\theta'_{w2}$ and $[\theta'_{w2}]$ is to correspond to the latent heat load $Q_L$. The relation between c and b is at given by the equation (16). The value of a becomes maximum when the load comes to the maximum, at which point, it is necessary not to overestimate the value of $Q_L$.

The values of $\theta'_{w1}$ and $\theta'_{w2}$ obtained from this simulation are not equal to those actually measured. But what is actually needed is the difference in temperature $(\theta'_{w2} - \theta'_{w1})$, and thus if near actual values are desired, they can be obtained by comparing the value of $\theta'_{w1}$ with $\theta'_{w1m}$ and adjusting $\theta_{w2}$ as much as the resultant difference.

Simulation of the automatic control performance at the early stage of operation is achieved as follows: The value $\theta_{f1}$ is changed toward the predetermined value of $\theta_{f1m}$, and $Q_{RB}$ is increased till $\theta_i$ reaches its predetermined value. With $Q_{RM}$ reaching $Q_{RBm}$, this state is maintained. After the room temperature reaches the pre-determined level, the difference in $\theta_i$ corresponding to the load variation is feed back to the heat source side with the medium temperature $\theta_{f1}$ and $\theta'_{w2}$, thereby controlling the output $Q_{RB}$.

The thermal relations in the heating operation can also be established merely by reversing the symbols for temperatures and heat given above.

### 3.4 Calculation of thermal relation in air conditioning system

Figure 7 shows a thermal relation in the air conditioning system based on a simulation model shown in figure 3, and the calculation procedure to obtain various values. To simplify and smoothly eccomplish the calculation, some assumptions are set as follows:

(a) Time required to transport heat medium is ignored.

(b) External heat load invading the heat medium transportation system is proportional to the heating or cooling load (this is obtainable by calculation as required).

(c) This calculation system includes no exact calculation method for relative humidities. Thus, the relative humidity of the air in the room is considered always maintained at 50%.

(d) Room air temperatures within one calculation unit (will be referred to later) at same time are all equal.

(e) Calculation is carried out at each step of time interval $\Delta t$. The calculation at $t = n.\Delta t + \Delta t$ subsequent to the completion of the cycle of calculation at desired time $t = n.\Delta t$ is started with room air temperature $\theta_{i(n.\Delta t + \Delta t)}$.

(f) As shown in figure 7, calculation is carried out in the order of room temperature, return air duct system, return water pipe system, supply water pipe system and supply air duct system, followed by calculation of room air temperature with time advanced as much as $\Delta t$.

(g) Although the heat flow of the cooling or heating load is reversible, the flow of heat transported by the heat medium transportation system is irreversible.

(h) The temperatures at various points of the air-conditioning system are calculated by the use of known related temperatures obtained when time is most advanced, as a general rule.

Today, these assumptions are within the permissible range of the overall calculation accuracy.

The following will outline the transfer of heat in the air conditioning system step by step.

(1) Return air system

The return air temperature changes with changing room air temperature. Assuming change in return air temperature between times $n.\Delta t$ and $n.\Delta t + \Delta t$ to be $\Delta\theta'_i(n.\Delta t)$, it can be expressed as follows:

$$\theta'_{i(n.\Delta t + \Delta t)} = \theta'_{i(n.\Delta t)} + \Delta\theta'_{i(n.\Delta t)} \qquad \ldots(17)$$

Suppose that the return air inlet temperature $\theta'_{i(n.\Delta t + \Delta t)}$ is changed to $\theta''_{i(n.\Delta t + \Delta t)}$ at the outlet. This temperature change can be caused to occur by external heat $q_{f2}$ invading the system and the heat capacity of the return air system. A relation between $\theta'_i$ and $\theta''_i$ can be obtained by the following equation, using the equations (8) and (9), with $q_{f2}$ token into account.

$$\theta''_{i(n.\Delta t+\Delta t)} = \theta'_{i(n.\Delta t+\Delta t-2m1.\Delta t)} + \sum_{m=1}^{2m1} \frac{2m1-m+1}{2m1} \Delta\theta'_{i\{(n-2m1+m)\Delta t\}} + \left(\frac{q_{f2}}{c.\gamma.v_2}\right) \Delta t \quad \ldots(18)$$

$$m1 = f_2/(c.\gamma.V_2.\Delta t) \qquad \ldots(19)$$

The temperature of a mixture of this return air and outdoor air is obtainable from the equation(12).

(2) Return water system

According to the given order of calculation in this paper, the calculation now comes to the return water system at time $(n.\Delta t + \Delta t)$. As shown in figure 3, the system contains a water pump, where the flow rate of water is p, water temperature at the outlet of the heat exchanger is $\theta'_{w2}$, and water temperature at the inlet of heat source equipment is $\theta_{w2}$. It is also assumed that all the thermal changes caused by the air at the inlet of the heat exchanger will be taken over by the return water system, which will join the effects dependent on the rate of invading heat $q_{w2}$, power load of the pump $Q_{KWP}$ and heat capacity of the system in the return water system.

The effects of the mixed air will be taken over by the equation (16), and by applying equations (8) and (9) to the above changes, the water temperature at the inlet of the heat source equipment $\theta_{w2(n.\Delta t + \Delta t)}$ can be obtained as follows:

$$\theta_{w2(n.\Delta t+\Delta t)} = \theta'_{w2(n.\Delta t+\Delta t-2m2.\Delta t)} + \sum_{m=1}^{2m2} \frac{2m2-m+1}{2m2} \Delta\theta'_{w2\{(n-2m2+m)\Delta t\}} + (\frac{q_{w2}+KWP}{p})\Delta t \quad ...(20)$$

$$m2 = \frac{w2}{p.\Delta t} \quad ...(21)$$

where $\theta_{f2}$ and $\theta'_{w2}$, and $\theta_{w1}$ and $\theta_{f1}$ used in equation (16) are values expressed at times $(n.\Delta t + \Delta t)$ and $n.\Delta t$, respectively.

(3) Supply water system

Thermal changes in the return waters system are carried over into the supply water system. The heat source equipment is located between the two systems and is subjected to the change in temperature depending on the output $Q_{RB}$. Assuming that the water temperatures at the inlet and outlet of the heat source equipment are $\theta_{w2}$ and $\theta_{w1}$, the relation of $Q_{RB}$ with them is $Q_{RB} = p(\theta_{w1} - \theta'_{w2})$. With $\theta'_{w1}$ assumed to be the water temperature at the inlet of heat exchanger and temperature change caused by heat invading the system $q_{w1}$ taken into consideration, $\theta'_{w1}$ can be expressed by the following equation in the same manner as above.

$$\theta'_{w1(n.\Delta t+\Delta t)} = \theta_{w1(n.\Delta t+\Delta t-2m3.\Delta t)} + \sum_{m=1}^{2m3} \frac{2m3-m-1}{2m3} \Delta\theta_{w1(n-2m3+m)\Delta t} + (\frac{q_{w1}}{p})\Delta t \quad ...(22)$$

$$m3 = \frac{w1}{p.\Delta t} \quad ...(23)$$

(4) Supply air system

Thermal changes in the supply air system are taken over by the supply air system via heat exchanger. Values for $\theta_{f2}$, $\theta'_{w1}$ and $\theta'_{w2}$ are all as measured at time $(n.\Delta t + \Delta t)$, as well as $\theta_{f1}$. In the supply air system is provided a fan. Assuming that the air temperature at the outlet of the heat exchanger is $\theta_{f1}$, average temperature of the diffused air is $\theta''_{f1}$, power load from the fan is $Q_{KWP}$, and heat invading the system is $q_{f1}$, we can obtain $\theta''_{f1}$ by the following equation in the same manner as above.

$$\theta''_{f1(n.\Delta t+\Delta t)} = \theta_{f1(n.\Delta t+\Delta t-2m4.\Delta t)} + \sum_{m=1}^{2m4} \frac{2m4-m+1}{2m4} \Delta\theta_{f1\{(n-2m4+m)\Delta t\}} + (\frac{q_{f1}+Q_{KWF}}{c.\gamma.v_1})\Delta t \quad ...(24)$$

$$m4 = \frac{f1}{c.\gamma.v_1.\Delta t} \quad ...(25)$$

The four systems mentioned above are connected by (a) air-conditioning room, (b) the heat exchanger for air-conditioning and (c) heat source equipment. The performance of these connected systems and heat medium transportation system will serve as various basic components of an air-conditioning system. Also, the air-conditioning system is available in other types using boost heater, dual duct system, three or four piping system, etc. These cases can be handled in the theoretically same manner as $Q_{RB}$ and $Q_{KWP}$, where subrutine is prepared for the calculation. By the combination of various basic models and subroutine as required, various types of air-conditioning systems and associated heat medium transportation systems can be made up.

## 4. Heating and Cooling Load in Room

When the material of wall surrounding the room, dimensions, position and service condition of the room, atmospheric condition and initial condition are given, and room temperature and humidity are designated, the heating and cooling load tends to be determined. In this Section, the calculation system whereby the transmitting course and thermal rate of the heating and cooling load can be obtained will be explained.

## 4.1 Preparation for calculation system

### (1) Calculation unit and area element

Unline the conventional method to calculate the load per each room, the calculation system being referred to in this Section is such that a plurality of rooms that little differ in temperature and are considered equal in temperature variation during the calculation are dealt with in one calculation unit. In case one room has two or more typical temperatures, as many calculation units as the number of typical temperatures are provided in the building. Thus, while the calculation unit has something related to the concept of zoning, it still depends on the typical room temperatures. Practically, rooms where the temperature difference is always within ± 1.5°C may be regarded as one calculation unit.

Heating or cooling load in the room is obtained by the calculation of transient heat transfer phenomena, except convection heat transfer, draft and latent heat that are directly obtainable. When the room air absorbs the heating or cooling load, its temperature changes at a rate dependent on its heat capacity. As the heat capacity of this room air, an apparent heat capacity $c.\gamma.V(1+\beta)$ is used. Walls, floor and ceiling forming a room play a leading role in the transfer of heat, radiant heat that has invaded the room cannot become a load without having been absorbed by the solid objects in the room.

The transmission course of the heating or cooling lead is formed as a thermal system chiefly by the surrounding structural objects, and its thermal characteristics depend largely upon the construction and composition of the structual surroundings, the load phenomenon occurs in largely different manner depending on whether the glass window is wide or narrow, or whether a curtain wall or concrete wall is used.

A calculation of heat transfer does not necessarily require simulation exactly to the thermal composition of the room, in this calculation system, the areas of structural objects equal in heat transfer phenoemonea are totalled for calculation and the results are proportionally divided per area. These area are called area elements for calculation.

### (2) Thick wall and thin wall

Disturbances with large changes are caused by the external atmospheric condition, and response depends on the properies of the outer wall. To estimate the rate of heat transfer from the wall body, thermal transmittance or thermal conductivity is used. The rate of temperature changes inside the solid can be determined according to the thermal diffusivity. The wall bodies are available in many types, such as concrete wall having a heat capacity with property of heat insulation, metal panels having a low heat capacity with poor heat insulating power, a combination of metal panels with heat retaining material that is low in heat capacity yet has proper heat insulation, and glass plates that let radiant heat penetrate through.

These wall bodies are classified by heat capacity per unit area, according to which those high in heat capacity are called a thick wall and those low in heat capacity are called a thin wall. The relation expressed by equation (6) is referred to for practical classification, whereby the divided measurement "d" relative to divided time $\Delta t$ is determined. Accordingly, the wall whose thickness is divided into two or less portions is called a thin wall, and that with three or more divisions of its thickness is called a thick wall. For the calculation on the thin wall, equation (1) or (48) is applied combined with such an expedient as including part of the heat capacity of the wall body in that of room air.

## 4.2 Course of heating or cooling load

The calculation of heating or cooling load in the room is aimed at clarification of the quantity of heat transfer through the analysis of the transfer course of the heating or cooling load. The flow rate and direction of heat always involves the temperature gradient on the course according to Fourier's low. With a temperature change of the heat medium on the course, the rate of its retained heat increases or decreases, and this behavior of the heat has an important bearing on the load. Kink of disturbance and intensity of the heat flow are given as the design considerations, and various equations shown in the appendix are applied for the calculation of heat transfer inside the building. The application of these equations is very simple, and will be outlined below in connection with the points particularly considered in this calculation method.

### (1) Heat transfer on interior structural bodies

The room has many pillars and beams in addition to the walls. While these structural bodies do not serve very much as the course of the external heat, they release or absorb the retained heat with changes in room temperature. Also the surrounding wall has a rugged surface, which has a similar thermal action. To compensate for these thermal actions, subrutines by the application of equations (5) and (36) through

(38) are prepared.

(2) Radiant heat transfer in the room

Let us assume a film at the boundary dividing the exterior and interior structural bodies. This assumed film represents a surface condition of the interior structual body, and its surface temperature (MRT) is $\theta_{Im} = \sum_n A_{In} \times \theta_{In} / \sum_n A_{In}$. The effective radiant heat transfer between the external surface and the film can be obtained by equation (42). If there is no marked temperature difference over the entire exterior structure, its average temperature (MRT) is $\theta_{om} = \sum_n A_{on} \times \theta_{on} / \sum_n A_{on}$. The radiant heat transferred to the assumed film from the exterior can safely be considered to be received evenly by the whole surface of the interior structure.

(3) Radiation from interior heating bodies

Interior heating bodies are available locally and in the objects evenly scattered on the floor, such as human bodies and lighting equipment. In the latter case, radiant heat is emitted evenly to the ceiling and floor.

### 4.3 How to set up load variation calculation system

To achieve calculation by computer, the design conditions and area element per room are applied as input data. Of the results from the calculation, necessary data are taken as output, and expressed with a proper time interval. The calculation system must be ready to take all kinds of possible input date conditions. If any special conditions occur, and the system is not prepared to permit their application thereto, an approximation method is emplyed in a form close to the existing method. If the effect of the approximation is not permissible, then the calculation system itself must be improved for higher accuracy. Such instances often occur in respect to multi-layer wall, or when the air-conditioning room is adjacent to a room under special condition.

## 5. Example of Calculation

The calculation system being dealt with in this paper is based on the simulation of a air-conditioning system, and is a computer program intended to obtain design data. The details of the setup, however, are the questions directly related to the programming, and thus are omitted in this paper. Already, this calculation method has provide to be able to provide permissible calculation data upon application to several existing buildings and comparison with actual values. The following is an example of the practical application.

### 5.1 Application of calculation method

(1) Preparation for calculation

Prior to calculation, the air-conditioning system is determined. Then, input data are prepared from the related materials. Since the capacities of the associated equipment are unknown prior to the designing, they are estimated from the actual statistic values. The capacity of heat source equipment for intermittent air conditioning $Q_{RBm}$ is obtainable from its relation with the time required to stabilize the room air temperature $t_4$, and accordingly values for p and $v_1$ can also be determined. For the operation of the calculation system, engineering decisions, such as initial conditions, are required. Various values estimated at first are corrected to proper values upon investigation of the calculation results, and are recalculated if necessary. For example, the temperature change in room air temperature in the adjacent part that will not be calculated is assumed and if this assumption is found far different from the calculation result obtained later, the calculation is repeated from the beginning.

(2) Application to building ( 6)

Location: Tokyo. Name: O Building. Purpose of use: to provide office spaces. Construction: SRC (curtain walls used as outer walls at the south and east sides). Scale: Nine storied, 3 basement floors and 3 penthouses. Total area: 7,260m$^2$. Air-conditioning area: 3,400m$^2$. Operation: Intermittent (8:30 to 17:30).

The application of this calculation method to the air conditioning of this building was attempted. In this application, actual values were emplyed, instead of exterior and interior design conditions. For the air-conditioning system, the model shown in figure 3 could be applied as it is, and the calculation was carried out in one calculation unit.

### (3) Results of calculation

By this calculation method, the temperatures and heat flows in the building and at the respective components of the air-conditioning system, could be calculated in detail over the given calculation perior. However, recording of all the data on the temperature and heat flow is tremendous and practically meaningless. Thus, what was considered necessary for the design was selected from among these data as the output, from which values for $Q_{RB}$, $\theta_i$ and only were picked up and diagrammed as given in figure 8 (2) with full lines and brocken line. The changes of the heat medium temperatures $\theta_{f1}$, $\theta_{f2}$, $\theta''_{f1}$, $\theta'_{w1}$ and $\theta'_{w2}$ were as shown in figure 9 with full lines. From these calculation results, the following were found out.

(a) To bring the room air temperature at the early stage of operation to a predetermined level, the heat source equipment is to be operated with full output for a time. According to the conventional design method, however, the flow rate of air in the supply air system becomes short. Thus, the output declines prior to arrival at the predetermined room air temperature level. As a result of a trial calculation with increased supply air rate to avoid the shortage, it was found that full output operation has to be continued till the room air temperature reaches the predetermined level, as shown in figure 10.

(b) The part indicated with A in figure 8 (2) at the initial stage of operation represents what is spent for the increase in the retained heat of the equipment (negative heat in the cooling operation). In contrast, the part B appearing after the operation is finished results from the discharge of this heat. Both rates of heat are not always equal, however.

(c) By the application of this calculation method to the air conditioning design, adequate value for $Q_{RBm}$ can be discovered. There are other advantages that rational heat medium transportation system suited to $Q_{RBm}$ can be designed and that a proper air-conditioning plan can be set up.

In this example, calculation was made using known values for $Q_{RBm}$, p and $v_1$ of the system set up by the conventional design method, thus given other results than those for rational equipment.

## 5.2 Comparison with measured values and evaluation

Changes of $Q_{RB}$ and $\theta_i$ obtained from the actual air conditioning measurement in this building are given in figure 8 (2) with dotted lines together with exterior conditions as figure 8 (1). Changes of heat medium temperatures are shown in figure 9 with dotted lines. The corresponding calculation results are as given in figure 8 (2) and 9 with full lines and brocken line. A comparison between both results snows that the change of $Q_{RB}$, Q and $\theta_i$ with time shows similar values and trends. Although the heat medium temperatures themselves are a little different because of difference in control method, the necessary temperature difference are in near agreement.

What calls for specific attention here is the fact that the existing air-conditioning system is the product of the conventional design method and not of a rational method. Also, when it comes to the actual construction, there were unexpectedly many causes of heating and cooling load, such as that from the heat medium transportation system exposed on the roof. With these points taken into consideration, a difference by 10% or so between the calculated and actual values for $Q_{RBm}$ is unavoidable.

For reference, the time and labor required for this calculation method are such that IBM 360 computer took 20 to 130 seconds with $\Delta t$ at 300 to 60 seconds per one calculation day, and a little more labor than for conventional method was required for input data preparation.

Judging from the above, we believe the practical value of this calculation method by degital computer is very high.

## 6. Conclusions

In the future, calculations of the type mentioned above will have to depend on computers by all means. As the calculation methods for solid heat transfer, there are weight function method, response factor method and analog method, in addition to the numerical method employed in our system. All these methods, however, pertain to the problem on less than 30% of the heating and cooling load, and as to remaining part of the load, they have little to differ. The gist of the heating and cooling load calculation should lie, not in the methodology of the solid heat transfer calculation, but in the systematization of the calculation method altogether.

Our calculation method introduced in this paper has been systematized since several years ago, independently of the progress in other methods. With the numerical method, too, it is possible to reduce the calculation time and memory capacity if proper consideration is given, and satisfactory results could be obtained in respect to accuracy.

Improvement of this calculation method itself and interchange with other methods are the problems remaining to be solved.

This paper limited clarification to the design problems ahead of the heat source equipment output, and has not dealt with the air conditioning energy. It, however, can be easily obtained using, the handling of the heat medium transportation system as a guide, provided the performance of the system components is already known.

While it is still premature to draw a conclusion because of insufficient data, the physical value for the wall body, a ( = $\frac{\lambda}{c.\gamma}$ ) is considered different between cooling and heating period. For such phenomenon, this calculation method is more advantageously applicable than other methods as in the case of intermittent air conditioning operation.

To conclude this report, the author expresses his profound appreciation to Dr. Kenichi Hiraga under whom he works for the opportunity of compiling this paper and very helpful advices and guidance.

## 7. Appendix

### 7.1 Various equations related to unsteady state heat transfer

Temperature in one dimensional object, see figure 11 (1)

$$\theta_{1.\Delta t} = P[\theta_2 + \theta_3 + (\frac{1}{P} - 2)\theta_1] \qquad ...(26)$$

One dimensional, surface temperature of object in contact with fluid with temperature of, see figure 11 (2)

$$\theta_{1.\Delta t} = 2P[\theta_2 + N.\theta_f + (\frac{1}{2P} - N - 1)\theta_1] \qquad ...(27)$$

$$\text{or, } \theta_s = \frac{N.\theta_f + 2\theta_1}{N + 2}, \text{ see figure 11 (3)} \qquad ...(28)$$

One dimensional, surface temperature of object subjected to heat flux q, see figure 11 (4)

$$\theta_{1.\Delta t} = 2P[\theta_2 + (\frac{1}{2P} - 1)\theta_1 + q\frac{d}{\lambda}] \qquad ...(29)$$

Heat produced at uniform rate inside object Q, see figure 11 (5)

$$\theta_{1.\Delta t} = P[\theta_2 + \theta_3 + (\frac{1}{P} - 2)\theta_1 + Q\frac{d^2}{\lambda}] \qquad ...(30)$$

One dimensional, surface temperature of object with heat produced at uniform rate inside Q, see figure 11 (6)

$$\theta_{1.\Delta t} = 2P[\theta_2 + N\theta_f + (\frac{1}{2P} - N - 1)\theta_1 + Q\frac{d^2}{\lambda}] \qquad ...(31)$$

One dimensional, heat conduction between two objects, see figure 11 (7)

$$\theta_{1.\Delta t} = P_1\left\{\theta_2 + \frac{2}{1 + \frac{\lambda_{II}}{\lambda_I}}\theta_3 + (\frac{1}{P_1} - \frac{2}{1 + \frac{\lambda_{II}}{\lambda_I}} - 1)\theta_1\right\} \qquad ...(32)$$

$$P_1 = a_1\frac{\Delta t}{d^2}$$

where, $\lambda_I$: Thermal conductivity of object with point 1, $\lambda_{II}$: Thermal conductivity of object without point 1.

Two dimensional, irregular shape surface temperature of object

In case of figure 11 (8):

$$\theta_{1.\Delta t} = 2P\left[\frac{\theta_2 + \theta_4}{2} + \theta_3 + N.\theta_f + \left(\frac{1}{2P} - N - 2\right)\theta_1\right] \qquad ...(33)$$

In case of figure 11 (9):

$$\theta_{1.\Delta t} = \frac{4P}{3}\left[\theta_2 + \theta_3 + \frac{\theta_4 + \theta_5}{2} + N.\theta_f + \left(\frac{3}{4P} - N - 3\right)\theta_1\right] \qquad ...(34)$$

In case of figure 11 (10):

$$\theta_{1.\Delta t} = 4P\left[\frac{\theta_2 + \theta_3}{2} + N.\theta_f + \left(\frac{1}{4P} - N - 1\right)\theta_1\right] \qquad ...(35)$$

Value for $\alpha$ used in equations (33) to (35) is a corrected heat transfer coefficient inversely proportionate to increase or decrease in area of the object surface.

### 7.2 Values of P and N

Because of the condition where multiplying coeffecient for $\theta_1$ in the right member of the equations for $\theta_{1.\Delta t}$ will not become minus, $\Delta t$ has a permissible maximum limit. For example, from equations (26) and (27) given in the latter paragraphs, we have

$$P \leq 1/2 \quad \text{inside object} \qquad ...(36)$$

$$P \leq \frac{1}{2(1+N)} \quad \text{on object surface} \qquad ...(37)$$

Also, the permissible range of P relative to any value of N is as follows: $P \leq 1/3$ ...(38)

### 7.3 Surface heat transfer ( 3)

$$Q = a(\theta_s - \theta_1) \qquad ...(39)$$

$$\alpha = \alpha_c + \alpha_r \qquad ...(40)$$

where, Q: Rate of heat transfer at surface, $\alpha$: Total heat transfer coefficient, $\alpha_c$: Convection heat transfer coefficient, $\alpha_r$: Radiation heat transfer coefficient.

### 7.4 Radiation (solar) temperature $\theta_s$

$$\theta_s = \frac{a_o}{\alpha} I + \theta_f \qquad ...(41)$$

where, I: intensity of radiation, a: absorption rate.

### 7.5 Radiation ( 3)

$$Q_{12} = A_2 \varphi_{12} C_{12} \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \qquad ...(42)$$

$$\mathcal{F}_{12} = \frac{1}{A_2} \int A_1 \int A_2 \frac{\cos \phi_1 \cos \phi_2}{\pi r^2} dA_2 dA_1 \qquad \ldots(43)$$

$$Q_{12} = - Q_{21} \qquad \ldots(44)$$

$$C_{12} = \frac{C_1 \cdot C_2}{C_b} = 4.88 \times \varepsilon_1 \cdot \varepsilon_2 \qquad \ldots(45)$$

$$a_1 = \varepsilon_1, \quad a_2 = \varepsilon_2 \quad \text{(in same wave length)} \qquad \ldots(46)$$

where, $Q_{12}$: Net rate of radiation from surface $A_1$ to $A_2$, $A_1$, $A_2$: Areas of surfaces exchanging heat by radiation opposite to each other, $\mathcal{F}_{12}$: Total shape factor between $A_1$ and $A_2$, $T_1$, $T_2$: Surface temperatures of $A_1$ and $A_2$, $dA_1$, $dA_2$: Small element areas in surfaces $A_1$ and $A_2$, $\phi_1$, $\phi_2$: Angles between normals on $dA_1$ and $dA_2$ and line connecting $dA_1$ and $dA_2$, $a_1$, $a_2$: Absorption factors of $A_1$ and $A_2$, $\varepsilon_1$, $\varepsilon_2$: Emissitivities of $A_1$ and $A_2$.

### 7.6 Equivalent out door temperature $\theta_e$ and equivalent temperature difference $\Delta\theta_e$ ( 4)

$$\theta_e = \theta_{sm} + f[\theta_{s(\tau)} - \theta_{sm}] \qquad \ldots(47)$$

$$\Delta\theta_e = \theta_{sm} + f[\theta_{s(\tau)} - \theta_{sm}] - \theta_i \qquad \ldots(48)$$

$$\theta_{sm} = \frac{1}{24} \sum_{t=o}^{23} \theta_s(t)$$

where, $\theta_i$: Room air temperature, f: Decrement factor, $\tau$: Time lag, $\theta_{s(\tau)}$: Value of $\theta_s$ as much as before time being calculated.

