Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

January through March 1950

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

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January 1 through March 31, 1950

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Projects and Publications
of the
NATIONAL APPLIED MATHEMATICS LABORATORIES

January through March 1950

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Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from January 1, 1950 through March 31, 1950.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss

Chief

E. U. Condon

Director
National Bureau of Standards
May 1, 1950

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

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Status of Projects

March 31, 1950

I. Institute for Numerical Analysis

(Section 11.1)

1. Research in Numerical Analysis

Key to letter symbols in project numbers:

AE - Algebraic equations
CM - Numerical methods in conformal mapping
ODE - Ordinary differential equations
PD - Partial differential equations
PM - Probabilistic methods in numerical analysis

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Project 11.1/1-49-AE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Work continued on the compilation of a more complete bibliography on computational aspects of finite matrix theory as well as a compilation of known methods of matrix inversion (see Projects and Publications, Oct-Dec 1949). Large scale numerical testing of the methods included in the general iteration scheme developed last quarter was pending, awaiting completion of the INA computer.

A certain "Monte Carlo" method of matrix inversion, devised by von Neumann and Ulam, was extended to deal with a wider class of matrices by using expectations instead of probabilities (see publication (1) below).

A manuscript is in preparation jointly by J. W. Tukey and G. E. Forsythe on the distribution of an n-rowed determinant whose n rows are independent unit vectors.

Publication: (1) "Matrix inversion by a Monte Carlo method," by G. E. Forsythe and R. A. Leibler, accepted for publication in Mathematical Tables and Other Aids to Computation.

NUMERICAL METHODS IN CONFORMAL MAPPING
Project 11.1/1-49-CM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The numerical procedure for testing Dr. Ahlfor's method was set up to calculate $U_k(Z)$ at points on the circular arcs at 10°

intervals (see Oct-Dec 1949 status report). The work was done with hand calculators and it was found that with such a large subdivision of the circular arcs the error in the numerical integration made it impossible to get more than 2-digit accuracy. However, it is evident that the procedure used is well adapted to automatic calculators and much greater accuracy can be obtained by taking a finer network of points on the arcs.

Publications: (1) "The construction and application of conformal maps: Proceedings of a symposium," edited by E. F. Beckenbach; to be published by the NBS. The volume will include the following papers written in connection with this project: (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A.M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A. Weinstein. (2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W. Seidel; IN MANUSCRIPT; publication under consideration. (3) "Numerical methods in conformal mapping," by L. Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Final editing and revision of publication (3) was completed by Mr. Jackson. Plans were underway to reissue a very useful article by Professor Rademacher on the accumulation of errors which appeared in the Proceedings of the 1947 Harvard Symposium and is now out of print. The article will be accompanied by a note on random round-off by Dr. Forsythe.

Publications: (1) "A note on the numerical integration of differential equations," by W. E. Milne; NBS J. Res. 43, No. 6, 537-542 (Dec. 1949). (2) "Note on the Runge-Kutta method," by W. E. Milne; to be published in the NBS Journal of Research. (3) "On the remainder of the Runge-Kutta formula in the theory of ordinary differential equations," by L. Bieberbach (edited and revised by A. M. Ostrowski and L. K. Jackson of the Institute for Numerical Analysis); IN MANUSCRIPT.

STUDIES IN NUMERICAL INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PD1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of the Symposium held at the Harvard Computation Laboratory in September 1949. (2) "Numerical solution of partial differential equations," by E. C. Yowell;

to be published in the Proceedings of a symposium on scientific computation held under the auspices of the I.B.M. Corporation in Endicott, N. Y., December, 1949.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The expected number of steps W(P) in a random walk starting from a point P in a domain B was investigated by Dr. Wasow. If the transition probability function F(P,Q) of the walk satisfies a very mild condition, W(P) is the unique solution of the integral equation

$$W(P) = \int_B W(Q)d_Q F(P,Q) + 1.$$

If F(P,Q) is sufficiently concentrated about P and sufficiently symmetric, W(P) can be shown to approximate the solution of a boundary value problem for a certain elliptic differential equation. Using this fact the inequality

$$W(P) = \pi^{-1} \Gamma^{2/n} (n/2 + 1) s^{-2} V^{2/n} [1 + \varepsilon(s)]$$

is proved for an important class of random walks in n dimensions. Here, s is the mean step length, V the volume of B, and $\lim_{s \to 0} \varepsilon(s) = 0$. The slow growth of W(P) with n is encouraging for computational purposes.

A rather complete treatment of probabilistic methods of solving the most general elliptic differential equation without a zero'th order term was prepared by Dr. Curtiss. New results include some explicit formulas for the mean length of a random walk with absorbing barriers, and for the degree of approximation of the solution of the corresponding difference equation to that of the differential equation, and for the standard error of the statistical estimate of the solution of the difference equation and its Green's function.

Publications: (1) "Random walks and the eigenvalues of elliptic difference equations" by Wolfgang Wasow; to be published in the NBS Journal of Research. (2) "Sampling methods applied to differential and difference equations, with special reference to equations of the elliptic type" by J. H. Curtiss; to appear in the Proceedings of a symposium on scientific computation held under the auspices of the I.B.M. Corporation in Endicott, N. Y., December, 1949. (3) "The Monte Carlo Method: Proceedings of a symposium" to appear in the NBS Applied Mathematics Series.

DETERMINATION OF EXTREMALS OF FUNCTIONALS
Project 11.1/1-50-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The revision of publication (2) was essentially completed. Many of the results given in a paper by Drs. Hestenes and Karush (see project 11.1/1-50-3) have been extended to quadratic forms in Hilbert space. This extension gives rise to methods of finding eigenvalues for differential and integral boundary-value problems; the methods appear suitable for machine computation.

Publications: (1)"Some elementary problems in the calculus of variations," by M. R. Hestenes; submitted to the Mathematics Magazine; also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A. (2) "Quadratic forms in Hilbert space, with applications in the calculus of variations," by M. R. Hestenes; accepted, subject to revision, by the American Journal of Mathematics.

DETERMINATION OF CRITICAL POINTS
Project 11.1/1-50-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The work accomplished in this quarter is reported under project 11.1/1-50-3.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Project 11.1/1-50-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Drs. Feller and Forsythe extended their work on the transformation of matrices (see July-Sept 1949 issue) to include the reduction of several axes at a time. A paper to be submitted to the Quarterly of Applied Mathematics was near completion. The methods of finding eigenvalues of symmetric matrices outlined in the Oct-Dec 1949 issue have now been written up in publications (4) and (5) below. Drs. Karush and Lanczos are making tests of their methods in connection with specific numerical matrices.

Certain numerical comparisons have been made between the lowest eigenvalue of the Laplace differential operator in two dimensions, and the lowest eigenvalue of the corresponding difference equation over the same region.

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C. Lanczos; accepted for publication in the NBS Journal of Research. (2) "Numerical determination of characteristic numbers," by W. E. Milne; accepted by the NBS Journal of Research. (3) "A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation,"

by M. D. Donsker and M. Kac; accepted by the NBS Journal of Research. (4) "The method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; to be submitted to the NBS Journal of Research. (5) "An iterative method for finding characteristic vectors of a symmetric matrix," by W. Karush; IN MANUSCRIPT at INA.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.1/1-50-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Publication (2) was completed and is to be presented at the Berkeley meeting of the American Mathematical Society, April 28-29, 1950. Publications (5) and (6) were also completed. Publication (11) concerns conditions which imply that a series Σa_n is convergent, if it is Abel summable. A simple proof is given for the following theorem: Let

$$U_n = \sum_{\nu=1}^{n} \nu \left(|a_\nu| - a_\nu \right); \text{ if } \frac{1}{n}(U_m - U_n) \to 0 \text{ as } \frac{m}{n} \to 1,$$

$n \to \infty$, and if Σa_n is Abel summable, then Σa_n is convergent. The paper is being submitted for publication in the Pacific Journal of Mathematics. If $\mu_{1,n}, \mu_{2,n}, \ldots, \mu_{n-1,n}$ are the successive relative extrema of $|P_n(x)|$, then $\mu_{r,n} > \mu_{r,n+1}$, for $n \geq r + 1$. This conjecture of John Todd for the Legendre polynomials was recently proved by G. Szego. Then Mr. Todd proved the same property for Laguerre polynomials. In publication (12) Professor Szasz proves an analogous property for ultraspherical polynomials.

Publications: (1) "On the Gibbs phenomenon for Euler means," by O. Szasz; accepted for publication in Acta Litterarum ac. Scient., Szeged. (2) "Gibbs' phenomenon for Hausdorff means," by O. Szasz; submitted to the Transactions of the American Mathematical Society. (3) "On a summation method of O. Perron" by O. Szasz; accepted for publication in Mathematische Zeitschrift. (4) "A generalization of S. Bernstein's polynomials to the infinite interval" by O. Szasz; accepted for publication in the NBS Journal of Research. (5) "On some trigonometric transforms" by O. Szasz; submitted to the Pacific Journal of Mathematics. (6) "On positive harmonic functions and ultraspherical polynomials," by W. Seidel and O. Szasz; submitted to the Journal of the London Mathematical Society. (7) "On subharmonic, harmonic, and linear functions of two variables," by E. F. Beckenbach; submitted to Revista, Universidad Nacional de Tucuman (Argentina). (8) "Certain Fourier transforms of distributions," by E. Lukacs and O. Szasz; accepted for publication in the Canadian Journal of Mathematics. (9) "Inequalities concerning ultraspherical polynomials and Bessel functions," by O. Szasz; accepted for publication by the Proceedings of the American Mathematical Society. (10) "Summation of slowly convergent series with positive terms," by O. Szasz; J. Math. Phys. 28, No. 4, 272-279 (Jan. 1950). (11) "On a Tauberian theorem for Abel summability," by O. Szasz; submitted to Pacific Journal of Mathematics. (12) "On the relative extrema of ultraspherical polynomials," by O. Szasz; IN MANUSCRIPT.

STUDY OF REAL ZEROS OF REAL DIRICHLET SERIES
Project 11.1/1-50-7

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: J. B. Rosser

Authorized 12/1/49
Completed 3/31/50

Objective: To determine for $k \leq 227$ that the corresponding series $L(s,\chi)$ has no positive real zeros.

Background: In the theory of the distribution of primes in arithmetic series, in assigning bounds in the three prime theorem, and in studying the class number of quadratic fields, a knowledge of the location of the real zeros of $L(s,\chi)$ is of value. A long standing conjecture is that there are no positive real zeros for any k. If proved, this result would be of value in each of the fields mentioned above. In the Bull. Am. Math. Soc. 55, 906-913 (1949), the project manager has given a computational procedure for proving the result for individual values of k, and treated each $k \leq 67$.

Status: COMPLETED. The method for dealing with individual values of k, which was referred to in the Background, failed for k = 163. Improved methods were developed, but still the case k = 163 appeared difficult. Finally a new formula for $L(s,\chi)$ was discovered which made it possible to treat many values of k simultaneously. By means of this formula, the difficult case of k = 163 was finally treated adequately. Results have been written up in a paper now being prepared for publication in the NBS Journal of Research.

Publication: "Real roots of real Dirichlet L-series" by J. B. Rosser; accepted for publication in the NBS Journal of Research.

AN ITERATIVE SOLUTION OF FREDHOLM'S INTEGRAL EQUATION
Project 11.1/1-50-8

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The investigation of the Laplace transform brought up the problem of inverting the Laplace transform. This investigation branched out into a new approach to the problem of electric network analysis and is discussed in project 11.1/1-50-9.

Publication: "An iterative solution of Fredholm's integral equation", by C. Lanczos; presented at the American Mathematical Society Meeting, Pasadena, California, November 26, 1949; final manuscript being prepared for publication.

NETWORK ANALYSIS BY LAGUERRE POLYNOMIALS
Project 11.1/1-50-9

Origin: NBS Authorized 3/31/50
Sponsor: Office of Naval Research, USN
Manager: C. Lanczos

Objective: To replace the traditional expansion theorem of Heaviside, which requires the knowledge of the complex roots of an algebraic equation, by a simple algorithm which obviates the evaluation of the roots.

Background: A Laplace transform is usually inverted by integrating along the imaginary axis and then closing the path of integration through the negative infinite semi-circle. The integration is then reduced to loops around the singularities, which in the case of an electric network are simple poles. To obtain the position of these poles one must get the (generally complex) roots of an algebraic equation, which occasionally may be of rather high degree. This is frequently a cumbersome task. The present method uses conformal mapping by reciprocal radii, transforming the infinite imaginary axis into the unit circle. However, in contrast to the traditional procedure, it is the right half of the complex plane which is mapped inside the unit circle. The Laplace transform is regular in this region and allows expansion into a Taylor series around the center of the circle. Inverting this uniformly convergent series term by term we find that the original function f(x), which has the significance of the transient response (Green's function) of the network, is now expanded into a series of orthogonal Laguerre functions:

$$f(x) = \sum_{k=0}^{\infty} c_k \frac{L_k(2x)}{k!} e^{-x}$$

The coefficients c_k of this expansion are made readily available by a simple division algorithm. The convergence is such that 10 to 12 terms of the series will usually give a good overall representation of the transient response. The "noise" part of the response, which shows up in small humps of the curve and which is usually of minor physical significance, is automatically smoothed out in the representation. This method obviates the solution of an algebraic equation of high degree, and is particularly useful if we want to see how modification of a network parameter influences the transient response. Moreover, this method gives a new solution of the problem of network synthesis, when a network of given order is to be designed which shall approximate a given transient response as closely as possible.

