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Oil and Gas Supply Modeling

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Oil and Gas Supply Modeling

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ABSTRACT

The symposium on Oil and Gas Supply Modeling, held at the Department of Commerce, Washington, DC (June 18-20, 1980), was funded by the Energy Information Administration of the Department of Energy and co-sponsored by the National Bureau of Standards' Operations Research Division. The symposium was organized to be a forum in which the theoretical and applied state-of-the-art of oil and gas supply models could be presented and discussed. Speakers addressed the following areas: the realities of oil and gas supply, prediction of oil and gas production, problems in oil and gas modeling, resource appraisal procedures, forecasting field size and production, investment and production strategies, estimating cost and production schedules for undiscovered fields, production regulations, resource data, sensitivity analysis of forecasts, econometric analysis of resource depletion, oil and gas finding rates, and various models of oil and gas supply. This volume documents the proceedings (papers and discussion) of the symposium.

Keywords: cost estimation; data collection; economic analysis; energy models; estimation; exploration; finding rates; forecasting; gas supply models; investment strategies; oil supply models; resource appraisal; sensitivity analysis.

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Goals and Purposes of the Energy Information Administration/ National Bureau of Standards Symposium on Oil and Gas Suppply Modeling

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This conference is convened to establish the state-of-the-art in oil and gas supply modeling: the initial step in improving the methodology behind the Energy Information Administration (EIA) projections. Also, to elicit critical comment on work performed within EIA, there are presentations of the EIA models. The conference concludes with a panel discussion on the directions needed for improving EIA forecasts.

The EIA is responsible for producing forecasts of the state of energy markets from now through the early part of the next century. These forecasts are widely distributed to inform the public about likely future energy costs, import levels, and domestic production and consumption rates.

Also, EIA is of service to the executive and legislative branches of government, providing independent analyses on the effects of energy programs. These analyses start with the Annual Report forecasts of energy supplies, demand, imports and prices with a representation of current programs. Then the models are altered to characterize the impacts of new programs within a new set of forecasts.

There have been significant advances in understanding oil and gas markets in the past several years. The most important advances have been in learning the right questions. When the government entered the energy forecasting business seriously, the questions about oil and gas were not well formed. One of the important goals of the analysis process is to try to make more precise the kinds of questions that the general public and policymakers ask about oil and gas. This function is aided by model building. Having the precise mathematical relationships of a model structures what the analyst can tell the customer. By the same token, a better understanding of the questions addressed comes from the debates on energy programs. This leads to alternative model formulations.

To understand what is now the state-of-the art, it is useful to see how perceptions of the issues evolved. The early questions were simpler than the current questions, allowing for smaller models to be effective. After the 1973-1974 oil embargo, the prominent questions were "Is there enough oil?" and "When would the Organization of Petroleum Exporting Countries (OPEC) cartel fall apart?" Except for pronouncements about the evils and unfairness of cartel pricing, little was said about the latter question except to state that all other commodity cartels had collapsed within a few years. This conference contributes to the international discussion only through the equation that imports equal consumption minus domestic production.

The first question, "Is there enough oil?" is imprecise. Efforts to turn it into a well-formulated statement led to the first modeling efforts. Why it is not precise is that it is a statement about supply while implicitly addressing supply and demand. If people never consumed any oil, there would be enough forever. What has now become abundantly clear is that there is not enough oil at prices people are willing to pay for the quantity they want to consume.

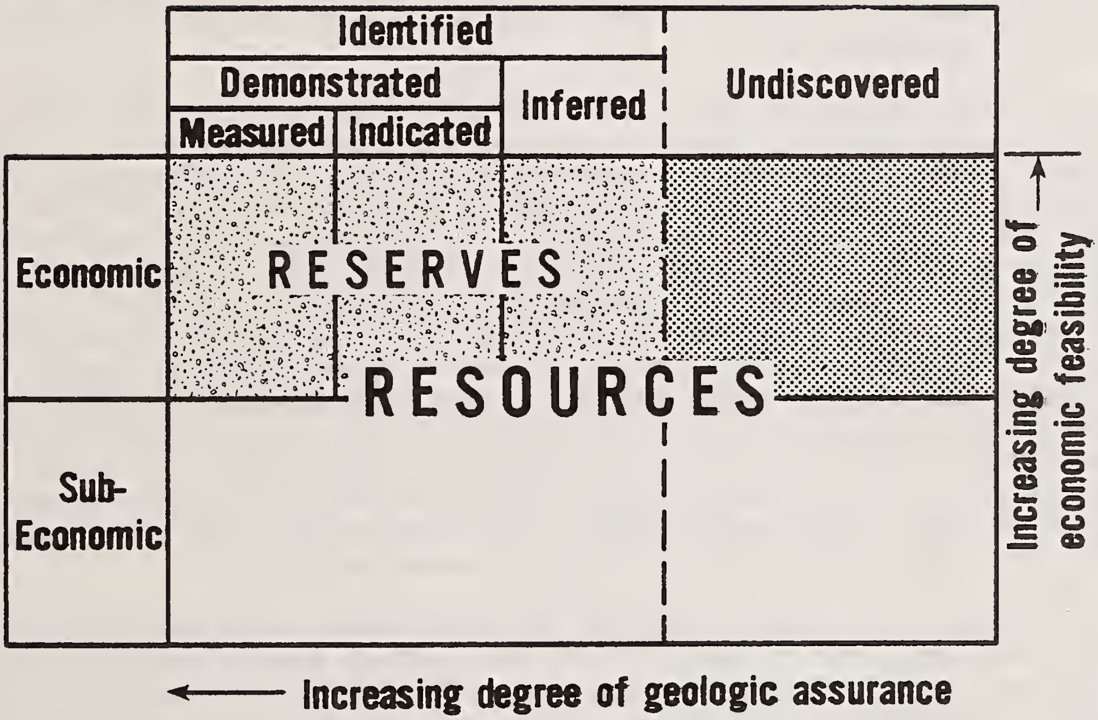
In a country where economic growth is matched by increased consumption of resources, one can alter "Is there enough oil?" to "When will oil production stop increasing?" A decline in production has always been inevitable because the resource base is finite. However, it was not until the work of M. King Hubbert that the point was clear. His remarkably accurate forecast of when production would start to decline is an example of answering a well-formulated question with a simple model that provided some insights.

The answers to the early questions essentially said that there is an energy problem and there will be an inevitable transition away from oil and gas. Given that the state of knowledge had gotten this far, the question became "How much oil is there?" Now, this is not a very precise statement either; it cost money to produce oil and gas, so there is a supply response to price.

The United States Geological Survey Circular 725 addressed the less precise question of how much oil there is while skirting the issue of price. It essentially gave the resource parameter for the Hubbert model, which also does not consider price.

Because the bulk of the oil discovered in a region is in very large fields, the supply response to price through new discoveries in that region, assuming uniform drilling costs, is not very great in terms of total resources for the region. This truth about the micro-level discovery process was extrapolated to the larger supply picture in Circular 725 by describing the resources using the imprecise notion of current economics. The standard diagram for representing petroleum resource classification (Figure 1) emphasizes this viewpoint by the sharp line dividing economic and subeconomic.

Figure 1. U.S. RESOURCE BASE



Diagrammatic representation of petroleum resource classification by the U. S. Geological Survey and the U. S. Bureau of Mines.

So, the correct characterization of the supply question is: "How much oil is economical to produce for a given range of prices?" It is convenient to address this subject in three parts, corresponding to this diagram. This partitioning leads to models, or modeling systems, with three components. For measured reserves, reservoir models can be used to estimate what is producible and how much oil can be produced at various prices. Next, there is the growth in reserves that comes from people learning while they are operating in existing fields. This constitutes the indicated and inferred reserves. As of yet, no one has studied the supply response to price for these reserves. The current modeling approaches use time-series analysis.

The third price of the partition is production from undiscovered resources. This is one of the areas where it is agreed that there is a supply response to price. The shape of the supply curve depends on the nature of what is left to be found and the character of the costs. If one uses the lognormal, field-size distribution model of Kaufmann or an Arps and Roberts model for a play, the total supply response to higher prices is not that great because the bulk of the oil is in a few, big pools that are found early. However, when one aggregates across plays, the supply response to price in a region is more elastic. This is a consequence of non-homogeneity across plays. In addition, for drilling in frontier areas, costs are high. Consequently, there are large fields that become economic only after prices have risen beyond what is necessary for finding the bulk of the resource in the easily explored areas.

The partition of the problem of reestimating supply curves for oil and gas into three prices has become more than an analytical device. Much of Federal policy is based on the nature of the supply repsonse to prices. If a supply curve is very steep, as with curve A in Figure 2, Congress might not allow the price to rise. There is not much of a supply response to price anyway, Congress can then either pass the lower costs onto consumer or use excise taxes to capture the profits for government. If there is more curvature, as in curve B, then the decision to control prices, keeping the extra profit, is more debatable because of the foregone production. When the supply curve gets very elastic, as in C, it does not appear to make any sense to control prices because the supply response is large, the "excess" profits are not as great, and the United States could achieve import independence.

So, not only is the economic quantity of oil and gas of interest, by the supply elasticity is as well. Even further, not only is the aggregate supply elasticity politically important, but also the supply elasticities for differing sources. This is shown in the legislation affecting oil and gas prices. Congress has imposed controls on what is perceived to be the less elastic portion of the supply curve and has allowed higher prices, or has decontrolled,

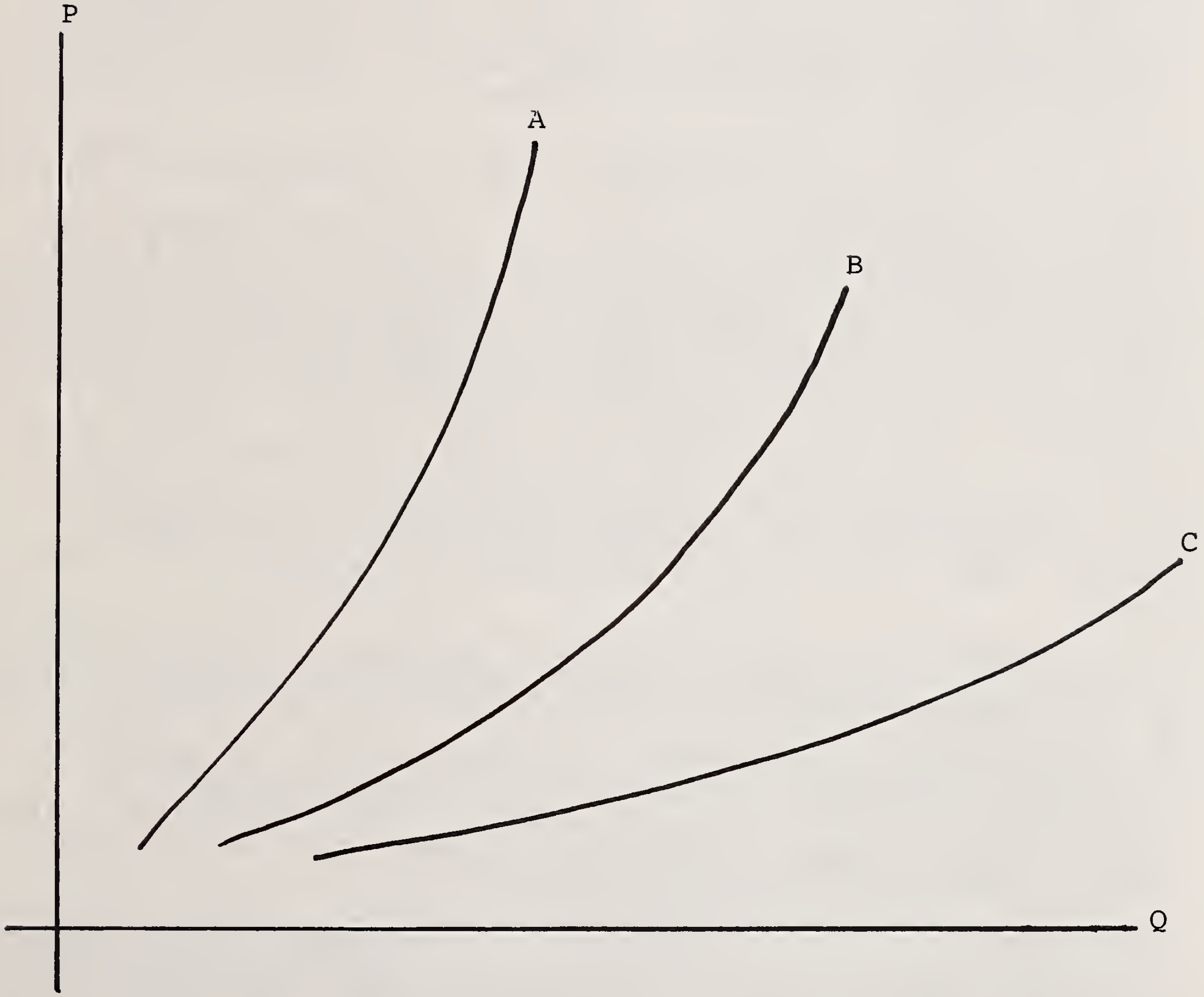


Figure 2. A Range of Supply Curve Shapes

the sources of supply that are more elastic. This presents a whole new category of more complex issues to analysts and their models. No longer are the explicit models of the anlysts, or the mental models of the policymaker, used just to understand the domestic supply position; they are used as an aid in determining who is paid what for their oil.

The discussion so far has emphasized the static aspect of the problem how many resources become economic with higher prices. The next question, first addressed by Hubbert, is "When will the resources be produced?" The timing of the development of these resources is important for determining when new technologies should be commercialized and how fast the country can achieve a modicum of energy independence.

The extent to which monopoly power exists affects the timing through the withholding of economic production. This issue is not addressed here because oil producers outside of OPEC are part of what is known as the competitive fringe, and the competitiveness of natural gas producers has been explored by the Federal Trade Commission in "The Economic Structure and Behavior in the Natural Gas Production Industry," February 1979. There is, however, a kind of withholding of production that could occur even if markets are competitive. With a depletable resource, a producer may hold on to it, rather than produce it, even if the cost of producing is less than the price. Knowing that the resource will eventually be depleted and its price will rise, producers can wait for the higher prices. This kind of withholding is not an example of monopoly power.

The last, and most complex, question addressed in the conference is "What are the impacts of all the Federal and State regulations on the industry in distorting the behavior of the participants by altering their incentives?" This is an important question because of the extent to which the oil and gas industry is regulated depends on those regulations. EIA has produced a study called Energy Programs, Energy Markets that broadly addresses some of these questions for all major fuels.

These last two issues of the timing of development and the effects of regulations are treated by economics, while the earlier issues associated with the static, ultimate recovery supply curve have been addressed by geologists and engineers. So, there is a range of questions that involves the work of geologist through the work of economists. This leads to the main purpose of this conference: to assemble the economists, operations researchers, and geologists in a room to talk about the aspect of questions on oil and gas supply they treat most effectively.

OIL AND GAS SUPPLY: PUBLIC PERCEPTION, MODELER'S ABSTRACTION, AND GEOLOGIC REALITY

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As some of you may recall, two years ago an article entitled "Oil and Gas Resources - Welcome to Uncertainty" appeared in RFF's publication Resources.[1] That was a "plain language" effort to present: the physical character of the search for and production of fluid hydrocarbons; the unavoidable narrowness of our approaches to examining oil and gas supply; and, the inescapable mystery of exactly how much oil and gas nature has to offer. This reflected what we had observed over the previous three years in trying to achieve a better public and governmental understanding of oil and gas resources.

Since a non-modeler has little in the way of technical expertise to offer a symposium of this type, your Chairman was kind enough to suggest that I as a consumer of your product might open the program with a few remarks. In effect I would like to re-examine "Welcome to Uncertainty" and comment on how much we have progressed in the past several years, and where shortcomings are still to be found. Perhaps I can be a bit challenging and provocative in the process.

As my title suggests, three viewpoints will be involved.

1) What is the current public perception of the oil and supply situation? Here we will regard the public as anyone who is not a professional supply analyst.

2) How well is the modeler in his abstractions or simulations capturing the essential ingredients in oil and gas supply, and, perhaps more important, how are those factors changing?

3) How much is the inescapable uncertainty about the ultimately retrievable quantity of oil and gas still hampering our work?

Public perception of oil and gas supply has gone through a metamorphsis over the past 6 years. In 1974, I arrived in Washington at the peak of concern over "how much is there". Resource estimates based on volumetric calculations vied against engineering projections in grabbing headlines in the Post or the Times. Each little divergence in someone's estimate of proved reserves on a lease or in the nation's reserves fueled the fire of public interest in the quest for who was telling the "Big Lie". The public was not yet convinced that there wasn't a lot down there, and felt "they" just weren't telling us how much.

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[1] Schanz, John J., Jr., "Oil and Gas Resource: Welcome to Uncertainty" Resources, Resources for the Future, Washington, March 1978. This article is reproduced in the Appendix.

Eventually this led to the transfer of the proved reserve calculations to the Federal government, and the resulting expensive and time consuming process of survey design, verification, and validation. It is of course somewhat disturbing to me that we have spent so much time on obtaining numbers that, while needed, have limited utility. Now that we have a public audit of proved reserves no one really seems too terribly interested. I doubt if a comparison of EIA proved reserves data with the old API/AGA numbers qualifies as the latest media sensation. Concurrently, we have waited five years for a contemporary assessment by the U.S. Geological Survey of the undiscovered oil and gas resources of the United States. We are akin to the driver in the desert who keeps eyeing his gas gauge when the real question is where is the next gas station.

This early concern over "how much", next shifted to a brief period of "they know where it is but won't drill". This involved public discussion of how much encouragement the driller needs, if any, to drill exploratory wells. And after oil and gas has been located, how much oil needs to be there to make it worth while to develop. Perhaps as a result of this, some of the public began to understand that "there are prospects- and then there are prospects". Some properties make the drilling agenda, some don't; and some won't make it unless the price goes up. Also, it became apparent that not every discovery was worth producing. The big hang-up was that not everyone could agree on how much had been found. Quite naturally the royalty interest, in many cases this was the government, was somewhat more bullish than the working interest.

The latest episode appeared about 1978. This was the period of "they are producing, but not as fast as they could", stimulated by the need for natural gas. This led to an examination of how many holes should you drill, and how fast can you "let'er rip" thereafter. The educational process of learning about the time configuration of investments and returns, recovery efficiencies, abandonment decisions, pay-out periods, and the flexibility or lack of flexibility in reservoir production schemes, became essential to consideration of proper diligence in the production of oil and gas from leases. Obviously this aspect of improving public perception was not a head-line grabber. But it was none-the-less an important aspect of public education. It is still going on. Adequate understanding is made difficult by the fact that an optimum development plan for the public may not necessarily coincide with that of the lease operator.

This seems to be where we are at the moment. I think in general we have a far more sophisticated public than six years ago - particularly among those that are close to the situation, as many public officials now have to be. I suspect that the man-on-the-street's perception is one of finally accepting that what is left to be found domestically will certainly not solve our problem. Unfortunately we may now find ourselves in the position of educational overkill. The apparent price has gone up dramatically and extensive drilling is reported, but we still seem to be no better off. This has led us away from the old "when the price goes up I bet they'll know where the oil is", to the view that "no matter how much the price goes up we won't find any more oil". As an economist I am intrigued that both arguments continue to suggest an increase in the price is not worthwhile.

This is a serious problem, because it threatens the necessary effort required to find and produce the remaining oil and gas resources of the United States. Regardless of judgments as to how much is left - 50 or 150 billion barrels, 700 or 1200 trillion cubic feet, that is still a lot of energy. We have left to produce more than the original endowment of many nations of the world. If the nation persists in this "our glass if half empty" viewpoint, or making futile comparisons with how much Saudi Arabia has compared to the U.S., we could very well end up not using what we still have at our disposal. While it is generally accepted that, short of dramatic technical change, the quantity left at the bottom of the glass is more expensive than the first half we consumed, oil and gas are still two of the better energy buys in town.

It appears that the next step in the continuing education of the public is for the modelers to show what is involved in emptying the last half of the glass. But most importantly, that the higher costs involved must be compared to the energy alternatives of today. The ghost-of-Christmas-past must be put to rest permanently. At issue is whether the pace of future depletion will be determined by past, present, or future costs and prices.

Let us now turn to the state-of-the art in our attempts to portray adequately the future response of oil and gas versus price. In 1978 in "Uncertainty" we called attention to the fundamental tunnel-vision of geologic appraisals, engineering projections, or econometric models. This was done in a kindly fashion, because we recognized those efforts were constructed for certain purposes and were limited by design. We can never fail to recognize that an oil and gas supply model, as a simplification, cannot capture all of the nuances and complexities of real world. If it did, we wouldn't need it, since we would be right back where we started. Without repeating all of the critique of several years ago, we said then, and can still say, that models have design limits. It is essential that the modeler clearly restrain the user by "red-lining" the model's guages. The modeler must make certain that he, as well as the user, knows what has been ignored, assumed away, or circumnavigated in accomplishing his purpose. I always like to be convinced that these things do not matter in achieving that purpose so long as the machine is not pushed beyond the red lines.

Looking back a few years ago, and comparing what I observed then with what I expect to hear in this symposium, I think we are finally beginning to recognize the need in many of our models to separate exploration from production, and that there are different kinds of explorers. Too many economic modelers in the past could not unlearn their first economic lessons of bushels of wheat versus price, or later on about manufacturing production functions. Exploration is a percursor of production. Its behavior is not inconsistent with economic theory, but allowing ourselves to be too quickly satisfied because the model appears to be consistent with the economic theory of price behavior and profit maximization can be a deception.

We have made progress in our oil and gas supply models by including the exploration sector and building in constraints. But I would suggest we should not become too smug. Some may be getting good initial responses in testing their model but not recognizing good results for the wrong reasons. While a economist can always live with a "catch-all" expectation that price in the long run captures all that is essential, I am not truly satisfied.

It is useful for everyone to occasionally go back and reread Grayson's Decisions Under Uncertainty.[1] It was very much a part of getting our present efforts started. Before he got involved in decision - trees and probabilities he described how in talking with oil managers he was able to identify the key factors in their decision process. We should remember the trio of; "goals", "economics", and "geology". I am quite content with development models built on a foundation of economics, because that is the way development occurs. But I continue to question a frontier exploration model that is solely profit-maximizing. If the modeler tells me that's the best he can do - so be it. I will accept that his model may be useful, but it is not an adequate representation of this key step in oil supply.

I focus on this because my own experience suggests that there are important non-price, non-profit factors in the exploration decision process. Given the fact that there is no reassuring knowledge of what is actually to be found in completely undrilled regions, the decision cannot be as calculating as some models may suggest - even given our growing sophistication in exploration technology and subjective, probabilistic analyses. As large enterprises manage large cash flows the decision process is not necessarily one of settling on the best menu of profit-maximizing projects. If this was the case, much of our exploration should be accused of ignoring opportunity costs - which an economic model should not tolerate. There is an ever present compulsion to explore because it needs to be done, and the discovery of physical oil is pursued by some with a degree of disregard about its relative profitability.

I would suggest that our exploration models can not adequately reflect governmental influence or the leverage of tax policies. Now that we are convinced that the world oil supply of the future is dependent upon the discovery of giant fields, I am concerned about how well we understand the consequences of the cash flow from past successes being diverted away from the companies that traditionally reinvested a major portion of past earnings back into frontier exploration for new fields. It now appears that much of the future earnings will move into the hands of governments, both U.S. and foreign. While I can see the government of an oil producing country seeking out profits from development, I cannot foresee how they will view a quest for the deferred profits from exploration. But more importantly, unlike multi-national companies who sought out geologic opportunities wherever they might be, recycled oil earnings in the hands of governments may be kept for the most part within national boundaries. This could restrict future investment to more exploration close to past discoveries. This is an ideal place to find a little more oil, but the worst place to look for another giant. In addition, the shift of oil income to non-petroleum purposes is already apparent.

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[1] Grayson, C.J., Decisions Under Uncertainty, Harvard University Graduate School of Business Administration, Boston, 1960, 402 pages.

The modeling of recent years seems to have laid to rest the economic expectation that raising the price can dramatically increase the recovery of oil and gas from old fields. The rewards from delaying abandonment are worth calculating but hardly of great consequence in the total picture. Also, we now recognize that an oil reservoir is not like a big factory where the production process can be changed like retooling a new car assembly line. While a lot of changes can be made before you develop, the economics can be very marginal if you decide to rework the reservoir after you have been producing it for some time with a given set of wells and practices. However, there is a lot of oil and gas left down there, and it well deserves continual economic and engineering examination of what we can do with it. Unfortunately, the evaluation and quantification of the rewards from advanced production technology has proved to be a frustrating exercise.

I would suggest that our governmental managers and members of Congress and their staffs-who have been close to this situation for nearly a decade- now recognize that in the absence of frontier discoveries a large increase in price will not produce a commensurate large increase in production. Also, they may now be convinced that the domestic industry's economics dictate, if demand is present, that reservoirs should be produced as rapidly as engineering prudence and regulatory rules will permit. Which, of course, is a domestic lesson we may now have to unlearn in that foreign governments may not be interested in the present value of future earnings in the same context as we are. Our international oil models will have to try to reflect a scheme of things which involves national income management, or the present value to current leaders of future social stability. That may prove difficult to put into a discounted cash flow model, and does not relate to traditional economic calculations. While our economists are now saying that foreign oil and gas may be worth more to foreign countries in the ground, this cannot be substantiated using a traditional economic analysis assuming conventional economic and political regimes. On purely economic grounds the anticipated increase in real price must be very large to justify taking a long-term risk on technical, market, and political uncertainties in preference to taking your profits as rapidly as possible. If, however, your strategy is not purely economic or conventional, then all signals are off.

As we view the progress we have made, we certainly must commend our modelers on the studies of the importance of field size distribution, the variation in the occurrence of oil and gas with depth, and the impact on exploration and production economics of investments in deep reservoirs, deep water, and hostile environments. We are also trying to recognize how we feed on exploration experience, with models now introducing a "play" concept.

The size of fields worth discovering in the frontiers of the world and the lumpiness of current investments have begun to be represented more clearly. There are lessons to be learned on how much the new game is a far cry from the modest investments of the old onshore wildcatter. In the past, the exploration game could turn on and off in the space of a few months, while the new world of oil does not work in that fashion. Also, the oil industry may be less likely to drill on the expectation of higher prices, so higher prices may have to be assured. This is in conflict with some who are inclined to continue to pay producers a low price for cheap oil from the past, and only offer a high price for more costly future oil after it has been found. Given the penchant for governmental authorities to perhaps change their minds after the heavy front-end billions have been sunk, there is a reluctance in some parts to play the game on a C.O.D. basis.

Our modelers have been recognizing, but still struggling with, various institutional restraints. In the final analysis the modeler must confess that leasing, environmental, and regulatory policies may have been ignored, and that political uncertainty is as difficult to deal with in the models as is geologic uncertainty.

I am concerned at the moment that there does not seem to be a public awareness of the rapidity with which oil exploratory drilling costs are going up. Much of this trend reflects the inflationary state of the economy, but there are also some pervasive factors relating to technology, locale, and depth that should be reflected in our analyses. A second trend is the shift of the proportion of exploratory drilling away from new field development toward other kinds of exploration. This must be highlighted now to defuse future public dissolutionment. Total domestric drilling has accelerated sharply and drilling rig availability may have been less restrained than we thought. As a consequence total wells drilled is now back at the 50,000 plus of bygone decades. But exploration in general has gone up less rapidly, and even more crucial is that new field exploration has responded least of all. The key question is whether or not new field exploration behavior is a reflection of the declining availability of new field opportunitites, or of public policy and other institutional restraints on new field exploration. It would seem important for our short and intermediate term models to both examine as well as reflect this trend.

In my last set of comments, I would like to turn to our basic geologic uncertainty. Obviously the modeler is driven to quantify, and the econometrician to place this in the context of economic theory. Unfortunately this must be accomplished with a few crude measurements and a host of uncertain estimates based on historical experience. When these are then coupled with a concept such as economic rent, the old rhyme about the purple cow comes to mind - "I have never seen an economic rent, I hope I never see one...". Perhaps my confidence in frontier leasing strategy built upon the concept of capturing the economic rent would be bolstered if I some day would overhear a chief executive officer brag to a colleague how he was able to keep X millions of dollars of economic rent out of the hands of the government.

One can be quite comfortable in the classroom discussing the disposition of surpluses earned in the market place that are over and above what is needed to entice the proper number of investors into the exploration and development of oil and gas resources. Yet in reality the estimates of the reserves on leases prior to drilling do not match very well with what is subsequently found. Nor does the size of the bid or the apparent sophistication of the company necessarily match up with ultimate discoveries. If our frontier leasing process is built upon the assumption that we can estimate oil and gas resources prior to drilling and thus calculate the economic rent, I find that a bit whimsical. So I remain unconvinced that a leasing system can be portrayed as a finely-tuned exercise in the capture of economic rents. This explicitly suggests that we in our ignorance will not alter the efficiency of our exploitation of our oil and gas resources. One can appreciate the concept, but that does not require the modeler or the economist to take the quantification of economic rent too seriously in the wrong contexture.

If we had a device that would actually detect and measure buried fluid hydrocarbons in situ, then the players in the game would know what is there to capture, have some idea of cost, and be able to decide what reward is needed to make it worth trying. Even without that device, after sufficient drilling we begin to approach that condition. At some point in the development process a prudent government can begin to set some limits concerning acceptable lease incomes. Prior to that, our discriminating powers about exploration targets remain a refinement of deciding where there are distorted marine sediments of the right age worth drilling. The process is not unlike a group of hunting dogs moving across a field sniffing in likely places. Once an actual scent is located, then the "play" is on and the whole pack goes charging off along a fresh trail.

To understand this process is important, because it suggests that much of our strict, new-field wild cat drilling may tend to be a process where sums of money are simple budget allocations to the game in contrast to being calculated investments. That does not suggest that the ante won't be increased when price conditions or expectations are bullish. Nor do we have to deny that the total process through end-product sale is justified on the basis of the eventual profitability. It merely suggests asking ourselves how much does bidding, competitive or otherwise, subtract from investment in frontier exploration drilling rather than being a simple transfer of rent which does not alter the exploitation process. If a billion dollars in lease bids actually translates into a reduction in exploratory drilling, then perhaps economic rents are more effectively retrieved after initial exploration rather than before.

Our knowledge of geology constantly entices us into viewing our oil and gas supply as a fixed inventory. But the subjective quantification of that inventory is a reflection of the limits of our geologic imagination and conventional practices. In effect, our mental classification scheme has semi-rigid boundaries. Over time there is a constant slippage with the appearance of oil and gas in new geologic settings once unknown, not understood, or too complex. Thus we now have the Overthrust Belt. A geologic opportunity not really reflected in our resource appraisals of a decade ago. It may not be a Middle East, but it is big enough to now show up on the charts.

A less dramatic example of this slippage is the slow evolutionary drift of unconventional resources across the boundary from known "other occurrences" into becoming a part of the estimate of conventional economic and sub-economic resources. During the Inter-Agency Study of the Permian Basin, one problem was trying to decide concerning these resources what had been counted, not counted, or counted twice. Here we must deal with such things as heavy oils or gas in sands and shales not previously producible, but located at the margin of opportunity. Higher prices and technology can gradually make a portion of these quantities usable - but how much and when?

So I find the reality of geology is having to cope with frustrating uncertainty. However, my statistically inclined colleagues tell me that once they unlimber their Monte Carlo wheels, apply their log-normal distributions, or introduce the nuances of Bayesian statistics, the mysteries of oil and gas resources begin to disappear. I watch with interest as they strip every bit of statistical inference from each new bit of drilling data. They have convinced me, like card-counters in a one-pack Black Jack game, that they have a handle on the future odds. Once a few cards are dealt, they already know that the Aces and high cards will be found in the top half of the deck, so we can now beat the House. Yet I have an uneasy feeling that if in estimating oil and gas resources each deal is a different deck, with a continual variation in the number and distribution of the cards, we still have some inadequacies in our models. We still don't know how many decks nature has left, nor is it out of the question that there are some kinds of decks we have never seen before. After all, the early Bradford Field drillers thought oil was distributed like ground water, and Captain Lucas probably never had heard of a salt dome when he drilled Spindletop. While I doubt if there are any major modes of oil and gas occurrence still unknown, one still wonders.

As we now turn to the various papers in this symposium, I am sure we will enjoy the magnificant display of "black boxes" and other things to fascinate those of us who like to probe the unknowns of oil and gas supply. I think we will be impressed with how much more knowledgeable we have become, but appropriately humble about what is still left unanswered. So I close with the same old punch line--continue to strive and you will one day have a truly satisfying oil and gas supply model. Let us always hope that it will be predictive rather than historic.

DISCUSSION

Dr. Parikh (Oak Ridge National Laboratory): Somewhere in the early part of your presentation, you mentioned that we have passed the 50 percent mark in oil production. I think you are referring to the Hubbert Curve. With or without a model, do you have a guess on what the cost of producing the last 10 percent is, or will be?

Dr. Schanz: Let me preface my comment that in a presentation of this type without being able to go back and examine my exact words, I hope that I presented that point as a likelihood that we are past the mid-point as a commonly held view.

I suspect, in my own judgment, that we are. But given the remaining in-place resources, we always can have a hope that the total presence in nature of unexploited oil and gas resources could provide us with a pleasant surprise. We would then find that we are not past the half-way point.

Obviously as an economist, but not currently involved in trying to quantify the last 10 percent through a model, I realize that everyone would like to identify what our energy alternative is in terms of what comes next as we move through the various energy resources. One would have to feel, in real costs, that the energy alternative to oil is somewhere to be found as we move through the $40, $50, $60 per barrel projections now being talked about. It would seem this kind of price level should trigger the alternative to oil, given a little lag time for its actual appearance in the energy market.

So intuitively I suspect that we cannot go much beyond what we are now paying for oil to start the energy replacement process for oil that all resource economists expect eventually to take place. Thus, not far beyond the current margin we can visualize that point where the remaining producible oil and gas resources will become the last 10 percent in our glass. Exactly where in the price spectrum, whether at $60 or $100 per barrel, we will cross that point of price versus quantity that defines the final 10 percent I am hard pressed to say. But going at the pace we now are, or expect go to, I would think we would reach the trigger point for alternative sources that will set the 10 percent pretty soon.

TECHNIQUES OF PREDICTION AS APPLIED TO THE PRODUCTION OF OIL AND GAS

M. King Hubbert*

Abstract

Techniques of prediction of future events range from the completely irrational to the semi-rational to the highly rational. Rational techniques of predicting the behavior of a system require first an understanding of its mechanism and of the constraints under which it operates and evolves. This permits the development of appropriate theoretical relations, which, when applied to the data of the system, permit solutions of its future evolution with varying degrees of exactitude. Such a theoretical analysis provides an essential criterion for what data are significant and necessary for the solution.

In the case of the production of petroleum in a given region, present geologic knowledge indicates that oil and gas accumulations occur in limited volumes of underground space, in porous rocks normally filled with water, within or immediately adjacent to basins of sedimentary rocks. These accumulations have resulted from plant and animal material accumulated in the sediments during the last 600 million years, at rates so slow that no significant additions can occur during the period of oil and gas exploitation. Therefore the exploitation of the oil and gas accumulations in a given region represents the continuous reduction of the amounts originally present and is a unidirectional and irreversible process, characterized by a definite cycle of events. Oil production in a given region begins with the first discovery at time t_o and ends finally at a later time t_k. Hence the cumulative production Q will have the value 0 at $t = t_o$, and a definite finite value Q_k at $t = t_k$. Thus, during the complete cycle of production, Q increases

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monotonically during the time interval t_o to t_k, being 0 for $t < t_k$ amd the constant Q_∞ for $t > t_k$. Also, during this cycle the production rate, dQ/dt, will vary as follows:

For

$$\left.\begin{aligned} t < t_o,\ dQ/dt = 0; \\ t_o < t < t_k,\ dQ/dt > 0; \\ t_k < t,\ dQ/dt = 0. \end{aligned}\right\} \tag{1}$$

Therefore

$$Q_k = \int_{t_o}^{t_k} (dQ/dt)dt = Q_\infty, \tag{2}$$

where

$$Q_\infty = \int_{-\infty}^{+\infty} (dQ/dt)dt. \tag{3}$$

Because of the definiteness of the limits of Q as compared with those of time, it is more useful to consider the production rate, dQ/dt, as a function of Q rather than of t. Then

$$dQ/dt = f(Q), \tag{4}$$

and the integration of this equation gives the corresponding functions of time, $Q(t)$ and $(dQ/dt)\ (t)$. Comparable relations pertain to other variables of this system such as the rates of discovery and cumulative discoveries, proved reserves, and the rate of exploratory drilling and cumulative drilling. By developing the appropriate equations among these variables, and supplying the data from petroleum-industry statistics, it becomes possible, after production in the region has passed through about the first third of its cycle, to determine with reasonable accuracy the principal constants of the equations: Q_∞, the ultimate cumulative production; various exponential growth constants; various critical dates of the cycle, such as that of the maximum rate of production.

Methods based upon this type of analysis, which have been developed and used by the present author during the last 25 years, have consistently given predictions of the future courses of oil and gas production in the United States which have agreed within narrow limits with what has subsequently occurred.

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Techniques of Prediction

Soothsaying, while probably not the world's oldest profession, can certainly offer a strong claim for being its second oldest. The techniques of soothsaying may be divided roughly into those that have some rational basis, and those that do not have; and the result of soothsaying, namely the prediction of some event, may be expressed in language whose meaning ranges anywhere from completely indefinite to precise and unambiguous.

The nonrational techniques of prediction are well exemplified by the activities of many of the priestcrafts of the ancient world - notably those of the famous Oracle at Delphi in Greece - and by those of the great variety of fortune tellers of today with their tea leaves, crystal balls, or astrological interpretations of the human consequences of various planetary configurations. These have usually been characterized by a combination of astute guesswork and ambiguous statement, conducted behind a facade of mystical rites.

From examples of the soothsayer's art which have been handed down from antiquity, it appears to have been learned very early in human history that a soothsayer's life expectancy could be considerably enhanced if his professional opinions, while appearing to convey useful information, were actually couched in language of such ambiguity as to cover all likely contingencies. For example, King Croesus of Lydia, in the sixth century B.C., was considering conducting a war of conquest against a neighboring state, but was doubtful as to the outcome. He decided that it would be prudent to consult the Delphian Oracle. The advice that he received was, "If you embark upon this campaign, a great empire will be destroyed." Thinking that to be a good omen, Croesus did embark upon the campaign and a great empire was destroyed, his own.

The technique of prediction by ambiguous statement is still by no means obsolete and it is not unknown to the petroleum industry. During the 1950-decade, one of the most widely quoted dicta released by the propaganda branch of the U.S. petroleum industry was: "The United States has all the oil it will need for the foreseeable future."

Rational techniques of prediction fall into two fairly distinct classes:

1. Techniques based upon empirical extrapolation of real data, with little or no theoretical guidance.
2. Techniques based upon the analysis of data with the theoretical guidance provided by a prior understanding of the mechanism of the phenomena investigated.

Prediction by trends and cycles.— Of the empirical methods, one of those used most commonly at present is based upon the extrapolation into the future of some variable which, during the recent past, has displayed an approximately linear variation with time. By extending this linear trend with a straight-edge, when the data are plotted graphically, a prediction of its future can be made. This is the simplest semi-rational technique to apply, and it probably is the one in widest use at present. But how reliable is it, especially when applied to the exploitation of an exhaustible mineral resource? As an example of the technique, the annual production of pig iron in the United States, plotted on semilogarithmic paper, for the 60-year period, 1850-1910, is shown in Figure 1.

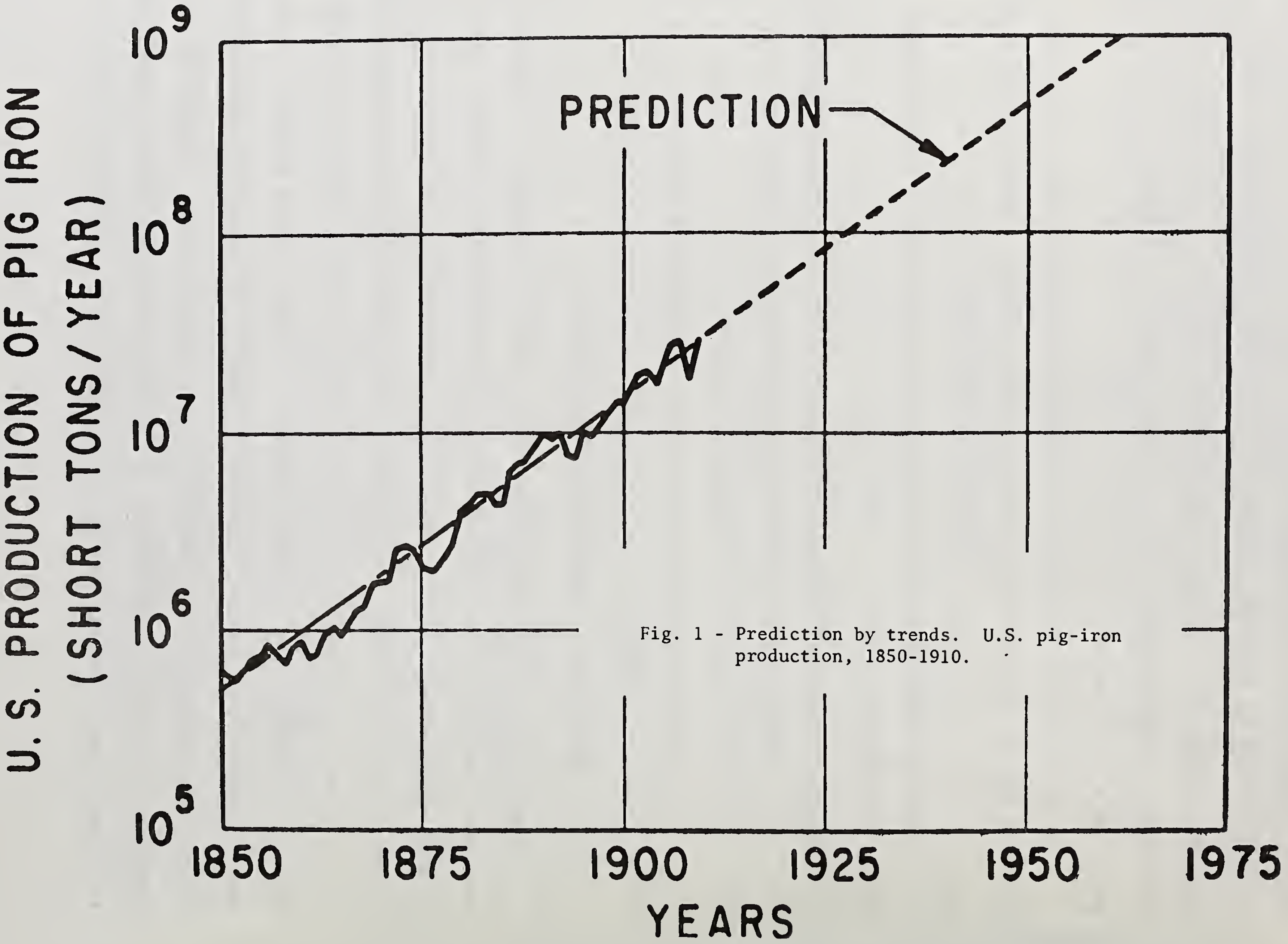

Fig. 1 - Prediction by trends. U.S. pig-iron production, 1850-1910.

The curve plots as an excellent straight line, corresponding to a uniform exponential growth at an annual rate of 6.9 percent. The dashed line shows the prediction of the growth of pig-iron production in the future. The steel industry has been reported to have based its long-range plans on this projection until 1925. Figure 2 shows what actually happened. Within a three-year period from about 1907 to 1910, this curve changed abruptly from a growth rate of 6.9 percent per year to another straight-line segment with a growth rate of only 1.8 percent.

In other instances the quantity of interest may vary in a more or less cyclical manner. In that case, the prediction of its future behavior consists in an extrapolation of this type of behavior into the future. In some cases - astronomical events within the solar system, for example - this can be done with great precision and solar or lunar eclipses can be predicted centuries in advance. Other cyclical or pseudo-cyclical events, such as the "business cycle," can be predicted by this means with much less assurance.

Necessity for knowledge of mechanism.—— The fundamental flaw in the use of blind empirical methods lies in the fact that such methods take a minimal account of the inherent constraints in the behavior of a system imposed by its mechanism and physical properties. Thus, when dealing with cyclical events of the solar system, the Newtonian laws of motion and of gravitation, in conjunction with the infinitesimal rate of energy dissipation in the motions of the planets, provides assurance that the cyclical nature of these motions will change extremely slowly over periods of many millions of years. On the other hand, a weight-driven pendulum clock can maintain a precise cyclical operation of its pendulum and its hands, but the fall of the weight is noncyclical, and when the weight reaches its lowest point, the clock stops.

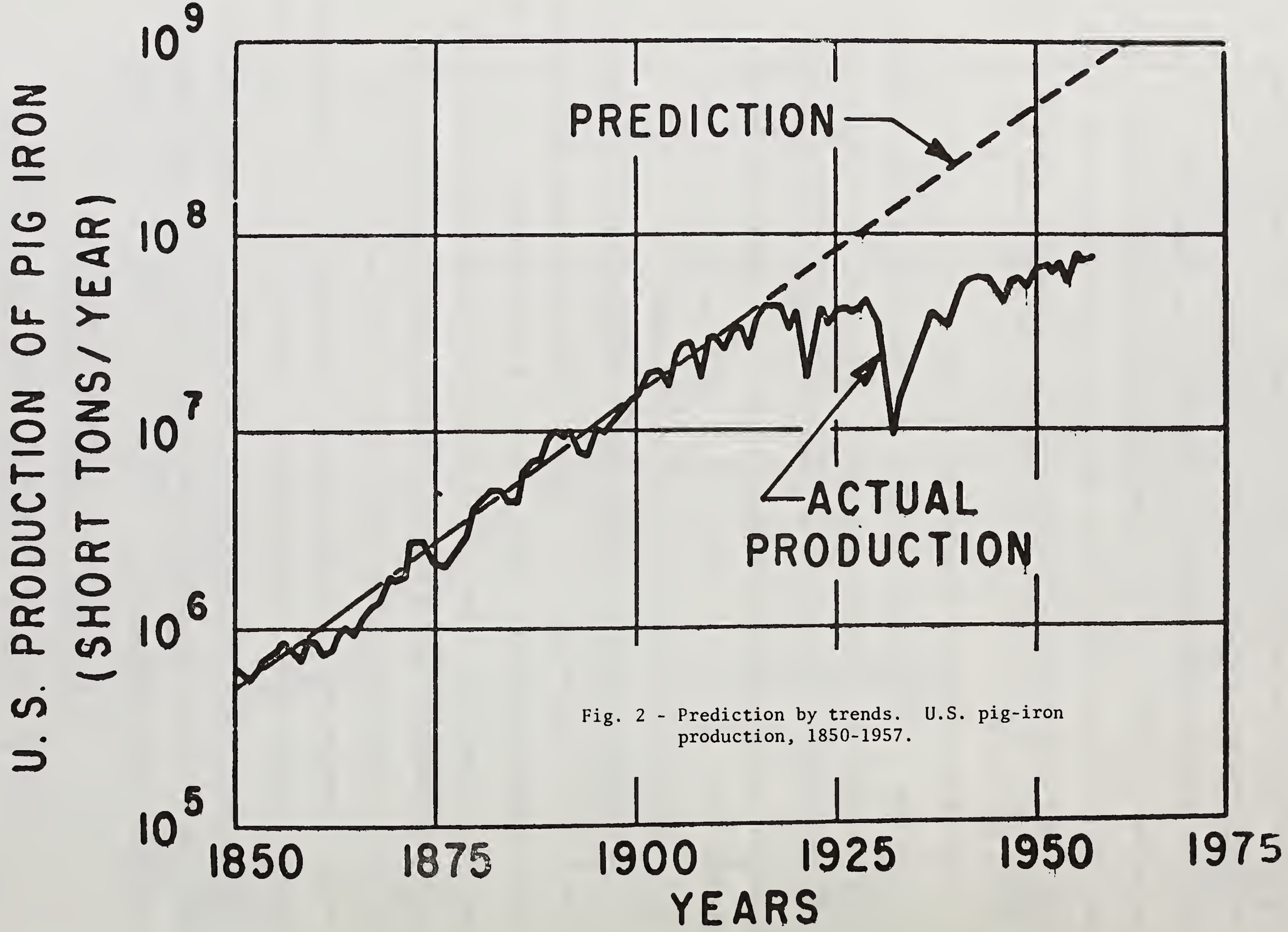

Fig. 2 - Prediction by trends. U.S. pig-iron production, 1850-1957.

A similar situation prevails in the rate of consumption of natural gas in a northern city. This curve consists of two parts, a base rate of consumption, which is noncyclical, upon which is superposed a roughly sinusoidal variation with a period of 1 year. This sinusoidal component is obviously related to the annual rise and fall of climatic temperatures, and hence upon the annual cycle of the earth's revolution. One cannot, however, predict the future of the gas consumption by its sinusoidal component for, say, another century, because by that time there may no longer be any natural gas.

The use of linear trends can be very reliable in cases where the mechanism is understood and does produce a sustained linear variation with time. One of the best known examples of this is the decay of a radioactive isotope. If N_o is the number of atoms of this isotope initially present in a closed system, then, it has been found, the number N of atoms remaining after a time t is given by

$$N = N_o e^{-at}, \tag{1}$$

where e, equal to 2.7183, is the base of natural logarithms, and a is a constant for the given isotope. Taking the natural logarithms of both sides gives the linear equation

$$\ln N = \ln N_o - at, \tag{2}$$

whereby $\ln N$ as a function of t plots as a straight line which may be extrapolated with confidence so long as N is large as compared with its statistical perturbations. This is the basic principle used in the radioactive dating of past geologic events.

Mechanism of Petroleum Occurrence

Similarly, any rational prediction of the future of petroleum production must depend upon a prior knowledge of the manner of origin of oil and gas, the geological situations where these fluids now occur, the procedures of petroleum exploration and production, and the respective time scales. As to their modes of occurrence, oil and gas at present are found as concentrations occupying the pore volumes of coarse-textured or fractured porous rocks in limited regions of space within or adjacent to basins of sedimentary rocks.

In petroleum geology, if a well is drilled at any point, it commonly will penetrate various thicknesses of porous sedimentary rocks - sandstones, shales or mudstones, and limestones. Eventually, if drilled deep enough, the well will reach the bottom of the sediments and will encounter the underlying older non-porous crystalline rocks which are commonly known as the *basement complex*, or the "basement." The upper surface of the basement complex is a continuous, world-wide surface. In certain large regions - a large part of Africa, most of the eastern half of Canada, and much of Scandinavia - the sedimentary cover is either very thin or else absent altogether. In these regions the surface of the basement rocks coincides with the surface of the ground. In other regions, the top of the basement is depressed and these depressions are filled with younger sedimentary rocks, having thicknesses ranging from zero at the edges, to a maximum in their interiors. Commonly, this maximum thickness may not exceed 2 or 3 kilometers, but in a few cases it has been found to be as great as 10 to 15 kilometers.

Unmetamorphosed sedimentary rocks are porous, with the pore volume ranging usually between about 10 and 30 percent of the total volume. Beneath shallow depths of a few tens of meters from the earth's surface, the pore space of these sedimentary rocks is filled with water which extends to depths below which the porosity becomes zero. Oil and natural gas, and also coal, are found in these sedimentary rocks. These are derived from the remains of plants and animals which lived at the times the sediments were being deposited. They became buried in the accumulating sedimentary sands and muds in an oxygen-deficient environment, and have subsequently become transformed chemically into the present fossil fuels.

Oil and gas thus occur as minority fluids immersed in an underground rock-water environment. Being fluids, oil and natural gas are mobile. Occurring originally in a dispersed state, the separate particles have been driven by mechanical forces or else transported in solution, into limited regions of space, corresponding to minimum levels of potential energy with respect to the local environment and lithologic barriers to further migration. The exploration for petroleum consists in determining as accurately as possible from less costly geological and geophysical data the most probable positions in three-dimensional space at which these accumulations may occur, and then drilling exploratory wells at the sites so selected. Most such exploratory wells are failures, but the probability of success is considerably higher than that of random drilling.

As regards time scales, the oldest industrial-sized gas field so far discovered appears to be one in Australia which occurs in rocks of late Precambrian geologic age - some 600 to 700 million years old. Oil and gas fields have been found in the United States and other parts of the world in rocks ranging in age from the Cambrian, 500 to 570 million years old, to sediments of Pleistocene age deposited in the Mississippi Delta of coastal Louisiana during the last million years. The exploitation of petroleum, on the other hand, did not begin on an industrial scale until as recently as the 1850-decade. Statistics of oil production in Romania are reported since 1857, and the initial oil discovery in the United States, by a well drilled for that purpose, was made at Titusville, Pennsylvania, in August 1859. The time required to essentially deplete the initial oil and gas resources of the world can hardly be longer than two or three centuries.

The disparity of these time scales, that for the accumulation of the world's supply of oil and gas, and that for its depletion, is very significant. Although the same geological processes are still operative by which the original oil and gas were accumulated, and at about the same average rate, any additional oil or gas that could accrue within the next few centuries would be infinitesimal as compared with that of the last 600 million years. Therefore, during the period of oil exploitation, the production of oil and gas must consist solely of the continuous withdrawal from a stockpile of an initially fixed and finite magnitude, to which no additions will be made. Therefore, the amount of oil or gas remaining can only decline monotonically as a function of time.

The So-called Geologic Methods of Petroleum Estimation

Petroleum geology and geophysics, which are fundamental to petroleum exploration, comprise the entire complex of existing knowledge regarding the origin, migration, and entrapment of oil and gas, and their present modes of occurrence. This involves of necessity the most detailed knowledge that can be obtained regarding the rocks filling various sedimentary basins, their spatial distribution, and their fluid contents, water, oil, and gas. This information is acquired jointly by surface geological and geophysical mapping, but eventually in most detail from the subsurface geological information provided by wells drilled into the sediments. It is a truism of the petroleum industry that the only tool that actually discovers oil is the drill. Hence it is the record of exploratory and production drilling in a given region that provides the most reliable information available regarding the occurrence of oil and gas, and the probable quantities of these fluids that a given basin may eventually be expected to yield.

However, during the last twenty years, a great deal of confusion has been introduced into the estimates of future petroleum production by the argument that "geological" methods of estimation must somehow be more reliable than so-called statistical methods based upon the cumulative information provided by drilling. By the advocates of this view, the scope of "geology" is seldom defined, but it apparently excludes or minimizes the importance of the information provided by drilling. The estimates obtained by these so-called geological methods during the 1960-decade for the ultimate amounts of crude oil and natural gas to be produced in the Lower-48 states and adjacent continental shelves of the United States were commonly about 600 to 650 billion barrels for crude oil and 2,500 trillion cubic feet for natural gas.

One of the more important uses of geologic methods is in a qualitative evaluation of the petroleum potential of an undrilled area. This is done principally by geological analogy. Suppose, for example, that two contrasting regions, *A* and *B*, have been explored and adequately drilled. Region *A* has been found to be a petroleum-rich region and Region *B* has been found to be barren. Two undrilled regions, *C* and *D*, are under consideration for future exploration. From preliminary geological and geophysical mapping, Region *C* is found to be geologically similar to barren Region *B*, and Region *D* to productive Region *A*. On the basis of this comparison, it would be inferred that Region *D* would merit further development, and that Region *C* should be given a lower priority.

In this connection, the United States is the most intensively explored major petroleum-producing region in the world. Consequently, it often has been used as a standard in the estimation of other less-developed or undeveloped potential petroleum-producing areas of the rest of the world. It is accordingly not surprising that estimates of the ultimate oil potential for the world have a strong correlation with estimates made by the same authors for the United States.

But how are the estimates for the primary areas to be obtained? It is easy to show that no geological information exists, other than that provided by drilling, that will permit an estimate to be made of the recoverable oil obtainable from a primary area that has a range of uncertainty of less than several orders of magnitude. To show this, consider the composite potential of all oil-bearing sediments of a primary region.

Let A be the surface area of the potential oil-bearing region, D be the average thickness of the sediments, and V the total volume of the sediments. Then

$$V = AD \tag{3}$$

Of the total sediments, oil and gas can be produced only from coarse-textured reservoir rocks, which are principally sandstones, and a lesser amount of porous limestones. Let V_{res} be the volume of reservoir rocks and λ the ratio between V_{res} and V. Let V_p be the pore volume of the reservoir rocks and ϕ their average porosity. Finally, let S_o be the average oil saturation of the reservoir rocks and V_o the volume of the oil. Let V_r be the recoverable oil and F the fraction of oil-in-place that can be recovered. Finally, let R be the richness of the region, defined as the ratio of the volume of recoverable oil to the total sedimentary volume. Then

$$V_{res} = \lambda V \tag{4}$$

is the volume of the reservoir rocks;

$$V_p = \phi V_{res} = \phi\lambda V \tag{5}$$

is the pore volume of the reservoir rocks;

$$V_o = S_o V_p = S_o \phi\lambda V \tag{6}$$

is the volume of oil in the reservoir rocks; and finally,

$$V_r = (F S_o \phi\lambda) V \tag{7}$$

is the volume of recoverable oil. Also, the richness R of the region is defined to be

$$R = V_r/V = (F\phi\lambda)S_o. \tag{8}$$

In equation (8), the factors F, ϕ, and λ are known within narrow limits. The recovery factor F may be taken to be about 0.4, the porosity ϕ of reservoir sands has an average value of about 0.15. According to F. W. Clarke, in his classical monograph, *The Data of Geochemistry* (Clarke, 1924, p. 34), shales comprise about 80 percent of the total volume of sediments, sandstones about 15 percent, and limestones and evaporites about 5 percent. Accordingly, we may take about 0.15 as the average value of λ. Hence the combined factor

$$\begin{aligned}(F\phi\lambda) &\simeq 0.4 \times 0.15 \times 0.15\\ &\simeq 0.009\\ &\simeq 10^{-2}\end{aligned}$$

$$R \simeq 10^{-2}S_o. \tag{9}$$

Then the total amount of recoverable oil that a given region will produce would be

$$V_r \simeq (10^{-2}S_o)V. \tag{10}$$

Therefore the accuracy of a geological estimate of the oil that a given region will produce depends almost entirely upon that of the oil saturation factor S_o. The factor S_o must lie between the limits 0 and 1.0 but how, except by the data of prior drilling can it be determined whether 10^{-1}, 10^{-4}, or 10^{-6} is the better value for this factor?

The cumulative results of exploratory and production drilling, on the other hand, in petroleum-producing regions in advanced states of development, do provide very good information as to the actual magnitudes of the richness, R, and of S_o, within a range of uncertainty of 2 or less for various regions. For example, consider the three following oil-rich regions, each of which is in an advanced state of exploratory and production development:

The Los Angeles basin, California

The Lower-48 states and adjacent continental shelves of the U.S.

The Arabian Gulf basin of the Middle East.

For the Los Angeles basin, about 97 percent of the oil discovered is found in upper Miocene and lower Pliocene sediments, having a volume of 6.67×10^{12} m^3. The cumulative discoveries amount to 7.65×10^9 bbl, or to 1.21×10^9 m^3. (Kilkenny, 1971, p. 170-173; Gardett, 1971, p. 278). From these data, the minimum value of the richness of this basin is

$$R = \frac{1.21 \times 10^9 \text{ m}^3}{6.67 \times 10^{12} \text{ m}^3} = 1.8 \times 10^{-4},$$

and

$$S_o \simeq 100\, R = 1.8 \times 10^{-2}.$$

In other words, the Los Angeles basin has about 180,000 m^3 of recoverable oil per km^3, or 4.7 million barrels per cubic mile, and an average oil saturation in the reservoir rocks of nearly 2 percent.

For the Lower-48 states and continental shelves, the area of potentially oil-producing sediments is 2.0×10^6 square miles, or 5.2×10^{12} m^2 (Cram, 1971, v. 1, p. 5). Then, with an average thickness of 2,500 meters, the total volume would be

$$5.2 \times 10^{12} \times 2.5 \times 10^3 = 13 \times 10^{15} \text{ m}^3.$$

The cumulative oil discoveries for the Lower-48 states amount to about 155×10^9 bbl, or to 24.6×10^9 m^3. From these data,

$$R = \frac{24.6 \times 10^9 \text{ m}^3}{13 \times 10^{15} \text{ m}^3} = 1.9 \times 10^{-6},$$

and

$$S_o = 100\,R = 1.9 \times 10^{-4}.$$

Thus the average oil richness for the potential oil-producing sediments of the Lower-48 states and continental shelves is only about 1,900 cubic meters per cubic kilometer, or 50,000 bbl per cubic mile, and the average oil saturation is only about 2 parts in 10,000 of the total pore volume of the reservoir rocks.

For the Arabian Gulf basin, about 460×10^9 bbl or 73×10^9 m^3 of oil has been discovered in a sedimentary volume of 2.5×10^{15} m^3 (Morris, 1978, Preface; Law, 1957, p. 60). This gives for the richness

$$R = \frac{73 \times 10^9 \text{ m}^3}{2.5 \times 10^{15} \text{ m}^3} = 2.9 \times 10^{-5}.$$

and

$$S_o = 100\,R = 2.9 \times 10^{-3}.$$

Thus the Arabian Gulf basin has a richness of about 29,000 cubic meters of recoverable oil per cubic kilometer, or about 760,000 bbl of oil per cubic mile, and an oil saturation of the reservoir rocks of about 0.3 percent.

From these comparisons, the Los Angeles basin, the richest known basin in the world, has a richness that is 95 times that of the entire Lower-48 states, and 6 times that of the Arabian Gulf basin. The latter has a richness 15 times that of the Lower-48 states.

From the foregoing discussion it should be clear that arguments over the relative superiority of the so-called geological estimates and those arrived at by other methods serve little useful purpose but can produce a great deal of confusion, especially when the geological estimates are several-fold larger than other estimates. Actually, a petroleum geologist or engineer, when studying a given region, makes use implicitly or explicitly of every kind of pertinent information that may be available. A large and significant part of this information has to be the cumulative knowledge provided by prior exploratory and production drilling.

Complete Production Cycle in Given Region

In any particular oil-bearing region, the production history of the region, or the complete production cycle, has the following essential characteristics. At some initial time t_o the first discovery well is drilled and oil production in the region begins. The first discovery is of a single field. Additional wells are drilled to develop the field and the rate of production increases. Further exploration in the region is thus stimulated, and, on the basis of geological and geophysical studies, further potential oil-bearing structures are drilled and new fields are discovered. However, since there are only a fixed number of fields in the region, as more and more fields are discovered, progressively fewer fields remain to be discovered, and these are the more obscure fields and commonly the smaller ones. As the undiscovered fields become scarcer, the amount of exploratory effort, including exploratory drilling, per unit quantity of oil discovered increases until eventually exploratory drilling becomes prohibitively costly and ceases.

The rate of oil production, dQ/dt, in the region begins at a near-zero rate at time t_o and thereafter commonly increases exponentially for a few decades. Eventually, as the rate of discovery slows down, the rate of production follows. It reaches one or more principal maxima, and finally goes into a slow negative-exponential decline. Then, at some definite time t_k, production ceases altogether. This sequence can be stated mathematically as follows:

For

$$\left.\begin{aligned} t < t_o, \quad dQ/dt = 0; \\ t_o < t < t_k, \quad dQ/dt > 0; \\ t > t_k, \quad dQ/dt = 0. \end{aligned}\right\} \qquad (11)$$

The period from t_o when production first begins until t_k when it ends comprises the complete cycle of oil production in the region. During that time, the cumulative production Q_t from time t_o to a later time t is given by

$$Q_t = \int_{t_o}^{t} (dQ/dt)dt$$

$$= \int_{t_o}^{t} Pdt, \qquad (12)$$

where $P = dQ/dt$ is the rate of production. Then for the complete cycle

$$Q_k = \int_{t_o}^{t_k} Pdt. \qquad (13)$$

Or, since for times earlier than t and later than t_k, $P = 0$, then

$$Q_k = \int_{-\infty}^{+\infty} Pdt = Q_\infty. \qquad (14)$$

Thus Q_∞ may be taken to be the ultimate amount of oil that will ever be produced in the region during an unlimited period of time.

This complete-cycle curve has only the following essential properties: The production rate begins at zero, increases exponentially during the early period of development, and then slows down, passes one or more principal maxima, and finally declines negative-exponentially to zero. There is no necessity that the curve, P as a function of t, have a single maximum or that it be symmetrical. In fact, the smaller the region, the more irregular in shape is the curve likely to be. The crude-oil production curve for the State of Illinois, for example, is shown in Figure 3. Here, oil was first discovered about 1900. Until 1905, the production rate increased very slowly. It rose sharply to a peak rate of about 35 million barrels per year during 1908 to 1910. This was followed by a long negative-exponential decline to about 1 million barrels per year by 1936. Then followed a new cycle of exploration and discovery based upon the use of the reflection seismograph capable of mapping geologic structures beneath the cover of glacial drift. With the new discoveries, the production rate increased sharply to about 145 million barrels per year by 1940, and then declined to 55 million barrels per year by 1953. Next followed a ten-year period of a third production peak of about 80 million barrels per year due to water flooding, and finally a decline to about 35 million barrels per year by 1971.

On the other hand, for large areas, such as the entire United States or the world, the annual production curve results from the superposition of the production from thousands of separate fields. In such cases, the irregularities of small areas tend to cancel one another and the composite curve becomes a smooth curve with only a single principal maximum. However, there is no theoretical necessity that this curve be symmetrical. Whether it is or not will have to be determined by the data themselves.

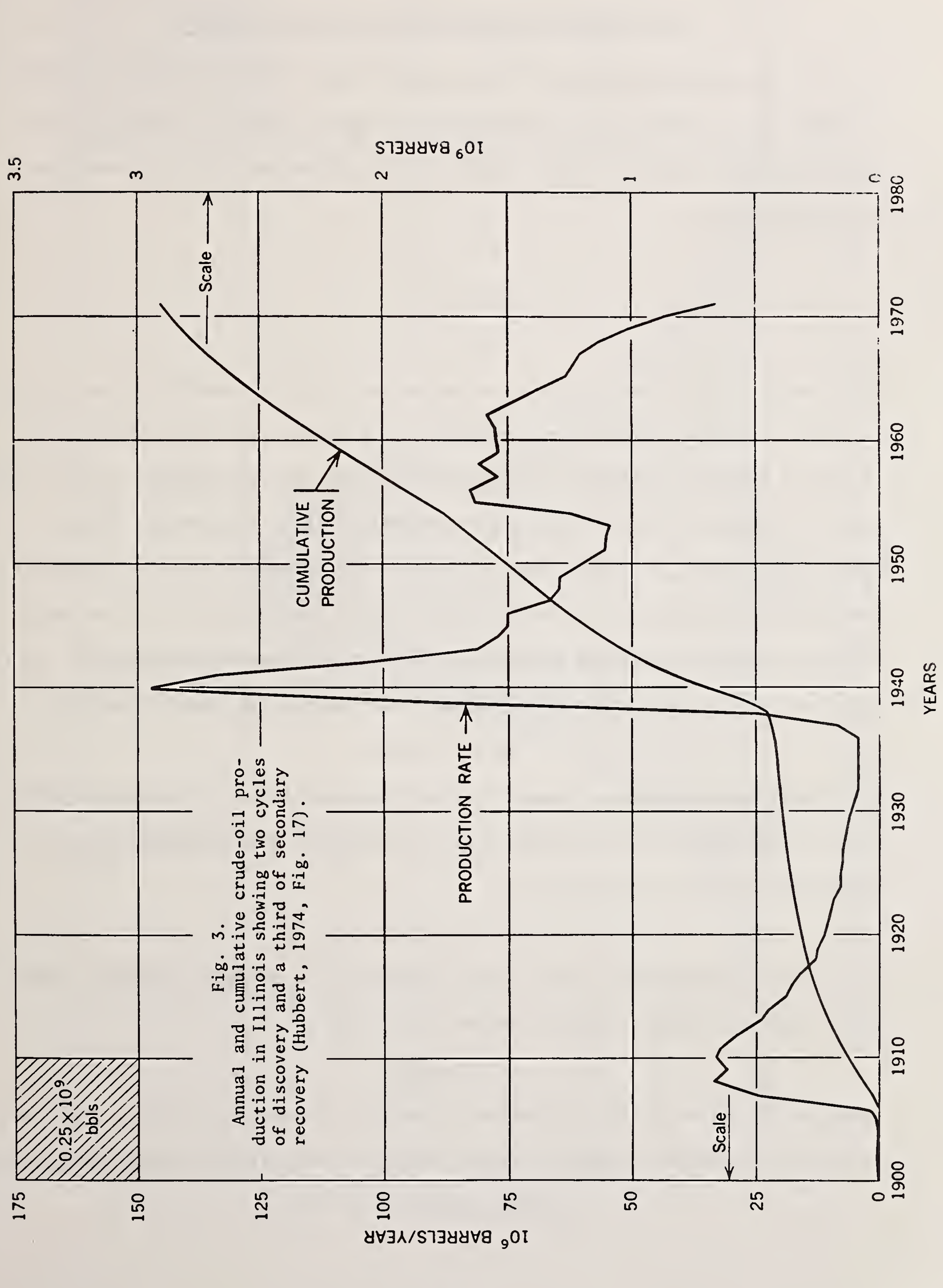

Fig. 3.
Annual and cumulative crude-oil production in Illinois showing two cycles of discovery and a third of secondary recovery (Hubbert, 1974, Fig. 17).

The Complete-Cycle Curve as a Function of Q_∞

The foregoing properties of the complete-cycle curve of production afford a simple but powerful means of estimating the future course of the production-rate curve as a function of Q_∞. After the completion of the cycle, as shown by equation (14),

$$Q_\infty = \int_{-\infty}^{+\infty} P dt.$$

When the curve P versus t is plotted graphically with arithmetic scales, as in Figure 4, the area between the curve and the t-axis to any given time t is a graphical measure of cumulative production. Then, for the complete cycle, the total area beneath the curve is a measure of Q_∞. Also, in plotting such a curve, scales must be chosen arbitrarily for the ordinate P, with a graphical interval ΔP, and for the abscissa t, with a graphical interval Δt. The grid-rectangle, $\Delta P \times \Delta t$, affords a graphical scale for cumulative production. At a constant rate ΔP, the quantity of oil produced during the time Δt would be

$$\Delta Q = \Delta P \times \Delta t. \tag{15}$$

Hence each grid-rectangle beneath the curve represents ΔQ of cumulative production. Therefore, if Q_∞ is known, then the number of grid-rectangles beneath the complete-cycle curve must be

$$n = Q_\infty / \Delta Q. \tag{16}$$

This is the inverse of the usual problem of the integral calculus, where one is given $y = f(x)$, and the problem is to find

$$A = \int y dx.$$

Here, we are given A and the problem is to find the curve $y = f(x)$. There are obviously an infinite number of curves that will satisfy this condition. However,

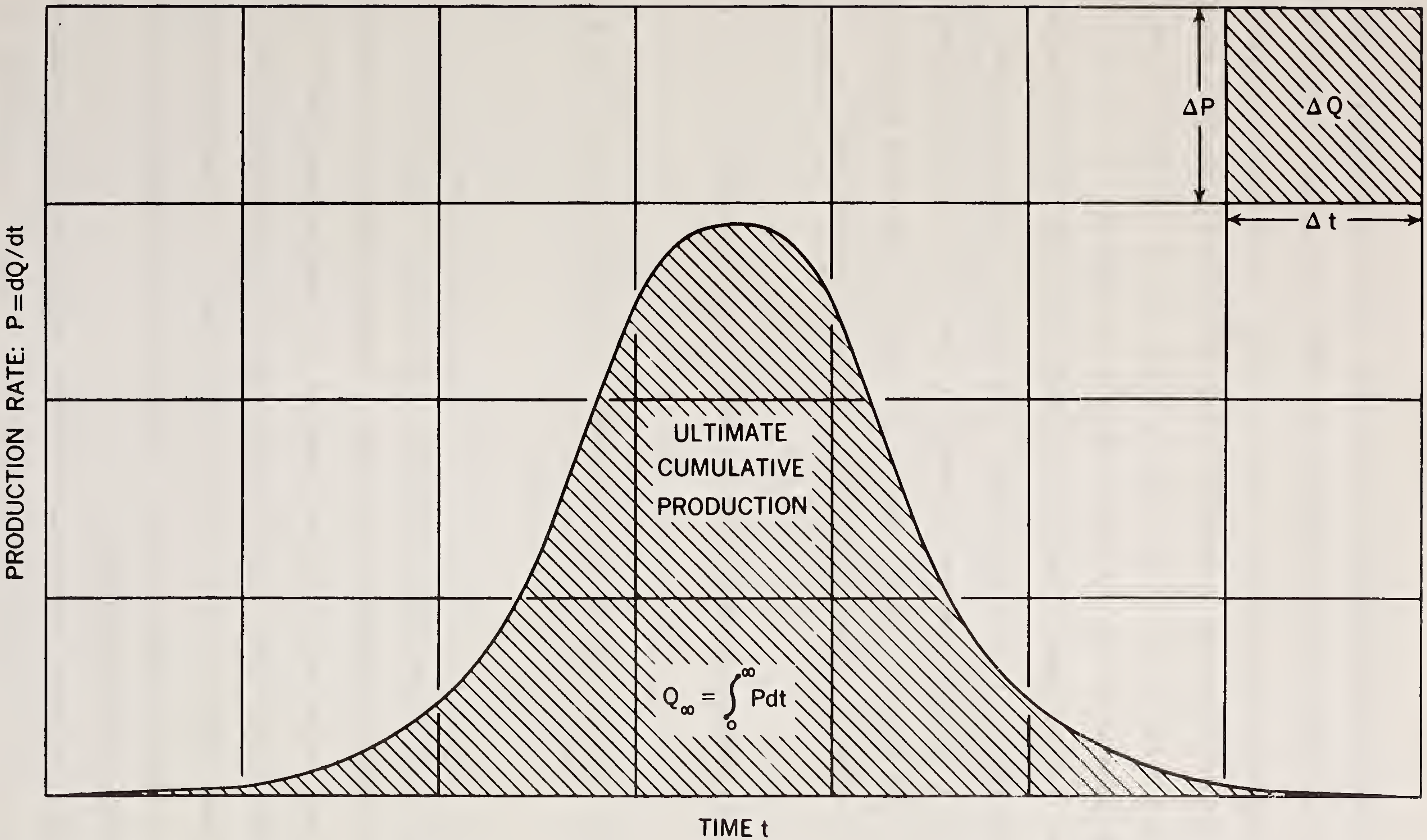

Fig. 4 - Mathematical relations involved in the complete cycle of production of any exhaustible resource (modified from Hubbert, 1956, Fig. 11).

when, in the oil-production case, the further constraints imposed by the technology of oil discovery and production are taken into account, the differences between separate solutions tend to become minor. Accordingly, although there may be an infinite number of complete-cycle curves corresponding to a single value for Q_∞, all of these curves have a strong family resemblance because the area subtended by each is the same, namely n rectangles.

When the petroleum discovery and production in a given region reaches a moderately mature state of development, the history of this development begins to provide a basis for reasonably good estimates of the approximate magnitude of Q_∞ for the region. Suppose that by the time t_1, cumulative production has already reached Q_1. Then the oil remaining to be produced would be

$$Q_2 = Q_\infty - Q_1. \tag{17}$$

Accordingly the area beneath the curve from t_0 to t_1 would be

$$Q_1/\Delta Q = n_1$$

rectangles, and the remaining area beneath the curve from t_1 to ∞ would be

$$\begin{aligned}(Q_\infty - Q_1)\Delta Q &= n - n_1 \\ &= n_2\end{aligned}$$

rectangles. Hence the future part of the curve must be consistent with the part that has developed already, and must also be drawn subject to the constraint that it can subtend only n_2 grid-rectangles. In case the peak production rate has not yet been reached by time t_1, then the curve for the future must rise to a maximum and then decline negative-exponentially. If the maximum production rate has already occurred by time t_1, then its future course will be principally the negative-exponential decline.

It must be borne in mind that at time t_1 any figure for Q_∞ is only an estimate, yet the foregoing technique provides a means of determining the approximate consequences in terms of future production history of any given estimated value for Q_∞. One of the most important results of such an analysis is the determination of the approximate date at which the peak production rate will occur. This is probably the most important event in the complete cycle of production because it is the dividing point between the first phase of the cycle during which the production rate steadily increases, and the second phase during which the production rate almost as steadily declines.

Application to the United States.— As an example of this technique, consider its application to the oil production of the Lower-48 states, onshore and continental shelves, in 1956 (Hubbert, 1956). Crude-oil production in the United States began with the initial discovery made at Titusville, Pennsylvania, in August 1859. By 1955 the production rate had reached 2.4 billion barrels per year. The curve of annual rate of production from 1900 to the end of 1955 is shown in Figure 5. By the beginning of 1956 oil had been produced in the United States for 96 years, and cumulative production amounted to 52.4 billion barrels. The problem was to estimate the future of the production-rate curve for the remainder of the cycle.

By 1956, petroleum exploration and production in the United States were sufficiently advanced that reasonably good estimates could be made of the approximate magnitude of the ultimate cumulative production Q_∞. Published estimates for this quantity by leaders of the petroleum industry fell principally within the range of 150 to 200 billion barrels. I was in research with Shell Oil and Shell Development Companies at the time, and further checking with production and exploration managers and other well-informed petroleum geologists and engineers showed an industry consensus that the unknown quantity Q_∞ would probably fall within the range of 150-200 billion barrels. Using these minimum and maximum

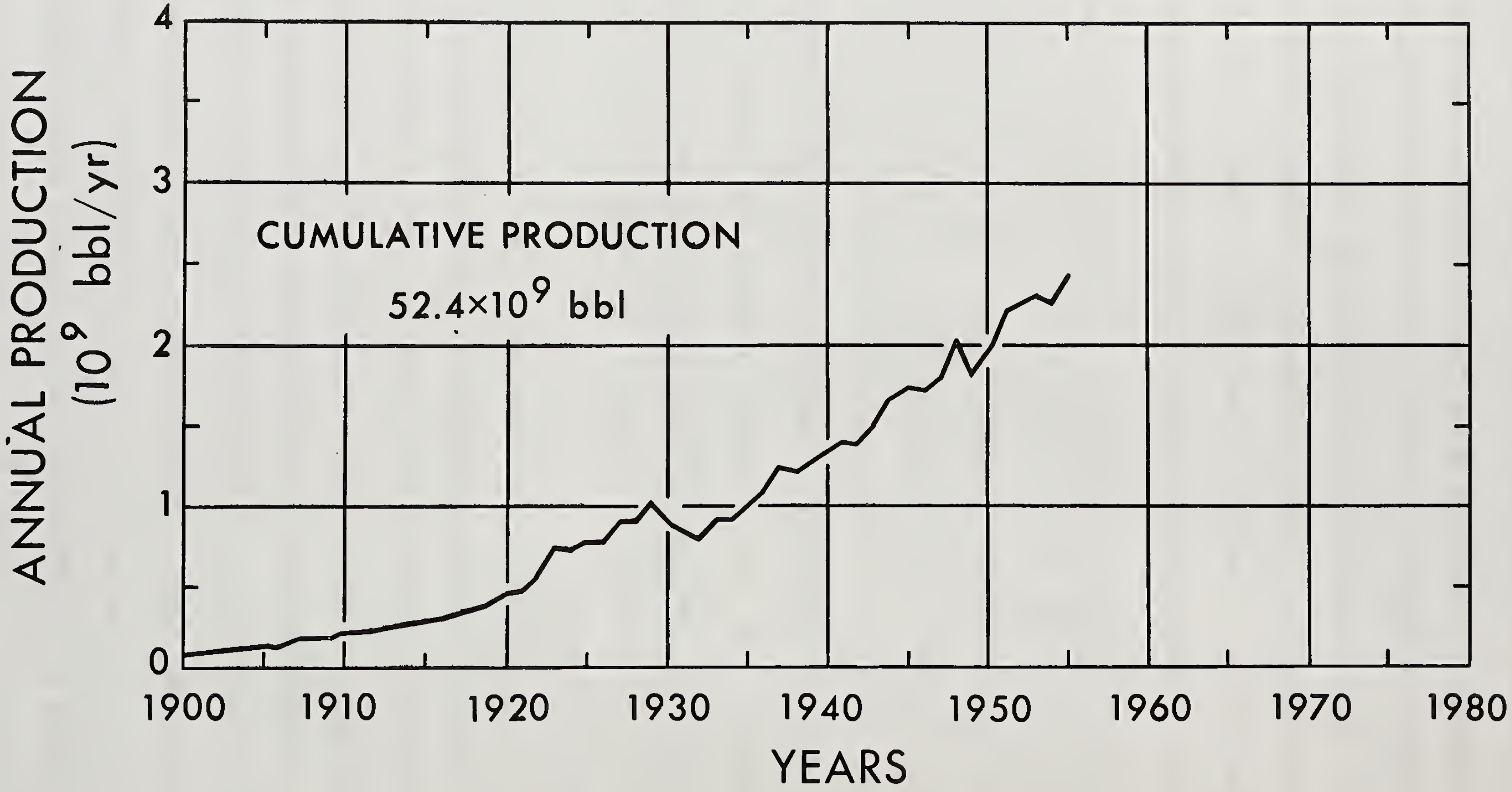

Fig. 5 - Annual U.S. crude-oil production, 1900-1955.

figures for estimates of Q_∞, in conjunction with the technique described above, gave the results shown in Figure 6. There one $\Delta P \Delta t$ grid-rectangle has the dimensions

$$\Delta Q = 10^9 \text{ bbl/yr} \times 25 \text{ years}$$
$$= 25 \times 10^9 \text{ bbl}.$$

For the lower figure for Q_∞ of 150×10^9 bbl, the total area beneath the curve for the complete production cycle would be 6 grid-rectangles. Of these, 2.1 corresponding to cumulative production of 52.4×10^9 bbl had already been developed, leaving 3.9 rectangles for future cumulative production of 97.6×10^9 bbl. The lower dashed-line curve of Figure 6 is drawn accordingly. To satisfy this condition in conjunction with a negative-exponential decline, it became impossible to draw this curve very differently from the way it is shown in Figure 6. From 1956, the curve would reach its peak rate of about 2.7×10^9 bbl/yr about 10 years hence, and then decline negative-exponentially back to zero.

Assuming that Q_∞ could be as large as the higher figure of 200×10^9 bbl would add another 50×10^9 bbl to the area beneath the curve of future production, or 2 more rectangles. This curve would rise a little higher than the first, and would reach its peak rate a little later, but the 2 extra rectangles would be the area between the two curves, principally during their decline.

By this analysis, if Q_∞ should be as small as 150 billion barrels, the peak in the rate of production should occur in about 10 years, or about 1966; for the higher figure of 200 billion barrels, the date of peak production would be delayed by about another 5 years, or to about 1971.

The curves drawn in Figure 6 were not based upon any empirical equations or any assumptions regarding whether they should be symmetrical or asymmetrical; they were simply drawn in accordance with the areal constraints imposed by the

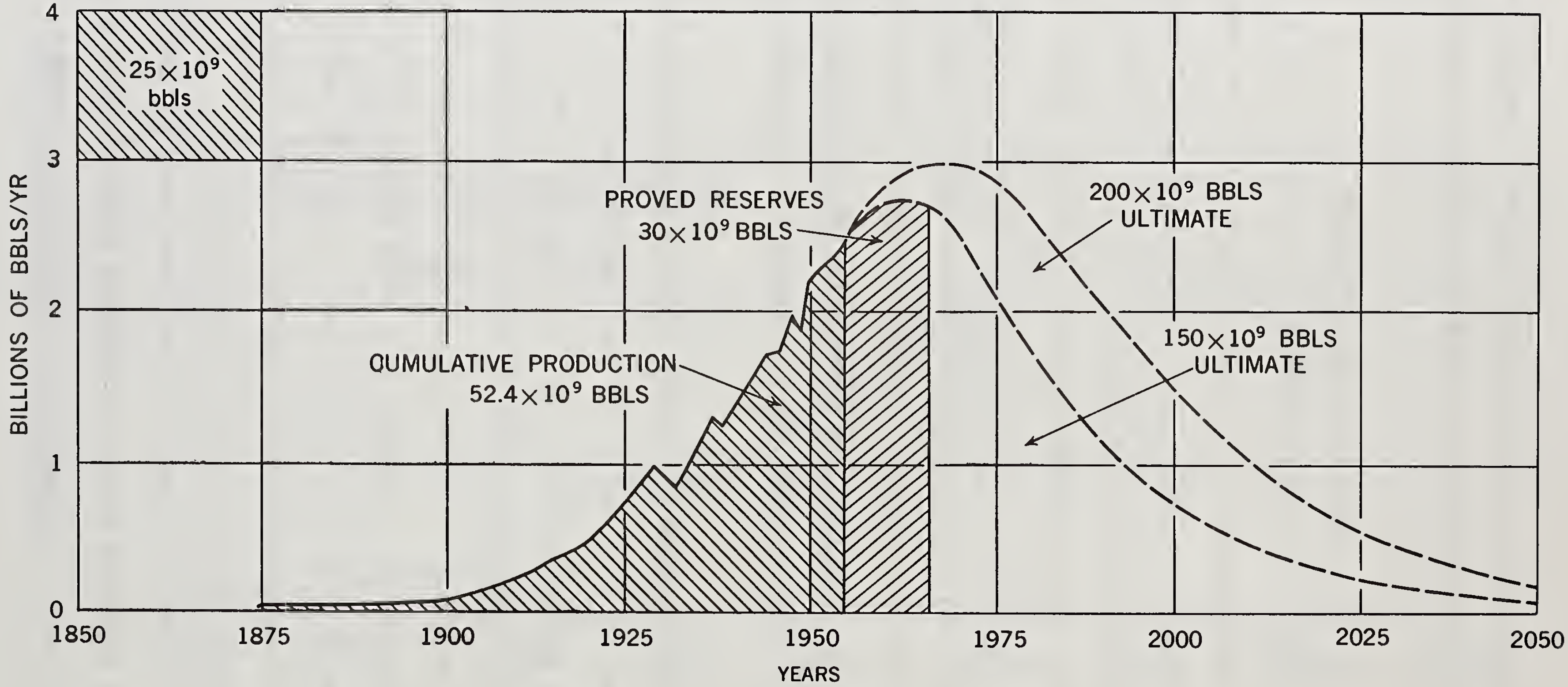

Fig. 6 - 1956 Estimates of two complete cycles of U.S. crude-oil production from Lower-48 states based upon ultimate recoveries of 150 and 200 billion barrels (Hubbert, 1956, Fig. 21).

estimates, and the necessity that the decline be gradual and asymptotic to zero. The strength of this procedure lies in the insensitivity of its most important deduction, namely the date of the peak-production rate, to errors in the estimate of Q_∞. As Figure 6 shows, an increase in the lower estimate of 150 billion barrels by one-third delays the date of peak production by only about 5 years, or to about 1971. If the lower figure were doubled to 300 billion barrels, the date of the peak-production rate would still be delayed only to about 1978.

Without this kind of analysis, the tendency is to extrapolate the production curve prior to 1956 into the future by the method of linear trends which would provide no information whatever regarding the imminence of the date of peak production. In fact, in 1956, the estimates that two or three times as much oil remained to be produced in the future as had been produced during the preceding century led to an attitude of complacency on the part of petroleum geologists, engineers, and oil-company officials alike, that no oil shortages were likely to occur in the United States before the year 2000.

The weakness of this analysis arose from the lack of an objective method of estimating the magnitude of Q_∞ from primary petroleum-industry data. The estimates extant in 1956 were largely intuitive judgments of people with wide knowledge and experience, and they were reasonably unbiased because of the comfortable prospects for the future they were thought to imply. When it was shown, however, that if Q_∞ for crude oil should fall within the range of 150-200 billion barrels the date of peak-production rate would have to occur within about the next 10 to 15 years, this complacency was shattered. It soon became evident that the only way this unpleasant conclusion could be voided would be to increase the estimates of Q_∞, not by fractions but by multiples. Consequently, with insignificant new information, within a year published estimates began to be rapidly increased, and during the next 5 years, successively larger estimates of 250, 300, 400, and eventually 590 billion barrels were published.

This lack of an objective means of estimating Q_∞ directly, and the 4-fold range of such estimates, made it imperative that better methods of analysis, based directly upon the primary objective and publicly available data of the petroleum industry, should be derived. Such methods, which encompass the cumulative records of discovery and production, drilling, and associated knowledge of petroleum geology, will now be developed.

Derivation of the Complete Cycle of Oil and Gas Exploitation from Primary Data

As we have noted heretofore, the complete production cycle of oil or gas, or of any other exhaustible resource, has the following general characteristics: At some initial time t_0 production begins. Subsequently, the production rate increases with time, passes one or more principal maxima, and finally goes into a negative-exponential decline until at some time t_k it ceases altogether. During this complete cycle, the cumulative production,

$$Q = \int_{t_0}^{t} P dt, \tag{18}$$

increases monotonically from 0 to a final value Q_∞. The curve of cumulative production is a generally S-shaped curve, being asymptotic to zero initially and to the limit Q_∞ as t increases without limit. If the curve of P versus t has only a single maximum then the cumulative curve Q versus t will have but a single inflection point, coinciding in time with the peak in the production rate. Earlier than that, the cumulative curve will be concave upward; subsequently, concave downward.

A difficulty in analyzing either P or Q as a function of time arises from the asymptotic approaches of these quantities to their respective limits as time increases without limit. On the other hand, Q itself has the definite finite limits 0 and Q_∞. It is convenient, therefore, to consider the production rate dQ/dt as a function of Q, rather than of time. In this system of coordinates, dQ/dt is zero when $Q = 0$, and when $Q = Q_\infty$. Between these limits $dQ/dt > 0$, and outside these limits, equal to zero. While it is possible that during the production cycle dQ/dt could become zero during some interval of time, for any large region this never happens. Hence we shall assume that for

$$0 < Q < Q_\infty, \quad dQ/dt > 0. \tag{19}$$

The curve of dQ/dt versus Q between the limits 0 and Q_∞ can be represented by the Maclaurin series,

$$dQ/dt = c_0 + c_1Q + c_2Q^2 + c_3Q^3 + ---. \tag{20}$$

Since, when $Q = 0$, $dQ/dt = 0$, it follows that $c_0 = 0$.

Then

$$dQ/dt = c_1Q + c_2Q^2 + c_3Q^3 + ---, \tag{21}$$

and, since the curve must return to zero when $Q = Q_\infty$, the minimum number of terms that will permit this, and the simplest form of the equation, becomes the second-degree equation,

$$dQ/dt = c_1Q + c_2Q^2. \tag{22}$$

By letting $a = c_1$ and $-b = c_2$, this can be rewritten as

$$dQ/dt = aQ - bQ^2. \tag{23}$$

Then, since when $Q = Q_\infty$, $dQ/dt = 0$,

$$aQ_\infty - bQ_\infty^2 = 0,$$

or

$$b = a/Q_\infty,$$

and

$$dQ/dt = a(Q - Q^2/Q_\infty). \tag{24}$$

This is the equation of the parabola shown in Figure 7 whose slope is

$$d(dQ/dt)/dQ = a - (2a/Q_\infty)Q, \tag{25}$$

which, when $Q = 0$, is $+a$, and when $Q = Q_\infty$, is $-a$. Also, the maximum value of dQ/dt occurs when the slope is zero, or when

$$a - (2a/Q_\infty)Q = 0,$$

or

$$Q = Q_\infty/2. \tag{26}$$

It is to be emphasized that the curve of dQ/dt versus Q does not have to be a parabola, but that a parabola is the simplest mathematical form that this curve can assume. We may accordingly regard the parabolic form as a sort of idealization for all such actual data curves, just as the Gaussian error curve is an idealization of actual probability distributions.

One further important property of equation (24) becomes apparent when we divide it by Q. We then obtain

$$(dQ/dt)/Q = a - (a/Q_\infty)Q. \tag{27}$$

This is the equation of a straight line with a slope of $-a/Q_\infty$ which intersects the vertical axis at $(dQ/dt)/Q = a$ and the horizontal axis at $Q = Q_\infty$. If the data, dQ/dt versus Q, satisfy this equation, then the plotting of this straight line gives the values for its constants Q_∞ and a.

Our problem now is to integrate equation (24) in order to determine how dQ/dt and Q each varies as a function of time. This can be simplified by substituting

$$u = Q/Q_\infty,$$

or

$$Q = Q_\infty u. \tag{28}$$

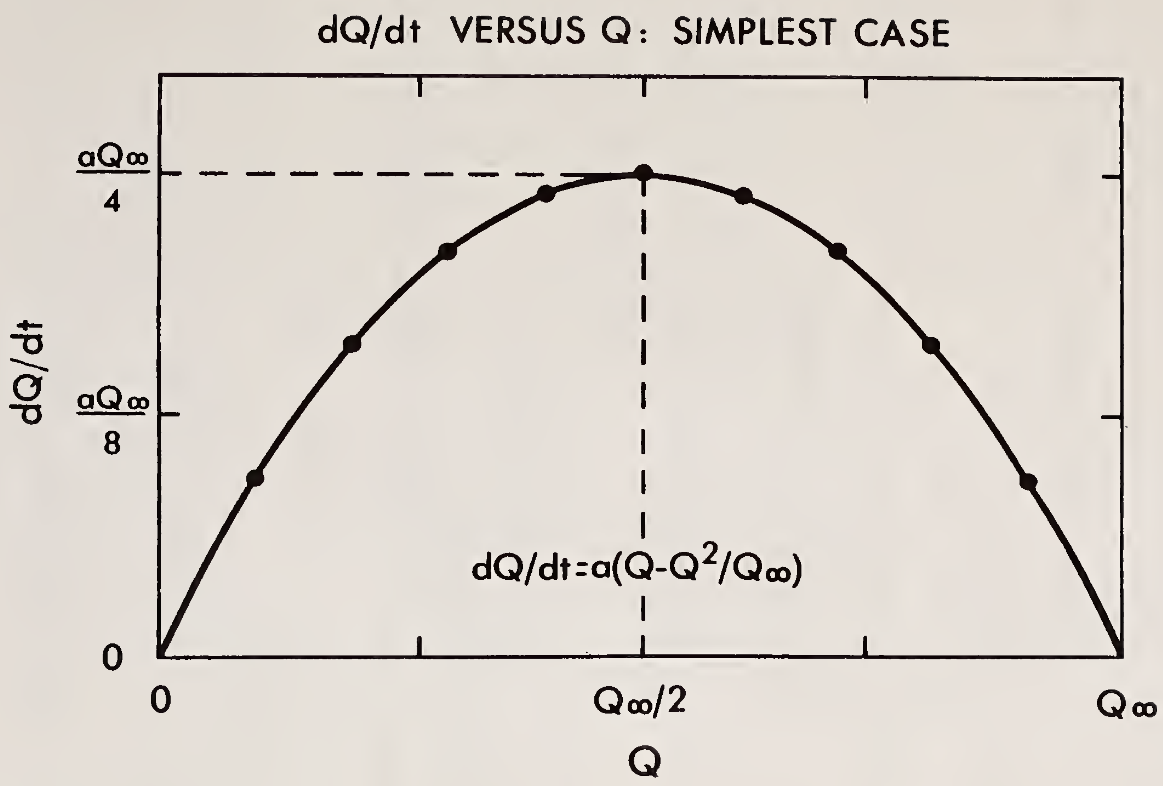

Fig. 7 - Production rate as a function of cumulative production for simplest mathematical case.

Then

$$dQ/dt = Q_\infty(du/dt)$$
$$= Q_\infty a(u - u^2),$$

or

$$du/dt = a(u - u^2),$$

and

$$du/(u - u^2) = adt. \tag{29}$$

This can be simplified by noting that

$$1/(u - u^2) = 1/u + 1/(1 - u).$$

Then the integral of equation (29) becomes

$$\int\frac{du}{u} + \int\frac{du}{1-u} = a\int dt + \text{const},$$

or

$$\ln [(1 - u)/u] = \ln c - at,$$

and

$$(1 - u)/u = ce^{-at}, \tag{30}$$

where c is the constant of integration.

Substituting Q/Q_∞ for u in the left-hand term of equation (30) then gives

$$(1 - u)/u = (Q_\infty - Q)/Q = ce^{-at} \tag{31}$$

Then, when $t = 0$, $Q = Q_o$, and

$$c = (Q_\infty - Q_o)/Q_o, \tag{32}$$

equation (31) becomes

$$(Q_\infty - Q)/Q = [(Q_\infty - Q_o)/Q_o]e^{-at}. \tag{33}$$

Then, letting

$$\left.\begin{aligned} N &= (Q_\infty - Q)/Q, \\ N_o &= (Q_\infty - Q_o)/Q_o, \end{aligned}\right\} \tag{34}$$

equation (33) simplifies to

$$N = N_0 e^{-at}. \tag{35}$$

Taking the natural logarithms of both sides of equation (35) gives

$$\ln N = \ln N_0 - at, \tag{36}$$

which is a linear equation between ln N and t. This plots graphically as a straight line which has the magnitude ln N_0 when $t = 0$, and a slope of $-a$. Or, if common logarithms are used,

$$\log N = \log N_0 - (a \log e)t, \tag{37}$$

which has a slope of $-a \log e$.

Solving equation (33) for Q gives

$$Q = Q_\infty/(1 + N_0 e^{-at}). \tag{38}$$

This is known as the logistic equation, which was derived originally by the Belgian demographer, P.-F. Verhulst (1838; 1845; 1847) in his classical studies of the growth of human populations.

In equation (38) the choice of the date for $t = 0$ is arbitrary so long as it is within the range of the production cycle so that N_0 will have a determinate finite value. It is seen by inspection that as

$$t \to -\infty,\ Q \to 0,$$

$$t \to +\infty,\ Q \to Q_\infty.$$

Also the curve of Q versus t is asymptotic to zero as $t \to -\infty$, and is asymptotic to Q_∞ as $t \to +\infty$. Likewise, the curve of dQ/dt versus t is asymptotic to zero as $Q \to 0$ and $t \to -\infty$, and again as $Q \to Q_\infty$ and $t \to +\infty$.

The maximum value of dQ/dt, from equations (24), (25), and (26), is

$$dQ/dt = (aQ_\infty)/4. \tag{39}$$

This coincides in time with the inflection point of the Qt-curve and occurs when $Q = Q_\infty/2$.

In equation (24), dQ/dt is given as a function of Q. If desired, this can be obtained as an explicit function of t by differentiating the logistic equation (38),

$$Q = Q_\infty/(1 + N_o e^{-at}),$$

with respect to time. This gives the result,

$$dQ/dt = Q_\infty \frac{aN_o e^{-at}}{(1 + N_o e^{-at})^2} . \tag{40}$$

If we then note from equation (35) that

$$N_o e^{-at} = N = (Q_\infty - Q)/Q,$$

and, from equation (38),

$$1/(1 + N_o e^{-at})^2 = (Q/Q_\infty)^2,$$

and substitute these into equation (40), we obtain, as we should, our original differential equation (24),

$$dQ/dt = a(Q - Q^2/Q_\infty).$$

Graphs of the logistic equation (38), and of its time-derivative, equation (40), in terms of the dependent variable, $u = Q/Q_\infty$, as functions of time are shown in Figure 8.

Determination of the Constants Q_∞ and a

Two properties of the logistic equation are of fundamental importance. These are represented by the linear differential equation (27),

$$(dQ/dt)/Q = a - (a/Q_\infty)Q,$$

and the linear equation (36) between ln N and t,

$$\ln N = \ln N_o - at.$$

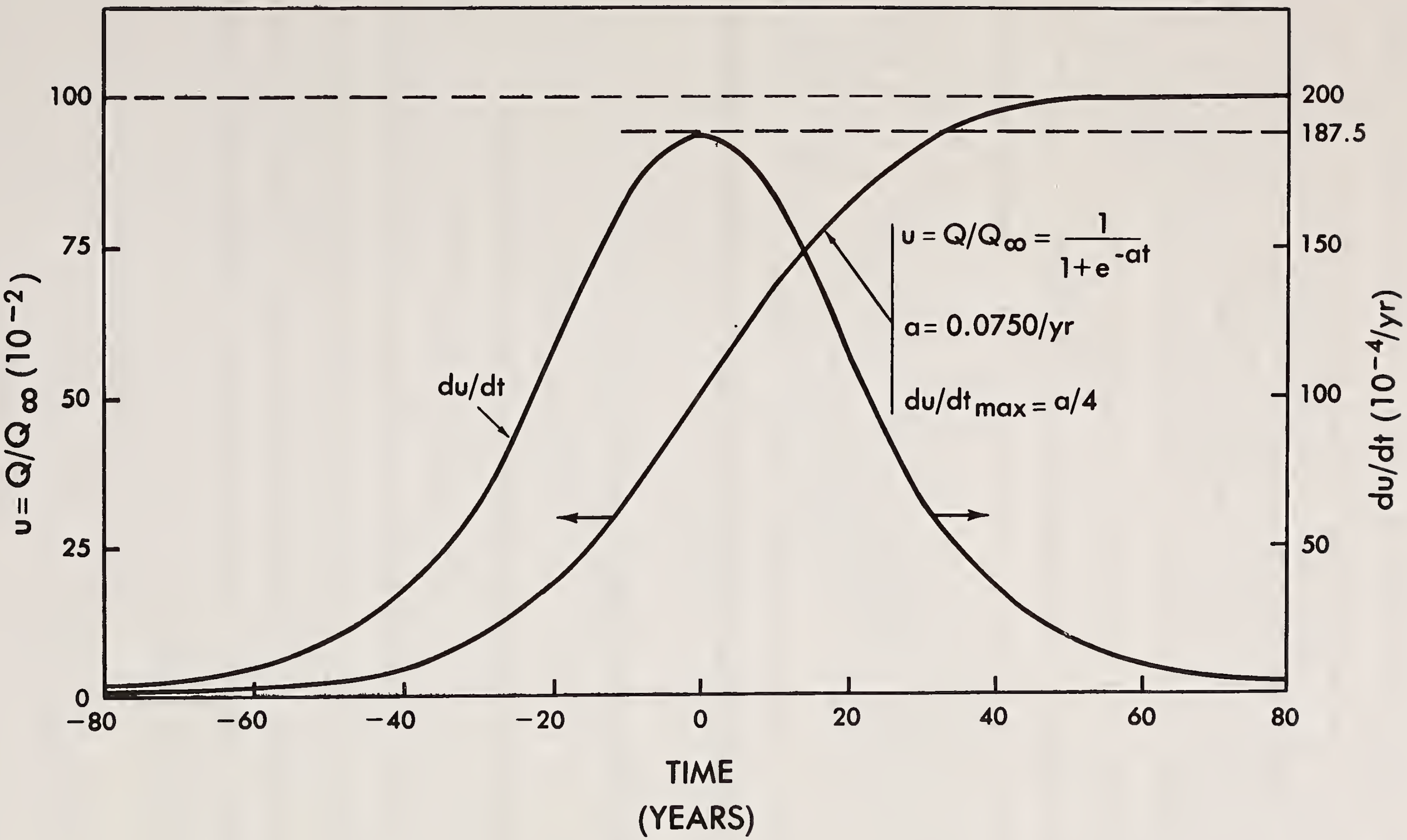

Fig. 8 - Graphical expressions of logistic equation and its time derivative as functions of time.

The virtue of the first of these two equations lies in the fact that it depends only upon the plotting of primary data, $(dQ/dt)/Q$, versus Q, with no a priori assumptions whatever. Using actual data for Q and dQ/dt, it is to be expected that there will be a considerable scatter of the plotted points as $Q \to 0$, because in that case both Q and dQ/dt are small quantities and even small irregularities of either quantity can produce a large variation in their ratio. For larger values of both quantities, as the production cycle evolves, these perturbations become progressively smaller and a comparatively smooth curve is produced. If the data satisfy the linear equation, then a determinate straight line results whose extrapolation to the vertical axis as $Q \to 0$ gives the constant a, and whose extrapolated intercept with the Q-axis gives Q_∞. However, even if the data do not satisfy a linear equation, they will nevertheless produce a definite curve whose intercept with the Q-axis will still be at $Q = Q_\infty$.

The use of the second linear equation,

$$\ln N = \ln N_0 - at,$$

is somewhat less direct than the first, because in this case

$$N = (Q_\infty - Q)/Q = Q_\infty/Q - 1.$$

Hence, before the linear graph can be plotted, Q_∞ must be known as a means of determining N and N_0. If the data satisfy the logistic equation, and if the correct value of Q_∞ is used for computing N and N_0, then the resulting graph of $\ln N$ as a function of t will continue as a straight line. If, on the other hand, an incorrect value,

$$Q_b = bQ_\infty, \tag{41}$$

is assumed for Q_∞, then

$$N_b = bQ_\infty/Q - 1, \tag{42}$$

and as

$$Q \to Q_\infty, \quad N_b \to (b - 1).$$

If the assumed value, Q_b, is greater than the correct value of Q_∞, then as t increases, the curve $\ln N_b$ versus t will approach the constant value, $(b - 1) > 0$, and will deflect to the horizontal. If the assumed value, Q_b, is less than Q_∞, then as

$$Q \to Q_b, \quad N_b \to 0,$$

and

$$\ln N_b \to -\infty.$$

In this case, as t increases, the curve deflects vertically downward. This is illustrated in Figure 9, where graphs of log N_b versus t are plotted for the three cases, $Q_b = 1.25\ Q_\infty$, $Q_b = Q_\infty$, and $Q_b = 0.75\ Q_\infty$.

These properties provide an iteration procedure for determining the correct value of Q_∞ and its associated constant a, provided the production cycle is far enough advanced for the deflections of the linear graphs to be perceptible. From the cumulative curve, Q versus t, a rough visual estimate of the value of Q_∞ can be made. Then a series of assumed values Q_{b_1} Q_{b_2} Q_{b_3}, etc. can be used for plotting a family of curves, $\ln N_b$ versus t. For the too large values of Q_b the curve will deflect toward the horizontal, and for the too small values, toward the vertical. The value of Q_b for which the curve continues as a straight line will be the correct value for Q_∞. The slope of that line will be $-a$.

A less cumbersome and more precise variation of this procedure consists in choosing three fixed times, t_1, t_2, and t_3. Let Q_b, regarded as a variable, be an assumed value for Q_∞. Then, at times t_1, t_2, and t_3,

$$\left.\begin{aligned} N_1 &= Q_b/Q_1 - 1, \\ N_2 &= Q_b/Q_2 - 1, \\ N_3 &= Q_b/Q_3 - 1, \end{aligned}\right\} \tag{43}$$

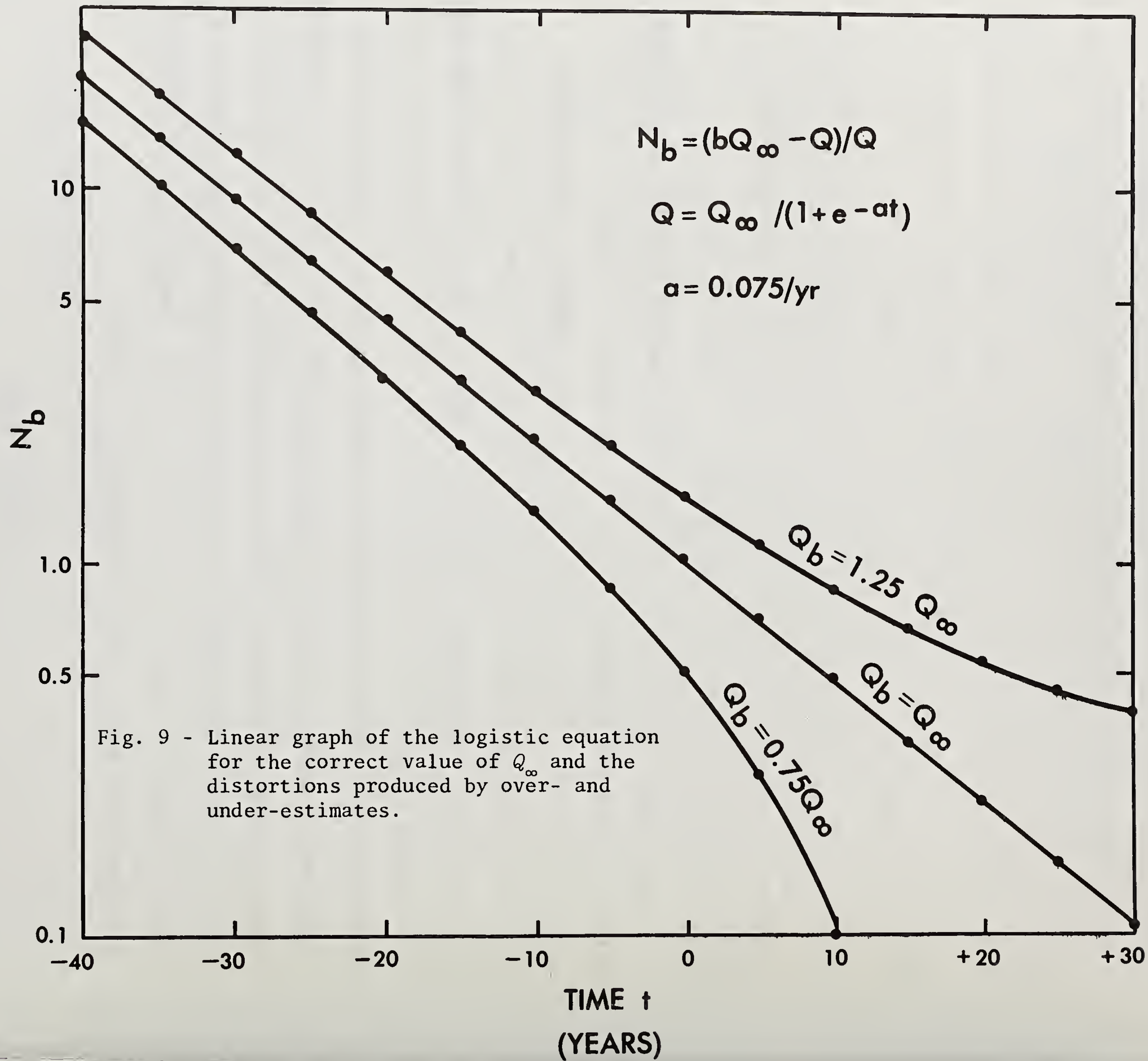

Fig. 9 - Linear graph of the logistic equation for the correct value of Q_∞ and the distortions produced by over- and under-estimates.

and the curve, $\ln N_b$ versus t, will consist of two line segments, one from t_1 to t_2, and the other from t_2 to t_3. From these respective line segments the corresponding negative slopes will be

$$\begin{aligned} -S_{12} &= \ln (N_1/N_2)/(t_2 - t_1), \\ -S_{23} &= \ln (N_2/N_3)/(t_3 - t_2). \end{aligned} \tag{44}$$

Hence $-S_{12}$ and $-S_{23}$ will each be a separate function of Q_b. When these two quantities are each plotted graphically as a function of Q_b, the point at which the two curves intersect one another will correspond to

$$S_{12} = S_{23},$$

for which the curve $\ln N_b$ versus t will be a straight line. The coordinates of that point will be

$$\begin{aligned} -S_{12} = -S_{23} &= a, \\ Q_b &= Q_\infty, \end{aligned} \tag{45}$$

which are the desired constants of the equation.

This is illustrated in Figure 10, based upon the following data for t_1, t_2, and t_3, and Q_1, Q_2, and Q_3,

Date (t)	1905	1945	1965
Q (10^9 bbl)	3.98	55.5	117.5

As shown in Figure 10, the values for the logistic constants corresponding to these data are

$$Q_\infty = 173 \times 10^9 \text{ bbl},$$

$$a = 0.0750.$$

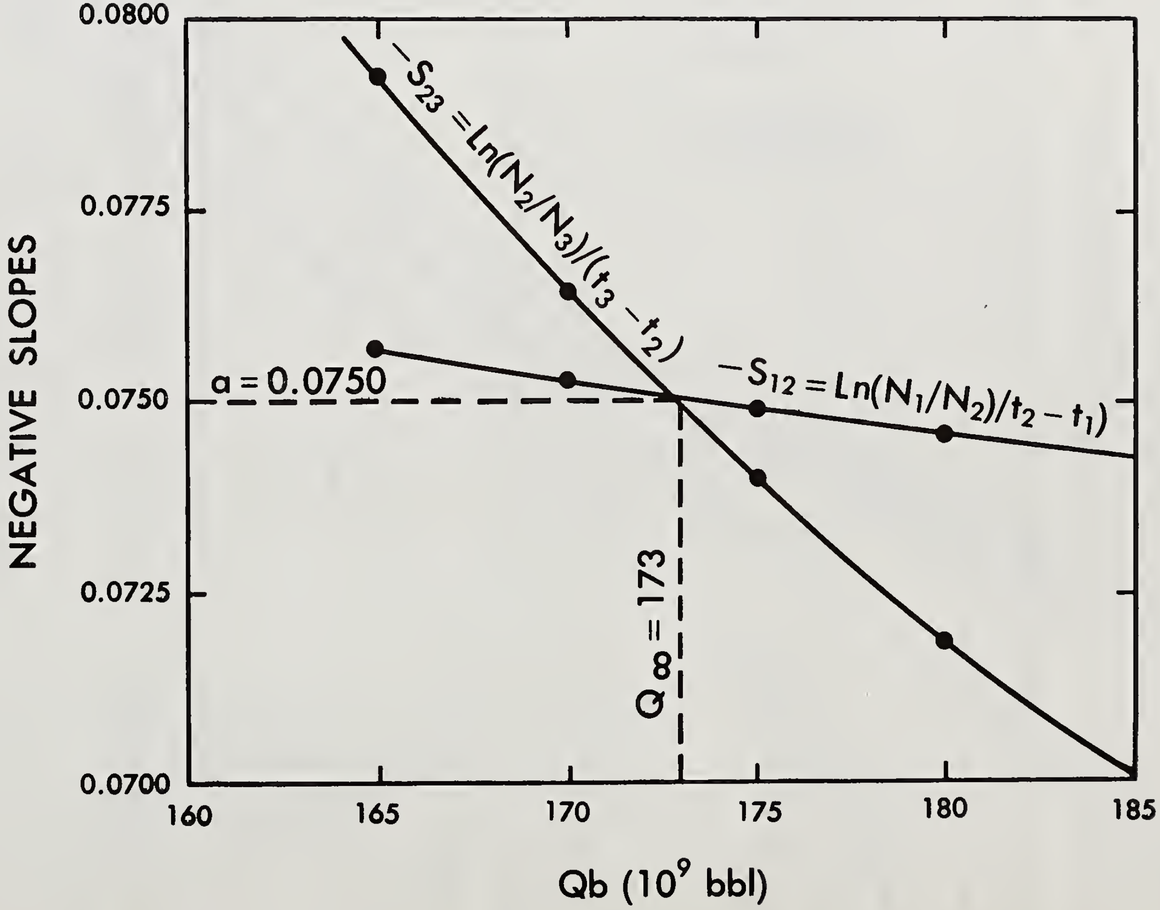

Fig. 10 - Graphical technique for determining the constants a and Q_∞ of the logistic equation.

An independent procedure for determining the constants Q_∞ and a is that based upon equation (27),

$$(dQ/dt)Q = a - (a/Q_\infty)Q.$$

If successive values of dQ/dt are known for successive values of Q, and if these data correspond to the logistic equation, then the curve of equation (27) will be a straight line intersecting the vertical axis as $Q \rightarrow 0$, at

$$(dQ/dt)/Q = a,$$

and the horizontal axis at

$$Q = Q_\infty.$$

This is illustrated in Figure 11, for the same constants as those for Figure 10.

After the best value of the constant Q_∞ has been determined, then the linear graph of equation (36),

$$\ln N = \ln N_0 - at,$$

can be constructed. From this, the time at which the inflection point on the Qt-curve, or the maximum rate of production dQ/dt, will occur can either be read from the graph or else computed from equation (36). According to equation (26), the peak production rate will occur when

$$Q = Q_\infty/2,$$

or when

$$N = Q_\infty/(Q_\infty/2) - 1 = 1$$

and

$$\ln N = 0.$$

Solving equation (36) for t when $N = 1$ then gives

$$t = [\ln (N_0/N)]/a$$

$$= (\ln N_0)/a. \tag{46}$$

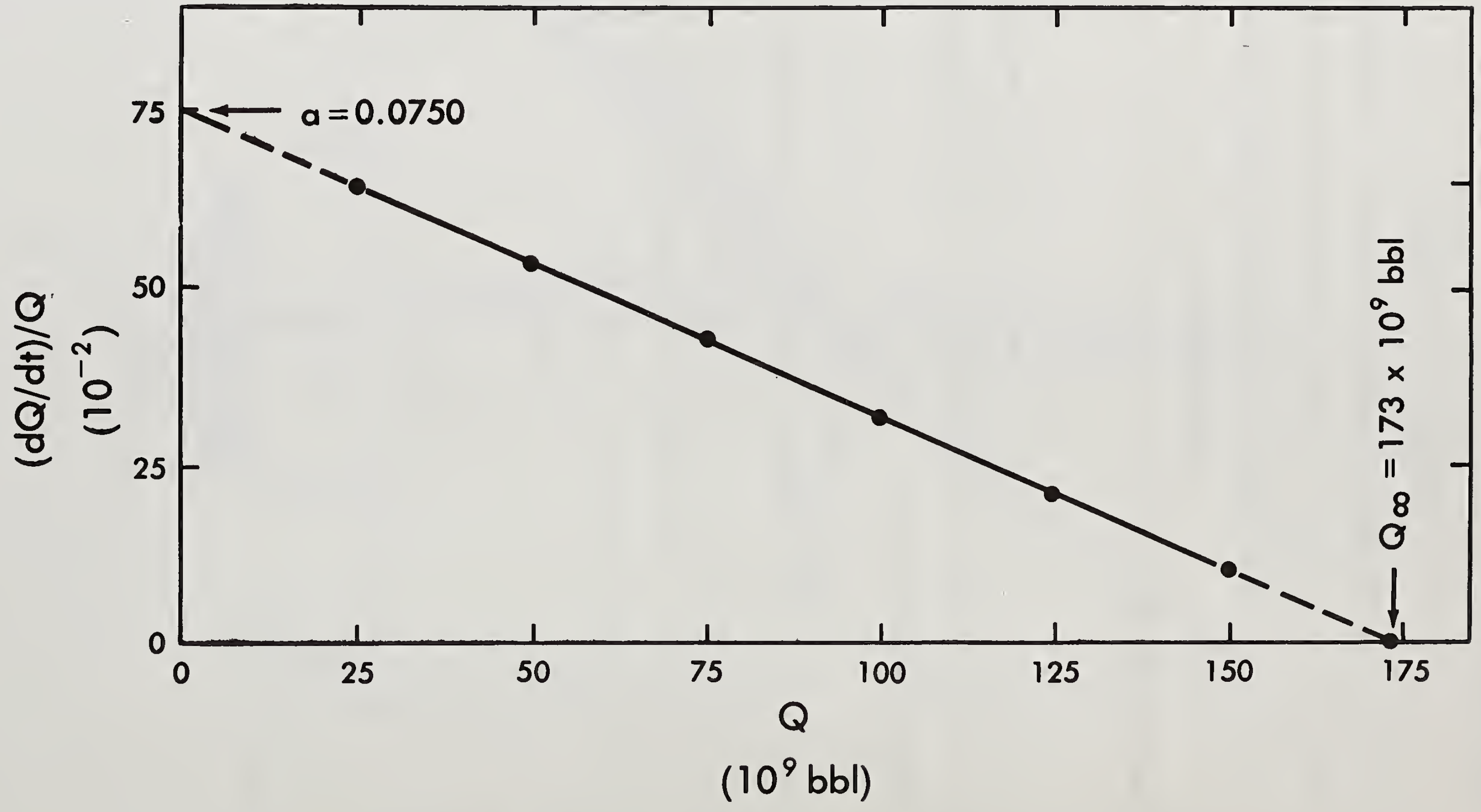

Fig. 11 - Determination of logistic constants by means of the linear graph of $(dQ/dt)/Q$ as a function of cumulative production Q.

Thus, if T be the date in years and T_o the arbitrarily chosen date for which $t = 0$, then the date of peak production rate would be

$$T_{peak} = T_o + (\ln N_o)/a. \tag{47}$$

Cumulative Production, Proved Reserves, and Cumulative Discoveries

The foregoing analysis pertains to a single quantity, such as cumulative production, and its variation with time during the complete production cycle. Actually, there is another important variable, based upon additional information, namely, proved reserves. Proved reserves, as defined by the Committee on Proved Reserves of the American Petroleum Institute, represents, essentially, oil in existing fields that has been proved by development drilling and is recoverable by existing installed equipment and technology. Estimates of proved reserves at the end of each year have been made annually for the United States since 1936 by the Proved Reserves Committee, and approximate figures, based upon various earlier estimates, are available back to 1900. Because additions to proved reserves are added annually only as new discoveries are made and older fields are developed, the figure for proved reserves is a conservative figure and is not intended to represent the ultimate amount of oil that the known fields will produce. Over- and under-estimates made in previous years are adjusted as new information becomes available by annual revisions. Proved-reserves estimates are therefore internally consistent and probably have a reliability within a range of a few percent.

A third significant quantity is that of cumulative proved discoveries. This does not represent independent data but is a derived quantity, defined in terms of the primary quantities, cumulative production and proved reserves.

If we let Q_p represent cumulative production, Q_r proved reserves, and Q_d cumulative proved discoveries, then Q_d is defined by the equation

$$Q_d = Q_p + Q_r. \tag{48}$$

In other words, all the oil that can be proved to have been discovered by a given time is the sum of the oil already produced plus the proved reserves.

The manner of variation of these three quantities during the complete production cycle, for a large region such as the United States, is indicated in Figure 12. The cumulative production curve, Q_p, will be a generally S-shaped, logistic-type curve, asymptotic to zero initially and to Q_∞ finally. The curve of proved reserves, Q_r, will be asymptotic to zero initially, and again at the end of the exploitation cycle, and will reach a maximum in the midrange of the cycle. The curve of cumulative discoveries, Q_d, will also be a logistic-type curve similar to that of cumulative production except that it will precede the latter in the midrange by some time interval Δt. This curve also will be asymptotic to zero initially and to the same value of Q_∞ finally as for the curve of cumulative production. This must be so because, as $t \to \infty$, $Q_r \to 0$, and equation (48) simplifies to

$$Q_d = Q_p = Q_\infty.$$

Taking the time derivative of equation (48) gives

$$dQ_d/dt = dQ_p/dt + dQ_r/dt, \tag{49}$$

the terms of which are the rates of proved discovery, or production, and of increase of proved reserves. When proved reserves reach their maximum value,

$$dQ_r/dt = 0,$$

and at that time equation (48) reduces to

$$dQ_d/dt = dQ_p/dt.$$

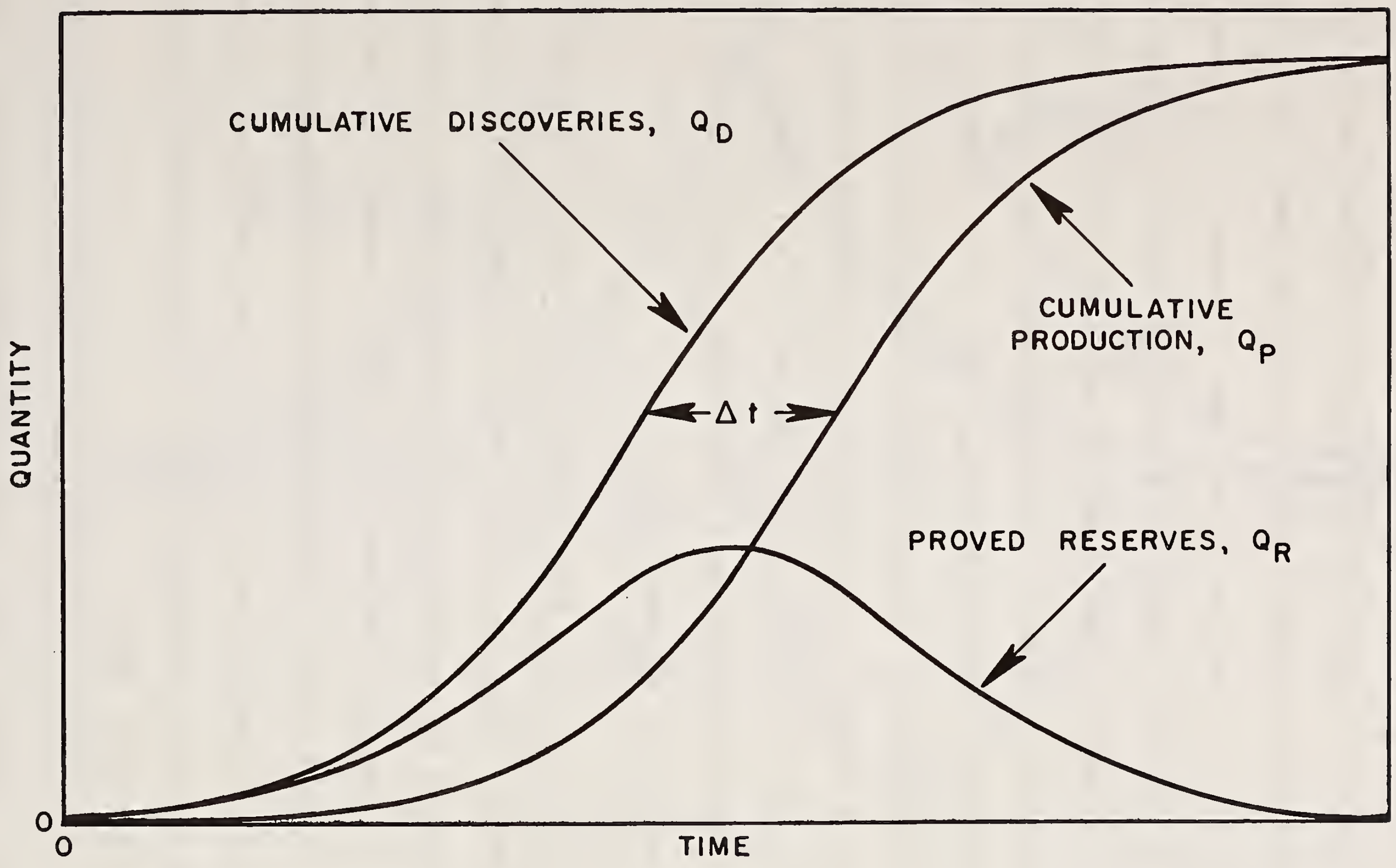

Fig. 12 - Variation of cumulative production Q_p, proved reserves Q_r, and cumulative discoveries Q_d during complete production cycle (Hubbert, 1962, Fig. 22).

This is the time at which the curve of the rate of production, which is still ascending, crosses that of the rate of discovery, already on its descent.

The curves Q_p, Q_r, and Q_d and their time derivatives, shown as functions of t in Figures 12 and 13 provide diagnostic evidence of the approximate stage of evolution in its complete cycle at any given time of the petroleum industry in a large area such as the United States. Because of the geometrical similarity between the curves of cumulative proved discoveries and of cumulative production, and the time lag Δt of the production curve with respect to that of discoveries, it follows that at any given time the discovery curve amounts to an approximate Δt-preview of the curve of production. Thus, if the curve of cumulative proved discoveries passes its inflection point, corresponding to the maximum rate of discovery, at a time t_1, then the curve of cumulative production will reach its inflection point and maximum production rate at a later time of approximately $t_1 + \Delta t$. The curve of proved reserves will reach its maximum at a time about halfway between, or at about $t_1 + \Delta t/2$.

Application to U.S. Petroleum Data as of 1962.---On March 4, 1961, President John F. Kennedy addressed a letter to the President of the National Academy of Sciences asking the Academy to advise him with respect to natural-resources policy. In response, the Academy appointed a Committee on Natural Resources to make the requested study and prepare reports for President Kennedy. I was a member of that Committee and directed the study and wrote the Committee's report on Energy Resources (National Academy of Sciences-National Research Council Publication 1000-D, 1962).

For this study, the technique used in 1956 was no longer appropriate because, during the intervening five years, the petroleum estimates of 1956 of 150 to 200 billion barrels had been progressively increased by various authors to 250, 300,

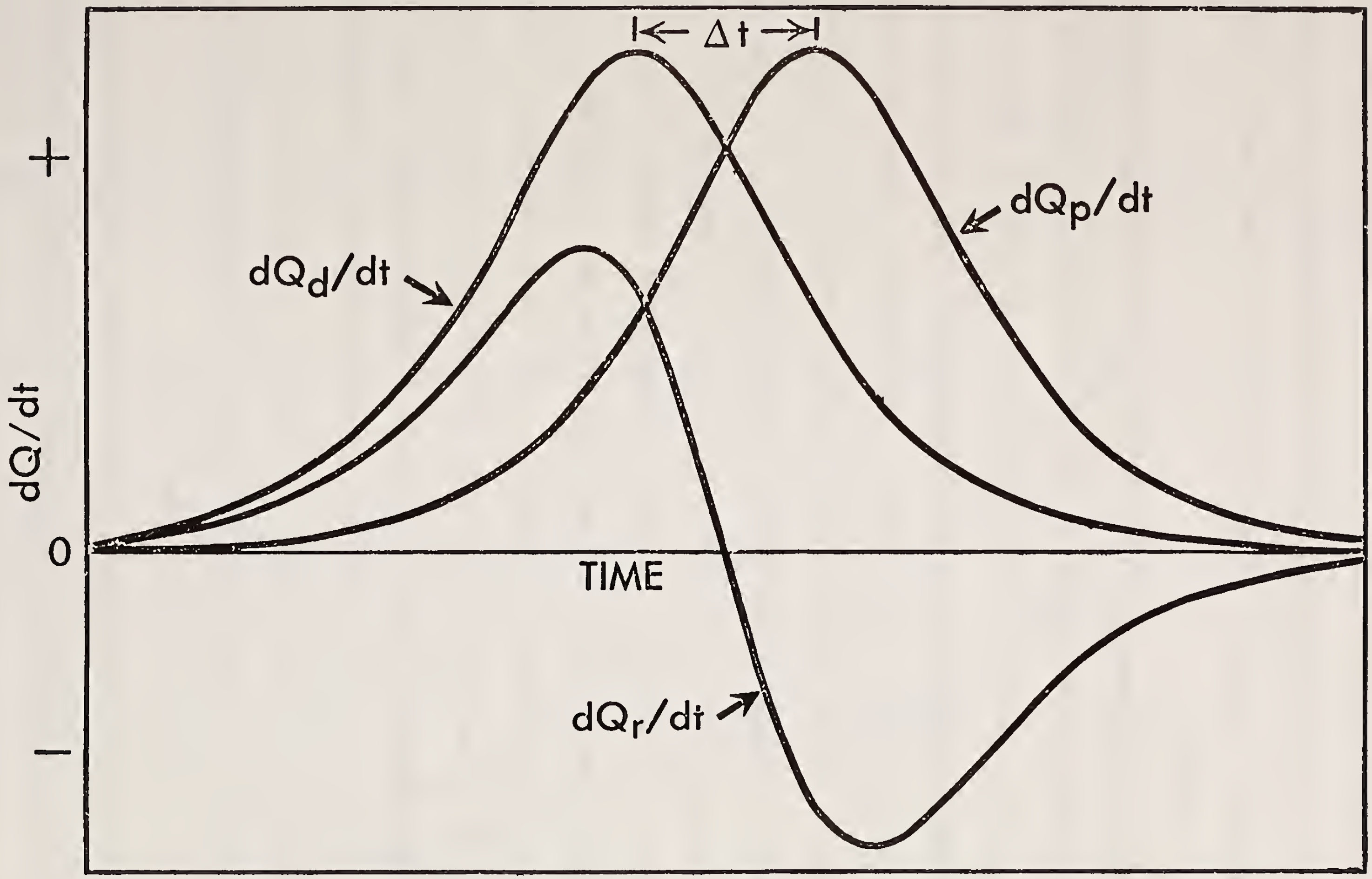

Fig. 13 - Rates of production and of proved discoveries, and of increase of proved reserves during complete production cycle (Hubbert, 1962, Fig. 24).

400, and eventually 590 billion barrels for the ultimate amount of crude oil to be produced in the Lower-48 states and adjacent continental shelves. We thus were confronted with approximately a four-fold range in the magnitudes of these estimates. For the lowest, the United States would reach its maximum rate of oil production at about 1965; for the highest, this would be delayed almost to the year 2000.

Accordingly, it became necessary to disregard the various a priori estimates of Q_∞, and instead let the historical data on discovery and production determine the approximate stage that the petroleum industry had reached in its evolutionary cycle. Of primary interest was the determination of such critical dates as those of the maximum rates of discovery and production, and of the maximum of proved reserves. From these data, as a secondary objective, an estimate of the magnitude of the ultimate production Q_∞ could be derived.

The theoretical basis for this analysis was that shown graphically in Figures 12 and 13. The actual data for the curves of cumulative production, proved reserves, and cumulative proved discoveries for U.S. crude oil from 1901 to 1962 are shown graphically in Figure 14. By visual inspection, the curve of cumulative proved discoveries had passed its inflection point at about 1957; proved reserves appeared to be at about their maximum in 1962; and the time delay Δt between the curve of production and that of discoveries was approximately 10.5 years, and had been so since 1925. Accordingly, the production rate should reach its maximum 10-12 years after 1957, the peak in the rate of discovery, or 5 to 6 years after the proved-reserves maximum in 1962.

For more precise calculations, the three curves of Figure 15 needed to be fitted by analytical equations so that analytical derivatives could be obtained with which to compare the actual annual rates of production, of discovery, and

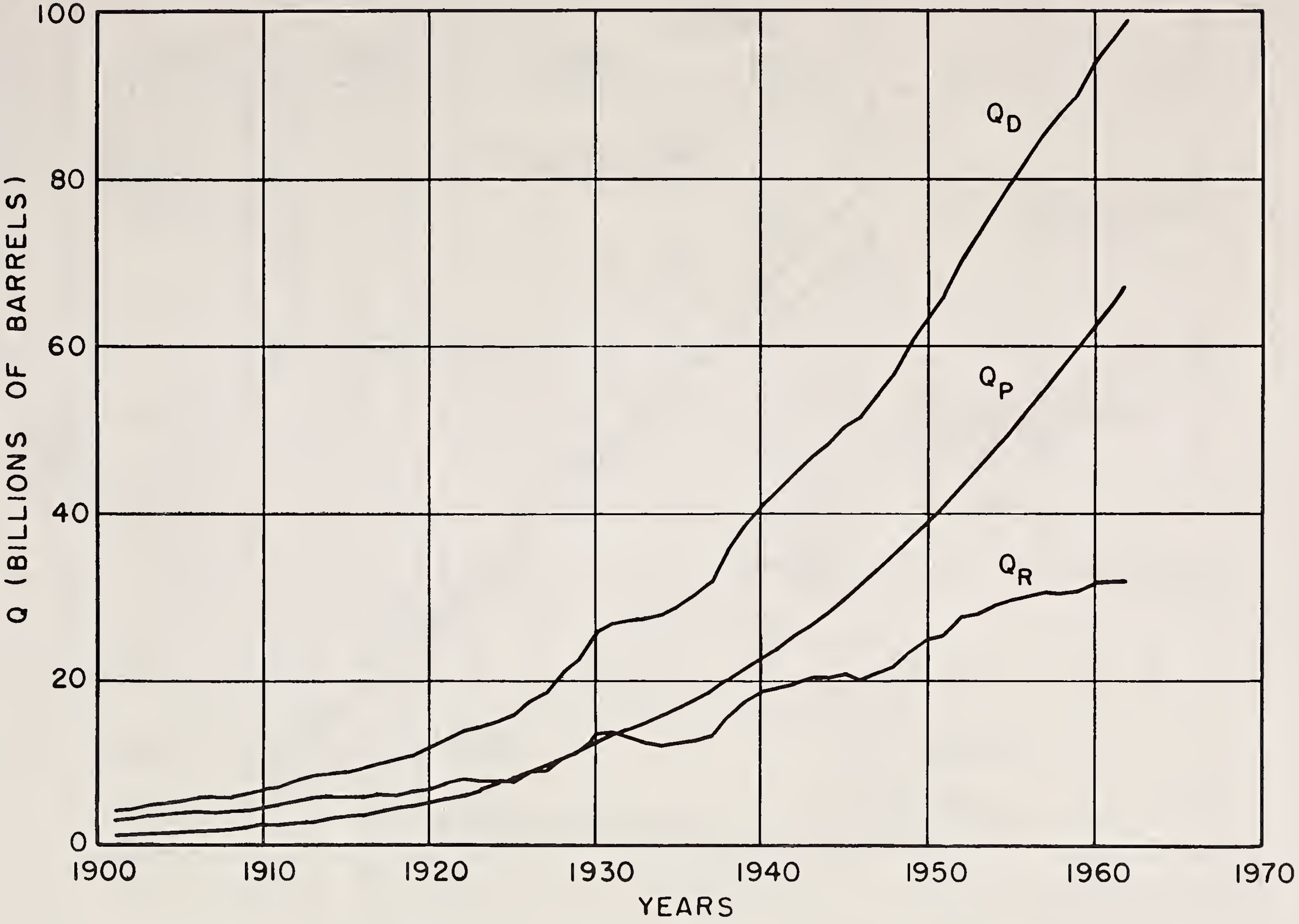

Fig. 14 - U.S. cumulative crude-oil production, proved reserves, and cumulative proved discoveries, 1901-1962 (Hubbert, 1962, Fig. 25).

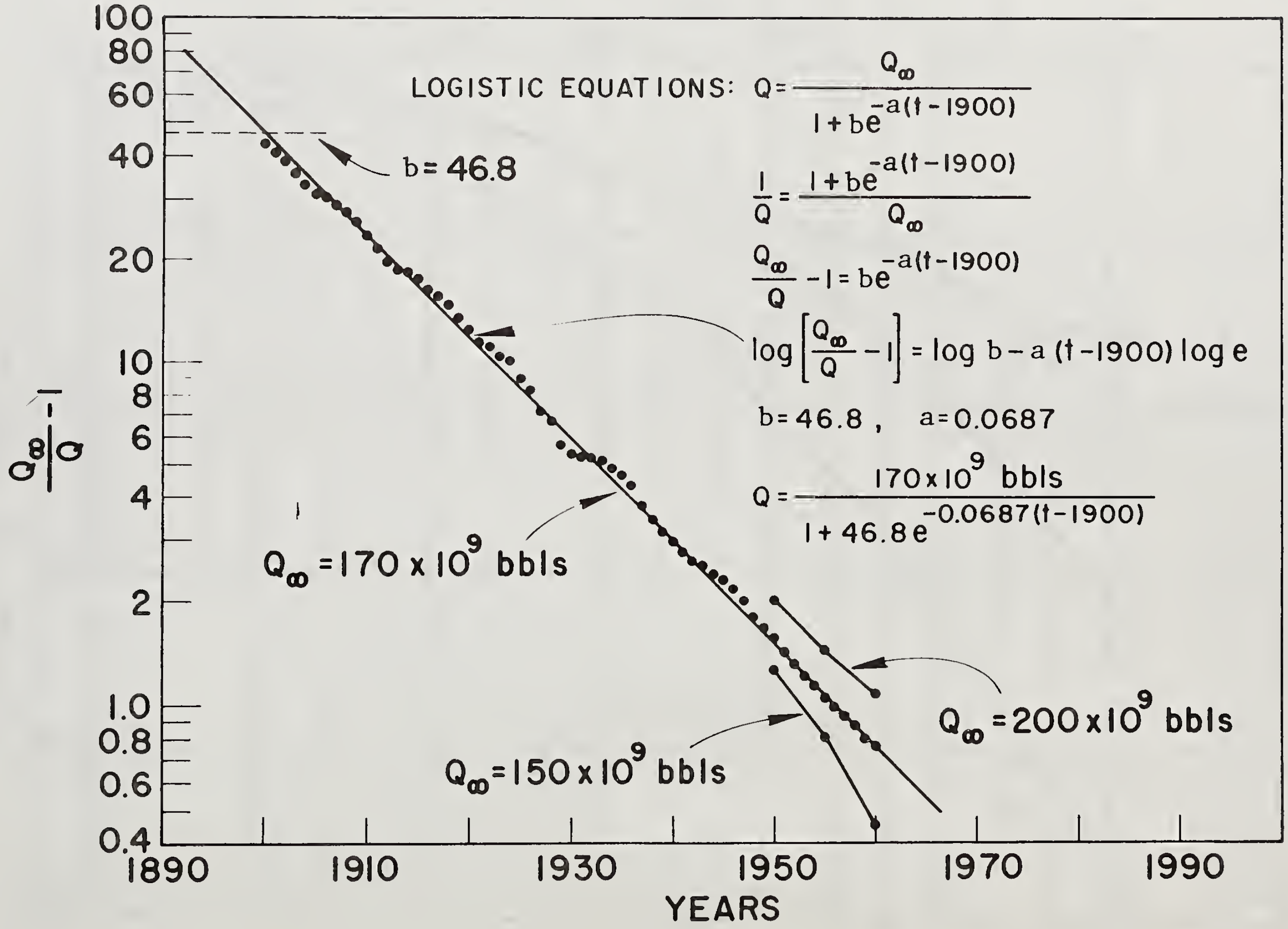

Fig. 15 - Graphical method used in 1962 for determining the constants of the logistic equation for U.S. cumulative crude-oil discoveries.

of increase of proved reserves. For this purpose, various forms of empirical equations were tested, but none gave satisfactory agreement with the data until finally the logistic equation was tried and found to fit the data with remarkable fidelity.

The curve of cumulative proved discoveries, having Δt more years of data than that of cumulative production, was fitted first. This was done by the iterative procedure of equations (41) and (42), as illustrated in Figure 9. The results, shown graphically in Figure 15, were

$$Q_\infty = 170 \times 10^9 \text{ bbl},$$

$$a = 0.0687/\text{yr}.$$

Then the year 1901 was taken for t_o, and

$$N_o = (Q_\infty - Q_o)/Q_o = 46.8.$$

With these parameters, the equations for Q_d, Q_p, and Q_r were

$$\left.\begin{aligned} Q_d^* &= (170 \times 10^9)/[1 + 46.8e^{-a(t-1901)}], \\ Q_p &= (170 \times 10^9)/[1 + 46.8e^{-a(t-1911.5)}], \\ Q_r &\simeq Q_d - Q_p. \end{aligned}\right\} \tag{49}$$

The graphs of the actual data for Q_p, Q_r, and Q_d superposed upon the theoretical curves are shown in Figure 16. Superposition of the annual increments of proved reserves upon the theoretical derivative curve is shown in Figure 17, and the corresponding superposition of the rates of discovery and of production upon their respective derivative curves are shown in Figure 18.

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*In the Academy report, t_o was given as 1900, but the data were as of the end of each year. Hence the end of 1900 is actually 1901.0

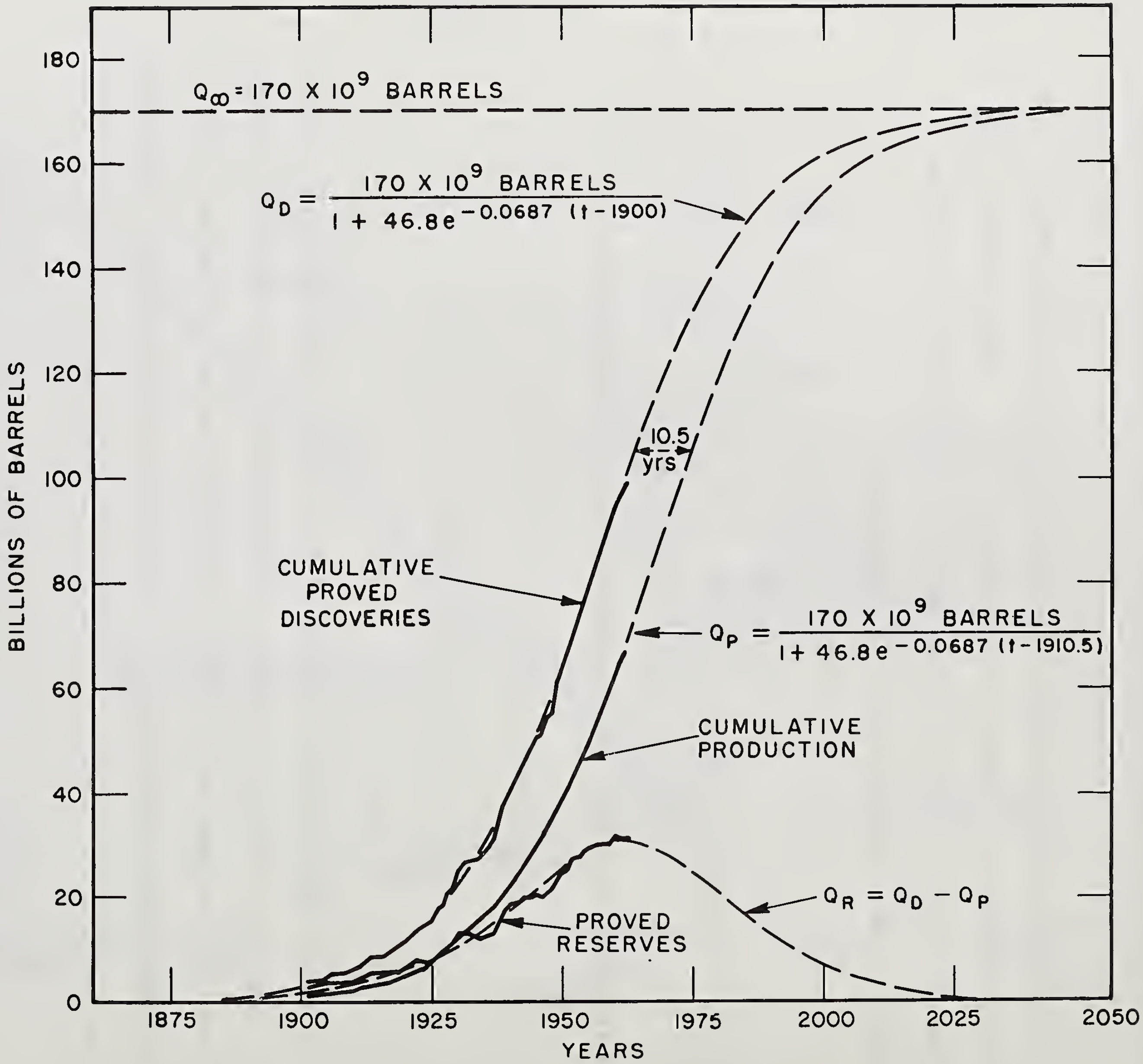

Fig. 16 - Cumulative proved discoveries, production, and proved reserves of U.S. crude oil to 1962 with graphs of the corresponding logistic equations (Hubbert, 1962, Fig. 27).

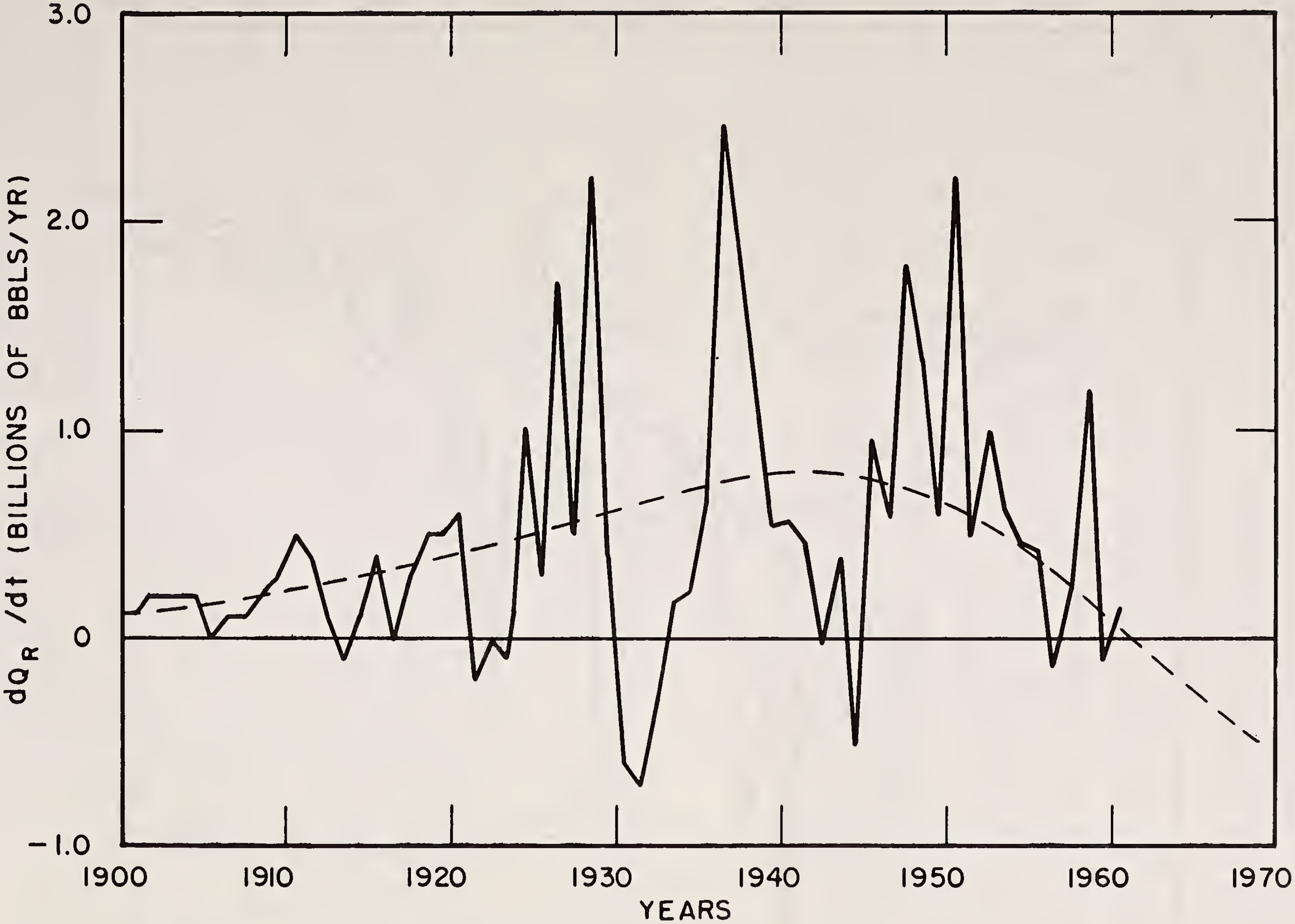

Fig. 17 - Annual increments of proved reserves of U.S. crude oil, 1900-1961, superposed upon the theoretical curve from the logistic equation (Hubbert, 1962, Fig. 29).

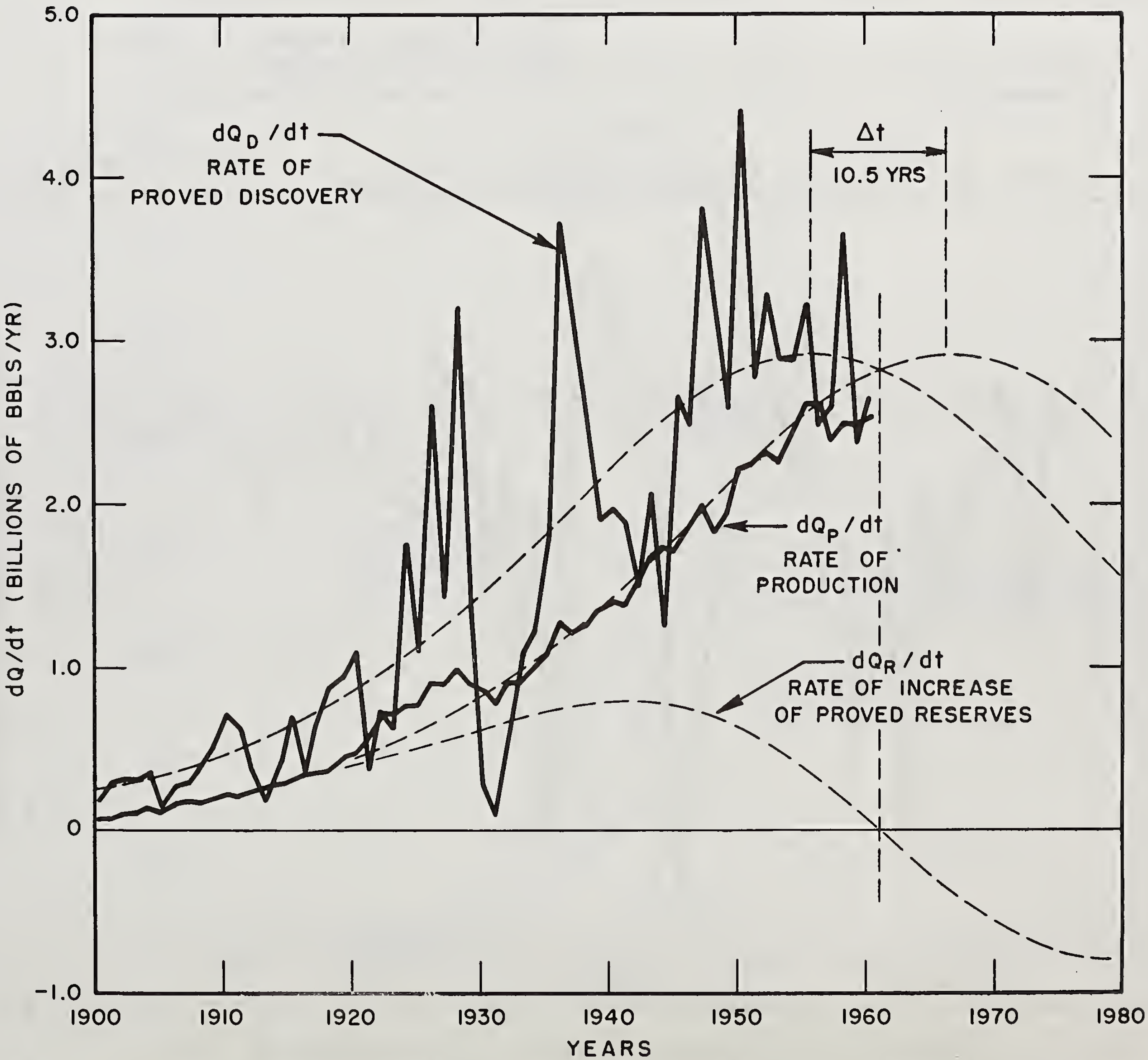

Fig. 18 - Annual rates of production and of proved discovery, 1900-1961, superposed upon derivative curves of the logistic equations (Hubbert, 1962, Fig. 28).

In Figure 17 it is seen clearly that the curve of increase of proved reserves had already gone through its positive loop corresponding to increasing reserves, and was crossing the zero line into its negative loop at just about 1962. This marks the date of the peak of proved reserves.

In Figure 18, the annual discoveries fluctuate widely, but their oscillations still follow the derivative curve and indicate that the peak region had been passed at about 1957. The peak in the rate of production can accordingly be expected to occur near the end of the 1960-decade.

These were the interpretations made graphically in the Academy report of 1962. Although this was not done at the time, more precise figures can be obtained from the equations (49). As shown in equation (46), the peak rate of discoveries occurs at the time

$$t \simeq t_o + \ln N_o/a.$$

Hence, from equation (49), the peak discovery rate should occur at the time

$$\begin{aligned} t &= 1901 + \ln 46.8/0.0687 \\ &= 1957.0. \end{aligned}$$

The corresponding date for the peak in the production rate would be 10.5 years later, or 1967.5, and the peak of proved reserves would occur at 1962.25. All of these figures are consistent with those obtained from the graphical interpretation of the data.

An even more informative procedure consists in plotting the linear graphs of the logistic equations of cumulative proved discoveries and cumulative production from 1901 to 1962 on semi-logarithmic graph paper. The corresponding equations for discoveries and production are

$$\log N_d = \log N_o - (a \log e)(t - t_o)$$

and

$$\log N_p = \log N_o - (a \log e)[t - (t_o + \Delta t)],$$

where $t_o = 1901$, and $\Delta t = 10.5$ years.

In these equations, the peak discovery and production rates each occur when $Q = Q_\infty/2$, or when

$$N = [Q_\infty/(Q_\infty/2)] - 1 = 1.$$

Hence the dates at which the respective linear graphs cross the line $N = 1$, or $\log N = 0$, are the dates of peak rates of discovery and of production. These graphs for the cumulative discovery and production data at 5-year intervals from 1901 to 1962 are shown in Figure 19. Note that by 1962 the cumulative-discoveries curve had already crossed the line $N = 1$ at 1957.0, and that the linear graph for cumulative production had been parallel to the discoveries curve since 1925 with the time lag of 10.5 years. Hence only a modest 6-year extrapolation of this curve was required to reach the line $N = 1$, corresponding to the peak in the rate of production at the year 1967.5.

Thus, the combined data on production, proved reserves, and proved discoveries of crude oil in the Lower-48 states were by 1962 sufficient to establish that the peak in the rate of production would have to occur at about the end of the 1960-decade with a range of uncertainty of not more than about 3 years. The corresponding maximum production rate from equation (39) would be about

$$dQ_p/dt \simeq aQ_\infty/4,$$

or

$$(0.0687 \times 170 \times 10^9)/4 = 2.92 \times 10^9 \text{ bbl/yr}.$$

The Decade of 1962-1972.— Although the reports of the National Academy Committee were released to the public by President Kennedy in January 1963, the influence on public policy of the Academy report on energy resources was essentially nil. The estimate of 590 billion barrels for the ultimate crude-oil production

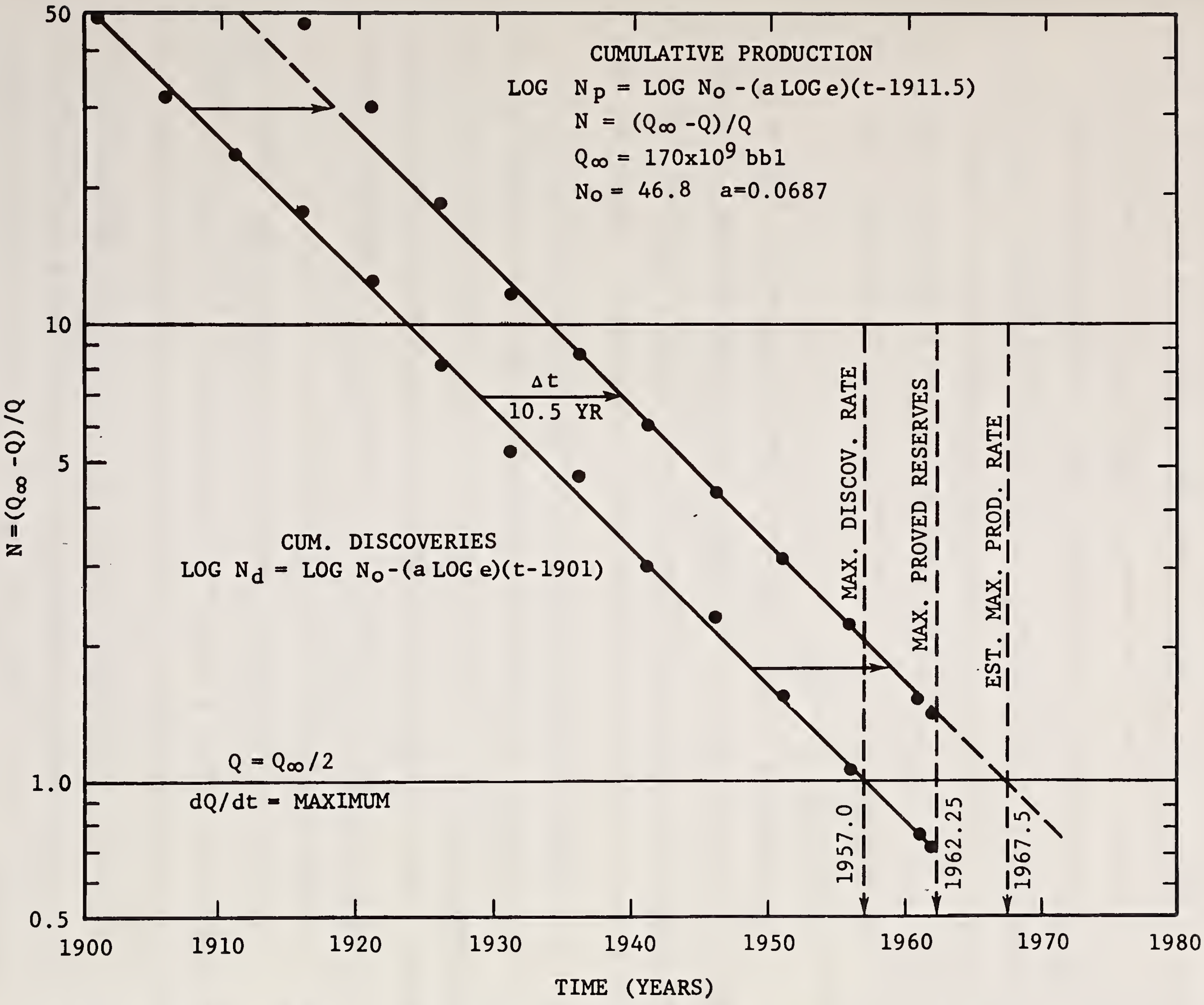

Fig. 19 - Linear graphs of the logistic equations of U.S. cumulative crude-oil discoveries and production, 1900-1962.

from the Lower-48 states had been given to the Academy Committee by V. E. McKelvey, Assistant Chief Geologist of the U.S. Geological Survey, as the official estimate of the USGS. It was cited in the Academy report but, because it could not remotely be reconciled with the petroleum-industry data, it had to be rejected. During the next 5 years, substantially the same figures, modified slightly from year to year, of about 600 billion barrels of crude oil and 2,500 trillion cubic feet of natural gas, were published repeatedly in U.S. government and other publications by McKelvey, either alone or in collaboration with D. C. Duncan (Duncan and McKelvey, 1963; McKelvey and Duncan, 1965; McKelvey, 1967).

In the meantime, the curve of annual crude-oil production from the Lower-48 states continued on the linear upward trend that had prevailed since 1932, except for a distortion from 1957 to 1970 associated with the first Suez crisis and successive Middle East disturbances and the Vietnam War. After 1957, the production rate fell below the linear trend, but by 1970 it rose to 3.24 billion barrels per year, which was just about on the trend. During 1971 it declined slightly to 3.18 billion barrels per year. Consequently, during the decade 1962-1972, there was no perceptible evidence from the curve of annual production alone of the imminence of an impending peak and subsequent decline of the annual rate of U.S. crude-oil production.

One of the first alarms over impending trouble came in the spring of 1969 when the annual report of the Committee on Natural-Gas Reserves of the American Gas Association was released, giving the proved reserves of natural gas as of the end of 1968. The report showed that by the end of 1968 natural-gas proved reserves for the Lower-48 states, which had been increasing steadily since 1947, had dropped 7.2 trillion cubic feet from 289.3 trillion cubic feet at the end of 1967 to 282.1 by the end of 1968. During the following year, the proved reserves dropped by 12.2 trillion cubic feet, as compared with the annual production rate of 20.7 trillion cubic feet.

By 1971, the evidences of impending declines in the rates of oil and gas production were sufficiently clear that the U.S. Senate Committee on Interior and Insular Affairs, under the Chairmanship of Senator Henry M. Jackson, began a new series of hearings on National Fuels and Energy policy. As of July 23, 1971, Senator Jackson addressed a letter to the Secretary of the Interior, Rogers C. B. Morton, requesting my assistance on statistical work for the Committee. What I was asked to do was to bring my earlier studies on energy resources up to date for the use of the Committee. The result was the report, *U.S. Energy Resources, A Review as of 1972*, which was released as a Committee Print in June 1974.

In Figure 20 the curves of cumulative proved discoveries, cumulative production, and proved reserves from 1900 to 1972 are shown as solid-line curves superposed upon the respective mathematical curves, shown as dashed lines. The respective logistic equations are also shown in the figure.

With 10 more years of data after the Academy report of 1962, the best value for Q_∞ was still 170 × 10^9 bbl and the growth constant a was still 0.0687/yr. The time interval Δt had been increased from 10.5 to 11.0 years, and 1930 was taken for t_o. The corresponding value of N_o became 6.17. By 1972, proved reserves are plainly seen to have passed their peak about 1962, and the curves of cumulative discoveries and of cumulative production are accurately following their respective logistic curves. From the logistic equations the dates for the maximum rates of discovery and of production, and of the maximum of proved reserves are found to be:

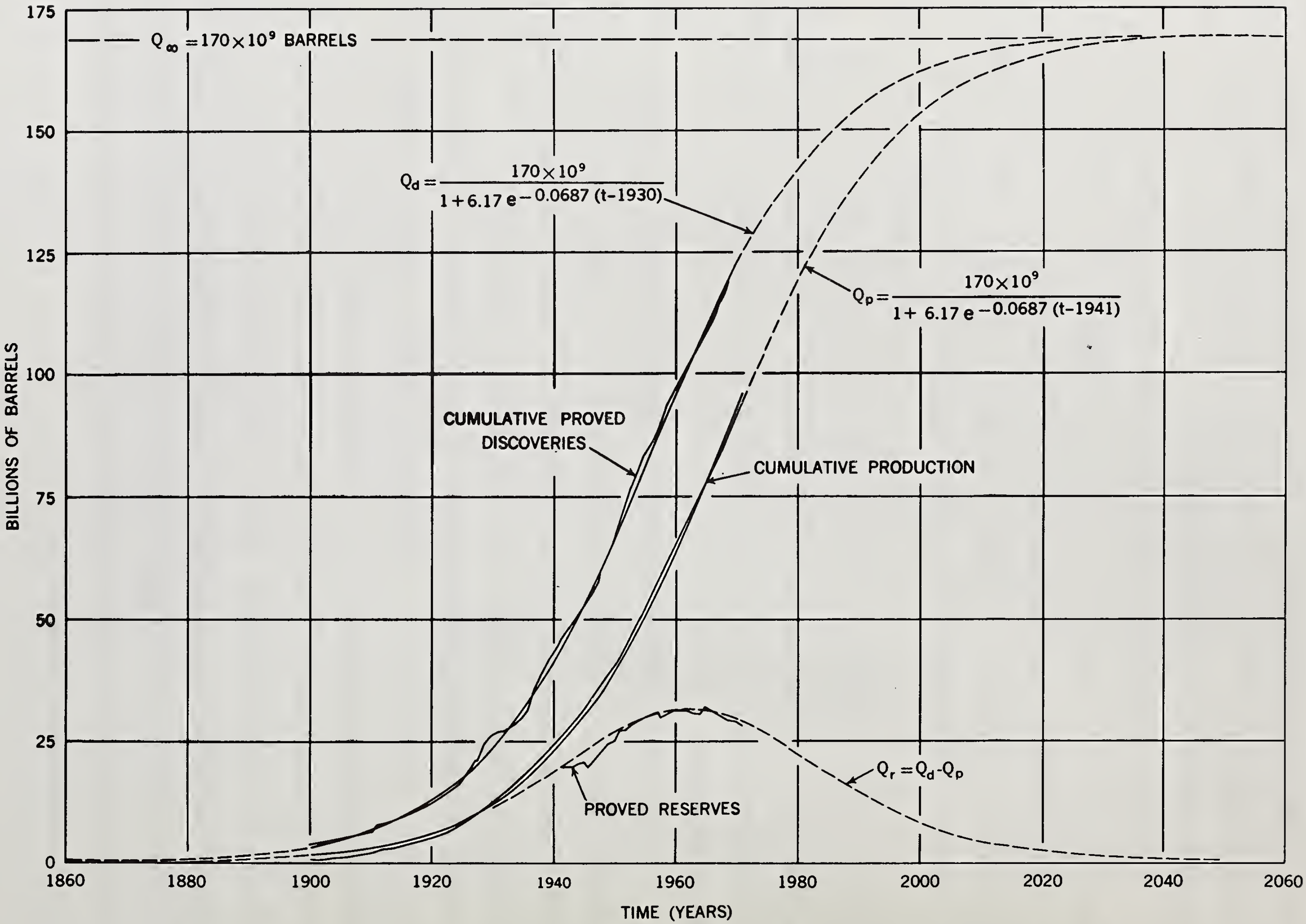

Fig. 20 - Cumulative production and proved discoveries, and proved reserves of crude oil from the U.S. Lower-48 states, 1900-1972, with the corresponding logistic curves (Hubbert, 1974, Fig. 36).

Discoveries	1956.5,
Production	1967.5,
Proved reserves	1962.0.

The time derivatives of these three curves, with the corresponding annual data, are shown in Figures 21 to 24, and the estimated complete cycle for crude-oil production, in Figure 25. Figure 21 shows the rate of increase of proved reserves superposed upon the mathematical derivative. The curve completed its positive loop and crossed the zero line in 1962, and by 1972 was approaching the low point of its negative loop. The rates of discovery and of production are both shown in Figure 22, and separately in Figures 23 and 24. In Figure 23, it is unmistakable that the discovery rate passed its peak before 1960 and is well advanced in its declining phase.

The curve of the rate of production in Figure 24 still shows no definite evidence that its peak has been reached. Instead of reaching a maximum about 1968, the curve fell below the mathematical curve after 1957 and then rose steeply from 1960 to 1970. Whether the slight reversal in 1971 represents the beginning of the decline is an open question. However, the composite evidence of all the data indicate that the reversal of the production-rate curve, if it has not already occurred in 1971, must happen in the very near future.

The estimate of the complete production cycle as of 1972 is shown in Figure 25. Cumulative production by 1972 amounted to 96×10^9 bbl and proved reserves plus additional oil in fields already discovered amounted to another 47×10^9 bbl, giving a total of 143×10^9 bbl for the ultimate amount of oil to be produced from fields already discovered. Then with 170×10^9 bbl for Q_∞, only 27×10^9 bbl are left for future discoveries. Another informative aspect of Figure 25 is the time span involved. The time required to produce the first 10 percent of Q_∞, or 17 billion barrels, was from 1860 to 1932. That required

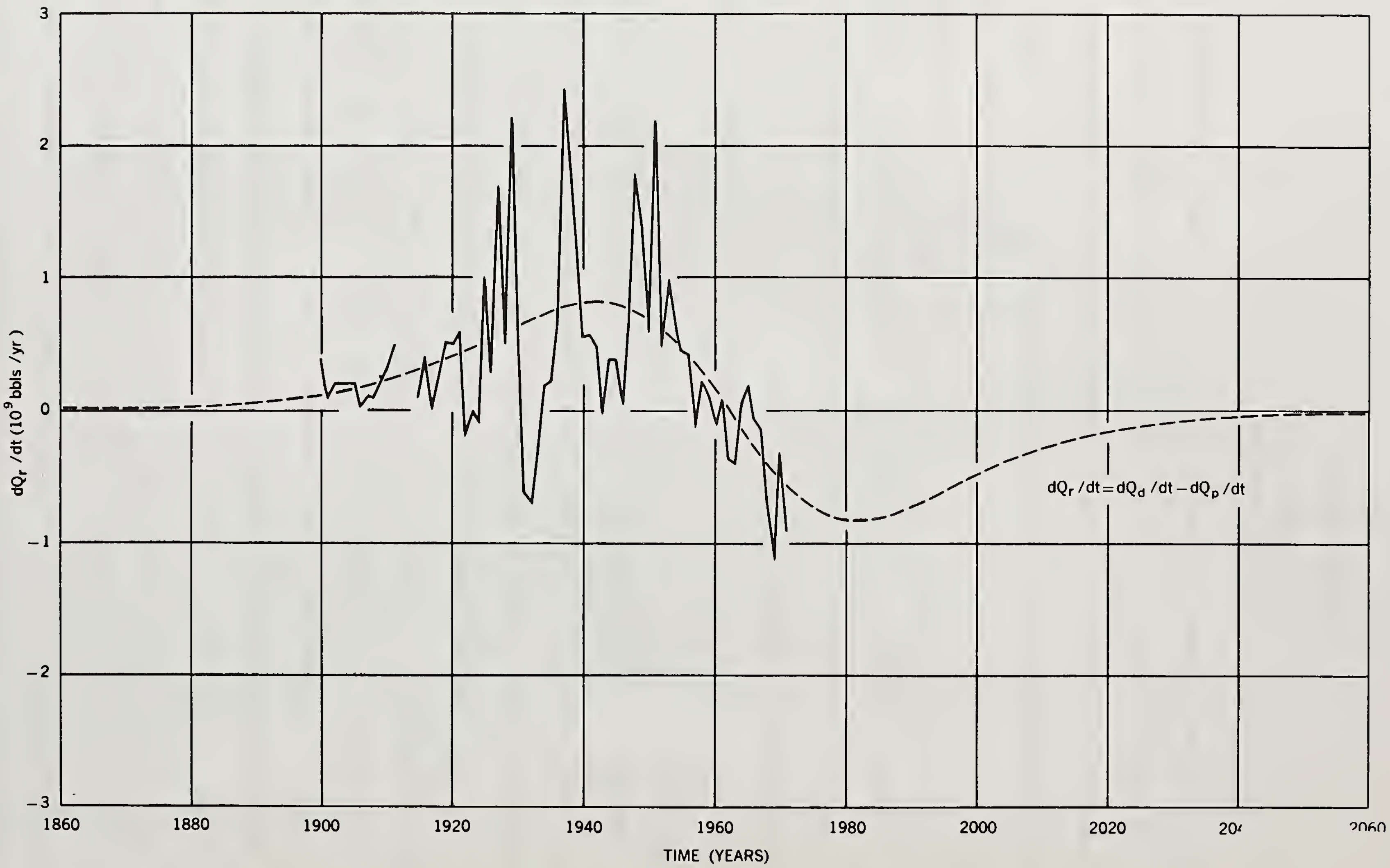

Fig. 21 - Annual increments of U.S. proved reserves of crude oil, 1900-1971, superposed upon the theoretical curve from the logistic equation (Hubbert, 1974, Fig. 40).

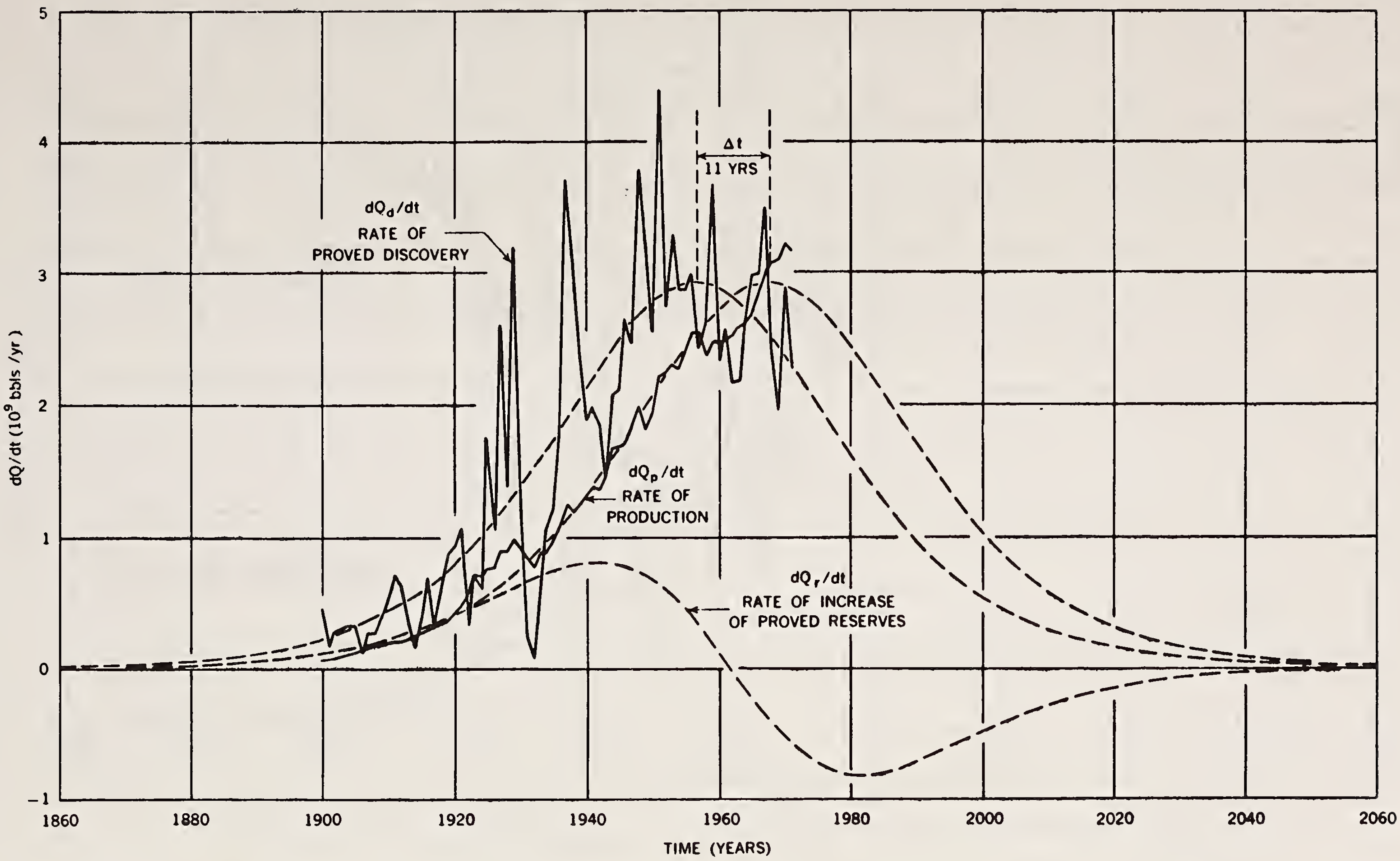

Fig. 22 - Annual production and proved discoveries of U.S. crude oil, 1900-1971, superposed upon curves of derivatives of logistic equations (Hubbert, 1974, Fig. 37).

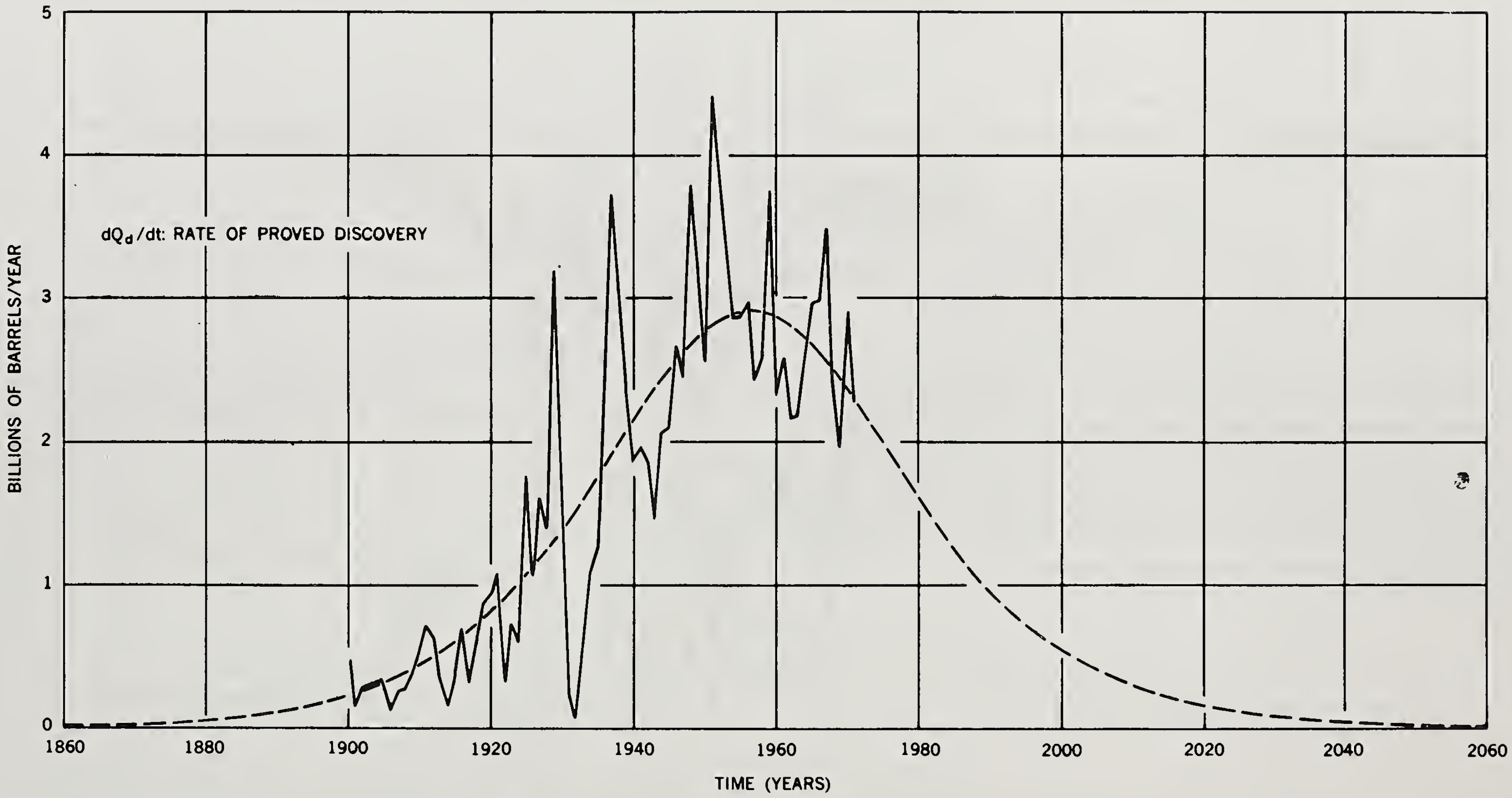

Fig. 23 - Annual proved discoveries of U.S. crude oil, 1900-1971, superposed upon curve of the derivative of the logistic equation (Hubbert, 1974, Fig. 38).

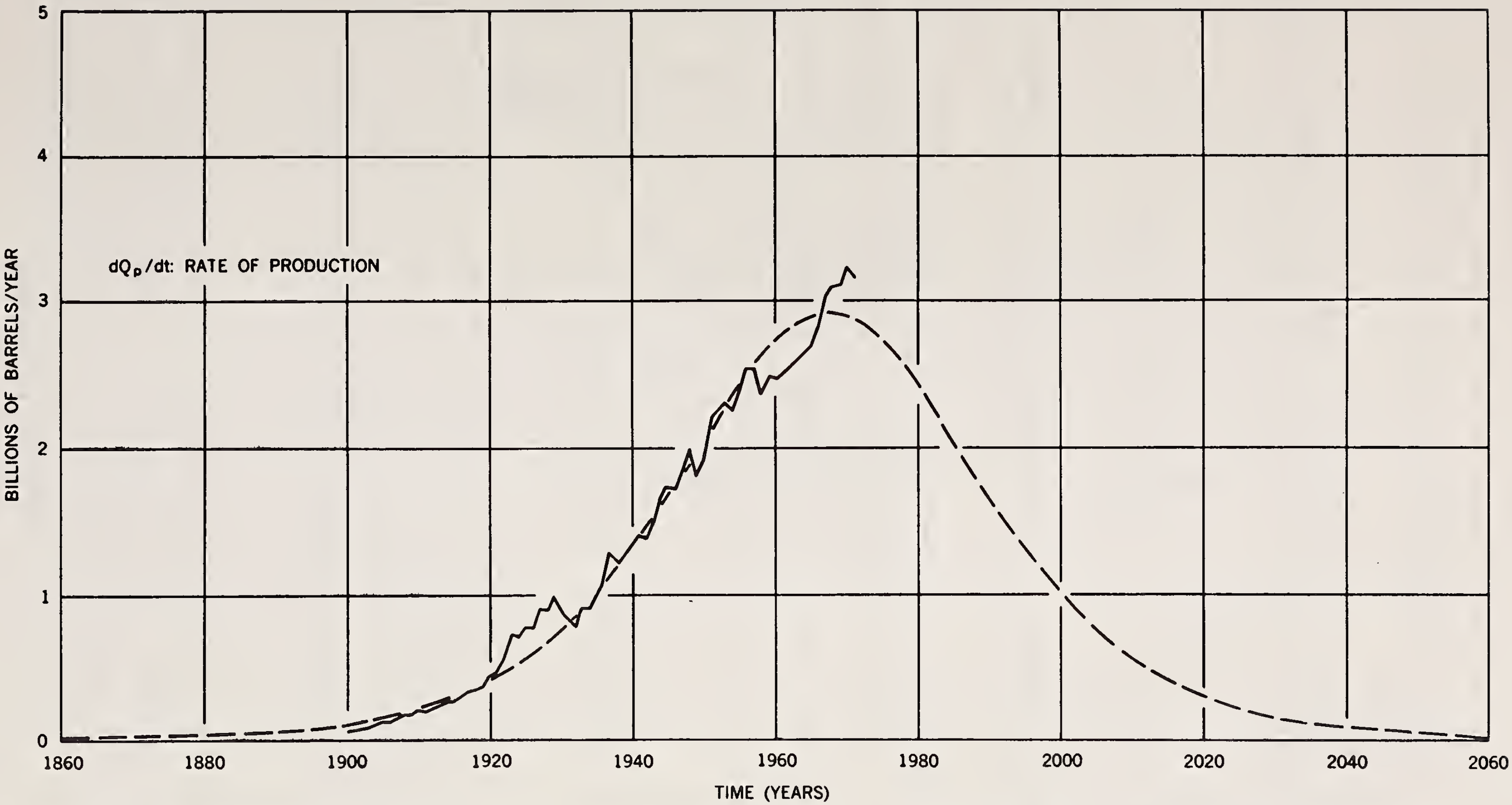

Fig. 24 - Annual production of U.S. crude oil, 1900-1971, superposed upon curve of the derivative of the logistic equation (Hubbert, 1974, Fig. 39).

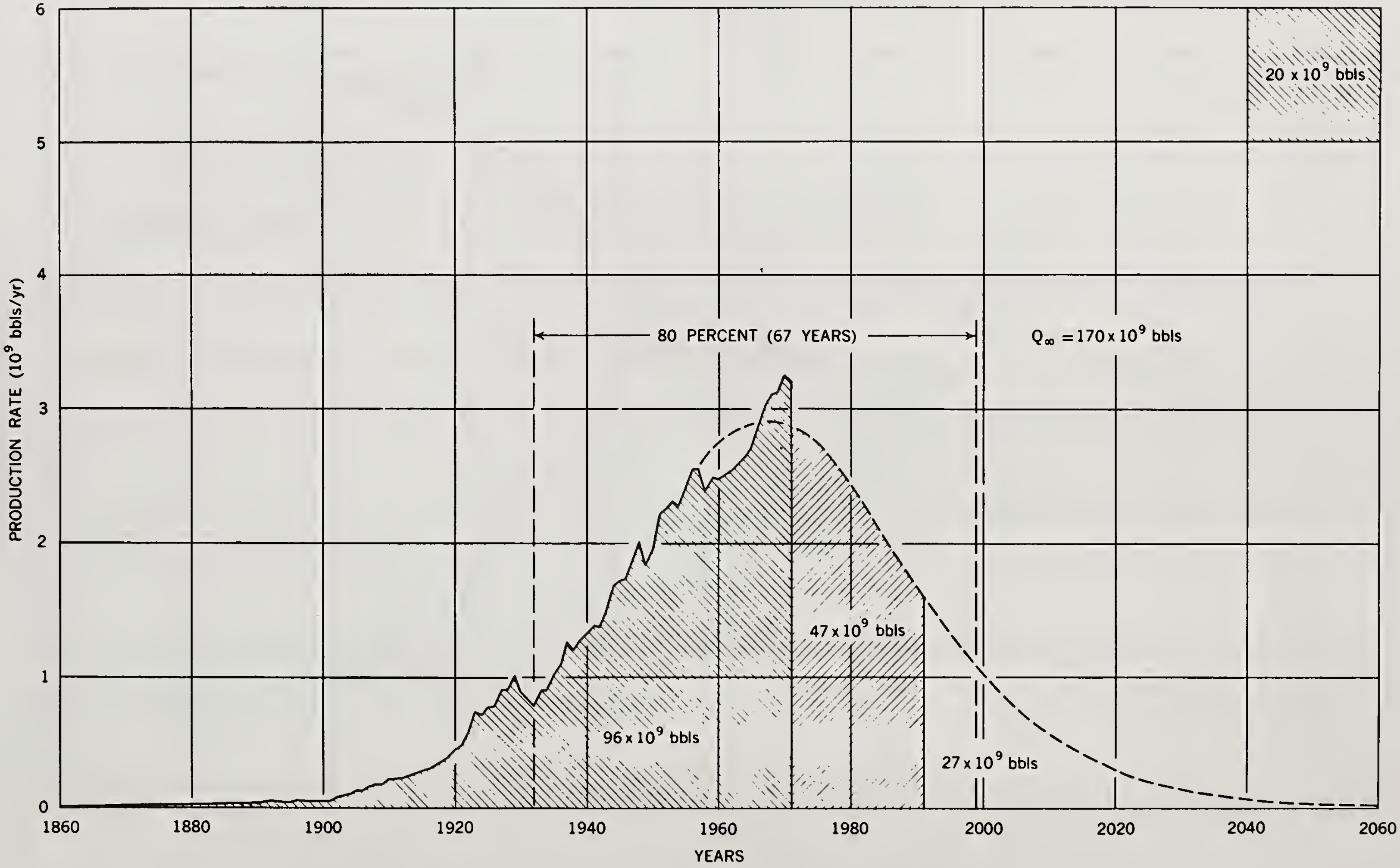

Fig. 25 - Complete cycle of crude-oil production from U.S. Lower-48 states as estimated in 1972 (Hubbert, 1974, Fig. 51).

for the next 80 percent, computed from the logistic equation, would be the 67-year period from 1932 to 1999, and the last 10 percent would occur after 1999.

The Period from 1972 to 1980.— Until 1972, our principal concern was the prediction of the future of the rate of U.S. crude-oil production. The mathematical curve of the rate of production passed its maximum about 1967-1968, but the production rate continued to increase sharply until 1970, yet all the evidence indicated that a decline was inevitable in the very near future. We now have 7 to 8 more years of data by means of which the pre-1972 predictions can be evaluated.

Figure 26 shows the linear graphs of the logistic equations for cumulative discoveries and production of crude oil in the Lower-48 states for the period 1925-1973. In this case a new determination of the logistic constants was made with the results:

$$Q_\infty = 170 \times 10^9 \text{ bbl},$$

$$t_o = 1925,$$

$$\Delta t = 10.7 \text{ yr},$$

$$N_o = 9.05,$$

$$a = 0.0674/\text{yr}.$$

Both curves by 1972 had crossed the line $N = 1$, corresponding to the dates of the respective maximum rates of discovery and production. The date for the discovery maximum rate was the year 1957.7, and that for the maximum production rate, 1968.4.

The curves of cumulative proved discoveries, cumulative production, and of proved reserves have been plotted to 1979 in Figure 27. These are superposed on the logistic curves of 1972 as a means of comparing the more recent developments

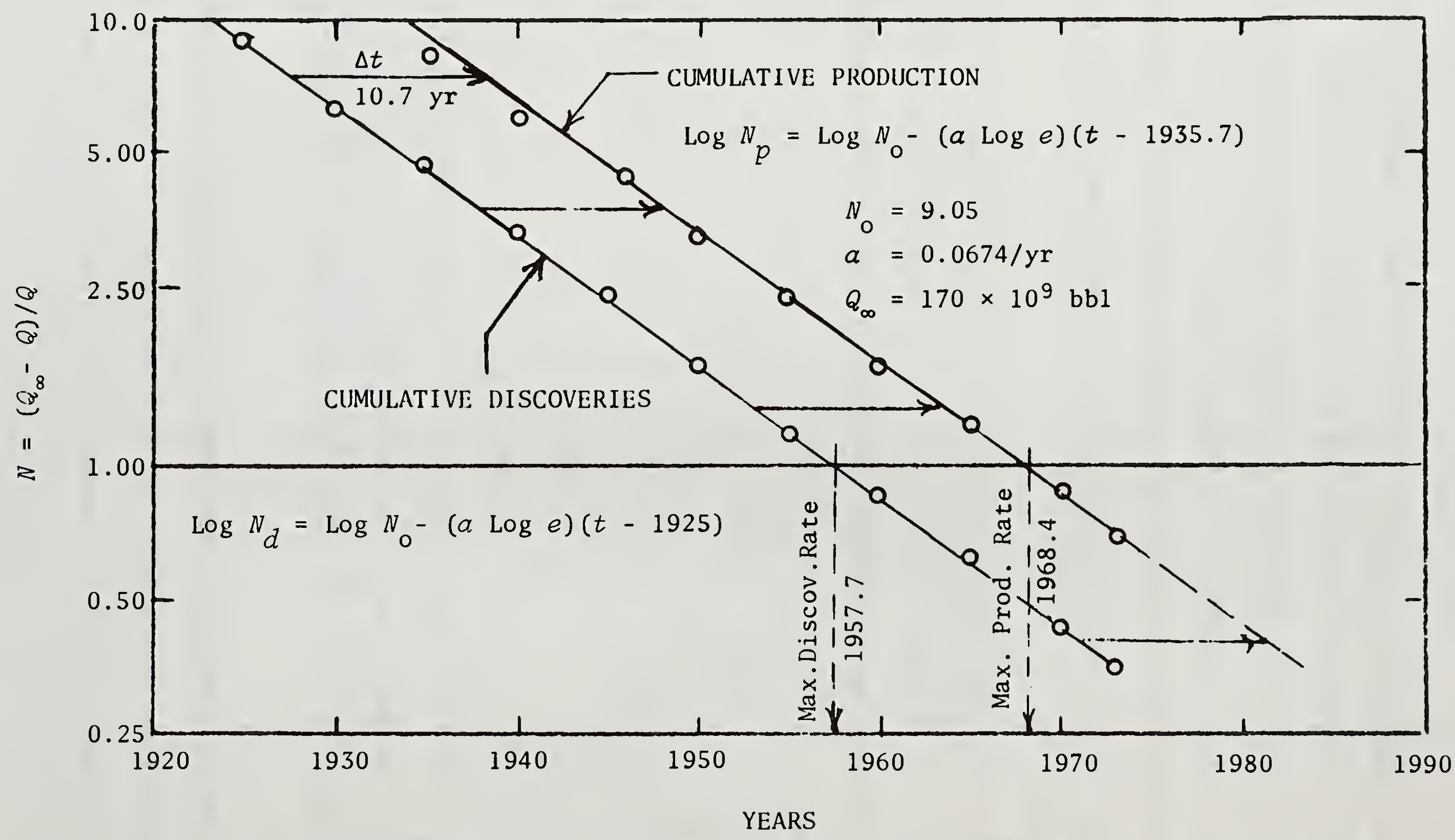

Fig. 26 - Linear graphs of the logistic equations for cumulative proved discoveries and cumulative production of crude oil from U.S. Lower-48 states, 1925-1973.