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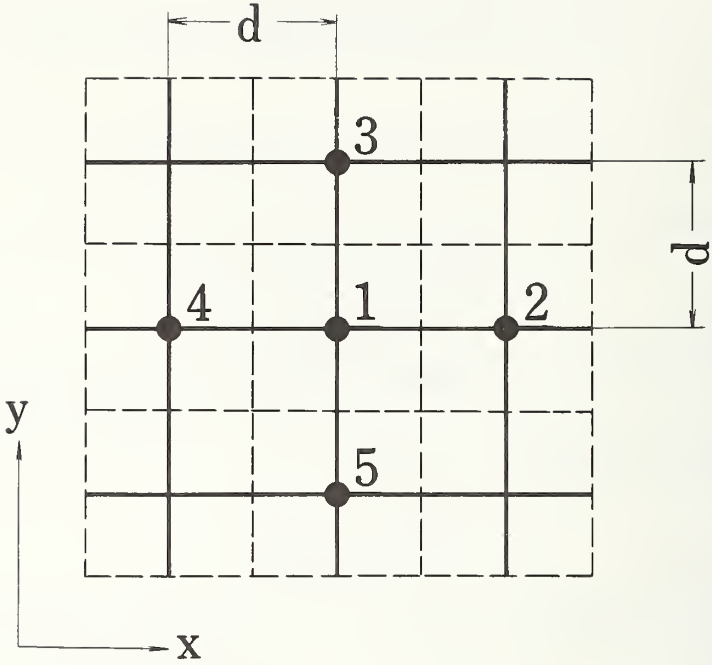


Fig-1 Heat Conduction in Solid, Two-Dimension

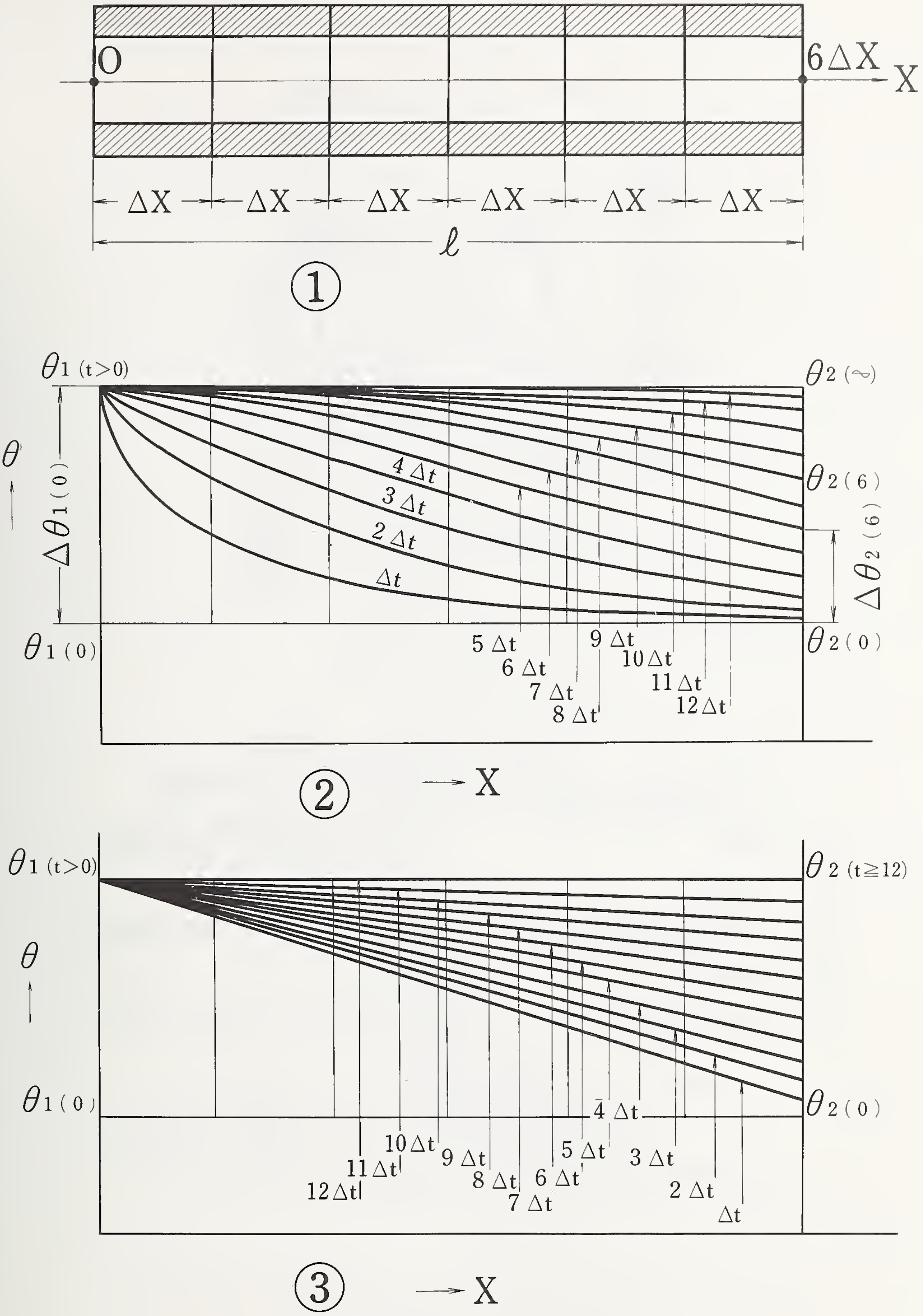
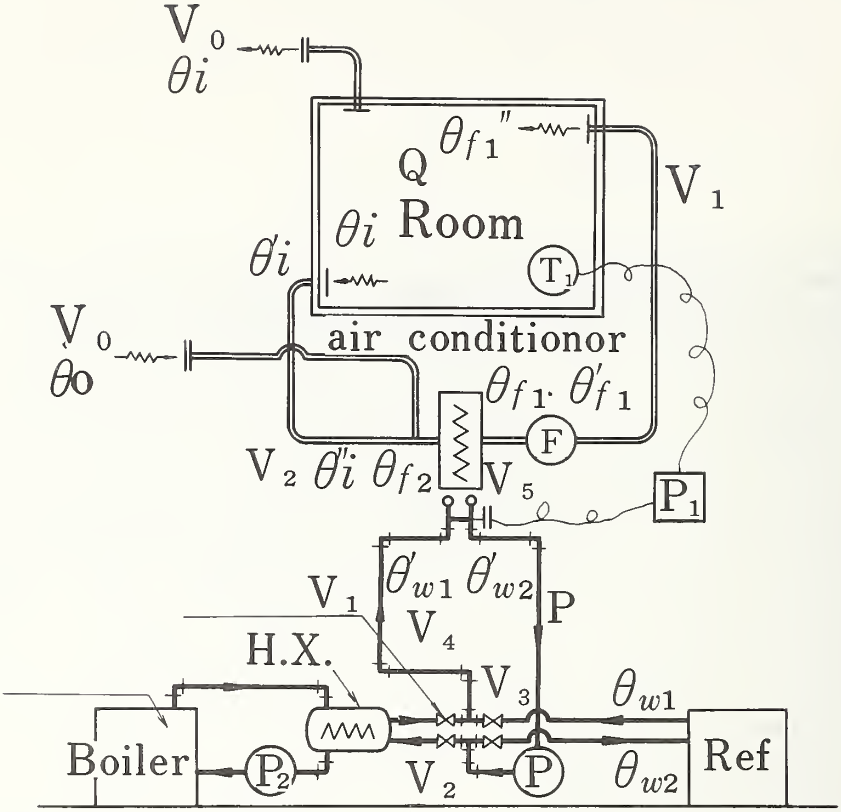


Fig-2(1)~(3)   Simulation of Thermal Medium Transportation System



| | | | |
|---|---|---|---|
| F | fan | $V_1$ | valve for heating |
| P | pump | $V_2$ | 〃 〃 |
| $P_1$ | control Panel. | $V_3$ | 〃 for cooling |
| $P_2$ | pump | $V_4$ | 〃 〃 |
| $T_1$ | thermostat | $V_5$ | three way Valve |

Fig-3 A Model of Air-conditioning System

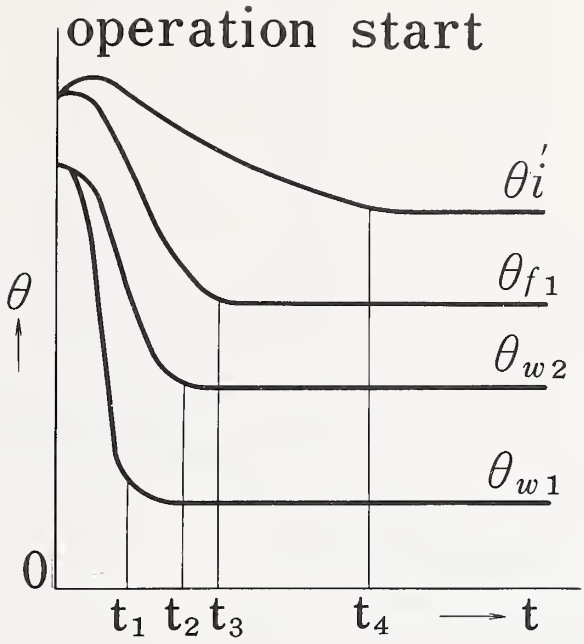


Fig-4 Change of Thermal Medium Temp. just after Operation Start

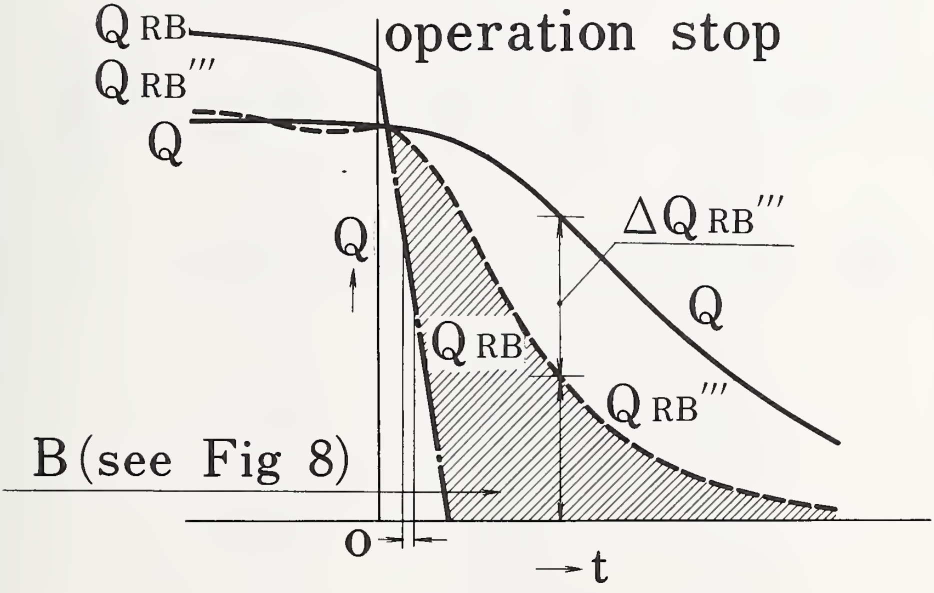


Fig-5 Change of Value of $Q_{RB}$, $Q''_{RB}$ and Q just after Operation Stop

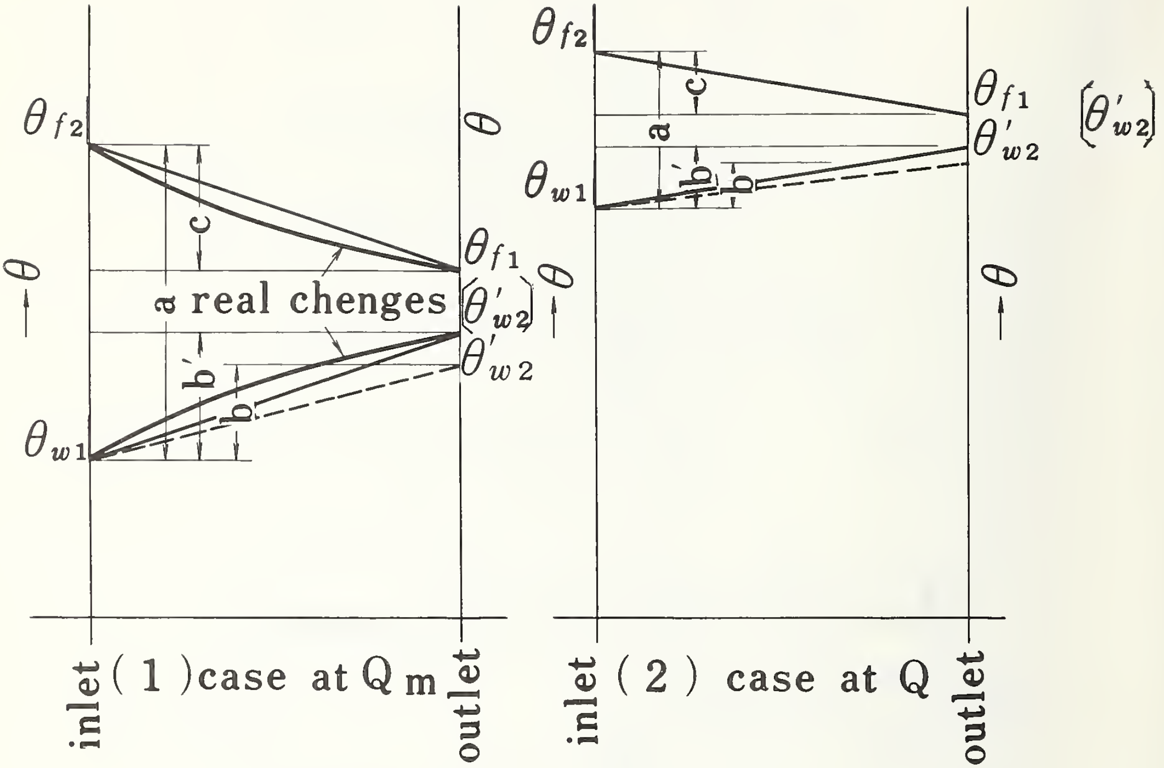


Fig-6(1),(2) Simulation of Heat-exchanger Performance

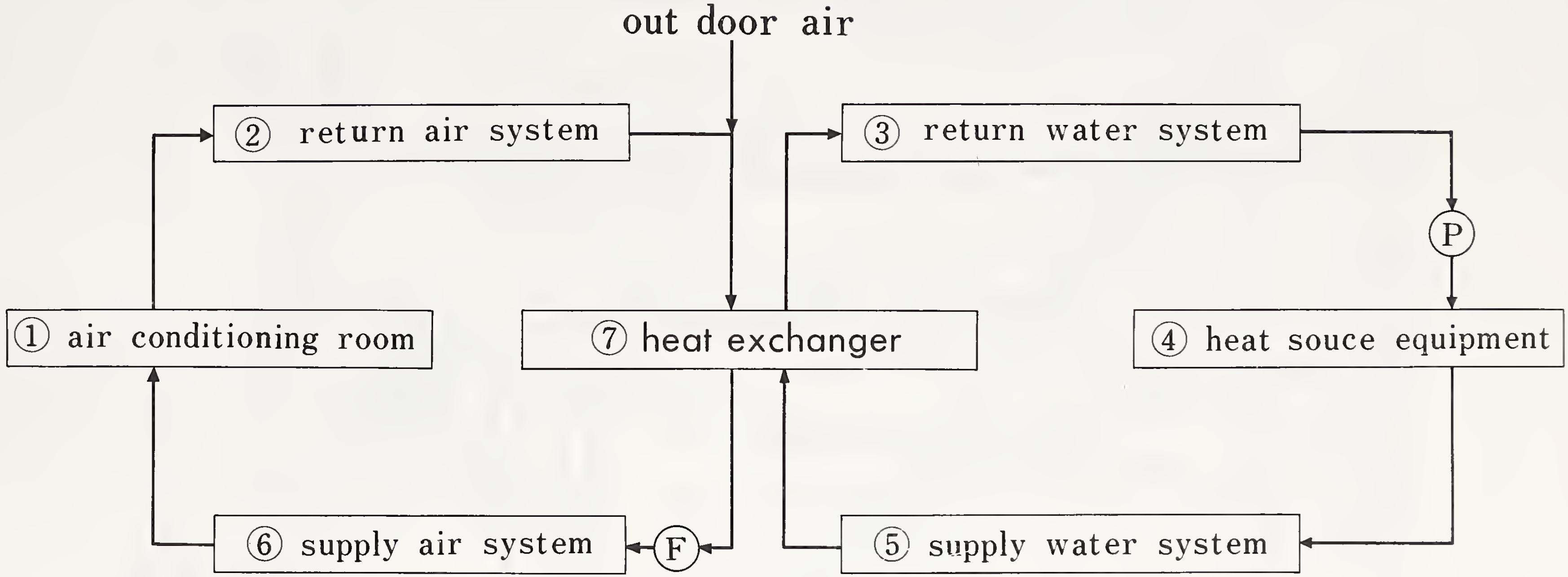


Fig-7 Calculation Procedure Diagram

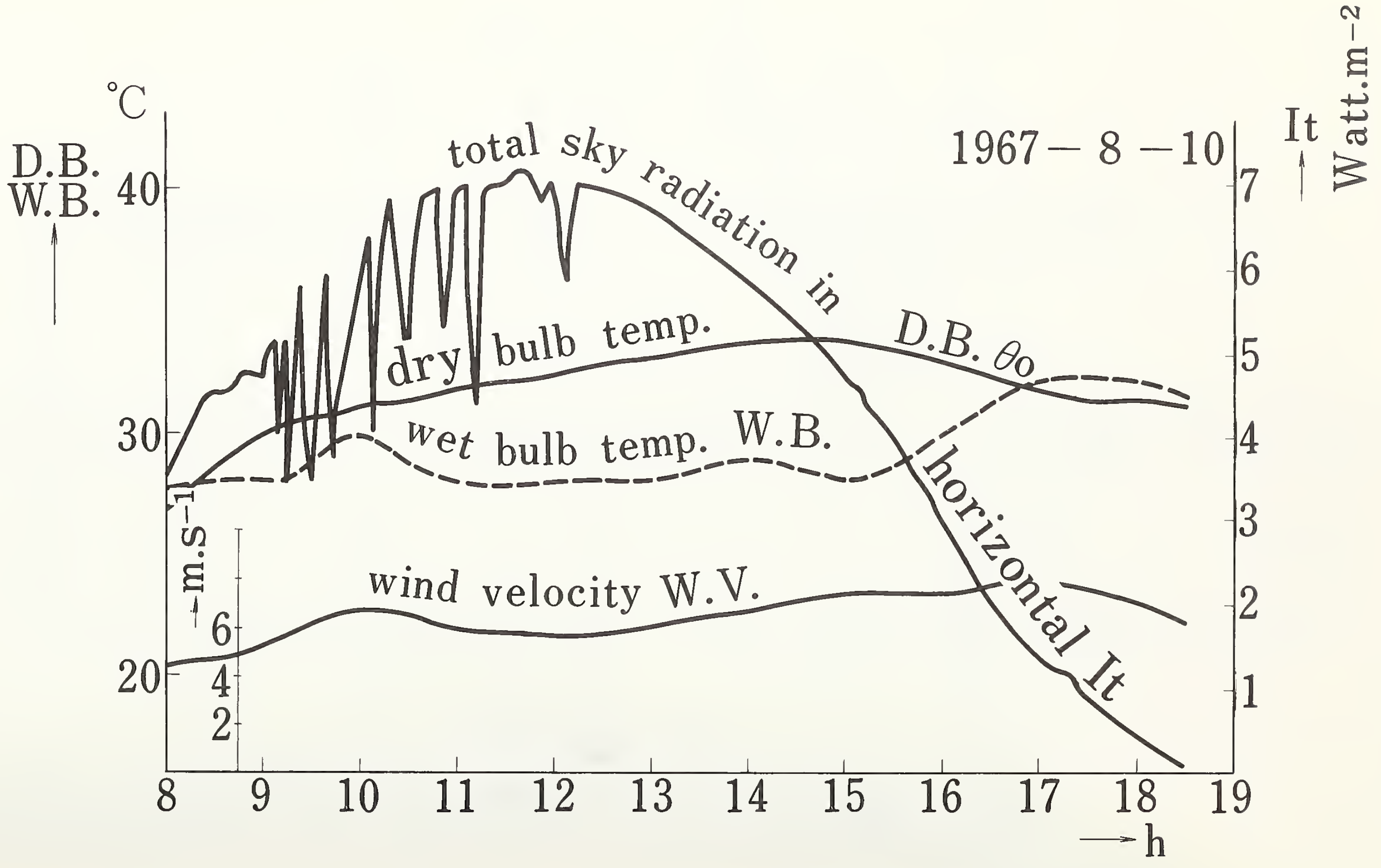


Fig-8 (1) Conditions of Weather

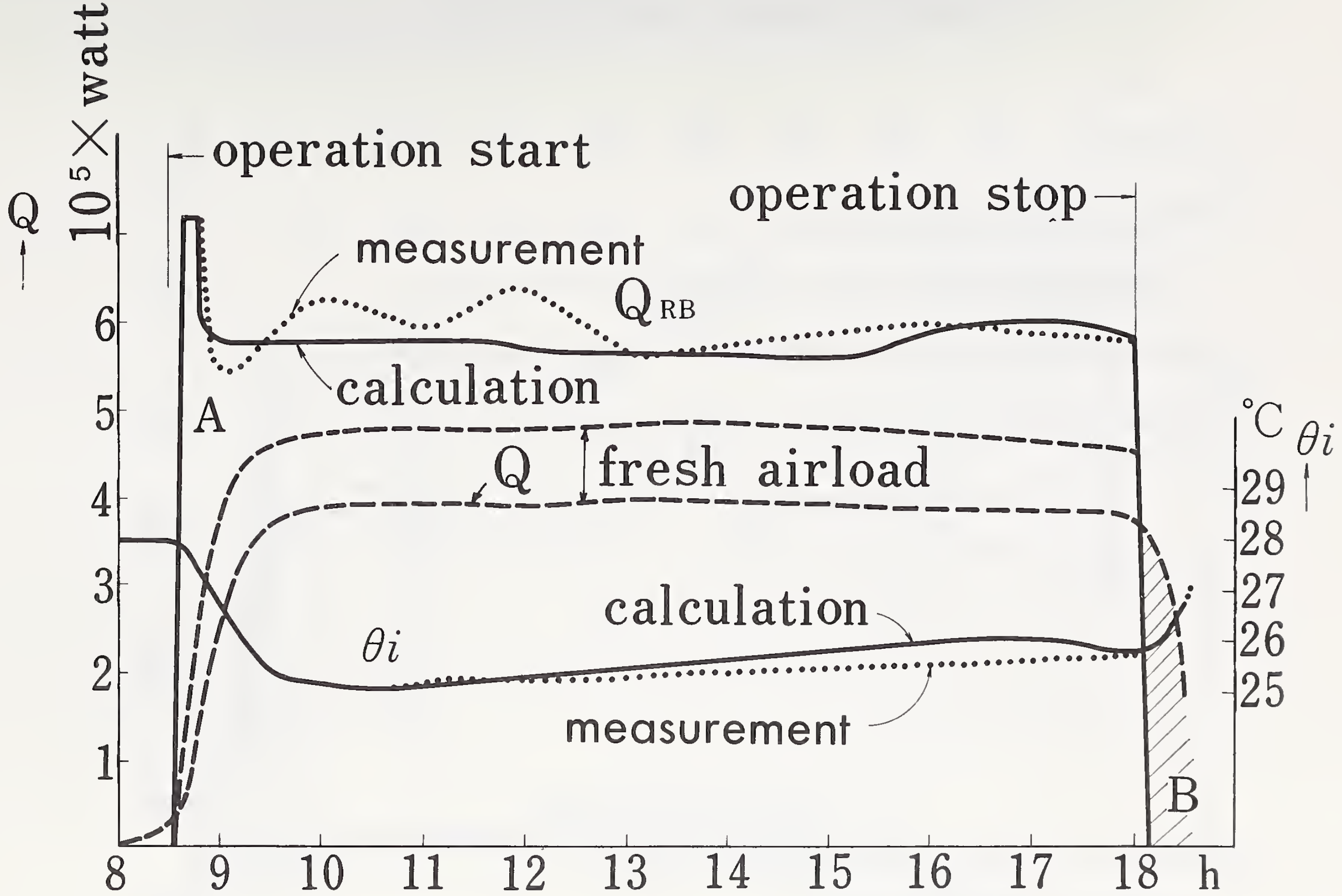


Fig-8 Cont. (2) Values of $Q_{RB}$, Q and $\theta_i$ obtained by Calculation and Measurement