Comment: This investigation originated from the study of the Laplace transform and its inversion, in connection with the iterative solution of Fredholm's integral equation; (cf. project 11.1/1-50-8).

Status: NEW.

Publication: "Inversion of the Laplace transform," by C. Lanczos; this material is to be presented to the American Mathematical Society Meeting, Berkeley, California, April 27, 28, 1950; a manuscript for the Proceedings of the Institute of Radio Engineers is in preparation.

2. Mathematical Tables

MATHIEU FUNCTIONS II
Project 11.1/2-45-1

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. All the coefficients received from the NBS Computation Laboratory have been checked. The function $Se_r(s,x)$ is about 70% completed.

SPECIAL TABLE OF BESSEL FUNCTIONS
Project 11.1/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication continued.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

ROCKET NAVIGATION TABLES
Project 11.1/2-48-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication continued.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLES OF $E_1(z)$, SECOND QUADRANT
Project 11.1/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. About 80% completed. The auxiliary functions $U_n(z)$ and $T_n(z) = U_n(z)/n$ were summed over n to form auxiliary functions R and P, from which $E_1(z)$ and other functions are directly obtainable. These functions R and P are being differenced in the z-plane.

PUNCHED CARD LIBRARY
Project 11.1/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following tables on punched cards were added to the library:

1. Tables of circular and hyperbolic tangents and cotangents. This table was punched at the North American Aviation Company from the "Table of Circular and Hyperbolic Tangents and Cotangents" computed by the Mathematical Tables Project of the Bureau of Standards in 1943 and published by the Columbia University Press. This table is being differenced and checked at the INA.

2. Table of offset circle probabilities. A table of the function Q(R,x) defined by

$$Q(R,x) = \int_R^\infty e^{-\frac{1}{2}(t^2+x^2)} I_0(t\,x)t\,dt$$

where $I_0(t\,x)$ is the Bessel function of imaginary argument $I_0(t\,x) = J_0(i\,tx)$ This table was computed in part by the INA and in part by Rand Corporation.

3. The integral function F(t,a) given by the equation

$$F(t,a) = e^{-\frac{t}{a} - 2\sqrt{t}} \int_0^t e^{\frac{y}{a}} E_0(y)dy$$

This table was computed and checked at the INA for use in the solution of a certain third order differential equation.

3. Development of Automatic Computing Machinery

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.1/22-49-1

Origin: Air Materiel Command, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. All of the chassis of the arithmetic unit were completed and 13 of the 37 digits have been operated satisfactorily. Two of the racks for holding these chassis have been fabricated and the remaining rack is now under construction. The completed racks were placed in position with respect to the computing machine. The sign digits have been completed and the M→C (M Register-Control) transfer drivers have been designed and are in the process of construction. The control leads running from the arithmetic unit to the

control unit are now being installed.

The memory rack was nearly completed and will soon be ready to be placed into its final position with respect to the machine. The cathode ray tubes were mounted in twenty-two of the memory chassis. These 22 chassis have been completely checked out and have been found to operate satisfactorily. As soon as the balance of the cathode ray tubes arrive from the supplier the remaining memory chassis will be completed and checked out. The deflection amplifier chassis have been built, and the rack to hold these chassis is now under construction.

The control unit has been in operation for some time and was operated satisfactorily in connection with the testing of the arithmetic and the memory units. The input and output circuits have been completely designed and are under construction at present. The Flexowriter Units to be used for input-output were delivered and will be modified for use with the computer.

The power supply has been installed, tested, and found to be satisfactory. The motor generator set has also been installed and has been operated for a few hours. A preliminary experiment with an electronic regulator for the motor generator set was conducted and details of the final design will be worked out at a later date. For the present the power supply will be operated directly from the main power lines instead of from the motor generator set. Arrangements were made with the University to supply the necessary room and power alterations. A contract for the ventilating system was awarded to the Universal Sheet Metal works of Los Angeles.

In summary, fabrication of all units was approximately 90% completed, and about 75% of the chassis have been individually checked and operated satisfactorily.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Project 11.1/22-49-2

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Conferences on the subject were held with several members of the staff of the Institute. Lists were prepared of the terminology and block diagram symbols used at the Institute in connection with automatic computing machines. These lists were distributed for comments and criticisms to members of the staff of the Institute, persons attending the course on automatic computing machinery given at the Institute (project 11.1/4-50-2), and to some other persons interested in the field of automatic computers, and as a result considerable response was received.

Publication: Preliminary lists of terminology and block diagram symbols in manuscript form are obtainable from the INA.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Project 11.1/22-50-1

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Further studies were made in the system of semi-automatic instruction for electronic computers. The results of these studies have influenced the coding being done in connection with the computing machine under construction at the Institute (project 11.1/22-49-1).

Publications: (1) "Semi-automatic instruction on the Zephyr" by H. D. Huskey; to be published in the Proceedings of the Symposium held at the Harvard Computation Laboratory in September, 1949. (2) "Characteristics of the Institute for Numerical Analysis Computer", by H. D. Huskey; to be published in the April 1950 issue of Mathematical Tables and Other Aids to Computation.

4. Computing Services

SEPARATION OF EXPONENTIALS
Project 11.1/31-48-2

Origin: NBS — Authorized 5/25/48
Sponsor: Air Materiel Command, USAF — Terminated 3/1/50
Manager: Roselyn S. Lipkis

Objective: To test a method of determining the unknown quantities n, a_1, and τ_1 from a known function F(t) and the relationship

$$F(t) = \sum_{1}^{n} a_1 e^{-t/\tau_1}$$

Background: An important problem in physics involves the task of identifying the decay constants of the components present in a mixture of radio-active substances. The measured total activity is represented by the above equation. Mr. Albert Cahn of the Institute has worked out a method of finding the unknown quantities, using two theorems from the problem of moments. It was desired to compute a numerical example. The project is being performed as part of the training program of the computation unit in section 11.1.

Status: TERMINATED.

CALCULATION OF AERODYNAMIC DERIVATIVES BY HASKIND'S METHOD
Project 11.1/31-50-5

Origin: Aircraft Laboratory, Engineering Division, Wright-Patterson Air Base, USAF — Authorized 8/1/49, Completed 3/31/50
Sponsor: Air Materiel Command, USAF
Managers: Gertrude Blanch, E. Yowell

Objective: To carry through the computations of aerodynamic derivatives for Mach number M = .7 and eleven values of the parameter μ, by the method outlined in the pamphlet by M. D. Haskind. (Russian; Translation by the Graduate Division of the Applied Mathematics Department of Brown University, A9-T-22, entitled "Oscillations of a Wing in a Subsonic Gas Flow".)

Background: This problem was sent to the Institute for Numerical Analysis via Dr. E. P. Little of the Air Materiel Command. The problem involves an intimate knowledge of Mathieu functions, and in the form submitted, it is suitable for hand calculators, with the aid of some work on IBM equipment.

Comments: Some of the tables relating to Mathieu functions, computed by the National Bureau of Standards and now in process of publication, will save several weeks of computing time for this problem.
The translated pamphlet of Haskind was found to have many errors. These are being corrected by L. L. Bailin, M. Howard, and G.Blanch.

Status: COMPLETED. Results were transmitted to the Air Materiel Command at Dayton, Ohio.

SOLUTIONS OF A DIFFERENTIAL EQUATION
Project 11.1/31-50-6

Origin: Hughes Aircraft Company
Sponsor: Air Materiel Command, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. About 80% completed. Results of the homogeneous equation for certain values of the parameter have been transmitted to the Hughes Aircraft Company.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Project 11.1/31-50-7

Origin: Lockheed Aircraft Corporation
Sponsor: Air Materiel Command, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. About 15% completed. Efforts are being made to cut down the computational chore of this problem, and it is currently being held on a stand by basis.

COMPUTATIONS RELATING TO A PROBLEM IN ATMOSPHERE DIFFUSION
Project 11.1/31-50-9

Origin: Meteorology Department, University of California at Los Angeles — Authorized 11/15/49, Completed 3/31/50
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Objective: Given a set of successive observations of different particles. Let $x_i(t), y_i(t)$ represent the positions of the i-th particle at time t. The objective is to calculate the angle

$$\theta = \text{arc tangent} \frac{\sum_i (y_i(T_i) - y_i(0))}{\sum_i (x_i(T_i) - y_i(0))}$$

where T_i is the time of the final observation of the i-th particle. Coordinates of the observations must then be calculated in a system rotated through the angle θ.

Background: This computation arose in connection with work being performed by the Meteorology Department of the University of California at Los Angeles under contract to the Watson Laboratory of the Air Materiel Command.

Comments: Some other calculations involving least square solutions were also performed in connection with this problem at the request of the originator.

Status: COMPLETED. Results were transmitted to the Meteorology Department, University of California at Los Angeles.

ANALYSIS OF RAM-JET DATA
Project 11.1/31-50-10

Origin: Marquardt Aircraft Company
Sponsor: Air Materiel Command, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. No requests for processing of data were received this quarter from the Marquardt Aircraft Company.

FLUTTER ANALYSIS, SWEPT WINGS, II
Project 11.1/31-50-11

Origin: Northrop Aircraft, Inc.
Sponsor: Air Materiel Command, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. About 20% completed.

SOLUTION OF A SET OF SIMULTANEOUS DIFFERENTIAL EQUATIONS
Project 11.1/31-50-12

Origin: Hughes Aircraft Company
Sponsor: Air Materiel Command, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. About 50% completed.

COMPUTATIONS RELATING TO CRITICAL SPEEDS
Project 11.1/31-50-13

Origin: AiResearch Manufacturing Co. Authorized 3/31/50
Sponsor: Office of Air Research, USAF
Manager: Gertrude Blanch

Objective: To find the roots τ of the equation

$$0 = \begin{vmatrix} -\alpha_1 - \frac{\tau}{\alpha_1^2} & 0 & -2\alpha_1 + \tau\left(\frac{1}{\alpha_1^2} - \frac{1}{r_1^2}\right) & 0 \\ 0 & -\alpha_2 - \frac{\tau}{\mu\alpha_2^2} & 0 & -2\alpha_2 + \frac{\tau}{\mu}\left(\frac{1}{\alpha_2^2} - \frac{1}{r_2^2}\right) \\ 3\alpha_1 + 2 & 1 & 3\alpha_1 + \frac{\tau}{r_1^2} & 0 \\ 1 & 3\alpha_2 + 2 & 0 & 3\alpha_2 + \frac{\tau}{\mu r_2^2} \end{vmatrix}$$

for various values of the parameters.

Background: This problem is in connection with research performed by AiResearch Mfg. Co. for the Air Materiel Command.

Status: NEW. About 70% completed.

COMPUTATIONS IN PROBABILITY THEORY
Project 11.1/31-50-14

Origin: Hughes Aircraft Company Authorized 3/31/50
Sponsor: Office of Air Research, USAF Completed 3/31/50
Manager: Gertrude Blanch

Objective: To evaluate numerically certain types of probability integrals and to maximize these integrals as a function of a parameter.

Background: This problem arose in connection with research performed by Hughes Aircraft under a contract with the USAF.

Status: COMPLETED. Results were transmitted to the Hughes Aircraft Company.

ROOTS OF FLUTTER MATRICES
Project 11.1/31-50-15

Origin: Douglas Aircraft Company, Inc. Authorized 3/31/50
Sponsor: Office of Air Research, USAF
Manager: E. Yowell

Objective: To find the eigenvalues of sixteen eighth-order matrices with complex elements.

Background: These solutions are required in certain flutter analyses conducted by the Douglas Aircraft Company in connection with a contract with the USAF.

Status: NEW. About 30% completed.

CALCULATION OF COEFFICIENTS OF POLYNOMIALS
Project 11.1/31-50-16

Origin: Hughes Aircraft Company Authorized 3/31/50
Sponsor: Office of Air Research, USAF
Manager: Gertrude Blanch

Objective: To find the coefficients of a fourteenth-degree polynomial when given the fourteen complex roots. To be performed for a number of such sets of roots.

Status: NEW. About 90% completed. All coefficients have been obtained, and evaluation of the polynomial for a given set of arguments will complete the problem.

METEOROLOGICAL MEANS
Project 11.1/31-50-17

Origin: Department of Meteorology, U.C.L.A. Authorized 3/31/50
Sponsor: Office of Air Research, USAF
Manager: E. Yowell

Objective: To calculate the time averages (daily readings for one month) and the space averages (readings, for a given day, around an entire latitude circle) of various meteorological elements, i. e. wind velocity, temperature, height, - of the 700 mb, 500 mb, and 300 mb pressure levels.