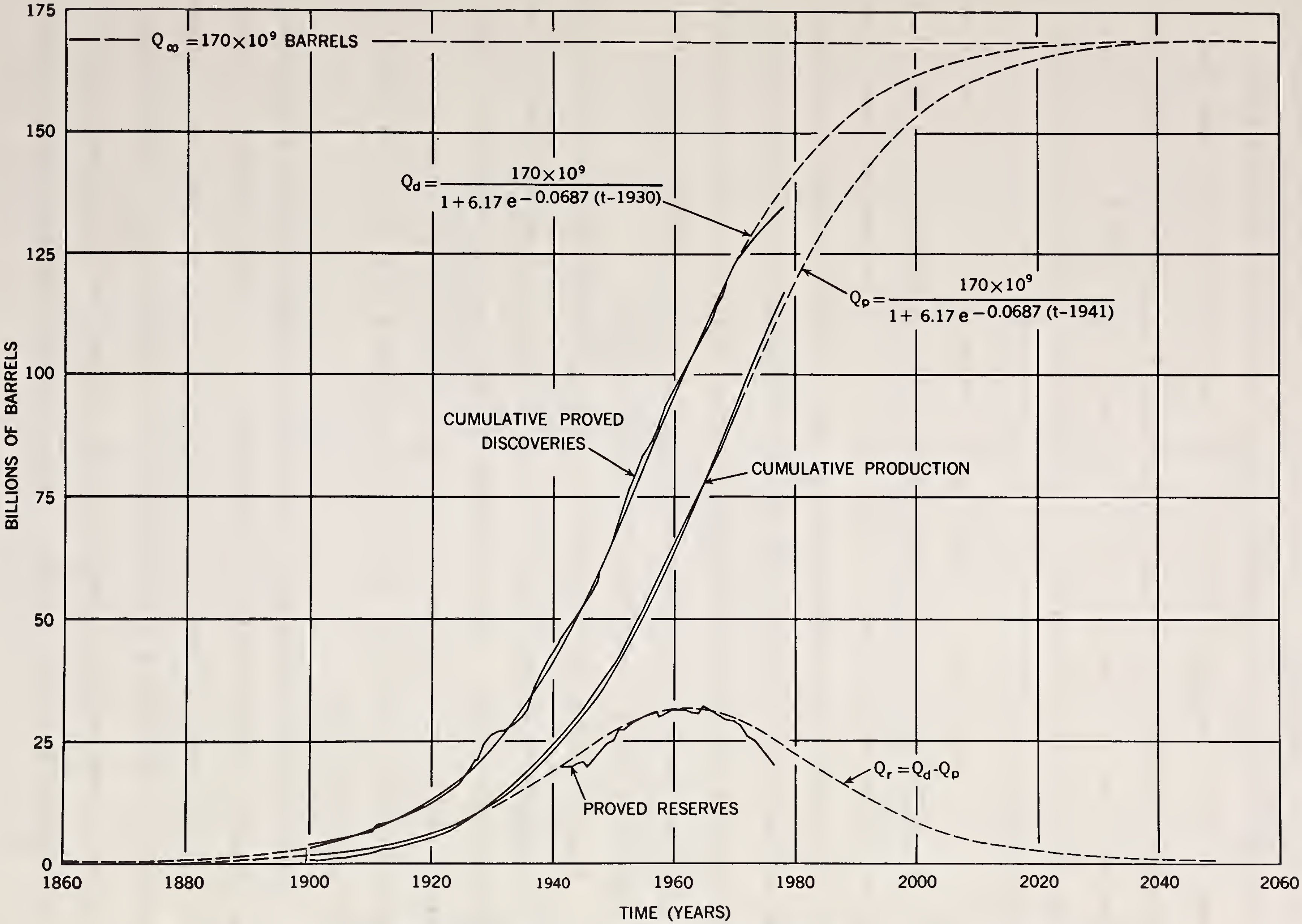

Fig. 27 - Cumulative production, proved reserves, and cumulative proved discoveries of U.S. crude oil, 1900-1978, superposed upon logistic curves of 1972.

with the 1972 predictions. It will be noted that the cumulative production is following the logistic curve rather closely, but both the curves of proved reserves and of cumulative discoveries have fallen significantly below their respective mathematical curves. From this figure, it appears that cumulative discoveries may fall short of the 170-billion-barrel asymptote for Q_∞ by as much as 7 to 10 billion barrels. Should this be so, cumulative production will have to do the same.

This is shown even more clearly in the derivative curves of Figures 28 and 29. Figure 28 shows that the annual rate of increase of proved reserves (which has been negative since 1962) has been well below the mathematical curve during the 1970-decade. The mathematical curve shows a minimum of about 0.8 billion barrels per year occurring in 1980 for the rate of decline of proved reserves. During the period 1973-1979 the average rate of decline was close to 1.5 billion barrels per year, indicating that about one-half of the production during that period was obtained by withdrawal from proved reserves.

Figure 29 shows that from 1972 to 1979 even the high points in the oscillatory curve of annual discoveries were below the mathematical derivative curve of the 1972 logistic equation. Figure 30 shows the corresponding comparison for the rate of production. The year 1970 was indeed the year of peak production, with the production rate falling steeply from its maximum of 3.24 billion barrels in 1970 to 2.45 in 1979.

In view of the departures of the curves of cumulative proved discoveries and of proved reserves during the 1970-decade, as is shown in Figure 27, from the logistic curves of 1972, new calculations have been made with the particular objective of obtaining a new estimate for the magnitude of Q_∞. One procedure has been to make new determinations of the constants of the logistic equation using the data from 1900 to 1980. A second procedure was based upon the linear

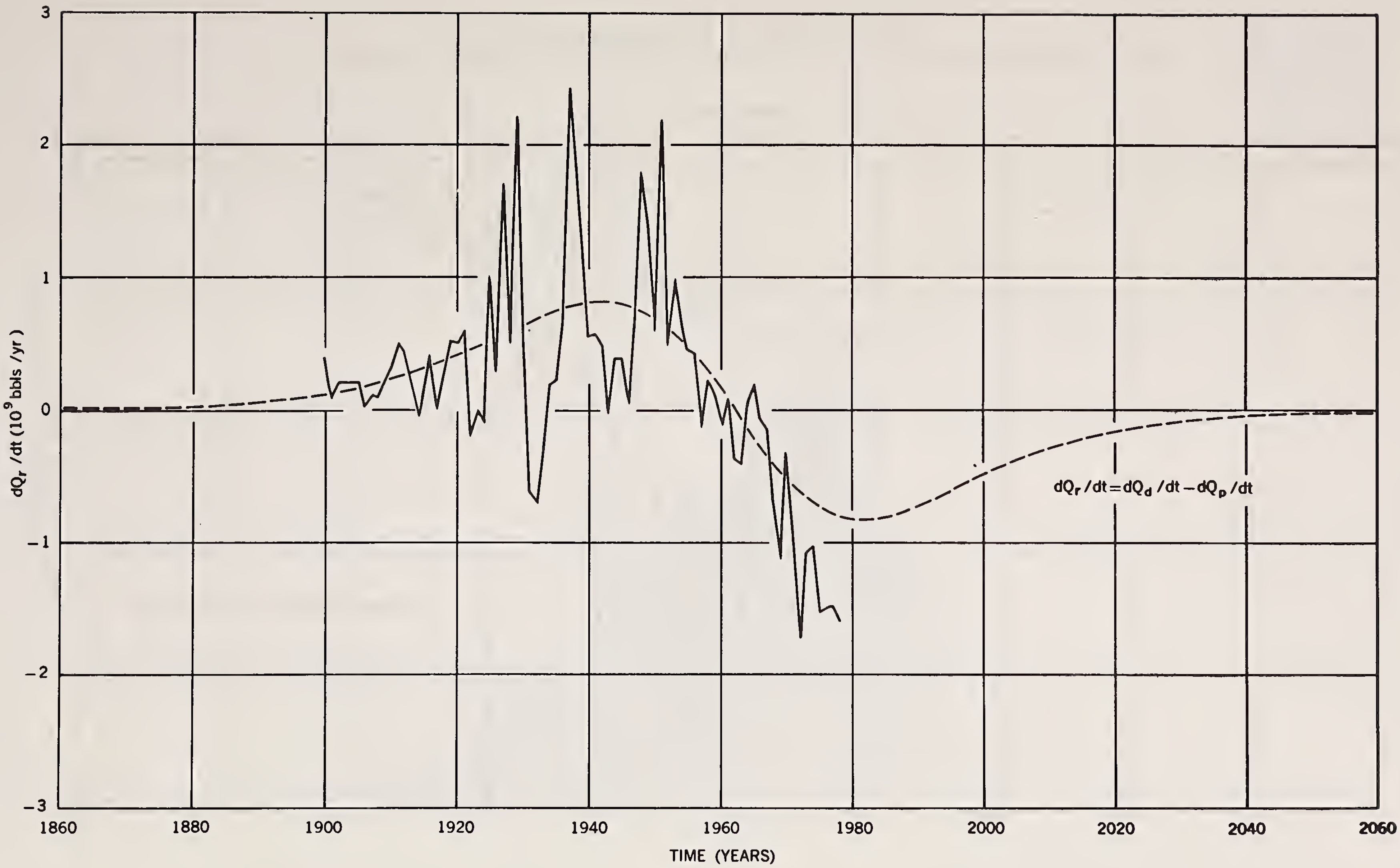

Fig. 28 - Annual increments of U.S. proved reserves of crude oil, 1900-1979, superposed upon theoretical curve of 1972.

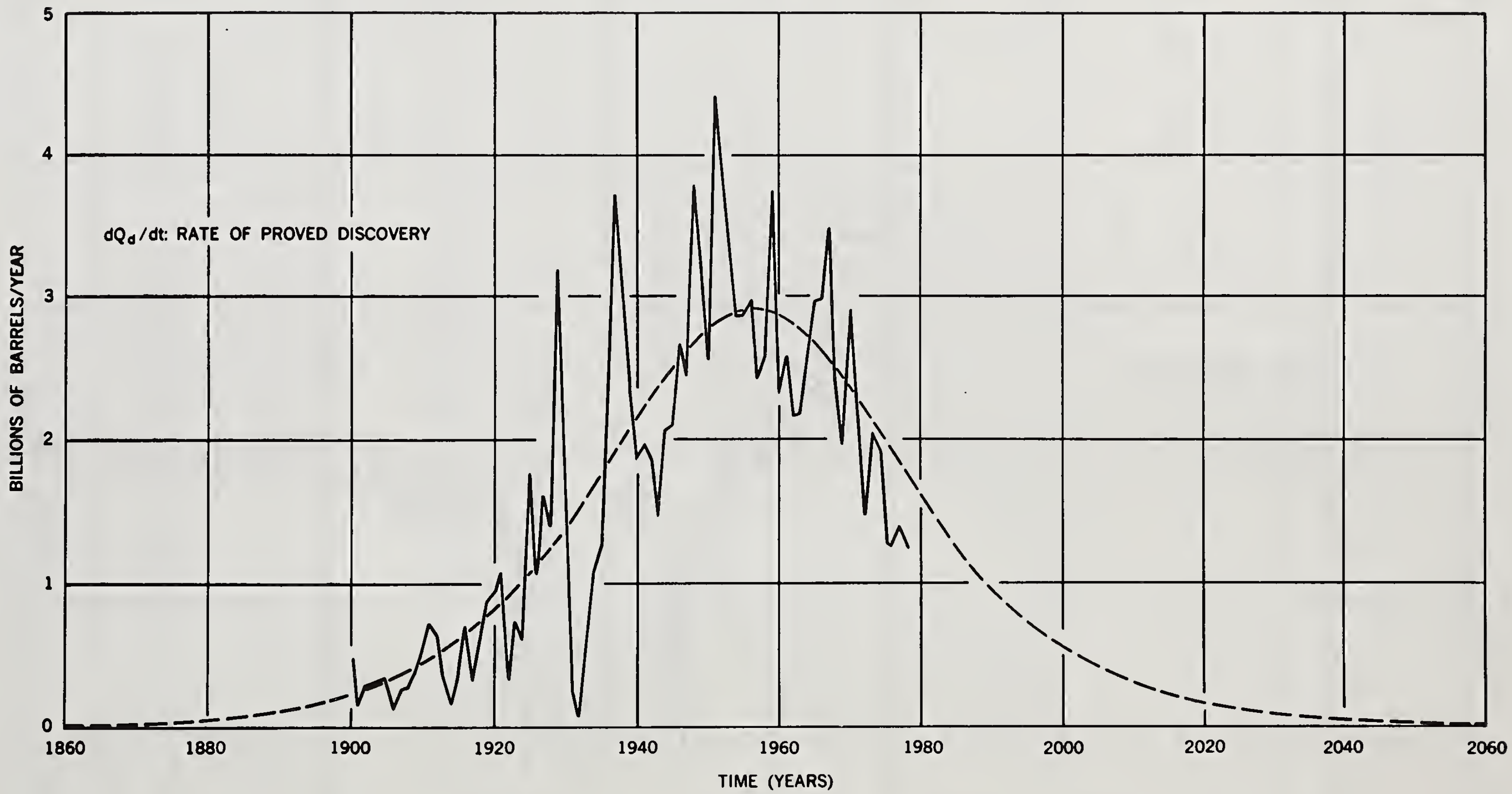

Fig. 29 - Annual proved discoveries of U.S. crude oil superposed upon derivative of logistic equation of 1972.

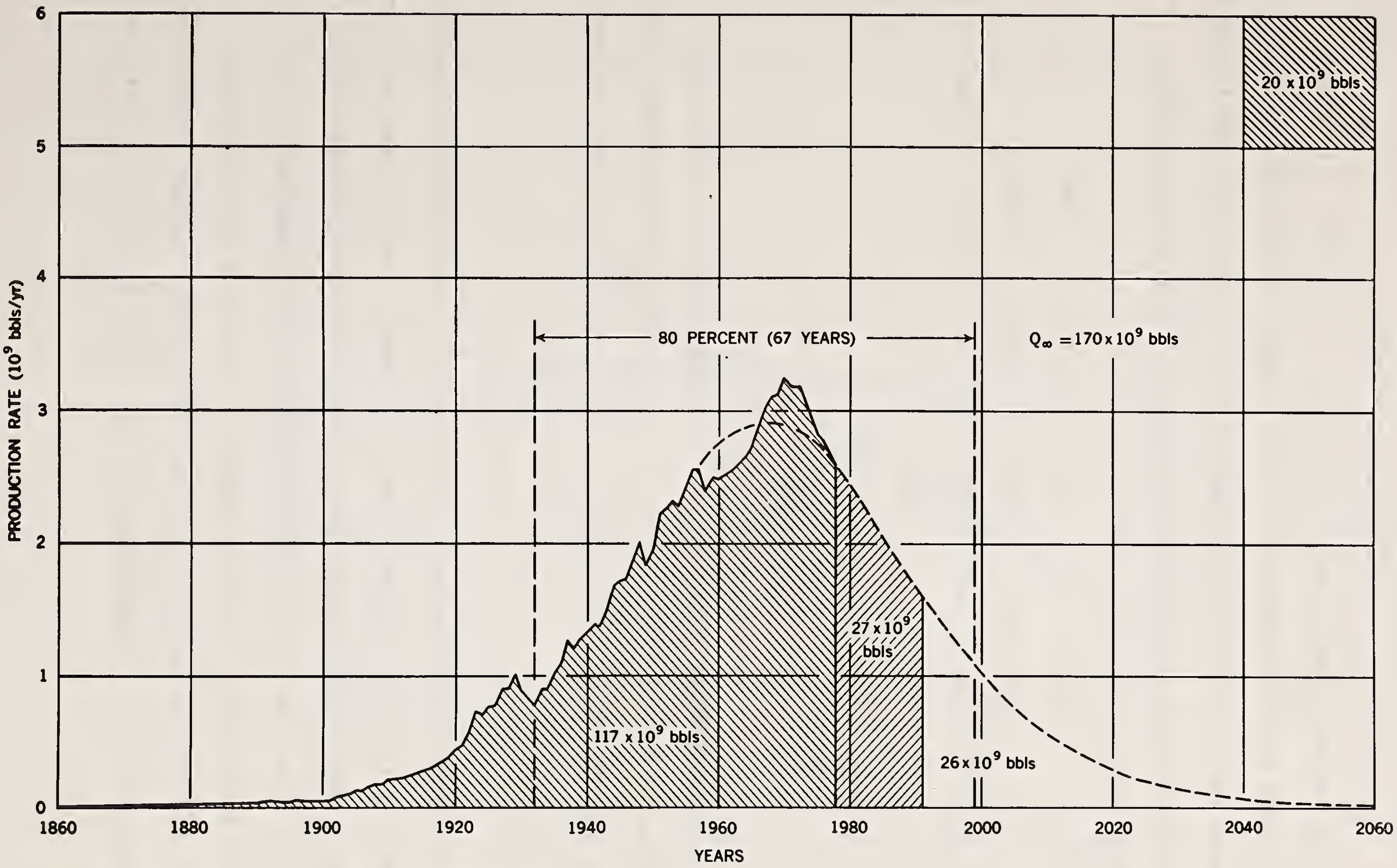

Fig. 30 - Complete cycle of U.S. crude-oil production as estimated in 1980 superposed upon derivative of logistic equation of 1972.

equation (27),

$$(dQ/dt)/Q = a - (a/Q_\infty)Q.$$

Using the method described in equations (43) to (45), the curve of log N_b versus t was constructed at 5-year intervals from 1900 to 1980, using an assumed value of 163.0 billion barrels for Q_b. From this, the following three dates and the corresponding cumulative discoveries were chosen as being points on the smooth almost linear curve:

Dates:	1915	1960	1980
$Q(10^9$ bbl):	8.74	92.61	136.90.

The constants obtained for the logistic equation, as shown in Figure 31, were:

$$t_o = 1915,$$

$$N_o = 17.570,$$

$$Q_\infty = 162.3 \times 10^9 \text{ bbl},$$

$$a = 0.0700/\text{year}.$$

A second calculation was made by the same procedure except that the curve of cumulative discoveries, Q_d versus t, was first smoothed by means of an 11-year running average except for the last 5 years. This gave the results:

$$Q_\infty = 161.8 \times 10^9 \text{ bbl},$$

$$a = 0.699/\text{year}.$$

The results of the estimate based upon the linear graph of $(dQ/dt)/Q$ versus Q are shown in Figure 32. In this case, the curve of cumulative discoveries versus time from 1900 to 1980, except for the last 5 years, was smoothed by an 11-year running average, and dQ/dt, at 5-year intervals, was based upon 5-year averages. For the earlier figures, as was expected, there was a wide scattering of the data points, but for the last 25 years from 1955 to 1980, the data points gave a very good straight line. Extrapolation of this linear trend to the Q-axis and to the

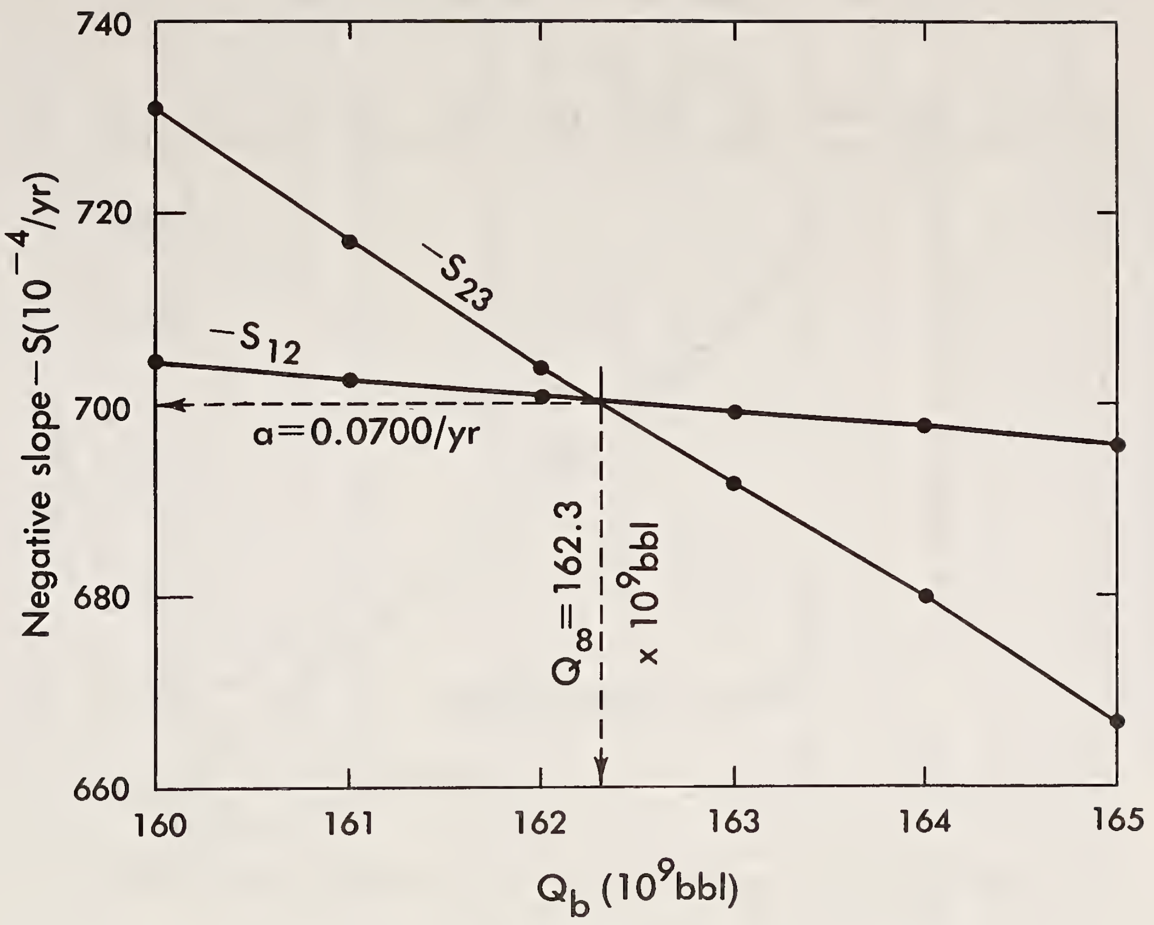

Fig. 31 - Determination of logistic-equation constants for U.S. crude-oil cumulative proved discoveries, 1900-1980.

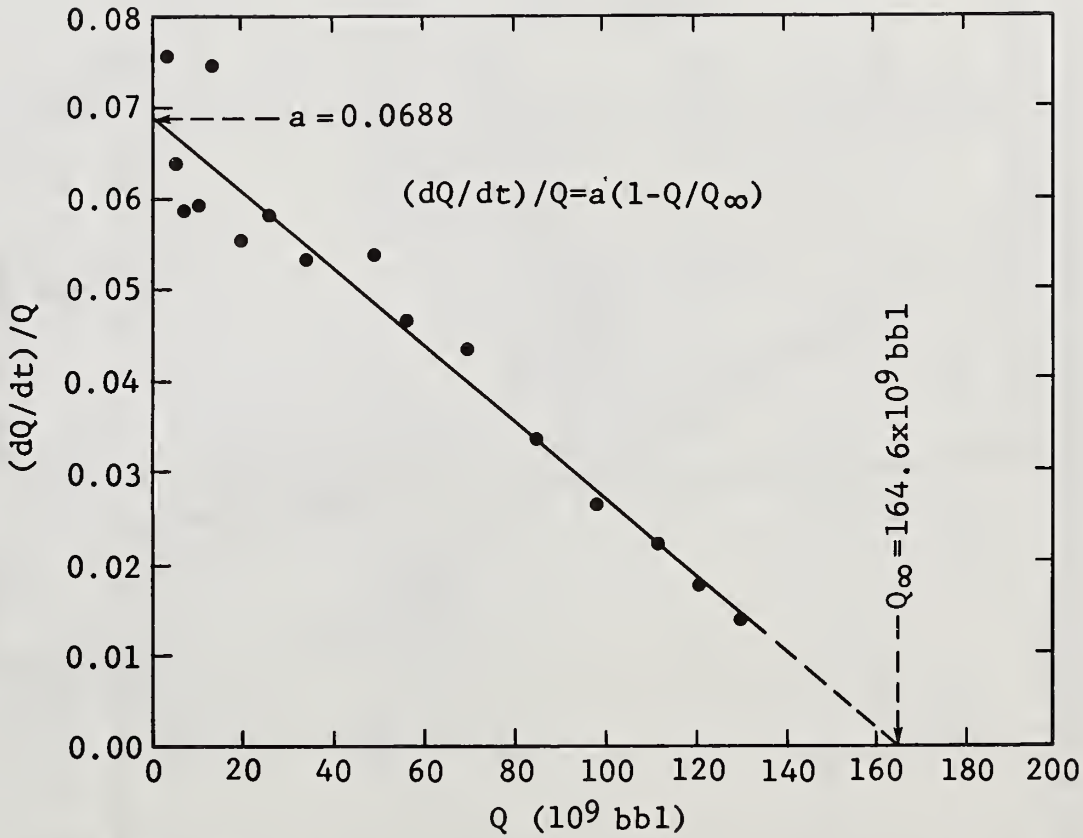

Fig. 32 - Determination of logistic-equation constants for U.S. cumulative proved crude-oil discoveries, 1860-1975, by the technique of $(dQ/dt)/Q$ versus Q.

vertical axis gave the following figures for the logistic constants:

$$Q_\infty = 164.6 \times 10^9 \text{ bbl},$$

$$a = 0.0688/\text{year}.$$

Estimates Based upon Discoveries per Unit Depth of Exploratory Drilling

Development of Theory.— Heretofore we have dealt principally with the variations with time of cumulative proved discoveries and production, proved reserves, and their derivatives with respect to time. One difficulty with variations with respect to time is that such variations are sensitive to economic influences such as fluctuations in prices. A different kind of variation, involving new data not previously used, is represented by discoveries per unit depth of exploratory drilling as a function of cumulative depth of drilling, or of cumulative discoveries. The rate of discovery of oil per unit depth of drilling is determined principally by the geological situation dealt with and by the technology of exploration and production; it is highly insensitive to economic influences.

Let h be the cumulative depth of exploratory drilling in a given region, and Q be the cumulative discoveries. Exploration in the region begins with $h = 0$, and $Q = 0$. Then as h increases without limit, Q tends to a definite finite limit Q_∞. The rate of discovery as a function of h will be dQ/dh.

Because of the indefiniteness of the upper limit of h, but not of Q, it is convenient to consider the variation of dQ/dh as a function of Q, as we have done previously when considering dQ/dt as a function of Q. Thus we consider the variation with Q of dQ/dh within the limits of the complete cycle as Q increases from 0 to Q_∞. During this cycle, when

$$\left.\begin{array}{c} Q = 0,\ dQ/dh > 0; \\ 0 < Q < Q_\infty,\ dQ/dh > 0; \\ Q = Q_\infty,\ dQ/dh = 0. \end{array}\right\} \tag{50}$$

The variation of dQ/dh with Q, can be expressed by the Maclaurin series,

$$dQ/dh = c_o + c_1 Q + c_2 Q^2 + --- + c_n Q^n. \tag{51}$$

The lowest degree and the simplest form of this equation that satisfy the conditions of equations (50) is the first degree,

$$dQ/dh = c_o + c_1 Q. \tag{52}$$

When

$$Q = Q_\infty,\ dQ/dh = 0,$$

and from equation (52),

$$c_o + c_1 Q_\infty = 0,$$

or

$$c_o = -c_1 Q_\infty \tag{53}$$

Substituting this into equation (52) then gives

$$dQ/dh = -c_1 (Q_\infty - Q),$$

and letting β be substituted for $-c_1$, we obtain

$$dQ/dh = \beta(Q_\infty - Q). \tag{54}$$

Thus dQ/dh varies linearly with respect to Q, as shown in Figure 33, having its intercept on the vertical axis, when $Q = 0$, at βQ_∞; and on the horizontal axis, when $dQ/dh = 0$, at Q_∞. The slope of this line is $-\beta$.

To obtain dQ/dh as a function of h we separate the variables and integrate,

$$\int \frac{dQ}{Q_\infty - Q} = \int \beta dh + c, \tag{55}$$

or

$$\ln(Q_\infty - Q) = -\beta h - c. \tag{56}$$

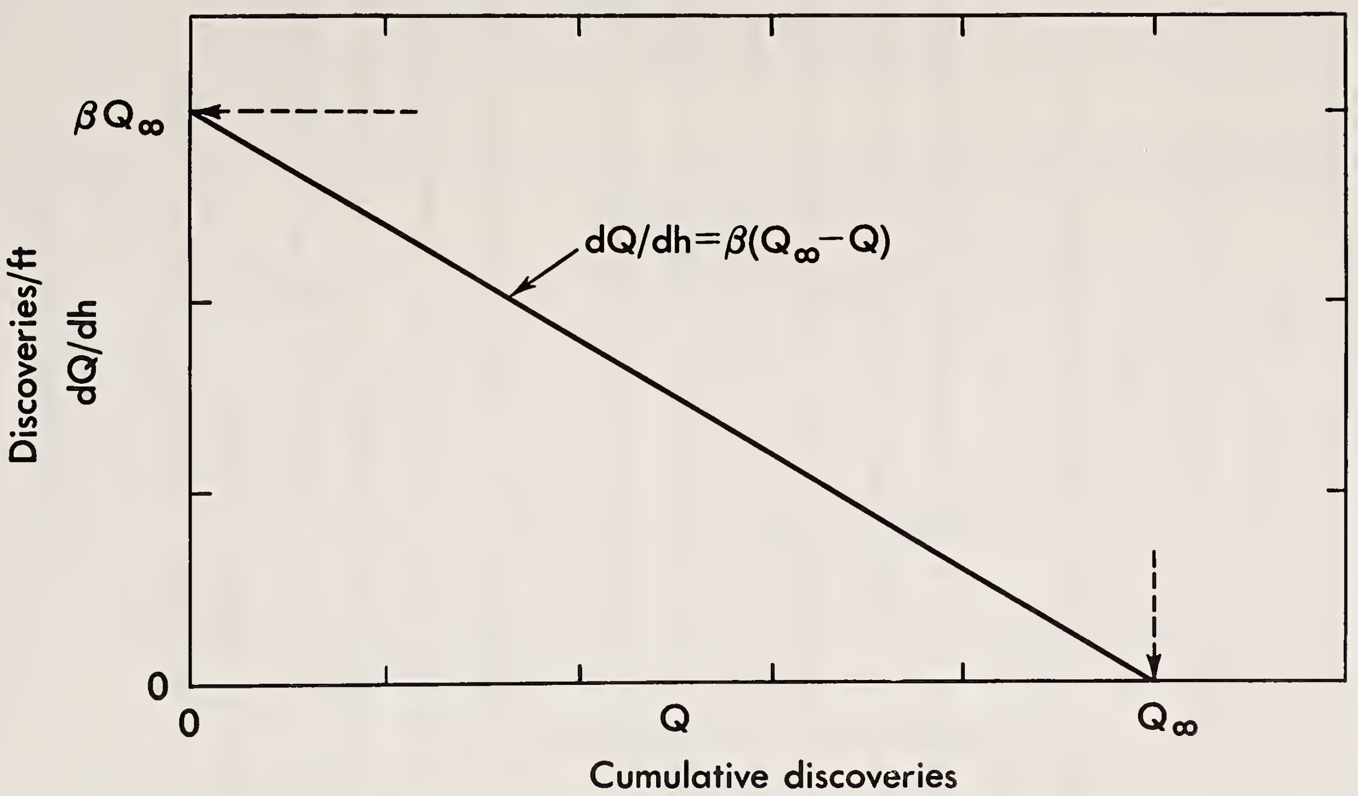

Fig. 33 - Linear relation between oil discoveries per unit depth of exploratory drilling and cumulative discoveries.

Then, when $Q = 0$, $h = 0$, and

$$-c = \ln Q_\infty, \tag{57}$$

Equation (56) then becomes

$$\ln(Q_\infty - Q) = \ln Q_\infty - \beta h \tag{58}$$

or

$$Q = Q_\infty(1 - e^{-\beta h}). \tag{59}$$

Differentiating equation (59) with respect to h then gives

$$dQ/dh = \beta Q_\infty e^{-\beta h}. \tag{60}$$

The manner of variation of both Q and dQ/dh as functions of h are shown graphically in Figure 34.

The foregoing results, as has been shown by Arps and Roberts (1958), Arps, Mortada, and Smith (1971), Menard and Sharman (1975), Root and Drew (1979), and Drew, Schuenemeyer, and Root (1980), are also consistent with the expectations of probability theory. The probability of the discovery of a given amount of oil by a fixed amount of exploratory drilling in a given area is roughly proportional to the undiscovered oil in the region,

$$Q_u = Q_\infty - Q.$$

Consider the idealized case of a region of exploration of total area S which contains fields of uniform oil content ΔQ occurring at the constant depth z. Let A_∞, which is small compared with S, be the total area of such fields, and let A be the area of the fields already discovered by cumulative depth of exploratory drilling h. Then

$$dQ/dh = (\Delta Q/z)[(A_\infty - A)/S]. \tag{61}$$

Then, if

$$\delta = Q/A \tag{62}$$

is the oil per unit area in the fields,

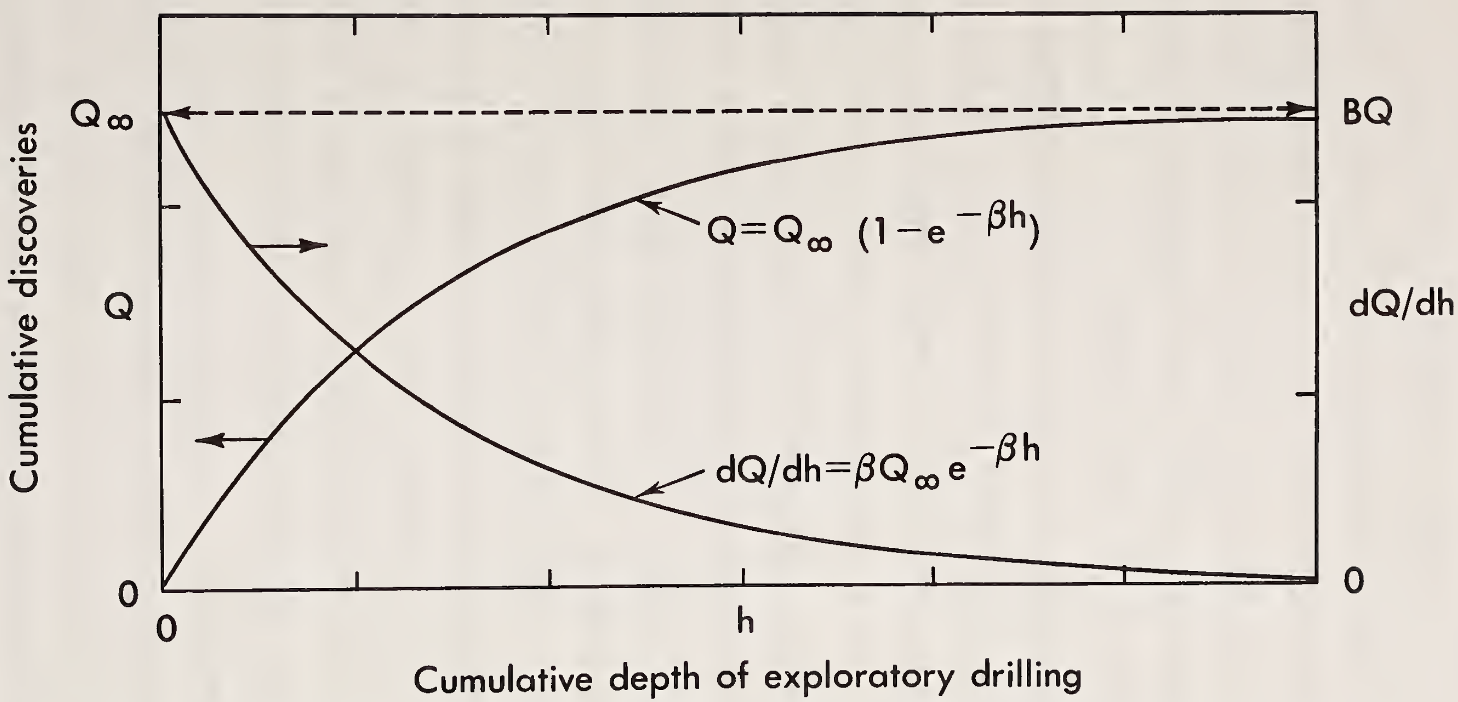

Fig. 34 - Variations of cumulative discoveries and of discoveries per unit depth of exploratory drilling as functions of cumulative exploratory drilling.

$$(A_\infty - A) = (Q_\infty - Q)/\delta, \tag{63}$$

which, when substituted into equation (61), gives

$$dQ/dh = (\Delta Q/zS\delta)(Q_\infty - Q). \tag{64}$$

This is of the same form as equation (54), where $\Delta Q/zS\delta$ in equation (64) corresponds to β in equation (54). Integration of equation (64) then gives for Q versus h an equation of the form of equation (59), and differentiation of that with respect to h gives an exponential decline of dQ/dh versus h of the same form as equation (60).

In an actual oil-bearing region these simplified conditions do not occur. There are commonly a small number of large fields which frequently contain most of the oil, and a large number of small fields. Also the depths of the fields range from a few hundred feet to as much as 20,000 feet. In addition, in a region such as the entire United States, the technologies of discovery and of production undergo progressive improvement during the entire cycle of oil exploitation. This favors the discovery of the larger and shallower fields during the earlier stages of the cycle with a rapid rate of decline of dQ/dh versus h, followed by a slower rate of decline as the sizes of the remaining fields decrease and their depths increase. Offsetting this somewhat is the steady improvement of the techniques of exploration and production which tends to increase the discoveries per foot with respect to the rates expectable by random drilling. The net effect, however, is still roughly an exponential decline of dQ/dh versus h.

In order to correlate the oil discoveries made in a given region with the corresponding exploratory drilling, a different definition of the term "discoveries" from that of "proved discoveries" used previously is required. In this case all of the oil that the fields discovered in a given year will ultimately produce must be credited to the exploratory drilling done during that year.

This involves estimates of "ultimate recovery" of oil from fields discovered in a given year t_i as estimated at a later year t_j. The ultimate recovery from fields discovered during the year t_i as estimated at the later year t_j is defined as the sum of cumulative production from those fields to the year of the estimate plus their proved reserves at the time of the estimate. At the end of the year of discovery the only oil credited to the discovery year is the item "New Field Discoveries," in the American Petroleum Institute annual report on proved reserves. This ordinarily is only a small quantity. During succeeding years, cumulative production from those fields steadily increases, and the sum of cumulative production plus proved reserves gradually approaches asymptotically the quantity δQ_∞, which is the true ultimate amount of oil those fields will produce.

The first study of this kind for the U.S. crude-oil production was that made during World War II by the Petroleum Administration for War (PAW) (Frey and Ide, 1946, Appendix 12, Table 10, p. 442; C.L. Moore, 1962, Table IV, p. 94). In effect, what was done in the PAW study was to combine the cumulative production to January 1, 1945, with the American Petroleum Institute estimate of proved reserves for the same date for all the fields discovered during each successive year from 1860 to 1944. This gave an estimate as of January 1, 1945, of the proved oil discoveries assignable to fields by their year of discovery. Two more such studies were made subsequently by the National Petroleum Council (1961; 1965). The first brought the PAW study up to the date of December 31, 1959, and the second to January 1, 1964. Each of the latter studies gave a lumped increase for all the discoveries made from 1860 to 1920, and then gave separate estimates for the fields discovered during each year from 1920 to the terminal date of the study.

In 1966, the name of the API "Committee on Petroleum Reserves" was changed to "Committee on Reserves and Productive Capacity" and the scope of its activities was expanded. One new item in the committee's annual reports was a table for the year of the report of estimations of ultimate recovery for fields discovered during the pre-1920 period and for individual years from 1920 to the year of the report. These reports are given from 1966 to 1979 in the annual publication, *Reserves of Crude Oil, Natural Gas Liquids, and Natural Gas in the United States and Canada as of December 31,* [given year], issued jointly by the American Petroleum Institute, the American Gas Association, and the Canadian Petroleum Association, for brevity, the API, AGA, CPA "Blue Books."

Using these data, the problem is to estimate the ultimate amount of oil that fields discovered during successive years will eventually produce. Two principal alternative procedures have been developed for this purpose. One, that developed by the present author (Hubbert, 1967; 1974), consists in following the fields discovered in a given year or group of successive years, and plotting their growth with increasing time following their year of discovery. If we let $(\delta Q)_1$ be the initial estimate of the new oil discovered at the end of the year of discovery, and $(\delta Q)_\tau$ be the estimate for the same fields τ years later, then we can plot a curve of

$$y_\tau = (\delta Q)_\tau / (\delta Q)_1 \tag{65}$$

as a function of τ.

Expressed in this manner, y_τ is dimensionless, and is independent of the absolute magnitude of the oil discoveries in any given year, and the time-delay τ is common to the discoveries of all years. Hence the data for all discovery years can be expressed in terms of y versus τ. This will be a curve that rises steeply initially and finally approaches the limit,

$$y \to y_\infty,$$

as τ increases

This was the procedure used in 1967 (Hubbert, 1967, Figs. 4 and 5) using the limited data then available, and again in 1972 (Hubbert, 1974, Figs. 45 and 46) with much more detailed information. In both instances, however, substantially the same growth curves were obtained, which were fitted by empirical equations of the form,

$$y_\tau = y_\infty[1 - e^{-\gamma(\tau+c)}]. \tag{66}$$

The data as of 1972 are shown graphically in Figure 35, which is reproduced from my report of 1974 (Hubbert, 1974, Fig. 45). From these the constants for equation (66) were:

$$y_\infty = 5.8,$$

$$\gamma = 0.076 \text{ per year},$$

$$c = 1.503 \text{ years}.$$

By means of this equation it is possible to estimate how much more the oil discovered in a given year will increase when its magnitude $(\delta Q)_\tau$ is given after τ years of production and development.

This is done by a correction factor α, defined by

$$\alpha = y_\infty/y_\tau, \tag{67}$$

which from equation (66) is

$$\alpha = 1/[1 - e^{-\gamma(\tau+c)}]. \tag{68}$$

Then, since

$$y_\tau = (\delta Q)_\tau/(\delta Q)_1,$$

and

$$y_\infty = (\delta Q)_\infty/(\delta Q)_1,$$

$$y_\infty/y_\tau = (\delta Q)_\infty/(\delta Q)_\tau.$$

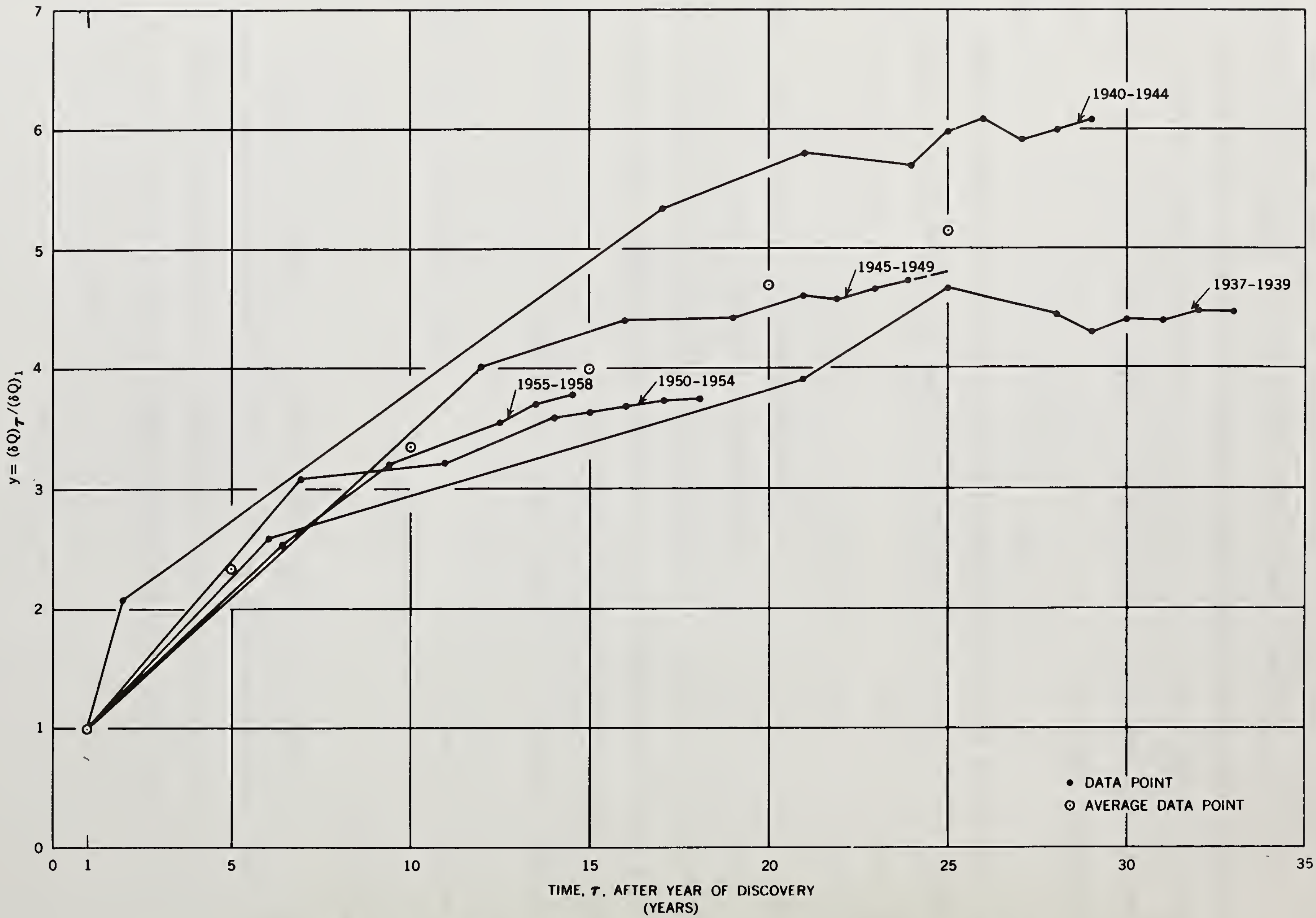

Fig. 35 - Growth in the estimates of the ultimate recovery of crude oil from fields discovered in given years as re-evaluated at successively later years (Hubbert, 1974, Fig. 45).

Hence

$$(\delta Q)_\infty = \alpha(\delta Q)_\tau . \tag{69}$$

This first procedure is based upon the growth during successive years, t_j, of the estimates of oil discovered in a given year, t_i. An alternative procedure is that described originally by Arrington (1960; 1966) and more recently by Marsh (1971). In this case, use is made of the API estimates for the two successive most recent years, t_j and t_{j+1}, for the oil discovered in previous years, t_i. In this case the fields considered are no longer the *same* fields, but instead are the different fields discovered during successive earlier years with an increasing value of the time-delay τ.

In this manner a ratio,

$$r_\tau = (\delta Q)_{\tau+1}/(\delta Q)_\tau , \tag{70}$$

is obtained as a function of τ for the oil discovered during each preceding earlier year. From these successive ratios,

$$r_1, r_2, r_3 \dots r_\tau ,$$

the growth factor y_τ is obtained by the product

$$y_\tau = (r_1 r_2 r_3 \dots r_\tau), \tag{71}$$

which tends to y_∞ as τ increases without limit.

As in the earlier procedure, the magnitude of $(\delta Q)_\infty$ for the discoveries made during any given year is given by

$$(\delta Q)_\infty = \alpha(\delta Q)_\tau ,$$

where

$$\alpha = y_\infty / y_\tau .$$

In practice, instead of dealing with the discoveries made during single years, the sum of the discoveries made during a sequence of 5 or so years may be used. In that case, for a common value of τ, the ratios must also be taken during a corresponding sequence of successive pairs of years.

Three Successive Studies of U.S. Crude-Oil Discoveries per Foot of Exploratory Drilling

The results of three successive studies of the U.S. crude-oil discoveries in the Lower-48 states and adjacent continental shelves, as a function of cumulative depth of exploratory drilling, are shown in Figures 36, 37, and 38. The first of these (Hubbert, 1967, Fig. 15) shows the discoveries made by the first 1.5×10^9 ft of drilling, which encompassed the period 1860 to 1966.7. The second (Hubbert, 1974, Fig. 49) gives the results obtained by 1.7×10^9 ft of exploratory drilling during the period from 1860 to just short of 1972. The third, by David H. Root of the U.S. Geological Survey (Root, 1980), gives the discoveries made by the 2.0×10^9 ft of exploratory drilling during the period 1860 to 1977.9. In the first two studies, the method used in estimating the amount of oil ultimately recoverable from fields already discovered was that of the present author, as described earlier. In the third study, Root used his own modification of the method of Arrington and Marsh, which was applied to the API, AGA, CPA "Blue Book" data from 1966 to 1978. For each successive study more and better data were available than for the one preceding.

For the graphical presentation of the data, a convenient unit for Δh is 10^8 ft. Hence, in the three figures, the cumulative drilling amounted to 15, 17, and 20 units respectively. For each unit of drilling, a vertical column is shown representing the quantity of oil discovered by that unit. The lower part of the column represents the proved cumulative discoveries as estimated at the date of the study. The shaded area at the top of each column represents the additional oil those fields are expected to produce as determined by the α-correction. The total amount of recoverable oil discovered by the first 15 units of drilling in the 1967 study was estimated to be 136.04 billion barrels, of which approximately 25 billion barrels were accounted for by the α-correction. In the

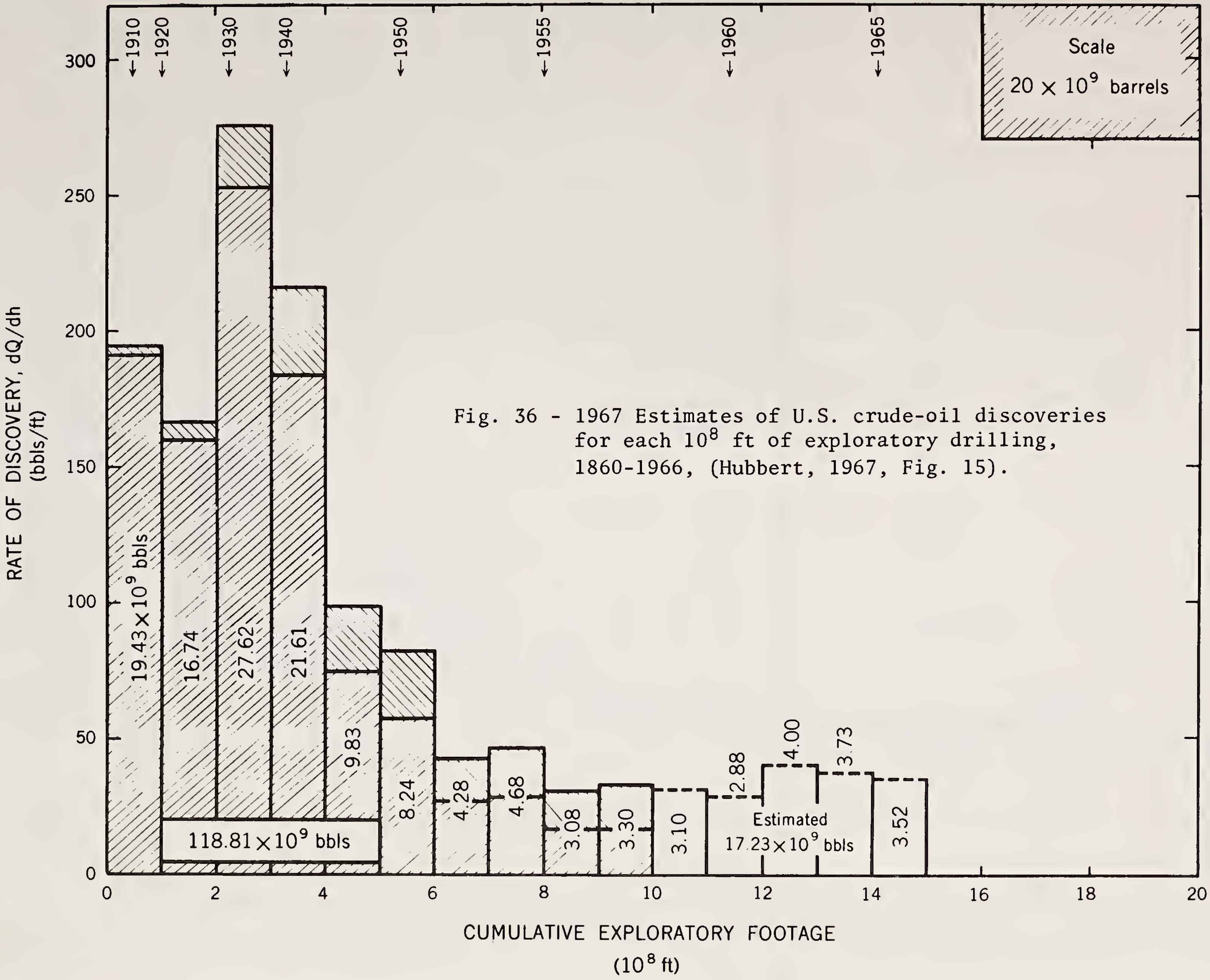

Fig. 36 - 1967 Estimates of U.S. crude-oil discoveries for each 10^8 ft of exploratory drilling, 1860-1966, (Hubbert, 1967, Fig. 15).

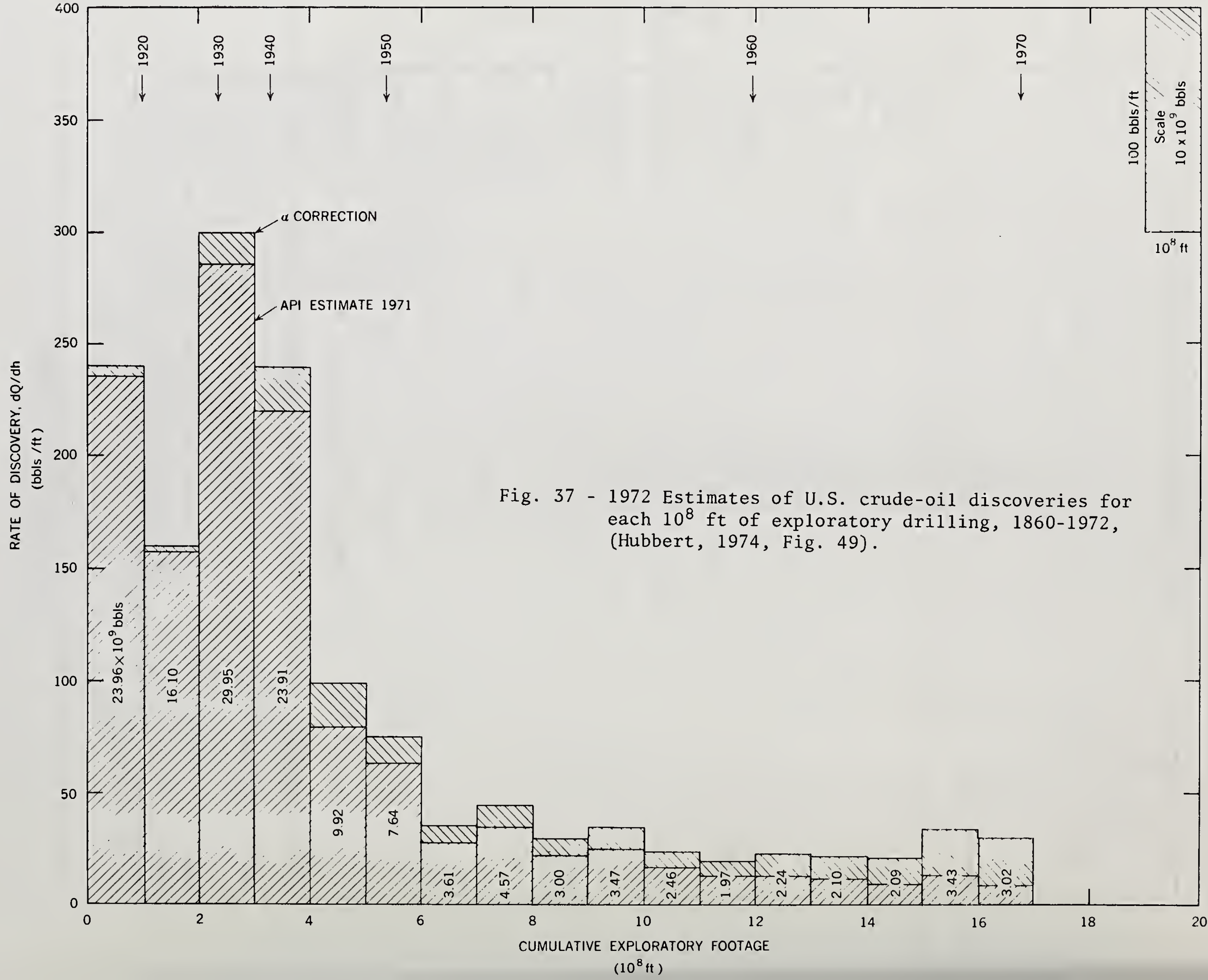

Fig. 37 - 1972 Estimates of U.S. crude-oil discoveries for each 10^8 ft of exploratory drilling, 1860-1972, (Hubbert, 1974, Fig. 49).

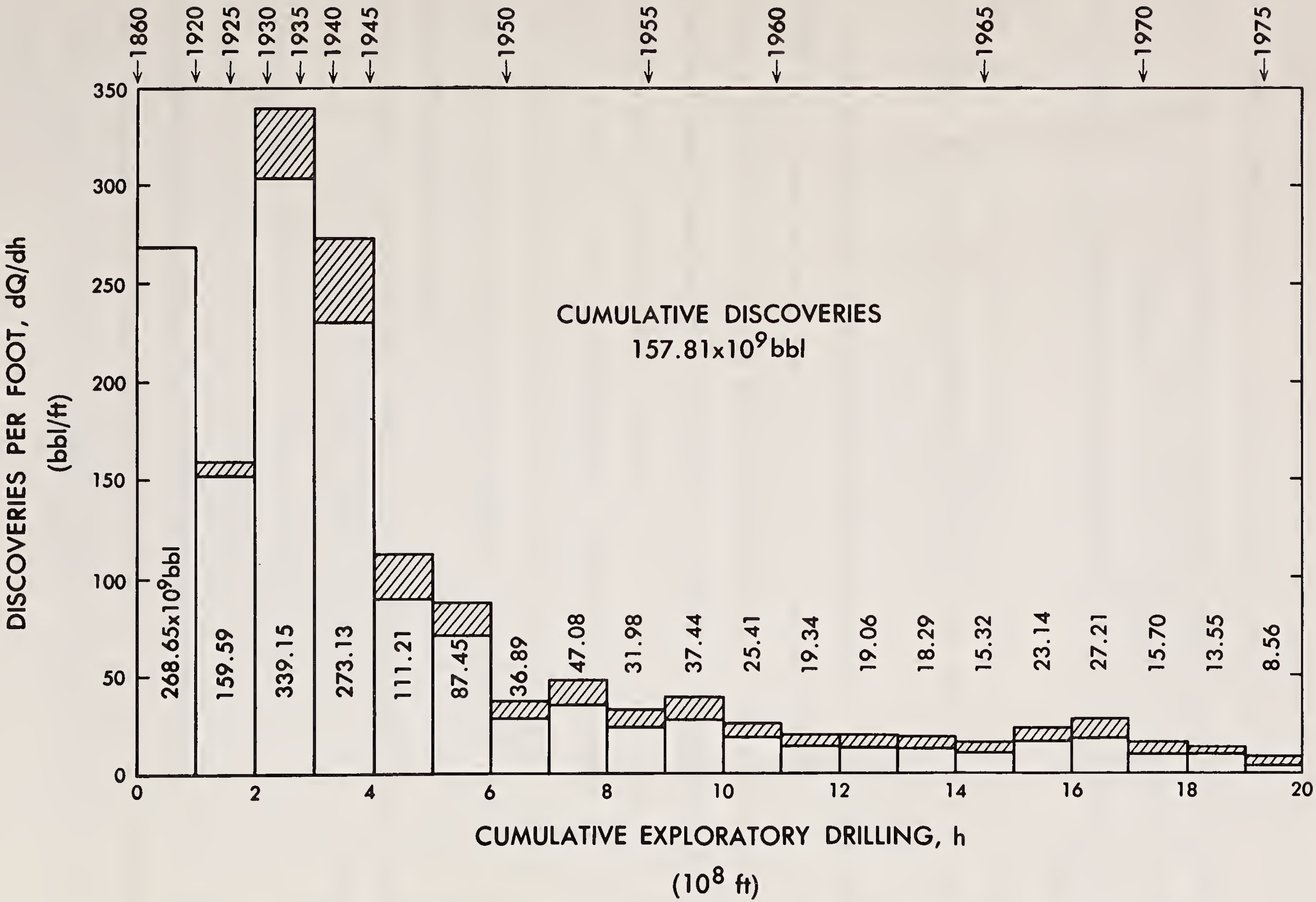

Fig. 38 - 1980 Estimates by David H. Root of U.S. crude-oil discoveries for each 10^8 ft of exploratory drilling, 1860-1977.9, (Root, 1980).

1974 study, 143.44 billion barrels of recoverable oil were estimated to have been discovered by the first 17 units of drilling. Of this, approximately 19 billion barrels were contributed by the α-correction. The total recoverable oil estimated in the third study as having been discovered by the first 20 units of drilling amounted to 157.87 billion barrels, of which 22.92 were due to the α-correction.

Because the results shown in Figures 36, 37, and 38 are all derived from different suites of data, and, in the case of the last study, by a different method of analysis, it is not to be expected that the results obtained would be in close agreement. Nevertheless all of the studies give results of strong similarity. All show high rates of discovery, averaging between 20 and 30 billion barrels per 10^8-ft unit of drilling (200 to 300 bbl/ft) for the first 4 units, followed by a precipitous decline. In Figure 36, the discovery rate had declined to only 3.52 (10^9 bbl/10^8 ft), or 35.2 bbl/ft, for the last or 15th unit; in Figure 37 this decline had reached 30.2 bbl/ft for the 17th unit; and in Figure 38, by the 20th unit, it had declined still further to but 8.56 bbl/ft.

All of these figures show a roughly negative-exponential decline in dQ/dh versus h during the entire cycle. In order to estimate the future, the negative-exponential curve best fitting the data needs to be determined in each instance. To simplify notation, let dQ/dh be represented by R.

Then

$$R = R_o e^{-\beta h} \tag{72}$$

will be the desired equation of which the two parameters R_o and β are to be determined from the data. The criterion for best fit will be

$$\int_o^{h_n} R dh = Q_n, \tag{73}$$

where Q_n represents the total discoveries made by the h_n units of drilling, with the curve passing through the last point (R_n, h_n) on the graph. In other words, we wish to determine the negative-exponential curve that equalizes the excesses and defects of the data, and passes through the last point.

Substituting the value of R from equation (72) into equation (73), we obtain

$$Q_n = R_0 \int_0^{h_n} e^{-\beta h} dh$$

$$= (R_0 - R_n)/\beta,$$

from which

$$\beta = (R_0 - R_n)/Q_n. \tag{74}$$

Also, taking the logarithm of equation (72), with $R = R_n$ and $h = h_n$, we obtain

$$\ln (R_0/R_n) = \beta h_n$$

or

$$\beta = \ln (R_0/R_n)/h_n. \tag{75}$$

Dividing equation (75) by (74) then gives

$$\frac{\ln (R_0/R_n)}{R_0 - R_n} \cdot \frac{Q_n}{h_n} = 1, \tag{76}$$

of which the only unknown is R_0. This can be solved for R_0 by an iteration procedure of substituting for R_0 an assumed value R_b whereby the left-hand term of equation (76) becomes $f(R_b)$. When $f(R_b)$ has the value of 1, $R = R_0$. The decline parameter β is then obtained from either of equations (74) or (75).

Once R_0 and β are known, the estimate of the ultimate cumulative discoveries, Q_∞, is given by

$$Q_\infty = R_0/\beta, \tag{77}$$

and that of the undiscovered oil, Q_u, by

$$Q_u = R_n/\beta. \tag{78}$$

The negative-exponential curves obtained in this manner from the data of Figures 36, 37, and 38 are shown in Figures 39, 40, and 41. For the study of 1967 (Fig. 36), the values of the parameters are:

$$R_0 = 18.63 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$= 186.3 \text{ bbl/ft},$$

$$\beta = 0.1111 \text{ per } 10^8 \text{ ft}.$$

From these parameters and the numerical data of Figure 36,

$$R_{15} = 3.52 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$Q_{15} = 136.04 \times 10^9 \text{ bbl},$$

$$Q_u = 31.7 \times 10^9 \text{ bbl},$$

and

$$Q_\infty = 167.7 \times 10^9 \text{ bbl}.$$

This value of 168 billion barrels for Q_∞ obtained from the 1967 study of discoveries per unit depth of exploratory drilling as a function of cumulative drilling, although based upon different data and a totally different method of analysis, is in very close agreement with the figure of 170 billion barrels obtained in the studies of cumulative production, proved reserves, and cumulative proved discoveries made in 1962 and 1974.

For the negative-exponential curve for the 1974 study of dQ/dh versus h shown in Figure 40, the values of the two parameters are:

$$R_0 = 18.154 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$= 181.54 \text{ bbl/ft},$$

$$\beta = 0.1055 \text{ per } 10^8 \text{ ft},$$

and

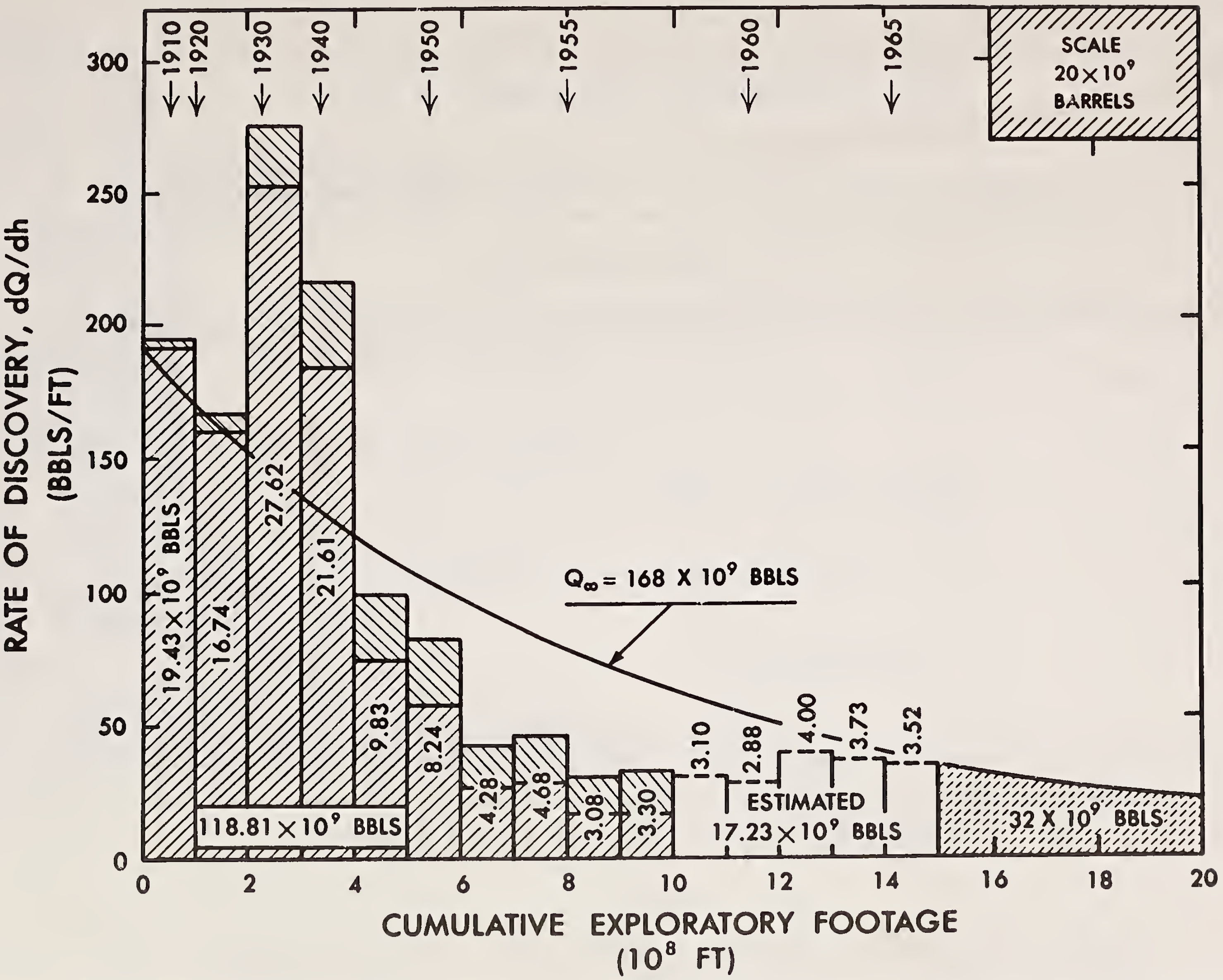

Fig. 39 - Exponential-decline curve superposed upon Fig. 36 giving 1967 estimates of future and ultimate crude-oil discoveries for the U.S. Lower-48 states.

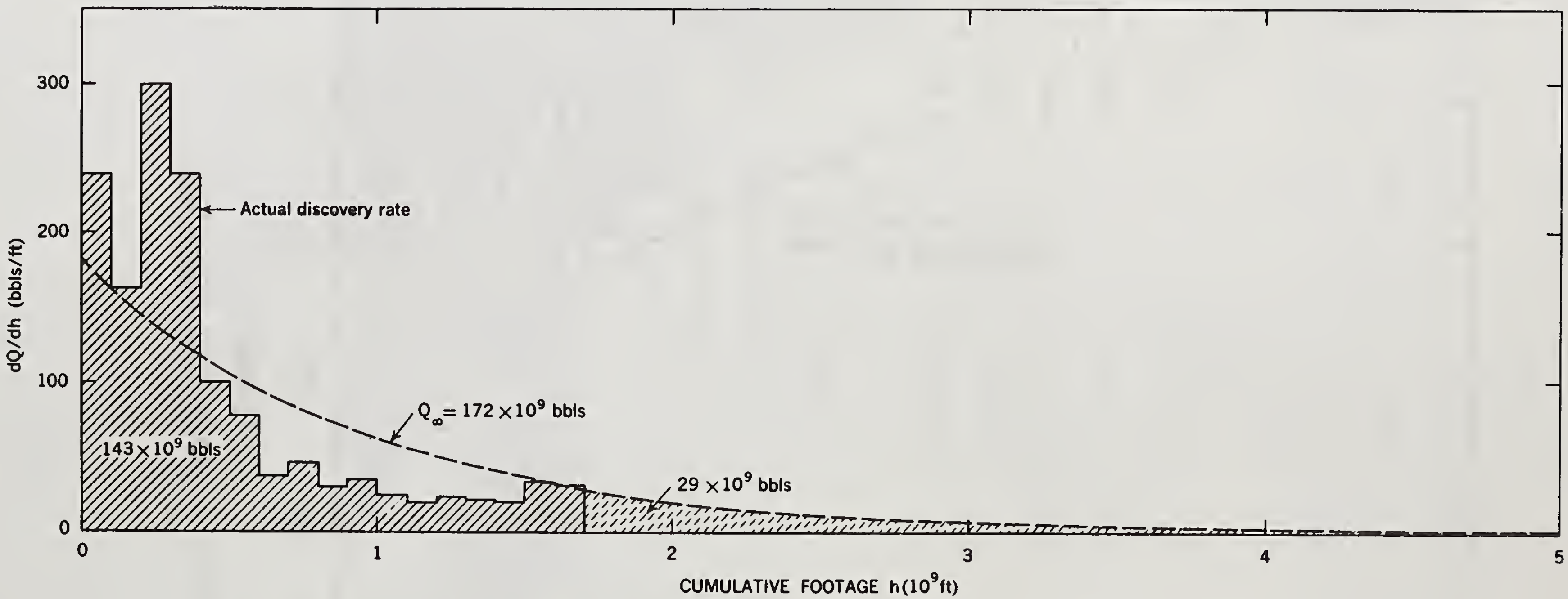

Fig. 40 - Exponential-decline curve superposed upon Fig. 37 giving 1972 estimates of future and ultimate crude-oil discoveries for the U.S. Lower-48 states (Hubbert, 1974, Fig. 50).

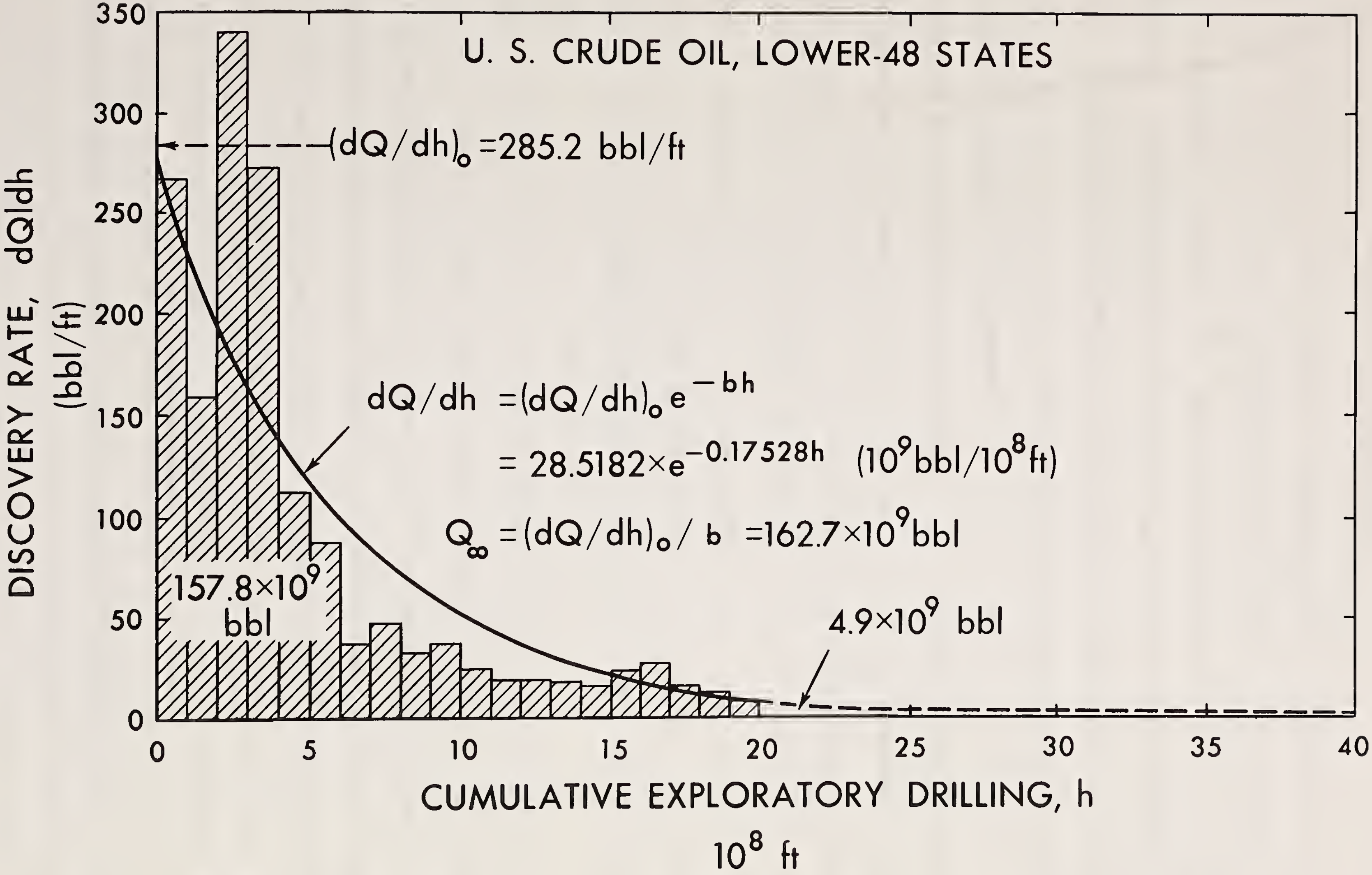

Fig. 41 - Exponential-decline curve superposed upon Fig. 38 giving 1980 estimates of future and ultimate crude-oil discoveries for the U.S. Lower-48 states.

$$R_{17} = 3.02 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$Q_{17} = 143.44 \times 10^9 \text{ bbl},$$

$$Q_u = 28.62 \times 10^9 \text{ bbl},$$

$$Q_\infty = 172.06 \times 10^9 \text{ bbl}.$$

Again, this is in very close agreement with the estimate for Q_∞ as of 1972 of 170×10^9 bbl obtained by the analysis of cumulative production, proved reserves, and cumulative proved discoveries.

The negative-exponential curve corresponding to the data of Figure 38 is shown in Figure 41. This is of especial interest because data extending to 1978 are included, and a still different method of analysis was used. The significant data of Figure 38 are:

$$R_0 = 28.518 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$= 285.18 \text{ bbl/ft},$$

$$\beta = 0.1753 \text{ per } 10^8 \text{ ft},$$

$$R_{20} = 0.8564 \ (10^9 \text{ bbl}/10^8 \text{ ft}),$$

$$Q_{20} = 157.81 \times 10^9 \text{ bbl},$$

$$Q_u = 4.89 \times 10^9 \text{ bbl},$$

$$Q_\infty = 162.70 \times 10^9 \text{ bbl}.$$

Again, this value of 162.7 billion barrels for Q_∞, as determined from data extending to 1978, is in very close agreement with that of 162.3 billion barrels shown in Figure 31, obtained from the logistic constants of the curve of cumulative proved discoveries to 1978. It also differs by only 1.9 billion barrels from the figure of 164.6 shown in Figure 32, based upon the linear-decline curve of $(dQ/dt)/Q$ versus Q. The average of these three figures is 163.2 billion barrels, with a range of uncertainty of only about plus or minus two billion barrels.

Estimation of Natural Gas

Estimations of the ultimate amount of natural gas to be produced in the Lower-48 states, and of the future rates of production, are more difficult than the corresponding estimates for crude oil. This is because the statistics of natural gas are less complete than those for oil until after World War II. Prior to that time a large amount of gas was burned (or "flared") in the fields as gas was produced as a by-product of oil and in excess of the pipeline capacity for collection and distribution.

Since World War II, this situation has greatly improved. Large pipelines were constructed for the transmission of gas from the producing areas to the industrial regions of the northeast and north-central United States and to the Pacific coast. Also, in the mid-1940s, the American Gas Association established its Committee on Natural Gas Reserves. From 1946 to 1979, this committee has issued annual reports on natural-gas proved reserves and production, in parallel with those of the corresponding committee on crude oil of the American Petroleum Institute.

Another serious difficulty is that the record of cumulative production, proved reserves, and cumulative discoveries of natural gas is much more irregular than the corresponding record for oil, which makes mathematical analysis of the data more difficult and of a lower level of reliability. Nevertheless, enough information exists to permit reasonably good estimates to be made of the approximate cumulative production and of future production rates.

Earlier Estimates

Estimate of 1956. —— In my 1956 paper, "Nuclear Energy and the Fossil Fuels" (Hubbert, 1956), the same technique was used for estimating the complete cycle of U.S. natural-gas production as was used for crude oil. The best current estimate for the ultimate amount of natural gas to be produced in the Lower-48 states and adjacent offshore areas was about 850 trillion cubic feet. Cumulative production by the end of 1955 amounted to about 150 and proved reserves to 224 trillion cubic feet. This gave the figure of 374 trillion cubic feet for cumulative proved discoveries, leaving 476 trillion cubic feet for future discoveries.

This is shown graphically in Figure 42, which is reproduced from Figure 22 of the 1956 paper. From this it was estimated that the maximum rate of gas production of about 14 trillion cubic feet per year would occur about 1970. As in the case of crude-oil estimates, the published estimates for the ultimate amount of natural gas to be produced began to escalate immediately after 1956 and, by 1961, the highest estimate had reached 2,630 trillion cubic feet, a figure three times that of 1956.

<u>National Academy Report of 1962.</u> — In view of the lack of agreement as to the approximate magnitude for Q_∞, it became necessary in the National Academy of Sciences report of 1962 to devise a new method of estimation. The statistical data on cumulative production, proved reserves, and cumulative discoveries, which had been available only since 1945, were insufficient for an estimate of Q_∞. To obtain this figure, the parallel study for crude oil was used in conjunction with the ratio of the discoveries of natural gas to those of crude oil during a given period of time. Thus,

$$Q_\infty \text{ gas} = Q_d \text{ gas} + G[(Q_\infty - Q_d)\text{oil}], \tag{79}$$

where G is the gas/oil-ratio.

At the end of 1961, cumulative proved discoveries of natural gas amounted to 474 trillion cubic feet. Cumulative proved discoveries for crude oil were 99.1 billion barrels, and the estimate for Q_∞ for crude oil was taken as 175 billion barrels, leaving 75.9 billion barrels for the undiscovered crude oil.

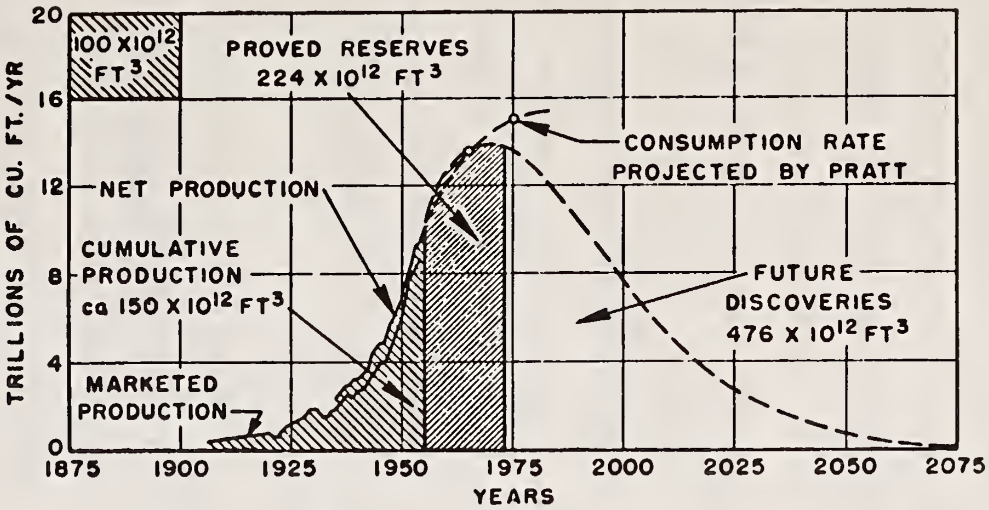

Fig. 42 - Complete cycle of natural-gas production in the U.S. Lower-48 states as estimated in 1956 (Hubbert, 1956, Fig. 22).

For the gas/oil-ratio, two figures were used. The ratio of gas discoveries to crude-oil discoveries during the most recent 20-year period, 1941-1961, was 6,250 ft^3/bbl. However, the possibility was also considered that in response to deeper drilling this ratio might increase in the future to as much as 7,500 ft^3/bbl.

Substituting these figures into equation (79) gave, for the ultimate amount of natural gas to be produced in the Lower-48 states, a low figure of 958 and a high figure of 1,053 trillion cubic feet, or roundly 1,000 trillion cubic feet. Using this figure for Q_∞ in conjunction with the limited data for Q_p, Q_r, and Q_d for natural gas, gave the logistic curves of Figure 43 and their derivatives in Figure 44. The two complete gas-production cycles, based on both low and high estimates of 958 and 1,053 trillion cubic feet, are shown in Figure 45. From these figures, the time delay Δt between cumulative discoveries and cumulative production was estimated to be 16 years. The maximum rate of discovery was estimated to occur at about 1961, the peak in proved reserves in 1969, and the maximum production rate of about 18 to 20 trillion cubic feet per year at about 1977.

Estimate of 1972. —— By 1972 (Hubbert, 1974), despite the fact that 10 more years of data were available, the natural-gas data on cumulative production, proved reserves, and cumulative proved discoveries were still so irregular as to make the use of the logistic equation of doubtful validity. However, by 1972, the proved reserves of natural gas had already reached their maximum in 1967, two years earlier than predicted in 1962. After 1967 they declined steeply. For estimates of Q_∞, two methods were used, that of the gas/oil-ratio in conjunction with the oil estimate, and the gas discoveries as a function of cumulative exploratory drilling. The first method gave an estimate of about 1,000 trillion cubic feet, and the second a higher figure of 1,103 trillion cubic feet.

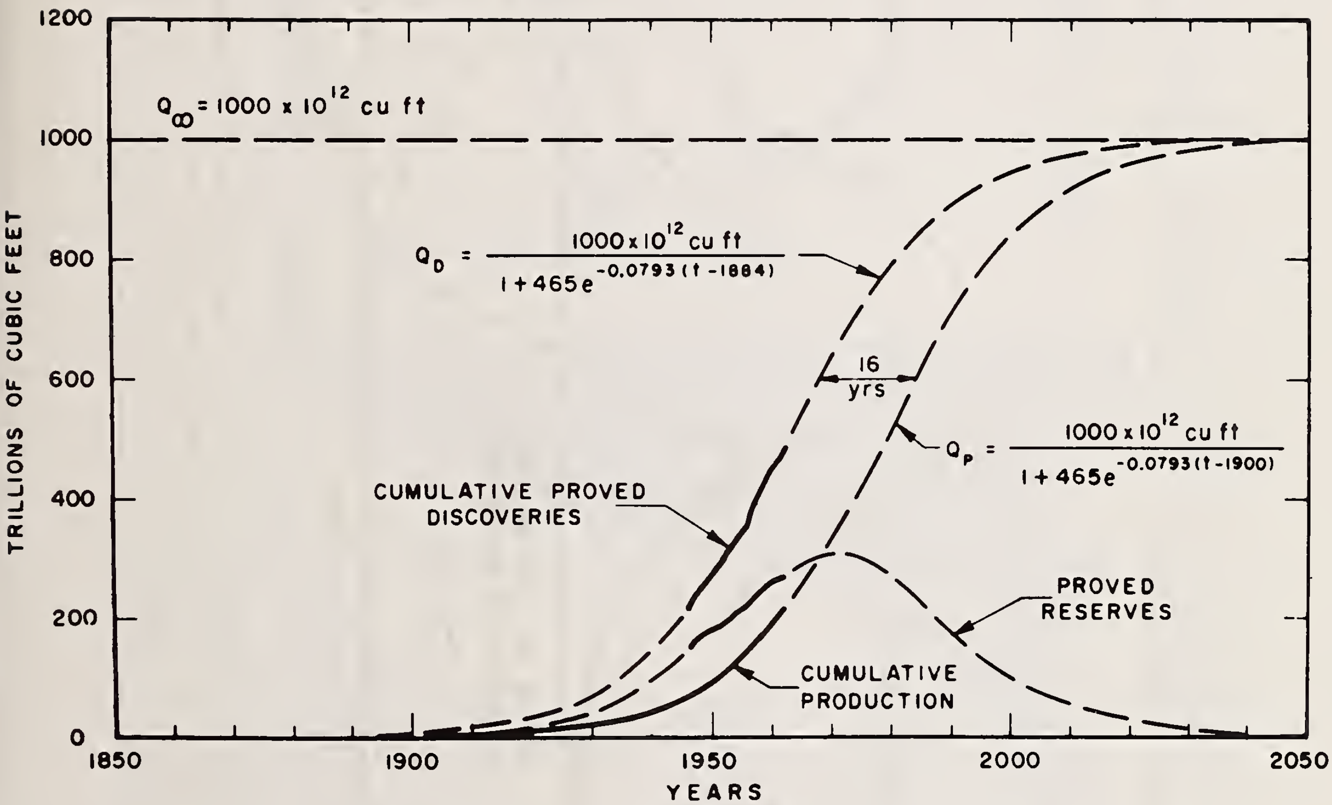

Fig. 43 - Logistic equations and curves for U.S. natural gas in 1962 (Hubbert, 1962, Fig. 45).

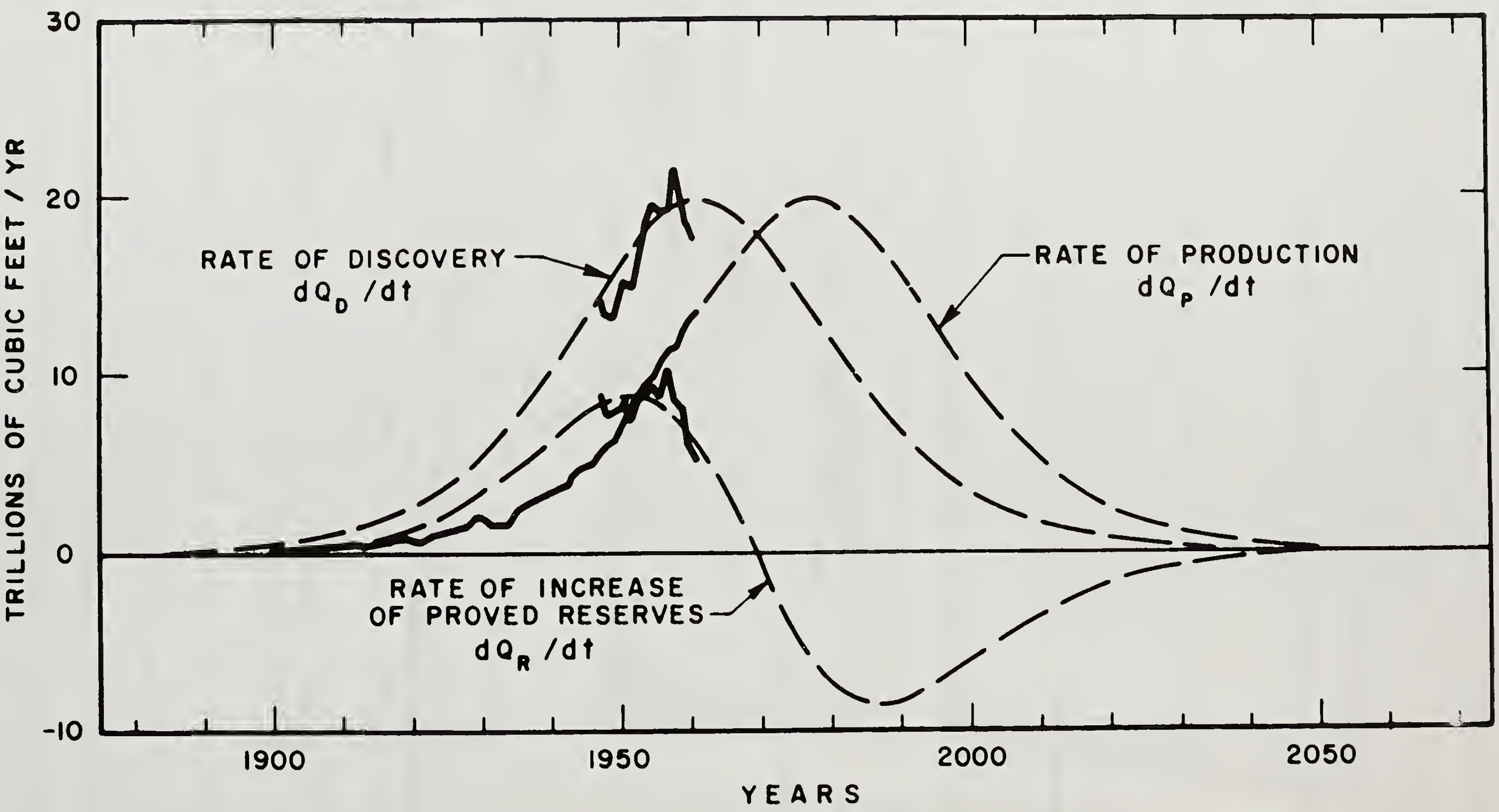

Fig. 44 - Rates of production, discovery, and increase of proved reserves of U.S. natural gas, 1900-1962, superposed upon derivative curves of logistic equations (Hubbert, 1962, Fig. 46).

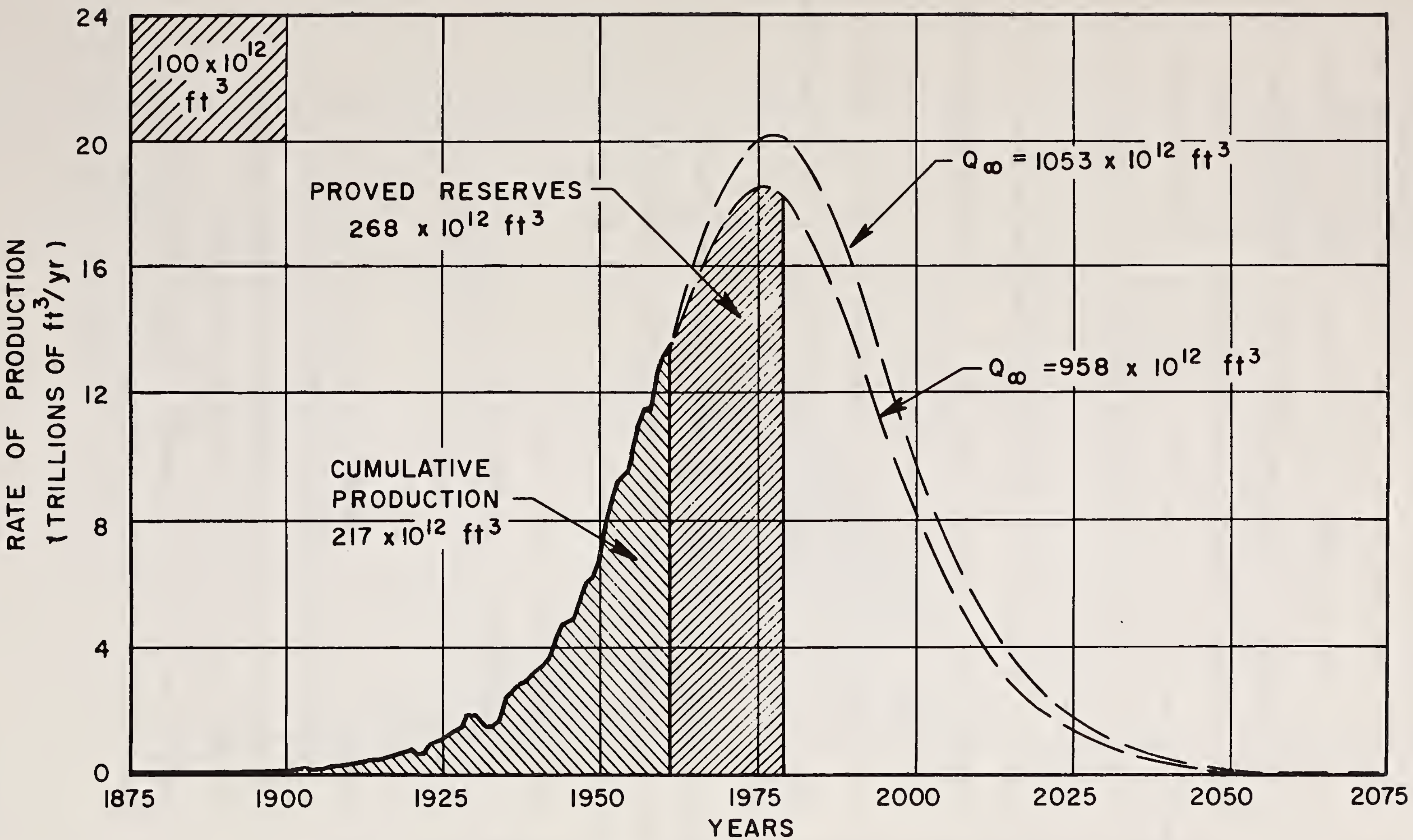

Fig. 45 - Two complete cycles of U.S. natural-gas production based upon high and low estimates of 1962 (Hubbert, 1962, Fig. 47).

The mean of these two figures of 1,050 trillion cubic feet was adopted as the value for Q_∞. This was then used, in conjunction with the statistical data for Q_p, Q_r, and Q_d, to construct Figure 46. Although in 1972 the maximum rate of natural-gas production had not yet been reached, all the evidence indicated that this would have to occur within the next two or three years. This was accordingly estimated to occur about 1975, with the peak production rate of about 24 trillion ft^3/yr. It actually occurred in 1973, with a peak rate of 22.6 trillion ft^3/yr.

The difficulty of trying to fit the cumulative data for natural gas with the logistic equation, using $1{,}050 \times 10^{12}$ ft^3 for Q_∞, is evident from inspection of Figure 47. The abrupt decline of proved reserves after 1967, and the corresponding downward deflection in the curve of cumulative discoveries combine to suggest that the actual figure for Q_∞ may be considerably less than the value of $1{,}050 \times 10^{12}$ ft^3.

Estimate of 1980. —— By 1980 the curve of cumulative proved discoveries is far enough advanced beyond its inflection point, which occurred about 1961, to permit estimates of the asymptote to which this curve is tending. For this purpose five different procedures have been used:

1. The linear regression $(dQ/dt)/Q$ versus Q.
2. The negative-exponential approach of the curve of Q_d versus t to Q_∞ as t increases.
3. Estimate of Q_∞ for gas based upon prior estimates for oil in conjunction with the gas/oil-ratio.
4. Estimate of the logistic constants for the curve of cumulative gas discoveries, Q_d, as a function of time.
5. Estimate based upon a new analysis by Root (1980) of gas discoveries per each 10^8 ft of exploratory drilling.

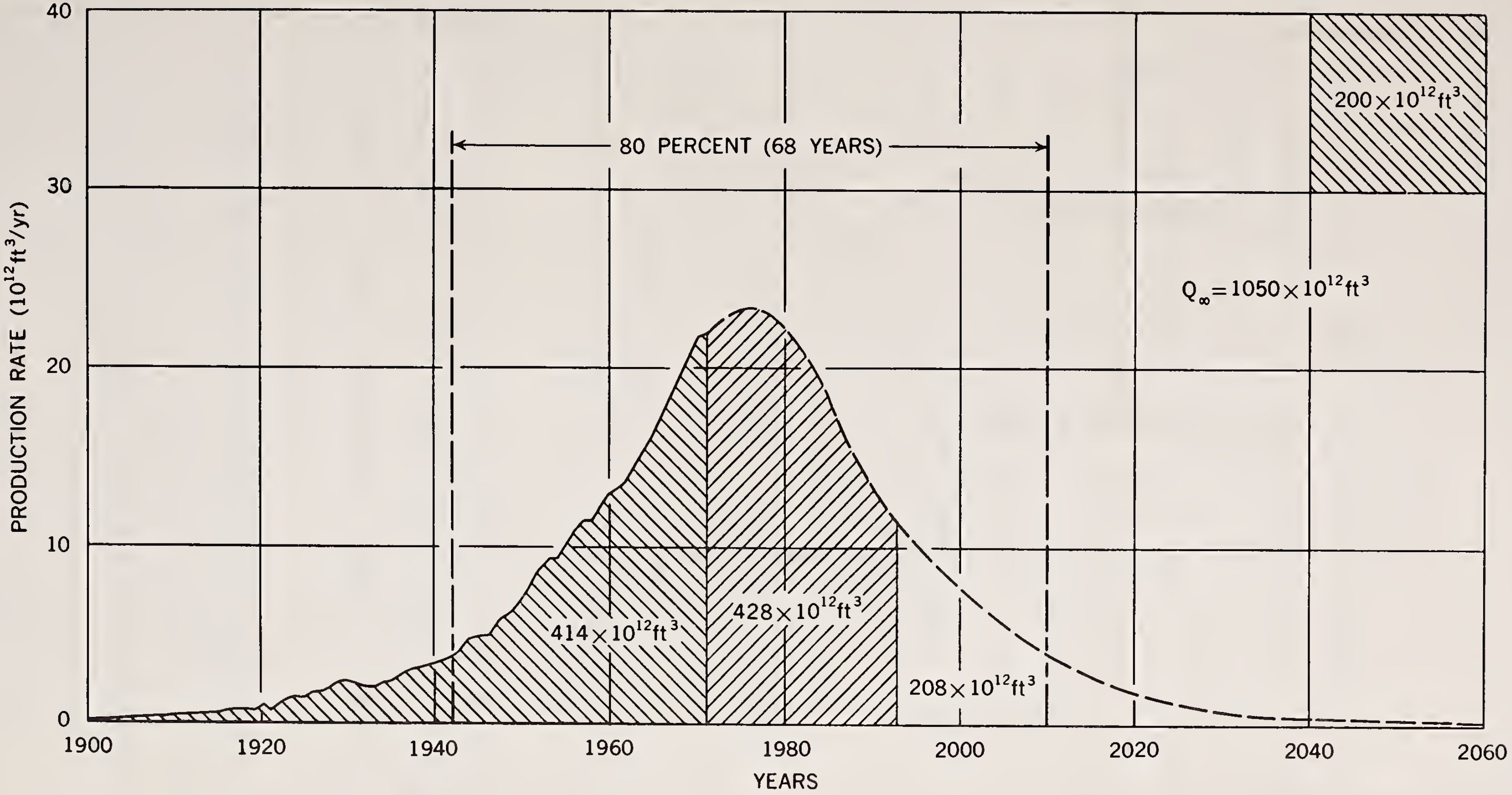

Fig. 46 - Complete cycle of U.S. natural-gas production as estimated in 1972 (Hubbert, 1974, Fig. 64).

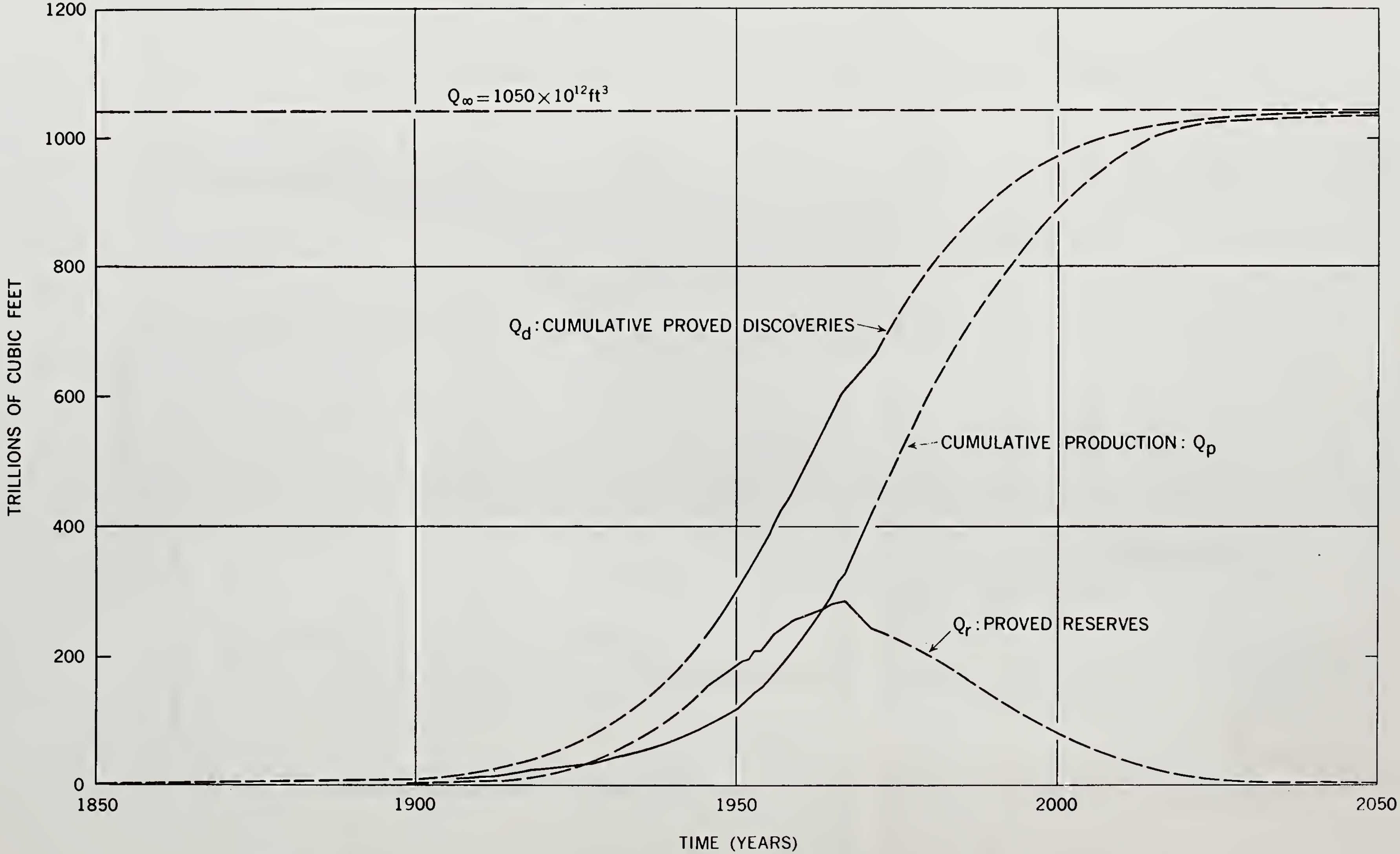

Fig. 47 - Cumulative production, proved reserves, and cumulative proved discoveries of U.S. natural gas, 1900-1972, with graphical estimates of future developments (Hubbert, 1974, Fig. 56).

Estimate based upon $(dQ/dt)/Q$ versus Q. —— The first of the foregoing procedures is based upon equation (27),

$$(dQ/dt)/Q = a(1 - Q/Q_\infty).$$

This is a linear equation between $(dQ/dt)/Q$ and Q, the graph of which intersects the Q-axis at the point $Q = Q_\infty$, and the vertical axis at $(dQ/dt)/Q = a$.

For data, the American Gas Association (May 1980) has recently published a table of cumulative proved discoveries of natural gas for the U.S. Lower-48 states corresponding to the end of each year from 1945 to 1979. Using these data, mean values for dQ/dt were computed for each year from 1950 to 1974 based upon a 10-year running average. Shorter periods of averaging were used for the years 1975 to 1978. For Q, the actual yearly figures were used. Plotting the data for $(dQ/dt)/Q$ versus Q gave a very good linear graph from Q = 480 to 720 trillion cubic feet, corresponding to the period 1960 to 1979. This line, as estimated visually, passed through the point, $(dQ/dt)/Q = 0.0500$; $Q = 400 \times 10^{12}$ ft^3, and intersected the Q-axis at 810×10^{12} ft^3, which is the estimated value for Q_∞. By backward extrapolation the line intersects the vertical axis at the point 0.099, which gives the value of the coefficient a of the corresponding logistic equation.

Although this figure of 810 trillion cubic feet is a surprisingly low figure for Q_∞, the data of the graph are sufficiently linear over the interval stated that very little latitude, possibly $\pm$ 10 trillion cubic feet, is allowable for the uncertainty of the point of intersection.

Estimate by the negative-exponential approach of Q_d versus t to Q_∞. —— The assumption that cumulative discoveries Q approach the asymptotic value Q_∞ in a negative-exponential manner during the later stages of the discovery cycle affords another means of estimation. At a fixed date t_o, let Q_o be the magnitude of cumulative discoveries. Then using this point as a new origin of coordinates, let $y = Q - Q_o$ be the subsequent increase in Q, and let $\tau = t - t_o$ be the subsequent time coordinate. Also let k be the asymptotic value of y. We then have the equation,

$$k - y = ke^{-b\tau}, \tag{80}$$

or

$$\ln[(k - y)/k] = -b\tau, \tag{81}$$

in which the two parameters, k and b are to be determined. Consider two points of the curve of y versus τ, (τ_1, y_1) and (τ_2, y_2). Introducing these values into equation (81) then gives the two equations,

$$\left.\begin{aligned} \ln[(k - y_1)/k] &= -b\tau_1, \\ \ln[(k - y_2)/k] &= -b\tau_2. \end{aligned}\right\} \tag{82}$$

By taking the ratio of the second to the first, b can be eliminated, and we obtain

$$\frac{\ln[(k - y_2)/k]}{\ln[(k - y_1)/k]} = \frac{\tau_2}{\tau_1}, \tag{83}$$

in which k is the only unknown. This can be solved by iteration if we substitute for k an assumed value k_a. Then the left-hand term of equation (83) becomes $f(k_a)$, and when

$$f(k_a) = \tau_2/\tau_1,$$

$$k_a = k.$$

After k is known, b may be determined from equation (81) by

$$b = \ln[(k - y_1)/k]/\tau_1.$$

Then finally, $$Q_\infty = Q_o + k.$$

The data of Q versus t and y versus τ are given in Table 1 at 5-year intervals from 1960 to 1980. Taking τ_1 at 10 years (date 1970), and τ_2 at 20 years (1980), and y_1 and y_2 equal to 169.05 and 267.50 trillion cubic feet, respectively, we find that

Table 1. — Estimates of future natural-gas production of U.S. Lower-48 states by negative-exponential approach to Q_∞.

Date	τ (yr)	Q_d	y	Q_d Computed	Date	τ (yr)	y	Q_d Computed
		(10^{12} ft^3)					(10^{12} ft^3)	
1960	0	466.35	0	466.35	2000	40	358.22	824.57
1965	5	553.52	87.17	562.23	2010	50	377.67	844.02
1970	10	635.40	169.05	635.40	2020	60	388.99	855.34
1975	15	680.17	213.82	691.24	2030	70	395.58	861.93
1980	20	733.85	267.50	733.85	2040	80	399.42	865.77
1985	25	-	(Computed) 300.02	766.37	2050	90	401.66	868.01
1990	30	-	324.83	791.13	∞	∞	404.78	871.13

Source for Q_d: Am. Gas Assoc., May 1980.

$$f(k_a) = \tau_2/\tau_1 = 2$$

when

$$k_a = k = 404.78 \times 10^{12}\ \text{ft}^3,$$

and that

$$b = 0.054066.$$

Then

$$\begin{aligned} Q_\infty &= Q_0 + k \\ &= (466 + 405) \times 10^{12}\ \text{ft}^3 \\ &= 871 \times 10^{12}\ \text{ft}^3. \end{aligned}$$

Estimates based upon the gas/oil-ratio. —— As shown in equation (79), an estimate of Q_∞ for gas is obtained by

$$Q_\infty \text{ gas} = Q_d \text{ gas} + G[(Q_\infty - Q_d)\text{oil}],$$

where the cumulative discoveries for both gas and oil are the most recent figures available, at present those for 1980.0, and G, the gas/oil ratio, is the ratio of gas discoveries to oil discoveries during a recent finite period of time.

By 1980.0

$$Q_d \text{ oil} = 136.9 \times 10^9 \text{ bbl},$$

$$Q_\infty \text{ oil} = 163 \times 10^9 \text{ bbl},$$

and

$$(Q - Q_d) \text{ oil} = 26 \times 10^9 \text{ bbl},$$

$$Q_d \text{ gas} = 734 \times 10^{12}\ \text{ft}^3.$$

For gas/oil-ratios during recent decades, cumulative discoveries of both crude oil and natural gas are given for 1960.0, 1970.0, and 1980.0 in Table 2. From these data three separate gas/oil-ratios are obtained for three different periods of time. For the decade 1960 to 1970, the value of G was 6,789 ft^3/bbl; for 1970 to 1980 it had declined to 5,472 ft^3/bbl; and for the 20-year period

Table 2. — Cumulative discoveries of crude oil and natural gas in U.S. Lower-48 states by 1960, 1970, and 1980, and gas/oil-ratios for 1960-1970, 1970-1980, and 1960-1980.

Date (Jan. 1)	a/ Q_d Crude oil (10^9 bbl)	b/ Q_d Natural gas (10^{12} ft^3)	Time interval Δt (yr)	ΔQ_d Crude oil (10^9 bbl)	ΔQ_d Natural gas (10^{12} ft^3)	Gas/oil-ratio G (ft^3/bbl)
1960	94.01	466.35	1960-1970	24.90	169.05	6,789
1970	118.91	635.40	1970-1980	17.99	98.45	5,472
1980	136.90	733.85	1960-1980	42.89	267.50	6,237

Sources:

a/ Am. Petrol. Institute, Am. Gas Assoc., and Canadian Petrol. Assoc., Annual "Blue Books," 1966-1979.

b/ Am. Gas Assoc., May 1980.

1960 to 1980 it had an intermediate value of 6,237 ft^3/bbl.

For use in estimating future gas discoveries the ratio of the last 10 years is evidently the best of the three figures, although the ratio for the last 20 years may also be considered. With the numerical data given above, the solution of equation (79), using the gas/oil-ratio of 5,472 ft^3/bbl, gives an estimate for Q_∞ for natural gas of

$$Q_\infty = 876 \times 10^{12} \text{ ft}^3.$$

Using the ratio of 6,237 ft^3/bbl of the last 20 years gives the slightly higher estimate

$$Q_\infty = 896 \times 10^{12} \text{ ft}^3.$$

Estimate based upon the constants of the logistic equation. —— Despite the fact that the curve of Q_d versus t for natural gas cannot be fitted accurately by the logistic equation, still a good approximation can be obtained using the data for Q_d from 1946 to 1980, during which Q_d increased from 233 to 734 trillion cubic feet. Expressing these data in the linear form of equation (36),

$$\ln N = \ln N_0 - at,$$

and then using the technique described in equations (43) to (45), approximate values of the parameters, Q_∞ and a can be determined. For this purpose the following data were used:

Date (t)	1946.0	1965.0	1980.0
Q_d (10^{12} ft^3)	233.18	553.52	733.85

The value of Q_d for which the two line segments, that from 1946.0 to 1965.0, and that from 1965.0 to 1980.0, have the same slope was found to be 840.0 × 10^{12} ft^3, and $-S$ = 0.0850/yr. Hence, the estimates for the logistic constants by this procedure are:

$$Q_\infty = 840 \times 10^{12} \text{ ft}^3,$$

$$a = 0.0850/\text{yr}.$$

<u>Estimate based upon gas discoveries per each 10^8 ft of exploratory drilling.</u> —— David H. Root (1980) has just completed a new study of natural gas discoveries in the Lower-48 states, based upon his own modification of the Arrington method of estimating the additional gas that fields discovered each year will ultimately produce. Root has estimated the ultimate amount of gas to be produced by each of the 20 10^8-ft units of exploratory drilling extending in time from 1860 to 1977.9. This is a parallel study for natural gas to Root's crude-oil study, the results of which are shown in Figures 38 and 41.

As in the case of crude oil, the discoveries made by the first 4 units of drilling, which extended from 1860 to 1945.2, were large, averaging slightly more than 100 trillion cubic feet each. However the discoveries per unit for the entire 20 units declined in a roughly negative-exponential manner to a final figure of 13.912 trillion cubic feet for the 20th unit.

Using the method developed in equations (72) to (78), the actual data for dQ/dh versus h can be approximated by a negative-exponential decline curve,

$$R = R_o e^{-\beta h},$$

whose integral from $h = 0$ to $h = 20$ units has the same value as the sum of the actual discoveries, and which passes through the last data point on the curve.

The significant data for this determination are:

$$R_{20} = 13.912 \ (10^{12} \text{ ft}^3/10^8 \text{ ft}),$$

$$Q_{20} = 844.406 \times 10^{12} \text{ ft}^3.$$

From these,

$$R_o = 95.04118 \ (10^{12} \text{ ft}^3/10^8 \text{ ft}),$$

$$\beta = 0.09608 \text{ per } 10^8 \text{ ft},$$

$$Q_\infty = R_o/\beta = 989.2 \times 10^{12} \text{ ft}^3,$$

$$Q_u = Q_\infty - Q_{20} = 144.8 \times 10^{12} \text{ ft}^3.$$

Summary of Estimates of Natural Gas

The foregoing estimates for the ultimate quantity of natural gas to be produced in the Lower-48 states and adjacent offshore areas are the following:

Method of estimation	Q_∞ (10^{12} ft^3)
$(dQ/dt)/Q$ vs. Q	810
Q vs. t	871
Gas/oil-ratio	876 896
Logistic equation	840
dQ/dh vs. h	989
Mean	880

What is most impressive about these separate estimates is the range from the lowest to the highest of 810 to 989 trillion cubic feet, or approximately 900 $\pm$ 90, with a mean value of 880 trillion cubic feet. If we omit the lowest and the highest estimates, each of which differs by a large amount from that next above or below, then the remaining four figures fall within the much narrower range of 840 to 896, or 868 $\pm$ 28, with a mean value of 871 trillion cubic feet.

In this series, both the lowest figure of 810 trillion cubic feet and the highest of 989 are anomalous, but the latter is especially suspect since it exceeds the average of 871 trillion cubic feet of the middle four estimates by

118 trillion cubic feet. This analysis by Root was a companion study to that of the crude-oil discoveries as a function of cumulative depth of exploratory drilling, the results of which are shown in Figures 38 and 41. In the crude-oil analysis the data used were the API "Blue Book" data on "ultimate recovery" of crude oil by year of discovery. In the natural-gas analysis the corresponding data were the AGA "Blue Book" figures for "ultimate recovery" by year of discovery. However, the natural-gas estimate of 989 trillion cubic feet shows the same inconsistency with the corresponding crude-oil estimate as it does with the other gas estimates given above.

This can be seen by using Root's data for crude-oil discoveries in conjunction with the gas/oil-ratio. In this case,

$$Q_\infty \text{ gas} = Q_{20} \text{ gas} + GQ_u \text{ oil}. \tag{84}$$

Using Root's figures of

$$Q_{20} \text{ gas} = 844.4 \times 10^{12} \text{ ft}^3,$$

$$Q_u \text{ oil} = 4.9 \times 10^9 \text{ bbl},$$

and the two values of the gas/oil-ratio from Table 1,

$$G = 5{,}472 \text{ and } 6{,}237 \text{ ft}^3/\text{bbl},$$

gives the following two estimates for Q_∞ for natural gas:

$$Q_\infty = 871 \times 10^{12} \text{ ft}^3,$$

$$Q_\infty = 875 \times 10^{12} \text{ ft}^3.$$

These figures are consistent with those ranging from 840 to 896 trillion cubic feet obtained by other methods. Combining the mean of the above two figures, 873 trillion cubic feet, with the previous estimates (omitting the low figure of 810), gives as our present best estimate for Q_∞ for natural gas,

$$Q_\infty \text{ gas} = (870 \pm 30) \times 10^{12} \text{ ft}^3.$$

Conclusion

The principal thesis of the present paper has been that the successful prediction of the future behavior of any matter-energy system must be based upon a prior understanding of the mechanism of the system considered, and upon a rational analysis of the data of the system in accordance with that mechanism. Also, the final arbiter of the reliability of any prediction is the future itself. So long as the predicted event is still in the future, whether or not the prediction is valid must remain to some degree uncertain. But after the time has been reached at which the predicted event was to occur, this doubt no longer remains.

In this paper, the results of the application of this philosophical view to the petroleum industry of the United States during the last 25 years have been reviewed. It is now evident that by the mid-1950s the cumulative data of the U.S. petroleum industry were sufficient to permit reasonably accurate predictions of its future development. With the passage of time, more and better data have permitted a refinement of earlier estimates, and also provided a verification of their degree of accuracy. By now, the peak in the rate of crude-oil production has already been passed in 1970, and that of natural gas in 1973, and the production rates of both are now in decline.

The present cumulative statistical evidence with regard to crude oil leads to a figure of approximately 163 ± 2 billion barrels for the ultimate cumulative production in the Lower-48 states. Less exact evidence for natural gas indicates that the ultimate cumulative production from conventional sources will probably be in the range of 870 ± 30 trillion cubic feet. However there still remain geological uncertainties regarding the occurrences of undiscovered oil and gas fields, yet those are being severely restricted by the extent of exploratory activity. In the case of crude oil, there is also the uncertainty regarding the magnitude of future improvements in extraction technology.

With due regard for these uncertainties, estimates for crude oil that do not exceed that given here by more than 10 percent may still be within the range of geological uncertainties; estimates that do not exceed this by more than 20 percent may be within the combined range of geological and technological uncertainties. Estimates for natural gas that do not exceed the upper limit of the range given above by more than 10 percent may likewise be regarded as possible although improbable. But estimates for either oil or gas, such as those that have been published repeatedly during the last 25 years, which exceed the present estimates by multiples of 2, 3, or more, are so completely irreconcilable with the cumulative data of the petroleum industry as no longer to warrant being accorded the status of scientific respectability.

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DISCUSSION

DR. HUBBERT (in reply to question by Samuel Kao - Brookhaven): Your statement that all of my curves are symmetric is not entirely correct. I have stated explicitly that the complete-cycle curve of production of an exhaustible resource in a given region has the following essential properties: The rate of production as a function of time begins at zero. It then increases exponentially during a period of development and later exploration and discovery. Eventually the curve reaches one or more maxima, and finally, as the resource is depleted, the curve goes into a negative-exponential decline back to zero. There is no requirement that such a curve be symmetrical or that it have only a single maximum. In small regions such a curve can be very irregular, but in a large area such as the United States or the world these irregularities tend to smooth out and a curve with only a single principal maximum results. If such curves are also approximately symmetrical it is only because their data make them so.

In my figure of 1956, showing two complete cycles for U.S. crude-oil production, these curves were not derived from any mathematical equation. They were simply tailored by hand subject to the constraints of a negative-exponential decline and a subtended area defined by the prior estimates for the ultimate production. Subject to these constraints, with the same data, I suggest that anyone interested should draw the curves himself. They cannot be very different from those I have shown.

DR. HUBBERT: As I have stated before, there is no theoretical necessity for the complete-cycle curve to be symmetrical. When such curves are symmetrical it is only because the data require that they be so. A critical test of whether such a curve is symmetrical or not is the linear equation,

$$\ln N = \ln N_0 - a(t - t_0),$$

where

$$N = (Q_\infty - Q)/Q,$$

Q = cumulative discoveries or production,

Q_∞ = the ultimate value of Q,

a = the growth constant.

This is the linear form of the symmetric logistic equation. If the quantity $\ln N$ plots as a straight line as a function of time, this is evidence that the cumulative data increase in accordance with the symmetric logistic equation.

For cumulative discoveries and production of crude oil in the U.S. Lower-48 states, during the period 1900-1973, the data plot as excellent straight lines in accordance with the above equation. For discoveries, the maximum rate occurred at about 1957. However, from 1973 to 1980, the discovery rate has been declining faster than the equation would predict.

DR. HUBBERT (in response to remarks by David Nissen - DOE): Your kind remarks with regard to my previous studies of the evolution of the U.S. petroleum industry are greatly appreciated. However, you suggest that my estimates of the ultimate amount of oil to be recovered is questionable for reasons of classification and because I have not taken into account the effect of the price of oil on ultimate recovery. You mention oil shale, coal, and the Orinoco heavy oils of Venezuela.

With regard to classification, if unintelligibility is to be avoided, it is essential that one define his terms and then adhere rigorously to those definitions. In the present study I have been concerned with the techniques of estimation as applied to conventional crude oil and natural gas in the U.S. Lower-48 states. This excludes consideration of shale oil, coal, Orinoco heavy oils, natural gas from unconventional sources, and also oil and gas from Alaska.

My analyses are based upon the simple, fundamental geologic fact that initially there was only a fixed and finite amount of oil in the ground, and that, as exploitation proceeds, the amount of oil remaining diminishes monotonically. We do not know how much oil was present originally or what fraction of this will ultimately be recovered. These are among the quantities that we are trying to estimate.

Your statement that the fraction of the original oil-in-place that will be recovered is a function of the price of oil is correct, but the effect may easily be exaggerated. For example, we know now how to get oil out of a reservoir sand, but at what cost? If oil had the price of pharmaceuticals and could be sold in unlimited quantity, we probably would get it all out except the smell. However there is a different and more fundamental cost that is independent of the monetary price. That is the energy cost of exploration and production. So long as oil is used as a source of energy, when the energy cost of recovering a barrel of oil becomes greater than the energy content of the oil, production will cease no matter what the monetary price may be. During the last decade we have had very large increases in the monetary price of oil. This has stimulated an accelerated program of exploratory drilling and a slightly increased rate of discovery, but the discoveries per foot of exploratory drilling have continuously declined from an initial rate of about 200 barrels per foot to a present rate of only 8 barrels per foot.

There is the further question of what fraction of the original oil-in-place is now being recovered. The conventional figure most frequently quoted is about one-third. However, a critical review of this question in a book entitled "Determination of Residual Oil Saturation" by a panel of nationally prominent petroleum engineers has just been published by the Interstate Oil Compact Commission (June 1978). In this study the average value of the residual oil saturation in the depleted reservoir sands of a hundred or so fields was found to be only 28 percent, as compared with previous estimates of 38 percent. According to this study, the recovery factor at present is evidently much higher than has been conventionally assumed, and the remaining oil correspondingly smaller.

CURRENT PROBLEMS IN OIL AND GAS MODELING*

William C. Stitt
ICF Incorporated

As a starting point, I assert that those of us here today agree that "problems" do exist in current oil and natural gas supply assessments. This assertion, of course, relies on the assumption that these problems, their critical importance to national policy and private sector investment, and the search for their solution motivates our professional interest in the subject of oil and gas supply.

I must, however, make clear at the outset the perspective which I bring to these problems. Those engaged in oil and gas supply assessments have been browbeaten regularly of late by our clientel and our critics. The former too often expect certainty where none can exist. The latter too often expect perfection, in some normative sense, where none is likely to exist until all of our oil and gas is used up. Both too often lose sight of what is possible, practical and feasible today; what great progress already has been made in improving oil and gas supply assessments; and what opportunities currently exist for intelligent use of our work. I therefore wish my comments about "problems" to be interpreted modestly and with a sense of opportunities for improvement rather than of condemnation of current practice.

The locus of my remarks also extend well beyond the term "modeling." The only strictly correct and precise model of the oil and natural gas supply is the actual resource base and myriad supply activities themselves. All practical oil and natural gas supply assessments are abstractions of this real model. As such, they represent ideas about the real world of crude oil and natural gas, only a small part of which set of ideas are exhibited as "formal" models of oil and natural gas supply. Consequently, the problems of interest here, I believe, are those related more broadly to oil and gas supply ideas, some of which are embodied in formal supply models.

In this context, oil and natural gas supply assessment is complex, difficult, and always challenging but often frustrating. Rich texture stems from the character of the resource as well as its extraction process and its economics. From a problem-solving perspective, this character is enhanced substantially by U.S. circumstances. Our resource base resides in both mature provinces and substantial frontiers. It consists of a large quantity of discovered oil and natural gas, only a relatively small fraction of which has been extracted. In addition, the surrounding marketplace and a highly dynamic price, technological and policy environment provide a wealth of situations of critical importance and intrinsic analytical interest.

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*This paper also appears in the book Energy Policy Planning, edited by B. A. Bayraktar, E. A. Cherniavsky, M. A. Laughton and L. E. Ruff, Plenum Press, New York, N. Y. 1981. Reprinted with permission

Given this complexion, one point hopefully will emerge from my remarks--sound assessments proceed from a thorough understanding of the physical facts which are unique to the oil and natural gas resource and supply process. Another hoped for point is that facts--data and a trustworthy empirical foundation for oil and natural gas supply assessment--are scarce. A large gap exists between elegant formulations, which pervade the literature relevant to oil and natural gas, and empirical tests of their merits. Concepts and potential techniques run far ahead of our abilities to put them to use in today's data environment. Insufficient evidence, in my opinion, is the major obstacle to progress in oil and natural gas supply assessment.

Nonetheless, the current oil and natural gas literature is rich. In addition, one or two dozen formal models of the U.S. oil and natural gas resource and supply process are operable. Many have been employed extensively, along with even more numerous informal models, throughout the U.S. domestic oil and natural gas supply policy debate which has endured since 1970. The sheer weight of this literature, models, and applications to policy questions defies detailed review in an abbreviated space or timeframe. Consequently, my approach here stands back from the details--lacking approbation and perhaps good sense--and raises a series of questions which the literature, these models, and the policy debate thus far suggest should be areas of emphasis and concern in improving oil and natural gas supply assessments.

FRAMEWORK

Oil and natural gas supply assessments, whether or not a "formal" model is involved, rely upon ideas about the actual resource base and supply activities. As such, all symbolize the "real model" which leads ultimately to extraction of oil and natural gas. Figure 1 shows a highly simplified and abstract notion of the ingredients of this supply system.

The components of the figure have the following meanings:

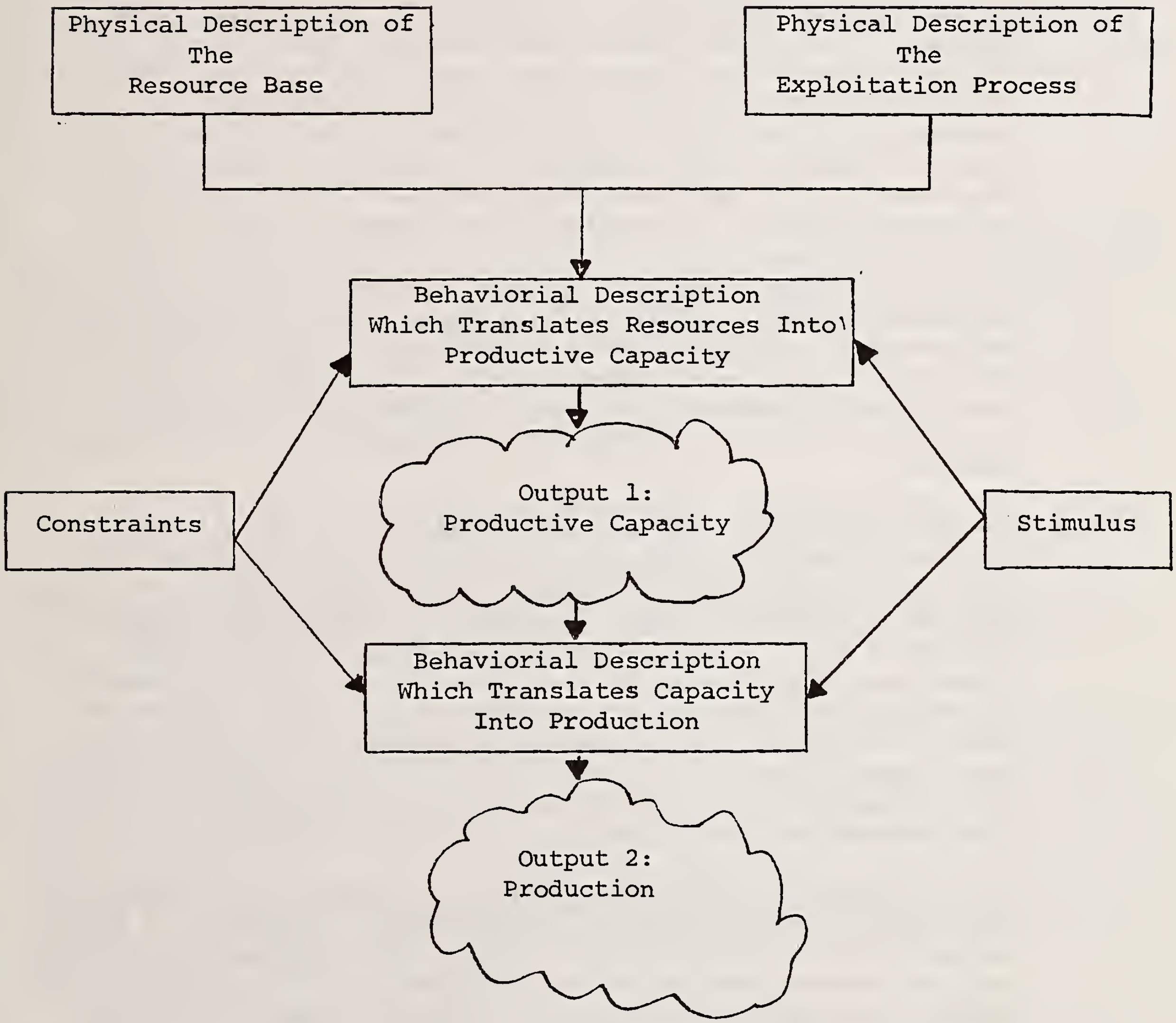
- Physical Description of the Resource Base: This part consists of a physical characterization of the resource base. For oil and natural gas, it ideally consists of a three-dimensional description of individual deposits; their locations and depths; the fraction of their volume which is hydrocarbons as well as the mechanical properties of the hydrocarbons (e.g., viscosity); and the mechanical properties of the overall reservoir (e.g., permeability, drive mechanism, pressure and temperature, etc.). Of course, these properties of undiscovered deposits are not known with significant certainty. And for those which have been discovered, these properties become known with a high degree of confidence only over a drawn out period during which the deposits' contents are exhausted through extraction.

- Physical Description of the Exploitation Process: This part consists of a description of the physical activities and "engineering" costs which lead ultimately to production. For oil and natural gas, three subparts of the exploitation process ideally deserve separate identification: exploration, development, and production. Exploration is the search for prospects, testing for the presence of hydrocarbons by drilling, and fuller delineation of the dimensions and mechanical properties of those which initially provide encouraging results. Exploration identifies oil- or gas-in-place and provides information relevant to deciding whether and how to develop and produce. Development installs the capacity to produce, initially through drilling and installation of surface equipment and, usually later, through augmentation of reservoir pressure and other measures which enhance the recovery of the deposit's contents. Production operates this capacity to yield a physical flow of the hydrocarbons for use or sale.

  Of special importance for oil and natural gas, a large number of alternative development programs can be pursued in order to create productive capacity for any deposit. Each might yield divergent time patterns and levels of costs and production and might alter the fraction of the hydrocarbons ultimately extracted.

FIGURE 1

COMPONENTS OF THE OIL AND NATURAL GAS RESOURCE ASSESSMENT PROBLEM



Finally, capacity to produce may not equate with production, even when measured in terms which envision normal downtime. For example, certain price expectations create incentives for under-utilization of existing capacity in the near term in order to create greater returns to production later on.

- Behavioral Description Which Translates Resources Into Productive Capacity: An oil and natural gas assessment must describe when and how extraction will occur over time. The idea, of course, is one of an objective function, but in the broadest sense of a description of what the agents which make extraction decisions seek to accomplish. Importantly for oil and natural gas, the objective(s) which motivates exploration may not be identical to the objective(s) which stimulates development or production.

- Constraints and Stimulus: From an assessment perspective, these ingredients are straightforward. Limited access to portions of the resource base may create a constraint, exemplified by leasing practices applicable to federal lands. The stimulus usually is price, although substantial debate surrounding U.S. oil and natural gas policy has centered on whether price is the primary stimulus.

- Productive Capacity: This component emphasizes the separate decision to actually produce. From a capacity standpoint, it connotes a production profile over time, composed of a production rate and a productive life. Inadequate distinction of these two components of the supply process--rate and life--may be the single, most important problem evidenced in existing assessments. A prevelant description of the output of the supply process consists of reserves--the integral of the production profile over time. Such a view of resources which provide a wealth of development alternatives, each emboding a unique production profile, overlooks an important feature of the supply process.

- Behavioral Description Which Translates Capacity Into Production: In a stable price environment--for example, one without an expectation of price jumps caused by either OPEC or domestic pricing policies--excess capacity will be avoided wherever possible.* But today, price instability and U.S. policies may make the urge to speculatively withhold an important consideration for U.S. oil and natural gas models.

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* Prior to 1970 in the U.S., numerous factors--resource finding and cost experience, tax subsidies, and oil import quotas--combined to consistently produce excess of oil production capacity, which was restrained artificially by market-demand prorationing.

A RESERVOIR LIFE CYCLE

This simple description of the supply process masks the individual activities involved over the life of a single deposit. For discussion purposes, a deposit means a generally contiguous and communicating unit within a geologic anomaly which contains hydrocarbons, typically labelled a reservoir. The anomaly or prospect is the micro-unit of principal interest for exploration decisionmaking; in contrast, the reservoir is the focal point of development and production decisionmaking. Nonetheless, exploration, development, and production decisionmaking are linked closely because the objective of the former is to identify concrete opportunities for the latter, through which the actual returns for exploratory investment are realized.

For oil and natural gas assessments, the distinction between exploration, development, and production probably should be emphasized. Exploration is governed by a complex probability law which combines the attributes of the natural processes which caused the deposition of hydrocarbons and of the search process which locates them. By comparison, development is deterministic and engineering-oriented, focused mainly on making capacity decisions which optimize in some fashion the returns available from investment in production capacity and subsequent production.

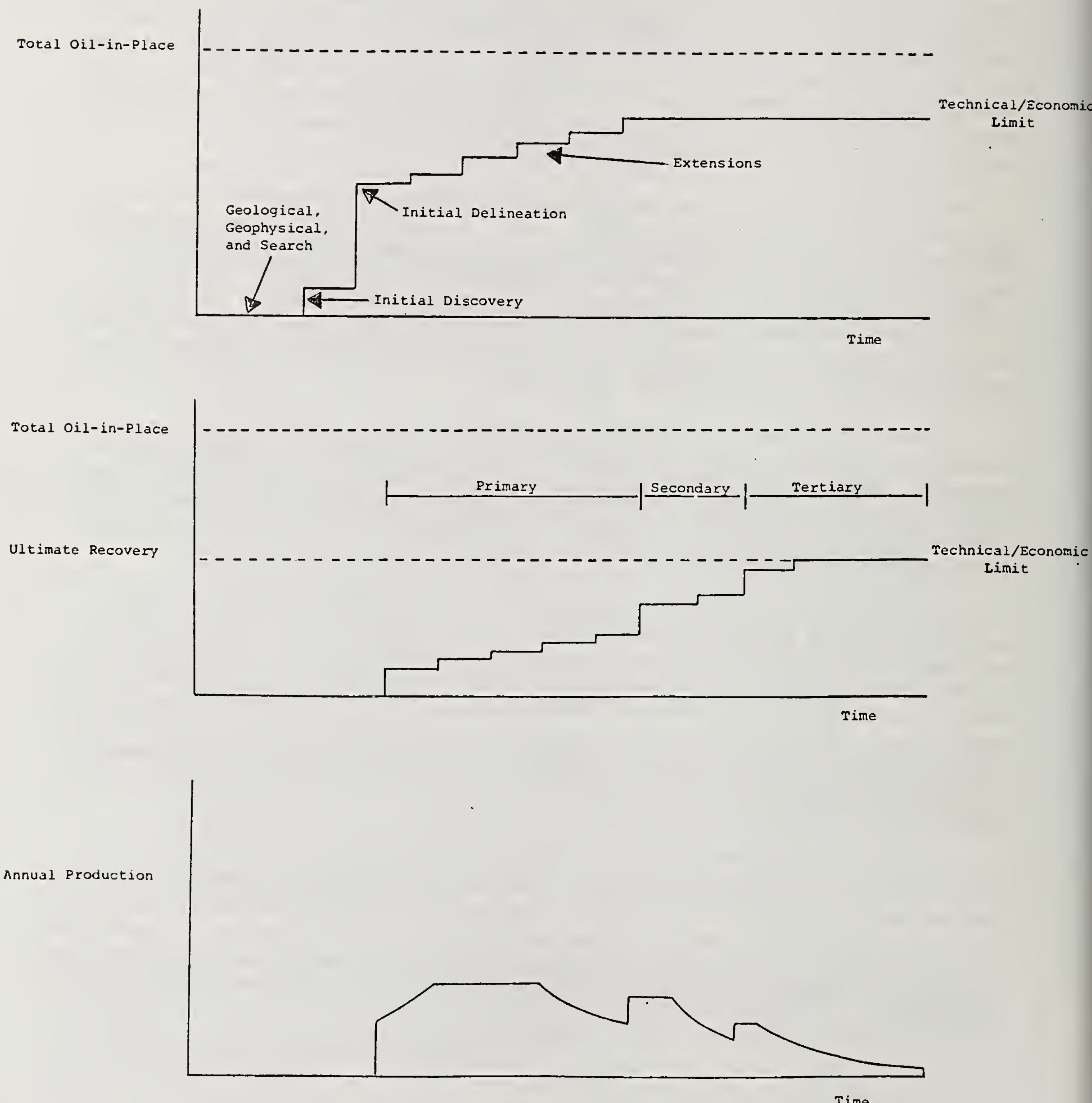
Figure 2 provides a simplified description of the lifecycle of an oil reservoir. It does not do justice to the real character of the exploration process. Rather it portrays an artificial situation, a play consisting of only one prospect consisting of one reservoir. The top part of the figure illustrates the sequence of activities which lead to the identification of oil-in-place in the deposit.

The middle of the figure portrays a three-part development program: a primary phase of production employing only the reservoir's natural drive; a secondary phase relying on augmentation of the natural drive, for example through the injection of water from an external source; and a tertiary phase depending on methods which might consist of the introduction of chemicals to ease migration of oil out of the reservoir pore space. These phases may not be distinct or sequential in practice; for example, a deposit of extremely viscous oil, so-called heavy hydrocarbons, might require the application of tertiary methods from the outset.

"Ultimate recovery" (shown on the middle portion of the figure) is an economic and technological artifact--one which introduces confusion into oil and natural gas supply assessments. In concept, ultimate recovery could equal total oil-in-place; literally, the reservoir rock could be mined, crushed and treated chemically to remove virtually all of the original oil contents. At any point in time, the limits on "ultimate recovery" stem from technology and economics.

FIGURE 2

LIFE CYCLE OF AN ILLUSTRATIVE OIL RESERVOIR



Importantly, my illustrative development program is silent about the intensity with which any phase is pursued. Subject only to intensities which would cause (economically) catastrophic reservoir damage or severely diminishing returns, development chooses the level of productive capacity to achieve at every point in time over the reservoir's life. U.S. petroleum engineering and conservation regulatory practices frequently evidence concern over rates of production which might "reduce ultimate recovery." These concerns are statements about technology and economics. Extraction rates which "reduce ultimate recovery" simply mean that more intensive development in the near term mandates new technologies and probably higher costs later in the reservoir's life (in order to achieve a level of recovery that otherwise could be achieved with less intense near-term development). Particularly in an environment of dynamic prices and technology, petroleum engineering jargon and conservation thumb-rules must be inspected carefully and used cautiously for supply assessment purposes. Fundamental doctrine of yesterday's engineering and conservation techniques is relevant only to the extent that they are justified by current and prospective economics and technology.

The bottom of Figure 2 illustrates a production profile that assumes that productive capacity is fully utilized. Over time the production profile consists of a super-imposition of new capacity added by each phase of development and a subseqent production decline as continuing extraction shrinks effective capacity.

Turning away from the illustrative and toward the "real" world, Figure 3 depicts the well classification scheme used by the American Association of Petroleum Geologists and the American Petroleum Institute. It illustrates the true richness of the oil and natural gas exploration and development processes. But it also provides an initial glimpse of the data problems facing oil and natural gas supply assessments.

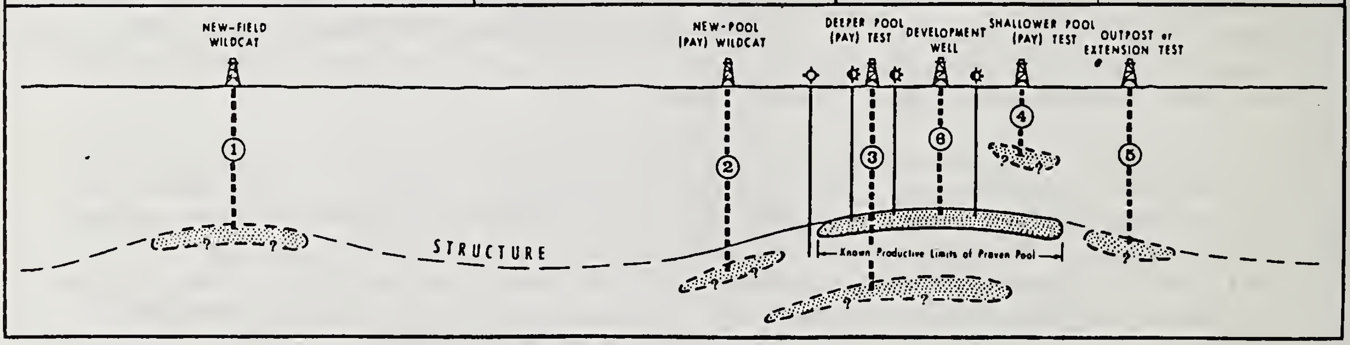
In the figure, unsuccessful wells are classified as "dry." "Dry" means either that insufficient hydrocarbons were present to justify development in the prevailing economic and technological environment or that no hydrocarbons were present. In a dynamic price and technological environment, these two different outcomes, hidden in the historical record of dry wells, suggest that a backlog of previously economically and technically submarginal identified prospects and discovered deposits may overhang the supply process at any point in time.

In Figure 3, development wells also are reported in a single, undifferentiated category. But the existence of development intensity alternatives suggests that there are two types of development wells. One type adds productive capacity by drilling into previously undeveloped portions of a partially developed reservoir, thereby adding so-called proved reserves. Another type adds productive capacity only by increasing the rate at which previously developed or proved reserves can be extracted. Lack of this distinction in the historical record can cause severe problems of interpretation and, in turn, biases in oil and natural gas supply assessments.

FIGURE 3

AAPG AND API CLASSIFICATION OF WELLS (1)

OBJECTIVE OF DRILLING				INITIAL CLASSIFICATION ϕ WHEN DRILLING IS STARTED	FINAL CLASSIFICATION AFTER COMPLETION OR ABANDONMENT			
					SUCCESSFUL ● ☼ ☼		UNSUCCESSFUL ⌖	
Drilling for a new field on a structure or in an environment never before productive				1. NEW-FIELD WILDCAT	NEW-FIELD DISCOVERY WILDCAT		DRY NEW-FIELD WILDCAT	
Drilling for a new pool on a structure or in a geological environment already productive	NEW-POOL TESTS	Drilling outside limits of a proved area of pool		2. NEW-POOL (PAY) WILDCAT	NEW-POOL DISCOVERY WELLS (Sometimes extension wells)	NEW-POOL DISCOVERY WILDCAT (Sometimes an extension well)	DRY NEW-POOL TESTS	DRY NEW-POOL WILDCAT
		Drilling inside limits of proved area of pool	For a new pool below deepest proven pool	3. DEEPER POOL (PAY) TEST		DEEPER POOL DISCOVERY WELL		DRY DEEPER-POOL TEST
			For a new pool above deepest proven pool	4. SHALLOWER POOL (PAY) TEST		SHALLOWER POOL DISCOVERY WELL		DRY SHALLOWER POOL TEST
Drilling for long extension of a partly developed pool				5. OUTPOST or EXTENSION TEST	EXTENSION WELL (Sometimes a new-pool discovery well)		DRY OUTPOST OR DRY EXTENSION TEST	
Drilling to exploit or develop a hydrocarbon accumulation discovered by previous drilling				6. DEVELOPMENT WELL	DEVELOPMENT WELL		DRY DEVELOPMENT WELL	



THE RESOURCE STOCK

Thus far, my framework has concentrated on the flows associated with the oil and natural gas supply process. Figure 1 connected physical knowledge, constraints and a stimulus with the behavioral ingredients of the process. Figure 2 portrayed the kinds of activities, time relationships, and decision problems associated with exploration, development and production decisions.

The stock aspect of the oil and natural gas supply assessment problem concerns the status of the resource base at each point in time. In principle, a fully correct and complete assessment should address the full assortment of "margins" along which the supply process can advance. Figure 4 displays the "McKelvey Box," which is a convenient expository device for identifying the various margins for supply activity which can exist at any point in time. Briefly, these margins have the following meaning:

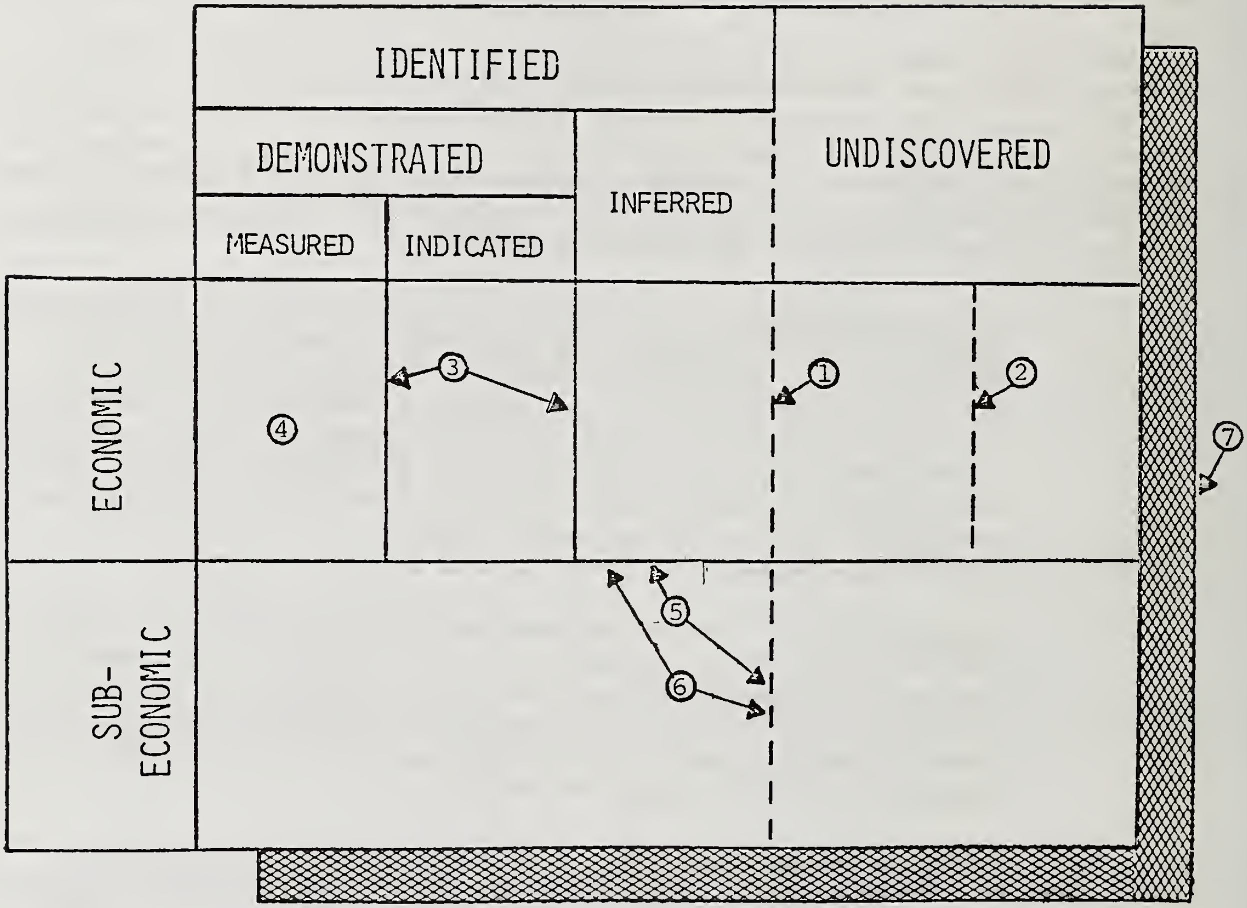
- Undiscovered Margin: this margin includes undiscovered deposits, outside of known fields. These are the target of exploratory tests labeled "New Field Wildcats" in the well classification scheme shown in Figure 3. Although the McKelvey Box separates the undiscovered margin into an economic (or recoverable) portion and a sub-economic portion (including non-commercial deposits and unrecoverable fractions of commercial deposits), the preferred assessment perspective encompass total oil- or gas-in-place potentially residing in undiscovered deposits.

- Access Margin: this margin locates the portion of the undiscovered margin where physical access is controlled by special institutional factors; examples include deposits on the U.S. Outer Continental Shelf subject to federal leasing and onshore lands owned by governments, some of which may never be accessible for reasons of environmental protection.

- Inferred/Indicated Margin: this margin encompasses deposits which generally have been discovered or inferred by geological and engineering work. By definition, the indicated portion resides within deposits which literally are known; it represents the expected results of the secondary recovery phase of development illustrated in Figure 2. The inferred portion represents the expected product of the kinds of exploratory drilling included in categories two through five in Figure 3. The approach to estimating the magnitude of inferred reserves typically consists of applying "growth curves" to measured reserves in known fields, based upon historical experience with the occurrence of extensions and new pays around known deposits.

FIGURE 4

(MODIFIED) MCKELVEY BOX (2)
CONVENTIONAL PETROLEUM RESOURCES
OF THE UNITED STATES



1 Undiscovered Margin
2 Access Margin
3 Indicated/Inferred Margin
4 Pure Intensive Margin
5 Uneconomic Conventional Resources Margin
6 Conventional Resources Technological Margin
7 Unconventional Resources Margin

- Pure Intensive Margin: this margin resides within the measured category (equivalent to the American Petroleum Institute definition of proved reserves). In my framework, this margin represents the addition of productive capacity through the intensification of development of existing proved reserves. Examples of trade terms for this kind of activity are "infill drilling" or "drilling for rate."

- Uneconomic Resources Margin: within the identified portion of the McKelvey Box, this margin represents the backlog of previously non-commercial deposits and unrecoverable portions of commercial deposits. In the undiscovered portion, it represents resources which might suffer the same fate in today's economic and technological environment. In the modified diagram, this margin represents those resources which might become economic mainly because of higher prices.

- Conventional Resources Technology Margin: this margin is similar in concept to the uneconomic margin, except that it responds mainly to improved technology. In the identified portion of the box, certain enhanced oil recovery technologies exemplify this margin; in the undiscovered portion, capabilities to explore and develop in deep water exemplify the technology margin for conventional resources.

- Unconventional Resources Margin: this margin--shown as a third dimension of the McKelvey Box--represents unconventional petroleum liquids and natural gas resources. With respect to U.S. resources and the historical emphasis of most previous U.S. resource appraisal work, this margin includes a long list and potentially massive quantities of resources: most heavy hydrocarbons and tar sands, oil shale, diatomaceous hydrocarbons, tight gas Devonian shales, geopressured methane, methane entrained in coal seams, and others.

IMPLICATIONS FOR OIL AND NATURAL GAS SUPPLY ASSESSMENTS

This framework suggests several things for oil and natural gas supply assessments:

- A satisfactory assessment should include physical information about the resource base and the exploitation process, a description of constraints, and some stimulus.

- In turn, this information should be integrated in a manner which describes how the supply process will respond to changes in the supply environment and the technology of the process.

- The physical information should be organized according to all of the "margins" which are represented in the stock of economically and technological feasible resources relevant to the time horizon of the model.

- At each of these margins, the model should recognize, explicitly or implicitly, the detailed processes which occur at the level of the micro-units of relevance to investment decisionmaking. For exploration, a logical unit of focus is the prospect, and the process description should recognize the complex probability laws associated with the deposition and exploration processes, particularly for resources well outside of known fields. For development, the most logical unit of focus is the reservoir, and the process description should recognize the flexible nature of the optimal development investment decision problem associated with selecting an extraction path over time for a single reservoir. For production, the possibility of speculative withholding should be recognized if (effective) prices may be "unstable."

- In order to appropriately accommodate the full range of relevant margins, the focus of the model should be on productive capacity, not reserves, and on the separate decision to operate available capacity.

- The model should look behind current data reporting systems and the geologic, engineering and conservation doctrine which may muddy the historical record and the forward assessment process with hidden economic, technical and institutional considerations.

CURRENT PROBLEMS WITH THE STATE OF THE ART

This framework, presented in the previous section, provides an outline for identifying problems and opportunities for improvement with respect to oil and natural gas assessment used for energy policy planning the U.S. My discussion of these problems and opportunities is organized into three sections. First, problems associated with the way models attend to the "margins" described in the preceding framework are discussed. Then, other problems related to the ingredient of the supply process labeled "behavior" (in Figure 1) are described. Finally, problems related to the use of resource models and reservations about current modeling trends are described. These three sections, however, are preceded by a brief discussion of several recent events in the U.S. oil and natural gas "record." Hopefully, these can illustrate the urgency associated with the subsequent problems.

SOMETHING HAPPEND TO OIL

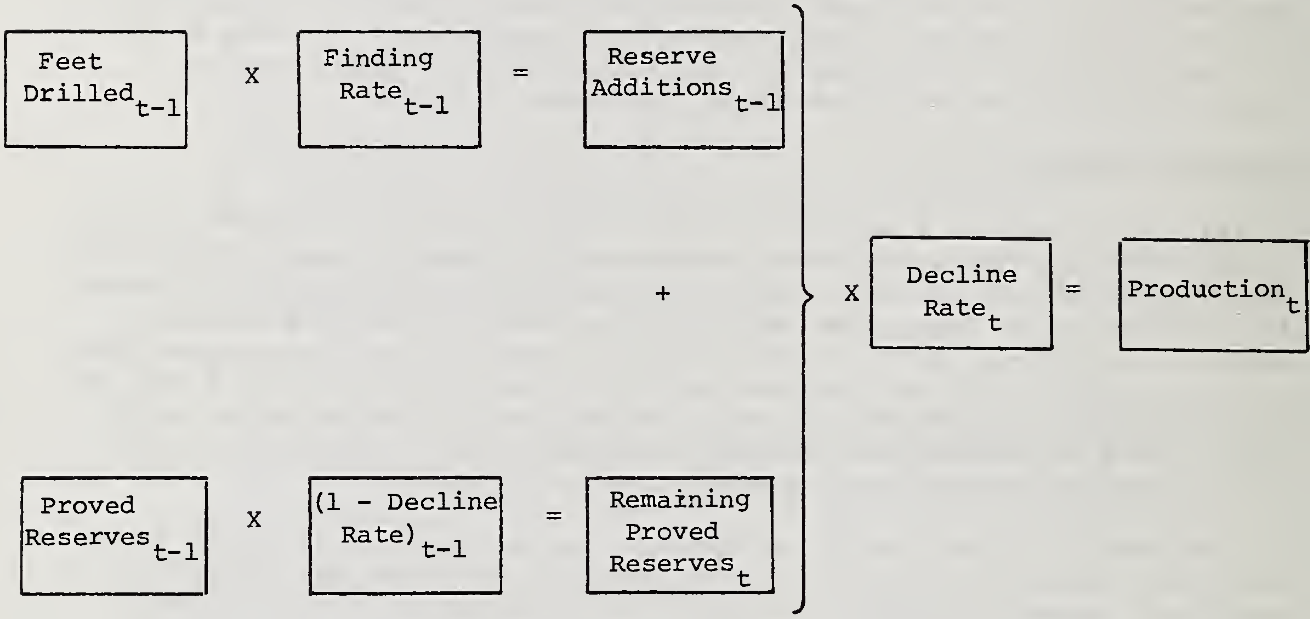
Figure 5 describes a simple oil supply model. The model consists of two parts. One is an accounting representation of the stock of previously proved reserves as they are reduced over time by production. The rate of production for this purpose is represented by a "decline rate," in modeling practice typically a constant ratio of production to proved reserves. The second part is a representation of the reserves addition process, consisting of a rate of drilling activity (measured in terms of either feet drilled or wells) and a finding rate (either per foot drilled or per successful well, often where a well success ratio is included in the formulation).

The rate of activity may be estimated exogenously or endogenously, sometimes as a function of prices and even sometimes as a function of industry cash flow. Where endogenous, the rate of drilling may be related to some expression of the costs of the drilling, the reserves added by the finding rate as a result, and prices. Finally, remaining reserves and reserve additions are summed and multiplied by a decline rate, again often invariant with respect to time or economic conditions, to calculate production.

To put my cards on the table immediately, many U.S. oil and natural gas supply assessments and models, although apparently more complicated on the surface, when stripped to their essentials, are represented reasonably accurately by this simplified model. Further, many of those which claim to develop the rate of drilling activity endogenously, as a practical matter, do not; instead, they simply track overtime exogenously specified drilling constraints. Almost all focus on reserves as the product of drilling and subsequently employ a static decline rate to estimate production. I hasten to add that formal models with which I have been involved heavily are not excluded entirely from these observations.

FIGURE 5

A SIMPLIFIED OIL SUPPLY MODEL



The finding rate portion of this simplified oil supply model is the primary focus of my example. A typical expression of the finding rate function is as follows:

$$\left\{\frac{\text{Oil Reserve Additions}}{\text{Total Oil Footage Drilled}}\right\}_t$$

The relationship typically is estimated by fitting a curve, often of an exponential form, to historical series of each statistic. The fitting also often is done to cumulative series of each, after which the first derivative of the cumulative function is used to estimate the finding rate over time or over successive increments of cumulative total footage drilled.

Figure 6 shows the historical behavior of the oil "finding rate" defined in this fashion. In order to avoid an issue of how to allocate "dry holes" between oil and natural gas, the figure displays the ratio of total reserve additions to successful oil footage drilled. In order to suppress year-to-year variations simply for discussion purposes, the figure plots cumulative reserve additions versus cumulative drilling.

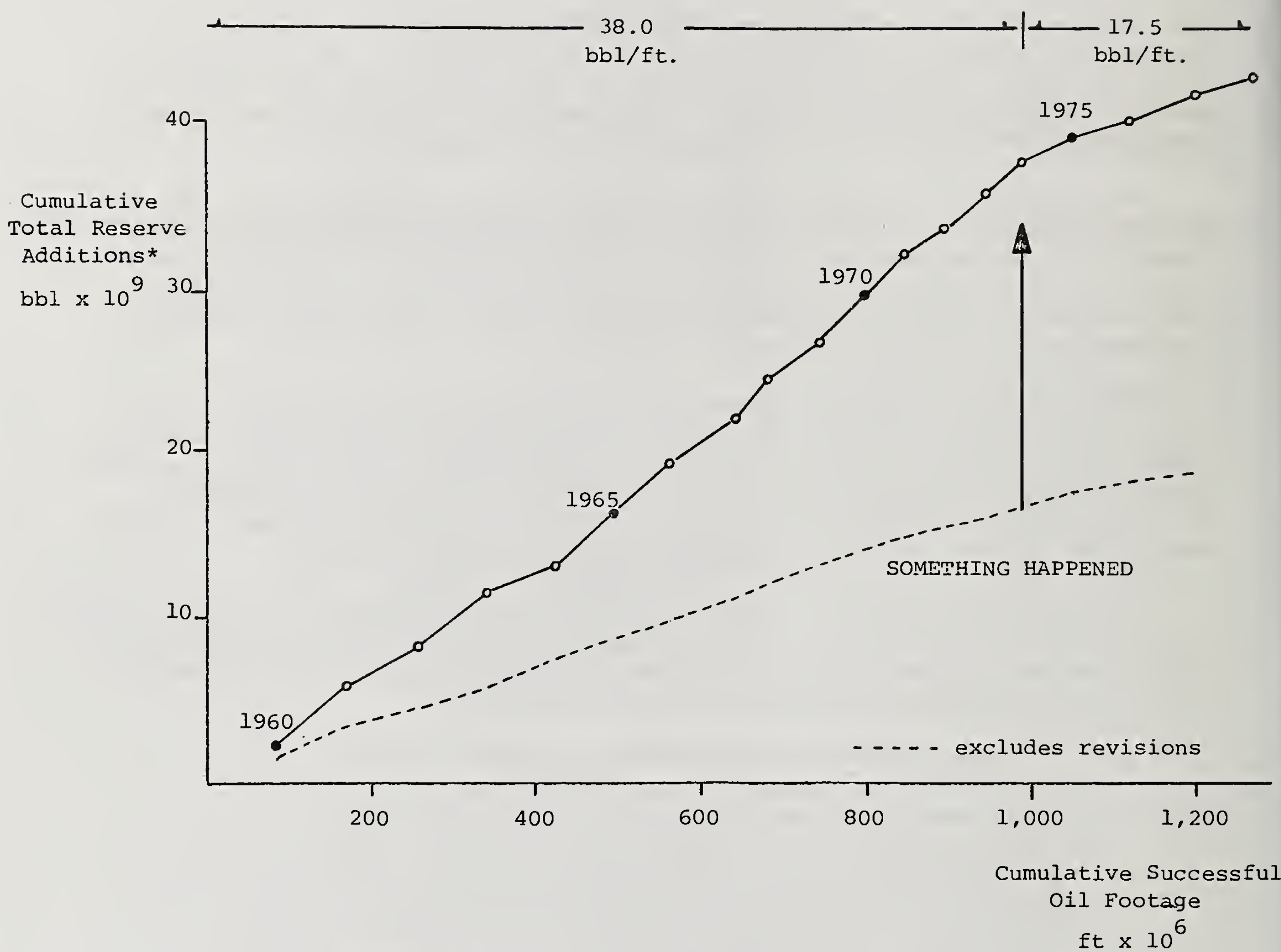
Prior to 1974, this rate appears to behave in a stable fashion, but during and afterward it appears to exhibit a downward kink. Measured crudely, a slope of 38 barrels per foot prevailed from 1960 through 1973 but was replaced by an average of 17.5 barrels per foot in 1974. This change, alledged to indicate abruptly declining productivity of U.S. drilling, has been a central feature of the U.S. oil policy debate during the last five years.

The coincidence of this "kink" and three other events--the 1973-74 oil embargo, much elevated oil prices, and imposition of price controls on crude at the wellhead in the U.S.--is enticing. The modeling problem associated with this simultaneity was summed-up well by Searl (3):

> "There appears to be an almost complete lack of attention to the manner in which historical data on which resource estimates are based have been conditioned by economic and institutional factors... At the micro-level, that is the deposit, reservoir and well level, statistics are clearly conditioned by economic and institutional factors... As a basic proposition, I would assert that all micro resource data, even when stated in physical units without economic parameters, have been tainted by economics and that projections using such data are implicitly projecting certain economic, institutional and technological conditions and constraints."

FIGURE 6

OIL FINDING RATE



* Excludes Alaskan North Slope

Because the interpretation of this abrupt change in apparent drilling productivity has profound implications for modeling and policy, its causes have been the subject of substantial discussion. At least four hypotheses have been offered:

- A New Geological Epoch: This hypothesis contends that the oil supply process encounters sequential epochs, each of which is distinguished by an abruptly lower finding rate. This view holds that the recent change will persist but that finding rates will hold steady at this new, lower level for a substantial period before descending once again.

- Changed Exploration Focus: This hypothesis resembles the geologic epoch concept, except that the abruptness is not caused by discontinuities in nature but rather by explorationists turning to the readily available backlog of submarginal prospects and previously non-commercial known deposits as a first response to changed prices and U.S. price regulations. This view holds that the low rate is temporary; soon the rate will return to a trend slightly below the pre-1974 level. The failure to return completely will be caused by the smaller-sized future discoveries which will be commercial under higher prices.

- Changed Drilling Mix: This hypothesis adjusts the preceding one to account for the changed economics of development under higher prices. It suggests that a short run emphasis on adding productive capacity at the pure intensive margin, without accompanying reserve additions, caused this aggregate "finding rate" statistic to yield misleading results. Simply, if the denominator of the finding rate were inflated by drilling clearly not intended to add reserves, the ratio of reserve additions to total drilling would fall. But because the backlog of infill drilling opportunities also is limited, the recent finding rate experience will be temporary and soon will rise to a level slightly below the earlier rate (unless even higher future prices justify additional intensification of development). Advocates of this hypothesis also note the peculiar incentives for drilling "stripper wells"--wells producing 10 or less barrels per day--contained in U.S. oil pricing regulations.

- The "Devil" Hypothesis: Because the historical data are provided by the oil industry, this hypothesis claims that they are false and are biased toward creating a case for abandonment of price controls.

In order to test these hypotheses in a thorough and logical manner, a necessary first step would be to decompose the historical record along the lines of the seven margins identified earlier in my framework. Unfortunately, publicly-available data may be inadequate to subject these hypotheses to this acid test. For various reasons, mostly related to a lack of detail and proper categorization, I believe that the historical record remains unresolved and unexplained. Somewhat catastrophically, the most fortuitous natural experiment of all time for aiding empirical understanding of the U.S. oil supply process remains obscured by data obstacles.

A major hint, however, resides in a category of reserve additions labeled "revisions," defined as follows by the American Petroleum Institute (4).

> "Both development drilling and production history add to the basic geological and engineering knowledge of a petroleum reservoir and provide the basis for more accurate estimates of proved reserves in years following discovery. Changes in earlier estimates, either upward or downward, resulting from new information (except for an increase in proved average) are classified as 'revisions'. Revisions for a given year also include (1) increases in proved reserves associated with the installation of improved recovery techniques; and (2) an amount which corrects the effect on proved reserves of the difference between estimated production for the previous year and actual production for that year."

In the period from 1960 through 1973, revisions accounted for 55 percent of cumulative reserve additions, averaging approximately 1.3 billion barrels annually. Since 1973, revisions have averaged .8 billion annually. A U.S. oil finding rate, excluding revisions, also is shown in Figure 7. Note that this exclusion improves the regularity of this function, but does not remove the "kink" entirely. But the definition of this category itself raises numerous, unresolved questions. Something happened. But what?

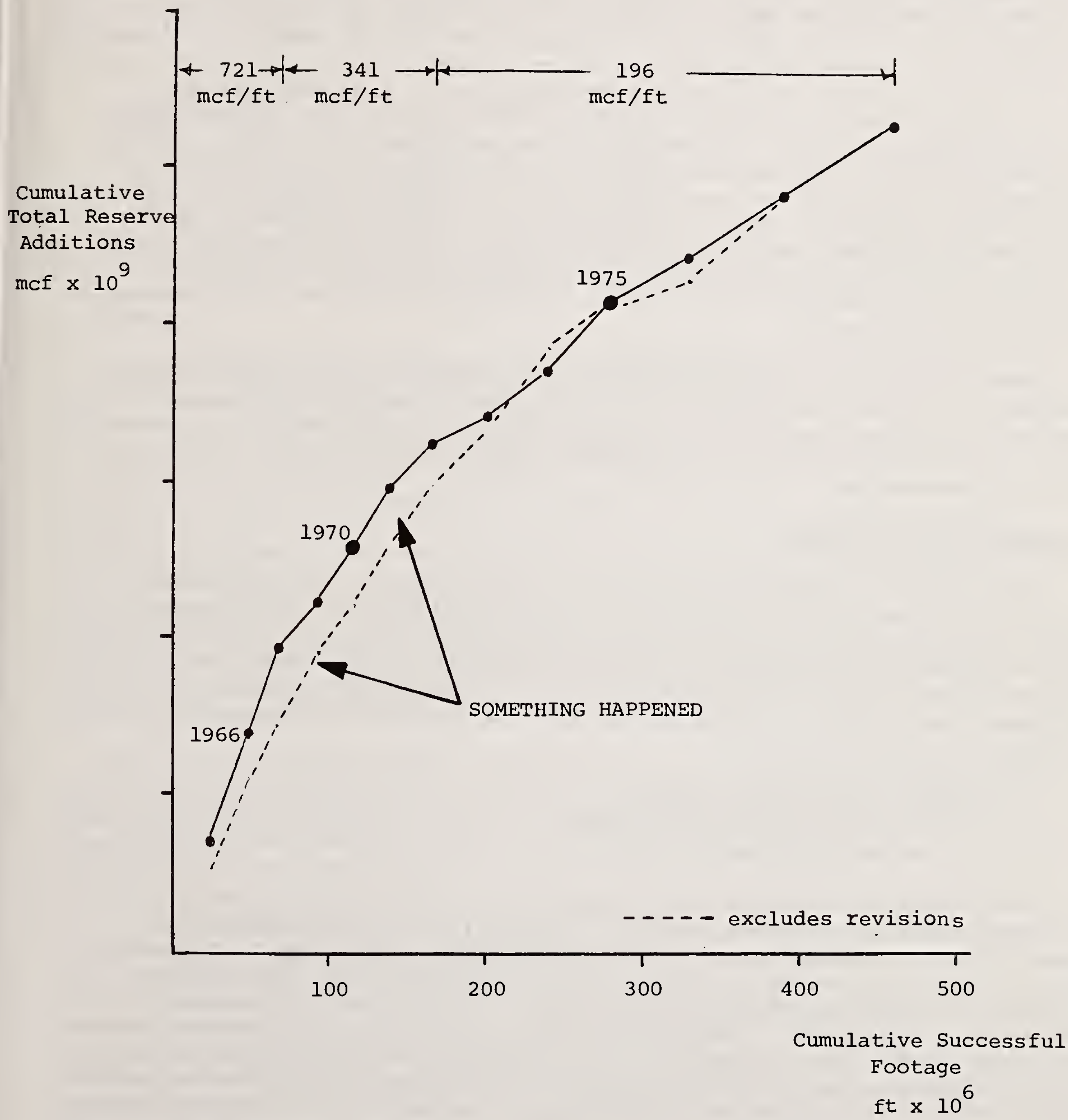
SOMETHING HAPPEND TO GAS

Figure 7 shows a similarly defined "finding rate" for non-associated gas, which is the product of drilling identified to gas in U.S. drilling data systems. This rate appears to evidence two "kinks," one at 1968 and another at 1972. The coincidence in this case is that 1969 is the point in time where U.S. interstate gas transmission systems initially became unable to obtain contracted values of natural gas from domestic producers.

The significance of these "kinks" for modeling U.S. natural gas supply and for domestic policy also is profound. Hypotheses identical to those put forth for oil have been offered to explain these kinks in U.S. gas finding experience. But an additional one related specifically to natural gas revisions also has been added to the list.

FIGURE 7

NON-ASSOCIATED GAS FINDING RATE



Traditionally, revisions have accounted for a much smaller share of annual reserve additions for natural gas than for oil. But beginning in 1969, a traditional, historical series of net positive natural gas revisions turned negative in a magnitude equal to their former, positive level. Suppose that, in reality, the positive side of the net revisions process remained unchanged. Then, from 1969 through 1978 the series of net negative revisions for natural gas (associated-dissolved and non-associated) would indicate that 30 trillion cubic feet of proved gas reserves somehow have been "written off." One hypothesis holds that these reserves actually were not "proved" in the first place. Rather, requirements to insure extended deliveries over long periods of time, associated particularly with contracts to sell gas to the interstate market, may have caused producers to include possible and even more speculative reserves in their reserve estimates. Of course, when it became impossible to deliver contracted volumes, it also became necessary to write down the earlier reserve estimates. If so, the record prior to 1968 is tainted with phantom proved reserves and the post-1968 record is tainted by the writing-off process.

Similar to oil, the true character of these kinks cannot be determined readily from publicly-available data. In Figure 7, however, excluding revisions from the aggregate finding rate appears to moderate the kinkiness. Yet, the modeling question still remains, simply because the pre-1969 record cannot be sorted out.

PROBLEMS AT THE MARGINS

The preceding examples suggest that a lack of attention to individual supply margins in U.S. oil and natural gas data collection systems defeat explanations of the historical record. Assessments of the U.S. oil and natural gas supply prospects tend to embody only the margins that the available data reveal. In descending order, according to the amount of emphasis they receive in existing models, as well as the literature, the prevelant margins are:

- The Undiscovered Margin
- The Inferred/Indicated Margin
- The Access Margin

Subjectively, I judge that order-of-magnitude differences in emphasis distinguish even these three margins in modeling and resource appraisals. Because of attention primarily to the undiscovered margins, most assessments focus on "proved reserves" as the controlling output of the supply process, thereby excluding the "pure intensive margin." And as will be discussed subsequently, development alternatives residing within the undiscovered margin generally are treated inadequately. Also missing generally are the uneconomic resources margin, especially in the identified portion of the resource base; the conventional resources technology margin; and the unconventional resources margin. Of course, there are exceptions. But I know of no model, or group of models used collectively, which are uniformly complete with respect to attention to these margins. For the U.S. resource base and for short-range, mid-range, and long-range estimates, this situation is less than ideal. The forecasting result may be an overly pessimistic oil and natural gas supply outlook, too little price responsiveness of supply, and perhaps misplaced emphasis in the policy process. What, then are the opportunities for improvement?

Undiscovered Conventional Resources Margin

Most advances in the state-of-the art appear to be focused on this margin. This singularity of emphasis itself may be problematic. Briefly, the trend is toward closer linkage of geostatistical models and economic models, accompanied by the use of stochastic modeling techniques, to grapple with the complex probabilistic laws associated with geological and search processes. Accompanying this is a parallel effort to provide a three-dimensional picture of deposits.

Even at this stage of advance, heroic estimating problems are encountered immediately, especially for immature petroleum provinces. But the full range of estimating problems goes well beyond estimating sizes, locations and depths of deposits. On the undiscovered resources margin, economic linkage typically is provided by an accompanying reservoir development model. In principle, to work out an optimal development and production profile for a deposit, much more must be known. The economics of development center around the development well. Since drilling costs dominate development investment, the commercial attractiveness of a deposit hinges on the time profile of production associated with each well. Estimating well performance requires knowledge of additional physical attributes of a deposit: permeability, pay thickness, viscosity, drive mechanism and others. Suddenly, the requirements for stochastic variables multiply, and data and estimating problems compound. Concerns about interdependencies between these myriad variables also crop up.

Perhaps as a result, improved models of the undiscovered resources margin tend to use a development model mainly to estimate minimum field sizes in a rough way, using some "base level" development program of static intensity. Consequently, the effects on supply of development alternatives at the undiscovered margin may require further work...data permitting.

A less sophisticated but also improved method of representing the undiscovered resources margin takes the form of a marginal extraction cost curve. The curve typically consists of a function relating cumulative barrels of reserves and minimally acceptable prices required to justify extraction. I have yet to encounter one of these curves which explains its meaning. If any deposit can be developed in different ways--the choice among which, for example, is the one which maximizes expected profits--no unique "extraction" cost may exist for any deposit. Further work is required to determine what these curves imply in terms of development and how, as a result, they should be used in models.

Access Margin

The access margin of major relevance to U.S. oil and natural gas centers on government-owned lands, principally in lower-48 offshore areas and in Alaska and its offshore waters. The usual approach to this margin employs a leasing schedule; a forecast of acreage offered per lease; a fraction of acreage sold per offering; and an estimate of the share of an aggregate resource base underlying the leased acreage. In reality, the specific acreage to be offered for sale seldom is known far in advance. Retrospectively, the economic and institutional factors affecting the choice of which acreage to offer, even at the basin level, and the fraction of the offered acreage actually leased usually are unknown.

These shortcomings typically are resolved by a modeling procedure which assumes that the best acreage in each offshore area is offered first and that the share of offered acreage leased remains constant over time, regardless of the economic and institutional environment. Review of U.S. leasing practices contradicts the first part of the procedure, and changed economic conditions probably contradict the latter. Consequently, there are clear opportunities for improved knowledge and modeling techniques on the access margin.

Inferred/Indicated Margin

Where treated explicitly, most resource appraisals and most models deal with this margin deterministically and in a highly aggregated fashion. In the U.S., the problems with this approach is signaled by the fact that the "growth curves" employed in U.S.G.S. 725 to estimate inferred reserves appear "shaky" when exposed to simple statistical significance tests. Consequently, the quantity and quality of resources residing on the inferred margin may not deserve a deterministic treatment, either for appraising the resource base or for modeling supply possibilities on this margin.

In addition, historically-derived "growth curves" may not be independent of the sizes of fields and reservoirs included in a truncated sample--only those of commercial worthiness in the past. If, as work concerning undiscovered resources suggests, larger deposits tend to be found first, a question can be raised about whether uniform growth curves estimated from experience with large deposits are appropriate for estimating inferred reserves, especially if deposit sizes decrease over time.

Pure Intensive Margin

Most existing assessments do little more with current proved reserves than extrapolate future production with a fixed decline rate, usually equal to an historical production to reserves ratio. Even in the most advanced, formal supply models, it always is interesting to observe one small "box" at the tail end of an otherwise elaborate model labeled "production from existing reserves."

Yet viewed at the most aggregate level, the U.S. production to reserves ratio for oil outside of Alaska has increased from approximately .08 to .13 since the effects of market-demand prorationing disappeared in 1971. This suggests that the U.S. oil supply process may be changing on the pure intensive margin in a way which existing data and oil supply models do not represent. Unfortunately, the situation for gas is sufficiently confused by negative revisions that it is impossible to say whether a similar change has occurred. Also unfortunately, the manner in which development drilling activity is reported--as a lump, undifferentiated between development purely to produce existing reserves faster and other activity which adds both new reserves and productive capacity--makes the importance of the pure intensive margin difficult to evaluate. This could be a worthwhile starting point for further work related to this margin.

Uneconomic Resources Margin

The supply assessment problems on this margin can be summed up succinctly: virtually nothing is known. The historical record of discoveries represents a severely truncated distribution. Possibly half of the deposits actually tested and found to contain hydrocarbons are excluded from the record because they previously were non-commercial. Perhaps the only possible next step in this category would be to improve the process of reporting dry holes in the future.

Conventional Resources Technology Margin

Three kinds of modeling issues exist on this margin. The one which thus far has received most attention concerns enhanced oil recovery, mainly achieved by tertiary recovery methods. This portion of the technology margin has been worked over reasonably well, considering the dilemmas associated with forecasting prospects for emerging technologies. With respect to opportunities for improvement, however, two areas may merit further work. First, a large share of today's assessments of enhanced recovery prospects consist of extrapolations from a relatively small number of major fields to a large universe. Because of the large role for enhanced recovery evidenced in many current forecasts, these extrapolations need closer attention. Secondly, current enhanced recovery estimates--even those related to the application of reasonably well-known methods--often employ development plans whose intensity is static. Consequently, the "pure intensive margin" within enhanced recovery may need additional attention.

Two other issues on the conventional resources technology margin center on drilling costs and current water-depth limits on drilling and production. U.S. drilling costs appear to have been escalating at a rate well in excess of the general cost of living. There are three potential explanations: costs of traditional materials and other inputs are rising because of conventional cost-push inflation; conversely, rising activity levels are causing inflation of a demand-pull variety; or a qualitative change in drilling inputs has occurred because of a higher oil price structure. In the past, drilling technology has exhibited substantial and continual improvement. Since drilling costs account for the bulk of oil and natural gas supply investment, a better understanding of what has caused costs to rise rapidly in the last decade and a better basis for estimating future rates of cost change would prove invaluable to the modeling process. Finally, the future for drilling and production in deeper waters, accompanied by a better assessment of deeper water prospects, could prove important and helpful.

Unconventional Resources Margin

This margin, of course, represents another dimension of technological advance and changed economic conditions. With due respect for the difficulties accompanying technological forecasting problems, the current situation might be improved in at least two areas. The first is a matter of emphasis, particularly with respect to resource assessment activity. Even for unconventional resources closest to commercial practicability, the extent and quality of the resource base largely are unknown. The resource appraisal attention they are receiving is much too small in relation to their potential.

The second area for improvement simply may be one of semantics or communication. Typically, potentials for unconventional oil and natural gas resources are quoted in terms of recoverable reserves and in the same manner as conventional crude oil and natural gas. What seems to be missing--and therefore confusing to policymakers--is the much more strenuous effort required to add an equivalent amount of productive capacity from these resources, simply because of low densities of energy material in the extraction streams (e.g., geopressured methane and oil shale) or low per well production rates associated with these resources (e.g., tight gas or Devonian shale). It may be that a new unit of measure is required to describe the different productive capacity potential of these resources compared to conventional crude oil and natural gas.

PROBLEMS WITH BEHAVIOR

For purposes of policy-related energy resource modeling, the appropriate objective for modeling behavior is to simulate the decisionmaking process of the industry or other agents under whose control investment and production decisionmaking reside. Hypothetically, the organizing principal for the behavioral ingredient of resource models is straightforward: capital and other inputs should be allocated to each of the individual supply margins such that their marginal products are equal on all margins at each point in time and at overall rates which yield extraction paths over time which satisfy some objective.

Three approaches toward implementation of this behavioral principal are exhibited in oil and natural gas models: econometrics, simulation, and optimization. The appropriateness of each technique has been the subject of extensive and sometimes heated debate, which will not be extended or amplified further here. In my opinion, these techniques tend to converge in the limit. Much criticism of econometrics appears to be focused more on an absence of explicit process-oriented structure in previous econometric supply models and on occasional inattention to depletion effects in some others. These are matters of physical structure rather than behavior. Similarly, simulation often is faulted for a lack of underlying theory; I leave it to economists to justify that current economic understanding of the behavioral aspects of the oil and natural gas supply process deserve to be elevated above the level of rough hypotheses. Finally, optimization is criticized for the overly normative nature of this approach. I have observed, however, that optimizing models often are made "realistic" with the ample use of essentially behavioral constraints, the origins of which should not leave econometricians or advocates of simulation embarrassed in their wake.

Problems on the Margins Again

Earlier, the modeling framework distinguished the decisionmaking focus appropriate for exploration from the one appropriate for development and production. Also, the different physical characteristics of the search process and the more engineering-oriented development process were noted.

In contrast to these physical differences, most models do not incorporate behavioral rules which distinguish exploration and development. The potentially special "gamblers ruin" aspect of the exploration process has been raised by Ramsey (5). Work thus far on the different behavioral properties is sufficiently suggestive that the proposition of a difference, empirical testing, and differentiation of behavioral tendencies between exploration and development may deserve enhanced attention on the part of oil and natural gas modelers.

Rig Constraints

A special dilemma regarding oil and natural gas supply faces today's optimization models. Almost without fail, models of this variety choose to drill-up vast quantities of the remaining U.S. resource base in the first period of their forecasts. The usual solution is to include drilling constraints in the model. The constraints usually are defined outside of the models and, as a result, an elaborate optimizing apparatus, in fact, forecasts simply by tracking exogenously specified drilling constraints. The constraints typically are a matter of the modeler's judgment.

If an analytical resolution of this phenomenon associated with optimization models is to be provided, there are at least three avenues for further work. One consists of the resource base descriptions and cost data which cause these models to see such a wealth of attractive drilling targets immediately. Another concerns the potentially different behavioral ingredients of exploration and development decisonmaking. The third is the potential, further optimizing problem--describing the investment decisionmaking of drilling equipment vendors and drilling contractors--which needs to be included in these models. Among these, a better understanding of the capacity formation process of the drilling industry would be especially helpful for improving oil and natural gas supply models.

Investment Criteria

Measures of the decisionmaking criteria of those who control oil and natural gas properties enter most supply models. Typically, the variable is a discount rate or an equivalent return on investment measure. In the recent U.S. oil price decontrol debate, alternative hypotheses concerning investment criteria were put forth. A recurring one concerns the relationship between industry cash flow from operations and rates of activity. The suggestion is that there is something special either about the capital markets as they apply to oil and natural gas or about the investment criteria which guide exploration (another variant of the behavioral issue raised earlier). The validity of this contention needs further inspection, if for no other reason than it has played a large role in U.S. oil and natural gas pricing policy and, if correct, would represent a new and different direction for many supply models.

For models which use an explicit discount rate, or its equivalent, much work needs to be done. To my knowledge, no solid and current empirical basis supports the rates used by most oil and natural gas supply models. Because of the capital-intensive, front-end loaded nature of drilling and the drawn-out production profile associated with oil and natural investments, supply models are strongly sensitive to their discount rate assumptions. Consequently, more work is required in this area.

Uncertainty, Foresight and Non-Competitive Markets

No commentary on opportunities for improving resource models would be complete without mention of these persistent, fundamental problems. Among these, however, the one receiving attention here is foresight, because it has become a major analytical question surrounding U.S. energy models used at the federal level. Also, it is of interest in the debate between econometrics, simulation and optimization--one sometimes humorously described as the choice between perfect hindsight and perfect foresight.

The prevailing solution to the problem appears to be "rational expectations." But from the perspective of implementing a forecasting model, what specifically are the expectations? More work is needed here, too.

RESERVATIONS AND OTHER PROBLEMS

Current trends in oil and natural gas supply modeling prompt a question about where the state-of-the-art is going and, if continued, another modeling problem that may be encountered. I would be pleased to see both receive serious comment and further work.

The current trends in oil and natural gas supply modeling are laden with overtones of ever increasing model detail and, in turn, models with ever expanding computing budgets. Most supply models cannot stand alone for many uses. Policy-relevant forecasting almost always requires linkage to other systems which affect oil and natural gas supply: transportation, conversion, distribution, and end-use. And because energy resources of all kinds compete in numerous end-uses, oil and natural gas supply models often become one subpart of a comprehensive, complex and large energy market model involving other fuels and often regional detail. If these energy market models are to be practical, how much of their computing budget can be consumed by the representation of the oil and natural gas supply process? And if not a large share, how are detailed, complex and large oil and natural gas supply models to be linked effectively to them?

Finally, the prospect of more detailed, more complex and larger oil and natural gas supply models deserves introspection by modelers and attention by others. How much expansion of detail is productive? How much is promoted by elaboration of modeling possibilities suggested by technique...without accompanying data to justify its use? And most importantly, will the subsequent results be an improvement from the perspective of the energy policy process?

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THE EVOLUTION IN THE DEVELOPMENT OF PETROLEUM RESOURCE APPRAISAL PROCEDURES IN THE U.S. GEOLOGICAL SURVEY

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INTRODUCTION

The state of the art for appraising petroleum resources has advanced rapidly during the past decade because of the growing awareness of the need for petroleum resource estimates for the formulation of reasonable energy policies and long-range planning.

Events triggered by the Arab-Israeli war of 1973 focused the attention of the world on energy problems and on the inherent uncertainty of the estimates made for petroleum resources. Many nations will need realistic forecasts of future petroleum supplies; these estimates of the distribution and magnitude of oil and gas resources throughout the world must be based upon the most reliable methods and data available. This situation calls for a high level of domestic and international cooperation among appraisers of petroleum resources.

Published appraisals of oil and gas resources in the United States date back at least 70 years (Thomsen, 1979). The first published estimates by the U.S. Geological Survey in 1909, by David T. Day, covered the known producing areas of the conterminous United States which at that time had a reported cumulative production of 2 billion barrels, 4 billion barrels of proved reserves and 4 to 18 billion barrels of potential supply. Since then, many published estimates have been made by the USGS, industry and individual researchers. In the 20-year period after 1955, the amounts reported from these appraisals varied widely, giving rise to great confusion and much controversy. Attempts to compare these estimates revealed that many of them were based upon inadequate data and were poorly documented. Each effort had utilized different assumptions, definitions, appraisal methods, geographic boundaries and data bases and therefore should not be compared. Increased efforts during recent years have been directed toward resolving some of these major problems, and there is evidence that progress is being made. This paper discusses the efforts made by the USGS within the last 6 years to improve upon its methods for petroleum resource appraisal.

METHODS

Many methods exist for estimating petroleum-resource potential with numerous variations in the basic techniques. Each method requires a certain level of knowledge or degree of available information on the area to be assessed. Each method, however, has recognized limitations. Problems arise because of misinterpretation of results and lack of recognition of these limitations on the methods and data used. Emphasize must be made that no single technique has universal application or appeal -- nor is there unanimity on the results. In 1974, the Oil and Gas Resource Appraisal Group was created by the USGS to devise and study resource appraisal methods and to apply these methods in assessing the nation's petroleum resources as a full-time responsibility.

The first nationwide appraisal of the undiscovered oil and gas resources [1/] for more than 100 geologic provinces was published by the Resource Appraisal Group in 1975 as USGS Circular 725, "Geological Estimates of Undiscovered Recoverable Oil and Gas Resources in the United States" (Miller et al. 1975). In this study, the appraisal methodology was applied on a broad scale and designed for the geologic basin or geologic province. The estimates of the undiscovered resources were made: (1) by reviewing and analyzing all available geological and geophysical information compiled on more than 100 geologic provinces; (2) by applying resource appraisal techniques, which included extrapolations of known producibility into untested sediments of similar geology for well-developed areas, and volumetric techniques using geologic analogs with ranges of yield factors; (3) by using group appraisals (in a modified Delphi procedure) determined by geologic experts applying subjective probability procedures; and (4) by reporting final results as probability ranges rather than as single number values.

1/ Undiscovered resources are defined by the USGS as follows:

Undiscovered Resources: Quantities of a resource estimated to exist outside of known fields on the basis of broad geologic knowledge and theory.

Undiscovered Recoverable Resources (Potential Resources): Those economic resources, yet undiscovered, which are estimated to exist in favorable geologic settings.

Original in-place Resources: Includes all discovered oil and gas reserves (produced and remaining), and the undiscovered resources believed to exist, both recoverable and nonrecoverable (Miller et al. 1975).

Since the organization and the first publication of the Resource Appraisal Group's work, the evolution in petroleum resource appraisal procedures has been significant. The methods developed and employed by the Resource Appraisal Group are designed to emphasize the compilation and evaluation of all available geological and geophysical data by geological basins or provinces. Resource estimates can be made on any level of data; however, the amount of data available will determine the method or methods to be used for the appraisal. The method or procedure used can change as the amount and nature of the available data change within a specific basin.