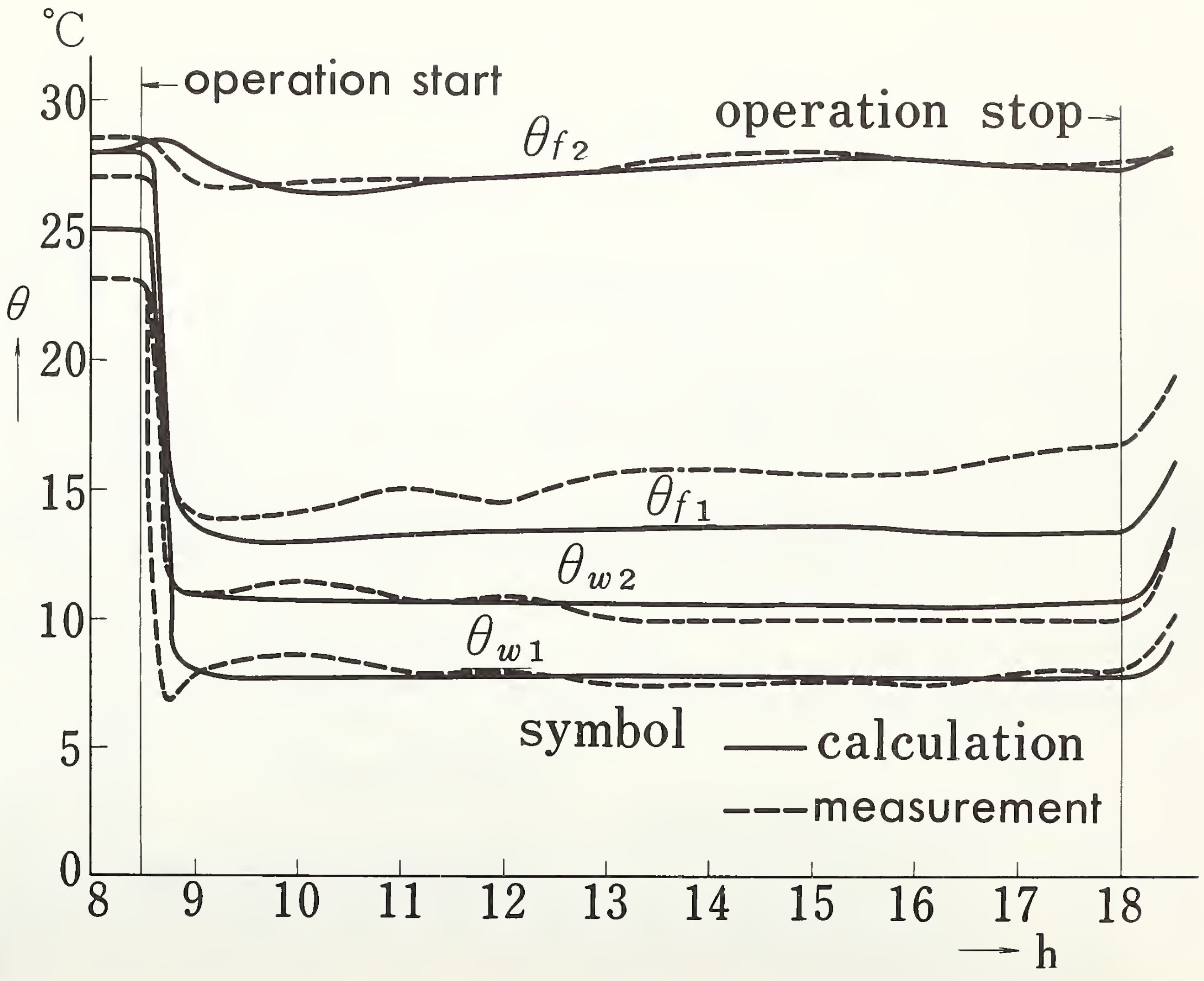


Fig-9 Thermal Medium Temp. obtained by Calculation and Measurement

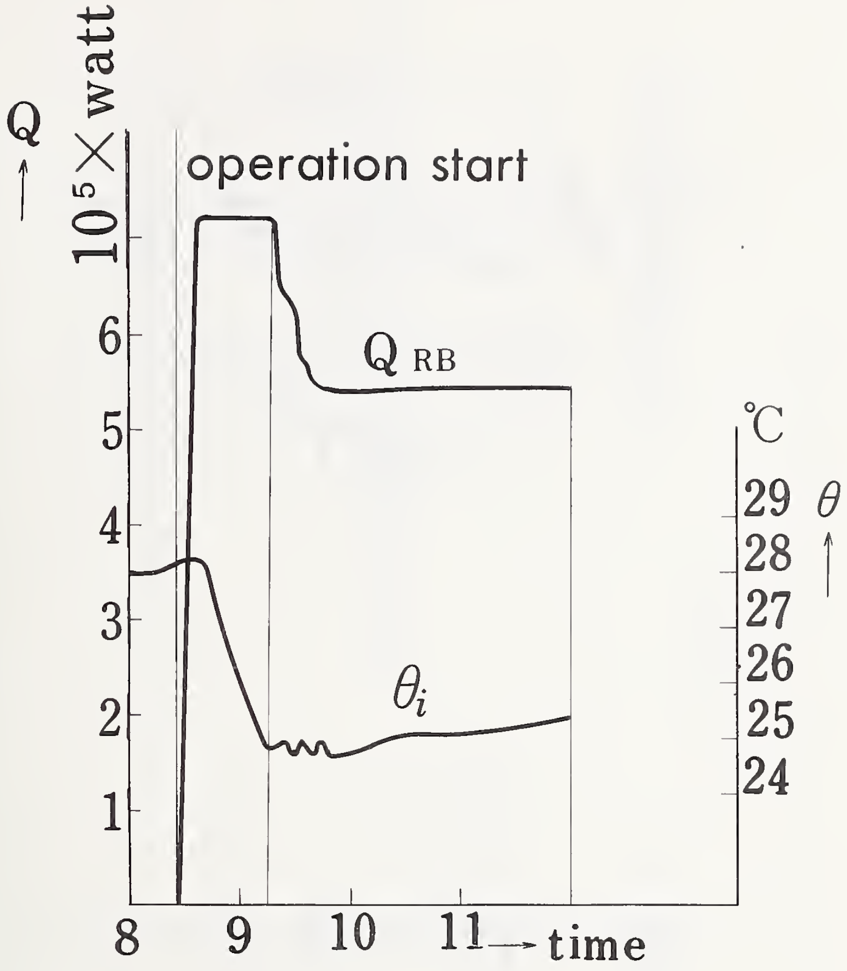


Fig-10 Results of Calculation in case of enough Air supply

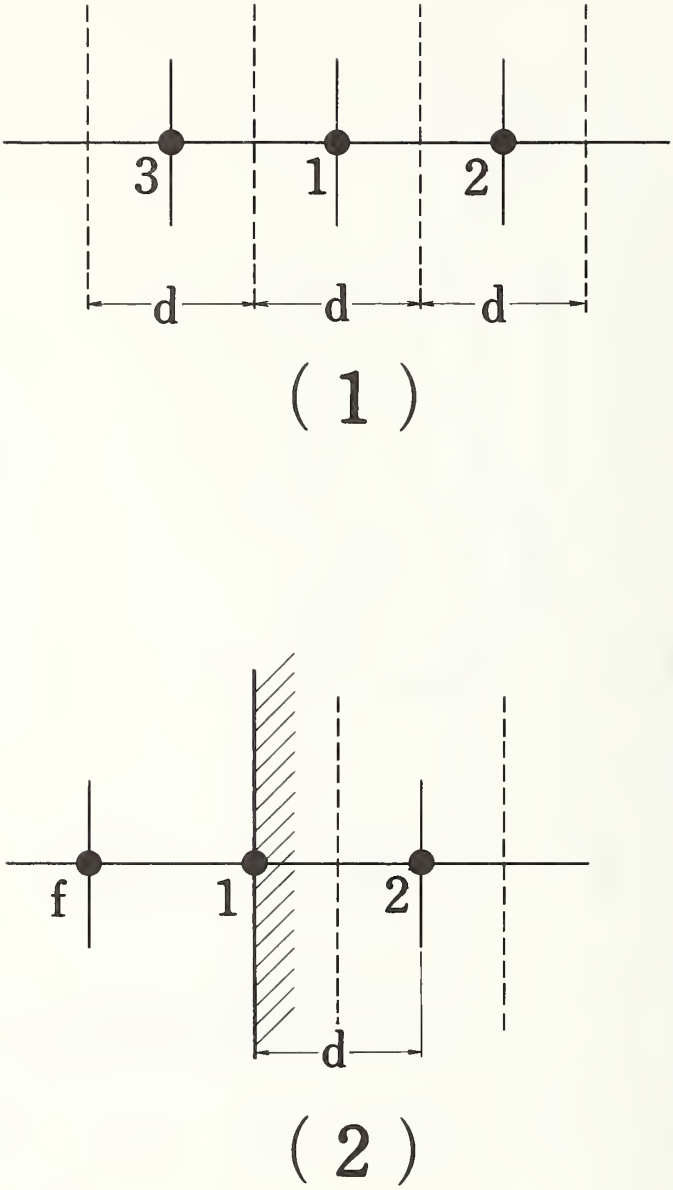
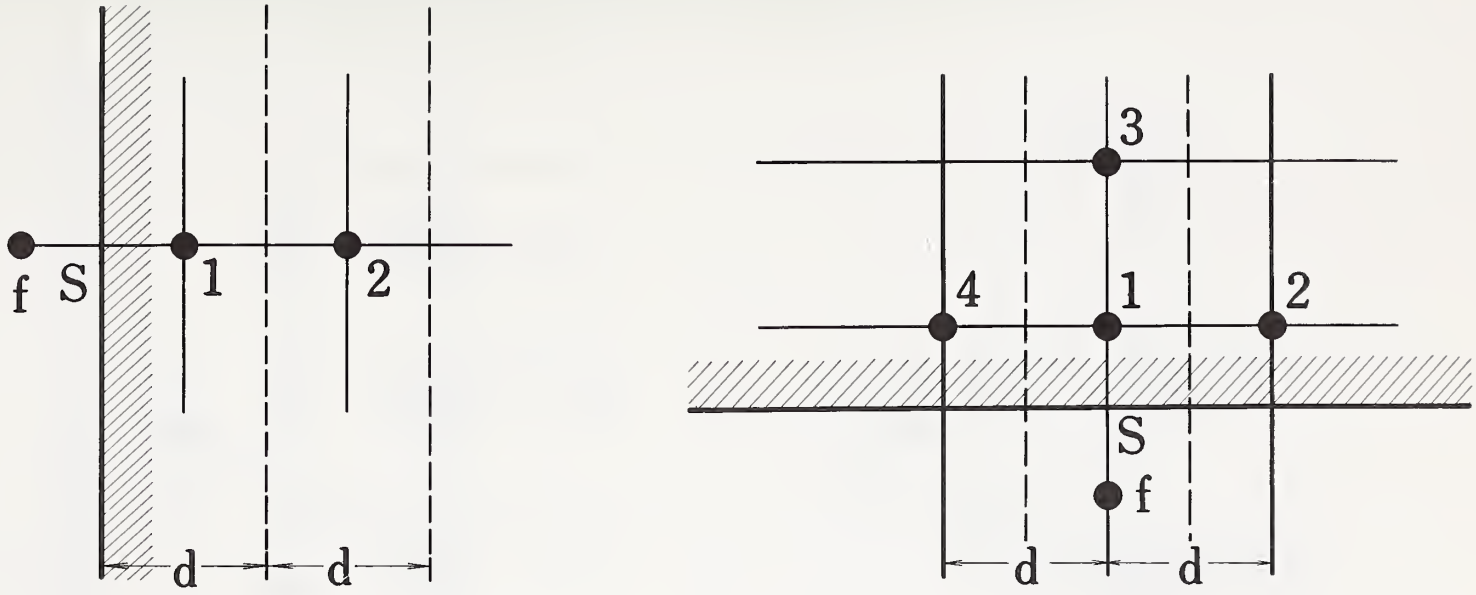


Fig-11 (1 and 2) Variation of Heat Transfer to be adopted

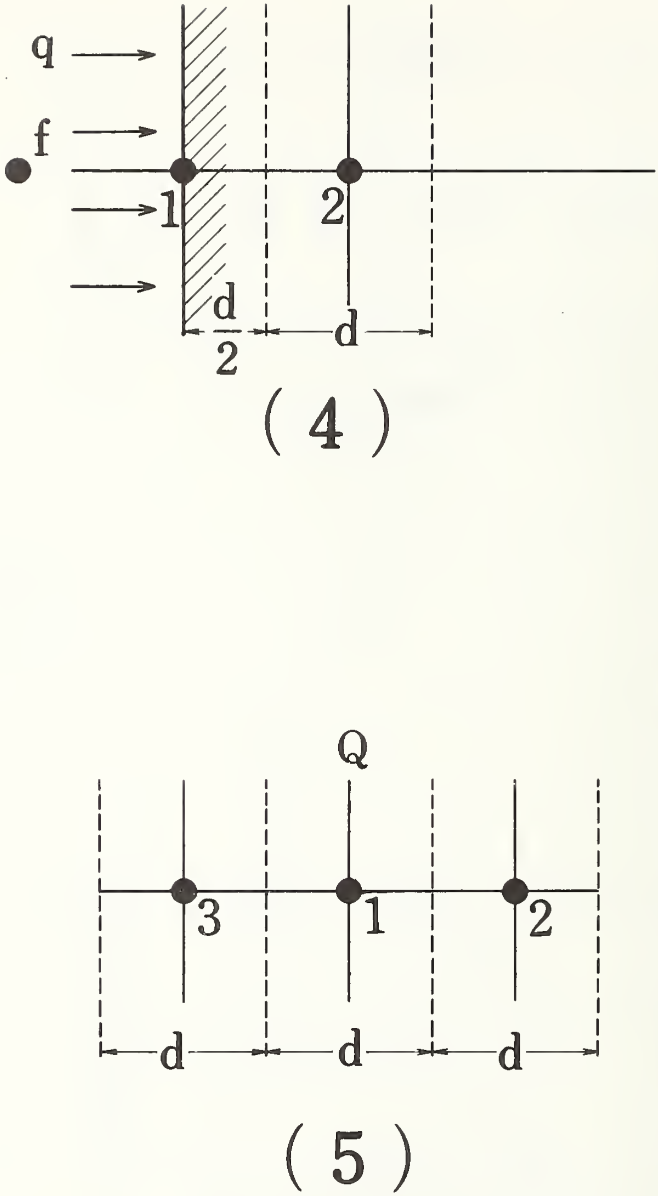


One dimensional

Two dimensional

( 3 )

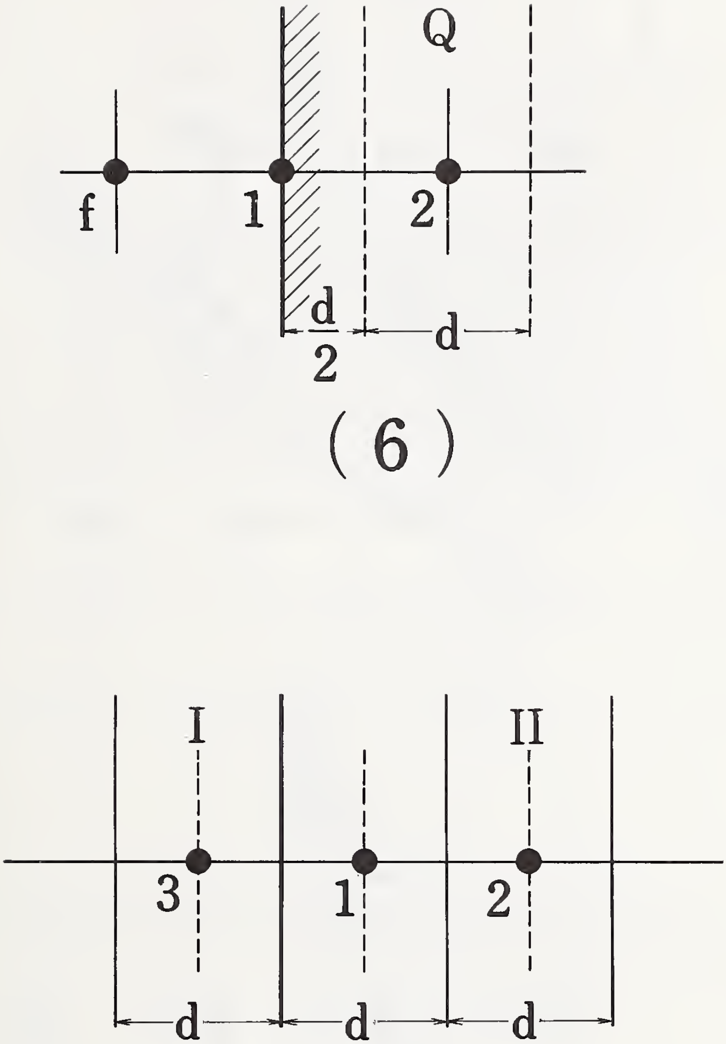
Fig-11 Cont. (3) Variation of Heat Transfer to be adopted



(4)

(5)

Fig-11 Cont. (4 and 5) Variation of Heat Transfer to be adopted



(6)

(7)

Fig-11 Cont. (6 and 7) Variation of Heat Transfer to be adopted

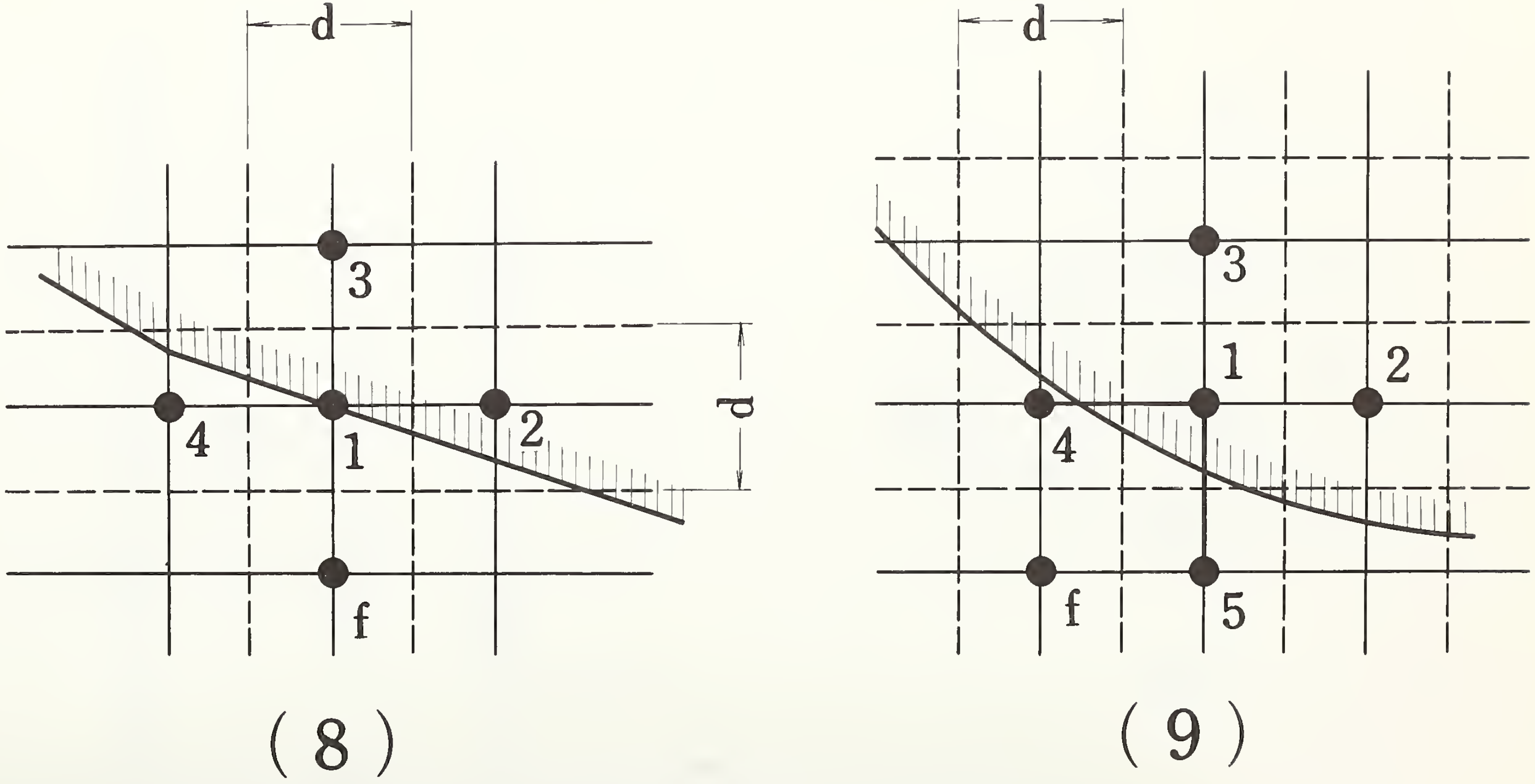


Fig-11 Cont. (8 and 9) Variation of Heat Transfer to be adopted

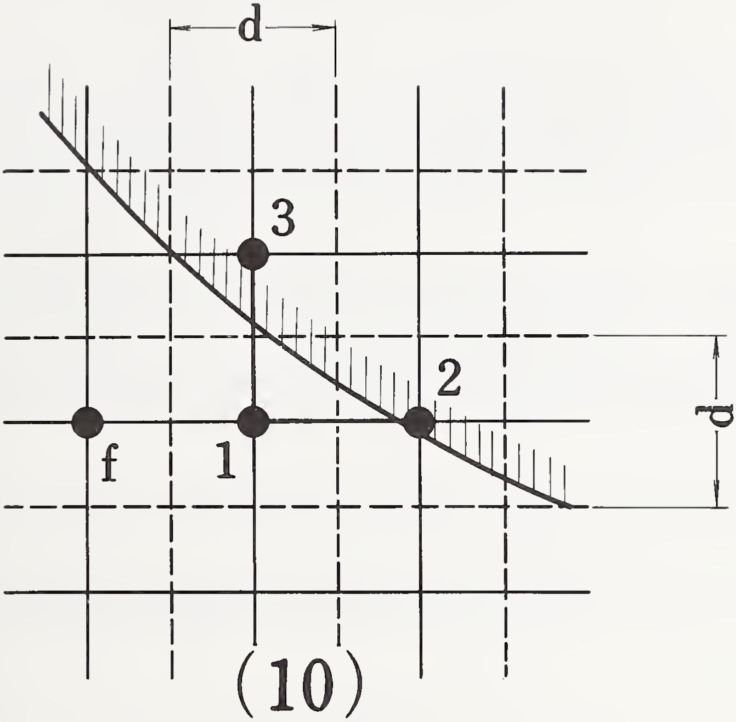


Fig-11 Cont. (10) Variation of Heat Transfer to be adopted

# Heating and Cooling Load Calculations by Means of Periodic Window Function

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One of the computer calculation procedures for the determination of heating and cooling loads, is presented in this paper. This method uses time series on assumption of linearity and stationerity. The concept of "window function" is introduced as combining function of time and time-series. The "triangular wave function" is considered as one of the periodic window functions that are practically easy to handle. The problem of heating and cooling load on building is assumed to be a periodic phenomenon, and this justifies the use of periodic window function.

The calculation procedure is first to determine the "heat flow matrix", of which elements consist of rate of heat flow emanated from each room for a given time, and this rate of heat flow assumed to be proportional to the room temperature. In the case of intermittent heating and cooling, the room temperature and rate of heat supply are alternately unknown. Then, both unknown temperature and heat supply rate is collected, and heat flow matrix is transformed into "thermal matrix". To determine the thermal matrix is to obtain the heating and cooling load and the fluctuation of temperature in each room at each time. In its principle, this method is readily understood and computer programing is also easy. In addition multi-room problem and completed time schedule of the indoor design condition can be easily handled.

Included in this report are an example of heating load evaluations for dwelling with five rooms and for building of which the whole space above the ceiling consists of a plenum chamber.

Key Words: Heating and cooling load calculation, window function, time-series, triangular wave function, response factor method, heat flow matrix, thermal matrix, warming-up period, duct-type room, multiple rooms, Gauss elimination, Gauss-Seidel method.

## 1. Introduction

Recently computer has come to be widely used in the field of architectural engineering. With respect to the calculation of thermal load of buildings, for example, ASHRAE's Task Group on Energy Requirements has developed a calculation procedure [1][2] based on the Response Factor Method [2], [3].

From the stand point of engineering usage, load calculation using computer will have to take account of the following points.

(1) Accuracy from engineering point of view.
(2) Applicability of commonly used computers.
(3) Short computation time.
(4) Simplicity in the preparation of input data and in the operation of partial modification of the data.
(5) Wide range of application.

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[2] Figures in brackets indicates the literature references at the end of paper.

It would be difficult to satisfy all these points. It is therefore desirable to prepare several calculation procedures having their respective characteristics features, so that designer may be able to choose a most efficient procedure according to his objective.

In this paper, a calculation procedure for periodical heat flow problem is prevented. The method of solution is based on the principle of superposition, which assumes linearity and stationerity.

## 2.1 Time-Series and Window-Function

"A time-series is just a series of numbers or quantities representing the values of a function at successive equal intervals of time." [3]

The time-series is assumed to be an approximation, using a finite number of information, of a time-function which varies with time. Thus, a concept of "window function" is introduced to relates arbitrary time-function to its time-series.

There are two types window function wf(t), i.e. <u>transient type</u> and <u>periodic type</u> which have the following properties

$$\text{Transient type} \begin{cases} wf(0)=1 \\ wf(N\cdot\Delta)=0 & ;\ N \pm (1,2,.......) \\ \int_{-\infty}^{+\infty} wf(t)dt=\Delta \end{cases} \tag{1}$$

$$\text{Periodic type} \begin{cases} wf(t)=wf(t+N.T) & ;\ N \pm (1,2,....) \\ wf(0)=1 \\ wf(N'\cdot\Delta)=0 & ;\ N'=1,2,....,\ T/\Delta-1 \\ \int_0^T wf(t)dt=\Delta \end{cases} \tag{2}$$

where $\Delta$ is the time interval of time-series and T is the period.

If a time-function f(t) represents a periodic phenomenon, it is convenient to use a window function of periodic type, and if f(t) represents a transient phenomenon it is necessary to use a window function of transient type.

If F(N) ; N=0,1,2,..... is the time-series of f(t), then the approximate time-function f'(t) represented by this time-series is given by the following equation.

$$f'(t)=\sum F(N).wf(t-N.\Delta) \tag{3}$$

where $\Sigma$ stands for $\sum_{N=-\infty}^{+\infty}$ if wf(t) is transient type and for $\sum_{N=1}^{T/\Delta}$ if it is periodic type. The term of the time-series is a time-function obtained by the product of the window function having time origin at that point and the value of the time-series.

From eq(3)

$$\begin{aligned} f'(N.\Delta)&=f'(N\cdot\Delta) & ;\ N=0,1,2,..... \\ f'(t)&\approx f(t) & ;\ t=N\cdot\Delta \end{aligned} \tag{4}$$

In general, $f'(t)$ is identically equal to $f(t)$ in the limit as time interval $\Delta$ approaches infinitely small. If $\Delta$ is a finite time interval, $f'(t)$ is an approximation to $f(t)$ at $t \neq (N \cdot \Delta)$.