Background: This computation arises in connection with research in the general circulation of the atmosphere being performed by the U.C.L.A. Meteorology Department for the Watson Laboratories of the AMC.

Comments: Raw data for this computation are furnished to INA on punched cards by the U.C.L.A. Meteorology Department.

Status: NEW.

SLOTTED ANTENNAE
Project 11.1/31-50-18

Origin: North American Aviation, Inc. Authorized 3/31/50
Sponsor: Office of Air Research, USAF
Manager: E. Yowell

Objective: Given a tabulated complex function $F(\theta)$. Required to find

$$S = F(\theta) + m_1 e^{i\psi_1} F(\theta - \tfrac{2\pi}{3}) + m_2 e^{i\psi_2} F(\theta - \tfrac{4\pi}{3})$$

for

$$\begin{matrix}\psi_1 \\ \psi_2\end{matrix} = 0\ (\tfrac{\pi}{4})\ \pi. \qquad \begin{matrix} m_1 \\ m_2 \end{matrix} = .25(.25)1$$

Background: The function $F(\theta)$ represents the radiation pattern due to a single slotted cylindrical antenna. The functions S represent the radiation pattern of a cylindrical antenna with three equally spaced slots fed with varying amplitudes and phases. It is sometimes necessary to use a large body of revolution, such as a fuselage, as an antenna. This work is necessary to find a configuration yielding a good radiation pattern.

Status: NEW.

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Project 11.1/32-49-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Among the more extensive computations during this quarter were: (1) Matrix inversion studies, for J. B. Rosser and C. Lanczos (see 11.1/1-49-AE2); (2) Studies in characteristic values of matrices, for M. R. Hestenes, W. Karush, G. E. Forsythe, and C. Lanczos (see 11.1/1-50-3); (3) Studies in the application of the Laguerre polynomials to electric network analysis and synthesis, for C. Lanczos (see 11.1/1-50-9); (4) Studies in finding the complex roots of an algebraic equation, for C. Lanczos (see 11.1/1-50-9); and (5) Studies in finding the roots of a J matrix for C. Lanczos (see 11.1/1-49-AE2).

REDUCTION OF THEODOLITE DATA
Project 11.1/32-49-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Three flights were evaluated during the quarter.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Project 11.1/32-49-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Project 11.1/32-49-6

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. About 35% completed.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Project 11.1/32-50-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. About 25% completed.

COMPUTATIONS ARISING IN THE THEORY OF HYPERSONIC FLIGHT
Project 11.1/32-50-4

Origin: Naval Ordnance Test Station, Pasadena Annex
Sponsor: Office of Naval Research, USN
Full project description appears in Oct Dec 1949 issue.

Status CONTINUED. About 70% completed.

MOMENTS OF ORDER STATISTICS
Project 11.1/32-50-5

Origin: University of Oregon
Sponsor: Office of Naval Research, USN
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. About 85% completed. Checking of certain inconsistencies being done on a standby basis.

DETERMINATION OF REYNOLDS NUMBERS
Project 11.1/32-50-6

Origin: Ames Aeronautical Laboratory, Moffett Field Authorized 3/1/50
Sponsor: National Advisory Committee for Aeronautics Completed 3/31/50
Manager: E. Yowell

Objective: To obtain the Reynolds Numbers by the following procedure: By numerical integration solve equations

$$(1) \qquad K\sqrt{\frac{\rho}{\rho_0}}\,du = \frac{-\sqrt{1-(\mu/\mu_0)p}\,dp}{p},$$

$$(2) \qquad K\sqrt{\frac{\rho}{\rho_0}}\,dy = \frac{-\sqrt{1-(\mu/\mu_0)p}\,dp}{p^2},$$

where $p = \frac{du}{dy}$ and $\frac{\mu}{\mu_0} = \frac{\rho}{\rho_0} = au^2 + bu + c$, to obtain

$$(3) \qquad u = f_1(p), \text{ and}$$

$$(4) \qquad y = f_2(p).$$

Equations (3) and (4) implicitly define u(y) and y(u).

Next, by numerical integration find

$$(5) \qquad g(x) = \frac{1}{x}\int_0^{y(x)} \frac{\rho}{\rho_0}\,u(y)\,\left[x - u(y)\right]dy, \text{ and}$$

$$(6) \qquad R = u_0^2\,g(u_0) - 2\int_0^{u_0} x\,g(x)dx.$$

The boundary conditions of equations (1) and (2) are indicated by the condition $R = 0$ at $u_0 = 0$.

This procedure is to be performed for 36 cases.

Background: These computations are in accordance with instructions outlined in a report entitled "An analytical investigation of the drag and heat transfer in the high-speed turbulent boundary layer of a flat plate" by M. W. Rubesin.

Status: COMPLETED. Results were transmitted to the Ames Aeronautical Laboratory, Moffett Field.

RADAR ERROR CALCULATIONS
Project 11.1/32-50-7

Origin: Naval Air Missile Test Center, Point Mugu Authorized 3/31/50
Sponsor: " " "
Manager: Gertrude Blanch

Objective: To compute the values of certain combinations of trigonometric functions.

Background: This work is in connection with a theory of tracking errors developed by the sponsor.

Status: NEW. About 80% completed.

CURVE FITTING
Project 11.1/32-50-8

Origin: California Institute of Technology, Jet Propulsion Laboratory, Pasadena, California — Authorized 3/31/50, Completed 3/31/50
Sponsor: Naval Air Missile Test Center, Point Mugu
Manager: Gertrude Blanch

Objective: To calculate

$$K = \frac{2\alpha}{\pi}\sqrt{\frac{1 - \left(\frac{2\alpha}{\pi}\right)^2}{\sin^2\alpha - \left(\frac{2\alpha}{\pi}\right)^2}}$$

for $\alpha = 5^\circ(5^\circ)90^\circ$, to 7S, and to fit a 3rd-order polynomial to the calculated points.

Background: This problem arose in theoretical investigations at the California Institute of Technology, Jet Propulsion Laboratory.

Status: COMPLETED. Results were transmitted to the Naval Air Missile Test Center, Point Mugu.

SUMS OF CROSS PRODUCTS
Project 11.1/32-50-9

Origin: Psychology Dept., University of Southern California — Authorized 3/1/50
Sponsor: Office of Naval Research, USN
Manager: E. Yowell

Objective: Given a set of thirty-four test scores for each of 284 subjects. Required to compute the cross products of each score with every other score in the set and sum these cross products over the 284 sets.

Background: These computations are required in connection with research being performed by the Psychology Department, U.S.C. under a contract with the Office of Naval Research.

Status: NEW.

5. Training

CHARACTERISTIC VALUES OF MATHIEU'S DIFFERENTIAL EQUATION
Project 11.1/4-49-5

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

COURSES ON AUTOMATIC COMPUTING MACHINERY
Project 11.1/4-50-2

Origin: NBS
Sponsor: Various agencies supporting the INA
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. A general course on automatic computing machinery meeting for one hour a week was given during the quarter by the project manager. Programing and coding details as well as explanations of electronic features of these machines were included in the course, with particular emphasis being placed on the machine under construction at the Institute.

The course was open to personnel of the University of California at Los Angeles and other neighboring scientific and industrial organizations; consequently, a number of persons not on the staff of the Institute attended the course.

At the end of the quarter, since all logical and circuit details had been covered and the Institute's machine was in a state of final assembly, it was decided to suspend the course temporarily until some operational experience had been acquired.

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Project 11.2/11-50-1

Origin: NBS Authorized 1/1/50
Sponsor: Office of Air Research, AMC, USAF
Manager: H. E. Salzer

Objective: To carry out research work on problems of classical numerical analysis, i. e., on numerical analysis mainly intended for use in connection with non-automatic machines.

Background: The direction and intensity of the research in these fields depends on the staff and facilities available and on the current needs of the Laboratory.

Status: NEW. Mr. Salzer worked on the following four projects:

1) Equal weighted quadrature formulas for semi-infinite and infinite intervals. A well-known series of quadrature formulas of Chebyshev, which are exact for polynomials of degree n over any finite interval, and which are equally weighted, have been extensively treated in the literature, both from the theoretical and computational standpoint. Although there have been treatments of other types of quadrature formulas over infinite intervals, there has been no investigation of the statistically and otherwise important type, which is taken over the intervals $[0,\infty]$ and $[-\infty, \infty]$, and where the weight factors are all equal. The present investigation is devoted to finding two kinds of equally-weighted n-point quadrature formulas for

$$\int_0^\infty e^{-x} f(x)dx \quad \text{and} \quad \int_{-\infty}^\infty e^{-x^2} f(x)dx,$$

n ranging from 0 to 10 in both cases, namely a) those which are exact for f(x) a polynomial of the nth degree and b) those which are the best possible in the sense of giving the highest degree accuracy while still allowing only values of f(x) within the interval of integration upon the real axis.

2) Formulas for calculating the complex error function. The calculation of the complex error function $\int_0^z e^{-u}\, du$ was treated by Miller and Gordon in 1931, and more recently by J. B. Rosser in 1945. The present simplification gives two formulas, both of which follow from a single formula of Dawson (which is a consequent of Poisson's formula). Their use enables one to obtain 16-place accuracy over the entire region of interest in the z-plane. This work will be submitted to the Journal of Research.

3) Note on powers of quaternions. A connection between powers of quaternions and powers of ordinary complex numbers is demonstrated, with application to the computation of the former. An alternative proof of the result has been supplied by Olga Taussky-Todd. This work will be submitted to MTAC.

4) Radix table for trigonometric and inverse trigonometric functions. Previous familiarity with various radix tables for calculating logarithms, including current work on Rosser's radix project, together with an interest in arc tangents stimulated by recent work on arc tangent decompositions, have combined to suggest an alternative radix method for

calculating tangents and arc tangents (and thence the other 10 direct and inverse trigonometric functions). A half page radix table gives nine-place accuracy. A practical table for eighteen-place accuracy would not occupy more than a single page. The work will be submitted to MTAC.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS

Project 11.2/11-50-2

Origin: NBS Authorized 3/1/50
Sponsor: Office of Air Research, AMC, USAF
Manager: R. R. Reynolds

Objective: To investigate the possibilities of Bergman's method for the solution of the Dirichlet problem for multiply connected domains by high-speed computers.

Background: The theory of the kernel function

$$K(z,\bar{z}) = \sum \psi_n(z) \overline{\psi_n(z)}$$

for a complete set of functions ψ_n orthonormal with respect to a certain metric j was developed by Bergman in a study of functions of two complex variables. It was observed that if ϕ_n were solutions of the boundary value problem for an elliptic equation $\Delta\phi = P\phi$ $(P > 0)$ and if

$$j = j(\phi_m, \phi_n) = \iint_B \left[\frac{\partial \phi_m}{\partial x}\frac{\partial \phi_n}{\partial x} + \frac{\partial \phi_m}{\partial y}\frac{\partial \phi_n}{\partial y} + P\phi_m\phi_n\right] dx\, dy$$

then

$$K_1(x,y;\xi,\eta) = \sum \phi_n(x,y)\,\phi_n(\xi,\eta)$$
$$= \tfrac{1}{2}\pi\left[N(x,y;\xi,\eta) - G(x,y;\xi,\eta)\right]$$

where N,G are the classical Neumann's and Green's functions for the domain B and the given equation.

A process was evolved for determining in terms of these ϕ_n the solution not only of this boundary value problem but also of the Dirichlet problem (which is the degenerate case $P = 0$). The method has already been successfully applied in solving certain torsion problems and computing harmonic measures. In the present study the first domains to be considered are the hollow rectangles: the area between $-1 \leq x \leq 1$, $-a \leq y \leq a$ and $-b \leq x \leq b$, $-ab \leq y \leq ab$ for a selected set of values of a and b between 0,1.

Status: NEW. Mr. R. R. Reynolds started work on an investigation of the possibilities of adapting Bergman's method, for the solution of the Dirichlet problem for a multiply connected domain B, to high-speed computing machines. As stated in the Background the first domains to be considered are hollow rectangles: the area between $-1 \leq x \leq 1$, $-a \leq y \leq a$ and $-b \leq x \leq b$, $-ab \leq y \leq ab$, for a selected set of values of a and b between 0, 1.

In the present study the first stage is orthogonalization of $z^n (n = 0, \pm 1, \pm 2, \ldots)$ and this requires the calculation of $\iint_B z^m z^{-n}\, dx\, dy$ $(m,n = 0, \pm 1, \pm 2, \ldots)$. This is in progress.

RESEARCH IN MODERN NUMERICAL ANALYSIS: CONDITION OF MATRICES
Project 11.2/11-50-3

Origin: NBS Authorized 1/1/50
Sponsor: Office of Air Research, AMC, USAF
Managers: John Todd, Olga Tuassky-Todd

Objective: To study the condition of systems of linear equations in particular those which arise in a natural way by replacing a differential equation by its finite difference analogue.