In the frontier stages of exploration when some information exists on the gross interpretation of basin geology, and, when the principles of petroleum occurrence from worldwide experience are applied, subjective judgment may be used with minimum amounts of data as a basis for the assumption of the presence or absence of potential hydrocarbons. As the data base grows, because of increased exploration, and as the results of geophysical surveys, drilling, and geochemical data become available, methods using more objective data should become increasingly dominant. The methods used in making estimates may evolve to the level of assessing exploration plays and may eventually focus on making estimates of undiscovered prospects, if this level of resource assessment is desirable. If abundant and detailed data are available, the choice of the method to be used may become more dependent upon the availability of the estimator's time, the effort involved, and the purpose of the resource estimate. The quality of the estimate is, however, dependent upon the quality of the geologic data and studies upon which the estimate is based (Miller, 1979).

DEVELOPMENTS IN THE USGS SINCE 1975

Since the initial studies and the resource-appraisal methods described in Circular 725, the older methods have been refined, alternate resource appraisal techniques have evolved, and new and more detailed oil and gas data have been compiled, particularly field and pool information, for stratigraphic units within specifically designated pilot areas in the United States. By using the new information increasingly available to the Resource Appraisal Group and the refinement in resource appraisal methodology, resource assessments can now be made for individual stratigraphic units and by depth increments within many basins. Results of the Permian Basin study conducted by the Resource Appraisal Group are reported in this paper to illustrate the use of these methods. Estimates can also be made on the probable size and number of fields in which the remaining undiscovered resources may be found within a semi-mature or maturely explored area and on the probable depth increments within which these fields are likely to occur. The Gulf of Mexico studies, completed by the Resource Appraisal Group are also used to demonstrate these methods. Additional refinements of resource assessment methods have been completed for the application of computerized geologic models using play analysis techniques for the appraisal of conventional petroleum resources in the National Petroleum Reserve of Alaska, and of unconventional natural gas resources in the Devonian black shales of the Appalachian Basin.

CURRENT METHODS USED BY THE USGS

The following discussion reviews the evolution and development of each of the basic resource appraisal methods currently being used by the Survey's Resource Appraisal Group. Specific applications from current studies by the Resource Appraisal Group are reviewed for each method. Although the Group works on both oil and gas resource estimates, and the methods are often similar for both, this paper will be directed primarily to the application of methods and results for natural gas resources.

METHODS USING VOLUMETRIC-YIELD ANALOGS

Volumetric-yield techniques have been used in a wide variety of ways in making petroleum-resource estimates. These techniques range from the use of worldwide average yields expressed in barrels of oil or cubic feet of gas per cubic mile of sedimentary rock, or per square mile of surface area (assuming constant thickness) applied uniformly over a sedimentary basin, to more sophisticated analyses in which the yields from a geologically analogous basin have been used to provide a basis of comparison. The pioneer works by Weeks (1950), Zapp (1962), and Hendricks (1965) are illustrative of early techniques.

In Circular 725, wherever yield factors were used,it was done in the context of a reasonably sound consideration of the geology of the basin or province and the selection of a geologically analogous basin or province. The records of 75 North American basins were compiled, the oil and gas yields being expressed per cubic mile of sediment as determined from well-explored areas within these basins, to establish a scale of hydrocarbon yields for geologically analogous basins. Figure 1 shows a frequency distribution of hydrocarbon yields for these basins. The productivity of a basin may range from less than 1,000 barrels per cubic mile of sediment to more than 3 million barrels per cubic mile of sediment, as in the exceptional case of the Los Angeles basin.

The accuracy of this method depends on the expertise of the geologist who compares the similarity of the geology of an unexplored basin with that of a developed basin, prior to the selection of a hydrocarbon yield used to make a forecast for the potential of the unexplored basin or unexplored part of a basin. A highly recommmended approach to this method is the selection of a representative range of analogous yields to which probabilities are assigned to determine a minimum and maximum estimate for the potential resource. The results obtained by this method can be useful on a broad regional basis or in a reconnaissance-type estimate of the resource potential, particularly in evaluating frontier or unexplored geologic areas.

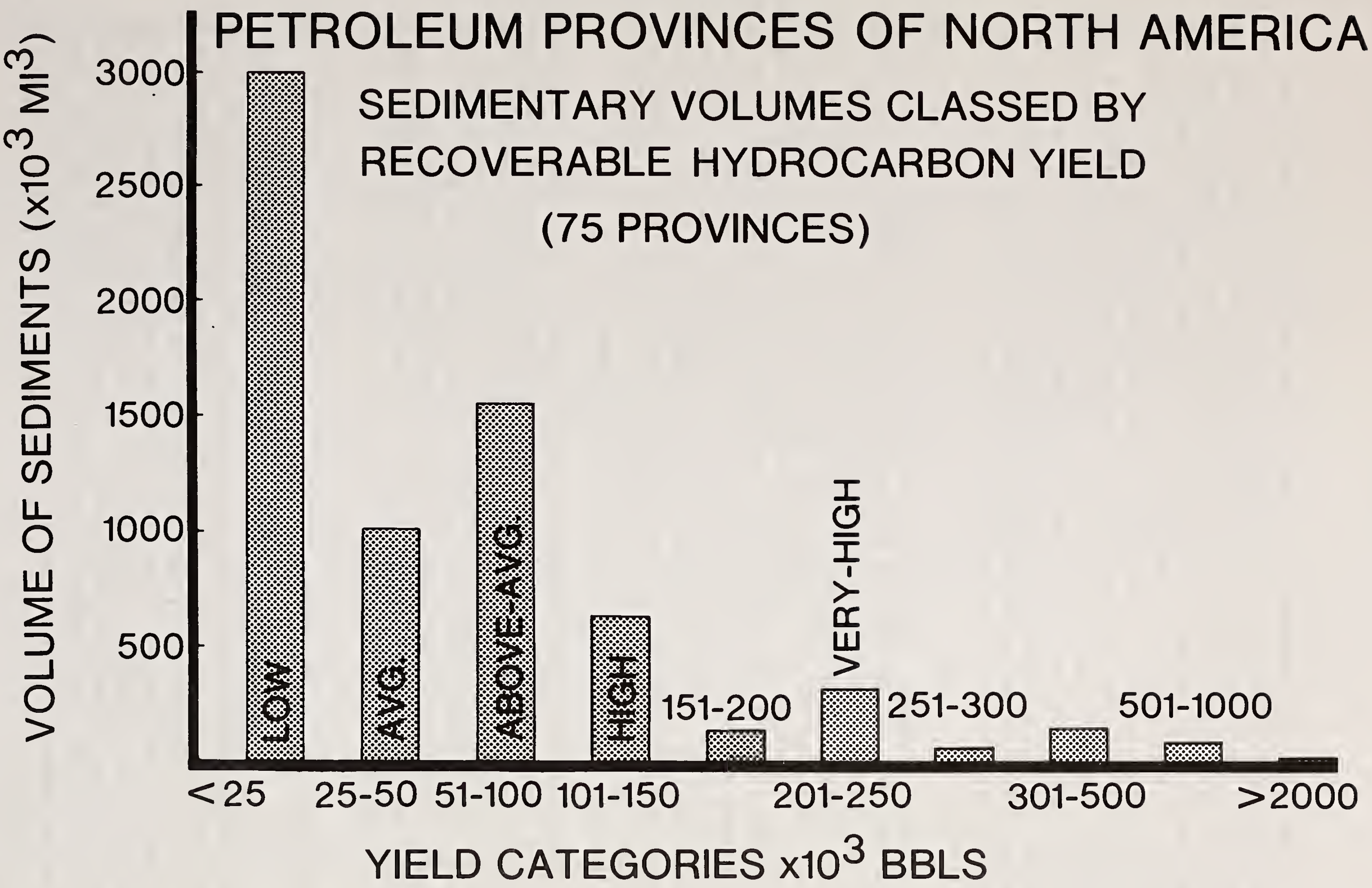


FIG. 1 - Frequency distribution for total sediment volumes from 75 petroleum provinces of North America versus their recoverable hydrocarbon yield.

Volumetric-yield methods have been refined recently by the Survey to categorize hydrocarbon yields for specific stratigraphic units, which may be characterized by lithology, environment of deposition, geologic age and basin classification, or by relation to geologic structures and basin tectonics. In this way, the potential of individual stratigraphic units in unexplored or partly explored basins may be evaluated by using analogous stratigraphic units from known basins. Hydrocarbon yields expressed as probability distributions are used for each potential stratigraphic unit because subjective judgments must be made on the various combinations of favorable geologic characteristics chosen for the analogs. Estimates of the total potential for the province are the sum of the individual stratigraphic units aggregated by using Monte Carlo simulation. The aggregated results of the resource estimates are reported in the form of a probability distribution.

A recent study by the Resource Appraisal Group that applied various aspects of this methodology has been completed on the Permian Basin of west Texas and southeastern New Mexico (Dolton et al. 1979). In this study, separate analyses were completed for the geologic units in the Permian, the Carboniferous, and the older Paleozoic, with individual appraisals made for each unit. In addition, the province was also evaluated at depth increments of 0-10,000 feet, 10,000-20,000 feet, and deeper than 20,000 feet. Drilling density maps penetrating each of these units as plotted from computerized well-data systems played an important part in these assessments. Figure 2 shows an example of the individual units independently analyzed for resource assessments of natural gas in the Permian Basin. Figure 3 shows examples of resource estimates for natural gas reported in the form of probability distributions for the lower Paleozoic of the Permian Basin.

The volumetric-yield method is a valid procedure for resource appraisal, providing care is taken in the selection of geologic analogs and in documenting and qualifying the results properly. There are more sophisticated resource appraisal methods, which, when closely analyzed, reveal a key volumetric element within their respective systems. This volumetric element usually consists of individual variables for either stratigraphic units, or exploration plays, or reservoirs, which, when mathematically manipulated, provide a volumetric estimate.

DISCOVERY-RATE OR BEHAVIORISTIC METHODS

Performance or behavioristic methods are based upon the extrapolation of past experiences from historical data such as discovery-rates, drilling rates, and productivity rates, and upon the fitting of past performances into logistic or growth curves by various mathematical derivations that are projected for the future. These techniques are not directly applicable to unexplored or nonproducing areas or to any area that is not a geologic and economic analog of the historical model. Generally speaking, they are most applicable to the later stages of exploration in a maturely explored area. Well-known examples of these models are: Hubbert's growth curve projections (1962, 1974); Arps and Roberts (1958); Zapp (1962); and the National Petroleum Council (1973).

Permian Basin Assessment Matrix

Estimates of Undiscovered Non-Associated Gas in-place (Trillion Cubic Feet) (95% - 5% Range, and Mean) *

	G e o l o g i c A g e			
Depth Interval (ft.)	Permian System	Carboniferous/ Systems	Older Paleozoic System	Total Ages
0 - 10,000	.17 - 1.33 .57	.27 - 3.81 1.40	.42 - 3.00 1.34	1.43 - 5.98 3.31
10,000 - 20,000	.01 - .08 .03	.12 - 1.38 .53	3.30 - 22.09 10. 14	3.73 - 21.90 10.70
20,000 - 30,000	Negligible	.01 - .20 .06	.71 - 4.94 2.23	.75 - 4.86 2.29
Total	.19 - 1.31 .60	.64 - 4.34 1.99	6.16 - 25.45 13.71	8.24 - 28.27 16.30

*Values correspond to the 95% and 5% probability that there is at least that amount. (Source, Dolton, et al, 1979)

Figure 2: An assessment matrix of the individual units analyzed for natural gas by geologic age and depth increments. The values represent the 95 - 5 percent range and the mean for undiscovered gas (in-place)

Permian Basin-Resource Appraisal
Undiscovered Non-Associated Gas In-Place

Lower Paleozoic by Depth

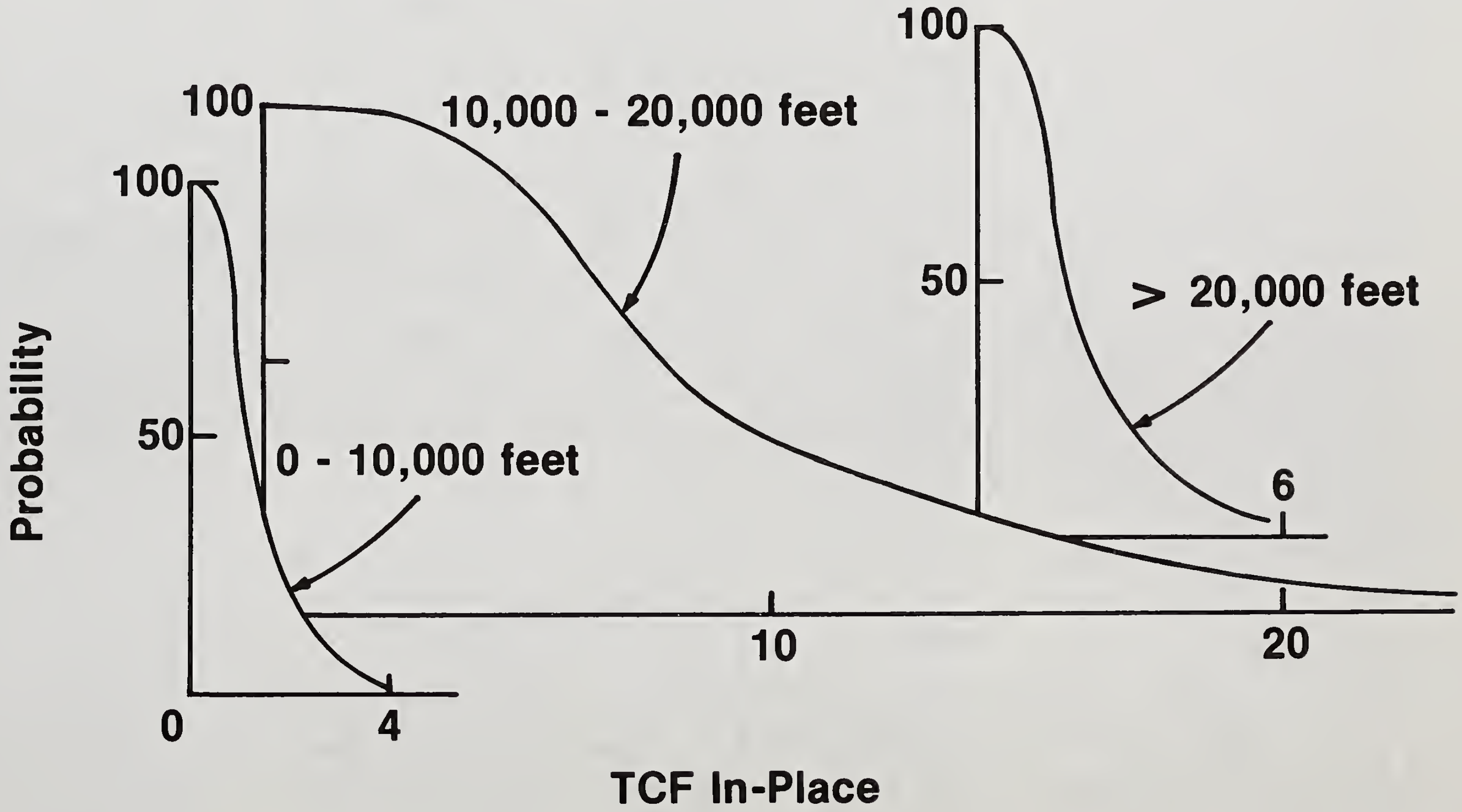


Figure 3. - Resource appraisal for undiscovered non-associated gas in-place for the Lower Paleozoic by depth increments. The resource estimates are reported in the form of three probability distributions.

Two aspects of the performance or behavioristic methods have been applied by the Resource Appraisal Group to resource assessment work. They are: 1) discovery-rate or finding-rate techniques projected for undiscovered fields, and 2) probability techniques for predicting the size of fields to be discovered with future exploration. Both techniques are discussed briefly below.

Discovery-Rate or Finding-Rate Methods

Since 1975, the Resource Appraisal Group has undertaken a continuing study of regional oil and gas finding-rate methods for the United States. The concept of finding-rate has been used at one time or another by most researchers in assessing and projecting resource availability. Terms and units of measurement used in defining finding-rate are variable; as a result, general finding-rate definitions have evolved. An understanding of the applications of finding-rate procedures is required before terms can be defined for any particular study. Regardless of the definitions used, the ultimate purpose of determining finding-rate is to permit statistically valid projections of resource availability based upon historical data within a given geologic and economic frame of reference.

As pointed out by Moore (1966), the fundamental concept of continuity of historic patterns and their validity for projecting future patterns must be assumed. Accordingly, most historical studies begin with empirical data and attempt to improve the projection of these data by being as quantitative as the limits of the data permit.

Many factors must be considered when predictions of undiscovered resources are made, but past studies indicate that the two most significant factors are drilling-rate and finding-rate. Drilling-rate is by far the single most important factor and the most easily controlled. Finding-rate, on the other hand, is difficult to control and is largely dependent upon the geologic characteristics of the area, field sizes, drilling-rate, and economic and technological factors.

In an attempt to minimize the effect of economic and political variables on finding-rates, some authors (principally Hubbert, 1974) have expressed finding-rate as a unit of oil or gas discovered per unit footage of exploratory drilling and as a function of cumulative exploratory drilling, determined from historical data.

Most published studies to date have made projections of resources based primarily on statistical studies of historical data and have included very little geological information. This emphasis upon historical drilling data rather than geologic data is due, in part, to the very large sample areas that have generally been evaluated, such as the entire conterminous United States, and the difficulty in assessing and quantifying the many and varied geologic factors that ultimately contribute to the control of resource occurrences over such large areas. The lack of essential data for more detailed finding-rate studies has also been a crucial element.

In order to improve finding-rate methods so that they can be applied to resource assessment work, the following procedures have been developed in the Resource Appraisal Group:

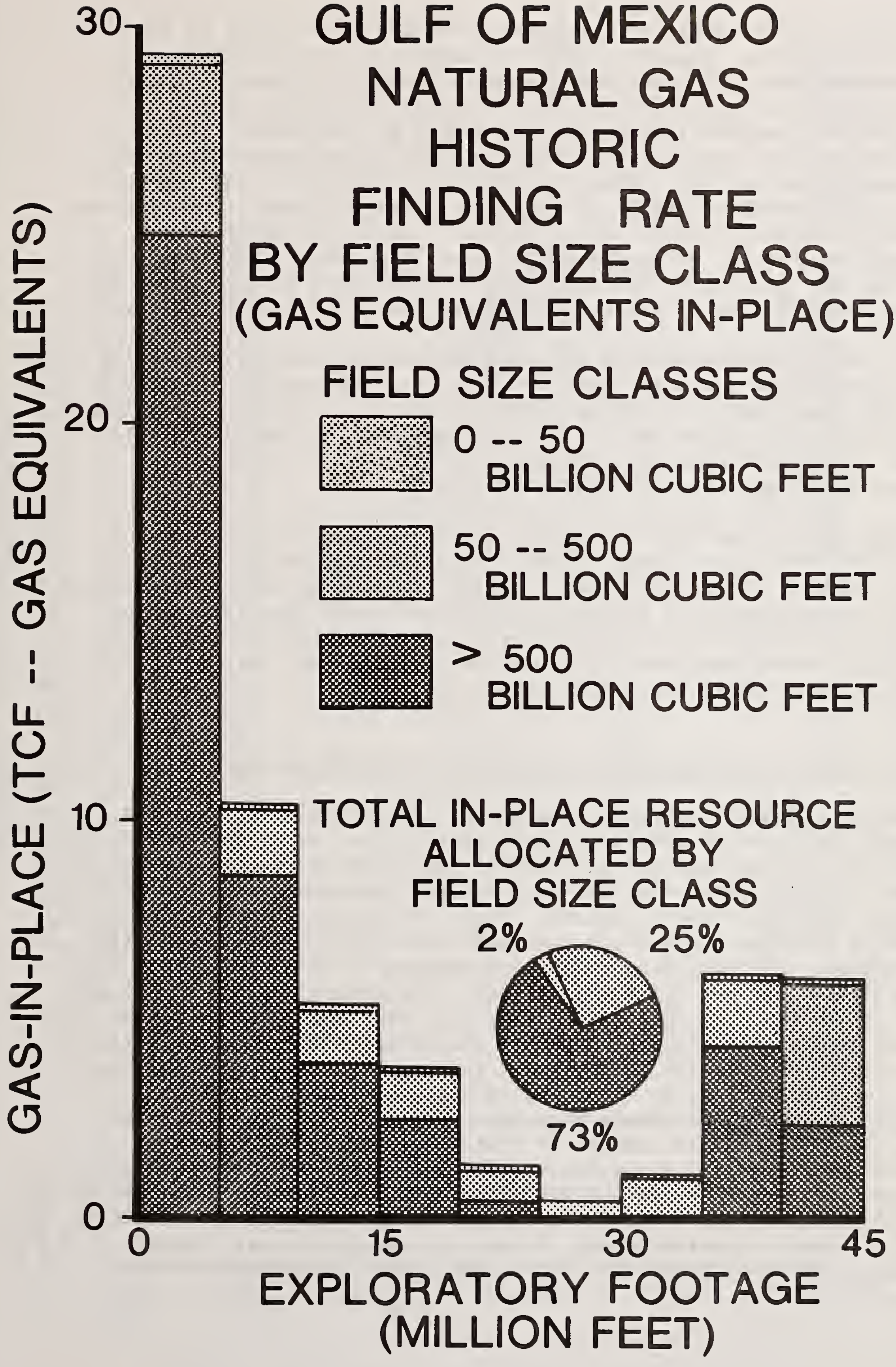
1. Directly relate geologic information to finding-rates, and limit the area of study to a well-defined geologic basin or to a specific stratigraphic unit or geologic section within a basin or province.

2. Separate the oil and gas discoveries by: year of discovery, geologic age of producing horizon and/or reservoir lithologies, depth increments for producing reservoirs, and field-size categories. If data are available, also identify the type of structural or stratigraphic trap for each field discovered.

3. Analyze the discovery-rate patterns for any of the above categories for which data are available to determine: whether there are any significant trends; whether these trends can be explained by the geologic data; and whether valid projections can be made that would contribute to an understanding of the remaining resource availability within a specified basin or province.

These finding-rate methods not only meet the requirement of being applicable to basins or provinces, they have also attained a refinement that will permit increased accuracy in analog comparisons between mature areas and frontier areas.

A study by the Resource Apraisal Group in which various aspects of the finding-rate methods have been applied has been completed on the offshore Gulf of Mexico (Miller, et al. 1978). In this study, all the oil- and gas-field data were compiled by size of in-place reservoir volumes, year of discovery, age of major producing reservoirs, and depth increments for major accumulations. Figure 4 depicts the historical finding-rate for all natural gas fields in the Gulf of Mexico, from 0 to 200 meters water depth, that are producing from the Miocene, Pliocene, and Pleistocene reservoirs. The discoveries, in trillions of cubic feet, are plotted with respect to the cumulative exploration effort in units of 5 million feet. The obvious decline in finding-rate from 1940, when the first major discovery was recorded, to 1975, ranges from 29 trillion cubic feet per 5 million feet of exploratory drilling to less than 6 trillion cubic feet per 5 million feet. One very useful method of projecting finding-rates consists of separating the known fields into field-size categories and using the historical finding-rates for each category to predict the amount of remaining resources yet to be found in each specific field size class. The finding-rates are consistently different for each field-size category in the Gulf of Mexico and in other areas in which these methods have been applied. The amounts for each class can be summed to



(Data through 1975)
Figure 4

obtain the total estimate of resources remaining to be found in the predictable future. Figure 5 depicts an example of the historical finding-rates for only those gas fields in the Gulf of Mexico in the greater-than 0.50 trillion cubic feet in-place field-size class. Hyperbolic and exponential decline curves fit by regression analysis to historical data show the best promise for finding-rate projections for these investigations. The "best fit" was selected by the highest index of determination and the F-statistic. The projected finding-rates are usually extended another 15 million to 25 million exploratory feet into the future, or approximately 5 to 10 years of additional drilling. All projected finding-rates are terminated if they reach the minimum field-size level set for the respective field-size category.

The major shortcomings of the finding-rate methods for projecting remaining resource estimates are: They can only be applied directly to semi-mature and maturely drilled producing areas, and they are considered a conservative technique for estimating resources, as they do not allow for any radical surprises in petroleum exploration, or significant improvements in exploration technology or economics.

Finding-rate techniques are very useful for providing a means of comparative checking on other resource-appraisal procedures used to analyze the same basin or province. Finding-rate procedures when applied to specific categories of geologic data often reveal some interesting exploration trends within the basins studied. Finally, finding-rate studies for known productive areas may be used with care as analogs for immaturely explored or frontier areas.

Field-Size Distributions for Estimating Undiscovered Resources

New developments in projecting the estimated field sizes of the remaining undiscovered resources hold great promise as another method of estimating remaining resources. Several techniques have been devised to estimate the field-size distributions for the remaining undiscovered fields in a maturely explored area. These techniques result from detailed studies on finding-rate methods that make use of historical field-size distributions. Seismic data on drilled and undrilled structures in the Gulf of Mexico are used by these methods to determine probable field size as related to the historical field-size distributions within the same or adjacent areas. (See Table 1.) Field-size distributions can be estimated by subjective probability procedures. The probable number of remaining undiscovered fields may be estimated in a similar manner. The total resource potential is determined, using an aggregation of the probable field sizes and probable numbers of fields, by means of Monte Carlo simulation techniques. A separate procedure is used whereby the resource assessment by some other method and the probable field-size distribution are used as input to a Monte Carlo simulation to determine the probable number of remaining undiscovered fields in a specific area. These techniques are still considered experimental and should be used with caution.

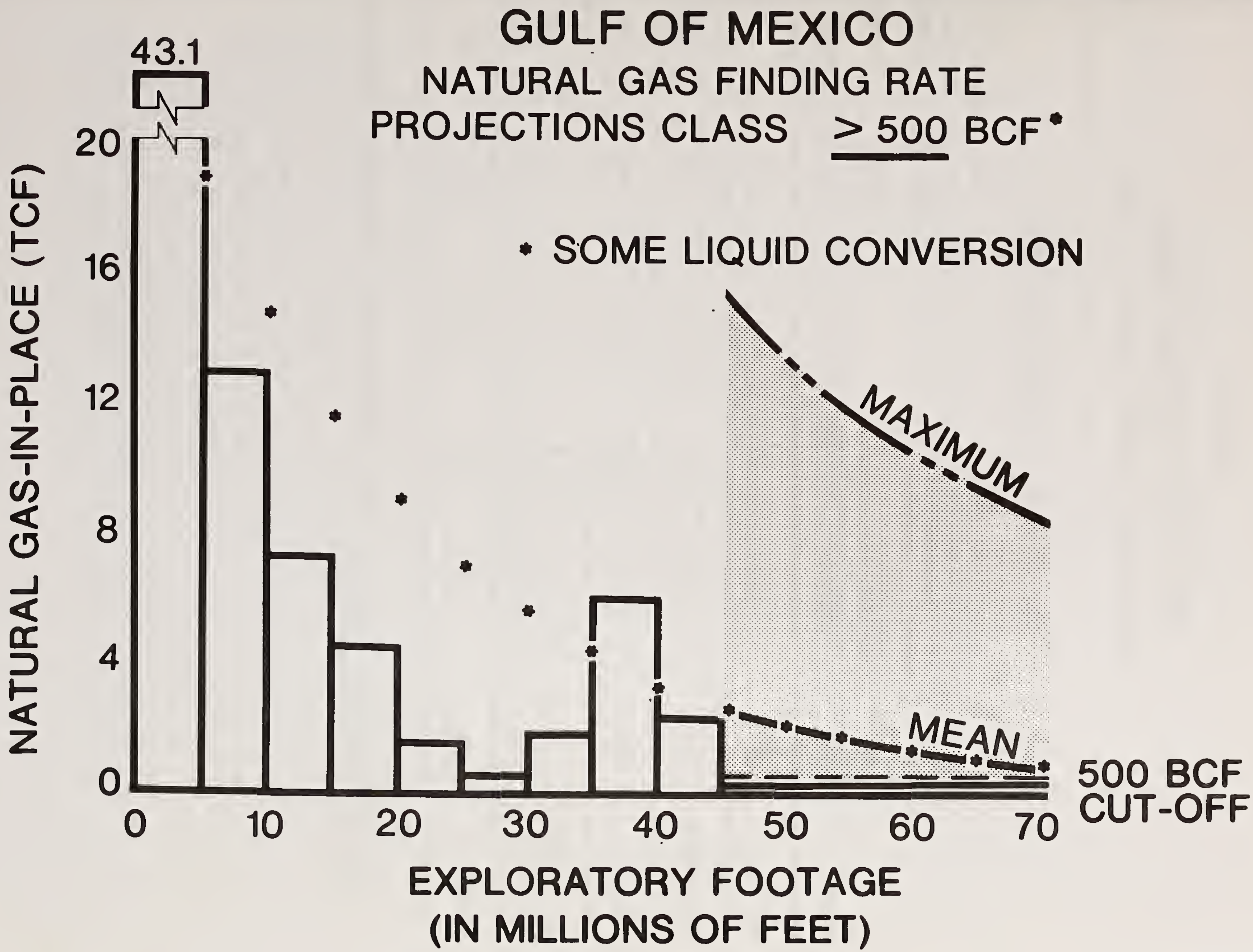

GULF OF MEXICO
NATURAL GAS FINDING RATE
PROJECTIONS CLASS ≥ 500 BCF*
* SOME LIQUID CONVERSION
43.1
20
16
12
8
4
0
NATURAL GAS-IN-PLACE (TCF)
0
10
20
30
40
50
60
70
MAXIMUM
MEAN
500 BCF
CUT-OFF
EXPLORATORY FOOTAGE
(IN MILLIONS OF FEET)

Figure 5

TABLE 1

Gulf of Mexico, Texas and Louisiana

Summary Table of Structure Count and Status

for

Total Shelf, 0 - 200 Meters Water Depth

	NUMBER OF STRUCTURES				Discovery Ratio	Percent Structures
Type	Total	Untested	Tested	Productive	(percent)	Productive
Fault	197	112	85	52	61	26
Piercement	158	51	107	61	57	39
Domal	225	78	147	96	65	43
No Seismic Control	6	1	5	4	80	66
Total	586	242	344	213 1/	61	36

1/ There are 59 total additional fields not identified on seismic structure count (Source: Miller et al 1978).

EXPLORATION PLAY-ANALYSIS METHODS

Conventional Petroleum Resources

Exploration play-analysis methods have been designed for identified or conceptual exploration plays within a basin or province for conventional petroleum resources. The basic definition of an exploration play is: a practical, meaningful planning unit around which an integrated exploration program can be developed. A play has geographic and stratigraphic limits and is confined to a formation or a group of closely related formations on the basis of lithology, depositional environment, or structural history.

There are, however, many variations to this definition and to play concepts that have been applied by various resource estimators using play-analysis techniques; these variations usually make the results noncomparable for any specific area.

Play-analysis methods are usually applied to smaller areas of appraisal than are the previously described methods, areas such as a geologic trend consisting of a reef-play or a channel or bar sand. However, in some studies the play-analysis procedure has been applied to an entire geologic horizon or stratigraphic unit, such as the total Cretaceous within a basin. Although the estimator may have called the procedure a "play-analysis," the basic concepts are no longer those of the original definition.

The basic technique requires more detailed data than the volumetric-yield methods, utilizing all the data used in the finding-rate approach and additional data concerning the individual fields within a play, plus the basic information on the reservoir characteristics in these fields.

An estimate of conventional petroleum resources is usually expressed as an equation relating a series of geologic and reservoir variables to the amount of potential oil or gas within the reservoir. Probability values may be assigned to the favorability of a play and usually to the probable success of the prospects within the play. The geologic and reservoir variables are described by subjectively derived probability functions based upon the judgment of the estimators or by use of selected analogs for many of the variables. The data formats are usually designed for sophisticated computer processing, probability distributions being assigned by the geologist for each variable. The estimates of the resource are derived for each play by means of the equation relating the variables to the potential resource by Monte Carlo methods. The procedure for processing the numerous variables evaluated by the geologist, and the accompanying probability distributions, is to use computer models that can rapidly process thousands of random samplings of the values of the variables needed for determining the resource appraisals which are shown as probability distributions. The total resource estimate for the area or basin is determined by aggregating the potential of all plays by using Monte Carlo simulation techniques. The output is in the form of a probability distribution for the total resource assessment.

Figure 6 shows an example of a simplified data format being used in a play-analysis technique currently under investigation by the U.S. Department of the Interior, Office of Mineral Policy and Research Analysis (OMPRA), and the USGS in a joint study to evaluate the petroleum resources of the National Petroleum Reserve of Alaska (NPRA). Geologic variables such as source rock, trapping mechanism, size of trap, thickness of reservoir bed, and porosity are described by probability distributions. A Monte Carlo technique is used to determine the size of the prospect or field and to solve the equation relating the geologic and reservoir variables to the resource assessments.

Table 2 shows the probability distributions for the resource estimates, completed in September 1979, of the undiscovered oil and gas in-place in the National Petroleum Reserve of Alaska (NPRA). The mean value of the total resources in-place for NPRA were estimated to be 7.10 billion barrels of oil and 14.12 trillion cubic feet of gas. These estimates were derived from the geology model in the play analysis system developed by the U.S. Department of the Interior. The basic geological input was provided by the USGS to OMPRA which, in turn, used the play-analysis technique for the resource appraisal and as the basis for the exploration, development, production, and economic evaluations for the NPRA studies published in the "Final Report of the 105(b) Economic and Policy Analysis" (U.S. DOI-OMPRA, 1979).

A major weakness of the play-analysis models is the assumption that all the variables assessed in each play, as used in the Monte Carlo simulation, are independent. Many of the geologic and reservoir variables are not independent, and this creates some confusion in the minds of the geologists who are to assign the values for each variable, for the degree of risk or success for the occurrence of a favorable play, and for a favorable prospect in that play.

One advantage of the play-analysis approach is that it simplifies, or appears to simplify, the geologist's task in evaluating the resources of an area by providing a fixed format for the variables he must evaluate; the actual resource assessment is determined by statistical and mathematical methods through the use of computer models. This method may also reduce the amount of time necessary to evaluate an area, provided ample data are readily available. However, such sophisticated computerized procedures do not necessarily mean that accuracy has been increased in the resource assessments resulting from this method over those evaluated using other resource appraisal methods. Geologists concerned over the results of their basic input into these programs must become increasingly concerned over the assumptions and mathematical manipulations within the computer system that are often designed by technical personnel who are not familiar with the basic assumptions and concepts concerning the geology of petroleum occurrence.

Oil and Gas Appraisal Data Form

Evaluator : ______________________ Play Name ______________________

Date Evaluated: ______________________

Attribute			Probability of Favorable or Present	Comments
Play Attributes	Hydrocarbon Source			
	Timing			
	Migration			
	Potential Reservoir Facies			
	Marginal Play Probability			
Prospect Attributes	Trapping Mechanism			
	Effective Porosity ($\geq$3%)			
	Hydrocarbon Accumulation			
	Conditional Deposit Probability			
Hydrocarbon Volume Parameters	Reservoir Lithology	Sand		
		Carbonate		
	Hydrocarbon	Gas		
		Oil		

	Fractiles / Attribute	Probability of equal to or greater than 100	95	75	50	25	5	0	Comments
Hydrocarbon Volume Parameters	Area of Closure ($x10^3$ Acres)								
	Reservoir Thickness/vertical closure (Ft)								
	Effective Porosity %								
	Trap Fill (%)								
	Reservoir Depth ($x10^3$ Ft)								
	No. of drillable prospects (a play characteristic)								
	Proved Reserves ($x10^6$ Bbl; TCF)								

Figure 6: Oil and Gas Appraisal Data Form

Table 2

Preliminary Distribution of Estimated

National Petroleum Reserve of Alaska Oil and Gas

Resources In-Place and Barrels of Oil Equivalent, as of September 1979

(%) Probability That Quantity is at least Given Value	Oil In-Place (Billions of Barrels)	Gas In Place (Trillions of cu. Ft.)	Barrels of Oil Equivalent in Place 1/ (Billions of Barrels)
95	1.04	3.51	2.08
90	1.35	4.25	2.66
50	6.03	12.52	8.57
25	10.01	17.54	13.26
10	13.72	28.29	17.33
5	16.45	34.97	20.35
1	24.80	40.17	30.00
Mean	7.10	14.12	9.60

1/ Barrels of oil equivalent are obtained by converting the estimated gas in place to the energy equivalent in oil and adding the resulting value to the estimated oil in-place. The values in this table are estimated independently, therefore, oil and gas estimates may not be added across percentiles to obtain BOE (DOI/OMPRA, 1979).

One of the most publicized of the play-analysis methods has been that of the Geological Survey of Canada (Department of Energy, Mines and Resources, 1977). Various modified versions of the Canadian approach and some of those used by industry are currently under investigation by the USGS. Ideally, a computerized procedure similar to that used in the play-analysis approach could be the ultimate goal in the idealistic world of resource appraisal. However, we have yet to achieve such a goal.

Unconventional Natural Gas Resources

The successful application of the exploration play-analysis model to the evaluation of conventional petroleum resources led to the experimental application of a modified play-analysis approach for an appraisal of unconventional gas resources for a pilot study of the Devonian black shales in the Appalachian basin.

In 1975, USGS was requested to aid the U.S. Energy Research and Development Administration (ERDA), now the U.S. Department of Energy (DOE), in their investigations for appraising the energy potential of the gas-productive petroliferous black shales of Devonian age in the eastern United States. To assist ERDA in achieving the goals of its program, the USGS was asked to perform a series of stratigraphic, structural, geochemical, and geophysical studies to establish a data bank and data retrieval system over a 5 year period, and to make an appraisal of the energy resources (predominantly unconventional natural gas) of the Devonian black shales in the Appalachian basin. The latter assignment involves the USGS Resource Appraisal Group working with all the groups in the project to evaluate the data at hand and then preparing an independent resource appraisal.

A review of the results of the basic stratigraphic, structural, geochemical, source-bed, maturation, and clay mineralogy studies soon reveals the complexity of the many lithologic units that compose the Middle and Upper Devonian black shale facies in the Applachian Basin, and the multivariate nature of the geologic characteristics that contribute to conditions favorable for the occurrence of producible gas from Devonian black shales. The basic geologic characteristics and degree of uncertainty as expressed by the geologists for explicit identification and substantial detail for each play are very similar to the circumstances encountered in the play-analysis approach used in appraising the petroleum resources of NPRA. Some significant differences may also be found between the geologic characterization for the conventional occurrence of petroleum and that of the unconventional natural gas occurrence in the black shales.

These similarities and differences can be summed up as follows:

The Devonian shales can be subdivided and delineated as distinct units or plays of approximately homogeneous geological and geochemical characteristics. The extent, geometry, and stratigraphic relations of each play can be defined and mapped the same way that they were in NPRA. Reservoir engineering variables such as porosity, permeability, reservoir pressures and temperatures, and methane compressibility can be measured, and a geochemical analysis can be made of types and amounts of organic matter within the black-shale facies.

One major difference between conventional gas and unconventional gas in shales is that the former occurs in well defined and mappable reservoirs, whereas the latter, when it is in commercial amounts, is usually concentrated in areas where the shales are naturally fractured, jointed, or faulted. The types of porosity in black shales are: 1) effective microporosity due to matrix porosity and microfractures, and 2) porosity due to macrofractures.

The Devonian shale resource appraisal includes only "movable" gas, i.e., gas that can leave the shale under "reservoir" conditions in response to the disequilibrium caused by a well penetrating the shale unit. "Movable" means that the gas can leave a core sample at surface conditions without grinding or heating. "Movable" gas is assessed only when in "black" facies, i.e., shale having an organic-matter content greater than 2% by volume (Schmoker, 1980).

In light of the significant differences between the geologic characteristics that identify the conventional natural gas reservoirs and those that define the unconventional natural gas in black shales, several changes must be made in the geology model for assessing the natural gas resources. The most important change is the basic equation used to calculate the amount of gas within each play and the essential parameters required for the equation.

The basic equation as designed by members of the USGS for assessing the amount of movable gas in the black shales is defined as follows:

$$G_m = (\phi_{e,mi})\,(Th_B)\,\frac{(P_{r1})}{(P_s)}\,\frac{(T_s)}{(T_{r1})}\,\frac{(1\ )}{(Z_1)} +$$

$$(\phi_{f,ma})\,(Th_f)\,\frac{(P_{r2})}{(P_s)}\,\frac{(T_s)}{(T_{r2})}\,\frac{(1\ )}{(Z_2)} + (Y_s)\,(TH_{ORG})$$

where

G_m = cubic ft of movable gas at standard conditions/square feet of land surface

$\phi_{e,mi}$ = effective microporosity due to matrix porosity and microfractures

$\phi_{f,ma}$ = porosity due to macrofractures

Th_B = thickness of black-shale facies (ft)

Th_f = thickness of fractured interval

P_r, P_S = reservoir pressure and standard pressure, respectively (PSI)

T_r, T_s = reservoir temperature and standard temperature, respectively (absolute)

Z = methane compressibility (gas-deviation factor)

Y_S = cubic feet of movable gas at standard conditions/cubic feet of organic matter

TH_{ORG} = net thickness of organic matter (ft) in the black shale facies.

The equation that calculates the amount of "movable" gas should be evaluated for each of the geologic plays as defined for the Appalachian Basin. The plays are defined so that within each unit the geologic and geochemical properties represented by the equation are relatively homogeneous. Values are determined using subjective judgment by the geologists and geochemists for the various probability distributions for the total "movable" gas, and the terms constituting the equation are computed for each play. These distributions when multiplied by the area of the play will give the expected volumes of movable gas. The probability distributions for the resource appraisal for the gas in each play will be aggregated statistically to give the total assessment of gas resources within the black shales of the basin.

In addition, the probability of each play being favorable, in terms of the existence of "permeable pathways" (such as a fracture system) that allow the movable gas to reach a well, is subjectively determined. A subjective ranking of the various plays can be made.

A Pilot Study in Devonian Black Shales of the Appalachian Basin

The modified play analysis approach, incorporating the newly formulated equation and the related assumptions and parameter values, was applied on an experimental basis to a pilot area within the Appalachian Basin. A five-county area within West Virginia was selected for the experimental trial runs to assess the unconventional gas within the black shales. Three plays were distinguished for an appraisal of the gas resources. Figure 7 is a map of the pilot area, showing the mapped boundaries of the plays within that area. Plays IA and IB are geologically defined in part by the structural aspects of the Rome Trough and in part by the characteristics of the organic matter within the black shale. Play II is outside the trough area and has different organic characteristics. Note that the county boundaries of the pilot area arbitrarily cut out only segments of each of the naturally occurring plays for assessment; thus, the resulting estimates do not represent a complete appraisal of any one of the three plays.

Figure 8 is an example of the data formats used to compile the essential information for each play (Miller, 1980). A team of six geologists and one petroleum engineer met to review and interpret available information prior to making the subjective judgments concerning the values for each of the geologic variables shown on the format. The reservoir data were compiled whenever available or analog data were used if specific information was not known within the boundaries of the pilot area.

Preliminary Findings of the Resource Assessment Procedures in the Pilot Study

The play approach was used on each of the three plays for several trial runs to remove the "bugs" from the newly modified geology model. The assessments were reviewed and analyzed in terms of the geochemical and technical information available in the pilot area. The following results should be viewed as preliminary and are shown for comparative purposes for the "gas richness" determined for the segments of each of the three plays.

Preliminary
Resource Estimates of Movable Gas
in Pilot Area, in Trillions of Cubic Feet

	Probability that Quantity is at least Given Value			Probability of Favorable "Pathways"
PLAYS	95%	5%	Mean Value	(%)
Play 1A	1.62	4.35	2.95	100
Play 1B	5.90	14.75	9.65	100
Play II	.74	1.77	1.23	100

Devonian Black Shale Pilot Area - Plays I A & B, II

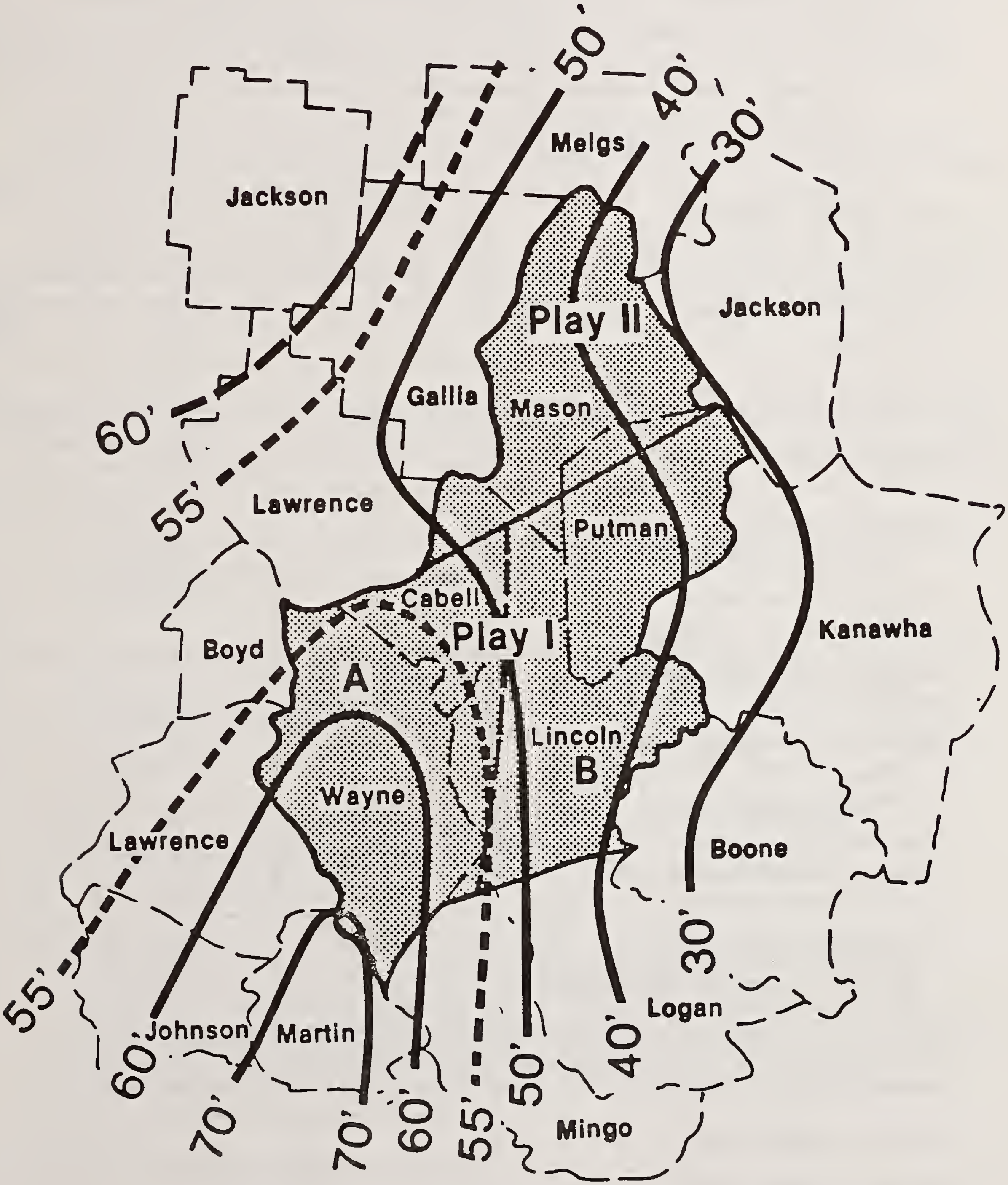


Figure 7: Average colume % Organic Content of "Black" Shales

Figure 8: Data format form for natural gas resource appraisal in Devonian Black Shales

PILOT AREA: DEVONIAN BLACK SHALE

Identify Play: __________

PLAY FAVORABILITY	PROBABILITY OF BEING FAVORABLE ON A SCALE OF 0 TO 1	COMMENTS
Existance of Permeable 'Pathways'*		

* 'Pathways' which allow the movable gas to reach a well, whether by fracture systems, porous lenses, etc.

GAS VOLUME PARAMETERS	PROBABILITY OF EQUAL TO OR GREATER THAN			MODE
	95%	50%	5%	
Effective microporosity $\phi_{E,MI}$ (%)				
Porosity due to macrofractures $\phi_{F,MA}$ (%)				
Thickness of black shale Facies TH_B (ft.)				
Reservoir Pressure P_R (PSI)				
Reservoir Temperature T_R (absolute)				
Movable gas/organic matter Y_S (cu.ft./cu.ft.)				
Thickness of organic matter TH_{ORG} = ORG* (organic content of Black Shale) X TH_B				
*Estimate ORG (% vol.)				
Depth of Black Shale Units (ft.) Assume mid-point of units				
Area of Play (sq. miles) If boundaries of play are fixed, give one value; if boundaries are uncertain, give range				

Standard Pressure P_S	
Standard Temperature T_S	
Compressibility of Gas, Z factor (Tables)	

Conclusions on the Use of the Play Analysis Approach to Black Shale Gas Appraisals

The play-analysis approach, using a modified geology model to determine the amount of unconventional natural gas within the Devonian black shales of the Appalachian basin, is considered by the geologists working on the project to have high merit. Research will continue on refining the method and the computer program and in checking out all the technical aspects related to the basic assumptions in the geology model, in particular, the geochemical concepts of "movable" gas and the relationships of sorbed gas to organic content in the black shales.

The USGS plans to continue the research and development of the play analysis approach for resource appraisal work both for conventional petroleum resources and for the assessment of unconventional petroleum resources.

A strong interest has been expressed by other government agencies for the Survey to use the application of the geology model to the play-analysis approach for assessing conventional petroleum resources which would provide input into those agencies various economic models.

A SUMMARY OF THE U.S. GEOLOGICAL SURVEY'S PETROLEUM RESOURCE-APPRAISAL SYSTEM

The resource-appraisal system used at this time by the Resource Appraisal Group within the U.S. Geological Survey is one that will achieve the following:

1. Resource-appraisal methods that emphasize the evaluation of all the available geologic and geophysical data by geologic basins or provinces.

2. The compilation of a comprehensive information data base containing all of the pertinent geologic and geophysical data, exploration statistics, field and reservoir data, and production and reserve data for each producing and potential petroleum province in the United States.

3. The application of at least two or more resource-appraisal techniques on each area to be assessed as a means of cross checking within reason the resource estimates, if time and conditions permit.

4. The review and analysis of the basic information and appraisal results by a team of geologists applying all the resource appraisal procedures feasible (see Figure 9). This team provides the final subjective probability estimates that are used as input in the various Monte Carlo techniques to aggregate the final resource assessments by basin, region, an entire Nation, or the World.

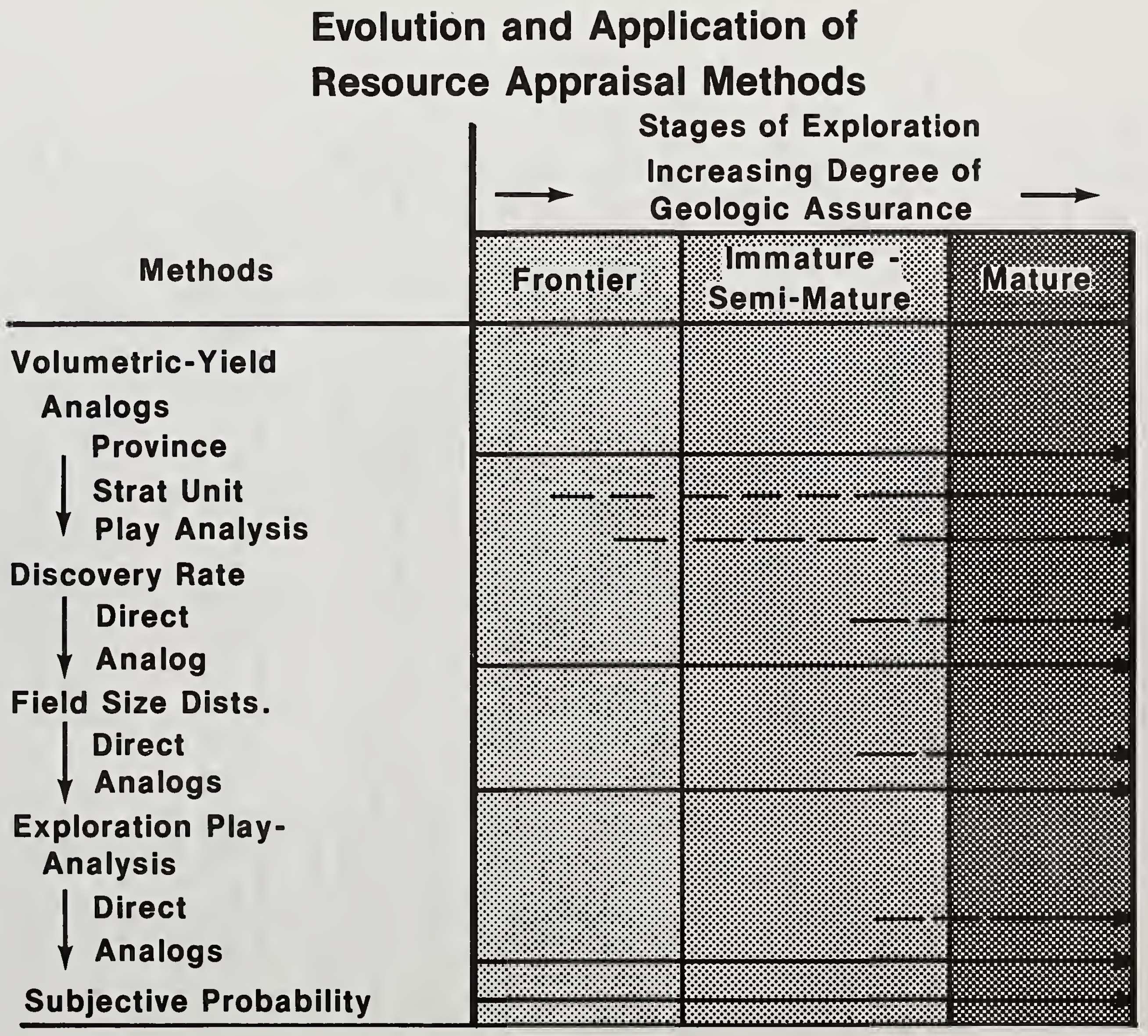


Figure 9: Resource Appraisal Methods Applicable for the Various Stages of Exploration in a Petroleum Province with an Increasing Degree of Geologic Assurance.

CURRENT ACTIVITIES IN THE RESOURCE APPRAISAL GROUP

Activities having priority in the immediate future for the Resource Appraisal Group are: (1) A revised, expanded, and updated version of Circular 725 which is currently in progress; this new assessment of the Nation's petroleum resources is to be completed late in 1980; (2) current updates on the petroleum resources of all Outer Continental Slope (OCS) basins for the United States; (3) continuing research on resource appraisal methods, particularly in the area of play-analysis methods; and (4) the initial development of a world petroleum resource-appraisal system for analyzing petroleum resources on an international basis.

CONCLUSIONS

This review of the evolution in the oil and natural gas resource appraisal methods used by the Resource Appraisal Group in the U.S. Geological Survey since 1975 covers the significant developments in the appraisal procedures for assessing our Nation's resources. We recognize that some major problems related to resource assessments have arisen primarily from a confusion in terminology and in the assumptions, qualifications and limitations related to various resource appraisal methods. These problems have created misunderstandings in the meaning and interpretation of resource information and in the application of resource estimates by the media, government and the public. Scientists making resource assessments must strive to reach some agreement on a system of resource classification, definitions, and basic assumptions for resource appraisal studies.

An understanding of the availability and distribution of the Nation's resources is a fundamental requirement for the formulation of a national energy policy. Great uncertainties are inherent in estimating undiscovered petroleum resources and will continue to plague the geologist trying to make these estimates. However, the limitations imposed by these uncertainties must be recognized, understood, and dealt with realistically. Geologists and other scientists devoting their expertise to making resource estimates must clarify the terms used, improve upon the resource appraisal methods applied to these studies, and keep up to date with the dynamic and everchanging petroleum data bases. Individual scientists, government agencies, and industry must use the best expertise available to estimate the amounts of undiscovered petroleum resources, both domestic and worldwide, that remain available for use, in order to plan for the rational exploration and development of these resources in the future.

The Resource Appraisal Group, as a part of the research program of the U.S. Geological Survey, will continue to meet its responsibilities to develop resource appraisal methods and to apply these methods for assessing the Nation's and the World's petroleum resources.

ACKNOWLEDGMENTS

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DISCUSSION

Dr. Murphy: On the last approach, you had the probabilities assessed for the various sets of parameters. How do you deal with the correlations across the categories, for factors you are assessing?

Dr. Miller: What you literally are doing is sampling the values of the parameters many times from the probability distributions that actually combine to form the reservoir (amounts) and calculate the amount of oil or gas in the reservoir and their distributions. The other probability values, such as the probability of a prospect and probability of a play, literally become the marginal probabilities or conditional probabilities which modify or "risk" what resources you have generated by the above calculations. These are used then to risk each of the elements in the play. There is also a probability assessed (in addition to the above) as to whether or not these plays occur in areas of fracture systems which would increase the favorability of their being within pathways of movable gas.

Most of your gas developed to date, in the shales, fall within rather unique fracture patterns. So, this also becomes a part of the assessment.

FORECASTING FUTURE OIL FIELD SIZES THROUGH STATISTICAL ANALYSIS OF HISTORICAL CHANGES IN OIL FIELD POPULATIONS

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ABSTRACT

This study involves analysis of historical changes in oil field sizes in Kansas, Wyoming and California. It is common knowledge that large oil or gas fields tend to be found early in the sequence of discoveries in a region, and that the sizes of fields tend to diminish progressively as exploration proceeds. This study has found that populations of oil (or oil and gas fields combined) tend to be more or less lognormally distributed, but in some regions or districts, the populations of fields discovered early tend to depart more severely from an ideal lognormal distribution than do populations discovered later. Comparisons between populations of fields discovered early, intermediate and late were made by segregating the presently known fields in each of the three states into intervals representing the first 20 percent to be discovered, the second 20 percent to be discovered, and so on. This method of segregating by discovery sequence also was employed for individual districts within each state.

The results are presented graphically and in tables, and may be used to predict the population parameters of fields to be discovered in the future. In most of the districts, as well as for each state, the forecasts of new-field discoveries are pessimistic. This pessimism stems from the general rapid decline in population parameters (median, geometric mean, and total volume) with the progression of discoveries. It is to be emphasized that these predictions pertain to the discovery of new fields, and exclude increases in oil and gas that may result from extensions of known fields, or to enhanced oil and gas recovery of existing fields. Furthermore, the forecasts pertain to the general regions in which fields discovered to date exist, and exclude provinces (such as offshore central and northern California) which have been relatively little explored.

In California, it is estimated that of the next 81 fields to be discovered (within the established oil and gas producing regions of California), the total volume of oil and gas (expressed as barrels of oil equivalent, or BOE) will be about 0.6 percent of the total hydrocarbons that ultimately will be extracted from California's total of 404 fields discovered from 1861 through 1974. Furthermore, this forecast population is estimated to be approximately lognormally distributed, with a median size of only 125,000 BOE. Given the graph of this forecast distribution, probabilities attached to individual field-size ranges (in BOE) can be estimated. Within the forecast population of 81 fields, for example, there is only a 9 percent probability that any particular field discovered will be between 10 and 100 million BOE. The probability of finding a field greater than 100 million BOE is only a small fraction of one percent.

In Wyoming, the forecast of the next 151 fields to be discovered is only slightly less pessimistic. It is forecast that this population of new fields will contribute only about 1.6 percent additional BOE relative to the total BOE extractable from the 754 fields discovered in Wyoming from 1884 through 1977.

The data for Kansas exclude gas and are based on cumulative production of oil through the end of 1978 for all fields discovered through the end of 1973. Thus, the oil field size distributions for Kansas (in contrast to Wyoming and California) are somewhat inadequate measures for forecasting purposes because they exclude estimates of remaining reserves. Thus, because many Kansas fields are still producing, the population parameters must be revised upward. Nevertheless, the forecast for new-field discoveries in Kansas is pessimistic. By comparison with the total of 2,992 oil fields discovered in Kansas from 1890 through 1973, if 598 new fields are discovered (20 percent more) they probably will contribute only 2 or 3 percent more to the oil discovered in Kansas through 1973.

INTRODUCTION

This study involves changes in the characteristics of oil field populations with their sequence of discovery. It is common knowledge that large oil and gas fields tend to be discovered early, and that the sizes of fields tend to diminish progressively as exploration proceeds in a region. If these changes are sufficiently regular, they should permit the characteristics of future oil field populations to be predicted, based on the historical shifts observed to date.

One of the most important aspects of any mineral resource assessment is an understanding of the statistical properties of the deposits that have been discovered. Unfortunately, inadequate effort has been expended in preparing an inventory of the United States oil and gas fields, and paradoxically, almost no effort has been spent in statistically analyzing the data that do exist.

This study involves a comparison of oil fields in Kansas, Wyoming and California. The data have been derived from publically accessible sources. We have analyzed the oil-field populations for each of these three states as a whole, as well as for individual geographic districts or sedimentary basins within each state. The population of oil fields within each of the states or subdivisions thereof has been segregated into five subpopulations according to sequence of discovery. The first 20 percent of fields to be discovered defines the first subpopulation, the second 20 percent discovered defines the second subpopulation, and so on. Comparison of the differences between these subpopulations provides a basis for prediction.

In California and Wyoming, data from both oil and gas fields have been used, and the field sizes have been expressed in barrels of oil equivalent (BOE). The field sizes in these two states involve the cumulative production per each field at the end of 1977 in Wyoming, and the end of 1978 in California. The cumulative production figure for each field is then combined with the estimated remaining reserves to yield an estimate of the total recoverable BOE per field.

In Kansas, only oil production data were used, and the oil field sizes are expressed solely as the cumulative production (to the end of 1978), data on reserves remaining in Kansas oil fields not being available.

It is important to realize that the predictions in this study are derived almost solely from historical changes and, with one exception, do not incorporate geological data other than the field volumes themselves. The predictions apply to new fields to be discovered, and do not pertain to increases in estimates that may arise from extension of existing oil fields, or from enhanced oil recovery. Furthermore, the predictions apply, more or less, to established provinces that have undergone exploration. Offshore central and northern California, for example, is not included in the prediction for California because this segment of the California offshore has generally not been explored and has not contributed to the existing resource base of proven oil and gas fields in California.

PROCEDURES

The procedures employed here involved transforming the estimates of oil and gas fields sizes as barrels of oil equivalent (BOE) for fields in California and Wyoming. A conversion factor of 5.7 MCF (thousand cubic feet) of gas equals one barrel of oil was used. In California the size tabulated for each field represents the cumulative production through the end of 1978, plus, the estimated reserves at the end of 1978. In Wyoming, the cumulative production has been tabulated through the end of 1977 and added to estimated reserves remaining as of that date. In Kansas, only oil production data were used (production from gas fields and production of gas associated with oil is not included). In Kansas, the cumulative production for each field through the end of 1978 was employed.

The oil field volumes, expressed as the total producible oil (in BOE) in California and in Wyoming, and cumulative oil production in Kansas, were segregated chronologically according to year of discovery for each field. Then, for each state, as well as for selected geographic districts or basins within each state, the fields were segregated into five classes according to discovery sequence, namely (A) the first 20 percent of the fields that were discovered relative to the total population of fields that had been discovered by a specific date (end of 1973 for Kansas, end of 1974 for California, end of 1977 for Wyoming), (B) the next 20 percent of fields to be discovered, (C) the third 20 percent to be discovered, and (D) and (E) the fourth and fifth 20 percent intervals to be discovered, respectively.

The frequency distributions for each of these five intervals was plotted, and certain population parameters were computed, namely the median, geometric mean, and either the BOE discovered through 1978 (for California) and 1977 (for Wyoming), or the cumulative production through 1978 (for Kansas).

The frequency distributions have been plotted on log-probability paper, a form particularly convenient because a perfect lognormal distribution plots as a straight line. Figure 1 provides a comparison between a lognormal distribution plotted in conventional form, with the same distribution plotted on log-probability paper. Part a of Figure 1 shows the lognormal distribution plotted as a histogram, to which a bell-shaped curve (S) has been fitted. The same distribution plotted in cumulative form (C) has been superimposed. The cumulative curve is sigmoidal, Please note that the cumulative percentage scale is linear and ranges from 0 to 100 percent.

If we now distort the cumulative percentage scale so that those parts of the scale that lie toward both the zero-percent end and toward the 100-percent end are progressively stretched, the cumulative percentage scale can be made to compensate for differences in the height of the normal curve. The normal curve is, of course, asymptotic towards its two ends, but if the cumulative percentage scale is stretched to compensate for this, the sigmoidal curve is transformed to a straight

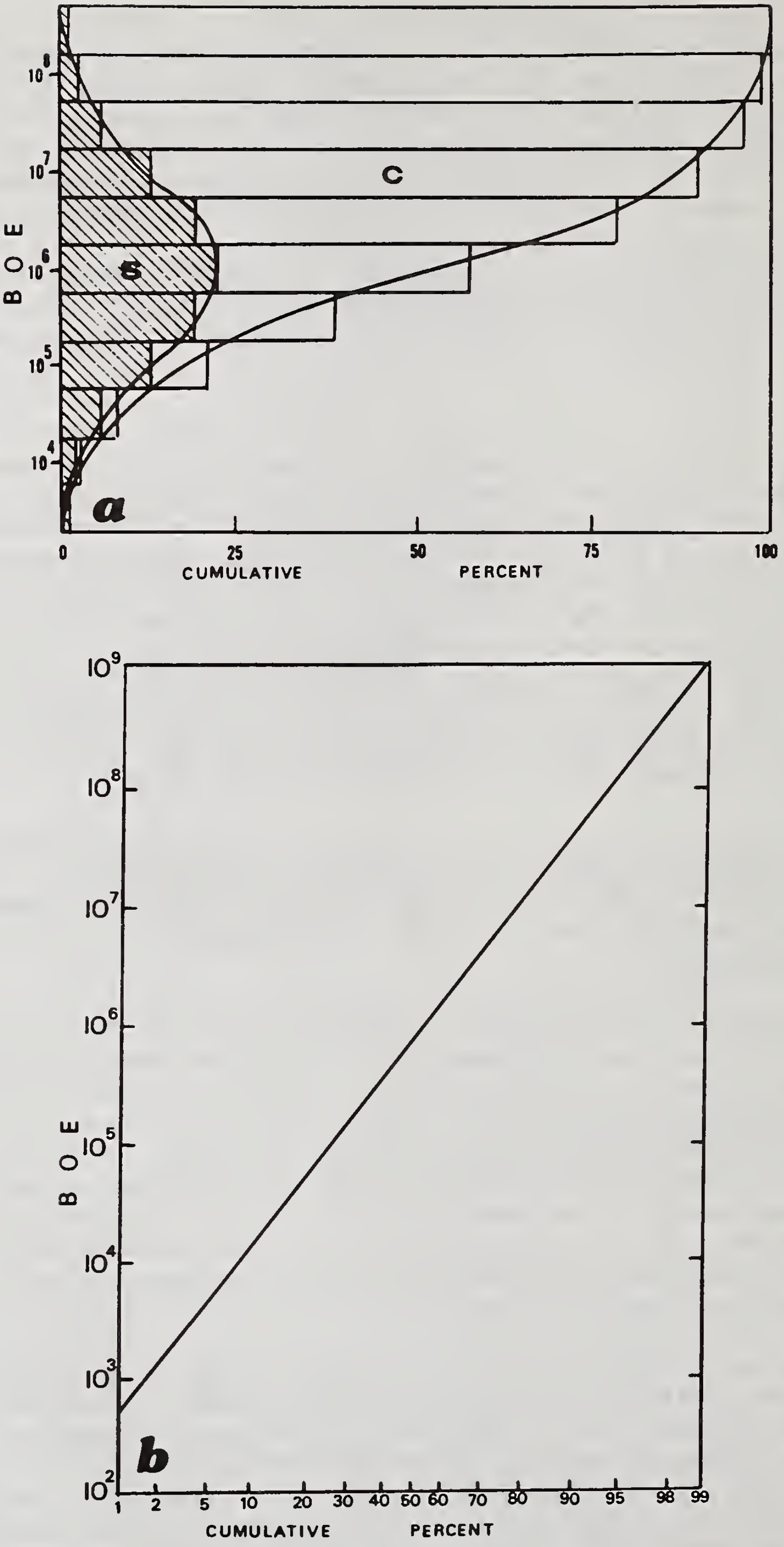

Figure 1. Diagrams illustrating forms in which a lognormal distribution may be plotted: (a) Histograms and fitted curves representing perfect lognormal distribution plotted in standard form(s), and same distribution plotted in cumulative form (c). (b) When same distribution is plotted on log-probability paper incorporating distorted cumulative percentage scale, distribution plots as a straight line.

line. Under these circumstances, 0 and 100 percent lie at an infinite distance, because the normal curve being asymptotic, is of infinitesimal height at these points. If an actual distribution deviates from a straight line when plotted on log-probability paper, the deviation provides a graphic measure of the degree to which the actual distribution differs from an ideal lognormal distribution.

The procedure for plotting a population on log-probability paper is simple. The objects (fields in our examples) are ranked in ascending order. A "fractile-percentage" is assigned to each field and the percentages are progressively accumulated. The fractile percentage is obtained by dividing 100 percent by the number of fields plus one. Thus, if there are 24 fields, the individual fractile percentage is 100/(24+1) = 4 percent, and the sequence of cumulative percentage values is 4, 8, 12, 16, . . ., 92, and 96. Thus, 0 and 100 percent are not represented because they cannot be accomodated on log-probability plots. By convention, the lower end of the cumulative percentage scale is plotted so that it corresponds with the lower end of the sequence of fields as ranked by size. The resulting plot thus extends from lower left to upper right, provided that the cumulative percentage scale is plotted horizontally.

Graphic Presentation of the Data

Most of the illustrations in this report, with exception of index maps and several other figures, involve use of a standardized graphic format. A single explanation will suffice for Figures 3 through 6, 8, through 17, and 19 through 26, all of which employ this standard format. Each of these figures contains four boxes labeled a, b, c, and d which contain graphs. Box a in the upper left contains a cumulative plot for the total population of fields within the area represented. For convenience, individual points at at 2, 5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 85, 90, 95, and 98 cumulative percent) have been plotted, and a curve then fitted manually. Since the plot is in log-probability form, the degree to which the plot approaches a straight line is a measure of the degree to which the overall population approaches an ideal lognormal distribution.

Box b, in the upper right also employs log-probability plots, but instead pertains to subpopulations which have been segregated according to discovery sequence. There are five such subpopulations, labeled A, B, C, D, and E, and which represent respectively, the first 20 percent of fields discovered, the second 20 percent discovered and so on. These curves are based on points plotted in a manner identical to that used in box a, but the individual points are omitted for simplicity.

Curves F and G in box b are shown with dashed lines and represent respectively, the forecast populations for the next 20 percent of fields to be discovered, and the 20 percent of fields to be discovered after that. The letters used to label these populations have been used consistently throughout. If we define the "present" total consisting of fields that had been discovered at the end of 1973 in Kansas, the end of 1974 in California, and the end of 1977 in Wyoming, as 100 percent, then the percentage ranges and identifying letters are as follows:

	Identifying letter	Percentage range of total fields presently discovered
Subpopulations of fields that have been discovered	A	0-20
	B	20-40
	C	40-60
	D	60-80
	E	80-100
Subpopulations of fields forecast to be discovered in the future	F	100-120
	G	120-140

In fitting curves F and G, the medians projected for these subpopulations were employed in manually fitting smoothed curves that conform, more or less, with the general trends observed in the progression of changes from curves A through F.

Box c in the lower left, is a plot of the medians of the subpopulations versus discovery sequence, the same letters being employed to label the subpopulations, A being the oldest subpopulation (the first 20 percent), and E the youngest (the last 20 percent).

A curve has been manually fitted to the five points, and in some plots, two or even three curves have been fitted, representing "optimistic" versus "realistic" projections. The extension of the fitted curve (the dashed portion) yields the projected medians for subsequent subpopulations F and G.

Box d, in the lower right, presents the cumulative volumes in the subpopulations, and as in box c, involves a projection (dashed part of the fitted curve) for the subsequent populations F and G. Both boxes c and d use a log scale along the vertical axis because of the very large ranges of volumes involved. The volumes may range as much as two orders of magnitude, making use of a linear scale impractical.

Tabular Presentation

Standardized sets of tables have also been employed. Tables 1, 3 and 5 contain data which pertain to the standardized graphs described above, plus other information. The subpopulation percentage ranges are arranged in rows and labeled A through G. By columns, information is provided, including the range of years, number of fields, median, geometric mean, total quantity discovered, and percentage of present total.

The geometric mean is computed by finding the average of the logarithms of the individual field volumes in a specific population, and then taking its antilog of this value.

Tables 2, 4 and 6 contain probabilities estimated for fields that remain to be discovered (subpopulations F and G). The number of fields in each forecast population is presented, as well as the probabilities attached to different field-size ranges expressed as a progression of powers of 10. Seven columns of field-size ranges are provided, that is $>10^3$ (less than 1000 barrels or BOE), 10^3 to 10^4, 10^4 to 10^5, and so on, the highest range being 10^8 to 10^9 BOE or barrels. These probability estimates are read from the curves F and G in box b for each population. They represent probabilities attached to the discovery of new fields within the specified area or district or basin. Table 7 provides a summary comparison of the three states.

CALIFORNIA

The data for California used in this study were taken largely from a report by the California Division of Oil and Gas (1979). This report contains information on a field-by-field basis for all fields in the state, and provides cumulative production of oil and of gas through the end of 1978, plus estimated reserves of oil and of gas as of that date. By combining the cumulative production figures with the reserves, and transforming gas to its equivalent in oil (BOE), a single figure was obtained for each field representing its estimated size (in recoverable oil and gas) per field.

The California Division of Oil and Gas has established six administrative districts in California (Figure 2). While these districts do not necessarily coincide with geologic province boundaries, District 6 essentially encompasses both the Sacramento Valley and the northern part of the San Joaquin Valley,which is a gas-producing province, whereas Districts 4 and 5 combined include the central and southern San Joaquin Valley, which is both an oil and gas-producing province. District 1 includes the Los Angeles basin as a producing province, but also includes part of the eastern extension of the Ventura basin (Newhall area). In our work we segregated the fields into only four geographic areas for simplicity, namely District 1, Districts 2 and 3 combined, Districts 4 and 5 combined, and District 6.

Frequency distributions for these four areas, as well as for California as a whole were tabulated and plotted in Tables 1 and 2 and Figures 3 to 8. With the exception of Figure 7, the results have been

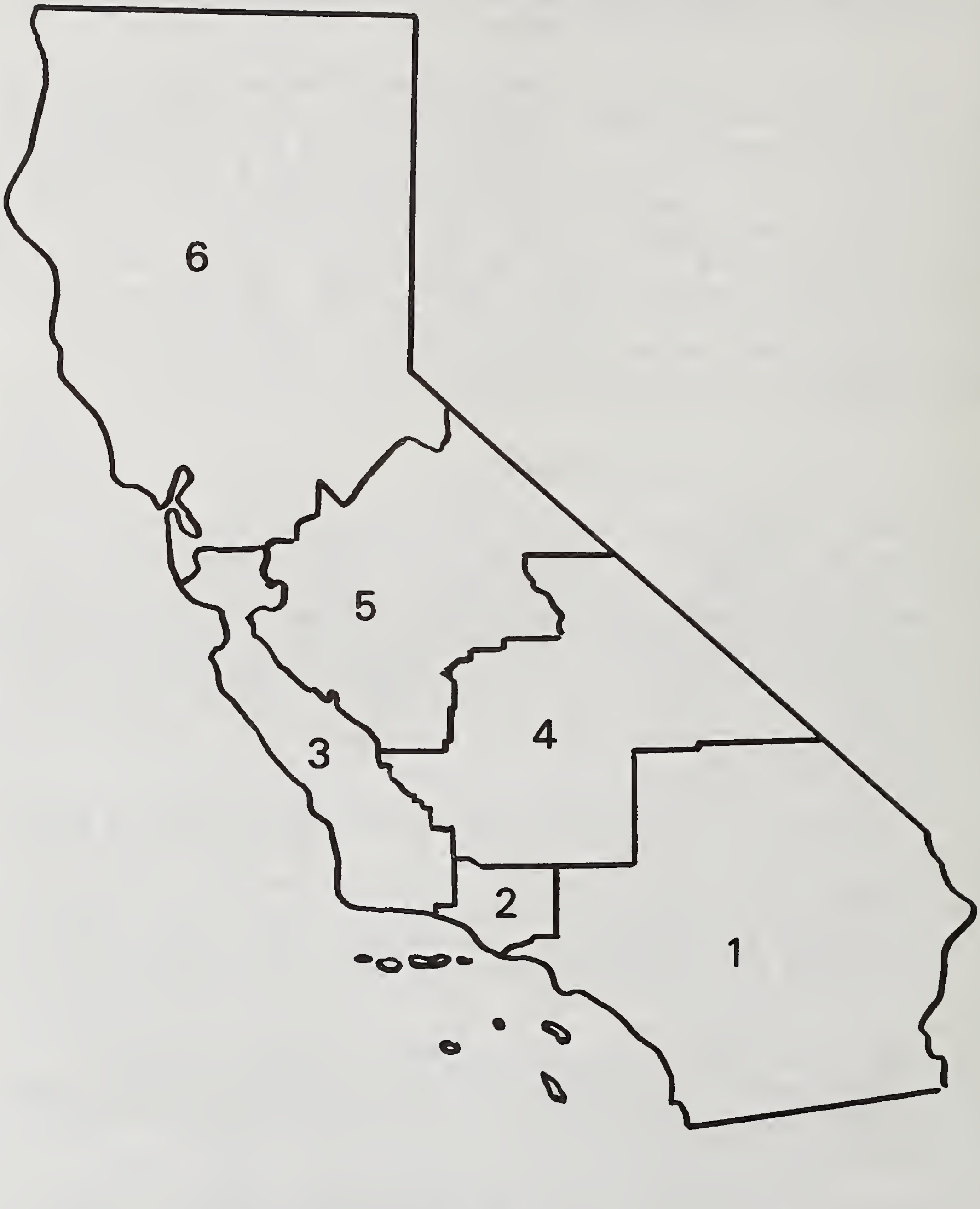

Figure 2. Index map of California showing six districts established by California Division of Oil and Gas for oil-field classification purposes.

plotted with an identical format for each area, as well as for the whole state, using the graphic format described in the section entitled Graphic Presentation of the Data. The preparation of the tables is discussed in the section entitled Tabular Presentation.

Entire State

California's overall population (Figure 3) of 404 fields approximates a lognormal distribution, although there is some skewness. The subpopulations segregated by discovery sequence reveal a drastic decrease in general sizes of the fields in the progression involving the first four discovery intervals (A,B,C, and D). As Table 1A indicates, there is nearly a 300-fold decrease in the median size from A to D, with corresponding large decreases in the geometric means (over 100-fold) and in the aggregate quantity of hydrocarbons discovered (over 50-fold). These large decreases are reversed, however, in the last 20 percent interval (E), which involves a dramatic rise in the median, geometric mean, and total quantity as compared with interval D. The explanation lies partially in a succession of discoveries of large gas fields in the Sacramento Valley, in District 6.

Projections for California as a whole are pessimistic. Of the population of the next 81 fields to be found in the state as a whole (this projection excludes areas which are not part of the area of California that had been explored as of the end of 1974), the population is forecast to have a median size of only about 125,000 BOE, and to yield roughly 185 million BOE, or about 0.6 percent of the BOE contained in fields discovered through the end of 1974 in the state. Probabilities attached to individual field-size ranges for this forecast population are shown in the first row in the body of Table 2.

It is probably of marginal value to consider California as a whole from an exploration forecasting standpoint, although the state's total outlook for new-field discoveries has strong relevance with regard to the nation's energy policy. Analyses of the individual districts are, however, more revealing from an exploration standpoint.

District 1

District 1 embraces fields of the Los Angeles basin as well as the eastern end of the Ventura basin (Newhall area). The plots (Figure 4) reveal an extremely large decline in the medians and geometric means following interval B (which ended in 1940). As Table 1-B details, the medians and geometric means declined on the order of 100-fold. Such a decline reflects the early discovery of very large fields, including Wilmington, Santa Fe Springs, Huntington Beach and Long Beach, discoveries which were not duplicated in size in later intervals.

The forecast for District 1 for new field discoveries is a guarded one. A "realistic" versus a "pessimistic" forecast is provided in Figure 4b and c, Table 1-A and Table 2. Two sets of curves, labeled F

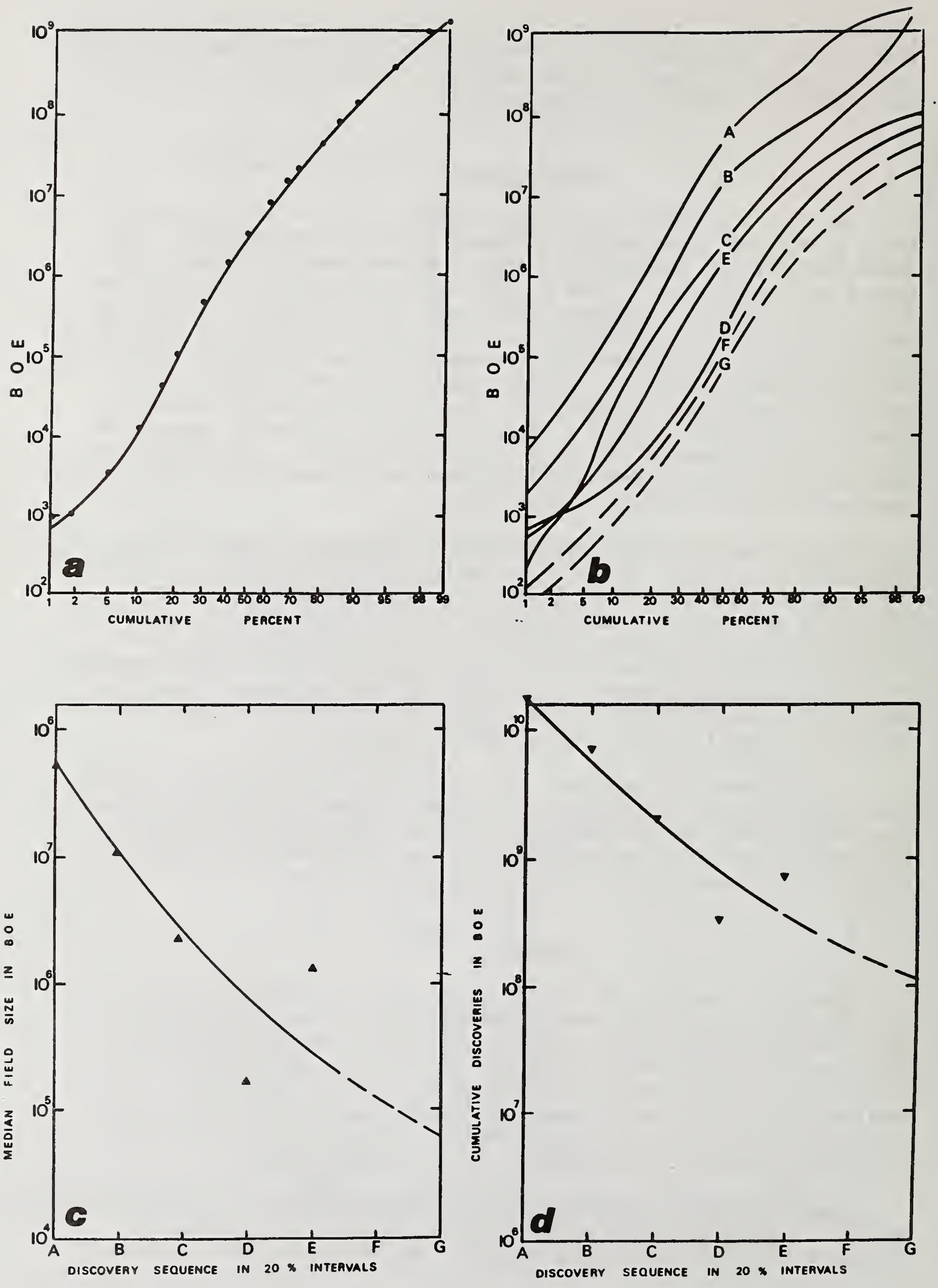

Figure 3. Combined oil and gas field distributions for entire California. See section entitled Graphic Presentation of the Data for explanation.

TABLE 1. CALIFORNIA PRODUCTION STATISTICS AND FORECASTS

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of BOE	Geometric Mean in Thousands of BOE	Total for Interval in Millions of BOE	Percentage of Present Total for Entire District
A: Entire State								
Entire District		0-100	1861-1974	404	3,040	2,032	28,643	100.0
Progressive Discoveries Through 1974	A	0-20	1861-1928	81	55,320	23,032	18,008	62.9
	B	20-40	1928-1943	81	12,460	6,066	7,460	26.0
	C	40-60	1943-1952	81	1,980	1,519	2,113	7.4
	D	60-80	1952-1959	80	190	204	342	1.2
	E	80-100	1959-1974	81	1,410	784	720	2.5
Forecast Future Discoveries	F	100-200		81	125		185	0.6
	G	120-140		81	70		118	0.4
B: District 1								
Entire District		0-100	1875-1967	89	8,320	2,919	10,940	100.0
Progressive Discoveries Through 1974	A	0-20	1875-1921	17	106,000	75,494	4,901	44.8
	B	20-40	1921-1940	18	94,570	42,618	5,501	50.3
	C	40-60	1940-1947	18	640	488	157	1.4
	D	60-80	1947-1955	18	1,550	482	265	2.4
	E	80-100	1955-1967	18	670	284	116	1.1
Forecast Future Discoveries	F	100-120		18	260		32	0.3
	G	120-140		18	108		17	1.1
C: Districts 2 & 3								
Entire District		0-100	1861-1967	91	3,498	2,074	4,626	100.0
Progressive Discoveries Through 1974	A	0-20	1861-1902	18	2,760	2,172	371	8.0
	B	20-40	1902-1932	18	38,423	19,962	2,659	57.5
	C	40-60	1932-1950	19	7,095	7,693	1,366	29.5
	D	60-80	1950-1958	18	190	213	119	2.6
	E	80-100	1958-1967	18	933	486	111	2.4
Forecast Future Discoveries	F	100-200		18	75		18	0.4
	G	120-140		18	15		6	0.1

Table 1 (cont'd)

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of BOE	Geometric Mean in Thousands of BOE	Total for Interval in Millions of BOE	Percentage of Present Total for Entire District
D: Districts 4 & 5								
Entire District		0-100	1890-1974	122	4,210	2,822	11,686	100.0
Progressive Discoveries Through 1974	A	0-20	1890-1929	24	158,500	42,862	9,147	78.3
	B	20-40	1929-1940	25	8,600	7,255	1,410	12.1
	C	40-60	1940-1946	24	7,140	4,186	482	4.1
	D	60-80	1946-1956	24	490	549	490	4.2
	E	80-100	1956-1974	25	140	186	157	1.3
Forecast Future Discoveries	F	100-120		24	52		100	0.9
	G	120-140		24	15		70	0.6
E: District 6								
Entire District		0-100	1890-1973	102	1,550	953	1,391	100.0
Progressive Discoveries Through 1974	A	0-20	1890-1944	20	1,950	1,528	800	57.5
	B	20-40	1944-1953	21	1,560	711	136	9.8
	C	40-60	1953-1960	20	1,130	386	61	4.4
	D	60-80	1960-1962	21	1,840	950	242	17.4
	E	80-100	1962-1973	20	2,640	1,185	152	10.9
Forecast Future Discoveries	F	100-120		20	900		35	2.5
	G	120-140		20	400		25	1.8

Table 2. Probabilities attached to field-size ranges for the next 20 percent (F) of fields to be discovered, and for the next 20 percent (G) to be discovered after that, in California

Area	Label on Curve	Number of Fields	Probabilities (in percent) attached to field-size ranges in BOE						
			$<10^3$	10^3 to 10^4	10^4 to 10^5	10^5 to 10^6	10^6 to 10^7	10^7 to 10^8	10^8 to 10^9
Entire State	F	81	7	19	22	21	22	9	
	G	81	12	19	21	24	19	5	
District 1 (realistic)	F	18	10	11	18	23	21	15½	1½
	G	18	14	8	23	24	21	9	1
District 1 (pessimistic)	F'	18	21	17	23	22	14½	2½	
	G'	18	39	24	21	13	3		
Districts 2 & 3 combined (realistic)	F	18	9	19	23	24	20	5	
	G	18	14	22	27	25	11½	½	
Districts 4 & 5 combined	F	24	12	18	23	25	14	7	1
	G	24	20	22	26	19	10	2½	½
District 6 (optimistic)	B	20	6	5	11	20	36	21½	½
		20							
District 6 (realistic)	F	20	6	8	14	25	32	15	
	G	20	9	9	14	27	34	7	

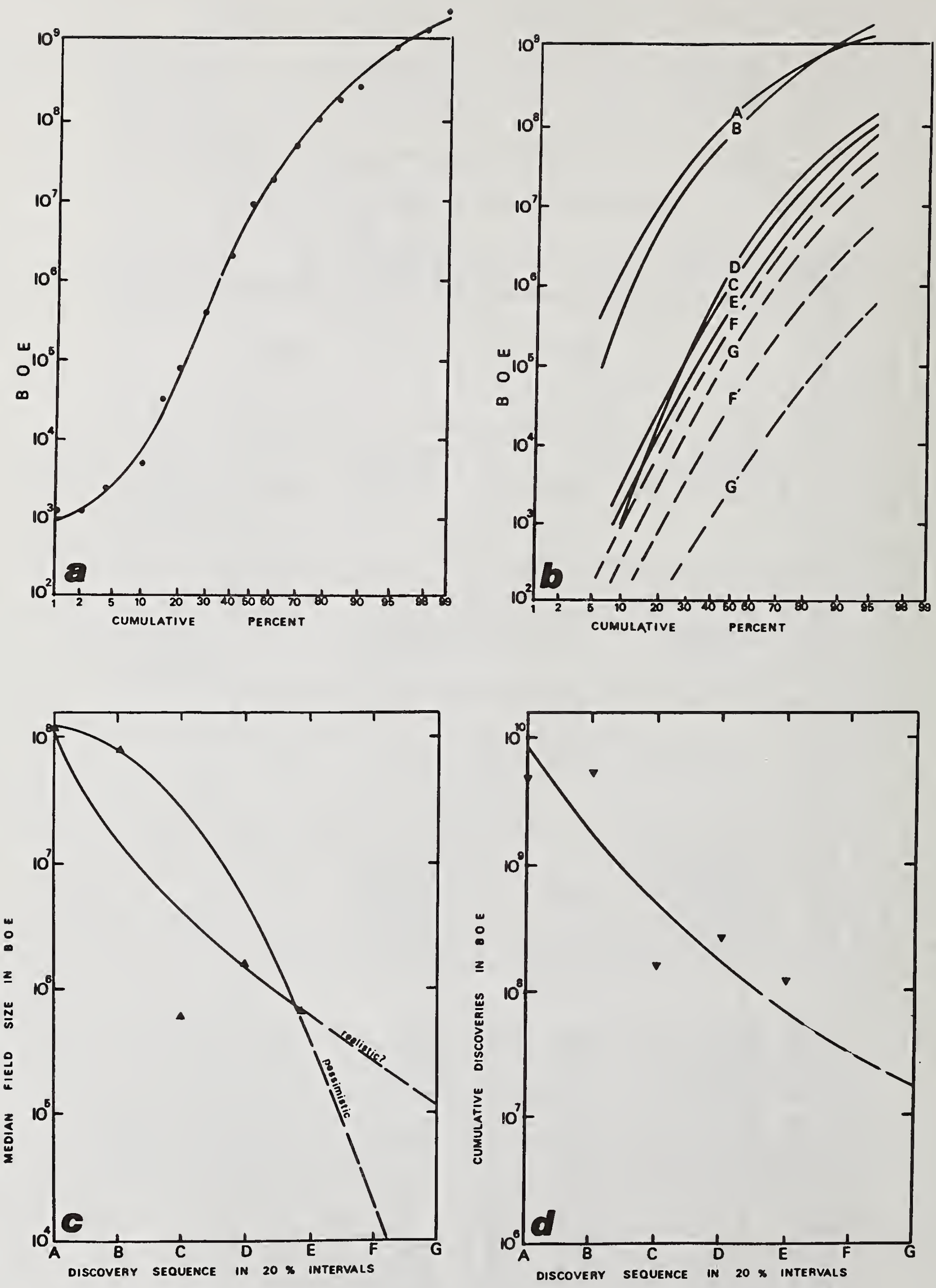

Figure 4. District 1, California.

and G, and F' and G' in Figure 4-b, represent the "realistic" versus "pessimistic" forecasts. The "realistic" forecast distribution, however, will yield only about 0.5 percent of the present aggregate BOE if 36 new fields are actually discovered.

As Table 2 reveals, the probability of finding a field greater than 100 million BOE is only about 1½ percent for any particular field among the next 18 fields to be discovered in District 1 (assuming 18 fields are to be discovered). On the other hand, a probability of about 21 percent is attached to a discovery of less than 10,000 BOE for each field to be discovered among these next 18 fields. Such small sizes are absurdly uneconomic for most of District 1, and may be discounted in advance as "non-discoveries". Nevertheless, they represent the forecast distribution of field sizes employing the "realistic" curve, F, for District 1.

Districts 2 and 3

Districts 2 and 3 combined are paradoxical in that initial interval A has a substantially smaller median (and geometric mean) than intervals B and C (Table 1-C). Thus, the usual sequence has been reversed (Figure 5). This is explainable, in part, by the large geographic expanse of the combined districts and their geologic diversity. Major discoveries, such as the Ventura field, occurred in interval B, accounting for its large median and geometric mean. Interval C, too, was blessed with large discoveries (Santa Maria and San Ardo fields, for example), accounting for its intermediate median and geometric mean.

Curves F and G of Figure 5-b represent the "realistic" forecast, and seem to be in accord with overall trends.

Districts 4 and 5

Districts 4 and 5 embrace the central and southern San Joaquin Valley, which forms a large and diverse petroleum-producing province. As Figure 6 reveals, the discoveries during interval A yield a population with an exceedingly large median and geometric mean. This is readily explainable by the early discoveries of a number of giant fields (Buena Vista, Coalinga, Elk Hills, Kern River, Kettleman Hills, Midway-Sunset, and South Belridge). Although some major discoveries were made in the next interval (East Coalinga Extension, for example), these subsequent discoveries did not keep pace in size. The decrease in field-size medians (Table 1-D) from interval A to E is impressive (more than an 1100-fold decrease). The decline in geometric means, and in gross BOE discovered, though less dramatic, is still very large. Based on these trends, the forecast for Districts 4 and 5 is not encouraging, as population F consisting of the next 24 fields to be discovered has a forecast median of only 52,000 BOE, with only an 8 percent probability that any field will be larger than 10 million BOE, and a probability of only about 1 percent that any field will be larger than 100 million BOE.

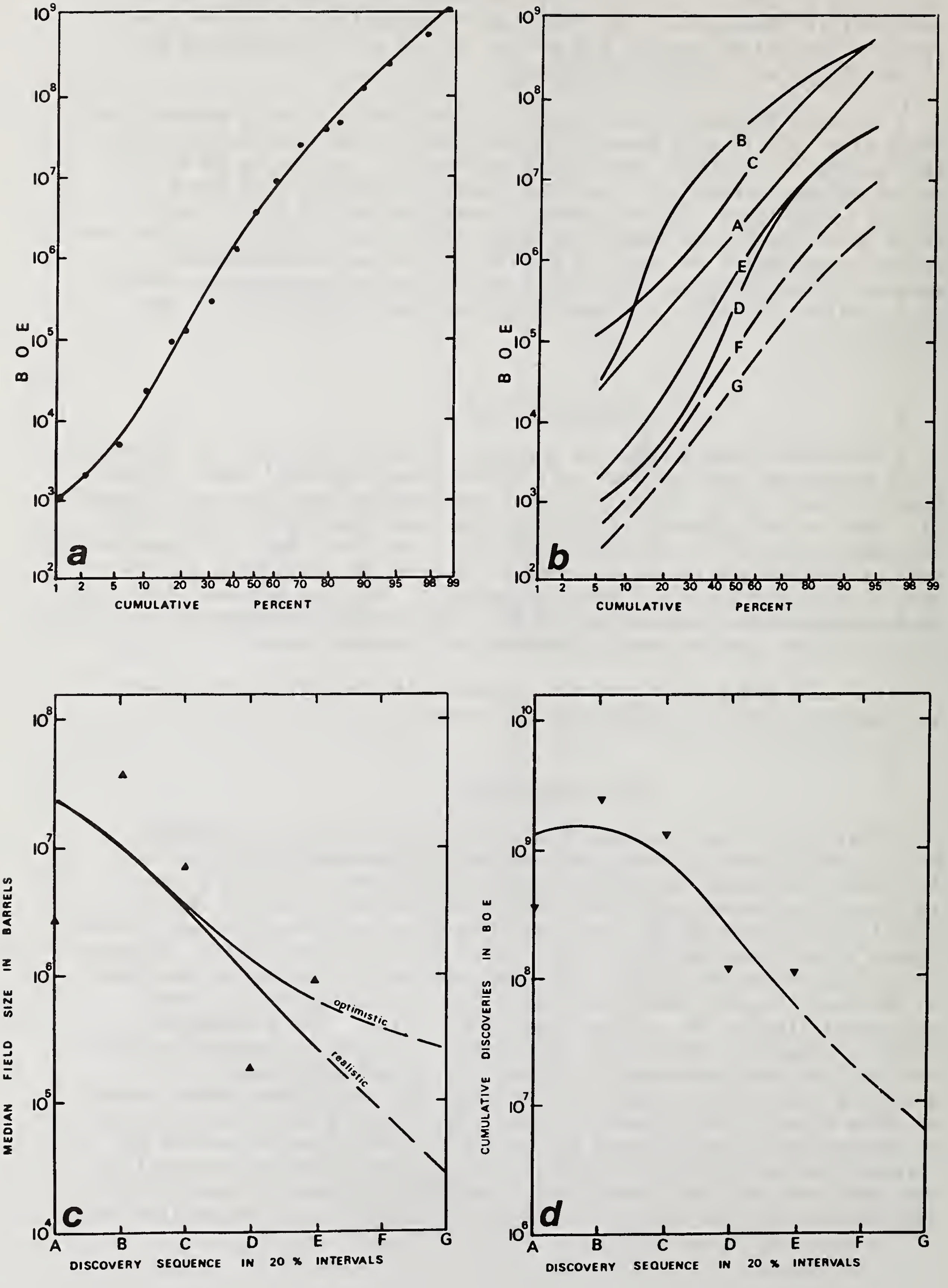

Figure 5. Districts 2 and 3 combined, California.

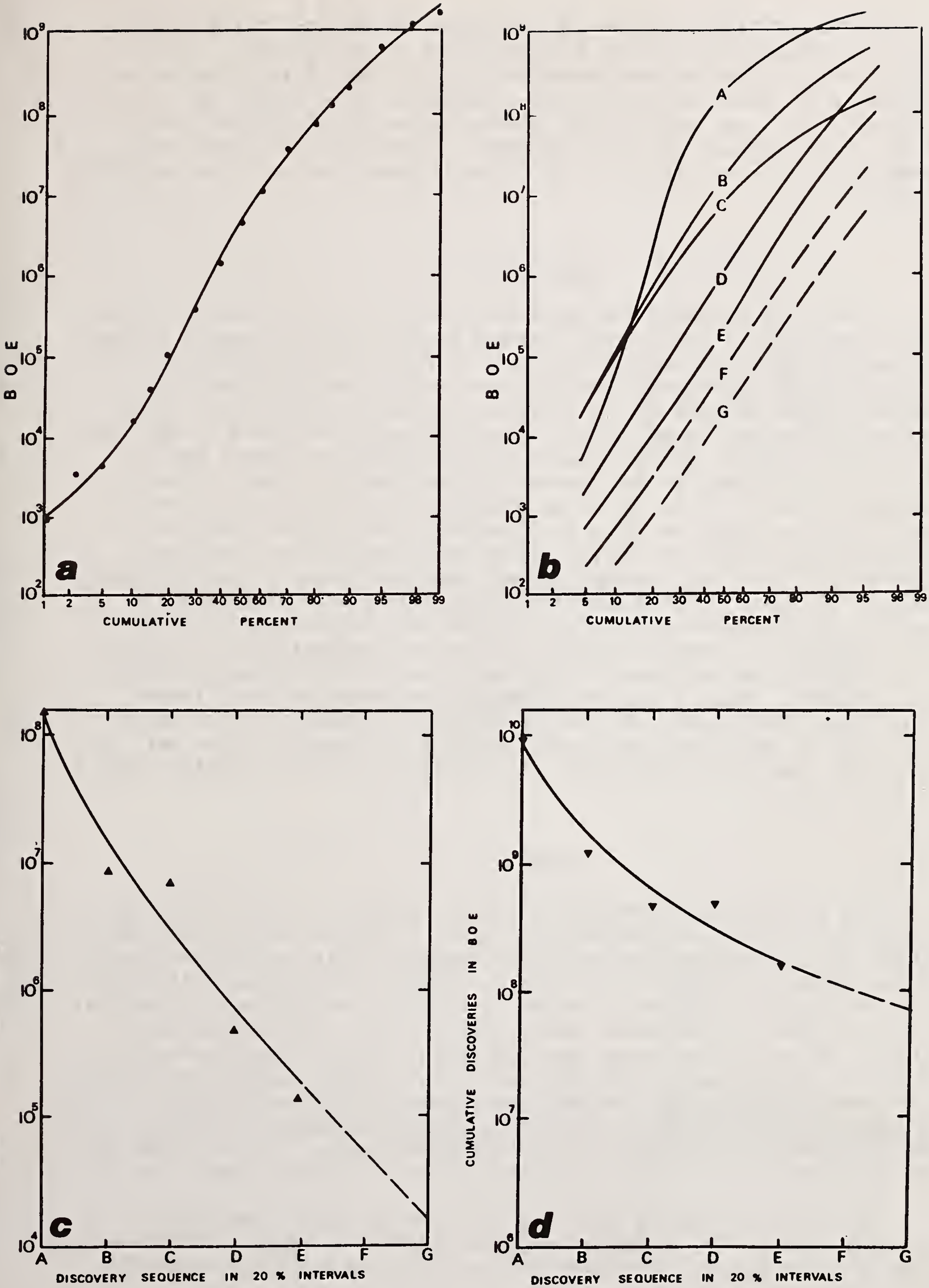

Figure 6. Districts 4 and 5 combined, California.

The population of 24 fields discovered during the initial interval (A) is strongly skewed, with a predominance of large fields. This is demonstrated by the extreme departure of the graph of this population (Figure 6-b) from a straight line. A smoothed curve fitted to a histogram of field sizes and plotted in conventional form (Figure 7) emphasizes this departure from the lognormal. Populations of fields discovered in later intervals, D and E, more closely approach the lognormal ideal.

District 6

District 6 embraces the Sacramento Valley and the central part of the Great Valley (that is, the northern part of the San Joaquin Valley). Virtually all of the production is gas. The overall population of 102 fields departs moderately from the lognormal (Figure 8-a), but the subpopulations defined by the succession of discoveries do not reveal the abrupt decline in medians (or geometric means) observed in the other districts. Indeed, both the medians and the geometric means decline from A to C, (Figure 8-c and Table 1-E) but they rise again in the succession from C to E. If we were to take a very optimistic view of the future, we might envision a progressive rise in the field-size parameters, as might be represented by the curve labeled "very optimistic" of Figure 8-c. However, a more realistic view is that the populations of fields to be discovered in the future will progressively decline. An estimated median of 900,000 BOE for the next 20 fields to be discovered (F) seem reasonable. Given the uncertainties in projection, however, we can take a view that an optimistic forecast also may be justified. Table 2 provides probabilities attached to different field sizes that accord with an "optimistic" projection (which coincides, more or less, with the curve labeled B in Figure 8-b), as well as with the "realistic" projection, which yields the curves labeled F and G in Figure 8-b.

WYOMING

The six principal sedimentary basins in Wyoming are outlined in Figure 9. Four of the basins (Green River, Big Horn, Wind River, and Powder River) have been analyzed in a fashion similar to that employed in California. In addition, fields in the Powder River basin also have been segregated according to whether they are associated with structural traps, or with stratigraphic traps. The two other basins (the Hanna-Laramie basin and the Denver basin) contain an insufficient number of fields to be analyzed in the same manner as the other basins, although a frequency distribution for each basin overall has been plotted. Please note that the Denver basin is a very large basin, but only a small fraction of its total area lies within Wyoming. The Colorado and Nebraska portions of the Denver basin are not considered here.

The data for Wyoming were obtained from an unpublished report prepared by the firm of Barlow and Haun (1978). Segregation of the oil-field data by basins seems more desirable than by arbitrary districts,

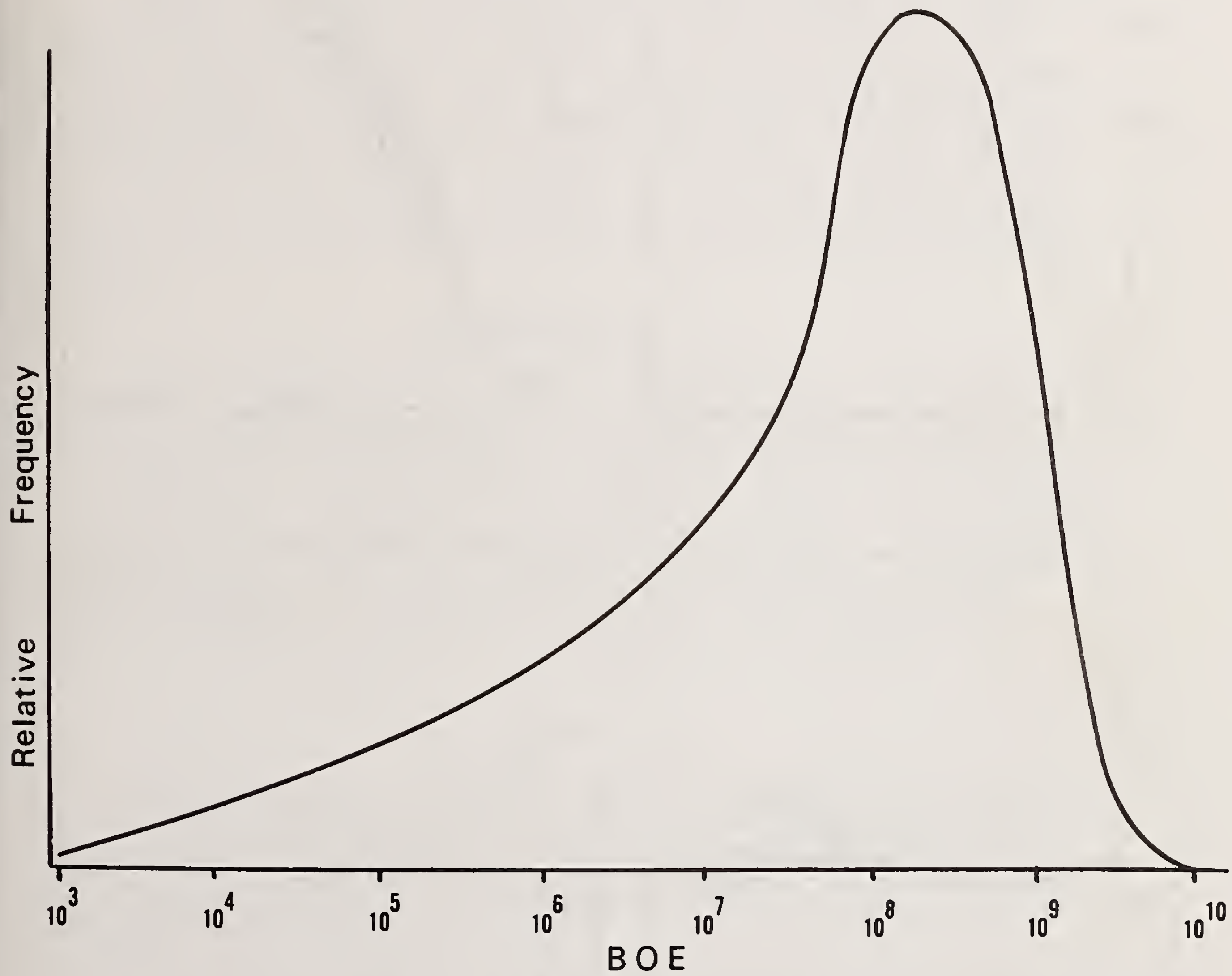

Figure 7. Plot of frequency distribution in standard form of curve A shown in Figure 6-b, representing first 20 percent of fields discovered in Districts 4 and 5 combined, emphasizing strong skewness of the distribution.

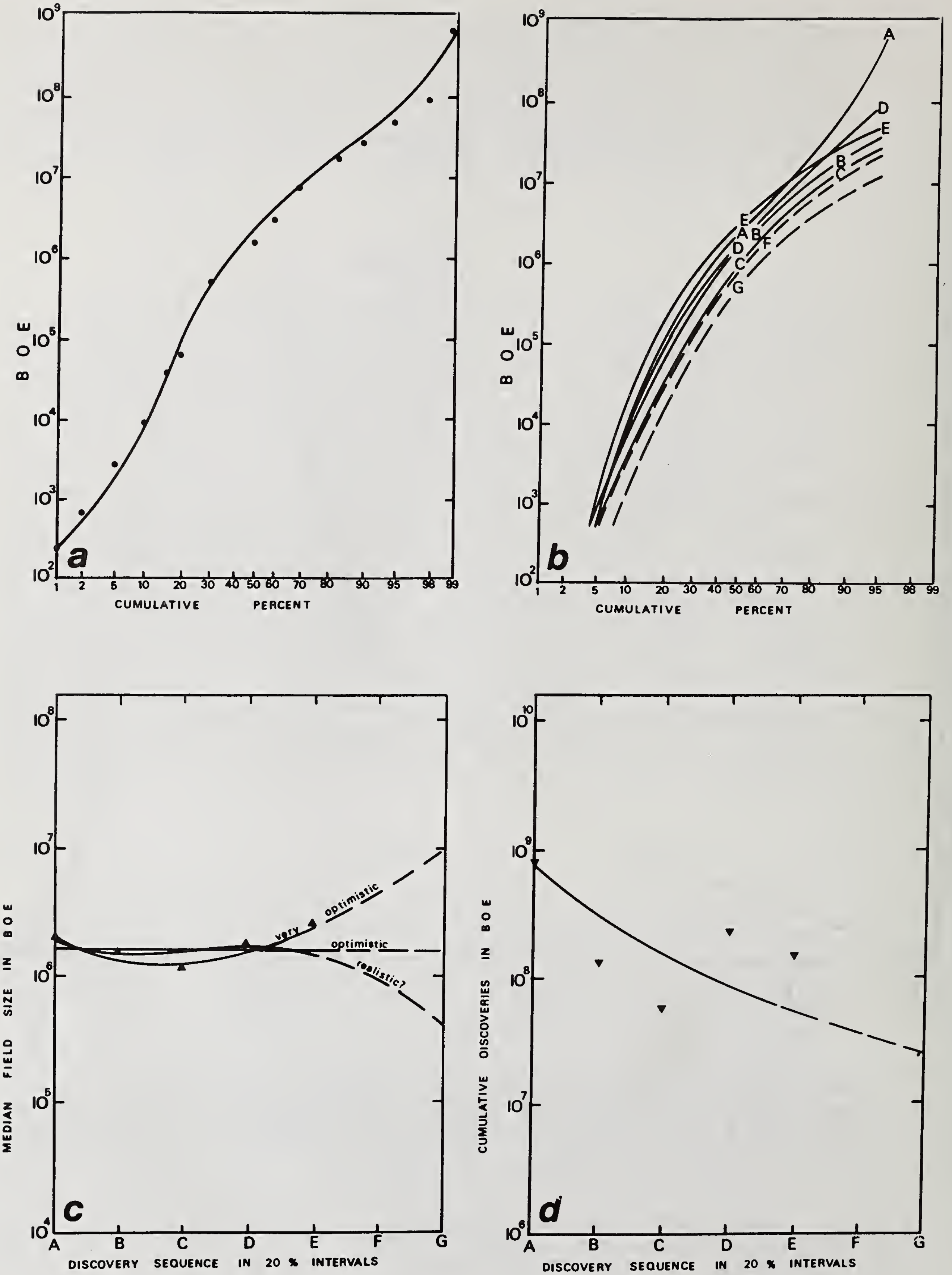

Figure 8. District 6, California.

particularly in Wyoming where the individual basins are geologically segregated from each other. Our objective in this study has been to determine whether the field-size statistics differ from basin to basin, perhaps reflecting underlying geological controls on petroleum occurrence within individual basins.

Entire State

Statistics for entire Wyoming are presented in Figure 10, Table 3-A and in Table 4. The distribution of field sizes, expressed in BOE, closely approximates an ideal lognormal distribution. When the 754 fields incorporated in this study are segregated by discovery sequence into 20 percent intervals, there is a general progressive decrease in medians, geometric means, and total BOE discovered. Although there is some overlapping of the distributions (curves A through E in Figure 10-b), the shifts are sufficiently regular so that future discoveries can be forecast by projection. The "realistic" projection of the medians (Figure 10-c) accords with the curves representing forecast populations F and G.

Green River Basin

The distribution of fields as a whole for the Green River basin approximates the lognormal (Figure 11-a) but the subpopulations, A through E, deviate considerably from the lognormal ideal (Figure 11-b). The subpopulation medians, geometric means, and total BOE (Figures 11-c and d, and Table 3-B) shift in somewhat erratic fashion. Curves F and G (Figure 11-b) representing the populations of fields to be discovered are based on the "realistic" projections of the medians. The probabilities attached to size ranges of fields to be discovered (Table 4) surpass those of the other basins in Wyoming, and so the Green River basin is relatively attractive from a statistical standpoint.

Big Horn Basin

The Big Horn basin (Figure 12, Table 3-C and Table 4) has an overall population that is virtually perfectly lognormally distributed. There is a very sharp decrease in field-size parameters between intervals C and D, with some improvement from D to E. The overall field-size population trends are not encouraging, and populations F and G are forecast to have small total volumes.

Wind River Basin

The populations of fields in the Wind River basin display a somewhat erratic behavior. The overall population (Figure 13-a) significantly departs from the lognormal ideal. Subpopulations A, B, D, and E also depart from the ideal lognormal, although subpopulation C is essentially lognormal (Figure 13-b). The populations to be discovered

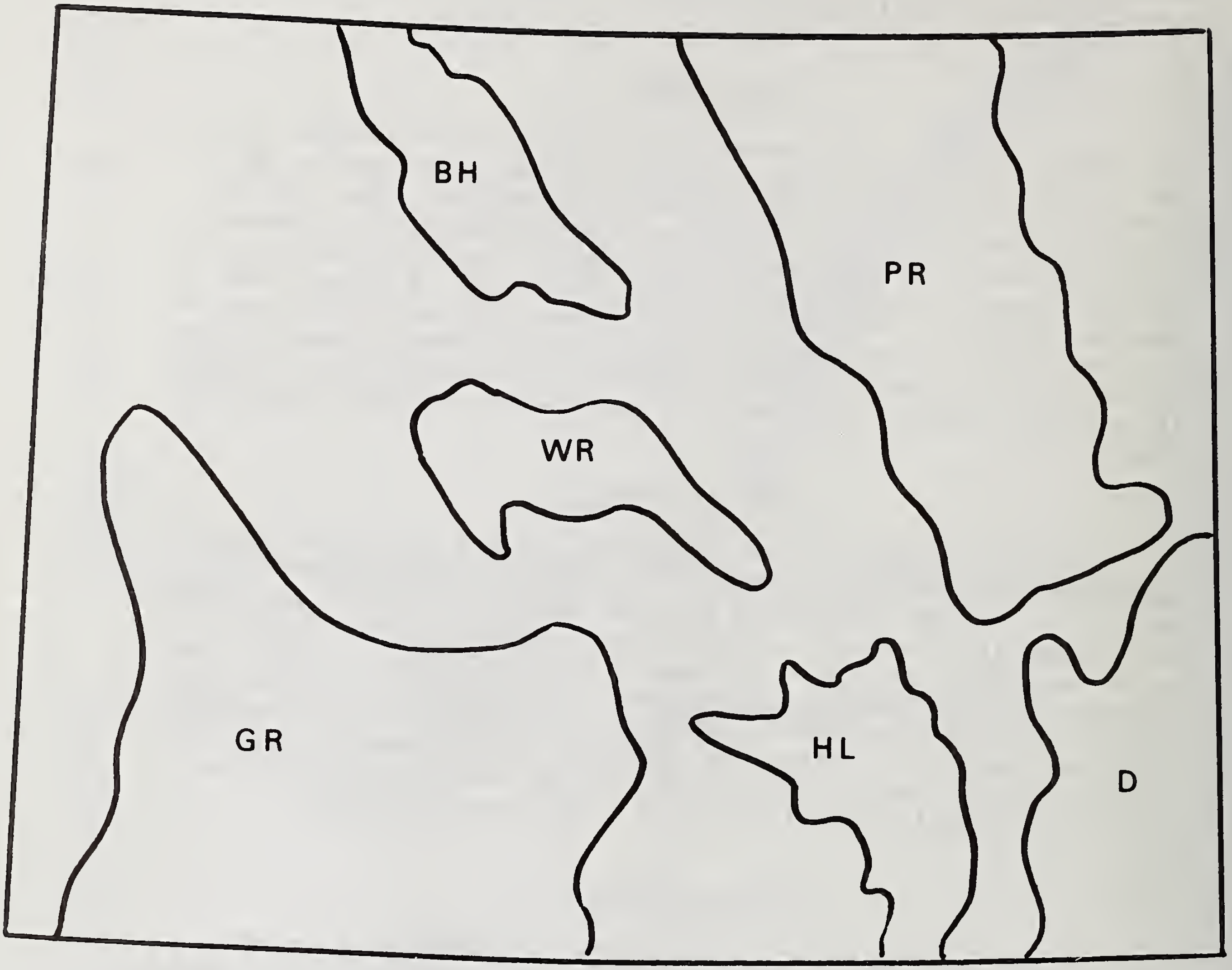

Figure 9. Index map of Wyoming showing principal sedimentary basins: GR = Green River basin, BH = Big Horn basin, PR = Powder River basin, HL = Hanna-Laramie basin, D = Denver basin.

TABLE 3. WYOMING PRODUCTION STATISTICS AND FORECAST

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of BOE	Geometric Mean in Thousands of BOE	Total for Interval in Millions of BOE	Percentage of Present Total for Entire District
A: Entire State								
Entire District		0-100	1884-1977	754	696	578	8,771	100.0
Progressive Discoveries Through 1977	A	0-20	1886-1948	151	2,259	2,320	5,698	65.0
	B	20-40	1948-1959	151	942	814	1,294	14.8
	C	40-60	1959-1966	150	551	417	670	7.6
	D	60-80	1966-1972	151	378	306	596	6.8
	E	80-100	1972-1977	151	358	279	513	5.8
Forecast Future Discoveries	F	100-200		151	200		140	1.6
	G	120-140		151	150		90	1.0
B: Green River Basin								
Entire District		0-100	1900-1977	139	902	835	2,582	100.0
Progressive Discoveries Through 1977	A	0-20	1900-1956	28	2,236	2,050	1,565	60.0
	B	20-40	1956-1961	28	2,632	2,022	677	26.3
	C	40-60	1961-1969	27	292	245	32	1.2
	D	60-80	1969-1974	28	877	958	230	8.9
	E	80-100	1974-1977	28	887	434	78	3.0
Forecast Future Discoveries	F	100-120		28	500		20	0.8
	G	120-140		28	450		11	0.4
C: Big Horn Basin								
Entire District		0-100	1906-1977	108	1,008	1,170	2,457	100.0
Progressive Discoveries Through 1977	A	0-20	1906-1926	22	8,617	7,210	1,787	72.7
	B	20-40	1926-1947	21	4,125	4,109	357	14.5
	C	40-60	1947-1953	22	2,527	1,494	221	9.0
	D	60-80	1953-1964	21	186	236	74	3.0
	E	80-100	1964-1977	22	340	233	18	0.7
Forecast Future Discoveries	F	100-120		21	125		5	0.2
	G	120-140		21	85		1	0.04

Table 3 (cont'd)

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of BOE	Geometric Mean in Thousands of BOE	Total for Interval in Millions of BOE	Percentage of Present Total for Entire District
D: Wind River Basin								
Entire District		0-100	1884-1974	76	405	604	904	100.0
Progressive Discoveries Through 1977	A	0-20	1886-1925	15	11,860	4,133	331	36.6
	B	20-40	1925-1954	15	4,314	2,007	374	41.4
	C	40-60	1954-1960	16	211	353	49	5.4
	D	60-80	1960-1966	15	388	265	41	4.5
	E	80-100	1966-1974	15	78	135	109	12.1
Forecast Future Discoveries	F	100-120		15	50		25	2.8
	G	120-140		15	30		11	1.2
E: Powder River Basin (all fields)								
Entire District		0-100	1887-1977	380	589	426	2,643	100.0
Progressive Discoveries Through 1977	A	0-20	1887-1954	76	1,414	973	1,649	62.4
	B	20-40	1954-1963	76	993	634	285	10.8
	C	40-60	1963-1968	76	423	356	277	10.5
	D	60-80	1968-1973	76	435	303	224	8.5
	E	80-100	1973-1977	76	185	211	208	7.9
Forecast Future Discoveries	F	100-120		76	125		80	3.0
	G	120-140		76	90		65	2.5
F: Powder River Basin (structural fields)								
Entire District		0-100	1887-1977	134	500	412	1,734	100.0
Progressive Discoveries Through 1977	A	0-20	1887-1941	27	1,000	895	1,220	70.4
	B	20-40	1941-1952	27	2,655	1,314	337	19.4
	C	40-60	1952-1960	26	587	347	72	4.2
	D	60-80	1960-1968	27	220	206	85	4.9
	E	80-100	1968-1977	27	112	116	20	1.3
Forecast Future Discoveries	F	100-120		27	65		10	0.8
	G	120-140		27	42		7	0.6

Table 3 (cont'd)

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of BOE	Geometric Mean in Thousands of BOE	Total for Interval in Millions of BOE	Percentage of Present Total for Entire District
G: Powder River Basin (stratigraphic fields)								
Entire District		0-100	1943-1977	246	607	431	905	100.0
Progressive Discoveries Through 1977	A	0-20	1943-1962	49	1,294	841	247	27.3
	B	20-40	1962-1966	49	501	417	139	15.4
	C	40-60	1966-1969	50	1,612	863	294	32.5
	D	60-80	1969-1974	49	377	245	78	8.6
	E	80-100	1974-1977	49	275	207	147	16.2
Forecast Future Discoveries	F	100-120		49	150		70	7.7
	G	120-140		49	100		58	6.4

Table 4. Probabilities attached to field-size ranges for the next 20 percent (F) of fields to be discovered, and for the next 20 percent (G) to be discovered after that, in Wyoming

Area	Label on Curve	Number of Fields	Probabilities (in percent) attached to field-size ranges in BOE						
			$<10^3$	10^3 to 10^4	10^4 to 10^5	10^5 to 10^6	10^6 to 10^7	10^7 to 10^8	10^8 to 10^9
Whole State	F	151	2	12	24	32	24	5	1
	G	151	3	14	25	33	21	3	1
Green River Basin	F	29	3	7	15	35	29	10	1
	G	29	3	7	19	34	28	8	1
Big Horn Basin	F	22	4	8	26	45	17		
	G	22	4	12	35	45	4		
Wind River Basin	F	16	6	14	39	32	9		
	G	16	7	19	41	31	2		
Powder River Basin (structural fields)	F	27	6	17	35	33	8½	½	
	G	27	6	30	42	19	3		
Powder River Basin (stratigraphic fields)	F	50	3	10	26	37	23	1	
	G	50	4	14	32	38	12		
Powder River Basin (all fields)	F	77	2	14	27	36	19	1½	
	G	77	7	20	36	24	12½	½	

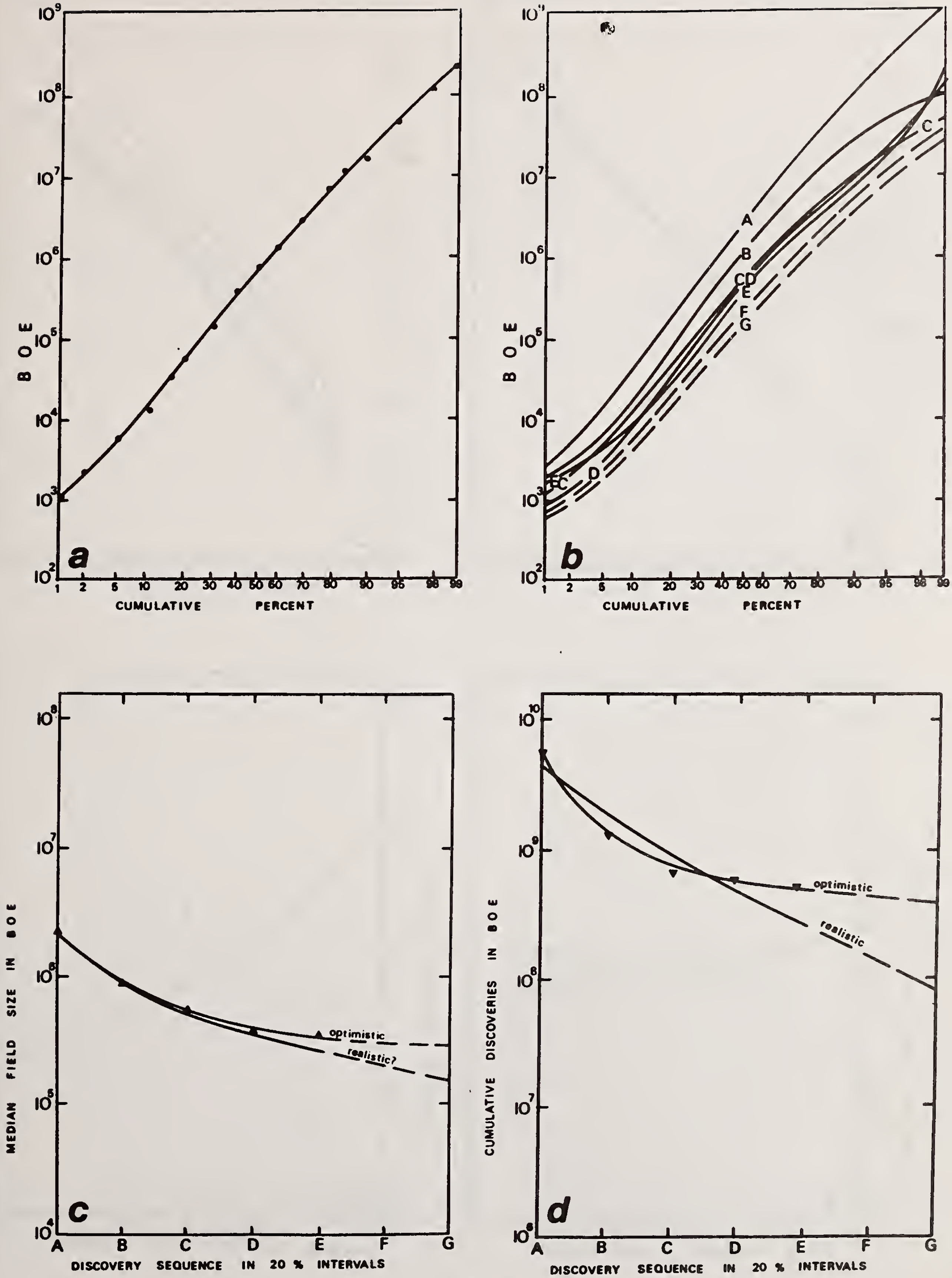

Figure 10. Entire Wyoming. See section entitled Graphic Presentation of the Data for explanation.

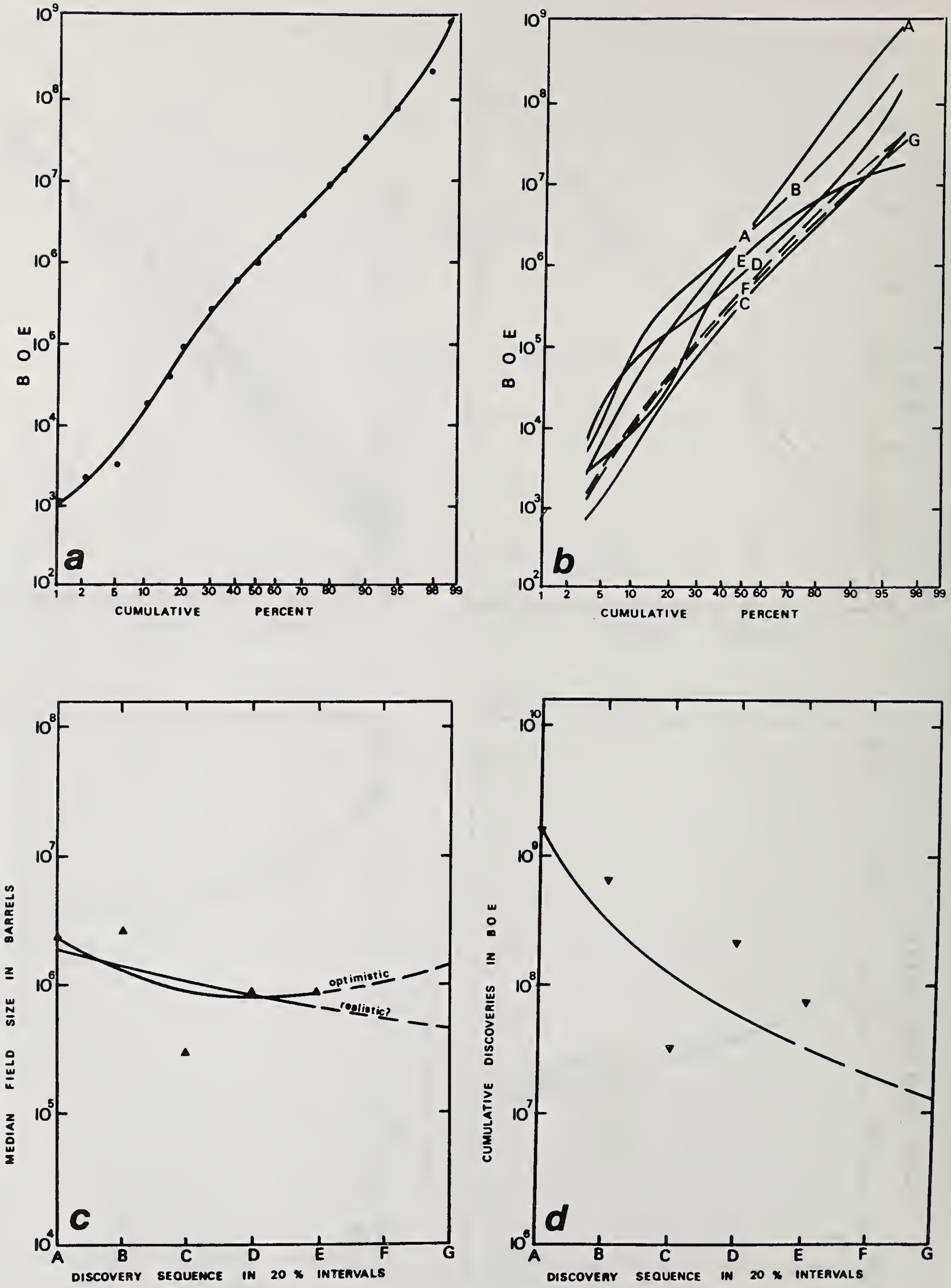

Figure 11. Green River basin of Wyoming.

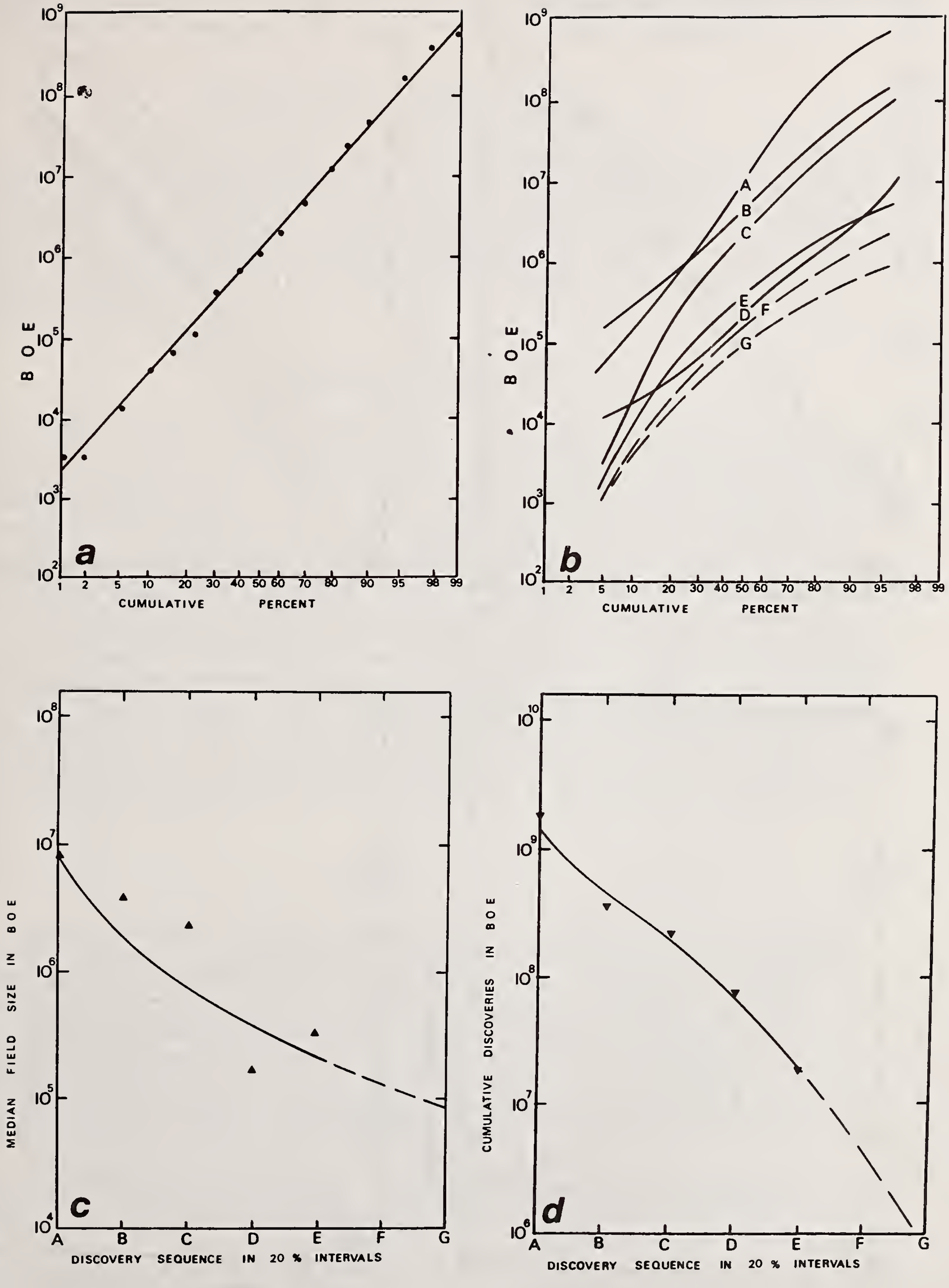

Figure 12. Big Horn basin of Wyoming.

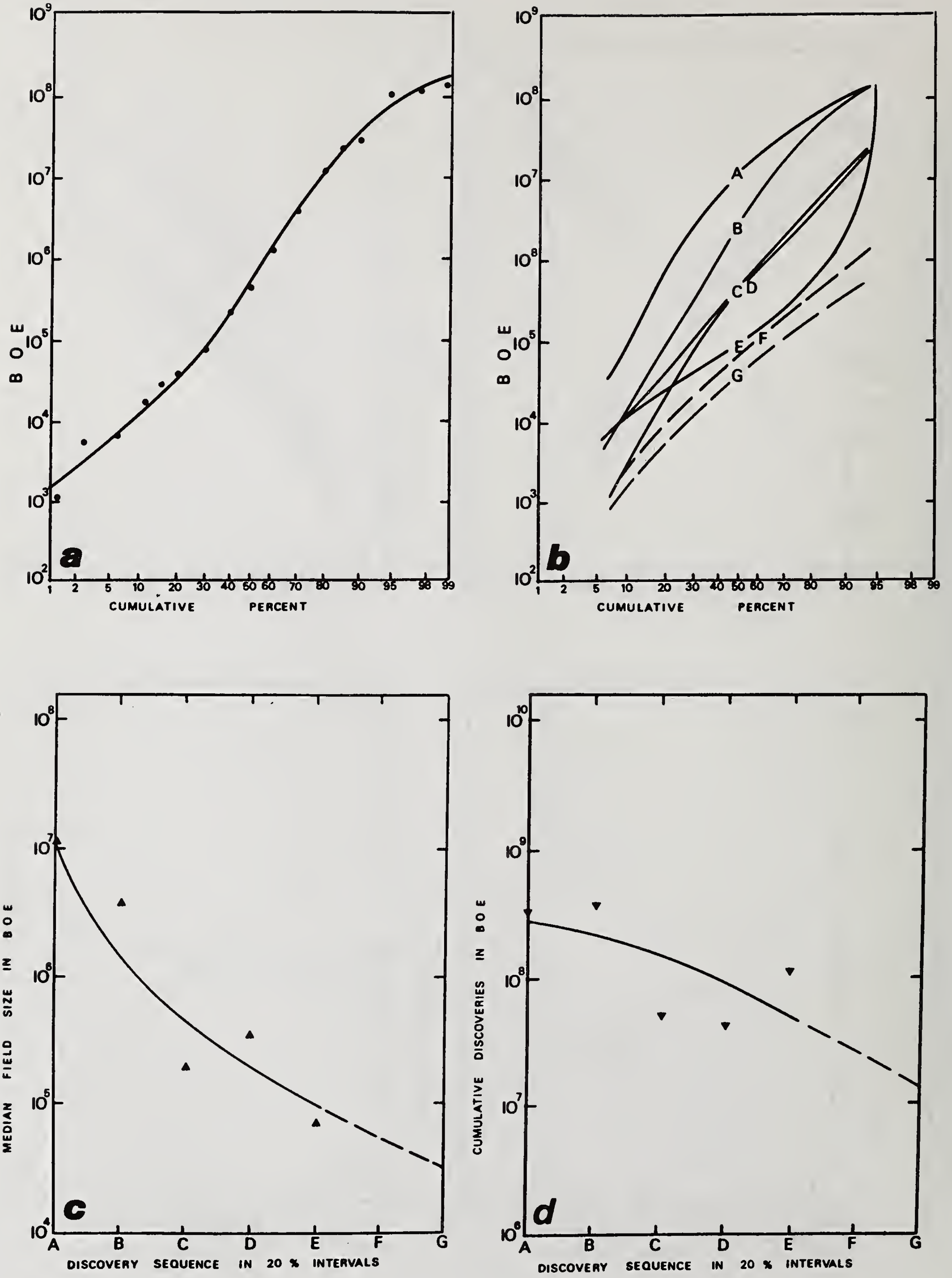

Figure 13. Wind River basin of Wyoming.

(F and G) offer some encouragement, particularly in view of their projected total volumes (Table 3-D and Figure 13-d), although the probabilities attached to the discovery of large fields are small (Table 4).

Hanna-Laramie and Denver Basins

Frequency distributions for the overall populations in each of these basins (Wyoming portion only of the Denver basin) are shown in Figure 14. Both depart from the lognormal ideal, particularly those in the Denver basin. Because of the small overall population (33 fields with recorded production in Hanna-Laramie basin, and 15 in the Denver basin), it is impractical to divide the overall populations into subpopulations.

Powder River Basin

Fields in the Powder River basin were placed in three classes, namely, all fields (380 fields, Figure 15), fields that are structurally controlled (134 fields, Figure 16), and fields that are stratigraphically controlled (246 fields, Figure 17). The structural field subpopulations (except for the last interval, E) depart substantially from the lognormal ideal, there being a pronounced tendency toward early discovery of medium-large fields (Figure 16-b). The stratigraphic fields, on the other hand, depart less from the lognormal ideal (Figure 17-b).

Forecasts for populations of fields to be discovered in the future differ markedly for structural versus stratigraphic fields. The decline in field-size parameters is much less for the stratigraphic fields than for structural fields. This relationship is not surprising, and probably reflects the fact that stratigraphic traps are much less obvious to explorationists than structural traps and therefore the bias toward early discovery of large fields is less for stratigraphic fields than for structural fields.

KANSAS

Kansas has been arbitrarily divided into seven districts (Figure 18) for purposes of this study. The districts do not coincide with officially defined districts, as in California. Districts 1 and 2 embrace much of western and northwestern Kansas. District 3 embraces much of the intensely explored and productive Central Kansas uplift. District 4 embraces part of the Hugoton embayment. Districts 5 and 6 incorporate much of relatively maturely explored, south-central Kansas. District 7 incorporates the remainder of the state, and embraces many older producing areas in southeastern Kansas.

The districts are defined in terms of townships and ranges as follows: District 1: T 1S-6S, R 16W-42W; District 2: T 7S-15S, R 22W-

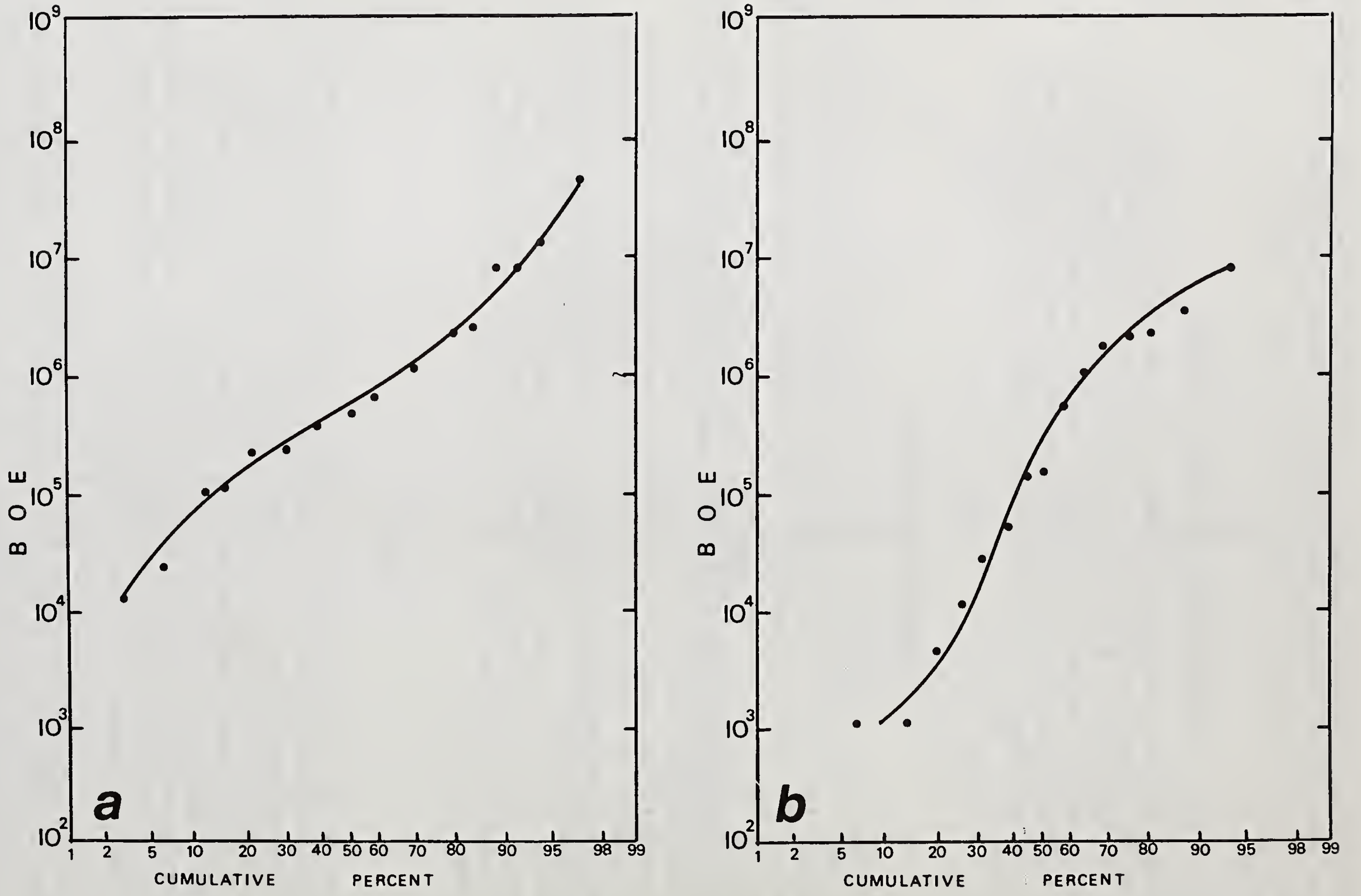

Figure 14. Distributions for all fields in (a) Hanna-Laramie basin, and (b) Denver basin, in Wyoming.

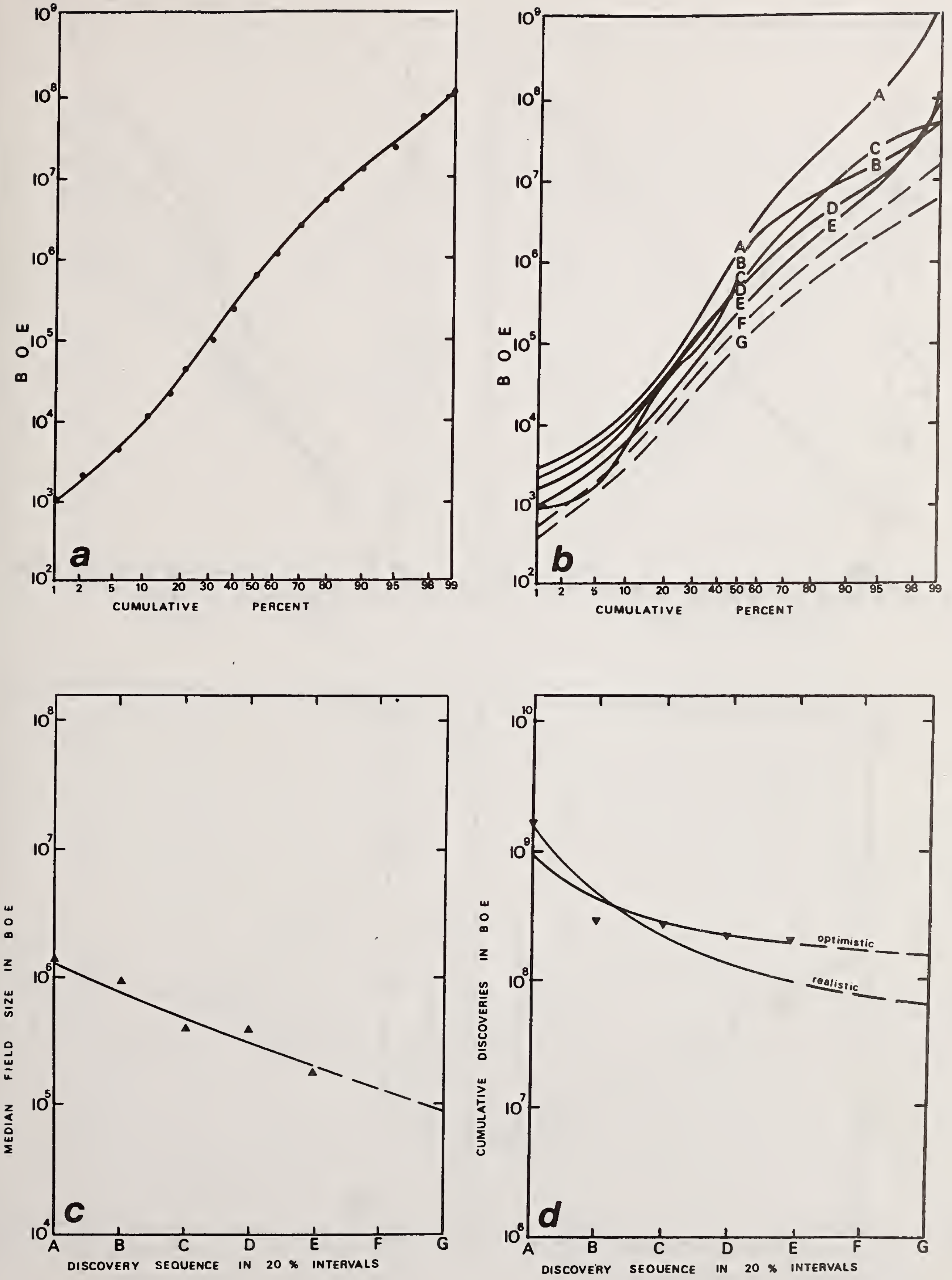

Figure 15. Powder River basin of Wyoming, all fields.

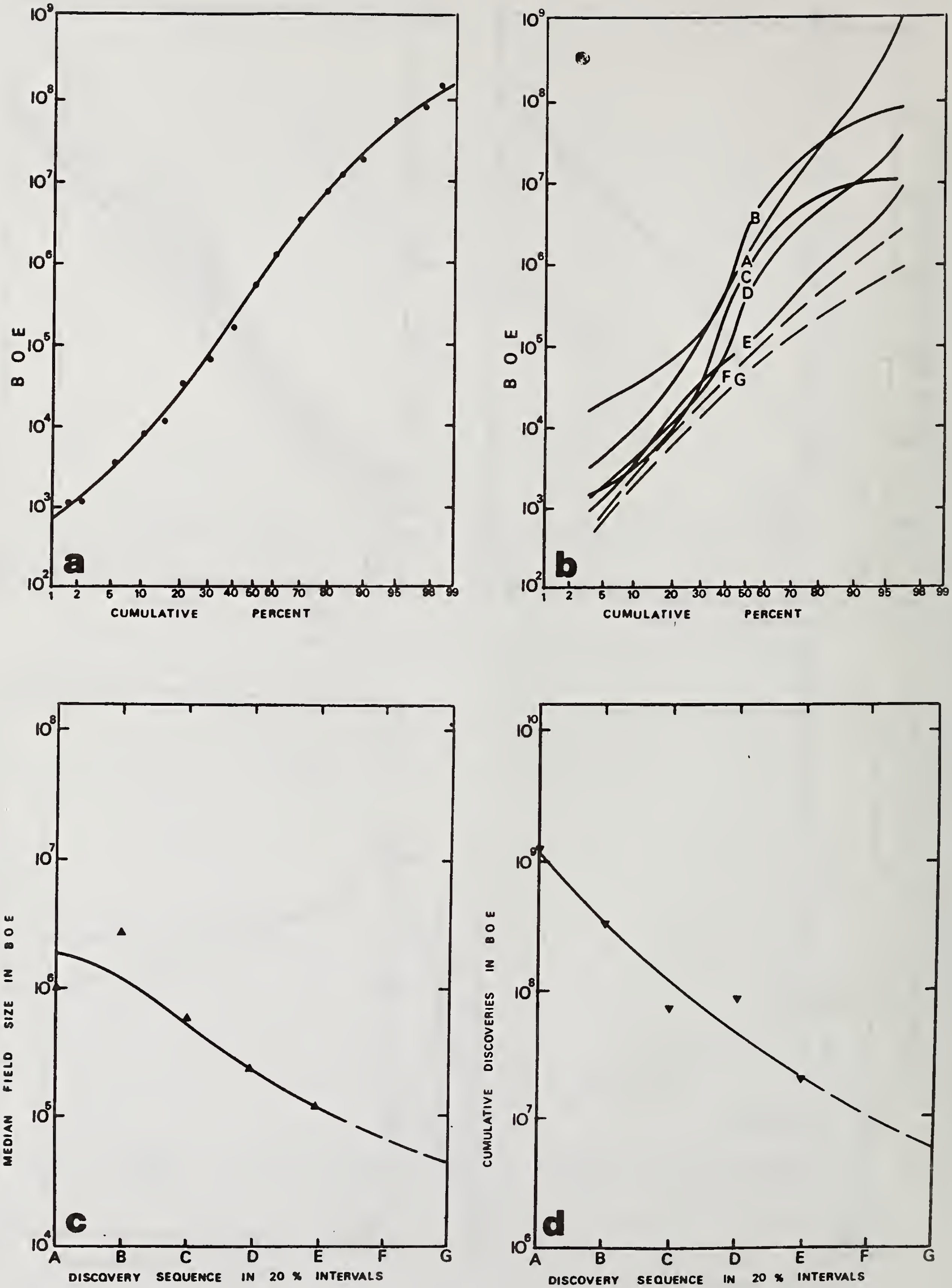

Figure 16. Powder River basin of Wyoming, structural fields only.

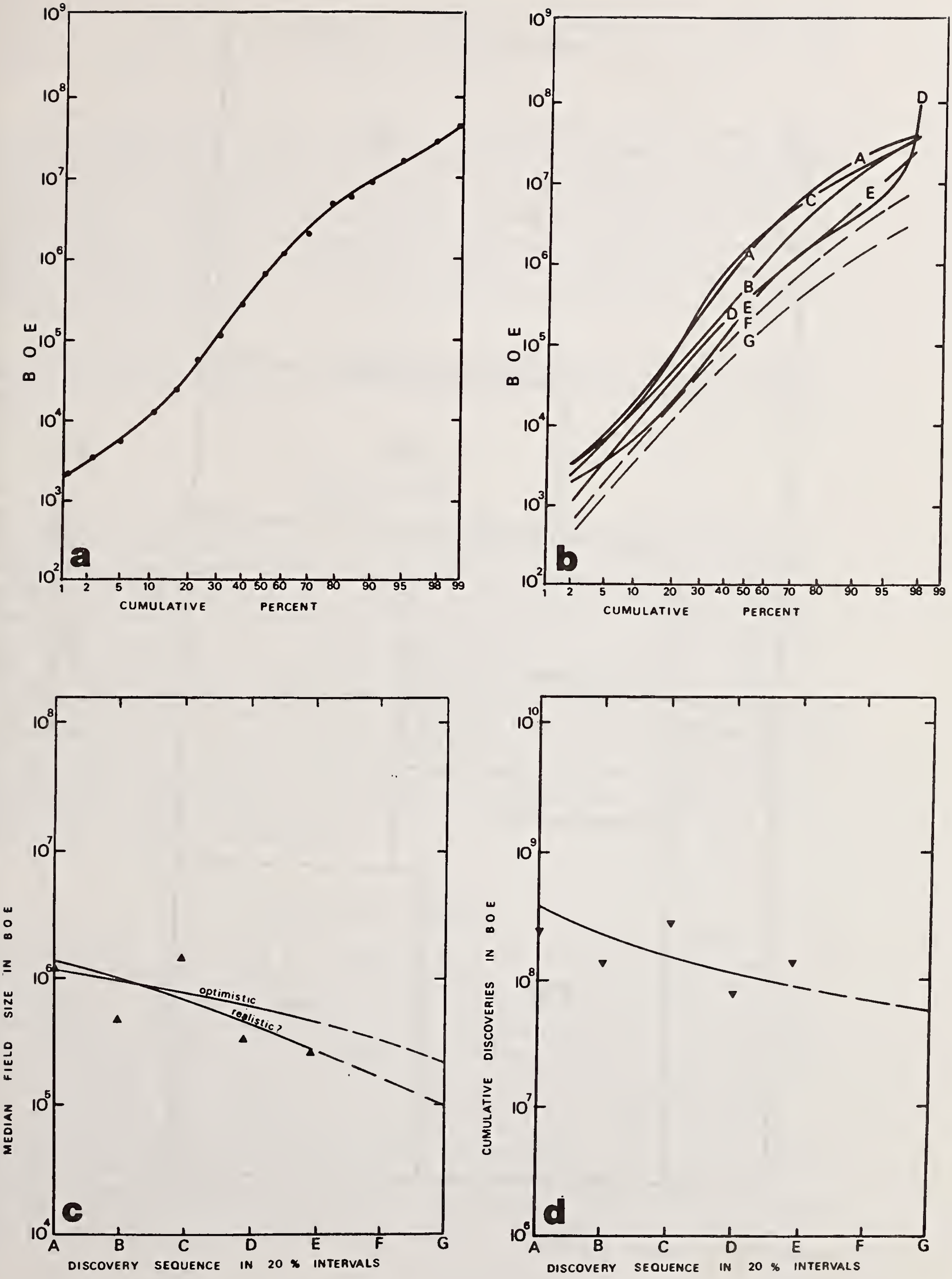

Figure 17. Powder River basin of Wyoming, stratigraphic fields only.

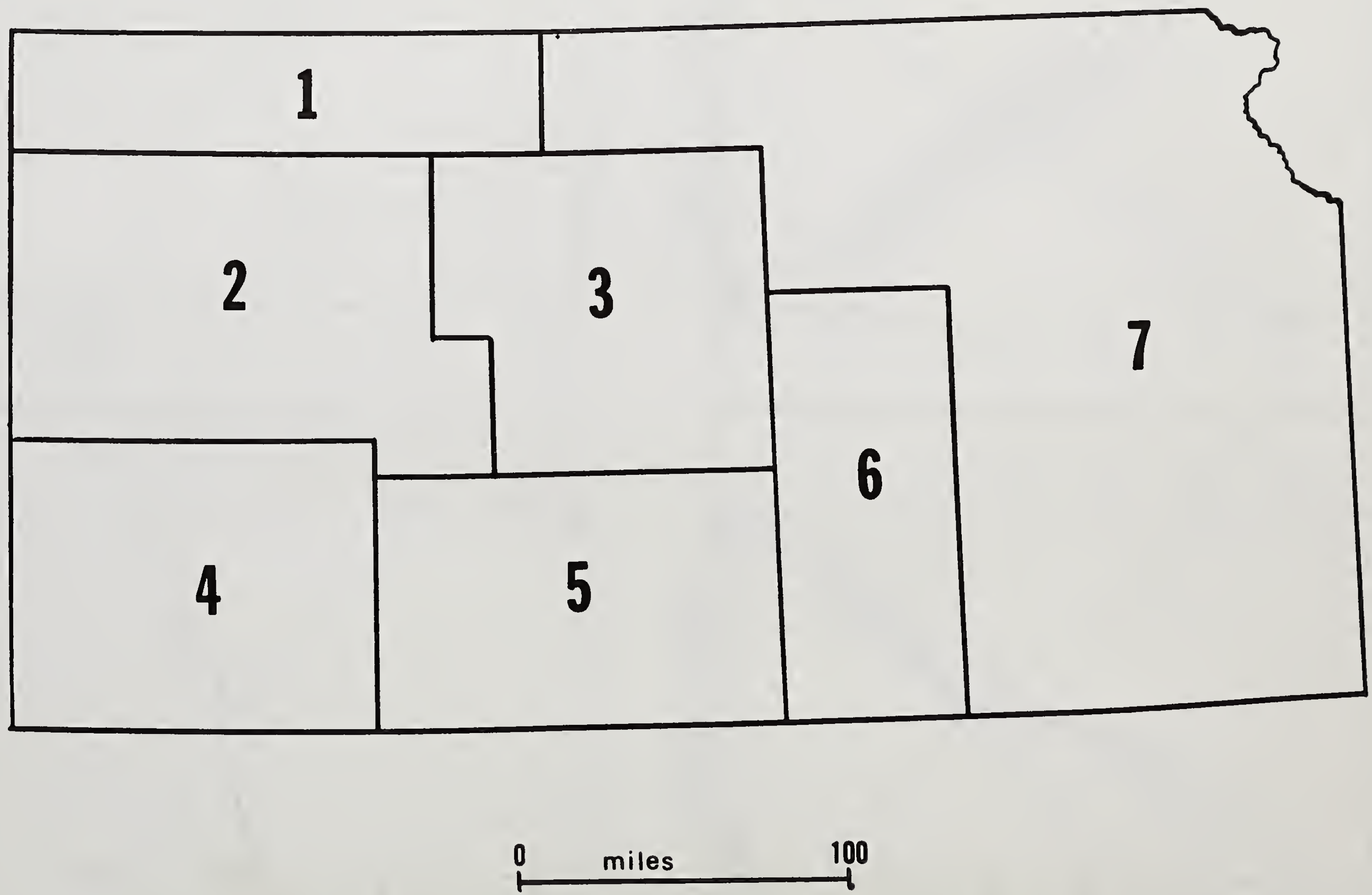

Figure 18. Index map of Kansas showing seven districts which have been established for this study.

42W; T. 16S-22S, R 19W-43W; District 3: T 7S-15S, R 5W-21W; T 16 S-22S, R 5W-18W. District 4: T 23S-35S, R 25W-43W. District 5: T 23S-35S, R 5W-24W. District 6: T 14S-35S, R 5E-4W. District 7: T 1S-6S, R 22E-15W; T 7S-13S, R 25E-4W; T 14S-35S, R 25E-6E.

The data presented for Kansas are based on cumulative production of oil (gas production is not incorporated, and the production statistics are in barrels and not BOE). The cumulative production data extend through the end of 1978, and include all fields discovered before the end of 1973. Reserve estimates are not included, so that the total field sizes (in barrels of producible oil) are necessarily less than they would be than if presented as an estimate of total recoverable oil. Thus, the data for Kansas are not directly comparable with those for California and Wyoming.

The data for Kansas were supplied in magnetic-tape form, but the Kansas Geological Survey has published an equivalent compilation (Beene, 1979).

Entire State

The oil field-size distribution for Kansas as a whole closely approximates an ideal lognormal distribution (Figure 19), and the subpopulation parameters decline consistently when the chronologically segregated subpopulations, A through E, are compared (Tables 5 and 6). Recall, of course, that the more recently discovered fields have had less time to produce, and therefore the population statistics reflect this influence as well as the bias toward early discovery of large fields. Note that the last 20 percent of Kansas fields included in this study (the 578 fields which define subpopulation E as segregated from an overall population of 2992 fields) were discovered from 1967 to the end of 1973, and it is obvious that they have had much less opportunity to produce than fields discovered earlier, as for example, those in subpopulation A (1890-1947), many of which have been benefitted from enhanced oil recovery operations.

Districts 1, 2 and 3

Districts 1, 2 and 3 are somewhat similar in their statistics (Figures 20 to 22). Because these districts have undergone extensive exploration in recent years, the rapid drop in medians, geometric mean, and aggregate cumulative production for the subpopulations (Table 5-B, C, and D) may be somewhat misleading, since these statistics will necessarily increase as existing fields continue to produce. Nevertheless, it is instructive to compare the percentages of the total production (the last column of Table 5) with similar statistics for California and Wyoming (where reserves remaining are incorporated). As a percentage of the total, per each district, those for subpopulations D and E in Kansas, although varying from district to district, do occur in the same general range as in California and Wyoming. This may imply that the bias toward early discovery of large fields is less in Kansas than in

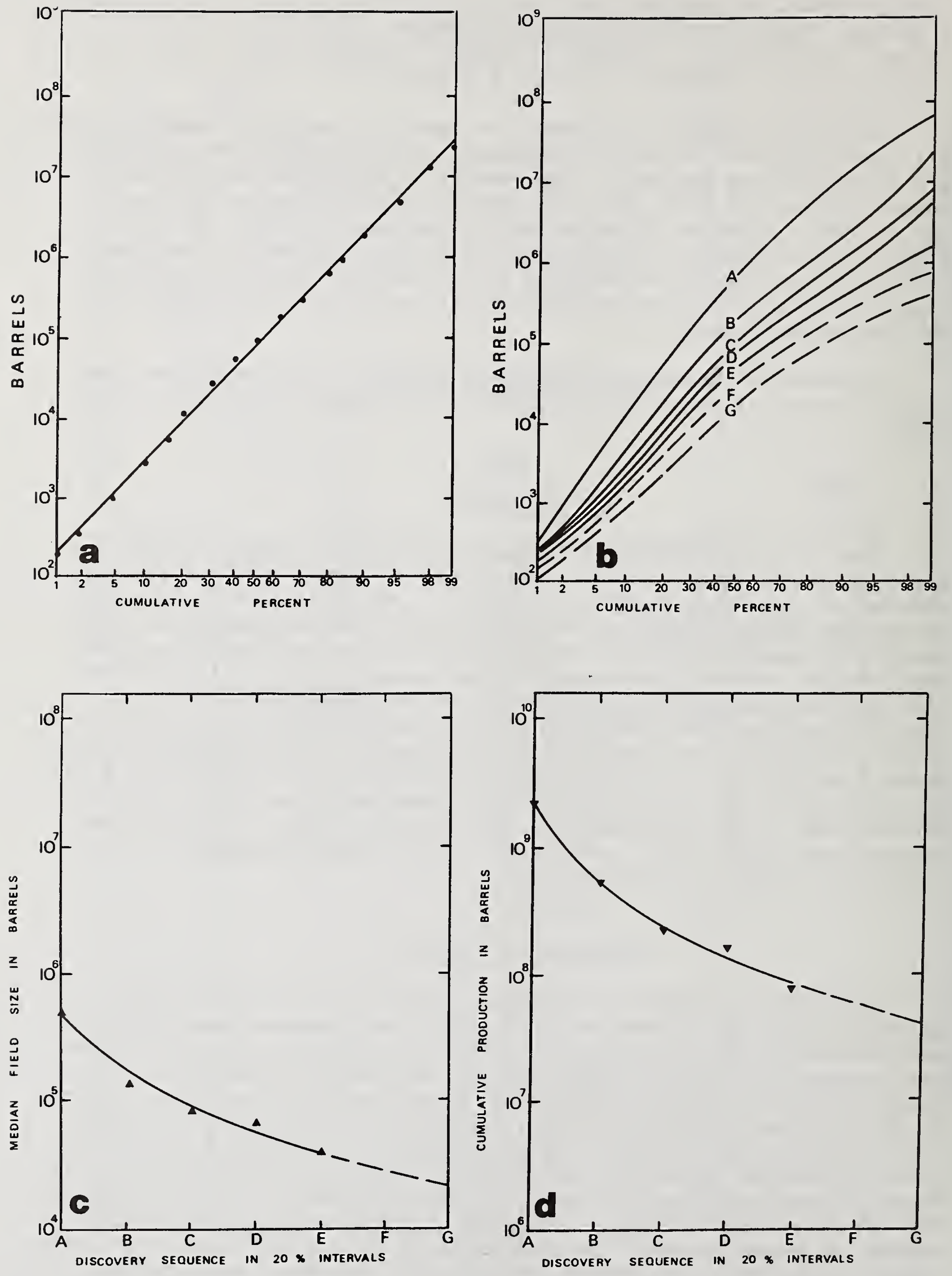

Figure 19. Entire Kansas. Oil field volumes are expressed in barrels and represent cumulative production through 1978, and exclude reserves. See section entitled Graphic Presentation of the Data for explanation.

TABLE 5. KANSAS PRODUCTION STATISTICS AND FORECASTS

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of Barrels	Geometric Mean in Thousands of Barrels	Total for Interval in Millions of Barrels	Percentage of Present Total for Entire District
A: Entire State								
Entire District		0-100	1890-1973	2,992	96	82	3,605	100.0
Progressive Discoveries Through 1973	A	0-20	1890-1973	598	533	436	2,503	69.4
	B	20-40	1947-1955	599	143	112	581	16.1
	C	40-60	1955-1960	598	87	64	261	7.2
	D	60-80	1960-1967	599	68	43	178	4.9
	E	80-100	1967-1973	598	39	28	83	2.3
Forecast Future Discoveries	F	100-120		598	26		55	1.5
	G	120-140		598	20		48	1.3
B: District 1								
Entire District		0-100	1936-1973	131	71	57	121	100.0
Progressive Discoveries Through 1973	A	0-20	1936-1952	26	395	322	86	71.1
	B	20-40	1952-1959	26	95	88	22	18.2
	C	40-60	1959-1965	27	38	27	6	5.0
	D	60-80	1965-1969	26	72	63	5	4.1
	E	80-100	1969-1973	26	50	26	2	1.7
Forecast Future Discoveries	F	100-120		26	23		1.8	1.5
	G	120-140		26	19		1.4	1.2
C: District 2								
Entire District		0-100	1929-1973	401	83	70	177	100.0
Progressive Discoveries Through 1973	A	0-20	1929-1956	80	220	176	92	52.0
	B	20-40	1956-1961	80	102	69	30	16.9
	C	40-60	1961-1966	81	105	73	30	16.9
	D	60-80	1966-1971	80	56	51	15	8.5
	E	80-100	1971-1973	80	40	34	10	5.6
Forecast Future Discoveries	F	100-120		80	25		8	4.5
	G	120-140		80	17		7	4.0

Table 5 (cont'd)

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of Barrels	Geometric Mean in Thousands of Barrels	Total for Interval in Millions of Barrels	Percentage of Present Total for Entire District
D: District 3								
Entire District		0-100	1922-1973	999	104	96	1,560	100.0
	A	0-20	1922-1949	198	865	659	1,235	79.2
Progressive	B	20-40	1949-1953	198	175	148	155	9.9
Discoveries	C	40-60	1953-1959	198	88	62	51	3.3
Through	D	60-80	1959-1966	198	77	50	99	6.3
1973	E	80-100	1966-1973	198	37	20	20	1.3
Forecast Future	F	100-120		198	25		19	1.2
Discoveries	G	120-140		198	20		14	0.9
E: District 4								
Entire District		0-100	1922-1973	124	120	86	131	100.0
	A	0-20	1922-1954	25	170	167	45	34.4
Progressive	B	20-40	1954-1957	25	203	101	33	25.2
Discoveries	C	40-60	1957-1959	24	128	111	18	13.7
Through	D	60-80	1959-1965	25	87	73	29	22.1
1973	E	80-100	1965-1973	25	40	35	6	4.6
Forecast Future	F	100-120		25	25		7	5.3
Discoveries	G	120-140		25	12		4	3.1
F: District 5								
Entire District		0-100	1926-1973	491	58	46	324	100.0
	A	0-20	1926-1952	98	345	265	236	72.8
Progressive	B	20-40	1952-1955	98	102	74	35	10.8
Discoveries	C	40-60	1955-1960	99	43	62	32	9.9
Through	D	60-80	1960-1965	98	27	21	15	4.6
1973	E	80-100	1965-1973	98	17	12	6	1.9
Forecast Future	F	100-120		98	13		5	1.5
Discoveries	G	120-140		98	9		4	1.2

Table 5 (cont'd)

	Label	Percentage Ranges	Range of Years	Number of Fields	Median in Thousands of Barrels	Geometric Mean in Thousands of Barrels	Total for Interval in Millions of Barrels	Percentage of Present Total for Entire District
G: District 6								
Entire District		0-100	1900-1973	446	128	119	872	100.0
Progressive Discoveries Through 1973	A	0-20	1900-1938	89	1,658	1,084	681	78.1
	B	20-40	1938-1951	89	199	166	81	9.3
	C	40-60	1951-1957	90	54	53	54	6.2
	D	60-80	1957-1962	89	117	86	45	5.2
	E	80-100	1962-1973	89	45	32	11	1.3
Forecast Future Discoveries	F	100-120		89	20		9	1.0
	G	120-140		89	14		7	0.8
H: District 7								
Entire District		0-100	1890-1973	409	126	96	420	100.0
Progressive Discoveries Through 1973	A	0-20	1890-1925	82	1,090	649	265	63.1
	B	20-40	1925-1938	82	118	106	74	17.6
	C	40-60	1938-1956	81	55	61	39	9.3
	D	60-80	1956-1965	82	99	51	25	6.0
	E	80-100	1965-1973	82	69	60	17	4.0
Forecast Future Discoveries	F	100-120		82	20		13	3.1
	G	120-140		82	15		11	2.6

Table 6. Probabilities attached to field-size ranges for the next 20 percent (F) of fields to be discovered, and for the next 20 percent (G) to be discovered after that, in Kansas

Area	Label on Curve	Number of Fields	Probabilities (in percent) attached to field-size ranges in BOE						
			$<10^3$	10^3 to 10^4	10^4 to 10^5	10^5 to 10^6	10^6 to 10^7	10^7 to 10^8	10^8 to 10^9
Entire State	F	598	8	23	43	25½	½		
	G	598	12	29	44	15			
District 1	F	26	17	23	34	26			
	G	26	19	23	42	16			
District 2	F	80	7	23	44	25	1		
	G	80	9	29	44	18			
District 3	F	198	6	24	50	20			
	G	198	9	29	50	12			
District 4	F	25	9	25	38	23	4½	½	
	G	25	13	32	36	16	3		
District 5	F	98	17	29	40	14			
	G	98	19	31	41	9			
District 6	F	89	9	25	48	18			
	G	89	12	31	49	8			
District 7	F	82	7	24	42	25	2		
	G	82	10	30	44	15½	½		

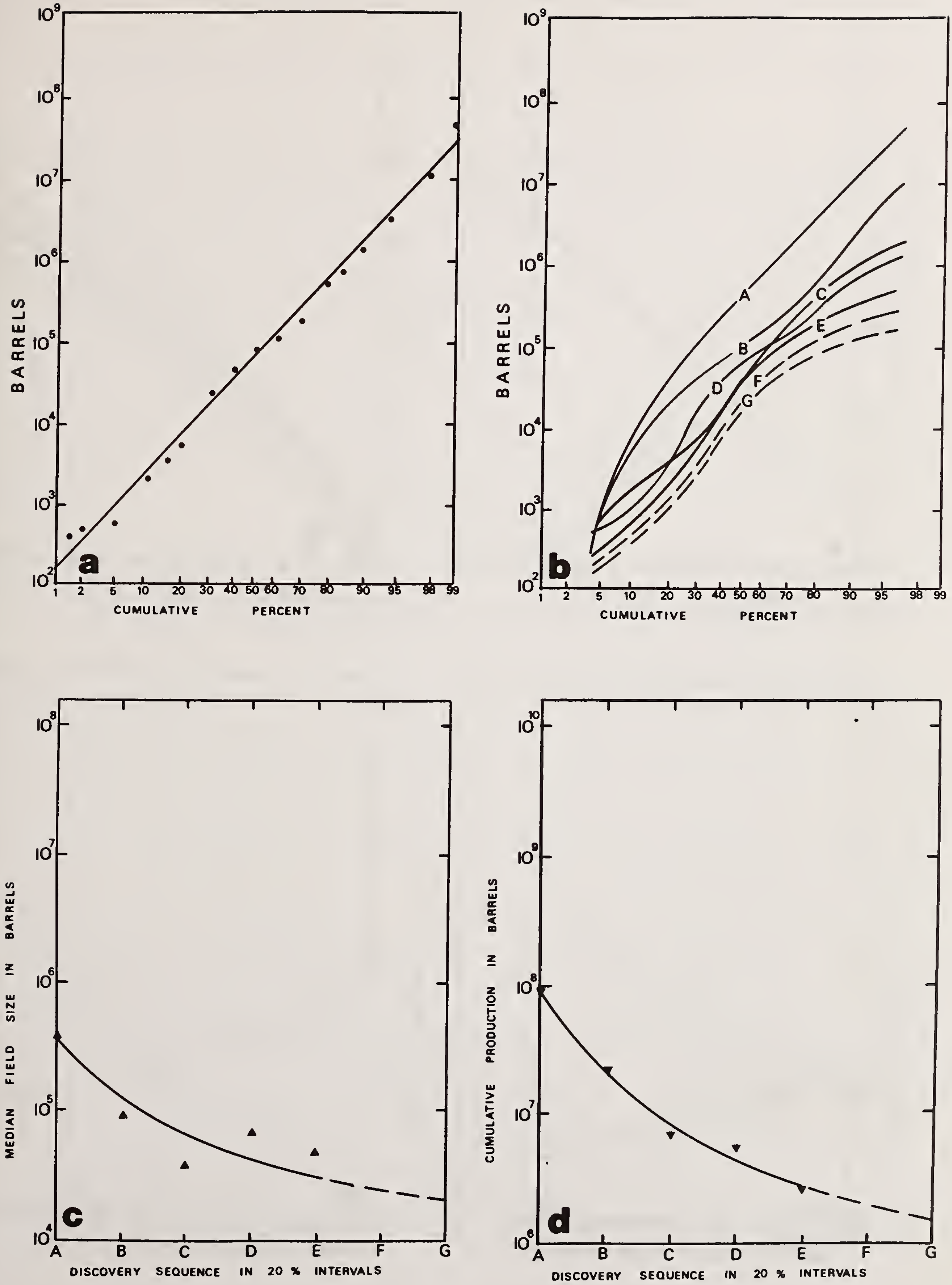

Figure 20. District 1, Kansas.

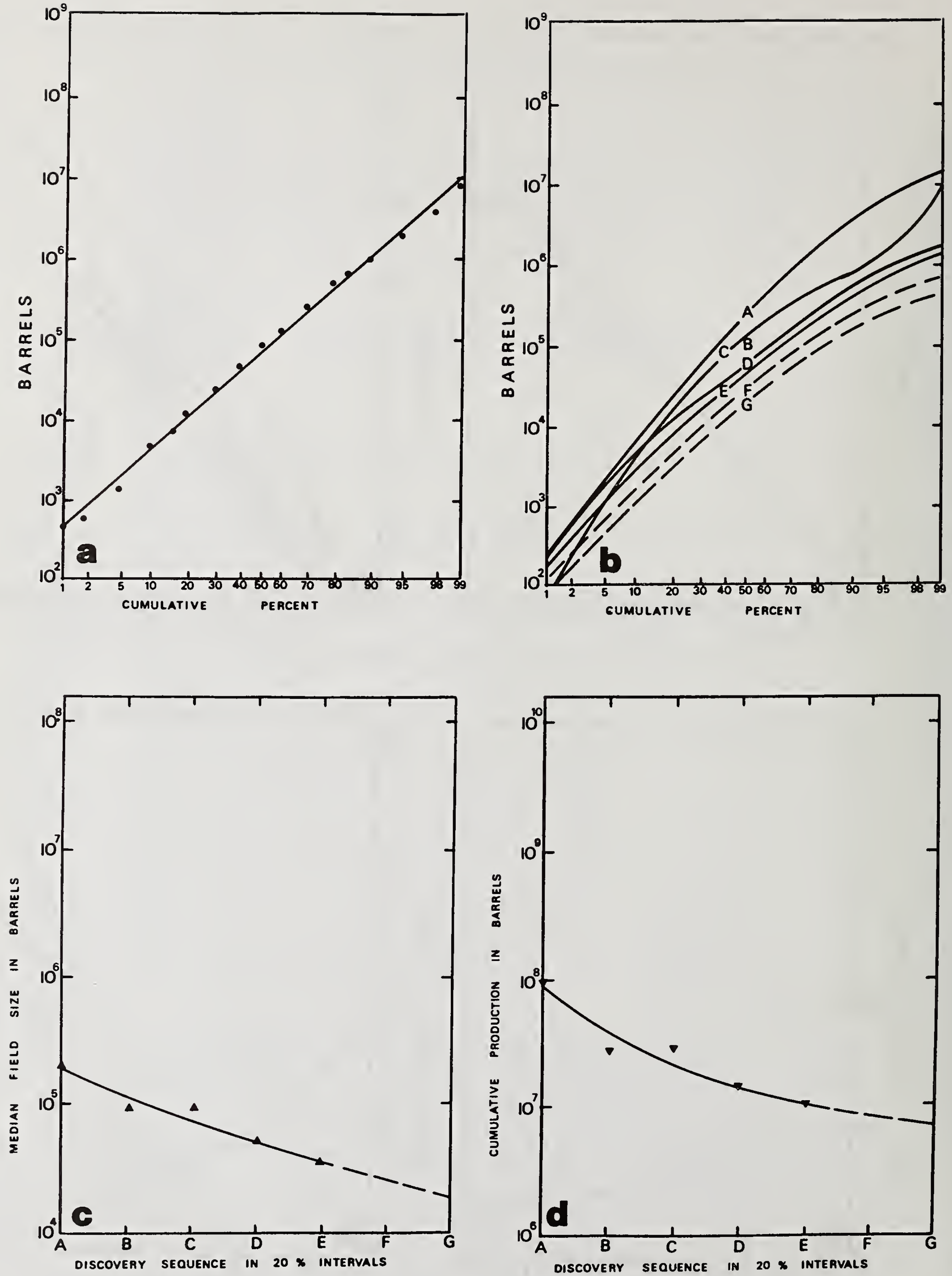

Figure 21. District 2, Kansas.

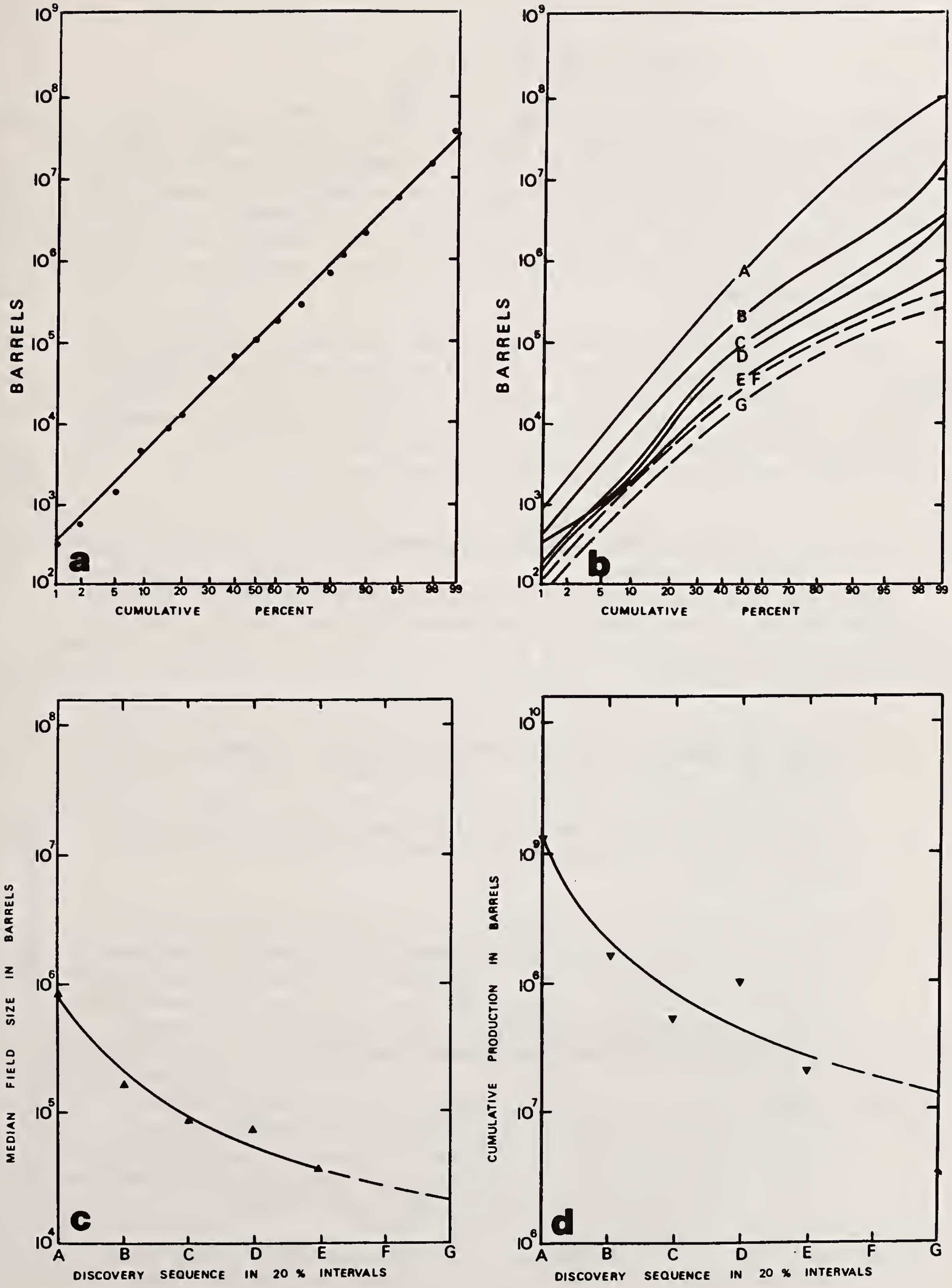

Figure 22. District 3, Kansas.

California or Wyoming, but this possibility cannot be convincingly determined unless reserve estimates are incorporated in the Kansas field-size data.

Districts 4, 5, 6, and 7

Districts 4 through 7 do not display orderly reductions in population parameters with discovery sequence. While the overall populations of each district are essentially lognormal (Figures 23 to 26) the graphs of the chronologically segregated subpopulation cross each other in an unpredictable manner. The medians and geometric means, however, exhibit somewhat more orderly arrangement (Table 5-E, F, G, and H), and seem to permit the extrapolation of future populations (Table 6) with some consistency.

Analysis of the Kansas data makes clear that we are dealing with populations of fields that have much smaller parameters than those of California or Wyoming. Table 7 presents a summary forecast new-field discoveries for all three states, with the proviso that the estimates for Kansas exclude gas (thus are not on a BOE basis), and must be adjusted upward to accomodate continuing production.

CONCLUSIONS AND RECOMMENDATIONS

This study was undertaken to achieve an understanding of the most elementary of petroleum resource-base considerations, namely, the frequency distributions of oil-field volumes. Some generalized conclusions may be drawn as follows:

(1) The lognormal distribution seems to be a very useful general model in dealing with oil and gas fields. Unless refuted by studies in other regions, it seems appropriate to assume that the populations of undiscovered oil and gas fields in frontier regions that have undergone little or no exploration will be essentially lognormal, assuming that such populations exist at all.

(2) The bias toward early discovery of large fields is a major influence, and statistically seems to be greater than many explorationists realize. A decrease in population parameters (median and geometric mean) of from one to as much as three orders of magnitude (powers of ten) appears to be common as a district or basin approaches maturity.

(3) Subpopulations of fields discovered early tend to depart more from the lognormal ideal than later subpopulations. However, these shifts in population characteristics vary widely from district to district, and generalized statements about these changes must await additional study.

(4) The "actual" distribution of oil and gas field sizes may not be lognormal, and instead may have the general form

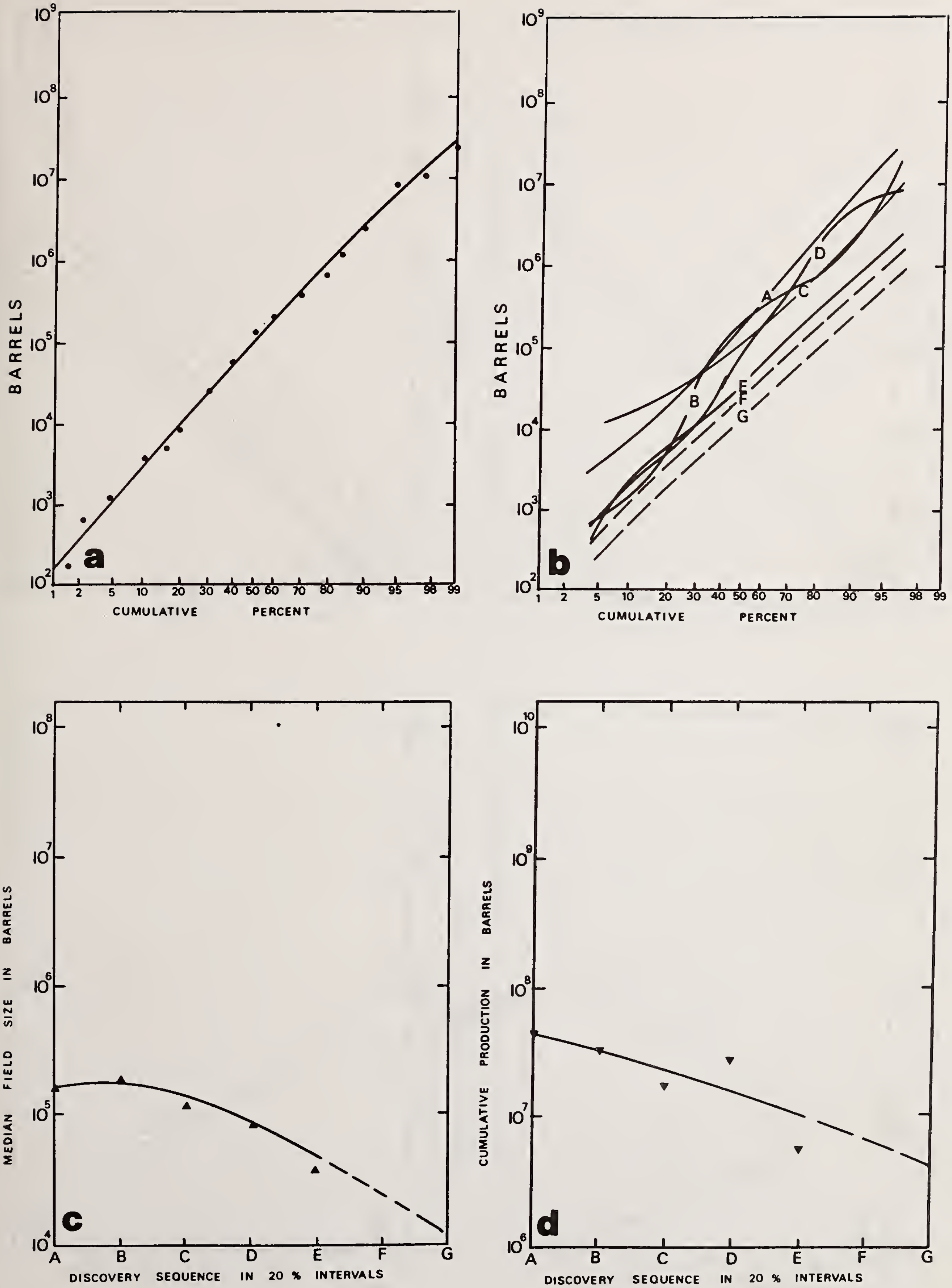

Figure 23. District 4, Kansas.

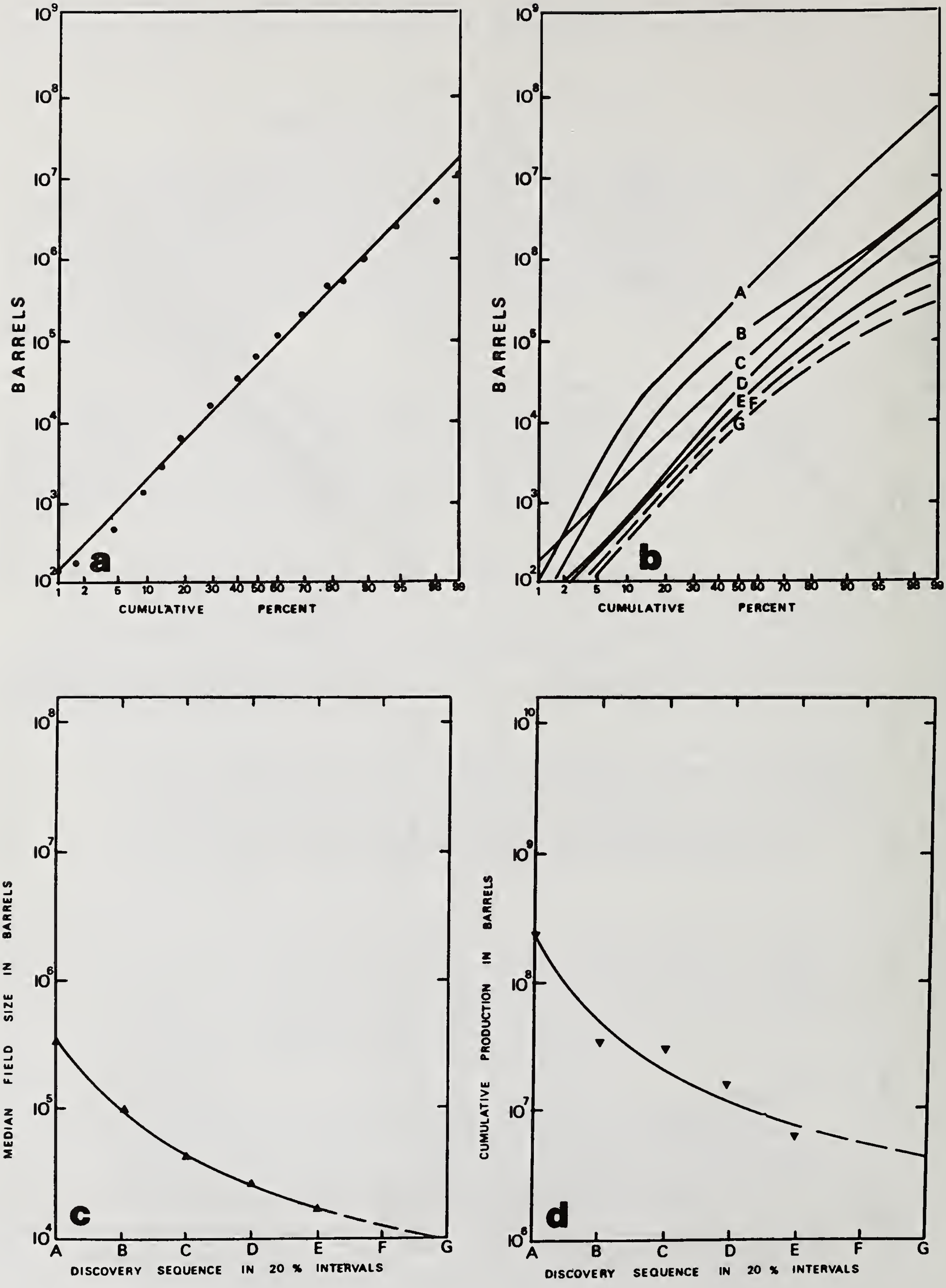

Figure 24. District 5, Kansas.