If $\Delta$ has a fixed finite value, the degree of approximation depends on the property of the window function as well as that of $f(t)$.

In response factor method [1], [2], [3], the triangular pulse is used for window function.

There exists infinitely many window function satisfying eq(1) or eq(2). However, it is desirable to choose one on practical consideration such as precision and ease of handling depending on the case. Figure 1 illustrates some examples of window function of transient type and periodic type.

## 2.2 Triangular Wave Function

Triangular wave function is a window function of periodic type which is represented in the form of Fourier series. Such functions are infinite in number, and the following is a relatively simple one

$$twf_{NN}^{(M)}(t) = (1/NN)\left[1 + (2/M)\cdot\sum_{J=1}^{(M\cdot NN/2)-1}\mathrm{COS}(J\cdot x)\left\{1 + \sum_{i=1}^{M-1}(2i/M)\cdot\mathrm{COS}(J\cdot\Delta\cdot(M-i))\right\} + (\beta/M)\cdot\mathrm{COS}(M\cdot NN\cdot X/2)\right] \tag{5}$$

where

$X = (2\pi/T)\cdot t$

$NN = T/\Delta$ (even number)

$M = 1, 2, \cdots$ (in the following, M is referred as "order")

$$\begin{cases} \beta = 0 & ; M = \text{even} \\ \beta = 1 & ; M = 1 \\ \beta = 1/M & ; M = \text{odd} \end{cases}$$

The degree of eq(5) as a polynomial in trigonometric function is $(M \cdot NN/2)$. The lowest degree of the triangular wave function is $NN/2$, which corresponds only to eq(5) with $M=1$, that is,

$$twf_{NN}^{(1)}(t) = (1/NN)\cdot\left\{\sum_{J=0}^{NN/2}\gamma\cdot\mathrm{COS}(J\cdot X)\right\} \tag{6}$$

where

$\gamma = 1$ ; $J = 0$ and $J = NN/2$

$\gamma = 2$ ; $J = 1, 2, \cdots, NN/2-1$

Substitution of the window function $twf_{NN}^{(1)}$ of eq(6) in eq(3) gives

$$f'(t) = \sum_{N=1}^{NN} F(N)\cdot twf_{NN}^{(1)}(t - N\cdot\Delta) \tag{7}$$

or

$$f'(t) = \sum_{K=0}^{NN/2}\gamma\cdot\left\{a(K)\cdot\mathrm{COS}(K\cdot X) + b(K)\cdot\mathrm{SIN}(K\cdot X)\right\} \tag{8}$$

where

$$\begin{cases} a(K) = (1/NN)\cdot\sum_{L=1}^{NN} F(L)\cdot\mathrm{COS}(K\cdot L\cdot\Delta) \\ b(K) = (1/NN)\cdot\sum F(L)\cdot\mathrm{SIN}(K\cdot L\cdot\Delta) \end{cases}$$

Equation (8) represents the result of harmonic analysis of NN time-series, and is the lowest degree Fourier series passing through NN points. Expressions in eq(8) are called Fourier coefficients. Hence, when we use $twf_{NN}^{(1)}$for window function, representation of periodic function f(t) by the time-series F(N) and representation by Fourier coefficients a (k), b(k) (k= 1,2,....., NN/2) are identical. Method of analysis using Fourier coefficients a(k), b(k) or using amplitude and phase constant obtained from them is called the frequency response method. Characteristic feature of the problem of heating and cooling load calculation which is different from the general engineering problem is that i) in the ordinary intermittent heating and cooling,the unknown time period for temperature and heat flow rate appear alternatingly, and ii) that the problem is not determination of response to a particular frequency but determination of the resultant temperatures and heating loads superposed over the whole frequency range. In this respect, the method of time-series which can treat the temperature and thermal load at each time period directly as unknown variable is more convenient that the frequency response method.

On the other hand, as for approximation of uniformly continuous curve, use of $twf_{NN}^{(1)}$ for window function is generally better than the broken line approximation (use of triangular pulse for window function) or the step function approximation (use of rectangular pulse for window function). However, in the thermal problem of buildings where discontinuity points appear in cases of beginning and termination of heating and cooling or on and off of illumination, approximation using the window function $twf_{NN}^{(1)}$ is not sufficiently good because of the appearance of overshoot immediately before and after the time period. Equation (5) is made up, considering these points and the overshoots are reduced by increasing the order M. The relation between M and overshoot is shown in (fig.2), where M values in even number are superior to those in odd numbers. With the order M gradually increased, eq(5) approaches a periodical Triangular pulse.

## 2.3 Response of System Elements

Thermal system of a building or a room is the set of basic elementary system such as heat conduction system of wall and floor and heat transfer system of ventilation. It is therefore required to determine the response of these basic system elements in the first place. The response time-series of the system corresponding to the window function input is denoted to be R(N).

The output time-series G(N) of the system corresponding to the input time-series F(N) is given by

$$G(N)=\Sigma F(M).R(N-M) \tag{9}$$

where $\Sigma$ stands for $\sum_{M=-\infty}^{N}$ if the window function is transient type and for $\sum_{M=1}^{NN}$ if it is periodic type. Thermal response of wall and floor corresponding to artificial function such as triangular pulse and rectangular pulse is generally an infinite exponential function. Thus in practical computation, it is necessary to produce approximation by cut-off or rounding, and the error produced by such approximation will be referred to as "internal error" in this paper. In contrast with this, approximation error in representing a time-function by time-series as previously described and the hypothertical error regarding input will be referrd to as "external error". When artificial function is applied to window function, the final estimation of errors are very difficult, as both of the internal and external errors are included in the result of calculations. While, the thermal response of walls and floors corresponding to the triangular wave function expressed by Fourier series being easily and accurately calculated, therefore, only external errors are considered.

# 3. Construction of Calculation Procedure

## 3.1 Heat Balance Equation of Room Air and Heat Flow Matrix

The following description in this paper adopts an approximation neglecting the energy interchange by radiation from enclosing wall surfaces. In other words, it is aussumed that the heat transfer between the surface of walls and floors and air is propotional to overall surface conductance and that overall surface conductance is invariant with respect to time. The problem is dealt with the multi-rooms problem and totaling number of the rooms is denoted by JJ. Heat flow matrix denoted by [TH]is a matrix whose element in row $\left({}^{J}_{M}\right)$ and column $\left({}^{I}_{N}\right)$ is the coefficient of the rate of heat flow flowing out of room J at time $M\cdot\Delta$ with respect to the room temperature $T\left({}^{I}_{N}\right)$ of room I at time $N\cdot\Delta$. Heat flow matrix is a square matrix whose dimension is given by the following equation

$$NXJ= JJ\cdot NN \tag{10}$$

where

$NN = T/\Delta$
$T$ = period
J and I = Integer denoting room number
M and N = Integer denoting time ( in the later description, $M.\Delta$ and $N.\Delta$ are replaced by M and N respectively for simplification)

Heating or cooling load $H\left({}^{J}_{M}\right)$ and the room temperature $T\left({}^{I}_{N}\right)$ are presented in the column matrix [H] and [T] respectively. Heat supply into room air caused by atmospheric temperature, solar radiation, illumination and heat generation of the room occupants, which can be calculated independently of room temperature $T\left({}^{I}_{N}\right)$ and thermal load $H\left({}^{J}_{M}\right)$, is denoted $HO\left({}^{J}_{M}\right)$, of which column matrix is symbolized [H].

Now, heat balance of room air is expressed in matrix form as

$$(TH)\cdot[T]-(H)=(HO] \tag{11}$$

Heat flow matrix (TH) represents the characteristics of thermal response of building with room temperature as input and heat quantity as output. If interchange of air by ventilation between the adjoining rooms, the heat flow matrix is a symmetric matrix. The inverse matrix $(TH)^{-1}$ of heat flow matrix (TH) will be named temperature matrix and shown as (HT). The temperature matrix (HT) represents the characteristics of thermal response of building with heat quantity as input and room temperature as output.

### 3.2 Elements of Heat Flow Matrix

The element of heat flow matrix $TH\left({}^{J}_{M},{}^{I}_{N}\right)$ is given by

$$TH = -WI + WO - VI + VO + HFA + HAA \tag{12}$$

The terms are defined as follows.

1) Heat gain by conduction from walls and floors (WI).

$$WI\left({}^{J}_{M},{}^{I}_{N}\right)_{I\neq J} = \sum_{K} WS(I,J,K)\cdot Y(K,M-N) \tag{13}$$

2) Heat loss by conduction into walls and floors (WO).

$$WO\left({}^{J}_{M},{}^{I}_{N}\right)_{I=J} = \sum_{I=1}^{JT}\cdot\sum_{K} WS(I,J,K)\cdot X \text{ or } Z\,(K,M-N) \tag{14}$$

where

WS(I,J,K) = area of wall No. K, that facing to both room I and J,
JT = JJ + numbers of atmospheric temperatures (room temperatures known at each time are considered as atmospheric temperature).

$$\left.\begin{matrix}X(K,M)\\Y(K,M)\\Z(K,M)\end{matrix}\right\} = \text{response time-series value of wall No.K at time M.}$$

Representation of X, Y, Z, are in accordance with response factor method (1), (2), (3).

3) Heat gain by ventilation (VI).

$$VI\left({}^{J}_{M},{}^{I}_{N}\right)_{M=N} = C\cdot V(I,J,M) \tag{15}$$

4) Heat loss by ventilation (VO).

$$VO\binom{J,I}{M,N}_{\substack{I=J\\M=N}} = \sum_{I=1}^{JJ} C\cdot V(J,I,M) \tag{16}$$

where

C = heat storage capacity of air per unit volume,
V(I,J,M) = rate of air flow from room I into room J at time M.

5) Heat storage by indoor objects such as furniture (HFA).

$$HFA\binom{J,I}{M,N}_{I=J} = \sum_{K} SF(J,K)\cdot XF(K.M-N) \tag{17}$$

where

SF(J,K) = surface area of No.K heat storage object in room J,
XF(K,M) = time-series of heat storage response of object K.

Wall and floor dividing two rooms having identical temperature fluctuation, is treated as heat storage object ; in which case,

$$XF = (X + Z - 2Y)/2 \tag{18}$$

6) Heat storage by room air (HAA)

$$HAA\binom{J,I}{M,N} = C\cdot RV(J)\cdot HA(M-N) \tag{19}$$

where

RV(J) = air volume of room J,
HA(M) = time-series of differential coefficient of window function Wf(t).

In case a triangular wave function is applied to window function, it is easy to determine HA(M). However, in case an artificial function such as triangular pulse is applied, an appropriate approximation is adopted [2].

## 3.3 Thermal Matrix

Heating and cooling of buildings are generally conducted intermittently. Thus, during the period when room temperature is specified, thermal load is the unknown variable, and during the period when the operation of heating and cooling system is stopped, room temperature is the unknown variable.

Let$\binom{I'}{N'}$ denote the time and the room for which room temperature is specified in eq(11). Since $T\binom{I'}{N'}$ is known, the product of each element of column $\binom{I'}{N'}$ of the heat flow matrix [TH] and the value of $T\binom{I'}{N'}$ is added to the right side of eq(10). Then, assuming $T\binom{I'}{N'}$ replaced by unknown $H\binom{I'}{N'}$, column $\binom{I'}{N'}$ of the heat flow matrix [TH] is 1 in row $\binom{I'}{N'}$ and 0 elsewhere. If$\binom{I''}{N''}$ is the room and the time for which heat supply from the heating and cooling system is specified (generally zero is specified), then $H\binom{I''}{N''}$ is added to the right side of eq(11) and $T\binom{I''}{N''}$ is left as unknown variable. The matrix obtained by transforming the heat flow matrix [TH] by the above operation is referred to as "thermal matrix" and is denoted by$\left[\begin{smallmatrix}TH\\H\end{smallmatrix}\right]$. The column matrix which is composed of unknown temperature and supply heat quantity is denoted by$\left[\begin{smallmatrix}T\\H\end{smallmatrix}\right]$. Furthermore, the column matrix on right hand side which is composed only of known valiables is denoted by$\left[\begin{smallmatrix}HO\\H\end{smallmatrix}\right]$. Then the eq(11) is transformed as follows

$$\begin{bmatrix}TH\\H\end{bmatrix}\cdot\begin{bmatrix}T\\H\end{bmatrix}=\begin{bmatrix}HO\\H\end{bmatrix} \tag{20}$$

Solving the said matrix eq(20) the unknown temperature and supplied heat value$\left[\begin{smallmatrix}T\\H\end{smallmatrix}\right]$are obtained.

## 3.4 Problem During Warming-Up Period in Intermittent Heating and Cooling

In intermittent heating and cooling operation, maximum load usually takes place in warming-up period, and the capacity of equipment of heating and cooling system is determined by this peak load. During the warming-up period, both room temperature T and thermal load H being unknown, the equations corresponding for unknown values can not be completed by heat flow matrix [TH] only. However, this problem is solved, in this paper, by approximation as follows.

Namely, the thermal load during warming-up period can be considered to be equal to the one at the starting time of the specified room temperature (i.e., at the time of termination of the warming-up period). According to this approximation, the number of unknown variables of thermal load H can be reduced so as to determine the thermal matrix $\left[{}^{TH}_{H}\right]$. Assuming M" to be the marginal time between warming-up period and specified temperature period, the column $({}^{J}_{M})$ of thermal matrix (J indicating room number while time M is within the warming-up period) is simplified by the following subtraction converting into $({}^{J}_{M})'$. (cf. table 1, 2, 3 )

$$\left\{\text{converted column } ({}^{J}_{M})\right\} = ({}^{J}_{M})' = \left\{\text{column } ({}^{J}_{M}) - \text{column } ({}^{J}_{M''})\right\} \qquad (21)$$

As a matter of course, similar subtraction must be conducted simultaneously as for column matrix $\left[{}^{HO}_{H}\right]$. By this transformation, the thermal matrix in the case including warming-up period takes the same form as in the case not including warming-up period. (cf. table 2, 3 )

### 3.5 Problem on Duct-Type Room

In case the whole space above the ceiling is occupied by plenum chamber, i,e, conditioned air is indirectly supplied into room under the ceiling, not directly but through the space above the ceiling, this space above the ceiling is referred to as "duct-type room" and the room under the ceiling (the room to be air conditioned) is referred to as "main room". In this case, the duct-type room is also considered a room to be calculated. In this duct-type room, both room temperature and thermal load are unknown during the period when specified temperature is maintained in the main room. However, in case heat is not directly supplied to the main room, there being no unknown variables in the main room during the period, the thermal matrix can be determined.

In case heat is supplied to both main room and duct-type room, this problem is indeterminate. However, even in this case, the thermal matrix $\left[{}^{TH}_{H}\right]$ can be determined by either one of the following two methods, i) the time-series of the temperature difference between the air temperature in main room and that in duct-type room are preliminarily provided (i,e, consequently the temperature in the duct type room is preliminarily specified so as to decrease the unknown quantities of temperature by one), or ii) the time-series of rate of heating quantity directly supplied to the duct type room and the main room plus duct-type room (i,e, consequently, the unknown quantities of thermal load are decreased by one).

### 3.6 Solution of The Thermal Matrix

From the viewpoint of practical application, it is required to solve the thermal matrix accurately and rapidly. Dimension of the thermal matrix is NXJ same as that of the heat flow matrix and as shown in the eq(9), it is determined by period T, time subdivision $\Delta$ and total number of rooms JJ. Thermal matrix is introduced for convenience sake to explain the relation between unknown variables and equations. The number of unknown variable is NXJ, but this does not necessarily mean that the dimension of the simultaneous equations to be solved is always NXJ.

In other words, assuming the period when room temperature is specified to be N', this period can be eliminated from the calculation of simultaneous equations and the dimension of the substantial simultaneous equations to be actually calculated are (NXJ-N'). (cf. table 2, 3)

Thus, all the unknown variables of the room temperatures can be determined by calculating the simultaneous equations of dimension (NXJ-N'), and using these quantities, thermal loads can be obtained by simple multiplications and additions. In case the dimension of the simultaneous equations (NXJ-N') is not too large, they can be accurately calculated by well known "Gaussian elimination".
In case the dimension is large, the "method of succesive displacement" (Gauss-Seidel method) is appropriate. In case of applying the method of succesive displacement, unknown variables are successively determined along the time elapsing, calculating the small matrix of dimension equal to the number JJ at each time. (c.f. table 16)

During the warming-up period, the small matrix expanded up to the terminating time of warming-up period (dimension of matrix is an integer multiples of JJ). (cf. table 17) In case duct type room is care must be taken in calculation, as principal diagonal of the matrix sometimes contains zero. In addition, response factor method can be considered to solve a matrix of infinite dimension by successive displacement. That is, since the transient response to the pulse input of wall and floor becomes almost zero, after a long time, calculation steps of the successive displacement method after this time period yield almost exact solution successively, if we proceed by arbitrarily setting up an initial condition.

Generally speaking, decision as to the choice between Gauss elimination and successive displacement method depends on the capacity of the computer and the dimension of the matrix to be solved.

## 4. Example Problem

Two examples are shown in the following.

Both have their principal objective to seek the peak load and the daily total heating load when the designeted time schedules of intermittent heating are changed. Periodic triangular pulse is used for window function and calculated with $\Delta=1$ hour and period$=24$ hour. Solar radiation, mutual radiation change between surfaces and heat storage of furnitures in room are neglected. Latent heat load is not included. Thermal matrix $\begin{bmatrix} \mathrm{TH} \\ \mathrm{H} \end{bmatrix}$ was solved by Gauss elimination and the computer used was TOSBAC-3400 (core memory 16 KW, magnetic disc 2400 KW)

### 4.1 Example Problem (I), Heating Load of Dwelling

Heating load for a dwelling unit of an apartment house as shown in (fig. 3) is calculated. Adjacent units up, down, right and left of the unit were assumed to be in the same thermal condition as the objective unit. The dwelling was divided into four rooms and one stair room (commonly used by dwellers in other houses). Arrangement of walls and floors, their composition and area, thermal constant of materials and air volume of the rooms are respectively shown (fig. 4) table 4, 5, 6. Assuming the life pattern of four family members as shown in (fig. 5), the heat quantities to be generated by lighting, electric appliance, cooking and human bodies roughly estimated by the said assumption are shown in table 7 (cooking heat is over estimated). The natural ventilation between rooms is classified by night time and day time and shown in (fig. 6). Assumption on five patterns concerning design temperatures in each room and their time schedule is shown in table 8. Designed outside air temperature adopted for calculation is shown in (fig. 7).

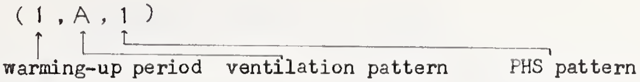
Results of calculation concerning the heating patterns A,B,C,D and E are respectively shown in (fig. 7-11). Total sum of the heating load for rooms $\boxed{1}$ - $\boxed{4}$ at each time is shown in (fig. 12) by heating patterns. The relation between peak load and daily total heating load is shown in table 9. Results of calculation indicates the big differences between heating patterns.

Since the temperature environment in room itself varies with the heating pattern, comparison by peak load and daily total load alone is difficult. Roughly speaking, however, programmed control (E type) is considered to be superior.

### 4.2 Example Problem (II), Office Building with Duct Type Room

Heating load and fluctuation of room temperature of an intermediate story in an office building with center core as shown in (fig. 13) are to be calculated. Upper and lower storey of this storey are assumed to be in the same thermal conditions as this storey. Heats generated from human bodies and lighting devices are neglected. Various values for calculation such as composition and area of walls and floors, thermal constant of materials, and air volume of rooms are respectively shown in table 10,11 and 12. Two kinds of warming-up period (2 hours and 3 hours) for calculation are shown in table 13. Three kinds of assumed ventilating patterns A,B and C are shown in table 14. Explanation of RHS patterns 0, 1, 3 for calculation is shown in table.15.

Here RHS is the ratio of "ratio of heat supply to duct type room" and "sum of heat supply ratio to duct type room and to main room" during warming-up and specified temperature period in main room. Combination of warming-up period, ventilation pattern and RHS pattern for calculation is represented as shown in the following example.



Calculations are conducted for nine combinations such as (1, A, 0), (1, A, 1), (1, B, 0), (1, B, 1), (2, A, 0), (2, A, 1), (2, B, 0), (2, B, 1), (2, C, 3). Result of calculation are shown in (fig. 14-22). The relation between peak load and daily total load of each calculation are shown in (fig. 23).

Although this is only one example of calculation, referring to the results of such caluculation, designers may be able to choose the rational and economic equipment and air conditioning system of building.

## 5. Conclusion

Solution by means of time-series is cleared by the concept of "window function". Window function can be chosen at liberty according to the problem, and when treating periodically fluctuating phenomenon use of "triangular wave function" is appropriate. In the problem to predict room temperature fluctuation and thermal load of a building, "heat flow matrix " plays a fundamental role. Unknown quantities such as room temperature and thermal load can be determined by solving "thermal matrix" which is obtained by a transformation of the heat flow matrix on the basis of indoor design condition (time schedules of intermittent heating or cooling and existence of duct type room) and outside condition (solar radiation, atmospheric temperature etc.). Two method for solving thermal matrix, i.e. Gauss elimination and method of succesive displacement are selectively applied in compliance with the nature of problems and the capacity of computer to be applied.

With the method of calculation described in this paper adopted, the problem of intermittent heating of multiple rooms can be treated as ordinaly one and the problem for single room only becomes rather special one. The periodical phenomenon is treated in the present paper, however, by extending the period T to infinity, the response factor method may also be considered to be included in the calculation method described here.

The effects in case time schedules of intermittent heating are changed, are mainly explained in the said examples of calculation. Ease of such calculation will be a great help to the designers and will facilitate them in their more accurate overall decision making.

## 6. References

1 Proposed procedure for determining heating and cooling load for energy calculations, The task group on energy requirements for heating and cooling, ASHRAE.,1968.

2 G. P. Mitalas, and D. G. Stephenson, Room thermal response factors, trans. ASHRAE., vol. 73, part I, 1967.

3 D.G. Stephenson, and G.P. Mitalas, Cooling load calculations by thermal response factor method, trans. ASHRAE., vol.73, part I, 1967.

4 K. Eguchi, A method of space temperature and and heating and cooling load calculations by window function and time series. trans. AIJ. Summaries of Tech.Papers of AIJ.,AUG.1669.

## 7. Appendix

### Solution of Thermal Matrix by
### Method of Successive Displacement

(1) The case in which all unknown variables are room temperature (problem of room temperature fluctuation)

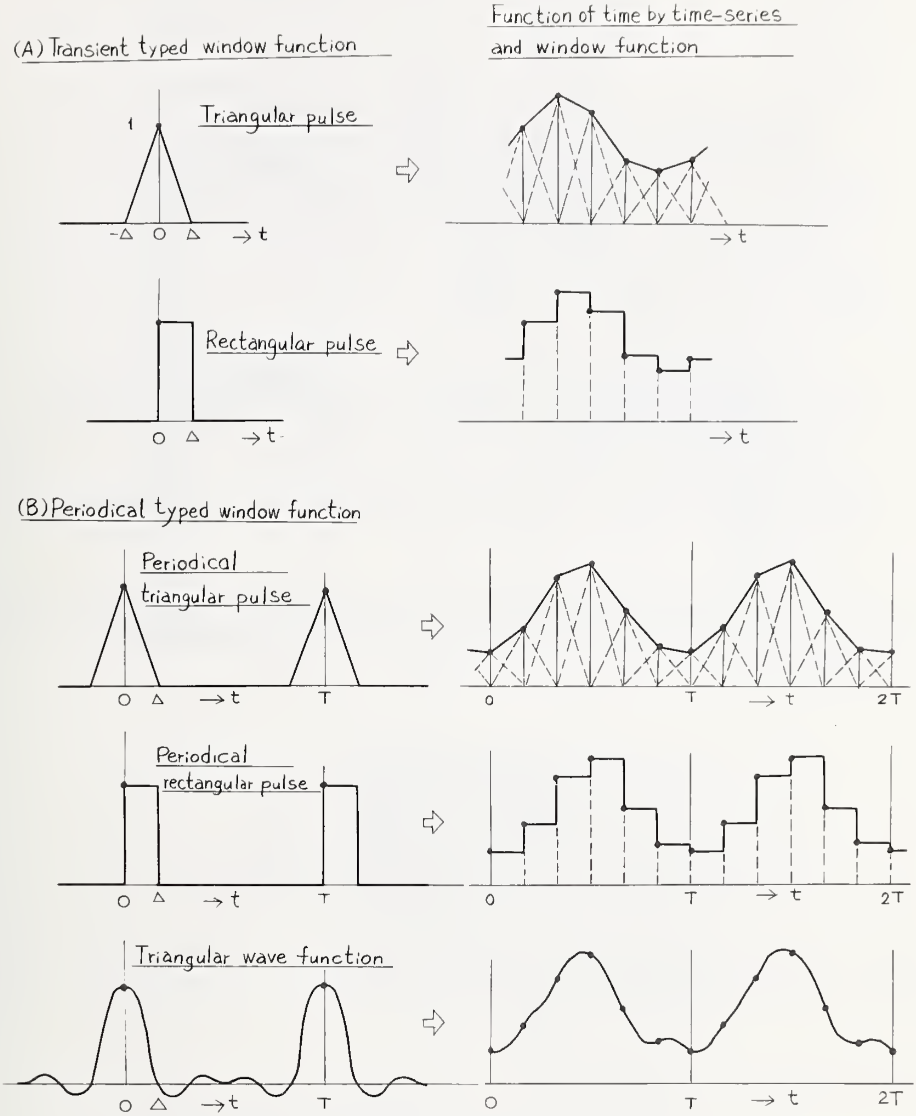
In this case, thermal matrix $\left\{\begin{matrix}TH\\H\end{matrix}\right\}$ is equal to heat flow matrix $\{TH\}$. (c.f. table 1) Small matrices at each time enclosed by bold line of table 16. are to be successively solved. In compliance with the initial conditions, start calculation and with the first small matrix solved, determine all room temperature at that time and advance to the next time. At the next time, treat the room temperature just determined (room temperature just one time segment before) as known quantity and with the next small matrix solved, determine the room temperature at the next time. (cf. table 1). Successively solving the small matrices, advance calculation.