Background: It is well-known that the solution to certain systems of linear equations, which have been called "ill-conditioned," have been found to be peculiarly sensitive to small changes in the coefficient, or to rounding-off errors. The investigation of the condition of the equation mentioned is essential in order to obtain information about the errors in the approximate solution of differential equations, and the difficulty of getting these solutions.

Status: NEW. Mr. Todd continued the preparation of a paper on the condition of the system of linear equations obtained by approximating the biharmonic equation

$$\nabla^4 w = 0$$

by its finite difference analogue.

Publications: (1) "Note on the condition of matrices," by Olga Taussky-Todd, to appear in the April 1950 issue of MTAC. (2) "Notes on modern numerical analysis, I," by John Todd, MTAC 4, 39-44 (Jan. 1950). (3) "The condition of a certain matrix" by John Todd, Proc. Cambridge Phil. Soc. 46, 116-118 (1949). (4) "The condition of certain matrices. I," by John Todd, Quart. J. Mech. and Appl. Math. II, Pt. 4, 469-472 (Dec. 1949).

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.2/11-50-4

Origin: NBS Authorized 1/1/50
Sponsor: Office of Air Research, AMC, USAF
Managers: John Todd, Olga Taussky-Todd, M. Abramowitz

Objective: To conduct various studies in those fields of pure mathematics on which the theory and practice of numerical analysis are based; also to carry out certain investigations in pure mathematics which require large amounts of computing and so may result incidentally in a contribution to the science of computing. Many of these studies will be devoted to the investigation of special functions. Some will be concerned with the exploitation of automatic digital computing machinery in number theory.

Background: The principal scientific purpose of this project is simply the general expansion of mathematical knowledge. It is important in conducting research work at the scientific level of that of the Laboratory to insure freedom of inquiry to the staff, and in particular, to avoid setting up barriers to research at the apparent (and probably quite ephemeral) boundaries of a specific program. However, there are certain secondary aims of this project which make it contribute tangibly to the program of the Computation Laboratory. Two of these may be inferred directly from the statement of the Objective. A third is that it is to the advantage of the Laboratory to have mathematical members of its research staff retain their proficiency in the techniques and logic of pure mathematics, as effective research in numerical analysis and applied mathematics nearly always makes use

of the methods of pure mathematics.

Status: NEW. Dr. Abramowitz, in connection with the tabulation of the Coulomb Wave Functions (11.2/2-47-2) obtained various expansions of these functions which are convenient for use outside the range of the tables. These results will be submitted to the Journal of Research.

Mrs. Todd has begun the preparation of a survey of the work in number theory of A. Scholz at the request of the editors of Crelles Journal für die reine und angewandte Mathematik. She has also been invited by the editors of the new edition of the Enzyklopädie d. math. Wiss. to complete the article on "Special Number Fields" which had been begun by A. Scholz.

Mrs. Todd has studied ideal classes in real and imaginary quadratic fields by means of matrix mathods, of project 11.1/1-48-2 (Oct-Dec 1948 issue). In particular, the classes of order 2 have been subdivided into those classes for which the corresponding matrix class contain a symmetric matrix and those for which it does not. Numerical examples have been obtained to illustrate the theory. A manuscript containing the first results of this study is being prepared.

Mr. Todd obtained proofs of certain conjectures suggested by tables of the Laguerre orthogonal functions. Proofs of a corresponding conjecture in the case of the Laguerre function was obtained by Professor G. Szëgo and his results were extended to the ultraspherical case by O. Szasz (11.1, see 11.1/1-50-4). (This work relates to an earlier project 48R1-6.) This work will be submitted to an Italian journal.

RESEARCH IN LINEAR PROGRAMING
Project 11.2/12-50-1

Origin: Air Comptroller's Office, USAF Authorized 3/31/50
Sponsor: " " "
Managers: John Todd, Olga Taussky-Todd, F. L. Alt, M. Montalbano

Objective: To develop new methods for the solution of problems in linear programing, with special reference to high-speed automatic computing machines.

Background: The class of problems referred to as "linear programing" arises in connection with management planning of procurement training, budgeting, etc. They consist in determining the amounts of various activities (such as procuring certain items of equipment, training personnel of certain specific skills, etc.) which have to be carried on in order to reach a stated objective. In some cases the objective determines the activities uniquely, cf. project 11.2/36-49-3. In other cases the objective can be reached in many ways, and the problem is to determine that solution which minimizes a given function of the unknown activities, e.g., reach the objective with the lowest possible cost, or in the shortest possible time, cf. projects 11.2/33-49-4 (Apr-Jun 1949 issue) and 48S2-15 (Apr-Jun 1948 issue). Mathematically speaking, these problems can be formulated as the solution of systems of linear equations (frequently with almost triangular matrix) or as the maximization of a linear function subject to side conditions which are in the form of linear equations or linear inequalities. Special methods are needed because the number of unknowns is very large and because in practical applications the solutions of many problems are needed in a short time.

Status: NEW. Attempts are being made to program the problem of finding "feasible" solutions by means of almost-diagonal coefficient matrices (cf. project 11.2/36-49-3), both for the NBS Automatic Computer I, and for the IBM Card Programmed Calculator. Because both machines have a low output speed (on the NBS Computer this is only a temporary limitation pending construction of a high-speed magnetic tape output),

only certain limited problems can be handled efficiently at present. Therefore, efforts are being directed at specifying the kinds of problems that are suitable for these machines.

For the more general "Minimization Problem," of which projects 48S2-15 (Apr-Jun 1948 issue), 11.2/33-49-4 (Apr-Jun 1949 issue), and 11.2/36-49-1 (Jan-Mar 1949 issue) are examples, no satisfactory method of solution suitable for mechanization is known. Experimental computations are being performed, using variants of the method of the "Transportation Problem" (11.2/36-49-1), in an effort to find ways of programing problems of this type for computing machines.

2. Mathematical Tables

TABLES OF $E_1(z)$, $(z = x + iy)$
Project 11.2/2-43-3

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Project 11.2/2-46-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations of $\log\Gamma(x+iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ were completed. Extension of the table to include values for $x = 0(.1)9$ was underway.

TABLES OF COULOMB WAVE FUNCTIONS
Project 11.2/2-47-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were completed for $L = 1(1)5$, $\eta = 0(1)5$, $\rho = 0(.2)5$.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLE OF ANTILOGARITHMS
Project 11.2/2-47-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of the manuscript by differencing was in progress.

Publication: To be published by the Columbia University Press.

TABLES FOR THE OCCASIONAL COMPUTER
Project 11.2/2-47-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1948 issue.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Project 11.2/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN HERMITE POLYNOMIALS
Project 11.2/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Roots for the first 11 polynomials were determined. Values of all polynomials in the neighborhood of zeros were computed, preparatory to finding the roots. Computation of weight factors was started.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Project 11.2/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Manuscript has been prepared and is awaiting final check. Introduction was drafted.

TABLE OF POWERS OF COMPLEX NUMBERS
Project 11.2/2-50-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: Irene Stegun

Authorized 9/1/49
Completed 3/31/50

Objective: To publish a table of the exact values of the first 25 powers of $z = x + iy$ for $x, y = 0(1)10$.

Background: The manuscript of such a table, which is basic for a great many computations, had been prepared several years ago by the Mathematical Tables Project, under Mr. Herbert Salzer. At the present time it is intended to check and publish this manuscript.

Status: COMPLETED.

Publication: Issued as No. 8 in the NBS Applied Mathematics Series. Available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 25 cts.

TABLES TO FACILITATE SEQUENTIAL t-TESTS
Project 11.2/2-50-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. Awaiting page proofs.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 7 in the NBS Applied Mathematics Series.

TABLE OF CHEBYSHEV POLYNOMIALS
Project 11.2/2-50-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Checking of galley proofs was completed.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 9 in the NBS Applied Mathematics Series.

PROBABILITY TABLES FOR EXTREME VALUES
Project 11.2/2-50-4

Origin: NBS, Section 11.3
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The calculation and first check of the function log(-log x) was completed. The table of the cumulative distribution and probability density of extremes was checked by recalculation. The tables of x log x and of the probability points for n-th extremes were subjected to a difference check. Checking of the table of distribution and density function for the range and preparation of inverse tables of these functions was in progress. This project is related to project 11.3/2-50-1.

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Project 11.2/2-50-5

Origin: NBS Authorized 3/1/50
Manager: H. E. Salzer

Objective: To prepare, and keep up to date in readily available form, information regarding a) mathematical tables, b) important publications in numerical analysis. The reference file is to indicate not only what publications exist, but also where they are available in this area. Also, c) to maintain a file of errata in mathematical tables, d) to maintain a reprint file on mathematical tables and numerical analysis, e) to maintain a file of reviews, comments, and errata of the publications of the Computation Laboratory.

Status: NEW.

3. Computing Services

GUST ATTACKS ON DELTA WING
Project 11.2/31-50-1

Origin: Aircraft Laboratory, AMC, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Of the 7 specific cases originally requested by the originator, three have been completed and results submitted to the AMC, three others have been calculated and await checking. Data on additional cases are being prepared by the originator.

HEAT CONDUCTION EQUATION
Project 11.2/33-46-1

Origin: Bureau of Ordnance, Department of the Navy
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Project 11.2/33-49-2

Origin: Naval Research Laboratory, USN
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameters a_1, m_1, were performed when requested.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Project 11.2/33-49-5

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Table of compressibility factors and relative density of CO_2 was started.

FERMI FUNCTION II
Project 11.2/33-49-10

Origin: NBS, Section 4.4
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Auxiliary tables for the Introduction were completed. Preparation of the manuscript was started.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Project 11.2/33-49-11

Origin: Lewis Flight Propulsion Laboratory, NACA
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The partial derivatives of the composition factors with respect to each of the parameters were being computed for all cases for which the composition has previously been determined.

SHOCK WAVE PARAMETERS
Project 11.2/33-49-13

Origin: Bureau of Ordnance, Department of the Navy
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

BASIC IONOSPHERIC DATA
Project 11.2/33-49-14

Origin: NBS, Section 14.1
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

Publications: Current data are published in the monthly CRPL-F reports of the NBS Central Radio Propagation Laboratory; copy prepared from punched cards will replace hand-made copy as soon as procedures and schedules are worked out. The results of statistical computations will be included in scientific papers where appropriate.

RADIO-TELEGRAPH INTERFERENCE
Project 11.2/33-49-17

Origin: NBS, Section 14.4
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

PERCENTAGE POINTS OF THE ARITHMETIC MEAN IN RANDOM SAMPLES FROM THE SECH AND SECH^2 DISTRIBUTIONS
Project 11.2/33-49-18

Origin: NBS, Section 11.3
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

TABLES OF CABLE FUNCTIONS
Project 11.2/33-49-21

Origin: David Taylor Model Basin, USN
Sponsor: " " "
Manager: S. Prusch

Authorized 6/25/49
Completed 3/31/50

Objective: To evaluate the functions

$$\log_e \tau = \int_0^{\phi} \frac{P}{Q}\, d\phi\,, \qquad \sigma = \int_0^{\phi} \frac{T}{Q}\, d\phi$$

$$\eta = \int_0^{\phi} \frac{\tau \sin \phi}{Q}\, d\phi\,, \qquad \xi = \int_0^{\phi} \frac{\tau \cos \phi}{Q}\, d\phi$$

where $P = f \frac{\cos\phi}{|\cos\phi|} + w \sin \phi$, $Q = - |\sin\phi| \sin\phi + w \cos\phi$,

and $w = \sec \phi_c - \cos \phi_c$, for $f = .01(.01).03$, $\phi_c = 0(5^\circ)90^\circ$,

$\phi = \left[\phi_c(1^\circ)\phi_c + 180^\circ\right]$.

Background: These functions facilitate the determination of the shape of and tensions in a flexible cable that is held in a uniform stream. The effects of the weight of the cable and frictional drag are included. The types of configurations that may be treated by means of these functions are most general.

Status: COMPLETED. Results were transmitted to the David Taylor Model Basin.

RATING OF WATER CURRENT METERS
Project 11.2/33-50-2

Origin: NBS, Division 6.5
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were performed as requested.

A PROBLEM IN MOLECULAR STRUCTURE, I
Project 11.2/33-50-3

Origin: Naval Research Laboratory, USN
Sponsor: " " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations for various values of the parameters were being performed as requested.

VIBRATION OF PLANETARY GEAR TRAIN
Project 11.2/33-50-5

Origin: David Taylor Model Basin, USN — Authorized 12/1/49
Sponsor: " " " — Completed 3/31/50
Manager: J. H. Levin

Objective: For a set of given matrices to find all real positive characteristic roots lying below a given boundary. The given matrices are of order approximately 15, are symmetric, and have the property that most, but not all, non-zero elements are close to the principal diagonal.

Background: The problem arises in the study of planetary gear trains whose vibrational frequencies are represented by the desired characteristic roots. It is desired to find a rapid mechanical computation procedure which will determine, for any given design of a gear train, those vibrations which fall within a certain dangerous frequency band. In order to make use of the almost diagonal character of the matrix, a generalization of the method of Prohl-Myklestad (J. Appl. Mech. 12, No. 3, Sept. 1945) is used. The latter method, as it stands, applies to matrices in which no non-zero element is more than one step away from the principal diagonal. This is not quite general enough for the present case.