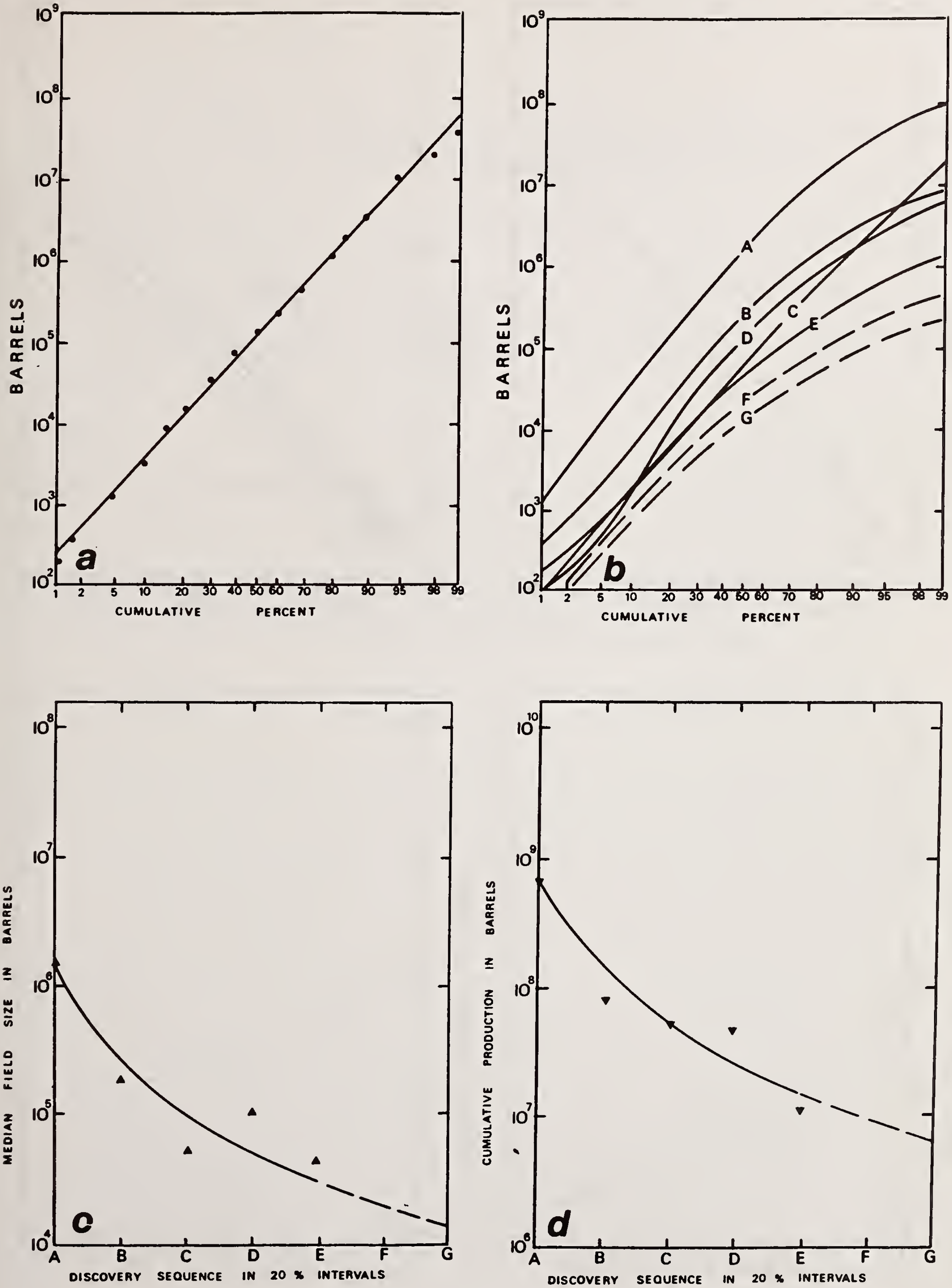

Figure 25. District 6, Kansas.

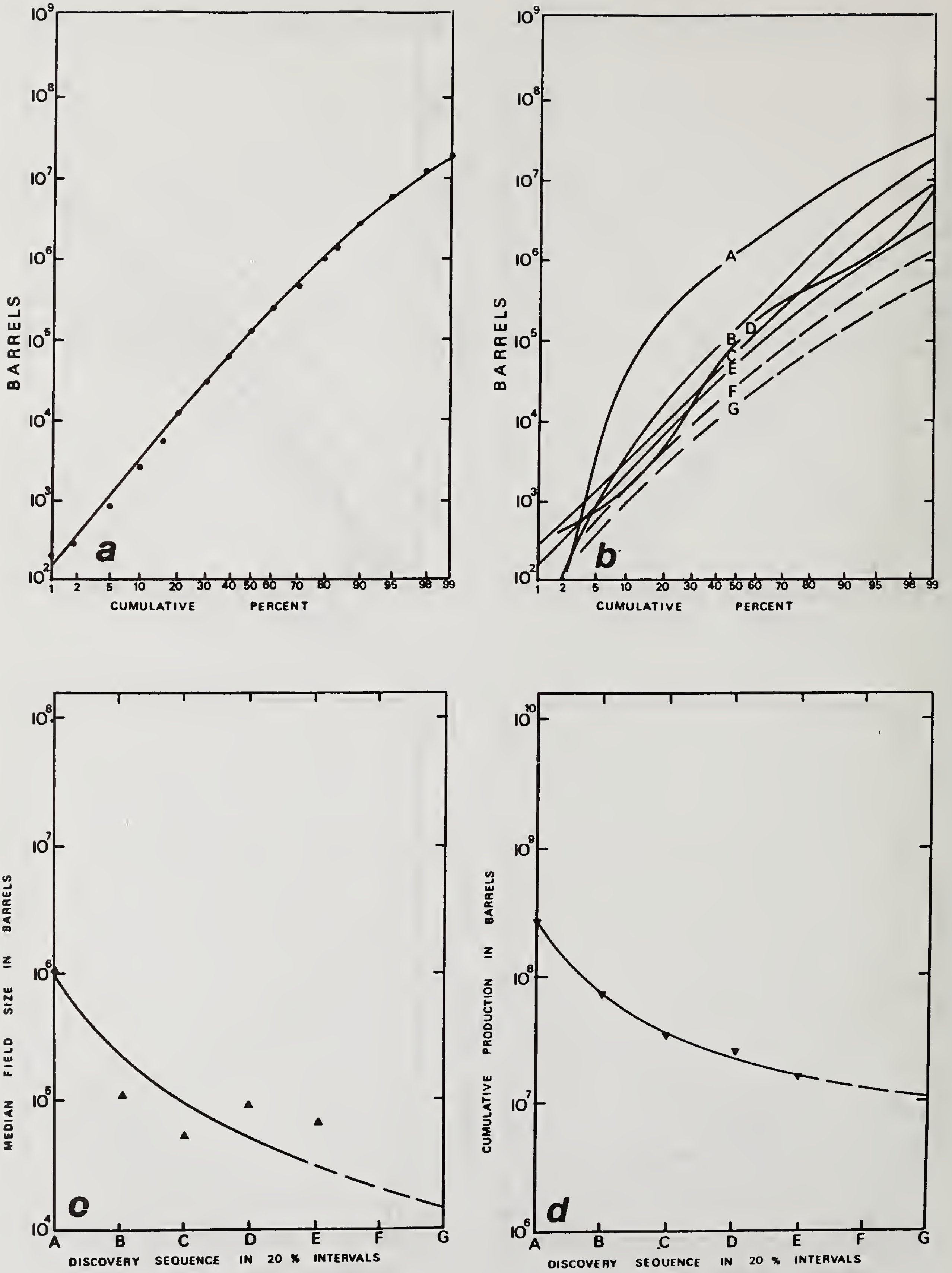

Figure 26. District 7, Kansas.

Table 7. Summary comparison of forecasts for the three states for the next 20 percent of fields to be discovered, and the 20 percent after that

	Next 20%		20% after that	
	Number of Fields	Millions of BOE[1/]	Number of Fields	Millions of BOE[1/]
California	81	185	81	118
Wyoming	151	140	151	90
Kansas	598	55	598	48

[1/] Volumetric estimates for Kansas exclude gas and involve predictions based on cumulative oil production through 1978, exclude reserves, and must necessarily be revised upwards as production continues.

suggested in Figure 27. The lower size limits of fields that have actually yielded oil or gas are generally set by either economic factors, or the ability to detect small accumulations, or both. It is possible, and even probable, that there is no definable lower field-size limit and that the form of the distribution is exponential with regard to very small accumulations. Thus, the actual distribution may be bimodal in the sense that there are two peaks, one of which represent the producing fields, and other (the exponential extension) represents a virtual infinity of accumulations, each of infinitesimal size. Obviously the definition an "oil field" becomes meaningless when extended to this extreme. It will suffice to say that we have almost no knowledge of the lower limit of field-size distribution. This shortcoming may be of minor consequence, however, since extremely small fields are of negligible economic importance.

(5) Studies of oil field populations should be conducted regionally. Furthermore populations should be segregated geologically. The data from the Powder River Basin suggest that the population parameters for structural versus stratigraphic fields may differ significantly in other regions.

(6) Extrapolation of parameters of chronologically segregated oil field populations is a useful predictive tool.

Acknowledgements

We thank James A. Barlow, Jr., of the firm of Barlow and Haun, for supplying information on Wyoming oil fields. Data on Kansas oil fields was made available to us by Douglas L. Beene and John C. Davis of the Kansas Geological Survey. Clifford Hayashi of Stanford University adapted the computer programs employed and assisted in their operation. The report was typed by Rita Hoffmann of Stanford University.

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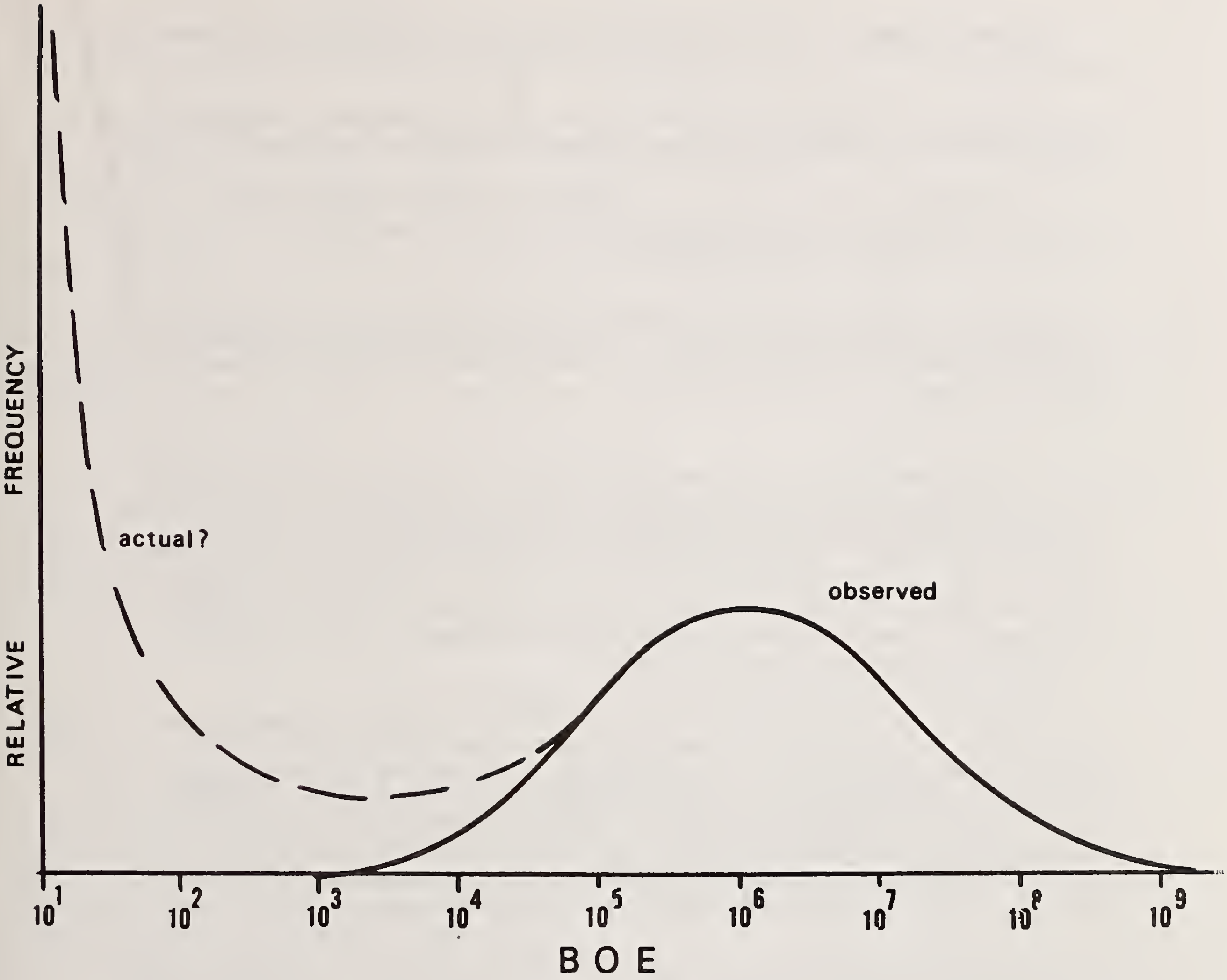

Figure 27. Possible alternative form of distribution of oil-field volumes. Observed distribution is lognormal, but actual distribution may be bimodal with long exponential tail toward lower end of individual volumes.

DISCUSSION

CHAIRMAN MURPHY: Are there any questions? Yes, come up to the microphone.

VOICE: In the field size data that you used, were there fields that had been discovered but which were not included in the study?

DR. HARBAUGH: This is partly a semantic problem posed by the difficulty in defining the boundaries of fields. In general, all relevant field-size data were included.

VOICE: The point that I am getting at is that in the curve that you showed, you said that the left or small-field side is determined by economic factors, whereas the right or large-field side is determined by geologic factors. It would seem that the right side is also influenced by economic factors?

DR. HARBAUGH: Well, indirectly it is, in that technology and economics affect the right side to some extent. But the counter-argument is that there are geologically inposed limits to the maximum size of fields which strongly affect the distribution of fields toward the large end. Whereas at the lower end, the distributions are unmeasured because we do not know what the smallest field is. In fact, the smallest field is incapable of being defined. It probably goes down to a teacup in size, with all possibilities in between.

VOICE: I am with the Department of Energy. I have two questions, one of which is statistical. Have you made goodness-of-fit tests for the lognormal distributions, and the other question was raised by your discussion. You were talking about the shifts in the median size of fields over the history of a region. Have you come up with possible predictor variables for estimating this median shift? Since you are able to come up with geological explanations for the samples you showed, have you found any consistent predictor variables as a result?

DR. HARBAUGH: The answer to the first question is that we have not made goodness-of-fit tests, but they could be made easily. However, the data for many of these distributions are so ample that you don't need goodness-of-fit tests to determine whether they approximate lognormal distributions.

The answer to the second question is also no, but it is one on which we are working. In California and Wyoming we intend to extend these studies from more geological standpoint, in which we aggregate the fields into geological categories such as structural versus stratigraphic and also as to whether the structures which have influenced accumulation were perceived early or late relative to their discovery.

DR. RICHARD MAST: I am Dick Mast, with the USGS. John, is there much depth variation in the Sacramento Valley gas fields?

DR. HARBAUGH: Yes, there are depth variations.

DR. MAST: This concept of large fields being found first has to do more with areal size of fields, and your conversion to a BOE basis turns that around. I would rather see you employ a volumetric basis for gas fields.

DR. HARBAUGH: Since the Sacramento Valley fields are all gas, the transformation to BOE is just a linear conversion, so the distributions are not going to be changed.

DR. MAST: No, because as you get deeper, you get more gas per unit of reservoir volume because of higher pressure.

DR. HARBAUGH: I see what you mean. The standard MCF is 1,000 cubic feet at 60 degrees F and 14.7 PSI, but at depth this amount of gas is compressed into a much smaller volume.

DR. MAST: Yes. The resulting distributions might turn out quite differently.

DR. HARBAUGH: I agree that there would be drastic differences if we tabulated gas field sizes according to actual volumes in the reservoirs, at depth.

CHAIRMAN MURPHY: Any other questions?

VOICE: I have a question about three things. But first, do you know of any good geological story, why the distribution of pools should be lognormally distributed?

DR. HARBAUGH: Why it should be so geologically?

VOICE: Yes, is there a geological model that would indicate that should be the case?

DR. HARBAUGH: Not a particularly good one. It is argued that processes that involve progressive splitting are involved. Take an entity, cut it in half, cut those halves in half and so on in random fashion. The result may approximate a lognormal distribution if we lose track of many of the progressively smaller fractions. This is not a very satisfying geological rationale.

VOICE: Okay.

DR. HARBAUGH: Perhaps someone else has a view on this matter?

VOICE: The other question is, it has been argued strongly by Kaufman that the distribution of pools within a play is lognormally distributed. You have taken it a step somewhat to the side, and I am not sure that it is clear to people here, that it is logically inconsistent for the distribution of plays to be lognormal, and then regard the distribution of pools representing the distribution of fields to be lognormally distributed within a basin.

There is a theoretical inconsistency. Do you feel uncomfortable about it, or do you feel that this is a problem that you need not deal with?

DR. HARBAUGH: I am not sure that I agree. Does your question involve the assertion that the distribution of oil and gas field volumes within plays is not lognormally distributed?

VOICE: I am claiming that some people say that the distribution of pools within a play is lognormally distributed, and some people say that the distribution of fields within a basin is lognormally distributed, and then even within an aggregation of basins. The second of the two statements that if the fields are lognormally distributed within a basin or within an aggregation of basins is not consistent, in that the lognormal distribution does not regenerate.

CHAIRMAN MURPHY: The sum of lognormals is not lognormal; is there an inconsistency?

DR. HARBAUGH: I agree; however, when we look at California or Kansas as a whole, versus individual districts in these states, we are in effect either merging or segregating sub-populations. In our experience, the lognormal model is a useful one throughout the ranges of population sizes that we have dealt with, although there are serious deviations from it.

VOICE: Okay. My last question is, it would be very useful to guarantee lognormality very early in the exploitation of a play, so that forecasting could be much more robust; however, examples can be found where the lognormal model is nowhere near correct.

Have you investigated or do you have opinions on the set of conditions that are sufficient to guarantee lognormality, and can those be recognized early in the exploitation of a play?

DR. HARBAUGH: That is a good question, and I wish I could answer it. I would say, the kinds of geological conditions that would most assure lognormality would be geological controls on accumulations that exclude large geologic structures, and include a large number of more subtle geologic features, such as permeability changes that form stratigraphic traps. For example, the Los Angeles basin contains very large geological structures, which may be the principal reason why the LA basin deviates so far from the lognormal ideal, at least in the fields discovered early.

ISSUES PAST AND PRESENT IN MODELLING OIL AND GAS SUPPLY

by

Gordon M. Kaufman
M.I.T.

1. Introduction

> "For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change in form."
>
> — J.B.S. Haldane (1928)*

Physical principles dictate the size of animals. Haldane points out that the human thigh bone breaks under about ten times the weight of a normal human. The strength of bone is in proportion to its cross-sectional area, while the weight of an animal is proportional to its volume. This lessened his respect for Jack the Giant Killer, for the Giant, a scaled up human sixty feet tall, would have broken his legs with his first few steps towards Jack.

If we replace the words "animal" and "size" with "policy problem" and "model" respectively, then models of oil and gas exploration, discovery, and production currently in vogue loosely adhere to Haldane's observation about the animal kingdom. At one extreme are disaggregated approaches to modelling that focus on individual deposits as the basic unit for analysis; at the other extreme are models that treat time series of data at the national level. As with the Giant, a direct scaling up of models for supply from individual petroleum plays or petroleum basins, "breaks the model's legs" in several ways. Yet there must be a logical connection between micro-models of these physical entities and a country-wide level of aggregation, for what happens at the national level to discovery and production of oil and gas is, after all, determined by what happens in over a hundred individual petroleum basins.

The territory that lies between aggregated and unit-specific disaggregated approaches is as yet uncharted. No logically tight methodology for aggregating projections of supply over different time frames from individual geologic units in varying stages of exploratory maturity is in sight. Ideally, we wish to have at our disposal a system of logically inter-related methods and models sufficiently flexible to allow economic supply functions to be computed for mature, partially explored, and frontier regions under a wide range of fiscal, regulatory, and technological alternatives at reasonable cost in time, human effort, and money. The current state of the art is very far from this ideal.

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* "Possible Worlds" by J.B.S. Haldane, Harper and Brothers, 1928.

An impressionistic snapshot of aspects of the current modelling environment has these salient features:

- No conceptual reconciliation of models of individual plays, stratigraphic units, and petroleum basins that explicityly incorporate individual deposits as components with models that don't,

- A movement towards process-oriented models that reflect key features of the physical processes of exploration, discovery, and production of petroleum,

- Few publicly available data sets that allow meaningful structural validation of highly disaggregated models,

- An increasing use of personal (or subjective) probabilities as a vehicle for representing expert judgements about uncertain quantities without a serious matching effort to train assessors to avoid cognitive biases that distort assessments,

and

- Policy issues as moving targets: an often rapid change in what policy analysts view as "important" policy problems places a heavy burden on modellers working with models, most of which are difficult to reconfigure rapidly.

This short list of observations about the modelling environment can be amplified many-fold, but it does set the stage for discussion of some current issues in probabilistic modelling of oil and gas exploration, discovery, and production. Wood (1979) comments in depth on the energy modelling environment and the policy research process, and Stitt (1979) gives an excellent review of problems in resource modelling with particular attention to the linkage of micro-economics and physical aspects of production and development of petroleum deposits.

2. Issues and Problems in Modelling Discovery At a Disaggregated Level

Formal modelling of exploration and discovery at a disaggregated level began with the work of Arps and Roberts (1958) and many of the issues and problems that their work suggested are still with us:

- Taxonomy: How to classify petroleum deposits in a basin into descriptively homogeneous sub-populations in accordance with the idea that elements of each such sub-population will possess statistically homogenous quantitative attributes like area, volume, hydrocarbons in place, etc.

- "Size Distributions: Given that such a classification is possible, are there particular functional forms that characteristize measurements of each of the principle quantitative attributes of individual deposits which play a role in supply analysis?

- Measurement and Observation: The peculiar nature of exploratory search methods and geologic interpretation of the measurements they yield combine with economic incentives and manifold sources of measurement error to complicate severely application of statistical methods: testing of hypotheses about size distributions, measurements of inter-relations between quantitative attributes of deposits, and prediction of returns to exploratory effort at a disaggregated level.

- Stretching of Assumptions: Models of exploration and discovery focused on individual basins and more particularly on plays within basins ought to be designed to reflect particulars of the evolution of discovery within such units. Attempting to "stretch" such models and apply them uncritically to arbitrary geographic regions or larger aggregates may be an exercise in mis-specification and lead to misleading predictions.

- Uses of Subjective Probability: When measurements are few as in frontier provinces or in a sparsely drilled stratigraphic unit within a partially explored or nature basin, methods currently used for elicitation of probabilities about simply described events bearing on occurrence or non-occurrence of petroleum and for deposit attributes conditional on the presence of hydrocarbons forces the geologist-assessor to integrate mentally and boil down a large quantity of spatially interrelated descriptive and quantitative data into a small set of numbers. Innovative approaches to use of both quantitative and non-quantitative data as conceptual aids to assessment are needed. Probabilistic dependencies among uncertain quantities tend to be ignored.

- Adaptive versus Non-Adaptive Modelling: When parameters of a model are not known with certainty a priori, learning about them from observational experience is possible. Revision of opinion about model parameters in light of new observations and a concomitant revision of predictions about yet to be observed quantities can in principle be done in a logical fashion. Few energy models incorporate adaptive learning. Should they?

That at least some of the above are contentious issues is evident in much recent literature on modelling of exploration, discovery, and production.

2.1 Taxonomy, Plays, and "Size" Distributions

Each deposit is unique; each prospect is unique. Each must be modelled in light of its particular attributes. This view leads modellers to a dead end. Too much data and too much modelling detail are demanded. The idea that deposits can be classified into homogenous collections of descriptively similar deposits enables us to move one rung up the ladder of aggregation; more parsimonious models of much simpler structure follow. Essential elements of the micro-economics of individual deposit exploitation are not sacrificed at this level of aggregation. A key concept at this level of aggregation is that of the size distribution of such a collection of deposits.

Properties of the distribution of sizes are obviously dictated by how membership in a particular collection is defined. It is up to the geologist to set the rules for inclusion and exclusion. Geologists do this. They define plays. Unfortunately, the rules for classification may vary from geologist to geologist and very seldom so crystal clear that non-geologist could do a meaningful classification. However, retrospective analysis of a very will explored area often leads to a classification scheme that provides an acceptable guide for modellers.

A challenge to the geologist: Can you provide modellers with a clearer description than is presently available of how classification useful for supply analysis should be done?

Modelling properties of collections of descriptively similar deposits in a petroleum basin, as opposed to modelling the particulars of each deposit and prospect greatly reduces computational cost, complexity, and amount of data required. This idea, while rubbing against the grain of many exploration geologists who keep detailed descriptive differences among deposits as well as descriptive similarities in focus, is behind the exploitation of the concepts of "play" and distribution of deposit "size" - volume, area, hyrocarbons in place, recoverable hydrocarbons - when modelling discovery, production, and making resource projections.

Some resoundingly disagree with this modelling tactic as evinced by the following comments by a senior geologist with a large oil company:

> "...I disagree ... that the first step in making resource projections should be to classify deposits of a basin into homogeneous subsets. This seems to be the essence of the current USGS-D.O.E. effort to make regional estimates on the basis of "plays". The first question that arises in the mind of any experienced exploration geologist is: "Who does the classifying"? The basis of the success of the U.S. oil and gas industry is its willingness and capability to consider and to test a wide variety of often contradictory geological concepts. When the D.O.E. undertakes to classify "plays", they must use a handful of geologists using a single concept of the geology and of the oil and gas distribution pattern of the area under consideration. The industry on the other hand, would make similar classifications by many different groups of geologists operating with diverse ideas about what the classifications should be based upon. A number of major oil fields have been found by people who were in fact ignorant of the accepted industry evaluation of a particular area or rock sequence.

> "Beyond the certain inaccuracy of classification by a single group of geologists there is the fact that few geological situations maintain any sort of uniformity over any large region. Oil and gas accumulation is controlled by stratigraphy, by structure, by formation fluid physics and chemistry, and by hydrocarbon source rock availability. Each of these factors varies independently so that the conjunction of any particular set of circumstances rapidly changes."*

Classification of prospects in a frontier areas into play types is an exercise in judgement based on little data. The idea gains more force when applied to partially explored and to mature provinces; plays are most crisply delineated by a retrospective analysis of data from a region where intensive drilling has taken place.

Much confusion arises if the concept of a "play" as a descriptive hypothesis entertained at the outset of exploration is not viewed as distinct from a classification of deposit types constructed from measurements provided by intensive drilling of an area. Many such descriptive hypotheses can be entertained at the outset of exploration of an area; one may possibly be demonstrated to be true after extensive drilling and the understanding so acquired used by a skilled geologist to classify deposits into types.

The more descriptively divergent the set of a priori hypotheses are, the greater one expects the "spread" of probabilistic predictions of sizes of deposits and possibly of aggregate amounts of hydrocarbons in place to be. This line of thought is in loose analogy with a Bayesian treatment of prediction when a set of structurally distinct models are envisaged as possible generators of observed data (cf. Zellner (1970) Chapter 10 of example). Figure 1 schematically displays the difference between procedures which require explicit evaluation of probabilities for a priori hypotheses and procedures which don't. In practice there is a difficult tradeoff to make: if a priori geologic hypotheses are explicitly introduced, then probabilities for occurrence attributes (source, timing, migration paths, etc.) and for individual deposit attributes must be assessed conditionally on each hypothesis. The number of assessments required increases linearly with the number of alternative geologic templates or hypotheses considered. Where is the balance between unwieldy assessment and explicit consideration of key geologic concepts?

To the author's knowledge, no publicly cited resource appraisal is based on a formal treatment of alternative descriptive geological hypotheses. It is left up to the geologist making an appraisal to "do it in his head." As a consequence, when subjective probabilities are used to express judgements about deposit attributes and resource potential, geology and probability are not as tightly coupled as they perhaps should be. I believe that a similar line of thought motivates the following observations, again from the same experienced petroleum geologist:

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* That D.O.E. "must use a handful of geologists using a single concept of the geology ..." is challengeable. Participants in the recent government study of NPR-A oil and gas potential will attest to the wide range of geological hypotheses considered prior to formulating play definitions and assessing potential resources within plays.

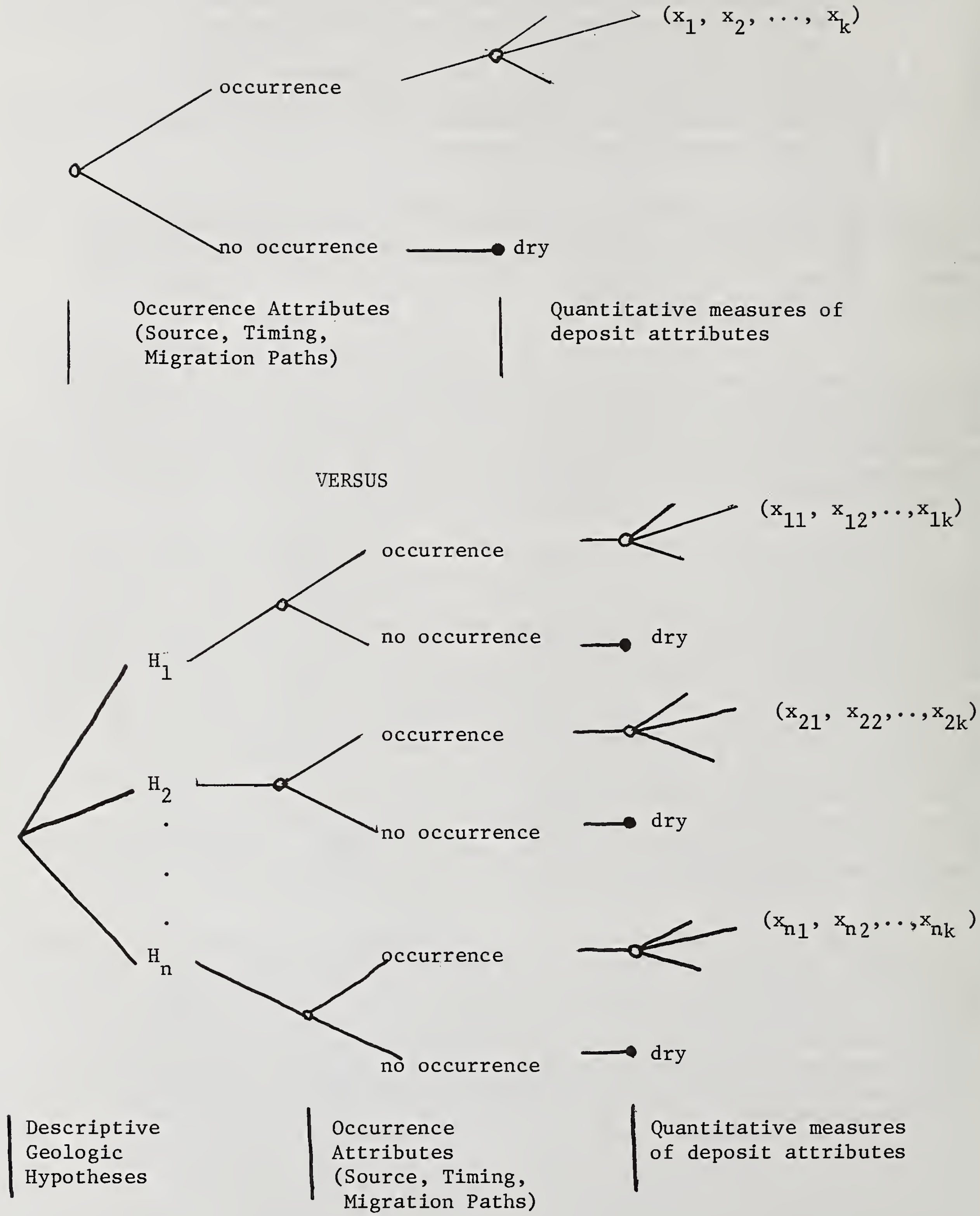

Figure 1. Trade-off between magnitude of assessments and explicit recognition of reasoning about alternative geologic templates.

"I remain convinced that most economic models attempting to describe oil and gas exploration are seriously incomplete in their recognition of the influence of geological concepts upon the location and nature of exploration targets. Alternate geological evaluations should be incorporated in the models in the same manner as cost, price, supply, and demand alternatives. I do not regard the probability estimates appended by the USGS to their resource estimates as filling this need."

2.2 Stretching Assumptions

Can predictive accuracy be maintained if the form of models designed for plays is stretched to apply to collections of plays in a basin and possibly to even larger geographic regions? Conversely, how well can models of arbitrary large geographic regions that do not incorporate physical and geological features of the exploration process predict? We are still fumbling for answers, but it is clear that much work on both structural and predictive validation of model types is warranted.

A small example illustrates the dangers in attempting to stretch the form of a model designed for individual plays to collections of plays. Let $\{A_1, \ldots, A_N\}$ be a collection of "sizes" (area, volume of hydrocarbons in place or recoverable) of deposits recognized to be targets for drilling. A parsimonious model of discovery in the absence of externalities restricting access to them (sequestered acreage, lease blocking) follows from the assumption that discovery proceeds as sampling without replacement and proportional to size (Kaufman, Balcer, and Kruyt (1975), Barouch and Kaufman (1978)).* Namely the probability of discovering $A_1, \ldots A_N$ in that order is

$$\prod_{j=1}^{N} A_j/(A_j + \ldots + A_N),$$

Bloomfield et al. (1979) propose a test of this model: in place of assuming that the probability of discovery of a deposit is proportional to its size, however size is defined, assume that it is proportional to a power α of size; use observed data to estimate the value of α. The model becomes

$$\prod_{j=1}^{N} A_j^{\alpha}/(A_j^{\alpha} + \ldots + A_N^{\alpha}).$$

---

* This sampling process appears in the statistical literature treating a problem known as the "coupon collector's problem". There it is called "successive sampling", a curiously uninformative name; cf. B. Rosen (1970), (1971).

If the estimated value of α is close to one, then the assumption that the probability of discovery is proportional to size is more or less reasonable, depending on the quality of the data and the size of the sample. Using as data all oil fields found in Kansas from 1900 to 1975, they found that "discoverability" as embodied in the parameter α was proportional to a "surprisingly low power of area", namely, $\alpha = .33$, and conclude that

> "... models assuming that discoverability is proportional to either area or volume should not be used on a regional basis without further study."

Stretching a simple probability model designed for individual plays to cover a region in which there may be many plays apparently doesn't work well.

In a commentary on this study, Kaufman and Wang (1979) argue that the authors findings are inconclusive and possibly misleading: the parameter α may or may not be different from one for individual plays, but in their study "... there is no recognition that deposits discovered in Kansas come from several descriptively distinct deposit populations and that as discovery effort grew, so did the number of deposit populations recognized as targets for drilling." They assert that

- the analysis of the Kansas data as they have done it does no more than confirm that a statistical model designed for data from a single population of descriptively homogeneous deposits in a petroleum basin may not be an appropriate model for discovery data drawn from a mixture of distinguishable deposit populations.

- the "surprisingly low power"* of the discoverability parameter α for the Kansas data as they estimate it ($\alpha = .33$) is possibly a result of use of an <u>incorrect</u> model: when data is generated by sampling without replacement and exactly proportional to size ($\alpha = 1.0$) from two or more distinguishable populations, each of which becomes a target for drilling at a different point in time, application of a model in which deposits from <u>all</u> populations are regarded as targets for drilling at the outset can yield an estimate of much less than 1.0.

- For samples of moderate size (30 observations or less) the maximum likelihood estimator for α as proposed by Bloomfield et al.can be <u>very sensitive</u> to the reported order in which fields are discovered. A shift of a single data point can cause large changes in a maximum likelihood estimate of α.

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* A value of $\alpha = 1.0$ corresponds to the probability of discovery of a field being exactly proportional to its size (area).

The evidence for the second assertion appears graphically in Figures 2 and 3 which summarize results of a Monte Carlo study of the effects of assuming that observed field sizes in a geographic region come from a single deposit population all of whose elements are recognized as drilling targets at the outset, when in fact they don't.

In these figures $\hat{\alpha}_{II}$ is a maximum likelihood estimator (MLE) for α if it is assumed that all observations from two distinct populations are targets for drilling at the outset, when in fact one of the two populations is recognized as a set of targets only after a fraction of elements of the first population have been discovered. The function $\hat{\alpha}_{III}$ is a MLE for α when this feature is explicitly taken into account.

The effects on estimation of α of sequestering acreage as opposed to allowing unrestricted search by drilling over the full areal extent of a single play are similar. (cf. Kaufman and Wang (1979)).

2.3 Uses (and Abuses?) of Subjective Probability

With few exceptions, subjective probability has been used in resource evaluation and supply projection in a very particular way: opinion is elicited, not about model parameters, but about observables — aggregate volume of hydrocarbons in place, generic deposit attributes like net sand thickness, porosity, etc. Said in another way, predictive distributions for deposit attributes are assessed; no statement of an <u>objective</u> probability model describing the process by which observables are generated is provided. As a result, when additional data — new discoveries and measurements of their attributes, additional measurements and revision of previous measurements of properties of previously discovered deposits — is provided, no explicit mechanism is available for a logical revision of the original assessments. Revision of the original assessments must be done <u>judgmentally</u> by "experts". No framework for appraisal of the coherence of <u>a priori</u> and <u>a posterior</u> assessments is in place.

Is this important? Admittedly sparse experimentation with small groups offers evidence that, without extensive training, humans may not correctly weight sample evidence and <u>a priori</u> probabilities for hypotheses about "states of nature", (Hogarth (1975), Tversky and Kahneman (1974) Edwards (1962)). While, given additional data, the geologist may provide an elaborate descriptive rationale for a shift in his or her assessments in, say, the amount of recoverable hydrocarbons in a stratigraphic unit, the correspondence between the <u>quantitative</u> shift in assessed probabilities from "before the new data" to "after the new data" is not easily fathomed.

Most practitioners of resource assessment art who use subjective probability are aware of the cognitive biases that may distort probability assessments. Assessment protocols for minimizing these biases have been suggested in the literature. However, in the heat of the battle to get "acceptable" assessments in a short period of time from busy geologists, self-protection aids often are dropped. Dominant personality effects and peer

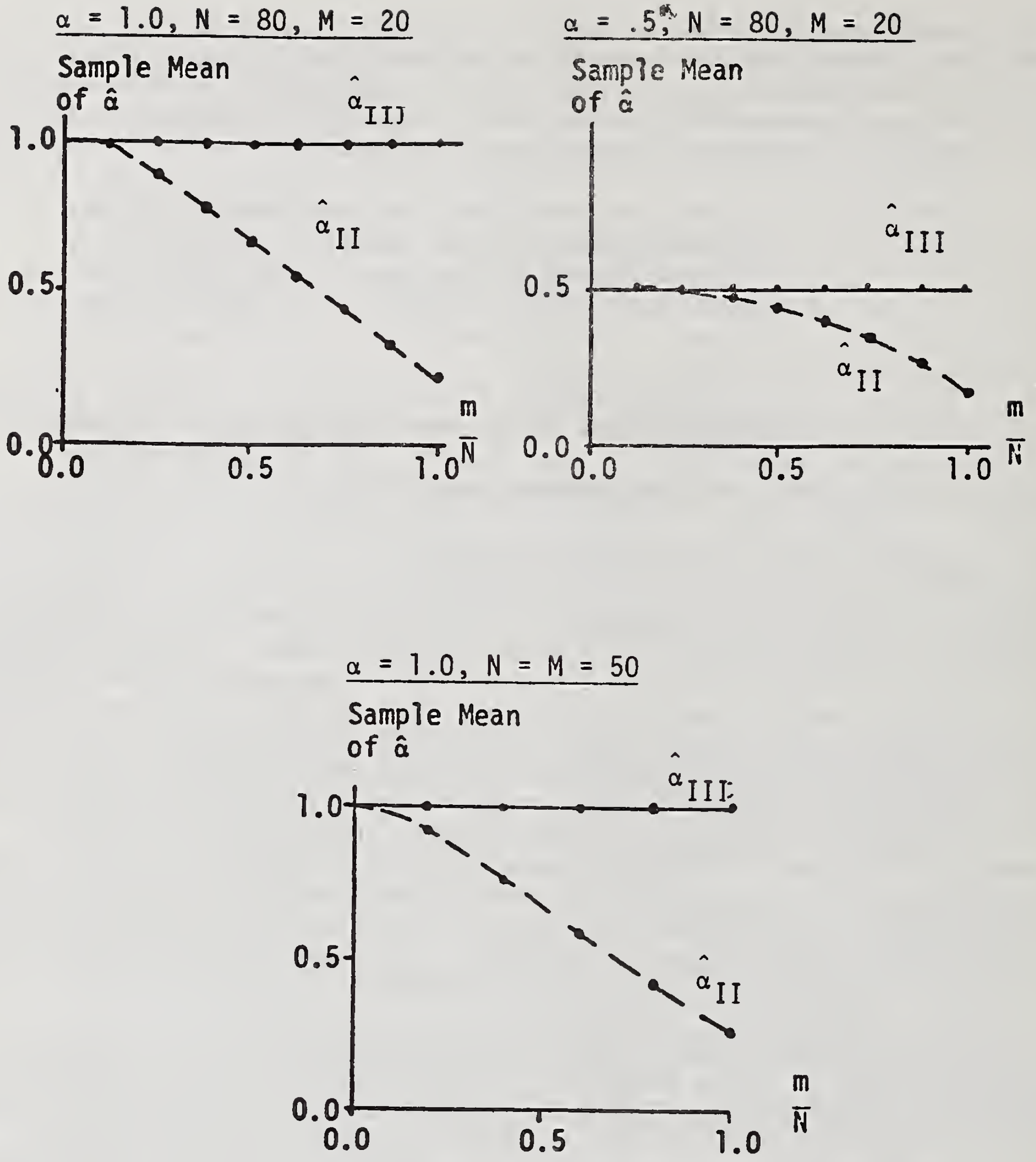

Figure 2. Comparison of Sample Means of $\hat{\alpha}_{III}$ and $\hat{\alpha}_{II}$ When Data is Generated by Model III*

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* Numbers of elements in first and second populations are N and M respectively. The second population becomes a target after a fraction m/N of the first population is observed.

(N = 80, M = 20)

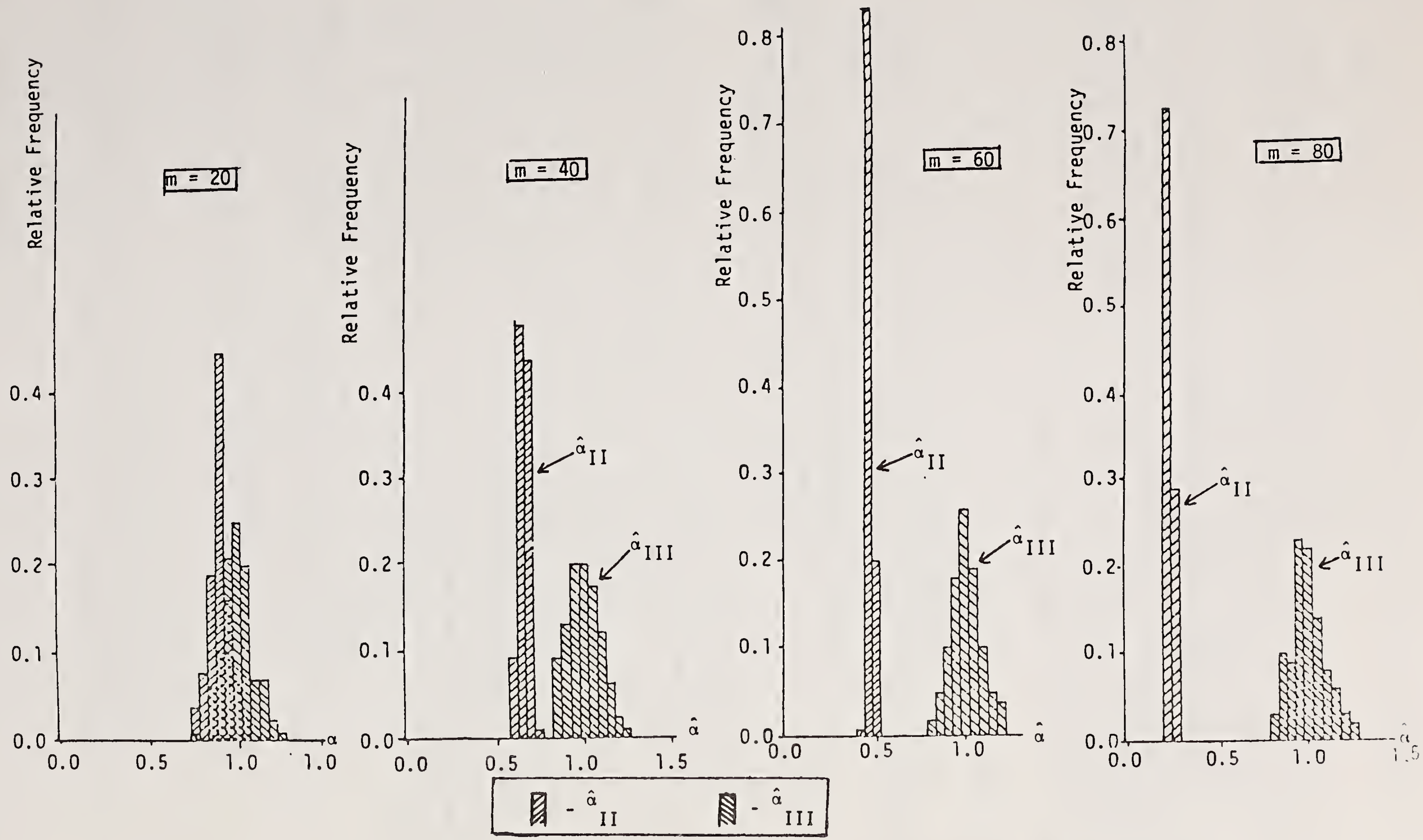

Figure 3. Monte Carlo Sampling Distributions of $\hat{\alpha}_{II}$ and $\hat{\alpha}_{III}$ When Data is Generated by Model III

group pressure come into play. While it is impossible to measure the ultimate cost of possible distortions in forecasts that this introduces, one is left with at best an uncomfortable feeling.

Probabilistic dependencies among uncertain quantities are almost always ignored and mutual independence implicitly or explicitly assumed. The time consuming nature of the assessment process and the tyranny of large numbers of assessments that may be required combine to justify a tactical dodge away from incorporation of dependencies among uncertain quantities. What is the character of dependencies between reservior attributes such as porosity, net sand thickness, etc. for a given set of deposits?

2.4 Data

Well measured data describing properties of deposits are an essential ingredient of any recipe for cooking up evidence bearing on many of the issues raised thus far.

For the modeller of supply it is important that the data display a recognition of the interplay between geologic features of deposition, reservoir attributes and recovery technology. A good example is the data gathered for the Interagency Oil and Gas Project Permian Basin study, several aspects of which have been presented by speakers at this conference.

Another excellent illustrative example is a recent study of the Lloydminster heavy oil play in Alberta and Satchkachewan done by the Geological Survey of Canada: the Manville section was "divided into twenty slices each comprising a sand-shale couplet representing one cycle of trangression and regression". Each of 1200 pools in the Alberta region of this play were reworked and attributes remeasured by MacCallum, Stewart and Associates. That all pools were reworked by the same group of experts using their particular approach to measurement is important: most publically available deposit data consists of data elements generated by more than one group of experts and different approaches to measurement of deposit characteristics may be employed across groups. In this particular instance "between group" heterogeneity is absent. Here are principal features of their study:

- net pay thickness, oil saturation, presence of and thickness of overlying gas and underlying water measured or estimated for each of twenty stratigraphic slices,

- probabilistic assessments of oil in place include "virtually all of the oil, even in the thinnest beds, with no *a priori* economic cutoff",

- individual reservoirs mapped and divided into segments according to applicable recovery method; nine thousand reservoir segments were processed,

- probabilistic assessments of recovery factors elicited for primary, water flood, fire flood and steam soak recovery,

- a probabilistic projection of technologically feasible recoverable oil generated for each type of recovery method.

The economics of recovery and production plays no role in this assessment; McCrossan, Proctor, and Ward probabilistically project amounts of oil technologically feasible to recover by primary, water flood, fire flood and steam soak methods. The amounts are large: the most likely quantity of oil in place is about 25 billion barrels with 2.5 billion barrels technologically recoverable, excluding unexplored areas. Of particular note for data analysts is their observation that

> "The apparent very large discrepancies between the existing booked reserves made by the regulatory agencies and the present study is a purely artificial one in that the booked reserves include only those pools on production or known to be capable of producing. The current study, on the other hand, should be looked upon as an inventory of the total resource with some reasonably realistic forecasts of its producibility but without consideration of the ultimate costs or timing, or any estimate of when the resource might become a component of Canada's supply."*

In addition, the authors present fractile plots on logarithmic probability paper for oil in place and for recoverable oil. Combining a visual evaluation of these plots with geological perspective they assert that:

> "It is clear from the geological work done to date that the oil has been pooled into discrete deposits which appear to be closely related genetically on the basis of the homogeneity of the pool size distributions. An examination of the distributions indicate that, as might be expected in the logarithmically distributed population, that there is a very large number of small pools and only a few approaching giant class. These distributions seem extremely orderly in spite of the fact that these deposits lie within an extraordinarily complex geological framework involving salt collapse, facies variations, changing structural attitudes over short periods of geologic time, abundant anastomosing channels cutting various stratigraphic horizons during various periods of time and highly variable fluid contacts with different amounts of underlying water and overlying gas."†

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* "Estimate of Oil Resources, Lloydminster Area, Alberta", by R.G. McCrossan, R.M. Procter and W.J. Ward.

† R.M. Procter and W.J. Ward, op. cit.

Their sample is large and suited to a study of distribution shape. In particular an examination of tail behavior along the lines suggested by Mallows and Tukey (1979) [Section 19, p. 19] and Hoaglin (1980) recommends itself. Both projective area and oil in place measurements are recorded for each deposit, so the relation between area and volume of oil in individual deposits can be investigated.

The Lloydminster data exemplifies what is needed for a meaningful retrospective study of

- size distributions
- probabilistic dependencies between reservoir attributes
- influence of volume and area on discoverability
- the impact of alternative economic scenarios on amounts of recoverable hydrocarbons and on the portfolio of recovery technologies that are economical to supply.

Unfortunately, the number of <u>publicly available</u> data bases of this type and quality are sparse.

2.5 Conclusions

There are impelling political and bureaucratic forces funneling great effort and money into building models that can address policy problems as currently conceived. By comparison relatively little effort is devoted to acquisition of data in a form truly appropriate for meaningful validation of the structure of disaggregated supply models.

Econometric and other stylized aggregate approaches to modelling petroleum supply ride rampant over structural detail. A gradual elision of disaggregated and aggregated approaches will probably evolve, with the former providing a process oriented framework for structuring the latter. In order to speed the process up, modellers need, as they have needed from the start,

(1) Much more carefully measured data in a form that allows meaningful structural and predictive validation of disaggregated models of exploration, discovery, and production,

(2) To place this data in the public domain at modest cost so as to encourage a larger component of the scientific community to work with us,

(3) More emphasis on a retrospective analysis of recurring problems in resource analyses unfettered by the need to apply results immediately to particular policy problems, and

(4) To promote a much closer liason between geologists and modellers so that geologic ideas are more concretely represented in model structure.

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ANALYSIS OF INVESTMENT AND PRODUCTION STRATEGIES FOR A PETROLEUM RESERVOIR

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I. INTRODUCTION

This paper is concerned with the optimum development planning and management of a petroleum reservoir. Rowan and Warren [40, p. 84] describe the problem as follows:

> For any oil reservoir, newly discovered or partially developed, the continuation of development drilling, superimposed on the natural decline of the reservoir's ability to produce, must eventually become unprofitable. Consequently it seems appropriate that there must be an optimum development program, drilling schedule and/or production rate which may be determined for a specific economic criterion when practical constraints are imposed.
>
> These observations are often phrased in the form of a question, "What drilling and/or production policy must be adopted to maximize the return from a given operation when certain practical limitations are present?"

In an offshore petroleum exploration process, one or more permanent production platforms are installed on the seabed (Figure 1). Production wells are drilled from the platforms into the reservoir rock. Petroleum flows through the wellbores and to the surface as a result of pressure differentials between the wellbores and the reservoir. The pressure differentials may be maintained by natural forces or artificially through enhanced recovery techniques. Production of oil or gas from a reservoir involves displacing the oil and gas from pore spaces in the reservoir rock. The primary mechanisms displacing the oil or gas are fluid expansion or depletion drive, fluid displacement (natural or artificial) or frontal drive, gravitational drainage, and/or capillary expulsion. Often some combination of drive mechanisms operate conjunctively.

The decision environment of the operator of an offshore reservoir system is extremely complex. In making development decisions, the operator is faced with uncertainties about almost every aspect of the problem from the parameters describing the reservoir to future economic and political conditions.

Several methodological approaches have been utilized to aid in reservoir development planning. The detail and level of complexity used in representation of the reservoir appears to be a key factor in any taxonomy. Models of petroleum reservoirs vary in their complexity from simple production decline curves (see Campbell [9]), in which it is assumed that

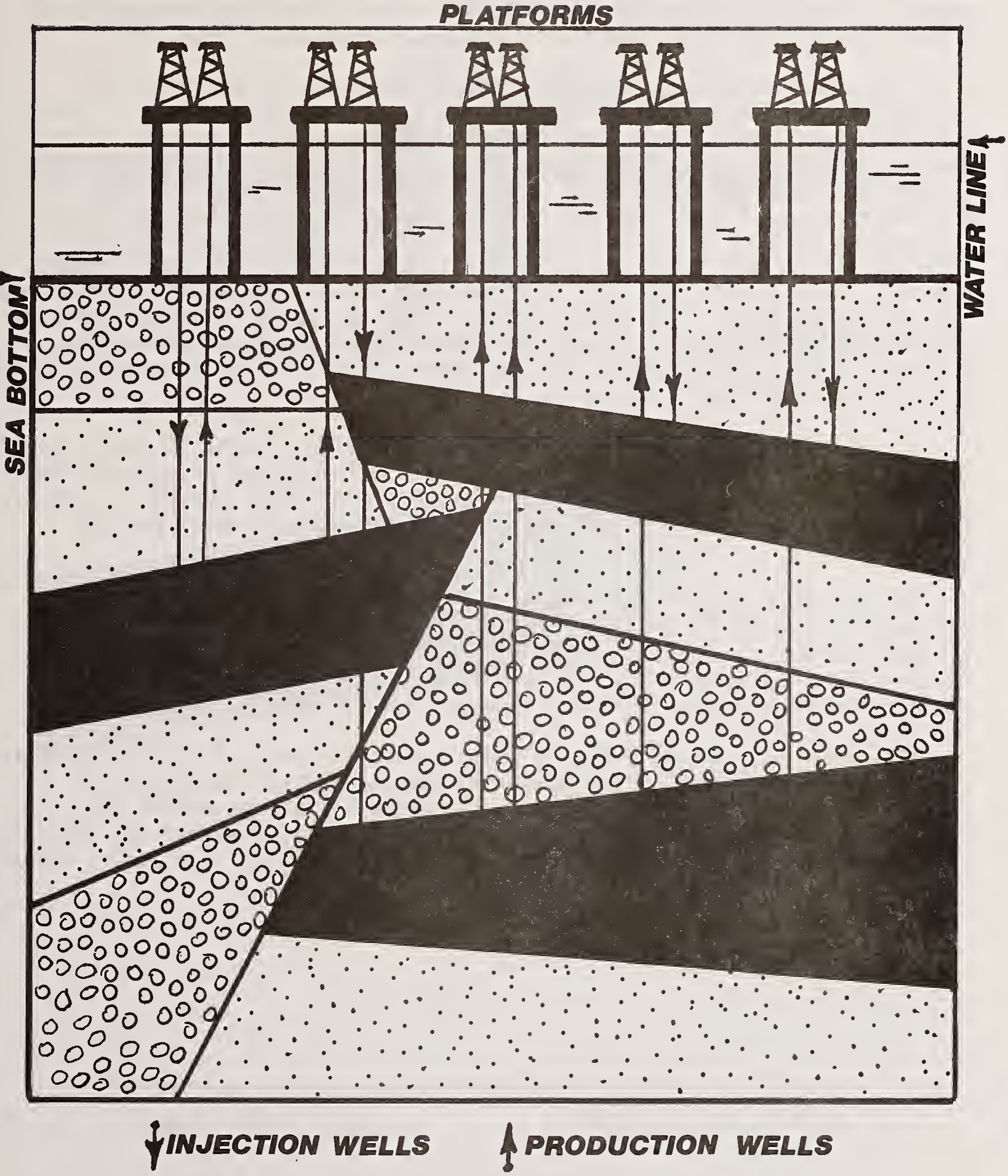

FIGURE 1: MULTIPLE PLATFORM
MULTIPLE RESERVOIR SYSTEM

production declines in some pre-specified way over time, for example, exponentially, to very complex three-dimensional, multiphase grid simulators of reservoir behavior (see Richardson and Stone [38] and Crichlow [13]).

The general modeling approaches appear to be of two general types: (1) optimization formulations that have been solved with mathematical programming techniques (linear, non-linear, or mixed-integer) and (2) reservoir simulation models of varying degrees of sophistication often combined with an economic discounted cash flow model. The first set of models have, in most cases, used a very simple reservoir representation. The second major approach is the use of reservoir simulators. Most of the studies using reservoir simulators focus on a specific part of the reservoir development problem. A case study approach is often used where a few selected development plans are evaluated. Some recent studies have attempted to bridge the gap between the optimization models and reservoir simulators by generating influence functions with the reservoir codes; these influence functions appear in constraints in the optimization problem.

Another important consideration in the modeling process is the level of aggregation that is assumed and the specific decisions and issues addressed; for example, the reservoir development problem might be a component part or subsystem within a larger field or multi-reservoir development problem (Figure 2).

A conceptual framework with which to view the reservoir development problem is depicted in Figure 3. Given information and data on the technological, economic, regulatory and physical environment for the operator, reservoir and economic subsystems are integrated to provide a mechanism whereby alternative development strategies can be evaluated.

The remainder of the paper is organized as follows. A brief literature review is given in Section II. A simple optimal control model for the analysis of production and investment decisions for a homogeneous gas reservoir with water drive is presented in Section III. Section IV contains results for a hypothetical example in the Gulf of Mexico. Concluding remarks are given in the last section of the paper.

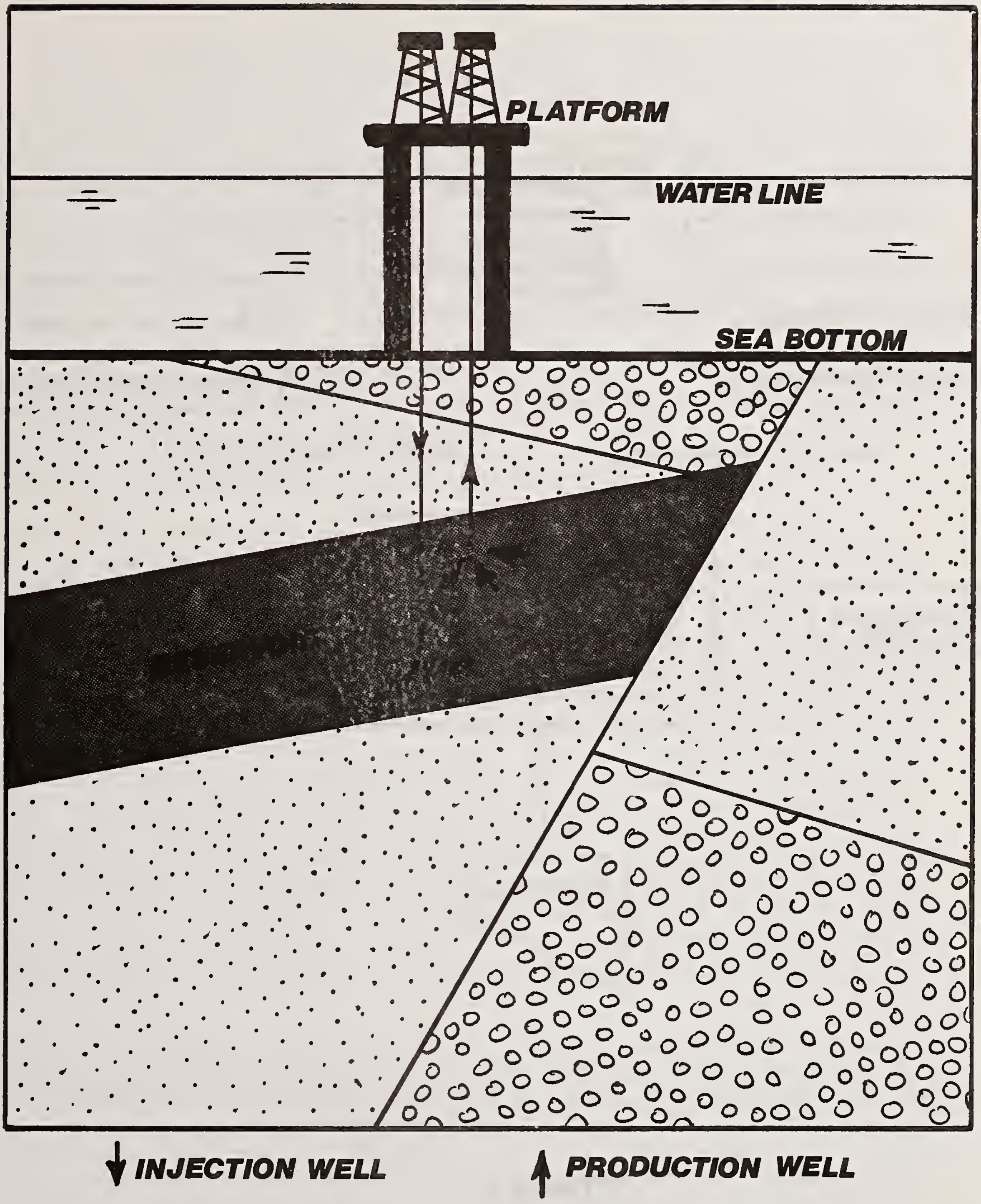

FIGURE 2: SINGLE PLATFORM-SINGLE RESERVOIR SYSTEM

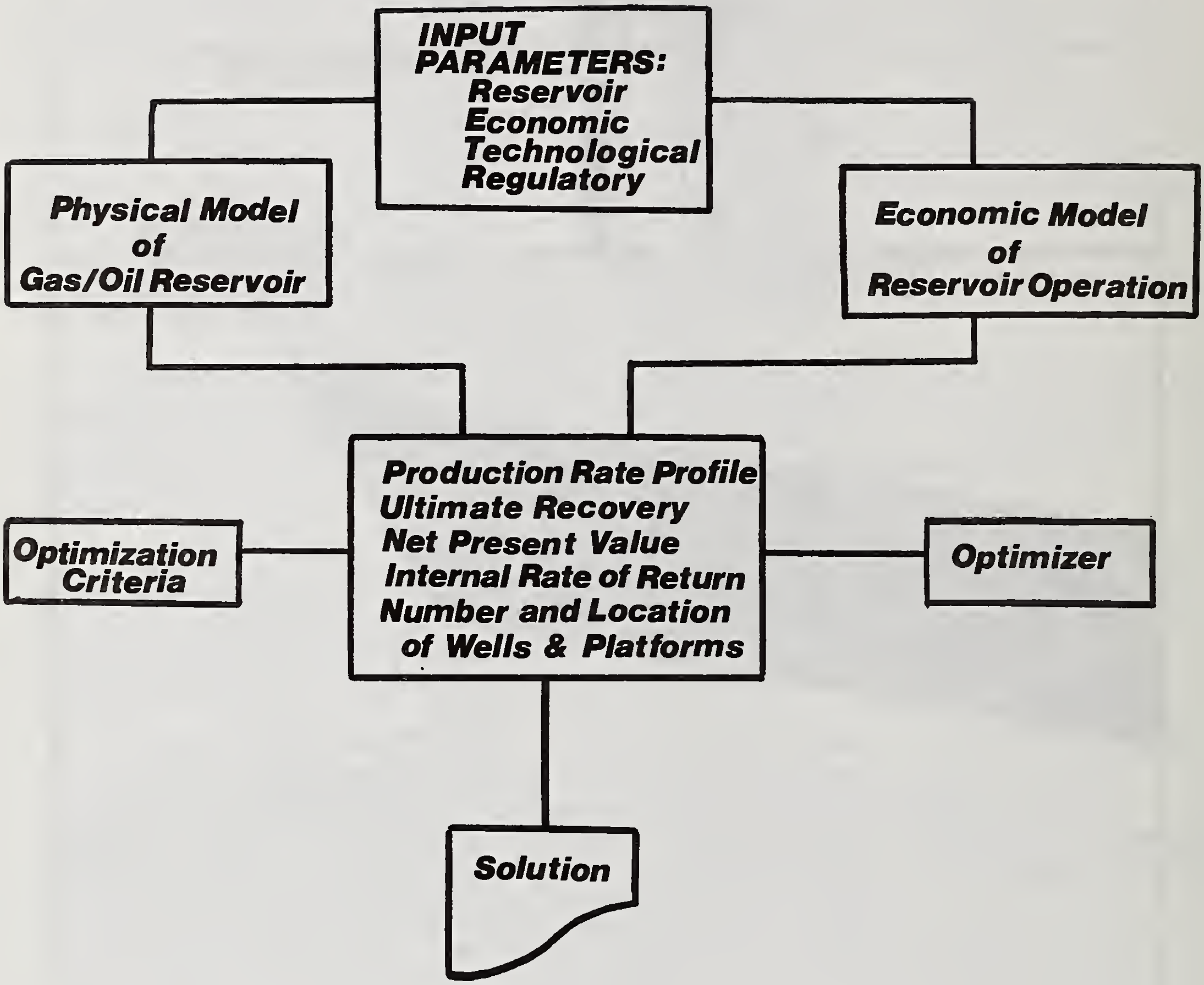

FIGURE 3:

Schematic Representation of Management Model and Data

II. BRIEF LITERATURE REVIEW

In 1958, Aronofsky and Lee [3] formulated a linear programming model for scheduling of production from a finite number of sources (reservoirs) so as to maximize profits. Each reservoir was assumed to be homogeneous with an infinite water drive. A Hurst-van Everdingen [45] model was used to represent pressure behavior in the reservoir. Using basically the same model, Aronofsky and Williams [4] studied two basic problems: 1) the optimum scheduling of production from a multi-reservoir system with no additional drilling or a fixed drilling schedule, and 2) the scheduling of drilling under fixed production schedules. Charnes and Cooper [10] formulated a one-reservoir, water injection model in which the objective was to minimize the cost of production, injection wells, and gathering station facilities subject to a fixed production schedule. Attra, Wise, and Black [6] were concerned with optimizing field operating conditions such that a field was produced at maximum oil rate, subject to well producing capacities, gas lift and pressure maintenance requirements, sales contract requirements, and gas compressor limitations. The production rate for each well was determined for a given set of field conditions.

Rowan and Warren [40] state the reservoir development problem and illustrate how the problem can be formulated in an optimal control framework. Solutions to the model are illustrated for special cases.

Bohannon [8] studies a "multi-reservoir pipeline system", that is, a system of many reservoirs producing into one or more gathering systems. A mixed-integer 0-1 linear programming model is specified to determine the annual production rate for each reservoir, the number of development wells to be drilled in each reservoir each year, and the timing of major capital investment projects such as secondary recovery and pipeline expansion, subject to constraints on reservoir production rates, pipeline capacities, and capital expenditures. Well production rates are assumed to decline exponentially with time. Odell, Steubing and Gray [37] used an optimization model for determining optimum field development and production scheduling from multi-reservoir gas fields. Decline curves are used to represent reservoir behavior over time. The model is structured to determine optimum completion times and production schedules; several optimization criteria are investigated.

Devine and Lesso [14], Friar and Devine [17], and Babayev [7] apply mathematical programming techniques to problems associated with offshore petroleum operations where their analyses are primarily concerned with the development and production of entire fields. Devine and Lesso and Friar and Devine are concerned with decisions relating to the number, size, and location of production platforms and the allocation of wells to platforms. Babayev's model focuses on the number of wells to drill in each layer of multilayer oil and gas fields and the transfer of wells between layers. In these studies the production rate profiles are assumed to be known or specified in terms of the number of wells (see, Babayev p. 1363). The effects of alternative operating strategies on such variables as ultimate recovery and reservoir pressure are subsumed within these relationships. The production rate versus time curves are commonly represented by either exponential or hyperbolic decline curves (see, e.g., Friar and Devine, p. 1372). Extensions of these models are considered by Devine [15] and Lilien [24]. Durrer and Slater [16] survey recent literature on the application of operations research techniques to petroleum and natural gas production problems.

Huppler [19] uses a single state variable dynamic programming model to examine the optimal well and compressor horsepower investment strategies for a homogeneous gas reservoir, given a desired gas delivery schedule and a specified peak delivery capacity. A tank-type reservoir model with a Hurst-van Everdingen unsteady state water influx is used. Huppler also applies nonlinear programming to the problem of production rate scheduling for a multi-reservoir gas field.

Kuller and Cummings [21] formulate an economic model of production and investment for petroleum reservoirs. Decision rules for the discrete-time optimal control problem are derived using the Kuhn-Tucker necessary conditions. The reservoir behavior is subsumed in the model in a maximum production constraint in which the stock of recoverable petroleum depends on the time paths of investment and production.

Wattenbarger [46] used a finite difference reservoir simulator to generate well influence coefficients. A well influence coefficient represents the pressure drop at one specific well site due to one unit of production at another well. These coefficients formed the basis for a set of constraints on production rates in a linear programming model to schedule production from a gas storage reservoir. The objective was to minimize the difference between desired and scheduled production. Rosenwald and Green [39], using influence functions, formulated a mixed-integer programming model to study the problem of optimum well placement. Also using the influence function approach, Murray and Edgar [35] developed mixed-integer algorithms to optimize the selection of well locations in a gas reservoir and the sequential optimization of flow scheduling from a multi-well gas reservoir.

Ali, Batchelor, Beale and Beasley [2] summarize four models that help in the management of Kuwait Oil Company with problems ranging from day-to-day operations through long term planning. A reservoir development model, based on the work of Rowan and Warren, is used to study investment policies and production and injection rates. Nonlinear programming techniques are used to provide optimum solutions to meet production targets given assumptions about costs, capacities and the behavior of reservoirs and wells during depletion.

The formulation, solution and application of grid-type reservoir simulators are discussed by Crichlow [13]. Richardson and Stone [38] provide a good historical perspective on the development, refinement, and use of these models. Henderson, Dempsey, and Nelson [18] and Coats [11] present results from applications of two-dimensional, single-phase models. Henderson, Dempsey, and Nelson are concerned with the problem of evaluating the effect of allocations of different flow rates among a set of wells. Coats focuses on the determination of an optimum drilling schedule for developing the remainder of a field. Coats uses complete enumeration to solve for the development scheme that maximizes discounted cash flow.

Ashiem [5] investigates offshore petroleum development in the North Sea using numerical simulation and optimization. A relatively detailed simulation model of offshore development operations is formulated. In his modular system, a two-dimensional reservoir simulator constitutes one component. A unidimensional search procedure is used to determine the optimum initial production processing capacity.

III. A SIMPLE MODEL FOR GAS RESERVOIR DEVELOPMENT[1]

Production and investment decisions are assumed to be made under the following conditions. A single operator exploiting the reservoir is postulated.[2] Leasing and exploration activities have been completed and the associated costs are known. External effects such as the production of brines are assumed to be controlled through production regulations and the costs are reflected in the production and operation expenses. There is no enhanced recovery.

It is assumed that the objective of the operator is to determine the production time path $\{ q(t), t_0 \leq t \leq t_1 \}$ and the investment time path $\{ I(t), t_0 \leq t \leq t_1 \}$ so as to maximize a stream of discounted profits over the life of the reservoir. In addition, the abandonment time t_1 and platform size $\hat{K}$ are decision variables. The objective function is:

$$\text{(1)} \quad \text{Maximize } J = \int_{t_0}^{t_1} [\pi(t)(1-\theta)q(t)-\phi(t)I(t)-\mu(t)K(t)]\,(1-\beta)e^{-it}dt-\psi(\hat{K})$$

where $K(t)$ = number of producing wells at time t;

$I(t)$ = number of wells drilled at time t;

$q(t)$ = reservoir production rate at time t;

$\hat{K}$ = number of well slots on the production platform;

$\pi(t)$ = wellhead price of gas;

θ = royalty rate[3];

$\phi(t)$ = cost per well (including completion and surface facility costs);

$\psi(\hat{K})$ = platform cost function;

$\mu(t)$ = operating, maintenance, and overhead cost per well;

β = tax rate[4]; and

i = discount rate.

To illustrate the methodology, a homogeneous gas reservoir with water drive is selected for study. Several rather basic tank-type, zero-dimensional models are often used by reservoir engineers in analyzing gas reservoirs (see, for example, Craft and Hawkins [12], Agarwal, Al-Hussainy, and Ramey [1], and Lutes, Chiang, Brady, and Rossen [27]). One of these simple reservoir models, based on the Schilthuis water drive assumption, is adopted for use in the optimal control formulation. Other reservoir characterizations and models could be incorporated into this decision framework.

The reservoir model depicts a permeable region in which gas is trapped above water from a large aquifer. Three state variables are used in describing the reservoir--the volume of the reservoir V, the pressure in the reservoir P, and the quantity of gas in the reservoir n. The initial gas in place is n_0 with an initial volume V_0. Initially the gas pressure P is at the aquifer pressure P_0. As gas is removed from the reservoir at some rate q, the pressure drops. It is assumed that the permeability of the reservoir is high enough that the pressure remains uniform throughout the gaseous region. Following Schilthuis [41], the water flow rate is assumed proportional to the pressure difference between the aquifer and gaseous region, that is, $\frac{dV_w}{dt} = \xi \frac{V_0}{P_0}(P_0 - P)$, where V_w is the volume of water which has invaded the reservoir, and ξ is the Schilthuis water drive constant.

The following differential equation for volume can be derived from the volume balance equation and the Schilthuis water drive equation.

$$(2) \quad \frac{dV}{dt} = -\xi \frac{V_0}{P_0}(P_0 - P), \quad V(t_0) = V_0.$$

From the mass balance equation and the ideal equation of state, the following equation can be derived for pressure,

$$(3) \quad \frac{dP}{dt} = \frac{P_0 V_0}{V}\left(-\frac{q}{n_0} + \frac{\xi P}{P_0}\,\frac{(P_0 - P)}{P_0}\right), \quad P(t_0) = P_0.$$

The quantity of gas in the reservoir, n, can then be determined from the ideal equation of state PV = nRT where R is the gas constant and T is reservoir temperature; $n(t_0) = n_0$.[5] Equations describing the reservoir are derived in McFarland et al. [29] and in Monash [34].[6] Equations (2) and (3) describe the effect of production on the reservoir through time.[7]

Let K(t) denote the number of producing wells at time t, and let I(t) denote the number of new wells drilled at t. Then dK/dt equals new investment in t, I(t), less the rate at which wells become flooded out due to water influx, $(1 - V/V_0)\,K(t)$. It is assumed that wells become flooded at a rate that is proportional to the reservoir volume reduction (see Agarwal, Al-Hussainy, and Ramey [1] and Huppler [19]). Thus,

$$(4) \quad \frac{dK}{dt} = I - (1 - V/V_0)K, \quad K(t_0) = K_0.$$

The reservoir production rate, q(t), is the sum of the individual well flow rates. It is assumed that the flow per well is a function of reservoir pressure squared.[8] Thus, using a homogeneous reservoir model, the reservoir flow rate is the flow per well times the number of wells,

$$(5) \quad q(t) = \alpha P(t)^2 K(t),$$

where α is the well flow constant.

A more general specification of (5) is to bound the production rate between zero and some maximum production rate $q(t)$, that is, $0 \quad \hat{q}(t) \leq \hat{q}(t)$. In addition to the limit on production imposed by well capacities and reservoir pressure, there are a number of possible considerations (maximum efficient rate regulations or well allowables, pipeline capacities, etc.) that might constrain production.[9]

Investments in wells are constrained to be between zero and some maximum rate $\hat{I}(t)$, that is,

$$(6) \quad 0 \leq I(t) \leq \hat{I}(t).$$

The upper bound $\hat{I}(t)$ could result from either physical or financial resource limitations. Drilling equipment and support services might not be adequate to permit drilling at a rate faster than indicated by $\hat{I}(t)$. In some applications these restrictions might be modeled using nonlinear cost functions on $I(t)$.

Decisions relating to the number, location, and size of production platforms are possibly more appropriately made when considering entire fields. In many cases the same platform may be used in producing more than one reservoir. Models such as those developed by Divine and Lesso, Friar and Divine, and Babeyev could be used to aid in making these decisions.

Given a platform with K well slots allocated to the reservoir, then the total number of wells drilled would be constrained by K.

$$(7) \quad \int_{t_0}^{t_1} I(t)dt \leq \hat{K}.$$

IV. APPLICATION

It is assumed that a relatively shallow gas reservoir with water drive in the Gulf of Mexico is being exploited.[10] Leasing and exploration activities have been completed.

It is assumed that the reservoir is of intermediate size with initial gas in place of $n_0 = 100 \times 10^9$ standard cubic feet (SCF) and an initial volume of $V_0 = 2.039 \times 10^9$ ft^3. The depth to the reservoir is assumed to be 2200 feet in a water depth of 200 feet, with an initial pressure P_0 = 1000 pounds per square inch (psi). The Schilthuis water drive constant, ξ, is set at 0.0025. A value for the well flow constant, α, of 0.00001 is used.

The economic parameters are based on 1977 data for the Gulf of Mexico. The regulated price of new gas, $\pi(t)$, is \$1.42 per thousand standard cubic feet (MSCF). The royalty rate, Θ, on Federal Outer Continental Shelf leases is 16 2/3 percent. The cost per well, which includes completion and surface facility costs, $\phi(t)$, is \$1,000,000.[11] Operating, maintenance and overhead costs, $\mu(t)$, are \$84,000 per well. The tax rate, β, is 0.48, and the discount rate, i, is 0.10. Platform costs are assumed to be given by $\psi(\hat{K}) = 3.6 + 0.3\,\hat{K}$.

Solutions to the model were computed using a generalized reduced gradient code for nonlinear programming developed by Lasdon et al. ([22], [23]). The code was run on a CDC 6600 computer at the Los Alamos Scientific Laboratory.

For the base case data given above, the optimal solution is to drill 10 wells[12] in year one, and none thereafter. The production time path for this investment strategy is platted in Figure 4. The reservoir is produced for 36 years. The time paths for the state variables, pressure and volume, are illustrated in Figure 5.

The number of wells, K, equals 10 in year one. Since no new wells are drilled after year one, the number of wells in operation gradually declines with time due to the water influx. Five wells remain in operation at abandonment in year thirty-six. The net present value of the reservoir is \$8.06 million.

By examining the necessary conditions for the problem, some insights as to the nature of the solution are obtained. The problem as stated can be classified as a "Problem of LaGrange," and the optimal controls can be characterized using the "Maximum Principle."

By forming the Hamiltonian and taking the appropriate derivatives, the decision rule for investment ($I > 0$) is to equate the discounted marginal cost of an additional unit of investment with the discounted marginal value (profit) of an additional unit of investment. $y_3(t)$ is the adjoint variable associated with the state equation for $K(t)$. The adjoint variable $y_3(t)$ equals the discounted marginal value from period t through the end of the planning horizon for an additional well drilled in period t.

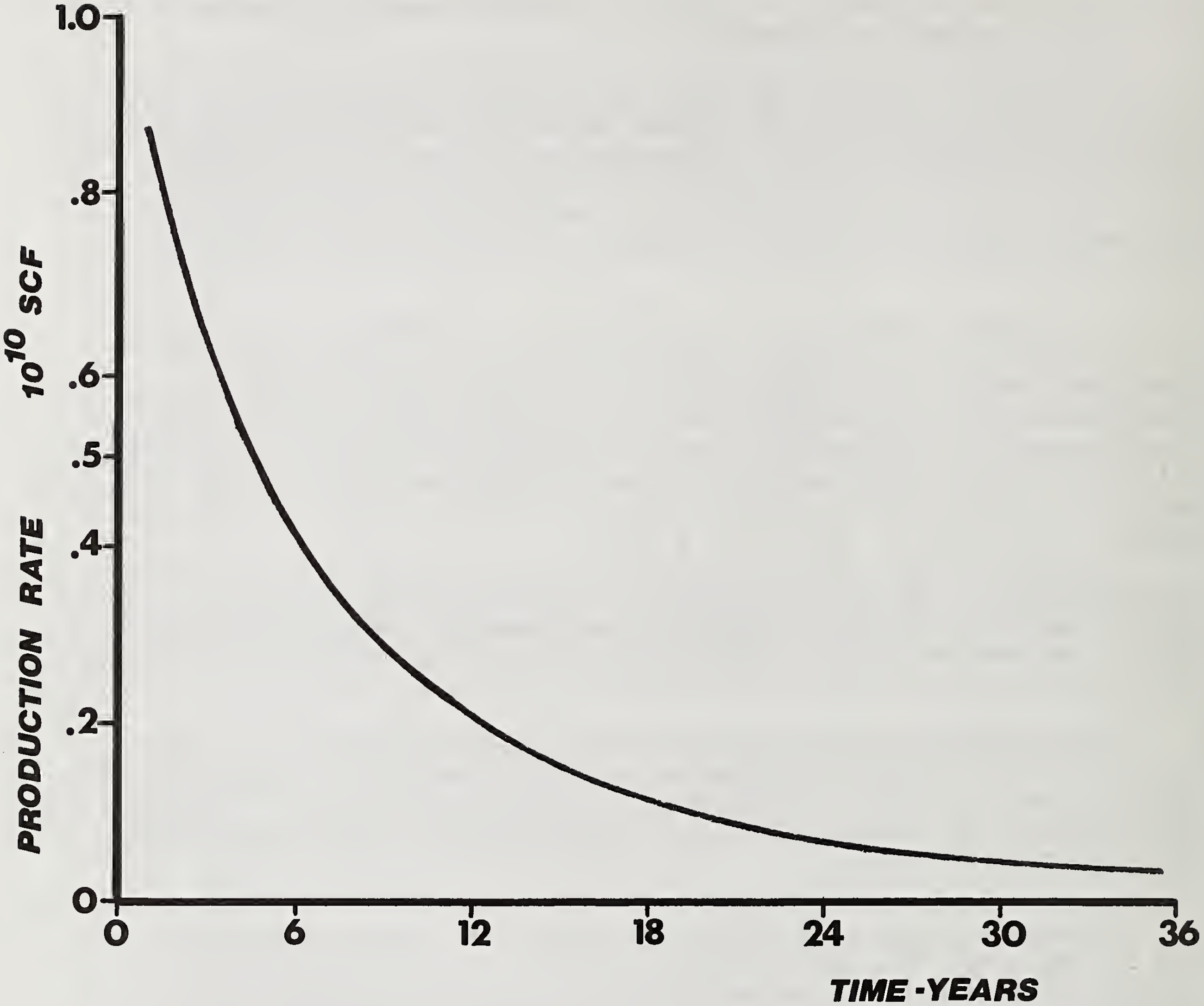

FIGURE 4: PRODUCTION RATE vs. TIME [BASE CASE]

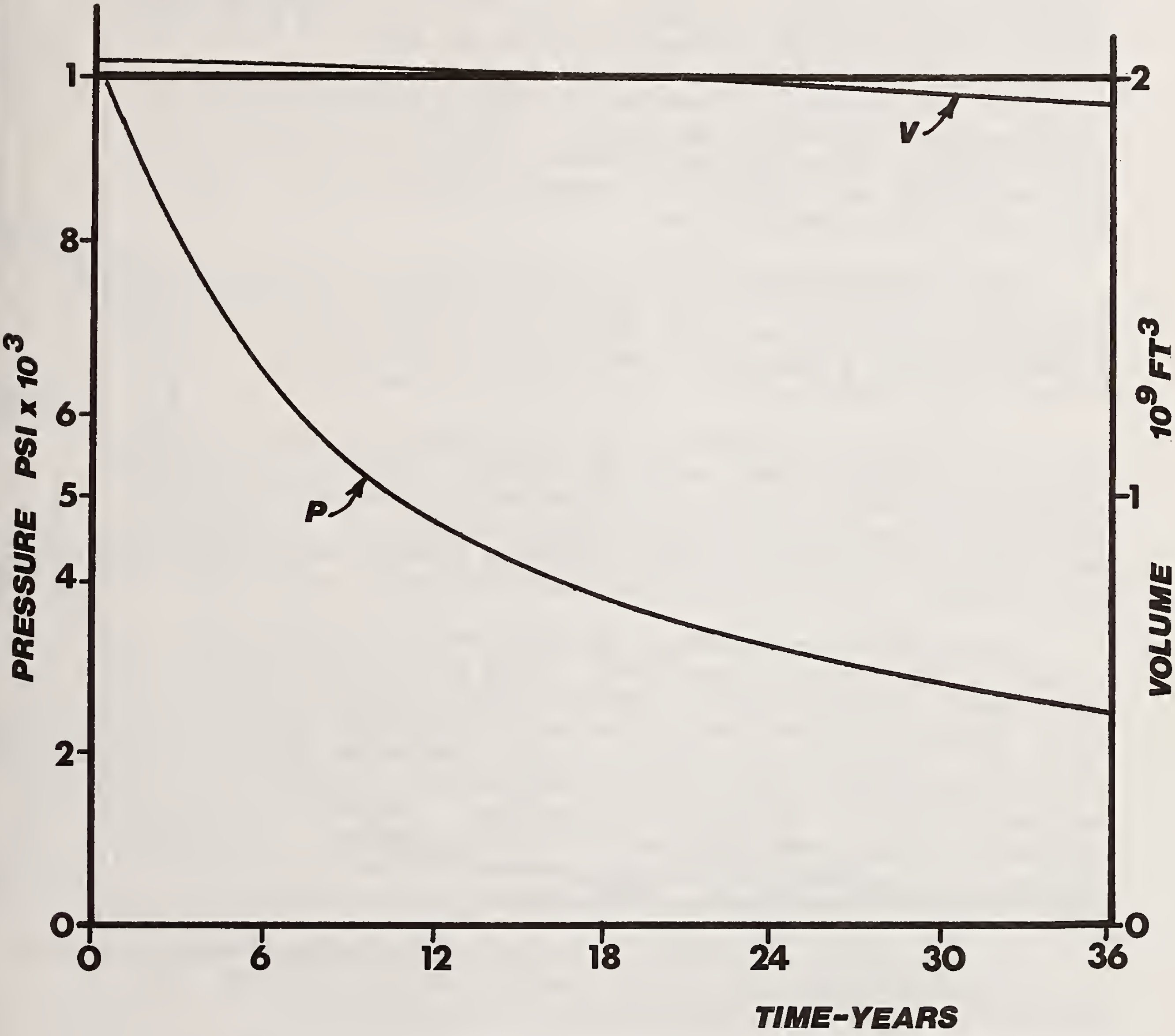

FIGURE 5: RESERVOIR PRESSURE and VOLUME versus TIME [BASE CASE]

The solutions for the adjoint variables $y_1(t)$, $y_2(t)$, and $y_3(t)$, corresponding to the state variables V, P, and K, respectively, are plotted in Figure 6. The discounted marginal value of reservoir volume, $y_1(t)$, is initially \$20.4 million per billion ft^3. $y_1(t)$ declines exponentially and equals zero at the end of the horizon. The discounted marginal value of reservoir pressure, $y_2(t)$, is initially \$34.9 thousand per psi; y_2t decreases to \$0 per psi. The discounted marginal value of wells, $y_3(t)$, is \$862 thousand per well at the beginning of the decision horizon; $y_3(t)$ declines rapidly through time and reaches \$0 per well.

Investment in wells in year one is pushed to the point where the discounted marginal cost of an additional well equals its discounted marginal value. After year one, however, the discounted marginal cost of an additional well exceeds its discounted marginal value. Thus, no wells are drilled after year one.

Several runs were made to test the sensitivity of the model to economic and reservoir parameters, including price, production and investment costs, the discount rate, the well flow constant, the initial conditions on pressure, volume, and quantity, and the Schilthuis water drive constant. Results from selected parameter variations are given in Table 1.

Increasing price, decreasing the royalty rate, or decreasing costs shifts the production path toward the present. The exploitation period is shortened, the final pressure is lowered, and the net present value and ultimate recovery for the reservoir are increased. A reduction in the discount rate yields a longer production horizon, lower initial investment, slightly lower ultimate recovery, and increased net present value.

Varying reservoir parameters can also have a significant impact on production and investment decisions. With a stronger water drive, these model results suggest that the reservoir should be produced at a faster rate. However, the net present value and ultimate recovery of the reservoir are slightly lower than for the base case. With higher initial pressure, the net present value and ultimate recovery from the reservoir are increased, and the reservoir is operated for fewer years. Varying other reservoir parameters affects model solutions as might be expected. For example, decreasing the size of the reservoir results in fewer wells being drilled and a lower net present value of the reservoir.

It is possible to simulate a reservoir supply response curve by varying price. At a higher price, a larger quantity would be supplied. The reservoir supply response function would be of the form:

(8) $$q_0 = f(\pi(t_0)/\text{reservoir and economic parameters})$$

The effect of changes in selected regulatory policies or other parameters on supply can be investigated through sensitivity analyses.

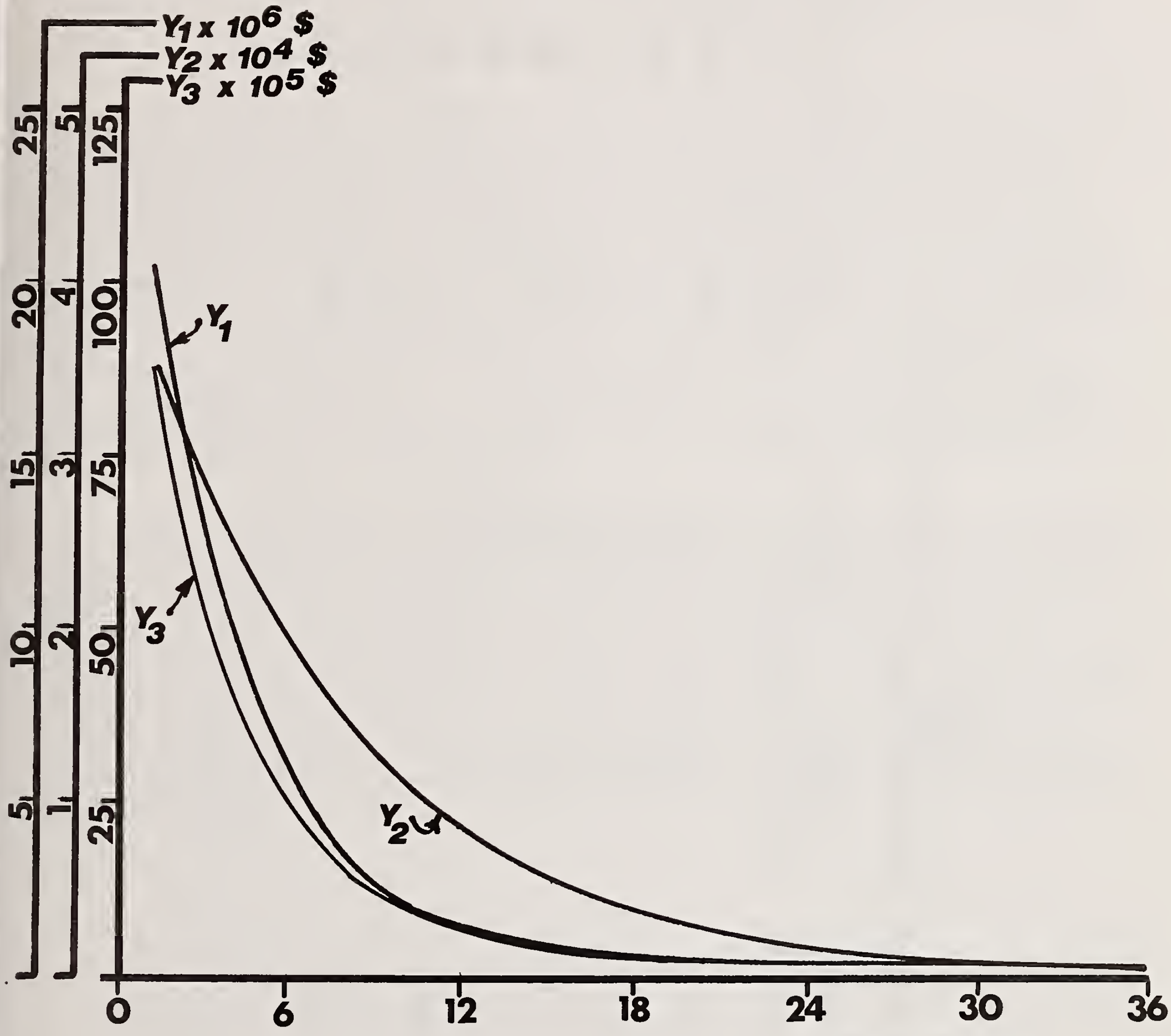

FIGURE 6: ADJOINT VARIABLES vs. TIME [BASE CASE]

TABLE 1

RESULTS FROM SELECTED PARAMETER VARIATIONS

	OBJECTIVE FUNCTION $ 10^6	INITIAL INVESTMENT (WELLS)	PRODUCTION HORIZON - YRS.	ULTIMATE RECOVERY 10^9 SCF	FINAL PRESSURE PSI
1. BASE CASE	8.06	10.	35.8	74.6	267.
2. PRICE = $1.75	12.21	12.	31.5	77.1	240.
3. PRICE = $2.25	23.33	16.	24.4	79.6	213.
4. PRICE = $1.42 x $(1.05)^t$	15.82	10.	50.0	78.2	236.
5. WELL COST = $1,500,000	5.94	8.	50.0	73.9	281.
6. DISCOUNT RATE = 0.05	13.43	8.	50.0	75.1	268.
7. WATER DRIVE = 0.025	7.94	11.	43.3	64.5	659.
8. INITIAL PRESSURE = 1500	17.44	10.	24.0	82.9	266.

IV. CONCLUDING REMARKS

The research concerned with petroleum reservoir development has focused on providing operators with methods to aid them in reaching better management decisions. These models also provide a framework that can be used to analyze the potential effects of government policies and regulations on investment and production decisions.

The Secretary of Interior was charged in the "Energy Policy and Conservation Act of 1975" [42] and "The Outer Continental Shelf Lands Act Amendments of 1978" [43] with determining "maximum efficient rates of production" and "maximum attainable rates of production" for oil and gas on selected Federal lands. The responsibility of establishing diligence requirements and setting production rates was transferred to the Department of Energy in "The Department of Energy (DOE) Reorganization Act (PL 95-91) Sec. 302" [44]. A methodology similar to that described in this paper has been used in analyzing MER regulations and the effects of alternative optimization criteria on oil and gas production rates. Some results from this research are reported in references [20], [28], [29], [30] and [32]. The determination of production rates is part of the larger problem of reservoir development and management.

There are two areas of future research that appear especially deserving of inquiry. The advantage of reservoir simulators over classical reservoir models is the ability to consider the geometrical complexity and detailed heterogeneity of the reservoir. In cases where heterogeneity plays a dominant role in reservoir performance, it is important that reservoir development and management models reflect these complexities. A shortcoming of most optimization approaches to date is that they have not, except in limited cases, incorporated this level of sophistication. Another area especially deserving of study is the introduction of risk and the stochastic nature of some of the underlying processes into the reservoir development planning problem.

FOOTNOTES

[1] Results from a similar model are given by McFarland [31].

[2] In the case of several firms exploiting the resource, either pooling, drilling, or unitization agreements would likely be required to avoid "common property" problems. There are numerous legal, technical, and administrative problems that often arise in operating a reservoir as a unit. A discussion of these is beyond the scope of this paper (see Kuller and Cummings [21], McDonald [33], and Lovejoy and Homan [26]).

[3] Variable royalty bidding systems are also being used in some OCS lease sales.

[4] The actual tax structure for oil and gas operations is considerably more complex than that used in this formulation. The tax rate may be viewed as an effective tax rate, a surrogate for this more complex tax structure. In extensions of this model, the refinement of the treatment of taxes is one area of interest.

[5] An ideal gas is assumed. For non-ideal behavior, the ideal gas law is modified to $PV = znRT$, where z is the gas deviation factor (see Craft and Hawkins). The gas deviation factor is the ratio of the volume actually occupied by a gas at a given pressure and temperature to the volume it would occupy if it behaved ideally.

[6] For results of matching studies using this reservoir model, see Lohrenz and Monash [25] and Nachtshiem and Siegel [36].

[7] Ultimate recovery, that is, the total quantity of gas recovered from the reservoir, is $U = \int_{t_0}^{t_1} r(t)dt$, where t_0 is the time production begins and t_1 is the time when production ceases. The relationship between ultimate recovery and production rate for alternative model specifications is investigated in McFarland, et al. [29].

[8] This function is usually specified as either a linear or quadratic function of reservoir pressure (Zaba and Doherty [47], Agarwal, Al-Hussainy, and Ramey [1], Rowan and Warren [40]).

[9] The model could be extended to investigate optimal compressor horsepower using an approach such as Huppler's. An additional state and control variable and associated constraints would be required.

[10] Gas reservoirs with water drive are fairly common in the Gulf. Some estimates suggest that approximately 15 percent of all gas reservoirs in the Gulf are water drive.

[11] Well costs vary with a number of factors (for example, well depth, water depth, drilling time, and geological conditions); however, it is fairly common to estimate well costs as a function of well depth or drilling time.

[12] The solutions for number of wells are rounded to integer values.

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A METHODOLOGY FOR ESTIMATING OIL AND GAS PRODUCTION SCHEDULES FOR UNDISCOVERED FIELDS

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INTRODUCTION

Oil and gas supply models disaggregated to the field level require some method of estimating the economic viability of a given size field under various conditions. These conditions might include the price of oil and gas, development and production costs, and regulations. Both the production costs and income stream from a given field depend in part on the production schedule that can be maintained. The production schedule in turn depends in general on the size, production technique, drive mechanism, and other physical parameters of a field. These general field parameters can be grouped by geographic region, depth and field size.

Today, I want to concentrate on the development of production schedules for undiscovered fields in the Permian basin. While many details are specific to the Permian basin, many of the procedures are applicable to other regions. This work was conducted as part of an Interagency Oil and Gas Supply Project which, in its initial effort, performed a study of the Permian basin. More general aspects of this study will be discussed in another paper tomorrow morning.

BACKGROUND

Every well in every field has a unique production history and it is a difficult task to develop accurate production schedules for fields for which reservoir parameters and drive mechanisms are reasonably well known. In this study, the only knowledge provided by the discovery model for a field is the depth bracket (within 5,000 feet) and the average size. Therefore, the production schedules that were developed for future fields were based upon the average of fields found in the past, of similar size and depth.

In the Permian basin study, field size was given in terms of barrel oil equivalent (BOE). Gas was converted to BOE on the basis on 5.27 thousand cubic feet of wet gas per barrel. There are 20 field size classes. The smallest size class is from 0 to 6,000 BOE. Each class's upper limit is double that of the previous class. The upper limit of class 20 is 3.1 billion BOE.

There are four depth classes: 0-5,000, 5,000-10,000, 10,000-15,000, and 15,000-20,000 feet. However, there were no historical oil fields deeper than 15,000 feet in the Permian basin.

Three types of undiscovered fields are considered: oil fields which would undergo secondary recovery, oil fields which would only be susceptible to primary recovery, and non-associated gas fields. For each type of field for each depth and class size, a production schedule was determined. Production schedules are determined for what is considered to be the average well in each field category. This has the advantage of ignoring the wide variations in behavior of individual wells within a field and allowing flexibility in the timing of field development. Field production was calculated by multiplying well production by the number of wells in a field.

Empirically, it has been found that oil production from almost all wells either actually declines exponentially for considerable periods of time, or that an exponential decline is a good approximation to production behavior. It is also a very simple function to work with. Therefore, exponential decline is used to describe the normal decline in production of all oil wells in the Permian basin area.

To describe a production schedule for an exponentially declining oil well, one needs to know the initial oil producing rate, QRO, the decline rate, D, and the economic limit rate, ELR, for a well. These quantities are related to the ultimate recovery, QWOE, by the following formula:

$$QWOE = \frac{QRO - ELR}{D}, \qquad (1)$$

where

QRO = initial oil producing rate,

ELR = economic limit rate,

D = exponential decline rate,

and

QWOE = expected ultimate oil recovery per well.

To determine the expected ultimate oil recovery per well, we first made a quick estimate of the expected ultimate recovery of every oil and gas field in the Permian basin excluding tertiary recovery. We also estimated the total number of wells in each field. From these data, we developed fitted values of a nominal expected recovery per well and field for each size and depth class. This required making estimates for 4,457 oil fields and 896 gas fields in the Permian basin which is located in southeast New Mexico and west Texas.

The results are shown in table 1. Note that there is a very large range in the size classes. The expected average recovery of a class one field is about 2,000 barrels, while a class 9 field is expected to recover about a million barrels and a class 19 field about a billion barrels.

Table 1. Expected Ultimate Oil Recovery per Well from Primary Oil Fields in the Permian Basin
(Thousand barrels)

Size Class (BOE)	0-5,000 (feet)	5,000-10,000 (feet)	10,000-15,000 (feet)
1	1.490	1.930	2.160
2	4.280	4.070	4.170
3	7.580	8.670	9.590
4	11.800	15.900	18.900
5	17.300	25.900	32.400
6	24.500	39.000	50.700
7	33.800	55.100	74.100
8	45.700	74.600	103.000
9	53.300	82.900	136.000
10	54.900	118.000	198.000
11	68.900	132.000	261.000
12	95.600	140.000	333.000
13	135.000	154.000	419.000
14	188.000	189.000	526.000
15	254.000	258.000	662.000
16	334.000	374.000	834.000
17	428.000	550.000	1,050.000
18	536.000	802.000	1,310.000
19	658.000	1,140.000	1,630.000
20	794.000	1,580.000	2,010.000

In the 0 to 5,000-foot depth bracket, a class 20 well would be expected to produce over 500 times as much oil as a class 1 well. A class 1 primary oil well would be expected to produce about 1,500 barrels, a class 9 well about 53,000 barrels and a class 19 well about 658,000 barrels.

There is also a higher expected recovery per well for each class of well as the depth increases. For example, in class 9 wells the expected recovery goes from 53,000 barrels to 83,000 barrels to 136,000 barrels respectively for depth brackets of 0-5,000 feet, 5,000-10,000 feet and 10,000-15,000 feet. While this could be due in part to changes in the geology of fields with depth, it is more likely that it reflects the relative economics of drilling deep wells compared to shallow wells. It cost about two and a half times as much to drill a well in the 5,000 to 10,000-foot depth bracket as in the 0 to 5,000-foot depth bracket and almost six times as much in the 10,000 to 15,000-foot depth bracket. The higher costs associated with deep drilling will always tend to push deep field development toward the minimum number of wells necessary for complete development.

Decline Rates

After determining the expected ultimate oil recovery per well, QWOE, we still have to determine the other three factors in equation one, D, ELR, and QRO. Primary production decline rates were determined for each size and depth category of oil fields. The decline rates are considered to be for average wells in the average field over the life of the field. In general, decline rates were determined from historical field production data, but there were several factors that made this task difficult; proration of fields which began in the late 1920's, continued drilling in old fields, changes in production methods, changes in decline rates over the life of a field, introduction of secondary recovery projects, wells being taken out of production, and variations between wells in a field. In addition, variations in the availability and quality of data led to different approaches for determining decline rates for different categories of fields.

The smaller fields (field size classes one through three) had very short production histories which made the yearly individual field data difficult to analyze. For these three classes, the average production of all fields discovered in a particular year was considered to be the production of an average field in that category. The decline in average production of these fields was then calculated for several successive years. The yearly averages were fitted to an exponential equation by the method of least squares. It was assumed that the average field was discovered at midyear and that in the following years, the instantaneous production rate at midyear was numerically equal to the annual production. This procedure was followed for fields discovered in several years and the values were averaged for each category of field size. There was considerable scatter in the yearly results.

Exponential decline rates were calculated individually for all declining primary fields in classes 4 through 20 for all depth brackets. The calculations were performed by a computer program utilizing the method of least squares on production data from 1970 through 1974. In classes 4 through 11 for the 0 to 5,000 and 5,000 to 10,000-foot depth brackets, and for classes 4 through 20 in the 10,000 to 15,000-foot depth bracket the average of the individual field decline rates were calculated and used. The general result is that as fields get larger and shallower, their decline rates get smaller. Decline rates for classes 7 through 11 in the 0 to 5,000-foot depth bracket were averaged as were classes 7 through 9 in the 5,000 to 10,000-foot depth bracket, classes 8 through 11 and classes 12 through 20 in the 10,000 to 15,000-foot depth bracket. This further averaging was done to smooth the calculated decline rates for those classes.

The larger fields (classes 12 through 20) in the 0 to 5,000-foot and 5,000 to 10,000-foot depth bracket had individual estimates made on a randomly selected sample from each category. A complete production history from 1937 or earlier, the number of wells drilled each year, number of wells producing by artificial lift each year and the number of wells flowing each year was prepared for each field. In addition, a literature search for published reports on these fields was made. Such factors as well top allowables, market demand factors, reaching marginal well status, and the onset of secondary recovery were considered. When data permitted, a number of years' production history was fitted to an exponential decline curve by the method of least squares.

The individual field rates were averaged for each field category and classes 12 through 15 in the zero foot depth bracket were averaged as were classes 12 through 14 in the 5,000 to 10,000-foot depth bracket to further smooth the data. It was assumed that classes 16 through 20 in the 0 to 5,000-foot depth bracket and classes 15 through 20 in the 5,000 to 10,000-foot depth bracket would have exponential decline rates of 0.2 per year.

Considerable engineering judgment went into the selection of the years chosen to represent the average primary production decline for a field. The larger fields often had continuous field development, a steadily changing mix of flowing and artifical-lift wells, a restrictive field allowable, and were prime candidates for early secondary recovery projects. Even though there are wide variations in decline rates among individual fields and individual wells in a field, an exponential decline rate of 0.2 per year gives a reasonable production schedule for an average well in a large oil field.' for example, testimony given before the Texas Railroad Commission in 1949 indicated that if the McElroy field (discovered in 1926) had been produced at full capacity during its life, it would have been abandoned in 1953, a primary producing life of 27 years. This is in reasonable agreement with the value of 30 years calculated for the average well in a field in that category. Another specific example would be the Loco Hills field in New Mexico for which a recent detailed study was available. Exponential decline rates of four individual wells over their primary productive life had an average exponential decline rate of .204 per year.

The exponential decline rates used for the Permian basin fields are shown in table 2. It should be noted that an exponential decline rate of one per year means that the annual production declines to 36.8 percent of the previous year's production or a production decline of 63.2 percent per year.

Nominal Economic Limit Rate

Nominal economic limit rates were calculated and used only for determining what the initial oil producing rate would be for a well. Once the initial producing rate has been determined, and annual oil production is being computed, the actual economic limit rate is computed for the well, depending upon the price of oil and gas, and when income is equal to out-of-pocket expenses. Table 3 shows the nominal economic limit rate for wells in the Permian basin. These rates were calculated for each depth bracket by using the estimated 1976 direct operating expenses for wells in each depth bracket and assuming an oil price of $14.00 per barrel.

Initial Producing Rate for Permian Basin Wells

An initial producing rate is calculated for wells in each depth bracket and BOE class by substituting the appropriate QWOE, D, and noninal economic limit rate, ELRN, into the following equation:

$$QRO = (D)\ (QWOE) + ELRN. \qquad (2)$$

This is just equation one solved for QRO.

Table 2. Exponential Decline Rate per Year for Oil Wells in the Permian Basin (Thousand barrels)

Size Class (BOE)	0-5,000 (feet)	5,000-10,000 (feet)	10,000-15,000 (feet)
1	1.50	1.50	1.50
2	1.00	1.10	1.10
3	0.90	1.00	1.00
4	0.35	0.51	0.61
5	0.29	0.38	0.40
6	0.24	0.34	0.35
7	0.22	0.32	0.35
8	0.22	0.32	0.29
9	0.22	0.32	0.29
10	0.22	0.24	0.29
11	0.22	0.24	0.29
12	0.20	0.21	0.20
13	0.20	0.21	0.20
14	0.20	0.21	0.20
15	0.20	0.20	0.20
16	0.20	0.20	0.20
17	0.20	0.20	0.20
18	0.20	0.20	0.20
19	0.20	0.20	0.20
20	0.20	0.20	0.20

Table 3. Nominal Economic Limit Rates for Wells by Depth in the Permian Basin

Oil Production	Depth Bracket, Feet		
	0-5,000	5,000-10,000	10,000-15,000
bbl/day	1.18	1.63	2.16
bbl/year	431	594	790

In this study, it is assumed that the initial oil producing rate of a well in a newly discovered oil field will not be allowed to exceed the Texas Railroad Commission 1965 yardstick allowable schedule. This allowable rate takes into consideration the depths of wells and spacing between wells. This schedule, in effect, established the maximum rate of oil production for an oil well. In most cases, the initial oil producing rate will be less than the maximum allowed. The maximum rates of oil production allowed for this study for wells in the Permian basin for the three depth brackets are shown in table 4.

Table 4. Maximum Oil Production, by Depth Brackets, for the Permian Basin

Oil Production	Depth Bracket, Feet		
	0-5,000	5,000-10,000	10,000-15,000
bbl/day	84	121	365
bbl/year	30,660	44,165	133,225

It was assumed that these rates would represent good field production practice or would be mandated by regulation. The rates are based on 40-acre spacing in the 0 to 5,000 and 5,000 to 10,000-foot depth brackets and on 80-acre spacing for the 10,000 to 15,000-foot depth bracket.

Those field size classes where wells would have initial producing rates that exceeded the 1965 Texas yardstick allowable production rate will be held to that rate until they have produced a sufficient quantity of oil so that they could no longer maintain the yardstick allowable. From this time on, they are allowed to decline at the calculated decline rate until their ELR is reached.

Example of Oil Production Schedules

Shown in figure 1 are the oil production curves for classes 10, 15, and 17, calculated for wells in the 5,000 to 10,000-foot bracket. Note that wells of classes 10 and 15 did not have an initial producing rate that exceeded the Texas Railroad Commission yardstick allowable and began to decline immediately. However, the initial producing rate for the class 17 well did exceed the yardstick allowable and, therefore, production was constant at the allowable rate for 7.5 years before the production decline started.

Associated Dissolved Gas Production

There are numerous factors that affect the production of associated and dissolved gas from a reservoir and it is impractical to attempt to take each, individually, into consideration. For the Permian basin, it was assumed that the associated gas will be produced as dissolved gas, thereby resulting in a higher overall gas production per oil well. Also, because no attempt was made to predict the drive mechanisms of reservoirs to be discovered in the future, it is assumed that dissolved gas production will be related to a depletion-type drive mechanism.

The methodology for a gas production schedule is based upon a relationship between cumulative gas produced and cumulative oil produced. This relationship was derived from a calculation of the Schilthius' form of the material balance equation for depletion drive reservoirs as shown in "Elements of Oil Reservoir Engineering" by Pirson. The production of oil and gas for a depletion drive mechanism, as reported by Pirson, was converted to percents of ultimate recovery and an equation relating the percent of cumulative dissolved gas production to the cumulative oil production was developed by the method of least squares. The theoretical gas-oil ratio and

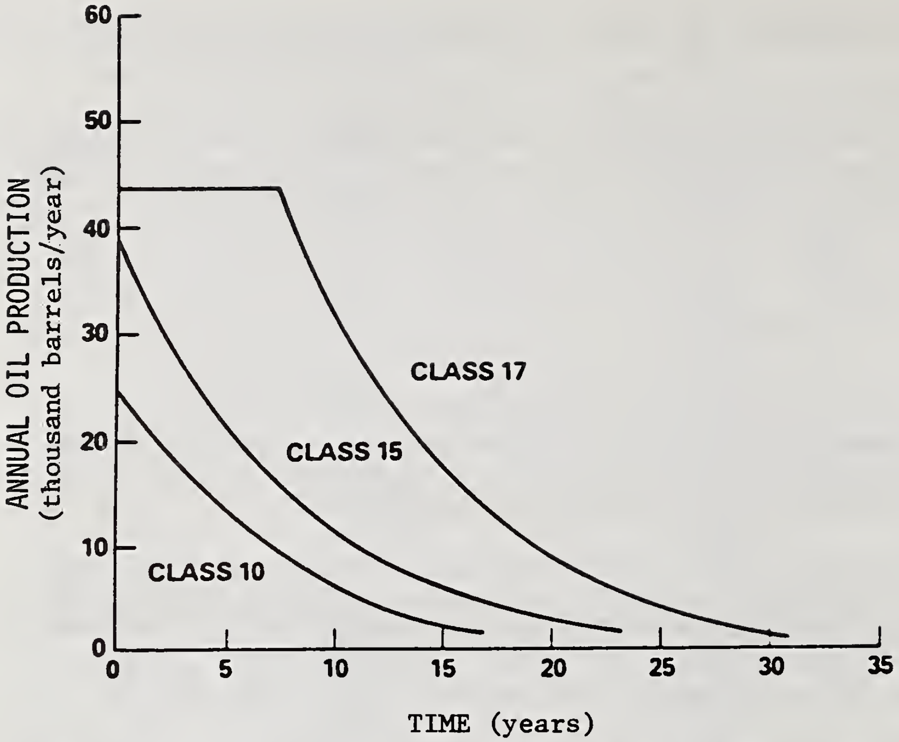

Figure 1. Oil Production Decline Curve for Primary Recovery at 7,200 feet in the Permian Basin

gas production curves resulting from the use of this relationship were compared to actual field performance curves and the curves are similar though not identical. Of necessity, this relationship will be used to represent composite oil reservoirs with a wide variety of drive mechanisms in the Permian basin. For a different basin, different assumptions would probably be made. In an area where water drives predominate, a nearly constant gas-oil ratio would be assumed.

Figure 2 shows the behavior of percent cumulative dissolved gas production as a function of percent of cumulative oil production. As you can see, about 20 percent of the expected gas is produced when 60 percent of the oil is produced leading to relatively low gas-oil ratios. From the 60 percent point on, the curve increases rapidly leading to higher gas-oil ratios. This is the type of behavior typically observed in the Permian basin.

In computing the annual gas production from oil reservoirs, the expected ultimate gas recovery per oil well, as shown in table 5, and the expected ultimate oil recovery per well are utilized. The percent cumulative gas produced to the end of a year is calculated as a function of the percent cumulative oil production, as was shown in figure 2. That is, for each year, the cumulative oil production is determined and the percent of expected ultimate recovery is computed. The percent ultimate gas recovery is determined and multiplied by the expected ultimate recovery shown in table 5. The cumulative gas production at the end of the preceding year is then subtracted from the resulting cumulative gas production at the end of the year under consideration. This difference will give the annual gas production.

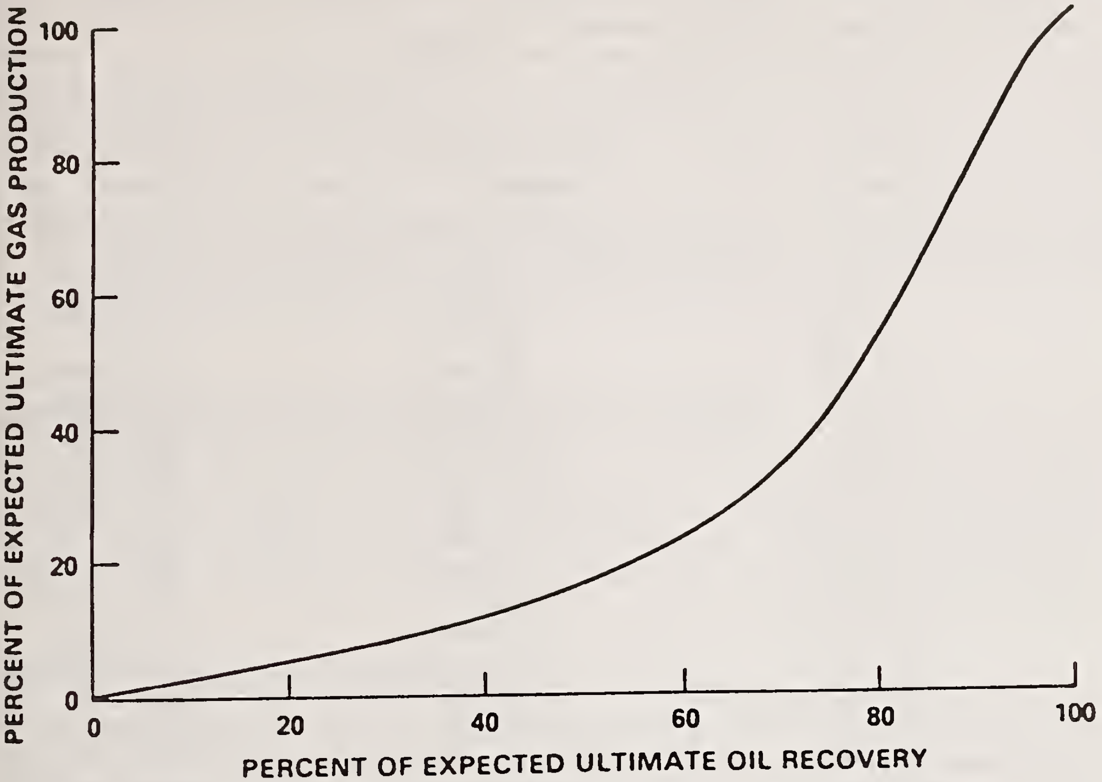

Figure 2. Percent of Expected Ultimate Gas Recovery as a Function of Percent of Expected Ultimate Oil Recovery in the Permian Basin

Production Schedule for Secondary Recovery Fields

Primary Phase Oil Production

Fields that are assumed to undergo secondary recovery have two distinct production phases. They first go through a primary phase which is basically the same as the production schedule for primary oil fields with the exception of expected ultimate recovery per well. The expected ultimate recovery for secondary and pressure maintenance fields per oil well that had ever produced oil was calculated. It was assumed that 60 percent of the expected ultimate field recovery could be produced by primary means.

In the 0 to 5,000-foot depth bracket, only 70 percent of the total oil wells were assumed to be drilled during the primary production phase. In the 5,000 to 10,000-foot depth bracket, the expected ultimate primary recovery per well was 60 percent of the expected ultimate recovery because it was assumed that all producing oil wells would be drilled during the primary development phase. The primary phase oil production schedule was then calculated with the same decline rates, nominal economic limit rates, and maximum oil production rates as for primary production.

Table 5. Expected Ultimate Associated-Dissolved Gas Recovery per Oil Well from Primary Fields in the Permian Basin (Million Cubic Feet at 14.73 psia and 60° F)

Size Class (BOE)	0-5,000 (feet)	5,000-10,000 (feet)	10,000-15,000 (feet)
1	1.490	4.320	7.470
2	4.740	9.210	14.400
3	9.050	20.100	32.000
4	15.200	38.400	63.000
5	24.100	65.700	110.000
6	36.500	103.000	175.000
7	53.200	153.000	261.000
8	75.100	216.000	371.000
9	81.900	266.000	506.000
10	117.000	327.000	657.000
11	164.000	389.000	806.000
12	223.000	453.000	1,070.000
13	293.000	518.000	1,460.000
14	374.000	584.000	1,960.000
15	467.000	652.000	2,580.000
16	572.000	721.000	3,320.000
17	688.000	1,080.000	4,170.000
18	816.000	1,570.000	5,150.000
19	955.000	2,240.000	6,240.000
20	1,110.000	3,100.000	7,450.000

Secondary Recovery Phase Oil Production

Primary oil production will continue until the annual oil production per well is less than the stripper stage (10 barrels of oil per day). It was assumed that a waterflood project will be initiated at the beginning of the following year. In general, each well that produces during the waterflood will be assigned an expected ultimate waterflood recovery, a variable fraction of which will be produced each year during the life of the waterflood. The specific production schedule depends on the size and depth of the field.

When the waterflood is initiated, some new oil wells may be drilled, some primary wells continue to produce, some primary wells are converted to injectors and some new injectors may be drilled. During the first year of the waterflood, all producing wells produce both primary production and a small amount of production that is due to the waterflood. After the first year, the production from each well is determined entirely by the waterflood production schedule.

It was assumed that for secondary recovery fields, 40 percent of the expected ultimate recovery for a field would be due to a waterflood. This oil plus the unrecovered expected ultimate primary production was divided by the number of oil wells that produce during the waterflood stage to determine the expected ultimate waterflood production per well.

The relationship between waterflood oil recovery and time is based on the extensive work of J. D. Walters of Sun Oil Company. His empirically derived curves were utilized to derive an equation representing the fraction of ultimate waterflood oil recovery as a function of the fraction of expected waterflood life. This equation was good only for fractions of expected waterflood life less than or equal to one. However, the expected ultimate waterflood recovery may not be reached or may be exceeded, depending on how each assumed oil price affects the economic limit rate per well. Therefore, the limits for life expectancy have been extended to values up to 1.5 times the expected waterflood life. The resulting curve is shown in figure 3. The expected waterflood life was assumed to be 10 years.

Primary Phase Associated-Dissolved Gas Production

The gas production procedure was basically the same as that for primary oil fields. It is assumed that all the expected ultimate associated-dissolved gas for a field would be produced if the oil wells that produce during the primary phase were produced down to their nominal economic limit rate.

Secondary Phase Gas Production

During the secondary phase, associated-dissolved gas that is not recovered under primary production schedules is recovered. It was assumed that wells producing under the secondary production schedule would have a constant gas-oil ratio. This ratio was determined by dividing expected ultimate associated-dissolved gas unrecovered under primary production schedules by the expected ultimate waterflood production of oil for each category of field. The annual gas production per producing oil well under the secondary production schedule was found by multiplying the annual waterflood oil production by this gas-oil ratio. The first year that the waterflood is initiated, gas is assumed to be produced by both the continuation of the primary production schedule and a small increment due to the waterflood. After the first year, gas production is assumed to result only from the waterflood production schedule.

Economic Limit Rate

In all cases, production is assumed to cease when an economic limit rate for production is reached. This occurs in the year in which annual operator income equals the sum of direct annual waterflood operating expenses and the operator's severence and ad valorem taxes. The operator is assumed to have a 7/8 working interest.

Pressure Maintenance Oil Fields

Oil Production

For depths greater than 10,000 feet, it was assumed that those fields which undergo a production process other than primary would have a pressure

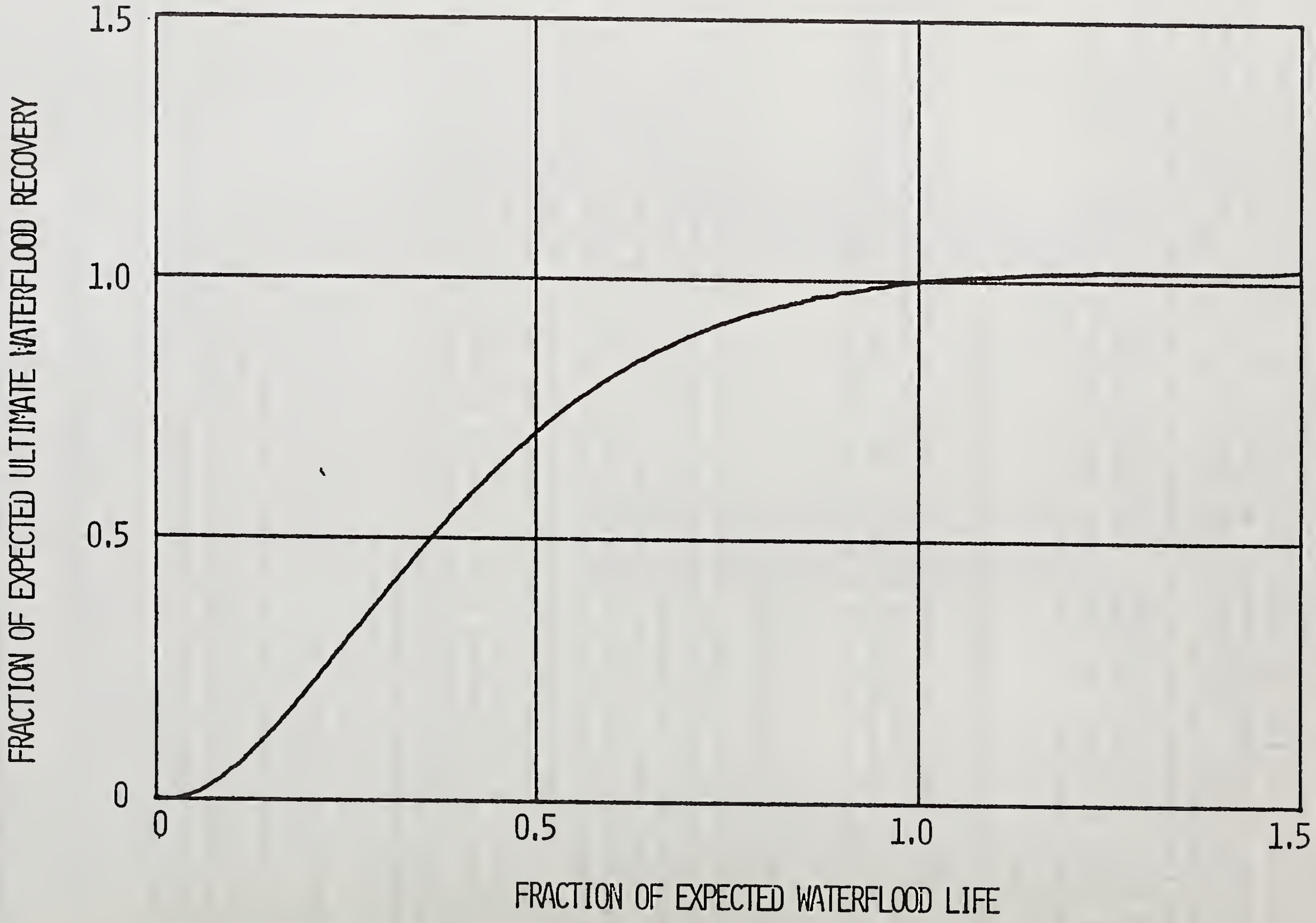

Figure 3. Fraction of Expected Ultimate Waterflood Recovery as a Function of Fraction of Expected Waterflood Life

maintenance program carried out from the initial stage of development. The oil production schedule for these pressure maintenance fields is basically the same as the production schedule for primary fields with the exception of the expected ultimate recovery per well. Wells in a large field would have a period of constant production and then decline to their ELR.

Associated Dissolved Gas Production

Due to the nature of pressure maintenance, it was assumed that there would be a constant gas-oil ratio during the life of these fields. The gas-oil ratio was found by dividing the expected ultimate associated-dissolved gas production per well by the expected ultimate oil recovery per well. The cumulative gas production at the end of each year was then found by multiplying the cumulative oil production by the gas-oil ratio. Annual gas production was found by subtracting successive cumulative gas productions.

Non-Associated Gas Production Schedule Per Well

Non-associated gas production schedules were calculated for the combined BOE classes 1 through 4 and BOE classes 5 through 18 for each of 4 depth brackets. The schedules were based on the average reservoir and gas characteristics of fields in southeast New Mexico and Texas Railroad Commission Districts 7B, 7C, 8, 8A, and part of District 1. The gas production schedules look similar to the oil production schedules, but they are calculated in a different manner. The oil fields made use of empirically determined decline rates while the gas field production schedules were based on equations which relate the average physical parameters of the fields. As with oil fields, the expected recovery per field, and number of wells per field for each size class and depth bracket along with gas in place were estimated.

A gas deliverability schedule was then calculated for each category of field by utilizing a computer program, which made use of the material balance equation with zero water influx and the back-pressure equation. The computer program required as input data the field gas in place, number of wells, gas gravity, absolute open flow rates, back-pressure equation slope, reservoir pressures, and gas production. Most of these data were obtained from a purchased computer tape which contained data obtained from state files that had already been manipulated into a suitable format.

The gas deliverability schedule for the representative well in a field was found by dividing the field gas deliverability by the number of wells in the field. It was assumed that the initial production rate for each of the larger fields would be limited to a daily contract quantity (DCQ) of approximately one million standard cubic feet/day for every 3 billion standard cubic feet of reserves.

The representative well for each category of field would than produce at this initial production rate until the breaking point (the time at which the well could no longer maintain the initial production rate) was reached. In classes 1 through 9, the well production capacity was extremely large for the amount of gas in place. For this reason, the representative wells in these classes were scheduled at a greater initial production rate which was obtained from a curve of BOE class size versus the initial producing rate for the larger classes.

After the annual breaking point was reached, the annual production predicted by the deliverability program was fit by an exponential decline curve. This was done to provide a convenient analytical form for production schedules. Figure 4 shows a production schedule for a well in BOE class 13 in the 0 to 5,000-foot depth bracket. The gas production from a well in this category was assumed to remain constant for about 6 years at a rate of 254 million standard cubic feet per year and then decline exponentially with a decline rate of 0.305 per year until it reached an economic limit rate (the production rate where the operator's income is equal to operating expenses). As with oil field production schedules, the initial production rates and recovery per well increased for larger and deeper fields.

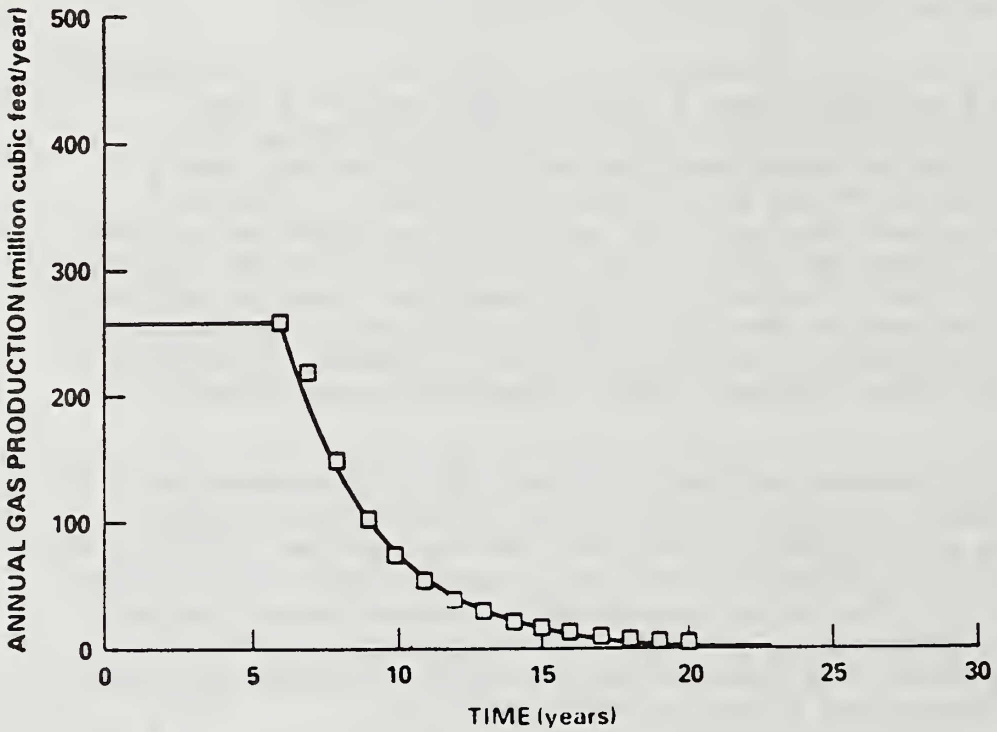

Figure 4. Non-associated Gas Well Production Curve for a Class 13 Field in the 0-5,000 foot Depth Bracket for the Permian Basin

DISCUSSION

Question: Have you looked at any data in terms of the economics of different gathering line pressures?

Dr. Wood: There was an assumed cut-off pressure. The pressure was built into the analysis for gas production curves.

Question: Can you tell me how you derived your equation one?

Dr. Wood: That is an intergral of an exponential production decline equation:

$$Q = (QRO)e^{-Dt},$$

where

$$Q = \text{production rate.}$$

The integral of Q with respect to time yeilds equation one.

Question: Wouldn't you get another exponential?

Dr. Wood: Yes, but you recall that the economic limit rate is, in fact, an exponential equation. You know it is a function which declines with time and what we did there was pick a production rate which was non-economical. You know that is the rate at which the operator income was equal to operating expenses, at an oil price of $14.00 per barrel. And then as the model was run, different prices were put into the model, and a different set of economic limit rates was calculated.

So, again, back to your question, that economic limit rate is in general an exponential function, but it declines each year, and when you talk about the expected recovery, you have to pick a point at which production will terminate. And the point we picked was the point where production will just meet the operating expenses.

Mr. Brashear (Lewin & Associates): John, I appreciate your work and we have used it a number of times, that and your offshore stuff too. You assumed that the gas-oil ratio, in the primary period of production was essentially a depletion type drive. Some of those then go on into a waterflood, and you get a standard gas-oil ratio. That all makes sense to me.

The fields that don't have a secondary flood, though, I am wondering if there is not an inconsistency there for those fields that never are slated to go to secondary, if that shouldn't be a standard gas-oil ratio, as the most likely case.

Dr. Wood: That is something one well might want to consider doing. There are other reasons why a waterflood is not put in. Well, I might just drop it at that. If one had, that those fields which were always eliminated or eliminated primarily because they had a water drive, then one might want to use a constant gas-oil ratio.

Mr. Brashear: I guess that is kind of my question. Do you have any feel for whether they are just too tight, too fractured, whatever?

Dr. Wood: In the deeper categories, they might have started tending more toward water drive fields. In the others, there was a variety of reasons why it wasn't done.

SOME MODERN NOTIONS ON OIL AND GAS RESERVOIR PRODUCTION REGULATION

John Lohrenz and Ellis A. Monash *

ABSTRACT

The historic rhetoric of oil and gas reservoir production regulations has been burdened with misconceptions. One was that most reservoirs are rate insensitive. Another was that a reservoir's decline is primarily a function of reservoir mechanism rather than a choice unconstrained by the laws of physics. Relieved of old notions like these, we introduce some modern notions, the most basic being that production regulation should have the purpose of obtaining the highest value from production per irreversible diminution of thermodynamically available energy. The laws of thermodynamics determine the available energy. What then is value? Value may include contributions other than production per se and purely monetary economic outcomes.

"The Fable of the Jones-Smith Apple Orchard"

Jones and Smith got some land as equal partners for the purposes of developing an apple orchard. However, before starting to develop the apple orchard, they learned they had a difference of opinion as to how to proceed.

Jones said he loved to eat apples, his family loved to eat apples, and all his friends just loved apples. Jones wanted to keep his family, friends, and himself well-supplied with apples. And Jones felt the more apples he got from the apple orchard for his family and friends and for himself, the more successful the apple orchard would be.

Smith said that was hogwash. If he wanted an apple for himself or family or friends, he would buy one. All Smith said he wanted out of the apple orchard was some money to take home after all the apples were sold. And then he could use that money to buy apples or anything else that pleased him. And Smith felt that the more money he got to take home from the apple orchard, the more successful the apple orchard would be.

So Jones and Smith argued and argued until, since this is a fable, they agreed on a precise definition of how to count being successful with their apple orchard. And they proceeded with the development of their apple orchard only to learn they had yet another difference of opinion.

Jones preferred a special kind of little, red apple which was very tangy and tasty. And in that part of their definition how to count being successful that involved bushels of apples, he was going to include only those little, red apples which were tangy and tasty. Smith said to wait just a damn minute! An apple was an apple, Smith averred, regardless of how tasty or tangy it was, whether someone ate it or baked it, or if it was red or yellow. But, an apple was good only if someone bought it. And Smith said that under their definition of how to count being successful with their apple orchard, all other things being equal, they should grow apples which make more bushels.

Now Jones became livid at Smith and his position. Jones recalled that Smith intended to give the unsaleable apples from the orchard to Smith's

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brother who ran a hog farm. Smith's brother would feed these rotten apples to his hogs because, according to Smith, they had no value and would have to be gotten rid of anyhow. Jones said that if those rotten apples have a use, and your brother does wish to use them, then they have value and should be counted as money to take home--and included in our definition of how successful we are--whether you get actual money for them or not.

So Jones and Smith argued and argued some more until, since this is a fable they agreed on the details of what should and should not be included in their definition of how success in the apple orchard should be counted. And they proceeded with the development of their apple orchard only to learn they had yet another difference of opinion.

Jones wanted to purchase an expensive species of apple tree for their orchard because this species was hardy and could withstand the occasional plagues of diseases and insects and very severe frosts. Jones said that way the apple orchard would still be there for his children, grandchildren, great great-grandchildren, etc.

Smith was aghast at that notion. Smith said neither he nor Jones would be around when their great- and greater-grandchildren were about and that it was unreasonable to worry about them now. Maybe they wouldn't even want apples or money from apples. So, Smith said, let them fend for themselves in their time as we must in our time. Smith said they should get some decent, less expensive trees while taking some reasonable chances with the diseases, insects, and frosts the next few years. And let our succeeding generations take care of themselves as it is presumptuous for us to even think we can.

So again Jones and Smith argued and argued until, since this is a fable, they agreed on what species of tree to buy. And they proceeded with the development of their apple orchard only to learn they had yet another difference of opinion.

Smith wanted to plant the apple trees very close together and fertilize them heavily. In this way, they would get a lot more apples and money to take home and, thus, be even more successful according to their previously agreed upon definition of how to count success.

Jones was pained at this notion. He said that it would be exceedingly troublesome to work around those closely planted trees and hauling all that fertilizer. Jones said it just wasn't worth all that trouble they would have to be that little bit more successful.

So again Jones and Smith argued and argued until, since this is a fable, they agreed precisely on the spacing of the trees and amount of fertilization that made the trouble balance out with the success. And they proceeded with the development of their apple orchard.

Now, during the development and life of the apple orchard, Smith and Jones had many other differences of opinion just as these four. But, since this is a fable, Jones and Smith quickly resolved all their differences of opinion with an agreement on the policy to apply to their apple orchard. Therefore, they always agreed on and had defined the quantitative objectives of their apple orchard business and did not deviate from those objectives--as this is a fable.

Thus, Smith, Jones, and the apple orchard lived most amicably and happily--not ever after--but throughout the planning horizon of this fable and because this is a fable.

*****

To put that fable in perspective, given our subject, the production regulation of oil and gas reservoirs, we consider a hypothetical situation and some questions:

> Suppose we discovered a huge oil and gas reservoir in this nation--so huge that it's production could free us of worries about oil and gas shortages for, at least, a generation or longer. What should we do with that reservoir? How should we produce it? How fast? Should we produce it at all?

It is a safe prediction that <u>if</u> that huge oil and gas reservoir did exist today, there would be arguments regarding how to answer the question. Some would be pleased with the existence of such a reservoir; among this group there would be arguments about how fast and exactly how to produce the reservoir. Some among this group might even argue that the best way to "produce" the reservoir is not to produce it at all, to "save" the reservoir for some future purposes. Yet others might be displeased with the existence of the reservoir at all. Perhaps, the reservoir's production would interfere with goals and purposes they believe of greater importance. Just like the bickering of Smith and Jones regarding the development of their apple orchard, there would be bickering regarding the development of this huge hypothetical reservoir. In fact, the only reason the story of the apple orchard is a fable is that Smith and Jones quickly and neatly arbitrated their arguments. The arguments regarding the apple orchard have the exact analogy in the plethora of arguments that would rage regarding the huge oil and gas reservoir. But, in the real world, the rapid and neat resolution of all these arguments yielding a decision for action is not so realistic an expectation.

The reason seems clear. In these arguments, basic cultural value judgments are exposed to harsh lights. To the extent that we disagree in these basic, even moral, judgments, we will likely disagree about what to do with a huge oil and gas reservoir. We might even disagree whether such a reservoir is good or bad. And we will continue to disagree as long as we do not share a common cultural value assessment regarding what, for example, we do and do not owe succeeding generations. Given disagreement over such fundamental judgments, true agreement is impossible and, even, rational bargaining to some concensus position is slow and difficult.

While the huge oil and gas reservoir is hypothetical, the thousands of smaller oil and gas reservoirs of the past, present, and likely future are not. For each of the past reservoirs, somehow the questions regarding whether and how the reservoir should be produced were answered. For each of the current reservoirs the questions are being answered and, no doubt, will be for reservoirs yet to be discovered. The business of oil and gas production regulation is answering these questions for the thousands of real oil and gas reservoirs.

We have already noted that for the non-existent huge oil and gas reservoir, the arguments involved in answering the questions regarding its production would center on fundamental cultural values and, therefore, would not be prone to quick resolution. Yet, these same arguments portend for the thousands of real, smaller reservoirs. The answers, right or wrong, to the questions regarding how to produce these reservoirs have been and are being made. The questions <u>are</u> answered for real oil and gas reservoirs one way or another.

No doubt, the difficulty or, even, impossibility reasonably conjectured about the difficulty of getting the production regulation questions answered for the imaginary huge oil and gas reservoir are also imaginary. If we had a real, huge oil and gas reservoir, as with smaller reservoirs, the production regulation questions would be answered with reasonable dispatch just as with all other reservoirs. As these questions are answered, we do have production regulation whether we want to or not. We cannot avoid having production regulation. Somehow, we answer the questions regarding how oil and gas reservoirs should be produced. Let no one be misled into thinking that answering the production regulation questions can be avoided. One can transfer the responsibility for the answers to some other place, but the answers are and will be made. The answers are and will be made either articulately or not, either with knowledge of choices or not, either with informed judgment or fiat.

The next Section, OIL AND GAS RESERVOIR PRODUCTION: PAST AND PRESENT, briefly reviews the past history. In effect, to address the modern notions of production regulations, we find we must dismiss some of the past and present and return to first principles. This we do in the Section, THE PRODUCTION RATE DECISION IN THE MOST SIMPLE FORM. Here, we consider the problem of what rate to produce an oil and gas reservoir in the most simple form we could contrive and yet show the basic technical issues. Thereby, we unmask some issues in the past history which do not turn out to be meaningful. Real reservoirs need not and, indeed, do not follow the simple form reservoir. The Section, PRODUCTION RATE DECISIONS FOR REAL OIL AND GAS RESERVOIRS, shows how, for real reservoirs, the problem can become much more complex than for the simple form reservoir; nonetheless, the underlying heuristics of the production regulation and rate decisions are exactly the same. Where previously we have adhered to the old notions of production regulations and rates which implicitly assume only recovery and economic outcomes can be involved in the decision, we expand the potential decision algebra in the Section, THE THERMODYNAMIC NOTION OF OIL AND GAS RESERVOIR PRODUCTION. Here, we show that how we define value obtained from oil and gas reservoir production may be defined considering not only production and economic contributions to value, but any other contributions such as strategic and intergenerational values. We conclude that the real purpose of production regulations is to obtain the highest value, however defined, per expenditure of the only irreversible quantity that is expended when producing a reservoir. That expenditure is thermodynamically available energy. The final Section, CONCLUDING REMARKS, summarizes to what extent we attained our purposes.

Throughout, our purposes are to delineate the arguments regarding production control based upon what we "know" about the laws of physics unburdened by past traditions and even myths. We shall not make choices, but we shall point to alternatives as clearly as we can. We hope to make the answers that will be made regarding the production regulations more articulate, more the result of knowledgeable choices and informed judgments.

OIL AND GAS RESERVOIR PRODUCTION REGULATION: PAST AND PRESENT

The history of oil and gas reservoir production regulation is long and so is the literature treating the subject[1*]. Unresolved controversies permeate the literature. There is controversy over the inclusion of economics in the public policy production regulations. Some have said it has not been included.

*Numerical superscripts refer to notes following the main body.

Others agree, but are aghast at the omission. Those who would include economics argue about how it should properly be considered. The objective that production regulation policies should restrict avoidable waste triggered long-standing controversies regarding what constitutes waste. Is waste physical or economic or both? The one thing we do know about that question is that it has been the wellspring of much rhetoric.

The extant Federal laws relating to oil and gas reservoir production regulation are the Energy Policy and Conservation Act of 1975 (Public Law 94-163, Dec. 22, 1975) which invoked a Maximum Efficient Rate (MER) and the Outer Continental Shelf Lands Act of 1978 (Public Law 95-372, Sept. 18, 1978) which defines a Maximum Attainable Rate (MAR). The definition in the Act of the MER is:

> "......the maximum rate of production of crude oil or natural gas, or both, which may be sustained without loss of ultimate recovery of crude oil and natural gas, or both, under sound engineering and economic principles."

The MAR is defined in its Act as:

> "......the maximum rate of production of crude oil and natural gas which may be produced under actual operating conditions without loss of ultimate recovery of crude oil and natural gas."

The definitions of these Acts leave some rather horrendous concepts undefined. Does a sustained maximum rate of production imply a <u>constant</u> rate? How ultimate is the ultimate in ultimate recovery? What are "sound engineering and economic principles?" What are their opposites? What are actual operating conditions? Certainly, there is enough vagueness, enough ambivalency in those definitions so that no equally expert practitioners in reservoir management will necessarily arrive at the same MER or MAR rate (or rates) or even nearly so. That is true even if the practitioners agree on all pertinent reservoir properties and economic parameters. There is no <u>quantitative</u> stipulation of what, exactly, is maximized by operating at the MER or MAR and no indication of what is lost or not. What McFarland (1976) concluded with regard to MER is still true and applies to MAR as well. He wrote:

> --"It appears that MER has actually been utilized to mean different things to different people. The use of MER as a regulatory tool also appears to have been very imprecise with considerable variability in its implementation." (p.19)

Our title presages some <u>modern</u> notions of oil and gas reservoir production regulation. Given the vague and non-quantitative nature of the past and current production regulations, one is, we think, well disposed to forget, at least for the moment, the past and what is current in the "arts" and return to first and basic questions, those being the questions already phrased in the introductory Section. Given one has a reservoir, of all of the ways and rates it could be produced, how should it be produced? How fast? Those are the basic questions of the business of production regulation and we shall endeavor to treat them directly paying only a modest, as seems fitting, obeisance to the past history of production regulation in this Section.

THE PRODUCTION RATE DECISION IN THE MOST SIMPLE FORM

Given an oil and gas reservoir, how does the decision with regard to what rates shall be produced proceed? Subject to any enforced production regulations how does the reservoir operator decide at what rates to produce? We consider, here, a most simple form of this production rate decision problem.

Suppose we have a known producible reserve of Q^{∞} that will be produced at an exponentially declining rate. Thus, the rate, q, at any time, t, is given by Eq. (1):

$$q=q_i e^{-Dt} \tag{1}$$

where q_i is the rate at t=0 and D is the rate of decline in reciprocal time. Q^{∞} is the integral of q from t=0 to ∞:

$$Q^{\infty} = \int_0^{\infty} q\,dt = \int_0^{\infty} q_i e^{-Dt} dt = \frac{q_i}{D} \tag{2}$$

Note the meaning of Eq. (2) which is depicted on Figure 1. If we produce to infinite time, we will obtain the entire producible reserve, Q^{∞}. The only choice we have with regard to producing this reservoir is the rate of decline, D. Figure 1 shows three possible choices for D or, more specifically, 1/D of 10, 20, and 50 years. Consistent with Eq. (2), the areas under all three curves of q versus t extended to infinite time are equal. But, if q_i is higher, then D must also be higher and 1/D lower, i.e., by setting q_i, we set the rate of decline, D.

Assume the development costs we incur to start production, V_D, are directly proportional to the production rate at time t=0 such that:

$$V_D = C_D\, q_i\, p\, X_D \tag{3}$$

Here, p is the unit selling price of the producible reserve, say in dollars per barrel, C_D is a constant of proportionality, and X_D is an adjustment factor for payments and credits resulting from direct development costs. These may be due to taxes and leasing contracts which, for example, involve profit sharing. If there are no taxes or other payments or credits affecting direct development costs, then X_D=1. Substituting Eq. (2) into Eq. (3):

$$V_D = C_D Q^{\infty}\, pDX_D \tag{4}$$

Further, in this most simple problem, assume that all development costs are incurred as a lump sum when t=0 at which time production starts. (This is a simplifying assumption, but not a limiting one for any costs incurred over a period of time can be lumped to an effective cost incurred at a specific time.)

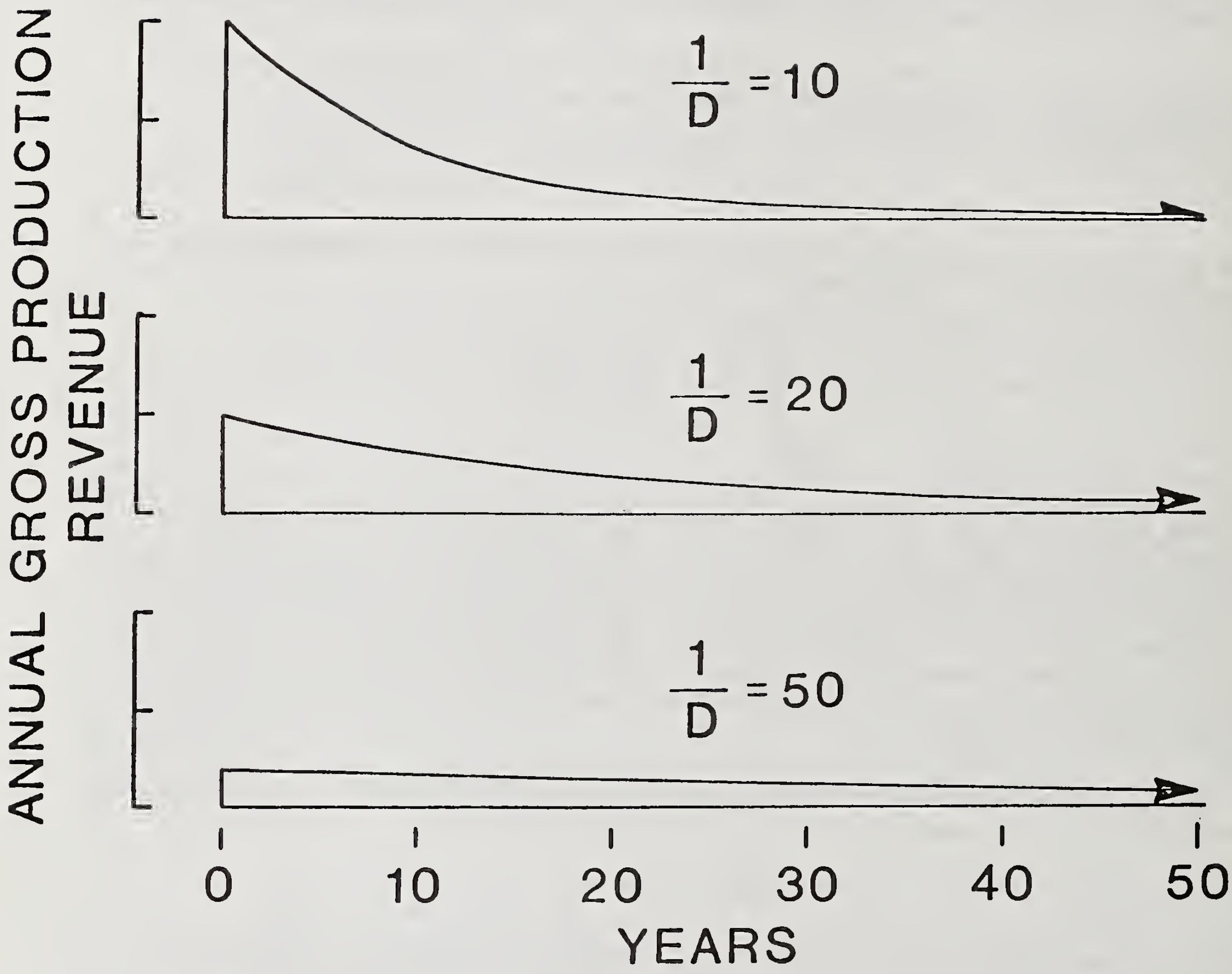

Figure 1

Three Exponentially Declining Production Rate Profiles with the Same Producible Reserves, Q^{∞}

Having considered the development costs, we now consider the net revenue being produced per unit time, v_R, which is:

$$v_R = [(1 - r - c_0)\, qp - C_0\,]\, X_R \tag{5}$$

Here, r is the fractional rate at which royalty is assessed on the gross value of production, c_0 is a constant of proportionality relating operating costs that depend on the amount of value of production, C_0 represents the fixed operating costs that do not depend on the amount of production, but would cease only if there were no production, and X_R (analagously to X_D) adjusts for payments and credits that result from the direct net revenue. Substituting Eqs. (1) and (2) into Eq. (5) yields:

$$v_R = [(1 - r - c_o)\, Q^{\infty}\, p\, e^{-Dt} - C_0\,]\, X_R \tag{6}$$

The aggregated potential net revenue from the production of Q^{∞} of producible reserves in the period from t=0 through t=∞ is the integral of v_R over the same time period. However, let us assume an operator averse to losing money out of pocket, i.e., operating with a negative v_R, and not constrained to continue production beyond that point. At that point the so called "economic limit" where the marginal rate of return becomes negative occurs. At that time, Δ_e:

$$(1 - r - c_0)Q^{\infty}Dpe^{-D\Delta_e} = C_0 \tag{7}$$

Solving Eq. (7) for Δ_e, yields:

$$\Delta_e = -\frac{1}{D}\,\ell n \left(\frac{1}{D} \cdot \frac{C_0}{(1 - r - c_0)Q^{\infty}p}\right) \tag{8}$$

Figure 2 graphs Δ_e versus 1/D with $[C_0/(1-r-c_0)Q^{\infty}p]$ as a parameter. Note that when there are no fixed operating costs (C_0=0), $\Delta_e = \infty$. Figure 2 also shows that Δ_e increases with 1/D until some maximum is reached after which Δ_e decreases with 1/D until some point where 1/D is equal to the reciprocal of the parameter, $[C_0/(1-r-c_0)Q^{\infty}p]$, and Δ_e=0 again, where the project would just break even considering operating costs, but not "pay back" any development costs.

Of course, if $\Delta_e < \infty$, then the actual reserves produced, $Q < Q^{\infty}$. Substituting Eq. (2) into Eq. (1) and integrating from 0 to Δ_e as defined by Eq. (8), one finds:

$$\frac{Q}{Q^{\infty}} = 1 - \left(\frac{1}{D}\right) \frac{C_0}{(1-r-c_0)Q^{\infty}\, p} \tag{9}$$

Eq. (9) defines the fraction of actual reserves produced of the reserves that would be produced if production were maintained to t=∞, Q/Q^{∞}, as a function of the rate of decline, D, and the parameter, $[C_0/(1-r-c_0)Q^{\infty}p]$. Figure 3 graphs Eq. (9) showing Q/Q^{∞} is a straight line function of 1/D with $[C_0/(1-r-c_0)Q^{\infty}p]$ as a parameter. Note that $Q/Q^{\infty} = 1$ only when there are no fixed operating costs,

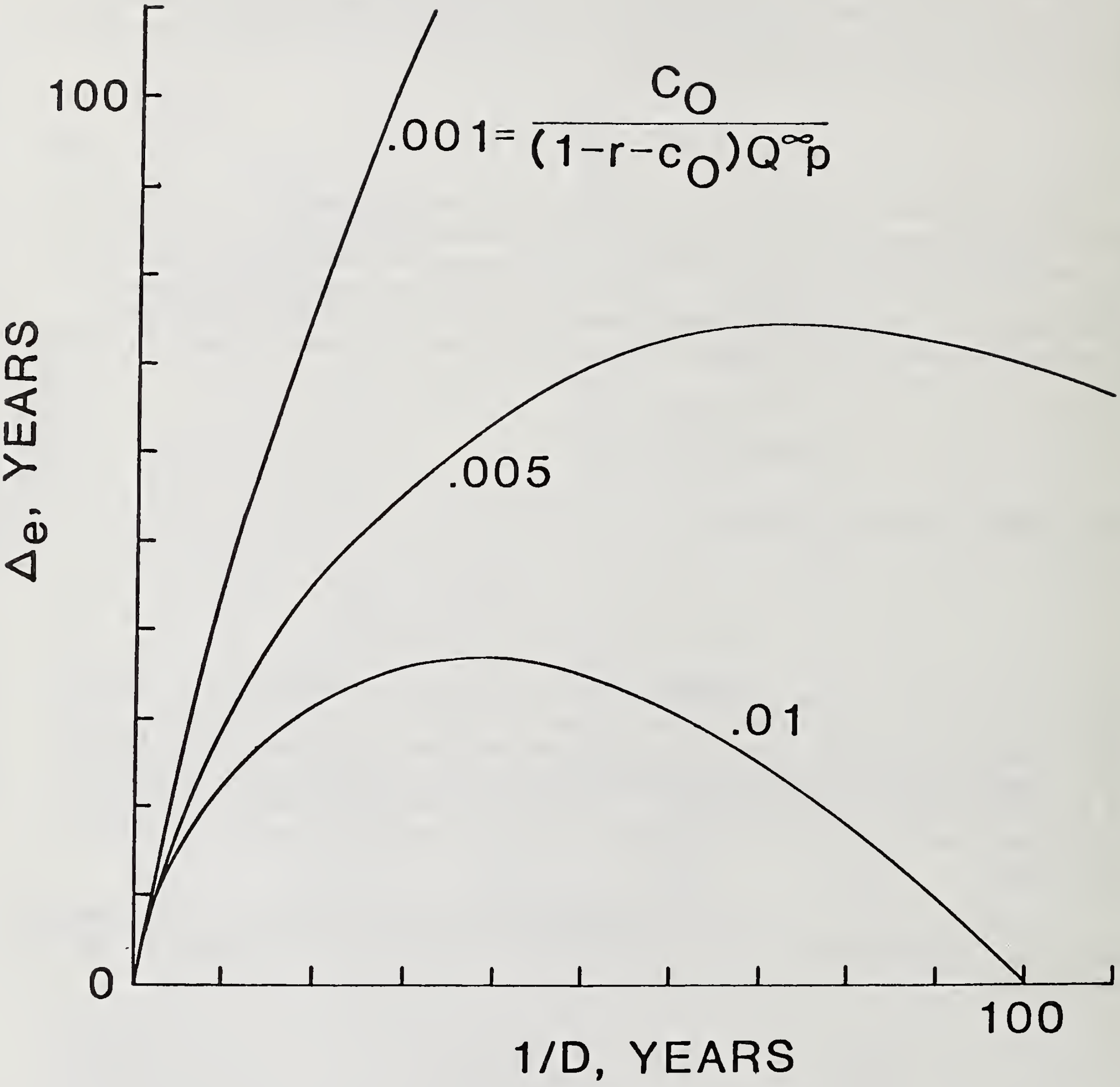

Figure 2

The Time After Production Starts When the Economic Limit Occurs as a Function of the Rate of Exponential Decline with the Fixed Operating Cost Parameter

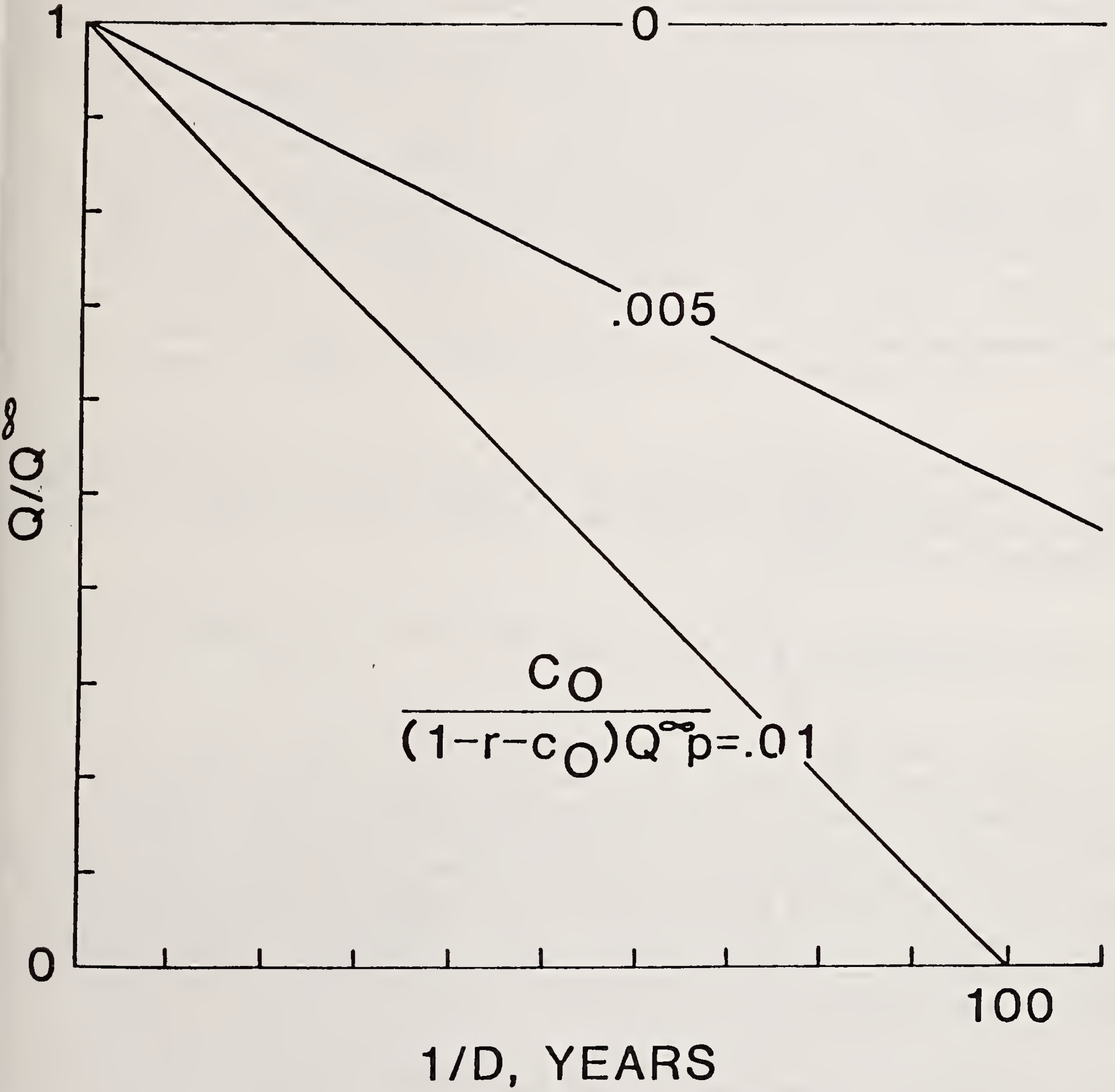

Figure 3

The Ratio of the Reserves Recovered at Economic Limit to Infinite Time as a Function of the Rate of Exponential Decline with the Fixed Operating Cost Parameter i.e., $C_0=0$. Otherwise, the difference between the amount of reserves which could have been produced until $t=\infty$ and would be produced until $t=\Delta_e$ increases as the parameter, $[C_0/(1-r-c_0)Q^\infty p]$ increases and as $1/D$ increases. The higher the initial rate of production, q_i, the higher that decline rate, D (according to Eq. (2)), but the greater the proportion of reserves, Q^∞, that will be produced before $t=\Delta_e$. As a specific example, if the parameter, $[C_0/(1-r-c_0)\, Q^\infty p]$ $= 0.01$ and the decline rate, $D=0.10$, then according to Eq. (9) and Figure 3, $Q/Q^\infty = 0.9$. Only 10 percent of the reserves that could have been produced until $t=\infty$ were lost by ceasing production where the economic limit is reached when $t=\Delta_e$ with $\Delta_e = 23$ years from Eq. (8) and Figure 3. Where the "loss" of 10 percent of the possible production may be acceptable, consider what happens with a slower decline rate, say $D=0.02$ and the same parameter, $[C_0/(1-r-c_0)Q^\infty p]$. Here, $\Delta_e = 35$ years; the time to reach economic limit has increased. However, $Q/Q^\infty = 0.5$; the amount of production that would actually be obtained from the same source, Q^∞, if produced until $t=\Delta_e$, has decreased. In other words, in the latter case with the lower rate of decline, production would be maintained over a longer period of time, but, overall, less production would be obtained over the longer production period compared to the former case with a higher rate of decline. And, where a 10 percent "loss" might be acceptable, a 50 percent "loss" is certainly less so. All this impinges on a technical point, frequently misunderstood, about the relationship between rates of production and ultimate recoveries from oil and gas reservoirs which we shall return to again.

The total potential present value of future profit, V^d, of producing the reserves Q of the producible reserves that could be produced until $t=\infty$, Q^∞, is the difference between the development cost, V_D, (Eq. (4)) and the net revenues, v_R, integrated from $t=0$ to Δ_e:

$$V^d = - C_D\, Q^\infty p\, DX_D + \int_0^{\Delta_e} [(1 - r - c_0)\, Q^\infty D p e^{-Dt} - C_0] X_R e^{-it} dt. \quad (10)$$

Note in Eq. (10), the term e^{-it} has been added in the integral of v_R. Here, i is the investor's discount factor, i.e., that fractional rate at which the investor discounts a present dollar compared to a future dollar. When $i=0$, the investor has the luxury of not caring <u>when</u> a dollar is spent or received while, of course, still preferring to receive rather than spend. Since development costs are, in this most simple problem, all deemed to occur when $t=0$ where $e^{-it}=1$, discounting of development costs is moot. The total potential profit, V^d, includes the superscript, d, to indicate Eq. (10) yields a value discounted at the rate, i, such that when $i=0$, $V^d=V$ where V is the undiscounted total potential profit.

Substituting Eq. (8) into Eq. (10) and integrating yields:

$$V^d = - C_D\, Q^\infty p\, DX_D + \frac{(1 - r - c_0)\, Q^\infty p\, DX_R\, (1-e^{-(D+i)\Delta_e})}{D + i} - \frac{C_0\, X_R\, (1-e^{-i\Delta_e})}{i} \quad (11)$$

Figure 4 shows characteristic curves described by Eq. (11) of V^d versus $1/D$ with the discount factor, i, as a parameter. Note the locus of the maxima of V^d with respect to i. Maxima exist for positive V^d in the range from i=0 to some $i=i_{max}$ at which the maximum in the V^d versus $1/D$ curve occurs just where $V^d=0$; i_{max} is generally called the internal rate of return.

An investor who can and who does select the initial rate of production, q_i, and, therefore, D, in such a way as to maximize V^d is a present value of future profit maximizer. In other words, such an investor would, either explicitly or implicitly according to an unseen hand posited by Adam Smith, look at a curve like that on Figure 4 for the economic parameters and discount rate deemed appropriate and choose that decline rate which maximizes V^d. If the investor's discount rate is greater than i_{max}, this investor will forgo the project. This investor's present value of future profit maximizing choices can be found by differentiating Eq. (11) to find (dV^d/dD), setting the derivative equal to zero, yielding a relation defining this investor's optimum selection of D which we will call D^o:

$$\frac{dV^d}{dD} = 0 = \frac{-C_D X_D}{(1-r-c_0)X_R}(D^o+i)^2 + i$$

$$-\left\{(D^o+i)\,\ell n\left[\left(\frac{1}{D^o}\right)\left(\frac{C_0}{(1-r-c_0)\,Q^\infty p}\right)\right] + i\right\}\left[\left(\frac{1}{D^o}\right)\left(\frac{C_0}{(1-r-c_0)Q^\infty p}\right)\right]^{\frac{D^o+i}{D^o}} \qquad (12)$$

Note that Eq. (12) really describes a function, ϕ, in 4 parameters as follows:

$$0 = \phi\left(D^o,\ i,\ \frac{C_D X_D}{(1-r-c_o)X_R},\ \frac{C_0}{(1-r-c_0)\,Q^\infty p}\right) \qquad (13)$$

Eq. (13) simply states that D^o is completely defined given the discount factor i and the dimensionless parameters, $[C_D X_D/(1-r-c_0)X_R]$ and $[C_0/(1-r-c_0)Q^\infty p]$. Consider these dimensionless parameters. The first is a ratio of the development costs with adjustments for taxes and other affected payments and credits required to get an annual dollar of revenue net of royalty and production-proportional operating costs similarly adjusted[2]. The second is the ratio of the annual fixed operating costs that are not production-dependent and would cease if production stopped to the revenue net of royalty and production-proportional operating costs that would be obtained if Q^∞ were produced. We call $[C_D X_D/(1-r-c_0)X_R]$ the development-revenue ratio and $[C_0/(1-r-c_0)Q^\infty p]$ the fixed operating cost-revenue ratio. The analogy of these dimensionless numbers to those used in engineering like the Reynolds number for the ratio of inertial to viscous forces if fluid flow is entirely apt. These are dimensionless numbers ratioing economic forces. For notational brevity, we will set $[C_D X_D/(1-r-c_0)X_R] = N_{dr}$ and $[C_0/(1-r-c_0)Q^\infty p] = N_{or}$.

Past and current values based on prevailing policies of N_{dr} have been estimated to be around 2[3]. Figure 5 presents solutions for D^o versus i with $N_{dr}=2$ and N_{or} at parametric values of 0, 0.01, and 0.02. Figure 5 shows that a present value of future profit maximizer with a discount factor in

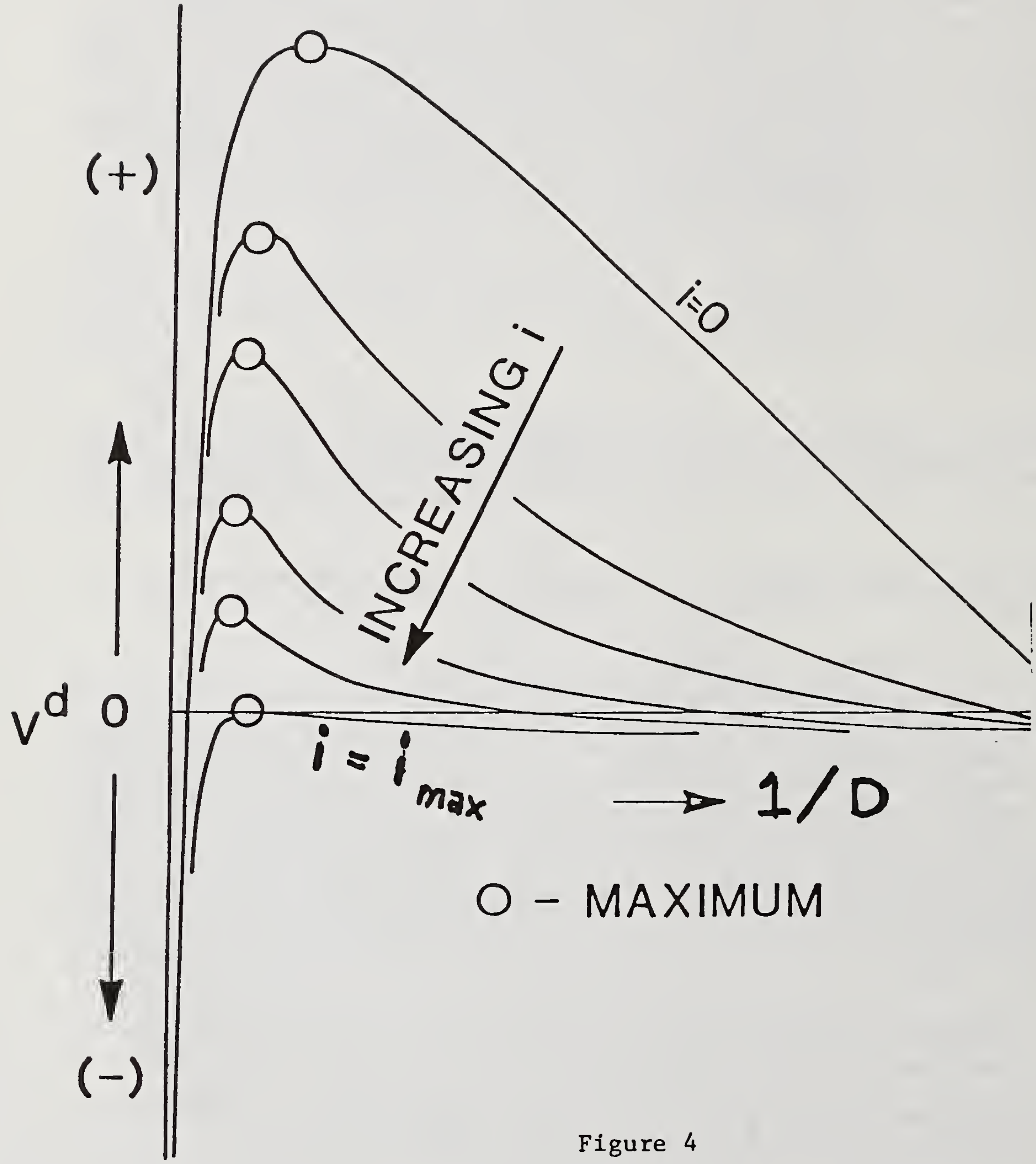

Figure 4

Characteristic Curves of the Present Value of Future Profits Versus the Reciprocal Rate of Decline with the Discount Factor as a Parameter

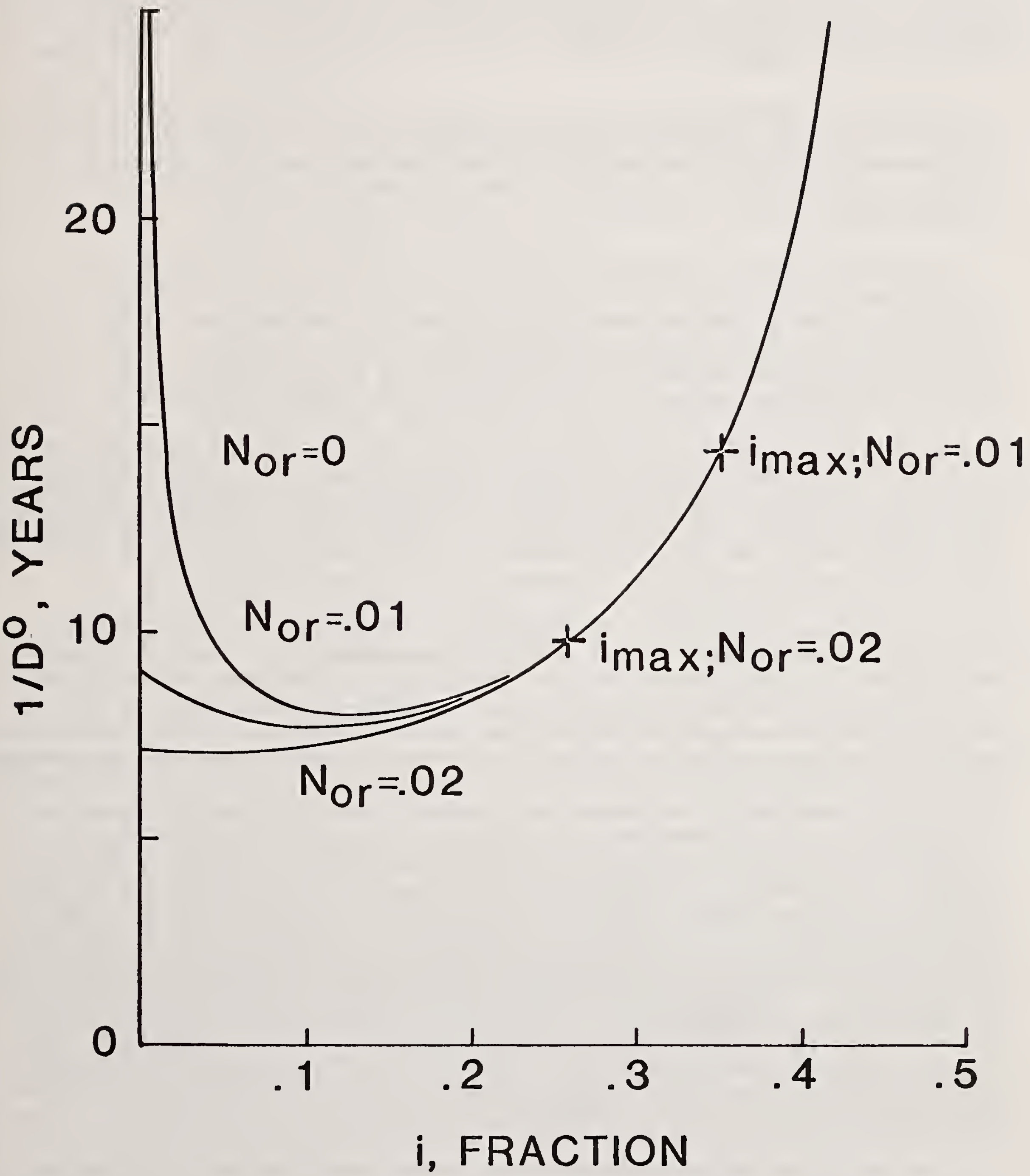

Figure 5

A Present Value of Profit Maximizing Operator's Selections of the Decline Rate as a Function of the Discount Factor with the Fixed Operating Cost Parameter the range between about 0.10 and 0.25 would select a value of $1/D^o$ from about 7 to 10 varying only slightly with N_{or}. This is equivalent to a D^o, the selected decline rate, of .10 to .14. Actual decline rates on aggregated Federal offshore oil and gas production have been calculated as 0.13 for liquid and 0.09 for gas (Lohrenz, Dunham, and Tomlinson, 1979) in good agreement. Certainly, the agreement is within the range of uncertainties about the exact economic parameters and the simplifications introduced by this most simple model.

Does this agreement mean that the actual production regulation policy--however stated and implemented--was to produce at those rates which maximized the present value of future profits? There can be no definitive answer; however, the results using this most simple problem are consistent with such a conjecture.

What the results from analyzing this most simple problem show is the speciousness of two notions sometimes adduced in the matter of production regulation. The first specious notion is that the decline rate of an oil and gas reservoir is determined by the mechanisms of the reservoir. Obviously, as we have just shown, this is <u>not</u> so. How fast the reservoir declines, how fast it produces are selections, realized or not, made by the operator within any prior production regulations enforced. The laws of physics do not constrain the operator's decision. The laws of physics would allow any selection of rates up to those approaching infinity. The particular mechanism of the reservoir affects the economics of the particular reservoir or, in the format of this most simple problem, the values of C_D, c_0, and C_0. But, these do not determine the decline rate, but merely influence its magnitude given some optimum objective such as the maximization of the present value of future profits.

A second specious notion is implicit in the assertion that there are reservoirs, some say many, which are rate insensitive, i.e., the ultimate recoveries from these reservoirs are invariant with the rates at which they are produced. For these reservoirs, the assertion continues, production regulation is moot since one will obtain the same ultimate recovery regardless of what rate that production is obtained. Analysis of the most simple problem and, particularly, Eq. (9) and Figure 3 show what is wrong with that assertion. Here we are dealing with a source that is truly rate insensitive when produced to infinite time and thermodynamic exhaustion. The most simple kinds of reservoirs, those involving only an expansion in-place mechanism sometimes called a solution-gas or depletion drive, actually are rate insensitive when produced to thermodynamic exhaustion. But, reservoirs which are rate insensitive when produced to thermodynamic exhaustion are <u>not</u> rate insensitive when production ceases at some time prior to thermodynamic exhaustion, prior to infinite time. <u>All reservoirs produced to some state less than thermodynamic exhaustion are rate sensitive</u>. Of course, the production of a reservoir to thermodynamic exhaustion is a hypothetical notion; real reservoirs are always produced only to some prior state. Therefore, <u>all real reservoirs are rate sensitive</u>. Thus, the realm of production regulation is not the least limited by being constrained only to reservoirs which are rate-sensitive.

We see, then, that production regulation would be an issue even for any reservoir which actually followed every assumption inherent in the most simple form problem with which we have just dealt. What happens when the assumptions are challenged by real reservoirs? This is the question considered in the next Section in which we show that real reservoirs and their development decisions are essentially variations, albeit with added complexities, of the theme provided by this most simple form problem.

PRODUCTION RATE DECISIONS FOR REAL OIL AND GAS RESERVOIRS

Let us enumerate the differences between the oil and gas reservoir development-production scenario treated in the most simple form of the production rate decisions problem of the previous Section and real reservoirs.

First, real reservoirs are rate sensitive. The ultimate recovery of real reservoirs when produced to thermodynamic exhaustion, as well as to some prior limit such as an economic limit, may depend on the rates at which the production is obtained. From a reservoir viewpoint, there are two mechanisms[4] which make a reservoir rate-sensitive:

(1) Influx of water from an adjacent aquifer with entrapment of hydrocarbons behind the invading water. The effect is that, _ceteris paribus_, ultimate recovery is increased by producing faster. Faster production prevents the invading water from occluding more hydrocarbons.

(2) A time-dependent segregation between the liquid and gas in the reservoir whereby gas evolved migrates to a gas cap. The effect is that, _ceteris paribus_, ultimate recovery is increased by producing slower. The slower production gives more time for the gas to migrate into the gas cap.

When both effects are present, water influx with ultimate recovery increased by faster withdrawals of hydrocarbons and time-dependent segregation with the opposite effect on ultimate recovery, then a physical maximum of ultimate recovery obtained at thermodynamic exhaustion occurs with respect to rates.

The two mechanisms cited are overall reservoir mechanisms prevailing in reservoirs which are homogeneous with respect to pressure as well as heterogeneous. The potential mechanisms which affect ultimate recovery because of pressure heterogeneities in reservoirs are profuse and preponderantly occupy the lore of reservoir engineering. These heterogeneities occur because reservoir withdrawals must be made, of course, through wells, i.e., discrete apertures in the reservoir rock matrix. The fluid flow patterns through the rock, which has heterogeneous properties itself, to the apertures can only be maintained with a pressure difference thereby invoking the rule, no producing reservoir is pressure homogeneous. In effect, by assuming a reservoir is pressure homogeneous, the assumption is that all wells are "perfectly" drilled, i.e., drilled and operated in such a way as not to disturb the assumption that the reservoir can be considered homogeneous with respect to pressure. The point is that wells and the pressure heterogeneities they cause, while adding complexity to the mechanisms in a real reservoir, simply perturb the overall reservoir mechanisms. These complicating perturbations can be modeled by the more complex models commonly available and usually are. Of course, to be cost-effective, one should justify that the cost of a more complex form of a model is really justified by the reservoir data at hand. The production rate decision can proceed either by assuming pressure homogeneity, and that all wells can be drilled "perfectly" or considering pressure heterogeneity which is, after all, only a more complicated variant of the pressure homogeneous assumption.

Either way, for each detailed development scheme, ultimate recovery to thermodynamic exhaustion versus rate parameter functions exist. The rate parameters may take various forms such as initial or peak production rates, well density, or, for enhanced recovery schemes, amounts or proportions of fluids injected. These functions can be developed given the appropriate physical reservoir data and model. Similar functions can be developed for ultimate recovery to cutoffs prior to thermodynamic exhaustion; these ultimate recoveries will necessarily be less than or, as a limit, equal to the recoveries to thermodynamic exhaustion.

What are the forms of these functions? The literature of production regulation is full of hypothesized forms almost entirely without quantitative analysis in support. Figure 6 shows some that have been given. All these forms of the function of the ultimate recovery, presumed to be Q rather than Q^{∞}, versus the rate or, more properly, a rate parameter have been advanced as a basis upon which to assess production regulations. Of these, only the one given by Carlson (1975) has some quantitative analysis in support[5].

What are ultimate recovery versus rate functions really like? Figures 7 through 10 show some[6]. Here we consider the case of an oil and gas reservoir which is drilled up in order to attain some initial rate of production, q_i. After production starts, no further drilling is done. Production declines proportional to the reservoir pressure. If production is allowed to continue until the reservoir pressure in the reservoir is zero (or all of the hydrocarbons in the reservoir not occluded by invading water disappear), the ultimate recovery is Q^{∞} depicted by the top curve on Figures 7 through 10. Below the curve for Q^{∞} versus q_i on each of Figures 7 through 10, lie the curves of Q versus q_i parametric in the cutoff production rate. The cutoff rate is that q_i for which Q=0. Figures 7 through 10 show Q^{∞} and Q through two orders of magnitude of q_i on a logarithmic abscissa.

Figure 7 depicts a reservoir solely under a hydrocarbon fluid expansion-type drive with no water drive and no time-dependent segregation. Q^{∞} is constant for all q_i; yet Q varies with q_i. Figure 7 graphically reinforces what we have concluded earlier--that reservoirs which are insensitive to rate with respect to Q^{∞} are rate sensitive with respect to Q.

A reservoir with a water drive with entrapment behind invading water is shown on Figure 8. Note that faster production uniformly increases Q^{∞}, the faster the more so. Faster production "gets" reserves before they are entrapped behind invading water.

The effect of a reservoir with a time-dependent segregation effect superimposed on the expansion-type drive that all reservoirs have is shown on Figure 9. Here, opposite to the water drive reservoir of Figure 8, Q^{∞} uniformly increases as q_i decreases. Slower production increases Q^{∞}; the mechanism being that slower production allows more time for the beneficial effects of the segregation to occur.

Figure 9 shows maxima occurring with respect to Q versus q_i. These maxima occur at some point where the ultimate recovery, Q, is optimum between high rates of production that allow little time for the segregation effect to act and low initial rates of production that leave little time for production before the cutoff rate is reached. Actually, maxima in curves of Q versus q_i can also

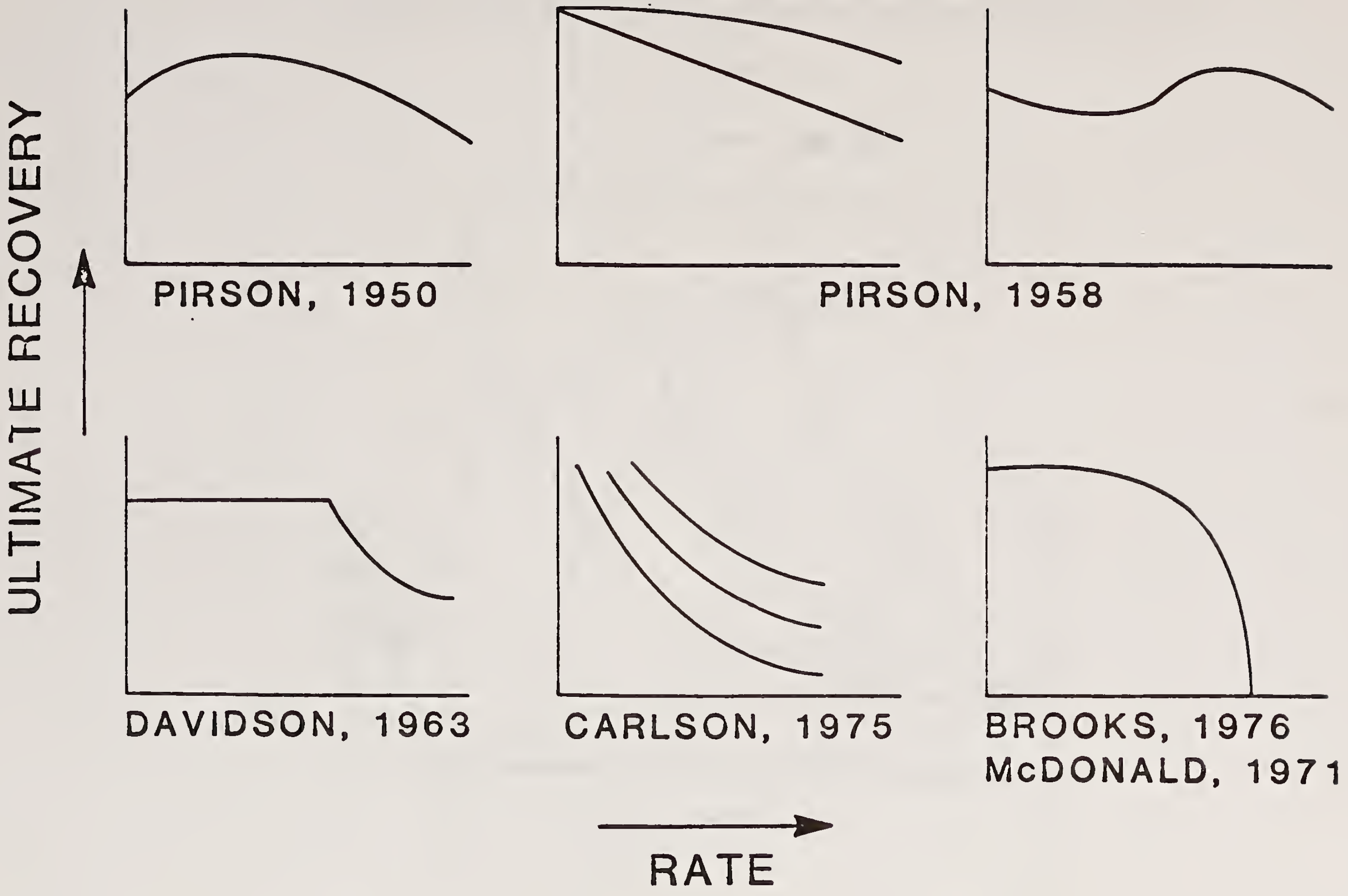

Figure 6 Some Ultimate Recovery Versus Rate Function That Have Been Hypothesized

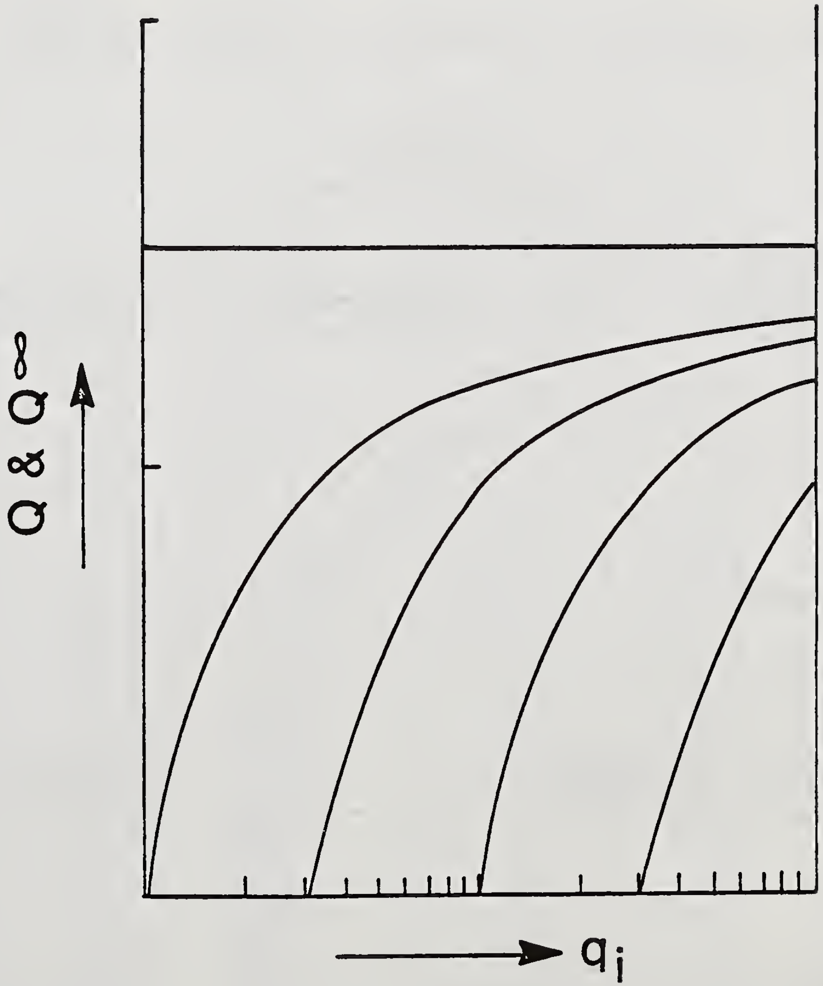

Figure 7

Ultimate Recoveries as a Function of Initial Production Rate
for a Reservoir with Only an Expansion-type Drive

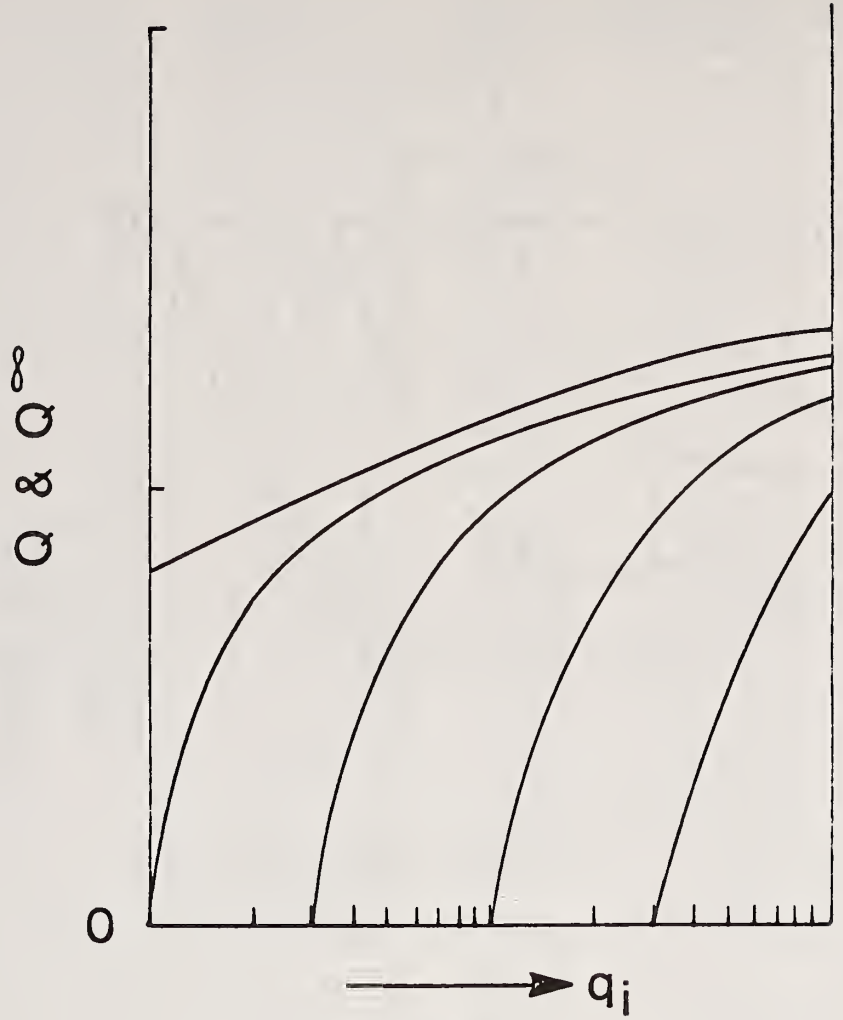

Figure 8

Ultimate Reoveries as a Function of Initial Production Rate for a Reservoir with a Water Drive (with Entrapment of Hydrocarbons Behind Invading Water)

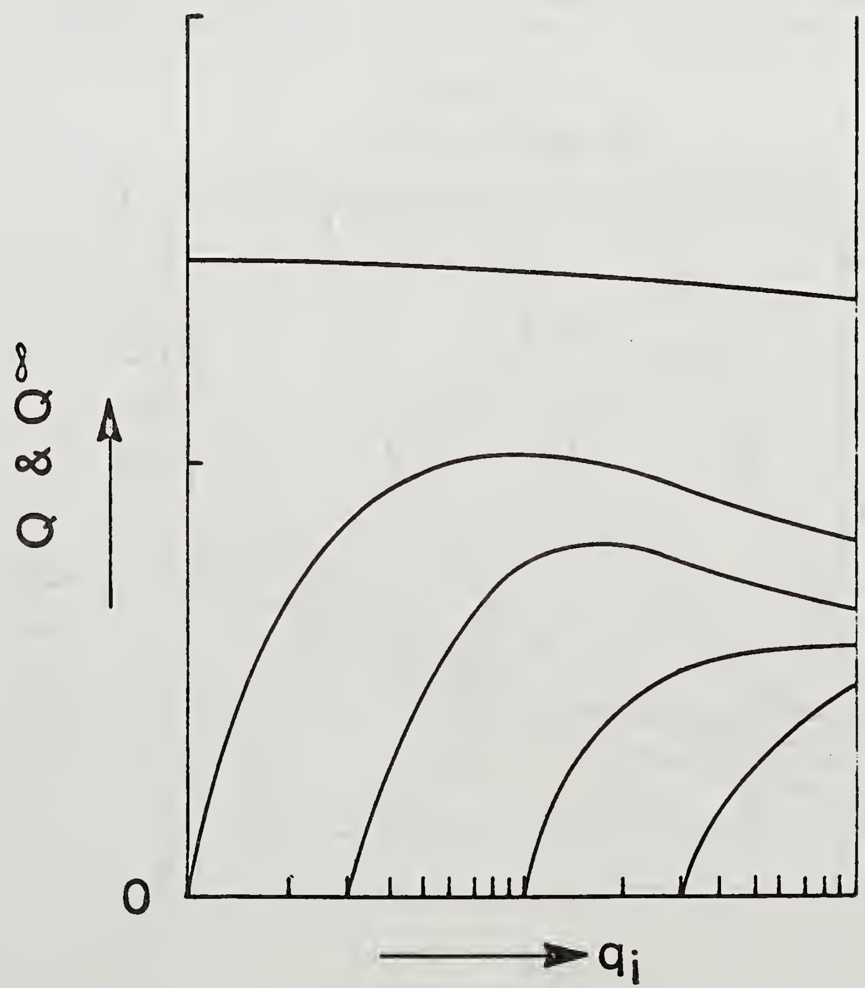

Figure 9

Ultimate Recoveries as a Function of the Initial Rate of Production for a Reservoir with a Time-dependent Segregation Drive occur for reservoirs with a water drive depicted on Figure 8. Depending on the range of q_i considered, maxima in Q versus q_i curves may or may not occur for reservoirs with a water drive and/or a time-dependent segregation mechanism.