(2) In case room temperature is specified (without warming-up period)

As shown in table 2, dimension of the simultaneous equations to be solved at that time decrease comparing with the case of 7.(1) (cf. table 16), however, the method is basically the same as above (cf. table 2, 3, table 17).

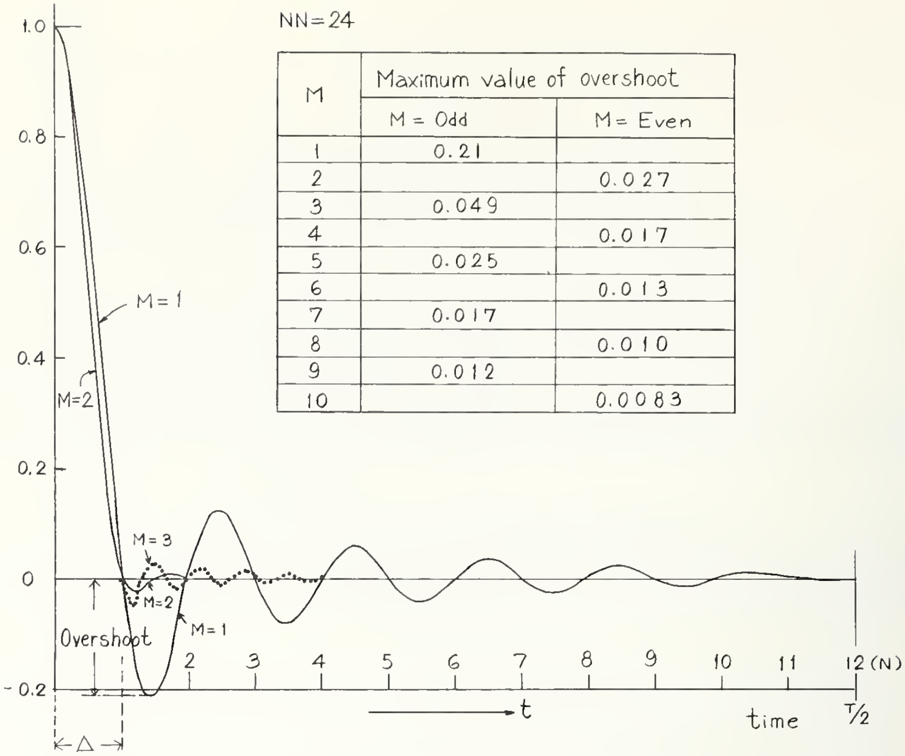
(3) In case warming-up period is provided

In a simple example shown in the table 17, small matrix expanded until the time when warming-up is finished must be solved. (cf. table 17) Considering the case when warming-up periods in different rooms and specified temperature period are overlapped, the said expression must be corrected as follows for making the expression more strict, i.e. "small matrix expanded from the time when warming-up period is started in any room to the time when warming-up periods are finished in all rooms". However, same as the case shown in table 3, dimension of the simultaneous equation decrease less than appearance.



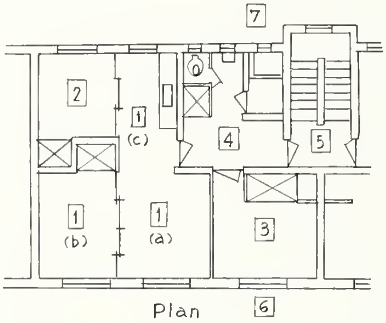
Notes : T ; Periodic time , Δ ; Time segment of time-series.

Fig. 1, Example of window function and function of time as represented through superposition of the former.



| M | Maximum value of overshoot | |
|---|---|---|
| | M = Odd | M = Even |
| 1 | 0.21 | |
| 2 | | 0.027 |
| 3 | 0.049 | |
| 4 | | 0.017 |
| 5 | 0.025 | |
| 6 | | 0.013 |
| 7 | 0.017 | |
| 8 | | 0.010 |
| 9 | 0.012 | |
| 10 | | 0.0083 |

Fig. 2, Example of trianguler wave function ($\mathrm{twf}_{NN}^{(M)}$ ) and relation between its order(M) and overshoot.

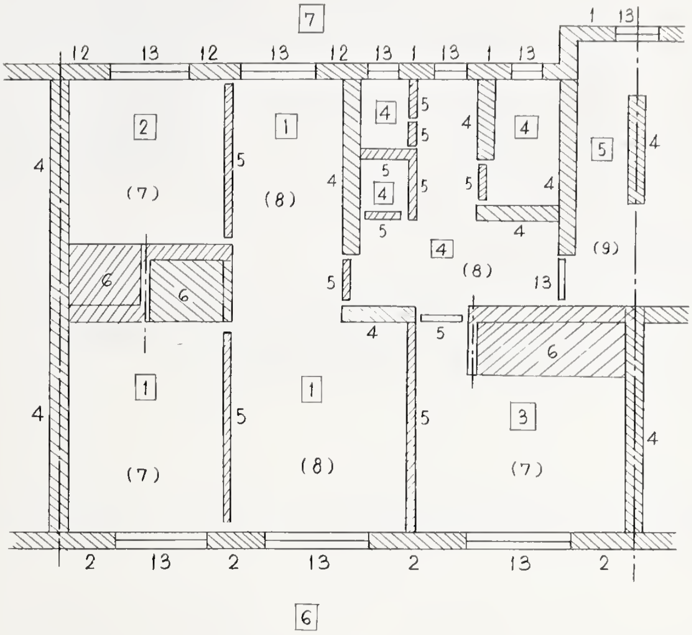


| Room No. | Description |
|---|---|
| 1 | Living rm.(a), Bed rm.(b), Kitchen (c) * |
| 2 | Bed rm. and study rm. for child ( I ) |
| 3 | Bed rm. and study rm. for child ( II ) |
| 4 | Entrance hall, bath rm., lavatory * |
| 5 | Stair-case (common use ) |
| 6 | Out side air ( South side ) } * * |
| 7 | ditto (North side ) |

*; Room(a),(b) and(c) are gathered and computed as room 1.

**; Assuming that temperature of 6 is equal to that of 7.

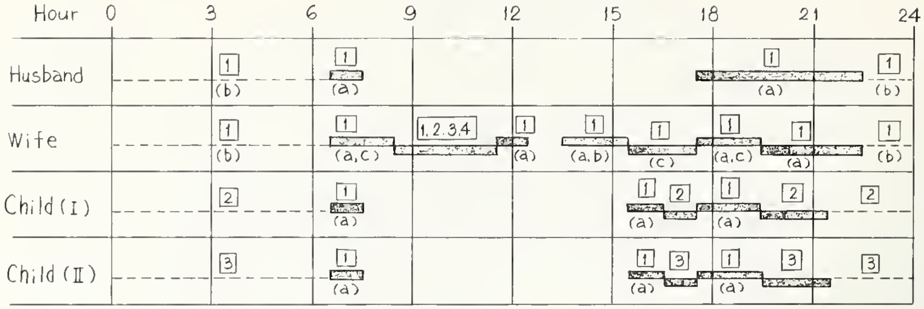
Fig. 3, Example problem (I), Heating load of dwelling. Sketch of dwelling used for calculation.



Notes; Figures in ☐ show each room No..

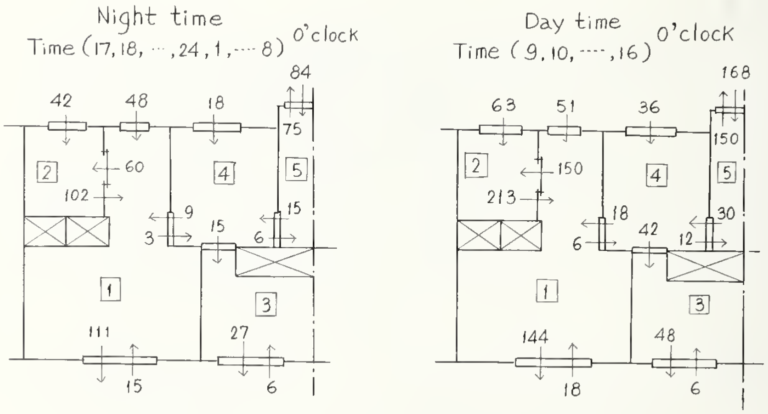
Figures in ( ) show No. of each floor. } (See Table 4)
Others show No. of each wall. }

Fig. 4, Example problem (I), Heating load of dwelling. Arrangement of wall and floor.



Note : Dotted lines indicate sleeping hour and figures in ☐ indicate the number of room.

Fig. 5, Example problem (I), Heating load of dwelling. Assumed living pattern of an occupant. (Number of family members : 4)



Unit : [ m³·h⁻¹ ]

Fig. 6, Example problem (I), Heating load of dwelling. Natural ventilation between rooms.

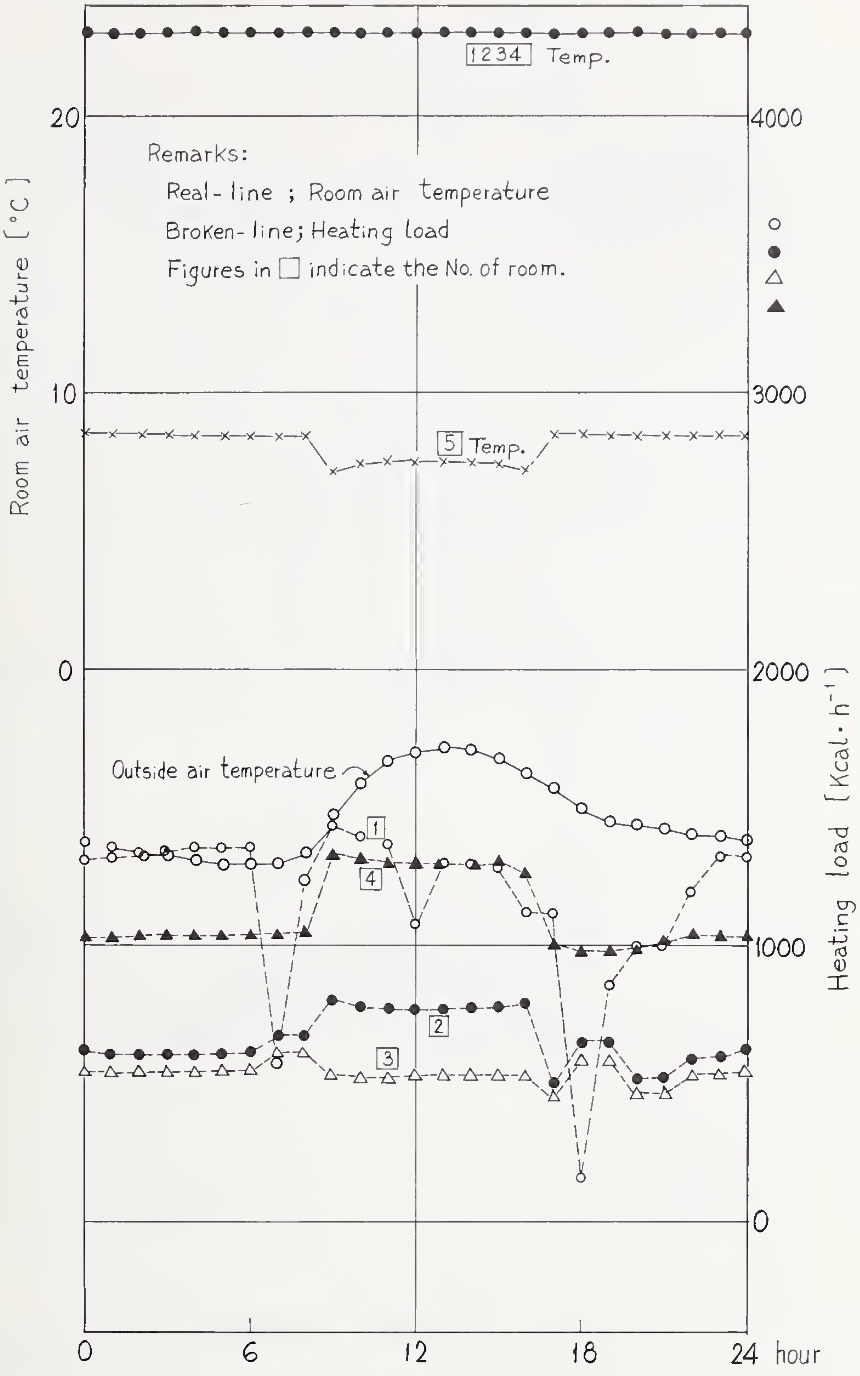


Fig. 7, Example problem (I), Heating load of dwelling. ( Heating pattern A )

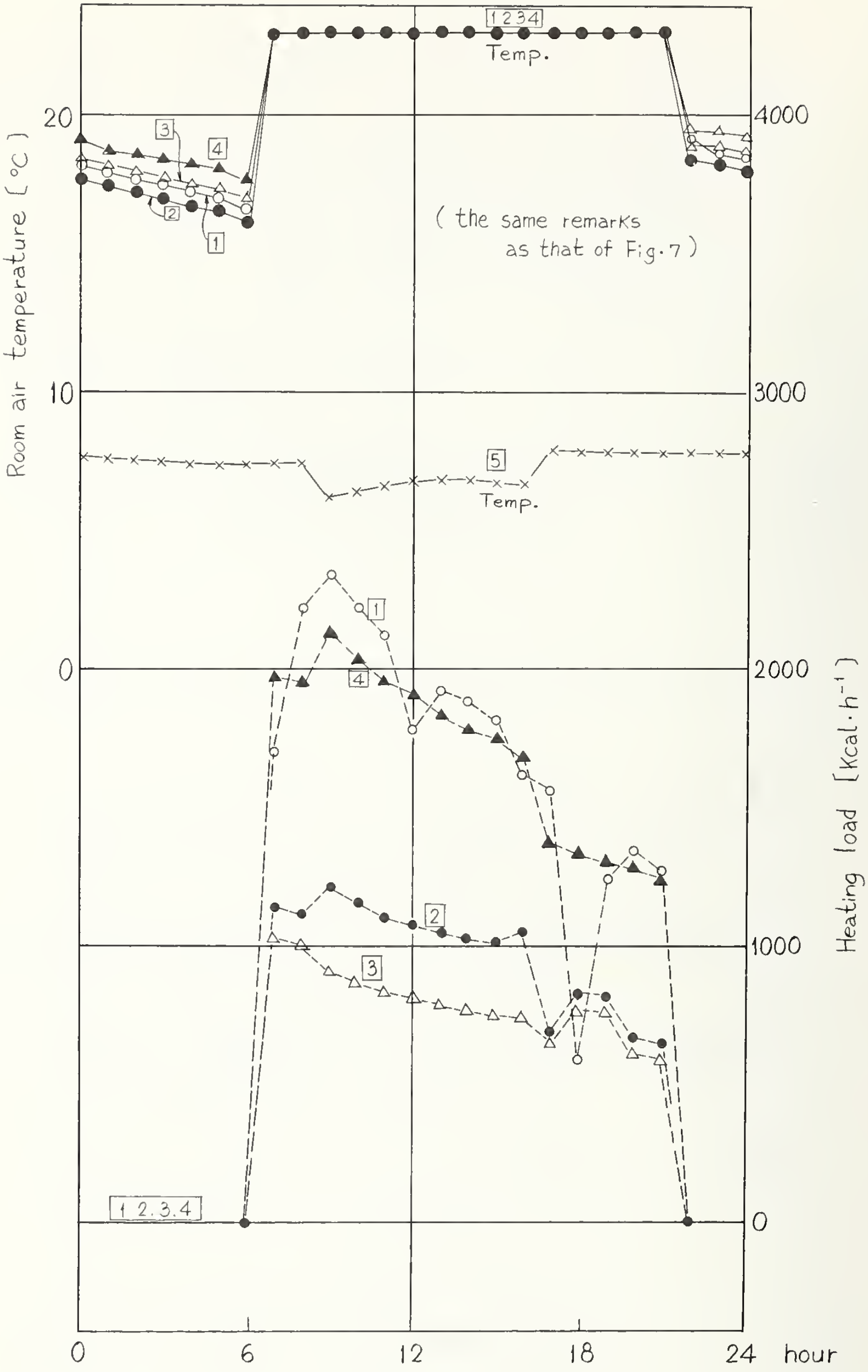


Fig. 8, Example problem (I), Heating load of dwelling. ( Heating pattern B )

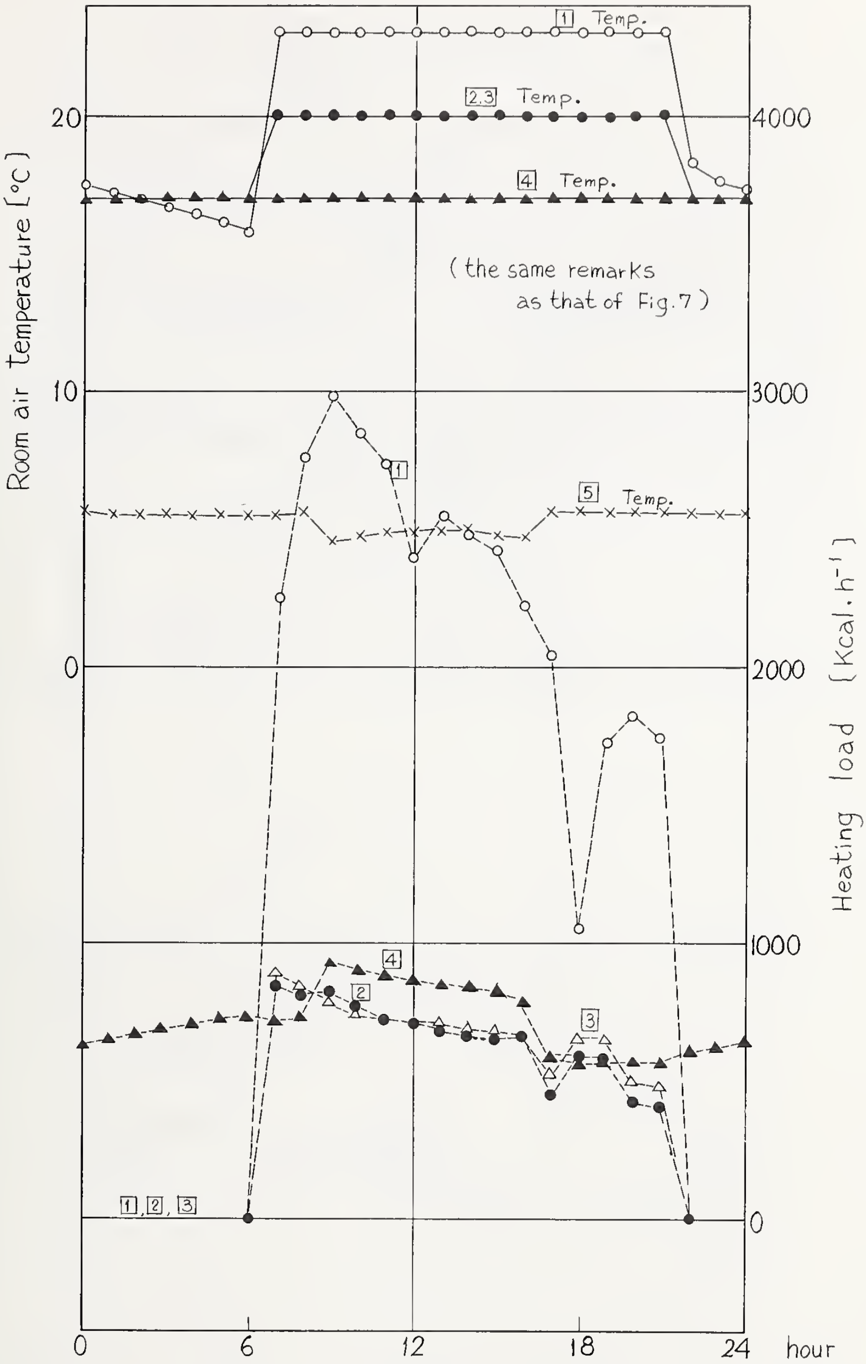


Fig. 9, Example problem (I), Heating load of dwelling. ( Heating pattern C )

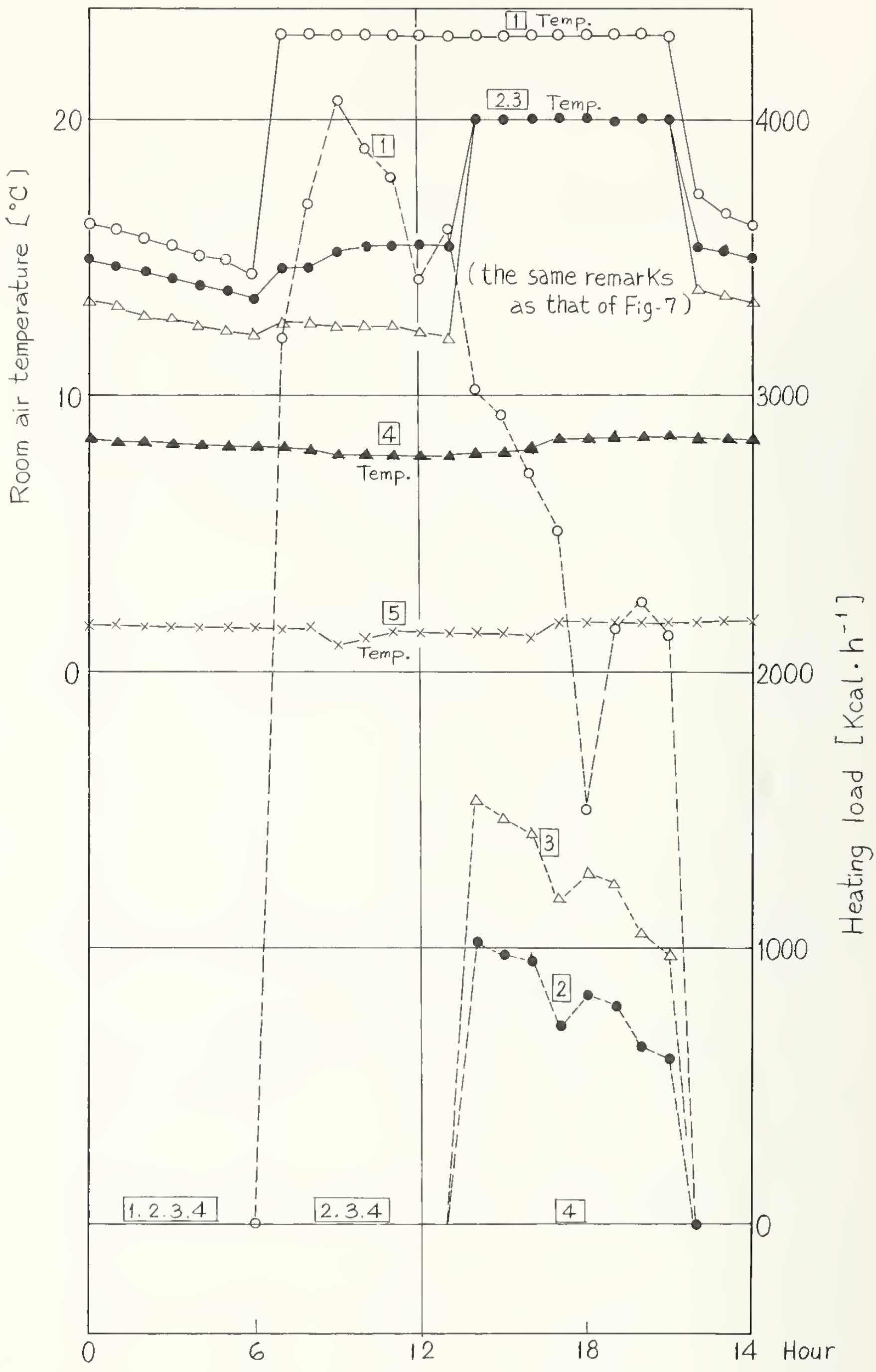


Fig.10, Example problem (I), Heating load of dwelling. ( Heating pattern D )

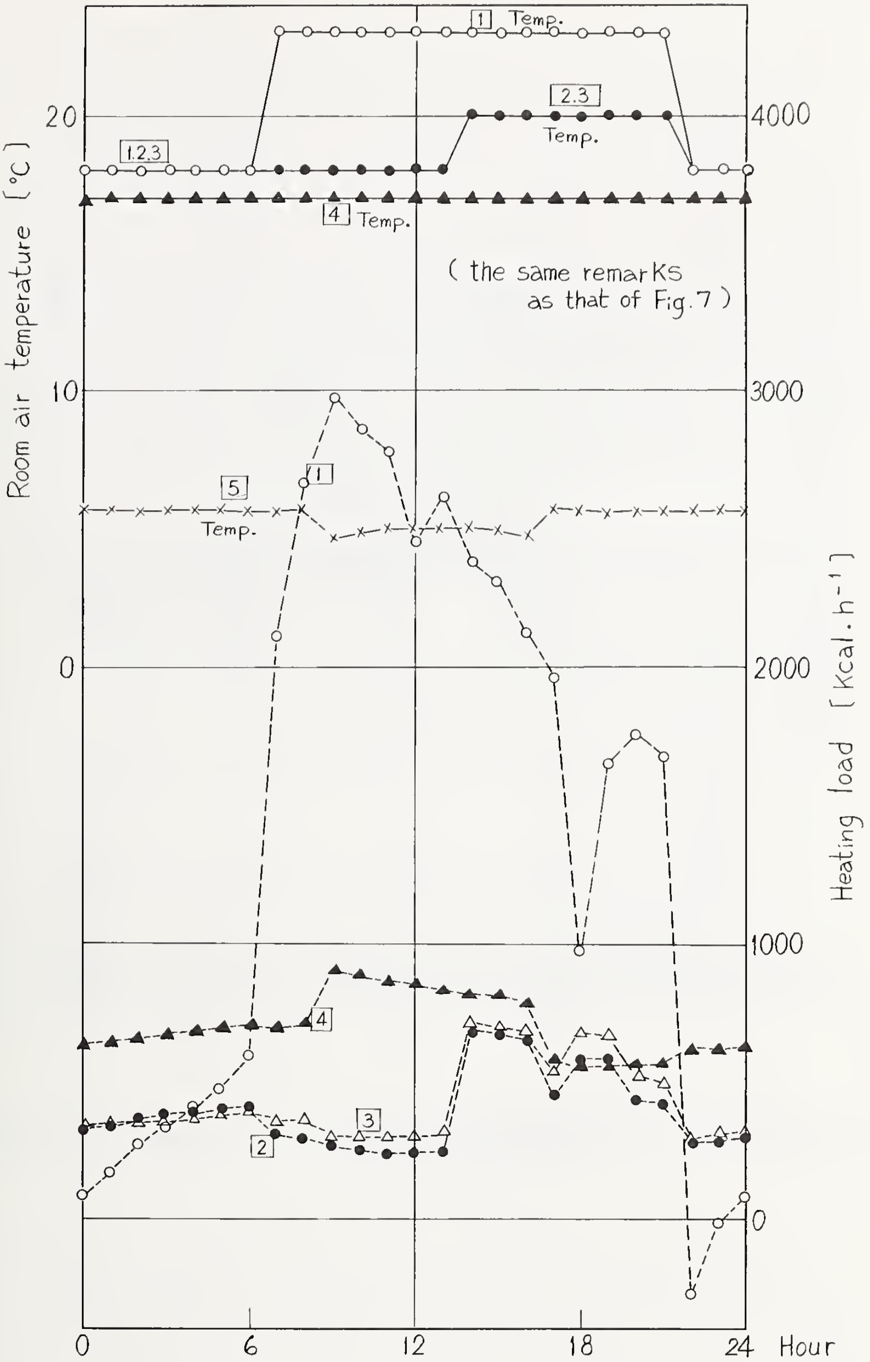


Fig.11, Example problem (I), Heating load of dwelling. ( Heating pattern E )