The problem and method of solution were requested specifically by Dr. A. Gleyzal of David Taylor Model Basin.

Status: COMPLETED. Results were transmitted to the David Taylor Model Basin.

IONOSPHERIC WINDS
Project 11.2/33-50-7

Origin: NBS, Section 14.1
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Data were being processed as submitted.

CRYSTAL STRUCTURES OF CEMENT COMPOUNDS
Project 11.2/33-50-9

Origin: NBS, Division 9, Portland Cement Research Associate Project
Sponsor: Portland Cement Association
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. Awaiting receipt of specifications of parameters.

WAVE RESISTANCE OF SHIPS, III
Project 11.2/33-50-11

Origin: David Taylor Model Basin,USN
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Work on evaluation of integrands on punched cards was in progress.

FREQUENCY DISTRIBUTION IN AN ENSEMBLE
Project 11.2/33-50-12

Origin: NBS, Section 5.2 Authorized 3/31/50
Sponsor: " "
Manager: Irene Stegun

Objective: To determine the cumulative frequency distribution, by weight, of glass beads of various sizes in a large ensemble of such beads, on the basis of stratified sampling.

Background: The ensemble of glass beads, which are of near-microscopic size, is to be used in the testing of sieves.

Comments: Requested by Dr. S. Carpenter of Section 5.2.

Status: NEW. Distributions for three samples were determined.

RAY TRACING
Project 11.2/33-50-13

Origin: NBS, Section 1.6 Authorized 3/1/50
Sponsor: " "
Manager: J. H. Levin

Objective: To compute the location and direction of light rays passing through a system of optical lenses.

Background: Numerous computations of this kind are required in order to evaluate the performance characteristics of a given system of lenses. The problem is mainly one of efficient organization of computations on a large scale.

Comments: Specifically requested by Dr. I. C. Gardner of the Optical Instruments Section (1.6) of the NBS. Methods employed are worked out in collaboration with Dr. Gardner and Mr. D. P. Feder.

Status: NEW. Several optical systems were evaluated by the method of "skew ray tracing" on the IBM Card Programmed Calculator. Computations by the method of third and fifth order abberations on the same machine are being planned.

WAVE PROPAGATION IN THE IONOSPHERE
Project 11.2/33-50-14

Origin: NBS, Section 14.1 Authorized 3/31/50
Sponsor: " "
Managers: M. Abramowitz, Gertrude Blanch

Objective: To solve a pair of simultaneous ordinary differential equations of order 4 with two-point boundary conditions, describing the propagation of electromagnetic waves of vertical incidence in a magnetic field of general intensity and direction.

Background: The equations arise in the magneto-ionic theory of wave propagation in the ionosphere.

Comments: Specifically requested by Drs. A. E. McNish and J. Feinstein of the Ionospheric Research Laboratory. The computations will be performed largely by the Institute for Numerical Analysis.

Status: NEW.

MOLECULAR STRUCTURE CALCULATIONS, II
Project 11.2/33-50-16

Origin: Naval Research Laboratory, USN Authorized 3/31/50
Sponsor: " "
Manager: B. Heindish

Objective: (a) Computation of radial distribution functions

$$F(r) = \sum_i A_i \sin s_i r$$

for specified r, where A_i and s_i are given.

(b) Computation of intensity of molecule scattering,

$$I(s) = K \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} \exp\left(-\frac{s^2 \overline{\delta \ell_{ij}^2}}{2}\right) \frac{\sin s r_{ij}}{s r_{ij}}$$

where N is the number of atoms in the molecule being studied, r_{ij} is the equilibrium distance between ith and jth atoms, $\overline{\delta \ell_{ij}^2}$ is the average of the squares of the projected displacement between the ith and jth atoms, c_{ij} is a constant equal approximately to the product $z_i z_j$ of the atomic numbers; K is a proportionality constant, and $s = (4\pi \sin\theta/2)/\lambda$ where θ is the angle of scattering and λ is the wave length of radiation.

Background: This problem is an application of punched card technique to the calculation of intensity and radial distribution functions involved in the determination of structures of gas molecules by the electron diffraction method. Briefly the procedure followed is first to compute the radial distribution function F(r) as in (a) where the A_i are obtained from an experimentally determined intensity function. The function F(r) is related to the probability of finding two atoms a distance r apart, and is used to obtain the interatomic distance r_{ij} for the calculation in (b). The intensity function computed in (b) should agree with the experimentally determined intensity function. The limits of precision of r_{ij} and $\overline{\delta \ell_{ij}^2}$ are gotten by

varying these parameters and comparing the resulting I(s) with the experimental I(s).

Status: NEW. Computations were being performed as requested.

STANDARD LORAN TABLES
Project 11.2/34-50-1: Gulf Coast Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. Awaiting receipt of base station coordinates.

EFFECT OF NUCLEAR RADIATIONS ON HUMAN BEINGS
Project 11.2/35-49-1

Origin: Operational Research Office, U. S. Army (Johns Hopkins University)
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computation of attenuation of radiation of varying energies penetrating body tissues is in progress.

TABLES FOR COMBAT CREW PLANNING
Project 11.2/36-49-2

Origin: Air Comptroller's Office, USAF — Authorized 3/21/49
Sponsor: " " — Completed 3/31/50
Manager: J. H. Levin

Objective: These tables pertain to combat losses among air crew personnel under specified conditions. Corresponding to assumed loss rates and tours of duty, the tables give: (1) chance of completing tour, (2) chance of not completing tour, (3) potential sorties per new crew, (4) average potential sorties per crew in combat in mature theater, and (5) effective loss rate per sortie (attrition plus retirement). "Tour of duty" is here defined to mean a given number of sorties for a crew to fly, at the completion of which it is automatically dropped from combat flying duty. "Mature theater" is a hypothetical theater in which the crew flow balances the losses and holds constant the proportion of crews at each experience level.

Background: These tables are an extension of the tables in AAF Manual 150-35-1. They are of importance, among other things, in the establishment of training programs. Thus, it is necessary to determine what attrition rates might be expected so that crews could be scheduled for completion of training in time to meet resulting replacement needs.

Status: COMPLETED.

Publication: Tables to be included by the Air Comptroller's Office in the next edition of the Air Force Technical Manual on Combat Crew Planning.

LINEAR PROGRAMING ON STANDARD PUNCHED CARD MACHINES
(formerly A Problem in Linear Programing)
Project 11.2/36-49-3

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Programs were being calculated as requested by the originators.

4. Training

TRAINING IN NUMERICAL ANALYSIS AND MACHINE COMPUTATION
Project 11.2/4-50-1

Origin: NBS Authorized 3/1/50
Managers: Senior Staff members of Section 11.2

Objective: To train members of the staff of 11.2 in the techniques of modern numerical analysis and in the use of existing and proposed computing machinery.

Comments: This project replaces project 11.2/4-49-3 (which is being discontinued) and broadens its objective.

Status: NEW. A training course on "Introduction to Numerical Computation" was given by Mr. John Todd. It was attended by 16 members of the Laboratory staff as well as by about 12 other students from other divisions of the Bureau and from the Bureau of the Census. Three members of the Laboratory staff were attached to the Machine Development Laboratory to receive training in programing and coding for the NBS Automatic Computer I equivalent to one month's full-time work. They, with senior staff of the Computation Laboratory, attended the weekly Seminar organized by Dr. R. S. Slutz of the Ordnance Division. Most members of the Punched-Card Unit attended a series of conferences on operation of the new IBM Card Programmed Calculator. On-the-job training of new staff members in the operation of standard punched card machines continued. Since December 1949, bi-weekly staff meetings have been held at which the background of various projects handled in all sections of the National Applied Mathematics Laboratories is discussed.

1. Research in Mathematical Statistics

THE MEAN DEVIATION, STANDARD DEVIATION, AND RANGE AS ESTIMATORS OF SCALE PARAMETERS (MEASURES OF DISPERSION) OF PROBABILITY DISTRIBUTIONS

Project 11.3/1-47-2

Origin: NBS — Authorized 7/1/47
Managers: Churchill Eisenhart, Lola S. Deming — Terminated 3/31/50

Objective: To evaluate percentiles and other features of the distributions of these estimators in small random samples from normal (Gaussian) and various non-normal (see project 11.3/1-47-1, Jan-Mar 1949 issue) populations.

Background: Previous studies of the relative merits of the mean deviation, standard deviation, and range as estimators of scale parameters of probability distributions have, in the main, concentrated on (a) evaluation of adjustment factors for rendering them unbiased estimators of, say, the standard deviation of the population; and (b) comparison of their "efficiencies" (as measured by the ratios of their sampling variances when so adjusted). Since their distributions in small samples are non-normal and generally differ in form, comparisons of their "efficiencies" in small samples may not truly represent their relative merits with regard to accuracy and precision in such cases. The approach via percentiles and other features (e.g., the probability of underestimating the true value of the relevant scale parameter) is expected to yield important new information.

Status: TERMINATED. The new project 11.3/1-50-1, Estimation of location and scale parameters, unifies and continues work begun under this and certain related projects.

Publications: (1) Probability center lines for standard deviation and range charts. Churchill Eisenhart. Industrial Quality Control VI, No. 1, 24-26 (July 1949). (2) The relative frequencies with which certain estimators of a normal population tend to underestimate its value. Churchill Eisenhart and Gelia S. Martin. To be submitted to the Journal of the American Statistical Association. An abstract appears in Annals of Mathematical Statistics XIX, No. 1, 600 (Dec. 1948).

STATISTICAL TESTS OF SIGNIFICANCE FOR 2 x 2 TABLES WHEN THE NUMBER OF OBSERVATIONS IS SMALL
Project 11.3/1-47-3

Origin: NBS
Manager: Churchill Eisenhart

Authorized 7/1/47
Terminated 3/31/50

Objective: To compare Fisher's "exact" test, Barnard's "C.S.M" test, and certain other statistical tests for data arranged in 2 x 2 tables with respect to (a) scope, i.e., conditions for which the respective tests are valid, and (b) operating characteristics (i.e., bias, power, etc.) under the conditions for which they are jointly valid.

Background: The project was undertaken in connection with an invited address given at the Symposium on 2 x 2 Tables, sponsored by the Institute of Mathematical Statistics at the New Haven, Connecticut, meeting on September 2, 1947.

Status: TERMINATED. There appears to be no prospect of reactivating research on this subject in the near future. This project is, therefore, considered terminated.

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Project 11.3/1-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Manuscript of the final paper, "Properties of statistics involving the closest pair in a sample of three observations", was typed. Examination and correction of typographical errors prior to submission to Division reader, was in progress.

Publications: (1) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. To be submitted to the NBS Journal of Research. (2) The fallacy of the best two out of three. For this summary of the results of the study and their practical implications see NBS Tech. News Bull. 33, No. 8, 1 (July 1949).

STATISTICAL PROCEDURES FOR INTERPOLATED MEDIANS
Project 11.3/1-49-2

Origin: NBS
Managers: Churchill Eisenhart, Julius Lieblein

Authorized 3/21/49
Terminated 3/31/50

Objective: To provide practical statistical procedures for employing interpolated medians in estimation and tests of significance for parameters of grouped populations, together with a rigorous mathematical development of the underlying theory.

Background: In response to a request from section 11.1 (see project 11.1/32-49-3), asymptotic standard errors and tests of significance were developed for interpolated medians in samples from grouped populations and form the basis of a note, "Standard errors and tests of significance for medians of grouped distributions," by Churchill Eisenhart and Miriam L. Yevick. It is the aim of the present project to provide a more complete mathematical treatment of the underlying theory, and a fuller treatment of the practical procedures.

Status: TERMINATED. The new project 11.3/1-50-1, Estimation of location and scale parameters, unifies and continues work begun under this and certain related projects.

Publication: An abstract of the note "Standard errors and tests of significance for medians of grouped distributions" by Churchill Eisenhart and Miriam L. Yevick appears in Annals of Mathematical Statistics 20, No. 1, 142-143 (March 1949).

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Project 11.3/1-49-3

Origin: NBS
Manager: Henry B. Mann. (Dr. Mann, who is responsible for technical aspects of this project, undertook the project during a temporary employment by the Bureau from March to June 1949, and is expected to return for brief periods until the work is finished. During his absence Dr. Eisenhart handles inquiries and other matters related to the project. Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Correspondence was exchanged with Professor Mann who is giving the first draft of the monograph a tryout in a course on stochastic processes at the University of California, Berkeley.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Project 11.3/1-50-1

Origin: NBS Authorized 3/1/50
Manager: Churchill Eisenhart

Objective: To compare statistical properties of alternative estimators of the location and scale parameters of particular probability distributions, from the viewpoints of (1) practical applications and (2) clarification of the aims and principles of statistical estimation.