Figure 10 shows a reservoir with both a water drive and time-dependent segregation superimposed upon the expansion-type drive. Here, the curve of Q^{∞} versus q_i exhibits a maximum as do the curves for Q versus q_i.

Returning our attention to Figure 6 , there appears little value in indulging in the exercise of judging the hypothesized curves compared to those that occur. The truth is that each reservoir has not only a curve, but a host of curves of Q versus q_i and all other rate parameters involved in the development plan. Any error of the hypothesized curves of Figure 6 is not so much in the curves, but that they were hypothesized without quantitative analysis of any reservoir.

A second group of differences between the simple form production rate decision problem of the previous Section and the "real world" are the mathematical assumptions imbedded in its formulation.

The simple form assumes that drilling occurs after which production immediately initiates. This is an assumption of mathematical convenience which likely does not approximate most actual cases. If there is an anticipated delay between when development costs are incurred and production proceeds, that will effect the economic outcomes and the operator's decision regarding rates of production[7].

No economies of scale are considered; the simple form assumes development costs are exactly proportional to the initial production rate. Some reduction in the unit development costs to obtain a unit of production rate as the rate increases would be expected[8].

The simple form mathematics may be considered as including inflation either by considering all costs and revenues adjusted to a constant-dollar adjusted for inflation or using a discount factor which is the same as the inflation rate and the inflation-free discount factor. Either way, the simple form assumption still is that unit costs and prices inflate equally. The simple form would have to be modified to include unequal inflation rates between costs and prices.

As long as the simple form inflation assumption holds, a given oil and gas reservoir either is an attractive investment at a given discount factor, i.e., has a V^d greater than 0 at the investor's i, or is an unattractive investment and will always be so unless there are technological, policy, or unit selling price changing affecting N_{dr}, or N_{or} making the formerly unattractive investment attractive. This raises the issue of speculations affecting the operator's decision regarding what rates to produce at[9] or whether to delay the start of production.

We have already mentioned the simple form assumption of a lump sum development cost. The simple form also considers no limitations on the external supplies in the development scenario. Sufficient drilling rigs, platforms, and other equipment are presumed available without physical constraint, at the cost

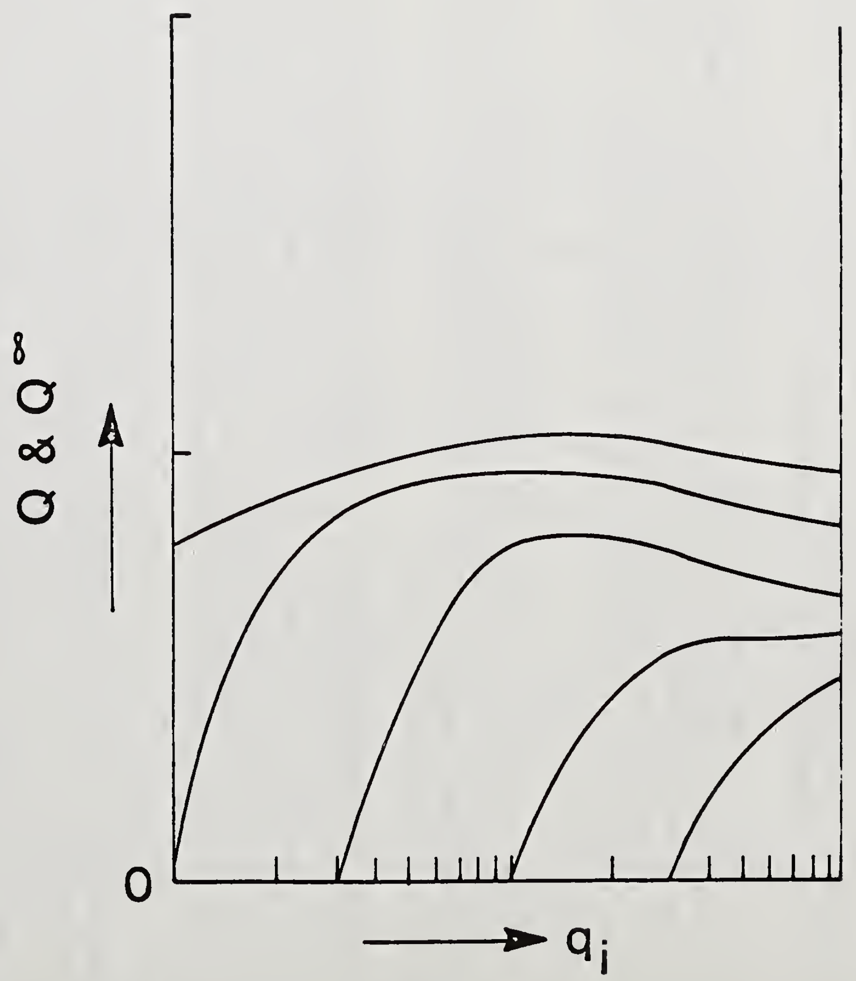

Figure 10

Ultimate Recoveries as a Function of the Initial Production Rate for a Reservoir with Both a Water Drive (with Entrapment of Hydrocarbons Behind Invading Water) and a Time-dependent Segregation Drive imputed, to implement the desired development plan. (One would expect any such constraint to be a temporary one.) The simple form considers no enhanced recovery schemes.

All or some of these assumptions and limitations of the simple form which, make it simple can, however, be removed. The "real world" can be more closely approximated by perturbing the simple form without changing what is required to arrive at the quantitative production rate decision by the operator or assessing the quantitative effects of any production regulations on those decisions[10].

A third and, perhaps, most burdening assumption inherent in the simple form development-production scenario is that no development drilling subsequent to the initial development is considered. The simple form boldly presumes an initial production "surge" followed by an exponential decline. An exponential decline is merely a mathematical form of a decline that has been empirically successful--possibly because practitioners have and use semi-logarithmic graph paper--in tracking declining production rates when they occur. There is no law of reservoirs requiring declines be exponential; other forms occur (such as the hyperbolic) and are used. Further, there is no reason why a decline must occur at all; additional drilling can, not only, arrest a decline, but yield a production rate increase. Admittedly, it is a severe restriction of the simple form to take the exponential decline following initial production as the only production rate profile available to the operator; this is clearly not so.

Yet, removing this assumption is another perturbation, albeit adding complexities, to the simple form. Bradley (1967) considered the case where individual wells decline exponentially[11]. McFarland (1979b) has implemented a linear programming model considering a gas reservoir with and without a natural water drive where as many wells are drilled to start and continue production as indicated by the operator's economic criterion. A similar model considering an oil and gas reservoir which may have a water drive and/or time-dependent segregation is under development (McFarland, 1980).

In summary, the simple form of the previous Section, where Q^{∞} was independent of q_i, yielded relationships between V^d, Q, and q_i, the only rate parameter. If the simple form were the "real world", the operator should select q_i to optimize whatever he or she chose, maximization of V^d being a reasonable presumption. The business of production regulation made as a matter of public policy would be to guide, influence, or, even, set those operator choices in conformance with the public policy, whatever it is. And that is still true in the more complicated case where the simple form does not occur. Here, Q is dependent upon q_i and all rate parameters involved in the development plan. One needs to consider, implicity at least, the function of Q with all these rate parameters. Further, one needs to consider this function for each and every technically feasible development plan, all types of form of enhanced recovery, for example. No doubt, the problem is not of the magnitude that may, at first reading, imply because reservoir engineering practitioners, given a reservoir and prevailing economics and policies, quickly hone in on those development plans which are technically feasible <u>and</u> also amenable economically and policy-wise.

THE THERMODYNAMIC NOTION OF OIL AND GAS RESERVOIR PRODUCTION REGULATION

In the production rate and regulation analyses of the previous Sections, we considered the trade-offs between ultimate recovery, Q, the rates of recovery, q, and the economic outcome measured as the present value of future profit, V^d. In so doing, without stating it, we have made an implicit assumption of rather startling magnitude. We have assumed that the only values that can possibly be obtained (and, of course, that can be wasted) are physical values, i.e., the oil and gas themselves, that could be produced from a reservoir and/or economic values that could be obtained from production of oil and gas from a reservoir. Once that assumption is exposed, it is then obvious why one has the arguments about physical versus economic waste in the rhetoric of production regulation. We need only to recall, once again, the rational arguments between Jones and Smith about their apple orchard to entertain the notion that values other than just the production of oil and gas themselves and profit in so doing are involved. Even if one does not agree, one could not dismiss those who believe other values are involved as irrational.

The thermodynamic notion of production regulation allows a completely general specification of the values one obtains from an oil and gas reservoir. The thermodynamic approach to assessing energy processes and policies can be broadly applied[12], but, here, we shall discuss the application to oil and gas reservoirs.

The immutable laws of thermodynamics which have no basis in mathematical logic[13] are the foundation of the thermodynamic notion. The laws of thermodynamics are "proved" because no one has been able to "by-pass" these laws. The laws are not positive laws; they do not state what can be done. The laws are negative laws; they state what can <u>never</u> be done. Compared to the laws of thermodynamics, other physical and economic laws are flimsy, i.e., these other laws can be overturned without affecting the laws of thermodynamics. For example, one could develop a society with a culture that abhors what is rare and prizes what is available in profusion. That society would certainly differ from our current, developed societies. That society would have revised economic laws of supply and demand. But, that society would still be constrained to the same laws of thermodynamics.

If we proceed from the laws of thermodynamics to the questions of how oil and gas reservoirs should be produced and regulated, we start with the recognition that there is only one real kind of waste that can occur. We can incur thermodynamic waste. In the course of incurring thermodynamic waste, we can incur wastes that others may call physical waste and/or economic waste (and, likely, will argue about which). But, these are secondary names for effects seen when thermodynamic waste occurs. One cannot have physical, economic, or any other kind of waste without the occurrence of thermodynamic waste.

What, then, is thermodynamic waste? Thermodynamic waste occurs when thermodynamically available energy is spent and value that could have been obtained was not obtained. Note, the laws of thermodynamics do not settle any argument between Jones and Smith or between operators and production regulators of oil and gas reservoirs regarding what value is, what comprises value. A mathematical statement of thermodynamic waste follows from the definition of the ratio, Φ, as follows:

$$\Phi = \frac{\int_{t_1}^{t_2} r_v \, dt}{[-w]_{t_1}^{t_2}} \qquad (14)$$

Here, t_1 and t_2 are the times, t, at the extremes of a planning horizon and r_v is the rate at which values, however defined, are produced in the planning horizon. The thermodynamically available energy that is expended to obtain those values is -w. (We continue thermodynamic sign conventions where a system that can do work on the surroundings has a negative available energy.) The denominator of Eq. (14) is the difference between the thermodynamically available energy of the reservoir at time, t_1, and at time t_2. We know from the second laws of physics that <u>any</u> withdrawal from the reservoir will occur only at the expense of a diminution of the thermodynamically available energy in the reservoir[14]. We know this because of the second law of thermodynamics (and failure to be able to build the perpetual motion machine we could build if there were no diminution). An oil and gas reservoir is, first of all, a source of thermodynamically available energy and irreversibly so. Once we have spent any or all of that available energy, we cannot get it back. We could, at best, only reconstruct the past conditions in the reservoir by spending more available energy than what we are replacing. The fundamental thing we "spend" when producing a reservoir, however it is done, is the available energy in the reservoir. It is this expenditure which is our choice to use well or waste[15].

The thermodynamically available energy is a state function, i.e., its magnitude depends only upon the contents of the reservoir. That is why the denominator of Eq. (14) is not an integral. On the other hand, r_v in Eq. (14) depends on the path of the production process and the numerator must be an integral.

Note that the numerator of Eq. (14) is the arena of the arguments about what value is produced by oil and gas reservoirs. Whoever is winning the arguments decides what r_v is. When the former winner loses, the new winner decides what r_v is and, most likely, will want to change the decision regarding what r_v is. This can be done. But, what neither can do is change the way the thermodynamic accounting is done in the denominator. Old winners and new winners (indeed, all of us) feed at a thermodynamic trough and the most fundamental measure of efficiency is whether we get value enough for what we irreversibly take from that trough. The best we can do is get the highest Φ, however r_v is defined, in Eq. (14). We avoid thermodynamic waste by maximizing Φ; thermodynamic waste occurs when Φ is not maximized.

The theoretical "gadget" conceived which would regulate the rate of production according to Eq. (14) would be a Φ-maximizing machine. Here, the production regulations policy maker would program the quantitative definitions and constraints by and within which r_v may be defined and these locked in. Then, the operator would program his or her objective function for which maximization is sought (if there are any options left). The Φ-maximizing machine then simply does what it's name implies and produces the reservoir so that Φ is maximized between times, t_1 and t_2. During this period, the production regulation policy maker can be satisfied that the reservoir is being produced to obtain the highest values per expenditure of thermodynamically available energy according to definitions and constraints deemed appropriate. During this same period, the operator can be satisfied that, subject to the constraints of the production regulation policies--regardless of whether the operator agrees with them or not, the reservoir is being produced in such a way to optimize what he or she has decided to optimize. Of course, when a time, t_2, occurs, another production regulations policy maker can program new quantitative definitions and constraints, but the Φ-maximizing machine adjusts[16] and, again, the policy maker and the operator can be assured that, given their definitions of value, the reservoir is producing such that the greatest value is being obtained per unit diminution of thermodynamically available energy.

The Φ-maximizing machine will, in the current planning horizon, remain a theoretical device, we project. We do not expect one to be build in some erstwhile inventor's garage or full-equipped modern laboratory. For one thing, the machine would have to be able to decide when and how to drill and continually operate wells in oil and gas reservoirs[17]. Machines, even the most highly automated, just cannot do that. The reason is that, even when we have a good information "fix" on the extent and nature of a reservoir, we truly know little compared to the vagaries of Mother Nature when the reservoir was constructed. Those who would like a Φ-maximizing machine built out of hardware will be disappointed and those who do not like the notion need not worry.

As a concept, the Φ-maximizing machine leads us to an interesting recognition. Arguments over how and how fast to produce a reservoir should and, eventually, will revolve around assertions like, "I get more value per diminution of available energy than you do", where value is defined by the production regulation and operator policies.

To make the notion of production regulation according to Eq. (14) and Φ-maximization more understandable, we consider an oil and gas reservoir as a buried tank with valves given choices with regard to whether we want oil or gas brought to the surface[18]. Figure 11 depicts the situation schematically. We will proceed under the constraint that the buried tank cannot be exhumed[19]. To make our illustration more meaningful, let us consider a specific buried tank which has a volume of 2000 cm^3 made up half of an ideal gas and half of an incompressible liquid. We shall consider the gas and the incompressible liquid mutually insolvent. Let the temperature of the buried tank be 300° Kelvin and let us consider there is so much thermal inertia that any process involving the tank proceeds isothermally. We shall take the original pressure inside the tank as 2 atm.; the pressure at the surface of 1 atm., so that when the pressure in the tank is 1 atm., the tank is thermodynamically exhausted. One can see the basic similarities between this buried tank and oil and gas reservoirs. One has choice of producing either gas or liquid from the buried tank of Figure 11; one does not have that either-or choice for reservoir, but one does have a constrained choice between producing more or less gas compared to liquid.

How shall we produce from this buried tank? That's the key question. Shall we produce liquid? Or, shall we produce gas? If we prize the liquid, we can produce all of the liquid after which the pressure in the tank will fall to 1 atm. and we will be able to produce none of the gas. If we prize only the gas[20], we could produce the gas only with no liquid until the pressure falls to 1 atm. Thereby, we will have produced half of the gas, but will be unable to produce any of the liquid. What should we do? The answer, of course, depends upon which we value, gas or liquid. Regardless of which of the ways we produce the tank, we expend the tank's available energy. We decide to open either the

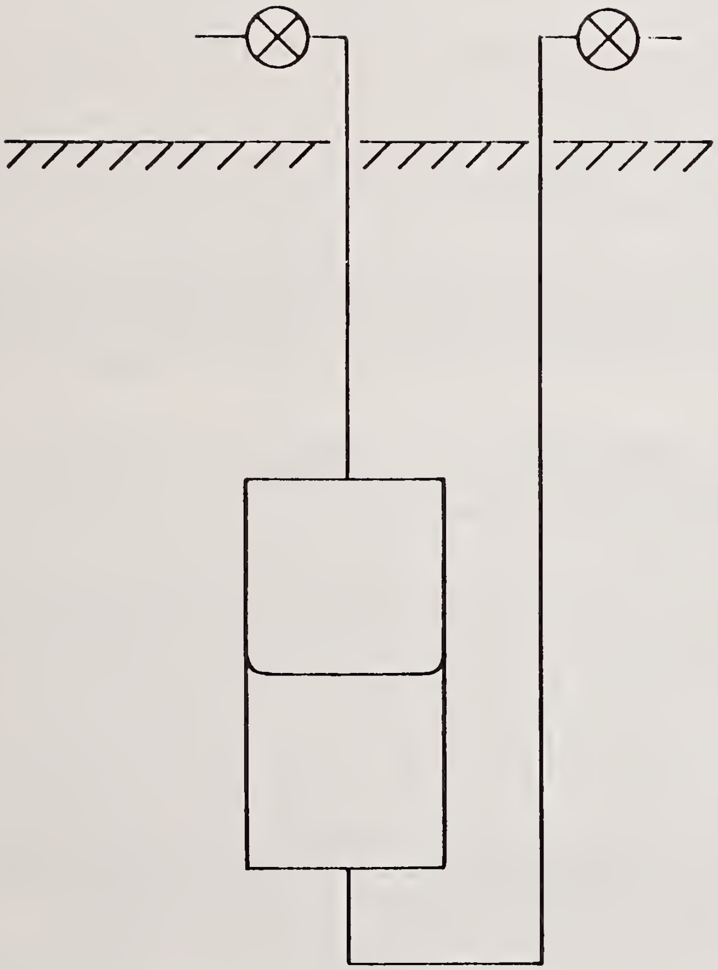

Figure 11

A Schematic Depiction of the Buried Tank liquid or gas valve in order to make that expenditure to maximize the value, however defined, we get for that expenditure. Eq. (14) operates and Φ is maximized even in this simple case.

Suppose now, we define t_1 and t_2 as the times between which one who values liquid only can withdraw exactly half of the liquid from the buried tank. In so doing, the pressure in the tank will decrease to 1.33 atm. Now, let the policy change and let some one who values only gas at the controls of the apparatus on Figure 11. Gas will be withdrawn until the tank pressure is 1 atm.; 25 percent of the gas will be withdrawn. The liquid valuator will get half of the liquid; the gas valuator will get one quarter of the gas.

But, now, let us change the order and give the gas valuator the first change at the controls to withdraw half of the gas. This will lower the pressure in the tank to 1 atm., and, when the liquid valuator gets a chance, he or she will get skunked! Here, the gas valuator will get half of the liquid; the liquid valuator gets nothing.

How can we preserve more equity in a situation like this? How can we prevent one set of production regulation and operator policies from closing out options to subsequent policies? Or, more precisely, how can we keep a score, an accounting of the effects of any production regulation and operator policies, whatever they are? The answer is, of course, to keep track of the expenditure of the thermodynamically available energy, -w.

Suppose now, we measure how much has been withdrawn from the buried tank by measuring -w. Let us assume that only the gas in the tank contributed available energy, -w, because without gas we could not even get any of the incompressible liquid[21]. The amount of gas originally in the buried tank is:

$$\frac{(2 \text{ atm.})\ (1000 \text{ cm}^3)}{\left(82.05 \dfrac{\text{atm - cm}^3}{\text{gm - mole - }^\circ\text{K}}\right)(300^\circ\text{K})} = 0.08125 \text{ gm - moles}$$

where 82.05 atm-cm^3/gm-mole-$^\circ$K. is the universal ideal gas constant, R. If we were to expand that gas or any ideal gas through a fractionless turbine under reversible conditions, i.e., maximum production of work by the turbine, the energy available would be:

$$-w = n_G RT \ln \frac{P}{P_S} \tag{15}$$

for isothermal operation. Here, n_G is the amount of gas in the tank, T is the absolute temperature, P is the pressure in the tank, and P_S is the pressure at the surface. The available energy in the buried tank originally is:

$$- w = (0.08125 \text{ gm-moles})\ \left(82.05 \frac{\text{atm-cm}^3}{\text{gm-mole-}^\circ\text{K}}\right)\ (300\ ^\circ\text{K})\ \ell n \left(\frac{2 \text{ atm.}}{1 \text{ atm.}}\right)$$

$$= 1386 \text{ atm - cm}^3$$

Now, let us see what happens if we let the gas and liquid valuators at the controls of the buried tank of Figure 11 in sequence, but who ever gets first chance at the controls can only withdraw half of the available energy, -w. The other valuator takes over the controls when -w=693 atm-cm^3.

If the liquid valuator is at the controls first, he or she will withdraw liquid until -w=693 atm-cm^3 withdrawing 41.42 percent of the liquid and reaching a pressure of 1.414 atm[22]. Now, the gas valuator can thermodynamically exhaust the tank by withdrawing gas until $P=P_S$ = 1 atm. The gas remaining in the tank after the gas valuator makes the withdrawal he or she can is:

$$\frac{(1 \text{ atm.})\ (1414.2 \text{ cm}^3)}{\left(82.05 \frac{\text{atm - cm}^3}{\text{gm - mole - }^{o}\text{K}}\right)(300\ ^{o}\text{K})} = 0.05745 \text{ gm-moles}$$

The gas valuator will get 0.02380 gm-moles (0.08125-0.05745) or 29.29 percent of the gas originally in the tank. Note that the gas valuator did somewhat better when given the second chance at the controls here than with second chance after the liquid valuator took half the liquid.

Suppose the gas valuator got first chance at the controls and could withdraw gas, but only until -w=693 atm-cm^3. The gas valuator would withdraw 0.01787 gm-moles of gas reaching a pressure of 1.560 atm[23]. Now, the liquid valuator can withdraw liquid until $P=P_S$=1 atm. Thereby, the liquid valuator will withdraw 56.01 percent of the liquid originally in the tank[24]. Here, unlike the case where the gas valuator took half the gas when given first chance at the controls and canceled the liquid valuator's opportunity for any production, the liquid valuator was guaranteed production and, indeed, could get more than 50 percent.

The reservoir's available energy, -w, is the fundamental thermodynamic measure of what a reservoir has originally or has left after production. The buried tank problem merely illustrates, in a grossly simplified way, the sense and equity of accounting reservoir production using -w.

Now, consider what decisions with respect to Eq. (14) are <u>hidden</u> in our treatment of the simple form reservoir production rate problem of an earlier Section. Effectively, we set $t_1=0$ and $t_2=\infty$ clearly implying that production regulation and operator policies once implemented are forever frozen admitting no subsequent changes in operating plans not previously anticipated. Further, the only possible contributors to r_v we considered were the ultimate recovery, Q or Q^∞, the rates of production, q, and the economic outcome, V^d. Even further, when we considered operators' maximization of V^d, applied to Eq. (14), we "decided" the policy was that the integrated numerator of Eq. (14) was equal to V^d. In that case, the Φ-maximizing solution to Eq. (14) is trivial, of course. We have defined a reservoir and an r_v which makes it so.

Solutions which maximize Φ in Eq. (14) are also trivial if we define $r_v=q$, even if we allow the constraint that production ceases when v_R, as defined by Eq. (5) goes to zero. Then the integrated numerator of Eq. (14) simply becomes either Q^∞ or Q. The trivial solution to Eq. (14) yields the edict: You will maximize Φ in Eq. (14) by producing as much stuff as possible before exhaustion of the reservoir.

Solutions to Eq. (14) are not generally trivial, however. There are two generic reasons. One reason is the consideration of other contributions to the values obtained from production from a reservoir, r_v. The other reason is that real reservoirs depart from the assumptions of the simple form reservoir as already noted in the previous Section and in other ways as well.

As an example of the first reason, suppose we wish to consider the policy of counting earlier production of greater value because of current pressing exigencies presuming upcoming solutions from other sources. Then one might define $r_v = qe^{It}$ where I is a positive value expressing how much more we would prefer a unit of production per se today rather than a year from now. (If I is negative, I indicates how much more we prefer a unit today than a year from now.) If Eq. (1) applies, then $r_v = q_i e^{(I-D)t}$ which basically defines decision algebra of a speculator who may be an operator or policy-maker. If (I-D)>0, then the speculator believes the future values, which may include non-economic contributions, are increasing so fast they over-ride any decline rate of production. Therefore, the speculator concludes not to produce, but hold the potential of producing. This Φ-maximizing solution implies that there is a definite contribution to r_v by having a reservoir capable of production and not producing from it. The contribution might be deemed appropriate for reasons of security and/or strategy. Or, the contribution to values of a reservoir conserved might be what economists call an intergenerational value--a gift we hand to succeeding generations. Such gifts require an overt policy decision. Certainly, the only things we spontaneously conserve for the future are things we do not know are there and things which are presently deemed grossly uneconomic. Eq. (14) and its use provides the quantitative policy flexibility of considering values from oil and gas reservoir production other than those having only to do with the production from the reservoir and/or profits made from that production.

A second generic reason why Eq. (14) Φ-maximization solutions are not trivial are departures of reservoir behavior from the assumptions of the simple form of Eq. (1). In the previous Section, two departures from an overall reservoir performance covered were:

- --Reservoirs having a natural gas water influx which entraps oil and/or gas behind the invading water such that entrapped hydrocarbons are no longer recoverable[25].
- --reservoirs having a time-dependent segregation with respect to the overall movement of reservoir liquid and gas relative to each other.

Note that optimization solutions of Eq. (14) using definitions of r_v that would be trivial for simple form reservoirs are not trivial for reservoirs with a water influx and/or time--dependent segregation. Likewise, solutions are not trivial for reservoirs with enhanced recovery schemes. And in cases where the reservoir mechanisms must consider local effects due to wells and their operations and other substantive reservoir homogeneities, then, most certainly, the Φ-maximization solution of Eq. (14) is non-trivial and becomes more complex. Yet, throughout, the objective function, Φ, is the same.

The thermodynamic approach to production regulation and rate decisions embodied in Eq. (14) recognizes that every oil and gas reservoir exists only once and irreversibly so. Once produced, any oil and gas reservoir is gone forever; no recipe exists for reconstructing that oil and gas reservoir. As far as the reservoir in situ is concerned, it is a source of available energy and the "score" of what we do with any reservoir is really kept in whether we husband or how well we expend the available energy of a reservoir.

How would the thermodynamic approach be adapted to production regulation? We have already invented an imaginary Φ-maximization machine, but concluded it will remain imaginary. But it need remain imaginary only as a piece of hardware; the machine can function in a dialogue between production regulation policy makers, operators, and the "people-ware" between. We have pointed to the advantage of policy flexibility and equity of the Φ-maximization machine to produce regulation and rate decisions. Another advantage, it appears to us, is the administrative simplicity with which the Φ-maximization non-hardware machine could be run. Basically, the policy makers' job would be to define quantitative rules, ways, and constraints for defining r_v. Presuming the policy makers do not completely define r_v, the operator can then make his or her own detailed specification within what the production regulation policies allow. The operator's obligation to the production regulations is fulfilled with a statement like the following:

> You have told me a way or rules that I must use to define r_v, the value of production from my reservoir (which I may or may not agree with). Where you did not already specify everything, I completed a definition of r_v within your specifications. But after all that, <u>I can now certify that the reservoir is being produced properly because it is being produced so that we are getting the greatest value the way you (and maybe I) defined it per unit of available energy being expended</u>.

Once the production regulation policy definition and constraints affecting r_v are made, then the <u>only</u> challenges to an operator's reservoir operations and rates can be (1) that the rules, definitions, or constraints are not being adhered to or (2) that there is disagreement with the operator's technical assessments and analyses regarding the reservoir. Here, of course, the arena for potential argument has been much delimited including only whether the rules are being followed and the technical area. The argument about what are true values that should be obtained from a reservoir have been resolved after, at least possibly, considering the full gamut of potential contributions to those values. Perhaps the arguments regarding value will not have been correctly resolved in the view of subsequent history, but, then, even historians are flawed. But, just as Jones and Smith disagreed while proceeding with the apple orchard, one can do whatever it is we do with oil and gas reservoirs in the same way.

CONCLUDING REMARKS

The title promised some modern notions about oil and gas reservoir production regulations. We note that no promise of <u>the</u> answer to how oil and gas reservoirs should be regulated was implied. Rather, we hoped to make the process of deciding production regulations more articulate with the issues better delineated.

We have delineated the issue of reservoir sensitivity and insensitivity to rates of production. We hope we have put to rest the myth that there are any reservoirs insensitive to rate such that production regulation is not an issue.

We have pointed out that the rate at which an oil and gas reservoir declines in production is most directly a choice that can be made by the operator and only indirectly influenced by the inter-relationship between economics and reservoir mechanisms. One can, as a matter of policy, choose to impose production regulations requiring certain decline rates or a certain recovery reserves to production rate ratio, but then the choice is made by the production regulations policy instead of the operator. It is still a choice made, not something defined by the laws of physics and reservoirs.

We have shown that concerns involving specific kinds of wastes such as physical or economic and the arguments that ensue about these kinds are really semantic chatterings about secondary effects. There is only one waste. That waste is thermodynamic. Waste occurs when we do not get the value we could have for what we expend thermodynamically.

Finally, and most importantly, we have pointed out that the definition of what we call value is a public policy perogative and necessity. And the definition of value is not restricted, necessarily, to considering only contributions of oil and gas per se from reservoir production or profit or only these two. Rather, the components of value used to define production regulations may include any of the entire array of what our society should value. As consumers, we are often counseled to get the most value for our money. As a society, we would seek the most value for the thermodynamically available energy we expend.

All these value decisions need to be made quantitatively to define production regulations. Jones and Smith did it so they could run their apple orchard. Whether we realize it or not, we are doing the same thing to run oil and gas reservoirs.

ACKNOWLEDGMENTS

The support of the Conservation Division of the U.S. Geological Survey and its management for this work is gratefully acknowledged. This is not, however, an official expression of the Survey, but a technical expression for which the authors are responsible. Of the many colleagues with the Survey who helped us, we want particularly to acknowledge Dr. R.G. Wayland, former Chief of the Division, who recognized the continuing importance of the technical issues of oil and gas production regulations early.

We want to express a special gratitude to the many colleagues with whom we discussed these matters. Many had or were employed by organizations who had definite policy views regarding oil and gas production regulations. We asked them to set aside their policy views and address only the technical issues with us which they did.

NOTES

1. We cite only some of the more recent treatments of the background of oil and gas reservoir production regulations as follows: Bishop, 1979; McDonald, 1971, 1979a, 1979b; McFarland, 1976, 1979; Schanz, 1976.

2. Lohrenz, Burzlaff, and Dougherty (1980) discuss the technological, economic, and policy matters that effect the parameter, $[C_D X_D/(1-r-c_0)X_R]$. Note that when $X_R=X_D$, the choice of D^o is not affected; however, V^d is proportionately affected.

3. Lohrnez, Burzlaff, and Dougherty (1980) present results for D^o for different values of N_{dr}.

4. These two mechanisms have been quantified for oil and gas reservoirs using a reservoir per se model by Lohrenz and Monash (1979). The special case of a gas (only) reservoir has also been treated. For a gas reservoir, segregation is moot and only the water influx mechanism is important from an overall reservoir point of view. Gas reservoir ultimate recovery is, *ceteris paribus*, always increased by faster production. Once production has started, one might as well produce the gas lest the water invade and occlude the gas. This special case has been treated extensively by Monash and Lohrenz (1979) who used a gas reservoir per se model to match data for real gas reservoirs including storage reservoirs for which the fluctuating pressure history puts models in severe jeopardy. (One side effect of these results is a challenge to the frequently used, more complicated reservoir models. Overkill in reservoir modeling, the substitution of complex mechanisms without examination to see if the data available actually justify the addition of a complexity, appears more prevalent than one might have supposed. The arguments about modeling seem many times to be decided upon the issue, "My model is more sophisticated and handles more mechanisms than yours", rather than, "My model efficiently handles the data that Mother Nature and her flawed helpers make available." This should not be so, but is is.) The case of waterflooding a reservoir has also been treated (Johnson, Monash, and Waterman, 1979). It turns out that, *ceteris paribus*, to increase ultimate recovery, one should initiate waterflooding after prior depletion of a gas reservoir to as low a pressure as possible. This result is entirely consistent with the natural water influx result.

5. The quantitative support is that due to Thachuk (1974) who studied reservoir cases using a sophisticated grid model. The curves were parametric with the density of the wells drilled.

6. The curves of Figures 7 through 10 were taken from Lohrenz and Monash (1979). Only the curves for oil are shown on Figures 7 through 10; curves for gas are depicted in the reference cited.

7. Lohrenz, Burzlaff, and Dougherty (1980) show that an anticipated delay causes a present value of future profits maximizing operator to decrease production rates.

8. Lohrenz, Burzlaff, and Dougherty (1980) treat economies of scale using the power scale-up law.

9. Under the constraint that the operator *must* start production, Lohrenz, Burzlaff, and Dougherty (1980) treat the case of the present value

of future profits maximizing operator's decision speculating on a price jump some years after production starts. Payoff matrices (2x2) are given showing the present values obtained by an operator making the production rate decision ignoring and considering the price jump and, then, if the price jump actually occurred and if it did not. The result is a break-even frequency such that the operator should believe the probability of the possible price jump is greater than the break-even frequency in order to base the production rate decision on the presumed price jump.

10. We cite three examples. McFarland, Springer, Monash, and Lohrenz (1978) treated gas reservoirs with and without a natural water drive drilled up then declining with the reservoir pressure and examined the relationship between Q, V^d, and i_{max}. Johnson, McFarland, Monash, and Lohrenz (1978), considering waterflooding of gas reservoirs, found that with current practices, economics, and policies, no region where $V^d>0$, a persuasive explanation why one finds waterflooded natural gas reservoir absent in the real world. McFarland, Parks, and Aggarwal (1979) have implemented a model treating an oil and gas reservoir which may have a natural water drive and/or time-dependent segregation examining the relationships between Q, V^d, and i_{max}.

11. Actually, the thrust of the work by Bradley (1967) was to track the required selling prices of the aggregation of reservoirs being developed in a geological region.

12. Georgescu-Roegen (1971, 1976, 1979) has written most articulately and ardently on this subject. Others we select to cite are Keenan, Gyftopoulos, and Hatsopoulos (1974), Ross (1978), Rotty and Van Artsdalen (1978) and an interesting Ph.D. thesis by Hertzmark (1978).

13. It is interesting to note that laws based on mathematical logic are vulnerable to fallibilities Gödel's Theorem finds cannot be avoided. The laws of thermodynamics are free of the burden of Gödel's Theorem.

14. This is true just as well for reservoirs with any kind of enhanced recovery where the reservoir system must include the reservoir and the enhanced recovery facilities. It is true that the thermodynamically available energy in the reservoir per se may increase with withdrawals, but, most assuredly, the available energy of the system including the enhanced recovery facilities will not.

15. In these times when the private and public worry is energy shortages and the high costs of energy, it may be, nonetheless, instructive to ponder what would happen in the "fortunate" happenstance that the energy problem would be solved by some surprising or miraculous discovery of new sources or processes. In other words, we have available energy, -w, without limits as far as we can see. Therefore, we can make withdrawals on that available energy to obtain our values, however defined, at any rate, r_v, without limit. This "fortunate" happenstance would, however, not just be moderated, but, essentially, be incinerated along with us because all use of energy, no matter how efficient, involves the dumping as energy in its lowest form, heat. If we used energy without limit, we would also create a limitless garbage dump for our heat. Thus, even if energy were freely available without limits and costs, constraints on its use would be necessary for survival if nothing else. (We will leave to non-technical disciplines, say ethics and theology, the considerations of the moral issues that may be involved if energy were "free".)

If that is true with unlimited available energy, then is it not reasonable that constraints also arise when energy is limited? The laws of thermodynamics tell us we cannot escape even when available energy is limitless. We still could only afford to use available energy within limits.

16. Any adjustment of a reservoir's operating strategy responding to a change in the definition of r_v would have to, of course, consider the cost of implementing the changed strategy. In real reservoirs, an operating strategy cannot likely be changed by re-programming a computerized algorithm or twiddling a few dials, of course, like our hypothetical Φ-maximizing machine.

17. Actually, one of the easier things that the Φ-maximizing machine would need done is the measurement of the available energy, -w, in the reservoir. Recall that -w is a state function defined only by the contents, pressure, and temperature of the reservoir and not dependent on any historical past by which those contents, pressure, and temperature were obtained. Admittedly, reservoir engineers only have uncertain estimates of the data needed to define -w, but they do make and do have those estimates in current technology.

18. The buried tank problem has been previously treated by Lohrenz and Monash (1979b) who treated a case where both the oil and gas had the same molal available energy.

19. This would amount to mining an oil and gas reservoir--something under active consideration (Maugh, 1980). The entire system including the reservoir itself and the mining process and apparatus would still fall under the tenets of Eq. (14) just as would any other enhanced recovery process.

20. Even the consideration of the possibility of prizing only the gas and assigning no value to potential liquid production may be anathema, at least at first, when considering oil and gas reservoirs. This means assigning no value to the condensate of gas condensate reservoir, for example. One of the authors was present at discussions regarding the large Prudhoe Bay reservoir where the alternative of producing gas only and eschewing oil production was listed. Umbrage was immediate and loudly offered only to be informed that the situation could develop that gas has by far the overwhelming value compared to oil. (The search for diamonds in the dung changes when the relative value of diamonds to dung becomes fractional.) Properly and not unreasonably chastened, the author recognized the thesis herein propounded, that how you should run an oil and gas reservoir depends on what you value in its production. We should note that the alternative of producing only gas from this Prudhoe Bay or any other oil reservoir is not being seriously considered, as far as we know. But, that is only because the policy decision is that gas values do not overwhelm oil values.

21. Here, we ignore effects on -w such as the potential, surface, and chemical contributions to available energy. In assuming these can be ignored, we are adding no new burden to our analyses beyond what is already in standard reservoir engineering which assumes thermodynamic equilibrium of the in situ reservoir fluids. Complex reservoir engineering models which consider reservoir heterogeneities do consider how potential and surface energy effects are determinants of how fluids flow within the reservoir. But, that is another matter. In a broader sense, one should consider the surface,

mechanical, and other available energies used in drilling, equipping, and operating wells. One might also consider the available energy used in the drilling rigs and other physical equipment between the time they are used on the reservoir and their use on subsequent reservoirs or salvage.

22.

$$693.14 = (0.08125)(82.05)(300)\ln P$$

Solving $$P = 1.414 \text{ atm}$$

The volume of gas then is $\frac{(0.08125)(82.05)(300)}{1.414} = 1414.2 \text{ cm}^3$ from the ideal gas law. The volume of liquid is 1000 cm^3 less than the volume of gas remaining.

23. From the ideal gas law, nRT is the product of the pressure and gas volume, 1000 cm^3. Substituting into Eq. (15):

$$693.14 = P(1000) \ln P$$

Solving $$P = 1.560 \text{ atm.}$$

The amount of gas left is $\frac{(1.560)(1000)}{(82.05)(300)} = 0.06338$ gm-moles. The gas produced is 0.08125-0.06338 = 0.01787 gm-moles.

24. The volume of gas remaining in the tank when P = 1 atm is:

$$\frac{(0.06338)\left(82.05 \frac{\text{atm - cm}^3}{\text{gm-mole-}^\circ\text{K}}\right)(300\ ^\circ\text{K})}{1 \text{ atm}} = 1560.1 \text{ cm}^3$$

Therefore, 1560.1 - 1000.0 = 560.1 cm^3 of liquid will be withdrawn.

25. Even a reservoir that is shut-in after production may not be preserved thermodynamically, but actually decline in available energy when water invades and entraps hydrocarbons. Whether a shut-in reservoir increases or decreases in available energy with water influx depends on the relative amounts of entrapment to water invasion. In terms of Eq. (14), this means that one can shut-in a reservoir (which is the thermodynamic equivalent of producing at a reversibly slow rate) and still obtain a diminution of -w. Obviously, the only time one would rationally do this is if there were values perceived in shutting in the reservoir that over-ride the thermodynamic expenditure of available energy in doing so.

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DISCUSSION

Q. How would you minimize the entropy in terms of the initial rates, based on what you said? Have you developed a formulation that would enable you to minimize entropy as a function of the initial rate?

A. No, that's not what I like to do.

Q. You statement implied, at least to me, that you wanted to minimize entropy. Maybe I misunderstood you. That is what I though you were driving at.

A. What we measure is the reversible maximum available energy.

Q. But you haven't formulated the entropy, as part of the determination of initial rates?

A. No.

Q. What about the Soviets? As you well know, they have said very often (that they) maximize short-term ultimate recovery.

A. That could be done. (Comment added post-session: For example, in the numerator of Eq. (14), one would simply define t_1 and t_2 consistent with the short-term value judgments or weight r_v such that earlier production is preferred in computing values.)

Historical Growth of Estimates of Oil- and Gas-Field Sizes

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Estimates of proved reserves of crude oil, natural gas, and natural-gas liquids in the United States have been published annually by the American Petroleum Institute (API) and the American Gas Association (AGA) since 1966. These estimates are updated periodically as the fields are developed and produced. The usual experience is that the estimated ultimate recovery (past production plus proved reserves) of all fields discovered in a given year tends to increase from one estimate to the next. Such an increase is to be expected from the conservative nature of the definition of proved reserves. The increases are proportionately larger for younger discoveries than for older. The purpose of this paper is to estimate how much this "growth of reserves" will add to the proved reserves of oil and gas fields that were discovered prior to 1979. Crude oil, including both recoverable oil and original oil in place, will be considered first, and then recoverable natural gas will be considered.

The estimate of the ultimately recoverable oil in fields discovered in a given year can change for several reasons. (a) Drilling could show that the field was physically larger or smaller than had been thought. (b) Production experience could show that the assumed recovery factor was too high or too low. (c) The application of water flooding or another improved recovery technique could change the anticipated recovery. (d) A field could be reported to the reserves committees for the first time several years after its discovery. (e) The discovery year assigned to a field could be changed, and this date change would shift the estimate of the field's recoverable oil from one discovery year to another. (f) New producing zones could be found in an old field. The available data are not sufficiently detailed to allow the estimation of the relative importance of these several factors, though by studying the changes in estimates of original oil in place, one can estimate how much growth in estimates of recoverable oil is due to improved recovery. The phenomenon of growth in estimates of the sizes of oil and gas fields has been studied by many authors (Arrington, 1960; Hubbert, 1974; Marsh, 1971; Pelto, 1973; White and others, 1975). These studies were usually carried out as a subsidiary part of an effort to estimate future discovery rates by extrapolating past discovery rates. In order to avoid serious underestimation of what the future discovery rate will be, the extrapolator must take account of the fact that estimates of proved reserves are conservatively made, especially for fields discovered near the end of the discovery series. The methods used by the different authors for estimating future growth are similar to that described here.

Figure 1 shows estimates made since 1966 through 1978 of the amount of recoverable oil discovered in the conterminous United States in each decade from 1920 to 1969. Except the curve for the decade 1920-1929, each of the curves shows an increasing trend, the most recent decade showing the largest percentage increase.

The future growth of estimates of ultimate recovery from fields discovered before 1979 is estimated under the assumptions a) that when a field has been known for 59 years, its estimated ultimate recovery will no longer change and b) that estimates of recoverable oil in recently discovered fields will show the same percentage growth year by year, as estimates of recoverable oil in fields that were discovered years ago have shown in recent years. Annual data go back only to 1920; hence, the choice of 59 years. Several estimates of ultimate recovery are available for fields discovered in the years 1966 through 1977; they include estimates made at the end of the year of their discovery and estimates made 1 year after that. From these estimates, one can calculate the expected percentage increase between the first and second estimate. Let w(i, j) be the estimate as of the end of year j of recoverable oil in all fields discovered during year i. The estimated 1-year growth factor from the first to the second estimate is then given by the ratio

$$\frac{\sum_{i=1966}^{1977} w(i,\ i+1)}{\sum_{i=1966}^{1977} w(i,\ i)} = r(1) \tag{1}$$

In general, the estimated 1-year growth factor from the n-1 year estimate to the nth year estimate is given by

$$\frac{\sum_{i=\max\text{ of }1967-n,\ 1919}^{1978-n} w(i,\ i+n)}{\sum_{i=\max\text{ of }1967-n,\ 1919}^{1978-n} w(i,\ i+n-1)} = r(n) \tag{2}$$

For the purposes of these calculations, all fields discovered before 1920 were credited to 1919.

Figure 2 is a graph of the average 1-year percentage growth of estimates made during 1971-78 of the amount of ultimately recoverable crude oil in fields of the conterminous United States versus years since discovery of the fields. Notice that positive changes are much more common than negative changes. The amount to which the estimates of ultimate recovery from fields discovered in a given year are expected to increase is obtained by multiplying the 1978 estimated ultimate recovery estimate by all of the r(n) from equation (2) where n is greater than the difference between 1978 and the discovery year and is less than 60. Define the growth factor for an estimate of recoverable oil in a field discovered n-1 years ago as of 12/31/78 to be

$$R(n) = \prod_{i=n}^{59} r(i) \tag{3}$$

This method of calculating growth factors could be carried out equally well if the API had begun publishing annual data series either before or after 1966. The only requirement is that data be available for at least 2 years.

Table 1 shows the 12/31/78 estimated ultimate recovery from oil fields discovered in the conterminous United States by year of discovery as well as the growth factors (from equation 3) for each discovery year. The growth factors were computed by using only a part of the available data series. The estimates of ultimate recovery from 12/31/71 through 12/31/78 were used, and those as of 12/31/66 through 12/31/70 were not used. The estimate of expected future growth of estimates of oil in fields discovered before 1979 is affected by the arbitrary choice of which part of the data series is used. Figure 3 shows how the anticipated growth during the 10-year period 1979-1989 varies as the length of the data series is reduced from 13 years (1966-1978) to 2 years (1977-1978). Figure 4 shows how the total anticipated growth of ultimately recoverable oil varies as length of the data series decreases from 13 years (1966-1978) to 2 years (1977-1978).

The same calculations for growth of estimates of ultimately recoverable oil can be carried out for original oil in place. The analogous table and figures (table 2, figs. 5-8) are presented; however, no accompanying text explains the calculations because the explanation would be completely redundant. The principal result is that the total growth projected for estimates of original oil in place is, proportionate to the 1978 estimate, smaller than that for recoverable oil, i.e., 6.9 percent versus 17.3 percent. This difference is to be expected because in estimating original oil in place, engineers need not consider the uncertainties of reservoir and fluid properties that affect recoverability.

Because the growth factors for estimates of original oil in place are smaller than those for estimates of ultimately recoverable oil, the fraction of oil in known fields that can be recovered is projected to increase. However, the increase is not large. Applying the growth factors for original oil in place and for recoverable oil shows the recovery factor for fields discovered before 1979 increasing from 0.3172 in 1978 to 0.3290 in 1990, or an average increase of one-tenth of a percentage point each year. Applying the growth factors out to the limit of 59 years shows the recovery factor increasing to 0.3480. These results indicate that about half the anticipated increase in estimates of recoverable oil from fields discovered before 1979 comes from improved recovery and about half comes from an increase in the estimates of original oil in place.

Estimates of recoverable natural gas in fields in the conterminous United States have been published annually by the American Gas Association. The recoverable natural gas figures are broken down by year of discovery, and associated gas and non-associated gas are tabulated separately. Figure 9 shows the estimates made at the ends of the years 1966 through 1978 of the total recoverable natural gas discovered in the conterminous United States in each decade from 1920 to 1969.

Growth factors for estimates of recoverable natural gas, like growth factors for estimates of recoverable oil and original oil in place, can be estimated by use of equations 1, 2, and 3. Figure 10 shows the 1-year growth factors for estimates of natural gas and is analogous to figure 2. However, frequent negative revisions are shown in figure 10, whereas only a few negative revisions are shown in figures 2 and 6; these negative revisions shown in figure 10 indicate that successive estimates of the amount of recoverable natural gas discovered in a given year approach the true value in a more erratic fashion than do successive estimates of crude oil. The erratic behavior of past estimates makes the forecasting of future estimates difficult and uncertain. The results of the growth factor calculations for natural-gas estimates are shown in table 3. The growth of estimates of non-associated gas and the growth of estimates of associated gas were calculated both separately and together. Those three calculations are only roughly consistent in that the growths of non-associated and associated gas do not sum to the calculated growth of all gas. The results in table 3 were based upon the use of a data series nine years long (1970-1978). The values of the total growth and the growth for ten years based upon data series of varying lengths are shown in figures 11 and 12. These figures are analogous to figures 3 and 4 and figures 7 and 8.

Conclusions

In the past, successive estimates of the amount of oil and gas discovered in any given year in the conterminous United States have tended to increase. This trend is a result of several factors including: late reporting of discoveries, the conservative nature of the definition of proved reserves, the application of improved recovery techniques, and the discovery of new reservoirs in old fields. Because there is available a series of estimates of the original oil in place and the ultimate recovery (past production plus proved reserves) from oil and gas fields in the conterminous United States by year of discovery, one can quantify this trend in growth of estimates. For both oil and gas fields, estimates of their contents grow each year by larger percentages for recently discovered fields than for older fields.

Estimates of the amount of crude oil and natural gas discovered continue to increase for many years after the field's discovery. At what year the growth ceases, of course, depends upon the particular field in question; however, for the purpose of analysis, growth was assumed to cease when a field became 59 years old. By using that assumption and the assumption that growth proceeds in the future as it has in the past, one can estimate that the quantity of recoverable crude oil discovered in the conterminous United States before 1979, which was estimated as of December 31, 1978, to be 135 $\times 10^9$ bbl, will ultimately increase by 23.4×10^9 bbl to 158.4×10^9 bbl. Natural gas is projected to increase from 721×10^{12} cf to 853×10^{12} cf. Implicit in these projections is the idea that the realization of the increases will require 59 years from the end of 1978. The amount of the increase that will appear in the first 10 years is 11.0×10^9 bbl for recoverable oil, 14.3×10^9 bbl for original oil in place, and 41×10^{12} cf natural gas. The long time span during which estimates increase means that the calculations were significantly affected by the growth of estimates of resources in the large fields discovered before 1950. Oil and gas fields discovered in recent years are smaller than the fields found in the 1920's, 30's, and 40's and they are developed more rapidly, so the prolonged growth (59 years) herein projected may never take place. For this reason, the estimated additions to reserves from fields discovered before 1979, that is 23.4×10^9 bbl recoverable crude oil and 132×10^{12} cf natural gas, are more likely to be overestimates than to be underestimates.

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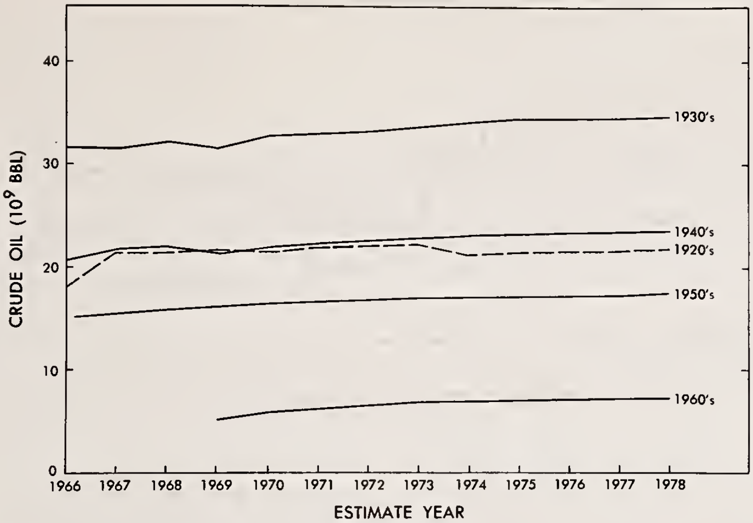


Figure 1.--Estimates made each year have 1966 through 1978 of the amount of ultimately recoverable crude oil discovered in the conterminous United States in each decade from 1920 to 1969. Estimates from American Petroleum Institute, American Gas Association, and Canadian Petroleum Association (1967-1979, v. 22-34).

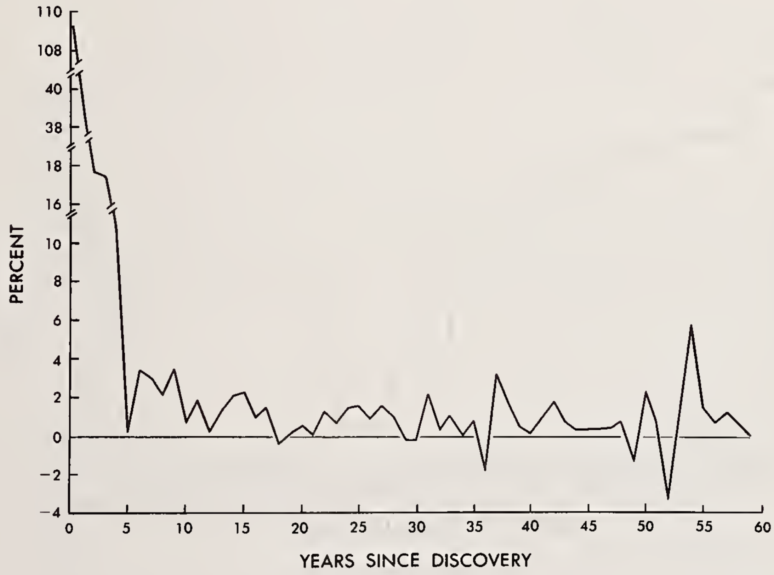


Figure 2.--Average 1-year percentage growth of estimates made during 1971-1978 of the amount of ultimately recoverable crude oil in fields of the conterminous United States versus years since discovery of the fields. Estimates were published by American Petroleum Institute, American Gas Association, and Canadian Petroleum Association, 1972-1979 (v. 27-34).

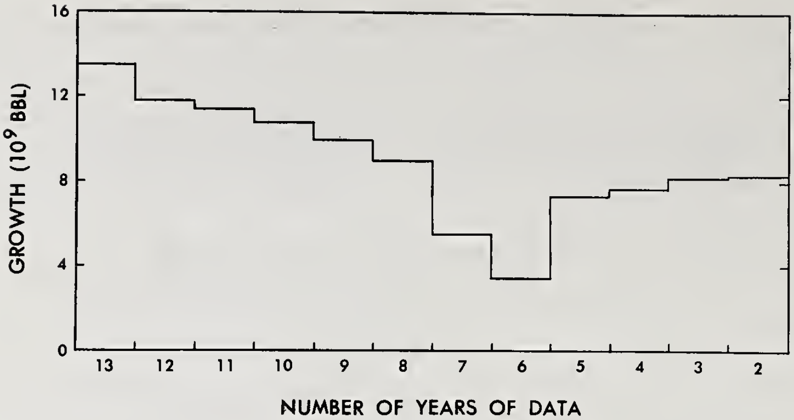


Figure 3.--Projected 10-year (1979-1989) growth of estimates of the amount of ultimately recoverable crude oil in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.

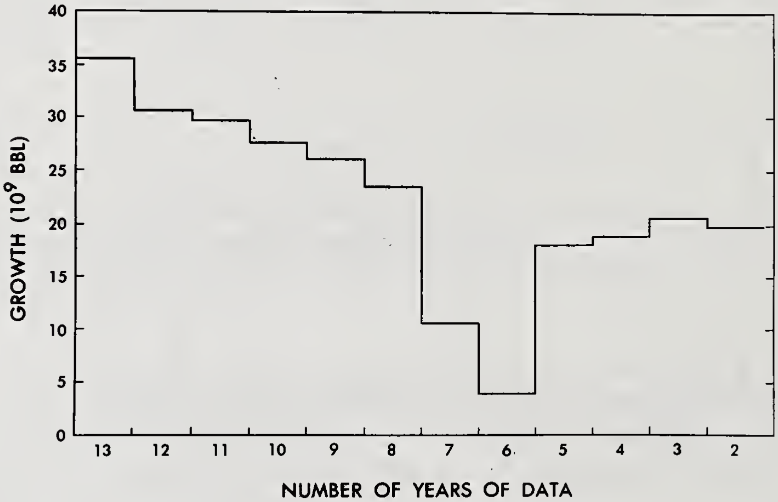


Figure 4.--Projected total growth of estimates of the amount of ultimately recoverable crude oil in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.

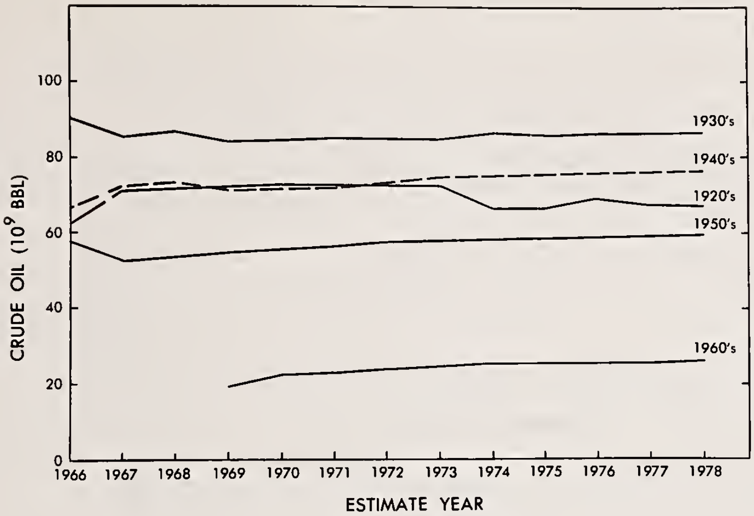


Figure 5.--Estimates made at the ends of the years 1966 through 1978 of the amount of original oil in place discovered in the conterminous United States in each decade from 1920 to 1969.

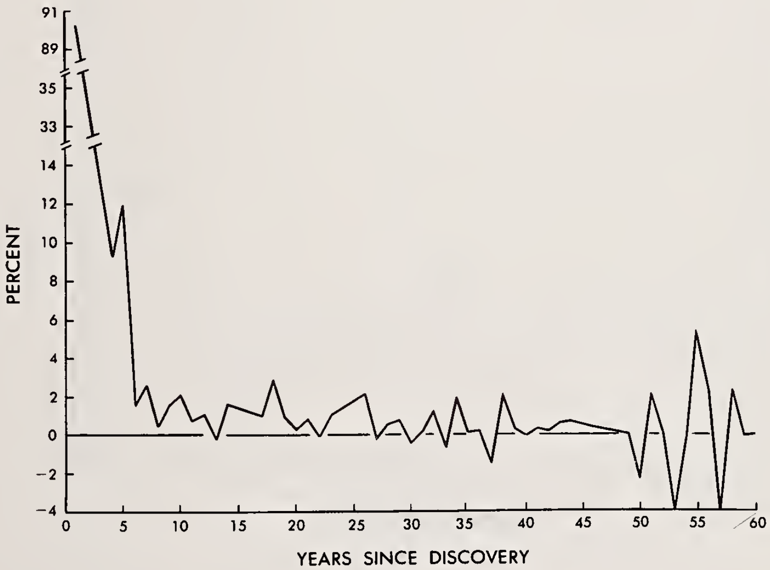


Figure 6.--Average 1-year percentage growth of estimates made during 1971-1978 of the amount of original oil in place in fields of the conterminous United States versus years since discovery of the fields.

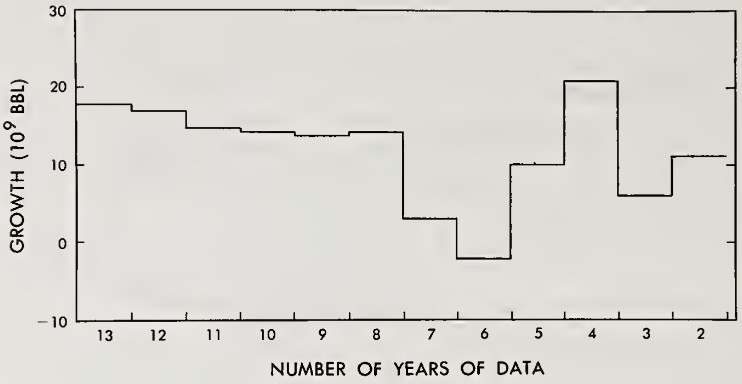


Figure 7.--Projected 10-year (1979-1989) growth of estimates of the amount of original oil in place in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.

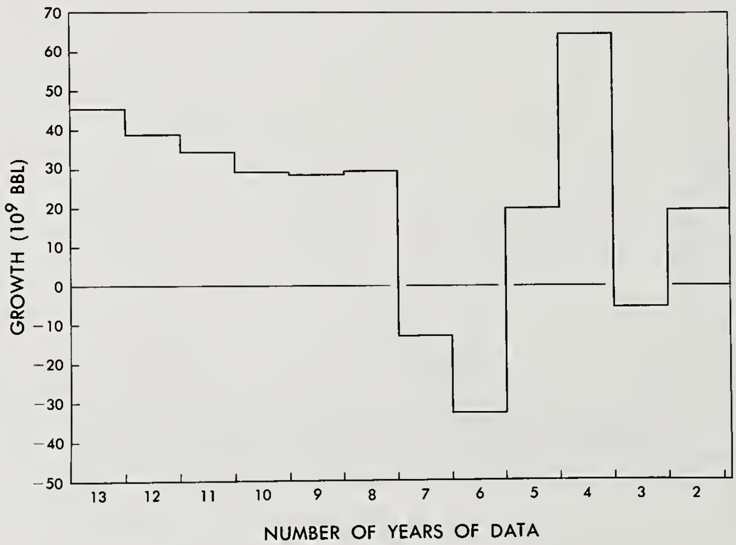
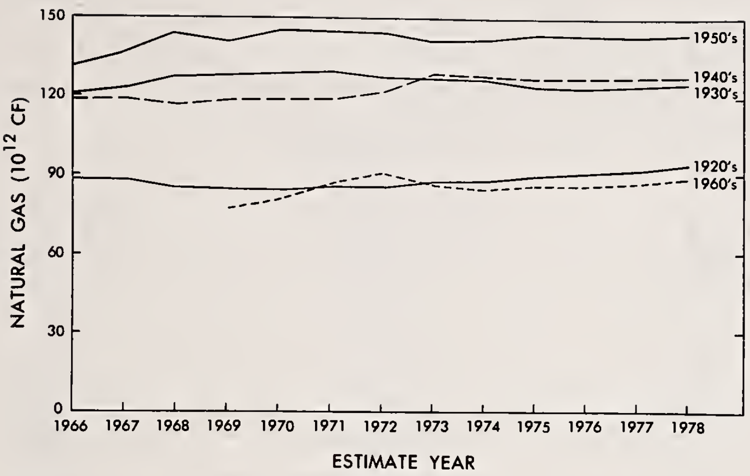


Figure 8.--Projected total growth of estimates of the amount of original oil in place in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.



igure 9.--Estimates made at the ends of the years 1966 through 1978 of the amount of ultimately recoverable natural gas discovered in the conterminous United States in each decade from 1920 to 1969.

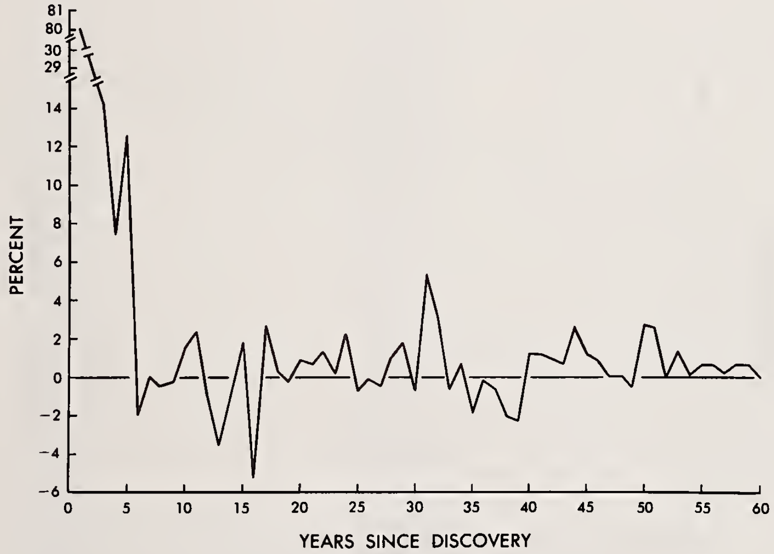


Figure 10.--Average 1-year percentage growth of estimates made during 1970-1978 of the amount of ultimately recoverable natural gas in fields of the conterminous United States versus years since discovery of the field.

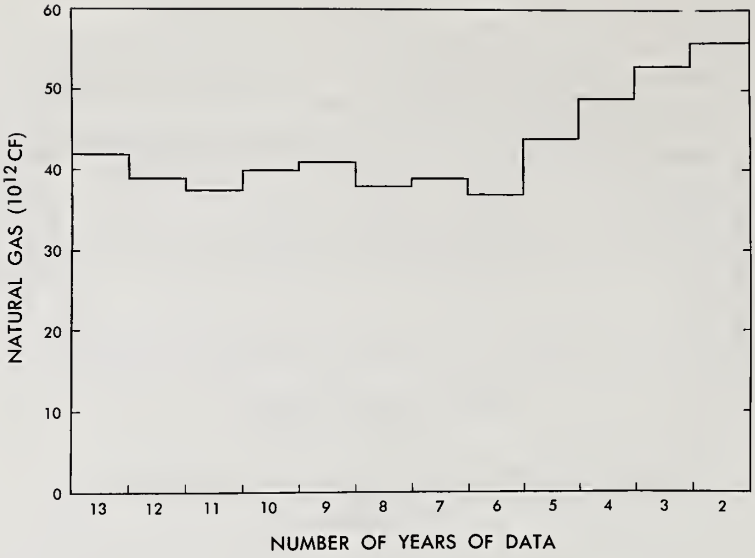


Figure 11.--Projected 10-year (1979-1989) growth of estimates of the amount of ultimately recoverable natural gas in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.

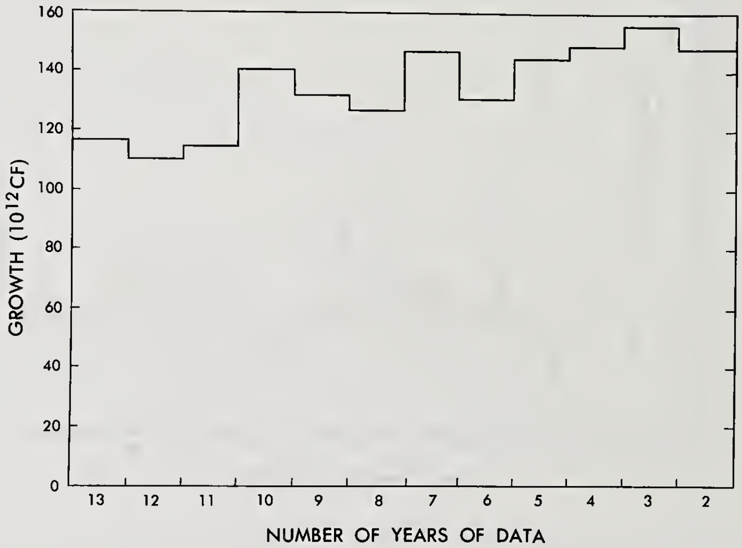


Figure 12.--Projected total growth in estimates of the amount of ultimately recoverable natural gas in fields discovered before 1979 in the conterminous United States versus the number of years of data used. Thirteen years of data are from 1966 to 1978; two years of data are from 1977 to 1978.

Table 1.--Discoveries of recoverable crude oil in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors. [Discovery estimates were published by American Petroleum Institute, American Gas Association, and Canadian Petroleum Association, 1979, v. 33]

Year	Discoveries 10^6 bbl	Discoveries times growth factor 10^6 bbl	Growth factor
1978	63	480	7.581
1977	136	494	3.624
1976	172	451	2.610
1975	344	763	2.216
1974	351	663	1.888
1973	513	876	1.708
1972	313	534	1.704
1971	595	978	1.643
1970	706	1126	1.595
1969	577	903	1.564
1968	967	1460	1.509
1967	700	1051	1.499
1966	502	739	1.471
1965	721	1059	1.468
1964	848	1230	1.450
1963	475	675	1.422
1962	962	1337	1.390
1961	498	686	1.377
1960	1005	1363	1.356
1959	740	1009	1.363
1958	1174	1597	1.360
1957	2062	2791	1.353
1956	1923	2601	1.352
1955	1524	2037	1.335
1954	2148	2851	1.327
1953	2155	2819	1.308
1952	1305	1682	1.288
1951	1733	2213	1.277
1950	2742	3451	1.258
1949	3425	4268	1.246
1948	3463	4327	1.249
1947	1569	1967	1.253
1946	1488	1825	1.226
1945	2098	2567	1.223
1944	2720	3291	1.209
1943	1339	1620	1.209
1942	1443	1734	1.201
1941	2247	2752	1.224
1940	3813	4520	1.185
1939	1963	2290	1.166
1938	4030	4678	1.160

Table 1.--Discoveries of recoverable crude oil in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors (continued).

Year	Discoveries 10^6 bbl	Discoveries times growth factor 10^6 bbl	Growth factor
1937	3425	3973	1.159
1936	6681	7678	1.149
1935	2493	2814	1.128
1934	3858	4322	1.120
1933	1578	1762	1.116
1932	579	644	1.112
1931	2477	2747	1.108
1930	7673	8471	1.103
1929	3770	4127	1.094
1928	2915	3237	1.110
1927	1770	1919	1.084
1926	4726	5083	1.075
1925	1062	1183	1.113
1924	896	988	1.103
1923	1082	1127	1.042
1922	1366	1404	1.027
1921	1934	1974	1.020
1920	2238	2252	1.006
Pre-1920	26857	26857	1.000
TOTALS	134962	158352	Growth 23390 x 10^6 bbl

Table 2.--Discoveries of original oil in place in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors. [Discovery estimates were published by American Petroleum Institute, American Gas Association, and Canadian Petroleum Association, 1979, v.33]

Year	Discoveries 10^6 bbl	Discoveries times growth factor 10^6 bbl	Growth factor
1978	255	1267	4.967
1977	613	1603	2.611
1976	786	1531	1.947
1975	1361	2333	1.714
1974	1312	2060	1.570
1973	1860	2608	1.401
1972	1334	1845	1.382
1971	1798	2422	1.347
1970	2375	3187	1.341
1969	2496	3296	1.320
1968	3146	4070	1.293
1967	2628	3376	1.284
1966	2161	2747	1.270
1965	2586	3295	1.273
1964	3050	3827	1.254
1963	1688	2088	1.237
1962	2700	3300	1.222
1961	2100	2543	1.210
1960	3436	4044	1.176
1959	2828	3297	1.165
1958	4072	4735	1.162
1957	6356	7333	1.153
1956	6368	7356	1.155
1955	5701	6515	1.142
1954	7330	8267	1.127
1953	8154	9036	1.108
1952	4670	5069	1.085
1951	6331	6884	1.087
1950	7624	8242	1.081
1949	18537	19890	1.072
1948	8122	8752	1.077
1947	5633	6065	1.076
1946	4227	4497	1.063
1945	6678	7153	1.071
1944	8051	8457	1.050
1943	4071	4283	1.052
1942	4433	4655	1.050
1941	6963	7421	1.065
1940	9568	9995	1.044
1939	5799	6035	1.040
1938	10374	10795	1.040

Table 2.--Discoveries of original oil in place in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors (continued).

Year	Discoveries 10^6 bbl	Discoveries times growth factor 10^6 bbl		Growth factor
1937	8557	8880		1.037
1936	19646	20352		1.035
1935	7015	7223		1.029
1934	9317	9529		1.022
1933	4229	4302		1.017
1932	1984	2012		1.014
1931	5930	6000		1.011
1930	13731	13885		1.011
1929	10657	10778		1.011
1928	8093	8382		1.035
1927	4901	4968		1.013
1926	13326	13428		1.012
1925	4069	4298		1.056
1924	3165	3358		1.060
1923	3617	3640		1.006
1922	4215	4148		0.984
1921	6176	6331		1.025
1920	9020	9017		0.999
Pre-1920	98277	98277		1.000
TOTALS	425527	455068	Growth	29541 x 10^6 bbl

Table 3.--Discoveries of recoverable natural gas in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors that are based in a data series from 1970 to 1978. (American Petroleum Inst., American Gas Assoc., and Canadian Gas Assoc., v. 24-33).

	Non-associated gas			Associated gas			Natural gas all kinds		
Year	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth factor	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth factor	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth
1978	1,704,545.	7,551,995.	4.4305	100,083.	409,771.	4.0943	1,804,628.	7,177,113.	3.971
1977	3,348,127.	8,309,106.	2.4817	300,647.	615,588.	2.0475	3,648,774.	8,037,533.	2.208
1976	3,938,727.	7,631,844.	1.9376	354,861.	533,195.	1.5025	4,293,588.	7,330,930.	1.704
1975	5,742,435.	9,785,282.	1.7040	797,785.	1,007,528.	1.2629	6,540,220.	9,758,898.	1.491
1974	4,843,853.	7,727,954.	1.5954	757,321.	863,388.	1.1401	5,601,174.	7,781,707.	1.383
1973	7,725,242.	10,948,601.	1.4173	945,605.	958,330.	1.0135	8,670,847.	10,702,233.	1.233
1972	6,797,879.	9,964,108.	1.4658	767,445.	742,463.	0.9674	7,565,324.	9,541,286.	1.262
1971	6,850,505.	9,974,690.	1.4561	1,103,143.	1,097,763.	0.9951	7,953,648.	10,024,296.	1.263
1970	3,310,152.	4,831,807.	1.4597	975,375.	990,886.	1.0159	4,285,527.	5,430,792.	1.262
1969	5,366,188.	7,756,882.	1.4455	889,705.	934,627.	1.0505	6,255,893.	7,905,275.	1.267
1968	5,030,856.	7,180,031.	1.4272	1,192,730.	1,219,857.	1.0227	6,223,586.	7,748,456.	1.240
1967	4,041,061.	5,559,318.	1.3757	1,007,671.	1,084,500.	1.0762	5,048,732.	6,136,766.	1.215
1966	7,179,337.	10,055,171.	1.4006	701,420.	731,702.	1.0432	7,880,757.	9,686,340.	1.221
1965	7,156,088.	10,410,675.	1.4548	930,521.	982,460.	1.0558	8,086,609.	10,285,126.	1.279
1964	7,416,777.	10,931,309.	1.4739	1,896,940.	1,979,059.	1.0433	9,313,717.	11,950,565.	1.281
1963	11,326,398.	16,255,966.	1.4352	1,228,005.	1,313,816.	1.0699	12,554,403.	15,823,529.	1.264
1962	8,966,887.	13,676,449.	1.5252	2,102,417.	2,310,032.	1.0988	11,069,304.	14,734,076.	1.331
1961	8,789,685.	13,035,351.	1.4830	877,919.	945,128.	1.0766	9,667,604.	12,530,275.	1.291
1960	10,257 514.	15,179,369.	1.4798	2,268,081.	2,420,872.	1.0674	12,525,595.	16,17,8547.	1.296
1959	6,127,987.	9,083,375.	1.4823	1,411,575.	1,519,187.	1.0762	7,539,562.	9,767,274.	1.295
1958	16,951,405.	25,025,570.	1.4763	2,906,873.	3,044,635.	1.0474	19,858,278.	25,501,978.	1.282
1957	11,241,993.	16,461,417.	1.4643	4,368,701.	4,571,883.	1.0465	15,610,694.	19,918,346.	1.279
1956	16,337,685.	23,446,501.	1.4351	3,305,791.	3,490,854.	1.0560	19,643,476.	24,727,872.	1.258
1955	7,953,806.	11,350,749.	1.4271	2,198,096.	2,349,444.	1.0689	10,151,902.	12,762,106.	1.251
1954	12,838,044.	17,796,183.	1.3862	3,307,818.	3,533,733.	1.0683	16,145,862.	19,850,284.	1.224
1953	9,343,772.	13,039,928.	1.3956	3,109,801.	3,352,273.	1.0780	12,453,573.	15,423,385.	1.235
1952	14,496,423.	20,209,011.	1.3941	2,221,071.	2,401,453.	1.0812	16,717,494.	20,705,554.	1.236
1951	9,014,687.	12,680,975.	1.4067	2,257,150.	2,432,429.	1.0777	11,271,837.	14,033,884.	1.240
1950	8,655,715.	12,100,860.	1.3980	5,439,260.	5,7353,51.	1.0544	14,094,975.	17,353,274.	1.232
1949	20,177,214.	27,715,743.	1.3736	5,028,075.	5,204,604.	1.0351	25,205,289.	30,481,478.	1.203
1948	5,518,900.	7,518,204.	1.3623	2,891,619.	3,099,129.	1.0718	8,410,519.	10,241,127.	1.217
1947	10,269,978.	13,444,429.	1.3091	2,709,695.	2,687,539.	0.9918	12,979,673.	14,989,181.	1.158
1946	4,051,050.	4,993,629.	1.2327	3,913,508.	3,954,223.	1.0104	7,964,558.	8,923,062.	1.123

Table 3.--Discoveries of recoverable natural gas in the conterminous United States by year of discovery estimated as of 12/31/78 and multiplied by the growth factors that are based on a data series from 1970 to 1978--continued.

	Non-associated gas			Associated gas			Natural gas all kinds		
Year	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth factor	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth factor	Discoveries 10^6 cf	Discoveries times growth factor 10^6 cf	Growth
1945	8,011,431.	9,967,658.	1.2442	6,109,918.	6,199,857.	1.0147	14,121,349.	15,939,179.	1.127
1944	6,659,156.	8,255,702.	1.2398	4,810,647.	4,815,857.	1.0011	11,469,803.	12,853,155.	1.126
1943	6,195,895.	7,842,955.	1.2658	2,364,609.	2,404,050.	1.0167	8,560,504.	9,776,743.	1.141
1942	5,562,352.	7,014,360.	1.2610	3,760,812.	3,864,465.	1.0276	9,323,164.	10,660,574.	1.145
1941	12,111,177.	15,384,573.	1.2703	3,800,191.	3,918,907.	1.0312	15,911,368.	18,302,096.	1.153
1940	8,647,100.	11,284,044.	1.3050	4,840,847.	5,044,422.	1.0421	13,487,947.	15,835,842.	1.171
1939	6,876,653.	9,123,076.	1.3267	5,281,228.	5,690,937.	1.0776	12,157,881.	14,608,325.	1.206
1938	9,400,355.	12,366,123.	1.3155	5,508,093.	5,823,083.	1.0572	14,908,448.	17,687,102.	1.184
1937	13,516,612.	16,796,771.	1.2427	7,945,800.	8,797,120.	1.1071	21,462,412.	25,136,460.	1.172
1936	18,105,863.	22,161,505.	1.2240	6,119,359.	6,754,206.	1.1037	24,225,222.	28,111,336.	1.164
1935	9,802,498.	11,705,105.	1.1941	4,019,587.	4,458,134.	1.1091	13,822,085.	15,922,741.	1.150
1934	5,469,848.	6,240,304.	1.1409	8,158,278.	8,920,851.	1.0935	13,628,126.	15,273,211.	1.127
1933	2,201,349.	2,572,671.	1.1687	1,870,491.	2,056,540.	1.0995	4,071,840.	4,625,485.	1.130
1932	3,051,587.	3,583,998.	1.1745	932,500.	1,006,406.	1.0793	3,984,087.	4,486,792.	1.122
1931	2,224,799.	2,609,126.	1.1727	3,249,815.	3,502,362.	1.0777	5,474,614.	6,156,479.	1.126
1930	3,841,032.	4,359,284.	1.1349	7,042,736.	7,741,605.	1.0992	10,883,768.	12,224,038.	1.121
1929	2,873,101.	3,217,981.	1.1200	12,265,464.	14,061,307.	1.1464	15,138,565.	17,094,376.	1.122
1928	3,188,101.	3,548,024.	1.1129	6,850,081.	7,163,254.	1.0457	10,038,182.	11,022,979.	1.091
1927	11,736,467.	12,624,764.	1.0757	1,447,775.	1,521,686.	1.0511	13,184,242.	14,106,599.	1.070
1926	2,127,685.	2,252,732.	1.0588	2,810,590.	3,110,190.	1.1066	4,938,275.	5,281,897.	1.066
1925	469,783.	487,989.	1.0388	528,428.	586,942.	1.1107	998,211.	1,052,508.	1.054
1924	1,148,182.	1,181,115.	1.0287	1,158,063.	1,263,598.	1.0911	2,306,245.	2,404,115.	1.044
1923	526,360.	537,333.	1.0208	1,302,715.	1,352,593.	1.0383	1,829,075.	1,875,948.	1.026
1922	37,936,162.	38,377,241.	1.0116	559,238.	581,513.	1.0398	38,495,400.	39,208,334.	1.015
1921	911,088.	918,320.	1.0079	4,197,268.	4,351,771.	1.0368	5,108,356.	5,185,044.	1.010
1920	592,538.	597,782.	1.0089	1,608,936.	1,614,362.	1.0034	2,201,474.	2,218,095.	1.006
pre-1920	70,263,018.	70,263,019.	1.0000	21,758,385.	21,758,385.	1.0000	92,021,403.	92,021,404.	1.000
Totals 10^6 cf	526,221,128.	686,906,016.	growth 160,684,888.	194,568,586.	206,882,112.	growth 12,313,526.	720,789,720.	852,513,384.	growth 131,723,664.

DISCUSSION

MR HERRON: Potter Herron from Drury Federal.

Dave, your gas numbers were 900 trillion cubic feet, and the oil numbers 154 billion barrels and that compares almost exactly with the ultimate recovery reserve Dr. Hubbert gave us yesterday. It's all over? Is that what you're saying?

DR. ROOT: Well, we all work from the same series of data, yes, I got the same answers.

MR. MOORE: Frank Moore of Lewin and Associates.

We've just completed an analysis of the blue book series for Canada, and we found similar general trends. I was wondering, in your various graphs that showed, there was wide variability in recovery as a function of the number of data sources (figs. 3, 4, 7, 8, 11, and 12). In view of the fact that the standard variable is a variance in sample means as a function of samples size, have you tried some procedure for using -- for every estimate you get of the growth ratio, there are different numbers that I take it you're averaging. Have you tried to incorporate a variability in that, perhaps using some sort of Monte Carlo, the selector growth factor, and using a large number of trials in some way, to incorporate variability and to give limit to uncertainty?

DR. ROOT: No, I really haven't. I don't know what causes the variation, but it doesn't seem to me that it's really a random error. I think it might be a change of personnel on the committees who put together the data. It really doesn't seem like the kind of thing I can compensate for, so I haven't tried. I'd be happy to get rid of the variability but I haven't seen any way to do it.

MR. MOORE: In our 1977 analysis, for ERDA, we did a Monte Carlo analysis, and the limits were fairly tight. The second question I had: Have you tried to, in any way, arrive at a behavioral explanation of the characteristic shape of the growth curve, tried to find any reasons or deterministic factors for the five-year precipitous drop in gas and the four-year drop for oil?

DR. ROOT: No, I don't know where it comes from. It might be that it just takes that long, once you find a field, to develop it to get it all reported. And by the time six years is up, if the field is worth anything, then that's been done. And thereafter it's just minor improvements in recovery. But I don't have any detailed explanation for the shape of the growth curves for oil and gas fields.

MR. BEPEK: John Bepek from Standard Oil.

The blue books show that some amount of oil is added from discoveries before 1920's. Did you take into account in your estimate of future recoveries, and if so, how do you do that?

DR. ROOT: Well, it is true that they do record oil discovered before 1920, and I did use it in my calculations, certainly for the totals. I assumed that growth would persist only for 60 years, which is admittedly a conservative assumption in that there are very large, very old fields that will probably still grow some. On the other hand, there are fields being discovered recently, which won't grow for anywhere near 60 years, because they'll be abandoned before they're 60 years old. So 60 years seemed like a compromise number. It was obviously to small for some and too large for others but I couldn't see any way to refine the choice more than that.

CHAIRMAN KEENE: Thank you very much.

I get a more uncomfortable feeling as people go through their analysis with the blue book, because we just published our first reserves report and probably in five years we'll have a total of eight white or yellow or however they're going to come out books, so save all your models and your charts, because you'll be able to redo them with different numbers -- and perhaps it'll explain some of the anomalies that you picked up.

Our variances from API and AGA are like in the state of California off by about 30 percent. So there are some differences.

The Economic Accounts of the Resource Firm

David Nissen*

I. SUMMARY AND ABSTRACT

This paper develops the economic accounts of the resource firm. The economic valuation of the firm's net cash flow, computed as the sum of the discounted cash flow (DCF), is analyzed in the format of conventional financial analysis. This permits consistent comparison of the economic valuation and the conventional financial analysis of the resource firm or resource project.

Economic net worth is defined as the DCF value of net cash payments to equity (dividends). The remainder of the economic balance sheet is completed with the economic valuation of the resource firm's assets: proven reserves, developing resources, and undeveloped resources.

The DCF valuation method is shown to contain implicitly a natural definition of economic income. Identification of income as the change in net worth (retained earnings[1]) plus dividends (distributed earnings) allows construction of the economic income statement that is consistent with the economic balance sheet. The key feature of the economic income statement, in general, is that it properly organizes and evaluates the imputed components of income: holding gains, allowance for finance during construction, tax expense discounting due to tax payment deferral (not treated here), and replacement cost expensing of depreciation, depletion, and amortization.

In this first report on research in progress, a highly simplified characterization of the resource production process is analyzed. In particular the tax regime, alternative production strategies, technical change, and uncertainty are assumed away.[2]

II. FINANCIAL AND ECONOMIC ACCOUNTING

A firm's management evaluates and chooses among investment projects. The financial community, viewing the firm as a managed portfolio of investment projects, evaluates and chooses among firms. These evaluations are accomplished using different, and more or less inconsistent, methods.

Discussion

The firm evaluates investment projects by the discounted cash flow method (DCF), summing the discounted net cash inflow and outflow of the project over time to develop a single "figure of merit" for the project. This figure can be interpreted as the excess or deficit present value of the project compared to a project yielding (and reinvesting) income at the discount rate used.[3]

The DCF evaluation of a firm's project is essentially forward-looking, evaluating the investment's <u>prospects</u>. By design, this method aggregates over the time intervals between cash outflows and inflows which embody much of the risk that is of interest to financial analysts.

The conventional financial accounts of the firm as a whole--the income statement, balance sheet, and flow-of-funds statement--also organize and present information about the cash flow of the firm. These statements are constructed with labels which evoke valuation: income, expense, assets, liabilities, and net worth; in fact, when the firm's activities occur entirely on current account, the entries beside these labels represent what economic intuition suggests. However, when production entails capital expenditures--when there are substantial lags between costs and consequent revenues--conventional accounting and economic valuation measures diverge.[4] In times of inflation in particular, the conservative bias of "generally accepted accounting practice" usually leads to undervaluation of income and assets.

The guiding principle of conventional financial accounting is to be "conservative" and "objective." Because the accounting profession regards its primary responsibility to be to present and potential stockholders rather than to managers and entrepreneurs, accounting practice is controlled by the goal of limiting the opportunity for dubious, self-serving, or fraudulent inflation of income and assets. As a consequence, the conventional accounting valuation of assets and liabilities is based on original cost rather than imputed market or replacement value, and the valuation of income is based on <u>realized</u> revenue net of the expiration (expensing) of <u>original</u> costs.[5]

Revenue realization and original cost expensing to some degree separate what is known from what is speculative about the firm. Thus, trends in cash flow and income realization implicitly embody the development of information about the firm's investments, as well as the development of the investments themselves. This, in turn, is implicitly recognized in the panoply of ratios between balance sheet, income, and funds statement items which financial analysts use to assess risk and performance of firms.

Thus the format of conventional financial accounts furnishes types of information, not directly available from the DCF analysis, which find widespread use for the analysis of risk and performance in the business community. Of course, because of differing measurement conventions, the content of conventional accounts differs from that of the DCF analysis as well.

Evaluation of the balance sheet at original cost (rather than replacement cost or market value) is consistent with the realization test and original cost treatment of the income statement. This treatment excludes four classes of imputed income and expense items which are central to the economic valuation of income and of balance sheet entries.[6]

These are:

- o unrealized holding gains on tangible or financial assets and fixed-interest-rate liabilities,[7]

- o imputed income allowance for finance during construction (AFDC),

- o imputed tax expense discount due to tax payment deferral, and

- o replacement cost recognition in depreciation, depletion, and amortization (DD&A) expense.

The production process of the resource firm, which comprises the ownership, discovery, development, and production of natural resources, is among the most capital-intensive economic activities. Although these processes employ large, durable facilities and equipment, as is true in heavy manufacturing, more important are the long lead times required to find and develop resources before production begins, and the long period required to complete production of developed reserves. Purchases of labor, materials, services, and property rights during the exploration and development phases are just as much capital expenditure as the purchase of tangible assets (though these purchases may be expensed for tax purposes). Valuation of these assets in economic rather than financial terms requires recognition of holding gains on marketable assets, increases in asset cost due to finance required during construction, reduced tax expense due to tax relief[8] and discounting of deferred tax payments, and recognition of replacement costs in depreciation, cost depletion, and amortization expense.

As assets change in economic value over time, consistent accounting requires that the economic measure of net worth change in step. Since the change in net worth flows from the retained earnings portion of income, the imputed income and expense items must be recognized in the determination of economic income.

This discussion suggests that the results of a DCF investment evaluation can be analyzed in the format of conventional financial analysis, thus constituting the economic accounts of the firm. This is true, and the purpose of this paper is to present a basic version of the analysis for a highly simplified depiction of the resource firm--that which is used in the optimal resource extraction (or Hotelling) literature. This is the first step in developing these accounts for the general case, with accurate representation of project investment timing, service life, technical change, and tax regime.

Motivation

The economic accounts, when developed for the general case, serve several purposes. They provide for cross-fertilization between financial and economic analysis. Economic information is made available to financial analysts in a familiar format. Economic valuations of balance sheet items are consistently presented. A consistent economic interpretation of income statement items--particularly imputed income and expenses--is provided.

Collaterally, the conventional financial accounting format--the allocation of value and income--becomes available to sharpen and refine the economic evaluation of the DCF method. The conventional analyses of cash flow, liquidity, coverage, and exposure can be carried out with economically meaningful values. At a time when the structure and terms of investment in resource industries, especially energy industries, are being resolved by the analysis and debate of operating business, financial business, government policy makers, and academics, this rationalization of measurement systems should sharpen understanding of costs, gains, and incentives.

An important and inflammatory issue in this debate is the question of "undue profits" in the resource industries, particularly the petroleum industry, where levels of profit, profits share of sales, and profitability are usually compared to manufacturing. Construction of the appropriate economic accounts can resolve this issue with regard to the comparative levels of economic profits and profitability.

In anticipation of this, two qualitative observations can be made. First, profits' competitive share of sales should probably be relatively higher in petroleum compared to manufacturing due to greater capital intensity. Since much of this capital accrues over the life of the resource project as holding gains, AFDC, and deferred tax discounting, the bias in conventional accounting measures of income, assets, and equity is to underestimate the petroleum figures relatively more than those of manufacturing.[9]

Second, since both the numerator and denominator of the return-on-assets and return-on-equity probably suffer relatively greater underestimation in the petroleum industry, the effects of the conventional accounting bias on the measurement of the absolute level of profitability in petroleum and profitability relative to manufacturing are unclear. In any case, measures of relative profitability taken from conventional accounts should be viewed with caution. I believe construction of the economic accounts with adequate representation of the investment timing, project service life, and tax regime in each industry is required to resolve this issue.

Further, the economic accounts directly provide a tool for use by and evaluation of management. Such a tool can be an important ingredient in the response to a pervasive theme in today's business press--that U.S. management, guided and evaluated by conventional accounting systems, behaves short-sightedly. Certainly, a manager must be especially articulate to prosper in the eyes of his board of directors, while following an investment strategy whose returns accrue economically but not financially during his tenure.

Finally, in these times of general inflation and the further relative escalation of resource and capital goods prices, the divergence between economic reality and accounting practice has induced a sense of unease in the accounting financial community, leading to a variety of proposed reforms: inflation accounting, replacement cost accounting, and the like. Neither the purposes nor the consequences of the proposals are well understood. Laying the economic accounts of the firm beside the conventional accounts in a common format should go a long way toward clarifying the relation of these measures to economic valuation.

III. FUNDAMENTALS OF THE ECONOMIC ACCOUNTS

The basis of the economic accounts of a firm (or project) is the definition of <u>economic net worth</u> as the DCF value (present value) of the firm's net cash payout stream, here defined as dividends.[10] Then, defining the change in net worth as retained earnings, it is natural to define <u>economic income</u> as equal to dividends plus retained earnings, that is, income distributed plus income retained.

The <u>balance sheet</u> is completed by allocating to each asset (liability) the present value of the net cash flow it generates (requires). In vertically integrated production processes this is accomplished by determining the appropriate transfer price for evaluating the flows between asset classes. In the integrated resource production process, for example, there are four prices characterizing transactions with external markets: the price of acquisition of property rights to undeveloped resources (the lease bonus plus royalty commitment), the prices of capital goods inputs, the prices of operating inputs, and the prices of finished products. The acts of exploration and finding, and development create intermediate assets, here distinguished as the developing resource and the proven reserve. Prices of these assets are constructed to reflect the resource acquisition cost, capital outlays, and the allowance for financing the required inventory of developing resources.

With income defined as the change in net worth plus dividends, and net worth equal to assets minus liabilities, the income statement is generated by classifying the components of changes in assets and liabilities.

The net cash flow generated by a class of assets can be separated into the value of the capital services generated (implicitly, the rental imputed to the capital service) minus capital expenditures. For intangible assets, those whose value cannot be decomposed into a measure of quantity and price, the change in the value of assets can be allocated to capital expenditure plus an allowance for finance required for holding the asset minus amortization of the capital services expired (expensed). For tangible assets, those whose value can be decomposed into measurements of price and quantity,[11] a sharper allocation of the change in asset value is available. Changes in the price of tangible assets generate holding gains, and changes in the quantity of assets held (or more generally, expiration of the value of remaining future services) generate depreciation plus depletion expense. Hence changes in the value of tangible assets held can be allocated to capital expenditures plus holding gains minus depreciation and depletion.

The whole point of the economic valuation of firms or investment projects or individual assets is that there is an intimate connection between their value as assets and the value of the capital services they furnish over time. Here we show that the economic value of an asset's capital services will cover depreciation and depletion plus a finance allowance net of capital gain while holding. Equivalently, the price of an asset equals the discounted, depreciated value of its capital services.[12]

To demonstrate these propositions we introduce the present value operator PV[ ]. Let X and r on $[t,\infty)$ represent cash flow and the discount rate between now (time t) and the indefinite future. Define the present value of X with discount rate r, say V, as:

$$V_t = PV_t[X;r] \triangleq \int_t^{\infty} F(t,\tau;r)\ X_\tau d\tau \qquad (3\text{-}1)$$

where the discount factor is

$$F(t,\tau;r) = e^{-\int_t^{\tau} r_z dz}$$

The change in V_t is given by

$$\dot{V}_t = \frac{d}{dt} PV_t[X;r] = r_t V_t - X_t. \qquad (3\text{-}2)$$

When there is no ambiguity, $PV_t[X;r]$ will be written, $PV[X]$, and $F(t,\tau;r)$ will be written $F(r)$.[13]

One technical note: throughout this paper, in all expressions of the form of equation (3-1) assume a <u>boundedness condition</u>, that the growth rate of X is bounded below r so that the improper integral in (3-1) exists and

$$\lim_{\tau \to \infty} F(t,\tau;r)\ X_\tau = 0.$$

<u>Relation of Net Worth and Income</u>

If we define net worth (NW) of a firm or project as the present value of its dividend stream, DIV,

$$NW \triangleq PV[DIV], \qquad (3\text{-}3)$$

and define income to be equal to dividends plus the change in net worth (retained earnings),

$$INC \triangleq DIV + \dot{NW}, \qquad (3\text{-}4)$$

then, since (from (3-2))

$$\dot{NW} = r\ NW - DIV, \qquad (3\text{-}5)$$

income must equal the discount rate times net worth,

$$INC = r\ NW. \qquad (3\text{-}6)$$

Note that this intuitively desirable result is not imposed but is derived from the natural definition of economic net worth and the inescapable accounting definition of income as income distributed (DIV) plus income retained ($\dot{NW}$). This is because the DCF valuation method implicitly compares the investment yielding DIV to an investment yielding r (both reinvesting at r) and, in effect, reinvests its excess value, NW, at r.

Asset Valuation and Balance Sheet

To permit an intellible discussion of the core results of the analysis, assume debt and interest, new equity, taxes, and working capital are all zero (generalization to accommodate these phenomena will be offered in subsequent papers). The only source of cash is operating income (OY) and the only uses for cash are capital expenditure (CE) and dividends (DIV). Thus the simple cash account might appear as in Figure 1. This can be expressed as:

$$DIV \triangleq OY - CE. \tag{3-7}$$

Suppose capital expenditure is divided between tangible and intangible assets,

$$CE \triangleq CE^T + CE^I, \tag{3-8}$$

and operating income is allocated between the services of tangible and intangible assets,

$$OY \triangleq CS^T + CS^I. \tag{3-9}$$

SIMPLE CASH ACCOUNT

<u>Sources</u>	<u>Symbol</u>
Operating income	OY
= Total sources,	Σ
<u>Uses</u>	
Dividends	DIV
+ Capital expenditure	+CE
= Total uses	Σ

Figure 1. Simple cash account

(The determination of the value of capital services from tangible assets, CS^T, is listed below; given CS^T, the residual from operating income, OY, is the imputed value of capital services from intangible assets, CS^I.) Defining tangible and intangible asset values:

$$TA \triangleq PV[CS^T - CE^T], \tag{3-10}$$

$$IA \triangleq PV[CS^I - CE^I], \tag{3-11}$$

we have (from Equations 3-3, 3-7, 3-8, 3-9, 3-10, and 3-11) the balance sheet identity,

$$NW = TA + IA, \tag{3-12}$$

net worth equals tangible assets plus intangible assets (recall there is no debt or taxes).

Income Statement

To construct the income statement, we must allocate the changes in assets. Define the allowance for finance on intangible assets,

$$AF \triangleq rIA, \tag{3-13}$$

and amortization of intangible assets equal to capital services expired,

$$AM \triangleq CS^I. \tag{3-14}$$

Then given (from Equation 3-11)

$$IA = PV[CS^I - CE^I] ,$$

we have (from Equations 3-2, 3-13, and 3-14)

$$\begin{aligned} \dot{IA} &= rIA - (CS^I - CE^I), \\ &= AF - AM + CE^I. \end{aligned} \tag{3-15}$$

Suppose the value of tangible assets can be written as price times net capital stock, say,

$$TA \triangleq pK, \tag{3-16}$$

and capital expenditure can be written as the value of net investment plus depreciation,

$$CE^T \triangleq p\dot{K} + DP. \tag{3-17}$$

Define holding gains as the capital stocks times price change,

$$HG \triangleq \dot{p}K. \tag{3-18}$$

Then

$$\dot{IA} = \dot{p}K + p\dot{K}, \tag{3-19}$$

and (from Equations 3-17 and 3-18)

$$\dot{IA} = HG + CE^T - DP. \tag{3-20}$$

Recall (from Equation 3-4)

$$INC \triangleq DIV + NW,$$

so (from Equation 3-12)

$$INC = DIV + \dot{TA} + \dot{IA}$$

and (from Equations 3-15 and 3-19)

$$INC = OY - CE + HG + CE^T - DP + AF - AM + CE^I, \tag{3-21}$$

so

$$INC = OY + (AF + HG) - (DP + AM). \tag{3-22}$$

(Economic income equals operating income plus imputed income items (allowance for finance plus holding gains) minus imputed expenses (depreciation and depletion plus amortization of intangible assets)).

This proves the income statement shown in Figure 2.

INCOME STATEMENT

Operating Income	OY
+ (Holding gains +	+ (HG +
Allowance for finance on intangibles)	AF)
- (Depreciation and Depletion +	- (DP +
Amortization)	AM)
= Income	= INC

Figure 2. Development of the income statement

We can understand the difference between economic accounting and conventional accounting by considering the three equations:

$$NW = PV[DIV], \tag{I}$$

$$INC = DIV + \dot{NW}, \tag{II}$$

$$INC = OY + (AF + HG) - (DP + AM). \tag{III}$$

Economic accounting takes I as the definition of net worth and II as the definition of income. Then III is derived as a consequence of the definitions. Conventional financial accounting takes III as the definition of income, given special rules about the determination of the non-cash items: finance allowance, holding gain, depreciation and amortization. Then II is taken as the definition of the change in net worth given conventions about initial and on-going capitalization. Of course in conventional financial accounts, Equation I is irrelevant.[14]

Capital Asset and Capital Service Pricing

To develop the relationship between the value of an asset and the value of its services, first note (from Equations 3-10 and 3-15)

$$TA = PV[CS^T - CE^T] = pK.$$

Then

$$\dot{TA} + rTA - CS^T + CE^T = \dot{p}K + p\dot{K}. \tag{3-23}$$

Recall (from Equations 3-17 and 3-18)

$$HG \triangleq \dot{p}K,$$

$$CE^T \triangleq p\dot{K} + DP,$$

and define

$$AF^T \triangleq rTA. \tag{3-24}$$

Then, combining the last four equations,

$$CS^T = AF^T + DP - HG. \tag{3-25}$$

This relation says the value of capital services from an asset equals the allowance for finance on the asset minus holding gains. It may be alternatively written to show that gains equal losses in this accounting system;

$$CS^T + HG = DP + AF^T,$$

which says current account yield (CS^T) plus capital gains (HG) equals depreciation and depletion expense (DP) plus carrying charges (AF^T). This result is at the heart of Jorgenson's development of capital theory [6].

Special Case: The Constant Rate of Depreciation of Net Capital

Jorgenson's treatment can be exploited to provide a computationally simple set of accounts in the case that the rate of depreciation of net capital is constant. Define economic depreciation as the replacement value of physical depreciation (δK),

$$DP \triangleq \delta pK. \tag{3-26}$$

Now the value of services from a unit of capital, called the capital charge factor or the user cost of capital services, c, is,

$$c \triangleq AF + DP - HG, \tag{3-27}$$

$$c = rp + \delta p - p.$$

This may be immediately integrated (given the boundedness condition) to give,

$$p_t = \int_t^\infty e^{-\int_t^\tau (r_z + \delta)dz} c_\tau d\tau \tag{3-28}$$

$$= PV[c; r + \delta].$$

Thus, the asset price equals the discounted and depreciated sum of its service values.

A similar relation between the value of tangible assets and the value of their services holds. Given:

$$TA = PV[CS^T - CE^T],$$

and

$$CS^T = cK = (rp + \delta p - \dot{p})K,$$

$$CE^T = p\dot{K} + \delta pK,$$

we have

$$TA = PV[rp + \delta p - p)K-(p\dot{K} + \delta pK)]$$

$$= PV[rpK - \dot{p}K - p\dot{K}],$$

$$= - \int_t^{\infty} \frac{d}{d\tau} (e^{-\int_t^{\tau} r_z dz} p_\tau K_\tau) d\tau \ ,$$

$$= -e^{-\int_t^{\tau} r_z dz} p_\tau K_\tau \Big]_t^{\infty} ,$$

$$= p_t K_t \text{ (given the boundedness condition).} \qquad (3\text{-}29)$$

This says the balance sheet valuation of tangible assets equals the market value of the net tangible capital stock.

In fact, with these assumptions we may now develop the balance sheet and income statement for the constant rate of depreciation case with complete computational operationality. Given time paths for,

- OY - operating income (revenue, RV, minus operating cost, OC)
- K - net tangible capital stock,
- CE^I - capital expenditure on intangible assets,
- r - the discount rate,
- p - the price of the tangkble asset,
- δ - the (constant) rate of physical depreciation of tangible capital,

define

$$NW = PV[DIV] = PV[OY - CE],$$

$$= PV[OY - CE^I - cK] + PV[cK - p(\dot{K} + \delta K)],$$

$$= IA + pK, \tag{3-30}$$

where

$$IA \triangleq PV[OY - CE^I - cK]. \tag{3-31}$$

Thus the intangible asset valuation is revealed as the present value of operating income minus intangible capital expenditure minus the appropriate charge for services of tangible capital.

This information may be calculated and presented in the balance sheet and income statement shown in Figures 3 and 4.

BALANCE SHEET

Assets	Notation	Calculation
Tangible assets	TA	pK (=$PV[cK - CE^T]$)
+ Intangible assets	IA	+ $PV[OY - CE^I - cK]$
Net Worth		
	NW	= $PV[DIV]$
		= $PV[OY - CE]$

Figure 3. The economic balance sheet -- constant depreciation rate case

INCOME STATEMENT

	Notation	Calculation
Revenue	RV	RV
- Operating cost	OC	$-OC$
= Operating income	OY	$=OY$
+ Holding gain on tangible assets	HG	$+\dot{p}K$
+ Allowance for finance on intangible assets	AF	$+rIA$
- Depreciation and depletion	DP	$-\delta pk$
- Amortization	AM	$-(OY - cK)$
= Income	INC	$= rIA+(c + \dot{p} - \delta p)K$
		$= r(IA + pK)$
		$= rNW$

Figure 4. Economic income statement--
the constant depreciation rate case

Finally the cash statement (more properly, the flow of funds statement or the statement of changes in financial position) in the traditional financial form, showing adjustments to income, is presented in Figure 5. (Net change in cash equals zero because there is no working capital and thus no changes in working capital).

To recapitulate, for the special case of a constant rate of depreciation for net capital stock (which allows analytic evaluation of the appropriate integrals), a computationally complete specification of the economic balance sheet, income statement, and cash statement is given in Figures 3, 4, and 5.

CASH STATEMENT

Sources	Notation	Calculation
Income	INC	rNW (=rIA + rpK)
- Non-cash income	- (HG + AF)	- ($\dot{p}$K + rIA)
+ Non-cash expense	+ (DP + AM)	+ (δpK + rIA)
= Total sources	Σ	= rpK - $\dot{p}$K + δpK + OY - cK
		= OY

Uses	Notation	Calculation
Dividends	DIV	OY - CE
+ Capital expenditure	+ CE	+ CE
= Total uses	Σ	= OY

Figure 5. Cash statement--constant rate of depreciation case

IV. ECONOMIC ACCOUNTS FOR THE RESOURCE PROCESS

We now address the main problem of this paper, construction of the economic accounts for the resource production process. The structure of the resource process is described in the next section. In the following section, the value of each asset which is distinguished in the process is analyzed. An asset's economic value is shown to have simultaneously a retrospective and prospective interpretation. Retrospectively, it represents replacement cost; prospectively it represents the present value of the operating income stream which the asset generates.

The accounts for the resource process are developed in the final section of this chapter. These accounts for developed by assuming optimizing competitive behavior for the process as whole and using the shadow prices on the constraints relating activities as the appropriate transfer prices between the activities.[15] Shadow prices on inventory requirement constraints are the unit production allowances for financing these inventories net of holding gains.

The Resource Production Process

The resource production process for which we develop economic accounts is depicted in Figure 6 and a glossary of the notation is in Figure 7. Figure 6 shows three aspects of the process. The diagram on the left shows value flows between the activities in the process and between the process as a whole and the outside world (generally, quantities are on the left and actual or imputed prices are on the right of an arc representing value flow).

The center column gives the dividend flow from each activity and from the entire process. These flows are implied by the conservation of value (and the absence of cash accumulation) in each activity.

The constraints characterizing each activity and the (imputed) prices measuring the value-added they induce are given on the right of Figure 6.

The process is divided into three vertically integrated activities: Production of the final resource product (say crude oil), Development (finding proven reserves from a pool of resources under development), and Ownership and Exploration (selling and exploring property rights to undiscovered resources for exploration).

There is a slightly unnatural aspect of this description. Capital expenditure is shown as an input to Ownership and Exploration. This is done to internalize the relationship between capital requirements and resource exhaustion. On the other hand, Development shows no inputs but developing resources and adds value only by holding them in inventory. But since Ownership and Exploration requires no inventory, this is equivalent to showing capital expenditures occurring at the beginning of the development activity.[16]

In the Production activity, proven reserves, R, are added to by finding, f, and are depleted by production, q, so

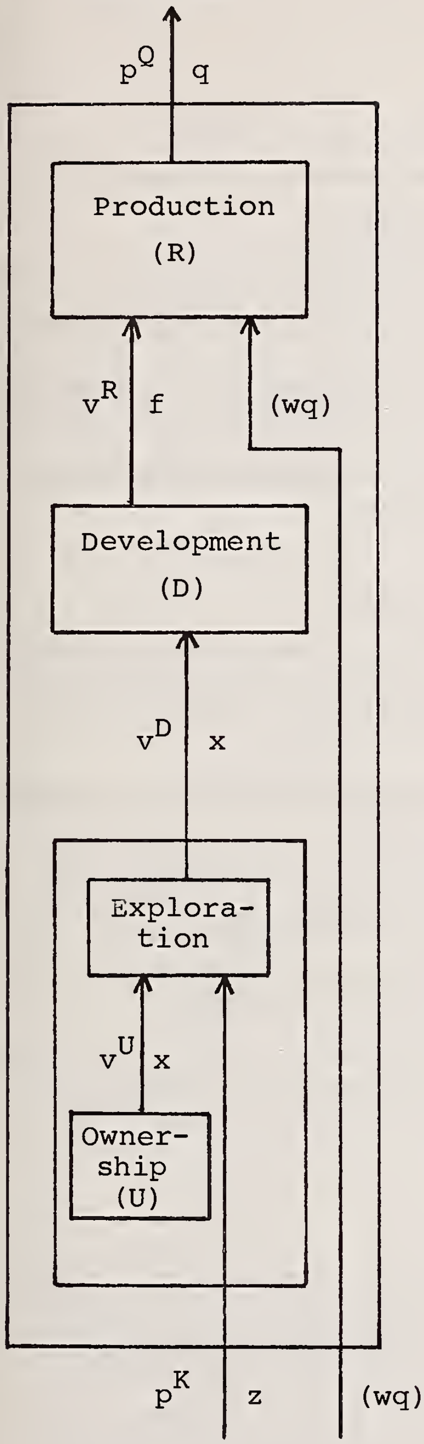
$$\dot{R} = f - q.$$

Production is constrained by the level of reserves divided by i^R, which is the reserve inventory required to produce one unit per period (the reserve-production ratio), thus[17]

$$q = R/i^R.$$

This inventory constraint adds value a^R to production as an allowance for financing the required inventory of proven reserves (net of holding gains). Thus the dividend flow from Production is

STRUCTURE	DIVIDENDS	CONSTRAINTS



$$DIV^R = p^Q q - wq - v^R f$$
$$= y^Q q - v^R f$$

$(v^R)\quad \dot{R} = f - q$

$(a^R)\quad q \leqq R/i^R$

$$DIV^D = v^R f - v^D x$$

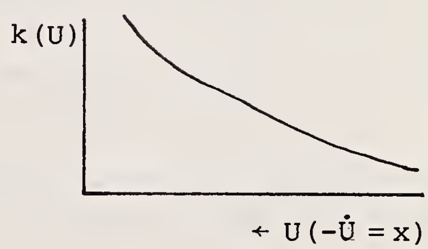
$(v^D)\quad \dot{D} = x - f$

$(a^D)\quad f \leqq D/i^R$

$$DIV^U = v^D x - p^K z$$

$(v^U)\quad \dot{U} = -x$

$z = k(U)\,x,\ k' \leqq 0$



$$DIV = y^Q q - p^K k(U)\,x$$

Figure 6. The resource production process

<u>Physical flows</u>	<u>Flow prices</u>
q - production (bbl/yr)	$p^Q w = y^Q$ operating income ($/bbl)
f - finding proven reserves (bbl/hr)	v^R - proven reserve value ($/bbl)
z - investment in undiscovered resources (units/yr)	p^K - capital goods price ($/unit)
x - acquisition and exploration of undiscovered resources (bbl/yr)	v^U - undeveloped resource value ($/bbl)
<u>Physical assets</u>	<u>Asset prices</u>
R - proven reserves (bbl)	v^R - proven reserve value ($/bbl)
D - developing resources (bbl)	v^D - developing resource value ($/bbl)
U - undiscovered resources (bbl)	v^U - undeveloped resource value[a] ($/bbl)
<u>Technical ratios</u>	<u>Finance allowances</u>
i^R - required inventory of reserves unit of production (yr)	a^R - finance allowance for proven reserve[b] ($/bbl)
i^D - required inventory of developed reserves/unit of reserves produced	a^D - finance allowance for developing resource[b] ($/bbl)
k(U) - unit capital requirement for developing the marginal undeveloped resource (unit/bbl)	

a. lease bonus plus royalty commitments

b. net of holding gain

Figure 7. Structured glossary of quantities and prices

$$DIV^R = p^Q q - wq - v^R f,$$

$$= y^Q q - v^R f,$$

where y^Q is operating income.[18]

There is a symmetric though slightly less familiar characterization of Development. Resources under development, D, are added to by exploration of new resources, x, and "depleted" by finding proven reserves, f, so

$$\dot{D} = x - f.$$

Finding is constrained by the level of developing resources and the required per-unit inventory i^D, so

$$f = D/i^D.$$

The inventory constraint adds value to proven reserves through an allowance for finance during construction, a^D. Thus the cash flow to Development is

$$DIV^D = v^R f - v^D x.$$

Finally Exploration, x, depletes undiscovered resources,

$$\dot{U} = -x,$$

and as the graph in Figure 6 shows, this depletion increases the required capital per unit of resources explored as the stock of undeveloped resources declines.

In summary, the resource production as a whole faces external prices for product, or operating factors for Production (normalized per-unit output) and for capital factors of Ownership and Exploration. The product of one activity within the process adds to the asset base of the next.

Exploration depletes Ownership's stock of undeveloped resources and adds to Development's developing resources. In addition to the exhaustion premium (the lease bonus) it adds value through the purchase of capital factors. Development adds value through the allowance for financing the required inventory of developing resources. Development's product, the finding of proven reserves adds to Production's inventory of proven reserves. The requirement for Production to hold inventory at at least a minimum reserve-Production ratio adds value through the allowance for financing the reserve inventory. The relationship among flows, stocks, constraints, and prices is illustrated in the layout of the glossary.

Resource Process Valuation

The task before us now is to derive the asset transfer prices and inventory holding costs for the activities in the resource process. Then with these prices we can evaluate the net worth of each activity in the resource process and construct the balance sheet and income statement recognizing the assets distinguished in the process description.

The analysis strategy is to derive the optimum behavior of the resource firm with the structure described. Characterization of this behavior provides shadow prices on the constraints describing interactivity transactions (asset transfer prices) and activity inventory requirements (finance allowances). (Note that the focus here is on prices--necessary relations between values when an optimum exists--rather than on quantities, the usual objects of attention in this kind of derivation.)

Figure 8 presents this derivation. In Equation 4-1 net worth is defined as the present value of the dividend stream qiven optimal behavior. Equations 4-2 through 4-4 represent the asset acquisition/depletion process, and Equations 4-5 and 4-6 represent the inventory requirements of the process depicted in Figure 6. Constructing the Hamiltonian, we find the conditions which obtain if optimum behavior exists (see e.g., Arrow [1]).

Equations 4-7 through 4-9 establish the necessary relations between product prices and input prices. Equations 4-10 and 4-11 quantify the inventory costs and Equation 4-12 quantifies the price of property rights to the marginal unit of undiscovered resources (the lease bonus). Equation 4-13 imposes the boundedness conditions which are required to make the present value calcualtion meaningful.[19]

Asset Prices

We now examine these pricing relationships in detail.[20] There are two main results characterizing asset prices. The first is that the prices of the products of an activity must cover replacement costs, including depletion and depreciation plus the

$$NW = \max_{f,q,x} PV[p^Q - w)q - p^K k(u)x] \tag{4-1}$$

Subject to:

$$(v^R)\ \dot{R} = f - q \quad , \quad q \geqq 0, \tag{4-2}$$

$$(v^D)\ \dot{D} = x - f \quad , \quad f \geqq 0, \tag{4-3}$$

$$(v^U)\ \dot{U} = -x \quad , \quad x \geqq 0, \tag{4-4}$$

$$(a^R)\ q \leqq R/i^R, \tag{4-5}$$

$$(a^D)\ f \leqq D/i^D, \tag{4-6}$$

given initial R, D, and U > 0.

$$\mathcal{H} = F(r)\ [(p^Q - w)q - p^K k(U)x + v^R(f - q) + v^D(x - f) - v^U x$$
$$+ a^R(R/i^R - q) + a^D D/i^D - f)].$$

$$\frac{\partial \mathcal{H}}{\partial e} \leqq 0 \Rightarrow y^Q = (p^Q - w) \leqq v^R + a^R, \text{ "<" } \Rightarrow q = 0, \tag{4-7}$$

$$\frac{\partial \mathcal{H}}{\partial f} \leqq 0 \Rightarrow \quad v^R \leqq v^D + a^D, \text{ "<" } \Rightarrow f = 0, \tag{4-8}$$

$$\frac{\partial \mathcal{H}}{\partial x} \leqq 0 \Rightarrow \quad v^D \leqq v^U + p^K k, \text{ "<" } \Rightarrow x = 0, \tag{4-9}$$

$$-\frac{\partial \mathcal{H}}{\partial R} = \dot{v}^R - rv^R = -a^R/i^R \text{ and } a^R > 0 \Rightarrow q = R/i^R, \tag{4-10}$$

$$-\frac{\partial \mathcal{H}}{\partial D} = \dot{v}^D - rv^D = -a^D/i^D \text{ and } a^D > 0 \Rightarrow f = D/i^R, \tag{4-11}$$

$$-\frac{\partial \mathcal{H}}{\partial U} = \dot{v}^U - rv^U = p^K k' x = -p^K k' U = -p^K \dot{k}. \tag{4-12}$$

$$F(r)v^R R \rightarrow F(r)v^D D \rightarrow F(r)v^U U \rightarrow 0 \text{ as } t \rightarrow \infty. \tag{4-13}$$

Figure 8. Optimal behavior - the derivation of the value of assets and inventories

inventory finance allowance. This is essentially a retrospective view of prices. The second result is prospective. It is that an asset's value is the present value of the operating income stream it generates.

The characterization of asset prices as replacement costs is presented in Figure 9. There are three activity products in the system: final product, proven reserves, and developing resources. For each product, its price (or unit operating income) must cover depletion cost (the "v" terms) plus the capital cost of production. For the first two products, capital costs take the form on inventory allowances (the "a" terms). These allowances equal the required inventory level (per unit of production) times the finance charge minus holding gain.

<u>Asset/product prices</u>

$$y^Q = v^R + a^R, \text{ if } q > 0, \tag{4-14}$$

$$v^R = v^D + a^D, \text{ if } f > 0, \tag{4-15}$$

(operating income) = (depletion) + (inventory allowance).

$$v^D = v^U + p^K k, \text{ if } x > 0, \tag{4-16}$$

(price) = (depletion) + (unit depreciation of capital).

<u>Inventory allowance</u>

$$a^R = i^R \ (rv^R - \dot{v}^R), \tag{4-17}$$

$$a^D = i^D \ (rv^D - \dot{v}^D), \tag{5-18}$$

(inventory allowance) = (inventory requirement) x

(unit finance allowance - holding gain).

Figure 9. Product prices at replacement cost

The second result is the forward valuation of assets. There are three assets in the resource production system: proven reserves, developing resources, and undeveloped resources. Each is worth the present value of the operating income it produces (or the cost it avoids). These results are presented in Figure 10. For example, the first equation (4-19) says that if future production is positive,[21] the value of a unit of proven reserves (v^R) equals the present value of its operating income (y^Q) times the decline rate (d^R, the level of initial production) discounted by the discount rate plus the decline rate. The derivation in Figure 10a shows this to be equal to the present value of the production stream from a unit of reserves times unit operating income. The second equation of Figure 10 (4-20) is a similar statement about the value of a unit of developing resources.

The evaluation of undeveloped resources (Equation 4-21) is in terms of opportunity cost rather than replacement cost (exhaustability implies uniqueness). When an additional unit of undeveloped resources is exploited, the world must make do with inferior remaining deposits. A time path of increased unit capital costs,

$$p_t^K \dot{k}_t dt = p_t^K (dk/dt) dt = p_t^K dk = p_t^K k' dU_t \quad ,$$

is imposed on reproducible capital by the exhaustion of an incremental unit of undeveloped resources. Thus the value of the marginal undeveloped resource is the present value of the profile of these costs.[22]

Balance Sheet

The economic balance sheet for the resource process is displayed in Figure 11. We partition the dividend stream for the process as a whole into the dividends (operating income minus capital expenditures) for each activity (from Figure 6). Then the net worth of each activity (the present value of the dividend stream equals the market value of the asset it possesses). Specifically, the net worth of the Production activity equals the value of the proven reserve inventory (this is proven in Figure 11a); the net worth of the Development activity equals the value of the inventory of developing resources; and the net worth of Ownership and Exploration equals the present value of the future stream of lease bonuses generated by exploration.

(The proof in Figure 11a collects the four necessary conditions for price and quantity behavior and shows they imply that the dividend stream can be integrated to give the initial asset value. Jorgenson's development of neoclassical capital theory [6] entails such a step.)

Given: $y^Q = v^R + a^R,$ if $q > 0,$ (4-7)

$a^R = (rv^R - \dot{v}^R)/d^R,$ (4-8)

then: $v^R = PV[y^Q d^R;\ r + d^R]$ if $q > 0$, a.e.[a] (4-19)

Given: $v^R = r^D + a^D,$ if $f > 0,$ (4-9)

$a^D = (rv^D - \dot{v}^D)/d^D,$ (4-10)

then: $v^D = PV[v^R d^D;\ r + d^D],$ if $f > 0$, a.e.[a] (4-20)

So: (asset value) = unit production discounted at the interest rate plus decline rate.

Given: $\dot{v}^U - rv^U = -p^K\dot{k},$

then: $v^U = PV[p^K\dot{k}].$ (4-21)

So: (undeveloped resource value) = (value of induced exhaustion).

a. This condition is discussed in Footnote 19.

Figure 10. Forward valuation of assets

Recall

$$y^Q = v^R + a^R, \qquad \text{if } q > 0, \qquad \text{(from 4-14)}$$

$$a^R = i^R(rv^R - \dot{v}^R), \qquad \text{(from 4-18)}$$

hence,

$$\dot{v}^R = rv^R = -d^R(y^Q - v^R),$$

$$\dot{v}^R = (r + d^R) = -d^R y^Q,$$

so: $$v^R = PV[y^Q d^R;\ r + d^R] \qquad \text{if } q > 0, \text{ a.e.},$$

$$v^R_{t_0} = \int_{t_0}^{\infty} e^{-\int_{t_0}^{t} (r_z + d^R)dz}\ y^Q_t d^R dt,$$

$$= \int_{t_0}^{\infty} e^{-\int_{t_0}^{t} r_z\, dz}\ y^Q_t (d^R e^{-d^R(t - t_0)})dt,$$

$$= PV[y^Q \hat{q}],$$

where

$$\hat{q}_{t,t_0} = d^R e^{-d^R(t - t_0)},$$

is the production at time t from a unit of reserves put into production at time zero.

Figure 10a. Derivation of proven reserve values

$$DIV \triangleq p^Q q = p^K kx,$$

$$= DIV^R + DIV^D + DIV^U,$$

where:

$$DIV^R \triangleq p^Q q - v^R f,$$

$$DIV^D \triangleq v^R f - v^D x,$$

$$DIV^u \triangleq v^D x - p^K kx,$$

then:

$$NW^R \triangleq PV[p\ ^Q q - v^R f] = v^R R, \tag{4-2}$$

$$NW^D \triangleq (v^R f - v^D x] = v^D D, \tag{4-3}$$

$$NW^U \triangleq PV[v^D x - p^K k] = PV[v^U x].$$

BALANCE SHEET

Assets	Liab. & NW
$v^R R$	
$v^D D$	
$PV[v^U x]$	NW

Figure 11. Balance Sheet

Recall:

$$R = q - f, \tag{4-2}$$

$$q = R/i^R, \text{ "<" } \Rightarrow a^R = 0, \tag{4-5}$$

$$y^Q = v^R + a^R, \text{ "<" } \Rightarrow q = 0, \tag{4-7}$$

$$a^R = i^R\,(\dot{v}^R - rv^R). \tag{4-10}$$

Then:

$$DIV^U = y^Q q - v^R f,$$

$$= (v^R + a^R)q - v^R f, \qquad \text{(from 4-7)}$$

$$= a^R\,R/i^R = v^R(q - f), \qquad \text{(from 4-5)}$$

$$= (\dot{v}^R - rv^R)\,R - v^R\dot{R}, \qquad \text{(from 4-10 \& 4-2)}$$

so,

$$F(r)DIV^U = -\frac{d}{dt}[F(r)v^R R].$$

Integrating both sides and imposing the convergence conditions gives,

$$NW_{t_o} = -F(r)v^R R]_{t_0}^{\infty} = v^R_{t_0}\;R_{t_0}\;.$$

Figure 11a. Proof of the balance sheet for production

Income Statement

The evaluation of income for each activity is given in Figure 12. Given activity dividends (from Figure 6) and net worth (from Figure 6) and net worth (from Figure 11), income is defined as dividends plus the change in net worth. Income can be equivalently expressed as

INC = operating income - depletion + holding gains,

or

INC = inventory allowance + holding gains,

or

INC = allowance for finance on net worth.

These relationships are derived for the Production activity in Figure 12a and are assembled in Figure 13. These activity income statements contain interactivity transactions which net out in the consolidated income statement or the resource process as a whole.

The consolidated income statement is described in Figure 14. There, income comprises [operating income minus depletion on production] plus [the inventory allowance on finding new reserves and the finance allowance on future lease bonuses] plus [holding gains on inventories of proven reserves and developing resources]. The associated cash statement is presented in Figure 15, revealing that the consolidated resource production process takes in operating income on production and pays out dividends plus capital expenditure. This is consistent with the process specification in Figure 6.

To recapitulate, this Section derives economic accounts and asset pricing for the resource process designated in Figure 6. The derivation depends on the imputation of the competitive value of the assets and asset inventories found in the resource process. This imputation of value is accomplished through the device of imputing optimality to observed behavior deriving the necessarily implied valuation (in Figure 8).

Once asset values and inventory allowances are in hand, the balance sheet and income statement follow from the valuation of assets and the principle that total income equals income distributed plus income retained.

Given:

$$INC \triangleq INC^R + INC^D + INC^U,$$

we have:

<u>Production</u>

$$INC^R = DIV^R + \dot{NW}^R \doteq y^Q q - v^R f + \dot{(v^R R)},$$

$$= y^Q q - v^R q + \dot{v}^R f \text{ (= operating income - depletion + holding gain),}$$

$$= a^R q + \dot{v}^R R \text{ (= inventory allowance + holding gain),}$$

$$= rv^R R = rNW^R = \text{(allowance for finance on net worth;}$$

<u>Development</u>

$$INC^D = DIV^D + \dot{NW}^D = v^R f - v^D x + \dot{(v^D D)},$$

$$= v^R f - v^D f + \dot{v}^D D,$$

$$= a^D f + \dot{v}^D D,$$

$$= rv^D D = rNW^D;$$

<u>Ownership and Exploration</u>

$$INC^U = DIV^U + \dot{NW}^R = v^D x - p^K kx + \dot{PV}[v^U x],$$

$$= rPV[v^U x] \doteq rNW^U.$$

Figure 12. Evaluation of income by activity

Given:

$$INC^R = DIV^R + \dot{NW}^R = y^Q q - v^R f + (v^R F),$$

the first condensation nets out net investment, i.e.,

$$(v^R R - v^R f) = \dot{v}^R R + v^R \dot{R} - v^R f,$$

$$= \dot{v}^R R + v^R (q - f) - v^R f, \qquad \text{(from 4-2)}$$

$$= \dot{v}^R R - v^R q,$$

leaving holding gains minus depletion as the "non-cash" change in capital stock.

So:

$$INC^R = y^Q q - v^R q + \dot{v}^R R.$$

Then since

$$y^Q - v^R = a^R \quad \text{or} \quad q = 0, \qquad \text{(from 4-7)}$$

we have $INC^R = a^R q + \dot{v}^R R$.

Finally since

$$a^R = i^R (rv^R - \dot{v}^R), \qquad \text{(from 4-10)}$$

$$i^R q = R \quad \text{or} \quad a^R = 0, \qquad \text{(from 4-10 \& 4-5)}$$

we have $INC^R = rv^R R$.

Figure 12a. Proof of income evaluation for production

Gross income statement	Production	E&F	Ownership	Total
Operating income	$y^Q q$	$+v^R f$	$+v^D x$	
- Depletion & depreciation	$-v^R q$	$-v^D f$	$-p^K kx$	
+ Holding gain	$+\dot{v}^R R$	$+\dot{v}^D D$	$+\dot{NW}^U$	
= Income	INC^R	INC^D	INC^U	$=INC$
Capital charges income statement				
Inventory charges against production	$+a^R q$	$+a^D f$	$+v^U x$	
+ Holding gains	$+\dot{v}^R R$	$+\dot{v}^D D$	$+\dot{NW}^U$	
= Income	INC^R	$+INC^D$	$+INC^U$	$= INC$
Allowance for finance on assets				
= AFDC	$rv^R R$	$+rv^D D$	$+rNW^U$	$= rNW$
Memo: Balance sheet				
Assets =	$v^R R$	$+v^D D$	$+PV[v^U x]$	
NW =	NW^R	$+NW^D$	$+NW^U$	$= NW$

Figure 13. Activity income statement and balance sheet

Recall

$$NW = PV[DIV] = PV[y^Q q - p^K kx] = v^R R + v^D D + PV[v^U x],$$

$$INC = DIV + \dot{NW}.$$

It can be shown that

$$INC = y^Q q - v^R q + a^D f + rPV[v^U x] + \dot{v}^R R + \dot{v}^D D,$$

so that we can form the:

CONSOLIDATED INCOME STATEMENT

Operating income	$y^Q q$
- Depletion	$- v^R q$
+ AFDC	$+ a^D f + rPV[v^U x]$
+ Holding gain	$+ \dot{v}^R R + \dot{v}^D D$
= Income	= INC

Figure 14. Derivation of the consolidated income statement

CONSOLIDATED CASH STATEMENT	
<u>Source</u>	
Income	$y^Q q - v^R q + AF + HG$
- Non-cash income	$-(AF + HG)$
+ Non-cash expenses	$+ v^R q$
= Total sources	$y^Q q$
<u>Uses</u>	
Dividends	$y^Q q - p^K kx$
+ capital expenditure	$p^K kx$
= Total uses	$y^Q q$

Figure 15. The consolidated cash statement

V. CONCLUSION

Section IV presents a detailed development of the economic accounts of the resource process. The derivation clarifies the role of non-cash income items (financial allowances and holding gains) and non-cash expenses (depletion and amortization) which are so prominent a part of valuation in this process. The method of Section IV, expanded to include the "intangible assets" treatment of Section III can be used to provide an economic accounting for any behavior associated with the management of any production process.

FOOTNOTES

*I am grateful to Vince Calarco, Ed Cazalet, Harvey Greenberg, Bill Hogan, Dale Jorgenson, Pat Keenan, George Lady, Rao Mangipudi, Fred Murphy, and Bob Pendley for discussions. The material developed here is an extension of the theory of capital developed by Jorgenson, described in [6]. In fact the basis of the balance sheet presented here is Equations 12 and ff. of [6]. I regard the contribution here to be the discovery of the appropriate definition of income and the exploitation of Jorgenson's "user cost of capital services" in elaborating the income statement. Errors and ambiguities are mine.

1. Retained earnings as used here include new equity financing. Negative dividends represent the investment of equity capital by owners.

2. An empirically applicable analysis admitting general treatment project investment lead structure, service life, and tax regime is being developed and programmed and will be reported later (see [8]).

3. This certainty-equivalent valuation may be an ingredient of a general analysis of risk and yield through a decision tree analysis, which is the formal economic version of risk analysis.

4. Leveraged by the long lead times involved, special tax treatment for resource firms, especially tax expensing of intangible drilling costs, enhances the divergence between economic and conventional accounting measures.

5. The "cost" and "realization" principles not only adhere to the principle of conservatism, they also follow the maxim: "Anticipate no gains; provide for all losses." The historical-cost approach, as modified (by the lower-of-cost-or-market rule), also is believed to result in objective figures free from personal, subjective bias. A consequence of this approach is that the accounting literature is filled with admonitions to refrain from connecting accounting measures with valuations:

    > The auditor's...report expresses an opinion that the balance sheet presents fairly the financial position of a company in conformity with generally accepted accounting principles. It is incorrect to conclude

from this, however, that a balance sheet is a statement of financial position in the sense of showing the value of a business (see [3, 23-24]).

Nor, it is fair to add, does the balance sheet, in general, show the value of anything the business owns or owes which is in the nature of a long-lived entity.

6. There is some recognition of these items in public utilities accounting where asset valuation is central to cost-of-service or rate-base/rate-of-return pricing.

7. Recognition of gains and losses is asymmetric in conventional accounting. Recognition of losses is encouraged by the "lower-of-cost-or market" valuation rule.

8. Tax relief includes investment tax credits and the excess of percentage depletion over cost depletion.

9. If you consider the oil production process to begin with the geological and geophysical evaluation of provinces and structures, and to end with the closing of the last stripper well, a process that can take 50 years or more, the average value of goods-in-process may amount to five to twenty years of production, where manufacturing typically has substantially less than a year's production of goods in process. (Reserve-production ratios for petroleum projects start at seven years and go up, and reserves are declared to be proven on average toward the end of the exploration and development process, so this measure excludes resources in prior stages of development. On the other hand reserves are worth less than production, so the quantity ratio overstates the value ratio.)

   The difference in the goods-in-process/production ratio between the petroleum and manufacturing industries is partly due to a classification difference and is partly intrinsic. The exploration and production of oil are considered as a vertically integrated activity, while the construction of, say, shoe factories and the production of shoes are not. On the other hand both the "construction" and production phase take longer in the oil business.

   The importance of the classification differences is that in the integrated petroleum industry, AFDC in the construction phase is never realized in the conventional account books. Similarly, in the longer-lived petroleum industry, imputed holding costs on the requisite reserve inventory are not capitalized.

10. We won't distinguish new equity as sources of funds, so negative dividends imply a capital levy on owners.

11. The nomenclature of the conventional accounting classification of assets as tangible and intangible is, I believe, misleading. Corbin [3, 28-29] says:

> The main types of <u>intangible assets</u> are: patents, copyrights, trademarks, tradenames, secret processes, leaseholds, licenses, franchises, corporate organization costs, stock promotion costs, and goodwill.

Whether these things differ from property, plant, and equipment in their tactile properties is not the point. What matters is the commensurability or fungibility of their quantity -- can the concept of usage or capital service expiration be made sufficiently and acceptably concrete so that a depreciation schedule for financial expensing can be published and adhered to. Intangible assets are those which by convention cannot be depreciated. The only measurement of quantity is value, and thus intangible assets are amortized, often on an admittedly arbitrary basis. Of course in the economic treatment, the measurement of depreciation and amortization is based on the expiration of the value of capital services (cf. Hotelling [1] and Jorgenson [7]), so from an economic point of view the difference between depreciation and amortization is unimportant.

12. This argument follows Jorgenson [6].

13. The demonstrations here are carried through in continuous time because differentiating is easier than differencing. For implementation as an accounting system a discrete time formulation is required. The following accomplished this conversion. (The machinations in what follows are required to get the dating in the change equation to be contemporaneous.) Define the (t-1) closing balance,

$$V_{t-1} = \sum_{\tau=t}^{\infty} F(t,\tau)X_{\tau}$$

where

$$F(t, t+1) = (1 - r_t)$$

$$F(t,\tau) = \Pi_{j=t}^{\tau-1} \ (1-r_j)$$ (The product over the null range is defined as unity.)

Then

$$V_{t-1} = X_t + (1 - r_t) \sum_{\tau=t+1}^{\infty} F(t+1, \tau) X_\tau ,$$

or

$$\Delta V_t = r_t V_t - X_t .$$

Thus the difference equation and its "integral" above can replace the differential equations and present value integrals of the text. The discount factor $(1-r_t)$ is not usually encountered in present value calculations; rather one sees $(1 + r_t^*)^{-1}$ as the one-period discount factor, where r_t^* might be called the coupon discount rate. That is, an instrument yielding a coupon r^*, and so worth $1 + r^*$, at the end of one period, discounted by $(1 + r^*)^{-1}$ to the present is worth 1 today. If r is the contemporaneous discount rate so that

$$(1 - r) = (1 + r^*)^{-1},$$

then

$$r = r^*/(1 + r^*),$$

the contemporaneous discount rate is the discounted value of the coupon discount rate.

14. My friend Patrick Keenan, a financial analyst, holds to the view that, "Income is what Price-Waterhouse says it is!"

15. The extent to which institutions and behavior differ from these assumptions can be captured in a residual "intangible assets" account as in Section III of this paper.

16. An empirically usable implementation would distinguish many stages of development and might show capital (pre-inventory constraint) outlays and operating (post-inventory constraint) outlays at any level.

17. This formulation finesses the "vintaging" that is required in more general treatment and suffices for our expositional purposes. The resource-finding constraint, like the reserve-production constraint, can be regarded as a simple

approximation to a deterministic time profile for extraction or for development. In fact the reserve decline rate

$$d^R = 1/i^R$$

can be interpreted as the probability that a given unit of reserves will be produced in the period; similarly, the developing resource-finding decline rate (discussed below)

$$d^D = 1/i^D$$

is the probability that a unit of resources under development will be proven in the period.

18. The price of operating factors, w, is per unit of production, subsuming the technical coefficient. The assumption here of independent operating costs, w, means that depletion is concentrated entirely in the quality decline of unexplored resources, not reserves. In reality, after discovery, reserves are declared proven (bankable is the operative word amongst independent producers) only if the production operating revenue covers development and operating costs. To represent this generally, depletion would degrade resource quality at all stages of production.

19. This is not strictly a necessary condition without further restrictions, which eliminate mathematical pathologies.

20. This is something of a diversion from the treatment of the accounts of an enterprise, but it shows that each unit of an asset can be considered an enterprise in itself (tax treatment, however, generates externalities) and justifies the use of these prices in the enterprise accounting of the next section.

21. If the future contains intervals where production, finding, or exploration are zero, the problem is more complex. The analysis would follow Arrow's treatment of irreversible investment [2] where, during "blocked periods," for speculative reasons the producers would wish to pump produced resources back into the ground. This happens when capital gains (plus "repletion") exceed the interest rate.

In the regulated natural gas industry of the U.S., this behavior is pejoratively called "withholding," an abrogation of service commitments. It was alleged that this behavior was rife when deregulation was in prospect. Though assiduously searched for, little evidence to support this claim surfaced. This implies either that American gas producers lack effective initiative or possess effective discretion.

22. The equation for resource value can be integrated to obtain an interesting equation for the value of a developing resource. That is,

$$v^U = PV[p^K\dot{k}]$$

integrates to

$$v^U = PV[k(rp^K - p^K k - \dot{p}^K],$$

which says that the value of a unit of developing resources equals the present value of the finance allowance net of holding gain on its acquistion cost through the future.

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Gulf Coast Undiscovered Resource Data Collection System

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INTRODUCTION

In the midst of the World's deteriorating energy supply, the Nation is increasingly relying upon conventional domestic offshore oil and gas resources to offset import dependency,. Investment in synthetic fuel projects are becoming more intimidating, with discouraging lead-times and escalating capital costs. A key element in federal energy policy analysis is the ability to estimate the conversion rate from petroleum resources to reserves.

The Energy Information Administration (EIA) within the Department of Energy (DOE) with Lewin and Associates, Inc. have been developing a model to analyze the impacts of policy changes on the rate of exploration, development and production of hydrocarbons from offshore regions.

This paper describes the data collection system applicable to 700 undeveloped prospects that provides model inputs concerning uncertainty in resources development. The model simulates the rate of exploration, development and production. A data collection protocol was devised to collect hard core and subjective estimates of undiscovered resource data for computer processing with probability distributions assigned key parameters. Objective resource element estimates were obtained on a prospect basis by means of standard engineering equations relating parameters to potential resources using Monte Carlo methods. Collection methodology and documentation are presented, stressing quantifying uncertainty inherent in the estimating process.

Use of conventional regional resource appraisals, distributional data, analogs and simulations have been inadequate to capture real-world site specific prospects containing exploratory and developmental criteria to estimate hydrocarbon potential and economic attractiveness. Purpose of the data collection project was to test the feasibility of constructing a disaggregated undiscovered resource prospect-specific data base for Gulf Coast to improve the validity of projections. The study is devoted to prospect geologic and engineering data developed from leasing tracts in water depths from surfline to 1000 meters or less. (Figure 1)

Company confidential files, records and maps of the Conservation Division of the U.S. Geological Survey (USGS) in Metairie, Louisiana provided the highly sensitive source data. Stringent security precautions were taken concerning data access, extraction and processing to maintain the integrity of the information. A team of experienced Gulf Coast geologists and consultants collected, transformed, interpreted and prepared subjective estimates for the Gulf Coast undiscovered resources. On site geological, geophysical, engineering and lease data were reviewed and used. In addition, consultations were held with USGS stratigraphic specialists regarding interpretation of data in deep water areas.

The data protocol is shown in Table 1, source of data element in Table 2, and the cover sheet used to bind fifty randomly numbered data sheet to maintain confidentiality in Table 3. The cover sheet and detached upper stub of data sheet remained with the USGS to protect against disclosure and to facilitate future data updating.

The prospect is the basic study unit. It is an exploration opportunity that would be evaluated as a single entity by an operator deciding to bid, explore or develop resources. It may consist of one or more potential closures, if more than one closure was considered, the relationships among respective closures was made explicit. That is to say, the prospect could be developed from a common platform and have common source rocks and/or were created by a common geologic phenomenom.

Data sources in the order appearing in Table 2 were as follows:

MAPS

Seismic:

(1) USGS prepared, evaluated for leasing areas adjacent to evaluated areas.

(2) Commercially prepared, of some leasing areas.

Evaluation:

Status map showing USGS sale number and leasing area.

Overall:

Large scale leasing area map of entire Gulf of Mexico.

Trend:

USGS prepared - depiciting aerial extent of recognized geologic age producing trends.

Status:

Section map for each of 26 official identified leasing areas commencing from the shoreline and extending to 1000 meters water depth. Tract data were superimposed showing if evaluated and noting lease sale number. Status of each tract was determined as to proved or dry, leased and drilled, leased and undrilled, or unleased.

Hazard:

USGS prepared indicating areas subsea stability of sediments regarding developmental hazards i.e., mudlumps, gas seeps, mudflows and deep seated faults that cut the shelf surface.

Bathymetric:

Bureau of Land Management, Department of the Interior prepared map indicating water depth distribution in Gulf of Mexico.

Interpretation/Calculations:

Data for tracts that had been evaluated by USGS, were transcribed from appropriate files. For areas adjacent to evaluated areas and those classed as other mapped areas, an appropriate evaluated analog was selected. These analogs were either near the prospect, of the same geologic age or at approximately the same vertical depth.

The following three columns in Table 2 refer to standard USGS forms in their evaluation files that contain engineering and physical parameters used as base data for the evaluated tracts and others estimated by analog.

In the interest of purposely modeling degrees of uncertainity the USGS leasing information (excluding lease sales 58 and 58-A, held in 1979) were assigned within four classes: (1) evaluated, (2) adjacent, (3) other mapped and (4) unmapped. Commencing with the most recent evaluated data, applicable relevant files and records were accessed for each tract. If a tract was considered a drainage or a downstructure tract of a proved oil or gas field, it was eliminated. These tracts are treated in a separate statistical analysis of inferred reserves and are not part of the undiscovered resources.

In adjacent tracts, USGS seismic maps were evaluated and tract data used as analogs for reservoir data. Closure areas were estimated from these maps using templates. For areas classed as other mapped, commercial seismic maps were used. The maps were covered with transparent paper; closures were identified; most likely areas drawn, and trap type assigned. The process was repeated for all horizons within the prospect. To determine the maximum reach of conventional platforms based on directional drilling to each horizon, a mathematical formula was derived. The radius of maximum reach was equated to the tangent of 55 degrees times the difference in average vertical depth and water depth. Number of platforms were determined on the basis of trap type and maximum drilling reach.

In the unmapped areas, interpretative data were assembled using analogs from evaluated or other mapped identified prospects.

Pipeline districts were established upon the assumption that each platform is equipped with 10 miles of gathering line.

Districts were assigned as follows:

1. A strip 10 miles wide from shore was defined and named minus one. If a platform is less than 10 miles from shore, it is assumed the pipeline will be built to shore.

2. The area demarcated by a 10 mile strip along both sides of existing trunklines was drawn. At the terminus of existing lines, a circle of 10 mile radius was drawn. This area was termed pipeline district "zero" (area served by existing trunk pipelines).

3. Areas not covered in (1) or (2) were divided into circles of about 20 miles radius or less, depending on the density of prospects. These designated areas were randomly numbered to prevent identification.

All pipeline district boundaries were made to conform to discrete tracts with each tract in one and only one pipeline district.

The data collection system described was applicable to approximately 60 percent of the Gulf offshore because of limitations in project time and resource availability. The remaining data were developed using a play-prospect analog method. For this approach, sub geographic areas are assumed to contain sufficient geologic similar counterparts. The geographic areas are termed "plays" of common geologic history. Analogs were assigned to these areas from evaluations and judgments of geologic trends designations and comparable exploration prognosis. These representative analogs were used in systemic sampling of the ratio of unproved areas of donor (leasing area for which data were available or could be estimated) and recipient (areas of common geographic and geologic composition for which no data are available) areas. Plans are to complete the Gulf sampling in fiscal year 1980.

The OCS Model concepts and implementation will be presented later in the symposium and also was presented at the 11th Annual Offshore Technology Conference in Houston, Texas on May 3, 1979. 1/ The approach uses these disaggregated resource data in a novel prospect by-prospect evaluation of costs with explicit treatment of risk.

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TEXAS - LOUISIANA
Gulf Coast Sampling Area

Figure 1

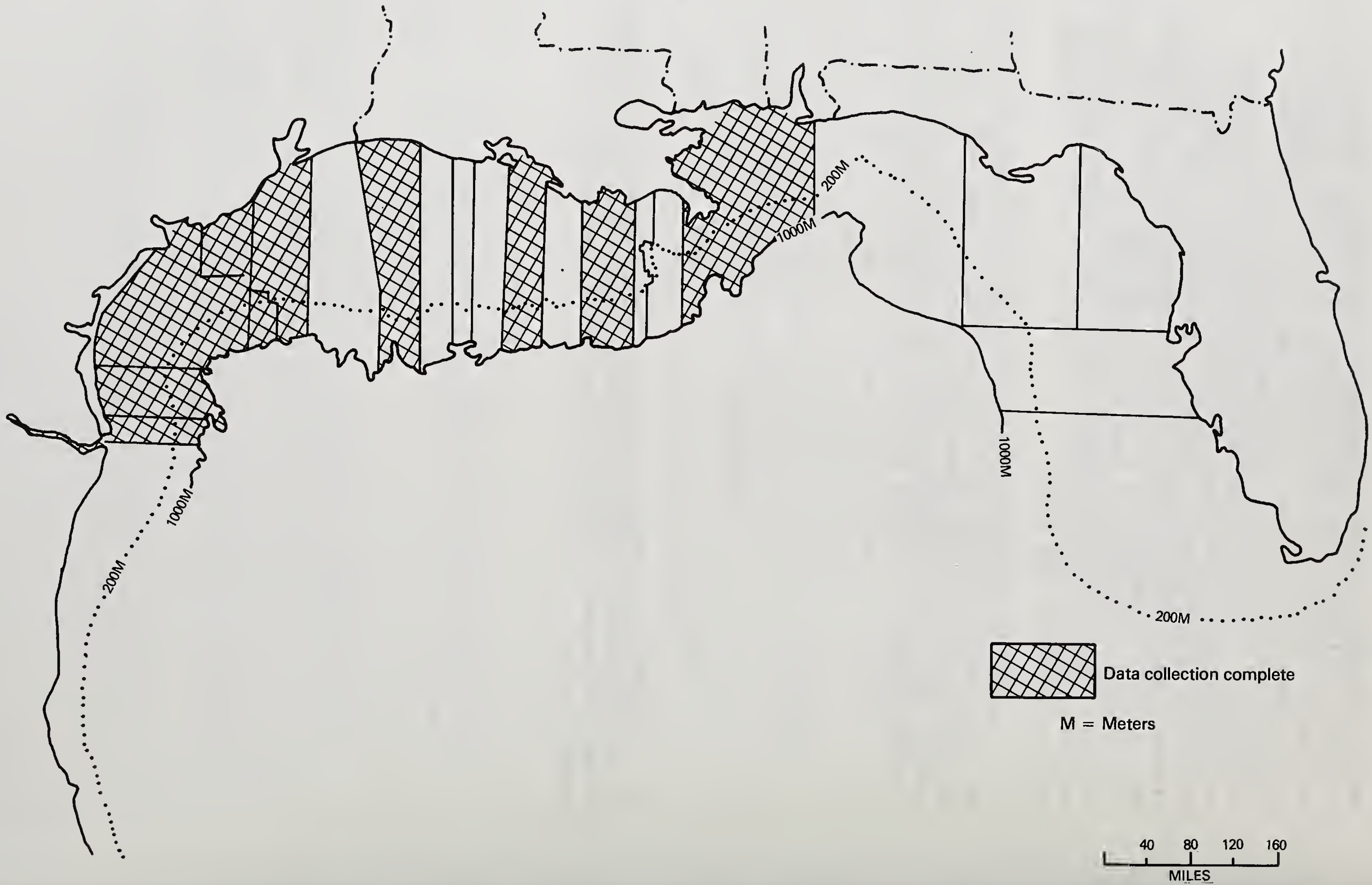

Table 1 - Data Protocol Sheet

FORM A
7/10/78

DOE/EIA OFFSHORE PROJECT

SOURCE DOCUMENT: SALE & PROSPECT ________ ________ MAP # ________

ID CODE ________ TRACT NUMBERS ________ HORIZON # ________

— — DETACH HERE — — — — — — — — — — — DETACH HERE — — — —

START CARD 1 ID CODE ________ SEQUENCE NO. ________

- OCEAN — 1 GULF, 2 ATL, 3 PAC
- SECTOR — SEE LIST OF SECTOR CODES
- TREND — 1 CRET, 2 MID, 3 PLIO
- PAY — 4 PLIO-PLEISTO, 5 PLEISTO, 6 OLIGO
- LEVEL — 1 UPPER, 2 MID, 3 LOWER
- PIPELINE DISTRICT — SEE PIPELINE DISTRICT MAP
- TRAP TYPE — 1 PIERCE, 2 DOMAL, 3 FAULT, 4 STRAT
- STATUS — 1 DELINEATED, 2 EVAL DRILLED, 3 EVAL UNDRILLED, 4 EVAL UNLEASED, 5 ADJACENT TO EVAL, 6 OTHER MAPPED
- TOTAL PAYS
- PAY # — VERT SEQ. NO.
- NO. TRACTS
- HAZARD CLASS

- LITH — 1=SS, 2=CB
- PROB OF HYDROCARBON — ESTIMATE
- PROB OF DRY — I-P (HYD)

END OF CARD I

START CARD 2

- NO. OF PLATS
- % UNDER 1ST PLAT
- % UNDER 2ND
- % UNDER 3RD
- % UNDER 4TH
- % UNDER 5TH
- % UNDER 6TH
- % UNDER 7TH
- % UNDER 8TH

END OF CARD 2

START CARD 3

- AVG VERTICAL DEPTH FT
- AVG DRILLED DEPTH FT
- WATER DEPTH FT
- DISTANCE TO GAS PIPELINE (MI)
- DISTANCE TO OIL PIPELINE (MI)
- DISTANCE TO NEAREST FIELD (MI)

- GAS COMPRESSIBILITY FACTOR (Z)
- OIL FORMATION VOLUME FACTOR
- GAS IN PLACE (MCF/AF) [G]
- OIL IN PLACE (B/AF) [N]

END OF CARD 3

START CARD 4

- GAS RECOVERY (MCF/AF) [G] — MIN, MAX, MOST LIKELY
- OIL RECOVERY (B/AF) [N] — MIN, MAX, MOST LIKELY
- OIL VOLUMETRIC FRACTION [PROB] — MIN, MAX, MOST LIKELY
- PRODUCING GAS OIL RATIO (MCF/B) — MIN, MAX, MOST LIKELY

END OF CARD 4

START CARD 5

- GAS YIELD (B/MMCF) — MIN, MAX, MOST LIKELY
- PRODUCTIVE AREA (ACRES) — MIN, MAX, MOST LIKELY
- NET PAY (FT) — MIN, MAX, MOST LIKELY

END OF CARD 5

NOTES:

Table 2 - Summary of Data Sources

SUMMARY OF DATA SOURCES

Item		Maps	Interpretation/ Calculation	"Engineering Parameters for Coding"	"Calculation of Rec. Hydroc. per AF"	"Factors Weighted for Coding	Other
STUB:	1. ID Code						Pre-Printed
	2. Tract Numbers	Seismic					
	3. Source Document	Evaluation/ Comml/Seis					File Structure
CARD 1:	1. ID Code						Pre-Printed
	2. Ocean	Overall					
	3. Sector	Overall					
	4. Trend	Trend					
	5. Pay	Seismic	Interpretation				
	6. Level	Seismic	Interpretation				
	7. Pipeline District	Pipeline	Defined on-site				
	8. Trap Type	Seismic	Interpretation				
	9. Status	Status (Eval+Base)					
	10. Total Pays	Seismic					
	11. Pay #	Seismic					
	12. No. of Tracts	Seismic					
	13. Hazard Class	Hazard					
	14. Lithology		Interpretation				
	15. Prob. of Hydrocarbons		Interpretation	1 - "Dry Risk"			
	16 Probability of Dry		Interpretation	"Dry Risk"			
CARD 2:	1. No. of Plats	Seismic	Calc/Meas.	"No. Plat."			
	2.-9. % Under Nth Plat	Seismic	Calc/Meas.	"Plat Well Dist."			
CARD 3:	1. Avg. Vertical Depth	Seismic	Interpretation	"Avg. V.D."			
	2. Avg. Drilled Depth	Seismic	Interpretation	"Well Depth-Dev."			
	3. Water Depth	Bathymetric	Read	"Water Depth"			
	4. Distance to Gas Pipeline	Pipeline	Measure				
	5. Distance to Oil Pipeline	Pipeline	Measure				
	6. Distance to Field	Base	Measure				
	7. Gas Compress Factor (Z)		Depth Correlation		"Z Factor"		
	8. Oil Formation Volume Factor		Analog		"B_o"		
	9. Gas in Place (Mcf/AF)		Anal.+Calc.		"G"		
	10. Oil in Place (B/AF)		Anal.		"N"		
CARD 4:	1. Gas Recovery (Mcf/AF)		Anal.+Calc.		"RECG"		
	2. Oil Recovery (B/AF)		Analog		"RECO"		
	3. Oil Volumetric Fraction		Analog	"Prob"			
	4. Producing Gas-Oil Ratio (Mcf/B)		Analog	"GOR"			
CARD 5:	1. Gas Yield (B/MMcf)		Analog	"Yield"			
	2. Production Area (A)	Seismic	Measurement			"Area"	
	3. Net Pay (ft)	Seismic	Anal.& Interp.			"Thick"	

Table 3 - Cover Sheet for 50 Data Sheets

CONFIDENTIAL

EXHIBIT 1

DOE/EIA OFFSHORE PROJECT

DATA COLLECTION FORMS
(Form A - 7/10/79)

Ocean ____________ Sector ____________ No. ________ Subsector ____________

Estimator(s) ______________________ Date of Estimate ____________

Page	I.D. Code	Tract No.(s)	Page	I.D. Code	Tract No.(s)	Page	I.D. Code	Tract No.(s)
1.			18.			35.		
2.			19.			36.		
3.			20.			37.		
4.			21.			38.		
5.			22.			39.		
6.			23.			40.		
7.			24.			41.		
8.			25.			42.		
9.			26.			43.		
10.			27.			44.		
11.			28.			45.		
12.			29.			46.		
13.			30.			47.		
14.			31.			48.		
15.			32.			49.		
16.			33.			50.		
17.			34.					

A METHODOLOGY FOR ESTIMATING COST OF
FINDING, DEVELOPING, AND PRODUCING UNDISCOVERED RESOURCES

Thomas M. Garland

John H. Wood

Dallas Field Office
U.S. Department of Energy
Energy Information Administration
Assistant Administrator for Applied Analysis

It's a pleasure to be here this morning, and I welcome the opportunity to give a brief explanation of the rationale utilized in developing the methodology in the Permian basin study. As most of you know, the Permian basin study was an interagency study by the Energy Information Administration (EIA) and the United States Geological Survey (USGS) with a primary objective of turning "resource" estimates into "recoverable resource" estimates with the cost required. The USGS was to develop the methodology for the resource estimates, and EIA was to develop the engineering and cost methodologies.

Before I get into the methodology details, I would like to quote Mr. Charles Masters of the USGS from a paper presented in the March 19, 1979, Oil and Gas Journal concerning resource estimates. He stated, "The reliability depends on (1) the estimator's ability to adequately organize and express the conventional wisdom and (2) the former assuming that, indeed, the conventional wisdom does reflect the factors that control petroleum occurrence. Its credibility depends on (1) the critic's perception as to the adequacy of the systematics of data manipulation in the assessment process and (2) the confidence in the data set."

This quote also truly applies in the development of a model for the engineering and cost factors related to undiscovered recoverable oil and gas resources. In the Permian basin study, our conventional wisdom for the engineering and cost model was that: (1) the production characteristics of undiscovered oil and gas fields of a given size would be similar to those of fields of comparable size , and (2) depth of all undiscovered resources was critical in estimating the cost of exploration, development, and production.

Therefore, with this as our conventional wisdom, we set about to develop both a data set that we had confidence in and a credible methodology for systematically manipulating the data set. The data set was developed by the Dallas Field Office by estimating the recoverable oil and gas from some 9,400 active and inactive oil and gas reservoirs in the Permian basin area. This data set was initially developed for the USGS for their resource appraisal study conducted in Denver and the field size distribution by depth study conducted in Reston. It was also utilized in our engineering and cost methodology. Basically, for each reservoir, this data set consisted of:

1. original oil in place or original gas in place,
2. recoverable oil and/or gas,
3. lithology,
4. depth,
5. number of wells,
6. seven years of production, and
7. secondary recovery process.

Using these data, a systematic data manipulation process evolved that was combined with engineering judgment and assumptions to develop the methodology for the Permian basin study.

With this as a background, let's now get into the methodology. The depth brackets used in the study were less than 5,000 feet, 5,000 to 10,000 feet, 10,000 to 15,000 feet, and 15,000 to 20,000 feet.

There were 20 barrel oil equivalent (BOE) size classes of oil and gas fields, with the smallest size class of 0 to 6,000 BOE and the upper limit of the largest class of 3.1 billion BOE.

As Bill Stitt mentioned yesterday, there's always one courageous individual in the study. In this Permian basin study I personally believe that the most courageous individual is my good friend Larry Drew, with the USGS. He is the one who had to come up with the finding rate for the whole study. He developed the discovery model which predicted the number of BOE fields by size class and depth for each 1,000 well increment in exploratory drilling. Knowing the number of BOE fields of each size class in each of the depth brackets, the logic of the study was then to develop an exploration model, a development and production model, and an economic model as shown in figure 1. Today we will be concerned only with the first two.

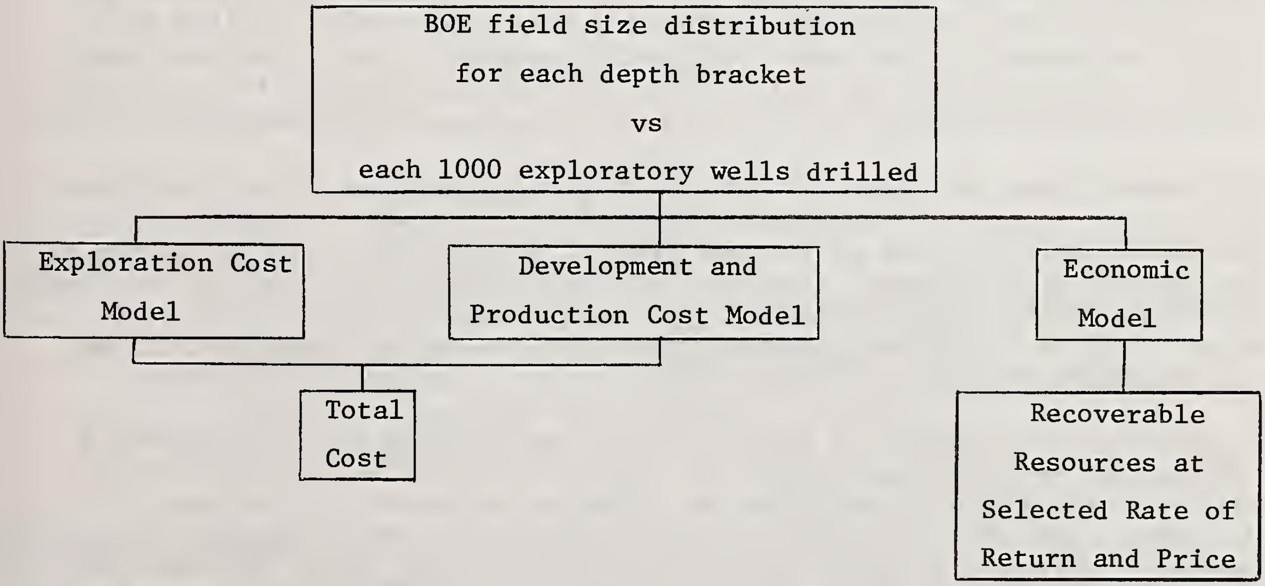

Figure 1. Logic of Study

However, figure 2 shows the overall schematic of the economic model as developed by John Wood in the Dallas Field Office and Emil Attanasi of the USGS in Reston,utilizing the price, rate of return (ROR), and cost. I would like to discuss briefly this economic model to show how the cost data that was developed fit into the economic model. As you can see, when you get the number of fields by size and depth, the first thing that has to be done is to determine whether they are non-associated gas fields, oil fields that only go through primary recovery, or oil fields that go through primary and secondary recovery.

Then the model generates a production schedule, as discussed yesterday by John Wood. It determines the cash flow using assumed wellhead prices in determining the present value for the assumed rate of return that was required, and if the present value was greater than or equal to zero, the deposit was developed and its reserves added to the total reserves. The process was repeated for each type of deposit for each size and each depth.

The model then adds up the net present values of economic deposits of oil and gas and determines the cost of exploratory wells. For the last 1,000 exploratory wells, if exploration cost is greater than the net present value of the resources found, then the model stopped exploration and subtracted the last calculated reserves from the total reserves. If there was a profit made from the last 1,000 exploratory wells, then it continued on.

The exploration model was developed in a separate study in the Dallas Field Office. It related the total exploration cost less lease bonus to the total cost of drilling exploratory wells. Therefore, for each 1,000 exploratory wells drilled, an average depth and average cost per well drilled at that depth was determined to calculate the total exploration costs.

Figure 3 shows the relationship between the average depth of future exploratory wells and the cumulative number of exploratory wells. This was determined by an extrapolation of a fitted function of average exploratory well depth versus the cumulative number of exploratory wells drilled since 1956. In this model, for each 1,000 well increment of additional exploratory wells, an average depth per exploratory well was calculated. For a value of 33,000 exploratory wells drilled the average depth was approaching 8,000 feet.

The average cost for drilling and equipping exploratory wells were expressed as a fitted function of average depth as shown in figure 4. Data used for estimating this function were found in Joint Association of Survey of the Oil and Gas Producing Industry (JAS) for 1975 published by the American Petroleum Institute (API). These costs were inflated to 1977 values for use in the model. For an 8,000-foot exploratory well the calculated average drilling and equipping cost per well was roughly $200,000 in the Permian basin.

To obtain total exploration cost per well, exclusive of costs of acquiring undeveloped acreage (lease bonus), a function of total exploration cost per well versus the cost of drilling and equipping exploratory wells was developed using data published in the JAS and in the Annual Summary of the Quarterly Review of Drilling Statistics for the United States published by API from 1966 through 1975. For our 8,000-foot exploratory well which had drilling and equipping costs of around $200,000, the total exploration expenditures would be roughly $400,000 as shown in figure 5. The exploration cost

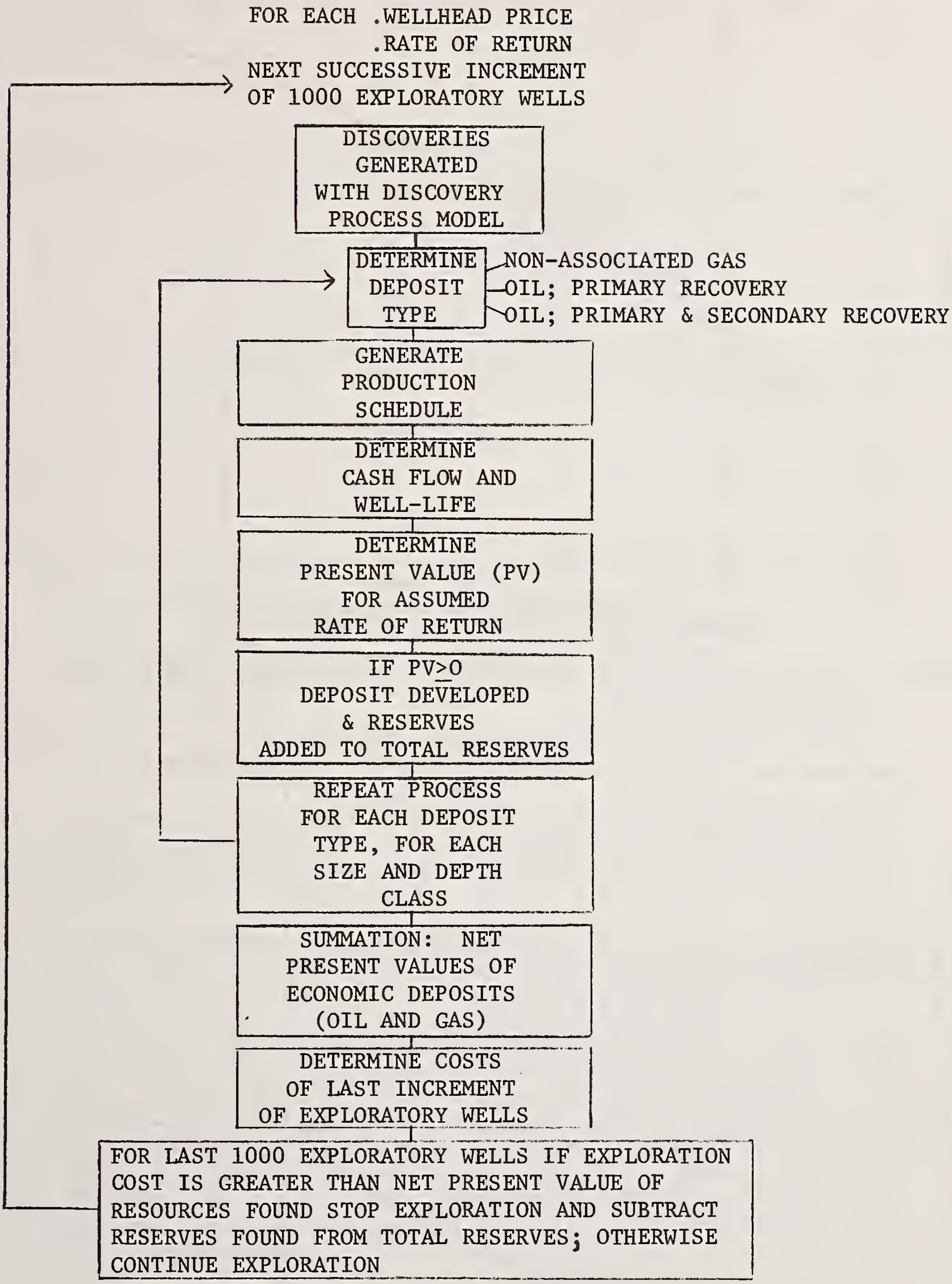

Figure 2. - Schematic of Costing Algorithm

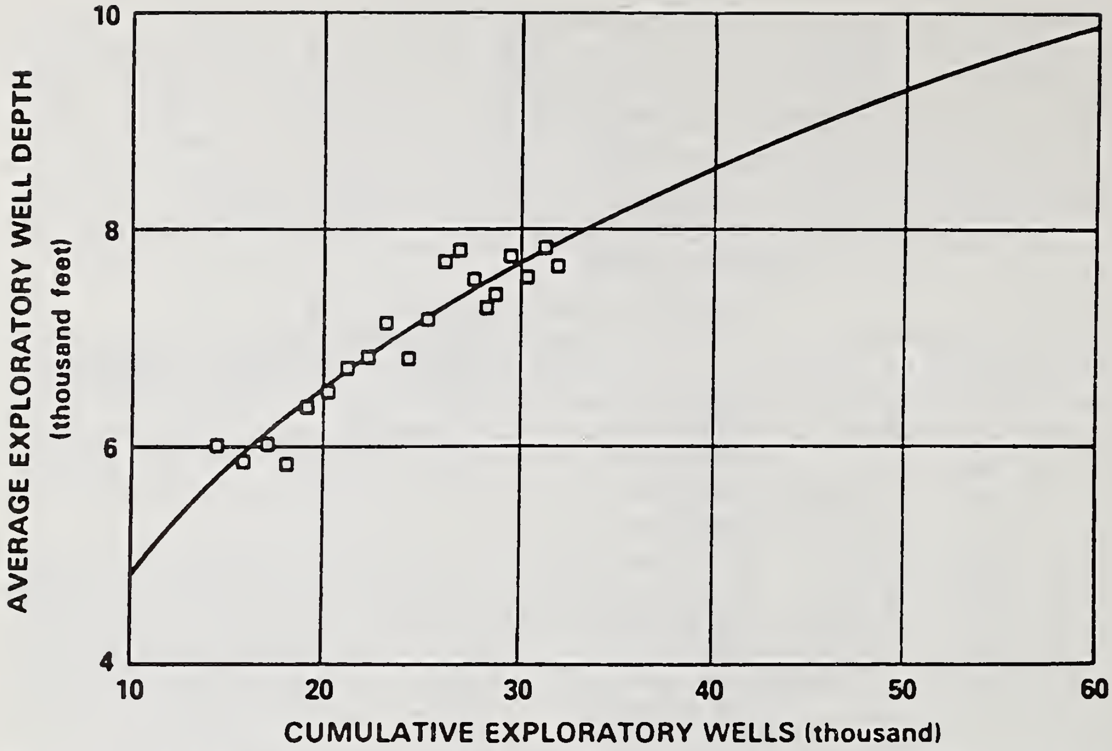

Figure 3. Average Exploratory Well Depth as a Function of Cumulative Exploratory Wells

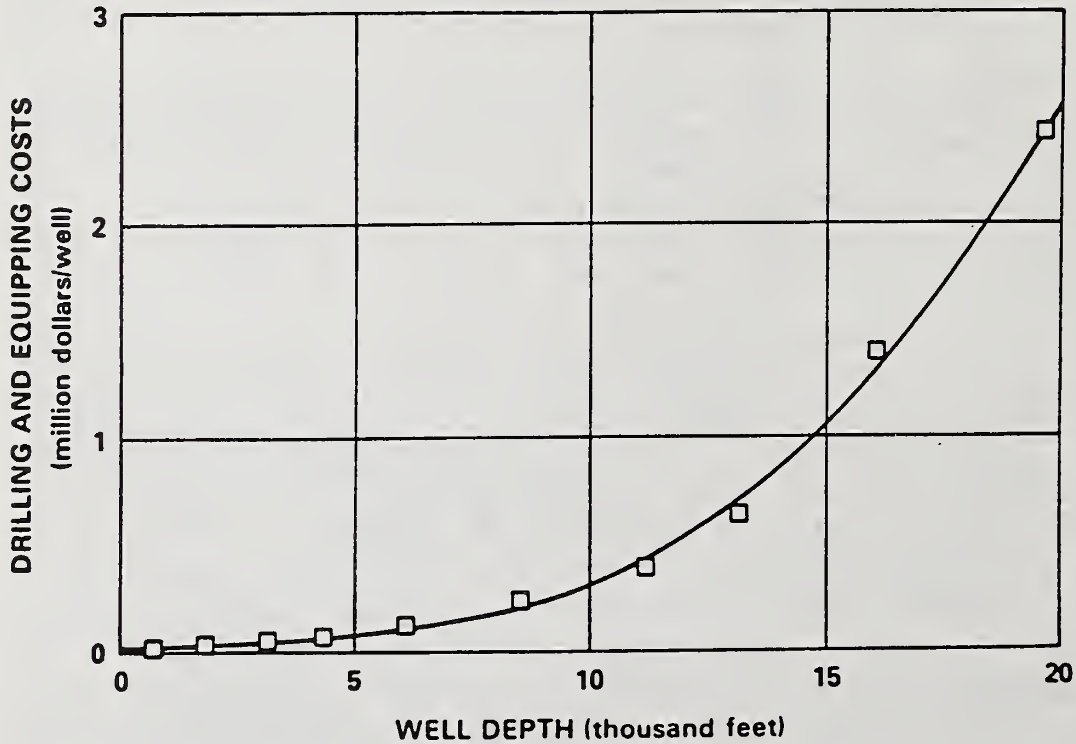

Figure 4. Exploratory Well Drilling and Equipping Costs as a Function of Well Depths

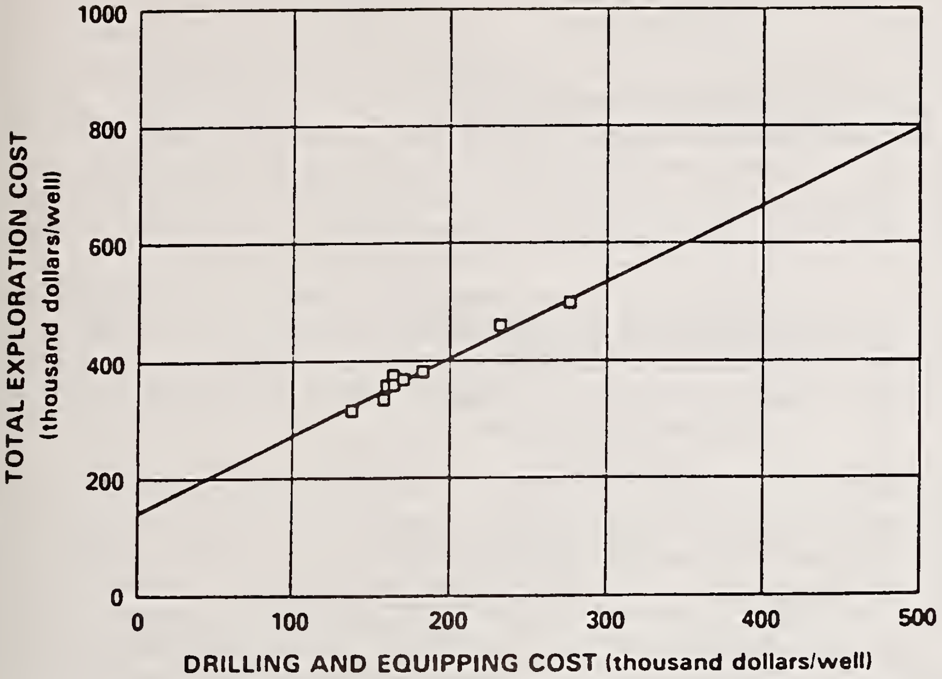

Figure 5. Total Exploratory Well Cost as a Function of Drilling and Equipping Cost

for each increment of 1,000 exploratory wells was then determined by simply multiplying the per well cost by 1,000.

Figure 6 is a schematic of the exploration cost model. Briefly, the procedure was to take the number of exploratory wells from the USGS discovery model, get the average depth and the cost for that depth, and determine the exploration cost less lease bonus.

For the benefit of those who are concerned about the lease bonus cost being excluded, it was excluded only in the exploration model. It was picked up in the economic model. In the economic model we assumed $50 per acre and 640 acres per exploratory well. We excluded the lease bonus cost from the exploration cost model because the data were very erratic. If you subtracted the lease bonuses that you knew were paid for offshore leases during any particular year from the U.S. total, then that would give you a negative lease bonus in that year from the onshore total for several years. The lease bonus data for the onshore United States were very poor. We couldn't find any data at all that would suit our needs so we just had to make reasonable assumptions for those costs. The lease bonus cost was brought into the economic model after the number of reserves (barrels of oil or thousand cubic feet of gas) had been calculated.

Note that we did make an allocation of the total exploration cost between the expenditures for oil and expenditures for gas. This has long been a controversial subject but we felt it was needed for proper determination of cost per barrel or per thousand cubic feet. Based upon a study of JAS cost data and API drilling statistics, we did develop a rationale that led us to make the assumption that 60 percent of the total exploration cost would be for gas and 40 percent for oil.

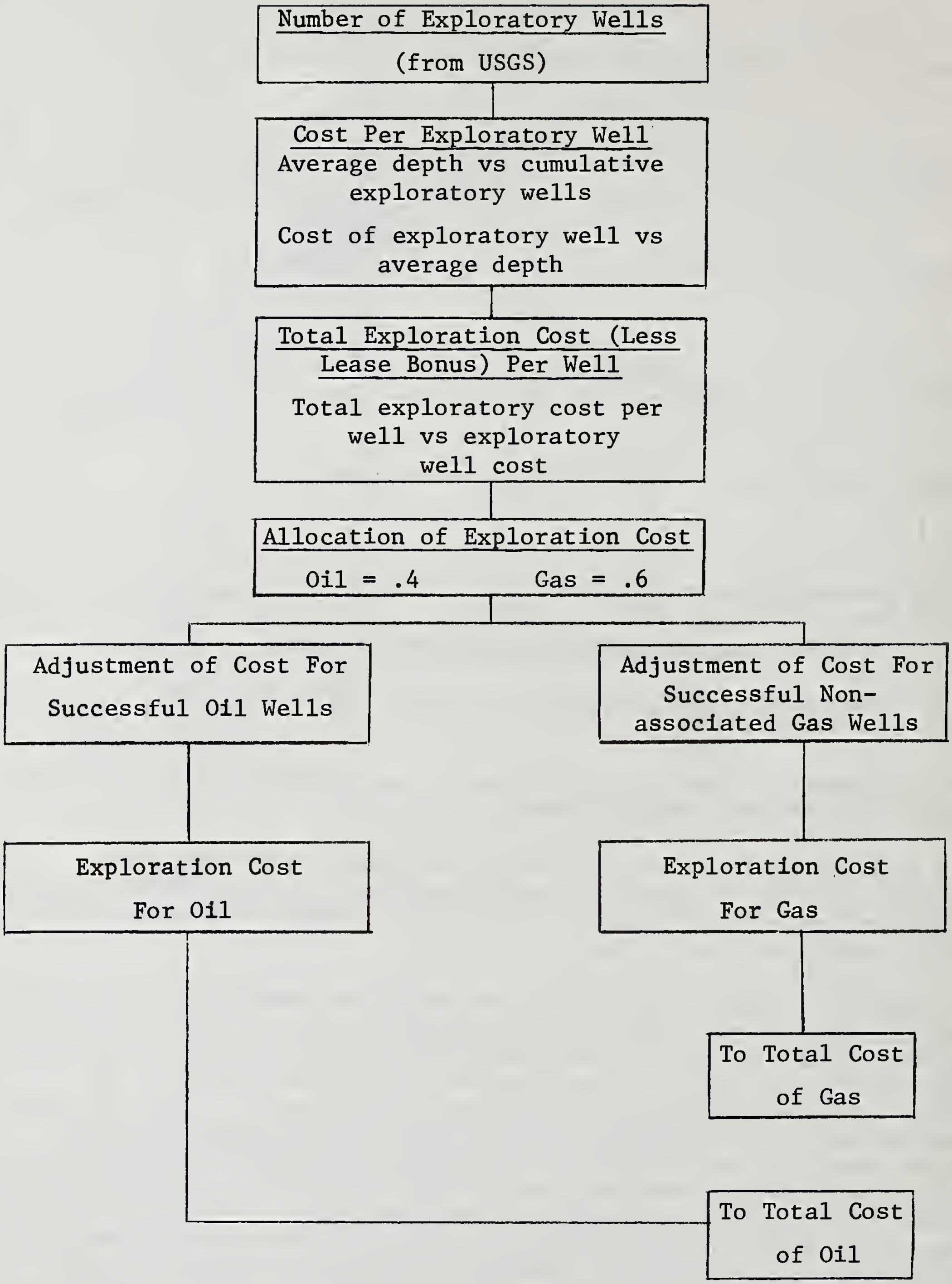

Figure 6. - Exploration Cost Model

Note, also, that an adjustment was made to the total cost for the successful exploratory wells. Because exploration costs are not included in the discounted cash flow calculations for successful oil and gas fields, an adjustment to exploration cost was made before exploration costs were brought into the picture for determining total cost per barrel or per thousand cubic feet. Otherwise, the cost of successful exploratory wells would be included in both the exploratory cost and development cost. This adjustment consisted of deducting from exploration costs, before combining with development costs, the cost of one exploratory oil or gas well for each oil and gas field found. The cost of these wells were included in the development cost.

In designing the "Development and Production" model, we first made an allocation of the BOE fields predicted by the discovery model as shown in figure 7. A further allocation of the oil fields was made between those that would undergo only primary recovery and those that would have both primary and secondary recovery or pressure maintenance.

Table 1 shows the ratio of oil fields to total fields based upon a fitted function of the time series of historical ratios developed from the data set discussed earlier. Note that a ratio was determined for each size class by depth bracket. It can be seen that the gas fields become more prominent with depth, especially below 10,000 feet. For size class 17, 100 percent of the fields shallower than 5,000 feet were considered to be oil. In the 5,000 to 10,000-foot bracket, only 89 percent were oil, and in the 10,000 to 15,000-foot bracket, 35 percent were oil. Below 15,000 feet there were no

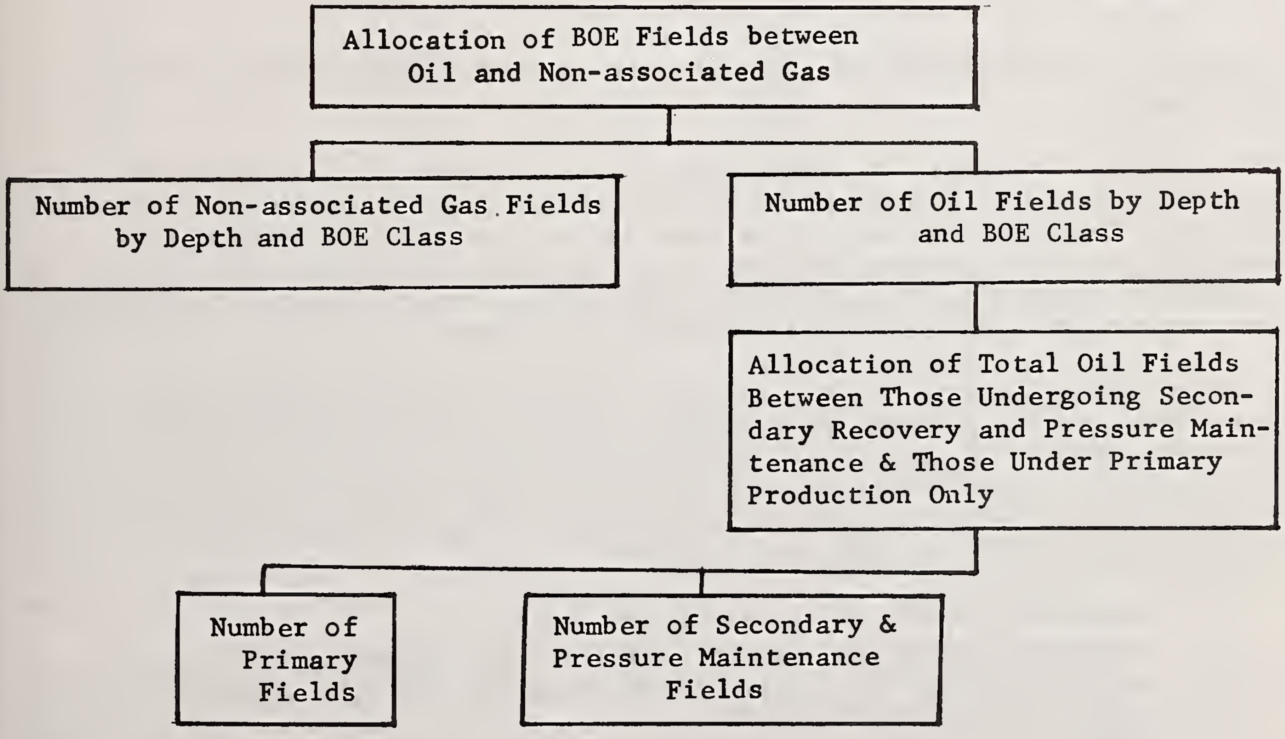

Figure 7. Development and Production Cost Model

Table 1. Ratio of Oil Fields to Total Fields in the Permian Basin

Size Class (BOE)	0-5,000 feet (depth)	5,000-10,000 feet (depth)	10,000-15,000 feet (depth)	Greater than 15,000 feet (depth)
1	0.86	0.84	0.54	0.00
2	0.78	0.78	0.51	0.00
3	0.72	0.73	0.49	0.00
4	0.68	0.69	0.47	0.00
5	0.65	0.67	0.45	0.00
6	0.65	0.66	0.43	0.00
7	0.65	0.66	0.41	0.00
8	0.67	0.67	0.40	0.00
9	0.70	0.69	0.39	0.00
10	0.73	0.71	0.37	0.00
11	0.77	0.73	0.36	0.00
12	0.81	0.76	0.36	0.00
13	0.86	0.79	0.35	0.00
14	0.90	0.82	0.35	0.00
15	0.94	0.85	0.35	0.00
16	0.97	0.87	0.34	0.00
17	1.00	0.89	0.35	0.00
18	1.00	0.90	0.35	0.00
19	1.00	0.91	0.35	0.00
20	1.00	0.90	0.36	0.00

historical oil fields in the Permian basin, therefore, in the model none are predicted.

Table 2 shows the ratio of primary oil fields to total oil fields for each size class by depth bracket. These were also computed from historical trends developed from the data set. Note that as the size class increases, more fields will have secondary recovery and/or pressure maintenance processes installed. However, for a given size class, the number of secondary recovery and pressure maintenance fields decreased with depth.

In determining the cost of developing oil and gas fields, the field design was determined by dividing the expected ultimate field recovery by the expected reserves per well to give the total number of wells to be drilled. These nominal values were discussed yesterday by John Wood, so I will not repeat them. In addition, the following field design data for each field size class and depth bracket were utilized:

1. expected ultimate oil recovery per field,
2. expected ultimate oil recovery per well for primary fields,
3. expected ultimate oil recovery per well for secondary recovery and pressure maintenance fields,
4. expected ultimate associated-dissolved gas recovery per oil well from primary fields,

5. expected ultimate associated-dissolved gas recovery per oil well from secondary recovery and pressure maintenance fields,
6. expected ultimate non-associated gas recovery per gas field, and
7. expected ultimate non-associated gas recovery per well.

Table 2. Ratio of Primary Oil Fields to Total Oil Fields in the Permian Basin

Size Class (BOE)	0-5,000 feet (depth)	5,000-10,000 feet (depth)	10,000-15,000 feet (depth)
1	1.000	1.000	1.000
2	1.000	1.000	1.000
3	1.000	1.000	1.000
4	1.000	1.000	1.000
5	1.000	1.000	1.000
6	0.984	0.986	1.000
7	0.958	0.983	1.000
8	0.891	0.972	1.000
9	0.781	0.940	0.981
10	0.636	0.881	0.962
11	0.474	0.792	0.938
12	0.318	0.679	0.895
13	0.188	0.554	0.818
14	0.097	0.429	0.693
15	0.048	0.315	0.515
16	0.030	0.213	0.307
17	0.025	0.119	0.116
18	0.013	0.032	0.001
19	0.000	0.000	0.000
20	0.000	0.000	0.000

Of course, the cost of drilling and equipping the wells in all size classes is dependent upon the depth. Within each depth bracket, an average depth was determined from the historical data set. For oil fields in the Permian basin, the average depths were 3,400 feet for the 0 to 5,000-foot bracket, 7,200 feet for the 5,000 to 10,000-foot bracket, and 11,400 feet for the 10,000 to 15,000-foot bracket. As perviously mentioned, there were no historical oil fields below 15,000 feet in the Permian basin. The average depths of gas fields were 3,400, 7,400, 12,000, and 17,700 feet in the 15,000 to 20,000-foot bracket. The cost per well versus depth was determined by using JAS cost data. Using these data, the total drilling and equipping cost was calculated by multiplying the total number of wells by the cost per well.