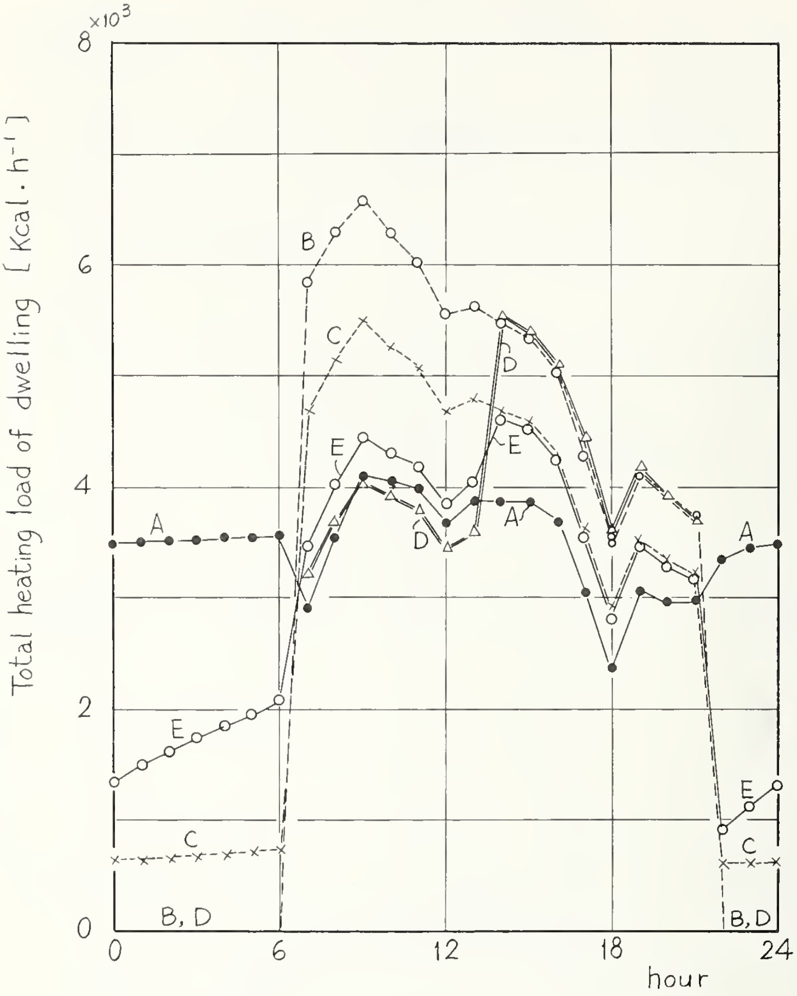
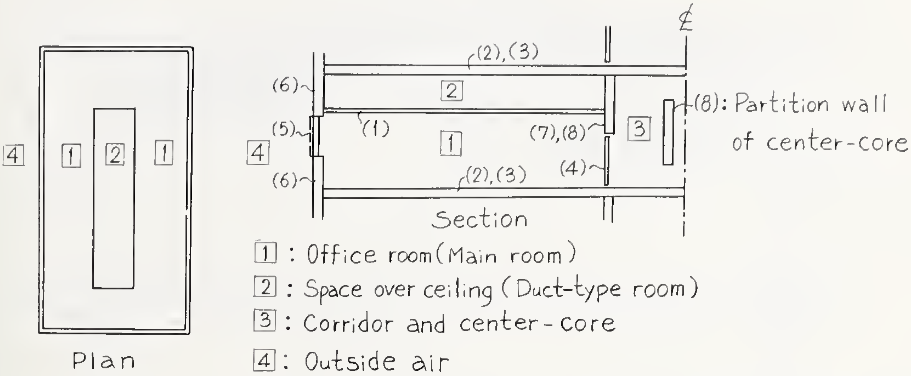


Fig.12, Example problem (I), Heating load of dwelling. Total heating load of dwelling on each heating pattern.



$\boxed{1}$ : Office room(Main room)
$\boxed{2}$ : Space over ceiling (Duct-type room)
$\boxed{3}$ : Corridor and center-core
$\boxed{4}$ : Outside air

Note : Figures in ( ) designate the number of wall and floor indicating their composition. ( See Table 10 ).

Fig.13, Example problem (II), Office building with duct-type room. Sketch of office building used for calculation.

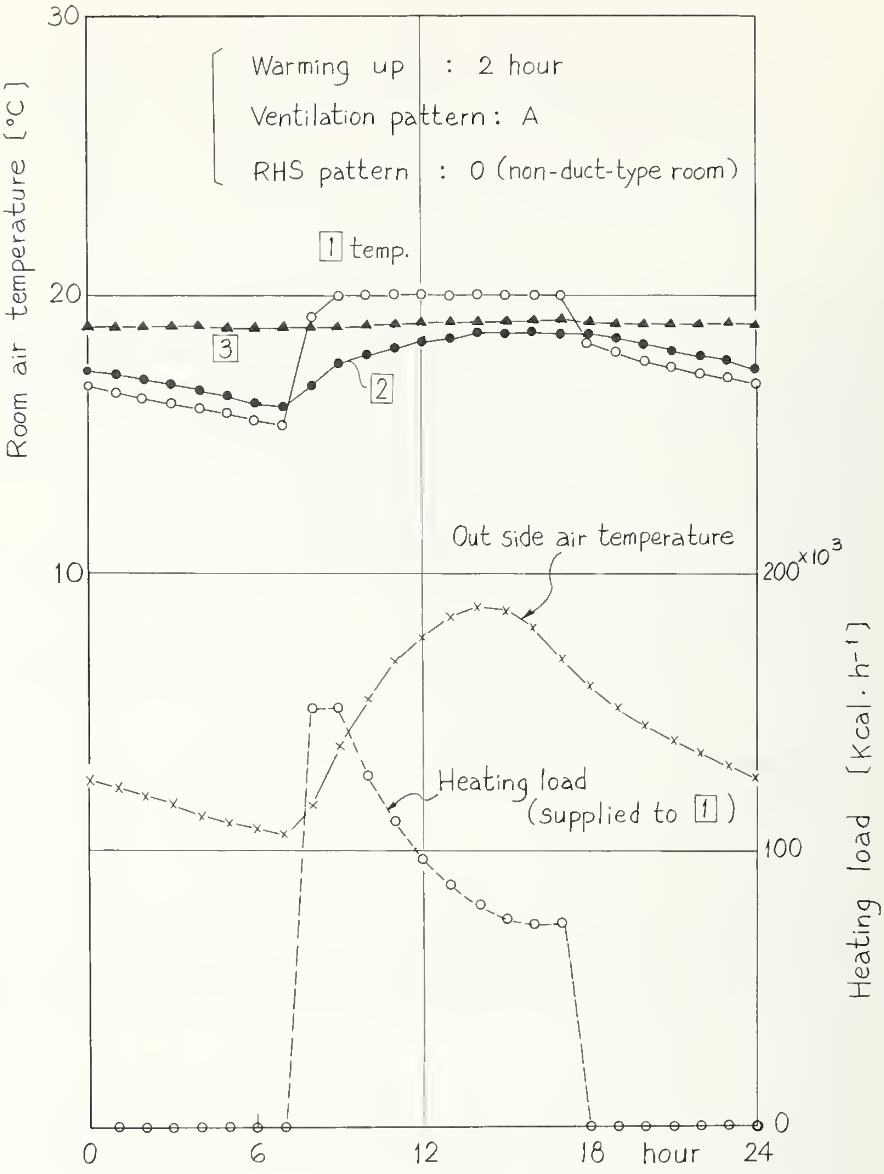


Fig.14, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 2,A,0 )

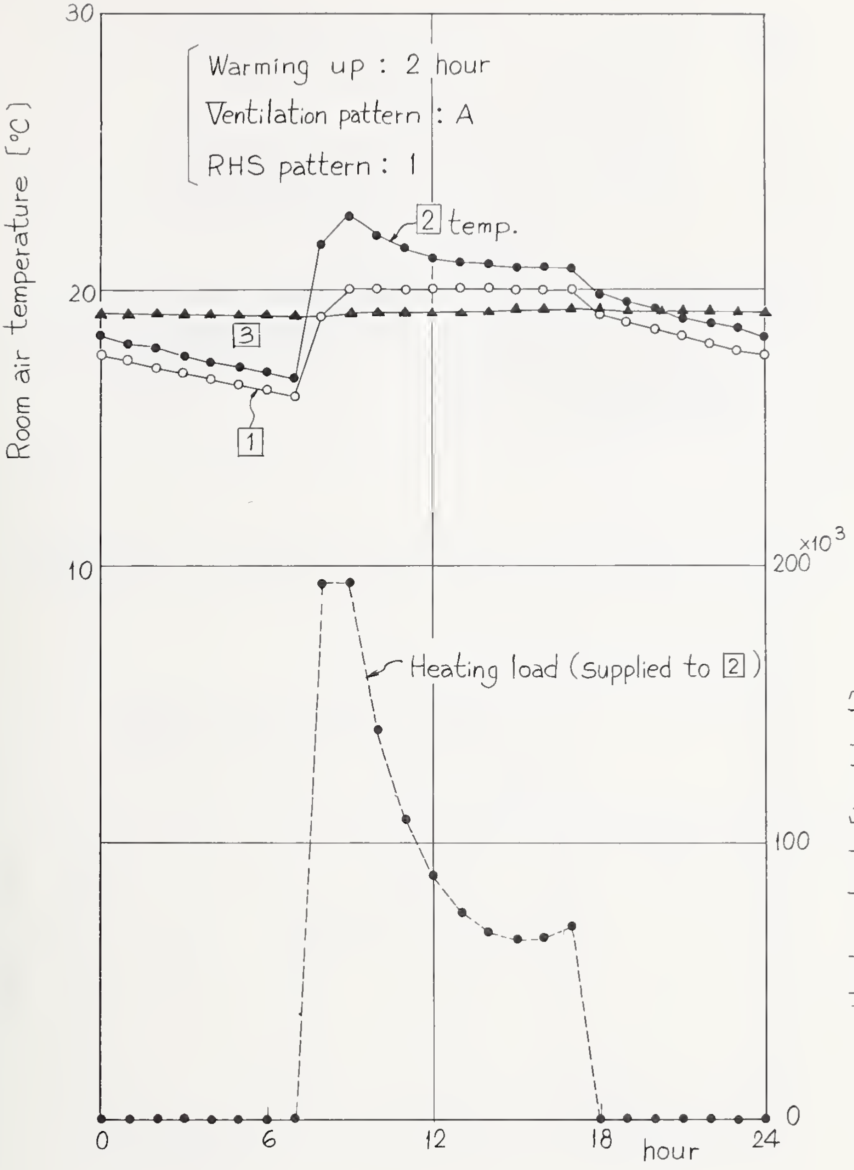


Fig.15, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 2,A,1 )

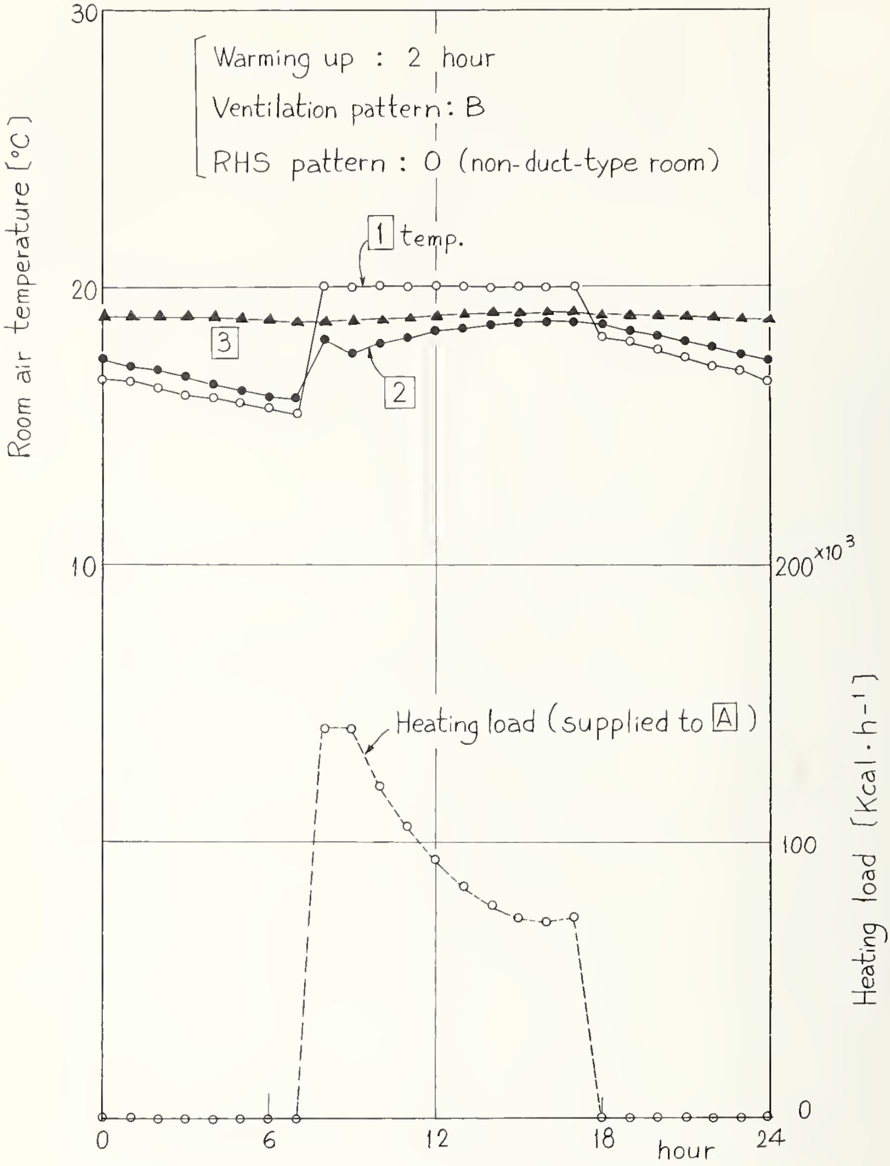


Fig.16, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 2,B,0 )

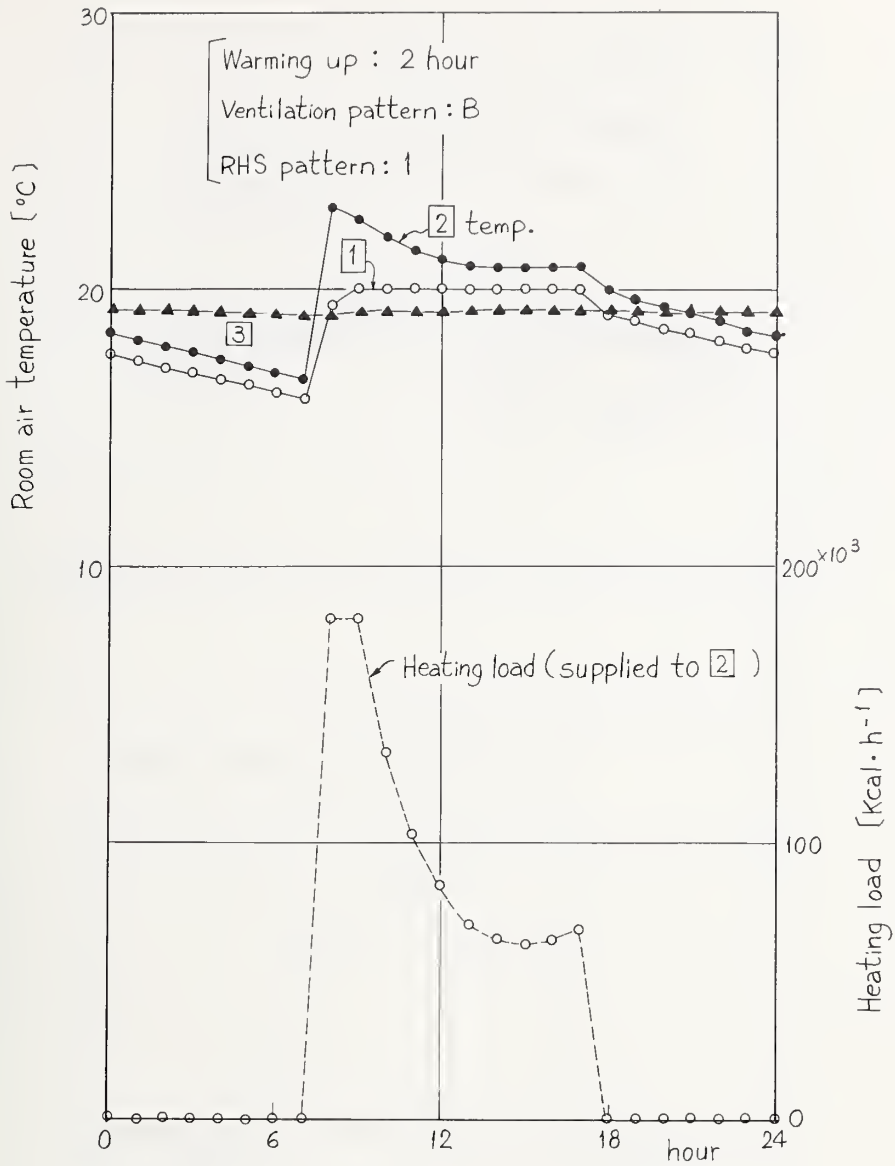


Fig.17, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 2,B,1 )

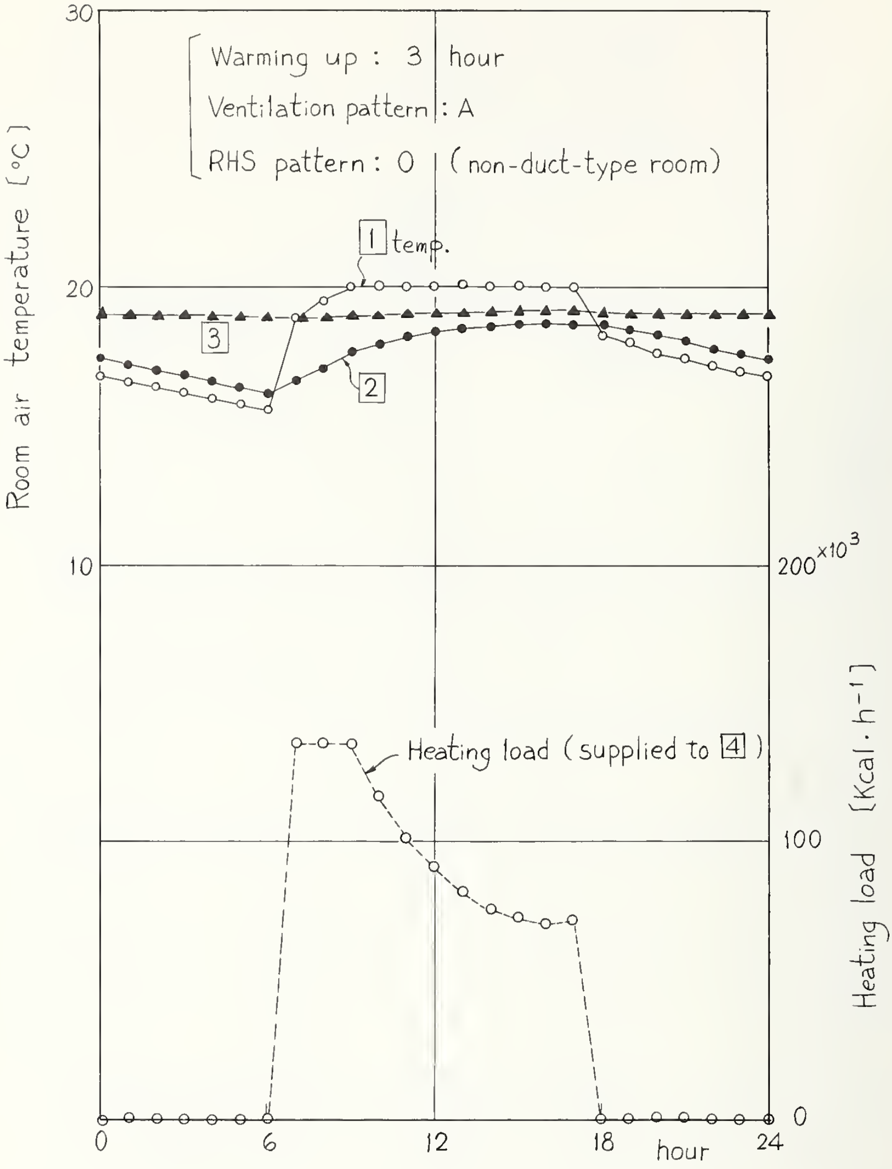


Fig.18, Example problem (II), Office building with duct-type room. Combination of design condition : ( 3,A,0 )

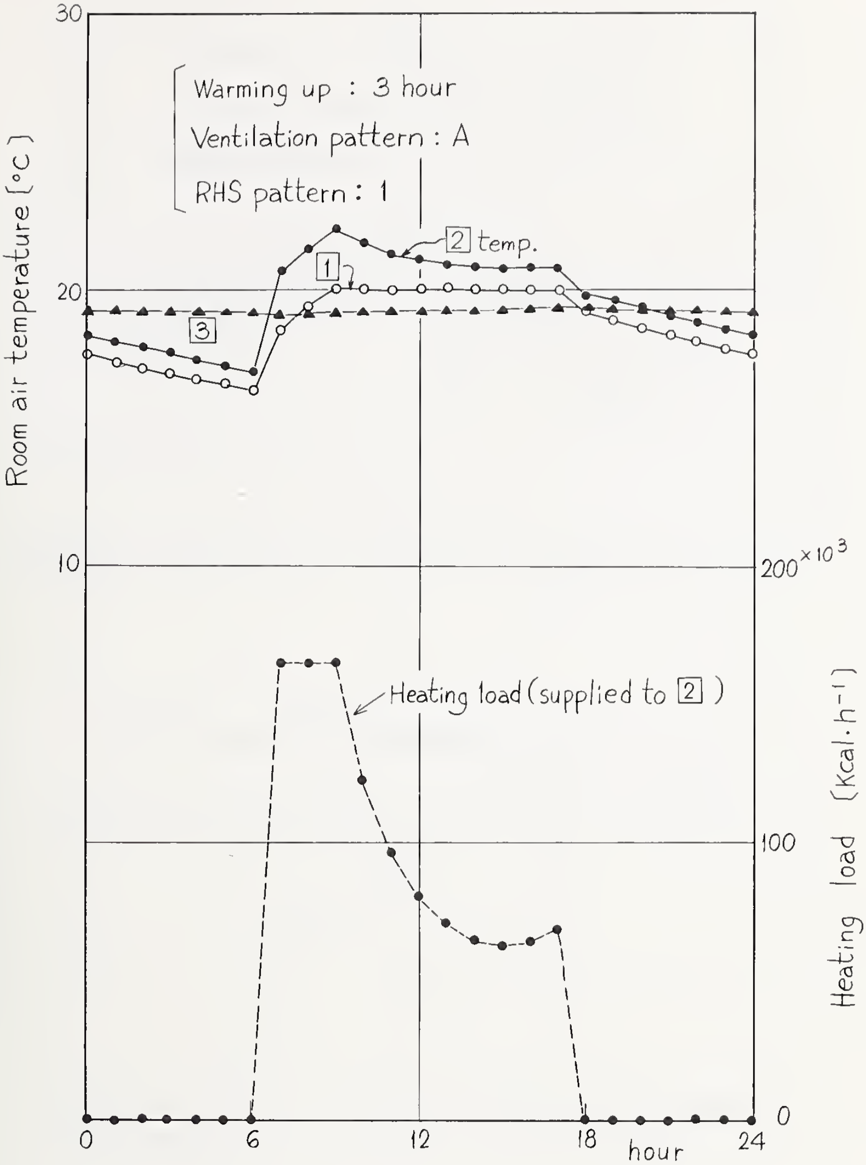


Fig.19, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 3,A,1 )