Background: Statistical estimation is a field of research in which there is considerable activity at the present time. One line of research is directed toward determining the "best" estimators that is, toward the development of principles and techniques for determining such estimators when they exist; and another line is directed toward the development of easier-to-compute estimators that sacrifice as little as possible of the desirable properties of the "best". Even in the definition of "best" there are some difficulties - thus, for purposes of combination of estimates from several independent samples, unbiased estimators are desired; whereas, for use in single instances, there is much to be said for closest estimators; but unbiasedness and closeness are not always compatible, e.g. in estimating the standard deviation of a normal distribution. Much of the work to date along these lines has concerned itself with the determination of asymptotically "best" estimators, which have certain optimum properties for infinitely large samples. The present project, on the other hand, aims to concentrate primarily on estimators of value in the case of finite samples, particularly "small" samples of ten values or less, such as are frequently met with in physical science and engineering measurement.

Comments: This project, conceived as a continuing study, unifies and broadens the scope of work initiated under projects 11.3/1-47-1, 11.3/1-47-2, and 11.3/1-49-2. The first of these was completed in March 1949 (for a summary of results obtained, see Projects and Publications Jan-Mar 1949). The other two are now terminated.

The numerical results of project 11.2/22-49-18 bear on the study of the arithmetic mean as an estimator of location being continued under the present project.

Likewise, the numerical results on moments of order statistics in samples from a normal distribution evaluated under project 11.1/32-50-5 promise to be of value in connection with the present project.

Status: NEW. As an aid to the use and evaluation of individual order statistics as estimators of location and scale parameters of continuous probability distributions, work was begun on a table expressing the .001, .005, .01, .025, .05, .10, .20, .25, .50, .75, .80, .90, .95, .975, .99, 995, and .999 probability points of all order statistics in samples ofsizes n = 2(1)10, in terms of the corresponding probability points of the underlying (continuous) distribution. The entries are to at least 5D in both F(x) and 1-F(x). For the probability levels from .005 to .995 inclusive, the table is complete and partially checked from n = 2 to n = 5; and about 70% complete from n = 6 to n = 10. A similar table for the median only, in samples of odd sizes, was prepared some time ago under project 11.3/1-47-1. A detailed description of this earlier table and an indication of the method of construction are given in Annals of Mathematical Statistics, 19, 598-599 (1948).

2. Manuals of Statistical Methods

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS
Project 11.3/2-47-2

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STANDARD SAMPLING-INSPECTION PROCEDURES
Project 11.3/2-48-1

Origin: Office of Naval Research, and Research and Development Division of the Department of the Army
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. A February 7, 1950 draft of MIL-STD-105A, the end product of the work of the Sub-Committee for Revision of JAN Standard 105, on which Dr. Eisenhart served from April-September 1949, is now circulating within the Department of Defense, for comment and approval. For a summary of the background of this document, see July-Sept 1949 issue.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Project 11.3/2-48-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The compilation and revision of definitions continued.

Dr. Eisenhart was appointed a member of Task Group No. 9 on Precision and Accuracy of Committee E-11 of the American Society for Testing Materials. The mission of the Task Group, of which General Leslie E. Simon is the Chairman, is to study the problems connected with the use of such terms as "accuracy", "precision", "reproducibility", "reliability", and so forth, to designate those terms of this character for which it seems necessary or at least desirable, to have standard definitions, and to prepare precise definitions of the terms so selected.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Project 11.3/2-49-1

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Work on the preparation of abstract cards was resumed.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Project 11.3/2-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. An introduction was prepared, and the manuscript was referred to a reader within the division.

EXTREME-VALUE THEORY AND APPLICATIONS
Project 11.3/2-50-1

Origin: NBS Authorized 3/31/50
Manager: J. Lieblein

Objective: To prepare for publication the draft manuscript of four lectures on the above subject delivered at the Bureau by Professor E. J. Gumbel.

Background: On October 18-20, 1949, Professor E. J. Gumbel delivered a group of four lectures "On the statistical theory and methods of extreme values and their practical applications", as item X in the Applied Mathematics Colloquium Series, NBS. It is felt that the substance of these lectures will be useful as a reference on the subject matter concerned. The lectures were recorded and a semi-final draft was prepared from this transcription by Dr. Gumbel. It is the aim of the project to bring this manuscript into form suitable for publication in the National Bureau of Standards Applied Mathematics Series.

Status: NEW. Early in the quarter Dr. Gumbel visited the Statistical Engineering Laboratory, bringing with him the last installments of the semi-final draft of his lectures mentioned above, together with supplementary charts, tables, and other ancillary materials in manuscript or other preliminary form. Certain theoretical points in the manuscript were discussed on this occasion, and have since been the subject of an exchange of correspondence.

Certain probability tables for extreme values being prepared for publication under project 11.2/2-50-4 will serve as an adjunct to the brochure of the present project. A number of conferences were held during the quarter between 11.2 and 11.3 personnel on matters of format, scope, and computation of these tables.

MANUAL ON FITTING STRAIGHT LINES
Project 11.3/2-50-2

Origin: NBS Authorized 3/31/50
Managers: W. J. Youden, J. M. Cameron

Objective: To survey the theory and practice of fitting straight lines, and to prepare a manual for non-statistical scientists and engineers on the fitting of straight lines to experimental data.

Background: There is at present no adequate compilation of techniques of fitting straight lines - adequate to meet the needs of scientists and engineers without extensive training in statistical theory and method. Members of Committee E-11 feel that a manual similar to the ASTM Manual on Presentation of Data would be extremely valuable to engineers and scientists.

Comments: The project is being coordinated with the work of a task group of Committee E-11 of the American Society for Testing Materials, which is developing a less technical manual on the subject similar to the ASTM Manual on Presentation of Data. The present project will be specifically directed at the needs of the NBS.

Status: NEW. (1) Material prepared by J. M. Cameron for his talk on "The fitting of straight lines when both variables are subject to error" before the Washington Section of the American Society for Quality Control, on 16 March 1950, (2) lecture notes for three lectures on "Curve fitting" to be presented by Churchill Eisenhart at the Bureau during April 1950 (see project 11.3/4-50-4) and (3) a set of lecture notes on "The theory and practice of fitting a straight line" prepared by J. H. Curtiss in the Spring of 1947 for a Bureau in-hours course on "Correlation and Regression" will serve as the starting point for the manual.

3. Statistical Services

WOOL CONTENT OF BLANKETS
Project 11.3/31-47-6

Origin: NBS Section 7.5, and Division of Statistical Standards, Bureau of the Budget — Authorized 7/1/47, Terminated 3/31/50
Manager: Churchill Eisenhart

Objective: To develop a procedure for sampling a lot of part-wool blankets and for taking one or more specimens from each of the sample blankets in order to determine the wool content and weight of the blankets with reasonable assurance.

Background: The present Federal Specifications for blankets gives no procedure, and the A.S.T.M. Standard, a very inadequate procedure, for sampling a lot of blankets. Neither specification gives any instructions for taking one or more specimens from each of the sample blankets for tests and analysis. The scanty instructions given in general specifications on the number of specimens to be subjected to any particular test apply only to the verification of the precision of the test procedures and leave variability of product out of consideration.

The omission of a proper sampling plan for blankets is serious as the specifications require a minimum wool content and a minimum weight without specifying whether these minima apply to the lot, sample, blankets or specimen. Therefore, the manufacturer will generally either have to furnish a large excess of wool content and weight over the requirement or else take a big chance that his blankets will be rejected. The purchaser has the same risk of obtaining a large quantity of material that is deficient in wool content and weight by accepting material on the basis of results obtained on a piece or pieces that are not representative of the material.

It is the aim of this project to determine the variation in wool content and weight from point to point within a blanket and from blanket to blanket within a lot. This will furnish a basis for a rational sampling procedure as well as for giving an operational meaning to the terms minimum "percent wool" and "weight" as commonly applied to blankets.

Comments: This project stems from a project completed in the fiscal year 1947 dealing with the components of the variance of a wool-content determination based on a small piece taken at random from a part-wool blanket.

Status: TERMINATED. This project aimed to provide, in the case of part-wool blankets, data on the variation in weight and percent wool content from point to point within a blanket and from blanket to blanket within a lot - information needed for the development of rational sampling procedures for taking specimens from blankets and drawing blankets from lots for acceptance-test purposes, as well as to give an operational meaning to the terms minimum "percent wool" and "weight" as commonly applied to blankets. In October 1948 preliminary arrangements were made to obtain such data from production-line tests of certain blanket manufacturers. The demands made upon the staff of the SEL during the succeeding months in connection with the baled-wool problem (Project 11.3/31-48-2) were such as to preclude work upon the present project. Subsequently, the aims of this project as applied to textiles in general were considered by a Task Group on Revision of Army Material and Test Method Specifications for Textiles, and procedures were suggested for acquiring the necessary data on various textile products in connection with the testing program of the Quartermaster Corps - see Projects and Publications for July-September 1949. Although no explicit studies have been carried out under this project in

the direction of its original aims, it seems desirable to terminate this project in view of the fact that there appears to be no prospect of resuming direct work upon it in the near future.

STATISTICAL STUDY OF THE FLOW OF CERTAIN STOCKROOM ITEMS
Project 11.3/31-48-4

Origin: NBS, Section M.4
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STATISTICAL ANALYSIS OF THERMOMETRIC MEASUREMENTS
Project 11.3/31-49-4

Origin: NBS, Section 3.1
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. An experiment involving aged, unaged, and aged-by-a-special-process thermometers of the three types - pear, stubby, and oral - is being conducted by personnel of the thermometry laboratory to determine the change in thermometers with time and the efficacy of the special aging process. Four out of the five sets of readings covering a period of 6 months have been taken and are now being analyzed.

Publication: "Comparative tests in a single laboratory" by W. J. Youden; to be published in the Bulletin of the American Society for Testing Materials.

WOMEN'S BODY MEASUREMENT STUDY
Project 11.3/31-49-5

Origin: NBS, Section 12.2
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Having studied and decided upon three stature groupings for women 18 through 29 years, 30 through 40 years, and 41 through 60 years, the next logical step in the analysis was to determine body types within these stature intervals. This was done by employing bust girth-hip girth bivariate frequency distributions for each stature group within each age group - nine distributions. A regression line was computed from the medians and drawn upon each distribution. It was decided that tolerance limits for hip and bust girths of 5 cm. and 7 cm. respectively would provide a reasonably good garment fit. Upon each regression line an arrangement of areas 5 x 7 cm. was constructed and regarded as the "medium" type. Keeping hip girths constant similar areas were constructed above (for the "full" bust types) and below (for the "small" bust types) the "medium" areas. Thus if we regard these areas as garment sizes, a particular figure may be located in the system if it falls within the tolerance limits established on four critical measurements - age, height, hip girth, and bust girth. Further analysis on the remaining body measurements significant to the correct fit of women's garments and patterns is proceeding.

The Sizing Committee of the Mail Order Association of America who requested this project met for discussion of its progress in Washington 13-16 March 1950.

PRECISION MEASUREMENTS ON STANDARD TEMPERATURE SOURCES
Project 11.3/31-50-2

Origin: NBS, Section 5.0
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

PROBABILITY THEORY OF A CERTAIN DOUBLE BRANCHING SYSTEM
Project 11.3/31-50-3

Origin: NBS, Section 4.5
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

GENERAL STATISTICAL SERVICES
Project 11.3/31-50-4

Origin: NBS Authorized 3/31/50
Managers: W. J. Youden, J. M. Cameron

Objective: To furnish statistical advice and assistance to members of the NBS scientific staff on the planning, conduct, and interpretation of experiments.

Background: The advances in modern statistics, particularly in the design and arrangement of experiments, have for some years been employed in agricultural experimentation. The introduction of modern statistical method in the physical sciences and engineering has been slow, even though remarkable examples of their effectiveness have been recorded. These modern statistical methods in planning experiments should in time become as much a part of the experimenter's "tool kit" as other forms of applied mathematics.

Comments: This project formalizes and provides a medium for reports on various statistical consulting and advisory activities of insufficient size and scope to warrant individual projects.

Status: NEW. During the quarter 33 services of less than NAML project size were performed, for example, (a) a factorial design was prepared for an experiment for determining the optimum combination of pH and concentration of acetic acid for lead plating, (b) a regression method was given for determining the error of each of two methods of measuring the same quantity when duplicate readings by either method are impossible, (c) the selection of the optimum division of time between sample and background measurements in radioactivity so that the sample readings corrected for background have the minimum error, (d) preparation of a sampling scheme for a specification on mica.

Two manuscripts were reviewed for outside technical journals; three, for the Research and Testing Editorial Committee; and eight for the Commodity Standards Editorial Committee.

Five conferences were held with personnel of other Government agencies on problems relating to the statistical design of experiments.

STATISTICAL SERVICES TO ARMY FIELD FORCES TEST BOARDS
Project 11.3/32-49-1

Origin: Logistics Division, General Staff, U.S. Army
Sponsor: Research and Development Group, General Staff, U. S. Army
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. During the quarter work was concentrated on the preparation of (1) an expository paper on experimental arrangements which may be expected to improve the precision of test procedures, and (2) a small handbook of useful statistical formulas.