We all know that development wells drilled are not always successful. A study of drilling data from 1970 through 1975 indicated that for every 100 producing wells drilled, 19 dry holes were drilled. Therefore, we added 19 percent of the cost of a dry development well to the cost of each producing well to obtain the total development drilling costs.

To obtain total development cost, we then added the cost of the lease equipment per well. The lease equipment cost data were also developed on the basis of well depth for primary operations, and for secondary recovery and pressure maintenance operations. For oil fields assumed to be susceptible to secondary recovery or pressure maintenance, additional assumptions were made:

1. For fields less than 5,000 feet below the surface, the number of producing wells during the primary stage was assumed to constitute 70 percent of all wells that produced oil. At the outset of the secondary recovery program, the remaining 30 percent of the wells that produce oil were drilled. Then, sufficient wells are assumed to be converted to injection wells so that one injection well existed for each producing well during secondary operations. This provides a means of allocating well cost to secondary recovery operations.

2. For fields in the interval from 5,000 to 10,000 feet, the number of newly drilled injection wells was given by calculating the number of wells needed to infill drill the centers of a square array of the wells that produced during the primary stage. Some primary producing wells are converted to injection wells so that the ratio of producing wells to injection wells would be 1:1 at the beginning of the secondary recovery program.

3. For depth intervals greater than 10,000 feet, a pressure maintenance program was assumed to be carried out from the initial stage of development. One injection well was drilled for every four producing oil wells.

Annual oil and gas operating costs were also determined on a producing well basis for each depth. These costs included the direct operating cost (or so-called lifting cost) and the indirect operating costs which included the general and administrative overhead costs, and severance and property taxes.

Using these development and operating costs for each size class oil or gas field and the production schedule from each size class field, as discussed by John Wood yesterday, the economic model was used to calculate the recoverable resources that could be developed at varying prices and discounted cash flow rates of return.

In summary, I have attempted to illustrate the rationale, in detail, that can be developed for modeling, when your data set is complete and properly designed to fulfill the requirements of the final product. As in most modeling situations, the data set is critical with respect to both model design and credibility.

Discussion

Mr. Brashear (Lewin & Associates): Tom, I greatly appreciate that methodology, as you well know. One question that occured to me as you were talking is on the development dry hole rate. Some of those, if we use historical data, were just lousy wells that weren't completely dry. It's more economic to write them off in taxes than to produce them. As prices come up real fast, though, some of those might be keepers. Did you attempt to plow that back into your developmental dry hole rate? We haven't figured out a good way of doing that.

Mr. Garland: No, we didn't look at it that way, Jerry. We looked at just the number of dry holes from our experience in the development of the wells at the time the secondary recovery went in. At that same time the price was probably constant and therefore, if the price had been higher, they may or may not have been completed.

I don't know of operators that would have plugged marginal wells when they could go ahead and complete them, and at least get some of the cost back. Whereas if they went ahead and abandoned them, all they get is just their dry hole charge-off.

Mr. Brashear: So you wouldn't think it was a major factor, then?

Mr. Garland: Beg pardon?

Mr. Brashear: You wouldn't think the historical data would have much effect?

Mr. Garland: I don't think so. I don't think we looked at it that way, but I don't think it would make a change.

Mr. Brashear: Thank you.

Chairman Keene: Thank you. You might be interested to know that data that the Dallas Field Office put together on this study are publicly available, and if anybody's interested in that, they should see Tom after the symposium.

You might also be interested in knowing that the case of the Permian basin is an on-going item of review, and we're matching data that we've received from individual companies by reservoir against the other results that were found earlier. Hopefully, we'll be able to use this to test theories that we've been unable to test previously.

THE OUTLOOK FOR OIL EXPLORATION AND DEVELOPMENT

By T. R. Eck*

I greatly appreciate this opportunity to be with you, and to share with you some of Standard Oil Company's thinking with respect to the energy challenges facing our nation. It is my conviction that through opinion interchanges such as this--involving representatives of the industry, government, and academia--we can begin to dispel many of the myths that have too long surrounded the entire spectrum of energy supply and demand.

Within the context of my comments, I hope to develop some coherent thoughts of the magnitude of our energy problems, my company's perception of their derivation, and the approaches we believe requisite to their solution. Within limits imposed by normal proprietary considerations, I should like also to discuss some of the approaches my company takes to oil-finding and the methodology we employ in predicting petroleum accumulations.

Should you be predisposed to predictions of imminent doom, I fear that I shall disappoint you. On balance, my company is reasonably optimistic about the prospect of providing adequate energy supplies, both for the immediate future and over the longer term. Our outlook applies pre-eminently to the U.S. energy base, but extends also to world supply.

To find solutions, one obviously must first define the problems. Our contemporary problems with respect to energy involve, in fact, a single energy source: crude oil. More specifically, our problems revolve about the availability of this commodity at prices we can afford to pay. The unprecedented drain on U.S. financial resources occasioned by a ten-fold increase in world oil prices over the past seven years has created a broad-based awareness of our dependence on this single energy source. Under free-market conditions, an escalating price pattern of this magnitude would have signalled sharply reduced consumption, increased production, and a scramble to develop alternative sources. Our unique problem here in the U.S. derives from the fact that, unlike other industrial nations, we have aborted this normal response by imposing price controls and complex regulations.

The consequences of our national predilection for substituting political decisions for economic reality have been as numerous as they have been disastrous. With respect to energy, these consequences have included the emergence of "conservation" as the favorite tool of government policy, and efforts to dampen demand without--at the same time--seeking to increase production or practicalize alternative fuels. Only in recent months have we, as a nation, arrived at the belated realization that mandated conservation would be necessary if oil and its products were allowed to seek their natural levels in the marketplace. Had this economic truism been recognized several years ago, our economy would quite likely have avoided much of its present disarray.

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*Chief Economist, Standard Oil Company (Indiana)

It is perhaps axiomatic that nations, like men, must reach their nadir before invoking some higher power to set their affairs in order. Having indulged ourselves in political palliatives, we now seem better prepared to allow a somewhat higher order of economic reason to prevail. Herein lies a significant, if somewhat nebulous, cause for my company's optimism vis-a-vis U.S. oil supply.

As this chart (fig. 1) shows, there is little cause to believe that the world petroleum base, proved and prospective, will prove inadequate to meet any predictable demands that may be placed upon it. Proved world reserves currently approximate 600 billion barrels, while those in the United States now total about 27 billion barrels. Prospective world reserves, including U.S., are roughly twice the 600 billion number.

Translating these reserves into years of supply at current production rates (fig. 2), we see that the U.S. has almost 9 years of proved reserves and more than 30 years of prospective reserves. OPEC reserves, proved and prospective, will sustain current production rates for another 40 years, while other free world reserves could conceivably last well into the Twenty-Second Century at today's production rates. I would caution that these numbers represent merely the energy base upon which we can base our predictions of future supply. Given today's international tensions, <u>availability</u> of supply is quite another matter.

For a somewhat different view of how the world petroleum base equates in terms of availability, let us look now at the CIA's interpretation of current OPEC capacity (fig. 3). Here we see that the maximum sustainable production rate is about 34.9 million barrels per day, while the politically available production rate is some 3.5 million barrels per day lower. At present, the "politically available" figure used for Iran is overstated by some 3 million barrels per day, as that unhappy nation struggles to regain a semblance of economic order. Even so, it is apparent that surplus OPEC capacity exists today, and we expect that situation to prevail for the balance of this century. Obviously, we must anticipate temporary supply interruptions in the future much as we have experienced them in the past, but these are more related to world politics than to reserves or productive capacity.

Because political instability is the norm rather than the exception in much of the Middle East, we see an infinite marketability for all non-OPEC crude oil, whether in the U.S. or offshore. We see a somewhat different picture emerging for natural gas, however, where we forecast a ready market for North American gas, but a surplus situation offshore. Particularly will this be true if the OPEC countries, and gas exporters in general, insist on anything approaching oil-equivalent parity for natural gas at the wellhead. The cost of liquefying and transporting gas is simply too great, even for peak-shaving purposes, to sustain export markets based on wellhead parity.

A secondary but important factor with respect to the U.S. gas market is the increased availability of this commodity from domestic sources at the higher prices permitted under provisions of the Natural Gas Policy Act. Simply put, America is unwilling to pay any price asked for natural gas; it must compete with other fuels including oil, coal, and nuclear power. We feel confident that the U.S. natural-gas resource base will be adequate to serve all residential demands for the balance of this century and, given the continued use of coal as replacement fuel, most industrial demand as well. Looking further ahead, we do not dismiss the possibility that passive energy sources will ultimately back out some fraction of residential gas demand.

I should like to turn now to the effect that realistic prices for crude oil and natural gas are having on exploration and production expenditures in the U.S. (fig. 4), and use that as a springboard for our relatively optimistic outlook for U.S. energy supplies. As you can see, the domestic petroleum industry invested $20 billion in E&P activities in 1978 and $28 billion in 1979. This year, the industry is expected to spend about $33 billion on finding and developing U.S. reserves, and we expect the six-year average--now through 1985--to be about $50 billion a year in current dollars. Total spending for the six-year period will be about $300 billion.

You will note, also, that we expect drilling activity to increase about 7 per cent through 1985, with a gradual decline--on the order of 2 per cent a year--in crude oil and natural-gas equivalents discovered per foot of hole drilled.

There is no legerdemain in these numbers. They are mathematical extrapolations of trends that have existed for some years, plus our interpretation of results that can reasonably be expected from advancements in oil-finding technology, completions from thinner and tighter pays, deeper drilling, and--of course--improved economic incentives. These are factors that I will develop in somewhat greater detail a little later.

I want to put another table (fig. 5) on the screen while I indulge in a bit of philosophical ruminating. For several years it has been popular to predict severe energy shortages looming just over the horizon, and some of today's predictions continue to reflect this view. My company tends to believe, however, that the parameters of the energy situation in the U.S. are undergoing profound and dramatic change, and that yesterday's thinking does not necessarily apply to today's realities. For several reasons, we reject the validity of the perennial shortage syndrome that has found its quintessential embodiment in a government-mandated conservation ethic.

Two things of far-reaching consequence have occurred. First, U.S. and world consumption forecasts have been scaled down radically; we are now looking at annual increases in world oil demand on the order of 1 or possibly 2 per cent, compared with predicted increases of 5 to 6 per cent only a few years ago. Second, we have developed a much improved definition of availability, of the magnitude of our resource base. We at Standard--and I hope others as well--are increasingly confident that our postulated reserve numbers are realistic, and that technology does indeed exist that will permit us to convert prospective reserves into proved reserves. This growing confidence is reflected to some extent in the API and AGA reserve estimates published last month. The upward revisions of previous estimates reflect two factors pre-eminently: increased price incentives, and the industry's rapidly accelerating efforts to improve recovery from existing fields through the application of higher technology and more intensive development practices.

I assume that as I have been talking you have been studying our predictions of U.S. reserve additions and withdrawals. Our assumption is that during the 1980-1985 period an average of 5 billion barrels of crude oil and natural-gas equivalents will be added to our reserve base, divided about equally between new discoveries and upward revisions in recoverability from existing reservoirs. Hence, with respect to reserves, we anticipate that higher activity levels on all fronts will allow the domestic industry to do at least as well as it did last year.

The other side of the coin, of course, is the rate at which we expect reserves to be depleted through production. Here we expect withdrawals in line with reduced demand, or at the average annual rate of about 6.5 billion barrels a year. Based on these assumptions, we expect our combined crude oil and natural-gas equivalent reserves at the end of 1985 to be some 9 billion barrels below current levels.

One area in which we may depart slightly from conventional wisdom is in lumping together domestic crude oil and natural gas and considering them, in effect, part of the same energy pie. We believe we are on a firm ground in doing this, however, because of the interchangeability of these two fuels for many--notably industrial--purposes. Moreover, as permitted by federal law, domestic gas already is being sold at or near parity with crude--a fact which greatly increases its desirability as a target for North American exploration.

As I observed earlier, we do not find it realistic to view the interchangeabilty of offshore crude oil and natural gas in the same light as we do domestic production. The notable exception is North Sea production, where markets for natural gas comparable to the U.S. are being developed. For the exporting nations, we are looking almost exclusively at crude-oil production when we consider the impact on world-energy supply.

I should like now to turn briefly to what we believe to be a realistic outlook for world oil production through 1985. This table (fig. 6) compares our production estimates with those prepared by the Central Intelligence Agency. As you can see, our numbers differ rather markedly from the CIA's and are considerably optimistic.

The CIA forecasts OPEC production declining by 2.9 to 5.4 million barrels a day by 1985, while we predict a decline of only 1.3 million barrels a day. They see production in the OECD nations remaining essentially static, while we anticipate an increase of 1.3 million barrels daily. Our outlook for production increases in other free-world countries exceeds the CIA's high case by 2 million barrels a day. And while they believe China and the Soviet block will experience a production decline of 2.3 million barrels a day by 1985, we forecast a relatively modest drop of some 800,000 barrels daily.

We possess no great insight as to how the CIA and some of our competitors have developed their numbers. We can only assume that they are making assumptions that exploratory activity will be significantly less than we postulate, or that their projected discovery rate is much lower than we predict. We have made what we believe to be conservative assumptions, yet we find ourselves on the high side as compared with government and some other company predictions.

Standard makes a conscientious effort to insulate its industry-wide predictions from internal biases. Still, it is conceivable that our outlook for U.S. oil supply is colored by our exploratory successes of recent years. To the extent that a large measure of our success can be attributed to our role as the nation's most aggressive wildcatter, and to our employment of increasingly sophisticated exploration and production technology, we believe it probable that other major companies will be similarly rewarded as they turn their attention to the U.S.

Due largely to the expectation of higher prices for domestic production, the upswing in U.S. exploratory activity is significantly higher than in the remainder of the non-OPEC free world. Despite our problems with the so-called "windfall profits" tax and multitudinous regulations, the U.S. environment is one of the best in the world at this time. Not the least of our domestic advantages is the fact that U.S. taxes--burdensome though they are--are generally assessed on a percentage basis, rather than the ad valorem basis favored by several foreign governments. There appears to be a lack of rationality in the governments that favor 95 per cent of nothing rather than 50 or even 70 per cent of whatever production can be established under less confiscatory taxing policies.

I question whether anyone outside the petroleum industry--and quite possibly many within it--are fully cognizant of the technological advances that have characterized oil finding and development in recent years. We are today finding, with near routine regularity, petroleum accumulations that eluded our best efforts half a dozen years ago. Rather than being characterized by isolated revolutionary breakthroughs, our progress has been of an evolutionary nature, requiring the honing of interdisciplinary skills in concert with advancements in computer technology and oil-finding tools. The nation's universities have been unable to keep pace with this technological explosion, much of which is still proprietary, and most if not all of the major companies are conducting intensive and extensive training programs of their own.

Quite obviously, the front-end costs for developing and applying petroleum technology must be factored into the replacement value of crude oil. Enhanced recovery, as but one example, is proving more difficult and expensive than anything yet envisioned within or without the petroleum industry. Yet the payoff, assuming successful efforts, will be billions of barrels of additional oil from secure domestic sources. The truly self-defeating thing about so much of our government policy, implicitly including the "windfall profits" tax, is the assumption that tomorrow's oil will be found and produced at yesterday's prices. Such is not the case.

Because of the high order of technology and front-end costs associated with current and future oil-finding and production, I am persuaded that the competitive edge will increasingly belong to the larger companies possessing requisite funding and cash-generating capabilities. Tax-law revisions and regulatory red-tape are factors, of course, but cold hard cash is the main reason why drilling funds and small independent operators are apt to find extremely rough sledding in the years ahead. Major field development now costs a billion dollars, minimum, and only the large, technologically oriented companies can be expected to undertake such ventures--particularly with respect to hostile and frontier environments such as found in the arctic, deep-water offshore, and onshore plays below 20,000 feet.

In closing, I should like to recite one example of complementary, interdisciplinary technology that I don't believe has been widely discussed outside my own company.

As perhaps you know, exploratory efforts off our Eastern seaboard in geologic province known as the Baltimore Canyon have not met with overwhelming success. So far, 19 exploration wells have been drilled. One rather modest gas field has been discovered and two other wells have tested what may prove to be commercial shows of gas. No oil has been found. Drilling costs incurred to date exceed $217 million, excluding acquisition costs of $1.13 billion paid to the Federal Government.

As perhaps you also know, Standard was one of the few major companies that declined to enter into the spirited and high-cost bidding that characterized the Federal Government's lease sale in the Baltimore Canyon. Instead, we were beavering away out in the Rocky Mountains, leasing every scrap of acreage we could find along the geologic trend that has since become well known as the Western Overthrust Belt. Before the Pineview discovery, on Amoco acreage in late 1974-early 1975, more than 500 dry holes had been drilled along the U.S. portion of the Belt. Since the Pineview discovery, 14 other commercial fields have been found. Some of the fields have multiple pays, and at least 10 separate formations have been proved productive.

Many months before drilling began in the Baltimore Canyon or along the Overthrust Belt, Amoco earth scientists had predicted that the latter province would be productive of both oil and gas, but that the former held far less probability of success. We based our predictions largely on source-bed evaluations having to do with the richness or content of organic matter in the rocks, the type of hydrocarbons this organic matter had probably generated, and the thermal maturity of the two provinces. Our application of this sphere of earth science, known as geochemistry, led to our conclusion that source beds in the Baltimore Canyon had been heated only to a marginal level and had just barely started the main phase of gas generation. We found that source beds in the Overthrust Belt, to the contrary, were well along in their maturity, and had been heated sufficiently to produce both oil and gas.

It was largely on the basis of our geochemical findings that we eschewed the Baltimore Canyon leases and concentrated our efforts in the Overthrust Belt. But having done so, we were still far from home. The geological complexity of the Overthrust Belt, consisting essentially of repeated thrust sheets and multiple trapping mechanisms, stymied exploration success until geophysical research developed new technology that enabled us to decipher the structural configuration of the reservoir beds.

I have several reasons for citing this example, none of which are intended to imply that Amoco is the only company in America that can find oil. Indeed, we are not. Rather, the observations I hope to leave with you are these:

My first observation obviously relates to the financial and technological resources that are being brought to bear throughout the entire spectrum of energy development. I have concentrated today on petroleum exploration and development. But my company and many others also are devoting increasingly large amounts of time and effort to developing technology associated with alternative fuels. Unless stifled by regressive taxing policies or regulatory harrassment, the job of supplying adequate energy can and will be done.

The second thought I should like to leave with you is the substantial amount of time and expense that precedes the finding and development of oil and gas resources. A review of my company's exploratory activities over the past 20 years reveals the average lead times of five to eight years, but sometimes as great as 13 years, are required between lease acquisition and initial discovery. Another four to six years are typically required for field development.

The obvious implication is the absolute and overriding need for access to public lands if the exploratory momentum now begun is to be sustained throughout this decade. A cloud of uncertainty surrounds the amount of Federal acreage that will be made available--and when. It is essential that governmental processes be speeded up.

My third and final observation has to do with the potentially massive petroleum resource base that remains to be developed in the United States. We see the oil and gas in place, we see the drilling, and developmental activity occurring, we see new technology coming on stream, and we see the impressive results being reported by our company and others. We simply do not share the pessimistic outlook that seems so much in vogue today.

I thank you for being a kind and attentive audience. If you have questions and time remains, I will be pleased to respond to any questions you may have.

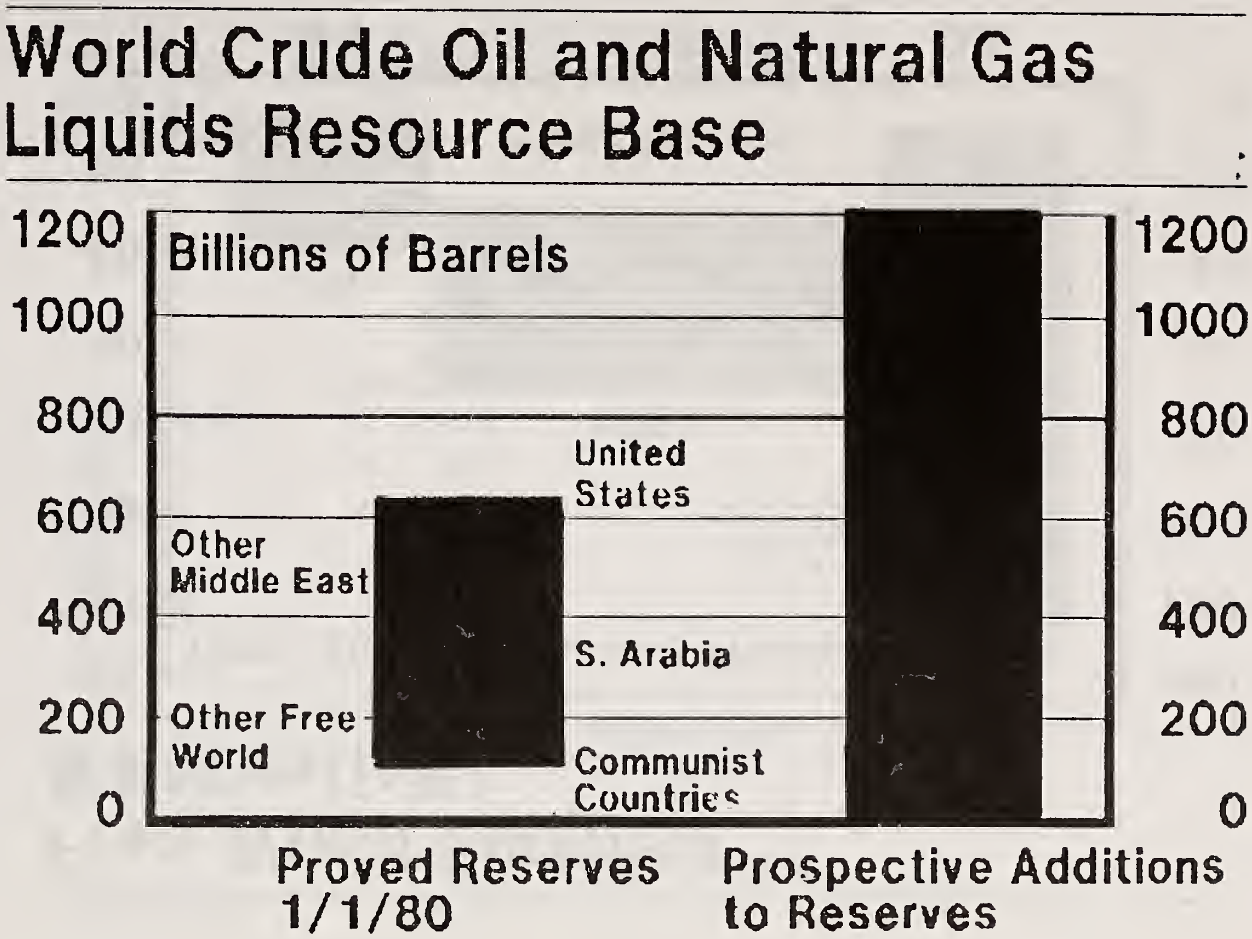
World Crude Oil and Natural Gas
Liquids Resource Base
Billions of Barrels
1200
1000
800
600
400
200
0
Other
Middle East
Other Free
World
United
States
S. Arabia
Communist
Countries
Proved Reserves
1/1/80
Prospective Additions
to Reserves

FIGURE 1

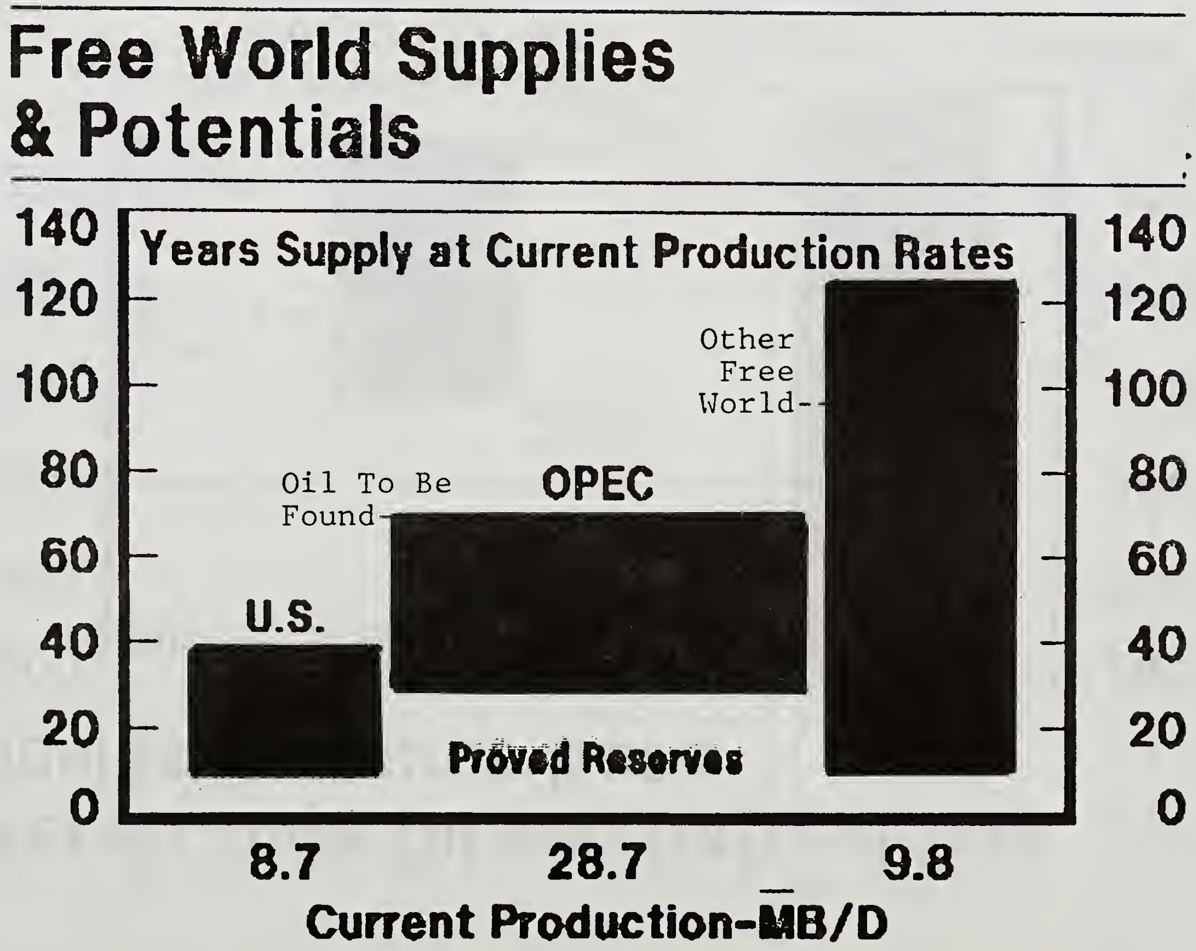
Free World Supplies
& Potentials
Years Supply at Current Production Rates
140
120
100
80
60
40
20
0
U.S.
Oil To Be
Found
OPEC
Other
Free
World
Proved Reserves
8.7
28.7
9.8
Current Production-MB/D

FIGURE 2

OPEC "Spare" Capacity
Million B/D

	CIA Estimate of Current Capacity	
	Maximum Sustainable	Politically Available
Saudi Arabia	9.5	9.5
Iran	5.5	3.5
Kuwait	2.5	1.5
UAE	2.4	1.8
All Other	15.0	15.0
	34.9	31.3
	Production	
Total OPEC	28.7	28.7
	Spare Capacity	
Total OPEC	6.2	2.6

FIGURE 3

U.S. Industry E&P Statistics

	1978	1979	1980-85 Avg.	1980-85 Period
E&P Expenditures (billion $)	20	28	50	300
Footage Drilled (million ft.)	230	240	300	+7%/yr.
Reserves per foot (crude eq. B/ft.)	16	21	16	-2%/yr.

FIGURE 4

U.S. Crude Oil & Natural Gas Volumes
(Crude Equivalent - Billion Bbls.)

			1980-1985	
	1978	1979	Avg.	Total
Reserve Additions	3.6	5.0	5.0	30
Production	6.9	6.8	6.5	39
Difference	-3.3	-1.8	-1.5	-9

	1970	1980	1985
Proved Reserves	94	62	53

FIGURE 5

Comparison of SOIND and CIA Estimates of Oil Production (Growth 1979-85, MB/D)

		CIA	
	SOIND	High Case	Low Case
OPEC	-1.3	-2.9	-5.4
OECD	1.3	0.1	0.1
Other Free World	4.3	1.8	2.3
Sino-Soviet Net	-0.8	-2.3	-2.3
Total	3.5	-3.3	-5.3

FIGURE 6

MODELS, UNDERSTANDING AND RELIABLE FORECASTS

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INTRODUCTION

While the interest of the Department of Energy, not to mention the general public, is strongly focused on the central question of how much oil and gas we can obtain at what price and how soon, the answers will inevitably depend on various models of supply and of discovery. It is also inevitable that the various models which purport to make such forecasts will all differ in their predictions and that the more models are consulted, the greater the number of differing opinions. The intent of the discussion in this paper is to indicate how one may reduce the plethora of alternatives to a manageable number and to give guidelines to models in which some modest reliance can be placed. Policies will be implemented; let us at least attempt to base these decision on model results in which some reasonable confidence can be placed.

There are five major sections to this paper. The first introduces the intuitively pleasing notion of "reliability of a forecast" and in the process indicates why black-box forecasting is dangerous. The second section extends the analysis in the first section by contrasting the concept of data fits with that of specification error analysis and the "maximization of residual entropy". The third section briefly discusses an often mentioned, but seldom analyzed, problem, that of "sampling" and the effects of different ways of obtaining samples on inferences about reservoir distributions, and so on. The fourth and fifth sections indicate the role that theory must play in trying to obtain reliable forecasts and in delineating the characteristics of reliable forecasts.

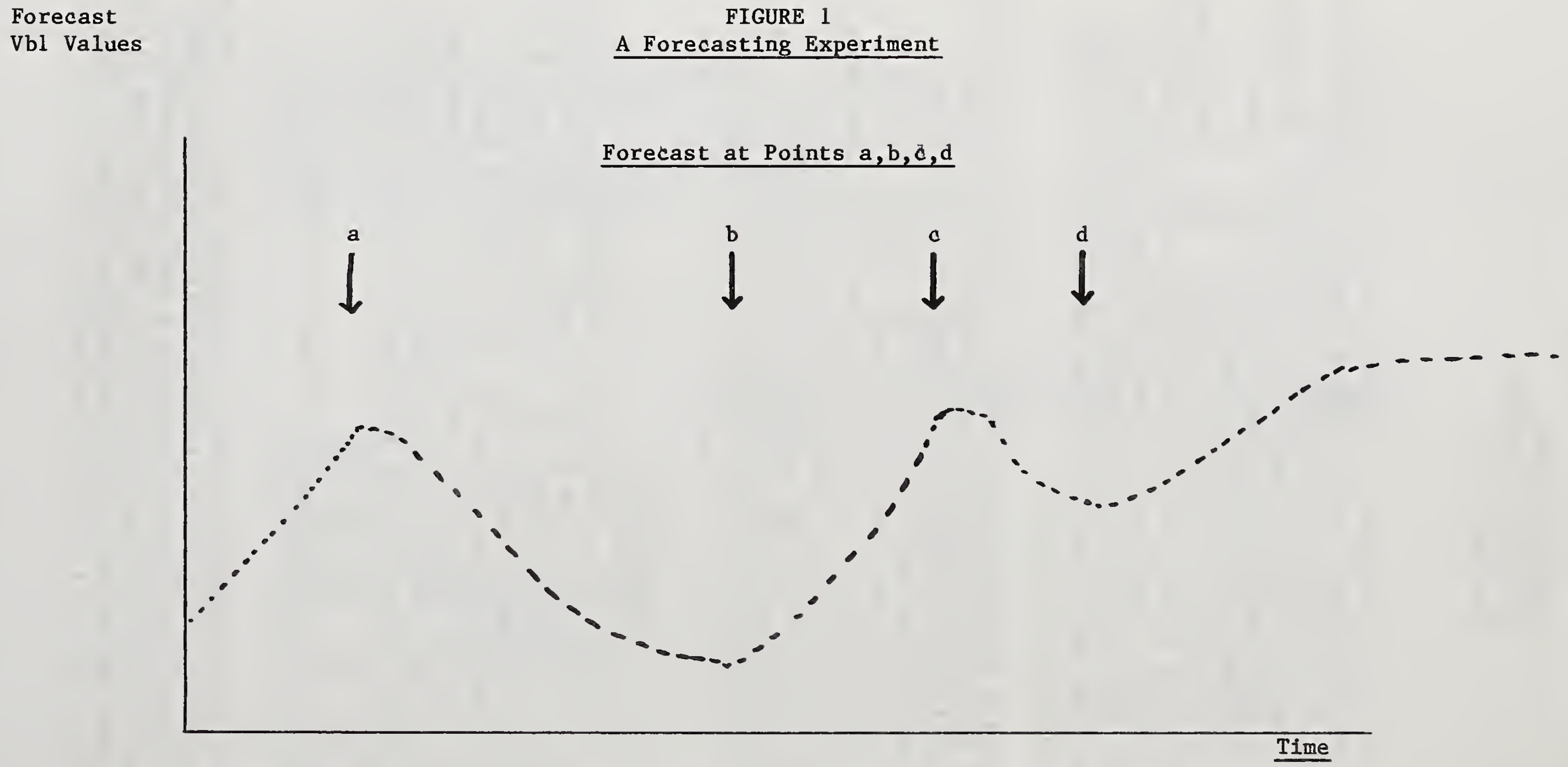
BLACK-BOX FORECASTING VS. "UNDERSTANDING"

The motivation for the main idea introduced in this section can be provided by performing the following hypothetical experiment; better still, the reader is requested to cover up the graph in Figure 1 <u>from</u> the <u>right</u> up to the line marked "a". Make a forecast based on the observed data up to point "a" <u>only</u>, recognizing that a near-perfect fit of an appropriate line can be obtained. Now move the covering card to "b", recognize your "forecast error", and forecast again; move the card to "c", discover your error, and then move to "d" to make your last forecast, and then evaluate your forecast record, even though at each stage you obtained a perfect fit to the historical data.

This simple exercise provides a number of important insights and several useful lessons.

Before beginning the discussion we need to recognize that the problems to be discussed have nothing to do with "random errors" and their unknown distribution since in the example above, there is a negligible amount of residual error. Secondly, the problems have nothing to do with the fitting procedure; the problems are as severe no matter whether one uses simple regression, Box-Jenkins, spectral analysis, or whatever. The fit to "the

FIGURE 1
A Forecasting Experiment



Experimental Procedure: Cover up figure to the right of point "a", make a forecast on basis of observed path, move to "b", reforecast, etc.

historical time path" in each case a,b,c, and d is perfect; but each forecast is less than perfect, it is in truth irrelevant.

Reasonable forecasts which the reader might well have made at each point are:

At "a", a straight line upward trend;
At "b", a curve which rises to a maximum and then approaches a lower bound (maybe zero) asymptotically;
At "c"; an oscillating curve;
At "d"; you may be convinced you now have it right, only to discover how wrong your latest forecast is.

The above curve and the series of forecasts from the observed historical time paths illustrate two basic but related lessons.

Fitting historical data in an economic time series without benefit of any theory inevitably leads to unreliable forecasts, a phrase to be defined below. "Good fits" to historical data, even perfect fits give no guarantee of reliability.

The question now arises - what is meant by the term "reliability"? This is a fundamental concept of importance in the inferential process prior to estimation and the choice of statistical methods as usually defined. Intuitively stated, the reliability of a forecast is the "confidence" one has in the maintained hypothesis or in the basic estimation model together with its required distributional assumptions. Essentially, reliability is the confidence one has that the model used to analyze the data is applicable to the situation being estimated and that the assumed model will continue to be relevant over the period of the forecast. Reliability, in short, is what is always assumed by a researcher whenever he begins to talk about standard errors of forecasts and setting probability limits on forecast values.

Indeed, the whole concept of probability statements attached to forecasts assumes implicity that the model used to generate these estimates is in truth relevant to the observed phenomena and that the researcher knows this with certainty. For if one entertains any doubts as to the validity of the model itself, then that doubt, expressed probabilistically, should be used to modify one's "estimated" probability statements based on the model.

Reliability, then, is pre-statistics, both in time and in that it is a concept which is essentially "discipline determined". This is to say, one gains confidence in a model in so far as the model provides an explanation for the phenomenon under examination and in so far as that explanation is consistent with other related theoretical ideas in which one has a high level of confidence.

Clearly, an "explanation" which is not in consonance with the observed facts is no explanation at all so that statistical rejection of a model itself, not just particular parameter values, is sufficient to reject the model and to state that the corresponding theory provides no explanation for the class of phenomena under examination. But in contrast the obtaining of a good fit to a set of historical data by some statistical expression not generated by a theory only reflects on the ingenuity of the data fitter and says nothing else.

To see the difference most clearly imagine we have a set of data on the time path of the volume of oil discovered within a year by year. Suppose further two sub-cases. In one, we have an interaction of economic and geological theories of discovery and market interaction which "explains" in terms of these theories why the observed time pattern is as it is. Further, suppose that the historical circumstances are such that the theoretical analysis reduces to a fairly simple model relating oil discoveries and time.

Now, in contrast, suppose someone tries a number of simple statistical models and finds one which seems to fit well; let this model so serendipitously discovered be exactly the same model as that generated by economic and geological theory. The striking fact in these two situations is that the results are not the same; the former model is "reliable", the latter is not. Standard errors of forecast and forecast probabilities can be quoted in the former case, but not in the latter. It is this paradox, if you will, which requires some explanation.

In the latter case our colleague's sum total of knowledge is that he obtained a good fit between a particular form of a model and the given historical data, but given some regularity in any data series, that can always be done. What has one learnt? Nothing, other than that a particularly convenient way to summarize or represent the existing data series is provided by the fitted model. It is true that there are a lot of matters our colleague wishes to know, such as, that the model does represent the data in an essential way and that this representation will apply in the future, that forecast probabilities can be assigned and are to be believed. But if wishes were truth, we would all be rich.

As opposed to our colleague, what do we know? At least something. First, if not most importantly, our theory will delimit those conditions and circumstances under which the historically determined simple relationship can be suspected to hold. This is important, especially in the subject matter of economics, wherein enormous bureaucracies, not to mention the rest of us, are dedicated to trying to change the existing economic structure.

This then is the real need for forecasting the outcome of human behavior, we are almost always having to extrapolate beyond our current experience and more importantly beyond the conditions under which our models were fitted. Unless we have some insight into when and when not extrapolations of the existing experience are useful and valid, we can never rely on our historical data fits. In the case with oil and gas there have been a number of large scale changes.

For example, the current U.S. situation reflects the joint facts that at approximately the same time that the availability of the very large low cost discovery and easily accessible fields was drying up, much greener exploration pastures were being discovered elsewhere, notably in Latin America, the Middle East, and even Canada. As a direct consequence seismic crew days in the U.S. fell dramatically during the mid to late '50's and remained at low levels until recently. We are now reaping our previous lack of effort.

Over the decades of this century, there has been a substantial shift in the overall orientation of the bureaucracy dealing with the oil industry from the encouragement and subsidy of exploration during the early period to the discouragement and indirect taxation of it today. Both the natural and governmentally induced shifts in demand which translate into pressures on supply are also neglected by mechanistic models. If oil had not been discovered to be an incredibly suitable fuel for locomotion, the time path of oil discovery would have had a vastly different shape. Similarly, a change in the technology of discovery and extraction (so far the technology of discovery and extraction has advanced relatively little) would have a significant effect on the time path of discoveries. All of this is "ignored" by mechanistic models, or rather, the assumption is that whatever determined optimal rates of effort, type and quality of inputs, and the relative pay-off in the past will remain constant in the future. But, if there is one thing we can know in an uncertain world, it is that whatever determined the course of events in the past, it will not be the same in the future. With certainty one can predict that mechanistic models will have to be "adjusted" to account for the change in the time path of discovery.

In contrast, theoretically based models gain evidence in so far as they constitute specific applications of generally accepted theories. Moreover, a theory which has already been subjected to a most extensive and intensive series of empirical tests outside of the particular situation in hand implies that one can have more assurance in relying on these particular results; in short, the testing of the general theory lends substantial evidence for the credibility of the results of any part.

Moreover, unless we are willing to act on faith alone, the fact that we can spin a plausible story which not only "explains" the path of our particular series, but more importantly relates that experience to more general events, then this fact in itself provides further evidence about the reliability of our forecasts.

Finally, if one reviews black-box forecasts (i.e. non-theoretically based, purely empirical forecasts) over any reasonable length of time, what soon becomes clear is that the forecasting model changes constantly over time as data accumulates and as new observations continually belie the guesses based on the old. Simply put, black-box forecasts do exactly what the reader did in making his forecast experiment at the beginning of this paper. Since we can see our errors in retrospect so easily, it should take little imagination to recognize that a retrospective view "in years hence" will also show how badly we went wrong. If then, we know with high probability that our retrospective view will show that we will have been wrong and likely to have been spectacularly wrong, we should view our mechanistic forecasts-without-understanding with considerable suspicion; i.e. black-box models do not provide reliable forecasts.

While mechanistic black-box forecasts are inevitably doomed to constant failure and revision by their proponents, this observation does not mean that even sound theoretically based models are perfectly reliable; clearly, they are not. But what we can say is that with theoretically based models, models wherein one understands why events were as they were, we can achieve our highest levels of reliability. And in this game, something is far better than nothing.

GOOD HISTORICAL FIT VERSUS ERROR ANALYSIS

We have already seen that if a data series has any observed regularities to it at all, it is no difficult matter to obtain a very "good fit" between some model and the observed data. Indeed, the objective of most data fitting procedures is precisely that, to produce a good fit, so that it is not surprising that good fits are obtained. But as we have now seen, producing a good fit does not reveal the truth whatever it might be. For the more statistically knowledgeable good fits are represented in terms of concepts like high R^2, or even high R^2, large t-ratios, big F's, and so on. Impressive language, but without the reliability provided by the knowledge that the model is relevant in the first place, these statistics are basically irrelevant.

Let us now suppose we have available to us a potentially reliable theory with valid and useful applications to our problem, but we do not know the precise form of our model. We are not too sure which variables we can safely ignore; we are not too sure of how far the model can be simplified without producing serious and inferentially significant errors.

In order to handle this perennial problem and aid us in our search for more reliable forecasts, I am going to make another apparently heretical statement. The analyst's objective should <u>not</u> be produce good fits, his objective should be to produce a set of residuals whose distribution is pure white noise; put more colorfully, his objective is to maximize the entropy of the residuals.

A less striking, but more cogent statement is that what we want for reliable forecasts are valid maintained hypotheses. And the empirical path to such a happy state is through specification error analysis. One may obtain R^2 values of 0.99 and higher and still have a seriously misspecified model; a model which has omitted an essential variable, uses the wrong functional form, is heteroskedastic, or has disturbances which are not independently and identically distributed. In order to begin the process of trying to detect these errors, one must engage in a series of careful analyses of the model, its properties, and its relationship to the data as exhibited in the observed distribution of the model's residuals.

In this respect specification error analysis of one type or another is a vital, but unfortunately, much neglected, tool; see, for example, Ramsey (1969), (1974), Hausman (1978), Leamer (1978), and Hale et al,(1980). Fortunately, the realization of the importance of specification error analysis is now beginning to grow at an ever faster pace. Inferences based on simple fits of data, while acceptable in the past, will no longer provide a suitable basis for policy in the future.

THE SAMPLING PROBLEM IN OIL SUPPLY FORECASTS

This is a topic to which much lip service is given, but little real analysis. The crucial issue is that the non-independent, non-random, truncated distribution of pools, reservoirs, and oil fields obtained by systematic sequential search is used to make inferences about distributions of reservior and pool sizes as if the data were a simple independently and identically distributed (i.i.d) sample. Nothing could be further from the truth. At least some researchers, Kaufman most notably, have taken into explicit account that the sampling within a basin is without replacement and is affected by the relationship between surface area of reservoir and volume discovered as well as the search process used to discover oil.

To give some idea of the effect of the search process on the characteristics of an observed sample and inferences to be drawn from it, imagine trying to determine the distribution of consumption of various goods without understanding the role of income and relative prices and where one's sampling procedure is to track down only people with gold American Express cards. Further, imagine that you only record credit-card expenditures and you throw out anyone who spends less than $100 per month on the card. The distribution of consumption obtained from this procedure makes as much sense as our current procedures in trying to determine the distribution of oil reservoirs.

SEARCH AND IMPLICATIONS FOR SUPPLY FORECAST MODELLING

Until very recently in economics the theory of search and exploration was an almost totally ignored subject. Consequently, it is not at all surprising that most so-called "econometric models" were based on very simple comparative static models with little recognition of the economic role of exploration and the crucial importance of understanding the nature, type, and degree of constraints imposed by geological factors. Consequently, such models in that they lacked an appropriately developed theoretical base were as unreliable as any other ad hoc modelling attempt. Economists still have not mastered the relevant economic theory in this branch of applications, but progress, albeit slowly, is being made.

In order to illustrate the importance of developing an economic theory of exploration consider some of the following examples. These examples illustrate the limitations of simple conventional models and point out that the use of such conventional models will produce significant specification errors and thereby lead to extremely unreliable forecasts.

First, one cannot usefully ignore the role of expected marginal information costs in determining the timing, location, and rate of oil exploration sampling. Essentially, we "search" where the anticipated marginal cost of information is low relative to the expected discovery gains. This type of procedure leads not only to a "success breeds success" view of exploration, but more importantly to a view that" non-success breeds neglect"; an area unsearched is guaranteed to produce no new discoveries.

Next, the economics of exploration theory developed by Ramsey (1980) indicates that mean (or log mean) size of fields discovered and success rates are in fact endogenous variables, not geologically predetermined exogenous variables. Mean field size discovered and drilling success rates are in fact a function of the distribution of firm sizes and degree of exploratory specialization as well as a function of conditions determined by geological factors. The distribution of field size and success rate can both change with economic conditions and with the rate of exploration.

The role of non-market forces in affecting the type, nature, distribution, and rate of exploration is usually ignored in "models of oil supply". Consider briefly the effects of government restrictions and sometimes subsidies on where, when, how, and by whom search is performed. Differential taxation between large and small firms will dramatically shift the distribution of type of exploration and the distribution of areas searched.

The analysis of the economics of exploration indicates that the distribution of exploratory effort and the role of exploratory specialization are far more important characteristics than a simple measure of the rate of exploration. Changes in the distributions of firms by size will change the distribution of fields (or reservoirs) discovered by size, see for example, Ramsey (1980).

For the economist trying to analyze exploration, ignoring the role of geological constraints is a serious lack in trying to build a model to produce reliable forecasts. The lack of an acceptable and unified geological theory in itself is a difficulty in trying to build a useful model of exploration. The uncertainty generated by a lack of a clear understanding of geological processes leads to a search behavior dominated, but not exclusively however, by searching in those types of areas which provided success before. Progress in exploration procedure is serendipitously made by the odd maverick who looks where every fool geologist knows one should never look.

These are merely some of the characteristics of the exploratory process which help to distinguish it from simple standard economic models of static equilibrium. Most attempts at building even a dynamic analysis of exploration have in the main ignored many of the above ideas. Economists have a long way to go before being able to claim we have sufficient understanding of the discovery process to be able to provide useful and reliable forecasts. Progress will be made in this respect, but only if a sufficient number of economists become interested and only then if the necessary funds for research are available.

SOME CHARACTERISTICS OF A RELIABLE FORECAST MODEL

My scattered comments can be most usefully summarized by making some recommendations for obtaining and recognizing a reliable forecast model for discovery.

A reliable model will be one which explains the empirically estimated relationships between the various observed values of the theoretically determined relevant variables. The conditions under which the simplified version of the theory generating the specific model is useful will be carefully delineated. The model will incorporate economic theory in explaining firm behavior and decision-making and geological ideas to determine the constraints on firm behavior.

A reliable model will not be a black-box of the simple mechanistic kind nor an *ad hoc* regression fit as characterized so many early "econometric" models. The relationship between the predicted behavior in the model under consideration and the rest of economic theory should be clear.

In the development of the model the economics of exploration as modified by governmental intervention should be recognized. The outcome will be a model which not only fits the historically observed data, but enables one to understand how the exploration market works and how it is affected by both governmental action and geological conditions.

Sample values of reservoir sizes or even geographic distributions of reservoirs will be recognized as the outcome of purposive search.

The estimation procedures should leave no doubt that the model in its basic form is not only relevant, but that the maintained hypothesis generated by it is acceptable with high confidence. The actual fitting of the model should give no indication of specification error and the estimated residuals should appear to be pure white noise.

If all of the above has been followed, one may claim to have a "reliable" model with which policy prognoses can be made with some confidence of success.

But, be humble in your forecasts, for nevertheless, you may still be wrong.

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DISCUSSION

QUESTION: I would only ask how close do you see us as being to this sort of, admittedly, optimal model which explains the features. It just seems to me that in the absence of good understanding of all these complicated processes, we are sort of like Galileo who dropped the two rocks and finds, consistently, that they land.

He may not understand the theory of gravity, but still may be willing to accept, on the basis of observations, a certain kind of prediction can be made. So, while I would agree with you that it is optimal to have this kind of theoretical model, I think we are more or less forced into less than optimal models. Would you agree with that assessment or not?

DR. RAMSEY: Clearly, we always have a compromise with reality, so that a fairly reliable forecast is possible, as I said.

The point is that given where we are, what is the degree of confidence that we place in what we do. If you are fully aware of the difficulties, you will be very sure of not engaging in a very sensitive and rash policy when you know that the outcome is very unsettled.

There is a big difference between getting a computer output, which is so common these days, and say here are the numbers; this tells me what I should do. Go ahead and let's implement it; then you may discover that you made a ghastly mistake.

It would be far better to be aware that you can make a ghastly mistake and be very cautious before moving. So, my point is to exercise caution and also to indicate what one can do to improve the situation. How does one improve it?

One, insure that more and more theory is utilized in the development of these models, to insure that the theory is consistent and that one recognizes that it is carefully and properly formulated; to insure that the estimation process is such as to meet the extraordinary demands on the use of the procedure, and to insure that the maintained hypothesis is a reasonable and useful approximation to the ideal.

On that basis, there are a whole battery of tests, or procedures, which can be utilized to achieve this result. There is very little evidence that this, in fact, is going on.

If you start looking under these logs, you will be horrified to discover the incredible number, degree, extent, and intensiveness of problems of all sorts, from simple computational arithmetic difficulties all the way through to the complete erroneous formulation of the hypothesis.

PHIL GLASNER (Standard Oil of California): Can you elaborate on how the process of search affects the distribution of reservoirs? You mentioned it; you did not go into it.

DR. RAMSEY: I think it would take longer than I have here. I will give you a copy of my paper on this and that will help to elaborate.

JOHN WOOD (EIA): Would you care to comment on what types of theory you think are ones that you should fundamentally understand? For example, whether econometric models are ever reliable and, let's say, resource estimate or supply, as opposed to certain underlying geological principles?

DR. RAMSEY: Let's put it this way. With the current models which I have seen to date, I would be extremely cautious in making any extrapolations from the historic data.

The Regulatory Framework in Oil and Gas Supply Modeling

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Ordinarily, supply modeling assumes that producers are free to respond to price and other economic incentives in both the short and the long run. This assumption cannot be made without qualification with respect to the oil and gas industry in the United States. Although gas production is relatively free, oil production is regulated in the name of conservation, and this regulation, anticipated by operators, significantly affects investment in exploration and development in the long run. Gas production in the long run is affected indirectly by oil regulation, due to the fact that oil and gas are joint products in the exploration phase and often in the development phase as well. In addition to production regulation proper, administratively determined leasing schedules on the Outer Continental Shelf and on some state lands influence the short- and long-run supply responses to economic incentives. This paper will be concerned with these regulatory restraints on supply.

Two systems of production regulation

There are two systems of oil production regulation in use today in the United States. The first of these is based on the concept of MER, or maximum efficient rate of recovery. MER is the basis of production regulation by the federal government on the OCS and by a number of states in their respective jurisdictions. In California MERs are estimated by the Conservation Committee of California Oil Producers for all reservoirs in which they are applicable and are recommended to operators for voluntary use as the basis of production restraint. Regulators in the Rocky Mountain states do not routinely estimate MERs in new reservoirs, but they do intervene to restrict production to something like MER when there is evidence of physical waste in the form of loss of ultimate recovery. In the Southwestern states, best known for market-demand prorationing in the 1950s and 1960s, MER is the basis of production restraint in a relatively few selected problem reservoirs.

The second system of oil production regulation, generally employed in Texas, New Mexico, Oklahoma, Kansas and Louisiana, is based on an instrument of market-demand prorationing, the depth-acreage allowable schedule. The schedules differ among the states named, but they have in common a table of maximum allowables per well per day, the allowables increasing exponentially with well depth and more nearly proportionately with the number of acres per well. They are designed so that the allowable per well increases with increasing drilling and production costs, as with increasing depth, so that all operators can "make a living" under production restraint. The schedules provide an administratively feasible method of restricting statewide output and allocating the total among wells in states where there are thousands of separate reservoirs. The allowables have little or no relation to MERs where such have been estimated.

The optimum rate of production

It will help to bring out the significance of production restriction if we have in mind the conditions under which the optimum rate of production would continually be sought by profit-motivated operators.

It is well known that the basic problem of unregulated oil production stems from (1) the rule of capture as the law of property in produced oil, (2) the fluid nature of oil in the reservoir, and (3) the inverse dependence of ultimate recovery on the rate of production. The first two of these mean that the operator who produces at a faster rate than his neighbors can drain oil (or gas) from beneath his neighbor's land. Every operator, then, has a motivation to drill wells densely and to produce at capacity. This is turn means that, given the third factor mentioned, competitive exploitation of an oil reservoir results in loss of ultimate recovery. Hence conservation regulation.

As indicated, the response to the conservation problem has been to restrict production (and incidentally to restrict the number of wells drilled on a given acreage). Another possible response is to require the unitization of reservoirs, so that the element of competition within reservoirs is removed. Under unitization, separate leases are pooled, costs and revenues are shared equitably, and the reservoir is operated as a unit by a single management. What benefits the operators as a whole benefits each individual lease-holder. On behalf of the operators as a whole, the unit manager would select the pattern of well spacing and the rate of production which promised to maximize the present value of the reservoir. As changes occurred in the rate of interest or the relation of present to expected future prices and costs, he would change the rate of production (and perhaps the number of operating wells) so as to track continually the rate that promised to maximize present value. The rate would not necessarily be that which maximized ultimate recovery, but any prospective loss would be taken into account in the present value calculation and weighed against the interest saving of speedier recovery.

Although it can be argued, as I have elsewhere done,[1] that unitization of oil reservoirs is a fundamental solution of the conservation problem and is in the social interest, that is not our primary concern here. Here we are concerned simply with the response of output to economic incentives, in the short and the long run, when unit operators are free to pursue the rate of output that maximizes present value-- the optimum rate of output in an economic sense.

We note first that under unitization short-run supply would have some price elasticity. Given expected future prices, a fall in present prices would induce a reduction in current output (and perhaps a reduction in the number of operating wells per reservoir), while a rise in present prices would induce an increase in current output (and perhaps an increase in the number of operating wells per reservoir), even though ultimate recovery might suffer somewhat. This result would follow simply from the continuous effort to maximize present value. Note that since capacity depends in part on the number of wells in a reservoir, the optimum rate of output and the optimum number of wells are jointly determined.

Second, if unit operators were free to maximize present value of reservoirs, investment in exploration and development would tend to be optimal also. If explorer-developers could anticipate that new reservoirs would be unitized and operated so as to maximize present value continuously, they would extend the margin of exploration to the maximum extent feasible under given expected price and cost conditions. Only some sort of subsidy would yield a more extended margin. Although the relative response of output to a change in price need not be affected, the absolute response to a level of price would be maximized, absent a subsidy, under a regime of unitized operation of reservoirs.

Before considering short- and long-run output response to incentives under different regimes, it is necessary to examine the economics of MER and depth-acreage allowable schedules.

MER: Definitions and implications

Among state regulators of oil production MER is typically defined as the rate of production from a reservoir which if exceeded will result in significant loss of ultimate recovery. It is an engineering concept which, as a tool of conservation, fits with the view that conservation consists of the prevention of physical waste. It is not an economic concept, although at least one authority admits that economics may have to be considered in estimating MER in cases where extremely low rates of recovery are indicated on grounds of engineering efficiency.[2] In any case, except by coincidence MER so defined does not correspond to the rate of production which maximizes the present value of the reservoir.

For purposes of oil production regulation on the OCS, current regulations define MER as:

> The maximum sustainable daily oil or gas withdrawal rate from a reservoir which will permit economic development and depletion of that reservoir without detriment to ultimate recovery.[3]

The Outer Continental Shelf Lands Act Amendments of 1978 appear to define MER in a consistent way as:

> The maximum rate of production which may be sustained without loss of ultimate recovery of oil or gas, or both, under sound engineering and economic principles.[4]

Both definitions refer to economics, but neither explains just how economics is to be used in estimating MER.

One possibility has been suggested authoritatively by a then official of the Department of the Interior. According to his suggestion, MER is the rate of production which results in the highest ultimate recovery consistent with a just-acceptable rate of return on the overall operation.[5] This definition implicitly takes into account the fact that in many instances maximizing ultimate recovery may be uneconomic and prevent the development of otherwise viable reservoirs. However, MER so defined does not necessarily correspond to the rate of production which maximizes present value. This may be shown by means of an illustration.

Suppose that a tentative rate of production has been adopted in a unitized reservoir which promises to maximize ultimate recovery; and suppose that at that rate of production the operators earn an acceptable rate of return on the overall investment. Now suppose that the unit operator calculates that if he drilled additional wells and sped up production he would reduce ultimate recovery but would earn a satisfactory rate of return on the incremental investment by virtue of speeding up receipts. If he were free to maximize value, the unit operator would take this step; the value of the reservoir to him and his associates would be increased. On the other hand, if the regulators used the indicated definition of MER as the basis of production restriction, he would not be permitted to take this step and the present value of the reservoir to him and his associates would be less than it might be.

There are two implications of the use of these definitions of MER in production regulation: (1) effectively zero short-run elasticity of supply, and (2) restrained investment in exploration and development at any given level of present and expected price.

As for the first, outside the Southwestern market-demand prorationing states operators have always been permitted to produce at the MER, or at an equivalent non-wasteful rate, even in the worst days of national over-capacity. The same is true, for the period since 1970, in the Federal OCS jurisdiction. In reservoirs with multiple operators the MER has also tended to be the minimum rate of production. This is because of the adverse drainage problem: the individual operator who restricts production below MER suffers drainage to any of his neighbors who do not similarly restrict the rate of output. Without cooperation, all operators are thus induced to produce continuously at the maximum allowable rate. Therefore, only in single-operator reservoirs would we expect to find any short-run supply elasticity. On the OCS, since 1973, even this source of elasticity is missing as pressure is put on all operators to produce at the maximum allowable rate so as to minimize imports.

It should be emphasized that this short-run supply inelasticity does not have to be; it is not in the nature of the industry as such. It is in the nature of an approach to conservation regulation that emphasizes prevention of physical waste, regardless of opportunity cost, and that fails to do anything directly about the problem of adverse drainage. As we have seen, operator freedom under a regime of unitized reservoirs would result in significant short-run elasticity of supply.

As for the second implication, production restriction based on MER causes all reservoirs, except by coincidence, to be less valuable to operators than they could be. Such restriction thus contracts the margin of exploration and development at any level of present and expected price. Long-run supply elasticity, the proportionate response of output to a proportionate increase in price, may be about the same as under unitization; but the entire long-run supply curve is farther to the left on the quantity axis. MER-based regulation thus keeps us from making the most of our national resources and increases our dependence on imports, with all that implies.

Again, it does not have to be. Under a regime of unitized reservoirs, the prospective value of reservoirs is maximized and the level of unsubsidized investment in exploration and development is correspondingly maximized. (I shall not go into the merits of subsidizing domestic exploration and development so as to reduce imports.)

<u>Production restriction based on depth-acreage allowable schedules</u>

As earlier noted, the predominant basis of production restriction in the Southwestern states is the depth-acreage allowable schedule, according to which maximum oil allowables are prescribed as functions of depth of wells and acreage per well. The general nature of the schedules, which differ from state to state, can be illustrated by means of the following excerpt from the 1965 Texas Yardstick, as it is called.

From the 1965 Texas Yardstick
Maximum Oil Allowable
in barrels per day

Depth (000 ft)	Acreage per well: 10	20	40	80	160
0.0-2.0	21	39	74	129	238
.	.	.	.	.	.
8.0-8.5	34	68	133	215	380
.	.	.	.	.	.
14.0-14.5	--	200	400	600	1000

It is readily seen that the allowables increase not quite proportionately to the acreage per well and exponentially with depth. (Drilling and operating costs per well tend to increase exponentially with depth.)

These are maximum allowables per well; that is, the allowables when the market-demand factor is 100%, as it is now and has been for a number of years. Subject to capacity, they also tend to be minimum rates of output per well, due to the adverse drainage problem; if any operator unilaterally restrains production below the allowable, he will suffer drainage to his neighbors' wells. The allowables have no systematic relation to MERs where the latter have been estimated. MERs may tend to increase with depth, due to increasing pressure and temperature in the reservoir, but every reservoir is unique and no one would suppose that every reservoir at 14,000 feet, for instance, would have an MER of exactly 1,000 barrels per day if developed at one well per 160 acres, etc.

A most important feature of the schedule is that the allowable depends on the acreage per well. By choosing a spacing pattern, which usually is uniform within a reservoir, the operators choose a maximum allowable. The present value of the reservoir depends on both of these, tending to increase with the total allowable (at least until loss of ultimate recovery becomes a significant factor) and to decrease with the number of wells. These relationships suggest that there is an optimum acreage per well at every depth, as indeed there is.[6] This optimum decreases with price of oil and increases with cost per well. But whatever the acreage chosen, there is only one effective rate of output. This means that in the joint selection of acreage per well and output per well operators are severely constrained, so that it would be a coincidence if the present value of any reservoir could be maximized.

Again, we have two implications: (1) short-run inelasticity of supply and (2) a contracted margin of exploration and development. With regard to the first, we have seen that the maximum allowable also tends to be the minimum rate of output, so that in response to a change in price there can be no change in output in the short run. There is conceivably an intermediate-run responsiveness. If there is sufficient rise in price it may pay the operators in a reservoir to secure permission from the regulators and drill additional wells and thereby increase the total reservoir allowable. Using the 1965 Texas Yardstick as an example, if one well 14,500 feet deep was drilled per 160 acres the allowable per 160 acres would be 1000 barrels per day; but if an additional well is now drilled on 160 acres the tract allowable would be 1200 barrels per day. The additional present value from speeding up recovery would, of course, have to be weighed against the additional cost of wells, cost of production and loss of ultimate recovery.

With regard to the second implication, the argument is essentially the same as that in regard to MER-based regulation. However, the arbitrariness of the depth-acreage allowable schedule suggests that regulation of production based on it has a more adverse effect on the margin of exploration and development than regulation based on MER.

It may be observed that most wells in the Southwest are now producing at capacity and that capacity, not the depth-acreage allowables, is limiting output in the region. This may be correct, but it does not alter our conclusion about short-run inelasticity of supply. As for contraction of the margin of exploration and development, capacity is not independent of the number of wells originally drilled in a reservoir, which depends in part on the allowable schedule. Also capacity output suggests the possibility of significant loss of ultimate recovery which operators cannot effectively do anything about since capacity tends to be the minimum as well as the maximum rate of output. In either case, our conclusion stands that production regulation based on allowable schedules tends to contract the margin of exploration and development.

<u>A word on natural gas</u>

The production of natural gas in the United States is not systematically regulated on the basis of MER or depth-acreage allowable schedules. Occasionally an MER may be applied, or reservoir output may be restricted to pipeline demand and allocated among lessees as a means of assuring ratable take and protecting correlative rights. So most of what we have said about oil production does not apply to natural gas.

However, in many situations natural gas is a joint product with oil, particularly at the exploration stage. If regulation of oil production tends to contract the margin of exploration for oil, it does likewise for gas. A rise in the expected price of gas stimulates oil/gas exploration, but due to the pattern of oil production regulation anticipated, the response is not as great as it would be if oil production were under a regime of unitization with operator freedom. Thus in the long run gas production is constrained by regulations that apply only indirectly to it.

The rate of leasing on the OCS

When the expected price of oil or gas rises, exploration is of course increased. But in the short run the industry's results are constrained by the inventory of prospects. These prospects vary in quality, and the industry's response to a price stimulus is to "dig deeper" into the bag of prospects, lowering the marginal and average quality of the prospects actually explored. Consequently, subsequent discoveries do not increase in proportion to exploration effort. If it were not possible to subject new land to pre-drilling exploration, and to secure new leases, the decline in the quality of prospects would severely limit the long-run response of supply to a price stimulus.

In general, leases are freely available to be bid for in the continental United States (onshore). The main limiting factor is the restricted stock of continental lands that have not already been thoroughly explored. But on the OCS, where perhaps the bulk of the oil and gas of the United States remaining to be found is located, the availability of leases is restrained by policy; and the leasing authorities cannot quickly react to a price stimulus. As present law is officially interpreted, the leasing authorities are required "(1) to assure orderly and timely resource development; (2) to protect the environment; (3) to insure the public a fair market value return on the disposition of its resources."[7] All of these requirements, but especially the second and third, stand in the way of a sudden marked increase in the rate of leasing in response to a price stimulus. This in turn restrains the oil and gas supply response to such a stimulus, particularly in the short and intermediate run.

Part of the problem is coordinating oil company preleasing exploration with the rate of leasing. If the industry was certain of the number and general location of leases to be granted in a given period of time, it could do the preliminary exploration essential to assuring "fair market value" bidding. Thus the leasing authorities could, with sufficient notice to the oil companies, raise the rate of leasing and sustain it for many years. But they still could not respond adequately to sudden and unexpected increases in prospective prices. The problem of protecting the environment alone, with all of the required impact studies, hearings and perhaps litigation, would seriously interfere with quick and flexible reaction. OCS leasing policy will probably continue to be a short- and intermediate-run restraint on oil and gas supply as the industry responds to price incentives.

Conclusion

I conclude that the continued regulation of oil production in the United States significantly constrains the oil and gas supply response to price stimuli, in both the short and the long run. Successful supply modeling must take into account the constraints we have discussed.

FOOTNOTES

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5. Statement of Jack W. Carlson, Assistant Secretary, Energy and Minerals, Department of the Interior, before the Subcommittee on Antitrust and Monopoly, Senate Committee on the Judiciary, Sept. 23, 1975, (mimeo) p. 20.

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7. Statement of Harrison Loesch, Federal Leasing and Disposal Policies, Hearings before the Committee on Interior and Insular Affairs, U. S. Senate (Washington, D. C., June 19, 1972), p. 38.

DISCUSSION

LARRY BUSH: My name is Larry Bush, and I am with the Texas Counsel of Public Accounts.

I have heard the concern expressed by two separate independent producers that there are occasions where large producers, primarily major producers, will over produce a field even beyond the state conservation rate in order to satisfy short-term cash flow needs.

Does that appear to be maximizing social value of the natural resource?

DR. McDONALD: Maximizing cash flow is not necessarily consistent with conservation as such. I define, as you know, Larry, conservation as action designed to maximize present value of the resource. Maximizing current cash flow will not necessarily do that.

Whether there are cases such as you mention, I don't know. I think it might well be more likely for a small independent who had a heavy bank debt and had to make payments on it to do that than a large company with good access to the capital market.

DR. HUBBERT: How do you maximize my present value? What is the technique?

DR. McDONALD: The technique is to rearrange the flow of your cash in such a way that when you discount it, it achieves the maximum present value. In the case of oil and gas production, what you would rearrange would be the time schedule of your production, that depending, of course, upon what the trend in expected prices is.

As I indicated, if you expect future prices to rise relative to present prices, you would shift production toward the future, and vice versa. But it is simply a matter of equating discounted marginal net revenues in all periods. That is the technique.

Firm Size and Performance in the Search for Petroleum

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Introduction

Crude-oil production in the conterminous 48 States of the United States peaked in 1970. At that time, few were concerned with the consequences of the rapidly increasing consumption of imported oil by the United States. Because of the oil embargo of 1973, by means of which OPEC (Organization of Petroleum Exporting Countries) became a functioning world oil cartel, attention was focused upon the structure of the domestic oil industry. Allegations have been made that the large integrated firms were engaged in monopolistic pricing within the United States. These allegations were based upon the fact that these firms spent a large part of their funds for exploratory drilling in foreign countries during the 1960's and early 1970's while at the same time they held vast amounts of undeveloped acreage favorable for the occurrence of petroleum within the United States. Industry critics (Blair, 1976) have pointed to this allocation of exploration as evidence that the large integrated oil firms were restricting potential domestic supplies by not exploring in the United States in order to drive up the price. Some critics (Engler, 1977) advocated that divestiture of the exploration and production units of the large integrated firms into regional units would increase domestic exploratory drilling and thereby increase the supply.

Before a major restructuring of the industry is attempted, the policy-makers must face the dilemma of predicting how the performance of the industry might be changed as a result of the suggested divestiture and regionalization of the exploration and production units of the major integrated firms. However, very few analyses have been made of the history of the efficiency of the performance of different sizes of oil firms in exploring for and discovering crude petroleum; two examples of such analysis were prepared by McKie (1961) and West (1977). In order to expand on the conclusions reached in these two studies, we undertook a detailed examination of the relative roles played by firms of different sizes in the exploration of a significant petroleum-producing province (the Denver basin). The exploration and production history of the firms operating in the Denver basin was examined (1) to compare the relative search performances of firms of different sizes and (2) to determine whether physical variables can explain the exploration strategies of firms of different sizes.

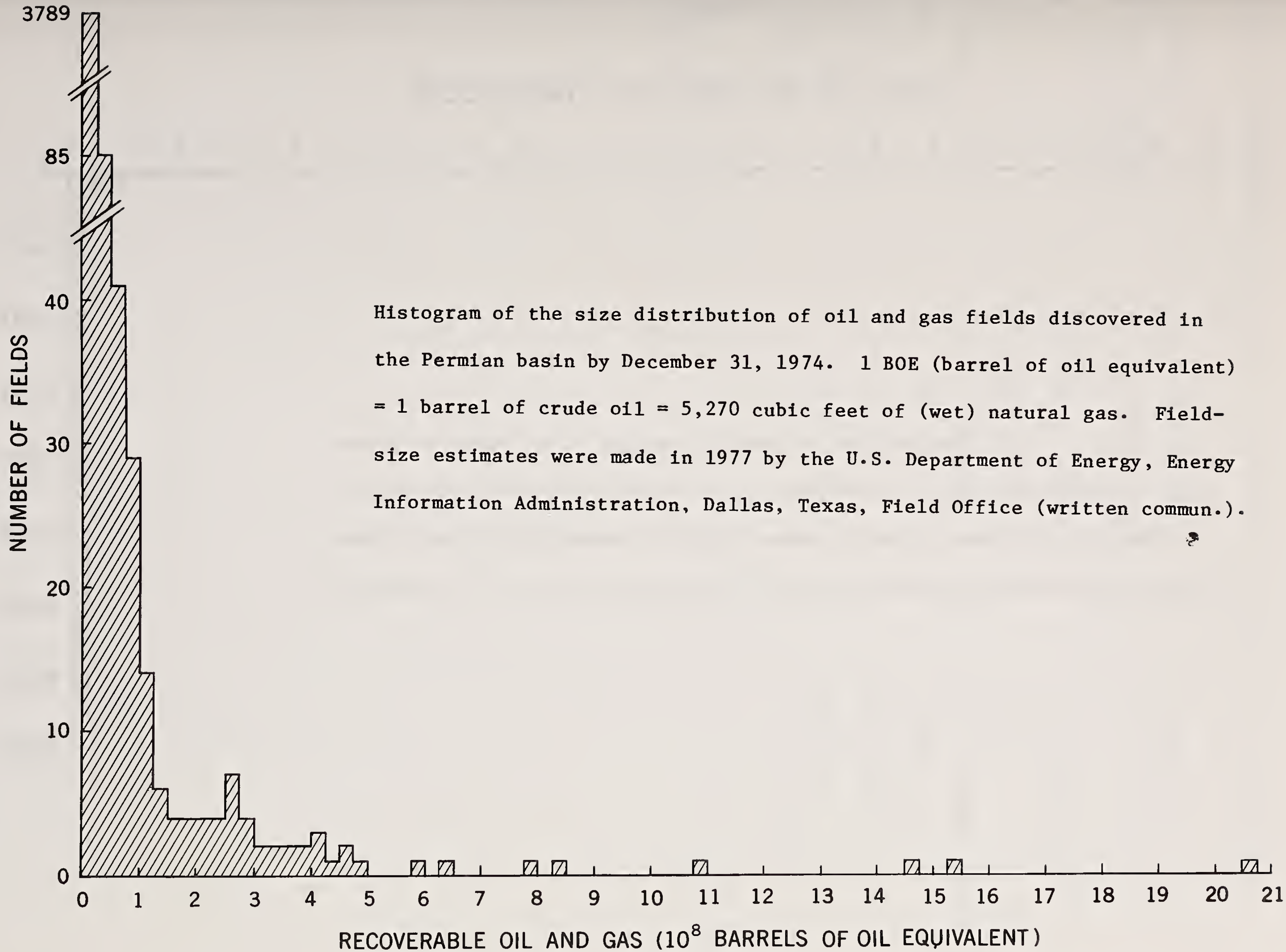
The exploration behavior observed in the Denver basin, we believe, is representative of how onshore exploration progressed in the United States. For example, in the United States from 1946 to 1953, the major firms drilled 19 percent of the wildcat wells (29 percent if those financed by them are included). Alternatively, from 1970 to 1976, the majors accounted for less than 10 percent of the wildcat wells drilled (West, 1977, p. 86). In the Denver basin during 1955, the major firms accounted for about 20 percent of the wildcat wells drilled, and by the mid-1960's, they had almost completely shut down their wildcat drilling in this area, thus, for the period 1949 to 1974, independent firms accounted for 88 percent of the wildcat wells drilled. Drilling costs in the Denver basin were cheap, and mineral rights were easily and regularly transferred. Consequently, the independent and small firms seem to have had a better chance here than elsewhere to play a significant role in the area's exploration.

Because our purpose is to demonstrate how physical characteristics of petroleum discoveries and the results of past exploration affected industry behavior the first part of the paper discusses physical characteristics of oil and gas fields that produce regularity in the petroleum-discovery process. This discussion is followed by a brief description of the exploration history of the Denver basin. A summary of the results is presented along with their implications in the concluding section.

Physical Characteristics Affecting the Petroleum-Discovery Process

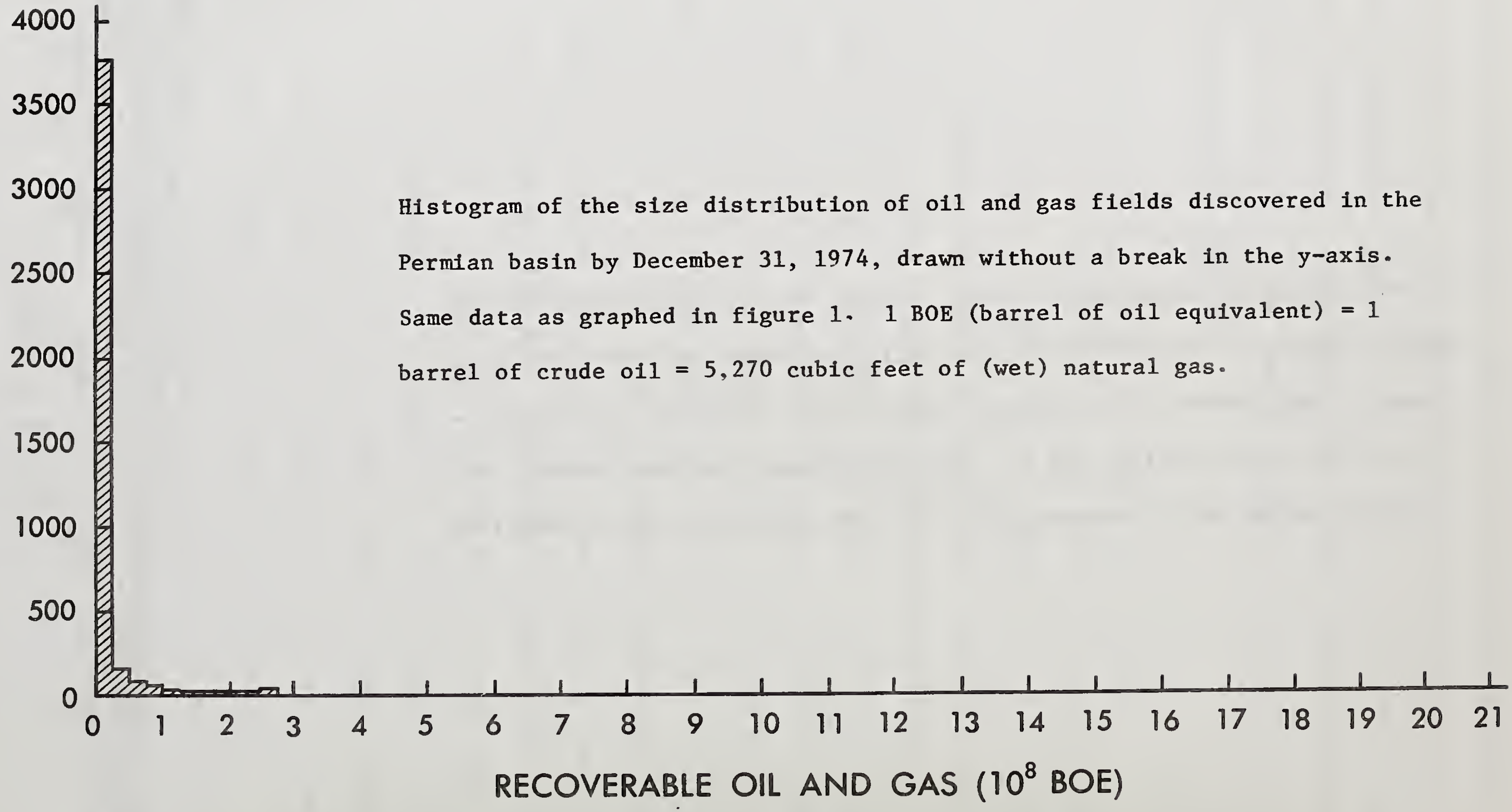
Petroleum fields occur in sedimentary rocks in geologic basins or provinces that extend over millions of acres. Figure 1 shows the size distribution of discoveries in the Permian basin, in western Texas and southeastern New Mexico. This field-size distribution is typical of other basins or petroleum provinces in that fields occur in a wide range of sizes, most fields are small, and most of the basin's reserves are contained in just a few large fields. The largest 38 fields, or less than 1 percent of the total fields, contain more than half the hydrocarbons whereas the 3,789 fields in the smallest class size account for only 16 percent of the hydrocarbons discovered. Figure 2 presents the same data as figure 1 but is drawn to scale without breaks in the y-axis in order to emphasize that small fields are much more common than large fields in the same basin. At the scale used in figure 2, the large deposits are too few to be graphed. The disparity in sizes between the smallest and largest size class is six orders of magnitude. Table 1, which lists pairs of items whose sizes differ by six orders of magnitude, is presented so that a vivid impression might be gained about the disparity in field sizes.

A major consequence of the wide disparity in field sizes is that the rate of discovery (volume of oil and gas discovered per unit exploration effort) declines as cumulative exploration increases. Because large fields generally have large



Histogram of the size distribution of oil and gas fields discovered in the Permian basin by December 31, 1974. 1 BOE (barrel of oil equivalent) = 1 barrel of crude oil = 5,270 cubic feet of (wet) natural gas. Field-size estimates were made in 1977 by the U.S. Department of Energy, Energy Information Administration, Dallas, Texas, Field Office (written commun.).

FIGURE 1.



Histogram of the size distribution of oil and gas fields discovered in the Permian basin by December 31, 1974, drawn without a break in the y-axis. Same data as graphed in figure 1. 1 BOE (barrel of oil equivalent) = 1 barrel of crude oil = 5,270 cubic feet of (wet) natural gas.

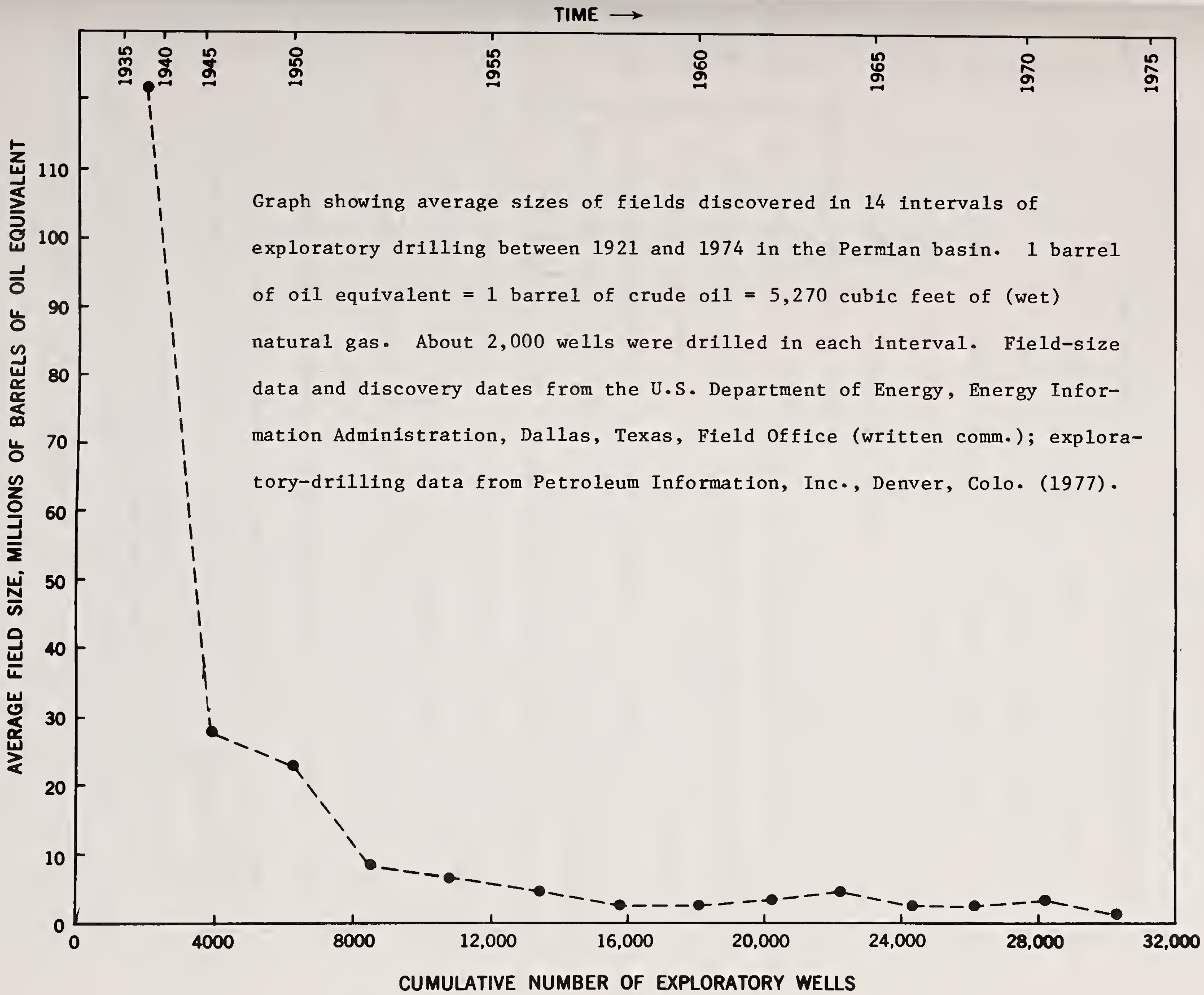
FIGURE 2.

Table 1.--Comparisons illustrating a difference of six orders of magnitude.[1]

Category	Small item	Large item
groups of people (number)	baseball team	New York City
price	used bicycle	new jumbo jet
time	half minute	year
length	soccer field	twice around equator
areas	Liechtenstein	Pacific Ocean
height	1/3 inch	Mt. Everest
foot race	4-cm. sprint	marathon
volume	1 drop	1 barrel of oil

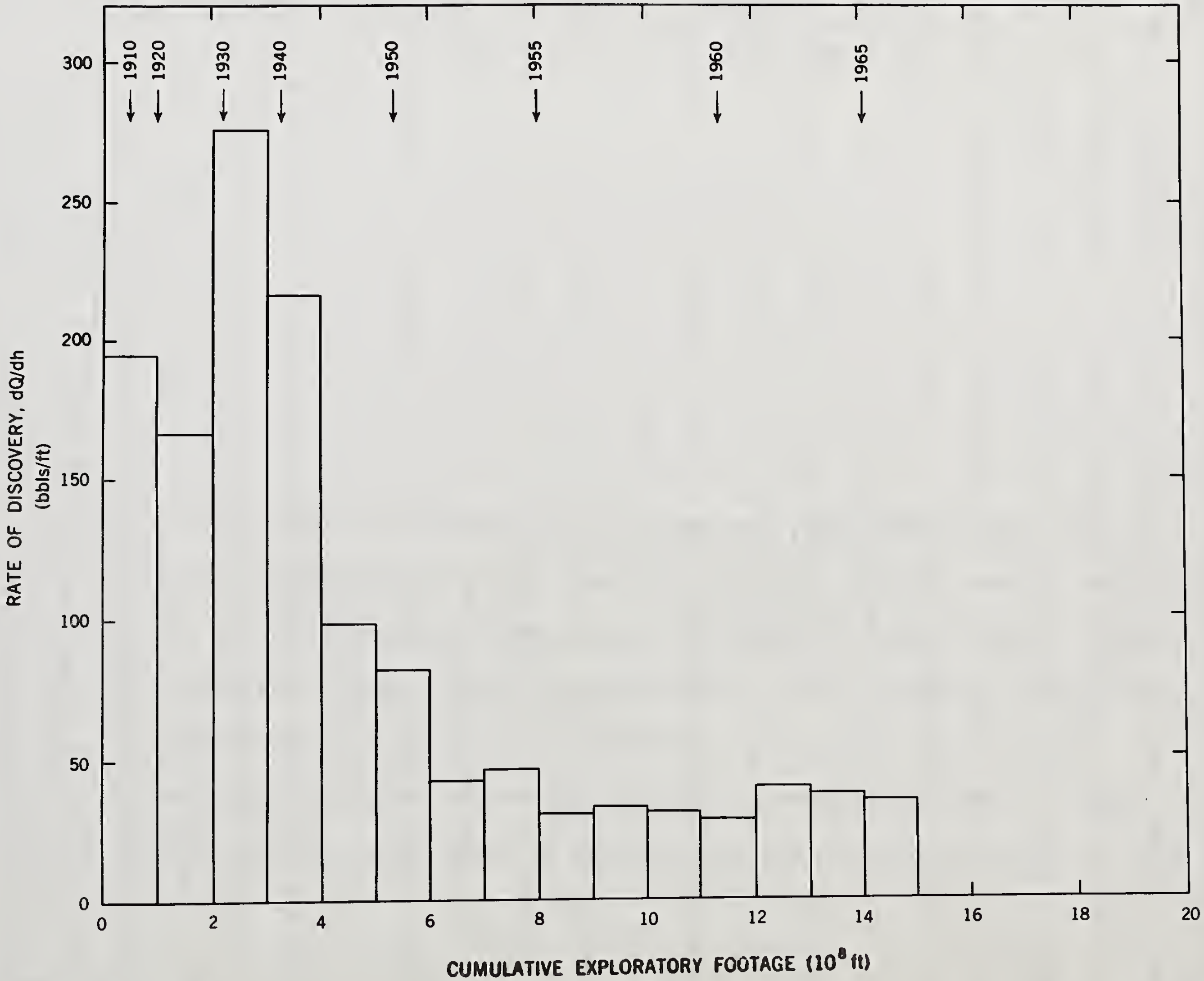
[1]From Root and Drew, 1979.

surface areas and are contained in large easily detected geologic structures, they are more easily discovered than small fields. In the Permian basin, the Yates field, which contains more than 2 billion barrels of recoverable oil equivalent has a surface area of about 50 square miles. The distribution in field sizes and the fact that large fields typically are more easily detected than small fields causes the rate of discovery to decline after the few large fields are found. Figure 3 presents the average field sizes for 14 drilling increments of approximately 2,000 wells each in the Permian basin. The rate of discovery in this region appears to have gone through three stages. These include (1) a short initial phase when the discovery rate was high, (2) a second stage when the discovery rate declined rapidly, and (3) the third stage when a low but stable discovery rate was maintained. These stages are also characteristic of the discovery rate for the conterminous United States as seen in figure 4 (modified from Hubbert, 1967). After World War II, the discovery rate dropped rapidly, and since 1953, the discovery rate has been fairly stable (Root and Drew, 1979). The rapid drop in the U.S. discovery rate was not immediately recognized at that time because generally, several years are needed to determine the size of a large oil field. However, the dramatic reduction in wildcat drilling in the United States and the shift of exploration by the major firms toward foreign areas of operation that began in the late 1950's clearly was induced by the precipitous drop in the U.S. discovery rate shortly after World War II. Major oil firms that had attractive prospects or opportunities in foreign areas because discovery rates were higher in these regions than U.S. discovery rates moved their exploration efforts to these areas. The regularity in the petroleum-discovery process for nearly all domestic areas, including the Denver basin, is the result of the physical characteristics of petroleum fields.



Graph showing average sizes of fields discovered in 14 intervals of exploratory drilling between 1921 and 1974 in the Permian basin. 1 barrel of oil equivalent = 1 barrel of crude oil = 5,270 cubic feet of (wet) natural gas. About 2,000 wells were drilled in each interval. Field-size data and discovery dates from the U.S. Department of Energy, Energy Information Administration, Dallas, Texas, Field Office (written comm.); exploratory-drilling data from Petroleum Information, Inc., Denver, Colo. (1977).

FIGURE 3.



Histogram showing crude-oil discoveries per foot of exploratory drilling in the conterminous United States (modified from Hubbert, 1967).

FIGURE 4

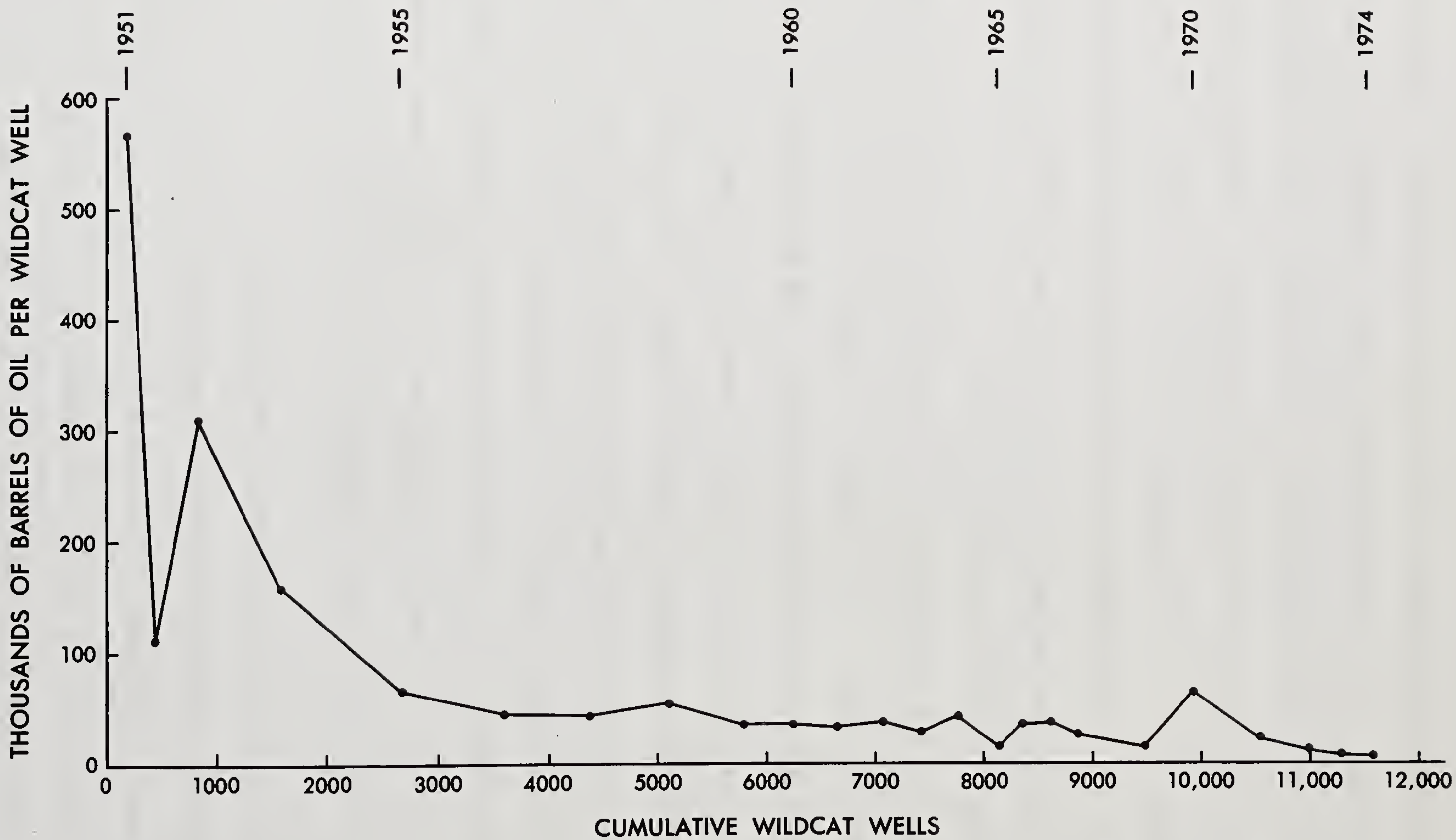
The Study Area

The Denver basin consists of approximately 40,000 square miles in eastern Colorado, southwestern Nebraska, and southeastern Wyoming. Data pertaining to the exploration history of the Denver basin were obtained from the Well History Control File of Petroleum Information, Inc., Denver, Colo. From 1949 to 1974, 11,577 exploratory wells were drilled in the basin, resulting in the discovery of 909 petroleum fields, which range in size from the Adena field (63.4 million barrels of oil) down to numerous one-well fields that produce only a few thousand barrels. The estimated amount of oil found in the 909 fields discovered between 1949 to 1974 was 742 million barrels of oil.

The skewed size distribution of fields present in the Permian basin also typifies the Denver basin. The largest of the 909 fields contains 8.5 percent of the oil found during the 1949-1974 period, and the smallest 659 fields accounted for only 7 percent of the oil found. Furthermore, the largest 31 fields accounted for 45 percent of the oil found. As a result of the wide range in field sizes, the discovery rate for the Denver basin (fig. 5) shows the same behavior as the discovery rates for the Permian basin (fig. 3) and the conterminous United States (fig. 4). The slight increase in the discovery rate in 1970 is accounted for by several significant discoveries made on acreage that was formerly withheld from exploration by the Union Pacific Railroad.

Firm Size and Search Performance in the Denver Basin

Firms operating in the Denver basin were classified into four groups. The first group consists of the major firms that are vertically integrated and are active in all stages of the petroleum industry from exploration and production on through transportation, refining, and product marketing. Table 2 presents a list of the major operators. The second group consists of large independent



Graph showing the wildcat-well discovery rate (barrels per wildcat well) for petroleum in the Denver basin during the 1949-1974 period. Data are from Petroleum Information, Inc., Denver, Colo. (1975).

FIGURE 5.

Table 2.--Numbers of wildcat and development wells drilled by the major integrated operators in the Denver basin 1949-1974.

[Data are from Petroleum Information, Inc., Denver, Colo., 1975.]

Firm	Unsuccessful wildcat wells	Successful wildcat wells	Producible crude oil discovered[1]	Successful development wells	Unsuccessful development wells
AMERADA	24	1	1.4	18	16
AMOCO	232	14	12.3	598	176
BRITISH AM.	241	28	58.2	333	180
CHAMPLIN	49	4	9.8	72	41
CHEVRON	11	3	6.7	37	6
CHICAGO	9	3	8.0	18	12
CONTINENTAL	45	4	9.3	67	44
GULF	32	2	0.5	40	29
KOCH	27	2	5.7	44	3
MOBIL	7	2	0.2	86	39
MONSANTO	37	3	1.3	117	34
OHIO	91	27	41.5	211	104
SHELL	276	25	17.4	250	113
SINCLAIR	54	4	7.2	99	46
SKELLY	47	10	7.8	107	63
SOHIO	37	0	0.0	24	13
SUN	20	1	1.8	35	15
SUNRAY	19	2	2.3	19	17
SUPERIOR	40	5	8.7	67	25
TEXACO	33	4	6.8	84	31
TOTAL	1331	144	206.9	2326	1007

[1]In millions of barrels.

firms that are principally involved in exploration and production. Drilling contractors are classified as the third group of operators; this group includes only firms whose principal business is the sale of contract drilling services to the petroleum industry. The fourth group, "small independents," consists of more than 3,000 individuals and small firms.

The performance of each group of operators is measured in terms of the wildcat success rates, development-well success ratios, oil discovered per exploratory well and the quantity of reserves owned. Because oil fields commonly encompass several leases, the firm that discovers a field generally does not own all the field's reserves. An estimate of ultimate oil recovery was assigned on a production-well basis in order to allocate reserve ownership to a specific class of operator. The method for apportioning expected field recovery to individual wells was based upon the assumption that a well's ultimate productivity is directly proportional to the initial producing rate of that well. For example, if a two-well field is estimated to have an ultimate recovery of 69,800 barrels and one well had an initial producing rate of 180 barrels per day and the other well has an initial rate of 120 barrels per day, the first well is assigned an ultimate recovery of 41,880 barrels, and the second well is assigned 27,920 barrels of reserves.

In table 3, the drilling records of the four groups of operators are presented. During the period considered, as a class, the major integrated firms discovered 206.9 million barrels of oil (28 percent of the total) by drilling 1,475 wildcat wells, of which 144 were successful. Per wildcat well, they discovered 140.3 thousand barrels, more than twice the amount discovered per well by any other group. Their wildcat success ratio was also the highest at 0.098.

Table 3.--Discovery success and volumes of petroleum discovered by operator class from 1949 to 1974 in the Denver basin.

[Data are from Petroleum Information, Inc., Denver, Colo., 1975.]

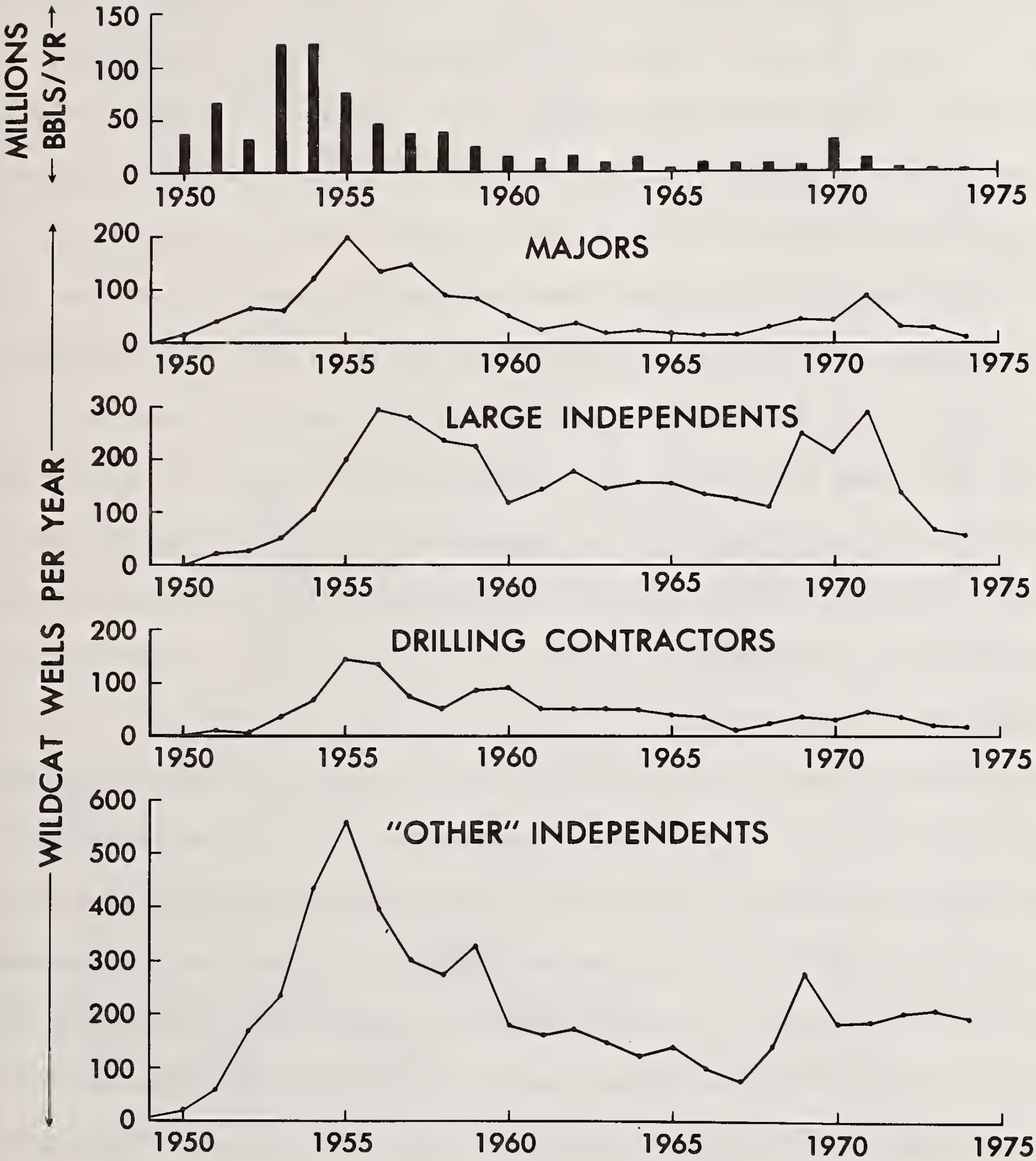
	Large majors	Large independents	Drilling contractors	Small independents	TOTAL
Number of successful wildcats	144	275	84	406	909
Number of unsuccessful wildcats	1,331	3,484	1,228	4,614	10,657
Wildcat-well success ratio	.098	.073	.064	.081	.079
Petroleum[1] discovered	206.9	185.6	80.8	268.7	742.0
Petroleum discovered per wildcat well drilled[2]	140.3	49.4	61.6	53.5	64.2

[1]In millions of barrels of producible oil.

[2]In thousands of barrels of producible oil.

Figure 6 presents the time profile of discoveries and wildcat drilling for the four classes of operators. The figure shows that the major integrated firms had essentially stopped drilling wildcat wells in the basin in the late 1950's. The reason why the major integrated firms left in the late 1950's is that the rate of discovery had fallen off sharply after 1955 and they moved on into other regions where high rates of discovery could be obtained, that is, offshore areas and foreign countries. The number of wildcat wells drilled by the majors closely tracks the total volume of discoveries per year when wildcats are lagged by 2 years. Attanasi and Drew (1977) modeled this effect by a distributed lag function. Drilling rates for each of the other three classes of operators show to a lesser degree the same lag effect. However, these other operators (combined) still drilled more than 200 wildcat wells each year throughout the 1960's and 1970's. This record of exploratory drilling obviously indicates that the major firms could not have been actively excluding smaller firms from exploring the basin.

The strategy that seems to have been followed by the majors was to assemble and hold large blocks of acreage during the long term. As a result of this strategy, the major firms can systematically evaluate and select for drilling what they regard as the higher quality acreage and then, in turn, farm out the poorer quality acreage to independent firms. In these arrangements, the major firm (usually holding a dominant acreage position on a particular prospect) provides the smaller operator with a parcel of its own lease or working interest in a parcel of acreage in exchange for information on the results of a specified number of exploratory wells that the smaller firm is committed to drill. This kind of exchange agreement may also be used when potential development acreage is farmed out, usually on the fringes of the field.

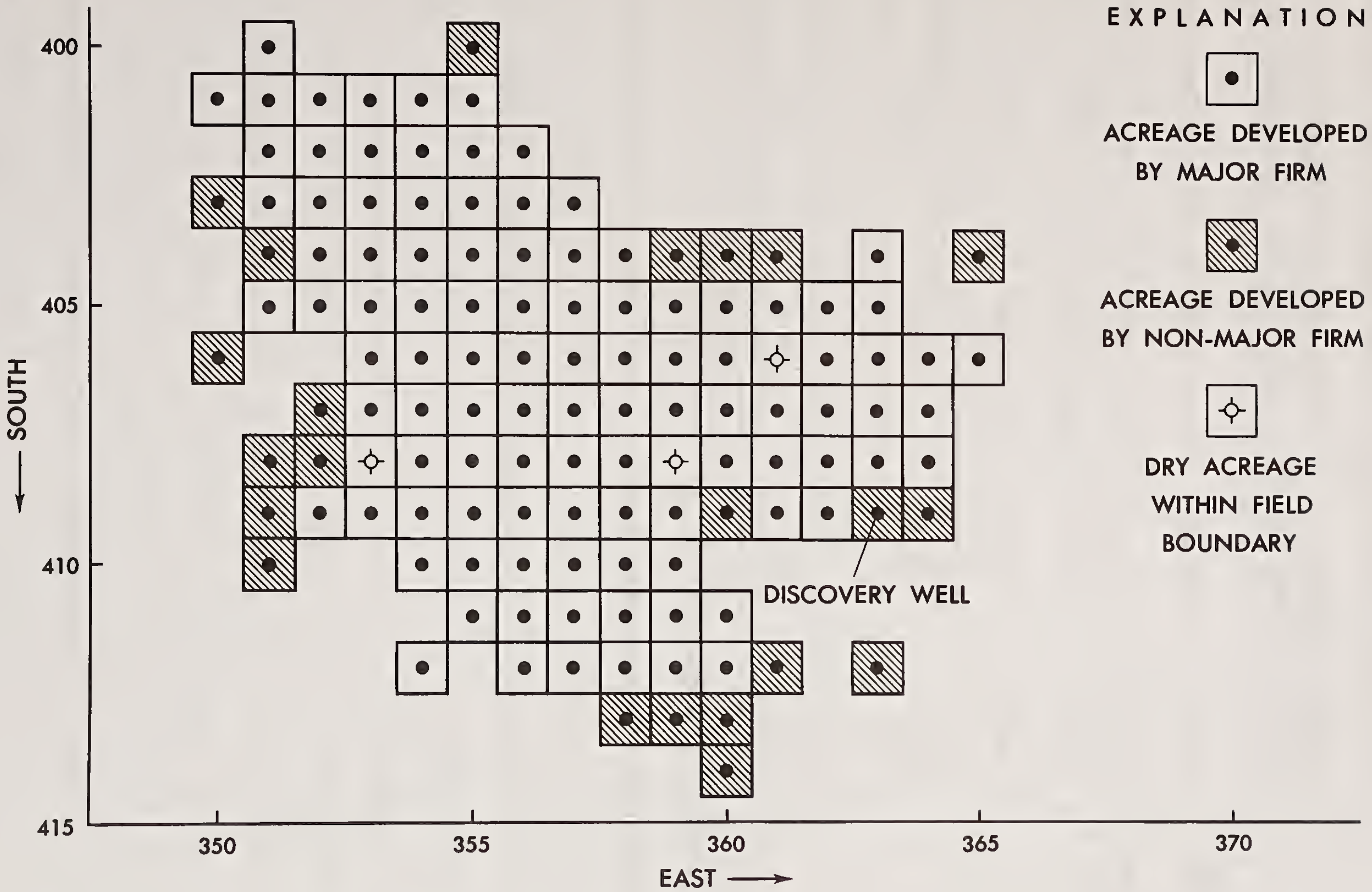


Graphs showing the time profile from 1949 to 1974 of oil discoveries and wildcat drilling for the four classes of operators. Data are from Petroleum Information, Inc., Denver, Colo. (1975).

FIGURE 6.

Figure 7 indicates the location of the discovery and development wells drilled by type of firm in the Sloss field, one of the largest fields in the Denver basin. First, the discovery of the field was credited to an independent at a site that ultimately proved to be on the edge of the productive limit of the field. Second, the major firms also drilled most of the normally more productive areas in the central part of the field, whereas areas drilled by the independent firms were generally on the margins of the field where petroleum yields are almost always inferior. Of the 31 largest fields in the basin (representing 45 percent of reserves found between 1949 and 1974), at most 13 were discovered by major operators. Field maps for the largest 20 fields show that these fields have the same pattern of discovery- and development-well ownership as the Sloss field (fig. 7). Apparently, even though the majors had identified the area as favorable to the occurrence of oil, they regarded it as marginal in terms of committing cash for exploratory drilling that could have been used elsewhere, and they were willing to farm out acreage to facilitate timely evaluation of the prospect.

Results of development drilling and reserve ownership by class of operator are compared in table 4. The major's development-well success ratio was substantially higher than those of any of the other classes of firms. Additional evidence that the major firms were able to retain the highest quality production acreage is that their reserves per development well were 46 percent higher than those for the large independent firms, almost twice as high as those for the drilling contractors, and almost four times the reserves per well of the small independent firms. Although the major firms were credited with the discovery of only 28 percent of the total value of oil discovered, they are expected to develop 47.3 percent of the reserves represented by these discoveries. Conse-



MAP OF ACREAGE OWNERSHIP WITHIN THE SLOSS FIELD
(THE WELL SPACING IS ONE WELL PER 40 ACRES)

Data are from Petroleum Information, Inc., Denver, Colo. (1975).

FIGURE 7.

Table 4.--Development-well success and volumes of petroleum reserves credited by operator class from fields discovered from 1949 to 1974 in the Denver basin.

[Data from Petroleum Information, Inc., Denver, Colo., 1975]

	Large majors	Large independents	Drilling contractors	Small independents	TOTAL
Number of successful development wells	2,326	1,106	309	1,926	5,667
Number of unsuccessful development wells	1,007	1,342	533	2,462	5,344
Development-well success ratio	.698	.452	.367	.439	.515
Reserves owned[1]	351.2	176.6	47.0	167.2	742.0
Reserves per development well drilled[2]	105.4	72.1	55.8	26.6	67.4

[1]In millions of barrels of producible oil.

[2]In thousands of barrels of producible oil.

quently, the major firm's strategy of acquiring and holding large blocks of undeveloped acreage for the long term permitted them to gain production rights to oil that was not discovered by their own wildcat wells. The strategy of farming-out exploration and marginal production acreage is consistent with the precipitous decline in the discovery rate in the mid-1950's and their subsequent movement of their own exploration activities to offshore and foreign areas where discovery rates were still high. Moreover, major firms typically like to use their large cash assets for exploratory activities in high-cost frontier and international areas where both the expected returns and contingency risks are likely to be very large.

What seems to be more difficult to explain is why the independent firms drilled so many poor-quality development acreage locations. In their attempt to develop reserves that represent just over half the oil in the Denver basin, the large and small independent firms and the drilling contractors drilled 4,337 (81 percent) of the 5,344 total dry development wells drilled. By definition, these wells were drilled in locations directly offsetting productive wells. However, they were just beyond the productive limits of the field. The behavior of the independents, particularly, the smaller ones is due in part to complex promotional devices used by the independent firms to finance both exploration and development drilling. These firms typically have few fixed resources; small or non-existent geologic staffs, few leases, and little if any drilling or production equipment. An important source of funds for these firms are individuals' speculative funds that would normally be taken as income taxes. A promoter for an independent may even get individuals to subscribe to the drilling of a well in a specific location. It is well known within the petroleum industry that certain independent firms can and do make a profit for themselves on a dry hole. The independent who has individuals subscribe to individual holes does not usually consider the opportunity cost of using these funds on other prospects. In contrast, major firms finance drilling from retained earnings generated by production or selling additional equity and they evaluate the opportunity cost of using capital in a whole range of areas and prospects. This explanation of the results leads to the suggestion that the major's principal objective is to develop production, perhaps because of the profit it can make on oil in its downstream (refining, retailing, and transporting) activities, whereas the independent firm who makes a profit on the act of drilling, has the objective of drilling as many wells as possible.

Conclusions

Industry critics argue for divestiture and perhaps regionalization of major oil firms by pointing to the facts that (1) during the last 30 years, the largest 20 integrated (or major) oil companies have owned more than 50 percent of the undeveloped favorable acreage in the United States and (2) starting in the mid-1950's and continuing for more than a decade, these same oil companies steadily decreased their level of exploratory drilling within the United States (Blair, 1976). These advocates claim that such evidence can mean only that the major integrated firms acted together to restrict exploratory drilling which, in turn, resulted in the restricting of the supply of domestic crude oil and ultimately forced up the price.

In this analysis of exploration of the Denver basin, which we think is representative of exploration of the onshore conterminous 48 States, no evidence was found to support the assertion that the major integrated firms had restricted supply of crude petroleum by restricting access to favorable acreage.

As long as major firms continue to evaluate prospects by using farm-out agreements they cannot restrict supply of oil and gas in an area. If a small firm discovers oil or gas on acreage obtained from a farm-out agreement, the owners of adjacent tracts (that also contain part of the new field) must immediately begin production on their property or risk losing these resources by a neighbor draining their tracts. The symbiotic relationship between large and small firms may accelerate exploration but will probably not significantly affect the region's or nation's petroleum supply. The willingness of the large firms to enter into such agreements implies that the prospect has been evaluated sufficiently to determine that the likely size of the discovery will be marginal when compared to the firm's other opportunities. The major firms reduced their own exploratory drilling in response to the sharp declines in the discovery rates in the United States and the Denver basin.

The practice of holding large amounts of undeveloped acreage is only one element in the overall exploration strategy used by the major integrated companies. Briefly, this land-holding practice gives these firms (1) a superior chance at obtaining significant parts of the larger, more profitable oil fields, which are for the most part, discovered early in the exploration of a region, and (2) an additional source of income to them, after the discovery rates have fallen below an acceptable level, from farming out parcels of acreage for continued exploration and development by the independent firms. These independent firms use a complex set of promotional devices to raise tax-sheltered and other types of speculative funds to finance their drilling activities. They normally deal with the major firms for acreage contributions, dry-hole money, and other assets. The essential idea is that an independent firm could make a profit on the act of drilling a well. Whether the well produces a net profit for its production may be only a secondary concern. In fact, the independents serve as instruments to divert speculative and tax-sheltered funds into the exploration industry, and because they can make a profit on the drilling of a hole itself, they can continue to drill even when the average rate of return from the petroleum produced does not justify such drilling. This behavior contrasts sharply with the major firm's primary objective in carrying out exploration; that is, to develop production for downstream activities.

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SENSITIVITY ANALYSIS OF FORECASTS FOR
MIDTERM DOMESTIC OIL AND GAS SUPPLY

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1. EXECUTIVE SUMMARY

This report describes a quantitative sensitivity analysis of midterm projections of United States crude oil and natural gas production for the Department of Energy's (DOE) EIA Annual Report to Congress, 1979 (1979 ARC).

1.1 The Project's Objectives

The specific target of this study is a set of projections of midterm oil and gas production generated by the 1979 version of the DOE analysis system MOGSMS (the Midterm Oil and Gas Supply Modeling System), in conjunction with the 1979 ARC. These forecasts apply to conventional oil and gas, on-shore and off-shore, in the lower 48 states, for the years 1985 to 1995, inclusive. The specific objective of this work was to employ analytic procedures to examine the sensitivity of the 1979 midterm oil and gas projections to some of its key elements.

The sensitivity analysis employed to achieve this objective draws heavily upon procedures and insights developed in model validation efforts recently carried out by the National Bureau of Standards of the United States Department of Commerce, for the Office of Analysis Oversight and Access of EIA.

1.2 Overview of the Study Design

The primary emphasis in this current study was the analysis of the sensitivity of the target projections to uncertainties in: (1) the (exogenous) estimates of regional finding rates and the original resource bases; and (2) various other significant data elements underlying the projections. Accordingly, three distinct statistical experiments served as the vehicles for the sensitivity analysis:

- Experiment 1: A Monte Carlo analysis of the USGS Circular 725 estimates of regional undiscovered (oil and gas) resources and their impact on MOGSMS finding rates and ultimate outputs.

- Experiment 2: A Monte Carlo analysis of other selected key elements (namely, recovery factors and decline rates) and their impacts on MOGSMS output.

- Experiment 3: A response surface analysis of various MOGSMS input data elements, including both physical and economic elements, to identify those in which uncertainty has the greatest effect on MOGSMS results.

Each of these experiments involved exercising MOGSMS in tests to determine the effects of input uncertainty or variations on the MOGSMS forecasts of conventional oil and gas production. We measured these effects primarily through computed product measures, which were time averages of production for crude oil, gas and NGL.

1.3 Key Results Obtained

The three experiments each yielded significant results, which can be summarized as follows. In Experiment 1, explicit treatment of this uncertainty via a Monte-Carlo analysis leads to higher production forecasts than the deterministic approach yielded for the 1979 ARC, especially for natural gas. The increment in gas production, as measured by the mean values of the Monte Carlo results, is 2.10 TCF in 1990 (17 percent higher than the 1979 ARC projection), increasing to 3.68 TCF in 1995. The average gas production increment is 1.91 TCF/year (16 percent) for the period 1985-1990-1995. Across all products, the Monte-Carlo analysis leads to the significant average production increment of 1.0 million barrels of oil equivalent per day, over the same period.

As anticipated at the start of the study, the probablistic treatment of resources keeps the spread between the "optimistic" and "pessimistic" projections of production relatively tight:

Monte-Carlo Ranges of MOGSMS Production Forecasts

	Time-Average	Percent of Mean Value
Crude Oil (MM Bbl/day)	0.40	8%
Natural Gas (TCF/yr)	1.10	8%
NGL (MM Bbl/day)	0.06	6%
	(Interquartile Ranges: 25th-75th Percentiles)	

Thus, in this experiment, explicit recognition and treatment of uncertainty in a key input data element leads to: (1) a fairly small estimate for the dispersion resulting from that random element; and (2) a small (but possibly significant) increase in the production forecast.

From Experiment 2 (the second Monte-Carlo analysis), we see that variability in either decline rates or recovery factors has a very major impact on ultimate production. The average interquartile widths here have been computed as a percentage of the base case totals to be:

Decline Rate Experiment

Crude oil average range = 13.8 percent
Natural gas average range = 29.4 percent

Recovery Factor Experiment

Crude oil average range = 12.9 percent

Experiment 3 - Response Surface Analysis - the response surface analysis - comprised a sequence of three sub-experiments, each relating MOGSMS oil and gas production projections to systematic variation in specific sets of input elements subject to uncertainty. Each sub-experiment led (via regression analysis) to a linear equation relating changes in the oil and gas product measures to changes in the values of the input data elements.

The experiments revealed broadly that the seven data elements originally suspected as vital were quite important. The only exception was the apparent minimal effect on natural gas production to be credited to the discount rate and drilling costs. Some key examples of the results are that:

- a 25 percent increase in the assumed discount rate induces about a 123,000 Bbl/day decrease in average oil production over the midterm;

- a 10 percent increase in the total-to-exploratory drilling ratio reduces production by an estimated 159,000 Bbl/day;

- a 1-year increase in the capital planning horizon leads to a decrease of approximately 92,000 Bbl/day.

2. BACKGROUND AND MOTIVATION FOR THE SENSITIVITY EXPERIMENTS

The detailed discussion of this study's analytical issues, experiments, and results begins in this section and continues for the remaining four sections. By way of background and orientation to this report, we begin by offering a brief summary of the objectives and context of the study.

The Energy Information Administration (EIA) has sponsored numerous activities to improve the inherent quality of EIA's Applied Analysis products. In that connection, its Office of Analysis Oversight and Access (OAOA) recently commissioned a set of reports evaluating various energy system projections and analyses published in the EIA Annual Report to Congress 1978, Volume 3 (referred to here as the 1978 ARC). A major element in this set is a study by Harris and Associates, and Hirshfeld and Associates (see Harris and Hirshfeld, 1980) for the National Bureau of Standards (NBS), U.S. Department of Commerce. That study's objective was to apply analytic procedures to determine and communicate the quality and usefulness of the EIA 1978 midterm oil and gas production projections. More specifically, the contract under which the study was performed (NBS No. NB80SBCA035) called for a "sensitivity analysis of DOE forecasts of midterm oil and gas supply for the 1978 Annual Report to Congress".

Since late 1978, the National Bureau of Standards has been one of the agencies conducting projects sponsored by OAOA to develop energy model assessment procedures and guidelines. The NBS work has focused on EIA's Midterm Oil and Gas Supply Modeling System (MOGSMS) as the vehicle for development of a sound assessment methodology. OAOA assigned to NBS the investigation of the 1978 ARC midterm projections of oil and gas production because they rest in large measure upon certain results generated by the 1978 version of MOGSMS.

Through their work to date, NBS staff and consultants have developed a considerable store of knowledge on MOGSMS's data, analytical structure and content, and performance (see Harris, 1979a; Harris, 1979b; Harris 1979c; Hirshfeld, 1979; Hoffman and Joel, 1979 , Gass and Joel, 1980; Gass, et al., 1979; and Gass, et al., 1980). The present study draws heavily on this knowledge and its documentation and especially on the methodologies established in Harris and Hirshfeld (1980). A major purpose thus is to extend the NBS and related efforts to a similar sensitivity study of the 1979 ARC forecasts of midterm oil and gas supply.

To clarify we repeat the following four basic groundrules for the current study:

(1) MOGSMS Results are the Object of the Analysis: Our study explicitly addresses only the MOGSMS output that leads to annual supply possibility functions for conventional oil and gas (excluding the Alaskan North Slope and other sources).

(2) Analysis Restricted to One Set of Price Increments : The supply possibility functions produced by MOGSMS are sorted tabulations (by year) of computed oil and gas production levels corresponding to numerous (exogenous) price trajectories (or "increments"). To reduce the size of our analysis without altering its relevance, we chose to work with just one set of those price increments:

- o Oil - Price Increment 9 (in 1979$)
  $32.00/Bbl in 1985, then increasing to
  $41.00/Bbl by 2008

- o Gas - Price Increment 1 (in 1979$)
  $2.42/MCF

These are the standard, user-supplied price increments (the others are internally generated by the addition or subtraction of various price increments uniformly over the planning horizon).

(3) Analysis Restricted to the Midprice Projection : The 1979 ARC contains a number of different midterm oil and gas supply projections, corresponding to various scenarios (see its Appendix B for a listing of assumptions). To further reduce the size of the sensitivity analysis, we deal exclusively with those MOGSMS results that entered the midprice, standard projection of the 1979 ARC. Applying our sensitivity analysis to the one central ARC projection yields findings that appear relevant to all the ARC's midterm oil and gas supply forecasts. The last basic ground rule is:

(4) Input Data Changes Are Permitted; Model Changes are Not : The issue here is an investigation of the sensitivity of MOGSMS output to changes in the input data, not to changes in the model structure. But the difference is not as clear as it may at first seem. For example, two of MOGSMS's key input data elements are the decline rates and the exploratory dry hole ratio for crude oil. One might wish to determine the effects of varying these data elements as a function of time over the model's planning horizon. However, MOGSMS's internal structure treats these elements as constants over the planning horizon. So, varying them as a function of time would require modification of the model's design philosophy and of the programs embodying MOGSMS's model structure. Such modifications can be very time-consuming and may unneccessarily complicate the sensitivity analysis.

Therefore this study deals only with systematic changes in MOGSMS input data items. Operationally, this means that: (1) we were free to vary any element in MOGSMS's input data sets; but (2) we did not touch the programs that implement the MOGSMS model.

Thus, in our example, we could indeed have varied the decline rates and the exploratory dry hole ratio, but only in their current form as constraints over the MOGSMS planning horizon.

This report is rather long, so some brief comments regarding its organization and content may be appropriate.

This section deals primarily with the study's objective and basis, and specifically cites that portion of the 1979 ARC containing the projections analyzed in the study. In addition, we introduce the main analytical features of our work and specify the major categories of uncertainty. Section 3 then discusses the nature of the sensitivity analysis experiments that the study comprised.

Sections 4, 5, and 6, respectively, describe the three sensitivity analysis experiments performed in the study. Each section treats the design, implementation, and results of a particular experiment.

2.1 Technical Issues

Explicit treatment of the uncertainty associated with energy supply and demand projections clearly enhances their usefulness for policy analysis. This study is part of an unfolding effort to develop and apply methods for the quantitative treatment of the uncertainty associated with the results generated by the energy models and analysis systems of the EIA.

In this context, we define "uncertainty" broadly, to include: (1) true randomness in input data and parameters; (2) errors in measurement of these exogenous factors; and (3) potential errors residing in model logic, model structure, or computational procedures. Sensitivity analysis, in turn, is the examination of the quantitative and qualitative effects of results of perturbations (random or deterministic) in a model's (or analysis system's) input data elements, parameters, or mathematical structure. Consistent with this definition, the present study is a sensitivity analysis of the oil and gas supply forecasts generated by EIA's MOGSMS analysis system.

This study is a direct outgrowth of the aforementioned NBS project for energy model assessment procedure development, sponsored by DOE's Office of Analysis Oversight and Access. We apply some of the major results of the NBS project to enhance the quality and usefulness of those MOGSMS results that are a part of the 1979 ARC's midterm forecasts of oil and gas supply. Specifically, our work is in the form of a sensitivity analysis (as defined above) of these MOSGMS results, with respect to uncertainty (or possible errors) in:

- o input data, and
- o logical structure.

Our sensitivity analysis does not address three other areas of uncertainty or possible error:

- o statistical methods,
- o mathematical formulations, and
- o computational procedures.

The first two of these three would have required a much larger effort than this study and the latter does not appear to be a critical aspect of MOGSMS.

Sensitivity analysis is especially pertinent for MOGSMS because its results are widely distributed, and these results appear to be sensitive to key input data elements, parameters, and assumptions. The 1979 ARC makes frequent reference to the effects of alternative input scenarios, uncertainties in parameters and variables, and even alternative procedures for making the forecasts.

This sensitivity is not surprising, because MOGSMS is a static model generating deterministic results that are to be forecasts of time-dynamic, highly uncertain phenomena. EIA has contracted for the development of a new family of oil and gas supply models that will capture more of the time dynamics and randomness in the target physical system. The present sensitivity analysis may provide some additional guidance for those modeling efforts, as well as improving the usefulness of the current MOGSMS results.

Many issues suggested themselves to be addressed in this sensitivity analysis. Our choice was influenced by a number of factors: the usual constraints on time and resources, a direct charge to address oil and gas resource estimates and finding rates, and the recognition that this study would be followed by others, perhaps broader in scope. These factors impelled us to concentrate on the impact of the estimation of the underlying resource base on MOGSMS results and on the explicit treatment and assessment of uncertainty in (at least) some of MOGSMS's input data elements. In particular, our quantitative sensitivity analysis comprises the following three distinct experiments, each addressing a specific issue affecting MOGSMS results:

- Issue 1: The effects on MOGSMS's oil and gas supply forecasts of the intrinsic uncertainty in the estimates of regional oil and gas resources (undiscovered recoverables).

- Issue 2: Likewise for other pre-identified key variables.

- Issue 3: The identification of the (relatively few) input data elements "critical" to MOGSMS - that is, those elements which are uncertain or random in nature and whose values strongly affect MOGSMS results.

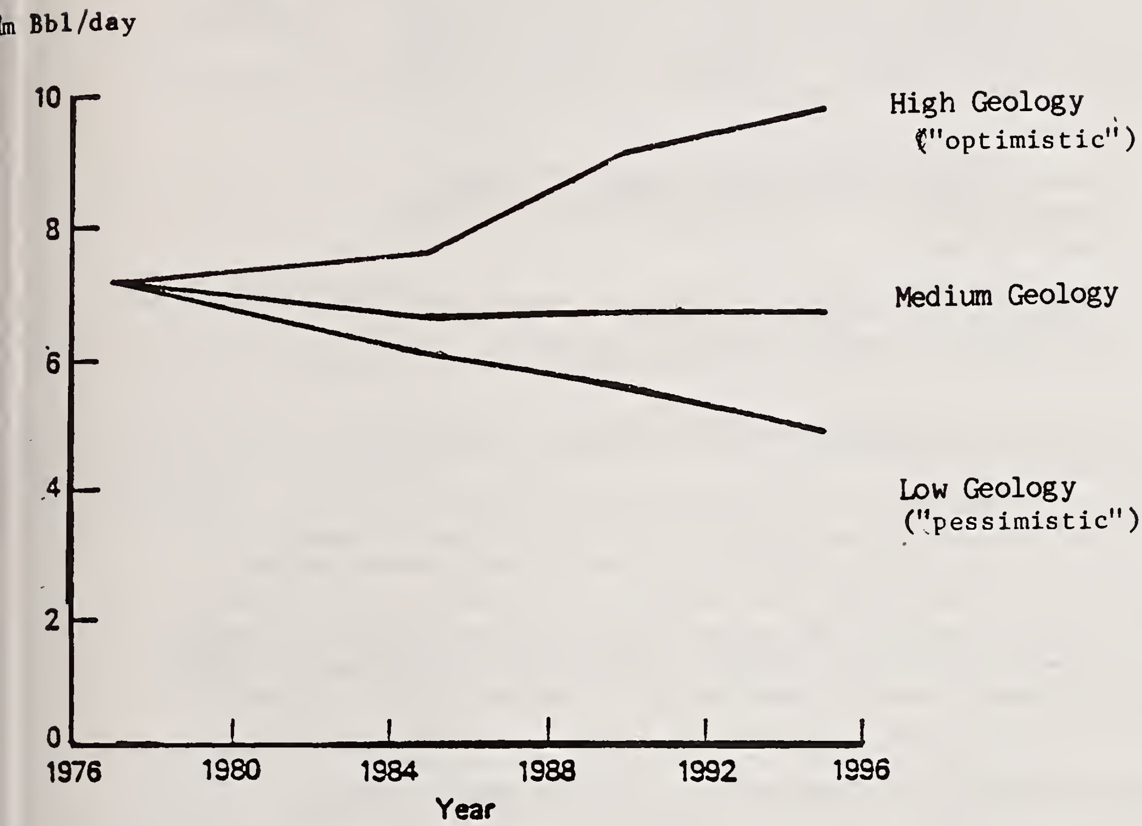
Analytical considerations associated with these issues are discussed in this section. We introduce that discussion with a few remarks.

A primary source of uncertainty in MOGSMS results is the inherent uncertainty in estimates of the regional oil and gas resource bases (reserves plus undiscovered recoverable resources). USGS Circular 725, source of the resource based estimates used in MOGSMS, sets forth individual probability distribution functions for the oil and gas resource base estimates in each NPC region. In an effort to delineate the effects of this particular source of uncertainty, the 1978 ARC, Vol. 3 showed (on pages 176 and 199) MOGSMS-generated responses to "optimistic" and "pessimistic" resource base assumptions. (See Figure 1, taken from page 199 of that ARC.) For crude oil, the difference between the extremes reached 5.0 million barrels per day by 1995.

The regional oil and gas resource base estimates are a particularly good target for a statistical treatment of uncertainty because: (1) they are of fundamental importance to the MOGSMS methodology (and, in particular, to the estimation of MOGSMS's finding rates); (2) they are inherently uncertain; and (3) widely accepted probability distribution functions exist for them (in USGS 725). Consequently, a Monte-Carlo analysis of the effects of resource base uncertainty of MOGSMS's oil and gas supply forecasts became the first element in this sensitivity analysis.

The second element of our study is a pair of Monte-Carlo experiments on two additional sets of input elements felt to be vital. These are the recovery factors and the decline rates.

Figure 1 1978 ARC Estimates of the Effect of Resource Base Uncertainty on U.S. Crude Oil Production (1977-1995)



More generally, many (if not virtually all) of MOGSMS's input data elements are uncertain - because of their fundamental randomness (e.g., the resource base estimates), because they pertain to future conditions (e.g., prices, costs, drillng distributions, etc.), or because they describe imperfectly understood phenomena (e.g., regional recovery factors, total-to-exploratory drilling ratios, etc.). Conceivably, explicit treatment of uncertainty in some MOGSMS data elements (in addition to the resource base estimates) would expose significant corresponding uncertainty in MOGSMS results. Conversely, MOGSMS oil and gas forecasts may be (relatively) insensitive to uncertainty in other input data elements. Thus a systematic effort to:

- o identify those input data elements in which uncertainty has an important effect on MOGSMS results, and

- o determine the magnitude of their effects on MOGSMS oil and gas supply forecasts

becomes the other key element in the sensitivity analysis.

The three issues addressed in our sensitivity analysis form a progression of increasing generality. The first issue is quite narrow: it deals strictly with a known set of data characterized by uncertainty and illustrating the application of Monte-Carlo methods for sensitivity analysis. The second does likewise, but for a broader class of elements. The third is more general yet, involving a search for the significant areas of uncertainty via a generally applicable analytical method.

2.2 Treating Estimates of Regional Oil and Gas Resource Bases

To motivate properly the discussion in this section and those to follow, we must recall a few definitions pertaining to the regional finding rates in MOGSMS. The finding-rate function employed in MOGSMS (for both oil and gas) follows from the assumption that undiscovered resource deposits (in an entire region) have a log-normal size distribution. This assumption leads to an exponential decline relationship (by virtue of the constant percentage depletion associated with log-normality) between cumulative oil (gas)in-place discovered and cumulative exploratory drilling, of the general form:

$$\left.\begin{matrix} COIP_k \\ \text{or} \\ CGIP_k \end{matrix}\right\} = Q_k \left[1-\exp(-b_k \cdot CMFT_k)\right] \qquad (1)$$

where

$COIP_k$ = Cumulative oil-in-place dicovered in NPC region k up to a reference year;

Q_k = Oil (gas) resource base, defined (for MOGSMS purposes) as discoveries from 1956 on plus the resources remaining to be discovered as of the start of MOGSMS's planning horizon (1 January 1979 for this ARC);

$CMFT_k$ = Cumulative exploratory (total) drilling assignable to oil (gas) in NPC region k from 1956 until the appropriate reference year (as described in Hoffman and Joel, 1980, MOGSMS relates oil discoveries to exploratory drilling assignable to oil, but relates gas discoveries to total drilling assignable to gas).

b_k = A finding-rate parameter that is region specific for oil and gas.

The regional finding rate, F_k, is then the incremental quantity of oil (gas) discovered per unit exploratory (total) drilling. Its formula can be obtained by differentiating Equation (1) with respect to CMFT :

$$\text{Oil:}\quad F = \frac{d(COIP)}{d(CMFT)} = b_k \cdot Q_k \cdot \exp(-b_k \cdot CMFT_k)\ \text{oil} \qquad (2)$$

$$\text{Gas:}\quad F = \frac{d(CGIP)}{d(CMFT)} = b_k \cdot Q_k \cdot \exp(-b_k \cdot CMFT_k)\ \text{gas} \qquad (3)$$

Thus F_k is in units of:

- o Barrels discovered/Ft. of drilling (oil)
- o MCF discovered/Ft. of drilling (gas)

(This functional form asserts that the oil and gas finding rates must decline exponentially with increasing drilling.) Taking the natural logarithm of both sides yields the convenient form

$$\ln F_k = \ln [(b_k \cdot Q_k)] - b_k \cdot CMFT_k \qquad (4)$$

Another log-linear version of this relationship is very useful in the subsequent data analysis:

$$\ln [Q_k - COIP_k] = \ln Q_k - b_k \cdot CMFT_k \qquad (5)$$

MOGSMS contains relationships similar to Equations (2) and (3) to calculate reserve discoveries year by year for oil and for gas in each NPC region as drilling proceeds through the planning horizon. Observe that the finding rate, F_k, depends upon uniquely specified values of Q_k (the resource base) and b_k (the slope). So MOGSMS runs require external determination of the values of each Q_k and b_k. The 1978 procedure for MOGSMS employed a (log-linear) regression analysis of historical data since 1956 (drilling and reserve additions, by region) on the functional form of Equation (5) to estimate b_k for each mature region. In this procedure, the Q_k for each region was taken as a constant, equal to the mean resource value presented in USGS Circular 725 for the given region. Both the b_k and Q_k values then become MOGSMS input. In 1979, direct nonlinear regessions where done on Equation (5), assuming the Q values to be at USGS levels and then finding the resultant $\{b_k\}$. 1/

1/ For the immature regions, namely, 1A, 2A, and 11A, MOGSMS utilizes the exploration experience (annual drilling footage and discoveries) as a direct estimate of the finding rate (Δ discovery/ Δ footage) in the vicinity of the origin (i.e., near zero cumulative drilling footage).

Consequently, one aspect of the issue at hand is to determine if a sound statistical strategy can be employed to provide a meaningful assessment of the range of uncertainty in MOGSMS's oil and gas supply forecasts given the probabilistic information available from USGS 725. Note that the treatment by MOGSMS of USGS 725's log-normal probability distributions on regional resource bases also affects the expected values (or central tendency) of future oil and gas supply. Recall that log-normal distribution functions have two parameters, the mean (μ) and the standard deviation (σ) of the originating normal population. If the log-normal variable is designated as Y and its logarithm (i.e., the parent normal) by X, then the following relationships obtain:

$$\mu = E[X] = \ln\sqrt{(Y_{.95})(Y_{.05})} \tag{6}$$

$$\sigma = S.D.[X] = [\ln Y_{.95} - \mu]/ 1.645 \tag{7}$$

$$E[Y] = \exp(\mu - \sigma^2/2) \tag{8}$$

$$\text{Mode}\ [Y] = \exp(\mu - \sigma^2) \tag{9}$$

$$\text{Median}\ [Y] = \exp(\mu) \tag{10}$$

Also note that the mulitplication of Y by a constant does not affect the underlying normal's standard deviation since the addition of the logarithm of such a constant to X yields another normal with the same standard deviation.

Considerable confusion has existed in the MOGSMS literature regarding means and medians. Most notably, page 195 of Volume 3 of the 1978 ARC contained a statement that "the projections in Series C are based on the median (statistical mean) assessment of undiscovered recoverable resources provided by the USGS in 1975 (see Table 11.11)" The USGS 725 values in question are, in fact, estimates of the expected or mean remaining recoverables- totally different from the median (and the mode for that matter). The median value is the 50th-percentile value, or that level both above and below which lies one-half of the probability distribution. The mode, the third major measure of central tendency, is merely the point (or points) with the highest probability (discrete case) or the variable value giving the largest value to the probability density function (continuous case). A footnote to these statements on page 195 adds that the "statistical mean" of the undiscovered recoverable resource in any NPC region is the sum of the 5th percentile value, the 95th percentile value, and the modal value, divided by three. This is a correct interpretation of the manner in which USGS derived the mean values for its log-normal distributions. However, this formula is only an approximation, and it creates some possible errors in MOGSMS results (though generally of a minor nature).

The first formal part of this study then deals with the relationship between the (probabilistic) regional resource base estimates offered in USGS 725 and the (deterministic) analysis to which these estimates have historically been applied in MOGSMS. This issue has two facets:

- the potential extreme values of U.S. oil and gas production, as delineated for example in the "optimistic" and "pessimistic" geology cases cases (Series A and E, respectively) of the 1978 ARC, and

- the verification of nominal values of U.S. oil and gas production, as provided by the nominal midprice case in the 1979 ARC .

Clearly, the nominal, midprice ARC supply forecasts alone do not offer a meaningful assessment of the effects of underlying resource uncertainty on possible levels of future oil and gas production. USGS Circular 725 sets forth estimated (log-normal) distributions of the undiscovered recoverable oil and gas resources in each NPC region. These distributions reflect the intrinsic uncertainty in any estimation of future oil and gas discovery prospects or of total resource-in-place. However, the MOGSMS-based analysis of oil and gas supply prospects has treated these resource estimates deterministically, and thus does not come to grips with the underlying uncertainty.

In particular, the 1978 ARC's optimistic geology case (Series A) assumes that the resource base of oil and gas in each of the NPC regions corresponds to the high values set forth for that region in USGS Circular 725 (properly adjusted for exploration experience subsequent to 1974). The USGS high values are the 95th percentile points for the assumed statistical distributions (all taken as log-normals). On the other hand, the pessimistic geology case (Series E) assumes that regional resource bases coresond to the USGS 5th percentile points. The estimated resource bases employed for the 1978 Series C projection are the expected values (according to the appropriate log-normals) for each region in USGS 725 .

If we assume that the USGS 725 frequency distributions are valid, it is extremely unlikely that all regions simultaneously exhibit either the upper or the lower level of resource base. In fact, we can compute the probability that all regions are on the pessimistic side (that is, have recoverable resources within the lower 5 percent probability tail) as $(0.05)^{15} = 3 \times 10^{-20}$, essentially zero. (All of this assumes that the resource base estimates are purely that and incorporate no other issues in their calculation. That is to say, no systematic factors, such as price responses, new technologies, etc., affect the independence of these probability functions.)

In summary then, the first element of our sensitivity analysis assesses the effects of uncertainty (or randomness) in resource base estimates (specifically as embodied in USGS 725's log-normal probability distributions) on MOGSMS-based oil and gas supply forecasts.

2.3 Additional Key Random Elements

For Experiment 2 we have carried out a Monte-Carlo analysis much like that of Experiment 1 on two sets of variables deemed to be of significant import to the model results. These are recovery factors (primary and secondary) and the decline rates.

In each case, subjective probability distributions were established and random draws carried out by computer. For the recovery factors, both primary and secondary factors were assumed to be uniformly distributed on a range from 80 percent of their nominal or base values up to 120 percent of nominal. For the decline rates, uniform distributions were also specified, but they ranged from 75 percent to 125 percent of nominal.

2.4 The Notion of Critical Data Elements and Its Application to MOGSMS

As we have noted, other MOGSMS data elements in addition to the resource base estimates are subject to uncertainty, for a variety of reasons. The nature and extent of the inherent uncertainty in MOGSMS has only recently come under any close scrutiny (see Harris and Hirshfeld, 1980).

Experience and judgement provide some guidelines in this regard. For example, MOGSMS's developers have indicated (EIA, 1978) a relatively small number of input elements. However, this assessment did not quantify the notion of "high sensitivity" or estimate the potential uncertainty in any of the indicated data elements.

Clearly an exhaustive analysis of the effects on MOGSMS results of uncertainty or change in each input data element is infeasible, as well as unwarranted. (The input data set for MOGSMS contains about 2,000 distinct numerical values.) What is feasible - and valuable - is an analysis to identify the "critical" input data elements in the current version of MOGSMS.

In this context, we define "critical" input elements to be those having two properties:

- Their values are highly uncertain (or random); and
- MOGSMS's results appear to be highly sensitive to variations in their values.

The first of these is related to inherent properties of the input data (sources, transformations applied, etc.) and the phenomena they are to describe. The second is related to the logical and mathematical structure of MOGSMS itself.

The nature of this study dictated the approach to identifying critical input elements. First, we specified candidate elements on the basis of judgement and experience. Second, we subjected these elements to quantitative analysis to determine the relationship between changes in the values and changes in MOGSMS results. Such an analysis is the object of the third element in our sensitivity analysis.

3. OVERVIEW OF THE STUDY

As indicated in Section 2, this study consisted mainly of a series of statistical experiments, focusing on the general nature and effects of uncertainty on MOGSMS results, and the individual analytical issues identified there. This section offers a brief overview of the study, in terms of these elements.

3.1 Statistical Experiments

The three statistical experiments addressed, in order, the analytical issues raised in Sections 2.2, 2.3 and 2.4.

- Experiment 1: A Monte Carlo analysis of the USGS Circular 725 estimates of regional undiscovered (oil and gas) resources and their impact on MOGSMS finding rates and ultimate outputs.
- Experiment 2 : A Monte Carlo analysis of the impact on MOGSMS output of subjective probability structures postulated for recovery factors and decline rates.
- Experiment 3: A response surface analysis of various MOGSMS input data elements, including both physical and economic elements, to identify those in which uncertainty has the greatest effect on MOGSMS results.

Each of these experiments involved exercising MOGSMS in tests to determine the effects of input variations on the MOGSMS forecasts of conventional oil and gas production. We observed these effects primarily through computed product measures (defined in Section 3.2) for crude oil, gas, and NGL.

The Monte-Carlo resource experiment treats the uncertainty inherent in the USGS Circular 725 estimates, through a two-stage procedure:

- o Generating one hundred vectors of estimates of of regional oil and gas resource bases, where each vector is made up of elements, each of which is an estimate of a region's undiscovered reserve of oil or gas. These resource realization vectors are randomly constructed according to a Latin Hypercube design.

- o Running one MOGSMS test for each of the one hundred resource realization vectors, with all other model inputs unchanged, to obtain the corresponding set of one hundred oil and gas production forecasts.

The second Monte-Carlo experiment assesses the effects on MOGSMS output of possible stochastic variability in recovery factors and decline rates. Forty runs of MOGSMS were performed for each set here. In each case, the values of the random variables were predetermined by sampling. For both the recovery factors and decline rates, the individual elements were combined as a Latin Hypercube as in Experiment 1. Finally, the response surface experiment seeks to identify the critical input data element for MOGSMS, through a three-stage procedure:

- o Creating a series of factorial experiments, involving systematic variations of MOGSMS input data elements, in each of two (arbitrary) categories: economics and drilling.

- o Running one MOGSMS test for each factorial combination of input values, to obtain the corresponding sets of oil and gas production forecasts.

- o Applying linear regression analysis to each set of forecasts to identify those input elements whose values (and changes therein) have the greatest influence on MOGSMS results.

Thus, all of the experiments involved repeated full executions of MOGSMS (always involving both the oil and gas routines). To drive these experiments, we developed simple computer routines to (1) create the modified input data sets, as needed for each experiment and (2) summarize and tabulate the test results for subsequent statistical analysis.

3.2 Product Measures

The output of any given MOGSMS run is physically voluminous and logically complex (embodying as it does the dimensions of region, time, product, and process), and the experiments involved many MOGSMS runs. Therefore, to facilitate (if not simply to permit) analysis of the experimental results, we chose specific element(s) of MOGSMS output to serve as descriptors of the full set of MOGSMS results. That is, we focused our analysis of the sensitivity of results to the specified measures associated with each MOGSMS run, rather than work with the entire set of output values. One might think of the output measures as the defined dependent variables of interest to us in each test (here, a test is simply a MOGSMS execution with a distinct input data set).

The primary measures that we established for this purpose correspond to six MOGSMS-computed production levels:

- Oil [Crude Oil Supply]
  [Associated-Dissolved Gas Supply]
  [Associated Natural Gas Liquids Supply]

- Gas [Non-Associated Gas Supply]
  [Non-Associated Natural Gas Liquids Supply]
  [Non-Associated Lease Condensate Supply]

Each of these is an element of MOGSMS output reported by the MOGSMS post-processors, OREPORT, and GREPORT, respectively. The brackets above - [ ] - denote that our defined measure corresponding to each of these elements is the average of the total U.S. value (taken over all regions) reported for four target years: 1985, 1990, 1995, and 2000.

The ultimate or product measures that we established are the three product aggregates of the primary measures:

			(OIL MODEL)		(GAS MODEL)
o	[Crude Oil]	=	[Crude Oil Supply]	+	[Non-Associated Lease Condensate Supply]
o	[Natural Gas]	=	[Associated Dissolved Gas Supply]	+	[Non-Associated Gas Supply]
o	[NGL]	=	[Assoicated Natural Gas Liquids Supply]	+	[Non-Assoicated Natural Gas Liquids Supply]

These three product measures (dependent variables) are, collectively, close indicators of the cumulative oil and gas supply over the entire planning horizon forecast by MOGSMS in response to a given input data set.

4. EXPERIMENT 1: MONTE CARLO ANALYSIS

Experiment 1 was a Monte-Carlo analysis of the effects on MOGSMS results of uncertainty in the USGS Circular 725 estimates of regional undiscovered (oil and gas) resources. In particular, it was aimed at the "establishment" of "High Geology" (optimistic) and "Low Geology" (pessimistic) forecasts of oil and gas supply for the 1979 ARC.

4.1 Design Considerations

Broadly speaking, the overall range of uncertainty in MOGSMS results is generated from the joint probability law governing the endogenous parameters (call them $\underset{\sim}{U}$), and exogenous input data elements (say $\underset{\sim}{V}$), together represented by the vector ($\underset{\sim}{U}$, $\underset{\sim}{V}$). Experiment 1 explores the underlying stochastic character of part of the $\underset{\sim}{V}$ set, namely, the regional resource base estimates. More precisely, we worked with the subset of input data elements corresponding to the (random) amount of original crude oil and natural gas still remaining to be discovered, by region, at the outset of the model's 1979 ARC planning horizon (January 1, 1979). Our specification of the distribution functions of these stochastic variables is that described in USGS Circular 725, with means adjusted according to subsequent revisions made since 1975 by USGS. As we have seen in Section 2, that document offered log-normal distributions for the undiscovered resource bases in each of the NPC oil and gas regions, assuming complete statistical independence between regions.

The properties of the resulting probability distributions on MOGSMS output (call it $\underset{\sim}{Z}$ with CDF F(z)) cannot be obtained by any closed-form analytic operations because of MOGSMS's complexity. To obtain such a distribution as the one measure of uncertainty in MOGSMS results we turned to Monte-Carlo methods.

In our experiment then: (1) values of the vector $\underset{\sim}{V}$ were drawn randomly according to its distribution (here equal to the product of the marginals in light of independence) according to a specific sampling plan; and (2) values of $\underset{\sim}{Z}$ were obtained by running MOGSMS tests with $\underset{\sim}{V}$ in the input set. When this procedure is carried out n times, it generated a sample of size n for $\underset{\sim}{Z}$. This sample permitted a detailed analysis of measures of central tendency, dispersion, and the distribution function itself. The generation of a complete Monte Carlo sample would have been rather expensive because of the complexity of the subject model and the relatively large number of regions (components of $\underset{\sim}{V}$). Therefore we employed a modified sampling procedure which reduced the cost and time required for the experiment, without sacrificing any statistical precision. The approach we chose uses a multi-dimensional version of the classifical Latin Square - called the Latin Hypercube - as described by McKay et al. (1979). Results presented by those authors indicated an approximate reduction in sampling of 75 percent vs. conventional methods, at comparable levels of precision.

The Monte-Carlo experiment thus comprised two main steps:

1. Generating one hundred vectors of estimates of regional oil and gas resource bases, where each vector is made up of elements, each of which is an estimate of a region's Q_k value, for oil and for gas. (We call such vectors resource realization vectors.) The vectors were randomly constructed according to a Latin Hypercube design, described shortly in Section 4.2.

2. Running one MOGSMS test for each of the one hundred resource realization vectors, with all other model inputs unchanged, to obtain the corresponding oil and gas production forecasts. The end result of this effort was one set of three product measures (see Section 3) for each of the one hundred resource realization vectors.

The results of the total set of runs can be summarized via a number of empirical statistical measures. We focus on: a) the mean, together with the interquartile range; and b) the empirical frequency function.

The key step in the experiment is the generation of the resource realization vectors. To be more specific on the starting points of this process, we begin with the assumption of log-normal distributions on the estimated sizes of the regional (oil and gas) resource bases. This assumption is the basis for the regional finding rates currently used in MOGSMS. Recall that a log-normal random variable is one whose logarithm - called the parent or underlying normal - is normally distributed. As a result of this defining variable transformation, a log-normal random variable thus is characterized by two parameters, the mean and the standard deviation (or variance) of the parent normal. With the means and variances of these normal distributions, one can derive random log-normal deviates of the estimated resource bases upon suitable transformation of the percentage points of the standard unit normal distribution.

A fundamental property of the log-normal distribution plays a key role in the subsequent interpretation of our results. The expected value of a log-normal random variable (say Y) is greater than or equal to the exponentiation of the mean of the parent normal (say X). In more precise notation,

$$E[Y] = E[e^X] \geq e^{E[X]}. \tag{11}$$

This is easily proved by a direct argument involving Jensen's inequality for the expectation of a convex function of a random variable (see, for example, Feller, 1966). Of course, when we replace the expectation operator by the median, equality is obtained; that is,

$$\text{Median }(Y) = e^{\text{Median }(X)}.$$

A log-normal distribution always exhibits the property that

$$\text{Mode} \leq \text{Median} \leq \text{Mean} \tag{12}$$

and thus the means and medians of log-normal samples are higher than one expects intuitively. This relationship is important to the results obtained in this experiment. (A typical log-normal density function is displayed in Figure 2).

Getting back to the derivation of the log-normal deviates for our experiment, let P_k denote the undiscovered resource base (for oil or gas, as the case may be) in the kth (NPC) region as of January 1, 1979, as estimated in USGS Circular 725 (adjusted for reserve additions reported after 1974). Also, let Q_k denote the cumulative resource base remaining as of January 1, 1956. Then P_k is a standard log-normal variable, while Q_k is a shifted one with

$$\begin{aligned} Q_k &= P_k + [\text{Cumulative oil (gas)-in-place found during 1956-1977, inclusive}] \\ &= P_k + R_k \end{aligned} \tag{13}$$

To proceed, we must next find the appropriate parameters of the log-normal distributions of the undiscovered resources: the means (μ) and standard deviations (σ) associated with the respective parent normals. To get these, we have proceeded in the following way. For σ we have used the 5th and 95th percentage points estimated for each region by the USGS. Since percentage points are preserved by any monotonic transformation,

$$X_{.05} = \ln(Y_{.05}) \tag{14}$$

and

$$X_{.95} = \ln(Y_{.95}) \tag{15}$$

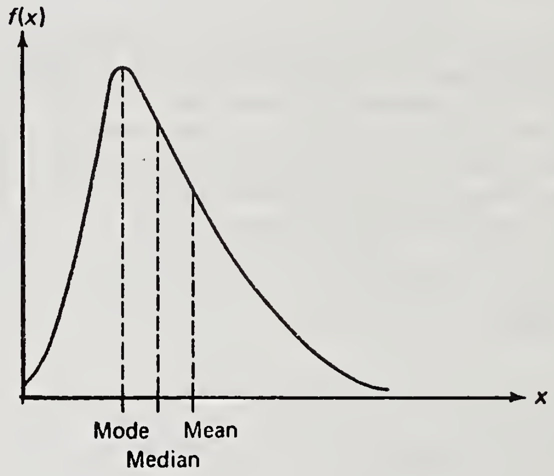
But (as is well known), the 5th and 95th percentage points of the standard unit normal are separated by approximately 3.29 σ. So

$$\sigma = \frac{\ln(Y_{.95}/Y_{.05})}{3.29} \tag{16}$$

By similar argument, we can find the mean as

$$\mu = \ln Y_{.95} - (1.645)\sigma . \tag{17}$$

Figure 2 Typical Log-Normal Density



However, as noted earlier, USGS erred in its computation of the mean and instead used an approximation formula. We decided to stay with the USGS (log-normal) means, despite the small errors so incurred. Accordingly, we find the parameter in the following way. Any log-normal variable Y has mean

$$E(Y) = e^{\mu + \sigma^2/2} \tag{18}$$

Therefore

$$\mu = \ln E(Y) - \sigma^2/2 \tag{19}$$

Now, with μ_k and σ_k in hand, it follows that

$$P_{kj} = \exp[\sigma_k r_{jk} + \mu_k] \tag{20}$$

where P_{kj} is the jth random variate of P_k, for the unit normal deviate r_{kj}. Likewise,

$$Q_{kj} = P_{kj} + R_k \tag{21}$$

where Q_{kj} is the jth random variate of Q_k, corresponding to r_{kj}. Table 1 shows mean regional resource levels used in the non-frontier regions for both the 1978 and 1980 reports to Congress. Table 2 then shows the mean Q for each region in addition to the standard deviations.

4.2 Implementation Considerations

In accordance with the foregoing reasoning, the procedure for Step 1 of our experiment, generating a set of (random) regional resource realization vectors for oil and gas, $\{Q_k\}$, is as follows:

(i) Divide the range of each random resource variable Q_k into 50 intervals, j, of equal probability (0.02) (j = 1, 2, ..., 50).

(ii) Let the probabilistic midpoint of each interval be Q_{kj}, when the corresponding r_{kj} is the jth odd percentage point of the standard unit normal.

(iii) Generate the order in which the 50 percentage points of each region are to be used in each of 50 MOGSMS runs by creating a sequence of 25 (for the 12 oil and then 13 gas regions) unique random permutations of the integers 1 to 50.

(iv) Form the required $\{Q_k\}$ vector for the first run by taking the leading percentage point from each of the 25 random permutations. Continue to do this by similarly matching the ith elements of each permutation to the ith resource realization vector, until all 50 are formed.

(v) Repeat Steps (i) and (iv) to obtain a total of 100 random resource realization vectors, to drive 100 test runs.
Each of these vectors then is a probabilistic estimate of the U.S. oil and gas resource base for the provinces covered in USGS Circular 725 .

Table 1. Mean Regional Resource Base (Q Values)

REGION		1978 ARC VALUES MOGSMS (1)	1979 ARC VALUES MOGSMS (2)
Oil (10^6 Bb)			
2		32,864	36,540
3		24,139	47,132
4		32,436	37,641
5		54,153	52,618
6		31,026	54,163
6A		28,924	27,567
7		19,734	34,290
8-10		13,834	19,515
	TOTAL	237,110	289,466
GAS (10^9 Ft^3)			
2		13,922	13,798
3		19,184	47,870
4		30,244	30,244
5		66,638	46,710
6		275,494	261,936
6A		182,625	170,156
7		114,090	93,112
10		20,739	30,742
	TOTAL	722,936	694,570

NOTES: (1) From USGS Circular 725 adjusted accordingly.
(2) New values obtained from the EIA Office of Oil and Gas Analysis.

Table 2. Parameters of Log-Normal Resource Distributions

Oil Region	(10^6Bb) Mean Q	(10^6Bb) standard deviation for Q (1)	Gas Region	($10^9 ft^3$) Mean Q	($10^9 ft^3$) standard deviation for Q
1A	24,806	30,481	1A	30,741	29,092
2	36,540	8,198	2	13,798	3,473
2A	19,198	5,564	2A	4,440	1,316
3	47,132	8,685	3	47,870	3,092
4	37,641	6,278	4	30,244	6,047
5	52,618	11,762	5	46,710	14,473
6	54,163	4,592	6	261,936	40,774
6A	27,567	5,471	6A	170,156	30,751
7	34,290	6,208	7	93,112	35,891
8-10	19,515	5,288	8-9	3,231	1,198
11	2,827	2,034	10	30,742	7,688
11A	7,500	3,196	11	120	179
			11A	10,080	1,619

(1) Note that the standard deviation for a Q is given in terms of the σ of its parent normal as

$$E[Q]\left(\sqrt{e^{\sigma^2}-1}\right)$$

Step 2 in the Monte Carlo experiment is the generation of a vector of regional finding rate slopes , $\{b_k\}$ (for oil and for gas), corresponding to each resource realization vector, $\{Q_k\}$.

Recall from Section 2 that the regional parameters, b_k, were estimated for oil by DOE for input to MOGSMS via an off-line nonlinear regression program for each region with the corresponding Q value at the USGS number. For gas, each Q was again fixed and b then found by using the most recent exploration experience.

For any oil Q_k specified as input via the first step in our experimental design, a (nonlinear) least-squares estimate b can be found. But, of course, we are treating the $\{Q_k\}$ as random variables in this experiment. By applying this process to the historical exploration and discovery data in each region, for each randomly selected value Q_{kj}, we can compute the corresponding b_{kj}. Hence we can readily construct the 100 vectors $\{b_k\}$ corresponding to our 100 vectors $\{Q_k\}$. The sequence is the same for natural gas except for the fitting process, as indicated in the prior paragraph.

Finally, we produce the numerical values of the elements in the 100 vectors $\{b_k\}$ in MOGSMS input format, and use each vector as the driving input for a MOGSMS test run. Each MOGSMS test generates output defining an oil and gas production forecast (over time), corresponding to the particular resource realization vector $\{Q_k\}$.

The experimental procedure described here is, of course, not restricted to 100 or any other number of MOGSMS tests. We specified 100 runs for our experiment, because it appeared to offer the most reasonable tradeoff between statistical precision and resource expenditure.

4.3 Results of Experiment 1

To facilitate our analysis of experimental results, we developed a small computer program to compute statistical measures and to plot these results. The accompanying graphs (Figures 3-7) are adapted from our computer-produced plots.

The results of this experiment are set forth in the following:

- o Table 3 - showing the mean production figures for the three key forecast years produced by both Experiment 1 and the ARC itself.

- Figures 3-5 - showing the relative frequency functions (histograms) of the three product measures over the 100 tests in the experiment.

- Figures 6-7 - showing the interquartile range (25th to 75th - percentiles) of production profiles for crude oil and natural gas obtained in the experiment - in comparison with the 1979 ARC base projections.

The most important aspects of these results can be summarized as follows:

- The frequency distributions of the product measures (Figures 3-5) are generally smooth and slightly skewed right, as might be expected from the log-normality of the regional resource distributions (the Q values). The bimodal distribution of the NGL results simply reflects the separate modes for crude oil and natural gas, both of which contribute to the total NGL. This suggests that, because of the Latin Square design, the 100 tests provide an adequate level of precision, in the sense that the primary statistical measures of the results of the experiments closely approximate their anticipated (theoretical) values. More generally this result suggests that EIA can achieve significant economies through the use of properly designed sampling techniques in its computer-based statistical analyses.

- Explicit treatment of the uncertainty in the USGS 725 estimates via the Monte Carlo analysis leads to higher mean production forecasts than the deterministic approach yielded directly for the 1979 ARC. As Table 3 indicates, the production increments are measured by the mean values of the Monte Carlo results are largest for natural gas, and smallest for crude oil (NGL, naturally shows an intermediate response). In general, the production increment is largest for 1985 and decreases uniformly with time.

- In particular, the Monte Carlo mean value of gas production is 1.85 TCF (11%) higher than the 1979 ARC value for 1985. The increment drops to 0.25 TCF (2%) for 1995. The average gas production increment is 1.12 TCF/year (7%) for the period 1985-1990-1995. Across all products, the Monte Carlo analysis leads to an average production increment of 0.62 MM BOE/day, over the same period.

- As anticipated at the start of the study, the Monte Carlo analysis leads to a fairly limited spread between the "optimistic" and "pessimistic" projections of production (Figures 6 and 7). We selected the 25th - 75th percentile (interquartile) range as the most meaningful measure of dispersion in the results of the Monte Carlo analysis. (The 5th and 95th ranges of the Monte Carlo results are not significantly wider than the interquartile ranges.) For crude oil, the maximum interquartile range of projections from MOGSMS is 0.8 MM Bb//day with an average of 0.4 MM Bbl/day, or only 8 percent of the mean value. The comparisons for natural gas and NGL are of roughly equal magnitudes.

Figure 3 DISTRIBUTION OF CRUDE OIL PRODUCT MEASURE

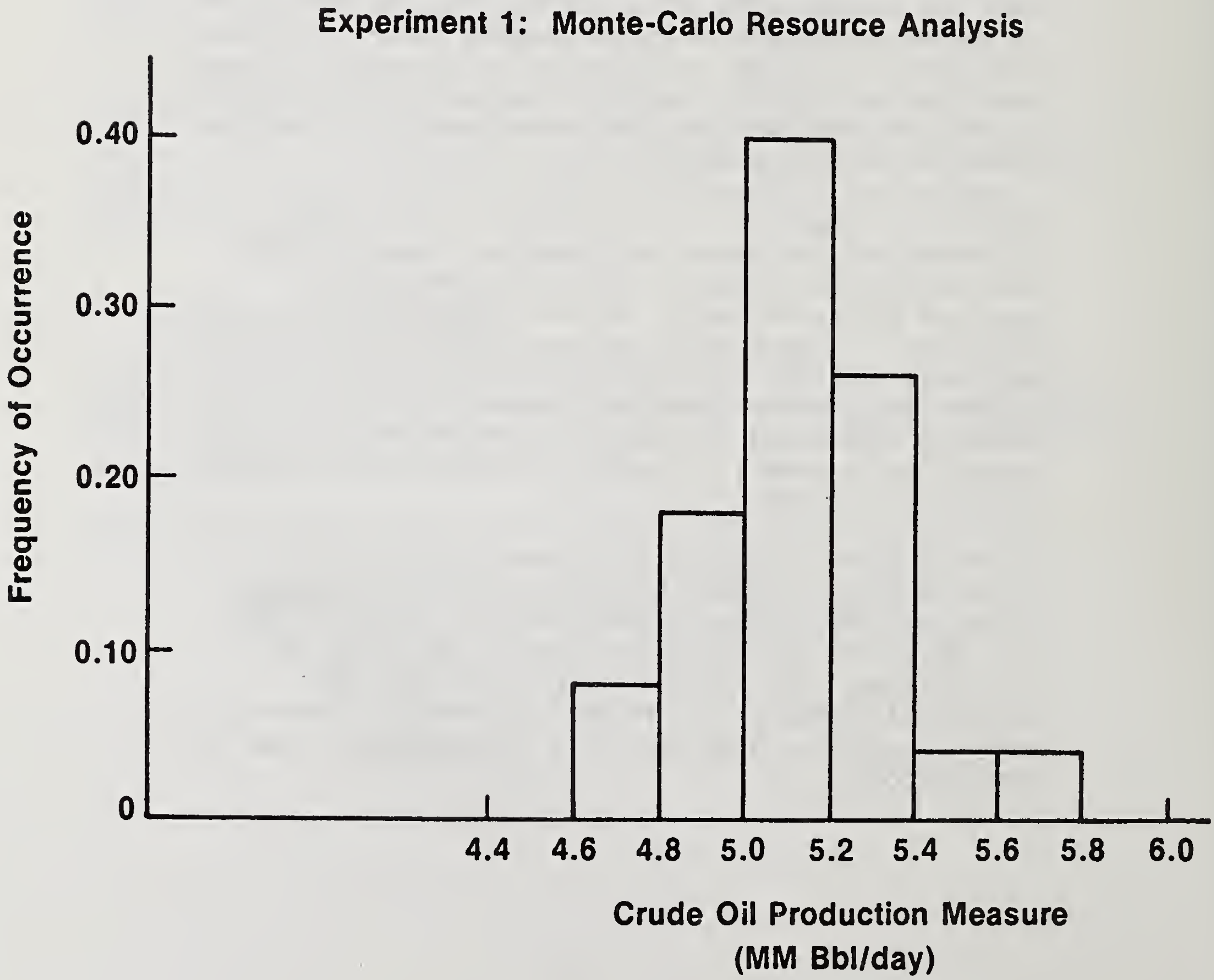


Figure 4 DISTRIBUTION OF NATURAL GAS PRODUCT MEASURE

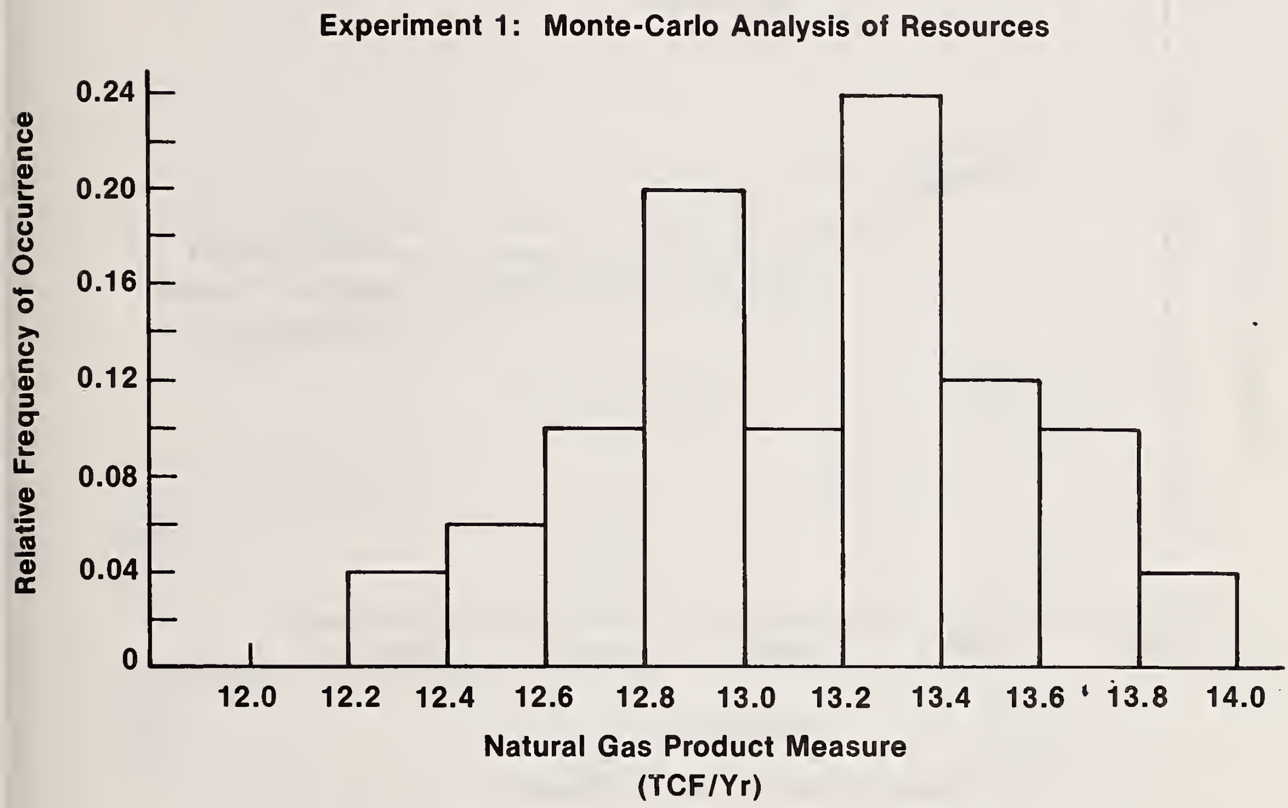


Figure 5 NATURAL GAS LIQUIDS PRODUCT MEASURE

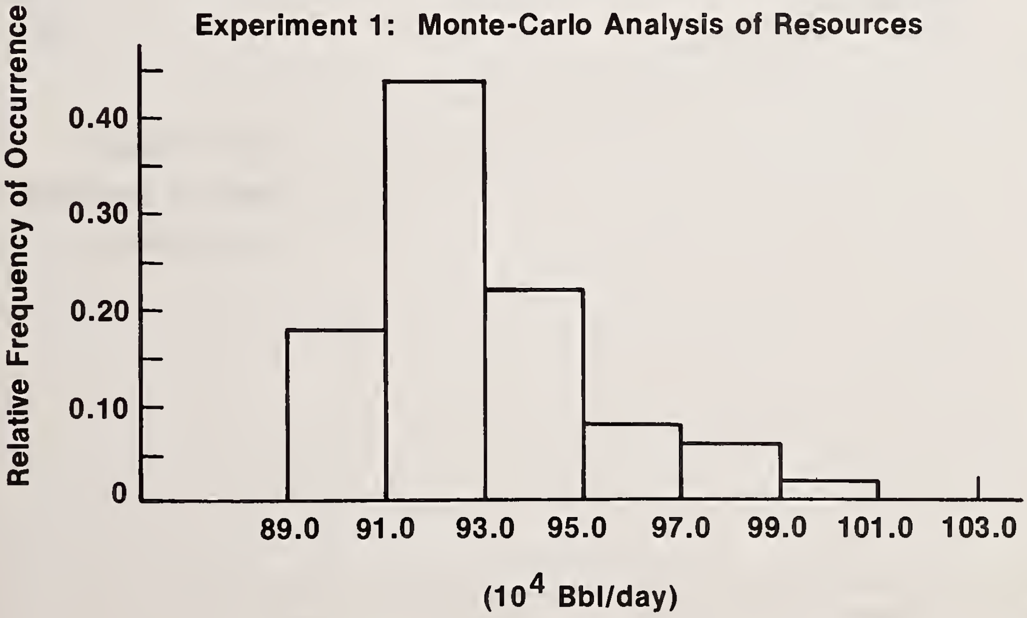


Figure 6

Effects of Resource Uncertainty on U.S. Crude Oil Production, 1978-1995

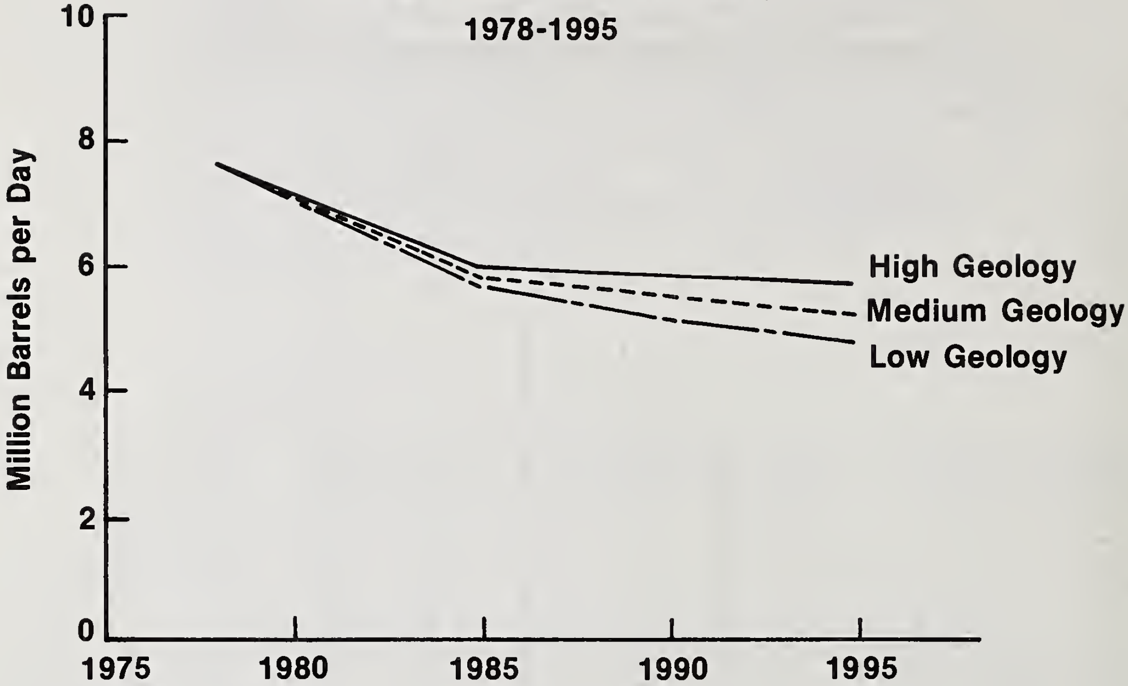


Figure 7

Effects of Resource Uncertainty on U.S. Natural Gas Production, 1978-1995

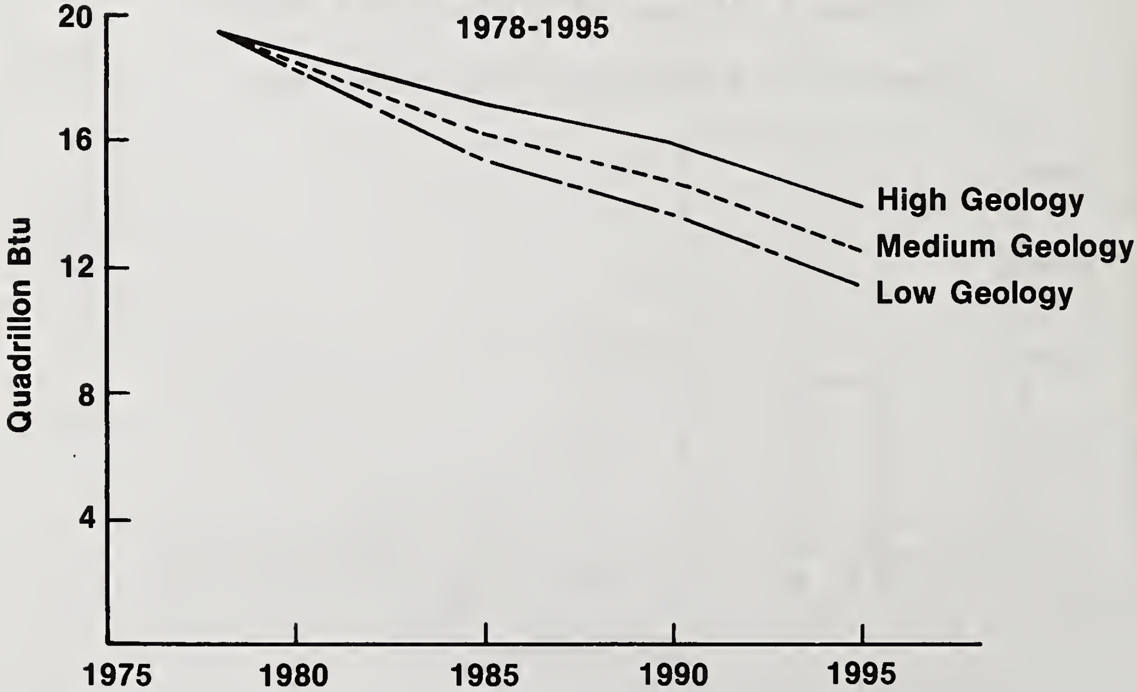


Table 3. MOGSMS Mean Production Forecasts
Experiment 1: Monte Carlo
Analysis of Resource Base Estimates
(Lower-48 States Ex. South Alaska)

	MOGSMS Values For ARC Base Series	Mean Values Experiment 1
Crude (In MM Bb/day)		
1985	5.742	5.663
1990	5.248	5.313
1995	4.922	5.016
Natural Gas (In TCF/Year)		
1985	15.755	15.699
1990	12.093	14.190
1995	8.751	12.429
NGL (In MM Bbl/day)		
1985	1.155	1.112
1990	0.934	0.985
1995	0.763	0.877

5. EXPERIMENT

The second major element of this study was a pair of Monte-Carlo experiments on two additional sets of input elements. These were the crude oil recovery factors (both primary and secondary) and the oil and gas decline rates. The issue here was to assess the effect on MOGSMS output of possible stochastic variability in these two variable sets.

Forty runs of MOGSMS were performed for each variable, as a distinct experiment. In each case, the values of the random variables were predetermined by sampling and then the individual elements combined as a Latin Hypercube as in Experiment 1. For the recovery factors, both primary and secondary factors were assumed to be uniformly distributed on a range from 80 percent of the nominal or base values up to 120 percent of nominal. For the decline rates, uniform distributions were also specified, but they ranged from 75 percent to 125 percent of nominal.

5.1 Results of Experiment 2

The results here will be presented in a form very much like those of Experiment 1. They are set forth in the following:

- Table 4 - showing the interquartile range (25 th and 75th percentiles) and the extreme (5th and 95th percentiles) production profiles obtained from the decline rate experiment, all expressed as percent changes from the base case.

- Table 5 - showing likewise for the recovery factor experiment.

- Figures 8-10 - showing the frequency histograms of the three product measures over the 40 tests in the decline rate experiment.

- Figures 11 - showing the crude oil histogram for the recovery factor experiment.

The most important aspects of these results can be summarized as follows:

- The frequency distributions of the crude oil product measure for each sub-experiment are generally uniform over a relatively tight range, as might be expected from the uniform nature of the input random variation. For crude oil, the interquartile range has widths ranging from 10.8 percent of the base case in 1985, to 15.5 percent in 1995, finally reaching 16.8 percent by 2000. The interquartile results for crude are a little different in the recovery factor experiment: they start from a width of 5.4 percent in 1985, hitting 16.5 percent in 1995, and then 18.7 percent in 2000.

- For natural gas however, the results appear somewhat mixed. In the decline rate experiment, there is an apparent (increasing) monotonicity. Here, the interquartile width starts at 31.9 percent of the base case for 1985, peaking at 38.6 percent in 1990, and finally receding to 18.1 percent by 2000. Thus we see the particular importance of decline in gas production.

- The natural gas liquids results fully reflect the fact that they come from the combination of oil and gas production.

Table 4. Results of Sensitivity Run on Decline Rates
(Percent Changes from Base Case)

	5th Percentile	25th Percentile	75th Percentile	95th Percentile
1985				
Oil	-12.3%	-5.1%	+5.7%	+9.8%
Gas	-29.4%	-16.2%	+15.7%	+30.3%
1990				
Oil	-12.9%	-5.9%	+6.4%	+11.2%
Gas	-34.6%	-17.9%	+20.7%	+27.4%
1995				
Oil	-15.5%	-8.1%	+7.4%	+14.4%
Gas	-33.0%	-13.6%	+15.4%	+17.4%
2000				
Oil	-17.3%	-8.4%	+8.4%	+15.7%
Gas	-28.1%	-8.4%	+9.7%	+11.2%

Table 5. Results of Sensitivity Run on Oil Recovery Factors
(Primary and Secondary)

	5th Percentile	25th Percentile	75th Percentile	95th Percentile
1985	-5.7%	-2.8%	+2.6%	+7.4%
1990	-10.4%	-6.0%	+5.2%	+13.1%
1995	-13.5%	-8.7%	+7.7%	+17.0%
2000	-17.3%	-9.5%	+9.2%	+19.7%

Figure 8

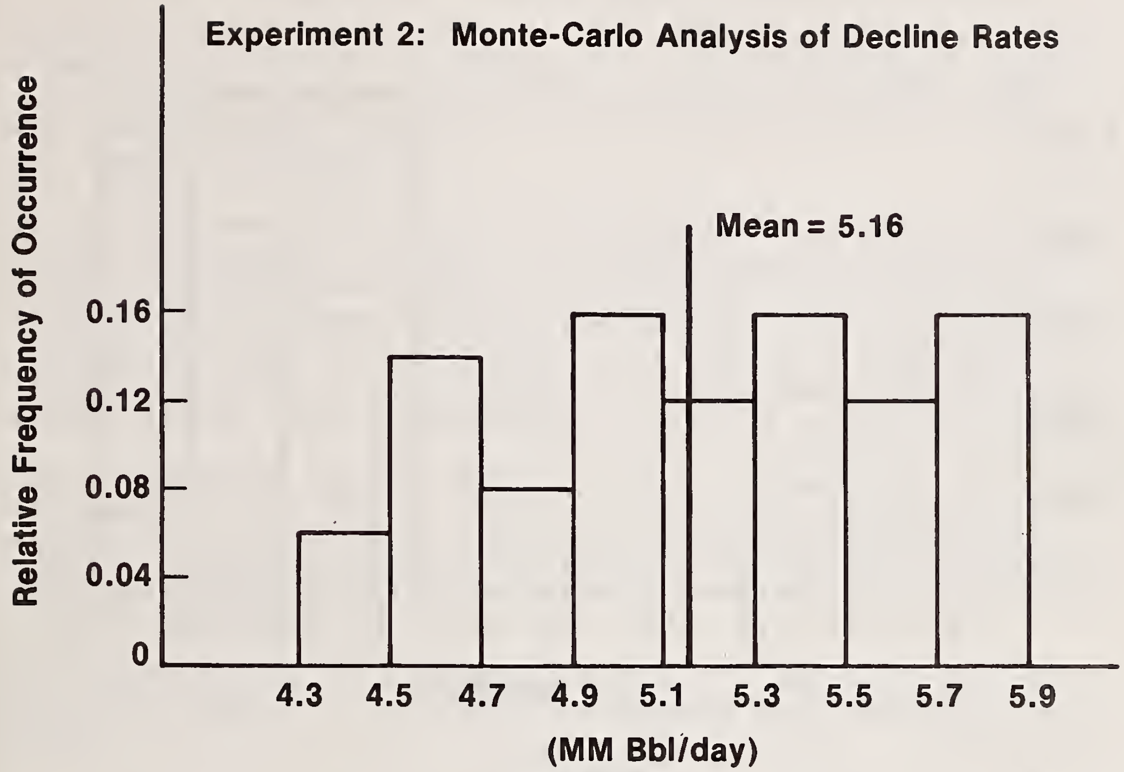
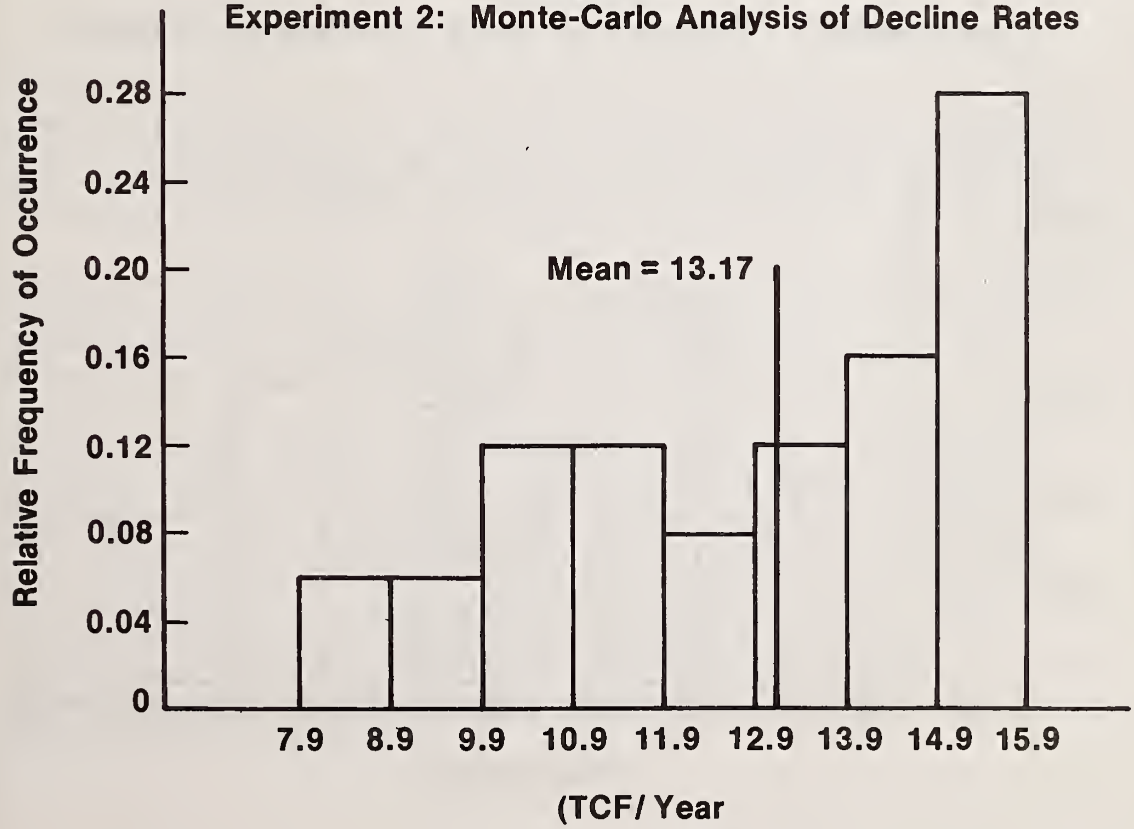


Figure 9



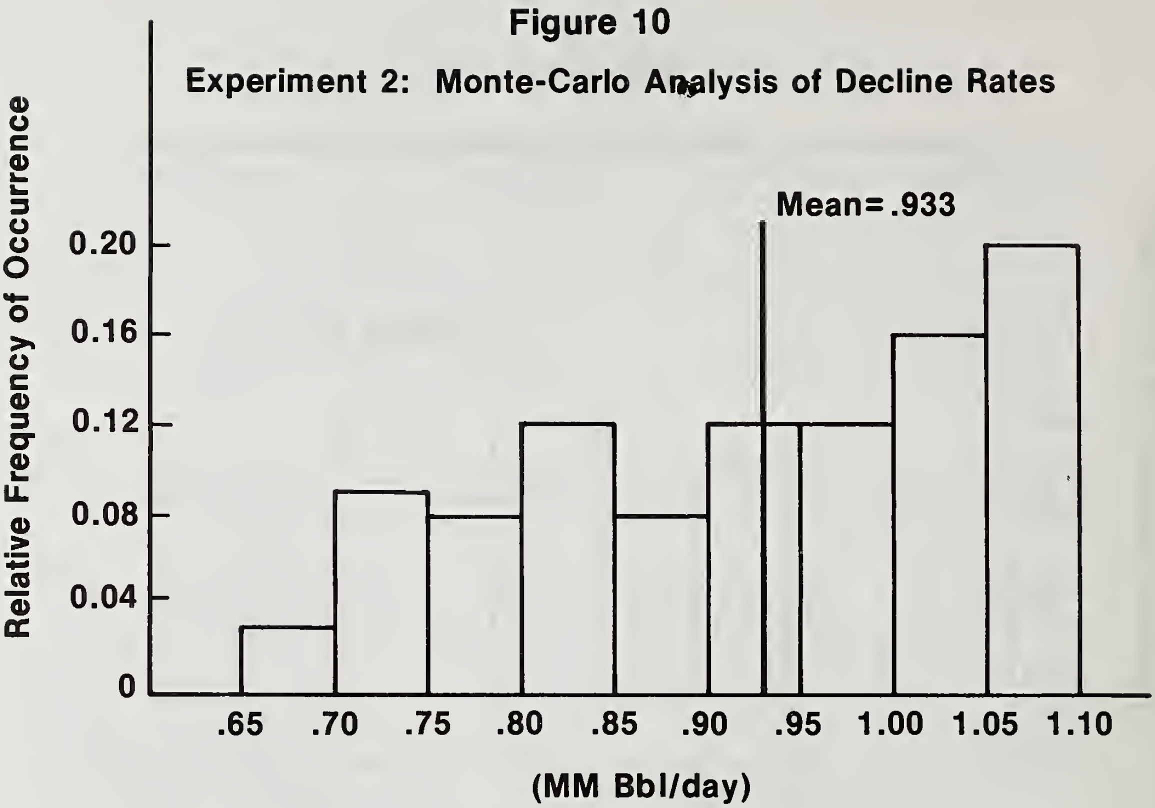


Figure 10
Experiment 2: Monte-Carlo Analysis of Decline Rates
Mean=.933
Relative Frequency of Occurrence
0.20
0.16
0.12
0.08
0.04
0
.65
.70
.75
.80
.85
.90
.95
1.00
1.05
1.10
(MM Bbl/day)

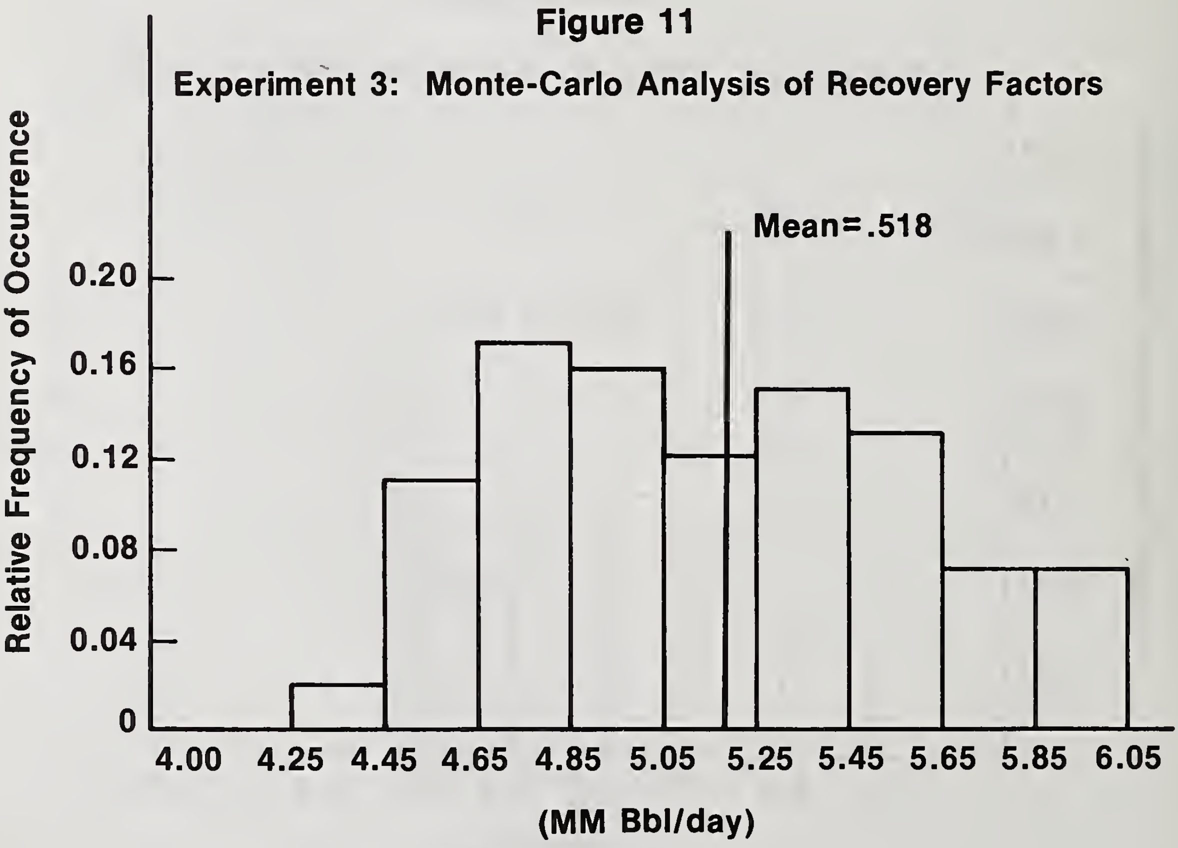


Figure 11
Experiment 3: Monte-Carlo Analysis of Recovery Factors
Mean=.518
Relative Frequency of Occurrence
0.20
0.16
0.12
0.08
0.04
0
4.00
4.25
4.45
4.65
4.85
5.05
5.25
5.45
5.65
5.85
6.05
(MM Bbl/day)

6. EXPERIMENT 3: RESPONSE SURFACE ANALYSIS

Experiment 3 was a first-order response surface experiment. It was aimed at identifying those MOGSMS input data elements whose values most strongly impact MOGSMS output values and at estimating the magnitude of these effects.