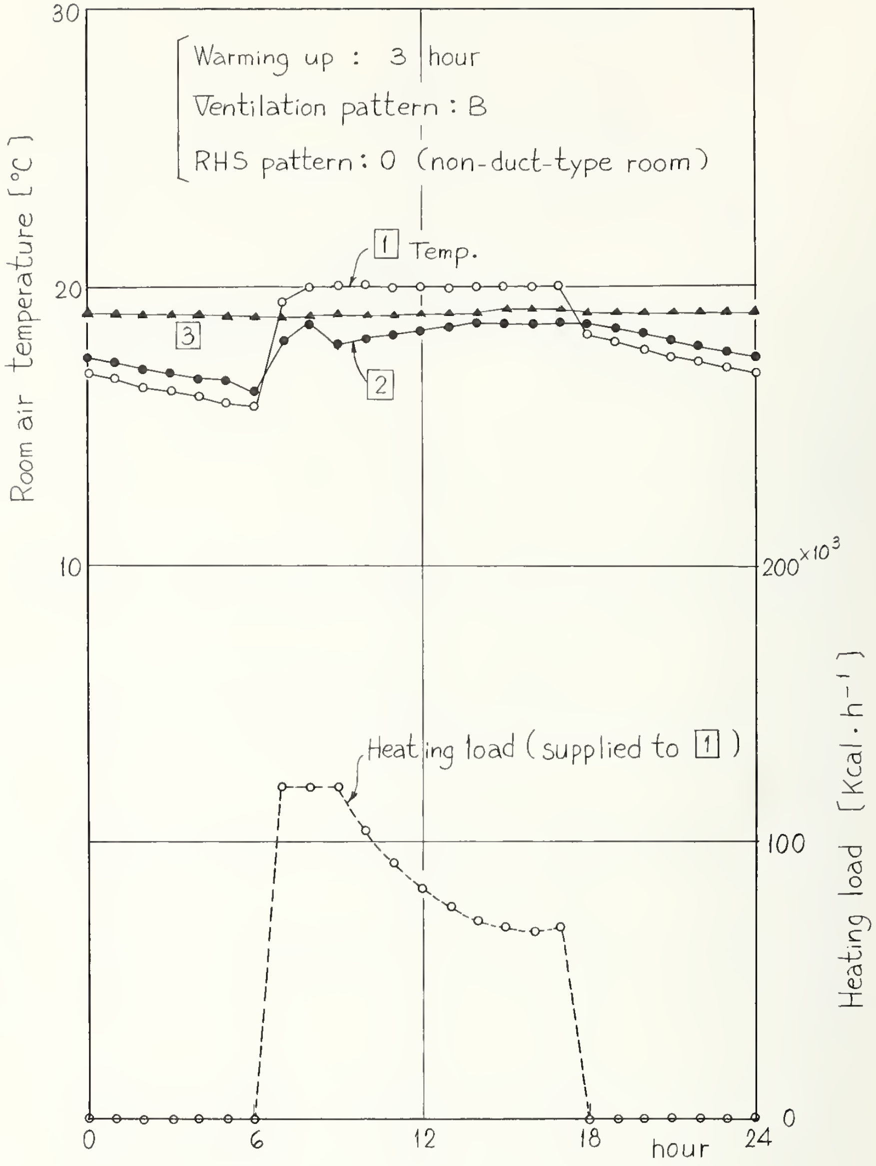


Fig.20, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 3,B,0 )

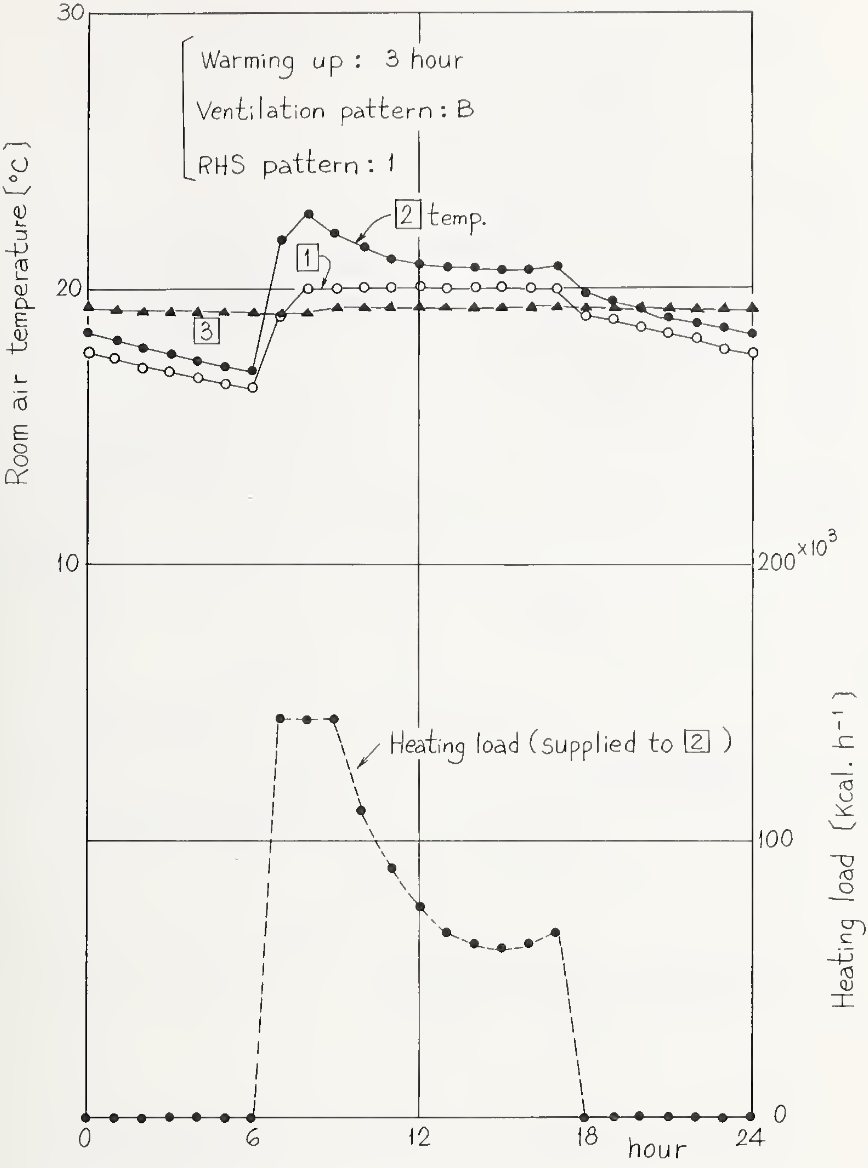


Fig.21, Example problem (II), Office building with duct-type room.
Combination of design condition : ( 3,B,1 )

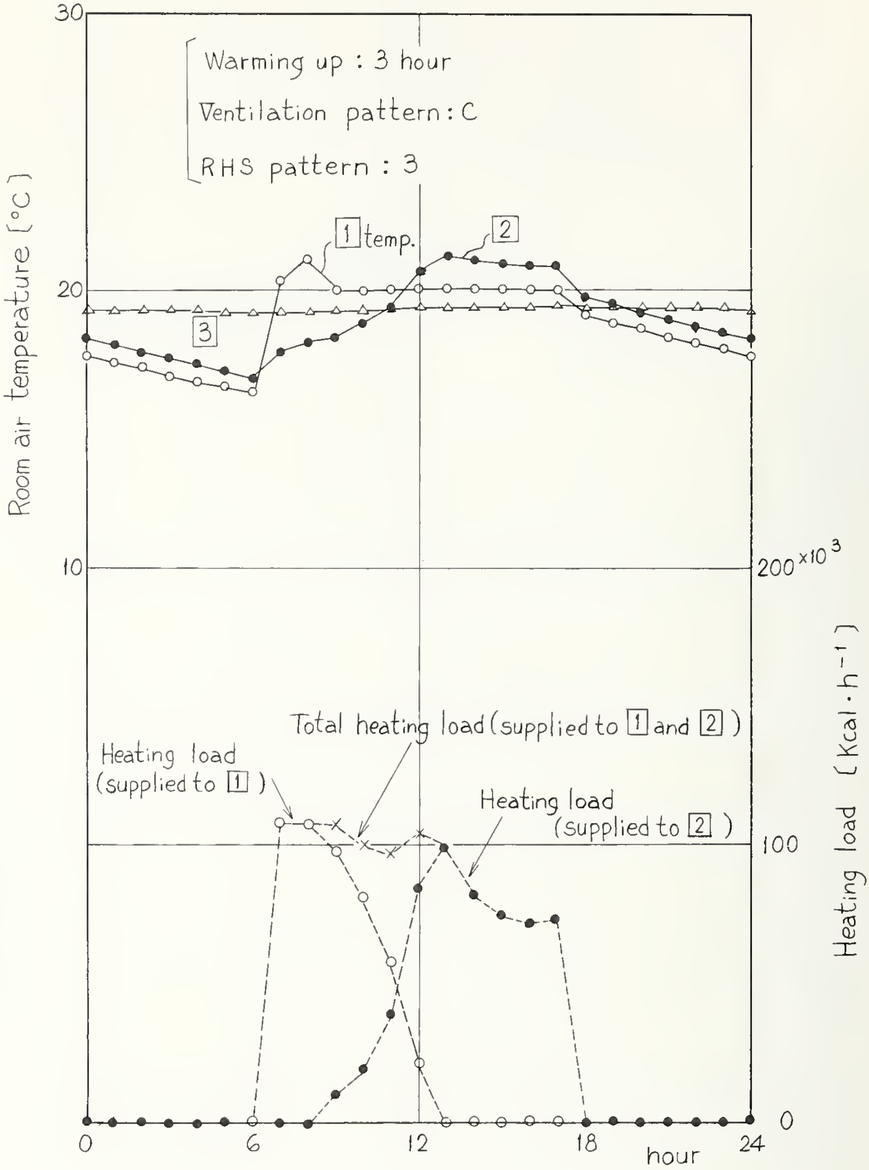


Fig.22, Example problem (II), Office building with duct-type room.
Combination of design condition ; ( 3,C,3 )

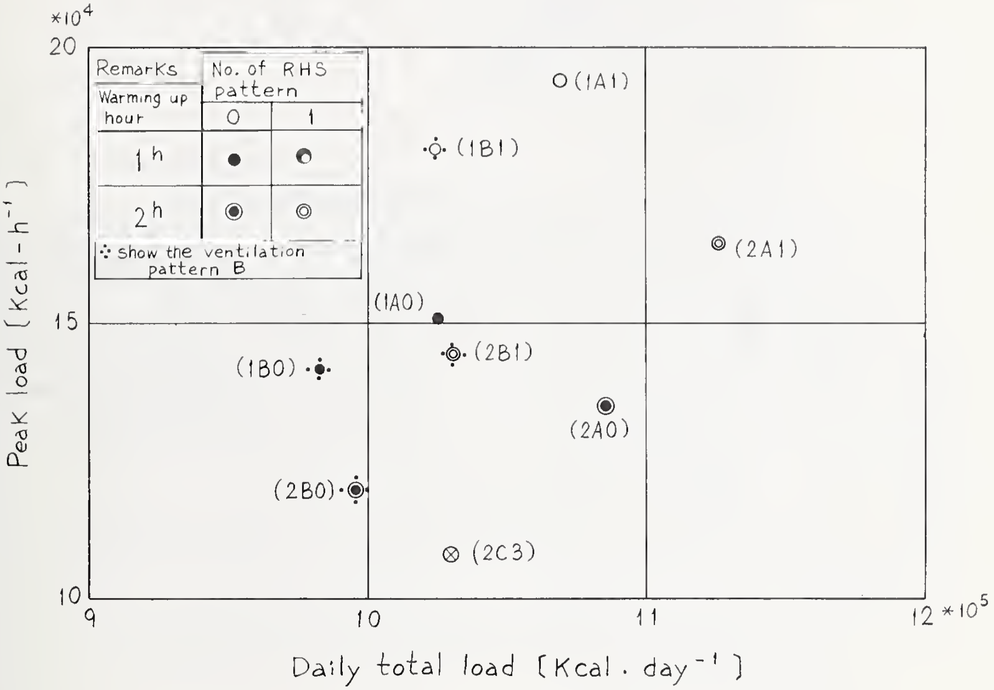


Fig.23, Example problem (II), Office building with duct-type room.
Peak load and daily total load.

Table 1. An example of heat flow matrix [TH]

| M \ N | J \ I | 1 | | 2 | | 3 | | 4 | | ← Time |
|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | A | B | A | B | A | B | ← Room |
| 1 | A | ▨ | ╳ | ╱ | — | ╱ | — | ╱ | — | |
| | B | ╳ | ▨ | — | ╱ | — | ╱ | — | ╱ | |
| 2 | A | ╱ | — | ▨ | ╳ | ╱ | — | ╱ | — | |
| | B | — | ╱ | ╳ | ▨ | — | ╱ | — | ╱ | |
| 3 | A | ╱ | — | ╱ | — | ▨ | ╳ | ╱ | — | |
| | B | — | ╱ | — | ╱ | ╳ | ▨ | — | ╱ | |
| 4 | A | ╱ | — | ╱ | — | ╱ | — | ▨ | ╳ | |
| | B | — | ╱ | — | ╱ | — | ╱ | ╳ | ▨ | |
| ↑ Time | ↑ Room | | | | | | | | | |

NN = 4

JJ = 2

NXJ = 8

Symbols used to show the composition of matrix element

| | | Symbol | Wall and Floor | | Ventilation | | Furniture | Room air | Description |
|---|---|---|---|---|---|---|---|---|---|
| | | | WI | WO | VI | VO | HFA | HAA | Detailed term of matrix element |
| I = J | N = M | ▨ | | ● | | ● | ● | ● | A matrix element shown in the symbol column left side represents summation marked terms on the righthand. |
| | N ≠ M | ╱ | | ○ | | | ○ | ○ | |
| I ≠ J | N = M | ╳ | ● | | ● | | | | |
| | N ≠ M | — | ○ | | | | | | |

Notes

● : Value of response to window function input when time is zero.

○ : Values of response to window function input when time is not zero.

Table 2. Thermal matrix $\left[\begin{smallmatrix}TH\\H\end{smallmatrix}\right]$ in which warming-up period is not included.

Time schedule

| | Period of designated temperature (β) |
|---|---|
| Room A | time 2, 3 |
| Room B | 〃 3, 4 |

| | $\left[\begin{smallmatrix}TH\\H\end{smallmatrix}\right]$ | | 1 | | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | A | B | A | B | A | B | A | B |
| | 1 | A | | | 0 | | 0 | 0 | | 0 |
| | 1 | B | | | 0 | | 0 | 0 | | 0 |
| (β) | 2 | A | | | 1 | | 0 | 0 | | 0 |
| | 2 | B | | | 0 | | 0 | 0 | | 0 |
| (β) | 3 | A | | | 0 | | 1 | 0 | | 0 |
| (β) | 3 | B | | | 0 | | 0 | 1 | | 0 |
| | 4 | A | | | 0 | | 0 | 0 | | 0 |
| (β) | 4 | B | | | 0 | | 0 | 0 | | 1 |

(Empty space is same as [TH] matrix element)

⇨ Exchange of rows and of columns

| | unknown → | ← T | | | → | ← H | | | → | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 1 | 2 | 4 | 2 | 3 | 3 | 4 | ← Time |
| | | A | B | B | A | A | A | B | B | ← Room |
| 1 | A | ▨ | ╳ | — | ╱ | | | | | |
| 1 | B | ╳ | ▨ | ╱ | — | | 0 | | | |
| 2 | B | — | ╱ | ▨ | — | | | | | |
| 4 | A | ╱ | — | — | ▨ | | | | | |
| 2 | A | ╱ | — | ╳ | ╱ | 1 | | | | |
| 3 | A | ╱ | — | — | ╱ | | 1 | | | |
| 3 | B | — | ╱ | ╱ | — | 0 | | 1 | | |
| 4 | B | — | ╱ | ╱ | ╳ | | | | 1 | |

(Portion enclosed with bold line is coefficient matrix of simultaneous linear equation to solved.)

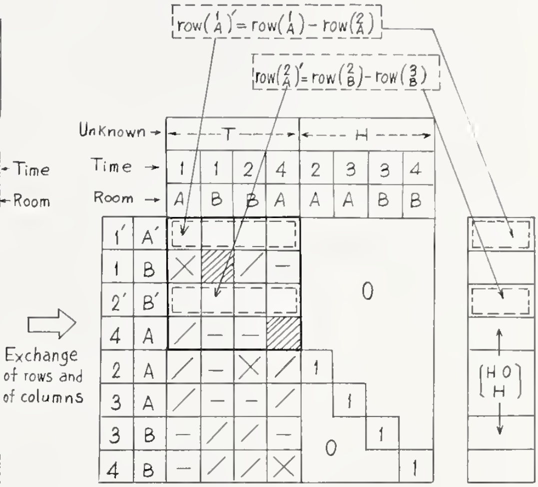
Table 3. Thermal matrix $\begin{bmatrix} TH \\ H \end{bmatrix}$ in which warming-up period is included.

Time Schedule

| | Warming up (α) | Designated (β) |
|---|---|---|
| Room A | Time 1 | 2, 3 |
| Room B | 2 | 4 |

| | $\begin{bmatrix} TH \\ H \end{bmatrix}$ | | 1 | | 2 | | 3 | | 4 | | ← Time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | A | B | A | B | A | B | A | B | ← Room |
| (α) | 1 | A | | | 1 | | 0 | 0 | | 0 | |
| | 1 | B | | | 0 | | 0 | 0 | | 0 | |
| (β) | 2 | A | | | 1 | | 0 | 0 | | 0 | |
| (α) | 2 | B | | | 0 | | 0 | 1 | | 0 | |
| (β) | 3 | A | | | 0 | | 1 | 0 | | 0 | |
| (β) | 3 | B | | | 0 | | 0 | 1 | | 0 | |
| | 4 | A | | | 0 | | 0 | 0 | | 0 | |
| (β) | 4 | B | | | 0 | | 0 | 0 | | 1 | |

(Empty space is same as [TH] matrix element)



(Portion enclosed with bold line is coefficient matrix of simultaneous linear equation to solved.)

Table 4. Example problem (I), Heating load of dwelling.
Composition and area of walls and floors (continued).

| No. | Building element | Composition of material and its layered | Thickness [mm] | Heat transfer coefficient [Kcal.$h^{-1}$.$m^{-2}$.°$c^{-1}$] | Facing rooms | Area [ $m^2$ ] |
|---|---|---|---|---|---|---|
| 1 | Outside wall | Air (outside surface) | | 15 | | |
| | | Concrete | 210 | | $\boxed{4}\leftrightarrow\boxed{7}$ | 9.9 |
| | | Insulation | 15 | | $\boxed{5}\leftrightarrow\boxed{7}$ | 4.4 |
| | | Ply-wood | 4 | | | |
| | | Air (inside surface) | | 6 | | |
| 2 | ditto. | ditto. (except insulation thickness 30) | | | $\boxed{1}\leftrightarrow\boxed{6}$ | 11.9 |
| | | | | | $\boxed{3}\leftrightarrow\boxed{6}$ | 8.2 |
| 12 | ditto. | ditto. (except insulation thickness 50) | | | $\boxed{1}\leftrightarrow\boxed{7}$ | 5.1 |
| | | | | | $\boxed{2}\leftrightarrow\boxed{7}$ | 6.8 |
| 13 | Window | Air (Outside surface) | | 15 | $\boxed{1}\leftrightarrow\boxed{6}$ | 6.4 |
| | | Glass | 4 | | $\boxed{1}\leftrightarrow\boxed{7}$ | 1.3 |
| | | Air space | | 5 | $\boxed{2}\leftrightarrow\boxed{7}$ | 1.9 |
| | | Glass | 4 | | $\boxed{3}\leftrightarrow\boxed{6}$ | 3.4 |
| | | Air (inside surface) | | 6 | $\boxed{4}\leftrightarrow\boxed{5}$ | 1.6 * |
| | | | | | $\boxed{4}\leftrightarrow\boxed{7}$ | 1.4 |
| | | | | | $\boxed{5}\leftrightarrow\boxed{7}$ | 0.7 |
| 4 | Partition (Heavy construction) | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{4}$ | 14.4 |
| | | Concrete | 180 | | $\boxed{4}\leftrightarrow\boxed{5}$ | 11.2 |
| | | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{1}$ | 13.5 |
| | | | | | $\boxed{2}\leftrightarrow\boxed{2}$ | 11.6 |
| | | | | | $\boxed{3}\leftrightarrow\boxed{3}$ | 11.9 |
| | | | | | $\boxed{4}\leftrightarrow\boxed{4}$ | 20.6 |
| | | | | | $\boxed{5}\leftrightarrow\boxed{5}$ | 5.8 |
| 5 | Partition, (Light-weight construction)<br>Door,<br>Japanese sliding screen | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{2}$ | 9.6 |
| | | Ply-wood | 4 | | $\boxed{1}\leftrightarrow\boxed{3}$ | 12.0 |
| | | Air space | | 5 | $\boxed{1}\leftrightarrow\boxed{4}$ | 1.6 |
| | | Ply-wood | 4 | | $\boxed{3}\leftrightarrow\boxed{4}$ | 1.6 |
| | | Air (Surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{1}$ | 23.8 |
| | | | | | $\boxed{4}\leftrightarrow\boxed{4}$ | 12.2 |

Table 4. Example problem (I), Heating load of dwelling.
Composition and area of walls and floors (concluded).

| No. | Building element | Composition of material and its layered | Thickness (m.m) | Heat transfer coefficient $[\mathrm{Kcal} \cdot h^{-1} \cdot m^{-2} \cdot {}^{\circ}c^{-1}]$ | Facing rooms | Area $(m^2)$ |
|---|---|---|---|---|---|---|
| 6 | Closet | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{2}$ | 4.35 |
| | | Concrete | 150 | | | |
| | | Air (surface) | | 5 | | |
| | | Air space (closet)<br>Sliding screen of closet<br>Air (surface) | Assuming → 5 | | $\boxed{3}\leftrightarrow\boxed{4}$<br>$\boxed{3}\leftrightarrow\boxed{5}$ | 6.5<br>3.5 |
| 6′ | Closet | ditto(except turn inside out) | | | $\boxed{2}\leftrightarrow\boxed{1}$ | 4.35 |
| 7 | Floor and ceiling (Tatami) | Air (above surface) | | 8 | $\boxed{1}\leftrightarrow\boxed{1}$ | 28.0 |
| | | Tatami (flooring) | 50 | | $\boxed{2}\leftrightarrow\boxed{2}$ | 23.2 |
| | | Ply - wood | 4 | | $\boxed{3}\leftrightarrow\boxed{3}$ | 32.8 |
| | | Air space | | 5 | | |
| | | Concrete | 120 | | | |
| | | Air space | | 4 | | |
| | | Ply - wood (ceiling) | 4 | | | |
| | | Air (under surface) | | 8 | | |
| 8 | Floor and ceiling (Asphalt tile) | Air (above surface) | | 8 | $\boxed{1}\leftrightarrow\boxed{1}$ | 46.4 |
| | | Asphalt tile (flooring) | 3 | | $\boxed{4}\leftrightarrow\boxed{4}$ | 34.4 |
| | | Ply- wood | 4 | | | |
| | | Air space | | 5 | | |
| | | Concrete | 120 | | | |
| | | Air space | | 4 | | |
| | | Ply - wood (ceiling) | 4 | | | |
| | | Air ( under surface) | | 8 | | |
| 9 | Floor | Air (above surface) | | 8 | $\boxed{5}\leftrightarrow\boxed{5}$ | 6.7 |
| | | Concrete | 180 | | | |
| | | Air (under surface) | | 8 | | |

*; The thermal response of entrance door is assumed as the same as that of wall No. 13.