In varied types of test methods the same basic principles are employed to reduce the errors of the measurements. The expository paper is concerned, therefore, with discussing these principles and illustrating them through a few of the most generally applicable arrangements which have proved highly successful and are most nearly the arrangements that are suitable for experiments on military problems.

SOLID PROPELLANT DEVELOPMENT TESTS
Project 11.3/32-49-2

Origin: ORDTU, Research and Development Division, Ordnance Dept., U. S. Army
Sponsor: Research and Development Group, General Staff, U. S. Army
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

PROBABILITY STUDIES OF MISSILE EFFECTIVENESS
Project 11.3/33-49-1

Origin: Bureau of Ordnance, Navy Department, via Electronics Division, NBS
Sponsor: Bureau of Ordnance, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Work continued, with some changes of direction and emphasis as a result of instructions received from those in charge of the over-all program.

TECHNICAL SYMBOLISM AND TERMINOLOGY
Project 11.3/4-50-1

Origin: NBS Educational Committee — Authorized 9/1/49
Manager: Lola S. Deming — Completed 2/14/50

Objective: To offer an in-hours "editorial course" designed especially for typists and clerks who are responsible for transcribing, typing, proofing, and editing scientific material. Topics covered to include: Greek alphabet (recognition, pronunciation, writing, and typing); graphs, charts, and tables (scales, legends, general format, etc.); exponents, subscripts, radicals, other mathematical and chemical symbols; vocabulary and spelling of common scientific terms and proper names; rounding (rules and practices); significant figures.

Background: Because the preparation of a large proportion of manuscripts at the Bureau involves transcribing, typing, and editing of mathematical and scientific material, a course of this nature seemed useful. It was offered for the first time last year and was received with sufficient enthusiasm to cause the Educational Committee to ask to have it repeated this year and extended to two terms to include more material with several sessions conducted by a chemist to cover chemical terminology and vocabulary.

Comments: This course is listed, with the above title, and designated F111.1, on page 17 of the Announcement of Courses for 1949-1950 of the Graduate School of the National Bureau of Standards.

Status: COMPLETED. 32 persons enrolled for this course which began October 4, 1949 and ended February 14, 1950. Of these 25 formally completed the course.

STATISTICAL DESIGN OF EXPERIMENTS
Project 11.3/4-50-2

Origin: NBS, Educational Committee — Authorized 3/1/50
Managers: J. Mandel, W. J. Youden

Objective: To offer a three-term in-hours course in the principles and devices of the statistical design of scientific experiments and engineering tests, and associated techniques of statistical inference.

Background: A repetition, at the request of the NBS Educational Committee, on a three-term basis with additional material, of a two-term in-hours course on the same subject offered last year (see Jan-Mar 1949 issue of this publication.)

Comments: This course is listed, with the above title and designated as A116.1-A116.3, on page 8 of the Announcement of Courses for 1949-1950 of the Graduate School of the National Bureau of Standards.
The first term and a half of the course was taught entirely by Mr. John Mandel, Section 7.5, NBS; and the remainder is being taught by Dr. W. J. Youden, Section 11.3.

Status: NEW. The second half of the course is now in progress with an enrollment of 16. Topics covered to date include: randomization and replication, completely randomized designs, randomized block designs, Latin squares, interactions, interpretation of analysis of variance for complex experiments and the factorial design.

ACCURACY AND PRECISION
Project 11.3/4-50-3

Origin: NBS, Personnel Division — Authorized 3/1/50
Manager: Churchill Eisenhart

Objective: To present a series of three lectures on (1) the operational meaning of accuracy and precision and (2) the practical evaluation and representation of the accuracy and precision of a process of measurement; and to prepare a set of lecture notes on the material presented.

Background: A repetition, with some modifications, of material on the same subject presented last year (see project 11.3/4-49-3) a full description of which appears in the Jan-Mar 1949 issue of this publication.

The lecture notes are to form a portion of "The Handbook for Professional Employees" being prepared by the Personnel Division of the National Bureau of Standards.

These lectures are to be offered as part of the general series entitled "Introduction to Research", one phase of the Junior Professional Training Program of the Bureau.

Status: NEW. The three lectures were presented on February 17th and 24th, and March 3rd, respectively, with 70 or more persons in attendance on each occasion. Mimeographed notes for one of the lectures, and certain supplementary materials were prepared for limited distribution to those enrolled in the Junior Professional Training Program. Work on the final draft of the notes for the other two lectures continues.

CURVE FITTING
Project 11.3/4-50-4

Origin: NBS, Personnel Division Authorized 3/1/50
Manager: Churchill Eisenhart

Objective: To present a series of three lectures on (1) the fitting of linear and curvilinear functions to quantitative data by the method of least squares and otherwise, (2) statistical techniques for judging the goodness of fit, and (3) evaluation and representation of the precision with which the parameters involved are estimated; and to prepare a set of lectures on the material presented.

Background: A repetition, with some modifications, of material on the same subject presented last year (see project 11.3/4-49-6) a full description of which appears in the Jan-Mar 1949 issue of this publication.

These lectures are to be offered as part of the general series entitled "Introduction to Research", one phase of the Junior Professional Training Program of the Bureau. The lecture notes are to form a portion of "The Handbook for Professional Employees" being prepared by the Personnel Division of the National Bureau of Standards.

Comments: The write-up of much of the substance of these lectures, together with other related material, in the form of a "Manual on fitting of straight lines", is the aim of project 11.3/2-50-2.

Status: NEW. The three lectures are now scheduled for April 7th, 14th, and 21st, respectively.

1. Design and Construction of Automatic Digital Computing Machines

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Project 11.4/21-47-1

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During this quarter the contractor, Eckert-Mauchly Computer Corporation, became a subsidiary of the Remington-Rand Corporation. Representatives of the Remington-Rand Corporation with Mr. Wistar Brown representing the Eckert-Mauchly Computer Corporation met with NBS and Bureau of the Census representatives to discuss the contract for the Bureau of the Census Computing Machine. It was the opinion of the Remington-Rand representatives that the delivery of UNIVACs according to the schedule given in the NBS-E-MCC contract would be achieved by the contractor.

THE NAVY COMPUTING MACHINE
Project 11.4/22-47-2

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The contractor continued work on mercury-line storage and tape-handling units, as authorized by the Bureau. A complete adder has been constructed, and tape-handling units of a type making the magnetic tape available as a slow-speed computer memory have been essentially completed. The adder and tape-handling devices were constructed for insertion in automatic electronic computers, and therefore are of the nature of full-scale prototyping for the computer being constructed under contract with the Bureau.

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.4/23-49-1

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Requests for bids, incorporating performance specifications which had been prepared during the previous quarter, were prepared and forwarded to interested parties.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Project 11.4/24-47-3

Origin: Office of the Air Comptroller, USAF
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

NBS AUTOMATIC COMPUTER I
(formerly NBS Interim Computer)
Project 11.4/24-49-1

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Construction by the Electronics Division proceeded according to schedule. The computer has been almost completely assembled, and is under test. The mercury memory was delivered by the Technitrol Company (Philadelphia, Pa.) and performed well enough to allow testing of the computer with a full-size mercury memory, rather than with only an eight-word acoustic memory as had been envisaged. Up to this time, the engineering testing of the computer has proceeded more smoothly than had been hoped, a fact which all those familiar with the development of complicated apparatus will realize should not be used as a basis for prediction of the duration of the testing.

ARMY MAP SERVICE COMPUTING MACHINE
Project 11.4/25-49-1

Origin: Army Map Service, U.S.A.
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

DEPARTMENT OF THE ARMY COMPUTER DESIGN
Project 11.4/26-49-1

Origin: Department of the Army
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Developments on this project consisted mainly of work common to this project and to project 11.4/24-49-1 (NBS Automatic Computer I).

2. Programing Studies

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Project 11.4/3-47-4

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED.

I. General programing for automatic computing machines.

1. A method for minimizing the loss of significant figures in the inversion of matrices of high order was investigated, preliminary to the coding of the method for use on electronic digital computers.
2. A preliminary survey was made of the Fourier method in crystal structure determination, with a view to determining whether the problems presented lend themselves to solution by automatic digital computers. The method was programed for the NBS Automatic Computer I, and considered for several other electronic digital computers under construction.

II. Programing for the NBS Automatic Computer I. The following routines and subroutines were completed and subjected to final checking during the quarter:

1. Conversion of double precision numbers (fixed binary point).
2. Subroutines for arc tangent, e^x, 2^x in floating form with fixed binary point.
3. Subroutines for the n-th root and arc tangent (single precision and fixed binary point).

A comprehensive and diagnostic test was diagramed and coded. This test will be used to check the computer during its initial operating period.

The analysis and preliminary coding have been completed for the following problems:

1. Solution of Laplace's equation for a square region.
2. Skew ray tracing through an optical lens system.
3. Linear programing for the Air Force.

CODING ON THE E.R.A. COMPUTER
Project 11.4/3-49-1

Origin: NBS
Manager: Ida Rhodes

Authorized 12/1/48
Terminated 3/31/50

Objective: To evaluate the performance characteristics of the E.R.A. design for an electronic computer having a magnetic drum internal memory.

Background: In order to check its appraisal of the suitability of magnetic-drum type computers for Office of Naval Research applications, the Bureau entered into a contract with the Engineering Research Associates for the preparation of designs of such computers and the evaluation of their mathematical capacity. Thus the Bureau's efforts to appraise fairly magnetic-drum computing machines were supplemented by intensive technical and mathematical work by E.R.A.

Comments: This project was essentially a development of project 11.4/22-47-2.

Status: TERMINATED.

CODING RELATED TO THE UNIVAC SYSTEM
Project 11.4/3-49-2

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Continued investigation of sorting procedures on the UNIVAC was conducted by the working group, on which were represented the Bureau of the Census, the Naval Communications Center, the Office of the Air Comptroller, and the National Bureau of Standards. As a result of this work, by special arrangement, representatives of the Office of the Air Comptroller and the NBS began the preparation of detailed UNIVAC instruction codes for the general sorting problem, to be used as a basis for further analysis by the entire working group.

CODING RELATED TO THE RAYTHEON COMPUTER
Project 11.4/3-49-3

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

3. Technical Reports on Computing Machinery

THE MTAC SECTION
Project 11.4/4-47-1

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for the Automatic Computing Machinery Section of the July 1950 issue of Mathematical Tables and Other Aids to Computation was assembled and edited. In addition to the bibliography and news items, one technical and two discussion articles were included in this issue. The technical paper is entitled "Planning and error considerations for the numerical solution of a system of differential equations on a sequence calculator" by F. J. Murray. The paper deals with the solution of a specific system of 14 ordinary differential equations of the form

$$z'_i = f_i\ (z_1 . . . . . z_{14},\ t).$$

This theoretical discussion gives the reasons for the choice of method, the detailed step-by-step process decided upon, the considerations which led to choice of interval length, and those which are concerned with choice of scale. There were three types of error in the solution process--the truncation error, rounding errors, the error due to the presence of non-analytic functions in the f's. It is shown that a plan of solution should be based on over-all error considerations rather than on error per step. In the case of the truncation error, a resonance effect (i.e., a build-up of error) is found: this is different in nature from instability. The rounding error is treated by probability methods. The non-analyticities require a special technique discussed herein.

The first discussion article by Ida Rhodes of the Machine Development Laboratory, is entitled, "The incorporation of subroutines into a complete problem on the National Bureau of Standards Automatic Computer I." Due to the limited memory capacity of this computer it is impossible to assign a fixed set of memory locations to each subroutine to be used in problem solutions. Hence, the addresses of each subroutine will have to be modified to fit each specific problem. In substance, each subroutine is coded to begin at address 400. When it is put into the machine, it is not recorded there but rather at its final location in the memory. The difference is found between the machine-memory designation and the address 400, and this difference is then applied to all addresses of 400 or above within the subroutine - thus modifying the orders to agree with the actual machine-memory designation.

The second discussion paper, "A note on 'Is' and 'Might Be in computers", was written by Dr. G. R. Stibitz. In particular, he points out the responsibility on the part of scientists and experts in exercising great care not to mislead the public by false analogies to computers and by extravagant and unwarranted claims as to their potentialities. It is important to present fair, accurate, and dependable statements. He cites references to the "brain" analogy as applied to computers. While this can be useful to the expert as an aid to invention, it can lead to grave misunderstanding on the part of the layman.

In addition to this material, the galley and page proofs for the January 1950 issue of the journal were corrected and returned to the editor.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Project 11.4/42-49-2

Origin: NBS
Full project description appears in Apr-Jun 1949 issue

Status: INACTIVE. For status to date see July-Sept 1949 issue.

Lectures and Symposia

Applied Mathematics Division Technical Meetings

ORDWAY, F. (Mineral Products Division). Fourier analysis of crystal structures. January 9, 1950.

CANNON, E. W. The NBS Computer program. January 16, 1950.

STEGUN, I. The conformal mapping of a unit circle onto an ellipse. January 30, 1950.