6.1 Design Considerations

The selection of the response surface approach draws upon an earlier survey of sensitivity analysis methodology (Harris, 1979c) and the earlier mentioned work of Harris and Hirshfeld (1980). That survey suggested that response surface analysis would be a powerful and economic method of screening MOGSMS input elements (independent variable) to identify the critical ones: those whose values have the largest effect on the MOGSMS output measures (dependent variables).

The term response surface refers to a formal mathematical relationship expressing the anticipated value of the dependent variable in an experiment as a function of the experiment's independent variables. The relationship is derived by statistical analysis of the results of designed factorial experiments.

More specifically, for a dependent variable, Y, say the crude oil production measure, we will assume a response surface having linear form:

$$Y = B_o + \sum_{i=1}^{n} B_i X_i \qquad (22)$$

where the $\{X_i\}$ are the set of independent variables in a particualr test, corresponding to a subset of the MOGSMS input data elements. A "test" in this context, as before, is simply an execution of MOGSMS for a given input data set, containing some systematic variations in X_i from the base (Midprice, Standard Projection) data set.

In general, each of the elments X_i in Equation (22) may be either a direct value of a data element or a transformed value. Transformations (i.e., logarithmic, exponential, etc.) may be necessary to preserve the validity of the linearity form.

In the response surface context, the effect on the MOGSMS output measure, Y, of a given input element, X_i, is measured by the magnitude of its coefficient B_i in Equation (22) as derived by a step-wise linear regression analysis. The larger the coefficient B_i, the more important is the variable X_i to the level of the product measure, Y. (This is true unless there are such wide variations possible in X_i, that the product $(X_i.B_i)$ can be "large" even for "small" B_i. Of course, the usual analysis of variance provides some additional clues in the search for the important input elements.)

For a reason that will become clear as the discussion proceeds, we have run not one, but three response surface experiments. Each sub-experiment addressed a specific subset of input data elements (as the independent variables in the experiment), and the first two experiments have no data elements in common. The third experiment (called the integrating experiment) is built up from the results of the first two.

With few minor exceptions (discussed in Section 6.2), the sub-experiments followed standard procedure, involving these steps:

(i) Define three response surface equations for the experiment, one for each of the product measures (as dependent variables:

$$Y_{OIL} = B_{k1} + \sum_{i=1}^{n} B_{ki1} X_i$$

$$Y_{Gas} = B_{k2} + \sum_{i=1}^{n} B_{ki2} X_i$$

$$Y_{NGL} = B_{k3} Y_{OIL} + B_{k4}$$

where the Y's are the product measures, the X_i's denote the same MOGSMS input data elements (independent variables) in each equation, and K denotes the sub-experiment.

(ii) Specify three numerical values for each independent variable: an upper value, a lower value, and a midpoint value. These three numbers were chosen purely for parsimony in the experimental design. They do not represent any assumption whatsoever of the stochastic behavior of the independent variables, inside or outside the stated range.

The upper and lower values constitute our estimates of the range of variation likely to occur or exist in nature for the given input data element. The corresponding midpoint value is either an (arithmetic) average or a "most likely" value for that element.

The upper and lower values need not be construed as absolute bounds on the corresponding independent variables. The response surface analysis seeks an approximation (linear, in this experiment) to the real functional from over some domain, defined here by the ranges of all the upper and lower values. If a close linear approximation within that domain can be found, one can resonably assume that the approximation would remain useful for some distance outside of the domain.

(iii) Define a factorial set of MOGSMS tests by assigning the extreme (or perhaps some midpoint) values to each independent variable inperiment, in all possible combinations. The MOGSMS run corresponding to the 1979 midprice forecast may be thought of as the "base case" test in each sub-experiment.

Since we have n independent variables in each sub-experiment and we work with the extreme values only, we have thus defined $2^n + 1$ MOGSMS tests. We split the overall experiment into sub-experiments simply to keep the total number of tests within reason.

(v) For each test run in Step (iv), record the values of the three product measures and the values of the subject input data elements.

(vi) Perform step-wise linear regression analyses on the experimental data recorded in Step (v) either "as is" or transformed, to obtain the coefficients in the response surface equations.

(vii) Sharpen the fit of the response surface equations by executing additional MOGSMS tests involving some of the (unused) midpoint values defined in Step (ii) above.

We ran a final, integrating sub-experiment, following the same procedure. The integrating sub-experiment had as its independent variables those input data elements revealed as most "significant" in the initial round of four sub-experiments. This final set of independent variables is thus the set of critical input data elements delineated by the response surface experiment.

- o Table 6 sets forth the input data elements that we selected for the first round of two response surface sub-experiments described above. Several comments regarding Table 6 are in order:

- o The selected elements are arranged in two broad categories, corresponding to the first two response surface sub-experiments.

- o Three types of input data elements are indicated:

    - Scalars are single numerical values in the oil or gas input data set;

    - Vectors are sets of numerical values in the input data, one per region;

    - Functions are more complex sets of values in the input data, varying both by region and by some independent parameter (total exploratory drilling footage or year).

- The input data elements pertaining to drilling were included in the analysis to produce systematic variation in the exploratory drilling trajectories (a key input element).

Our selection of input data elements for the response surface experiment was based on the judgement of various model reviewers (including the authors) and the experience of MOGSMS's developers and users (EIA 1978). An element's appearance in Table 6 is not an assertion that we consider the underlying MOGSMS data specious. Rather, it is a reflection of our prior belief that MOGSMS results were likely to be sensitive to the stipulated input data element, and that some uncertainty is connected with the values of these elements.

Table 6. Input Data Elements (Independent Variables) for the Response Surface Experiments

Category	Data Element	Submodel Oil	Submodel Gas	Type Scalar	Type Vector	Type Function
Economics (SE-1)	Discount rate	X	X	X		
	Drilling costs, producing wells	X	X			X
	Drilling costs, dry wells	X	X			X
Drilling (SE-2)	Total-to-exploratory drilling ratio	X				X
	Lease acreage footage offshore	X				X
	Years of foresight, rig.mgrs.	X	X	X		
	Rig and plant life	X	X	X		

6.2 Implementation Considerations

As we have seen, Experiment 3 comprises three sub-experiments (SE-1, SE-2 and SE-3), each a response surface analysis of a specified subset of MOGSMS input data elements:

- SE-1 Economics data elements
- SE-2 Drilling data elements
- SE-3 Critical elements identified in the first round of SE's

SE-1 and SE-2 involved the same experimental procedure, just described in Section 6.1. The essence of each sub-experiment was the specification of the numerical values of the X_i data elements for each test. Collectively, the set of tests in each sub-experiment define a modified "Latin Hypercube" in n-dimensional space, where n is the number of target data elements. Normally each test (but one) corresponds to a unique vertex in that hypercube. There are two types of exceptions to this: the first corresponds to the center point of the hypercube, while the second type is any edge point not completely out at a vertex. Table 7 and 8 define the tests comprising SE's 1 and 2, respectively. The definitions are in terms of the values (high, medium high, medium low, low, or center) assigned to each X_j in each test.

Tables 9 and 10 further define SE's 1 and 2, respectively. These exhibits set forth the rules we used for calculating the values for each of the X_j's in the corresponding SE. In most instances, the experimental values were calculated as a multiplicative factor times the corresponding values in the base projection run for the 1979 ARC, reflecting our (subjective) estimates of the potential ranges for the values of the various data elements.

Inspection of the results of SE-1 and SE-2 led us to combine all of their input data elements for the integrating response surface analysis (SE-3). SE-3 involved one modification to the standard experimental procedures. Specifically, in lieu of executing steps (ii), (iii), and (iv) of the procedure, we defined SE-3's test set to be the union of all of the tests originally defined and run in SE-1 and SE-2. That is, we did not define run a new set of MOGSMS tests to drive the subsequent regression analysis. Instead, we used the test results we already had and simply redefined the set of independent variables (X_j) to be union of those included in SE-1 and SE-2.

This tactic created a set of 251 tests, all having the same independent variables. We performed the usual regression analysis on this (derived) set of tests to obtain SE-3's response surface equation. Thus we were able to carry out sub-experiment 3 without executing any new MOGSMS tests.

Table 7. Tests for Sub-Experiment 1

Economics			9 Tests
		Independent Variables (Xj)	
		DiR	DC
	OIL:	X	X
Test	GAS:	X	X
1		Hi	Ce
2		Med Hi	Ce
3		Med Lo	Ce
4		Lo	Ce
5		Lo	Hi
6		Ce	Med Hi
7		Base Case	Base Case
8		Ce	Med Lo
9		Ce	Lo

DC: Drilling Costs, producing Wells & Dry Wells

Table 9 defines the high, low, and center (base values) for each of the X j's and specifies the the precise location (by card image) of the Xj's in the MOGSMS input data set.

Table 8. Tests for Sub-Experiment 2

Drilling						243 Tests
		Independent Variables (Xj)				
		TER	DRC	YF	RPL	CAPL
	OIL:	X	X	X	X	X
Test	GAS:			X	X	X
1		Hi	Hi	Hi	Hi	
2		Hi	Hi	Hi	Hi	
3		Hi	Hi	Hi	Hi	
4		Hi	Hi	Hi	Lo	
5		Hi	Hi	Hi	Lo	
6		Hi	Hi	Hi	Lo	
7		Hi	Hi	Hi	Hi	
8		Hi	Hi	Hi	Hi	
9		Hi	Hi	Hi	Hi	
10		Hi	Hi	Lo	Lo	
11		Hi	Hi	Lo	Lo	
12		Hi	Hi	Lo	Lo	
13		Hi	Hi	Lo	Hi	
14		Hi	Hi	Lo	Hi	
15		Hi	Hi	Hi	Hi	
16		Hi	Hi	Lo	Lo	
17		Hi	Hi	Lo	Lo	
18		Hi	Hi	Lo	Lo	
etc...........						

Key to the Xj's

TER: Total-to Exploratory Drilling Ratio
DRC: Oil Drilling Constraints
YF: Years of Foresight, Rig Mfgrs.
RPL: Rig and Plant Life
CAPL: Planning Horizon

Table 10 defines the high, low, and center (base) values for each of the Xj's and specifies the precise location (by card image) of the Xj's in the MOGSMS input data set.

Table 9. Specification of Numerical Values in Sub-Experiment 1

Category: Economics

Data Element	OIL	GAS	Input Data Set/Cards OFRIS	OTAIL	GFRIS	GTAIL	Type	Vertex Values HIGH	LOW	BASE
Discount Rate (DiR)	X		227-229				Scalar	.15	.05	.10
		X			211-212			.15	.05	.10
Drilling Costs, Producing Wells	X	X	176-199				Function	1.5 X (Base)	0.5 X (Base)	Mid Price Input
Drilling Cost, Dry Wells (DC)	X	X			18-73					

Table 10. Specification of Numerical Values in Sub-Experiment 2

Category: Drilling

DATA ELEMENT	OIL	GAS	Input Data Set/Cards OFRIS	OTAIL	GFRIS	GTAIL	Type	Vertex Values HIGH	LOW	BASE
Total to Exploratory Drilling Ratio (TER)	X		55-66				Function	0.8x (Base)	1.2x (Base)	Mid Price Input
Oil Drilling Constraints (DRC)	X						Scalar	1.5x (Base)	0.5x (Base)	Mid Price Input
		X					Scalar			
Years of foresight rig manufacturers (YF)	X			1			Scalar	6	0	Mid Price Input
		X				1	Scalar	6	0	
Rig and Plant Lift (RPL)	X			1			Scalar	15	5	0
		X				1	Scalar	7.5	2.5	5
Planning Horizon (CAPL)	X			1			Scalar	15	5	10
		X				1	Scalar	7.5	2.5	5

6.3 Results

The primary results of Experiment 3 appear in Tables 11-13. The most important aspects of these results can be summarized as follows:

- Sub-experiments 1 and 2 deal with those input data elements that we deemed to be potentially the critical ones. Table 12 (oil and 13 (gas) summarize the results of these experiments and highlight the key explanatory variables or critical elements identified by this (screening) around of sub-experiments. The tables show the change in oil and gas production (dependent variables) for a given change in each of the data elements (independent variables), as specified by the coefficients in the derived response surface equation (Table 11).

- All response surface equations emerging from the sub-experiments show high R^2 values (>0.82 in all cases). This result implies that the true response surfaces are well-approximated by a (non-transformed) linear form. By way of verification, the equations all closely fit the corresponding center points (which were not used in deriving the equations, but reserved for calibration purposes).

Table 11. Response Surface Equations: Sub-Experiments 1, 2, and 3

Indendent Variables	SUB-EXPERIMENT 1: Economics Oil	Gas	2: Drilling Oil	Gas	3: Integrating Oil	Gas
Dir	-4930.35	0			-4930.35	0
DC	- 772.08	0			- 772.08	0
TER			-317.59	-114.30	-317.59	-114.30
DRC			107.18	54.40	107.18	54.40
YR			102.58	543.58	102.58	
RPL			-440.84	-1252.86	-440.84	-1252.86
CAPL			-230.97	-425.75	-230.97	-425.75
R^2	0.9402		0.8515	0.9513	0.8242	0.9337

Table 12. Sensitivity of (Average) Oil Production to Changes in Key Input Variables

For This Input Element	This Amount Of Change	Leads To This Change (In MBl/Day)
Total-to-Exploratory Drilling Ratio	10% (2)	-159
Drilling Constraints — Regional:	10% (2)	
Drilling Constraints — Total:	7% (2)	+107
Years of Foresight	1 Year (1)	+34
Rig and Plant Life	1 Year (1)	-176
Cap. Planning Horizon	1 Year (1)	-92
Discount Rate	25% (2)	-123
Drilling Costs	25% (2)	-193

[1] Changes in numerical values.
[2] These are percent changes from nominal or base values.

Table 13. Sensitivity of (Average) Gas Production To Changes in Key Input Variables

For This Input Element	This Amount Of Change	Leads to This Change (In MCF/Day)
Total-to-Exploratory Drilling Ratio	10% (2)	-57
Drilling Constraints — Regional:	10% (2)	+54
Drilling Constraints — Total:	7% (2)	
Years of Foresight	1 Year (1)	+181
Righ and Plant Life	1 Year (1)	-501
Cap. Planning Horizon	1 Year (1)	-170
Discount Rate	25% (2)	Infinitesimal
Drilling Costs	25% (2)	Infinitesimal

[1] Changes in numerical values.
[2] These are percent changes from nominal or base values.

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NATURAL RESOURCE DECISIONS
INVOLVING UNCERTAINTY

by

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1. INTRODUCTION

With the increasing scarcity of natural resources in recent years there has been a growing interest in problems of optimal management of these resource stocks. Starting with the classic paper by Hotelling (1931), the more recent extensive literature on the economics of exhaustible resources is represented by Solow's (1974) exposition of the basic theory, essays in the 1974 Symposium of the Review of Economic Studies and the monograph by Dasgupta and Heal (1978).

In general, however, only a few of the studies have explicitly incorporated the crucial element of uncertainty in their analysis. These studies may be broadly classified into three categories. Models in the first category are concerned with optimal resource extraction decisions when the total resource stock is unknown and may be suddenly exhausted, as in Kemp (1976, 77) Cropper (1976), Loury (1978) and Gilbert (1979), or it may be expropriated as in Long (1975). The second category of models involves uncertainty regarding the time at which a perfect producible substitute becomes available so as to eliminate the dependence of the economy on the natural resource. Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1976) analyze optimal extraction decisions when the probability distribution of the uncertain timing of innovation of a substitute is specified exogenously, while Dasgupta, Heal and Majumdar (1977) and Kamien and Schwartz (1978) also permit

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the innovation process to be controlled through R & D expenditures. In a related model Hoel (1978) assumes that the time of innovation is known but the cost of producing the substitute is uncertain. In the third category of models the uncertainty is regarding the discovery of additional resource stocks through search and exploration. Arrow and Chang (1980) and Deshmukh and Pliska (1980) have studied optimal consumption and exploration decisions that control the uncertain timings and magnitudes of discoveries; see also MacQueen (1961, 64) and Heal (1978) for related models involving stochastic discoveries and Pindyck (1978) for the certainty case.

In this paper we present a general model of natural resource decisions in the presence of uncertainty regarding the time of occurrence of some significant event. The decisions involve selection of extraction (consumption) and exploration (search or R & D expenditure) rates and the uncertain event corresponds to resource exhaustion or discovery of additional stock or invention of a substitute. In addition to characterizing optimal decisions under uncertainty, we also consider the behavior of the resource price over time. The general model is presented in Section 2 and its special cases are studied in the subsequent sections in light of the related literature outlined above.

2. THE MODEL

The distinguishing characteristic of a natural energy resource (such as oil or natural gas) is that it is nonproducible and nonrenewable. Consequently, the future supply of the resource cannot be controlled or determined with certainty. In an extreme event, the resource may be exhausted, thereby imposing a severe hardship on the economy. At the other extreme, a perfect producible substitute may become available, rendering the natural resource inessential. Between these possibilities of extremely unfavorable and favorable events, an interesting intermediate case is the one in which an additional stock of the same resource is discovered. In this section, by an "event" we mean exhaustion of the resource stock or discovery of an additional stock or development of a producible substitute. We assume, for simplicity and consistency, that only one type of event may occur and that it can occur only once. The time at which the event takes place is a random variable, and its probability distribution can be controlled through the extraction and exploration rate decisions. Resource extraction (consumption) yields social utility but depletes the stock on hand and hastens exhaustion. On the other hand, exploration involves (search or R & D) expenditures but also expedites the occurrence of a desirable event (i.e. discovery of an additional stock or development of a producible substitute). The problem then is to determine optimal extraction and exploration policies in face of the uncertain timing of occurrence of the event of interest.

Let the nonnegative random variable X_t denote the state of the natural resource in the economy at time $t \geq 0$. For instance, X_t may be the size of proven reserves on hand at time t or it may be the cumulative amount extracted and consumed by t. Suppose the central planner's decision variable $c_t \varepsilon [0, \bar{c}]$ denotes the consumption (extraction) rate at which the resource stock is depleted at time t. This yields a social utility (net of extraction costs) to the economy at rate $U(c_t)$, which is assumed to be increasing and concave in c_t. Denote by $\alpha > 0$ the rate at which future utilities and costs are discounted.

In addition to the resource state X_t, let the binary valued random variable Y_t denote the occurrence or nonoccurrence of the event of interest (i.e. exhaustion, discovery or innovation) by time t. Suppose $Y_t = 0$ means the event has not occurred by time t and $Y_t = 1$ corresponds to the occurrence of the event prior to t (so that $Y_t = 1$ implies $Y_s = 1$ for all $s \geq t$). Equivalently, we let the nonnegative random variable T denote the (Markov) time of occurrence of the event, i.e. $T = \min \{t \geq 0;\ Y_t = 1\}$. While c_t advances the date of exhaustion, the exploration expenditure rate $e_t \ \varepsilon \ [0, \bar{e}]$ expedites the discovery of an additional stock or a substitute through search or R & D activities. In general, let $\lambda(x, c, e)$ denote the hazard rate (success or failure rate) associated with the event time T, i.e. $\lambda(x, c, e)$ is the probabilistic rate of occurrence of the event at t, given that $T > t$, $X_t = x$, $c_t = c$ and $e_t = e$. Roughly, $\lambda(x, c, e)\, dt$ is the probability that the event will occur during (t, t+dt), given that it has not occurred by time t, the resource state is $X_t = x$ and the consumption and exploration decisions are $c_t = c$ and $e_t = e$. We assume that λ is increasing in (c,e) in order to reflect the advancing of exhaustion through c or of discovery through e.

Once the uncertain event occurs at time T, the planner's problem then becomes a relatively easy one of determining optimal consumption pattern under certainty. Let $W(x)$ denote the maximum attainable total discounted utility over $[T, \infty)$, given $X_T = x$ and $Y_T = 1$. For instance, in the event of exhaustion, $W \equiv 0$; in the case of discovery of size z, $W(x + z)$ is the total utility from consuming the stock $x + z$ optimally, as in Hotelling (1931); in the case of substitute discovery, $W(x)$ is the optimal value of the program as in Dasgupta and Heal (1974).

The planner's problem is then to determine $\{(c_t, e_t);\ 0 \leq t < T\}$ so as to

$$\text{maximize } E\{ \int_0^T \exp(-\alpha t)\ [U(c_t) - e_t] + \exp(-\alpha T)\ W(X_T) | X_0 = x\}.$$

Let $V(x)$ denote the <u>optimal value</u> of this program starting in the resource state $X_0 = x$ and $Y_0 = 0$. Selection of decisions (c, e) in $[0,t]$ yields net utility $[U(c) - e]t$ and the resource state changes to $X_t = x - ct$ (if X is the stock on hand) or $X_t = x + ct$ (if X is the cumulative consumption). Also, the uncertain event occurs in $(0,t)$ with probability $\lambda(x, c, e)t$ (so that the optimal value is determined by $W(X_t)$) and with probability $[1 - \lambda(x, c, e)t]$ the event does not occur (so that the optimal value is $V(X_t)$). The dynamic programming argument then yields

$$V(x) = \max_{c,e} \{[U(c) - e]t + \exp(-\alpha t)[\lambda(x,c,e)t\ W(X_t) + (1-\lambda(x,c,e)t)V(X_t)]\}$$

Using $\exp(-\alpha t) = 1-\alpha t + o(t)$, and the Taylor's expansion of $V(\cdot)$ and $W(\cdot)$ around x, dividing by t and letting $t \downarrow 0$ yields the <u>optimality equation</u>

$$(1) \qquad \alpha V(x) = \max_{c,e} \{U(c) - e - c V'(x) + \lambda(x,c,e)[W(x)-V(x)]\}, \ x \geq 0$$

prior to the occurrence of the event, if $X_t=x$ represents the stock on hand (with $V'(x)$ replaced by $-V'(x)$ if X_t is the cumulative consumption). Upon occurrence of the event, exploration is unnecessary (i.e. $e_t \equiv 0$), and the optimal value function $W(\cdot)$ satisfies the optimality equation

$$(2) \qquad \alpha W(x) = \max_{c} \{U(c) - cW'(x)\}, \ x \geq 0.$$

Optimal decision policies specify, as functions of the resource state $X_t=x$ at any time $t<T$ those consumption and exploration rates $c^*(x)$ and $e^*(x)$ that attain the maximum in (1). Similarly, given $X_t = x$ at any time $t \geq T$, $c^*(x)$ attains the maximum in (2) and $e^*(x) = 0$.

The optimality equations (1) and (2), which characterize the optimal policies $c^*(\cdot)$ and $e^*(\cdot)$, may be written in a comprehensive fashion in terms of a more general optimal value function as follows. Let $\bar{V}(x,y)$ denote the optimal value function starting in $X_0 = x$ and $Y_0 = y \varepsilon \{0,1\}$ so that $\bar{V}(x,0) = V(x)$ and $\bar{V}(x,1) = W(x)$. Consider the <u>infinitesimal generator</u> $A_{c,e}$ of the Markov process $\{(X_t, Y_t) ; t \geq 0\}$ defined by

$$A_{c,e}\bar{V}(x,0) = \lim_{t \downarrow 0} \{E[\bar{V}(X_t, Y_t) | X_0 = x, Y=0, c_0 = c, e_0 = e] - \bar{V}(x, 0)\}/t$$

$$= - c\, V'(x) + \lambda(x,c,e)\, [W(x) - V(x)],$$

which is the expected rate of change in the value of $\bar{V}$ starting in $X_0 = x$, $Y_0 = 0$ and selecting (c,e) decisions. Similarly $A_{c,e}\, \bar{V}(x,1) = A_c W(x) = - c\, W'(x)$. With this additional notation, (1) and (2) may be rewritten compactly as

$$(3) \qquad \alpha\, \bar{V}(x,y) = \operatorname*{Max}_{c,e} \{U(c) - e + A_{c,e}\bar{V}(x,y)\},\ x \geq 0\ y \varepsilon \{0,1\}.$$

Recall that, given $t < T$ and $X_t = x$, $V(x)$ is the maximum long-run expected net utility over $[t,\infty)$. Therefore, $V'(x)$, the marginal improvement from an incremental unit of the resource stock, corresponds to the imputed (shadow) price P_t of the resource prior to the occurrence of the event. (Similarly, if $X_t = x$ and $t \geq T$, $W'(x)$ is the resource price after the event occurs.) The rate of change in the resource price without taking into account the possibility of occurrence of the event is then the time derivative $\dot{P}_t = -c_t\, V''(X_t)$. However, the expected rate of change in the resource price, allowing for the possibility of occurrence of the event (and the consequent change in the price from V' to W'), is obtained by considering the infinitesimal generator A evaluated at $V'(x)$. Therefore,

$$(4) \qquad A_{c^*,e^*}V'(x) = -c^*(x)\, V''(x) + \lambda(x,c^*(x),e^*(x))[W'(x) - V'(x)]$$

is the expected rate of change in the resource price prior to the event and

$$(5) \qquad A_{c*} W'(x) = -c*(x) W''(x)$$

is the corresponding price dynamics after the occurrence of the event.

In the rest of the paper we shall study the optimal policies $c*(x)$ and $e*(x)$ and the dynamics of prices $V'(x)$ and $W'(x)$ in a number of special cases. In Section 3 we first consider the benchmark case of certainty, while in the three subsequent sections we consider the uncertain events of exhaustion, discovery of new stock, and development of a substitute respectively. We shall also discuss in each section the related literature.

3. KNOWN, FIXED RESOURCE STOCK

In this classic case studied by Hotelling (1931), Heal (1973) and Solow (1974), the given initial stock X_0 is to be consumed optimally over $[0, \infty)$ when no additional stock or a substitute is anticipated. If the resource is essential, its exhaustion occurs only asymptotically (under the typical assumption of $U'(0) = \infty$). While the usual approach to analysis of this case employs variational calculus, our optimality equation (1) with $\lambda = e = 0$ specializes to

$$(6) \qquad \alpha V(x) = \operatorname*{Max}_{c} \{U(c) - cV'(x)\}, \ x \geq 0.$$

It can be shown that its unique solution V is concave and increasing in resource stock size x. Thus, the shadow price of the resource, $V'(x)$, is positive and decreasing in the amount on hand. As to the optimal consumption rate $c*(x)$, an interior optimum in (6) requires $U'(c*(x)) = V'(x)$. Thus, the marginal utility of consumption is equated with the marginal worth of unit consumption postponed. Since, by concavity, $U'(\cdot)$ and $V'(\cdot)$ are decreasing, optimal consumption rate $c*(x)$ is increasing in the stock size.

Also note that $U'(c*(x))$ is the shadow price of consumption and $V'(x)$ is the shadow price of holding reserves. Thus, $X_t=x$ and $t < T$ yields the price $P_t = U'(c*(x)) = V'(x)$. The rate of change in the price is then

$\dot{P}_t = - c^*(x)\, V''(x)$. Now optimality of $c^*(x)$ in (6) implies

$$\alpha\, V(x) = U(c^*(x)) - c^*(x)\, V'(x).$$

Differentiating and using the optimality condition yields

$$\alpha\, V'(x) = [U'(c^*(x)) - V'(x)]c^{*\prime}(x) - c^*(x)\, V''(x)$$
$$= - c^*(x)\, V''(x)$$

i.e. $\alpha P_t = \dot{P}_t$, so that we have

(7) $$P_t = P_0\, e^{\alpha t}.$$

Thus, the resource price (net of extraction costs) rises at the rate of discount, which is the fundamental theorem of economics of exhaustible resources (Hotelling (1931), Solow (1974)). In the competitive resource market this may be interpreted as the arbitrage condition as follows. In the flow equilibrium, in order for the resource holders to be indifferent between supplying at different points in time, the discounted prices must be the same at each point in time, i.e. $e^{-\alpha(t+s)} P_{t+s} = e^{-\alpha s} P_s$, so that $\dot{P}_t = \alpha P_t$; otherwise it would be preferable to change the supply pattern. Alternatively, the suppliers hold the resource stock as an asset and the stock equilibrium in the assets market requires that all assets yield the same rate of return (dividends plus capital gain) equal to α. Since the resource stock in ground yields no dividends, its value (price) must grow at rate α. If it grows slower, more will be supplied earlier and the resource will be exhausted too quickly. If the price grows at a rate faster than α, then it is better for the resource holders to hold the stock as an investment that yields a rate of return higher than α. The welfare economics implication of the above theorem is that, in a socially managed economy, the imputed price (net of extraction costs) of the resource rises at the social rate of discount. Finally, if the resource is owned by a monopolist, the corresponding statement is that his marginal profit must rise at the rate of interest.

Given the above price dynamics, the optimal consumption pattern over time can be derived. Since $c^*(\cdot)$ is decreasing in the stock size which is depleting over time (in absence of new discoveries), the optimal consumption rate declines through time. More precisely, we have $P_t = U'(c^*_t)$ so that $\dot{P}_t = \dot{c}^*_t\, U''(c^*_t)$, which together with $\dot{P}_t/P_t = \alpha$ yields

$$\dot{c}^*_t \,/\, c^*_t = \alpha/[c^*_t\, U''(c^*_t)/U'(c^*_t)]$$

i.e.

(8) $\dot{c}^*_t / c^*_t = -\alpha/\eta(c^*_t)$, where $\eta(c) = -c\, U''(c)\, /U'(c)$

is the elasticity of marginal utility. For example, if $U(c) = c^{(1-\varepsilon)}$ for $0 < \varepsilon < 1$, then $\eta(c) = \varepsilon$ and hence $c^*_t = c^*_0\, e^{-\alpha t/\varepsilon}$ i.e. optimal consumption decreases exponentially. The initial rate of c^*_0 is then chosen so that $\int_0^\infty c^*_t\, dt = X_0$ i.e. $c^*_0 = \alpha\, X_0/\varepsilon$.

4. EXTRACTION OF FIXED UNCERTAIN STOCK SIZE

This is the case of optimally "eating a cake of unknown size" studied by Kemp (1976, 77), Cropper (1976), Loury (1978) and Gilbert (1979). The total stock size is a random variable S with the distribution function $F(\cdot)$ and the density function $f(\cdot)$, and no additional discoveries of the resource or a substitute are expected. The resource state X_t is the cumulative amount consumed by time t, the "event" corresponds to exhaustion (so that $Y_t = 0$ if $X_t < S$ and $Y_t = 1$ if $X_t = S$) and the post-event return is $W(x) = 0$. Given $X_t = x$ and $Y_t = 0$ (i.e. $S > x$), the conditional density function of S is $\lambda(x) = f(x)/[1 - F(x)]$. It can be seen that, if the consumption rate is $c_t = c$, the hazard rate of the time of exhaustion is $\lambda(x,c) = c\, \lambda(x)$. Thus the optimality equation (1) specializes to

(9) $$\alpha\, V(x) = \max_c \left\{ U(c) + cV'(x) - c\lambda(x)\, V(x) \right\},\ x \geq 0.$$

Note that the certainty case of the previous section is obtained by taking $\lambda(x) = 0$ for all $x < S$, the known resource stock, and $\lambda(S) = \infty$. Note also that $V(x)$ is the optimal value over $[t, \infty)$ given that $X_t = x$ <u>and</u> that the exhaustion has not occurred by t. Equivalently, using $\lambda(x) = f(x)/[1-F(x)]$ and multiplying both sides of (8) by $[1 - F(x)]$ we get

(10) $$\alpha\, v(x) = \max \left\{ U(c)\, [1 - F(x)] + c\, v'(x) \right\}$$

where

(11) $$v(x) = V(x)\, [1 - F(x)]$$

is the expected optimal value over $[t, \infty)$ if the cumulative consumption over $[0, t]$ is $X_t = x$ (but it is not known whether the exhaustion has occurred by t). Optimality of $c^*(x)$ in (10) requires

$$(12) \qquad U'(c^*(x))\,[1 - F(x)] = - v'(x),$$

which is the resource price P_t. The expected rate of change in the resource price is then $\dot{P}_t = A_{c^*}v(x) = - c^*(x)\, v''(x)$. Also, since $c^*(x)$ is the maximizer in (10), we have $\alpha\, v(x) = U(c^*(x))\,[1 - F(x)\,] + c^*(x)\, v'(x)$, which upon differentiating with respect to x and using (12) yields

$$(13) \qquad \alpha\, v'(x) = - U'(c^*(x))\, f(x) + c^*(x)\, v''(x).$$

Hence, from (12) and (13), we have

$$(14) \qquad \dot{P}_t/P_t = \alpha - \lambda(x)\, U(c^*(x))/U'(c^*(x))$$

i.e. the price is expected to rise at a rate slower than the rate of discount. In fact, if $\lambda(x)$ or $c^*(x)$ is very high, the price may even fall through time.

As to the consumption pattern, note from (12) that

$$P_t = U'(c^*_t)\,[1 - F(x_t)]$$

so that

$$\dot{P}_t = - U'(c^*_t)\, f(x_t)\, c^*_t + U''(c^*_t)\, \dot{c}^*_t\,[1 - F(x_t)]$$

yielding

$$(15) \qquad \dot{P}_t/P_t = U''(c^*_t)\, \dot{c}^*_t/U'(c^*_t) - \lambda(x)\, c^*_t.$$

Equating the right hand sides of (14) and (15) and rearranging we get

$$(16) \qquad \dot{c}^*_t/c^*_t = \left\{- \alpha + \lambda(x)\, c^*_t\,[U(c^*_t)/c^*_t - U'(c^*_t)]/U'(c^*_t)\right\}/\eta(c^*_t),$$

where again $\eta(c) = - cU''(c)/U'(c)$ is the elasticity of marginal utility. To interpret (16) note that deferring a unit consumption at time t is like an investment that costs $U'(c_t)$ but delays the exhaustion instant (which is t with probability $\lambda(x)\, c_t$) by $1/c_t$ during which additional utility can be earned at rate $U(c_t)$. Thus $\lambda(x)c_t[U(c_t)/c_t - U'(c_t)]/U'(c_t)$ is the expected net rate of return per unit investment at t (which is positive, due to the concavity of U). It is the conservation motive in favor of delaying consumption; consumption tends to rise over time (or is postponed to reduce the probability of exhaustion) due to this factor. On the other hand, $-\alpha$ represents the time preference in favor of current consumption. As a combined effect of these two conflicting factors, the consumption may be rising or falling through time, in constrast with the certainty case of Section 3 wherein ($\lambda(x) = 0$ and hence) the consumption falls over time. For example, if the exhaustion probability $\lambda(x)$ is very high, the consumption may rise; if $\lambda(x)$ is increasing in x, the consumption may fall or first fall and then rise through time; if the discount rate α is small (i.e. if the plan is more future-oriented), then the consumption rises (it is postponed); see Kemp (1976), Cropper (1976) and Loury (1978) for details.

We close this section by considering a special case in which $\lambda(x)$ is a constant λ or, equivalently, when the distribution of the resource stock size is exponential, i.e. $F(x) = 1 - e^{-\lambda x}$. From the memorylessness property of the exponential distribution it is clear that, given no exhaustion yet, the optimal value V(x) is independent of the cumulative consumption x. Hence, (9) becomes $\alpha V = \underset{c}{\mathrm{Max}} \{U(c) - c\lambda V\}$ and the optimal consumption rate is a constant c^* which satisfies $U'(c^*) = \lambda V$. Also $V = U(c^*)/(\alpha + \lambda c^*)$, which is the expected discounted utility from the constant consumption rate c^* until the moment of exhaustion. The resource uncertainty may thus be viewed as raising the discount rate from α to $\alpha + \lambda c^*$.

5. EXPLORATION AND UNCERTAIN DISCOVERY OF NEW STOCK

In the previous section, learning about the uncertain stock size was accomplished through extraction alone; the probability distribution of the stock size was then updated over time merely using the fact that the true stock has to be at least as large as the cumulative amount already extracted. In this section, exploration is considered as a distinct activity of learning that involves expenditures to search for and discover the existence of additional stocks. Pindyck (1978) has considered the exploration activity under certainty, MacQueen (1961, 64) and Heal (1978) have studied related models involving uncontrolled stochastic discoveries, while Arrow and

Chang (1980), and Deshmukh and Pliska (1980), have analyzed optimal consumption and exploration decisions when the latter controls the uncertainty about timings and/or magnitudes of discoveries.

In this section, X_t denotes the size of proven reserves on hand at time t and the "event" refers to the discovery of a new stock. We assume that only one discovery is possible and it occurs at a random time T which can be controlled through the exploration expenditure rate $e \in [0,\bar{e}]$. The probabilistic rate of discovery $\lambda(e)$ is assumed to be increasing in e, with $\lambda(0) = 0$. Let the nonnegative random variable Z denote the size of the stock discovered at T and suppose $G(\cdot)$ is the probability distribution of Z.

If the resource stock just before the discovery is $X_{T-} = x$ and if the discovery is of size Z=z then the post discovery problem is that of optimally consuming the total resource stock $X_T = (x + z)$ on $[T, \infty)$. This problem was analyzed in Section 3 to yield the maximum discounted utility $V(x + z)$, where $V(\cdot)$ is the concave increasing function that is the solution of (6). Consequently, the terminal reward at T for the problem in the present section is

$$(17) \qquad W(x) = \int_0^\infty V(x + z)\, dG(z),$$

which is also concave and increasing in $x \geq 0$. With this $W(\cdot)$, the optimality equation (1) now becomes

$$(18) \qquad \alpha V(x) = \operatorname*{Max}_c \{U(c) - cV'(x)\} + \operatorname*{Max}_e \{-e + \lambda(e)\, [W(x) - V(x)]\},\ x \geq 0,$$

with the boundary condition

$$(19) \qquad \alpha\, V(0) = \operatorname*{Max}_e \{-\, e + \lambda(e)\, [W(0) - V(0)]\}.$$

It can be shown that (18) and (19) have the unique solution $V(\cdot)$ which is concave and increasing in the stock size; $V(x)$ is the optimal value of the stock size x prior to the occurrence of the favorable event of discovery of a new stock. From (17) and monotonicity of V it follows that $W(x) \geq V(x)$. Also concavity of $V(\cdot)$ implies that $[W(x) - V(x)]$ is decreasing in x, i.e. $W'(x) \leq V'(x)$, so that the marginal value of the original resource stock falls after the discovery of an additional stock. (Strictly speaking, these conclusions hold when an infinite sequence of discoveries is permitted). Note that $\lambda(e) \equiv 0$ (i.e. impossibility of new discoveries) yields the case of Section 3.

The optimal consumption rate $c^*(x)$ is a maximizer of the first term on the RHS of (18). By concavity of $U(\cdot)$ and $V(\cdot)$, it can be seen, as before, that $c^*(x)$ is increasing in x. Consequently, as the resource stock depletes on $[0,T)$, the consumption rate decreases and the shadow price of consumption $U'(c^*(x))$ rises through time. When the stock level jumps by amount Z at T, the consumption rate increases and the price falls, depending on the discovery size z.

Prior to the discovery, the optimal exploration cost rate $e^*(x)$ maximizes the second term on the RHS of (18), namely,

$$(20) \qquad f(x, e) = -e + \lambda(e)\,[W(x) - V(x)].$$

Since $\lambda(e)$ is increasing in e and $[W(x) - V(x)]$ is decreasing in x, it follows that $e^*(x)$ is decreasing in x. To see this suppose $x_2 > x_1$ but that $e^*(x_2) \geq e^*(x_1)$. Now

$$[f(x_2, e) - f(x_1\ e)] = \lambda(e)\,\{[W(x_2) - V(x_2)] - [W(x_1) - V(x_1)]\}$$

is positive and increasing in e. Hence

$$[f(x_2, e^*(x_2)) - f(x_1, e^*(x_2))] \geq [f(x_2, e^*(x_1)) - f(x_1, e^*(x_1))]$$

i.e. $[f(x_1, e^*(x_1)) + f(x_2, e^*(x_2))] \geq [f(x_1, e^*(x_2)) + f(x_2, e^*(x_1))]$.

But this contradicts the fact that $e^*(x_1)$ maximizes $f(x_1, \cdot)$ and $e^*(x_2)$ maximizes $f(x_2,\cdot)$. Hence we must have $e^*(x_2) < e^*(x_1)$, i.e. the exploration effort rate should be higher when the resource level is lower. Thus, initially very little investment is made in exploration (and more stock is consumed); as the resource stock depletes, more search effort is expended (and less stock is consumed), until new stock is discovered, at which point the exploration effort drops (to zero if no more discoveries are anticipated).

To determine the expected rate of change in the shadow price prior to T note that

$$A_{c*,e*}V'(x) = - c*(x)\ V''(x) + \lambda(e*(x))[W'(x) - V'(x)]$$

while

$$\alpha\ V(x) = U(c*(x)) - c*(x)\ V'(x) - e*(x) + \lambda(e*(x))[W(x) - V(x)]$$

so that

$$\begin{aligned}\alpha\ V'(x) &= c*'(x)\ [U'(c*(x)) - V'(x)] - c*(x)\ V''(x) \\ &\quad - e*'(x)\ \{1 - \lambda'(e*(x))[W(x) - V(x)]\} + \lambda(e*(x))[W'(x) - V'(x)] \\ &= A_{c*,e*}\ V'(x),\end{aligned}$$

where the last equality follows from optimality of c*(x) and e*(x) and the definition of $A_{c*,e*}\ V'(x)$.Thus, we have

(21) $$A_{c*,e*}\ V'(x)/V'(x) = \alpha$$

i.e. given that a discovery has not occurred by t and that $X_t = x$, the price is expected to rise at the rate of discount. This is a stochastic analog of the Hotelling's (1931) result discussed in Section 3, wherein no additional discoveries were possible. At the instant T of discovery if $X_{T-} = x$ and if the discovery is of size z, the stock price decreases from V'(x) to V'(x + z) and then it increases again at the rate of discount, as before. For the analysis involving multiple discoveries see Deshmukh and Pliska (1980).

6. R & D AND UNCERTAIN DEVELOPMENT OF A SUBSTITUTE

In the previous section, the occurrence of the favorable event of discovery of new stock relaxed the resource constraint temporarily. In this section we consider the possibility of an extremely favorable event (technological change) that permanently eliminates the resource constraint as a result of the development of a producible perfect substitute. The substitute development process may be expedited by allocating higher R & D expenditures. The special case of uncontrolled development was analyzed by Dasgupta and Heal (1974), Dasgupta and Stiglitz (1976) and Hoel (1978), while Dasgupta, Heal and Majumdar (1977) and Kamien and Schwartz (1978) have permitted the development process to be controlled endogenously.

Let X_t be the size of the natural resource stock on hand at time t and suppose T corresponds to the random time at which the perfect substitute becomes available. If the substitute can be produced then on at a unit cost of k and if $X_T = x$, the planner's problem on $[T, \infty)$ is to determine the substitute production rate $s_t \varepsilon [0, \bar{s}]$ and the resource consumption rate $c_t \varepsilon [0, \bar{c}]$, $t \geqslant T$, so as to

$$\text{maximize} \quad \int_0^\infty e^{-\alpha t}[U(c_t + s_t) - k\, s_t] \text{ subject to} \int_0^\infty c_t dt = x.$$

Let W(x) be the optimal value of this program, given $X_T = x$. Then, as in (2), the dynamic programming argument yields the following optimality equation

$$(22) \qquad \alpha\, W(x) = \underset{c,s}{\text{Max}} \left\{U(c + s) - ks - cW'(x)\right\}, \ x > 0$$

with

$$\alpha\, W(0) = \underset{s}{\text{Max}} \left\{U(s) - ks\right\}.$$

It can be shown that the optimal value function W(x) is concave increasing in x. Optimal consumption and production rates $c^*(x)$ and $s^*(x)$ are then obtained as the maximizers in (22) and can be characterized as follows. If $W'(x) < k$ (i.e. the imputed price of the resource is less than the cost of producing the substitute) then $s^*(x) = 0$ and $U'(c^*(x)) = W'(x)$, so that $c^*(\cdot)$ is increasing. As the resource stock depletes over time, consumption rate decreases and shadow price rises at rate α (as in Section 3) until $U'(c^*(x)) = W'(x) = k$. At that time the resource stock is just exhausted and the optimal production rate s^* then on is determined by $U'(s^*) = k$ with $W(0) = [U(s^*) - ks^*]/\alpha$.

Prior to the development instant T, the control variables are c (the resource consumption rate) and e (the R & D expenditure rate), the former depletes the resource and the latter increases the rate $\lambda(e)$ of discovery of the substitute. The resulting optimal value function V then satisfies the following specialization of the optimality equation (1).

$$(23) \qquad \alpha V(x) = \underset{c}{\text{Max}}\left\{U(c) - c\, V'(x)\right\} + \underset{e}{\text{Max}}\left\{-e + \lambda(e)\ [W(x) - V(x)]\right\}$$

Given that $W(\cdot)$ is concave increasing in x, it can be shown that $V(\cdot)$ also has the properties. Moreover, since the event of a substitute discovery is a beneficial one, we have $W(x) > V(x)$; otherwise it is clear from (23) that $e^*(x) \equiv 0$. Moreover, it turns out that $W'(x) < V'(x)$, so that the marginal value of the resource falls after the discovery of the substitute.

Prior to the substitute development, $c^*(x)$ satisfies $U'(c^*(x)) = V'(x)$, while after T we have $U'(c^*(x)) = W'(x)$. Concavity of U, V and W implies that $c^*(x)$ is increasing in x, before as well as after the development. Moreover, since $W'(x) < V'(x)$, it follows that, given the same stock size x, $c^*(x)$ prior to the discovery is smaller than after the discovery, i.e. the conservation motive is stronger prior to the discovery, as to be expected.

The optimal R & D expenditure rate $e^*(x)$ is a maximizer of the second term on RHS of (23), which is similar to (20) of the previous section. Since $\lambda(\cdot)$ is increasing and $W'(x) < V'(x)$, as in the previous section, it follows that $e^*(x)$ is decreasing in x.

Similarly, one can show that

(24) $$A_{c^*,e^*}V'(x)/V'(x) = \alpha$$

i.e. given that the substitute has not been discovered by t and that $X_t = x$, the price is expected to rise at the rate of discount, as before. At the time of invention of the substitute if $X_T = x$, the price falls from $V'(x)$ to $W'(x)$. Then it rises again at rate α until it becomes $k = W'(0)$ and stays at that level from then on.

Thus, as the resource level falls over time, the stock price rises, the consumption rate is reduced and the intensity of the R & D activity is increased until a substitute is discovered. At that time the stock price drops, consumption rate is increased and R & D expendutures become unnecessary. From then on the price rises, only the resource is consumed until it is exhausted, at which point the price equals the cost of producing the substitute. In the final phase, the constant rate of consumption is sustained only through the substitute production.

7. REMARKS

We have presented a general model of natural resource decisions that involves uncertainty regarding the occurrence of some significant event of interest. The consumption rate decision depletes the resource stock and the exploration rate decision expedites the occurrence of a favorable event. Dynamic programming was employed to characterize the optimal value function, optimal decision policies and the behavior of prices. The model was then specilized to the analysis of three cases involving the events that are most unfavorable (exhaustion), somewhat favorable (discovery of a new stock) and most favorable (development of a substitute). The analysis was mostly heuristic and the emphasis was on intuitive arguments and interpretations rather than on the technical details involved. The related literature was also reviewed within the context of the general model and its three special cases.

The model could be extended along two significant directions. It may be important to allow for the possibility of occurrence of multiple random events, such as a sequence of discoveries of new stocks or a sequence of partial substitutes developed. Secondly, the probabilistic rate of occurrence of the events should depend not only on current decisions but also on some aspect of the past history (such as the time elapsed, the cumulative amount of stock discovered or the cumulative R & D expenditures), which may expedite or delay the occurrence of the event.

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The Depletion of U. S. Petroleum Resources: Econometric Evidence

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Introduction

Econometric models of exhaustible resource supply have tended to focus on characterizing the near term path of supply of the resource. Depletion, if it is captured at all, has been impounded in a time trend. This leads inevitably to unsatisfactory long term forecasts; no credible estimate of ultimate resource recovery is available from such models. By contrast, geological resource base assessments result in estimates of ultimate recovery without a characterization of the time path by which this ultimate recovery will be realized.

We have developed an econometric modeling strategy capable of integrating the problem of estimating ultimate recovery and the problem of forecasting the time path of recovery. The dependence of both ultimate recovery and the time path of recovery on economic variables is captured by the model. Results of applying this methodology to exploration and development of U.S. oil and natural gas reserves are discussed in this paper.

Overview of Modeling Approach

In this section we provide a brief summary description of the derivation of our model. More detailed discussion of our modeling strategy and its application to oil and natural gas supply may be found in Epple and Hansen (1980a, 1980b).

We view the producer of exhaustible resources as choosing the time path of exploitation of the resource to maximize the expected after-tax net present discounted value of the resource. The cost function for the resource is assumed to depend on both current and cumulative exploitation. A quadratic specification for this cost function is adopted to permit formal derivation of the econometric equations from the producer's objective function. Uncertainty enters this problem in two ways. Uncertainty about extraction cost is represented by a random shock to the cost function. Future prices are also assumed to be uncertain, and the producer is assumed to use past and present data to forecast future prices. Forecasting equations for prices are thus a part of the model.

Application of this approach to oil and natural gas exploration and development is accomplished as follows. New discoveries result from exploration decisions made by oil and natural gas producers. Price forecasts and exploration costs affect the exploration decision. The producer is assumed to form expectations of future oil and natural gas prices based on past and present information including past and present oil and natural gas prices. Exploration costs are assumed to depend on current and cumulative discoveries as well as random shocks. The resource exhaustion feature of the model derives from the appearance of cumulative discoveries in the exploration cost function which is expressed as follows:

1. $$[\Delta y_{o,t}, \Delta y_{g,t}] \left\{ \begin{bmatrix} \phi_{oo} \\ \phi_{gg} \end{bmatrix} + \begin{bmatrix} \theta_{oo} & \theta_{og} \\ \theta_{og} & \theta_{gg} \end{bmatrix} \begin{bmatrix} \Delta y_{o,t} \\ \Delta y_{g,t} \end{bmatrix} + \begin{bmatrix} \pi_{oo} & \pi_{og} \\ \pi_{go} & \pi_{gg} \end{bmatrix} \begin{bmatrix} y_{o,t} \\ y_{g,t} \end{bmatrix} + \begin{bmatrix} \mu_{oo,t} \\ \mu_{gg,t} \end{bmatrix} \right\}$$

In equation (1), $\Delta y_{o,t}$ and $\Delta y_{g,t}$ are current oil and natural gas discoveries while $y_{o,t}$ and $y_{g,t}$ are cumulative discoveries. Hence $\Delta y_{o,t} = y_{o,t} - y_{o,t-1}$. Random components are $\mu_{oo,t}$ and $\mu_{gg,t}$. There are nine parameters in equation (1). The off-diagonal terms θ_{og}, π_{og}, and π_{go} permit interaction between oil and gas in the determination of exploration costs. This gives rise to the potential for "directionality" in the exploration process. The producer determines the amount of exploration to undertake to maximize discounted expected after tax profits using the price forecast and exploration cost equations.

Exploration adds to the inventory of known reservoirs. These increases in the cumulative amounts discovered feed back via the cost function in (1) causing an upward shift in the exploration cost function for subsequent periods. The inventory of known reservoirs serves as the input to the development process. As with exploration, development decisions are affected by price forecasts and the costs of development. The cost functions for oil and gas reserve additions are:

$$\text{a)} \quad \Delta a_{o,t,\tau} \left[\emptyset_o + \frac{\theta_o}{\Delta y_{o,\tau}} \Delta a_{o,t,\tau} + \frac{\pi_o}{\Delta y_{o,\tau}} a_{o,t,\tau} + \xi_{o,t,\tau} \right]$$

2.

$$\text{b)} \quad \Delta a_{g,t,\tau} \left[\emptyset_g + \frac{\theta_g}{\Delta y_{g,\tau}} \Delta a_{g,t,\tau} + \frac{\pi_g}{\Delta y_{g,\tau}} a_{g,t,\tau} + \xi_{g,t,\tau} \right]$$

Note that current oil reserve additions $\Delta a_{g,t,\tau}$ and cumulative oil reserve additions $a_{g,t,\tau}$ affect the cost of oil development. Subscript t denotes the current date while τ denotes the date of reservoir discovery. The random component of oil development cost is $\xi_{o,t,\tau}$. The cost of development per unit of reserve additions is an inverse function of the amount discovered in a particular year, $\Delta y_{o,\tau}$. Thus, the appearance of $\Delta y_{o,\tau}$ in (2) links development cost to the output of the exploration process. The development cost function for natural gas (2b) has exactly the same form as for oil. Note too that equations (1) and (2) are quite similar in form. The difference in the functional forms of the exploration and development cost expressions is the absence of interaction terms between oil and gas in the cost functions for development. The output of the development process is proved reserves. The amount of proved reserve additions feed back to affect future development costs via the cumulative component of cost in (2a) and (2b).

The remainder of the model is relatively mechanical. Production is presumed to be proportional to proved reserves, and the unit cost of production is assumed to be constant in real terms.

Parameters in the econometric model arise from two sources. First, there are parameters in the cost functions (1) and (2). The other source of parameters are the equation by which producers forecasts future oil and natural gas prices. In implementation of the model thus far, we have assumed that producers employ static price expectations. As a result, no additional parameters are introduced from the price forecasting equations.

It is useful to discuss the development model first. Maximization of expected discounted profits leads to the following equation for reserve additions:

$$3. \quad \Delta a_{t,\tau} = \left(\frac{\psi_1}{\sqrt{\beta}} - 1 \right) a_{t-1,\tau} + \frac{(1-\beta)\Delta y_\tau}{\psi_o(1-\psi_1\sqrt{\beta})} \left\{ \frac{(1-\epsilon)}{(1-\epsilon\,\beta)} [p_t T_{r,t} - c_p T_{p,t}] - \emptyset T_{d,t} \right\} + \frac{\xi \Delta y_\tau}{\psi_o T_{d,t}}$$

The form of the development equations for oil and gas are the same. Therefore, only a single equation is presented in (3) to avoid the redundancy of presenting two equations differing only in subscripts. Parameters ψ_o, ψ_1, and $\emptyset$ are functions of the cost function parameters in (2). The discount factor is β, and tax parameters are $T_{r,t}$, $T_{p,t}$, and $T_{d,t}$. Unit production cost is c_p and the ratio of production to reserve additions is $(1-\epsilon)$.

The exploration model derived from producers optimization problem is:

$$4. \quad \Delta y_t = \left[I - \frac{\underset{\sim}{\psi}_1}{\sqrt{\beta}} \right] y_{t-1} + (1-\beta)\, \underset{\sim}{\psi}^{-1}\, T_{xt}^{-1}\, (I - \sqrt{\beta}\, \underset{\sim}{\psi}_1')^{-1} \cdot (q_{x,t} - \underset{\sim}{\emptyset}\, T_{x,t}) + \underset{\sim}{\psi}_o^{-1}\, T_{x,t}^{-1}\, \mu_t$$

In equation (4), Δy_t is a vector, $\Delta y_t = [\Delta y_{o,t}, \Delta y_{g,t}]$, as is y_t. $\underset{\sim}{\psi}$ and $\underset{\sim}{\psi}_1$ are two-dimensional square matrices of parameters and $\underset{\sim}{\emptyset}$ is a two-dimensional parameter vector. A (~) beneath the parameter matrices and vectors is used to distinguish the parameters of the exploration model from the scalar parameters in the development model. $\underset{\sim}{\psi}_o$ and $\underset{\sim}{\psi}_1$ are functions of the parameters in equation (1). In this equation, $q_{x,t}$ is the after tax discounted present value of revenues per unit discovered net of production and development costs.

The econometric equations are (3) and (4). Equation (3) is estimated separately for oil and natural gas reserve additions. The equations for oil and natural gas discoveries in (4) are interdependent and are estimated jointly. The results of estimating these equations are presented in Epple and Hansen (1980). Work on estimation of the model is continuing and results available to date are still somewhat tentative.

Equations (3) and (4) can be used to simulate future time paths of reserve discovery and development. When $a_{t-1,\tau}$ is added to both sides of equation (3), the equation is converted into a linear difference equation in cumulative reserve additions. Similarly, when y_{t-1} is added to both sides of (4), it is converted into a linear difference equation in cumulative discoveries. By solving these difference equations and taking the limit as $t \rightarrow \infty$, we obtain equations for forecasting ultimate resource recovery.

While the obvious use of these equations is to forecast ultimate recovery, they also have a second and perhaps more interesting application. Estimates of ultimate recovery are available from a variety of sources. Rather than simply comparing our forecasts to those made by others, we can formally test whether forecasts made by others are significantly different from those resulting from our model. To accomplish this, we impose the restriction that the parameters estimated by our model be consistent with the ultimate recovery predictions to which comparison is being made. We do this by setting our expression for ultimate recovery equal to the estimates of interest.

In the case of oil and natural gas, two restrictions result because there are two resources. Imposition of these restrictions reduces by two the number of free parameters to be estimated. By estimating our model with and without the resource base constraints, we obtain separate likelihood function values. A likelihood ratio test can then be employed to determine whether the constraints significantly reduce the quality of fit of the model.

Results

The procedure outlined above has been applied using the resource base estimates provided by the U.S. Geological Survey. U.S. Geological Survey Circular #725 contains resource base estimates for the United States for 1974. We have estimated our model subject to the restriction that ultimate recovery be consistent with mean values estimated by the USGS. In addition, we consider a high estimate in which the mean USGS values are doubled and a low estimate equal to one half of the mean USGS estimates.

Resulting likelihood and chi-square values are presented in Table 1. Both the high and mean estimates are rejected at a high level of significance. The low oil-low gas case is not rejected at the 5 percent level but is rejected at the 10 percent level. The final result in the table tests the low oil-mean gas case. This proves to yield a likelihood function value quite close to the unrestricted case.

These results suggest that the low oil and mean natural gas values of the U.S. Geological Survey are quite close to those estimated by our econometric model. We emphasize that these results should be

TABLE 1

TESTS OF RESOURCE BASE RESTRICTIONS
FOR CRUDE OIL AND NATURAL GAS IN THE U.S.

Resource Base Assumption*	Likelihood Function Value	Chi-Square Value**
Unrestricted	.18	
High Oil, High Gas	-11.49	23.34
Mean Oil, Mean Gas	-7.02	14.40
Low Oil, Low Gas	-2.24	4.85
Low Oil, Mean Gas	-0.06	.48

*High is double the mean USGS value.
Low is half the mean USGS value.

**The χ^2 statistic with two degrees of freedom at the .05 significance level has the following percentile values:

Percentile	Value
10	4.61
5	5.99
1	9.21
.5	10.60

considered somewhat tentative as we are continuing work on estimation of our model. However, the results demonstrate that our approach is operational. We believe that it provides a promising avenue for integrating econometric analyses and geological resource bas estimation.

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DISCUSSION

DR. KAUFMAN: Dennis, in particular with respect to gas, the recent change in world price structure and following change on the domestic market has instigated a rather enormous burst of drilling activity to leak gas, because of a rent window that this affords within our tax structure, so you have enormous rates of drilling in the Tuscalossa trend, in the overthrust belt, and elsewhere, which looks as if it's going to discovery very, very significant amount of gas that will be brought on line. There are typical sections which have 37 trillion cubic feet in place, reminding that you consume 20 trillion cubic feet a year, currently -- 18 to 20.

Where does that appear in the context of this kind of model? One would expect that what's going to happen is you'll get bumps or peaks as a result of that impetus, and then if the time rate of production of gas -- how do you get that into the model?

It's certainly not this kind of phenomenon, which is an interaction of economics and geology. Certainly it's not in the Geological Survey estimates. Circular 725 came out circa 1975-1976.

MR. EPPLE: In our model the amounts of oil and natural gas ultimately recovered depend upon their prices. We are thus estimating equations which can be used to generate schedules of oil and natural gas recovery as functions of prices. When we impose restrictions during estimation, those restrictions require that the price-ultimate recovery schedules go through specified points. For example, in one case, we impose the U.S. Geological Survey mean estimates of oil and gas recovery as restrictions during estimation. Those restrictions assure that our model will predict mean oil and gas recovery as specified by the USGS at the prices assumed by the USGS. The USGS estimates are based on prices prevailing in 1974. Hence, for this particular case, our price-ultimate recovery schedules yield mean USGS ultimate recovery estimates at 1974 prices. If higher prices are substituted into our model, then ultimate recovery values predicted by our model will be higher as well.

Our model does not simply predict ultimate recovery. It also predicts the time rate of discovery of new reservoirs, the time rate of development of proved reserves in those reservoirs, and the rate of production from proved reserves. If a jump in price occurs, then a bulge in discoveries will be predicted by the model. That bulge will work its way through the development and production components of the model in subsequent years.

In my presentation, I have emphasized the ultimate recovery predictions of the model. However, our model provides an integrated treatment of the time path of recovery and the problem of predicting ultimate recovery. Both the time path of recovery and ultimate recovery depend on prices. The link between geology and economics that we attempt to make is, as I've indicated above, the use of USGS estimates in fixing particular points on the ultimate recovery schedule generated by our model.

MR. COMER: I'm Dan Comer from EPA. I'd like to suggest that you look at the financing, as well as the other scenarios. And why don't you get bigger bumps?

MR. EPPLE: Well, I'm not quite sure in what sense we're missing them. I've shown you what the historical data are that we used for estimation. The model has been fitted to that data, and it fits quite well the large bumps in the historical senses.

MR. COMER: You alluded to a steady state solution for the model.

MR. EPPLE: Ultimate recovery predictions from our model depend on the values to which ultimately converge as the limit is taken with respect to time. Hence, the transient characteristics of the model do not matter for purposes of determining ultimate recovery. The time path of recovery is, of course, dependent on the actual path of prices. Simulations of the model to generate paths of discovery, reserve development, and production, are based on specific price paths, and those simulations do reflect any transient responses that result from price changes.

OIL AND GAS FINDING RATES IN PROJECTION OF FUTURE PRODUCTION

W. L. Fisher[1]

Future levels of production of crude oil, natural gas, and natural gas liquids are critical elements in domestic energy supply forecasts. These domestically produced commodities constitute more than 50 percent of our total energy supply and some 65 percent of our total domestic production of energy. The U.S. currently produces about 19.4 mmboed of oil, gas, and NGL.

Oil production amounts to 8.5 mmbd, or 44 percent of the total; NGL, 1.6 mmbd, or 8 percent; and natural gas, 19.7 TCF, or 48 percent. This production comes from a year-end, proven reserve base of 27.1 billion barrels of oil and 195 TCF of natural gas. At the current level of contribution, slight percentage changes in future levels of production translate into significant supply volumes. Production of these commodities is so much a part of our total supply that a difference of one percentage point in average annual decline rate amounts to 2.0 mmboed by 1990, or a total of some 4 billion barrels of oil and oil equivalent.

Yet despite the critical importance of future production levels, there is a significant range in forecasted production over the next decade. If one surveys some 14 separate projections made over the past 2 years, including 7 projections made by oil companies and 7 made by government, academic, and other non-industry entities, one sees the following range in 1990 levels of projected production: Crude oil and liquid levels range from 7.2 to 11.5 mmbd, or from 29 percent less than last year to 14 percent more. Average of the forecasts was 9.6 mmbd for 1990. The range of estimates, as well as averages, was exactly the same for industry and non-industry projections.

The ranges in natural gas forecasted production by 1990 are 14.3 to 17.9 TCF, or a decline of 9 to 27 percent from last year's production; average of the various forecasts was 16.3 TCF. Industry forecasts ranged from 14.3 to 17.6 TCF, with an average of 15.9 TCF; non-industry forecasts were 16.3 to 17.9 TCF, with an average of 16.7. The average of the various oil and liquid production forecasts indicates an average annual

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decline through 1990 of about 0.5 percent; this compares with the actual average annual decline from 1973 through 1979 of 1.8 percent, or 3.7 percent if North Slope Alaska production is excluded. The average annual decline of natural gas production, as shown by the average of the several forecasts, is 1.5 percent; this compares with the 1977-79 average annual decline of 2.3 percent.

One may note that the range between low and high estimates for oil and liquids is a rather wide 60 percent. The range in forecasted natural gas production is significantly tighter, about 25 percent.

Nearly all the forecasts assume continued increases in total drilling activity with an average annual increase (AAI) of 6 to 7 percent through the 1980's; most assume pricing and taxing policy to be essentially the same as that now in place; and nearly all assume that North Slope Alaska gas production will be on stream in 1990. But beyond these assumptions, variations in forecasts hinge on three main variables: (1) degree of optimism relative to potential major discoveries in the frontier areas, notably onshore and offshore Alaska, as well as availability of the lands to exploration and development and necessary lead times; (2) assumptions relative to unconventional oil and gas production--be it synthetics, tertiary recovery, or unconventional infill drilling; and (3) assumptions as to future rate of finding and its behavior.

Variations in projected unconventional recovery, including synthetics, were not very great nor significant in the forecasts. The higher forecasts, especially for oil production by 1990, generally assume significant additional production from frontier areas. If the volumes based on these assumptions are backed out, the spread in range of oil forecasts becomes about 30 percent, approximately that shown for ranges in natural gas production. Accordingly, one may infer that the range in forecasts from low to high of 25 to 30 percent is largely attributable to assumptions relative to oil and gas finding rates over the next decade.

With more than 50 years of statistical record of finding in most of the U.S. non-frontier basins, and with a reasonably sophisticated understanding of the geology and hydrocarbon occurrence of these basins, one might assume a bit more precision in our ability to calculate reserve additions as a function of the level of future drilling. Unfortunately, this appears not to be the case, and although several efforts in calculating and projecting oil and gas finding rates have been made, we can certainly say that no consistent methodology has been obtained.

Variations mainly include: (1) how finding rate is calculated, that is, what kind of drilling should be related to what kind of reserve additions; (2) how the calculated values are interpreted, such as, are changes related to changes in the resource base or are they a function of changes in drilling mix and exploratory targets; and (3) finally, of course, how finding rates are projected into the future.

Basically, finding rate is nothing more than hydrocarbons discovered as a function of drilling. But what drilling and what reserves added? Drilling, as classed by The American Association of Petroleum Geologists (AAPG), consists of exploratory drilling, including highest risk drilling new field wildcats (NFW) along with outposts, new-pool wildcats, deep-pool tests, and shallow-pool tests, and developmental drilling. Additional minor classes of drilling include service wells and stratigraphic tests. The other half of the equation: reserve additions are subdivided into revisions, extensions, new field discoveries, and new reservoir discoveries in old fields.

A common simple calculation of finding rate involves dividing footage into volumes of oil and gas added as reserves. In some cases, this involves dividing total footage of all drilling into total reserve additions, including revisions, extensions, and discoveries. In other cases, barrels per foot have been calculated, leaving out revisions. Other calculations have considered total annual reserve additions, with or without revisions, as a function of total annual exploratory and/or new field wildcats drilling. Still other calculations consider only new field discoveries as a function of new field wildcats drilling. Obviously any combination of drilling and reserve additions will give a statistical value expressed in barrels per foot, and when plotted give historical trends in the values. It is important to relate appropriate classes of reserve additions to appropriate classes of drilling, to appreciate growth factors in new discoveries and the time lag in additions as a function of development drilling.

What is obvious in even casual observation and calculations is that decrease in discovery per unit of drilling began as early as the early 1950's, was subsequently followed by a decline in drilling in the late 1950's, followed by a decline in reserve additions, then a decline in proven reserves, and in the early 1970's, a decline in production.

Since about 1950, drilling additions of oil have been steadily declining (Fig. 1); this trend has been due to both declines in drilling and declines in reserve additions per increment of drilling. Only in the past 7 years, with a steady upsurge in drilling, have total drilling additions begun to level off. One component of reserve additions--that attributable directly to wildcat drilling--has remained relatively stable since the early 1950's (Fig. 2). Reserve additions from other exploratory drilling stayed reasonably stable from the early 1950's to the early 1970's. Since 1972, additions from that component of drilling, as measured on a per well or per foot basis, have declined sharply. Additions as a direct function of development drilling have likewise declined, and, although for years much higher than for wildcat drilling, have now dropped below new field wildcat drilling on a per well or per foot basis. Oil revisions have played a changing historical role in contribution to reserve additions (Fig. 1). From the late 1940's until about 1960, revisions represented about one-third of total reserve additions. Since 1960, oil revisions have contributed between 50 and 65 percent of total additions. In the early 1960's, significant increases in revisions, due in my judgment to active development in the offshore Gulf of Mexico and certain secondary recovery projects on land, actually raised total additions. Since the middle 1960's, revisions have been in decline and in particularly sharp decline since 1970.

Associated gas has behaved, as expected, in much the same fashion as oil. The situation with non-associated gas has been somewhat different (Fig. 3). Sharp declines in total additions as well as exploratory additions occurred in the late 1960's. Since then exploratory additions have remained relatively stable, with sharply increased drilling since 1972 generally sufficient to offset declines on a per well or per foot basis. Development drilling additions have declined, and revisions have fluctuated widely.

If one calculates new field discoveries of total hydrocarbons as a function of new field wildcats, one sees a relatively stable rate of finding over the past 15 to 20 years (Fig. 4). By contrast, if one considers non-wildcat drilling and additions other than new discoveries, one sees a steady deterioration in rate of finding.

With increased prices for domestic oil and gas that began in the early 1970's, there has been a strong upsurge in total drilling effort. Nationwide that increase has been greater for development and non-wildcat drilling than for wildcat drilling. This has led some to argue that the reduced finding rate experienced during most of this decade is attributable to the relatively greater increase in lower yielding, but less risky, non-wildcat drilling, and further that a significant number of previously known small prospects, uneconomic at lower prices, have been appropriate targets, at the expense of true high-risk wildcat drilling. Certainly the increased success ratio of both oil and gas wells in all areas of drilling, in the face of declining finding rates, would support this point. But do the shifts tell the whole story?

The crux of the situation is this: (1) Are we facing a lower quality resource base and will recent trends in finding persist into the future?, or (2) Are recent trends anomalously low, and if used as a basis for formal projecting, do they understate future reserve additions? How one interprets and judges these data becomes significant in projecting future oil and gas additions as a function of drilling.

While the recent shift in drilling mix has had some effect on overall finding rate, I doubt that it has been that significant. For one, the rate of finding for non-wildcat drilling, while declining rapidly, has been on the average only about 1 barrel per foot lower than wildcat drilling over the past few years. Further, in areas of the country where intrastate demand has been high and where intrastate prices for gas were at market rates during the 1970's, the percentage of wildcat drilling of total drilling has increased over a comparable period of time preceding increased prices. Previously known small prospects may have indeed been drilled, but these were wildcats and previously untested nonetheless.

Statistics on average field size are difficult to develop for fields discovered over the past 6 years. But, longer term trends, established long before the price increases of the early 1970's, do indicate that the average size of fields and drillable prospects is decreasing. Accordingly, the quality of the resource base is declining as measured by rate of finding, even though the total volume of estimated and discovered oil and gas is still substantial.

Another aspect pertinent to finding rate calculation and interpretation is that of reserves growth, that is, the ultimate reserves of a field compared with reserves based on initial discovery. A number of statistical calculations have been made, and while the calculated volumes of reserves growth vary, these data indicate that the growth potential of larger fields, through subsequent non-wildcat drilling, was, and is, significantly greater than for smaller fields, and that the growth factor for fields older than 48 years (i.e., larger fields) is greater than for more recent smaller fields.

A recent analysis by Professor John Haun, Colorado School of Mines, is worth note. Professor Haun made an extensive calculation of reserves growth, calculating ultimate growth of fields discovered this century. He then calculated rate of finding by crediting calculated ultimate reserve to discovery wildcat drilling. His plot of finding rates for oil, gas, and total hydrocarbon shows steady declines since the late 1940's (Figs. 5-10). As calculated, total ultimate barrels of oil and oil equivalent per foot of wildcat drilling steadily declined from 352 barrels per foot in the late 1940's to 53 barrels per foot in the late 1970's. His projections put finding rate of 23.5 barrels per foot in 1990. His method of calculation tends to rule out such effects as changes in drilling mix in that all ultimate reserve is credited to original discovery.

Conclusions

1. The business of oil and gas projection is something less than an exact science even when reduced to the statistical components.

2. Variation in assumption of future rates of finding has significant impact on projected volumes of reserve additions and production per unit volume of drilling.

3. Nevertheless, in my judgment, the facts are sufficient to indicate that recent rates of finding are valid for future projection in that they are consistent with long-term declines, basically the result of decrease in size of prospect targets, and a decrease in growth potential through subsequent drilling. This is shown basically by the fact that while wildcat finding rate has remained stable, long-term finding rates of non-wildcat drilling are declining sharply.

4. If these trends in finding of different kinds of drilling are valid and relate primarily to a universe of smaller prospects and fields, we should pay particular attention to prospect- and field-sized populations. The distribution of size could have significant bearing on the behavior of finding rates through time. Decline through time may not be a continuous function, but rather periods of relatively stable, but progressively lower plateaus. This could be a positive aspect to the overall decline in oil and gas finding in mature basins.

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DISCUSSION

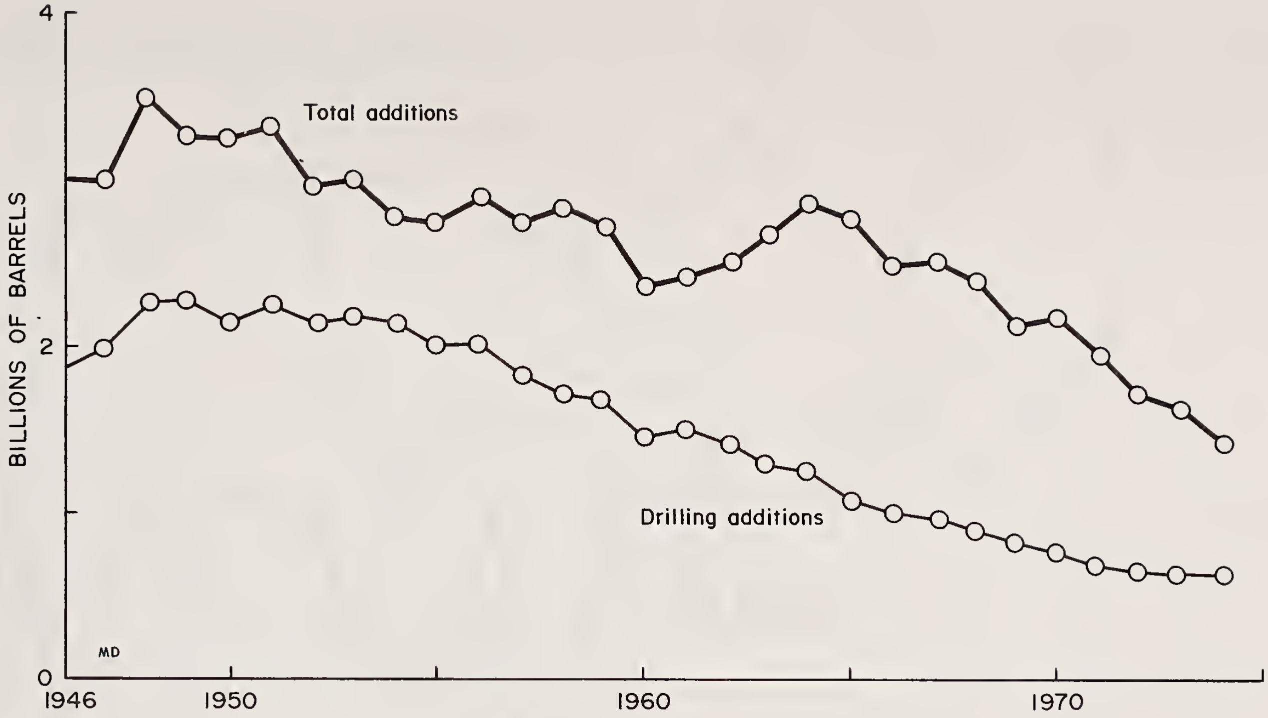
DR. GORDON EVERETT: One of the things I wanted to ask you about was the fact that you concentrated on the statistics that Haun had on a per foot basis. But if you take a look at some of those finding rates on a per well basis, because we're tackling the situation of having to continually look at deeper and deeper strata; therefore, that footage has got to increase on new finds. So you've got a factor in this per footage basis that tends to give some of these curves a somewhat increased downward trend versus plotting on the per well basis, and as an example, for instance, when we take a look at the drilling in the west side of the Williston Basin, several years ago, at 8,000 feet, the success ratio per well was fairly low--it was a fairly mature horizon, and we were largely drilling on the flanks of known fields.

When that shifted to 11,000 feet the first year, it was a 100-percent success ratio for wildcats, because it was a different horizon that hadn't been explored with a successful exploration model we had a high number of hits.

So I wonder if perhaps we shouldn't be taking a look at some of these things on a per well effort basis, rather than a footage basis, because of the effect that this footage basis is going to have on forcing this thing down continually.

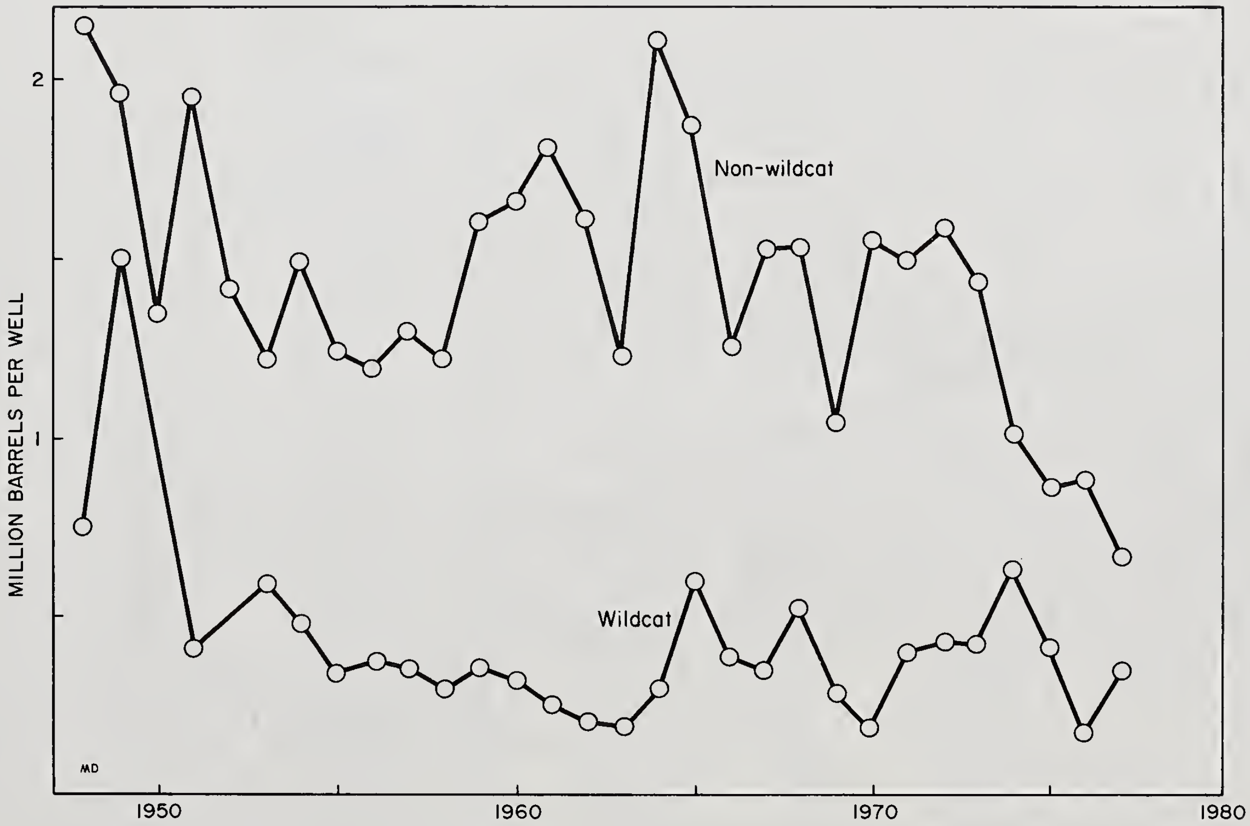
DR. FISHER: Gordon, I think that is a very good point. At this point the trends are quite similar irrespective of whether plotted on a per well or per foot basis.

I suspect though, particularly with the stimulus for deeper drilling, which we are seeing in a number of areas for gas, that there will be over the next 10 years more disparity in the trends on a per foot and a per well basis than there has been up to now.



Oil Reserve Additions, U.S. General Accounting Office, 1969.

Figure 1

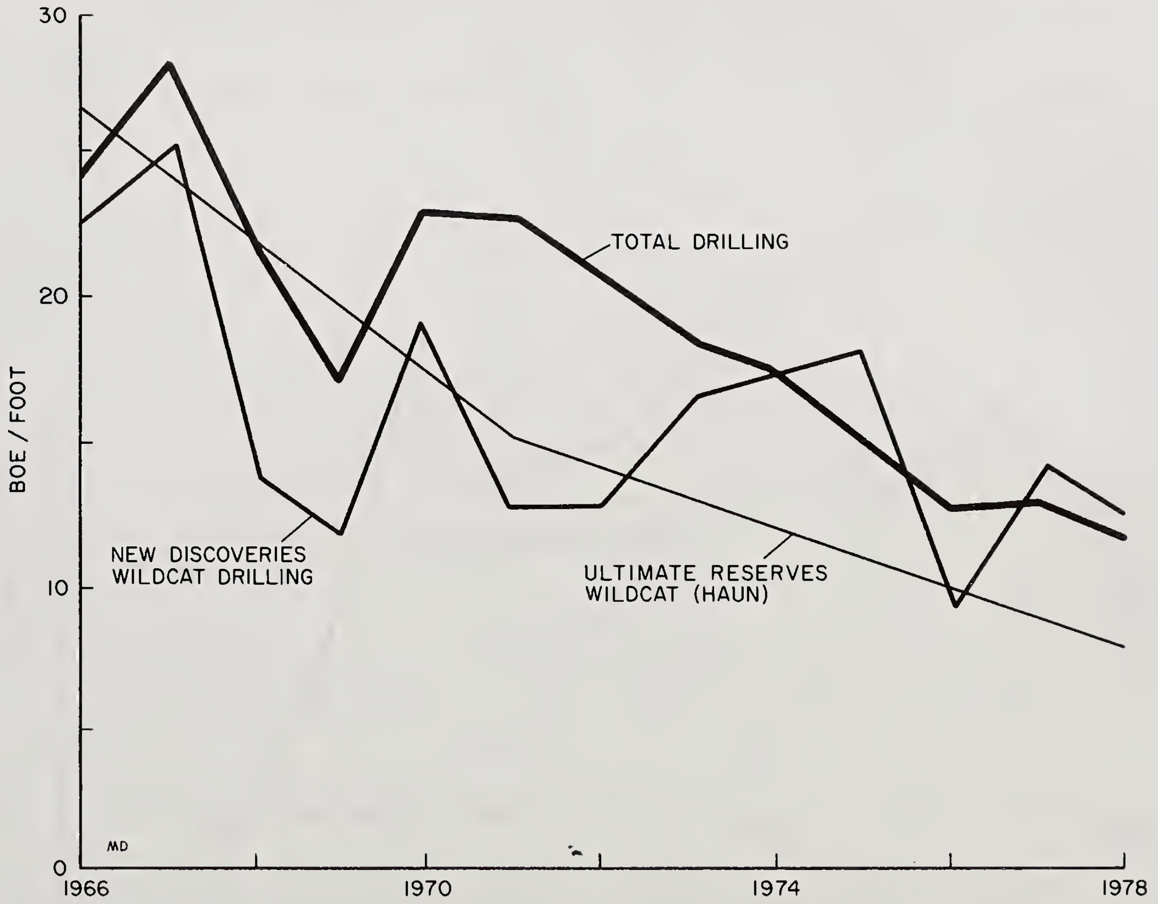


Average Oil Reserve Addition per Successful Exploratory Well, Wildcat versus Non-Wildcat Drilling, U.S. General Accounting Office, 1979.

Figure 2

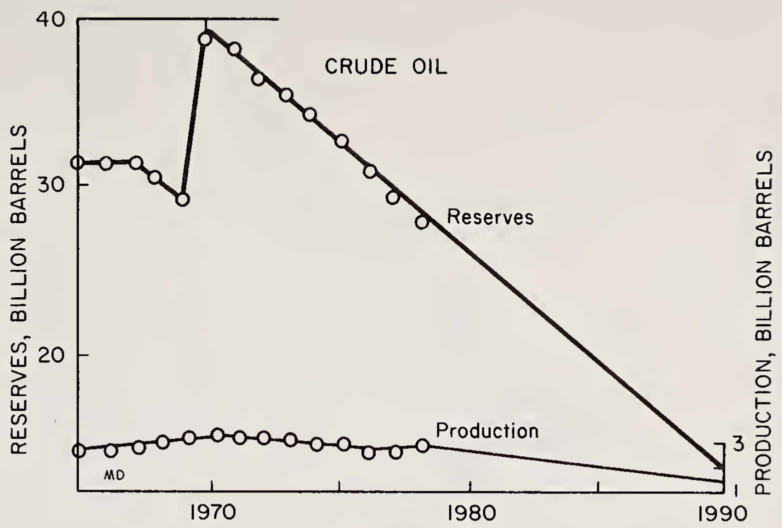
Reserve Addition of Non-Associated Natural Gas, in TCF,
U.S. General Accounting Office, 1979.

Figure 3



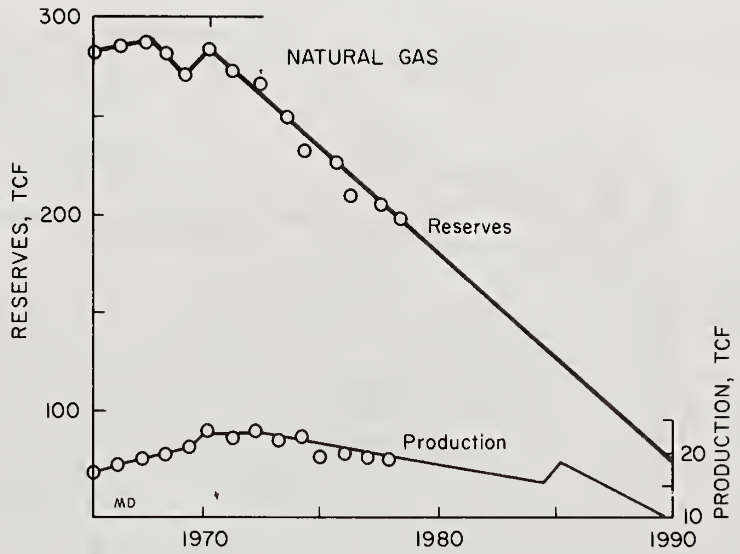
Finding Rates for Oil and Natural Gas, Barrels of Oil Equivalent Per Foot Drilled.

Figure 4



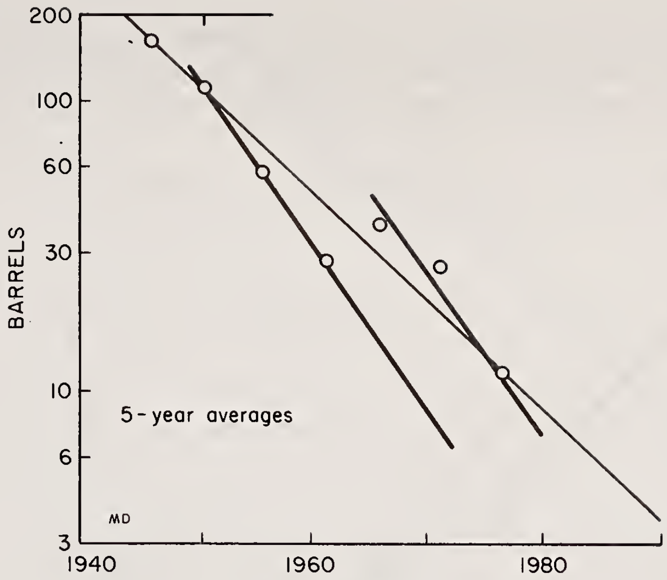
Trends in Annual Reserves and Production of Crude Oil, from Haun, 1979.

Figure 5



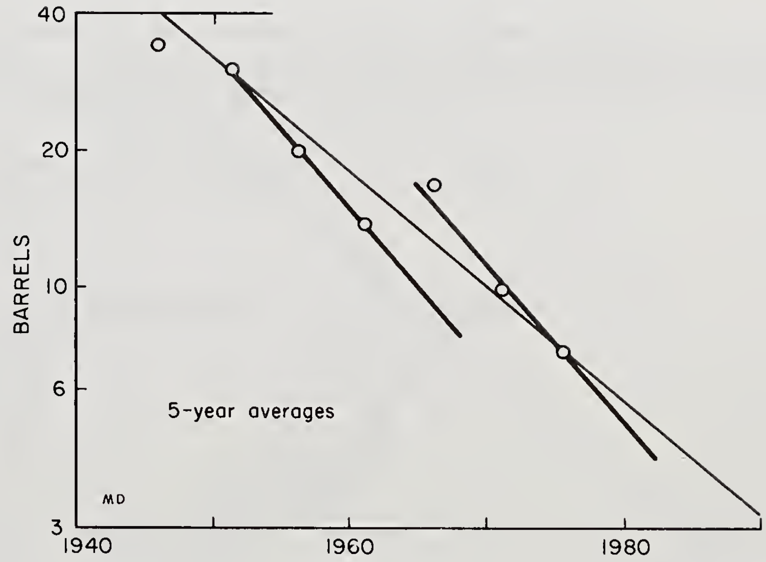
Trends in Annual Reserves and Production of Natural Gas, from Haun, 1979.

Figure 6



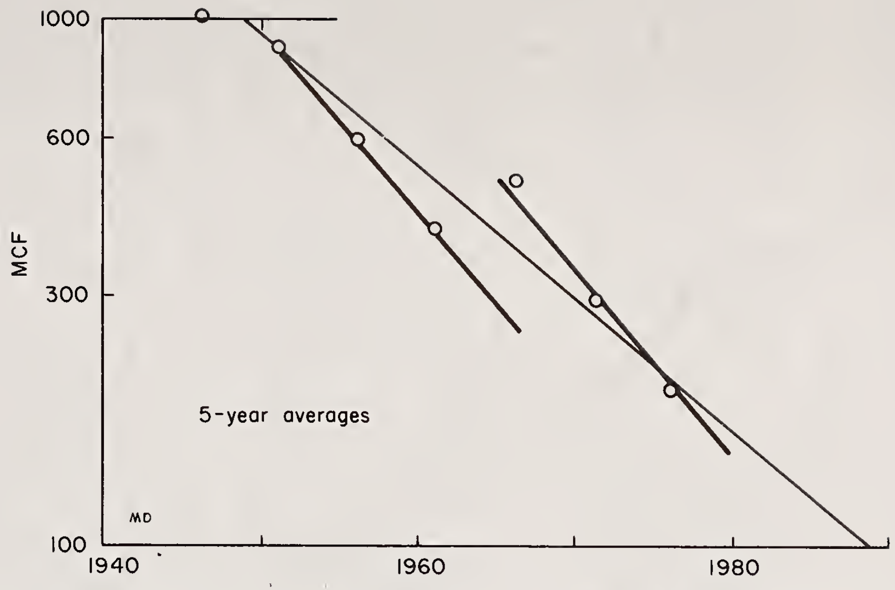
Crude Oil Discovered Per Foot of New Field Wildcat Drilling, Ultimate Reserve Added by Subsequent Development Drilling Credited to Original Discovery, from Haun, 1979.

Figure 7



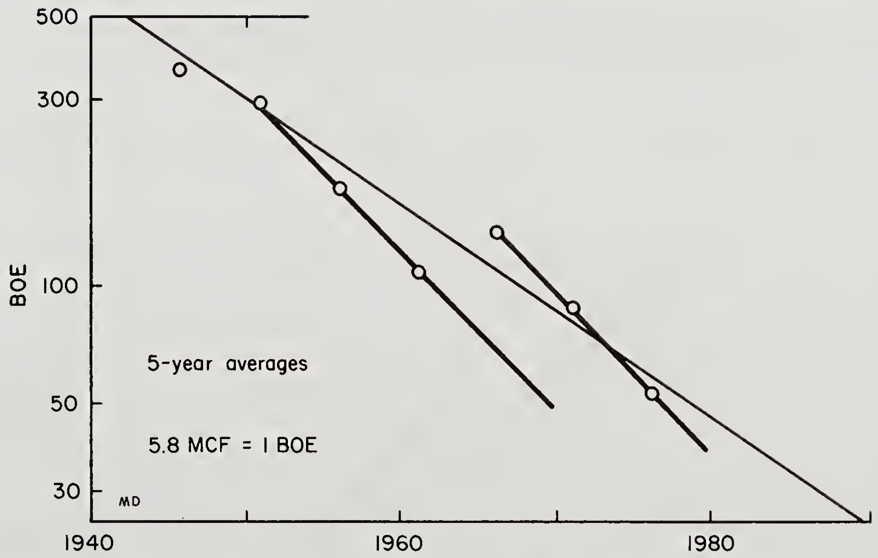
Natural Gas Liquids Discovered Per Foot of New Field Wildcat Drilling, Ultimate Reserve Added by Subsequent Development Drilling Credited to Original Discovery, from Haun, 1979.

Figure 8



Natural Gas Discovered Per Foot of New Field Wildcat Drilling, Ultimate Reserve Added by Subsequent Development Drilling Credited to Original Discovery, from Haun, 1979.

Figure 9



Barrels of Oil Equivalent (Oil, Natural Gas, and Natural Gas Liquids) Discovered Per Foot of New Field Wildcat Drilling, Ultimate Reserve Added by Subsequent Development Drilling Credited to Original Discovery, from Haun, 1979.

Figure 10

ISSUES IN FORECASTING CONVENTIONAL OIL AND GAS PRODUCTION

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"AN IDEALIST IS ONE WHO ON NOTICING THAT A ROSE SMELLS BETTER THAN CABBAGE CONCLUDES THAT IT WILL MAKE BETTER SOUP." - H.L. MENCKEN

1. INTRODUCTION

In the United States, petroleum (crude oil and natural gas) is being produced and used at a faster rate than it is being found. Since 1970, proven petroleum reserves, which include crude oil, natural gas, and liquids, have been consumed at an annual rate 4 percent higher than the rate at which they have been expanded. The most important question is: Will this trend continue, and if not, when and how will it change?

This paper addresses the issues in and approaches to forecasting conventional supply in the onshore Lower-48 states. These projections are critical since Lower-48 production is the "pivot point" for Government policy. After conventional Lower-48 production, the remaining supply from imports, unconventional sources, and frontier areas is strongly affected by Government policy. Further, the issues and approach to Lower-48 conventional production are basically different from those regarding other sources.

2. THE EXPLORATION-DISCOVERY-PRODUCTION PROCESS

Formation, Migration, and Entrapment of Petroleum

No one knows for sure how petroleum is originally formed. The petroleum formation process, like other occurrences of eons ago, rests on theory. Basically, two different theories attempt to explain the origins of crude oil and natural gas: the inorganic theory and the organic theory. The inorganic theory -- not widely accepted outside the Soviet Union -- holds that methane was inorganically formed deep in the Earth's crust, migrated upward, and accumulated in traps (reservoirs). The organic theory, for which a preponderance of evidence exists, is that living organisms were the starting point in petroleum's formation. As these organisms died, their remains were carried to and deposited in the sediment beds of ancient rivers and seas. Gradually over long periods of time and under appropriate temperature and pressure conditions, bacterial, chemical, and physical processes transformed these remains into petroleum (for more detail, see [66], chapter 11).

Rarely, however, is the concentration of petroleum in the source beds of rivers and seas sufficient for commercial viability. Large accumulations are believed to be the result of migration. Since petroleum is insoluble and not as dense as water, gravity causes it to partially separate from water and draws it from the source rocks through permeable sedimentary rocks toward the Earth's surface. This migration ends when the petroleum encounters impermeable rock, forming a trap, or when the petroleum reaches the surface, at which point the lighter fractions evaporate and the heavier fractions form a natural tar desposit called a seep.

There are two distinct types of geological traps, although most traps are a combination of the two. The first type, structural traps, are formed by abrupt changes in geology, caused by faulting, the intrusion of salt domes, or more gradual deformations, such as anticlines. The second type, stratigraphic traps, on the other hand, result from more subtle changes in rock permeability. In summary, the four necessary factors for commercial petroleum accumulation are: (1) Source sediments containing once living organisms, (2) Appropriate subsurface environmental conditions, (3) Migration opportunity, and (4) Existence of a trap formed prior to the migration.

Search for Petroleum Prospects

That petroleum occurs at surface seeps has been known since early recorded history. Petroleum seeps were often an important part of the religious, medical, and economic life of societies in many parts of the world. In the United States, E. L. Drake drilled the first intentional oil well in 1859 (the Chinese had been drilling them for centuries) near an oil seep, discovering an oil bearing anticline (a structural trap) at Oil Creek, Pa. He was offered $20 per barrel (in 1859 dollars).

For many years thereafter, decisions to drill were based on intuition and even dowsing rods were used. Eventually, surface geology was studied to assess subsurface potential. Geophysical methods -- indirect measurements of subsurface rocks -- were introduced around 1920. Today sophisticated equipment, including satellites and high speed computers, are used to gather and process huge quantities of information to aid exploration. These methods demand high levels of technical skills and substantial financial investment.

Even with all this highly developed exploration technology, two classical problems remain. The first is the identification of promising geological conditions, and the second is the determination of the presence of petroleum in a prospective trap. The detection of stratigraphic traps is especially difficult because there are only subtle changes in the subsurface geology. Moreover, the presence of a trap is only one of four necessary conditions for the existence of a petroleum reservoir. Drilling is the only conclusive test to determine if petroleum is available underground in commercial quantities. Once a prospect is identified the right to drill must be obtained through a lease.

The Leasing Process

The leasing process, beyond the standard legal rituals, is probably the facet of the mineral extraction industry that is least understood by energy analysts. Obtaining the rights to explore and drill for oil and gas is the job of a landman. This activity has developed to the point that a professional organization, the American Association of Petroleum Landmen, has been formed.

Since reservoirs and fields often extend over large areas with many landowners involved, a firm usually attempts to acquire a "position" by leasing as much land as possible in a specific area. A typical private lease contract specifies bonus and rental payments, requirements for exploration and production activity, and a royalty payment, if commercial production is established.

In "hot" areas (where activity is brisk), bonus and rental payments usually rise dramatically -- an indication that some of the "economic rent" is captured by the landowner. Otherwise, such fees are usually nominal. A standard royalty payment is one-eighth of the production, but can be negotiated. The contractual requirements concern the term of lease, which is automatically extended as long as the property is producing, and the obligation to drill and continue drilling to maintain the lease. In leases contracted recently, drilling usually must commence within a year and continue, with periods of inactivity of less than a year, until a producing well is found or the lease expires.

The U.S. Government has its own system of leasing onshore Federal lands. If the property is on a "known geologic structure (KGS)" it is leased by sealed bids, just as with offshore areas. If it is not on a KGS, only a $10 request to explore is filed, and when 2 or more requests are filed, a lottery is held to grant rights. (This process has been suspended recently because of suspected irregularities.) These leases are harder for oil and gas producers to acquire mostly because absentee leaseholders (who also do not live near the leased land) hold a large portion of the property leased in this manner. The United States Geological Survey (USGS) estimates that approximately 50 percent of the undiscovered petroleum remaining in the United States is on government land.

Exploration and Development

After surface exploration and lease acquisition, a well may be drilled. The drill bit resolves the most uncertainty in the process of exploration, discovery, and production. If drilling finds no petroleum, the well and the formation is declared dry. (Petroleum may have been present at one time, but it may have migrated through the area before the structural trapping conditions were formed.) If petroleum is discovered, commercial viability must be determined which involves determination of product quality and the capacity of the reservoir to produce. If the petroleum is not considered commercially producible, the well also is declared dry. Otherwise, it is declared successful.

Wells are classified initially according to the intentions of the driller into one of several categories (see Appendices B and C). Most risky of these is the new field wildcat, which historically has had about a 10 percent success rate. A field is a geographically distinct area consisting of one or more reservoirs, each characterized by its own pressure system. Next in level of risk are exploratory wells for new reservoirs and boundary extensions to existing reservoirs in already discovered fields. These have an historical success rate of about 25 percent. The last category is development wells; these are drilled within the existing boundaries of the reservoir, and have a success rate of more than 90 percent.

Development and Production

Once a new field has been discovered, development and production operations begin. Development wells are drilled and there is additional exploration to confirm and enlarge the proved area and discover new pools. Development wells (using the Lahee definition; see Appendices B and C) "exploit or develop a hydrocarbon accumulation discovered by previous drilling," and are successful more than 90 percent of the time. Production from newly discovered fields is often shut-in (i.e., not producing) initally, waiting for the installation of the equipment and facilities necessary for continuous production, including storage facilities and gathering pipelines. Initially, some oil wells produce because of natural forces (for example, gas pressure and water drive) in the reservoir. However, about 90 percent of producing oil wells eventually require artificial lift supplied by pumping. Usually less than 25 percent of the oil in the reservoir can be recovered in this primary stage. By contrast, gas wells have primary recovery rates of 60 to 80 percent or higher and do not require application of artificial production methods.

Secondary recovery techniques, applicable only to oil reservoirs, augment the drive mechanism of the reservoir by injecting water or reinjecting natural gas. About 460 billion barrels of oil have been discovered in the United States to date, but only about 33 percent is currently recoverable by primary and secondary methods. Tertiary, or enhanced, recovery processes seek to overcome natural forces trapping the remaining oil. These techniques include steam injection, in situ combustion, gas injection (other than natural gas), and surfactant/polymer and caustic flooding. It is estimated that between 8 billion and 50 billion barrels of the remaining 310 billion barrels of discovered oil could be recovered with tertiary techniques.

Regulation and Taxes

Regulation and taxes in the oil and gas industry have a long history dating from the early 1900's. Since the turn of the century, pipeline operations, imports, and prices have been regulated by the Federal Government. Since then, States have imposed conservation regulations and severance taxes on production, and both State and Federal Governments have imposed excise taxes on petroleum products.

Crude Oil Regulations, Price Controls, and the Windfall Profit Tax

When government originally intervened in oil and gas markets, it was to the benefit of the producer. Eventually, the regulatory environment worked to reduce returns to producers. Since 1906, oil pipelines have been subject to regulation. Passage of the depletion allowance, and well-spacing and prorationing regulations in the 1920's and the Connally Hot Oil Act of 1935, which nationalized prorationing, gave producers higher prices and lower costs. In 1959, the Mandatory Oil Imports Program limited imports. However, in the late 1960's and early 1970's, the regulation and tax milieu that had been beneficial to the producers started to turn against them. Price controls were established and fees were imposed on imports. Since the early 1970's oil prices have been controlled under the Economic Stabilization Act, the Emergency Petroleum Allocation Act of 1973, and the Energy Policy and Conservation Act (EPCA) of 1975.

In September 1973, the Cost of Living Council (CLC) established a two-tiered pricing system. Old oil was price controlled and new oil was not. The essential distinction between old oil and new oil was based on a base production control level (BPCL) for a "property". Under CLC's definition of "property," oil discovered on a producing property was considered old oil if the BPCL was not being maintained. Such old oil was released to market prices on a barrel-for-barrel basis with new oil produced. This created an incentive greater than the market price (i.e., adding the difference between the old oil price and the market) for new oil up to the BPCL. Once the base level was reached, the incentive price was the market price alone. Stripper oil (oil from a property averaging 10 barrels or less per well per day) was also raised to the market price. From 1973 to 1975, numerous adjustments were made in the regulations (for more details, see [83]).

In 1976, the property and base level definition changed in a significant way and a three-tier system was created. The definition of property was changed, allowing for more newly discovered oil to be classified in the middle ("upper") tier. For the remainder of the 1970's, many adjustments continued to be made. When price controls expired, there were at least 11 categories of oil. Price controls were followed by the Windfall Profit Tax.

The Windfall Profit Tax (WPT) of 1980 allows crude oil to be sold at the market price, but taxes the producer on a portion of the difference between the market price and a base price. The first two tiers under controls constitute Tier 1 of the WPT. Tier 2 is stripper oil and oil from national petroleum reserves. Tier 3 is newly discovered, heavy, and incremental tertiary oil. The tax varies for each tier (30 to 70 percent) and gives a lower rate to oil companies classified as "independents" in Tiers 1 and 2. Also, there is a provision that one may not be taxed on more than 90 percent of his profits. The tax phases out when the U.S. Treasury has collected $227.3 billion or in December 1987, whichever comes latest, but no later than December 1990.

Natural Gas Price Regulations

Federal regulations for natural gas were initiated for the benefit of the producers. Federal regulation of interstate natural gas pipelines began with the Natural Gas Act of 1938. In 1954, the United States Supreme Court decision in Phillips vs. Wisconsin created wellhead price controls. This decision immediately created separate markets in each producing state for intrastate and interstate gas. In the 1970's, the interstate system had low wellhead prices and supply shortages, while the intrastate system had almost the opposite higher market clearing prices and no shortages. After several regulatory adjustments, the Natural Gas Policy Act (NGPA) of 1978 was passed as the legislative response to the problem.

The NGPA created no fewer than 12 price ceilings for the wellhead price of natural gas. These categories include new (on and offshore) gas, gas dedicated to interstate commerce before 1978, gas under existing interstate contracts, gas under rollovers of existing contracts, high cost gas, stripper gas, gas dedicated to intrastate commerce, Prudhoe Bay area (north Alaska) gas and miscellaneous gas categories (for more details see [29]).

3. MODELING THE FUTURE

Objectives in Modeling the Future

The approach to modeling the future is highly dependent on the purpose of modeling. Since very few analysis groups can afford the luxury of maintaining more than one modeling system, a choice must be made. The choice should be based on the main purpose of the analysis; for example, forecast accuracy, the ability to perform "what if" analysis, or general understanding.

Forecast accuracy may be defined or measured in several ways. One way is the mean square error, a retrospective statistic, composed of the sum of the squared deviations of the forecasted values from the actual historical values. A second way is to count how often the actual values fall into some constructed confidence band around the forecast, rewarding the capture of the actual value. The pitfall in the second statistic is understanding the penalty for constructing intervals that are too large. One way of identifying this type of forecasting is when the independent variable is time. The examples of this type of forecasting are Box-Jenkins time series forecasting and other extrapolations based on time. The theory behind these methods is analogous to Newton's first law of motion: Things in motion tend to stay in motion. That is, large inertial forces exist in the system that will not deter it from its inertially ordained future. These methods often perform reasonably well in short-term forecasting where seasonal and secular trends must be sorted out. Sensitivity or "what if" analysis is not possible.

Models that have a theoretical premise capable of doing "what if" analysis constitute a second class of models. Besides giving forecasts based on some theoretical foundation, these models are often used to examine first differences in the forecast. In cases where there appears to be a systematic bias, such models can perform poorly at absolute forecasting and do well at examining differences. Econometric models often fall into this class. This approach is closer to Newton's second law of motion: Each action has an equal and opposite reaction. This approach may sacrifice absolute accuracy to obtain levers with which to analyze the effects of certain "driving" variables. In oil and gas forecasting these variables are typically prices, costs, rig availability and funds availability.

A vaguer but often more important objective is modeling for understanding. The hard measures of accuracy almost disappear. An example of this approach is exploratory data analysis where the objective is to play in a statistical "sandbox" until the analyst is satisfied with some statistical relationship that improves his or her understanding or insight.

In most situations the luxury of having all of the above systems is not feasible. The last system should always be available. The choice is usually between the first and second. The rigor that is produced by forcing the analyst to produce consistent numbers cannot be overemphasized. Numbers can return to haunt an analyst, but numbers force the analyst to look deeper, examine definitions, and understand the input data. Also, reproducibility allows for future improvement of the process.

Theories of Behavior and the Politics of Forecasting

Forecasting by its very nature involves many assumptions and simplifications. It encompasses everything from prognostication by informed and less well-informed analysts to the output of complicated hierarchical modeling systems. Much debate has been devoted to the minimum threshold of "tests" the model must pass before it can be used for forecasting. Creating an arbitrary absolute threshold does not address the issue properly. Forecasts will be produced. The issue is whether one method improves on the other and whether the addition of another forecast illuminates the discussion about the future. Further, it is difficult to determine where the modeler stops and the model starts. In most analyses they are inseparable. Modelers and the sponsor should be part of the model assessment process.

Perhaps the most important questions that must be addressed in building a model are: What are the critical aspects of the problems being modeled and how should they be modeled? Several theories are presented below that demonstrate different points of view about what is or is not important in modeling the discovery/production process. There is a high correlation between the background of the group and what the group considers important.

Geologists tend to focus on remaining resources and finding rates. Finding rates are measured in terms of resource-in-place or reserves found as a function of drilling, usually wells or footage. Geologists prefer to work with new field discoveries and new field wildcat wells. Many believe that sheer system inertia will determine the production level for about 10 to 15 years. Also, most prefer to work on a disaggregate basis.

Economists, on the other extreme, are more comfortable working at more aggregate abstract levels. They believe that profitability is the determining factor in any market, and for finite resources, they add some concepts about eventual exhaustion or "backstop" technology. Economists also tend to be concerned about the industry structure, both vertical and horizontal. Bankers and financial analysts, a special breed of economist, place a strong emphasis on investment or cash flow as a determinant of production. They usually give less attention to the technological, since the capital markets and cash flow are considered to be the critical aspects.

Many oil and gas companies believed that if they were just "left alone," they would supply the United States with energy. "Left alone" must be qualified. They would like to be free of what they view as poorly devised and implemented environmental regulations and have more government land available for exploration. Additionally, some tax incentives and subsidies for risky capital intensive ventures are considered desirable. The independent (non-integrated) operators are often behaviorially characterized as "macho," "high rollers," or brinksmen who love high risk ventures and pay less attention than the majors to scientific information and economic analysis. By definition, they must dispose of the oil and gas they find. On the other hand, the drilling fund investor is said to invest for tax shelters and cocktail conversation.

Policymakers believe that rules can be written to change the behavior of the agents in the process and protect the public interest. The general theme of policy as contained in the WPT and NGPA is to extract all the "economic rent" above some nominal level and create income transfers. Nevertheless, the public's collective characterization of the petroleum industry has been one of distrust of the majors.

None of these characterizations are necessarily right or wrong. The question is which are more important to model?

A Simple Model

The next step is to build a simple conceptual model of the discovery/production process. The model in Figure 1 contains several parts. (The annotation on the arcs refers to the organization that collects information on the process.) The figure will be used as a scheme to organize the presentation that follows. The next step in the modeling process is to examine the data.

4. THE DATA

There are basically four sources of petroleum data: The Federal Government, the State governments, the petroleum industry, and private data collection firms. Within the Federal Government, the U.S. Geological Survey (USGS), the Energy Information Administration (EIA), and the Bureau of the Census collect most of the available data. Each State has its own unique system to collect information mainly for taxation and "conservation" purposes. Within the industry, the two principal sources of data are the American Petroleum Institute (API) and the American Gas Association (AGA). Both are industry trade groups. In the remaining group, there are a number of private firms - for example, the Petroleum Data System of North America (PDS) and the Petroleum Information Corporation (PI) (both are partially funded by Federal agencies) - that supply information primarily to explorers.

Much of the available data is a transcript of some primary source, but it is often very difficult to determine the primary source. Additionally, some of the data are intelligence data, since various types of information give a competitive advantage to an explorer. Consequently, accuracy often is compromised for timeliness. Often data collected for one purpose has questionable value for another activity. For example, data that are collected for regulatory purposes often becomes rosy "museum" data for use by lawyers. To follow H.L. Mencken's reasoning, it is sniffed but never used in the analytic soup. Models need cabbage not roses.

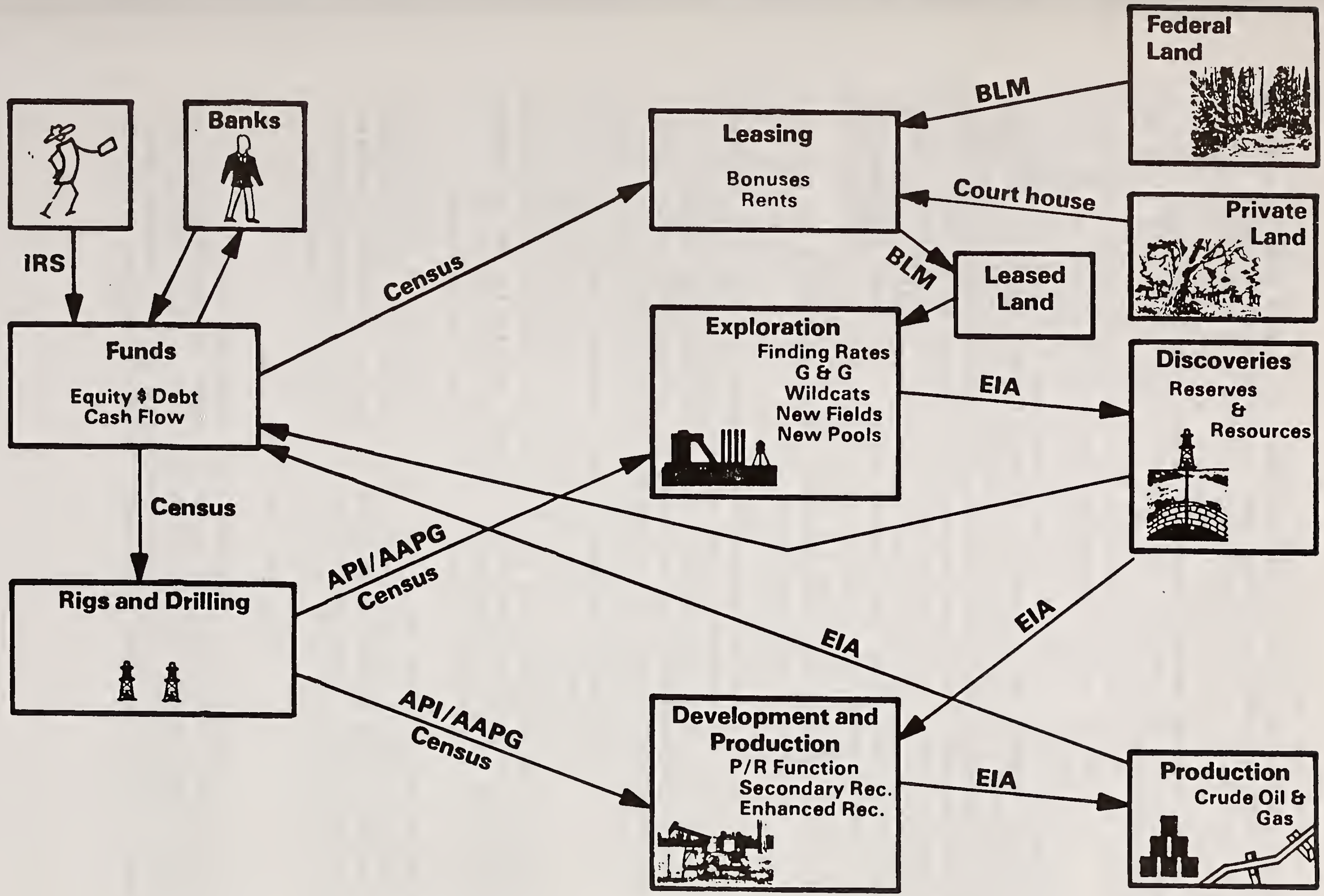

Figure 1. The Exploration-Discovery-Production Process

What are primary, basic (i.e., raw) data and what are not is a difficult question. What many consider to be basic physical data (e.g., reserves or drilling) has already been through the human judgment process (as opposed to basic physical measurement). In this paper, the arbitrary distinction will be that historical information is raw, or primary, data.

Financial Data

Financial data can be divided into four groups: costs, prices, taxes, and financing. Most of the costs of finding and producing petroleum are associated with drilling. The basic and most complete source of drilling cost data is the Joint Association Survey on Drilling Costs [4]. This information is reported by region and depth interval. Until 1978, this survey also contained other expenditures for finding, developing, and producing oil and gas. However, it has been discontinued because similar data are collected by the Census Bureau's Annual Survey of Oil and Gas. Additional cost data are collected and published by EIA's Dallas Field Office (see [30]).

The IRS Income Tax Return publications ([58], [59]) present information on revenues, taxes, and profitability of proprietorships, partnerships, and corporations. Most industry statistics are reported on a regional basis. The Census and IRS report by company size and type.

Seismic Activity and Lease Data

Data are collected at various points in the exploration process. The exploration process begins with the leasing of property and seismic activity. There is little or no central collection of lease activity, although each lease is recorded in the local (county or parish) court house.

Seismic activity is collected by the Society of Exploration Geophysicists [95] and measured by number of active seismic crews and line miles of seismic activity. Seismic activity is considered a leading indicator of the discovery process, but has been used very little in quantititive analysis.

Rigs and Drilling Data

After the analysis of seismic and other geologic information, the decision to drill must be considered. The leading indicators for drilling are the average number of rotary rigs in use, which is collected by the Hughes Tool Company [55], and the rig count, which is collected by the

Reed Tool Company [87]. Unfortunately, these two series are not compatible since the regional boundaries for the rig counts are Reed sales districts are different from the more standard API subdivisions used by Hughes.

Before drilling can commence, state agencies must issue a drilling permit. These agencies also require well tickets to be filed when the well is completed. The American Petroleum Institute (API) and the American Association of Petroleum Geologists (AAPG) collect a well ticket that contains requests for information such as the depth, location, success status, and Lahee class of the well (see Appendices B and C). Since this information is supplied voluntarily, it is often not complete. Space for two volumetric quantities is provided on the well ticket for the estimated ultimate yield for oil and gas. For some reason, they are requested only for new field wildcats and deeper pool tests. The yield information is based on the "judgment of the AAPG Committeemen" and it is not stated explicitly whether it is for the well or the entire reservoir being discovered.

Currently, there are four sources that collect drilling data for the entire United States: the Energy Information Administration (EIA), Petroleum Information Corporation (PIC), API, and the Bureau of Census. PIC's Well History Control File is probably the most comprehensive and detailed source with information on more than one million wells. The Census Bureau collects drilling information as part of its _Annual Survey of Oil and Gas_ [25]. Table 1 illustrates a basic problem between the data collected by the Census Bureau's survey and that collected by API as presented in EIA's _Annual Report to Congress 1979, Volume 2_.

Table 1. Wells Drilled in 1977-1978
(In Thousands)

	1977 Census	1977 API	1978 Census	1978 API
Exploration				
Total	8.95	9.96	9.77	10.68
Oil	2.15	1.21	2.12	1.13
Gas	1.73	1.48	2.07	1.60
Dry	5.07	7.28	5.58	7.95
Success Ratio	0.43	0.27	0.43	0.26
Development Ratio				
Total	21.68	35.02	23.16	36.38
Oil	11.12	17.70	11.45	16.65
Gas	6.65	9.70	7.48	47.06
Dry	3.91	7.42	4.23	8.27
Success Ratio	0.82	0.79	0.82	0.77
All Wells				
Total	30.63	44.98	32.93	47.06
Oil	13.27	18.91	13.57	17.78
Gas	8.38	11.38	9.55	13.06
Dry	8.98	14.70	9.81	16.52
Success Ratio	0.71	0.61	0.70	0.65

Sources: Bureau of Census, Annual Survey of Oil and Gas, 1977, 1978; Energy Information Administration, Annual Report to Congress 1979, Volume 2, Table 17, Tables 13 and 14.

As shown in Table 1, the most striking difference is summarized in the exploration success ratio. Any time trend is completely blurred by the marked difference between these series. In both years, the Census reports about 30 percent fewer wells drilled, and reports more successful exploratory wells than API. Moreover, successful exploratory oil wells differ by a factor close to 2. Wells are classified as dry if they are not capable of commercial production. Often, wells will produce for a short time and then become uneconomical because of a high water/oil production ratio, for example. How these wells are classified presents a problem.

An additional complicating factor can be seen in Table 2. EIA collects "net successful (in finding) natural gas footage" as a part of its Annual Survey of Domestic Oil and Gas Reserves. In the Lahee classification taxonomy (see Appendix C) the first five classes of wells are exploratory and they are the only wells (by definition) that can discover new reserves. Therefore, the amount of "net successful" footage from the EIA Annual Survey should be the same as exploratory drilling reported by API and the Census. One conclusion that can be drawn from this data is that development well drilling must be producing reserves. Another conclusion is that no one pays attention to definitions.

Table 2. Natural Gas Footage
(in millions of feet)

	1978			1977		
	Census	EIA	API	Census	EIA	API
Exploratory	11.4	NA	9.7	13.7	NA	10.8
"Net Successful"	NA	30.6	NA	NA	32.8	NA
Total	47.3	NA	59.5	66.0	NA	70.2

NA= Not Available.

There are several possible reasons for the discrepancies shown in Table 2. First, since API reports 50 percent more wells, the sampling frame for the Census is probably biased and not complete. Second, the determination of an exploratory well in the API/AAPG series is determined by the AAPG. The Census data is from the company. Companies keep separate records for external reporting (e.g., annual reports), tax purposes, internal decisionmaking, and possibly one or more for non-IRS Government reports. The external financial reporting process is governed by the statement of Financial Accounting Standards No. 19 [40] and has vague definitions for classification of wells. In some companies exploratory wells are defined as those wells drilled by the exploration department and the development wells are drilled by the production department. Further, the companies may never know how the well was classified by AAPG, which appears to be more stringent in its classification of exploratory wells. A third reason may be the treatment of the well for tax or regulatory purposes. There is no difference in the intangible drilling expense for exploration vs. development, but there is a difference in the pricing if a well finds a new reservoir. The definition of new oil prices includes oil from new reservoirs, creating a strong economic incentive to classify a well as exploratory whenever possible. A well that to the

AAPG looked like infill drilling may be classified by the company and the regulatory authority as finding "virgin" pressure and hence a new reservoir.

Reserves and Resource-In-Place

Once a successful exploratory well has been completed, the question is how much petroleum did it find? This is a difficult question to answer. Since 1946, API/AGA/CPI have been reporting information on reserves in an annual publication called the Blue Book [4]. Recently, API/AGA announced that in 1979 they would discontinue the publishing of the Blue Book since EIA-Form 23 was now collecting and publishing the same reserve information. Reserves are the volumes of crude oil and natural gas that are currently economical to bring to the surface. But it is not reserves that are found; rather, what is found is a reservoir containing fluid. This fluid (or fluids for multiple phase reservoirs) consists of hydrocarbons, water, and various other compounds and molecules. Along with other physical properties of the reservoirs, each discovery is unique.

The amount of petroleum that can be economically produced is a function of reservoir energy, technology, quality of the product, prices, and costs of production. With no change in what is physically in the reservoir, its reserves can change. One quantity that cannot change is the amount of petroleum initially in the reservoir -- original resource-in-place. The ratio of reserves to resource-in-place is called the recovery factor (historically 33 percent) for crude oil. Reserves are more important to the economics of the operation since, for example, banks will loan money on reserves. Resource-in-place by date-of-discovery has more modeling importance. At this point EIA's reserves survey has not addressed resource-in-place and date-of-discovery information which many modelers, analysts, and engineers consider more useful and critical information.

Resource-in-place can be measured by an estimate of the reservoir size and data on porosity and water saturation. Reserves are measured using methods ranging from rules of thumb or analogy to complex reservoir simulation models. Since engineers, accountants, and bankers are taught to be conservative when dealing with reserves, the announced reserves after a discovery tend to underestimate the amount actually discovered (although in small reservoirs this bias is usually less). To accommodate this approach the API/AGA reserve additions are reported in four categories: revisions, extensions, new reservoirs in old fields, and new field discoveries.

Reporting of Discoveries

The reporting of new field discoveries is something like a strip tease - after a new field is found (which at some later date may even be declared not to have been a new field due to results of further drilling and testing) only part of what was found is revealed (reported) to the public. But as time goes on, more and more is revealed. This "dance" includes the drilling of confirmation and extension wells, discovery of new reservoirs, analysis of data, and principles of engineering and accounting conservatism. The revelation sequence may take five years or more.

The problem of determining when discoveries are made is illustrated by the discovery and reporting of reserves for Prudhoe Bay. In 1970, the <u>Blue Book's</u> Table II reported reserve additions in Alaska of 9.85 billion barrels. From the information, one may assume that Prudhoe Bay was discovered in 1969 or 1970. Upon turning to Table III-2, which contains original resource-in-place and ultimate recovery by year of discovery, one finds 9.4 billion barrels (ultimate recovery) discovered in 1968 and 320 million in 1969, but nothing in 1970. One must conclude that it took 2 years to report the discovery of Prudhoe Bay as reserves. Is this a standard operating practice? Do all new fields take this long to report? Will the date of discovery be lost forever, when EIA is the only organization reporting?

Extensions are used to report additions to proved areas. Revisions are a catch-all category that includes positive and negative revisions due to errors in calculation, changes in economics, new data often from infill drilling, and installation of secondary or tertiary recovery techniques. Revisions always create serious problems for three reasons. First, they are a category for reporting anything that does not fit nicely into other categories. Second, excluding secondary and tertiary recovery, revision should be attributed to estimates made in the past, but currently only ad hoc methods exist for matching these data to the data they correct. Third, development drilling can produce new reserves. New pools can be discovered, better information on the reservoir characterisitics is obtained, or better flow is established in a poorly connected reservoir. No one has attempted to estimate the magnitude of these phenomena separately.

From 1968 through 1978, revisions in natural gas reserves have been mostly negative. This problem was so serious that the resulting total associated-dissolved natural gas reserves were negative. Without being able to assign these revisions to past data, extrapolating trends becomes difficult.

The Ideal Data Base

In order to properly understand the domestic exploration and production industry, a good data base must be constructed (cabbage for the soup). The first step in building that data base is to define the reasonable micro units of the process. For the physical discovery process, these units are the reservoir and the well. The reservoir is important because it characterizes the target of exploration drilling and the natural unit of oil and gas occurrence. Reservoirs aggregate into plays, and fields aggregate into basins, and then provinces and regions. Wells are important because wells are the final determinant of discovery and production. Without complete and compatible data for wells and reservoirs, any analysis of the past starts with a severe handicap. The data needed for this data base differs only slightly from that currently being collected.

The ideal data base would have three parts linked by common identifiers. The first part would be the well data file. This part would contain information very similar to a current API/AAPG well ticket with the addition of a reservoir code. The second part would contain geologic, resource, and production data by reservoir, and would include information on the discovery well and number of wells and well months of operation in the reservoir. Part of the reservoir identifier would be the field, basin, and region of which it is a member. Additionally, production would be reported by regulatory category. The third part would contain financial information reported on a gross operator basis for each field and would include a breakdown of ownership of production and profit distribution. Additionally, companies could be asked for financial information, including land acquisition, lease inventory, and geological and geophysical costs. This information could serve as a crosscheck on the gross operator data and the sampling frame.

5. RESOURCES AND WITHHOLDING

Resource Distribution

Many authors have made estimates of remaining petroleum resources using a number of different approaches ([46], [52], [53], [54], [76], [60]). Recently, the most visible and detailed assessment is the USGS Circular 725 [78]. The critical dimensions for the resource distribution are depth (both water and actual), the size of the reservoirs or fields, the cost of discovery, and availability of the potential resource. The size distribution is generally accepted to be lognormal. This conclusion has resulted from empirical analysis and deductive inference using geologic theories ([95], [11]). Additionally, the AAPG classification scheme (see Appendix B) reflects this assumption.

The cost dimension covers many facets. Among them: the depth of deposit, the above and below ground environment, characteristics of the reservoir and the amount and type of petroleum in it, and the trap detection mechanism. The trap detection mechanism includes the geology and geophysics used to discover the trap, and the geochemistry used to contribute to the estimation of the presence of petroleum. After all predrilling information is assembled, the number of holes necessary to explore the geologic prospect becomes a very important determinant of the costs. For example, to detect and discover stratigraphic traps usually requires more drilling than structural traps.

Leasing, Withholding, and Intertemporal Rent

In 1931, Hotelling demonstrated the existence of intertemporal "economic rent" that, if not paid to the owners of exhaustible resources, would encourage the owner to withhold the resource from production [48]. The U.S. Government withholds land for a variety of economic and non-economic reasons.

Oil and gas leases transfer the mineral ownership rights. There is no doubt that some speculative withholding exists in the production process. Whether it is massive or minor is a question that has not been fully answered. Today, leases are harder to acquire because the owners are smarter and more knowledgeable. Consequently, landowners are demanding and getting more "upfront" or bonus money and higher rent. The bonus money is extracting some of the "economic rent." In general, the term of the lease is getting shorter. The old "sample" leases were for 10 years or longer and were usually renewed by a rent payment and with no requirement to drill. Today, leases are being written for 1 to 5 years. In "hot" areas, leases also contain a provision to drill within 6 months. In addition, lease bonuses have gone from $3 to $5 per acre to $75 to $100 per acre. In a 1980 Permian Basin lease sale by the University of Texas, up to $11,000 per acre was paid in bonus money in addition to a 25 percent royalty.

Also, lease terms are much more variable today than in the past, an indication that owners of expected marginal properties are not getting much, if any, intertemporal "economic rent." Whether anyone actually understands what they are withholding, or that they are allocating their resources where the expected marginal cost is lower, is difficult to assess.

6. INVESTMENT, RIGS AND DRILLING

Investment and Funds Flow

Drilling for oil and gas requires investment. There are three general groups of agents that operate in the petroleum exploration business: (1) the majors, (2) the independents, and (3) the fund investor. Each has separate motivations. As a general policy, banks have refused to loan money for exploration. Nevertheless, when a large bank loans a large oil company money, no specific collateral is required. Even if the money is borrowed for a new refinery plant and equipment, it frees other funds for possible use in the exploratory program. The majors are vertically integrated with massive investments in refining and in the distribution and retailing systems. To make their operations profitable, they a need large flow of crude. They can either find it or buy it. For example, in 1979 Shell purchased Belridge Oil Company for $3.6 billion, increasing its reserves by 44 percent.

The independents are in the business of finding petroleum. The motivation behind this group is conjectured to be a complex combination of the high-stakes gambler, who goes broke at least twice in his career, and the small businessman trying to make a fortune. To remain solvent this group must find oil in profitable quantities, but usually its only use for the oil, once it is found, is to sell it.

Another source of funds is from the private investor or a drilling fund. This activity is very attractive to those in high tax brackets because the tax laws allow a large portion of the investment to be expensed by the investor. Private investment is a function of the general well-being of the high-tax brackets and the commodity markets. The major companies, who have a large share of the oil and gas leases, contract with an independent financed by a drilling fund to drill its poorer prospects on the property leased by the major. The major retains the right to the crude. The independent receives cash and the drilling fund gets a tax write-off. This type of operation is called "farmout." According to the IRS, proprietorships and partnerships in oil and gas exploration have been operating collectively at a loss at least since the early seventies ([58], [59]).

The timing of cash realization for the various explorationists differs drastically. An integrated oil company receives cash resulting from a discovery only after the refined product is sold, although there are transfer payments for bookkeeping purposes. On the other hand, the independent can borrow on the reserves found as soon as they are proved. In the middle is an operator who sells the crude as it is produced. In the extreme cases the timing of the actual final cash realization from a discovery differs by 20 years or more. Further, the tax structures of the majors and independents differ substantially. For example, the majors have foreign tax credits (credits to their United States income tax) in the billions of dollars. The independents have lower excise taxes and still have a depletion allowance.

The above description of the industry is obviously oversimplified, but serves to highlight the complexity of the funds-flow problems. Some models use a "top-down" finding approach and are driven by the industry "exploration budget." After the "budget" is determined, the number of exploratory wells drilled is the budget divided by the average well cost. A projected budget for each year would need to address the reallocation of the major's internal budgets, the reinvestment of the independent's profits, and the size of private investments. A simple recycling of the previous period's profits may seriously understate the budget. Unless surrogates are found for the various components of the "exploration budget," it is very difficult to forecast.

The "bottom-up" approach is to use the micro technique of discounted cash flow analysis. Using project costs, finding rates, price, and production profiles, each region is then treated as a project. With this approach alone production forecasts would be very high because it could over-state the funds available to the projects.

Rigs

Projecting the amount of exploratory or developmental drilling is a perplexing task. The drilling industry can be viewed as two separate industries - the rig builders and the drilling contractors. There are about 20 rig building companies in the United States. Several are subsidiaries of steel companies. The market for rigs is international and no hard data exists on the number of rigs that are shipped abroad or returned each year. The raw material for building rigs is mainly tubular steel. In periods of high demand for steel, this market can become tight. The price of a rig varies according to its depth rating. For example, a rig rated for 5,000 feet costs about $500,000; for 10,000 feet it costs about $2 million; and for 20,000 feet it costs about $5 million. The well costs increase at a highly nonlinear rate as a function of depth.

The composition and size of the rig fleet has changed dramatically since 1973. From 1973 to 1979 the number of rigs (as reported in the Reed Census) grew at an annual rate of 10 percent. Rigs rated below 10,000 feet grew only at a rate of 7 percent while those rated at or above 10,000 feet grew at a rate of 13 percent, indicating a shift to deeper drilling potential. Nevertheless, at the same time, the average depth of wells was less in 1979 than in 1973.

The drilling industry has changed its basic composition over the last 40 years. In 1939, about half of the drilling was done by the oil companies. Since then oil companies have gradually divested their drilling operations and currently they account for only 1 percent of drilling. The other 99 percent is done by 680 contractors, none of whom is large enough to dominate the market. The majors now contract for most of their drilling. Drilling contractors occasionally drill holes for themselves.

One of the conjectured reasons for the majors' divestiture is the inability to move crews as fast as their prospect analysis and corporate strategy dictate. This raises the question of rig mobility. That is, in a model that is regionally disaggregated, how should the migration of rigs be represented? Certainly, premiums must be paid to move rig crews long distances from their homes. Further, in periods of high demand, it is probably more difficult to move rigs and their crews. Two simplifying approaches exist. One approach disregards mobility and allows drilling increases only by the introduction of new rigs in each region. The other approach allows a percentage of the rigs to migrate to other areas based on relative regional profitability. After the number of rigs has been determined, the next question is what type of hole will be drilled.

Drilling

Although developmental well drilling has been increasing at an annual rate of 13 percent since 1973, exploratory well drilling has only increased at a rate of 6 percent, and it declined in 1979. These changes have been taking place under a changing and complicated series of price controls. In addition, the proper decisionmaking unit for drilling should be the reservoir. However, regulations, taxes, and controls are based on a legal property definition, which often contains many reservoirs or subdivides a reservoir, and distort optimal economic decisions. How should these trends be extrapolated over the next 10 years?

The fraction of successful exploratory wells increased steadily from 16 percent in 1972 to 29 percent in 1979. The significance of this increase is difficult to assess since a combination of factors could account for this change. First, due to price increases, the minimum prospect size (dry hole threshold) is lowered; therefore, more wells are successful. Second, better exploration techniques help avoid the drilling of dry holes. Third, some "exploration" is simply the redrilling of geologic structures that were drilled, plugged, and abandoned as dry holes in previous years. Fourth, there has been a general conservative shift in the broad category of exploratory drilling (e.g., from new field wildcats to extension wells).

Whether, when, and how current trends will change are critical issues. For example, if the recent increase in the proportion of exploratory wells declared successful were to continue, by 1990 60 percent of exploratory wells would be successful, a dramatic historical change.

Producers, adjusting to the higher prices, soon may have drilled the inventory of prospects in existing fields. If the prices continue to rise, the infield inventory of new viable prospects will increase, allowing current trends to continue. If prices do not rise, the infield prospects soon decline. As infield drilling declines, rigs are idled, reducing rig demand and shortage rents (currently, there is at least a 1 year wait to drill a well) which may provide a further incentive to drill the riskier wildcat wells.

Intentional Drilling

Does the driller have prior intentions of finding oil or gas or is the search simply for petroleum? Most people in the industry believe that a large part of exploratory drilling is "intentional." That is, before drilling a well the driller has specific intentions of finding either oil or gas. When a successful hole is drilled, it usually produces a mixture of liquids and gases. The decision whether it is declared an oil or gas well is based on the gas to liquid ratio.

From a modeling viewpoint, it is often convenient to treat discoveries of oil and gas as independent events (except for the coproducts, associated natural gas, and natural gas liquids). There are several reasons why this choice is made by most modelers. Natural gas and crude oil differ in the way they are processed and distributed. Further, the prices on a heat value basis have never been the same and they are substitutes in the market place on a limited basis. Additionally, the constituencies of each product are different and the sponsor of the model usually wants more attention paid to one of the products.

An important question regarding intentional drilling is how to allocate dry holes. Since there can be no information on what was not there, the easy choice and the one pursued by many modelers is to allocate dry holes (or footage) in proportion to successful oil and gas wells (footage). A second choice is to allocate them in a more sophisticated way, for example, by historical regression of successful oil well and successful gas wells on dry holes. The results are presented in the table below.

Table 3. Dry Hole Allocation

	All Exploratory Drilling		New Field Wildcat	
	Wells (Thousand)	Feet (Millions)	Wells (Thousand)	Feet (Millions)
Oil	1.14	7.50	0.49	3.31
Gas	1.78	11.77	0.67	4.71
Dry	7.46	43.22	5.16	32.50
Ratio				
(Oil/Gas)	0.64	0.64	0.73	0.70
Dry Oil	2.91	16.86	2.17	13.38
Dry Gas	4.55	26.36	2.98	19.12
Regression				
Ratio	3.65	2.90	NE	NE
Dry Oil	5.85	32.14	NE	NE
Dry Gas	1.60	11.08	NE	NE

NE = Not Estimated.

The difference between these two approaches is significant. Almost twice as many dry holes are allocated to oil and less than half are allocated to gas if the regression analysis is used. But the regression masks two important historical trends. While the ratio of successful oil to gas wells changed from approximately 3 in the 1950's to .67 in the late 1970's, the average depth of all holes steadily increased by more than 50 percent. An additional confounding factor is that advances in technology were made concurrently with these trends.

In 1978, about 1 in 3 exploratory wells drilled deeper than 7,500 feet were successful, while approximately 1 in 4 of those drilled less than 7,500 feet were successful. Since deeper wells cost more to drill, good economics dictates that more care and analysis be exercised in drilling deeper, and hence, fewer dry holes. Gas wells have historically been deeper than oil wells.

If the intentional drilling approach is abandoned, the statistical problem of allocating dry holes vanishes, but in its place is the problem of modeling the economic and regulatory incentives that change relative profitability of liquids and gas.

7. DISCOVERY AND FINDING RATES

What is a finding rate? In a broad sense a finding rate for petroleum is the amount of petroleum found measured against some unit of activity or factor input needed to find it. This section will examine several approaches to modeling the discovery process. The choice of numerator and denominator of the finding rate has been the subject of much debate, political and scientific.

For example, on November 4, 1977, the Governor of Texas, Dolph Briscoe, sent a letter to James Schlesinger, Secretary of Energy, stating that finding rates were a "critical issue" and suggested that "about 15 barrels per foot" would be appropriate for Lower-48 onshore production. In his response, Schlesinger stated that finding rates are "the paramount determinant of future supply" and the data must be "meticulously studied." He stated that Briscoe's definition of finding rate was not "clear." In a second letter, Governor Briscoe stated that "I concur with your observation that forecasting of finding rates is a paramount determinant of future supply." He went on to address the question of finding rate definitions and the use of recent vs. longer term trends.

The range of possible choices for the numerator includes new fields or discoveries measured as reserves or resource-in-place of crude oil, natural gas, or the net equivalent amount of crude oil. The choices for the denominator include time, total footage (or wells) drilled, total exploratory footage (or wells), total successful exploratory footage for oil(or gas), total footage (or wells) for oil (or gas), new field wildcat footage (or wells) rank, wildcat footage (or wells), dollars invested, or cash flow.

The Matching Problem

Depending on the numerator of the finding rate calculation, an appropriate denominator must be chosen. For example, if new field discoveries are to be used in the numerator, new wildcats (wells or footage) must be used in the denominator. A very serious problem arises when an attempt is made to determine what was found in a new field wildcat. When reserves are reported (in the past by the API and AGA; now on EIA Form 23), there is no mention of the wells that discovered them. Further, reserves are not reported by date of discovery, but often up to 4 years later. To estimate finding rates, drilling must be matched with the amount found by that drilling. Currently, although the reporting of reserves lags behind the reporting of the discovery wells, only anecdotal evidence exists for any estimate of this lag. The problem is what reserves or resources-in-place are found by which wells -- the matching problem. In addition to the timeliness of reporting, the relationship between drilling categories and reserve addition categories is worrisome. Table 4 presents drilling and reserve reporting categories and several different finding rates.

Table 4. Reserve/Drilling Finding Rates

Drilling	Reserves
1. New-field Wildcat	A. New field discoveries
2. New-pool Wildcat	
3. Deeper-pool test	B. New reservoir discoveries in oil fields.
4. Shallower pool test	
5. Outpost (extension) test	C. Extensions
6. Development	D. Revisions

Possible Finding Rates

Name	Calculation
New Fields	[(A)]/[(1)]
New Reserviors	[(A) + (B)]/[(1) + (2) + (3) + (4)]
New Discoveries	[(A) + (B) + (C)]/[(1) + (2) + (3) + (4) + (5)]
Reserve Additions	[(A) + (B) + (C) + (D)]/[(1) + (2) + (3) + (4) + (5)]
All Drilling	[(A) + (B) + (C) + (D)]/[(1) + (2) + (3) + (4) + (5) + (6)]

There are several basic approaches to modeling the discovery process. One approach estimates a finding rate for new fields, a measure of long-term, high-risk exploration. Subsequent to finding a field, the extensions, revisions, and new pool discoveries must be estimated. This approach has two drawbacks. First, the data reported as new field discoveries frequently underestimates the amount found. Second, the remainder of the discovery sequence and the staging of production must be estimated adding a second complicated estimation process. The finding rate for new reservoirs is a more conservative measure of discoveries, but suffers from the same problems as the finding rate for new fields.

The finding rate for new discoveries has appeal since all discoveries and exploratory drilling are included, but any revisions must be estimated separately. When the finding rate for new dicoveries is calculated on a yearly basis and plotted against time, its behavior is a reasonably smooth decline. When the same plot is generated using finding rate for reserve additions (i.e., including revisions) the behavior exhibits considerably more variation Figure 2.

The reserve additions and all drilling finding rates differ by the inclusion of developmental drilling. They are not theoretically pleasing, but allow the modeler to neglect many of the staging and data definition problems since they are all inclusive.

Finding Rate Categorizations

It has been proposed that finding rates be differentiated by depth of holes, trapping mechanism, search type (e.g., surface and subsurface geology and geophysics), company type (e.g., majors vs. independent), regulatory category, and geographical region. Additionally, questions of using long-term (e.g., from 1940's), midterm, or short-term (from 1974) trends in performing the analysis are debated religiously.

Even though information to categorize finding rates is available only on a limited basis and is not done on a national level, the research into and arguments for the additional stratification are interesting. For example, depth stratification allows for three additional refinements. First, the success ratio increases with depth. Secondly, the well costs increase nonlinearly with depth. Third, depth can be considered a "third dimension" of discovery process.

There are two basic types of trapping mechanisms for petroleum: structural and stratigraphic. Structural traps are easier to find and usually present very definitive targets. On the other hand, stratigraphic traps are harder to detect and are often detected accidently or as the result of a more random type drilling. Success rates and amount found, it is hypothesized, could be markedly different for reservoirs with different trapping mechanisms. Categorization by the field or reservoir size is also desirable, since for example, the costs of finding and developing 100 million barrel fields is different from a 100 million barrel field.

The Arps-Roberts Model

In 1958, Arps and Roberts published a paper on the economics of drilling in the Denver-Julesburg basin which contained finding rates differentiated by the size of discovery [7]. The paper proposed the following model of discovery:

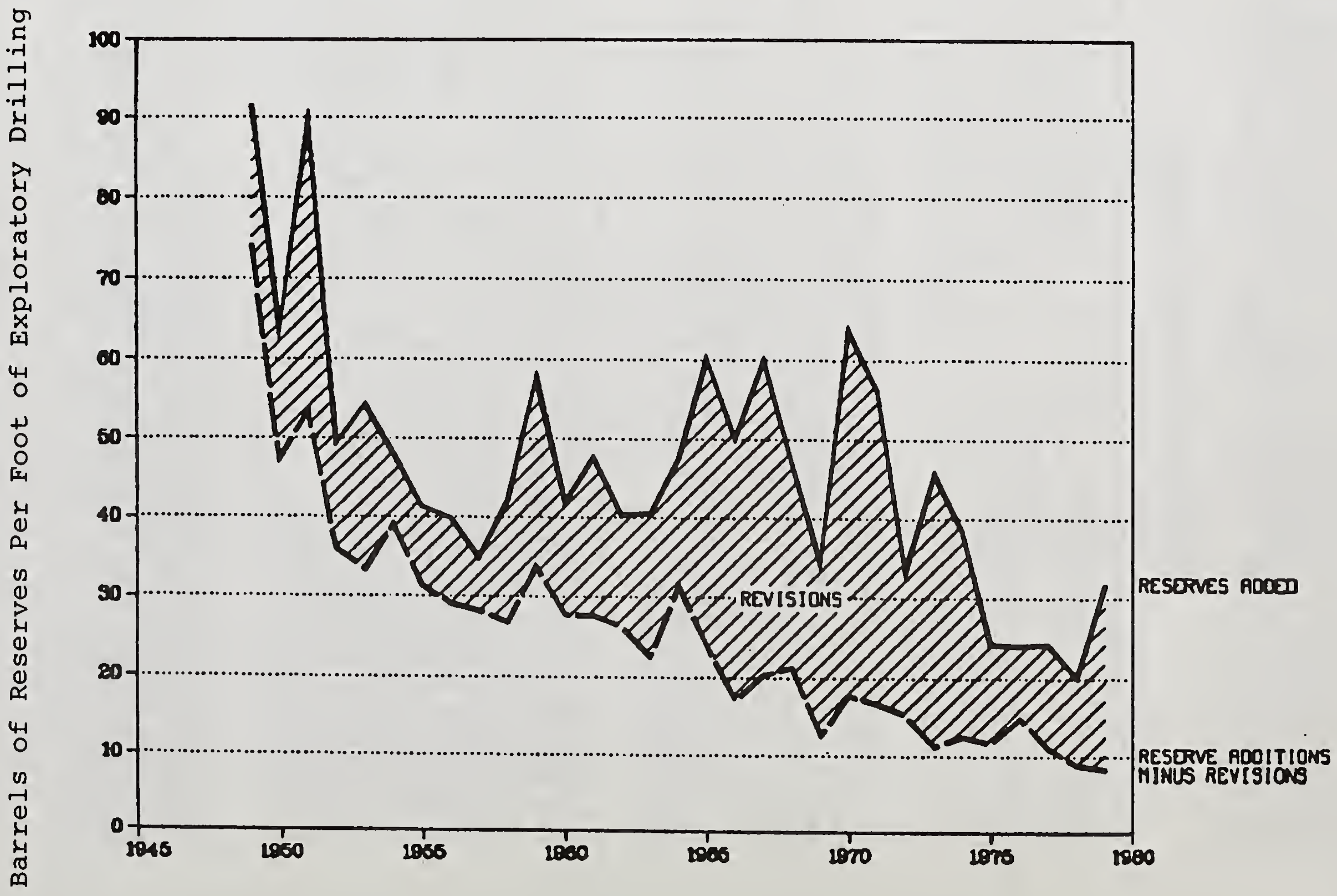

Figure 2. Finding Rates

$$F(W,A) = FU(A)*(1-\exp(C(A)*W))$$

where W is the cumulative number of wildcat wells drilled, A is the area size of a field, FU (A) is the ultimate number of fields of size A, C(A) is an "efficiency of drilling" parameter, and F(W,A) is the number of fields of size A discovered after W wildcats.

FU(A) and C(A) need to be estimated from data on F(W,A) and W. Arps and Roberts describe three different cases for deducing C(A): (1) For random drilling, C(A)=A/B where B is the basin or search area; (2) For pattern drilling, C(A) = infinity or C(A) = A/B depending on the well spacing; and (3) For drilling based on geological or geophysical analysis, C(A) = 2*A/B. That is, geology and geophysics in effect increase the search efficiency or decrease the search area by a factor of two. Since more than two decades have passed since the initial work, it would be worthwhile to reexamine this model and its predictive ability.

Play Analysis

A play is a series of discoveries resulting from one geologic idea or concept usually geographically confined. For more than 20 years, Kaufman and his collegues have been modeling plays ([12], [13], [14], [15], [60], [61], [63]). The discovery model is based on four postulates:

1. The size distribution (in barrels or million cubic feet) of petroleum deposits in pools within a subpopulation is lognormal.

2. Within a subpopulation, the probability that the "next" discovery will be of a given size (in barrels or million cubic feet) is equal to the ratio of that size to the sum of sizes of as-yet-undiscovered pools within the subpopulation.

3. The probability that an exploratory well will discover a new pool is a function of the number of non-dry and dry drilling targets.

4. Interarrival times between successive plays are uncertain quantities. The mean time between two successive plays, measured on a scale of cumulative exploratory wells drilled, (a) increases with an increase in the proportion of wells drilled extensively subsequent to the beginning of the first of these two plays, and (b) increases as the volume of unexplored sediment in the province decreases.

The most successful part of these studies has been the ability to verify the lognormal assumption (Postulate 1) and the sequencing of discovery size (Postulate 2). Two significant aspects that these studies do not address are the time sequencing and the aggregation of plays. United States data in general are not available in a form in which the above postulates can be tested. Only in a few instances are data available in a form amenable to proper analysis. Further, whether the analogy between the United States and other countries can be made is questionable. Plays in the United States compared to other countries normally involve more drilling and smaller, hence more, leases. Consequently, subplays could be created based on lease holdings. Further, the rules for sharing of exploration information differ among countries.

The question of aggregation of plays was addressed inadvertently by Benjamini et al. when attempting to test the Kaufman model on Kansas discovery data [18]. Although aware that the data contained the aggregation of 3 to 5 plays, Benjamini et al. set out to test Kaufman's single play model and found that it did not fit the data. Kaufman and Wang then showed that by the aggregation of 3 to 5 plays using the Kaufman model, the results of Benjamini et al could be obtained [62]. Although not the original purpose of the research, these results demonstrated that the aggregation of plays cannot be modeled as a play.

Permian Basin Study

In mid-1976, the Interagency Oil and Gas Supply Project was established to extend the work of USGS Circular 725 by taking a closer look at several basins: the Permian in west Texas and southeast New Mexico; the Gulf of Mexico; and the Atlantic's Baltimore Canyon. The Permian Basin study has been completed [57]. The report contains detailed assessments of what has been found, what is yet to be found and the costs of extraction. The Arps-Roberts model was chosen as the discovery process model and enhanced by adding depth categories. Detailed cost estimates were made for drilling and operating in the basin and cost curves were developed terminating at $40 (in 1977 dollars). Methods and potential of enhanced recovery were also examined. As a result, the USGS estimate of recoverable resources in the Permian Basin was reduced by more than half. Since this is the only onshore region that was or is planned for study, several caveats should be issued. First, the Permian Basin is only part of the United States, and is not necessarily representative of other geologic regions. Second, alternative methodologies and new data could change the assessments.

Long-Term vs. Short-Term Finding Rates

In the early 1960's, the domestic petroleum production industry started to "close up shop." In 1970, the average number of seismic crews hit an over 20-year low of 195. Levels three times higher were sustained in the fifties. In 1971, the average number of rotary rigs in use, the number of exploratory wells, and total wells drilled hit record lows. Levels two or more times higher were sustained in the fifties. In real terms the price of crude oil had been dropping from more than $4.50 (1972 dollars) in the fifties to about $3.50 in the early seventies. The domestic industry was depressed. In 1973, the embargo hit. By 1974 the real price of upper tier crude oil had more than doubled. New price controls were instituted.

Throughout the 1970's, the seismic crew and rotary rigs in use grew at more than 8 percent per year. Total drilling footage grew at more than 6 percent per year. Exploratory footage grew at less than 4 percent per year. The real cost of drilling a well grew by more than 6 percent per year. The real price of a barrel of crude oil grew at more than 8 percent per year, but after the 1974 jump, the growth was less than 2 percent per year. What will happen in the 1980's? In September 1980, rotary rigs hit a record high of 3,138. Total drilling in 1980 was 40 percent higher than for the same period in 1979.

The post embargo finding rate (measured in reserves additions per total exploratory footage) for crude oil has been in the twenties. In 1979, the rate was up to 35. There is virtual unanimous agreement among analysts that due to the depletion effect, the finding rate should decrease as exploratory drilling increases (with appropriate caveats) over the long run, but in the short run, there are hills and valleys. One of the most hotly debated questions of the past several years is: should the 1975-1978 low finding rates be used as a basis of a long-run trend, or should they be considered a temporal anomaly? When a simple model such as

$$R = RU*(1-\exp(-b*CEFT))$$

(where RU is the ultimate amount of resource, CEFT is the cumulative exploratory footage, b is a coefficient representing the depletion effect, and R is the cumulative resource discovered as a function of RU and CEFT), is used to estimate finding rates, the residuals are almost always serially correlated. The recent finding rates are well below the long-term trend estimated by a nonlinear least squares fit of the equation. The next section will present several reasons why the post-embargo trends could be a temporal anomaly.

Risk Aversion, Prices, and Success Rates

In 1975, Drew hypothesized the existence of two components of wildcat drilling: ambient and cyclical [32]. Using data from the Powder River Basin, ambient drilling was defined as the long-term systematic exploration program and was found to be characterized by low success rates and high finding rates (measured in reserves of crude oil per wildcat well). Cyclical or transient drilling was associated with the initiation of plays (e.g., the Minnelusa Sandstone play of the late 1950's and early 1960's, and the Muddy Sandstone play a decade later). This type of exploratory drilling is initiated by a large discovery, is transient in nature, and usually lasts about 3 to 5 years. It was found to be characterized by a surge in activity close to the time of discovery, higher success rates (34 percent higher) and lower finding rates (about 1/3 of ambient). This is evidence of strong risk aversion and/or the "herd" instinct. With some modifications this theory can be adapted to exploratory drilling for the entire country.

When a play is initiated by a discovery, almost immediately an inventory of prospects associated with the play is upgraded and becomes less risky. Similarly, when the price (net of tax) of crude oil is increased, the inventory of prospects that are considered economic (in a probabilistic sense) grows. This inventory addition is lower risk since it is usually associated with known fields and geologic horizons. But it results in low reserve additions, that is, this inventory consists of the small pools and extensions to the thin sands of the reservoir. Analogously this risk aversion process would be characterized by high success rates and low finding rates, exactly what has occurred since 1973. This theory argues that finding rates in the short term fall as the price rises. Under this hypothesis, this trend will continue as long as the price continues to rise and rigs are in short supply. With the phaseout of price controls, the eventual phaseout of the windfall profit tax and the recent forecasts of imported oil prices, the price should rise through the remainder of the century.

Majors and Independents

Exploration strategies differ depending on the motivation and objectives behind the search. Perhaps the most discussed difference is between the majors and the independents. Depending on how a major is defined, there are 8 to 50 companies. The independents are a collection of everyone else. The majors need crude to keep their downstream activities operating at high levels. Further, the majors, because of size and personnel, are in a better position to undertake high-risk and high-technology ventures. A successful venture that could double the company size of an independent may be insignificant to a major. Tax differences, risk aversion, and the utility of the smaller payoffs, therefore, differentiate the independents from the majors.

If the above hypothesis is true, the majors would be drilling high-cost, high-risk (low success rates) prospects and finding at high rates when compared to the independents. Data presented in Table 5 support this hypothesis (neglecting the difference of opinion between the Census Bureau and the API/AAPG on well classification). These statistics are basically the same as those of the 1977 Census survey. The larger companies drill the deeper and more costly wells and have lower success rates. To fully test this hypothesis, it is necessary to know the amount found. Unfortunately, this information is not easily obtained. The majors assess risk and profitability on an international basis and have in the past turned their operations away from the United States when the opportunity presented itself. As a caveat, however, the same data also could lead to the conclusion that the large companies are not as efficient or as cost effective as the smaller independents.

Table 5. Companies Ranked By Sales of Crude Oil, Condensate, and Natural Gas

	Number of Companies			
	First Eight	First Fifty	Ninth thru Smallest	U.S. Total
Exploration Wells	676	1,725	9,095	9,771
Success Rate	0.30	0.36	0.47	0.44
Average Depth (feet)	9,750	8,920	6,490	6,723
Average Cost (Thousands of Dollars)	1,837	1,225	379	452

Source: Annual Survey of Oil and Gas, 1978, Bureau of the Census, Tables 4B and 17.

8. PRODUCTION

Once a field has been discovered and initial operations are established, the operator must manage the property with sound engineering, economic judgment, and according to State (or other government) regulations. One objective is to establish a level of production that fully utilizes the capacity of the lease equipment. That is, the decision to drill additional development wells during the mid-life of a field is usually taken when existing well production is declining. The proper decision is to examine the discounted cash flow of each additional well. Further, the operator must decide whether and when to drill for new pools and extensions that are riskier, or whether to simply increase the production rate by infill drilling.

Once a "find" occurs on a property, the strategy changes to delineation and development. There are two court rulings that apply. They are the "rule of capture" and the requirement to produce "in paying quantities." Unless the mineral rights to the entire reservoir are clearly in possession of a single owner, there is a great incentive to drill (and produce quickly) due to the "rule of capture." Unitization and production regulation can control overproduction, but there are numerous State regulations about unitization that tend to deter withholding of production (see, for example, McDonald [73]).

Production from a well is usually modeled using a exponential, hyperbolic, or harmonic decline function. The most common is the exponential that is represented by a constant production to reserves (P/R) ratio. For the Lower-48 the P/R ratio has been climbing from .08 in the early 1960's at a rate of about 4 percent a year. If this trend were to continue the P/R ratio would be about .21 by 1990. This statistic has been largely taken for granted by modelers, but needs careful study. Aggregate models usually do not address the P/R ratio as a function of price, but on a micro basis the discounted cash flow is an important determinant in infill drilling. Ideally, this process should be modeled at the reservoir level and aggregated, but would require significant attention to detail.

Secondary recovery techniques such as waterflood and natural gas reinjection are now standard practice. The question arises whether post-1973 environmental and technological advances have changed the timestaging of these methods. Wells are required for injection. Hence, the demand for wells comes from several sources and the operator must choose properly between the various opportunities based on discounted cash flow. The reporting of reserve additions and production by recovery method would be helpful in analyzing this problem.

Double Counting and Undercounting

The various approaches to projections require estimation of the potential resource base. This information can often subtly appear twice when production from different sources is estimated independently. For example, the resource potential for gas from tight sands may be counted in the resource base for conventional gas. In another example, the reserves from steam drive may be included in proved reserves and at the same time projected by an enhanced recovery methodology.

There are two basic approaches to enhanced oil recovery (EOR) forecasting. A simple, but not very appealing, approach is simply to increase the recovery factors in the conventional model and call the resulting increase enhanced oil recovery. A second, but more complex, approach is to develop a separate data base and model for enhanced recovery. This model would allow for analysis of the recent enormous change in incentives created by the WPT and the front-end incentives program. Care must be taken not to include proved reserves from successful EOR projects (e.g., steam drive) in the conventional model and the EOR model. Currently, it is difficult to determine which portion of the revisions are due to the EOR methods, although EIA will collect this information beginning with its 1980 survey.

The reporting of natural gas statistics has similar definitional difficulties. Natural gas is reported on a "wet" (before liquids extraction) and a dry basis. Liquids include crude oil, natural gas liquids, and condensate, but the latter two are not always counted. The difference between natural gas liquids and condensate is now defined by regulations. It is often difficult to compare forecasts due to these definitional problems.

Secton 107 of the Natural Gas Policy Act (NGPA) contains a provision for deregulating gas from wells deeper than 15,000 feet, and from geopressured brine, coal seams, or Devonian shale high-cost gas. There are estimated large quantities of natural gas in tight sand reservoirs, coal seams, shale, and geopressured brines. Since technological and financial problems have inhibited high-cost gas production there is little historic precedence for production. These sources are referred to as unconventional. Studies of these categories are being undertaken to assess resource and production potential. Whenever the results of a specially focused study are combined with a more general forecasting technique, the risk of double counting becomes great. For example, there may already be reserves or drilling from these sources embedded in the data series used to forecast conventional production.

Co-Product Effects

Natural gas and liquid petroleum emerge from the ground together. The costs of producing gas and oil are joint until the two products reach the surface. At the surface, the two receive different treatment. Natural gas can be flared, reinjected for pressure maintenance, or marketed via pipeline. Petroleum liquids generally are shipped to refineries via pipeline, water, or ground transportation. Due to regulatory, physical, and safety restrictions, liquids and natural gas are only partial substitutes. Natural gas has always been cheaper than any of the liquids on a heat value basis because natural gas does not compete with liquids in many markets -- for example, for transportation fuel and for residential and commerical heating outside the proximity of natural gas pipelines. When liquid prices are high, the joint cost effect lowers natural gas prices. Care must be taken to include the price of liquids in the price of products emerging from a gas well and vice versa.

Regulatory Crosswalks

How should regulatory impacts be assessed? Since data series giving historical evidence of behavior under new or proposed regulations and taxes do not exist, impacts must be assessed by postulating models of behavior on <u>a priori</u> grounds.

To project impacts, a matrix of "sharing" coefficients (same properties as a Markov matrix) is often constructed so that forecasted quantities can be converted into regulated quantities (using historical data). More importantly, the incentives are almost always changed by regulations and taxes.

9. SOFTWARE AND MODELING

Different supply models and model structures have been compared in several studies [23], [24], [56]. There has been much debate over the technique and categorization of the models (i.e., linear programming, econometric, structural, probabilistic) and how they perform. The arguments at the overall model level generate a good deal of heated discussion, but miss the point. The proper perspective is to look at which techniques are better for what components of the model. Almost all models for oil and gas forecasting are implemented in Fortran on digital computers. Often these models are reviewed as black boxes. That is, the modeler or analyst, unless participating in coding the model, seldom examines the code to discover exactly what the model does. Usually reading the code is considered a long and arduous task or virtually impossible, since much of the coding and design is behind the state-of-the-art in programming. Further, when the analyst wants to make a change to the model, the change often requires a coding modification. In order to test various hypotheses and examine alternative model structures, a modular hierarchical software system needs to be designed where, for example, changing the form of the finding rate or the production function is only slightly more difficult than changing input parameters such as the discount rate. In this way many theories can be quickly examined and evaluated.

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Appendix A

Definitions

This appendix is provided to establish definitions for terms that are often ambiguous.

<u>Barrel</u> — Standard barrel of 42 U.S. gallons, used as an oil measure, abbreviated as Bbl.

<u>Oil or gas in-Place</u> — The total amount of oil or gas contained in a reservoir, a portion of which will remain in the reservoir upon abandonment for economic or technological reasons.

<u>Oil or gas pool</u> (reservoir) — A discrete unit of porous, permeable rock containing oil and or gas and distinguished by a single pressure system so that withdrawal of fluids from any part of the reservoir affects the pressure in all other parts. The terms "reservoir" and "pool" are synonymous and are used interchangeably.

<u>Oil or gas field</u> — Any area underlain by one or more oil and/or gas pools (reservoirs) that are recognized as being part of a common geologic or production unit. Where only one reservoir is involved, the terms "field" and "pool" (or "reservoir") may be used interchangeably to designate the same unit.

<u>Basin</u> — A large, bowl-shaped subsurface geologic feature formed by downwarping of the underlying basement rock and filled with sedimentary rocks. Large basins such as the Permian Basin may be divided after initial formation by uplifts and platforms which in effect create other basins (such as the Midland and Delaware Basins) within the original structure.

<u>Province</u> — A rather loosely defined term implying a region of common geologic character that contains one or more basins.

<u>Resource</u> — A concentration of naturally occurring solid or liquid petroleum or petroleum-like

material, or natural gas, in or on the Earth's crust in such form that economic extraction is currently or potentially feasible. The resource includes all the material in-place in a deposit.

<u>Discovered resources</u> Resources, and reasonable extensions thereof, whose location, quality, and quantity are known from drilling and geologic evidence supported by engineering measurements.

<u>Undiscovered resources</u> Resources surmised to exist on the basis of broad geologic knowledge and theory.

<u>Reserve</u> That portion of the resource base from which a usable mineral and energy commodity can be economically extracted at the time of estimation. Such commodities include but are not necessarily restricted to petroleum, condensate, natural gas, tar sands, and naturally occurring asphalt, without regard to mode of occurrence. In terms of the resource classification nomenclature, this includes proved, indicated, and inferred categories.

<u>Proved reserve</u> Material for which estimates of the quality and quantity have been computed from analyses and measurements from closely spaced and geologically well-known sample sites.

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STATISTICS DEPARTMENT
DRILLING STATISTICS SECTION

Appendix B

INDIVIDUAL WELL TICKET

API - AAPG(CSD)

B 48217

STATE													
COUNTY													
STATE CODE		COUNTY, PARISH OR AREA CODE			WELL CODE ORIGINAL HOLE					DIREC-TIONAL SDTR CODE		CSD DIST. CODE	
2	3	4	5	6	7	8	9	10	11	12	13	14	15

☐ CHECK ONLY IF CORRECTION OR ADDITIONAL DATA ON PREVIOUSLY REPORTED WELL

◀ API WELL NUMBER - SEE API INSTRUCTION MANUAL
DUPLICATE COLUMNS 2-15 IN EACH CARD CODED ON THIS PAGE.

DISTRIBUTION

1-WHITE - TO API HEADQUARTERS
2-BLUE - ORIGINATOR RETAINS
3-PINK - TO API HEADQUARTERS AFTER CLASSIFICATION BY CSD
4-YELLOW - TO CSD DISTRICT FILE

API DATA

CARD NO	1-OFFSHORE	COMPLETION DATE MO.		COMPLETION DATE YR.		MONTH REPORTED		FOOTAGE DEEPEST TOTAL DEPTH					FOOTAGE BYPASS-SDTR OR DEEPER DRLD. FOOTAGE					BOTTOM OF DEEPEST PRODUCING INTERVAL-FT					WELL CLASS	NO. OF COMP. OIL	NO. OF COMP. GAS	OPER-ATOR CODE			TYPE LEASE	OPERATOR NAME - WELL NUMBER - LEASE NAME
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45–80
1																														

WELL LOCATION DATA

CARD NO	CDS	X COORDINATE OR NORTH LATITUDE							Y COORDINATE OR WEST LONGITUDE								SEC-TION			TOWNSHIP					RANGE					OTHER LOCATION DATA SUCH AS OFFSHORE AREA AND BLOCK, QUARTER CALL, FOOTAGE CALL OR OTHER WHERE DATA FOR COL. 16-44 NOT AVAILABLE
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45–80
2																							—					—		

AAPG (CSD) DATA

CARD NO	GEOLOG-ICAL BASIN CODE			WELL CLASS INITIAL	WELL CLASS FINAL	WELL CLASS OWDD	TOTAL EXPLORATORY FOOTAGE DRILLED					DEEPEST FORMA-TION CODE			DEEPEST FORMATION NAME					DEEPEST PAY CODE			DEEPEST PAY NAME					EST. UTL. YIELD OIL	EST. UTL. YIELD GAS	NAME OF FIELD	REMARKS
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45–59	60–80
3																															

AAPG (CSD) DISTRICTS

01 - ARKANSAS
02 - SOUTHEASTERN ATLANTIC STATES
03 - CENTRAL EAST COAST
04 - GULF STATES, SOUTHERN
05 - ILLINOIS
06 - INDIANA
07 - NORTHEAST U.S.
08 - KENTUCKY
09 - LOUISIANA, NORTHERN
10 - LOUISIANA, SOUTHERN & OFFSHORE
11 - MICHIGAN
12 - MID-CONTINENT, NORTHERN
13 - NEW MEXICO, EASTERN
14 - OHIO
15 - OKLAHOMA
16 - ROCKY MOUNTAINS, CENTRAL
17 - ROCKY MOUNTAINS, NORTHERN
18 - ROCKY MOUNTAINS, SOUTHERN
19 - TENNESSEE
20 - TEXAS - DISTRICT 1
21 - TEXAS - DISTRICT 2
22 - TEXAS - DISTRICT 3
23 - TEXAS - DISTRICT 4
24 - TEXAS - DISTRICT 5 & 6
25 - TEXAS - DISTRICT 7B
26 - TEXAS - DISTRICT 7C, 8 & 8A
27 - TEXAS - DISTRICT 9
28 - TEXAS - DISTRICT 10
29 - WEST COAST INCLUDING ALASKA
30 - WEST VIRGINIA
34 - GULF OF MEXICO NORTH
35 - ATLANTIC COAST OFFSHORE

CODE EXPLANATION

CARD 1: COL. 28-32 SEE API INSTRUCTION MANUAL
COL. 38 E - EXPLORATORY
D - DEVELOPMENT
C - STRAT. (CORE)
R - SERVICE
X - OWDD-EXPLORATORY
V - OWDD-DEVELOPMENT
Y - OWDD-SERVICE
COL. 44 1 - FEE
2 - STATE
3 - FEDERAL
4 - CONTESTED OR UNDETERMINED
5 - UNIT OR OTHER

CARD 2: COL. 15 C - STATE PLANE X-Y COORDINATES
D - LATITUDE AND LONGITUDE IN DEGREES TO 5 DECIMALS
S - LATITUDE AND LONGITUDE IN DEGREES MINUTES-SECONDS TO 1 DECIMAL OF SECONDS

CARD 3: COL. 15-18 SEE AAPG (CSD) INSTRUCTION MANUAL
COL. 19-21 SEE "AAPG (CSD) WELL CLASSIFICATIONS" ON BACK OF YELLOW COPY
COL. 27-29 SEE AAPG (CSD) INSTRUCTION MANUAL
COL. 35-37 SEE AAPG (CSD) INSTRUCTION MANUAL
COL. 43-44 SEE "ESTIMATED ULTIMATE YIELD" ON BACK OF YELLOW COPY

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Appendix C

AAPG (CSD) WELL CLASSIFICATIONS

CODE
NO.

1 — NEW-FIELD WILDCAT - A new-field wildcat is a well located on a structural feature or other type of trap which has not previously produced oil or gas. In regions where local geological conditions have little or no control over accumulations, these wells are generally at least two miles from the nearest productive area. Distance, however, is not the determining factor. Of greater importance is the degree of risk assumed by the operator, and his intention to test a structure or stratigraphic condition not previously proved productive.

2 — NEW-POOL WILDCAT - A new-pool wildcat is a well located to explore for a new pool on a structural feature or other type of trap already producing oil or gas, but outside the known limits of the presently producing area. In some regions where local geological conditions exert an almost negligible control, exploratory holes of this type may be called "near wildcats." Such wells will usually be less than two miles from the nearest productive area.

3 — DEEPER-POOL TEST - A deeper-pool test is an exploratory hole located within the productive area of a pool, or pools, already partly or wholly developed. It is drilled below the deepest productive pool in order to explore for deeper unknown prospects.

4 — SHALLOWER-POOL TEST - A shallower-pool test is an exploratory well drilled in search of a new productive reservoir, unknown but possibly suspected from data secured from other wells, and shallower than known productive pools. The test is located within the productive area of a pool or pools, previously developed.

5 — OUTPOST (EXTENSION) TEST - An outpost is a well located and drilled with the expectation of extending for a considerable distance the productive area of a partly developed pool. It is usually two or more locations distant from the nearest productive site.

6 — DEVELOPMENT WELL - In general, a development well is a well drilled within the proved area of an oil or gas reservoir to the depth of a stratigraphic horizon known to be productive. If the well is completed for production, it is classified as an oil or gas development well. If the well is not completed for production, it is classified as a dry development hole.

7 — STRATIGRAPHIC TEST - A stratigraphic test is a drilling effort, geologically directed, to obtain information pertaining to a specific geological condition that might lead toward the discovery of an accumulation of hydrocarbons. Such wells are customarily drilled without the intention of being completed for hydrocarbon production. This classification also includes tests identified as core tests and all types of expendable holes related to hydrocarbon exploration.

8 — SERVICE WELL - A service well is a well drilled or completed for the purpose of supporting production in an existing field. Wells of this class are drilled for the following specific purposes:

Gas injection (natural gas, propane, butane, or flue gas)
Water injection
Steam injection
Air injection
Salt water disposal
Water supply for injection
Observation
Injection for in-situ combustion

OWDD - An old well drilled deeper is a previously drilled hole which is reentered and deepened by additional drilling. Such wells are reported as either oil or gas wells if completed for the production of oil or gas; or as dry holes if sufficient quantities of oil or gas are not found to justify completion of the greater depth.

ESTIMATED ULTIMATE YIELD

This item must be filled in for all New Field Wildcat discoveries and successful Deeper Pool Tests. The yield indicated should reflect the possible overall significance of the discovery, based upon the best professional judgement of the CSD committeemen. Letter values are: A= over 50 million barrels or 300 billion cubic feet; B=25 to 50 million barrels or 150 to 300 billion cubic feet; C=10 to 25 million barrels or 60 to 150 BCF; D=1 to 10 million barrels or 6 to 60 BCF; E=less than 1 million barrels or less than 6 BCF; F= abandoned as non-profitable.

OIL/GAS SUPPLY MODELING CONSIDERATIONS IN LONG-RANGE FORECASTING

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Oil and gas supply modeling may not only generate forecasts on a "stand-alone" basis, but may provide input data and assumptions to large scale, long-range integrated energy-economy models. In such a framework, parameters that may not have seemed especially crucial in the stand-alone formulation may be found to have an unexpectedly great influence on the results. Additionally, it may become necessary to incorporate considerations that are usually omitted from conventional oil and gas supply models. This paper discusses two examples of this: the decline rate, or more generally the production profile, and the treatment of resource exhaustion.

This investigation arose in connection with a project for the Energy Information Administration (EIA). One part of the project was to incorporate the assumptions and input data of EIA's Midterm Oil and Gas Modeling System (MOGSMS), which has already been discussed in this symposium, into EIA's Long-Term Energy Analysis Program (LEAP).

LEAP is a large scale general equilibrium model of energy economy interactions. Its time horizon spans 50 years. In each time period, the energy-economy system is represented by a network of process nodes. At the "bottom" of the network are the resource process nodes. They accept quantities as inputs, and output the prices required to bring forth those quantities. (Fuller description of the basic general equilibrium model structure may be found in (1) while more detail on the resource submodels appears in (2)). The prices and quantities are then passed up the network. At the "top" of the network are demand process nodes which take prices as inputs and output quantities. The new quantities are passed back down through the network. If they are the same as those previously input to the resource process, convergence has been achieved, and the general equilibrium sought has been found. In the course of incorporating MOGSMS oil and gas assumptions and input data into the LEAP oil and gas resource processes, it became apparent that the decline rate plays a major role.

It is typically assumed, in oil and gas supply models, and also in LEAP, that the production from proved reserves will follow an exponentially declining pattern. Equivalently, a fixed fraction of reserves is produced each year. This fixed fraction, or production to reserves ratio, is called the decline rate. In LEAP, the decline rate, by determining production, implies levels of commitments (i.e., reserves that must be discovered and proved) that must be achieved. The cumulative commitment level is related to the capital investment required by an increasing function, reflecting the assumption that costs of discovery will increase as the resource base is depleted. Finally, the price is computed using discounted cash flow techniques and adding a bonus for scarcity rent; the decline rate determines the behavior of the revenue stream. Figure 1 depicts the role of the decline rate on the production schedule.

The importance of the decline rate can be illustrated by considering its impacts on commitment levels and the differential between minimum acceptable price and operating cost. First, the commitment levels vary as the inverse of the decline rate, which is a relatively small number, on the order of ten percent, so that small absolute differences in estimates of its value cause required commitments to vary widely. Suppose d is the decline rate, and R is initial reserves. Then production in year n is

$$R*d*(1-d)^{n-1}.$$

If q is the quantity required in a given year, then C, the commitment level required, is given by

$$C = q/d.$$

Comparing commitment level C when d = 0.11 with commitment levels C' when d = 0.08, one finds

$$\frac{C' - C}{C} = \frac{q*(1/d' - 1/d)}{q/d} = \frac{d}{d'} - 1 = 0.375.$$

This is a substantial difference, which is even more important when one considers the implications for oil-in-place discovered; normally only a third of oil-in-place is ultimately recovered. The potential difficulty is somewhat alleviated by the fact that the LEAP time frame is five years. Given q, one must determine the commitment level required to produce at an average quantity q over five years:

$$1/5 * C * \sum_{n=1}^{5} d*(1-d)^{n-1} = q$$

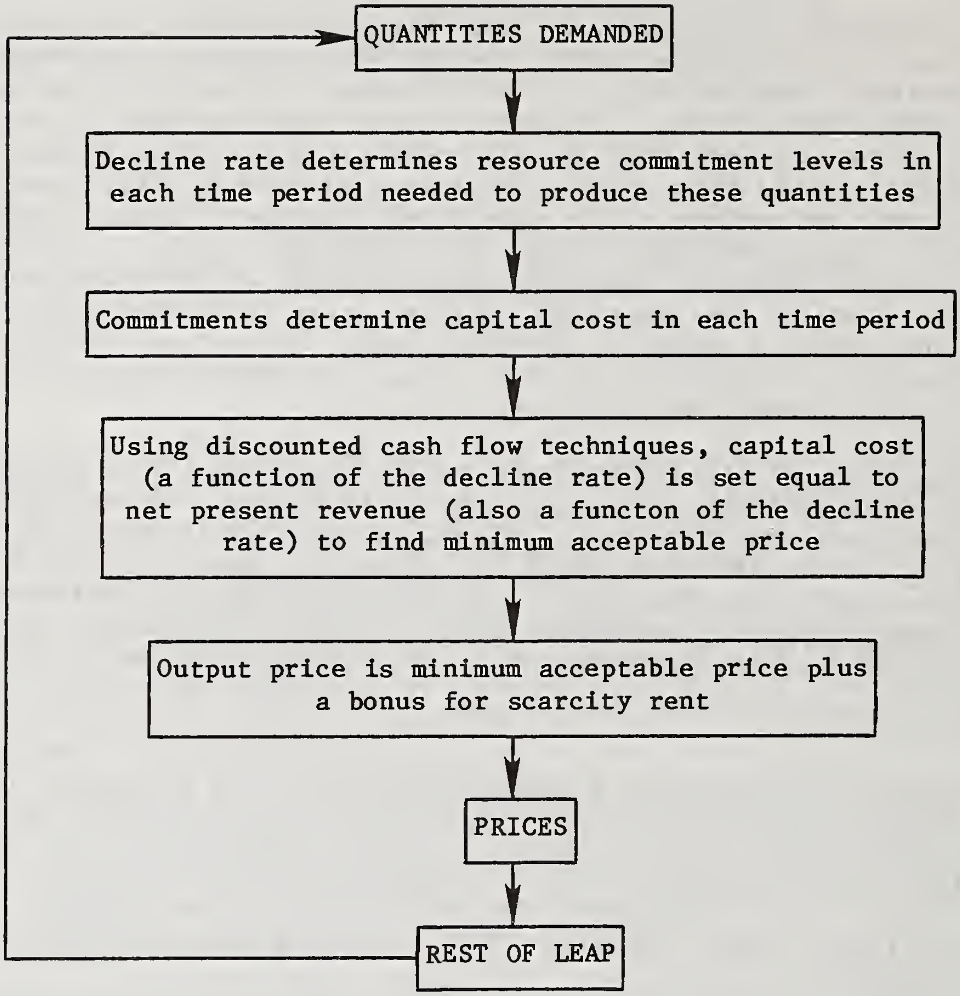

Figure 1
Functions of the Decline Rate in LEAP

or

$$C = 5q/[1-(1-d)^5].$$

Again comparing the commitment levels,

$$\frac{C' - C}{C} = \frac{[1-(1-d')^5]^{-1} - [1-(1-d)^5]^{-1}}{[1-(1-d)^{-5}]^{-1}}$$

$$= \frac{1-(1-d)^5}{1-(1-d')^5} - 1 = 0.295$$

Given this potential sensitivity, two questions arise: first, is there that much variation in the data, and second, does it matter in the long-run.

The answer to the first question is that the decline rate has not remained constant over time, as is seen in Table 1. These data are taken from the Energy Information Administration (EIA) Annual Report to Congress (Ref. 5, pp. 32 and 39). The behavior of this parameter has been erratic: it has not been monotonically increasing, as one might expect due to technological progress, and even between two consecutive years the difference in decline rates can be as much as 34 percent. Of course, the discontinuity in 1970-1971 is due to the Prudhoe Bay discovery, production from which could not begin until completion of the Alaskan pipeline. Cumulative production from Prudhoe Bay up to the beginning of 1978 was only 129.4 million barrels (Ref. 6, p. 216). When Prudhoe Bay reserves are removed from the calculation, the decline rate continues to rise (although not monotonically) to nearly 14 percent.

When one looks at regional data, for south Alaska and the Lower-48, the picture is even worse. Data collected for the 1977 update of the Midterm Oil and Gas Supply Modeling System(4)and listed in Table 2 show regional decline rates ranging from 9.9 to 19.1 percent. The weighted average is 13.4 percent. (These are the National Petroleum Council Oil and Gas regions, shown in Figure 2.) The differences in regional characteristics could greatly affect the future behavior of the average decline rate, since some regions have far better prospects than others. The conclusion is that decline rate data show a significant amount of variation, both regionally and over time.

The next question is, what effects does this uncertainty have over an extended time horizon? Will it all even out in the end?

Table 1
U.S. Production to Reserves Ratios, 1949-1978

Year	Reserves 10^9 bbl	Production 10^6 bbl/day	Ratio
1949	23.3	5.05	0.0791
1950	24.6	5.41	0.0803
1951	25.3	6.16	0.0889
1952	27.5	6.26	0.0831
1953	28.0	6.46	0.0842
1954	28.9	6.34	0.0801
1955	29.6	6.81	0.0840
1956	30.0	7.15	0.0870
1957	30.4	7.17	0.0861
1958	30.3	6.71	0.0808
1959	30.5	7.05	0.0844
1960	31.7	7.04	0.0811
1961	31.6	1.18	0.0829
1962	31.8	7.33	0.0841
1963	31.4	7.54	0.0876
1964	31.0	7.61	0.0896
1965	31.0	7.80	0.0918
1966	31.4	8.30	0.0965
1967	31.5	8.81	0.1021
1968	31.4	8.66	0.1007
1969	30.7	8.78	0.1044
1970	29.6	9.18	0.1132
1971	39.0	9.03	0.0845
1972	38.1	9.00	0.0862
1973	36.3	8.78	0.0883
1974	35.3	8.38	0.0866
1975	34.2	8.01	0.0855
1976	32.7	7.78	0.0868
1977	30.9	7.88	0.0931
1978	29.5	8.67	0.1073

Table 2
Regional Decline Rates

Region	Reserves	Decline Rate
1	386	0.154
2	2,728	0.099
2A	862	0.099
3	356	0.178
4	1,259	0.150
5	5,948	0.130
6	5,719	0.138
7	1,700	0.138
8-10	571	0.154
11	40	0.191
11A	0	0.155
	21,537	

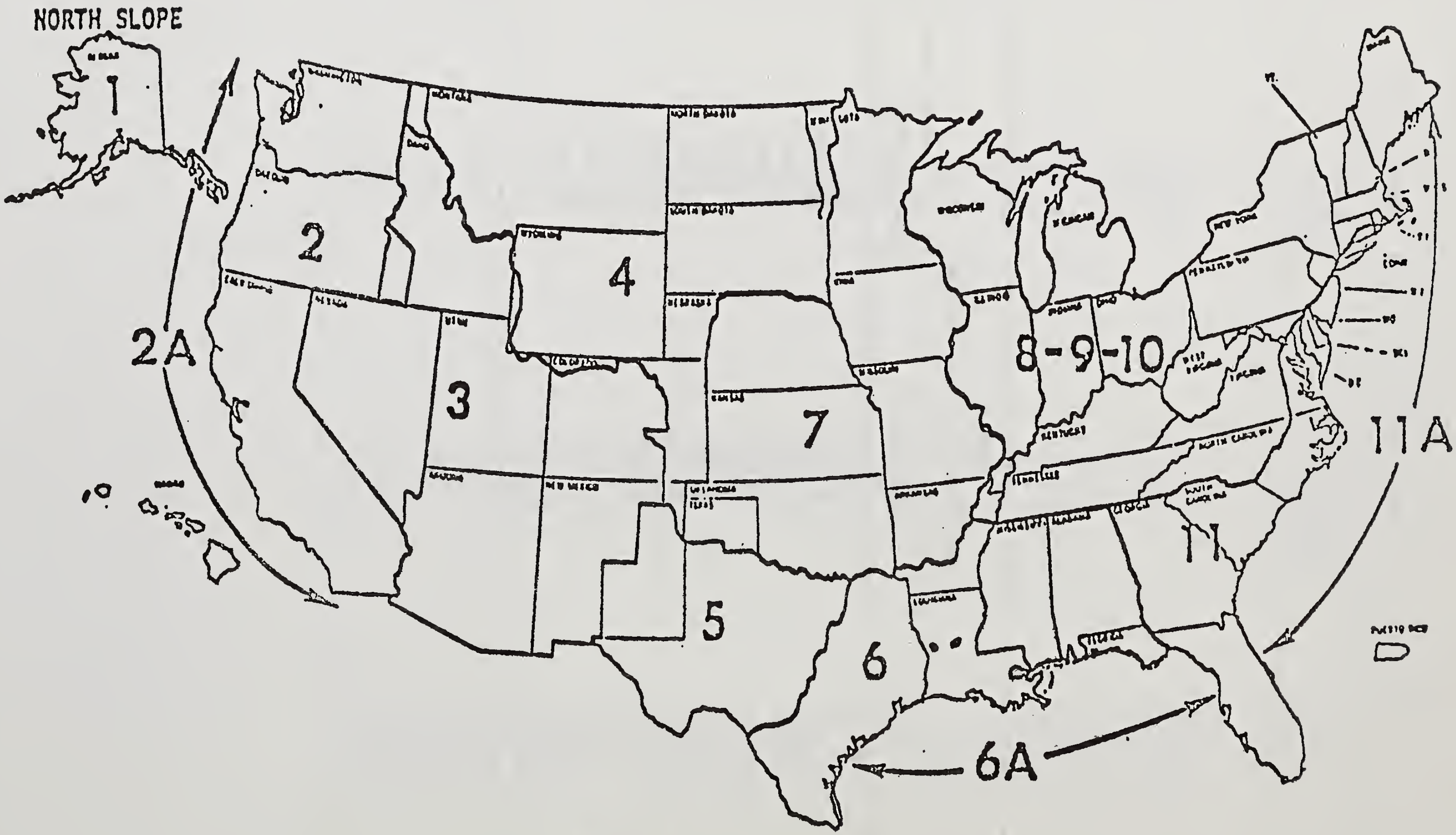

Figure 2

Oil and Gas Regions
(National Petroleum Council)

Consider cumulative production over a 30 year well life for different decline rates, as shown in Table 3. The differences are especially marked in the early years, e.g., a 50 percent difference after five years between cumulative production given decline rates of eight and 15 percent. One can see that production levels each year will be higher for some years given a higher initial decline rate. For the first nine years, yearly production with d = 0.13 exceeds that at d = 0.08. One can calculate the "break-even" year n at which production at the lower rate, d_1, equals or exceeds production at the higher decline rate, d_2:

$$d_1 (1-d_1)^{n-1} \geq d_2 (1-d_2)^{n-1}$$

when

$$n \geq 1 + \ln(d_1/d_2)/\ln[(1-d_2)/(1-d_1)], \text{ for } d_1 < d_2.$$

Comparing rates of 0.10 and 0.13, one finds $n \geq 8$ years; this holds even for $d_1 = 0.12$, $d_2 = 0.13$.

The immediate implication is that for normal discount rates, the minimum acceptable price, using discounted cash flow techniques, will be higher for the project with the lower decline rate. (In MOGSMS, total yearly discoveries in a region are treated as a single project.) How much higher can be estimated by looking at the expression for present value of discounted cash flow as a function of discount and decline rates, PV(d,δ), assuming constant prices and operating costs or a constant differential.

$$PV(d,\delta) = \sum_{k=1}^{L} (p-\pi)d(1-d)^{k-1} \delta^{k-1}$$

where

p = minimum acceptable price,
π = operating cost,
d = decline rate,
δ = discount factor,
L = lifetime.

This can be rewritten as:

$$PV(d,\delta) = (p-\pi) \left\{ d* \frac{(1-[1-d)\delta]^{L})}{1-(1-d)\delta} \right\}.$$

Table 3
Cumulative Production, $1 - (1 - d)^n$

N	d=.08	d=.10	d=.11	d=.13	d=.15
1	0.08	0.10	0.11	0.13	0.15
2	0.1536	0.19	0.2079	0.2431	0.2775
3	0.2213	0.271	0.2950	0.3415	0.3859
4	0.2836	0.3439	0.3726	0.4271	0.4780
5	0.3409	0.4095	0.4416	0.5016	0.5563
10	0.5656	0.6513	0.6882	0.7516	0.8031
15	0.7137	0.7941	0.8259	0.8762	0.9126
20	0.8113	0.9284	0.9028	0.9383	0.9612
25	0.8756	0.9282	0.9457	0.9692	0.9828
30	0.9180	0.9576	0..9597	0.9847	0.9924

The term in brackets is the present value of production,

$$PVP(d,\delta) = d_*[1-((1-d)\delta)^L]/[1-(1-d)\delta].$$

Values of this present value of production for different decline rates are shown in Table 4 with a lifetime L=30. For $\delta = 0.10$, the present value of production when d = 0.13 is about 1.3 times the present value of production when d = 0.10. In general, at the same discount rate the present value of production is proportional to the decline rate, while for the same decline rate the percentage difference in present value of production is proportional to the percentage difference in discount rate (e.g. increasing δ from 0.05 to 0.10 causes roughly a five percent increase in present value of production).

Table 4
Present Value of Production

Decline Rate	δ = 0.05	δ = 0.10	δ = 0.15
0.08	0.0839	0.0881	0.0928
0.09	0.0943	0.0990	0.1042
0.10	0.1047	0.1099	0.1156
0.11	0.1151	0.1207	0.1269
0.12	0.1255	0.1316	0.1382
0.13	0.1359	0.1424	0.1495
0.14	0.1463	0.1532	0.1607
0.15	0.1567	0.1639	0.1719
0.16	0.1670	0.1747	0.1831
0.17	0.1774	0.1854	0.1942
0.18	0.1877	0.1961	0.2052
0.19	0.1980	0.2067	0.2163
0.20	0.2083	0.2174	0.2273

The proper value to use for the discount rate has often been debated, so one might wonder whether uncertainties in the decline rate are overshadowed by uncertainties in this parameter. This seems not to be true. For example, if the decline rate is 13 percent and the discount rate changes from 10 to 15 percent, the decline rate that would yield the same present value of production as before is 12.4 percent (L = 30). There does not exist a positive discount factor which equalizes present value of production with a 10 percent discount factor and 10 percent decline rate to present value of production with a 13 percent decline rate.

In some energy models, what happens in future periods is discounted: for example in linear programming models where the objective is to minimize total discounted cost over the time horizon. But in LEAP, although prices are partly computed using discounted cash flow techniques, each time period is equally important. Within the LEAP resource submodel framework, the decline rate has a more profound effect than the discount rate, though the latter, being thought a more judgemental and political parameter, tends to be exposed and discussed to a far greater extent.

Pursuing the investigation of the effect of the decline rate on minimum acceptable price, if one equates the present value of cash flow for two different decline rates, one finds that the inverse of the ratio of present values of production equals the ratio of differentials between minimum acceptable price and operating cost.

$$(P_1 - \pi) * PVP(d_1, \delta) = (P_2 - \pi) * PVP(d_2, \delta)$$

$$\left(\frac{PVP(d_2, \delta)}{PVP(d_1, \delta)}\right)^{-1} = \frac{P_2 - \pi}{P_1 - \pi}$$

$$\left(\frac{PVP(d_2, \delta)}{PVP(d_1, \delta)}\right)^{-1} * \left(\frac{PVP(d_2, \delta)}{PVP(d_1, \delta)}\right)-1 * \frac{P_1 - \pi}{P_1} = \frac{P_1 - P_2}{P_1},$$

Ratio of present values of production	percentage change in present value of production	ratio of profit to price	percentage change in price

so if the decline rate is 10 percent instead of 13 percent, the differential between price and operating cost must rise by about 30 percent. Even a change from 13 to 12 percent involves an increase in the differential of 8 percent.

The decline rate affects the minimum acceptable price in another way. Within the LEAP methodology, it is assumed that an input curve relates capital investment to cumulative commitments. This capital investment then enters the minimum acceptable price computation (adjustments for technological change over time may be made). Suppose quantities $q_1, q_2, \ldots$ are demanded, and the decline rate is d. One can calculate commitment levels $R_1, R_2, \ldots$ assuming, without loss of generality, no initial reserves.