Table 5. Example problem (I), Heating load of dwelling.
Thermal constant of materials.

| Materials | Thermal conductivity [$Kcal \cdot h^{-1} \cdot m^{-1} \cdot °C^{-1}$] | Thermal diffusivity [$m^2 \cdot h^{-1}$] $* 10^4$ |
|---|---|---|
| Concrete | 1.4 | 25.0 |
| Insulation | 0.04 | 8.0 |
| Ply-wood | 0.14 | 4.0 |
| Tatami | 0.13 | 7.0 |
| Asphalt tile | 0.28 | 5.8 |
| Glass | 0.7 | 14.0 |

Table 6. Example problem (I), Heating load of dwelling.
Air volumeof each room.

| Room No. | Air volume [$m^3$] |
|---|---|
| 1 | 84.8 |
| 2 | 23.6 |
| 3 | 30.6 |
| 4 | 41.1 |
| 5 | 17.1 |

Table 7. Example problem (I), Heating load of dwelling. Time series of heat generation from lights, electric equipments, cooking and human bodies.

| Time | Room | | | |
|---|---|---|---|---|
| | 1 | 2, 3 | 4 | 5 |
| 23 ~ 6 | 133 | 60 | 13 | 0 |
| 7 | 922 | 0 | 13 | 0 |
| 8 | 248 | 0 | 0 | 0 |
| 9 ~ 11 | 45 | 16 | 0 | 0 |
| 12 | 320 | 0 | 0 | 0 |
| 13 | 90 | 0 | 0 | 0 |
| 14 | 93 | 0 | 0 | 0 |
| 15 | 114 | 0 | 0 | 0 |
| 16 | 287 | 0 | 43 | 0 |
| 17 | 274 | 132 | 43 | 0 |
| 18 | 1250 | 0 | 56 | 0 |
| 19 | 565 | 0 | 56 | 0 |
| 20 ~ 21 | 435 | 132 | 43 | 0 |
| 22 | 242 | 60 | 13 | 0 |

Unit : [$Kcal \cdot h^{-1}$]

Table 8. Example problem (I), Heating load of dwelling. Daily heating pattern ( Designated temperature and its time schedule ).

| Sign of heating pattern | Room No. | Designated temperature | Period in which temperature is kept as disignated | Description |
|---|---|---|---|---|
| A | 1 | 23 | 0~24 (Continuously) | Every room is kept continuously at 23 °C. |
| | 2 | | | |
| | 3 | | | |
| | 4 | | | |
| B | 1 | 23 | 7~21 | Every room is periodically heated to keep temperature at 23°C during 7-21 o'clock. |
| | 2 | | | |
| | 3 | | | |
| | 4 | | | |
| C | 1 | 23 | 7~21 | Room $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ are periodically heated during 7-21 o'clock, while the temperature in $\boxed{2}$ and $\boxed{3}$ is specified (20°C) lower than that in $\boxed{1}$ (23°C).<br>Room $\boxed{4}$ is kept at 17°C continuously. |
| | 2 | 20 | 7~21 | |
| | 3 | | | |
| | 4 | 17 | 0~24 | |
| D | 1 | 23 | 7~21 | Room $\boxed{1}$ is same as above.<br>Room $\boxed{2}$ and $\boxed{3}$ are heated at 20°C only when children use them.<br>Room $\boxed{4}$ is not heated. |
| | 2 | 20 | 14~21 | |
| | 3 | | | |
| | 4 | not heated | | |
| E | 1 | 23<br>18 | 7~21<br>Other hours | Automatic temperature control for varying temperatures in different time zones is assumed. |
| | 2 | | | |
| | 3 | 20<br>18 | 7~21<br>Other hours | |
| | 4 | 17 | 0~24 | |
| Note; Room $\boxed{5}$ is not heated any heating patterns. | | | | |

Table 9. Example problem (I), Heating load of dwelling. Peak load and daily total load in each heating pattern.

| Heating pattern | Peak load [Kcal·h$^{-1}$] | Daily total load [Kcal·day$^{-1}$] * 10$^{3}$ |
|---|---|---|
| A | 4 100 | 83.5 |
| B | 6 590 | 77.8 |
| C | 5 510 | 71.5 |
| D | 5 560 | 61.7 |
| E | 4 600 | 72.2 |

Table 10. Example problem (II), Office building with duct-type room.
Composition and area of walls and floors.

| No. | Building element | Composition of material and its layered | Thickness [m.m] | Heat transfer coefficient [$Kcal.h^{-1}.m^{-2}.°C^{-1}$] | Facing rooms | Area [$m^2$] |
|---|---|---|---|---|---|---|
| 1 | Ceiling | Air (above surface) | | 8 | $\boxed{1}\leftrightarrow\boxed{2}$ | 2855 |
| | | Rock wool | 18 | | | |
| | | Air (under surface) | | 8 | | |
| 2 | Floor | Air (above surface) | | 8 | $\boxed{1}\leftrightarrow\boxed{2}$<br>$\boxed{3}\leftrightarrow\boxed{3}$ | 1048<br>474 |
| | | Light-weight-concrete | 155 | | | |
| | | Air (under surface) | | 8 | | |
| 3 | Floor | ditto (except thickness of Light-weight concrete 80) | | | $\boxed{1}\leftrightarrow\boxed{2}$<br>$\boxed{3}\leftrightarrow\boxed{3}$ | 1807<br>816 |
| 4 | Door | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{3}$ | 29 |
| | | Steel plate | (negligible) | | | |
| | | Air space | | 10 | | |
| | | Steel plate | (negligible) | | | |
| | | Air (Surface) | | 6 | | |
| 5 | Window | Air (outside surface) | | 15 | $\boxed{1}\leftrightarrow\boxed{4}$ | 320 |
| | | Glass | (negligible) | | | |
| | | Air (inside surface) | | 6 | | |
| 6 | Outside wall | Air (outside surface) | | 15 | $\boxed{1}\leftrightarrow\boxed{4}$<br>$\boxed{2}\leftrightarrow\boxed{4}$ | 324<br>340 |
| | | Metal skin | (negligible) | | | |
| | | Air space | | 15 | | |
| | | Insulation | 30 | | | |
| | | Air space | | 10 | | |
| | | Insulation | 30 | | | |
| | | Ply-wood | 5 | | | |
| | | Air (inside surface) | | 6 | | |
| | | | | | | |
| 7 | Partition | Air (Surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{3}$ | 403 |
| | | Plaster | 15 | | | |
| | | Air space | | 5 | | |
| | | Plaster | 15 | | | |
| | | Air (surface) | | 6 | | |
| 8 | Partition | Air (surface) | | 6 | $\boxed{1}\leftrightarrow\boxed{3}$<br>$\boxed{3}\leftrightarrow\boxed{3}$ | 135<br>1279 |
| | | Light-weight-concrete | 120 | | | |
| | | Air (surface) | | 6 | | |

Table 11. Example problem (II), Office building with duct-type room. Thermal constant of materials.

| Material | Thermal conductivity [$\mathrm{Kcal \cdot h^{-1} \cdot m^{-1} \cdot {}^{\circ}C^{-1}}$] | Thermal diffusivity ($\mathrm{m^2 \cdot h^{-1}}$) $* 10^4$ |
|---|---|---|
| Rock wool | 0.064 | 4.9 |
| Light-weight concrete | 0.58 | 14.0 |
| Insulation | 0.13 | 11.3 |
| Ply-wood | 0.1 | 3.6 |
| Plaster | 0.55 | 2.0 |

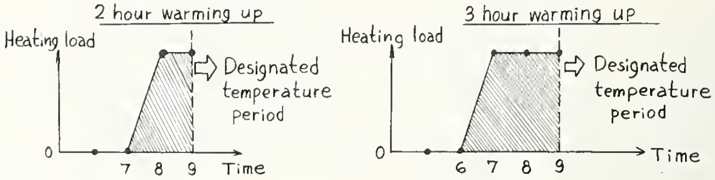
Table 12. Example problem (II), Office building with duct-type room. Air volume of each room.

| Room No. | Room | Air volume [$\mathrm{m^3}$] |
|---|---|---|
| $\boxed{1}$ | Office room | 7310 |
| $\boxed{2}$ | Space above ceiling | 3654 |
| $\boxed{3}$ | Corridor and center-core | 2477 |

Table 13. Example problem (II), Office building with duct-type room. Daily heating pattern ( Designated temperature and its time schedule ).

| Room $\boxed{1}$ | Period of non-designated temperature (Temperature change) | Period of warming-up (Constant load) | Period of designated temperature (Load change) |
|---|---|---|---|
| 2 hour warming-up | Time 18, 19 ----, 0, 1, ----, 6, 7 | 8 | } 9, 10, ····, 17 |
| 3 hour warming-up | 18, 19, ---, 0, 1, -----, 6 | 7, 8 | Designated temperature is 20°C |

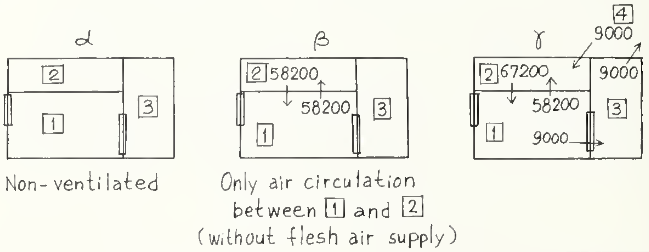
It should be noted that the term "warming-up hour" is meant as such shown in the following charts, on account of the periodical triangular pulse being used as window function.



Room $\boxed{2}$ and $\boxed{3}$ are continuously non-designated temperature.

Table 14. Example problem (II), Office building with duct-type room. Daily pattern of ventilation between room.

| Symbol of ventilation pattern | Period of non-designated temperature | Period of warming-up | Period of designated temperature |
|---|---|---|---|
| A | α | γ | |
| B | α | β | γ |
| C | α | | γ |



Unit: [m³·h⁻¹]

Table 15. Example problem (II), Office building with duct-type room. Daily pattern of RHS.

$$\mathrm{RHS} = \frac{\text{Ratio of heat supply to room } \boxed{2} \text{ (Duct-type rm.)}}{\text{Ratio of heat supply (to room } \boxed{2} + \text{to room } \boxed{1} \text{ (Main rm.))}}$$

<table>
<tr><th>No. of RHS pattern</th><th>Description</th></tr>
<tr><td>0</td><td>Necessary heat amount is directly supplied to $\boxed{1}$ (main rm.).<br>(There's not duct-type room)</td></tr>
<tr><td>1</td><td>$\boxed{1}$ is main room and $\boxed{2}$ is duct-type room.<br>Necessary heat is directly supplied to $\boxed{2}$ (duct-type rm.).</td></tr>
<tr><td>3</td><td>$\boxed{1}$ is main room and $\boxed{2}$ is duct-type room.<br>Necessary heat is directly supplied to $\boxed{1}$ during warming-up period of $\boxed{1}$.<br>During the designated temperature period of $\boxed{1}$, the necessary heat is supplied to both $\boxed{1}$ and $\boxed{2}$ according to following assumed RHS values in varying hours.
<table>
<tr><td>Time</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13 ~ 17</td></tr>
<tr><td>RHS</td><td>0.1</td><td>0.2</td><td>0.4</td><td>0.8</td><td>1.0</td></tr>
</table>
</td></tr>
</table>

Table 16. A case where all unknown is room temperature.

| N | | 1 | | 2 | | 3 | | 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| M | J \ I | A | B | A | B | A | B | A | B | ← Room |
| 1 | A | ▨ | X | | | | | | | |
| | B | X | ▨ | | | | | | | |
| 2 | A | / | – | ▨ | X | | | | | |
| | B | – | / | X | ▨ | | | | | |
| 3 | A | / | – | / | – | ▨ | X | | | |
| | B | – | / | – | / | X | ▨ | | | |

← Time

Table 17. A case where warming-up period is included.

Time schedule

| Time | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A rm. | Non-designated | Warming - up (α) | designated(β) | |
| B rm. | ------- Non-designeted -------- | | | |

| N | | | 1 | | 2 | | 3 | | 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| M | J \ I | | A | B | A | B | A | B | A | B | ← Room |
| 1 | A | | ▨ | X | | | | | | | |
| | B | | X | ▨ | | | | | | | |
| 2 | A | (α) | | | ▨ | X | 1 | 0 | | | |
| | B | | | | X | ▨ | 0 | 0 | | | |
| 3 | A | (β) | | | / | – | 1 | X | | | |
| | B | | | | – | / | 0 | ▨ | | | |
| 4 | A | (β) | | | | | | | 1 | X | ← (A case of Appendix −(2)) |
| | B | | | | | | | | 0 | ▨ | |

← Time

(Unknown is heating load)

## BANQUET ADDRESS

# Computers and the Building Industry

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Computer technology has made amazing advances within a short period of the last two decades. The computers are being utilized in many phases of business and industry--from guidance of missiles to operation of meat packing plants for optimization of meat mix for hot dogs. Considering the popular acceptance of the computer in today's technological society, one would expect to find extensive utilization of the computer throughout the building industry. Surprisingly, the degree to which computers are actually used in the building industry is very slight. Even the design disciplines do not make any significant use of it.

Compared to other industries, the building industry is just starting to utilize the computers. Several engineers have developed impressive problem-solving programs. Most of you are already familiar with programs developed by APEC. Some large-scale computer systems have been developed and implemented for structural analysis and construction project management. However, all the computer utilization at the moment does not replace even 5 percent of the total design manpower involved in the building industry, and therefore is insignificant and somewhat discouraging.

## Why the building industry is not using computers at the moment

There are several reasons for lack of utilization of computers in the building industry.

We are proud of our building industry here. People all over the world are fascinated by our high-rise buildings which are ever leaping skyward. Visitors are always anxious to see the latest projects of our architects and engineers.

While we are proud, we should also be realistic.

If the building industry is compared with any other automated industry, such as the manufacture of automobiles or television, the problems will be evident. The construction industry has scarcely reached the level of mass production. Strangled by lack of organized research, outdated building codes and conservative trade unions, it has remained, in a large measure, a pre-industrialized craft. The industry's rate of technological advance is far below that of other industries. We know the more advanced the technology, the easier it is to automate.

The building industry is large but consists of several forces pulling in different directions and therefore lacks a sense of direction. The professional talents required in financing, design and construction of a large building are amazingly diverse. For the sake of explanation, these can be classified into one of four general sectors: Management, Design, Construction and finally, Operation and Maintenance.

Let us consider the problems associated with the design disciplines, though extensive computer usage could also be made in the other three sectors.

The Architect has been the leader of the design team--and safeguards the owner's interests during design and construction. Due to the widely divergent and highly technical nature of many aspects of building design, the architect has to depend on professional consultants in the engineering areas, such as the structural engineer, the foundation engineer, the mechanical engineer, the electrical engineer. The design team may also have the cost estimator, the interior designer and the specialists in acoustics, illumination and landscaping.

This diversity of intellectual disciplines involved in the building design generally fragments the building team in several ways, the most important being the intellectual fragmentation. The architect does not appreciate the mechanical engineer's goal. The mechanical engineer does not care about the electrical engineer's objectives; and so on. The design profession is like a universe with several planets, each merrily surviving in its own orbit.

The most damaging consequence of this fragmentation is the problem of communication. As we know, communication is the binding force of any social unit, be it design or construction of buildings. Various disciplines cannot be integrated without communication and professional interaction.

The lack of communication always results in suboptimization of design. Conceived without communication, the best structural design can force a suboptimal mechanical design. A solution based on initial cost may be viable on the basis of the operating cost. The optimized building design is more a matter of communication than of linear programming.

The last consequence of lack of communication is expensive duplication of effort. The same information is handled over and over again by different persons involved in the process of design.

Another reason for lack of use of computers in the building industry is due to the mis-match in the requirement of the building industry and the present applications of the computer. While the major problem in the building industry, in design and construction, is that of communication, the present computer applications are geared mainly to engineering problem solving. In addition to problem solving, what we need is a system which will consolidate all the information about the project in a central file of data where it will be available to all the members of the design team at the same time. The availability of data to all designers will allow for analyzing the impact of design decisions made in one design area on the other aspects of the overall design. Thus, engineers could develop integrated designs in terms of overall project goals instead of being limited to their own discipline.

Most of the computer efforts in engineering to date have been aimed at problem solving capabilities. As you know, the major effort of engineering is not problem solving but integration of the solutions to form a project. Therefore, concentration on the problem solving capabilities has fragmented the work that can be done on a project with the computer, and has slowed down application of computers to solution of communication problems.

The problem solving approach to engineering has resulted in two handicaps which have again affected utilization of the computer.

First of all, there is duplication of effort since each problem solving program contains repititions or large parts of some other programs. Secondly, data from problem to problem, even within the same discipline, has to be transferred manually. The situation is similar to having a road system without intersections for transferring from one road to the other road. When you reach an intersection you walk off from one vehicle and take the other vehicle on the other road.

The manual transfer of data combined with extensive initial data gathering and input preparation involve much mundane work on the part of the engineer. This tends to discourage the engineer from application of the existing programs unless the problem is complex or sufficiently large. Several problems require more than one run of the program to enable the engineer to zero-in on the solution. Repeated effort in data transfer, data gathering and input preparation adds to the cost of the program utilization and often the result is more expensive than the old fashioned slide rule solution.

This brings us to the problem of economics as related to the cost of computers.

In spite of all the progress in programming, it is still time-consuming and expensive. Actually at present programming has not advanced at the same rate as developments in hardware. We are unable to estimate the magnitude of effort involved in coding and algorithms for all the building disciplines. As a point of reference, the structural design and analysis system in ICES called STRUDL comprises almost ten million characters of coding and data. As you know, the structural component is just a fraction of the entire building design. We do not know how many characters of data would be required to describe completely a building design if it were stored in a computer. When we consider a complete computer-aided design system, no one is prepared to answer questions such as:

"What is the cost of suitable hardware?"
"How many man years will it cost to take to develop suitable software?"
"How much will it cost?"

The reason for 'no' or vague answers is that based on the present capabilities, the cost of such a computer-aided design system will be out of this world; beyond the economical reach of most of the design organizations.

Why should we be using computers in the building industry?

"The computers are not economical at the moment".
"Programming is difficult and time consuming".
"The building industry is too disorganized to be automated".

Having said all that, the question is "why should we be using computers in the building industry?".

The answer to this question involves awareness of the challenges of the future rather than the handicaps of the present.

The key factor of the challenge of the future is the expected growth of population. Next to food, shelter is a necessity for human survival. The building industry has to plan to meet the critical demands of the future.

The world population is expected to almost double by the year 2000 and have a 30% increase by the year 1985. The following figures for increase in population are based on conservative estimates:

| | (Figures in millions) | | |
|---|---|---|---|
| | 1970 | 1985 | 2000 |
| World | 3,700 | 4,600 | 6,900 |
| U.S.A. | 210 | 240 | 310 |
| Developed Countries | 1,060 | 1,350 | 1,570 |
| Less Developed Countries | 2,640 | 3,250 | 5,530 |

(Developed countries include: North America, Soviet Union, Australia, New Zealand, Europe and Japan. Less developed countries include: East Asia (excluding Japan), South Asia, Africa, Latin America and others.)

Now you can translate this population growth in the demand for housing, schools, areas of business, hospitals and recreation facilities. All of this adds up to a heavy demand or may be critical shortage for building construction facilities.

One can look at this problem from a different perspective. In 1850 only four cities had a population of one million or more. By 1900, the number had increased to nineteen cities. By 1960, 141 cities had a population of more than one million people. The past trend has shown that the population expansion has taken place around the cities. If this trend continues and if population doubles by the year 2000, by that time we will have to double the existing cities. We will have to build one more London, one more Rome, one more Tokyo, one more New York, one more Chicago, one more Los Angeles and so on.

It is expected that by the year 2000 the United States will probably have three megapolises, one on the East Coast which will extend from Boston to Washington; maybe from Portland, Maine to Portsmouth, Virginia and might contain almost 25% of the United States population, that is about 80 million people.

The second megapolis will be concentrated around the Great Lakes area which may stretch from Chicago to Pittsburg and possibly also north to the Toronto region of Canada--thereby including Detroit, Toledo, Cleveland, Akron, Buffalo and Rochester. This megapolis seems likely to contain more than one-eighth of the United States population, about 40 million people.

The third megapolis on the Pacific Coast would probably stretch from San Diego to Santa Barbara, ultimately from San Francisco to Santa Barbara and would contain one-sixth or about 20 million people.

Let us continue to look at this problem from a different perspective. In the last 70 years in this country, the consumption and therefore production of goods and services has almost doubled every 15 years. If this rate of increase continues, we will see one more doubling by the year 1985 and another doubling by the year 2000. As you well know, both production as well as consumption of goods and services will require additional construction--that is factories, stores, warehouses and offices. With progress in technology, we can automate production of goods, introduce efficiencies in production and services, but up to now we have no plans for automating the consumption.

Let us continue to look again from a different perspective, this time the growth in educational facilities.

In spite of extensive educational facilities here, there are estimated to be about 25 million adults, 18 years of age or over, in the United States with less than eighth grade education. This has created the problem of unemployable unskilled labor. 15% of the unemployed in the three major ghetto areas in New York City have never had a white collar job at any time. On the other hand, 50% of job vacancies in the New York City area are for white collar workers. It is estimated that in 10 years only 4% of the employment market will be open to the unskilled. We will need more schools, colleges and universities to educate people, not for the sake of education alone, but to enable them to hold jobs and provide services for the future.

In some of the less developed countries, the educational system is either non-existent or minimal. We have educational problems here, but our problems are similar to a shot from a toy pistol as compared to the educational explosion which is going on in countries such as Africa, Asia, Arabia, and Latin America. These nations feel that it is crucial to their development to have a basic educational system. Well over half a billion people must be taught to read, write and master simple computational skills. Hundreds of millions of people must be brought to accept new practices in agriculture, health and home management.

The occupational skills of millions must be brought up to the requirements of new industries, public works and service occupations which are waiting to be created. These educational facilities will again require buildings and more buildings.

Medical facilities are being improved constantly, which have increased the life span for people throughout the world. We will need more hospitals, not only because of increased population, but also for more people, as, hopefully, people will be living longer.

In the Colonial times, the buildings were planned for a life of no less than 100 years. The recent construction, it is hoped, will last at least 30 years. In other words, a building completed today, will have to be replaced by the year 2000 if not sooner.

Finally, in this respect we know that as societies become more industrialized, people work less and less on their job, and we tend to have a leisure-oriented society. It is expected that by the year 2000, working hours will be reduced from the present, about 1900 hours per year, to 1100 per year. In such a leisure-oriented society, one could spend 40% of one's day on vocation, 40% on avocation and 20% on neither, just relaxing. The increased availability of time for leisure, avocation and relaxation will require increase in buildings for recreation and travel.

At present, apart from lack of money, the second critical shortage is of design and construction personnel for the building industry. If we are planning to meet the critical needs of the future, we have to utilize efficiently all the available resources. Our plans will have to include the potential of computers to solve the communication problems of the building industry.

## What should be done to plan for the future?

In order to plan for the future and avoid making mistakes, we must begin to anticipate earlier than we have in the past the problems of the future. Some of them already are becoming quite clear, and their impact can be expected to be so enormous as to require a long lead time for assessment and preparation.

A look at the history of scientific and technological change will help us understand that inadequately perceived goals may be of greater significance than any possibilities we can foresee. We must alter our standard approach to the future in a way that will enable us to cope with what cannot be anticipated. At the same time, however, we must also try to anticipate as much as possible in order to provide a rational framework for our expectations.

First of all, we do not have to be discouraged by the present cost and capabilities of the computer.

In the past, computer performance has increased by a factor of 10 in every two or three years, and this is considered a conservative estimate. If computer capabilities were to continue to increase by a factor of 10 every two or three years until the end of the century, then all current concepts about computer limitations will have to be reconsidered. Even if the trend continues for only the next decade or two, the improvements over current computers will be factors of thousands to millions. If we add the likely enormous improvements in input/output devices, programming and problem formulation and better understanding of the basic design process, the estimates of improvement will be conservative. Even if the rate of change slows down by several factors, there will still be room in the next thirty years for overall improvement of some 5 to 10 orders of magnitude. Therefore, it is necessary to ignore often meaningless or non-rigorous statements such as "we are at present limited by the computer".

The computer costs will continue to decrease while the manpower cost will increase rapidly. What might seem uneconomical today, will be economical tomorrow, and a necessity for survival day-after-tomorrow.

With improvement in the computer hardware and reduction in cost, one can foresee that the computer utility industry will become as fundamental as the power industry. A large central processor will handle information at a low unit cost just as a large central generator produces electricity at a low unit cost. It will be cheaper to make use of this central utility than it is for each individual to have his own generator.

With the development of telecommunication for computers whereby engineers can use low cost terminals in their offices, it will be possible to help in the matter of locational fragmentation existing at the moment.

We have already established the critical needs of the building industry. We hope that the computer technology will progress fast enough to enable us to satisfy our requirements. While suitable hardware is being developed, we will have to work on development of a Computer-Aided Design System to solve problems of communication. Such a system will have several subsystems, but all of these will share a common data file for the entire project stored on a secondary storage.

It is now up to us to develop such a system based on our imagination, expectations and capabilities. From the stage of problem solving we have to progress to the concept of communication as a key to computerized building design system.

We also have to recognize the necessary components or requirements of such a system.

The most important component of the system is people: People or designers. Because of present inadequate understanding of the design process the computer-aided building system must allow the designers to make all the critical decisions involved in the use of the system. The real danger in developing a computer-aided design system is not that it will produce poor design, but that it may fail to consider the human element of the system and force the user into rigid procedure in its use, thereby either discouraging use of the system or stifling any creative aspects of design.

Most important, we have to analyze and increase our understanding of the basic design process in order to utilize full potential of the computer-aided design system.

When computers first became available, the tendency was to program all of the approximate methods of analysis that have been designed for human users. It was only when reformulating the process that engineers realized that the appropriate way to attack the problem of computerized analysis was to use all the abilities of the computer and program rigorous method of analysis.

As you can appreciate, development of a computer-aided design system will be an expensive and time consuming task. We cannot afford any duplication since we have limited resources and are racing against time. It is therefore necessary that all of you who are interested in the building industry should pool your resources and join in a common effort in developing such a computer-aided design system.

The sheer intensity of the future demand makes it ridiculous to compete in this task on a professional, regional, or national basis. We can utilize our resources more efficiently by establishing international cooperation on what appears to be a common goal. You can achieve this objective by joining, supporting and leading existing cooperative efforts.

How much progress will be made to meet the problems of tomorrow will depend on you.

# Candid Views of the Participants at the Symposium Banquet

P.R. Achenbach, NBS, Symposium Chairman, welcomes the delegations.

F. H. Bridges, President of ASHRAE, Master of Cermonies of the banquet.

Banquet head table from left to right: A. T. Boggs, ASHRAE; Mrs. P. R. Achenbach; F. H. Bridgers, ASHRAE; S. Daryanani, APEC; P. R. Achenbach, NBS; J. M. Ayres, President, APEC; Mrs. F. J. Powell; and T. Kusuda, NBS. (Not shown at the left end of the table are Mrs. T. Kusuda and F. J. Powell, NBS.)

Japanese delegation with Dr. T. Kusuda, Chairman of the Program Committee and Mrs. Kusuda. K. Kimura, Head of delegation, is shown at the center of front row.

G. L. Gupta, Building Research Institute, India; Mrs. G. L. Gupta; T. Y. Sun, J. M. Ayres, and K. Parks of Ayres, Cohen & Hayakawa and Julia Szabo, ASHRAE (left to right).

I. Hogland, Royal Institute of Technology, Sweden; A. T. Boggs, ASHRAE; P. G. Down, Oscar Faber and Partners, England; C. W. Phillips, NBS; J. B. Chaddock, Duke University; and E. N. Van Deventer, National Building Research Institute, South Africa (left to right).

J. J. Anquez and L. G. Bertolo, CSTB, France; K. Fitzner, LTG Lufttechnische Gmbh, West Germany; W. K. Thomas, Thomas-Young Associates; R. F. Mehnert, B. H. Silberstein Associates; and J. L. Norris, Long Island Lighting Company (left to right).

F. J. Powell, NBS, Vice Chairman, Symposium; Mrs. F. J. Powell; R. H. Tull, ASHRAE; P. R. Achenbach, NBS; and Mrs. P. R. Achenbach (left to right).

Mrs. G. L. Gupta; N. Gulati, D. C. Government; G. L. Gupta, Building Research Institute, India; Mrs. N. Gulati; Mrs. A. Chawla; A. Chawla, D. C. Government; Valerie Butler, Nash Love Associates; and N. K. Khosla, Enviro-Management & Research (left to right).

C. W. Phillips, NBS, Chairman of Arrangements Committee, (left), and F. Clain, COSTIC, France.

E. N. Van Deventer, National Building Research Institute, South Africa; Sarah Torrence, NBS; and E. Christopherson, Danish Building Research Institute (left to right).

E. M. Barber, NBS; A. R. Paradis, Dynamic Graphics Inc.; Mrs. A. R. Paradis; N. La Courte, ASHRAE; B. W. Ward, Britt Alderman, Jr.; L. G. Spielvogel, Inc.; and H. K. Varma, North Carolina A & T State University (left to right).

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This proceedings of the First Symposium on the Use of Computers for Environmental Engineering Related to Buildings contains all of the technical papers and invited addresses presented at the symposium, which was held November 30 - December 2, 1970, at the National Bureau of Standards.

The fifty-nine papers deal with the application of the computer to such environmental engineering problems as building heat transfer calculations, heating and cooling load calculations, system simulations, energy usage analyses, computer graphics, air and smoke movement inside buildings, and weather data analyses for load and energy usage calculations.

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