LIEBLEIN, J. On certain integrals arising in the analysis of samples of three observations. February 13, 1950.

FANO, U. (Atomic and Radiation Physics Division). The Fermi function. February 27, 1950.

ANDREW, M. Automatic digital computing machinery. March 13, 1950.

Numerical Analysis Colloquium Series
(Los Angeles, California)

KARUSH, W. Characteristic vectors and roots of symmetric matrices. January 9, 1950.

HIRSCHFELDER, J. O. (University of Wisconsin). The theory of steady state propagation of one dimensional flames. January 23, 1950.

SZËGO, G. (Stanford University). Membranes and plates. January 31,1950.

WASOW, W. Random walks and eigenvalues of differential equations. February 13, 1950.

FEYNMAN, R. P. (Cornell University). A "new" calculus of operators. February 23, 1950.

LEFSCHETZ, S. (Princeton University). Some recent research on non-linear differential equations. March 7, 1950.

FISHER, R. A. (University of Cambridge, England). Exact test of significance for a Poisson series. March 11, 1950.

Statistical Engineering Seminars

EISENHART, C. On the precision of the mean of a group of similar experiments. January 20, 1950.

CAMERON, J. M. The fitting of straight lines when both variables are subject to error. February 3, 1950.

EISENHART, C. and DEMING, L. S. On the randomness of the digits of π and e to 2000 decimal places. February 17, 1950.

LIEBLEIN, J. (1) On a linear fit to extreme value data. March 3, 1950.
(2) On a linear fit to extreme value data. March 10, 1950.

Talks Presented by Members of the Staff at other Scientific Meetings at the National Bureau of Standards

CURTISS, J. H. Probability methods in the solution of differential equations (the Monte Carlo method). Presented at the NBS Scientific Staff Meeting, January 27, 1950.

HANDY, B. F., and D. P. FEDER (Electricity and Optics Division). Optical ray tracing problems and the card-programmed calculator. Presented at the Electricity and Optics Division Staff Meeting, March 21, 1950.

YOUDEN, W. J. Experimental arrangements that improve the precision of measurements. Presented at the NBS Scientific Staff Meeting, March 24, 1950.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J. M. The fitting of straight lines when both variables are subject to errors. Presented at a meeting of the Washington Section of the American Society for Quality Control, Washington, D. C., March 16, 1950.

CANNON, E. W. Large-scale computing machinery: A series of lectures presented to the Air Materiel Command, Wright Field, Dayton, Ohio, January 25-27, 1950.

CURTISS, J. H. (1) Probability methods in the solution of elliptic partial differential equations. Presented at a Conference on Automatic Computing Machinery held by the Association for Computing Machinery, at Rutgers University, New Brunswick, New Jersey, March 28, 1950. (2) Probability methods in the integration of differential equations. Presented at a Joint Stanford-University of California Seminar on Mathematical Statistics, Palo Alto, California, February 17, 1950. (3) A survey of the development of automatic digital computers. Presented to the Mathematics Department, Stanford University, Palo Alto, California, February 16, 1950.

EISENHART, C. On certain aspects of accuracy and precision. Presented at a meeting of the Western Massachusetts Society for Quality Control at Springfield, Massachusetts, February 2, 1950.

HESTENES, M.R. A gradient method for finding eigenvalues of differential equations. Presented to the Peripatetic Society at the University of Southern California, March 6, 1950.

HUSKEY, H.D. Various aspects of automatic digital computing machines. Presented at Wright Field to the Air Materiel Command on January 25-27, 1950.

LANCZOS, C. Iteration methods in the solution of Fredholm integral equations. Presented to the Peripatetic Society at the California Institute of Technology, January 9, 1950.

RHODES, IDA. Planning mathematical problems for electronic computation. Presented at the meeting of the New York Section of the American Statistical Association, New York, N. Y., March 30, 1950.

ROSSER, J. B. (1) Iterative schemes for solution of linear equations. Presented to members attending the Rand Research Seminar at the Rand Corporation, Santa Monica, California, March 10, 1950. (2) Iteration methods for inverting matrices. Presented to the Mathematics Department of Stanford University, Palo Alto, California, January 26, 1950. (3) Transformations to speed the convergence of series. Presented to the Mathematics Department of the University of California at Berkeley, California, January 27-28, 1950.

TAUSSKY-TODD, O. Hypercomplex systems with real coefficients. Presented to the Mathematics Department, Maryland University, January 19, 1950.

YOUDEN, W. J. (1) Statistics in biological experimentation. Presented at the Food and Drug Administration, January 23, 1950. (2) Yale University Lectures in Applied Statistics: (a) Determining the precision of measurements without a large number of replicates, February 14, 1950, (b) Improving the precision of scientific and engineering measurements, February 16, 1950, (c) The analysis of samples of three, presented at a seminar in the lecture series, February 16, 1950. (3) Design of experiments and applications. Presented at a meeting of the Baltimore Section of the American Society for Quality Control, Baltimore, Maryland, March 20, 1950.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables of the binomial probability distribution. NBS Applied Mathematics Series 6. Individual terms $\binom{n}{r} p^r q^{n-r}$ for $p = .01(.01).50$, $q = 1-p$, $n = 2(1)49$, $r = 0(1)n-1$, 7D; Partial sums $\sum_{s=r}^{n} \binom{n}{s} p^s q^{n-s}$ for $p = .01(.01).50$, $q = 1-p$, $n = 2(1)49$, $r = 1(1)n$, 7D. Available from Superintendent of Documents, Government Printing Office, Washington 25, D. C., $2.50.

(2) Tables of powers of complex numbers. NBS Applied Mathematics Series 8. Exact values of z^n for $z = x + iy$, $x, y = 0(1)10$, $n = 0(1)25$; exact values of x^n for $x = 0(1)10$, $n = 0(1)25$. Available from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 25 cts.

1.3 Technical Papers

(1) A class of mean value functions. E. F. Beckenbach. Am. Math. Mo. LVII, No. 1, 1-6 (Jan. 1950). Reprints available.

(2) Metric differential geometry. E. F. Beckenbach. Math. Mag. 23, No. 3, 143-152 (Jan-Feb 1950). Reprints available.

(3) Table of modified Bernoulli polynomials. G. Blanch and R. Siegel. NBS J. Res. 44, No. 1, 103-107 (Jan. 1950). Available as RP2060 from Superintendent of Documents, Washington 25, D. C., 5 cts.

(4) Acceptance sampling by variables, with special reference to the case in which quality is measured by average or dispersion. J. H. Curtiss. Appeared in "Acceptance Sampling: A symposium given at the 105th annual meeting," American Statistical Association, 1603 K Street, N. W., Washington 6, D. C., March 1950. $1.50.

(5) Solution of the telegrapher's equation with boundary conditions on only one characteristic. G. E. Forsythe. NBS J. Res. 44, No. 1, 89-102 (Jan. 1950). Available as RP2059 from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 10 cts.

(6) 1949 Ephemeris of Jupiter's ninth satellite. S. Herrick. Publications of the Astronomical Society of the Pacific 61, No. 360, 136 (June 1949).

(7) A note on the numerical integration of differential equations. W. E. Milne. NBS J. Res. 43, No. 6, 537-542 (Dec. 1949). Available as RP2046 from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 5 cts.

(8) Summation of slowly convergent series with positive terms. O. Szasz. J. Math. Phys. 28, No. 4, 272-279 (Jan. 1950). Reprints available.

(9) A recurring theorem on determinants. O. Taussky-Todd. Am. Math. Mo. LVI, No. 10, 672-676 (Dec. 1949). Reprints available.

(10) Notes on modern numerical analysis, I. John Todd. MTAC IV, No. 29, 39-44 (Jan. 1950). Reprints available.

(11) The condition of a certain matrix. John Todd. Proc. Cambridge Philos. Soc. 46, Pt. 1, 116-118 (Jan. 1950). Reprints available.

(12) The condition of certain matrices, I. John Todd. Quar. J. Mech. and Appl. Math. II, Pt. 4, 469-472 (Dec. 1949).

(13) Index for rating diagnostic tests. W. J. Youden, Cancer 3, No. 1, 32-35 (Jan. 1950). Reprints available.

(14) Statistics in analytical chemistry. W. J. Youden. Annals of the New York Academy of Sciences 52, Art. 6, 815-819 (March 10, 1950). Reprints available.

1.4 Reviews and Notes

The Institute for Numerical Analysis. An unsigned item prepared by J. H. Curtiss and published in the University of California Faculty Bulletin (Berkeley) 19, No. 7, 67-68 (Jan. 1950).

1.5 Miscellaneous Publications

(1) Misuse of the average deviation. NBS Tech. News Bul. 34, No. 1, 9-10 (Jan. 1950). Reprints available.

(2) Statistical Engineering. NBS Tech. News Bul. 34, No. 3, 39-41 (Mar. 1950). Reprints available.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION MARCH 31, 1950.

2.1 Mathematical Tables

(1) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. In press, Government Printing Office

(2) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(3) Tables for conversion of X-ray diffraction angles to interplanar spacing. H. Swanson. In press, Government Printing Office.

(4) Table of Arctangents of rational numbers. J. Todd. In press, Government Printing Office.

(5) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. In press, Columbia University Press.

(6) Tables relating to the Mathieu functions. In press, Columbia University Press.

(7) Errata in "Tables relating to Hankel integrals of order zero," by L. Schwarz, published in Luftfahrtforschung, 20, No. 12, 341-372 (1943), and translated by J. Lotsof, Cornell Aeronautics Laboratory, May 1946. A.H.Rosenthal Submitted to Mathematical Tables and Other Aids to Computation.

2.3 Technical Papers

(1) Tables of integrals of Struve functions. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. Submitted to the Bulletin of the American Mathematical Society; also to appear in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(3) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E.F.Beckenbach, W. Seidel and O. Szasz. Submitted to the Duke Mathematical Journal.

(4) On subharmonic, harmonic and linear functions of two variables. E. F. Beckenbach. To appear in Revista, Universidad Nacional de Tucuman (Argentina).

(5) A method for reducing the amount of inspection. J.M.Cameron and W. J. Youden. Submitted to Industrial Quality Control.

(6) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Submitted to the Duke Mathematical Journal.

(7) Forced oscillations in non-linear systems. M.L.Cartwright. Accepted for publication in the NBS Journal of Research.

(8) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(9) A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation. M. D. Donsker and M. Kac. Accepted for publication in the NBS Journal of Research.

(10) Second order determinants of Legendre polynomials. G. E. Forsythe. Submitted to the Duke Mathematical Journal.

(11) Matrix inversion by a Monte Carlo method. G.E.Forsythe and R. A. Leibler. Accepted for publication in the July 1950 issue of Mathematical Tables and Other Aids to Computation.

(12) Generation and testing of random digits at the National Bureau of Standards, Los Angeles. G. E. Forsythe. To appear in "The Monte Carlo method: Proceedings of a symposium held June 29, 30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office.

(13) Some elementary problems in the calculus of variations. M. R. Hestenes. Submitted to the Mathematics Magazine; also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A.

(14) Quadratic forms in Hilbert space, with applications in the calculus of variations. M. R. Hestenes. Accepted for publication in the American Journal of Mathematics.

(15) The method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix. M. R. Hestenes and W. Karush. Submitted to the NBS Journal of Research.

(16) Semi-automatic instruction on the Zephyr. H. D.Huskey. To appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory in September, 1949.

(17) Characteristics of the Institute for Numerical Analysis Computer. H. D. Huskey. Accepted for publication in the April 1950 issue of Mathematical Tables and Other Aids to Computation.

(18) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Submitted to the Bulletin of the American Mathematical Society.

(19) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. C. Lanczos. Accepted for publication in the NBS Journal of Research.

(20) Numerical determination of characteristic numbers. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(21) Note on the Runge-Kutta method. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(22) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Submitted to the Bulletin of the American Mathematical Society.

(23) Note on an infinite in egral. A. M. Ostrowski. Submitted to the Duke Mathematical Journal.

(24) Note on Vincent's theorem. A. M. Ostrowski. Accepted for publication in Annals of Mathematics.

(25) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to the American Mathematical Monthly.

(26) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

EXPLANATION OF PROJECT DESCRIPTIONS

The project descriptions appearing in this report are reproduced from the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With a view toward making this report more useful, an explanation of certain terms used in the Project Forms is given here.

Date of Authorization. This is the date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Status. Here is given the narrative of the progress to date on the project. Certain descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to which the entry applies. These terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the quarter covered by the report to work on the project.

"CONTINUED" means that the work was initiated prior to the quarter covered by the report, and was in progress during the quarter.

"INACTIVE" means that the Laboratories made a commitment prior to the quarter covered by the report to work on the project, but no work of any consequence was performed on the project during the quarter.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof is to undergo extensive mathematical checks, the designation "Completed" is employed only after these checks have been performed.

"TERMINATED" means that, although all aspects of the objective were not achieved, it was necessary to terminate the project due to circumstances beyond the control of the Laboratories.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "In Manuscript" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "Completed" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under *Status* or *Publication* to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.