In the first year, one must have

$$q_1 = R_1 d.$$

In the second year, $R_1(1-d)$ of those reserves remain, so

$$q_2 = R_1(1 - d)d + R_2\, d.$$

In general,

$$q_r = \sum_{k=1}^{r} R_k\, (1 - d)^{r-k} d.$$

By induction,

$$R_1 = q_1/d$$

$$R_n = [q_n - (1 - d)\, q_{n-1}]/d,$$

so cumulative commitments at year n are

$$\sum_1^n \quad R_j = \sum_0^{n-1} q_j + q_n/d.$$

In any year n, the difference in commitment levels with decline rates d' and d is

$$q_n*(1/d' - 1/d).$$

With $d' = 0.10$, $d = 0.13$, the factor multiplying q_n is 2.3. In LEAP, the time period is five years, so the aggregation magnifies differences by a factor of five. Depending on the steepness of the capital cost curve, small changes in the decline rate mean large changes in commitment levels; for conventional oil and gas this curve becomes steep in the LEAP time because these resources are nearing exhaustion.

The preceeding discussion of the impacts of the decline rate may be generalized by considering production schedules which need not be exponentially declining. In practice, production from an oil well does not strictly follow this pattern, because of secondary recovery efforts and developmental drilling. The question is whether it is necessary to represent the production schedule explicitly. An alternative is to find a decline rate which yields the same present value of production. It is easy to see that this can be done, since

$$\begin{aligned} f(x) &= PVP(x,\delta) \\ &= \left(1-[(1 - x)\ \delta]^L\right) * \ x/[1-(1 - x)\ \delta\ ] \end{aligned}$$

is the product of two non-negative functions increasing in x for x in [0,1] and is therefore increasing for x in [0,1]. f(0) = 0, f(1) = 1 and f is continuous in [0,1], therefore if

$$0 < \sum_{k=1}^{L} q_k\, \delta^{\,k-1} = Q < 1,$$

where q_k is fraction of ultimate production in year k ($\Sigma\ q_k = 1$) then some (unique) value of x exists such that f(x) = Q. In fact, under not very stringent conditions, it is possible to find a decline rate which, given future prices and operating costs, will yield the same present value of net revenue as an arbitrary production schedule. One might anticipate that as long as the equalizing decline rate value is near the first year's production fraction, meaning that first year commitment levels will be about the same in either case, the approximation will suffice.

First, consider the case where production is constant over a 30 year life. Then d = 1/30 = 0.0333. The equivalent decline rate, at a 10 percent discount rate, is d' = 0.0334. If the time periods are aggregated to five years, the LEAP time step, d = 0.167 and d' = 0.17. The equivalent decline rate does turn out to be near the first period's production fraction. The resulting production patterns are shown in Table 5. Although cumulative production levels are reasonably close for the first two to three time periods, the ultimate production using the equivalent decline rate is only two thirds of the true value. This implies that commitment levels will be higher than if the actual production schedule was used. Prices will be higher and production lower than the "true" values. So while the decline rate formulation is mathematically tractable, it appears to yield a poor approximation in these cases.

Table 5
Production Patterns

Cumulative Production - 30 year life

Year	Constant Production	Equivalent Decline Rate
1	.0333	.0334
2	.0667	.0658
3	.1000	.0970
5	.1667	.1564
10	.3333	.2884
20	.6667	.4936
30	1.0000	.6396

Cumulative Production - 6 period life

Time Period	Years	Constant Production	Equivalent Decline Rate
1	5	.167	.170
2	10	.333	.310
3	15	.500	.428
4	20	.667	.525
5	25	.833	.606
6	30	1.000	.673

Although a constant production level pattern is more characteristic of coal mines than oil and gas reservoirs, the investigation above is relevant because enhanced oil recovery (EOR) projects follow a pattern similar to that of coal. A typical EOR schedule (Ref. 3, p. 31) is given below.

EOR Production Schedule
Steam Drive

Year	Percent of Incremental Recovery
1	12
2	22
3	22
4	20
5	14
6	10

In conclusion, the outputs of the LEAP resource submodel are quite sensitive to variations in the decline rate. The evidence is that this parameter's influence in the LEAP framework outweighs that of the discount rate, which is usually assumed to play a major role in long-term models.

Quantifying the decline rate is not a straight-forward matter. The national data show significant variation over time; regionally the dispersion is even greater. To some extent reservoir production schedules can be altered by drilling additional wells; one can expect this to happen if the economics provide appropriate incentives. An important future direction for oil and gas supply model development is investigation and representation of the regional and economic factors affecting oil and gas production schedules.

Another modeling consideration that deserves more attention is the effects of depleting the resource base. It is expected that conventional sources of oil and gas will be exhausted in the long-term time frame. What happens in the model when resources run out?

In a linear programming formulation, one can always impose constraints. Then no matter how high the marginal, or shadow, prices rise, no more of the resource will be forthcoming. Economists criticize this sort of result on the grounds that in the real world higher prices will in fact lead to reserve additions.

Another approach is to use a supply curve that rises very sharply as the resource nears depletion, with the hope that increasing prices will dampen demand. This mechanism is used within LEAP; when the resource is exhausted, the price is set to a level that represents some multiple (the actual value is an input parameter) of the last price on the supply curve.

There are two possible problems here, first, one ends up with artificially high prices being passed through the network, perhaps impeding convergence; second, if the ultimate price is not high enough, the model will behave as though an infinite quantity of the resource is available at this fixed price. On the other hand, the supply curves are probably higher than they should be because the sub-economic resource base is not considered. Data are lacking on how extensive it is now; in most oil and gas models, drilling results either in success or failure. There's no in-between. Either the discovery is economically exploitable or it is classified as completely dry. This is not realistic. In the long-term, with greater depletion and higher prices wells that were previously sub-economic will be brought into production. For this reason, current models probably overstate exploratory drilling requirements.

Like subeconomic deposits, enhanced recovery will become increasingly attractive. This option should be integrated into oil and gas supply submodel of a large scale energy-economy framework in order to maintain consistency between price and quantity available for resources extracted in primary, secondary, or tertiary phases of reservoir development. For oil, imposing a constraint on ultimate recovery that is lower than oil-in-place is open to question. After all, the resource is known to be present. Here, the economics set the limitations.

To summarize, two important areas for oil and gas supply model development and data collection are representation of the decline rate or production schedule from reserves, and the treatment of resource depletion including the subeconomic resource base and enhanced recovery. Both aspects receive relatively little attention in oil and gas supply models, but play key roles when the assumptions of these models are integrated into a long-term model such as LEAP.

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DISCUSSION

MR WOOD: John Wood, EIA. There are two things I'd like to ask. The first one would follow some of the remarks made yesterday. Am I right in assuming you're applying that analysis to the entire United States on a supply curve, or a major subdivision of the United States?

MS. CHERNIAVKSY: Yes. Usually the analysis is done regionally, using the National Petroleum Council regions, because the data are available for those regions. At other times, the data have been aggregated to a national supply curve.

MR WOOD: Okay. Well, I would suggest to you that you'd better look at that with extreme care because it looks like a case of badly stretching the assumptions that were mentioned yesterday. You looked at a discounted cash flow type analysis which is applied to an individual well, sometimes to a lease, perhaps with care to a field, and you applied it to a reigon, which isn't allowable.

My second question is, in the model, you use what you call the decline rate, which is really a production to reserve ratio. You know, that doesn't correspond to the actual production decline rate in any of those regions, necessarily, which may be higher or lower, depending on how new discoveries come into the production strain. So, it seems like you're ignoring an awful lot if you make assumptions that apply to a well -- and then extend them to an entire region of the United States.

MS. CHERNIAVSKY: Yes. This is a problem that has been mentioned, as you say, yesterday, that when you go from the smaller to the larger, it does tend to stretch the assumptions. [However these are the assumptions in the Midterm Oil and Gas Supply Modeling System, which were to be incorporated into LEAP].

MR. ALEXANDER: Alexander,DOE. You said you were concerned about the assumption of infinite resources [When using a high price to depress demand]. Well, it's possible that we might have infinite resources in the form of hydrogen. In time we'll be able to get hydrogen down to a competitive price. Why would you assume infinite resource availability is an unreasonable assumption?

MS. CHERNIAVSKY: It might be an unreasonable assumption for hydrogen; but for oil and gas, as I say, the coventional wisdom is that these are finite and unrenewable.

AN INTEGRATED EVALUATION MODEL OF DOMESTIC CRUDE OIL AND NATURAL GAS SUPPLY

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BACKGROUND

This paper describes a system designed, developed and implemented by MATHTECH, Inc., for the Electric Power Research Institute (EPRI) to evaluate factors influencing future supplies of crude oil and natural gas in the Lower 48 onshore and offshore provinces. The models and associated data bases described were installed during the past year on the Boeing Computer Services (BCS) time-sharing network and are currently being exercised and maintained by the Supply Program staff of EPRI's Energy and Environmental Analysis Division.

Over the past four years, MATHTECH had conducted for EPRI comprehensive state-of-the-art assessments of both natural gas and crude oil supply models and modeling techniques.* In these two assessments a combined total of twenty-two models and methodologies were thoroughly analyzed with respect to technical, data and policy-related evaluation criteria.

Based on this assessment, it was concluded that major new model development appeared unwarranted either on methodological or informational grounds inasmuch as (i) no innovative theoretical approaches could be identified which had not as yet been applied to this problem, but which might offer the potential for fruitful application in the future; and (ii) no major data sources were identified which had been untapped by previous research in this area.

Rather, it was recommended that the most cost-effective way to proceed would be to adapt, integrate and synthesize existent methodologies into a single comprehensive evaluation system of oil and gas supply. It was on this premise that MATHTECH embarked upon the development and implementation of the integrated supply model discussed below.

It is our belief that the model finally delivered to EPRI in this effort does incorporate what we perceived to be the most refined techniques available at the time for handling each of the various aspects of supply forecasting and evaluation.

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*See References (1) and (2)

SYSTEMS OVERVIEW

The effort resulted in the installation of a fully integrated, systems-oriented process model consisting of the following three principal component modules:

- The Onshore Module models future exploration, discovery and production of oil and gas in the onshore portion of the Lower 48.
- The Offshore Module performs the same task for the offshore regions.
- The GASNET2 Module is a detailed model of the domestic natural gas transmission system.

Among the modeling techniques used are:

- Geostatistical analysis - expected discoveries as a function of exploratory activity is based on statistically derived "finding rate functions."
- Engineering cost analysis - costs of exploration, development and production were derived from a variety of sources.
- Econometrics - onshore exploratory activity is modeled as an econometrically derived function of a "profitability index."
- Economic decision analysis - regional and directional (oil vs. gas) allocation of exploratory activity is based on economic decision analysis.
- Nonlinear optimization - offshore exploratory activity is modeled using nonlinear optimization with a "short term" finding rate function.
- Linear optimization - the "GASNET2" component is a generalized network model which is solved by a special linear programming procedure.
- Deterministic simulation - both the onshore and offshore components simulate oil and gas industry behavior over time.

The model is accessed via a series of interactive rountines (controlled by a single master control program) which leads the user through a series of questions to allow him to create, store and use sets of control and data variables (prices, expected total reserves, tax rates, report options, etc.) which run the model. (See Exhibit 1.) Each of the three components can be run independently, or all three can be run together, with the onshore and offshore components providing input to GASNET2.

Each module is structured in "breadboard" fashion from a series of smaller components (i.e., programs, subroutines and data files) which can easily be replaced in future revisions. Thus, for example, if a new costing model for exploratory wells is developed, only minor program changes would be needed to incorporate it in the existing model structure.

ONSHORE EXPLORATION AND DISCOVERY MODULE

The Onshore Module forecasts exploratory effort, reserve additions and production of onshore oil and natural gas in each of the ten NPC Petroleum Provinces in the contiguous Lower 48 United States. Production includes both output from new discoveries forecast by the model as well as from existing proven reserves previously booked.

Two key concepts drive the model: (i) a "profitability index" is used to determine the total level of exploratory effort and its allocation to oil and gas exploration in each province; and (ii) a province-specific "discovery curve" that is used to estimate the new reserves found by a given level of exploration.

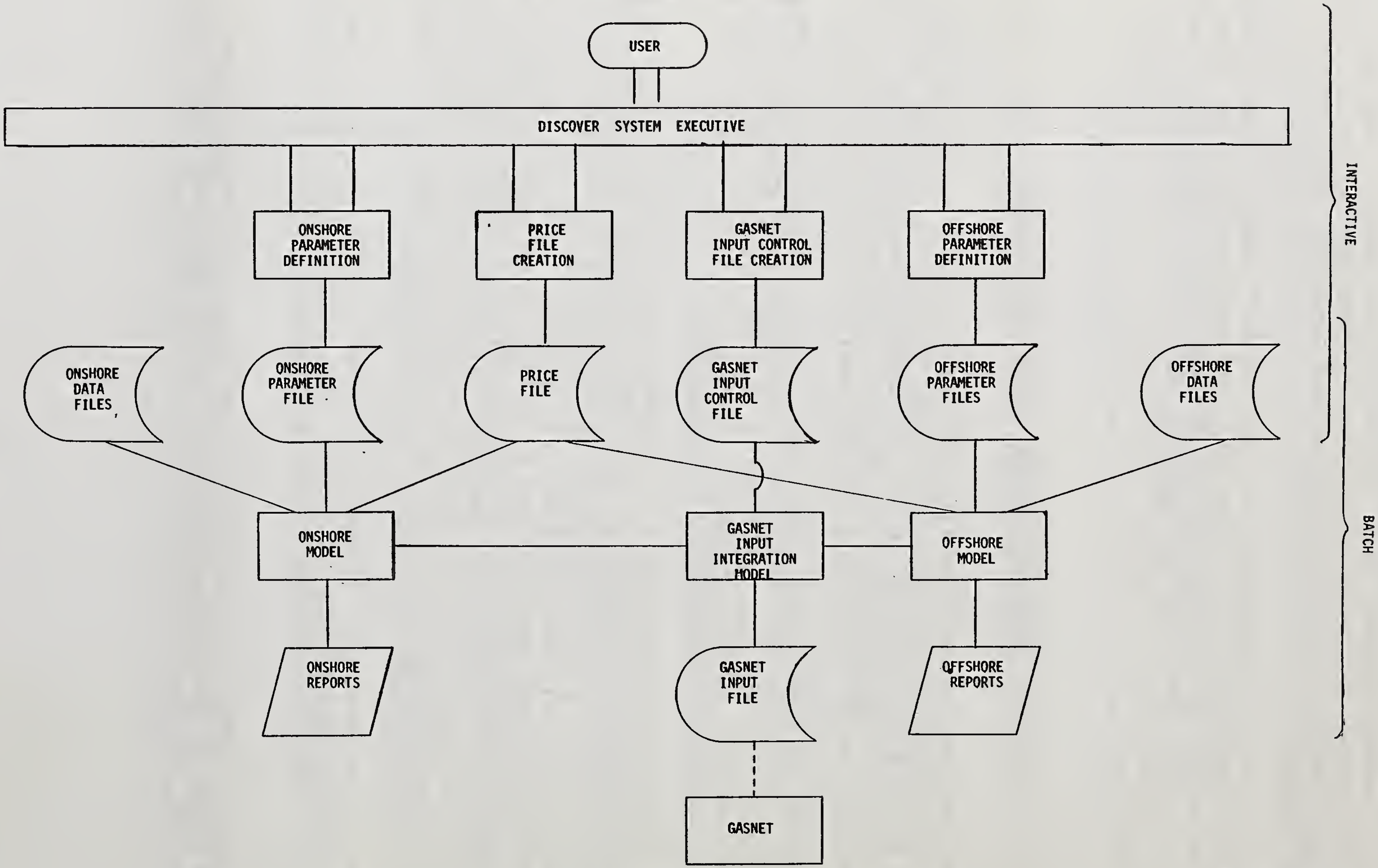
A typical discovery curve is derived from empirical data relating to cumulative reserve additions and cumulative exploratory footage drilled as well as estimates of total discoverable, recoverable resource-in-place. A set of actual data for sixty (60) such curves (oil and gas in ten (10) provinces for each of three (3) total resource level scenarios) was compiled and a logistic-type curve fitted to the data so as to asymptotically approach the hypothetical total reserves. Using this sort of curve, new reserves discovered can be estimated as a function of new future drilling and cumulative prior drilling. Appropriate corrections are made for the fact that the last actual data points probably do not fall on the fitted curve.

The three scenarios correspond to the low, mid and high total resource level estimates published in U.S.G.S. Circular 725. In any run of the onshore or offshore models, the user selects the desired scenario.

The profitability index (PI) is defined as the expected net present value of future post-tax cash inflows as a fraction of the expected net present value of all future cash outflows associated with

EXHIBIT 1

System Overview



a given incremental level of exploratory drilling effort. Each year, the PI is computed for the prior year's total exploration and discovery effort. A preliminary estimate of the total number of exploratory wells in the current year is made using the actual (or computed) profitability indices for the prior two years. In order to simulate the effect of capital and materiel limitations, the annual growth rate of total exploratory wells may also be constrained to a maximum pre-set by the user.

The user may specify a preliminary allocation to each region of a certain percentage of its prior year's exploratory effort, with additional exploratory wells allocated to oil and gas in each region based on the expected PI for each hydrocarbon in each region.

Exploratory wells are allocated in sets of a predetermined size of "N" wells (e.g., 10 or 100, but modifiable at the user's option), as follows: For each of the 20 (region, hydrocarbon) pairs, an expected PI is computed for the case where all N exploratory wells are drilled for that pair; the expected discoveries for the N wells are estimated from the discovery curves. The N wells are then allocated to (in proportion to the PI's) those pairs whose PI's are positive. This process continues until all expected exploratory wells for the year have been allocated, or other constraints halt further exploratory effort. (In order to simulate regional materiel limitations, the growth rate of exploratory wells for each region is constrained to the maximum pre-set by the user or the largest compound annual growth rate observed in that region for any three-year period since 1950.)

The new reserves discovered each year are then translated into annual production by applying typical wellhead deliverability profiles for oil and gas.

Optionally, the gas production for a given year can be allocated to the 144 GASNET2 "nodes" for analysis in the GASNET2 model. The significance of these nodes is described later.

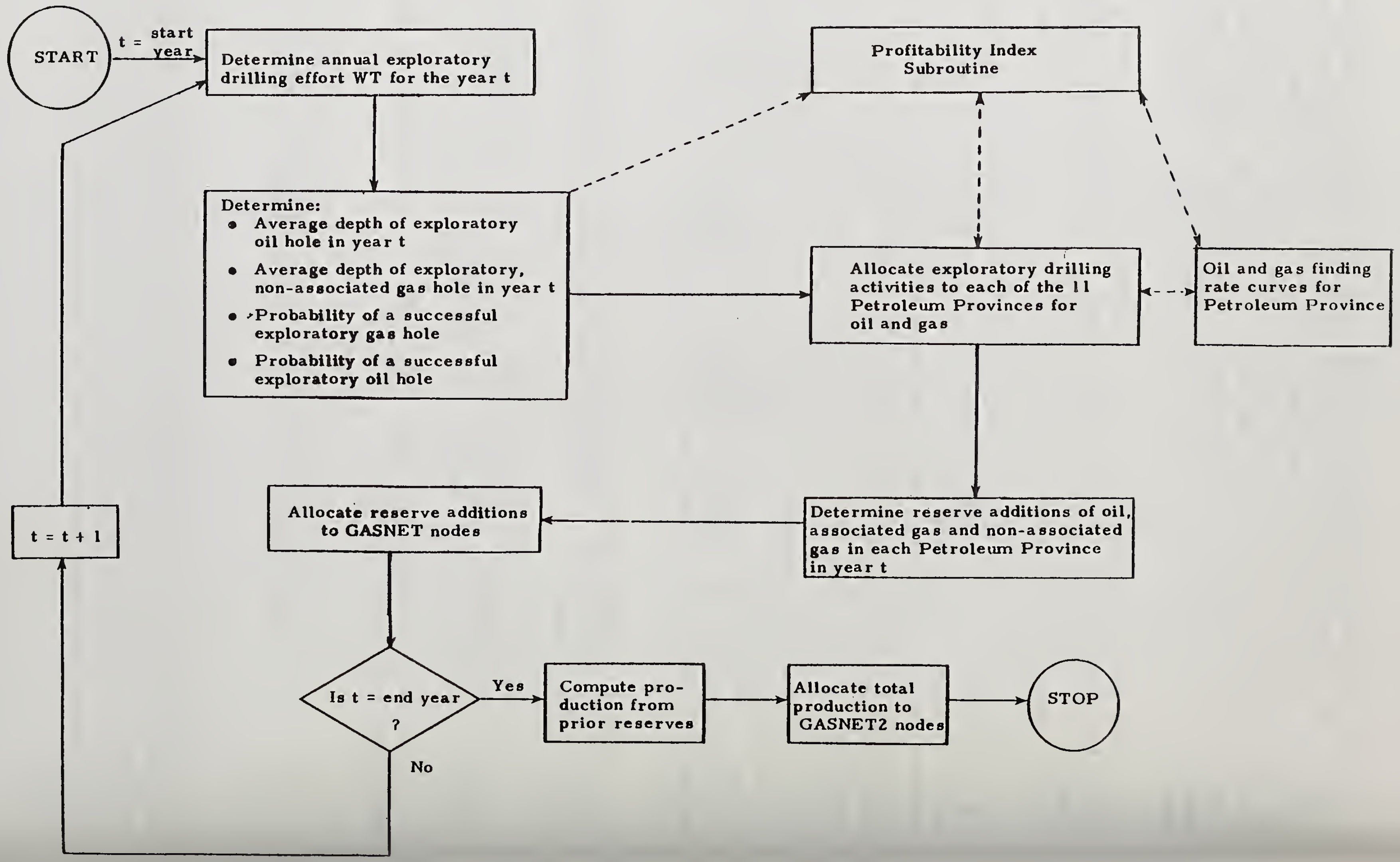
Exhibit 2 displays the sequence of interrelated steps that are employed in this module.

OFFSHORE EXPLORATION AND DISCOVERY MODULE

In the Offshore Module, aggregate offshore drilling decisions are simulated in the context of a user-designated scenario defined on various policy, economic and physical factors. Based upon these assumed conditions, the model projects expected future exploratory drilling, developmental drilling, gas and oil discoveries, reserve additions, and production for each U.S. Outer Continental Shelf (OCS) region.

EXHIBIT 2

The Onshore Module



The user has control over a wide variety of factors in the design of a scenario to be simulated by the model. The model is formulated to consider policy, economic and technologic/geophysical variables and constraints such as:

- wellhead oil and gas prices
- acreage leasing schedules
- acreage bid and acceptance rates
- in-place resource base and ultimate discovery potential
- resource finding rates
- production and deliverability profiles
- royalty and income tax rates
- drilling, well completion and O&M costs for exploration delineation and development
- investment credits and intangible tax treatment
- capital depreciation rates
- rig fleet and materiel constraints
- density of drilling constraints
- lease-discovery-production phased time lags
- minimum acceptable rate of return on capital investment
- estimated maximum and minimum feasible bonus payments

The intended purpose of the Offshore Module is to determine additions to reserves and production levels under various alternative policy sets and geological states of nature. The general framework developed is adaptable to the Gulf of Mexico, Atlantic, Pacific Coast and Alaskan Gulf OCS areas provided the specification of the required geologic parameters can be made. Except for the Gulf of Mexico, these regions are largely unexplored. Thus, assumptions regarding the resource levels and ultimate recovery in each area are of critical importance.

The basic structure of the model entails the determination of the optimum level of exploratory drilling activity for any lease offering. This requires an analysis of the expected revenue and cost streams, with the bonus level determined by the minimum required rate of return on the investment.

The discounted revenue stream is functionally related to the finding rates for oil and gas, ultimate recovery, deliverability profiles and the time trajectory of oil and gas prices. The discounted expected cost stream is determined by the royalty rate, income tax and intangibles rates, and unit exploration, delineation, development and production costs net of bonus payments. The bonus range is then determined as the difference between the expected revenues and expected costs including the entrepreneurial return.

Major activity-influencing factors in the model are the leasing schedule, the prices of oil and gas, finding rates (related to ultimate recovery) and rig fleet and materiel constraints.

The model simulates aggregate offshore drilling decisions annually for each outer continental shelf by optimizing expected net present value after taxes through an algorithm which structurally simulates established industry accounting and decision-making practices. For each year in each region, the model selects a number of trial drilling levels. Given a trial drilling level and the set of user-specified conditions, the time-series of expected costs is computed and discounted to net present value, the time-series of expected revenues is computed and discounted, and their net difference calculated. The optimum drilling level in each region is determined by comparison of net present values of the alternative drilling levels.

The goal was to develop a general formulation whose applicability would be equally valid to as yet largely unexplored areas (e.g., the Atlantic OCS) as to maturing areas (e.g., the Gulf of Mexico). Thus, the main effort was directed at developing a formulation which considers in a detailed and logical fashion, the relationships among the host of policy variables which influence the decision to bid on, acquire, explore and develop offshore acreage given postulated states of economics, geology and technology. The result of the exploration activity undertaken offshore is subject to a much greater degree of uncertainty than corresponding activity onshore because the regions considered are, for the most part, "frontier areas."

Thus, the model strives to reliably emulate the decisions which will most likely be made by those who will actually undertake these ventures and to forecast exploratory and development effort based on a rationale which roughly corresponds to the manner in which the industry itself views these risks and benefits. However, in translating this effort into actual reserve additions and production levels, one can expect to encounter a much greater degree of uncertainty and corresponding margin of error.

In describing the detailed workings of the module, it is convenient to segment the computational process into four distinct phases:

(1) the specification of physical parameters;

(2) the specification of economic parameters;

(3) the optimization process; and

(4) the feedback, recycle and adjustment process.

Exhibit 3 contains a highly simplified diagrammatic overview of the model structure which illustrates the four-stage computational sequence discussed above.

GASNET2 TRANSMISSION MODULE

GASNET2 consists of programs and data which together define a detailed model of the U.S. natural gas delivery system. This model consists of three fundamental parts: a regionalized supply (production) file, a regionalized and sectoralized demand (requirements) file, and a network model of the system used to deliver the gas from producers to consumers.

Regionalization of production is defined by a breakdown of the U.S. into 144 substate areas. These substate areas have been carefully chosen to reflect the various franchise areas of natural gas distributors and the sales areas of natural gas pipelines. In this disaggregation, for instance, Texas has thirteen different areas, New Mexico, Wyoming, Ohio and New York have four, many have three or two and a few states are not subdivided at all. In addition, Canada and Mexico are included because of their current and future roles in natural gas imports into the U.S.

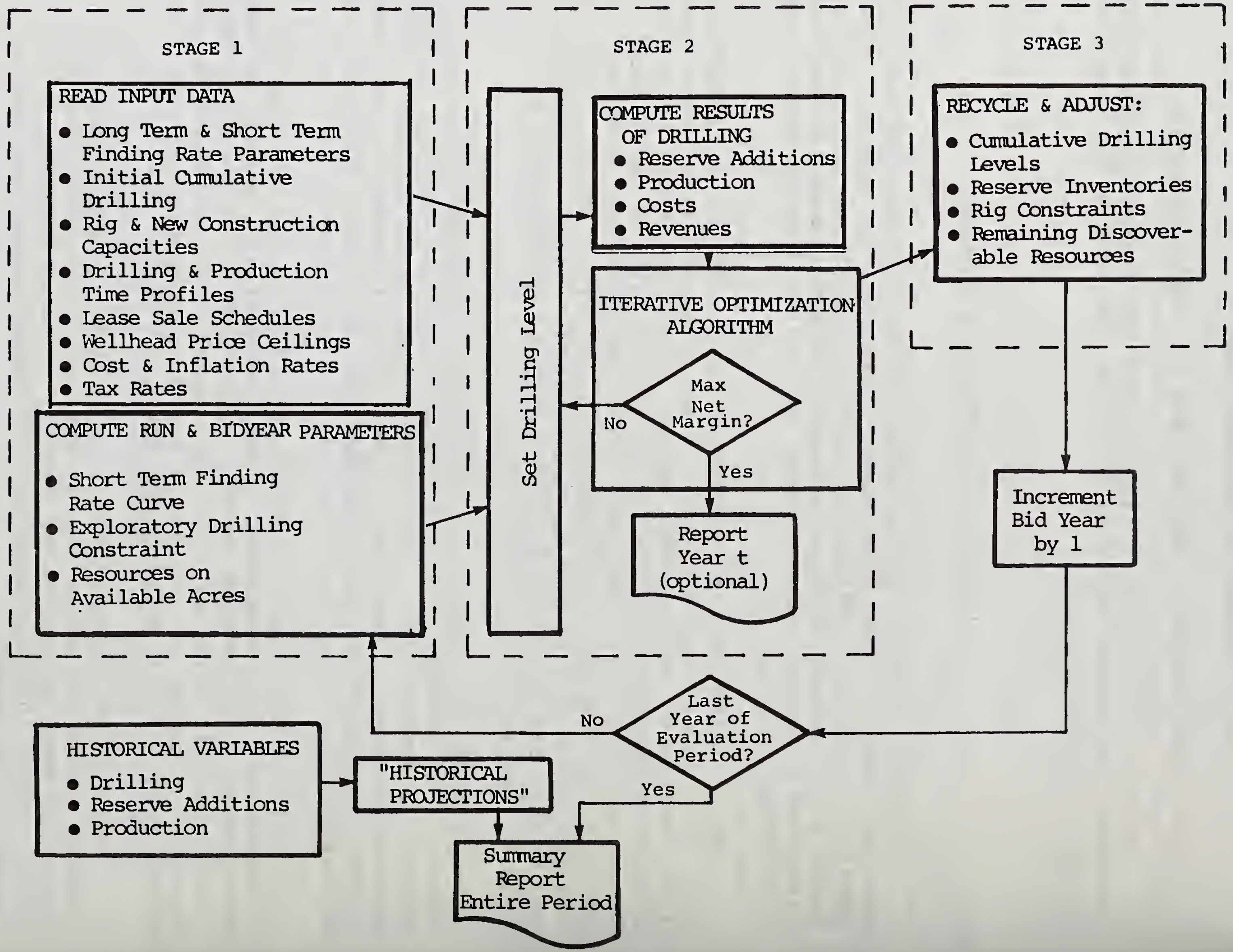
Natural gas consumption is divided regionally and by end-use category in GASNET2. Each production area in GASNET2 is also a potential demand area. Each demand area is an existing or potential production area. Thus great flexibility in modeling new sources of alternative gaseous energy production on a local level is possible with GASNET.

The end-use categories defined in the GASNET2 base model include "residential," "commercial," firm industrial," "public authorities," interruptible industrial and/or commercial," "miscellaneous other," "electric utilities and interdepartmental," and "undifferentiated." The last category is necessary for some companies for whom historical sales data is not separated by end-use class.

The GASNET2 network model is a highly disaggregated generalized network model of nearly all of the major interstate natural gas pipelines and most of the major natural gas distributors in the country today. Over 100 pipelines and 240 distributors are explicitly represented in the model.

EXHIBIT 3

The Offshore Module



Each pipeline is modeled as a subnetwork of nodes and arcs. Each node corresponds to one of the GASNET2 substate areas which is served by the pipeline either through purchases from producers of other pipelines, sales to distributors, end-users, or other pipelines or simply delivery of gas through the area. All of the pipeline's transmission and/or distribution plant located in this area is included in this node. Each arc in the submodel represents either (i) a connection between two areas (nodes) by natural gas transmission lines crossing the boundary between them, or (ii) a transaction between producers, pipelines or distributors. Each node is parameterized by a "loss factor" which represents gas lost in transmission, distribution, storage, or unaccounted for, as well as gas used by the pipeline to power transmission or distribution line compressor stations. Each transmission arc is parameterized by a capacity which defines an upper limit to the quantity of gas deliverable over an inter-area boundary on the pipeline's transmission lines in a given time period and a unit cost of transmission between "centroids" in two connected areas. These centroids are defined individually for each pipeline in a particular region and are based on the location of its transmission lines rather than on some "average" centroid for all the pipelines. Each transaction arc is parameterized by a cost which represents the average unit "markup" by the seller on that transaction.

Each distributor in the model is defined by the areas and demand sectors it serves. A distributor is allowed to receive gas from interstate pipelines, intrastate producers, and its own manufacturing plants. A distributor which operates in more than one area cannot communicate directly between these areas. This defines the logical difference between a "distributor" and a "pipeline": a "pipeline" can sell gas to other companies for resale, a "distributor" cannot. This does not pose a real problem to the modeler, however, since companies which both transport and distribute gas can be modeled as two separate company divisions with the transmission division delivering gas to the distribution division for resale.

In the network model there are six basic "transactions":

(1) deliveries by producers to pipelines (XS);

(2) deliveries by producers to distributors (XI);

(3) deliveries by pipelines to distributors (XD);

(4) deliveries by pipelines to other pipelines (XX);

(5) deliveries by pipelines by transmission line between contiguous areas (XT); and

(6) deliveries by distributors to consumers (XC).

Each of these deliveries is represented as a flow on an arc in the model. These flows are constrained by upper and lower bounds representing capacities and/or contractual arrangements between seller and buyer. These flows are also subject to reduction due to losses and use of gas as pipeline fuel in compressor stations. As discussed previously, these losses are considered to occur in the nodes. In the computerized implementation of GASNET2, however, it is more convenient (and standard) to define losses as occurring on arcs. Thus in GASNET2, each node's loss factor is applied to all arc flows leading into that node. Thus each arc's flow corresponds to the amount delivered after losses have been accounted for in the "upstream" node and prior to losses in the "downstream" node.

GASNET2 is essentially a linear network model and can be solved by linear programming routines. It is fairly large in its complete form (2,200 equations and 5,300 variables), however, and rather expensive to solve using standard general purpose LP codes. Tests were run using IBM's MPS360 general purpose LP routine and a new state-of-the-art special purpose network code in order to establish the savings in costs which such a code could achieve for the model. The results indicate a savings on the order of about 30/1 using the special purpose solution algorithm.

SUMMARY

Recall that the EPRI model was developed by adapting and (where necessary) extending "preferred" concepts and techniques found in other oil and gas models to form an integrated system. As such, the resultant model has several important strengths, as well as remaining limitations, which indicate the need and directions for future model improvements.

Among the noteworthy strengths are:

- An integrated evaluation framework which allows a non-programmer to easily run the model to see the effect of different price scenarios, tax policies, cost estimates, etc.
- A modular structure which provides for easily installed improvements in model modules and input data, as well as integration with other supply evaluation systems.
- Explicitly considered relationships among public policy, economic, geological, technological and institutional factors which impact on oil and gas supply.
- An attempt to utilize the methodological procedures best suited to the behavioral and physical phenomena being modeled.

Some of the model limitations:

- Finding rate curve parameter estimates are based on (among other things) the resource base estimates in

U.S.G.S. Circular 725; these should be recomputed when better data is available.

- Technology representation and costing can and should be improved; e.g., exploration costs might be estimated as a function of region and average drilling depth instead of the overall "per hole" average now used.

- Constraints need to be more directly linked to models of public policy planning and macro/micro economic activity with corresponding feedback.

- Structural changes in the industry (caused by OPEC and U.S. government policy as well as macroeconomic conditions) not embedded in the historic on which the model is based are methodologically well represented, but poorly measured:

  - directionality of drilling (oil vs. gas)
  - regional allocation of effort
  - drilling success rates and prospect inventories

During the period of performance, three other major efforts were undertaken in the area of oil and gas supply which, due to their concurrent development, could not, except for some minor features, be adapted into the EPRI model. These were:

- DOE-EIA OCS Oil and Gas Supply Model (Lewin-MATHTECH)

- Inter-Agency Task Force on Resource Base Reappraisal (USGS-RAG)

- Nehring Geostatistical Assessment of Major Lower 48 Reservoirs (RAND)

The EIA effort is especially noteworthy because, for the first time, structure-specific data permitted a level of disaggregated analysis heretofore impractical to consider. Even at this date, however, EIA is still testing and evaluating the output of this model.

The RAND effort is really a retrospective rather than a prospective view of oil and gas supply, but with some very interesting and potentially quite transferable application of geostatistical techniques. Similar work along these lines is also being pursued at U.S.G.S.-Reston.

In summary, except for the three efforts just described, we believe the model developed for EPRI does, in fact, incorporate those state-of-the-art features uncovered in MATHTECH's extensive examination of previous oil and gas supply modeling. However, current prospects for its use by EPRI as a forecasting and evaluation tool remain clouded in the context of the changing role of the Supply Program as a "producer of forecasts and supply projections" to a sponsor of information systems and planning models which can be adapted by the utility industry to assist in securing adequate supplies of fuel in an uncertain decision-making environment.

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AN EVALUATION OF THE ALASKAN HYDROCARBON SUPPLY MODEL

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Introduction

Alaskan oil and gas supply forecasts are a key component of the forecasting of energy markets by the Energy Information Administration (EIA) of the Department of Energy. Earlier EIA efforts to forecast Alaskan oil and gas supplies were generally inadequate. The forecasts were simple aggregates based almost entirely on the expectations of the industry. While this approach is not without merit, it is too rigid and cannot be adjusted to reflect responses to varying economic stimuli. The Alaskan Hydrocarbon Supply Model (AHSM) was developed to enhance EIA's capability for estimating future supplies from this region. The model incorporates engineering and economic factors that affect both resource extraction and transportation.

The approach to estimating future supply consists of segmenting the model into components. The first part describes the extent and quality of the resource base of crude oil and associated and dissolved natural gas. The next part focuses on the non-associated natural gas resource base. The third piece of the model deals with the transportation possibilities for all hydrocarbons within Alaska. These three components are combined to provide a forecast of supply in future years using trajectories of domestic hydrocarbon prices.

In estimating the resource base for both oil and gas, the model samples possible geological outcomes and produces tables that classify potential reserve additions by the exploratory drilling needed to prove them and by the average total cost of finding, developing and producing the reserves. These tables reflect the results of the resource evaluation and contain, in effect, regional families of supply curves. Each curve represents the expected volume of reserves distinguished by the level of exploratory effort; in other words, the drilling required to discover the supply of reserves.

These supply tables become part of the data base for the "integrating" model--the portion of the AHSM that uses linear programming to maximize the present values of profits from production and transportation investments over time, linking the individual sources of supply. Market and policy parameters that affect the solution are prevailing market prices for oil and gas, leasing constraints, and limitations on the potential for pipeline network expansion.

The AHSM in its current form represents a significant improvement in the forecasting of Alaskan oil and gas production. It is a dynamic modeling system which incorporates a large degree of flexibility while still retaining a reasonable level of computational efficiency. Nonetheless, AHSM contains certain shortcomings in its structure. The major problems within AHSM include the inability to deal directly with the nonlinear relations that affect pipeline design and the conceptual difficulties associated with the handling of uncertainty in the current model design.

This paper is organized as follows. The next section describes the linear programming system of the model. The description is brief but provides sufficient detail to aid in the evaluation of the model. The resource submodel in AHSM is not described in this paper with any detail because it is not germane to this evaluation. The problems associated with pipeline costing in the linear program are presented, followed by a description of the conceptual problem of dealing with uncertainty.

Mathematical Formulation of the Integrating Model

This section describes the linear programming submodel. The integrating model chooses the timing and extent of exploration, development and transportation activities to maximize the present value of profits in light of expected selling prices of oil and natural gas at the Alaskan border. The model that represents this choice process consists of a time-staged, linear program that depicts five activities: exploration, development, transportation, transportation network expansion, and sales. The specification of the equation system is presented next, followed by discussion of the activities and relations.

In addition to the objective function, there are five basic relations in the linear program: a reserves inventory equation, a material balance equation, a pipeline capacity equation, and convexity constraints on both exploration activity and pipeline expansion. The entire set of equations can be found in Table 1 of Appendix A. Before presenting the entire system, each relation of the system is presented in turn.

Reserves Inventory Balance

The first class of equations is the reserves inventory balance. Relation (1) transfers unproduced inventory of a given fuel to the succeeding time period. The inequality simply ensures that beginning stocks plus new reserve additions must be at least as great as the production plus remaining stock; that is, hydrocarbons may be "wasted," but they cannot be "created" in the solution process.

Mathematically,

$$-I(f,s,p,t)-\sum_{k} e(f,s,k,p)*E(f,s,k,t)$$

$$+D(f,s,p,t) + I(f,s,p,t+1) \leq 0 \qquad (1)$$

where

f = fuel type
s = supply region
p = supply price
t = time period
k = unit of exploratory drilling

E(f,s,k,t): exploration unit k in region s for fuel f in time t.

e(f,s,k,p): yield of fuel f from exploratory unit k in region s at supply price p.

D(f,s,p,t): development activity for fuel f at price p in region s at time t.

I(f,s,p,t): unproduced inventory of fuel f with supply price p in region s at time t.

The entire system of equations along with subscript and variable definitions appear in Table 1 of Appendix A.

The I(f,s,p,t) variable represents the inventory of fuel f in supply region s with a supply price p that has not been developed by time t. The supply price p does not refer to the exogenously entered market price that is received for all sales. It measures instead the average total costs for producing that component of the total regional reserves. For reserves with p exceeding the product price in every time period, there is no established production plan. These reserves are retained from one time period to the next until the market price exceeds p. Thus p may be interpreted as a minimum acceptable price. In the implementation of the model, p is not continuous. The values for p constitute a set of discrete price categories. The predetermination of possible cost or price levels serves to limit the size of the overall equation system and the corresponding computational burden.

The D(f,s,p,t) term represents the development activity for fuel f in supply region s. The development plan differs for different supply prices p and is initiated at time t. Each D activity "develops" the found reserves leading to a production schedule for future time periods. The representation of actual production appears in the next equation, and so discussion is deferred until then.

The last element of Relation (1) measures the reserves additions from units of exploratory activity. Total exploratory drilling for a region is the accumulation of successive blocks of drilling feet. Yields of reserves decrease with increasing drilling. As the k-th unit of exploratory activity occurs, reserves from all supply price categories are both possible and likely to result. The E variable represents the particular unit of exploratory activity and ranges from zero to one where a value of one indicates the exhaustion of the entire exploratory unit. The e coefficient marks the yield from the given exploration unit for a particular supply price level. Thus, for the total reserves discovered in a particular cost category, the product of these two values must be summed across all k exploratory units. In other words, $\sum_k e(f,s,k,p)*E(f,s,k,t)$ measures the reserves of fuel f with a supply price of p from region s in time t.

The value of e is determined through the resource simulation model. The resource evaluation is modeled by successive sampling of the geology base in a Monte Carlo framework. Each particular realization of the geology base is evaluated to determine the amount of exploratory drilling and the production costs to develop the deposits within that realization. Then the complete set of results for all realizations are used to determine expected values for reserves yielded at each level of exploratory drilling within all cost categories. A unit of exploratory drilling typically yields reserves across all price categories. The e coefficient is the expected value for reserves of a given fuel in a given supply region differentiated by drilling effort required and the unit costs of production. The reserves inventory balance must be imposed on both fuel categories (f) in all supply regions (s), for each time period (t).

Material Balance

The next set of inequalities, material balance relations, accounts for sources and uses of fuel at all activity points. All sources and uses of fuel are traced, precluding the occurrence of artificial gains or losses within the system. The accounting occurs for all fuels in all time periods at all geographic entities within the model: supply regions, demand regions, and transshipment points.

Within the context of the current model, only certain activities may occur at any activity point. For example, sales do not occur at supply regions. The three representative forms of the inequality appear below. However, for conciseness, the entire relationship is provided in Table 1. The intent of the inequality is quite straightforward: the amount of fuel used for sales or further shipments must not exceed the total receipts from production and shipments received (less transportation loss).

Receipts consist of production at and shipments to the node. Production is a flow of product over time, a consequence of earlier development activity. Production of fuel f from region s in time period t is denoted by: $\sum_{z=1}^{t} D(f,s,p,t)*d(f,s,p,t-z)$. The coefficient d(f,s,p,t-z) measures the amount of fuel f at a supply price p yielded from region s in time t given the initial development of the field in time z. Note that in the development of an oil deposit, associated-dissolved gas is possible. In such a case the yield variable, d(f,s,p,t-z), reflects both oil and natural gas. In the supply region inequality, supply region s is linked to transportation node n.

Shipments of product occur at all points in the network. The amount of shipment of fuel f from point m to point n in time t is measured by T(f,(m,n),t). As a notational convention, the summation over the first (second) link parameter indicates the summation of all upstream (downstream) flows into (out of) the activity point (e.g., $\sum_{q} T(f,(n,q),t)$ represents the total shipments from point n). The summation over a link parameter is designed so that this occurs only for links recognized as flowing to or from the transshipment point. Shipments received must be adjusted for transportation losses. The proportionate transmission loss is represented by L(f,(m,n)).

Sales of fuel f at demand region j in time t are represented by S(f,j,t). In the demand region inequality, demand region j is linked to transportation node n. The three inqualities accounting for the above activities follow.

Supply regions:

$$\sum_{p}\sum_{z=1}^{t} D(f,s,p,z)*d(f,s,p,t-z)-\sum_{q}T(f,(n,q),t) \geq \emptyset$$

Transshipment points:

$$\sum_{m}T(f,(m,n),t)*(1-L(f,(m,n)))-\sum_{q}T(f,(n,q),t) \geq \emptyset$$

Demand regions:

$$\sum_{m}T(f,(m,n)t)*(1-L(f,(m,n)))-S(f,j,t) \geq \emptyset$$

Where:

f: fuel type
t: time period
p: supply price (or equivalently, average total costs of production)
s: supply region
j: demand region
(m,n): shipment link from point m to point n
z: for development only, z represents the initiation year for development

D(f,s,p,t): development activity for fuel f at price p in region s at time t

d(f,s,p,t-z): proportion of fuel f at price p in region s produced in time t given the initiation of development in time z

T(f,(m,n),t): transportation of fuel f from point m to point n in time t

S(f,j,t): sales of fuel f at demand region j in time t

L(f,(m,n)): proportionate loss incurred in transmission of fuel f from point m to point n

Pipeline Capacity

Each of the major pipeline links connecting transshipment points in the system has an explicit capacity that imposes an upper limit on the volume of product that may travel along the route at any instant. The capacity, location and cost data concerning these projects are exogenously fed into the system. The occurrence and timing of the projects, however, are determined as part of the LP solution.

The concern here is only for the links between transshipment points. These links constitute the major pipeline system bringing the product to market. Between the supply regions and the major pipeline network are the gathering systems for the producing fields within a region. Gathering systems are represented within the model by "spurs." Spurs are similar to the major links in that they move products. The construction schedule and capacity are assumed, however, to accommodate the field development schedule and requirements. The essential feature in this discussion is that only the major pipeline links have a capacity constraint.

The capacity of a pipeline link for fuel f from point m to point n existing in time t is given as $\sum_{y}^{t}\sum_{i} X(f,(m,n),i,y)*K(f,(m,n),i)$. The K parameter is the capacity of the given(m,n) link from project i. The X variable measures the degree of completeness for the particular project. In the mixed integer formulation for the model the X variables are constrained to equal zero or one. As a continuous variable LP, the X variables are bounded by zero and one. The summation over i is required to account for multiple projects being possible between two given points. The i subscript serves as an identifier for each project. The summation over all earlier time periods is required since a pipeline once constructed is available for all future time periods.

The pipeline capacity constraint is imposed at each link for all fuels in every time period. It requires the capacity to be at least as great as the flow. The specific inequality is:

$$\sum_{y=1}^{t}\sum_{i} X(f,(m,n),i,y)*K(f,(m,n),i)-T(f,(m,n),t) \geq 0 \quad (3)$$

where

f: fuel
y,t: time
i: project identifer for pipeline expansion
(m,n): shipment link from point m to point n

T(f,(m,n),t): transportation of fuel f from point m to point n in time t

X(f,(m,n)i,t): pipeline expansion for fuel f, project i, from point m to point n in time t

K(f,(m,n),i): pipeline capacity for fuel f from point m to point n, project i

Convexity Constraints

There are variables that measure bounded activities within the system: pipeline expansion, X(f,(m,n),i,t) and exploration, E(f,s,k,t). By definition of each variable, the associated event can be allowed to occur only once even though segments of the activity may occur in different time periods. To ensure this result, the sum of these activities over time has an upper bound of one as represented in the following relations.

$$\sum_t X(f,(m,n),i,t) \leq 1 \qquad (4)$$

$$\sum_t E(f,s,k,t) \leq 1 \qquad (5)$$

f: fuel
t: time
s: supply region
(m,n): shipment link from point m to point n
k: exploratory drilling unit
i: project identifier for pipeline expansion

X(f,(m,n),i,t): pipeline expansion for fuel f, project i, from point m to point n in time t

E(f,s,k,t): exploration unit k in region s for fuel f in time t

The Objective Function

The objective of the linear program is to maximize the present value of profits by optimally timing the occurrence and extent of exploration, development, transportation, network expansion, and sales.

The objective function is statement (6) in Table 1 of the Appendix. All activities present in (6) have been discussed earlier. The additional variables are either product prices for sales, or costs of the other activities. An additional feature is that all value figures are discounted over time. Thus, the optimization is over net present value.

Summary of the Linear Programming Integrating Structure

In the linear program, exploration activities yield additions to reserves in each development cost class. The additions are joint; that is, each increment of exploratory activity leads to discoveries potentially distributed across all cost categories. The yields from exploration activities appear in Relation 1. The convexity constraint ensures that each unit of exploratory activity is conducted only once, regardless of the relative profitability.

In turn, development activities "consume" the reserves included within a particular cost category and yield production over time of both primary products (crude oil and non-associated gas) and associated-dissolved gas. The shapes of the "production profiles" conform to the base-level plan used in the resource model to calculate minimum-acceptable supply prices and thus to grade potential reserves into cost categories. Consequently, these profiles and cost categories serve as the critical linkage between the stand-alone resource model and the integrating model.

Downstream from production, liquids and gas are transported to terminals located at the Alaskan boundary. The transportation network consists of three kinds of activities. Network expansion adds capacity to the links of the network. These expansion activities are "comprehensive" projects in the sense that capacity is added jointly to any number of links in the network. Each transportation activity moves a product between pairs of regions. The transportation activity consumes capacity of a link. A spur also moves products from supply regions to transshipment points but does not consume capacity. Spurs connect supply regions to the network at a unit cost based upon an exogenously estimated pipeline size.

All of the production and transportation activities involve cash expenditures discounted in relation to the time of their initiation. Sales activities at the Alaskan terminals provide the discounted revenues driving the system. These activities are simply variables that "consume" the delivered product at an exogenously supplied price.

One additional note is warranted here. The subscript for time is presented in the above equations without qualification. However, in the actual implementation of the model, all of the exploratory activities are provided with early start dates. Early start dates are also provided for some network expansion projects. These dates are provided in the latter case to prevent unreasonably early initiation periods and in the former case to reflect the anticipated leasing schedule.

This completes the presentation of the mathematical formulation of the model. The entire specification is provided in Table 1. The discussion now turns to the treatment of transportation charges in the system and the possible distortion resulting from this treatment.

Pipeline Design

The following discussion focuses upon certain formulation issues embedded within the Alaskan Hydrocarbon Supply Model (AHSM). The specific problem concerns the effects of linear approximations on the nonlinear pipeline design relationships. As a general conclusion, the costs of transportation within the postulated pipeline network are understated. This statement is valid for both the major pipeline links and the spurs, even though the computation for them differs substantially.

Pipeline design relations are nonlinear with respect to the planned flow rate. Specifically, pipeline diameter increases at a decreasing rate with respect to increases in the planned flow rate, other things being equal. (This statement is made with extreme caution. For a good treatment of the subject see Petroleum Transportation Handbook, Harold S. Bell, ed., McGraw-Hill Book Company, Inc., 1963; or Pipeline Design for Hydrocarbon Gases and Liquids, American Society of Civil Engineers, 1975.) This nonlinearity in design is propagated in the equations used to calculate the pipeline costs for both oil and gas. The equations used in AHSM to scale the pipeline cost estimates are provided in Table 2 of the Appendix. The approach to cost determination relies on initial estimates for total pipeline costs, less overhead, for a link of a given capacity. (2 million barrels per day is the reference size for oil, 2.4 billion cubic feet per day for gas.) These reference cost estimates are then divided by the reference sizes to provide cost estimates per unit of volume for the new link. It is sufficient to evaluate the scale equations alone since the reference cost estimates are constant for a particular link transporting a given fuel. The scale equations are referred to as the cost equations from now on for convenience.

The decision to expand the pipeline network to a given supply region is made on the basis of the region's production potential. The chief determinant of this potential is the level of the region's reserves. One may therefore say that the decision to construct the new pipeline link is based on the expected reserves that may be tapped for production. The cost equations are clearly nonlinear in the flow rate. Nevertheless, the methodology treats the cost of new capacity as a given, based on an a priori estimate of the flow rate. The linear program accepts this information, then computes the optimal expansion strategy based on these costs as one part of the overall profit maximization problem.

The evaluation of the error due to linear approximations of pipeline costs requires a more detailed look at the equations listed in Table 2. Table 2 contains the normalized equations based on a flow rate of 2 million barels per day and 2.4 billion cubic feet per day for oil or gas, respectively. These reference capacities were not chosen arbitrarily. They are the respective capacities of the Trans-Alaska Pipeline System (TAPS) and the Alaskan Natural Gas Transportation System (ANGTS). TAPS and ANGTS have been studied in depth and constitute the only knowledge the industry has relevant to large scale transportation of oil and gas in this area.

The cost equations in Table 2 are both nonlinear in Q, the flow rate. The first derivative of the equation, denoted by S', is strictly positive throughout, indicating increasing costs with higher flows. The second derivative, S", is strictly negative for natural gas for all positive values of Q. The second derivative for the oil equation is negative with Q less than 1 million barrels per day and positive where Q exceeds this figure. (The actual inflection point occurs at roughly 1.006, but the conclusion will be the same.) Coincidentally, 1 million barrels per day is the largest pipeline capacity proposed for any of the new pipeline links. With the relevant range for new oil pipeline capacity confined to flows between zero and 1 million barrels per day, each of the cost equations can be characterized as concave over the relevant range of flows.

A representative graph of the scale, or cost, function appears in Figure 1 of Appendix A as curve S. OA measures a hypothetical design capacity for a new pipeline link. The value of the cost function for flow OA is represented by segment AB. As a mixed integer program, total costs are either zero with no flow, or AB with flow OA. When AHSM is run as a linear program, the entire project need not be undertaken, and the actual cost is approximated by OB in Figure 1. How closely the linear approximation compares to the nonlinear estimates is considered next.

The comparison of the two functions requires some preliminary discussion. Obviously, the total costs for a new pipeline link as computed by either function will be close to AB only as the flow value approaches OA. However, by varying the value of Q and comparing the results, it is possible to approximate the difference between the two estimators. Table 3 contains a listing of the results of the comparison. The equations for both oil and gas are used to derive the values in the SCALE column. In the case of oil, the SCALE values must be normalized. The 0.737 value of the normalized SCALE for Q=0.2 means that a pipeline designed for a flow of 0.2 million barrels per day would cost 73.7 percent of the cost for a 1 million barrels per day pipeline. (The implicit working assumption of this paper is that the costing equations are valid. The quality of these relations is the subject of a future paper. The current study is focused upon the merits of the linear approximations in AHSM.) With the linear function, a line carrying 20 percent of 1 million barrels per day would cost only 20 percent of the total for the larger flow. Thus the figures from the nonlinear equation is 368 percent of the figure from the linear function. If oil flowed at 50 percent of the 1 million barrels per day design capacity, the incurred cost in the linear case is not even 60 percent of the nonlinear estimate.

In the case of natural gas, the table is slightly different. For gas, the SCALE equation yields normalized values. That is, the nonlinear SCALE equation yields figures from zero to one. However, the SCALE values for the linear equation require normalization. The balance of the Tables remains the same in character and intent. The ratios of nonlinear to linear estimates are slightly less severe. However, the gas comparison uses a pipeline capacity of 2.4 billion cubic feet per day. With larger capacity values, the linear approximation is better. The proposed values in AHSM for expansion of new gas pipeline links are only 0.8 or 1.0 billion cubic feet per day. So the test results listed in Table 3 do not reflect the degree of distortion actually encountered in the implementation of AHSM.

The results of the comparison listed in Table 3 demonstrate that the linear approximation employed within AHSM performs well only within a limited neighborhood of the design capacity. Unfortunately there is no method to control the value of the flow variable within its range without sacrificing the flexibility embodied within the LP. In fact, a survey of results used in the 1979 edition of the Energy Information Administration's Annual Report to Congress shows that most expansion projects are well below their design capacities. It appears as though AHSM may be grossly understating transportation costs along the major pipeline links in its system.

Unfortunately, underestimation of transportation charges also occurs in the case of the spurs linking supply regions to the major pipeline links. The underestimation arises for a different reason, however, as the costs of the spurs are handled differently within AHSM.

Spurs in AHSM are proxy variables for the network that is required to transport products from the supply region fields to the major pipeline links. The intrafield gathering system is included as a part of the production process, and so that cost is incorporated into the total costs of production. The spurs represent the connecting lines between the intrafield gathering systems and the major transportation network. Since one spur generally represents multiple pipelines, detailed specifications such as capacity are omitted for spurs. The costs for this transportation segment, however, must still be recovered. AHSM charges a single per unit cost for all transportation along spurs for one supply region. The computation of the unit cost is based on postulated flow rates along the spur for both oil and gas. The scale equation of Table 2 employs these rates to adjust the reference cost estimates. The reference cost estimates are $7 million/mile and $8.35 million/mile for a 2 million barrels per day oil line and 2.4 billion cubic feet per day natural gas line, respectively. The postulated flows are 100 thousand barrels per day for oil and 1.3 billion cubic feet per day for natural gas. The derived cost estimates are $3.58 million/mile and $6.4 million/mile for oil and gas lines, respectively. These figures would then be increased by computing the product of the unit cost and an estimate for mean distance. The 1.3 billion cubic feet per day figure is not documented; it is the figure that most closely yields the $6.4 million total cost per mile for gas pipelines.

The above procedure incurs inaccuracies due to the very large flows postulated for the spurs. Proposed flows of 100 thousand barrels per day and 1.3 billion cubic feet per day are quite optimistic. The survey of representative AHSM runs generated for the 1979 Annual Report to Congress demonstrated that these estimates are entirely unrealistic. The largest gas flow along a spur in the 15 year forecast horizon is 0.15 billion cubic feet per day. This figure is not even 12 percent of the 1.3 billion cubic feet per day as postulated. Oil fares somewhat better. Flows along spurs range as high as 333 thousand barrels per day in one instance. However, each region consists of large tracts, covering hundreds of square miles. Given such a vast area, it would seem unreasonable to assume the existence of centralized groups of deposits to support the massive spur lines. In fact, a 100 thousand barrels per day flow of crude implies a reserve of 730 million barrels. (Maximum production from reserves is 5 percent per year.) It is likely that there would be multiple spurs, each less than 100 thousand barrels per day. If this is so, transportation charges along these spurs are underestimated. In addition, the other figures were generally not even one-half the 333 thousand barrels per day recorded for the single instance cited above.

The above problem can be ameliorated to some degree by simply choosing a smaller flow value for the calculations. A strong candidate for selection is the expected value for the flow. Two problems work against this choice however. First, computing the expected value for production constitutes an extremely complex, if not intractable, problem. The expected production depends upon the entire set of interrelations within the model. These parameters, of course, include the transportation costs. Altering that set of parameters changes the initial equation system. Thus, the expected production as calculated initially would not necessarily match the expected production values of the respecified system.

The problem could be formulated as a mixed integer program (MIP). The MIP imposes integer values of zero or one on the pipeline expansion activities [the X(f,(m,n),i,t) of Table 1]. This approach is intuitively appealing since pipeline expansion occurs as a discrete event: one either constructs a line or not. Multiple projects may be possible, each one of a different size. The range of sizes would then allow for the selection of a possible size most appropriate for its use. In the current context, "most appropriate" is the lowest cost alternative for the proposed flow over all possibilities.

Even if the system were guaranteed to converge, the advantages of repeatedly solving such a massive problem may not justify the cost. Because the cost structure is nonconvex, there is no easy solution to the resulting mathematical program.

The Problem Of Representing Uncertainty

Even if the MIP could be made to work, problems arise with the uncertainties in the resource estimates. The resource evaluation model employs stochastic variables within a Monte Carlo framework to compute expected volumes of reserves for increasing drilling effort and costs of production (i.e., the e(f,s,k,p,) of Table 1). The effect of this uncertainty is contingent upon whether the model has been run as a continuous variable linear program (LP) or as a mixed integer problem.

If the MIP approach is used, the expected, or average, outcome of the final decision may differ markedly from the final decision based upon expected input data. A simple example, valid for both oil and natural gas, clarifies the point. Consider a model with a supply region containing one potentially productive site. The probability of hydrocarbon occurrence is discrete; one out of three times, a reserves level of R will be present, zero the other times. Assume that production is proportional to the level of reserves. If R is the level of reserves, then production flows at a rate of X units per time period. Assume the associated economics of the problem are such that the transportation costs can be recovered only with a flow of at least Ø.5X. The problem is to determine the expected flow rate, if any, from this supply region.

The final answer depends upon the approach used to solve the problem. The AHSM methodology first computes expected reserves and then forms its decision based on that data. The expected level of reserves is Ø.33R. The resulting production would be Ø.33X, which would be inadequate to recover all transportation costs, while a production rate of Ø.5X provides a sufficient return for production to be profitable. The level of production determined by using the AHSM approach is therefore zero, whereas expected production should be .33X.

The above argument demonstrates that there is a clear potential for distortion when AHSM is solved as a MIP. Activity levels are underestimated whenever the expected level of reserves is inadequate to support production yet the range of possible values for reserves includes levels sufficient to support profitable production. As a practical matter, however, the equation system is so massive that it is quite a burden to solve as an MIP. In providing forecasts to the Annual Report to Congress AHSM has been solved as a continuous variable LP.

Presented so far have been an example of overestimating supply due to underestimating costs, and an example where supply would be underestimated if a MIP formulation were used. The following example shows supply would be underestimated if the correct pipeline cost function is combined with expected flow.

The problem is presented graphically in Figure 2. Suppose for a given spur, the production flow based upon the expected reserves equals OZ. The OZ flow incurs a spur cost of ZC. The OZ flow is based upon a given level of expected reserves treated as a certain value. However, the conceptually correct procedure to derive the expected total cost would be to employ an iterative process to realize successive possible geologic states and compute the expected value of total costs based on the complete set of outcomes. (This agrees with the earlier argument concerning the execution of AHSM as a MIP.) Suppose that successive runs based on single realizations of the geology yield positive points ranging from OX to OY. Zero values are likely, but they are ignored in the present discussion. The argument deals only with the flow values, conditional upon the presence of petroleum. The total cost figures from each iteration would lie along arc ACB. If the event space were defined as the set of iterations yielding positive flows, the probability of any of these flows occurring would be 1/N, where N equals the number of iterations when the flow is positive. The expected value for total cost conditional upon the presence of a positive flow equals the simple average of all positive total cost values. Since the expected value function is a convex combination of the points along arc ACB, its value would lie in the region bounded by arc ACB and line AB. The value would lie below arc ACB and be less than ZC. Thus, computing total cost based on the expected flow would overstate the expected total cost based on the presence of a positive flow. Naturally, this difference is greater if one were to compute the expected total cost over all iterations, including those when the flow and total cost are zero. Thus, within the current structure of AHSM, correctly calculating spur costs for the average flow values may well lead to overestimates of costs, the opposite problem of the underestimates previously discussed.

Conclusions

The Alaskan Hydrocarbon Supply Model (AHSM) is a useful forecasting tool. For most of the midterm forecast period, having the accounting structure of the model and a leasing schedule is sufficient to produce a credible forecast. However, the model is not without its faults. AHSM's major deficiency is in the treatment of transportation costs. This area is critical due to the extremely high costs for all phases of industry operations in the harsh conditions of Alaska. Accurate computation of transportation costs is necessary to evaluate fields that are on the margin of profitablity. As Alaska enters a more mature state of exploration, any existing giant field such as Prudhoe Bay will have been discovered. As exploration proceeds, the smaller, less profitable fields will serve increasingly as the source of production. Reliable estimates of Alaska's productive potential depend as much on accurate estimates of transportation costs as they do on precise estimates of production costs and the geology base itself.

The cumulative effect of the AHSM treatment is probably to understate transportation costs. The unit transportation charges imposed on shipments from supply regions to major pipeline links are calculated as if these activities may enjoy the economies of scale afforded by an unreasonably large pipeline. The shipments along the major pipeline links similarly enjoy the economies of scale inherent in a large diameter line regardless of whether the entire project is constructed. This understating of pipeline costs along major pipeline links occurs when AHSM is executed and solved as a continuous variable LP. If AHSM is treated as a MIP, this understatement of costs is avoided, but the activity levels in the solution may then be underestimated.

Estimating the exact measure of distortion within the activity levels in the AHSM solution is computationally burdensome. Some insight into the degree of error in the cost calculations is provided in the figures listed in Table 3. The large divergence between the estimates of the nonlinear equations and the associated linear approximations is cause for concern. Although the resulting degree of distortion in the LP solution is uncertain, it has the potential to be significant.

These conclusions are based on an argument that implicitly assumes the correctness of the two pipeline cost equations. This assumption allowed the study to focus on the conceptual issues in the AHSM system. The equations themselves might require corrections to specific parameter values, but the nonlinear properties of the relations would be the same in all likelihood. Thus, the conceptual argument and its conclusions would still apply; only the intensity of the problems would vary.

Alternatives to the Current Version of AHSM

In light of the preceding statements, the last issue to address concerns the direction of future work. There are two alternative paths to be considered. One path involves an overhaul to the existing AHSM framework. The conceptual difficulties would be eliminated by incorporating the linear programming equation system into the iterative Monte Carlo procedure that evaluates the resource potential and then solving the constrained optimization problem as a MIP. This would avoid the conceptual difficulty inherent in basing the LP solution on expected values of resources. The user could compute the expected activity levels correctly. Solving the equation system as a MIP would correctly estimate the total pipeline costs for the major links. However, transportation charges for spurs would continue to be estimated incorrectly. Also, the computational burden of numerous executions of such a large LP is impractical. In fairness to the developers of AHSM, it is noted in the model documentation that the LP should be incorporated into the Monte Carlo procedure. They, too, noted the computational considerations and dismissed it as impractical.

The second alternative is the option being pursued currently at the Department of Energy. That alternative is to retain the best features of AHSM while developing an alternate methodology that avoids the problems inherent in AHSM. The AHSM work includes valuable data on costs of operation at all stages of operation: exploration, development, and transportation. There is a complete set of stochastic variables that depicts the geology base of northern Alaska. These data represent the product of a many hours of research. Perhaps the most valuable feature of AHSM is the insight gained from the total experience. This insight will provide a useful foundation for the new model. The new model is currently under development.

The rough framework for a single iteration of the new model appears in Appendix B. The model also employs a Monte Carlo framework that iteratively determines possible geologic states. Within the geologic state present in any iteration, all decisions are represented explicitly. The explicit representation of decisions has a dual advantage.

First, the nonlinear pipeline relations can be incorporated directly into the simulation of the decision process. Second, the data available for decisionmaking can be controlled. The true state of geology, for example, is unknown to the exploration decision, and becomes known only through exploratory drilling.

The process depicted in Appendix B is oversimplified but conveys some of the model's flavor. Each iteration begins with the initialization of a state of geology and expected values for exploration and development costs and product prices. The former constitutes the true state of geology. The latter set of data is used in the decision-making of the operators. Since virtually any decision concerns the determination of activity through "future" years, the operators need to have economic expectations to calculate the expected profitability of a proposed project. As presented here, the operators acquire a set of economic expectations for all years in the initialization. An alternate choice is to develop a dynamic adjustment process that would occur at the start of each time period.

After initialization, the model proceeds with successive time periods. In each time period, three major activities are conducted--exploration, development and transportation. Exploration activities are modeled as a search process based on limited, uncertain information. Development activities convert the known reserves into flows of production over time. Development requires the drilling of wells and the construction of surface equipment. Transportation activities include the shipping of petroleum along the existing system and the construction of both new links and expansion to existing capacity. Any decisions result in implementation only if the project is expected to be profitable. The decisions are made on the basis of available information. Thus, although the operators work to attain the best possible outcome, the lack of perfect knowledge may result in losses on projects undertaken. The uncertainty resulting from the lack of foresight is exacerbated in the exploration process since the true geologic state is unknown.

A complete description of the model will be available when it becomes operational, currently expected in the summer of 1981. The model will avoid the omniscient character of the decision process inherent in the linear programming framework of AHSM and rely on decisionmaking under uncertainty. The model can accommodate the nonlinear relations required for the calculations of pipeline links. Also, the output from the numerous iterations lends itself to statistical analyses of the results. Each of the three advantages is considered to be a significant improvement over the modeling characteristics of AHSM.

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APPENDIX A

Table 1. Mathematical Formulation Used in the Alaskan Hydrocarbon Supply Model

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<u>Equations</u>:

Reserves Inventory Balance

for all f,s,p,t:

$$-I(f,s,p,t)-\sum_{k} e(f,s,k,p)*E(f,s,k,t)$$
$$+D(f,s,p,t,)+I(f,s,p,t+1) \leq 0 \qquad (1)$$

Material Balance

for all f,t,s,j,(m,n):

$$\sum_{z=1}^{t}\sum_{p} D(f,s,p\ ,z)*d(f,s,p,t-z)+\sum_{m} T(f,(m,n),t)*(1-L(f,(m,n))$$
$$-\sum_{q} T(f,(m,q),t)-S(f,j,t) \geq 0 \qquad (2)$$

Pipeline Capacity

for all f,(m,n),t:

$$\sum_{y=1}^{t}\sum_{i} X(f,(m,n,),i,y)*K(f,(m,n),i)-T(f,(m,n),t) \geq 0 \qquad (3)$$

Convexity Constraints
-Pipeline Expansion

for all f,(m,n),i:

$$\sum_{t} X(l,(m,n),i,t) \leq 1 \qquad (4)$$

-Exploration

for all f,s,k,:

$$\sum_{t} E(f,s,k,t) \leq 1 \qquad (5)$$

Objective Function

Maximize $\sum_{f}\sum_{j}\sum_{t} S(f,j,t)*a(f,j,t)$

$$\sum_t\sum_k\sum_s\sum_f CE(f,s,k,t)*E(f,s,k,t)-\sum_f\sum_s\sum_p\sum_t D(f,s,p,t)*CD(f,s,p,t)$$
$$-\sum_f \sum_{(m,n)} \sum_t T(f,(m,n),t)*CT(f,(m,n),t)$$
$$-\sum_t\sum_f \sum_{(m,n)} \sum_i CK(f,(m,n),i,t)*X(f,(m,n),i,t) \qquad (6)$$

Subscripts:

- f: fuel type
- t: time period
- p: supply price (or equivalently, average total cost of production)
- s: supply region
- j: demand region
- (m,n): shipment link from point m to point n
- k: exploratory drilling unit
- i: project identifier for pipeline expansion
- z: for developement only, z represents the initiation year for development

Variables:

E(f,s,k,t): exploration unit k in region s for fuel f in time t

e(f,s,k,p): yield of fuel f from exploratory unit k in region s at supply price p

CE(f,s,k,t): present value (PV) of the k-th unit of exploration for fuel f in region s at time t

D(f,s,p,t): development activity for fuel f at price p in region s at time t

d(f,s,p,t-z): proportion of fuel f at price p in region s produced in time t given the initiation of development in time z

CD(f,s,p,t): PV of the development cost for fuel f in region s at time t with suppy price p

T(f,(m,n),t): transportation of fuel f from point m to point n in time t

CT(f,(m,n),t): PV of transportation charge for fuel f from point m to point n in time t

X(f,(m,n),i,t): pipeline expansion for fuel f, project i, from point m to point n in time t

K(f,(m,n),i): pipeline capacity for fuel f from point m to point n, project i

CK(f,(m,n),i,t): PV of constructing pipeline project i for fuel f from point m to point n in time t

S(f,j,t): sales of fuel f at demand region j in time t

a(f,j,t): PV of selling price for fuel f at demand region j in time t

I(f,s,p,t): unproduced inventory of fuel f with supply price p in region s at time t

L(f,(m,n)): proportionate loss incurred in transmission of fuel f from point m to point n

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Table 2. AHSM Pipeline Cost Equations For Oil and Gas

Oil:

S=0.149+0.58Q**0.2+0.046Q**2

S'=0.116Q**(-0.8)+0.092Q

S"=-0.093Q**(-1.8)+0.092

Natural Gas:

S=0.17+0.416Q**0.4+0.1Q

S'=0.166Q**(-0.6)+0.1

S"=-0.10Q**(-1.6)

Variables: S: scale factor for total costs for new pipeline links

Q: flow rate; MMB/D (oil), or BCF/D (gas)

Notation: ** denotes exponentiation
S' and S" represent the first and second derivatives of the original equation, respectively.

Table 3. Comparison of Cost Factors for the Linear and Nonlinear Functions

<u>Oil</u>: SCALE=0.149+0.58Q**0.2+0.046Q**2

Q	SCALE	Normalized SCALE	Linear SCALE	Ratio (Normalized: Linear)
.2	.571	.737	.2	3.68
.4	.639	.825	.4	2.06
.5	.665	.859	.5	1.72
.6	.689	.889	.6	1.48
.8	.733	.946	.8	1.18
1.0	.775	1.0	1.0	1.0

<u>Natural Gas</u>: SCALE=0.17+0.416 Q**0.4+0.1Q

Q	SCALE	Normalized Linear SCALE	Ratio (Nonlinear: Linear)
.4	.498	.167	2.98
.8	.630	.333	1.89
1.0	.686	.417	1.65
1.6	.832	.667	1.25
2.0	.919	.833	1.10
2.4	1.0	1.0	1.0

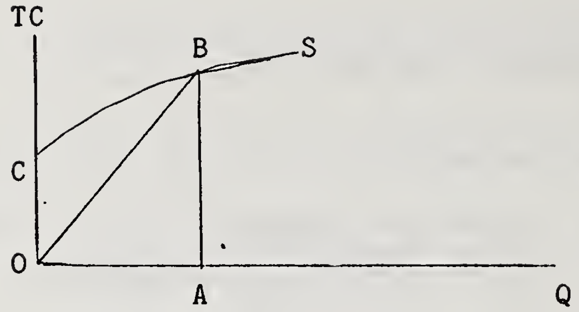

Figure 1. Total Cost Function: Major pipeline Links

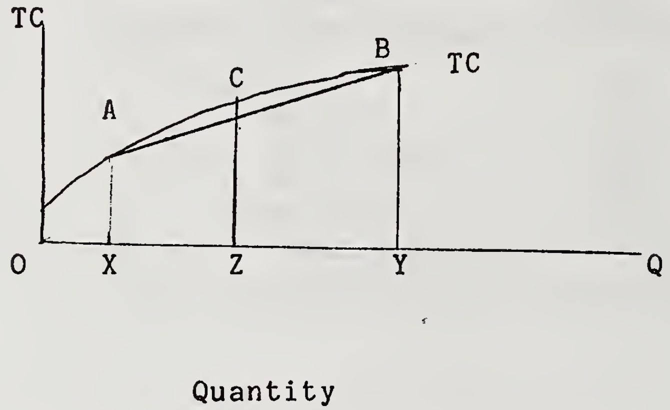

Figure 2. Total Cost Function: Spurs

Appendix B

Program Flow for Proposed Model: Single Interation

1. Initialization:

    A. Establish a state of geology - number of structures and their associated area.

    B. Generate the expected value for exploration and development costs to be used for reference.

2. Exploration:

    A. Calculate the expected unit costs for selected sites in a given region.

    B. 'Drill' successive sites until limit on exploratory drilling is met or the portfolio of profitable projects is exhausted.

3. Development:

    A. Drill wells for field development.

    B. Construct:

        1. Surface equipment.
        2. Larger pad(s) or island(s).
        3. Flowlines.

    C. Determine the total reserves and production rates.

4. Transportation:

    A. For existing lines: transportation activities will use up available capacity.

    B. Construct new pipelines for known reserves lacking transportation facilities, if profitable.

A PROSPECT SPECIFIC SIMULATION MODEL OF OIL AND GAS EXPLORATION IN THE OUTER CONTINENTAL SHELF: METHODOLOGY*

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Hydrocarbon resources in the Outer Continental Shelf (OCS) regions are an important but uncertain component of the nation's energy future. This paper describes the methodology used in constructing a model and database to forecast reserve additions and production from the OCS as a function of price and Federal policy in the context of explicit geologic drilling prospects and engineering costs.

The model is based on microeconomic analysis of individual exploration prospects as identified from seismic maps. A prospect was defined as one or more traps that, if productive, would be developed as a new field. A resource distribution is built for each prospect by including uncertainty concerning geological and engineering properties of the prospect. Exploration and development decisions are based on deterministic cost and engineering information for each phase of the project. Bidding and exploration decisions are based on expected net present value incorporating full costs and resource uncertainty. Development decisions occurring after successful exploration and delineation drilling, are based on present value given known field size with lease acquisition and exploration costs sunk. The model incorporates Federal leasing schedules and leasing policy (variations on bonus, royalty, and profit share bidding), user-specified, market price tracks and constraints on rigs, platforms and pipelines.

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**** Mathtech, Inc.

1. Introduction and Summary

Domestic offshore oil and gas production, especially from the deep water frontier areas, offers the possibility of reducing America's near-term dependence on imports. The rate at which these resources are explored is highly sensitive to the policies of the Federal Government, especially leasing schedules, leasing system (bidding on bonus, royalty, profit-share, or mixed systems), oil and gas prices, and tax policies. The rate at which the resource is developed is influenced, however, by the number and size of hydrocarbon deposits actually found, and costs of development, and the available market price.

The Energy Information Administration of the U.S. Department of Energy sought to model these relationships as the basis for improved production forecasting and analyses of Federal policy options. To take advantage of valuable data sources with a minimum of data preparation or interpretation, a disaggregate analytic strategy was adopted. The resulting model simulates the behavior of the individual oil and gas explorationist who must make two discrete decisions: (1) whether to acquire and explore the prospect given full geologic uncertainty and full engineering costs and (2) whether to develop and produce the prospect given the marginal costs of platforms, developmental drilling, operations, and transportation for a field whose size has become known through prior exploration, and delineation. A prospect, as used in this study, consists of one or more potentially productive traps (as shown on seismic interpretation maps) that, if productive, would be developed as a new field. The data available to the model were obtained by inspection of the records of the USGS Conservation Division at Metarie, Louisiana which approximate the type of geological information that would be available to the individual explorationist.

This paper describes both the methodology and the supporting database required for this analytic strategy. The goal of the effort was to implement the strategy in a computer model with the capacity to simulate future exploration, development and production from Outer Continental Shelf (OCS) regions of the lower 48 states. The model produces estimates of oil and gas supply from 1980 to 2000. Projections are made individually for the Gulf, Atlantic and Pacific OCS from the surfline to 1000 meters water depth and are subdivided into estimates of proved reserves, extensions (and revisions) to proved reserves, and undiscovered resources.

The supporting database contains information collected by geologic interpretation of seismic and engineering data. Approximately 65% of the undeveloped prospects in the Gulf were believed to be covered by this process; for the remaining 35%, sampling based on geographic proximity and common geologic trend was used to complete the working database. "Prospects" in the Atlantic and Pacific were arbitraily created by sampling of field sizes from assumed distributions to match the total regional resource estimates published in USGS Circular 725 [2] and its updates. Consistent with the analytic approach of the model described here, these synthetic data should eventually be replaced with data developed in a manner similar to that used in the Gulf.

The supply of oil and gas from undiscovered resources is the major emphasis of the model; it is evaluated by means of four serially executed submodels:

- o Geology: The uncertainty expressed by the geologist in specifying the value of specific reservoir parameters is estimated through Monte Carlo* simulation to produce a prospect-specific resource distribution.

- o Technology: Engineering options, production profiles, and costs associated with the exploration, delineation, and development phases are evaluated for selected points on the resource distribution of each prospect.

- o Field Sampling: The prospect-specific resource distributions are sampled to supply a set of field-size values that represent a potential geologic outcome of exploration, for each Monte Carlo trial of the analysis submodel.

- o Analysis: Given a time series of assumed market prices, leasing schedules, and leasing policy specifications, the phases of exploration, delineation, development and operations are simulated for each prospect, based on the geologic outcomes. A number of Monte Carlo trials are run to incorporate uncertainty in the resource base.

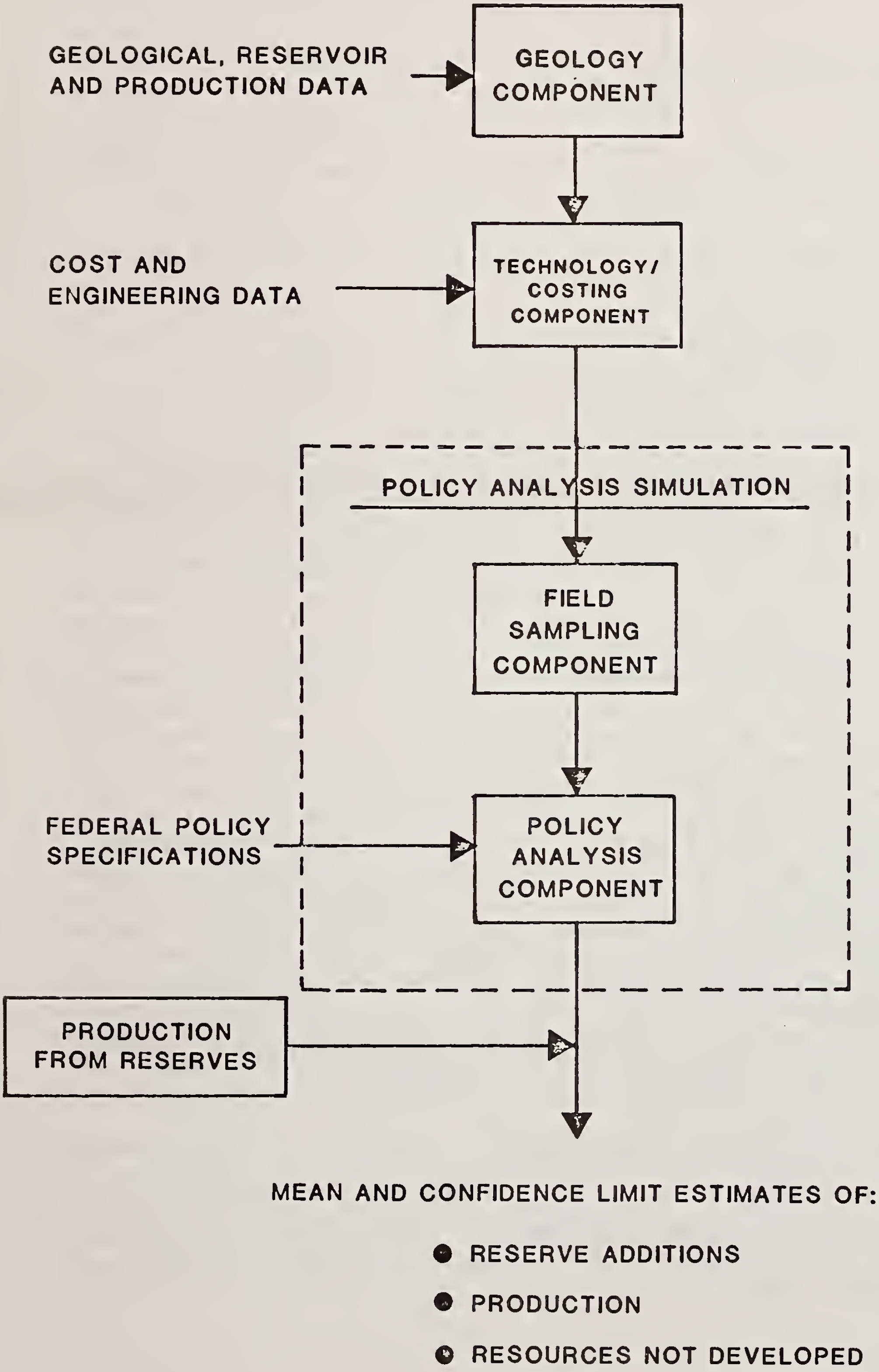
Production from currently proved reserves and their likely future extensions and revisions are estimated by constant ratio of reserves to production which is calculated from historical trends. Additions to proved reserves through extension drilling in existing fields (inferred reserves) are computed by methods similar to Hubbert [3].

The model is depicted in Figure 1. The technology, geology and discovered reserves models are used only to initialize the model, as data is updated. The field locator (sampling) and policy analysis models are used whenever new policies are considered.

*Pseudo-random sampling over a large number of trials.

Figure 1:

OVERALL SYSTEM FLOW



2. Model Approach in Contrast with Previous Studies

The model forecasts production from discovered fields as a constant percentage of reserves. A Hubbert-type analysis is used to model reserves additions through extension drilling (the "inferred reserves" of USGS Circular 725). The analysis of inferred reserves is based on the prior work of Hubbert [3], Root [8], and Mast and Dingler [6];and achieves notably closer precision in "backcasting" existing data series than preceding studies. See [10] for the comparison.

For undiscovered resources, the model represents a departure from earlier studies (see Kalter, [4]; Mansvelt-Beck, [5]; EIA Midterm Energy Forecasting System, [11]). In general, previous models had been forced to employ aggregate data which may suffer from internal confounding of technology, policy and economic factors.

The disaggregate approach of the present work is aimed at improving prediction by eliminating the use of aggregate resource estimates, historical "finding rates," and assumed field-size distributions. For example, all prior studies referenced above use area-wide resource estimates derived from a process of group-decision making that considers geological evidence only in an indirect fashion. All three previous models yield forecasts of economically recoverable reserves only. The use of these models to forecast the quantities of resource <u>foregone</u> resulting from Federal policy variations or from technical, geological or logistical constraints is virtually impossible, requiring a large number of subjective assumptions. The EIA Midterm model [11] is based on the use of "finding rates" which represent historical trends in the marginal quantity of discovered resource attributable to yearly increments of drilling activity. The data series upon which finding rates are based does not permit the unraveling of the tangled effects of changed Federal policy, improved technology or economics. The Kalter [4] and Mansvelt-Beck [5] models represent a further refinement by explicitly representing Federal policy effects by use of decisionmaking based on microeconomic analysis of "prospects" (sampled from the Circular 725 resource distributions). However, neither model makes a separate assessment of oil and gas production. Further, since costs are assessed on a "per BOE" basis derived from historical averages, these two models cannot represent the specific engineering decisions which must be faced in the development of oil versus gas prospects.

This model provides an improvement over the previous work in five areas:

- o Uncertainty in the resource base is a function of the interpretation of individual of prospect specific seismic and engineering data.

- Exploration and development costs are based on actual engineering studies applied to prospects of known extent in known water depths.

- The detailed geologic data base enables the model to make specific estimates of the production of oil, associated gas, nonassociated gas, and natural gas liquids.

- Because the model begins with a prospect-specific resource distribution, it is possible to provide a full accounting of the undiscovered resource including that which is not produced due to economics, constraints and leasing schedule. (The full accounting is shown in attached Figure 2).

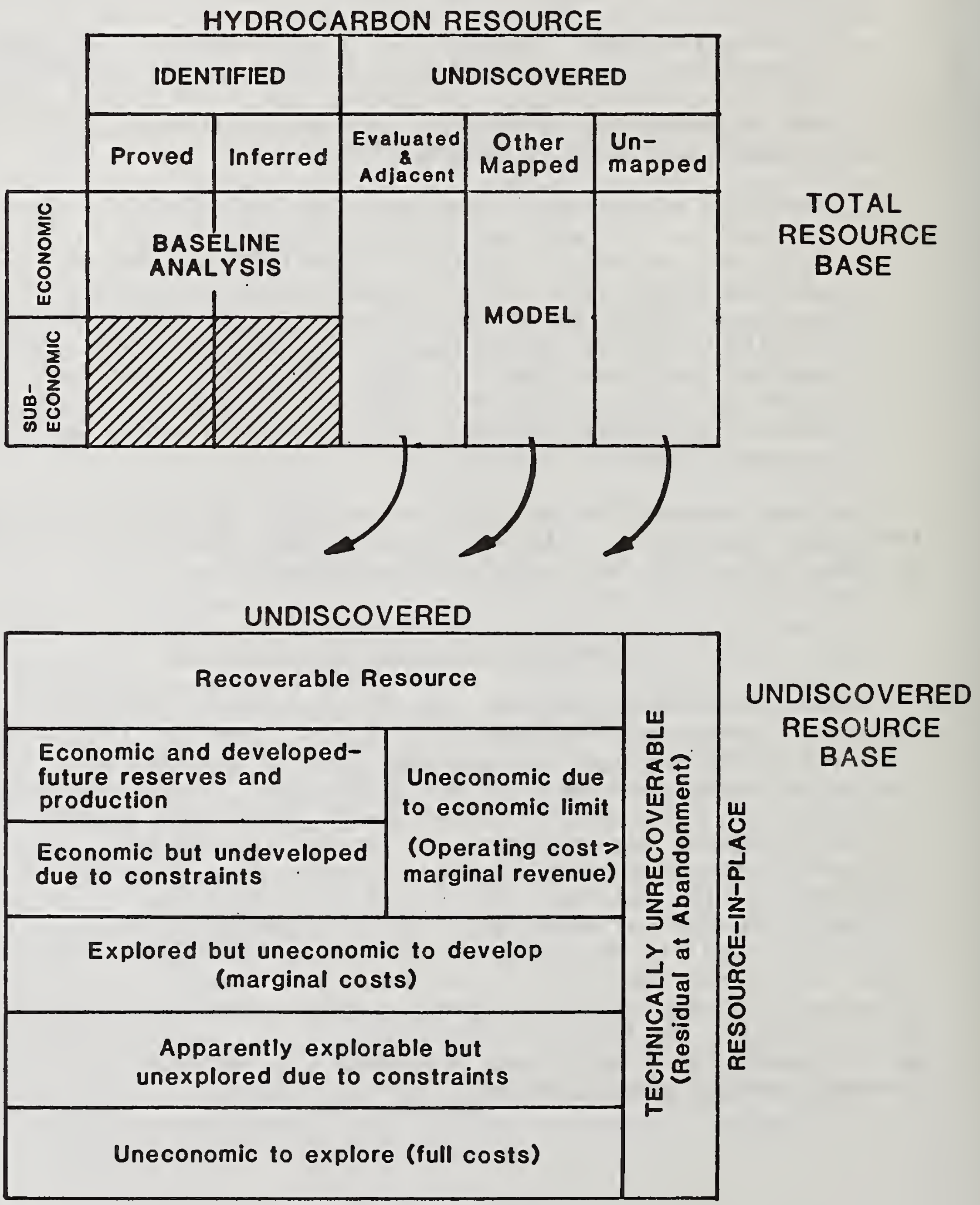
- The model may be easily updated as new geologic and engineering information is acquired. As more seismic exploration is conducted, additional prospects (especially smaller and deeper) will be identified. Other prospects will be drilled, thereby becoming fields or abandoned leases -- no longer prospects. Finally, offshore technology is rapidly changing. Updated geology and technology requires only minor modifications or additions to the database. Other models require costly, time consuming area-wide resource estimates which cannot be readily updated as experience accumulates.

In sum, through the detailed engineering and geologic database this model represents a significant advancement over previous effort by reducing the need for "empirical generalization".

3. Production from Identified Reserves

While the primary emphasis is on production from undiscovered resources, production from proved and inferred reserves (which sum to the "identified reserves" of Circular 725) is included in order to provide a complete estimate of OCS production. Proved reserves are the portion of the resource in known fields that can be produced under current technology and economics. (Circular 725 refers to these as the sum of "measured" and "indicated" reserves). Inferred reserves represent future additions to proved reserves due to extension drilling, revisions of estimates, and the discovery of pools in known fields (too small to be considered significant new finds). Production from "identified" reserves is assumed to be insensitive to Federal policies affecting the undiscovered portion resource; estimates of this production are based on projections of the separate time series of (a) production from proved reserves, (b) additions to inferred reserves, and (c) production from inferred reserves.

THE MODEL CAN PROVIDE A FULL ACCOUNTING OF THE UNDISCOVERED RESOURCE BASE BECAUSE IT IS BASED ON MICROECONOMIC ANALYSIS OF INDIVIDUAL PROSPECTS



Proved reserves are assumed to be produced at a constant fraction of the stock of remaining reserves; the rate of production was based on recent reserve decline trends. Inferred reserves are added by a Hubbert-type model [3] which uses the time series data of the American Petroleum Institution/American Gas Association (API/AGA) [1]; calculations are based on the year of discovery of the reservoirs that constitute proved reserves.

Each year, API/AGA [1] report both estimates of ultimate recovery by year of discovery of the reservoir and estimates of new reserves added during the year. According to API/AGA definitions, this change cannot come from "new" discoveries -- these would be recognized as discovered in the subject year. Hence, the change implies additional reserves added by means of additions, revisions, extensions and discovery of new pools within existing fields - i.e., inferred reserves. Subsequent production from inferred reserves employs the same decline rate used for proved reserves.

The approach used for estimating additions to reserves from revisions, extentions, and new pools is as follows:

Let
u = the number of years since the resource was added to reserves,
f(u) = the yearly percentage change in ultimate recovery from reserves discovered "u" years previous to the current year.

For example, f(10) would be found by averaging the change in estimates of ultimate recovery detected between their ninth and tenth years; f(20) would be the change in estimate for fields between 19 and 20 years and so-on. Since the API/AGA data is available for the period 1967-1979, there are 13 estimates for values of f(u) for $1 \leq u \leq 48$. The value of f(u) used by the model is determined by averaging the percentage changes in ultimate recovery at age "u".

It is possible to use f(u) in combination with the most recent API/AGA estimates to forecast yearly changes in reserves by the following:

Let
Q(t,d) Represent the ultimate recovery of nonassociated gas from fields discovered in year "t" (e.g., 1947) as reported in year "d" (e.g., the API/AGA time series for 1978)
A(t,d) = Age of reserves discovered in year "t" at time "d"
= d-t

Then Q(t, d+1) = (f(A(t, d+1)) + 1)*Q(t,d)

Production data for offshore areas are reported by the API/AGA, USGS, DOE, and a variety of state and private sources. Reserves data, however, are reported separately only by API/AGA [1] and these only for the Gulf of Mexico. Production estimates for the Atlantic and Pacific OCS regions are made by using ratios keyed to Gulf of Mexico figures.

Proved and inferred reserves and production of natural gas liquids are proportional to natural gas reserves and production; similarly, associated/dissolved gas production is proportional to crude oil production. In all cases, gaseous and liquid hydrocarbons are reported separately.

This component of the model provides base supply estimates, representing the likely future contribution to production from measured, indicated, and inferred reserves using current technology and economics.

4. Geological Data Base

The data used to forecast production from undiscovered reserves are derived by four basic processes:

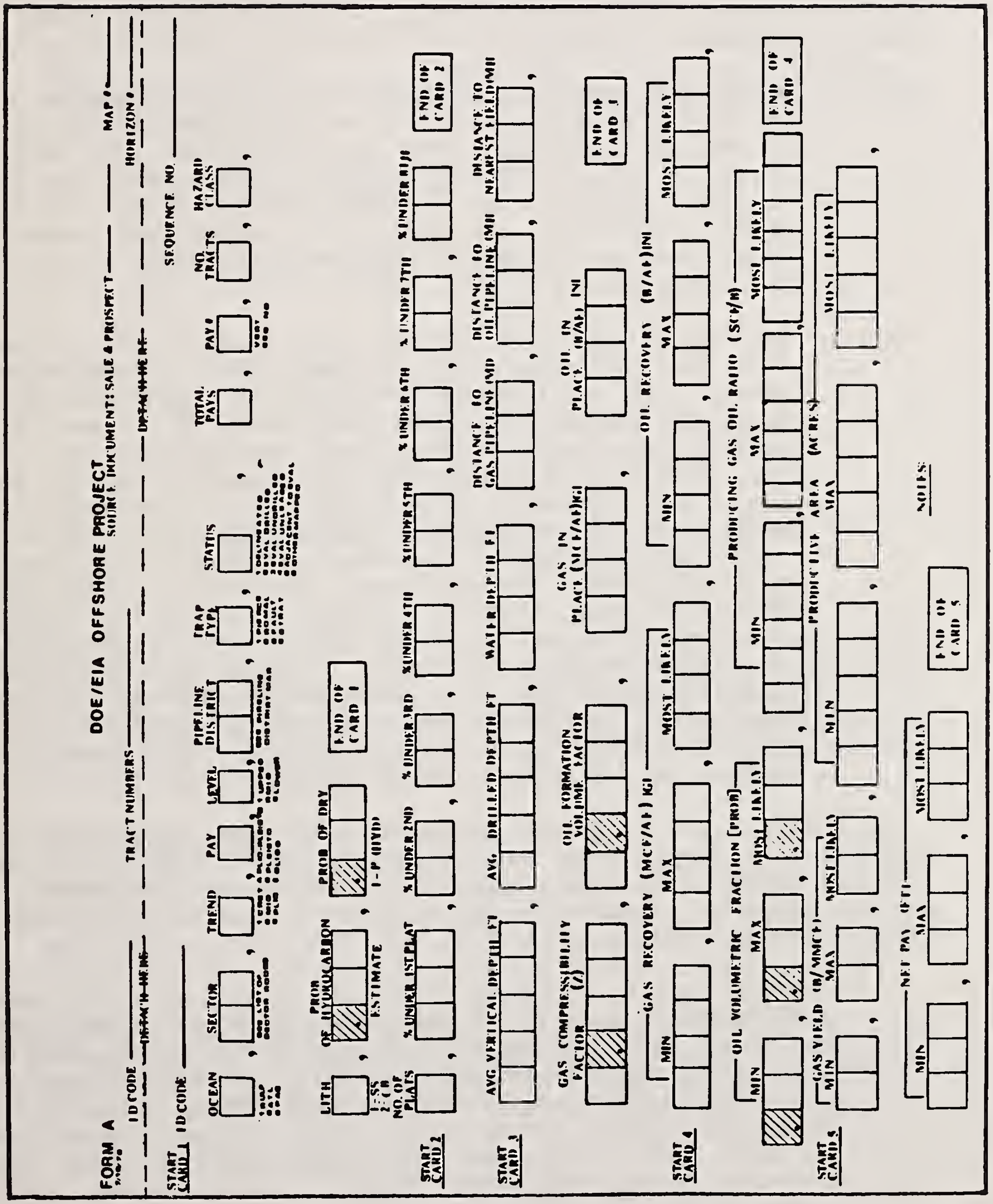
Class A: Data are taken directly from the files of the Conservation Division of the United States Geological Survey, which evaluates each tract nominated for sale. This evaluation includes seismic interpretation, engineering analysis, and economic evaluation. In addition, all structures in tracts adjacent to nominated tracts have been seismically interpreted. In the Western Gulf of Mexico, virtually all tracts with prospects of trapped hydrocarbons to a water depth of 200 meters have been mapped seismically and the majority have been evaluated.

Class B: Commercially prepared low-resolution seismic data for areas not covered by "Class A" methods -- essentially between 200 and 1,000 m in water depth -- were examined by USGS geologists and other geologists under contract to DOE. These reviews identified the existence of prospective traps and outlined three potential areas of closure depending on estimates of the location of the hydrocarbon-water contact. Engineering parameters were taken from an analogue prospect selected from the "Class A" data group based on geographic proximity and common geologic characteristics.

Figure 3 shows the data collection form used in this activity. The form was designed to protect the confidentiality of the USGS data by detaching all explicit references to location. Note that the evaluating geologists estimated a number of engineering, logistical, and geological factors as point-estimates. Geological uncertainty was

FIGURE 3

FORM USED IN COLLECTING PROSPECT-SPECIFIC DATA



introduced by estimates of the "minimum" (defined as the 0.95 probability), "maximum" (defined as the 0.05 probability), and "most likely" (or 0.50 probability) for parameters controlling resource in-place and technically recoverable resource, as discussed below.

The intention is to eventually employ methods of Class A and B for all lower-48-state OCS waters. Resource limitations in the study restricted coverage to about 65 percent of the Gulf of Mexico area. Accordingly, two additional techniques were used to approximate the remaining prospects in the Gulf, and in the Atlantic and Pacific Ocean.

Class C: Within the remaining portions of the Gulf, geographic areas of similar geological history, near to areas with Class "A" or "B" data, are designated as "recipients." The sampling frame for "donors" (i.e., prospects having detailed data) was the "play" defined by common geologic trends and water depth. Individual prospects were sampled from donors in proportion to the area of the recipient zone. Details of this approach may be found in [12].

Class D: For the Atlantic and Pacific, only broad, basin analogues from USGS Circular 725 are available. Prospects were sampled from the lognormal distributions, aggregating to the basin-wide resource estimates of Circular 725. A map of the Atlantic and Pacific areas under consideration was ruled into blocks approximately 20 miles square. Sampled prospects were randomly assigned to blocks with probability proportional to the number of nominated tracts in the block; water depth and depth of pay were estimated from bathymetric maps and regional geologic maps, respectively. This procedure helps insure that the prospects have a close analogue relationship to existing physical properties, field sizes and total resource estimates. Appropriate correlations (c.f., Standing [9]) and known relationships based on depth were used to assign reservoir engineering parameters to the sampled prospects. All of the specific data elements and methods are described in detail in [12].

For the Class A and B data collection activities, many of the data elements were simply read from seismic or other maps (e.g., true vertical depth, trap type, water depth, distances from shore, to nearest pipeline). Estimates of reservoir properties were made by using the following techniques:

<u>Reservoir Area</u>: Based on seismic structure maps, the geologists measured the area of apparent traps based on the assumption of an oil - or gas-water contact at the point where the trap was 5%, 50%, and 95% percent "full." Multiple traps could be included in the prospect if they had a common seismic occurrence and common source rock. When multiple traps were included, their respective areas were added for the three estimates because they were assumed to vary together due to common geologic history.

Reservoir Thickness: The net reservoir thickness was estimated based on nearby producing fields in the same geologic trend and the shape of the trap.

Fraction of Reservoir Volume Containing Oil (Gas): Assigned by analogue from the nearest field with similar geologic history. Oil-proved and gas-proved areas are relatively well established in the Gulf.

Oil (Gas) In-Place per Acre-Foot: These values were calculated volumetrically based on engineering parameters, using the following equations:

$$\text{Oil in Place per acre-foot} = \frac{7758\ 0\ (1-Sw)}{B_o}$$

$$\text{Gas in Place per acre-foot} = \frac{43.56\ 0\ (1-Sw)}{B_g}$$

Where:
- 7758 = Barrels per Acre foot
- 0 = Bulk porosity (%)
- S_w = Connate water saturation
- B_o = Oil formation volume factor (ratio of reservoir volume to volume of surface conditions
- 43.56 = Thousands of cubic feet per acre-foot
- B_g = Gas formation volume factor

Nearby analogue fields of common geologic history and depth provided estimates of porosity, connate water saturation, and oil formation volume factor. Gas formation volume factor was calculated from the pressure, temperature and gas compressibility based on the actual prospect depth using engineering correlations (See Standing [9]).

Recoverable Oil (Gas): Based on nearby analogue fields, both the expected drive type (natural water drive or pressure depletion) and its efficiency were considered. Based on this, three percentage "recovery factors" (representing min, max and most likely values for oil (gas), respectively) were multipled by the oil (gas) in place to estimate the portion recoverable.

Recovery of Associated Products: Producing gas-oil ratio (GOR) and natural gas liquids yield (NGL) were assigned on the basis of the nearest analogue field, as corrected for pressure, temperature and compressibility as a function of measured depth.

Number of Platforms: Platforms were assigned by estimating the radius of a circle that could be drained by directional drilling at 55° after the first 1,000 feet of hole. The number of circles of this radius necessary to cover the full prospect was estimated based on the size and shape of the prospect. The number of these circles is the number of platforms required.

Percentage of Resource Under Each Platform: Was assigned by estimating the relative volume of traps under each platform template.

Probability of Hydrocarbons: Was defined as the likelihood of hydrocarbons in any quantity being in the trap, and was assigned by the geologist as a subjective function of his own experience and drilling success in nearby fields.

5. Geology Model

The purpose of the Geology Model is to combine individual distributions of separate uncertain reservoir parameters using Monte Carlo sampling, into an overall resource distribution. As described above, the input data reflect uncertainty about specific geologic and reservoir parameters that determine the type and amount of hydrocarbon resource which is "in-place" and which may be produced from a given prospect. For each prospect, the geologists made three point estimates for the following:

- o Fraction of bulk sediment occupied by oil;
- o Recovery of oil (bbl/Ac-ft);
- o Recovery of nonassociated gas (Mcf/Ac-ft);
- o Yield of natural gas liquids (bbl/MMcf);
- o Productive area (Ac);
- o Net pay, or effective thickness (ft);
- o Producing gas-oil ratio (Mcf/bbl);

The minimum, maximum, and most likely estimates are used to determine a triangular distribution for each parameter; 1000 psuedo-random sample trials are used to combine these individual distributions (in conjunction with basic engineering relationships) into overall distributions.

The calculation sequence for each pseudo-random sample trial is:

1. Bulk sediments = productive area x net pay

2. Oil producing volume = bulk sediments x fraction occupied by oil

3. Gas producing volume = bulk sediments x (1 minus fraction occupied by oil)

4. Oil-in-place = oil producing volume x unit volume oil-in-place

5. Associated-dissolved gas in place = oil in place x gas-oil ratio

6. Recoverable oil = oil producing volume x oil recovery factor

7. Recoverable gas = gas producing volume x gas recovery factor

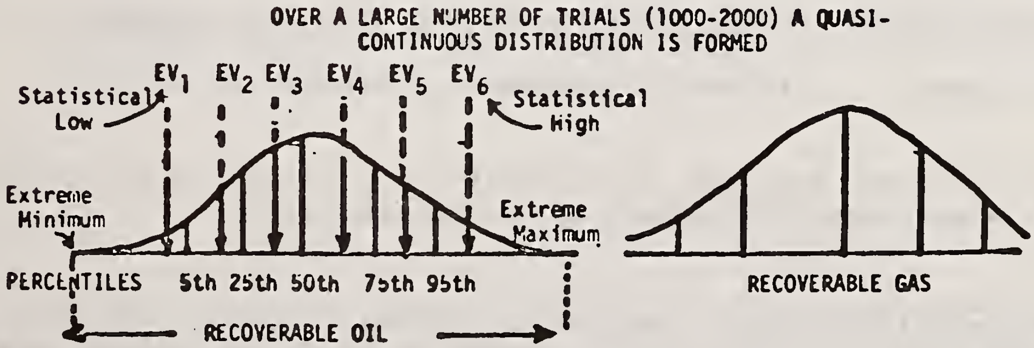
8. Recoverable associated/dissolved gas = recoverable oil x gas-oil ratio

9. Recoverable NGL = recoverable gas x NGL yield ratio

The distributions for items 4 through 9 are saved for later combinations into total liquids and total gases, in-place and recoverable, to complete the accounting for resources and reserves.

By combining over 1,000 pseudo-random samples trials, a continuous distribution is formed as shown in Figure 4. The distribution is summarized for later use by six points corresponding to the expected values of outcomes lying within the 0th and 5th, 5th and 25th, 25th and 50th, 50th and 75th, 75th and 95th, and 95th and 100th percentiles.

FIGURE 4



The full operation of the geology model, including a detailed statement of the data and equations may be found in [14].

6. Technology and Costing Model

The purpose of the Technology/Costing Model is to evaluate the cost of exploraton, delineation and development for each prospect in the data file. For each of the six field sizes derived from the oil and gas resource distributions and for the condition "dry," (i.e., no hydrocarbons at all in the trap) for each prospect, the Technology/-Costing Model estimates the investment costs and discounts them to their net present values. Thus, in general seven cases are cost-estimated for each prospect. When more than one platform is required for some field sizes, the costs of alternative development schemes are also estimated for later optimization, as described below. The model is based on an extensive study of the costs and availability of off-shore technology.

The development of a typical offshore project is conducted according to the following schedule:

Phase	Activity/Decisions
Exploration	First year that a prospect becomes available for exploration; time of decision whether to explore.
Delineation	End of exploration phase; dry prospects are abandoned and delineation drilling begins in "non-dry" prospects to determine field size.
Development	Delineation drilling completed, defining field size; time of decision on whether to develop; development begins in fields that are economic at the margin viewing exploration costs as "sunk".
Production	Development completed; production operations commence.
Economic Limit*	Economic limit is reached; production ceases.
Abandonment	Platform is abandoned and removed.

Cost data were developed in conjunction with the Dallas Field Office of EIA and through independent engineering analyses.

* Because the economic limit is a function of price, this phase of each prospect's development is treated in the policy analysis model.

The exploration costs are those expenditures required to determine whether the prospect is "Dry" or "Not Dry". Exploration is generally conducted from a mobile drilling rig. Exploration costs are based entirely on the external physical parameters of the prospect; the size of the field which may be contained in the prospect does not influence exploration costs. The following costs are estimated:

- o Geological and geophysical costs (function of surface area);
- o Number of exploration wells (function of surface area and type of trap);
- o Cost per well (function of water depth and drilled depth);
- o Drilling rate per year (function of total drilled depth, allowing for movement of rigs); and
- o Business factors (overhead, rate of return).

Where exploratory wells fail to find hydrocarbons, the sequence is ended. Given the field has been determined to contain hydrocarbons in the exploration phase, however, delineation costs are expendited for additional exploratory wells to determine the size of field contained in the prospect. Delineation drilling commences with the termination of exploration. Delineation wells are drilled using the same technology (and per well cost) as exploratory wells. All exploratory and delineation wells are assumed to be plugged and abandoned due to lack of production facilities. When delineation drilling determines the field is too small for commercial development, the prospect is abandoned. When the field size is sufficient to justify further investment, development costs are estimated.

Some development costs depend only on the physical properties of the prospect and not the field-sizes including:

- o Per-Well Development Drilling Costs: Including the cost of a single successful production well and a single developmental dry-hole. (Function of water depth and average drilled depth).
- o Expected Fraction of Developmental Dry Holes: A function of trap-type, recognizing that various traps require inherently different patterns (and risks) of development.

- Pipeline Costs: If the prospect is within 10 miles of a pipeline trunk or the shoreline, the actual amount of connect line needed for each product (oil and/or gas) is costed. If the prospect is not within 10 miles of a trunk, only 10 miles of 2 inch connect line are built for each product. (A separate algorithm "builds" trunk pipelines).

- Distance to Shore: A regression equation is used to yield distance to shore as a function of water-depth. This equation is made possible by the gently sloping bottom of the shelf; it is made necessary by the requirement that the confidentiality of the data be preserved, and that actual distances could not be included in the database.

Once these costs are calculated, development costs that depend on field extent and size are estimated. In the data collection process, the geologists estimated the number of platforms required to drain the entire prospect based on the "reach" of a directionally drilled well (to a maximum of 56° from vertical from each platform). For prospects requiring more than one platform, the proportion of the resource under each platform was estimated based on the estimates of the area of closure. This procedure allowed large and/or unusually shaped prospects to be incorporated into the analysis, with a pattern of development which would approximate that which would be followed in actual exploration.

For prospects requiring only one platform, estimation of the remaining development costs is straightforward.

Number of wells in prospect: a function of field size and flow rate.

Platform Cost: a function of water depth, and number of wells. Overall total wells are assigned to platforms according to an apportionment of resource to platforms. The platform count was obtained by laying circular templates representing radius of drilling on the volume of closure on the seismic map (as interpreted by the staff geologist). The number of platforms is determined by the number of "circles" required to cover the prospect. Percent of resource attributable to each platform is based on the per cent of volume drained by the platform.

Platform Construction Time: a function of water-depth and number of platforms.

Peak Production: a function of product (i.e., oil or gas) and field size.

Equipment Costs: a function of peak production, product, and distance to shore.

Yearly Drilling Costs: a function of number of wells, dry hole rate, rate of drilling and per-well costs.

Yearly Platform Development Costs: a function of platform cost and platform construction rate.

When more than one platform is required, optimization is necessary. The model estimates the costs of a number of development options, including additional platforms and use of subsea satellite wells, and mixes of these. The shape of the prospect may be such that a portion of the prospect lies too far from the platform to be reached by directional drilling but is too small in itself to justify an additional platform. In such cases, subsea satellite wells can be used to obtain production from portions of the prospect that might be uneconomic if each portion had to bear the full cost of platform development. This development system requires at least one permanent platform, but, production from the satellite wells uses a subsea wellhead with remote control metering and safety devices. Maintenance is accomplished by a "through flowline" system requiring one dedicated 2-inch flowline to transport special tools; production is transported to the platform by a second 2-inch flowline for metering and eventual sales to a trunk pipeline.

All conventional technology costs are discounted to the time at which platform construction starts. The development sequence, is as follows:

When more than one platform is needed, the model calculates the investment cost of all relevant permutations of additional platforms and subsea completions.

Subsea satellites are used when the marginal cost of a subsea well is less than the prorata share of platform costs attributable to each platform-based well. The marginal cost of a subsea or satellite well is:

- o The difference in drilling costs between wells drilled from an exploration-type temporary rig and those drilled from a platform (including dry hole costs), plus
- o The costs of the subsea or satellite wellhead completion system (installed) plus
- o The costs of flowlines to the platform.

The costs of the satellite completions vary with the water depth. For water depth less than 60 feet, caisson completions are used. At greater depths, full subsea completions with "through flowline maintenance" are used, requiring two 2-inch flowlines. The length of the flowlines is set at two times the maximum drilling radius from the fixed platform

The net present value of all relevant development options is calculated and passed to the Analysis Model. In that model, the net present value of the revenues from each option is compared with the costs, permitting selection of the optimal development option on the basis of marginal revenues and marginal costs.

All equations, and coefficients, and procedures used in this section may be found in [13].

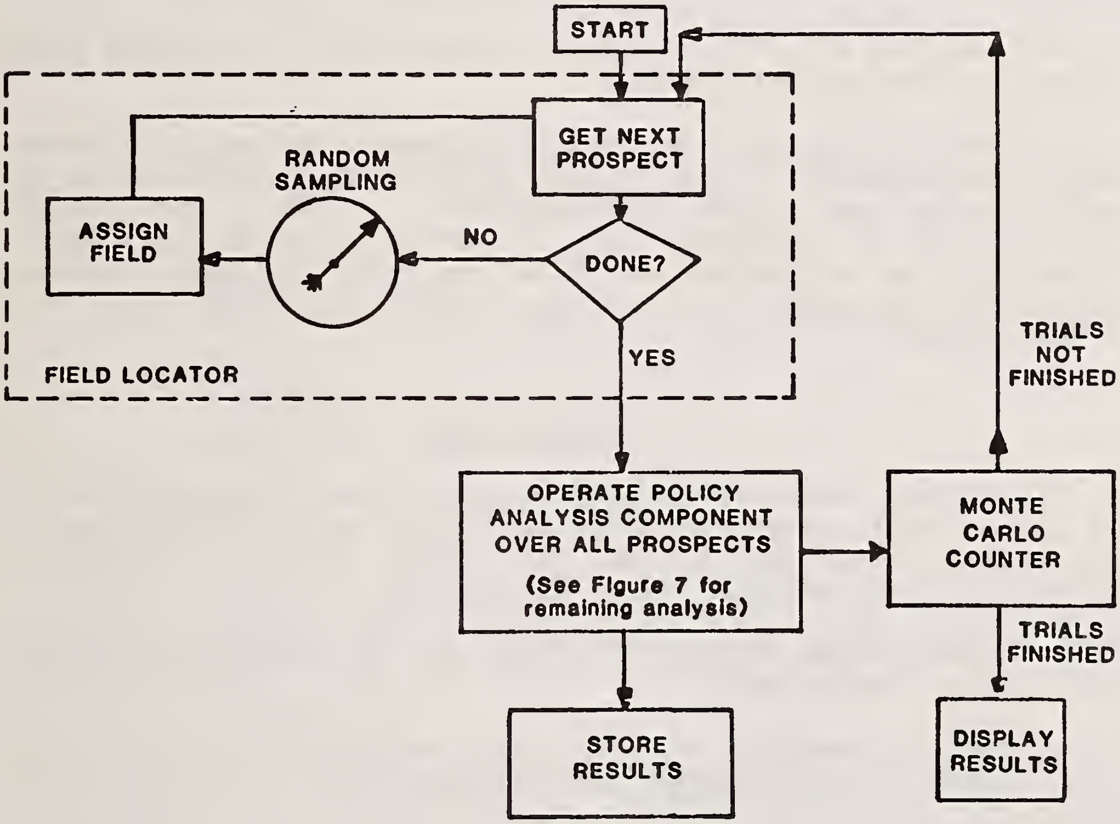
7. Field Sampling Component

The Geology Model and Technology Costing Model need be run only after updating because they create large datasets for the remaining two modules. The Field Sampling Routine and analysis model are the two portions of the OCS model that are run routinely for forecasting and policy analysis. The purpose of the Field Sampler is to provide a particular field-size assignment to each prospect in the data-base for each Monte Carlo trial (see Figure 5). This serves the role of linking uncertainty in the resource base with the deterministic engineering and business decisions in the Analysis Model. As shown in Figure 6, the unit interval is first partitioned into "dry" (i.e., no measurable hydrocarbons in the prospect) and "not dry" on the basis of the geologist's estimate in the data collection phase. The "not dry" portion is further subdivided according to the percentiles of the six key points used to characterize the prospect's resource distribution. This creates a seven-fold partition of the unit interval; sampling from this interval is accomplished by selecting a random fraction on (0, 1].

Figure 5:

POLICY ANALYSIS MODEL (RESOURCE UNCERTAINTY)

A SPECIALIZED PROGRAM CALLED THE FIELD LOCATOR ASSIGNS FIELDS (OR "DRY") TO EACH STRUCTURE BASED ON THEIR LIKELIHOOD (i.e., P(Dry), P(EV1) ... P(EV6))



OVER A NUMBER OF MONTE CARLO TRIALS, THE UNCERTAINTY IN THE RESOURCE IS INTRODUCED TO THE DETERMINISTIC ECONOMIC ANALYSIS

Figure 6 Sample Outcomes and Their Probabilities

	Dry	Not Dry					
		Class Size					
Possible Outcome		1	2	3	4	5	6
Probability	P	(1-P).05	(1-P).20	(1-P).25	(1-P).25	(1-P).20	(1-P).05

The sampling of all prospects is repeated for each Monte Carlo iteration of the analysis submodel.

Each Monte Carlo trial requires an assignment for each of approximately 1800 prospects. For a 'suite' of 100 Monte Carlo trials of the analysis model some 180,000 assignments would be made. These assignments are kept in a computer file, for access by the Analysis Model as the costs of exploration are paid, the model "discovers" whether hydrocarbons are present; as costs of delineation are paid, field size is "determined".

8. Analysis Model

The Analysis Model is designed to account for the disposition of the undiscovered resource as a function of (1) geologic, engineering, and economic factors and (2) specific Federal policies. The Federal policies which may be specified by the user are:

- o Assumed market prices for oil and gas at the wellhead for the period 1980-2000.
- o Leasing schedules in acres per year by the USGS leasing districts (e.g., Georges Bank, Western Gulf).

- o Bidding rules, including (a) lease bonus -- fixed or variable; (b) royalty system--fixed or declining for the life of the prospect; (c) profit share--fixed or variable.

- o Corporate tax rates, investment tax credits, etc.; and

- o Institutional delay (in yr.) between delineation and development to provide for permitting.

The Analysis Model simulates decision-making under uncertainty for the industry as a whole annually from the present through the planning horizon (e.g., 1990, 2000). Figure 7 presents a general flowchart of this model.

The simulation of exploration, development and production for each prospect is the core of the model including:

- o Prospect evaluation and ranking;
- o Bid determination;
- o The "go," "no-go" and "defer" decisions associated with

  - -- bidding,
  - -- exploration,
  - -- development; and

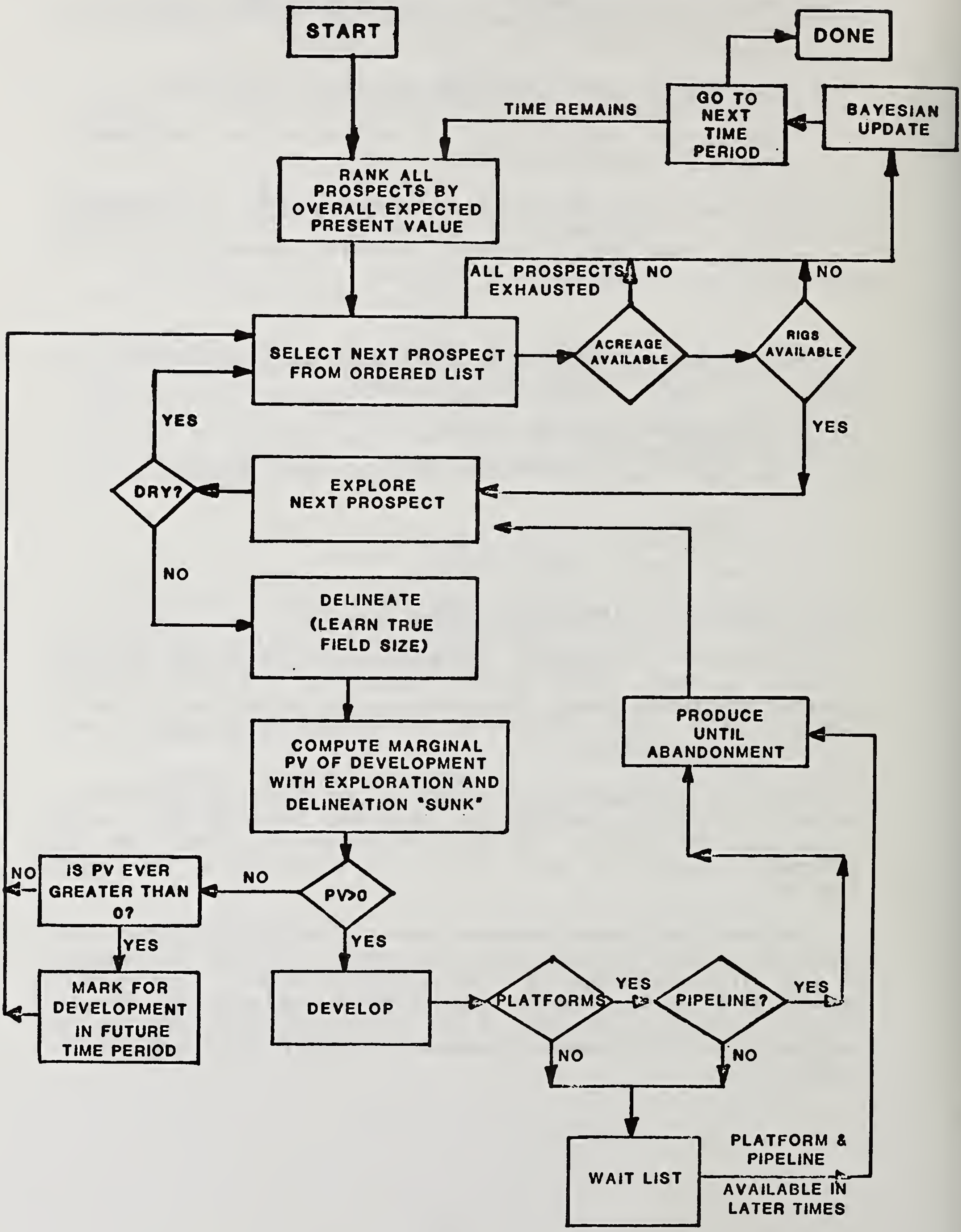
- o Production to the economic limit.

All decisions in the model are made using a discounted cash flow analysis. Economic attractiveness is determined only on the basis of expected net present value, as opposed to other motives such as secure supply or other issues ancillary to the venture itself, such as denying access to competitors.

Given the availability of prospects expected to be profitable, exploratory drilling activity is determined by both the leasing schedule and rig capacity. The model can be set to constrain the rate of growth of capacity to drill after it is initialized at the current capacity.

The model recalculates expected present value for each prospect in each time period to accommodate (a) changing prices and (b) evidence obtained from exploratory drilling in prior periods. The expected present value is the statistically weighted combination of the net present value (NPV) of each of the possible outcomes that can be selected in the field sampling component. The NPV for each outcome is based on:

Figure 7:
POLICY ANALYSIS MODEL (ECONOMICS) FLOW OF ONE MONTE CARLO TRIAL



- o A cost stream representing exploration, delineation, development, and abandonment (generated by the Technology/-Costing Model) and is discounted appropriately.

- o A revenue stream composed of production multiplied by price and adjusted to reflect the effects of taxation and operating costs up to the economic limit.

- o Appropriate relations between cost and revenue suggested by the bidding strategy.

The net present values representing all possible outcomes are combined to form an overall expected present value by using probabilities based on two cojoint factors:

- o The relative likelihood of each field size for each prospect given an estimate of the probability of a "dry" structure.

- o Four states of nature representing various possibilities for the dry structure rate with a priori likelihood for each state.

The a priori likelihood of each state of nature is a function of initial geologic uncertainty and accumulated experience. These are specific to a "play" (a confined geologic and geographic area) defined as all prospects of a given trend within the east-west width of the USGS nominclature districts (e.g., Galveston, Eugene Island). These probabilities are updated in part by a Bayesian process at the end of each time period. This process was adopted to simulate "play" behavior often observed -- i.e., a few discoveries tend to revise the explorationist's probabilities upwards, whereas disappointing exploration results tends to discourage further exploration. These adjustments often appear to be larger than simply a Bayesian reconsideration of earlier data.

In establishing bid values, the Analysis Model treats the industry as a unitary entity, not attempting to model intercompany competitive bidding. Of the various bidding options (lease bonus, royalty, profit share, etc.) it is assumed that (1) the Federal Government fixes the rates for all components except for one and (2) the maximum, economic bid (the one which would produce an expected net present value of zero) is then calculated. This bid is not necessarily the bid that would happen in practice. Depending on assumptions about risk aversion and distributions of expectations in firms, the actual bids in an auction could be higher or lower. Once the bids are calculated, they are compared against a minimum bid, if the model is run with this option. The prospects with positive bids that (a) exceed the minimum bid (if specified); and (b) are available for lease, are ranked according to expected profitability.

The unexplored prospects are rank-ordered according to the overall expected net present value. All profitable prospects are scheduled for exploration, subject to acreage and rig availability. To maintain close correspondence with current leasing practice, the model assumes that the number of prospects that can "be drawn" from undrilled structures in any time interval is constrained by (1) lease schedules; (2) remaining exploration "commitments" carried over from earlier decision points; (3) remaining delineation "commitments" generated by successful exploration efforts in earlier periods; and (4) rig availability after these prior commitments are satisfied. This ensures compliance with the acreage and drilling industry capacity restrictions imposed upon the system.

A user-specified lease schedule is defined in terms of acres per year by leasing area. Before exploration is simulated, the amount of drilling capacity not previously committed is calculated. Based on this and total rig capacity, the uncommitted rigs are assigned to each structure to be drilled and an accounting of rigs is kept by type (i.e., appropriate to water depth ranges) and "ocean" location (Gulf, Atlantic and Pacific). Rig assignment is made in three passes:

- Wherever possible, the economically most attractive prospects are assigned available rigs of an appropriate type currently in the same ocean as the prospect.

- Next, rigs capable of drilling in deeper water, in the appropriate ocean, are assigned prospects in shallower water. This rig is then committed to shallower water for the duration of the exploration and delineation of this prospect.

- Finally, a limited number of rigs may be transferred from other oceans. This transfer is permanent unless conditions warrant a return.

Prospects are then explored and delineated according to their ordering on expected net present value within the constraints upon the availability of rigs. Structures that cannot be explored during the present time period due to rig constraints are deferred until future periods.

As the costs of exploration and delineation are paid, the Model simulates "discovery" by referencing the Field Sampler file to determine the "true contents" of the prospect for that Monte Carlo trial (i.e., both presence of hydrocarbons and the field size).

Prospects that have been determined to contain hydrocarbons are then evaluated for development on the basis of the marginal costs of development. The present value <u>given</u> known field size governs the decision to initiate development or to continue production and is based solely on incremental costs. Lease acquisition and exploration costs, for example, are sunk and do not influence the development decision.

Given that exploration yields hydrocarbons, a prospect need not be developed immediately. A "prospect inventory" is kept, where development that is currently uneconomic may be deferred to some later period. For example, development may be postponed until there is a sufficient future increase in real oil and gas market prices (as specified in terms of an input growth in the real price trajectory for each) or if other nearby prospects justify the construction of a pipeline to that region. The marginal venture would be developed when net present value, using development and operating costs only, is positive. The "prospect inventory" is re-evaluated on each cycle, and deferred prospects must compete with prospects newly discovered in the cycle. The model does not allow for the speculative withholding of currently profitable structures.

The prospects that pass the marginal cost evaluation are scheduled for development in conjunction with constraints on:

- Pipeline availability. Each structure is assigned to a pipeline district. If a trunk line serves the district, there is no pipeline constraint for that structure. Each district without a pipeline is assigned a list of "nearby" pipeline districts (including the shore) along with the minimum reserves* required in that district to justify construction of a trunk pipeline into that district from a neighbor. Unproduced but proved reserves available in each district are summed each year; when they reach the minimum level, a pipeline is assumed to be built, and all economic reserves in that district then become eligible for development. The cost and benefits of building the trunk pipeline are assumed to be borne by an external pipeline service company and do not directly enter exploration or development decisions.

- Development platform availability. A maximum number of development platforms is specified for each year as an input variable. Structures with the highest dollar value of reserves are given the available platforms. Since platform building constraints are not currently expected to materialize, the constraints have been set to be inoperative. Those prospects that cannot be developed due to constraints during the current time period remain available for development in future years.

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* A separate subprogram computes the investment costs of a main trunk pipeline based on water depth, pipeline length and pipe diameter. Operating revenues are assumed to be derived from a tariff on the flow of oil and gas. Minimum reserves are calculated as the amount needed to provide a flow sufficient to repay the investment costs plus rate of return and operating costs of the pipeline over a 20 year period.

The model schedules production from each developed prospect according to the capacity that was estimated in the Technology/Costing Model associated with the field size that resulted. Year-by-year production, is aggregated across prospects and resource-accounting bookkeeping is kept over the period 1980-2050. Results are saved for each Monte Carlo iteration. The volume of recoverable resource for each of several categories (developed, unexplored due to economics, unexplored due to constraints, undeveloped due to economics, undeveloped due to constraints), is recorded by ocean. Cumulative reserve additions, cumulative reserve adjustments, cumulative and incremental production by year and ocean are also tallied.

Total production is derived by adding production levels associated with currently identified reserves (see Section 2) to those from undiscovered reserves evaluated in the Analysis Model. The final resource and production accounting is performed at the conclusion of the complete simulation. Variability across Monte Carlo trials is reported.

9. Limitations of the Present OCS Data Base and Model

As with any model, the OCS model can be improved with several enhancements. First, the database needs to be completed for the missing portions of the Gulf and for the Atlantic and Pacific. In addition, systematic procedures should be developed to maintain and update the files as new seismic prospects are identified, interpretations change, and as real exploratory drilling either "condemns" as dry or "proves" as reserves the prospects in the file. Moreover, routine updating of the technology specifications and costs as these evolve should occur. However, the existing database is sufficient for sensitivity analysis and experimentation with policy variables. As a forecasting tool, the missing data are a handicap, but even with this limitation, the treatment of Class D data (Atlantic and Pacific) represents a more precise analysis of the subjective resource estimates and is well structured for updating as new prospects are included, "old" prospects are explored, and/or technology or costs change.

As data for the remaining identified seismic anomalies are entered as prospects, a further limitation must be noted. Maximum precision and validity of the OCS analytic strategy lies in explicitly limiting the forecasts to production from seismically identified prospects. Certainly for the near term, and very probably for the intermediate term (through 1995), this poses no difficulty. When completed, the database will, in fact, represent exploration targets for this time horizon. For the longer term, however, current limitations of seismic technology require acknowledgement that additional, future prospects will be identified over time as pre-drilling exploration technology improves. The OCS analytic approach will always understate the ultimate potential of the offshore regions by the amount of the prospects not yet identified. For very deep prospects and very large, stratigraphic prospects, this future cannot be estimated. Two possible solutions to this limitation appear feasible. First, limit the forecast period to, say, twenty years or so, without attempting to represent the "all-time" ultimate resources or reserves. Alternatively, acquire subjective resource assessments of these unidentified prospects and simulate them using the methods employed for "Class D" data described in Section 4. This latter approach would permit reconciliation with the "overall" estimates of USGS and others while incorporating some of the precision of this approach: if one knows where the identified prospects <u>are</u>, by deduction, one also knows where the unindentified are not. Thus, they must be in unmapped areas (generally in deeper water or further from existing pipelines) or at greater total water depth. This knowledge alone permits insightful economic calculations. If this second approach is used, it is recommended that the resulting estimates be segregated according to the class of source data so that the user of the analysis will recognize the distinction.

A model of rig capacity and rig movement needs to be developed to eliminate arbitrary constraints as well as provide more realistic estimates of cost changes in drilling activity. Reduced form, econometric models may be sufficient.

The method of revising the <u>a priori</u> probability of hydrocarbon occurrence should be reconsidered. A straightforward approach would be to take each of the prospects and group them by the geologists' evaluations of the probability of success within geologic trends. Each group could have its associated prior distribution updated whenever a member of the group is chosen for exploration. Establishing the prior distribution for the initial year would require, however, further data gathering and analysis which tends to be very expensive. Grouping all prospects in a region together and applying a pure Bayesian update on the aggregate is unsatisfactory because the best (i.e., highest ENPV) opportunities tend to have higher probabilities of success. This approach would bias the model to drilling beyond the point at which it is economic to continue exploration. Finally, there is the theoretic question of what the update means since the initial geologic assessment is what is used to sample the states of nature. It would appear that the notion of "learning" from exploratory experience is necessary to the realism of the model -- vis, "play" behavior in industry -- but the specific theory and calculation deserve further development.

As currently structured, the model requires over 6 CPU minutes (on an IBM 3033 MP system under MVS) to prepare a forecast. However, substantial efficiencies might be achieved through elimination of repetitious calculations through judicious assumptions and restructing of the simulation accordingly.

10. Conclusions

The OCS model provides a rigorous tool to analyze many important, sometimes subtle, questions of Federal policy on the basis of geophysical and other disaggregate data. In this, the experimental analytic approach adopted by the OCS study is very promising.

The majority of the data exists in readily accessible form in USGS files. Both the database and models are readily updated in modular form, obviating massive new studies as conditions change or new, more precise policy questions are raised.

Forecasts and analyses from this model can serve the needs of many users. Such users would include leasing-policy officials in DOE and the Department of Interior, market planners for offshore technology in private industry, policy-makers attempting to provide special offshore incentives, public and private decision-makers seeking to relax logistical constraints, and others. In particular, this modeling approach serves to bridge the gap between overall subjective appraisals of undiscovered resources and detailed accounting for proved reserves, permitting the monitoring of the flow from highly speculative undiscovered resources to cumulative production. Models requiring more assumptions and less data than this approach can produce widely differing conclusions with reasonable choices for aggregate parameter settings, thereby confusing the policy analysis process.

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CONCLUDING SESSION

DR. GASS: Good afternoon: the panel will be run by Dick O'Neill and Fred Murphy from EIA. Charles Everett and Wally Keene will be joining them. We would like to have an interchange between the panelists, the audience, and try to develop some ideas and raise some questions from the point of view of DOE's future activities in the development of oil and gas supply models.

DR. MURPHY: We spent two and a half days with people presenting their work; showing us what they perceive to be the state of the art and what they are doing.

This morning, we spent time looking at models that are built to provide complete oil and gas forecasts. It is useless to have complete knowledge about a single aspect of oil and gas supply if you can't bring that knowledge to bear on national oil and gas supply issues. Most of the models being presented this morning are models that are used by EIA to develop national forecasts.

What we hope to achieve from this panel discussion is advice from the participants in the audience as to where the Energy Information Administration ought to use its resources to improve the state of the art in oil and gas supply forecasts.

Dick, do you want to say something?

DR. O'NEILL: Yes, I'd just like to continue and possibly be a little more specific, with some questions to the audience. We've been telling you how we do our modeling, this morning. One question that should be addressed is: What kind of data should we collect?

The Energy Information Administration has the opportunity to collect energy data that it can justify as being important in this area. It has to consider costs and the burden on the respondent. One of the biggest problems that I mentioned in my presentation this morning, is that people want to talk about finding rates; but, in fact, the way the data are gathered and put together, you are almost hopelessly lost from the beginning, in obtaining good data for the estimation of finding rates.

Another question is: What level of aggregation should our models take?

Something that has been mentioned infrequently in this discussion, but that has been causing headaches for the Oil and Gas Division, Fred and myself is the use of models for policy studies, which leads to the question: How do you get the physical model correct, and then on top of it, do policy analysis? If you model new discoveries, new field discoveries, extensions, or revisions in one way, Congress and the various regulatory agencies create legislation and regulations that often do not map well onto the data collection categories.

If you look at the original definitions of leases and properties for old oil, lower tier oil, upper tier oil or the three tiers oil for windfall profit tax you find that they don't map conveniently into physical quantities that have been discussed in a modeling framework here.

DR. MCFARLAND: I'm James McFarland, University of Houston. It appears to me that people have not been asking the questions that need to be answered, and that should be what's driving the data collection and model building phases. There seems to be a lot of concern for large models without that driving mechanism, and it's not at all clear to me that very simple models could be be used to give you much better insights to some very basic policy issues.

But it would appear that you should state what the policy question is, and then ask what data and what models might be used to try to gain insights into that.

It's not at all clear to me that the very complex models buy you that much; especially, when you're not putting the questions to be answered at the top.

DR. O'NEILL: I agree with you. Could you be more specific? Remember, you don't always have the time to build a model from scratch to answer a question. On occassion, you have less than 24 hours.

DR. MCFARLAND: Well, I think it has to start at the front end. You don't build a general purpose model to answer all questions.

There's no general policy model. It just doesn't exist, and it doesn't appear to be that DOE is going to be making decisions about where Exxon is going to explore, or how are they going to develop, except through certain policies and regulations. That will control it to some extent, but Exxon will be building models to make these decisions, or they will have decisionmakers making these decisions, as long as we have the free enterprise system.

And so I think there's a real question about what DOE should be modeling. I would think that you primarily should be developing models to assist the government policymakers in reaching better decisions; and hopefully, providing insights into what some of the regulations that are being formulated will have--from the producer up.

DR. MURPHY: Could you also give us your idea of what you think a large model is?

DR. MCFARLAND: Well, several have been described.

For example, the work that's been done at Brookhaven involving process-type models that are driven by final demands, and I assume that what was being discussed was some type of exhaustible resource model that acts as a constraint on that model. I consider that a very large model. The model that was discussed as being developed at EPRI; I would consider that a large model. I would consider some of the models at FEA large models, whether they're linear programing or whatever.

I question whether or not some of the policy issues that need to be answered can't be answered with simpler models. Before beginning model building phases, questions should be asked such as, How much do I really need to know about this? Do I want to be specific enought to know to drill a well, a 100 miles off Houston, or am I just looking for general--broader insights into some of these policy issues?

I don't have the answers. The model building seems to be evolving before some of the key questions are asked.

MR. EVERETT: Let me take a shot. I'm somewhat sympathetic to some of the things you're implying and stating.

EIA has a model simplification program of sorts. It is directed towards the simplification of existing EIA models. It is hoped that as a result of the program some models will be more directly updatable and will consume less resources to operate and document. In the process we will have to choose public policy issues that are most important and the models that apply.

I think one of the major problems with many models currently is the lack of clear documentation and precision in definitions at the data element level. (A data element being crude oil production or-natural gas liquids production, gas/oil ratios, etc.) A second problem which is related involves a clear classification of what information is required to discuss policy issues.

I think a lot of people in this room, certainly the speakers and a great majority of the audience, know what issues they're interested in--can speak and communicate on oil and gas supply modeling in particular.

But you'd be surprised how many times the same people really say something where they don't mean exactly that. They're talking about crude oil and you have to beg the question, "Do you mean total petroleum liquids? Are natural gas liquids to be included?"

When they say, "Gas" you have to beg the question, "Is it non-associated gas only, or are you talking about total gas production; from all sources, or simply conventional sources?"

One of the programs in EIA, which is not resident in the Office of Applied Analysis, but is very closely aligned, is a top-down approach to data requirements analysis. It is being done by the Office of the National Energy Information System.

Step Number 1 is to answer the question: What issues and what public policy areas need information? This information might be provided by a simple data collection mechanism--a survey, the development of an indicator or an index from existing data or, a model; but, a couple of things have to happen. Step Number 2, a vocabulary has to be established. Step Number 3, the information holdings of the EIA must be described with this vocabulary. These holdings include models, survey forms, publications (tables in publications), and frequently used data not generated by EIA. Finally all of this has to be indexed and classified, and to be blunt, put into some kind of form like yellow pages.

When we have the yellow pages there won't be nearly as much of a problem in trying to get to the root cause of certain definitional problems. The yellow pages ought to be indexed by major issue-area, as well as by a simple energy source and function scheme (e.g., crude oil transportation, or oil refining).

Currently the yellow pages are missing, but it is being developed. The Department of Energy and the EIA have only existed for a reasonably short period of time with respect to the time that agencies like Census, or BLS, or BEA, or other groups have been approaching their subject area.

DR. MCFARLAND: I would be concerned that people develop information systems for the sake of developing information systems. I can understand why you need data for various uses, but I'd really be concerned about large information systems just for the sake of having large information systems.

I've worked on very few projects where you could get the data that you needed to use in trying to address some issue exactly in the form you wanted.

I think people dream about having data bases and some great computer program where you push a button and it's going to give you all sorts of answers to questions as you ask them. I'm just afraid that's not likely to happen. In fact, I'd be very surprised if that ever happens.

MR. EVERETT: Well, people don't dream of the yellow pages, and it's a useful device. I'm working on it at the present and would be glad to take into account any suggestions you might have.

DR. MCFARLAND: Let me give one example. Let's suppose you're looking at the offshore development problem in the Gulf Coast. You're concerned with petroleum reservoir. development. You can take one company's data, just on one reservoir that has a fairly long lifetime, and you're going to find that they have a tremendous amount of data of various kinds.

If you look at what's reported to federal agencies, it's really a small percentage of data that the company has, and most of the time it's not in a very useable form if you're really concerned about the problem of petroleum reservoir development.

In fact, as best I can determine, companies seldom report pressure data, which is probably the most critical variable in terms of the reservoir behavior. You can have volumes and volumes of data that's totally irrelevant for managing a petroleum reservior.

So I really question whether or not we are ever going to get to the point where DOE can have data to answer some of these basic questions. There's no way you can collect enough data to answer some of these questions, and that's the reason I say I think you've got to start with the basic question that you're trying to look at, if it's a policy issue or whatever, you take the best models you can get, the best data you can get, and you try to provide policymakers with the best answers that you can provide them.

MR. EVERETT: I'm being more modest than that. Once you define your issues, and I think that you would agree they're fairly straightforward (as is petroleum development off the Gulf Coast), I just simply would like to index the information that we (the people at this conference, for example) currently feel comfortable with. I'm not for annexing private property without due process. We don't want all the oil company information, as near as I can tell. (Moreover, pressure data may currently be reported to the USGS.)

DR. HARBAUGH: John W. Harbaugh, Stanford University. I'd like to comment on geological resource base data. I touched on data sources when I gave my presentation, and one of my recommendations involve the creation of a national inventory of oil and gas fields within the United States. Such an inventory could be created at very small cost relative to its value. Many of the data are already available, such as oil and gas field cumulative production, and in some states, estimates of remaining reserves are available. The cumulative production data and the reserve estimates could be updated annually and maintained in a computer file. It could be made available on computer tape, and printouts, which would probably aggregate about the size of the D.C. phone book, could be produced for convenient manual reference.

MR. EVERETT: I agree, and I think--Wally Keene, who's our Director of Oil and Gas Information, has a program that's moving in that direction quickly.

DR. O'NEILL: Well, there is a problem with that. Wally's main thrust is to reproduce the reserve estimates. The future of oil(gas)-in-place ultimate recovery and date of discovery information is not clear at this time.

VOICE: Well, that might be work that could go on for years--estimated reserves in place is an enormous undertaking.

DR. O'NEILL: EIA is only collecting reserve estimates, not producing the estimates.

MR. KEENE: Let me just address a piece of that. First off, we sample in our survey, so that has a certain amount of problemswith it as it stands. In the next cycle--between now and January 1981, I guess, it would safe to say our goal is to come up with a composite list (of fields). We started with the old FPC field code list, if you're familiar with it. We want to come up with a composite list from several sources, including the University of Oklahoma's Petroleum Data System, and the information submitted to us by operators on fields.

The field level data, which we've already received from our '77 and '78 surveys, probably hits around between 92 and 95 percent of domestic crude oil and natural gas production in the United States. So, while we can't say we have totally complete information on all fields, between the information which we currently have in other systems, I think it's probably achievable that between now and January of next year we would have a pretty good index: One, of all of the fields in the United States; and two, probably relatively complete information by field for those reported.

DR. HARBAUGH: What information will be contained by fields?

MR. KEENE: I don't know that we would publish reserve information by field. We would probably publish information that is accessible to the public, if you had enough time and money to dig it up, like production information, or cumulative production, or perhaps if we have other sources we're integrating--maybe original hydrocarbon in place, I'm not sure.

We have the sources I mentioned, plus we also have information from API and AGA that underlie the Blue Book, which they have published for '77. We have to get their permission to be able to release it. So there are some proprietary data questions and some data-access questions. However, I think some general information and certainly an index of the fields is achievable probably by January.

DR. HARBAUGH: It would be desirable, as a longer term effort, to incorporate certain basic geological data in the information base, as for example, the names and ages of the reservoir horizons. In California, which has less than 500 fields (although many of them are very large), relatively thorough descriptions of the geology of most of the fields are available through the reports of the California Division of Oil and Gas. Equivalent agencies in other states, however, have not documented the oil and gas fields within their boundaries in such detail. To do so nationally would be desirable albeit a large undertaking. Information that could be treated systematically in such a descriptive inventory could include volume of structural closure and the type of trap present. Such information would be useful for statistical classification purposes, and would permit various kinds of Bayesian relationship to be extracted which would bear strongly on resource forecasts.

MR. KEENE: I'm a little reticent to go into how much we might be able to achieve in integrating all that. I mean you can go to Bartlesville and they can tell you the amount of carbon residue involved in various oil samples. What we're planning to do is use the Permian Basin Study, as we mentioned several times, and try to see if we can integrate into an engineering framework, by reservoir and field, enough information so that you would get a feel for what might be achievable.

But we're only going to do a small piece of it. We may even only wind up doing a piece of the Permian Basin; but, there's information available from Bartlesville, there's other information--a ton of publicly available information--it's just never been integrated.

DR. HARBAUGH: Yes, I have a few comments on data. One of the ironies is that the USGS's Conservation Division has accumulated an immense mass of geological, geophysical, and production information for all OCS regions. This information could be used for statistical forecasting purposes. The data repose in four locations: here in Washington for the Atlantic OCS, Metairie for the Gulf Coasts, Los Angeles for the Pacific Coast except Alaska, and Anchorage for Alaska. The information includes seismic data, borehole logs, well-engineering data, and production data. So, within the Federal government, there is a very large file of data useful for post mortem analysis and for resource forecasting. Unfortunately, most of this information has a high security classification and is relatively inaccessible. For example, in the Louisiana and Texas OCS, there are a number of tracts that have not been nominated for leasing and do not form part of the proven resource base. Some of these tracts, however, have potential which could be estimated on the basis of the available seismic data and other data.

For example, in the Louisiana OCS and Texas OCS regions, there are a number of tracts that have not been nominated--come up for testing, in other words, which do form part of the resource base. Now the resource assessment people could most effectively estimate potential for these untested tracts on the basis of the available seismic data.

DR. MURPHY: That Metairie data is the basis for the Outer Continental Shelf model.

Remember the discussion on the data for that model, yesterday. What was done was to gather all of the pre-lease evaluations that were done--a tract has to be evaluated if it's going to be leased or if it's adjacent to something that's going to be leased. This means there's almost complete coverage of the Gulf, to 200 meters, for a tract-by-tract evaluation. The USGS evaluations formed the core of the data for the OCS model.

DR. BRASHEAR: What we did was to look at estimates of undiscovered resources. In order to define a specific point in the process so that our data collection did not overlap with inferred reserves or proved reserves, we just asked whether a tract entered the proved reserves stream yet. I think what Dr. Harbaugh is suggesting, and I would certainly endorse it, is that there is also the proved reserve data files.

If someone took the effort and felt it were useful, it would be possible to start with something that's a gross regional resource appraisal, such as USGS Circular 725, in truly frontier areas, where we don't know anything. As seismic data begin to build, the equivalent of the OCS data could be collected, so that you can begin to say, "Here are real prospects. This is what their economics look like."

The next step is to stay on top of information on prospects that were drilled, tracking into the reserve system. At that point you would have a complete inventory of the onshore and offshore prospects, right to the point of abandonment. It's a matter of integrating it. It's not a matter of magic. It's all there.

DR. MURPHY: There is, however, the problem of how best to update the prior estimates for the prospects that remain to be drilled.

DR. BRASHEAR: Yes. Update with real data.

The interesting point is that USGS gets logs on every well that's drilled. So, by watching those logs, you literally could keep that data up to speed where things that are currently estimated could suddenly be made clear, such as thickness of the reservoir and hydrocarbon content.

DR. LOHRENZ: May I make a brief observation about what McFarland said and tie something together with what you're saying now.

Is there anyone saying here there's an inconsistency in what Dr. McFarland said and what Dr. Brashear is saying: "Boy, there's all kinds of data." Dr. McFarland said, "Lousy data."

We are talking undiscovered oil--future undiscovered oil. You say there's a lot of that. But you are also talking about something that's intangible. I can't touch undiscovered oil.

Dr. McFarland said the data's lousy. But he was talking about pressure data. You have some in the ground, and know how much is there. What can you do with it? This is where pressure data is important. What these gentlemen are saying is not inconsistent. There is a consistency, isn't it true?

VOICE: It's always harder to get good data on real things, than it is on imaginary things.

DR. BRASHEAR: Just for clarity, in the offshore, there are periodic pressure tests that are public and in the USGS files. Those files are confidential, but they can be made available, with proper safe-guards, for legitimate purposes of DOE. Pressure data are not a problem in the offshore.

Where pressure data and other reservoir data becomes very problematic is in those states that have not put together data collection systems. California is a superb example that it can be done. They have an excellent data set without being a major burden to the operators. Oklahoma is the opposite extreme. They don't bother the operators for anything. They, in fact, buy their production data, or it's given to them by Dwight Service. Between those two extremes, major producing states are everywhere along the line.

So, the answer is the data are terrible, in some places. The data are very good in other places. There are some data on existing reservoirs. It does need to be swept up and put together, and there are a lot of people looking at that. Intercomp is looking at it. PDS is constantly updating it. Tom Garland's office has constantly got a sweep going on to clean up these data and update those files. Several private data bases are around. Already the government has bought into Petroleum Information Corporation (PIC), and Well History Control System (WHCS). It's there. It's a big job to integrate it.

DR. MCFARLAND: The point is though, so what if you have a few pressure observations? What are you going to do with those few pressure observations?

DR. LOHRENZ: I think that's the point. What are you going to do with it? Can you estimate reserves with it?

VOICE: We're talking about reservoir pressure data, not pressure build-up, data. The people that you have, have a lot of pressure build-up, but not reservoir pressure data.

VOICE: Oh, Okay.

MR. KEENE: When we described--when I described the kind of data that we're pulling together, I didn't want to lead you into saying--"that's great," and "Gee, shouldn't we have more geologic data?" We can go in that direction, or we could even go to the direction where you say, "We're going to get rid of the AAPG and API well ticket, and we're going to come out with a Federal well ticket; everytime somebody even steps on land, they're going to log it in."

DR. HARBAUGH: We should be cautioned that mere collection of raw data will not provide much information that is directly usable for resource forecasting. For example, you can buy PI's (Petroleum Information Incorporated) file of about 1.2 million wells, which consists principally of scout ticket information placed in machinable form. PI's well data must still be interpreted geologically before they are useful for resource forecasting purposes.

MR. KEENE: Now, we just can't--we have to draw a line someplace to the kind of information we're going to capture, the cost of that data, and the burden that it imposes on the industry. And I'll tell you one thing further, if you have some specific ideas about how data could specifically be used in analysis and the benefit to the American public, if you'd write it down, I'll make sure that we get it into some kind of system or pulled together, if the trade-off is to the benefit is to the American public.

That's a battle we have to fight every time we go to the Office of Management and Budget, and it's a battle, both from a forms clearance standpoint and also to collect money. So we could probably use your insight as to why you would need an additional data element. Just to collect API gravity is a nightmare. To get the approval of Office of Management and Budget to do that, is a nightmare.

DR. MURPHY: Let me say something about the problems of using small models. Everyone knows the problems of large models. They have a 100 percent probability of coding error or operating error for any run. They consume vast amounts of resources to maintain and they do not necessarily have increased information because of increased size.

By the same token, small models have problems.

The smallest model of oil and gas would be to specify the elasticity of cumulative supply as a function of price. Reasonable choices would be .1 or .2.

I have seen this kind of small model used with the elasticity of .1 when a person wanted to show that a piece of legislation was bad. I saw the same person use an elasticity of .2 when we wanted to improve the merits of another piece of legislation. The problem with small models is that they use aggregate parameters with a wide range of reasonable estimates. They have the problem of calibration.

So the question is, how do you get from the large masses of data to a form that you can operate a small model? One alternative that I've been looking at is building a big model but treat it as something that creates pseudo data, from which you can estimate a small, usable policy analysis model.

But that task is hard. I do not know of a truly successful one, although several attempts have been made.

MR. EVERETT: Before the next question is asked, let me just point out one of the themes that I think has run pretty strongly through this conference.

The models are a way for organizing the data collection, or as I like to call it the "mining" of existing data. (I am not for collecting more information until we're down to a point where we really can't satisfy a lot of our analytic requirements with existing EIA data.) This also includes data in state regulatory agencies, and data from other agencies of the Federal Government like USGS. I just think there's an awful lot of guidance that can be derived from reviewing these supply models.

If somebody wants to use a straightforward elasticity computation to predict supply in some future year, let that person go out and measure elasticities. That's impossible or, at best very hard to do.

DR. MCFARLAND: I want to make my position clear on the large versus small model thing. I'm not opposed to large models. I don't particularly favor small models, unless they will do the job. I don't see it as a large versus small model issue, necessarily.

I do think that if you're doing policy analysis where a decision is going to be made, there may not be time to implement a large model. So if response time is critical, and you're looking for just qualitative answers to policy questions, the small model may be the only approach that's available, if you're going to use a modeling approach. So I'm not arguing small versus large in terms of models. You do what it takes to adequately represent the problem.

Let me say something about supply response that is the topic of the symposium. If we think about an econometric model, a supply model, suppose we are trying to estimate a supply curve for oil, where we have price and quantity data, historical price and quantity data. There's a classic identification problem in that we would like to be able to say how much supply is going to change as a result of a change in price.

Unfortunately, we do not have data so that we can identify the supply curve, because what we have got is a series of points that would be considered equilibrium points. So we really don't have the data necessary to identify that curve. So if we collect all the data we wanted, we cannot--it's not possible to have the economic experiment that generates the data points to estimate the function. I think that's another kind of classic example where--regardless of how much data we have, we cannot--we don't have an experiment that we can replicate to give the data that we would actually need to estimate that function and get the simple elasticity coefficient.

MR. EVERETT: Your position is that it's hard.

DR. MCFARLAND: That's right.

DR. LOHRENZ: What you're talking about is a very difficult problem.

I want to identify one general caution with regard to, I guess, energy supply modeling, but actually all kinds of modeling and then give one specific example.

It must have been about ten years ago when I first heard the phrase--Latin phrase. Had to look it up, what it meant, and subsquently found out that it means "assuming all other things stay equal," but at least I only use it when I know that they aren't going to.

Now, case in point, right now the Department of Energy is talking about different bidding alternatives for the offshore, like profit sharing. Higher royalty rates are already in effect, either set at a third, or due to royality bidding, or to sliding royalties, they're higher royalty rates. Their work commitment is allowed under the law, but they are proceeding with this.

All I'm saying is that all of the work and all the data that you have on previous energy supply is based on historical, traditional royalties of an eighth, and a sixth, and something like that. And the models show that if they're rational--people, deciding how fast to produce what you've got, it's going to change it immensely.

Hubbert's difference between discovery and subsequent production was ten and a half years. That's ten and a half years, because of traditional, past royalty rates in general overall economic principle.

When we play with the bidding alternatives, that whole thing can be upset. The ceteris paribus assumption goes down the drain. I'll cite that only as a specific, but I'm sure there are many others.

DR. BRASHEAR: I just want to make a couple of general comments. Things that I found missing in our discussions, or they were alluded to in passing.

One of the things, because we have done a fair amount of our work in unconventional resources, example of unconventional resources are tight gas sands or enhanced oil recovery, things like that--we are very sensitive to the notion of technology change within a finite foreseeable period, and it doesn't just have to happen in the "unconventional resources." We're seeing a significant increase in drilling speed, for example. Whether it's reflected in reduced costs over time, we have yet to see that. But drilling speed is going way up.

Some of the things--certainly in frontier areas--deeper water, and the miserable cold environment of Alaska--we're going to see are new technology ideas. Some of them are on the drawing boards now, some are in tests. It'll change a lot of the economics and timing.

It's awfully difficult for me to see how one can incorporate technology change into a finding rate supply curve kind of an approach.

One of the reasons that we have always found a more disaggregated approach to be helpful is that our other objective is to assist in R and D planning, so we had a separate issue.

Second general point: by the way this town works and the kind of laws that Congress writes and the kind of demands that come up within the Administration, models very quickly get put into use in policies. We're all happy to see that happen, and that makes us feel good that we're making a contribution and we believe very much in our results; but, we all know that we have some limitations in our results.

I think it's incumbent on us as modelers, when there is a policy decision--policy issue at stake, is to look at the implications, maybe just in terms of what's the opportunity cost of the wrong decision. Our models might be able to help us there through some sensitivity analyses. They might not. They might just be able to say we should do this. On the other hand, if that's wrong, what fallbacks have we got? How will we know we've made a mistake, whether it's data, or further analysis, or restructuring of a model. I don't know--the answer would be different in each case.

But we do very often get to the point of debating one model against another model, bcth of which have limitations and probably use enough difference in definitions and standards, that they're really not comaprable at all.

And somehow the impact of models hasn't been brought to the surface in the symposium. I think we're all aware of it. It's sort of like walking around with a loaded gun; you have to be careful with it. And I suspect all of us are insufficiently aware of that in the heat of the discussion or the press of a deadline.

DR. O'NEILL: Most of the people who are modelers, believe that the key result of modeling is insight into the process that is being modeled.

We've gotten into the habit of publishing hard numbers which imposes rigor on the modeling system, but the people who aren't modelers have a tendency to take them more seriously than we do. We're not really sure how to put proper confidence limits on forecasts. The standard 95 and 90 percent confidence ranges, are the result of one man's (Fisher) thinking about agricultural problems a long time ago. Another very well-known statistician in time series analysis, James Durbin, has stated that 50 percent confidence intervals are good enough, and when asked why, he said he didn't have to explain because after all, Fisher didn't.

MR. PARKER: I'm Jerry Parker from DOE's Office of Oil and Gas, and at the outset of the panel discussion, you framed a few questions, Dick. And one of them dealt with enhanced oil recovery. And as you are aware, my office has been involved with promoting the sort of framework in which we can optimize and maximize enhanced oil recovery.

I have not been able to attend many of the sessions, but in looking over the symposium schedule and program, I didn't notice any specific topics on EOR and I was wondering if during the symposium, or at this time, you might summarize what you're doing. Because, in our view, we're in a whole new ballgame, and your using historical data on finding rates and ultimate recovery rates and the like does not acknowledge that we're looking for a big payoff from enhanced oil recovery.

DR. O'NEILL: I think enhanced oil recovery has all the classic problems of a new technology. By changing the rate of penetration of a new technology,you can make projections look very good, or very bad.

VOICE: It would be nice to have a data base that highlighted, on a field-by-field basis, including abandoned fields, what was originally found or expected to have been in place, what was ultimately recovered, and what some final estimate of residual oil in place is.

That kind of a well-organized, well-documented data base would be something that could be useful to policy--a set of scholarly analyses that have very little to do with economics, or to a specific regulatory program in DOE. I think that's what's happening in that area, that's all. Maybe not quickly enough.

MR. PARKER: As you're probably aware, we have set in place a regulatory framework from a financial incentive standpoint, and we are dealing with the environmental problems. In our recent meetings with the majors, they seem to have firm--as firm as they can be--mid and long-range plans for their enhanced oil recovery production, and perhaps the time is here to get that kind of information and try to crank it into your forecasting.

DR. O'NEILL: Given the problem of data collection, we'll probably have the data base constructed properly when EOR starts to decline after a peak.

MR. EVERETT: Whether it's the Resource Applications Office within DOE, or the Economic Regulatory Administration (ERA), the people with the carrots, or the people with the sticks in DOE, both groups should think about what information they require. A very simple question usually lays waste to anyone who thinks he or she wants all the data in the world about everything, and that is, "What do you want to do with it?"

I think if you're going to operate a regulatory program you will want to track its performance--how much money--(public money) was pushed in and how much oil came out. That is a straightforward information requirement. EIA services the regulatory functions of DOE, and that kind of data collection should fit in.

This year there's a burden budget, just like there's a dollar budget for the Federal Government. And I'm not sure how adequately or fairly it's going to be divided up with regard to oil and gas supply. I would hope a major portion of it gets directed to that end, but it's encumbent upon the program offices within DOE to identify and defend data requirements to run their programs. If they can't do that, they probably don't have to defend their programs in the first place. This strikes me as an unreal situation. And as Wally pointed out, everything should fit in its place.

ERA has proposed a form, I forget the number of it, that collects some information on prospective EOR projects (from operating companies). I doubt if it's particularly oriented to be plugged into a supply modeling framework; certainly none of those we heard about during this conference. This conference mainly addressed conventional oil and gas supply, except for the resource classification and resource appraisal discussion earlier.

MR. KEENE: Perhaps on a little more straightforward engineering approach, on the principal data collection form which we use, Form 23, for an annual survey, we received information on about 40,000 oil reservoir units, and we didn't get as much data as we'd like. We have a letter from Bartlesville that lists the additional data elements they'd like to see tacked on to such a form going back to operators. We're currently in the process of working on a task jointly with Bartlesville that would identify the burden--the best place to take that information, whether it's from an operator, or refinery, or wherever, the burden associated with doing that--do we want a point estimate or point value or can we come up with a range, which would certainly be a lot less burden, and what would the EOR program do to individual data elements?

Now in theory, we could wind up at the end of this year going back out with a form to anybody that operates an oil field in the United States, and collect all the information they need for EOR.

Probably somewhere between that and collecting no reservoir data is where we'll wind up. And it's a function of the kind of engineering input Bartlesville puts into the job that we're looking at right now.

So it's not that it's not being given any thought. There's money being spent right now with petroleum engineers putting together the information that we need. Whether it's timely I don't know, whether or not OMB will buy a budget--a burden budget and allow us to collect it, that's not really clear. There are some issues that have to be resolved, but it's not something that's being ignored.

VOICE: I'd like to ask about--well, while we're talking about data, I think there are two basic distinctions in data that we've tried to draw out. There are two kinds of information that's needed.

One is, what regulatory information is needed to see whether a law is being implemented as planned, and the other kind of data are the research type data. The kind of data that have an error component. Regulatory-based data are generally affirmed and generally, you know, the respondent assures that data are correct under some penalty. That's one type of data that's collected for a very specific purpose.

Then there's another kind of data base. The kind of data base where for some reason or another you're willing to tolerate a little epsilon on the end.

Because regulatory information serves a legal purpose, it tends to dominate. And I think if you look at the corpus of data that EIA maintains, a lot of it is regulatory based, and it's concerned with a certain number of small issues. It's like the Eskimos that have essentially one word for the whole non-white spectrum of colors and a thousand different words for white.

It's the same sort of thing within the Government. The regulatory information draws you to make finer and finer distinctions, until you get to the end where there are vast whole areas that don't even have data points.

There's a need for some sort of pro-active data collection. For example, if you want to characterize the geologic resource of the United States, probably the best possible study would be to drill one core on every square mile. Now you can't do that, because of constraints.

And so to move away from the simple solution, you apply some heuristics. You apply some theory and some values that lead you to just put wells where there is sedimentary deposits or something else like that.

In the data planning, is there not a need to specify in some way, to maybe convene a blue ribbon panel of experts, or some technique, to specify the heuristics that you would use to either put an item in your data base or take it out and to look at the relative driving or regulatory function versus pro-active anticipatory functions, or evaluate a function sort of to keep the machine on keel, to see if there's not a need to go outside the boundaries of the system?

It seems that every time I go through a data series, either the API,AGA, or EIA's own data series, the good stuff, the stuff that you're looking at to answer today's questions, isn't there. There needs to be a new categorization. Or you'd say, "Gee, I'd like to have this cross-categorized by this."

The existing data base is based on today's questions, and what I get is, "How can we possibly anticipate tomorrow's questions in a Federal framework where you have all the lags?"

MR. EVERETT: All right. Seriously, that's right. The first job is to try to identify what we have in this "EIA Data Base," and try to put it in perspective with what others have. Like what, for example, the current industrial reports out of the Department of Commerce, Bureau of the Census, mean with regard to Wally's survey on oil and gas reserves.

Cost data are sort of another open area where a lot of people have done important work, but it hasn't been tied together--no concordances have been drawn between certain prominent series. Various things still have to be done.

To summarize to this point I can say that the models are the vanguard of helping one organize information in a useful way. One of the things that is also necessary is access to information (this conference helps to some degree). As many people in as many classes of endeavor as possible deserve access. I think that in the next several years major inroads will probably be achieved in these areas, with just the emphasis that you mentioned.

MR. EVERETT: One other brief point on this. Then we'll take the next question.

For the economists in the room, remember the Unemployment Insurance Statutes and IRS have a great deal to do with economic information that currently is in place in other agencies in town. Census is a special case, in which Congress decided a long time ago that it was necessary to count heads. Regulations are the best way in the world to ensure that data collection satisfies some minimum degree of verification and validation.

DR. HARBAUGH: I'd like to comment again on the data acquisition policies. Geophysical data are of great importance in assessing the potential of frontier regions. If we appraise the Atlantic OCS at the moment, much of the useful information consists of seismic data. These data reside with the USGS's Conservation Division here in Washington, but they are inaccessible to the public, and only with difficulty accessible to other Government agencies. A change in policy might be considered by the Federal Government, whereby permits for seismic and other geophysical surveys in OCS regions would be granted with the provision that the data later become accessible to other agencies for purposes of analysis and forecasting.

MR. EVERETT: You're talking mainly about the USGS. The

Conservation Division holds most of the seismic information on public lands. Several of the papers delivered here dealt with EIA's attempt to join with the Conservation Division to interpret these data, and put them into a data base. Now, whether or not the data can be manipulated so that the Office of Primary Control (the Conservation Division) is happy with the release of that data to the public, is another question. I think that it is possible with careful aggregation and masking.

DR. HARBAUGH: The Conservation Division probably won't be very happy because of agreements with the oil companies regarding security of the data. What has been lacking, however, is appropriate policy with regard to subsequent utilization of these data by various Federal agencies as a condition for granting of permits to obtain the data in the first place.

DR. O'NEILL: You've made a very important point. The United States, as a property owner, probably has one of the poorest information systems concerning the resources it manages. If you were to look at the majors and the value they place on information (for example, seismic and well data), the Federal Government, by comparison, has a very poor assessment of what it owns.

The Department of Agriculture has recently realized that forestry may not be as important as oil and gas and other minerals on the lands it manages. There is no central point in the United States Government where the management of its total resource is considered. I'm sure that oil companies has much better information on property they have leased.

DR. HARBAUGH: Release of information is obviously a very controversial issue, and one that would provoke anguish within the oil industry simply by raising the question. However, it is appropriate to review existing policies regarding the acquisition and release of oil and gas data that involve Federal lands, both onshore and offshore.

MR. KEENE: We've been attempting, for well over two years, to establish a policy--I say we, the Energy Information Administration--to establish a policy just for the Energy Information Administration, on the handling of proprietary data--if that helps give you an idea of how difficult that issue is. We've been in court battles. We've been in Congressional hearings, and it's a very, very, difficult problem.

I don't know--even if we started today, my best guess is before you could even get a preliminary opinion, it would probably take two years--another two years.

DR. MURPHY: Let me present a straw man, to get at some of the modeling issues. The first statement that was made is that it's important to have models that deal with policy questions. You have to know your question first, before you build your model.

Let me pose a question, and I'd like to ask the audience what the right way is to deal with the question.

The Natural Gas Policy Act has many categories of gas that are controlled at various price levels. How do you deal with a supply response to all of these different price levels?

We make the bold assumption that for any of the tiers that are controlled, there is no increased supply at higher prices beyond the control prices. For oil say that the supply elasticity is entirely in new discoveries and EOR.

DR. O'NEILL: Let me add that, if you say that you can't address an issue because you cannot apply all the proper methods of science someone else in this town will. The process of the scientific method, as explained in the freshman textbooks, does not exist anywhere and has never existed.

DR. LOHRENZ: That's a fundamental conundrum. You can initially say, "Well, let's assume the guy will behave rationally." But then the question is how rational is rational. I want to talk about the Windfall Profit Tax where the same thing comes. Now here's a guy--let's say I'm a major, I have a producing property, but the tax, you see, is higher on me than on the other guy. So when I model, shall I say that "Okay, this is a major, and he's going to maximize the present value of future profits, based on that tax." Or, the other thing I can do is say, "Hi, buddy, you non-major. I got this property that I can sell you at an incremental difference and let's split the pie." Shall we crank that in too?

The question I'm getting at, how--where does rationality stop? I'm not answering here how to do it, I'm just saying I don't know how. You don't either, do you?

DR. MURPHY: No

DR. GASS: The rational decision right now would be to adjourn. Is that correct?

(Whereupon, at 3:30 p.m., the symposium was concluded.

APPENDIX

OIL AND GAS RESOURCES -

WELCOME TO UNCERTAINTY

by

John J. Schanz, Jr.

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GLOSSARY

A number of oil and gas terms are used differently by different people. Below are some short, nontechnical definitions that may be helpful in reading this issue as well as other reports about oil and gas resources.

Commercial accumulation an occurrence of oil and gas that meets the minimum requirement for size and accessibility to be of commercial interest to a company. The term commercial is frequently synonymous with economic.

Deliverability the amount of natural gas that a well, field, pipeline, or system can supply in a given period of time. Only valid for that period.

Discovered resources that portion of the oil and gas in the earth whose presence has been physically confirmed through actual exploration drilling.

Indicated reserves known oil and gas that is currently producible but cannot be estimated accurately enough to qualify as proved.

Inferred reserves reserves that are producible but the assumption of their presence is based upon limited physical evidence and considerable geologic extrapolation. This places them on the borderline of being undiscovered. The accuracy of the estimate is very poor.

Inplace all of the oil and gas in the reservoir, combining both the recoverable and nonrecoverable portions.

Maximum efficient rate (MER) when used in a practical or operational sense, it is the optimum rate, as of a specific time, at which oil and gas should be drawn from a developed field in order to balance cost, percentage recovery, and speed of withdrawal. To exceed this rate for the reservoir or to produce individual wells too rapidly can lead to loss of oil and gas recovery from the reservoir.

Occurrence a physical accumulation of oil or gas or related hydrocarbons in the earth regardless of size and physical or economic characteristics.

SPECIAL ISSUE

From time to time, RFF will publish a special issue of Resources *that focuses on a single, timely topic. This, the first such issue, written by RFF Fellow John J. Schanz, Jr., embodies the results of a series of workshops.*

Oil and Gas Resources— Welcome to Uncertainty

Until 1973, the American public was accustomed to glad tidings about U.S. oil and gas resources. If you read the business sections of newspapers or followed the trade and professional publications, you were aware that the forecasts became increasingly optimistic over the years (see table 1).

There were some less sanguine estimates from those who looked at the ever-declining curves of oil field production and projected rising costs through time. But these more cautious projections appeared to be overshadowed by the upward path of U.S. oil production. As the world's leading oil producer, the nation passed the 1 billion-barrel level in 1929, 2 billion in 1948, 3 billion in 1966, and reached the 3 and one-half billion level in 1970. The perennial optimism of the wildcatter—"Give us an incentive and we'll go find you some oil and gas"—was well supported by over 100 years of production history. The United States seemed a permanent fixture as the world's number one producer of oil and gas.

The undercurrent of concern during the 1960s over declining exploratory activity in the United States elicited little real attention outside of the oil and gas industry itself and a small circle of petroleum specialists. It was easy for others to treat these worries as merely the customary background noises that accompany an industry's efforts to encourage favorable treatment by Congress on taxation, incentives, or protection from foreign competition. However, the major disturbance caused by the Organization of Petroleum Exporting Countries (OPEC) oil embargo in 1973 brought an immediate end to this lack of public attention.

In 1975, a report by the Committee on Resources and the Environment (COMRATE) of the National Academy of Sciences, based on a review of contemporary estimates, stated that, of the original stock of crude oil and natural gas liquids (249 billion barrels), only 113 remained to be discovered. For natural gas 530 trillion cubic feet (of an original 1,227 trillion) remained.[1] This marked the end of general optimism both in industry and government about the future U.S. oil and gas resource position. For the public and Congress, whose ears are normally more receptive to good news, it was a shocking revelation to learn that instead of over 400 billion barrels of liquid hydrocarbons there might be much less. To have this unwelcome news appear in the midst of the oil and gas industries' post-embargo clamoring for high prices resulted in both public confusion and distrust. With respect to natural gas, the winter crisis of 1976–77 caused renewed

[1] M. King Hubbert, whose work received considerable attention in the press, was among the COMRATE participants. His estimate was reported as 72 billion barrels of oil and 540 trillion cubic feet of gas.

Oil basin a large basin-like geologic structure in which oil and gas fields will be found.

Oil field a geologic unit in which one or more individual, structurally and geologically related, reservoirs are found.

Oil region a large oil-bearing area, often encompassing several states, in which oil basins and fields are found in close proximity.

Production or decline curve (S curve) the annual production of an oil or gas reservoir through time is a dome-shaped profile with its peak usually to the left of center. The progress of the production from its peak toward depletion is called the "decline curve." If this is plotted as cumulative production it follows a gradual S-shape as it approaches the total, or ultimate, production of the reservoir.

Productive capacity the amount of oil that can be withdrawn each day from existing wells with available production facilities. Only valid at one point in time.

Proved reserves an estimate of oil and gas reserves contained primarily in the drilled portion of fields. The data to be employed and the method of estimation are specified so that the average error will normally be less than 20 percent. May also be called measured reserves.

Recoverable that portion of oil and gas resources that can be brought to the surface, as distinct from the oil and gas found in place in the reservoir.

Reserves oil and gas that has been discovered and is producible at the prices and technology that existed when the estimate was made.

Reservoir a continuous, interconnected volume of rock containing oil and gas as a hydraulic unit.

Resource base the total amount of oil and gas that physically exists in a specified volume of the earth's crust.

Resources the total amount of oil and gas, including reserves, that is expected to be produced in the future.

S-curve. See production or decline curve.

Subeconomic resources oil and gas in the ground that are not producible under present prices and technology but may become producible at some future date under higher prices or improved technology.

Undiscovered resources resources which are estimated totally by geologic speculation with no physical evidence through drilling available.

Table 1.
CHANGING PERSPECTIVES OF U.S. OIL AND GAS RESOURCES

Forecast in Year	Original supply of recoverable reserves[a] Oil (billion barrels)	Natural gas (trillion cubic feet)
1948	110	
1952		400
1956	300	856
1965	400	2,000
1969		1,859
1970	432	
1972	458	1,980
1975	249	1,227

[a] Unfortunately, any sampling of estimates encounters variations in the treatment of past production, recoverability, and the inclusion of natural gas liquids. These have been chosen, or adjusted when possible, to make the totals roughly comparable regardless of year of estimate.

doubts and confusion among the public, the media, and members of government.

In the three years since the COMRATE report, several staff members at Resources for the Future have been looking into questions about oil and gas reserves and resources. It, therefore, seems appropriate at this time to distill from these recent RFF efforts as much understanding about oil and gas resources as possible. We have no intention of producing a new set of resource estimates; there are more than enough of these. Rather, we hope to show why we keep getting different signals about the status of our oil and gas resources. If we can reduce some of the confusion, our efforts will be well rewarded.

Obviously, it will not be possible to explore in these few pages all of the problems in the collection and use of oil and gas statistics. Our attention is directed toward the different methodologies and perspectives employed by the various analysts who produce conflicting estimates.

The following discussion draws heavily upon a number of recent RFF activities, including: a workshop on oil and gas resources sponsored by the National Science Foundation, a study on resource terminology sponsored by the Electric Power Research Institute, a workshop on Maximum Efficient Rate (MER) of oil and gas production sponsored by the U.S. Department of the Interior, and a workshop which reviewed the Federal Energy Administration's *National Energy Outlook, 1976* sponsored by the National Science Foundation. In addition, members of the RFF staff have participated on a regular basis in the work of the committees and boards of the National Academy of Sciences, the Gas Policy Advisory Council of the Federal Power Commission, and the oil and gas resource appraisal groups of the American Association of Petroleum Geologists and the U.S. Geological Survey. The contribution to this summary report of Dr. John C. Calhoun, Jr., Vice-Chancellor of Texas A&M University, who directed the oil and gas resources workshop, is especially acknowledged.

A Matter of Definition and Classification

An oil or gas reservoir is not a subterranean cavern filled with oil and gas, which we empty like a huge storage tank. During geological time, various mixtures of crude oil, natural gas, and salt water were formed and moved about in the interconnected minute pores of certain kinds of rock where they have remained trapped. When the driller's bit penetrates the rock, natural pressures cause a slow migration of the fluids toward the well bore. The well operator may decide initially, or eventually, to give the flow of oil and gas an assist through the application of the sucking action of a pump, or by fracturing the rock around the well, or by injecting water, chemicals, heat, or gases into the rock. To understand this production process, three things must be kept in mind: 1, the flow of fluids through rock pores is a function of the physical forces at work; 2, the quantity resulting from additional effort gradually diminishes, just as wringing a wet rag produces less and less water; and, 3, the only actual measurement that can be made is of the oil and gas produced at the surface—all other information about the reservoir is estimated.

An oil and gas reservoir or pool is basically a hydraulic unit where all the interconnected pores holding the oil and gas in the rock behave as a single fluid system. Theoretically, a well drilled at precisely the right place would be all that would be needed to produce all of the oil or gas the reservoir will ever produce, given sufficient time for the fluids to move through the rock to this one point. In one geographic area, encompassing a few or thousands of acres, there may be a series of reservoirs or individual traps containing oil and gas that are geologically related but not physically interconnected. To find and produce all of the oil and gas requires additional wells, dispersed either vertically or horizontally. A single isolated reservoir or a group of reservoirs related by physical proximity and geological origin are identified as an oil or gas field. Once the oil and gas exploration teams have found a specific bed of rock that contains oil and gas accumulations, they will tend to follow this "play" by drilling down to that bed or zone over an extended area. A discrete geological environment having a large number of oil and gas fields is known as a basin or province. In the United States, there are over 100 basins found clustered in five major regions. Within the basins there are thousands of fields and many thousands of individual reservoirs. Over 2 million wells have penetrated the earth in the vicinity of these oil and gas traps, and more than 500,000 successful ones are still producing oil and gas. Approximately 10,000 wells are abandoned each year.

Once the physical characteristics of an oil and gas resource system is appreciated, the complexity of the question "how much oil and gas do we have?" becomes more apparent. Any response can be no more than a judgmental estimate. Intelligent communication about oil and gas resources becomes exceedingly difficult unless both the questioner and respondent understand what kind of data they are using. A start in this direction is to use a diagram becoming common in governmental circles (see figure 1).

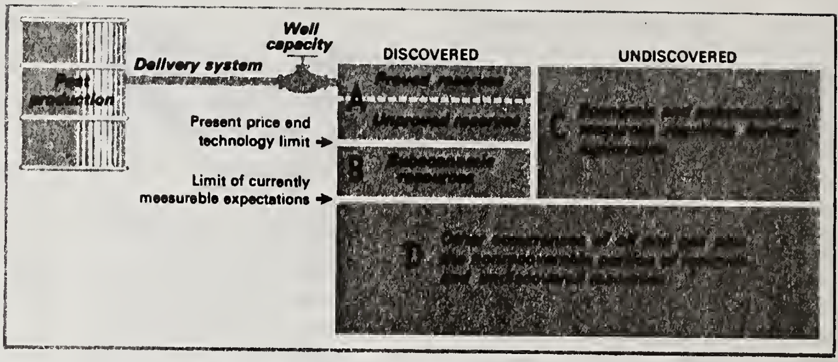

Figure 1. Diagram of Reserves and Resources

A complete oil and gas resource diagram is a pictorial representation of all unproduced natural oil and gas hydrocarbons that may exist. We can also visualize a valve attached to the diagram representing the oil and gas wells through which oil and gas is removed from the reservoirs that have already been penetrated by the drill. Beyond this valve there is a conduit which represents the pipelines, tankers, barges, railroad cars, and trucks used to move the oil and gas through processing and on to the final user. It is worth repeating that past production, that is, the quantity already delivered by this system, is *our only actual measurement*. That quantity of oil and gas is gone forever. References to *original oil and gas in place* mean the sum of both the remaining oil and gas plus all that has ever been produced.

The *productivity capacity* of the United States is the amount of oil and gas that can be produced from existing wells during a specified period of time under specified operating conditions. The totality of physical oil and gas in the earth but not yet produced from the continental crust to a depth of perhaps 50,000 feet is sometimes called the *oil and gas resource base*. There are four kinds of oil and gas found in this *resource base*. The first kind (segment A in figure 1) consists of oil and gas which has already been found and is considered producible under present prices using current technology. These quantities are customarily known as *reserves*. The immediately producible portion of these reserves—the oil and gas that will flow from wells in developed reservoirs, the quantity of which can be estimated with considerable accuracy—is classified as *proved reserves*. The balance, or *unproved reserves*, has been discovered but cannot be estimated with as great accuracy and may require additional drilling and development (see figure 2).

In segment B we find oil and gas that has been discovered but in the judgment of the operators cannot be produced under current prices with existing technology. These quantities are known as *subeconomic resources*. There are two kinds of subeconomic resources. First, the unrecoverable, high-cost portion of oil and gas currently left behind in producing reservoirs. Second, oil and gas in other reservoirs that have been found but are not now producing or have been abandoned because they would cost too much to produce due to size or other problems.

Segment C of the resources diagram encompasses the oil and gas that remains to be discovered. Exploratory drilling has not proceeded to a point where there is physical evidence of the actual presence of this oil and gas. There is only expectation, and estimates of undiscovered oil and gas are based solely upon geologic and engineering extrapolation. This requires the use of geological and geophysical data rather than using physical data based upon the actual *existence* of the oil and gas. It is possible to subdivide undiscovered resources into economic and subeconomic quantities, but to do so requires the analyst to make some sort of assumption about prices and technology conditions. Present prices and technology are frequently assumed despite the fact that the oil and gas, when actually discovered, will be produced under future conditions of price and technology.

The final portion is segment D—*other occurrences*. This includes any oil and gas left behind that is not expected under any future circumstances to be worth the effort or cost of production, as well as deposits which are considered too small to either find or produce if found. Finally, this category is a convenient place to account for other forms of oil and gas hydrocarbons about which either little is known, or production technology is so immature that economic and technologic judgments cannot be made, even though large quantities may be involved.

Estimation of Reserves

As the drill bit penetrates a rock reservoir for the first time and finds oil and gas, the first questions asked are how much has been found and can it be produced economically. The initial well provides limited information about the rock strata that have been penetrated and nothing about strata that are below the bottom of the well. Once a layer of rock containing oil and gas has been found, the approximate thickness of the bed at that point—anything from a few to hundreds of feet—is known. A core of rock is usually taken from the bed.

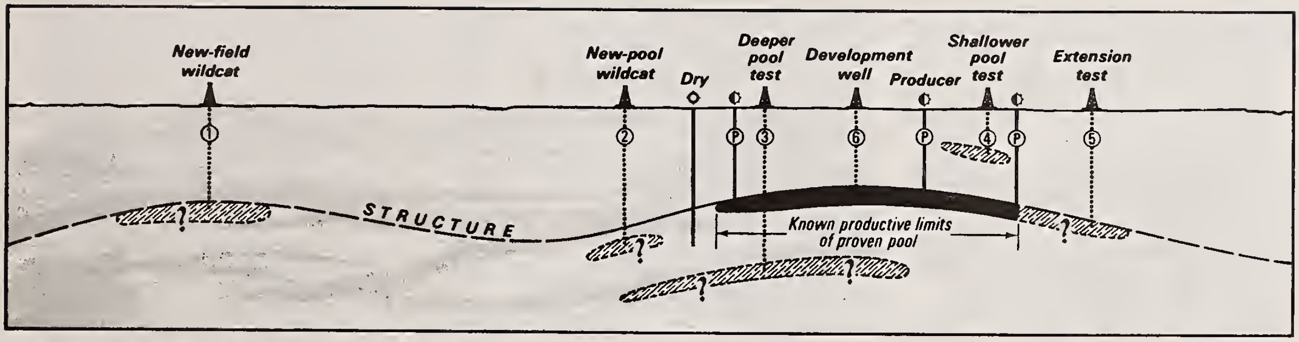

Figure 2. The Classification of Wells by Geologists

Note: Proved reserves are established by the producing wells (P). Unproved reserves in the field will require additional drilling by wells 3, 4, 5, and 6. Wildcat exploratory drilling can find undiscovered resources in adjacent pools or in separate fields (1 and 2).

Electrical and other measurements are taken inside the hole. All of these data provide information about the porosity and permeability of the rock and the amounts and kinds of fluids it contains. If the reservoir seems to justify production on the basis of this preliminary information, then the drilling equipment is removed, production pipe is put in place, and the well prepared for production. The initial flow of a new well provides information about the production rate, pressure, and other physical data.

At this point, a preliminary judgment on how much oil and gas have been found can be made based on: the flow from a single well; a rock sample a few inches in diameter of a multiacre reservoir; a map of the surface geology; and a seismic "shadow picture" of the structure holding the oil and gas thousands of feet below the surface. Obviously, this first estimate cannot be very precise. Yet based on this one well and past experience with the kind of reservoir that *appears* to have been found, the engineer makes a judgment. This estimate may range from the least amount of oil and gas that appears to have been found to the outer limit of what the reservoir might ultimately produce if the buried structure is entirely filled with oil and gas.

The scientific guesswork about a reservoir hundreds of acres in size is useful but extremely crude. It is akin to going to an unfamiliar supermarket on a foggy night and trying to estimate the total amount of asphalt used in paving the parking lot, with no other data than a cubic inch sample of the blacktop used. How uncertain these judgments about reserves can be was illustrated in a study published by the National Academy of Sciences in 1976.

The study concerned the amount of gas reserves under lease in certain fields in the Gulf of Mexico. Previous estimates had been made by the technical staff of the Federal Power Commission (FPC), but there was disagreement about their accuracy. The Academy suggested that two consulting firms, experienced in the Gulf fields, should make independent estimates using the same geological survey data that was used by the FPC. This was done for a random sample of nineteen (out of a total 168) leases. All the estimates by these firms proved to be lower than those of the FPC staff, but a comparison between the estimates made by the two firms was more interesting. For one lease, the difference between their estimates was only 10 percent. But for nine of the leases, the upper estimate was from two to eleven times higher than the lower estimate.

Even before a well is drilled, companies will appraise the potential of a new region to help them determine whether or not exploratory wells are worth drilling (see figure 3). Once a well has been successful in finding oil and gas, two new estimates can be made: first, an estimate of the minimum amount (the proved reserve) that seems to be producible by that well and, second, a less certain estimate of what might be the ultimate potential of the entire field. As more wells are drilled and additional production data are gathered, the proved reserve estimate may be revised up or down. The expectation of ultimate production can also change upward or downward, usually over a much wider range than that of the proved reserve figure—several multiples are common. For a typical field it takes approximately five or six years before the proved reserve estimate of remaining oil plus past production begins to approach a true estimate of ultimate production. In other words, it takes a number of years before there emerges a reasonably accurate estimate of what has been discovered *in toto* in a reservoir. The exact amount of producible oil or gas is not known until the field is permanently abandoned and that oil or gas has been measured as past production.

In addition to a company's estimates of proved and ultimate, other appraisals may be made by a producing company during the life of a field for various purposes. Estimates based on well logs and other data are commonly used by banks for making loans. Information is also released to the press about the importance of new discoveries. The Securities and Exchange Commission expects that companies will release information to stockholders about their holdings and expectations. And, finally, the many kinds of information required by government agencies lead to a number of estimates being provided by a variety of federal offices. Considering the array of purposes for which reserve estimates are made and the constant revision of most of these through time, it is not surprising that various reserve reports may appear to be in conflict.

The proved reserves of oil and gas represent only a small portion of the total oil and gas resource base that remains unrecovered in the United States. Yet, these proved reserve data sometimes receive more attention, and in recent years have prompted more controversy, than the more significant resource estimates of undiscovered oil and gas.[2] For crude oil, proved reserves represent the stock of immediately producible oil from existing wells. The oil

[2] Despite the apparent clarity of the generalized concept of proved reserves the actual estimation requires certain judgments to be made concerning the amount of extrapolation to be used, whether to include oil and gas from known reservoirs not actually being produced, or how to account for oil resulting from secondary stimulation. As a consequence, there are at least nine "official" definitions from various professional, industrial, or government agencies. Each one leads to some variation in the estimates made.

producer knows that the amount of oil or gas that can flow in a given year from producing wells is physically linked to the number of wells available and the quantity of oil and gas still remaining in the reservoir. Thus, proved reserves for many years have been the industry's empirical indicator of current capability, not a measure of the total supply of oil and gas left for the future. Equating proved reserves with "years of supply" is particularly misleading.

Since proved reserves plus past production are normally less than we might reasonably expect to produce from known oil and gas fields, do we have any estimates of this undeveloped and less certain, part of our discovered oil and gas reserves? Unfortunately, there are no regular government or industry-wide efforts to report on what is known as *indicated* or *inferred* oil and gas.[3] The American Petroleum Institute (API) does report on additional reserves of oil that could be produced from secondary recovery projects but that are not yet fully evaluated at the time of the proved reserve estimations. An industry-sponsored effort, called the Potential Gas Committee, has included this portion of the gas reserves in its occasional reports on gas resources. The Federal Energy Administration in its 1975 survey of operators had hoped to go beyond merely proved reserves, but its final report only included oil from secondary and tertiary projects not the less certain oil and gas quantities. Currently, the new Department of Energy is again considering how to define and request data on oil and gas reserves that are not reported as proved.

The U.S. Geological Survey in its 1975 Circular 725 relied upon the use of a statistical ratio devised by M. King Hubbert to account for indicated reserves. This ratio is based on the historical relationship of the amount of oil and gas that has been added through extensions and revisions to proved reserves during the typical life of reservoirs. The relationship shows that approximately 80 percent more oil and gas will be produced from known fields than is currently being reported as proved. Although proved reserves data are considered to be accurate within plus or minus 20 percent, this refers to the oil and gas expected to flow from existing wells. On the average, almost twice as much oil and gas will ultimately be found in these fields once their true size or limits become fully identified. This is not an intentional understating or "hiding" of reserves, but merely a reflection that the definition of "proved" limits the estimators to the drilled portion of the field.

[3] The terms probable and possible are also used to describe discovered reserves that cannot qualify as proved or measured.

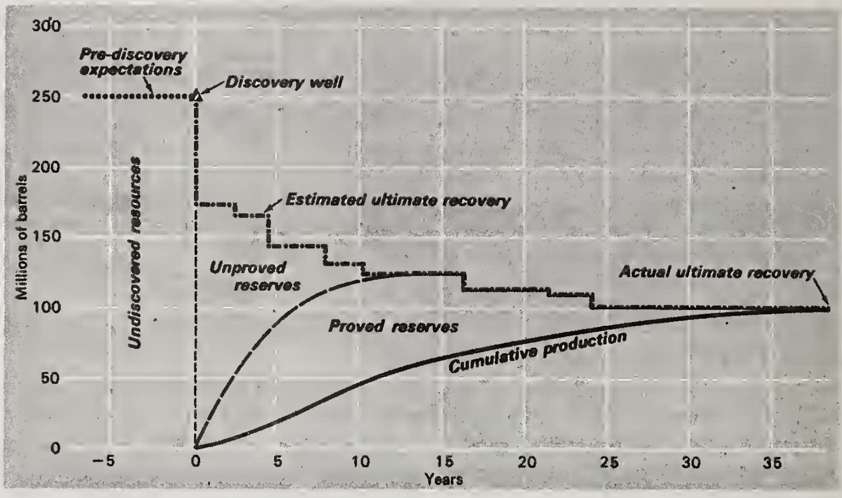

Figure 3. Life Cycle of Oil Reservoir Estimates (adapted from G. C. Bankston, API Reserves Seminar).

One must be aware that when estimates go beyond proved, accuracy deteriorates rapidly, with errors of perhaps 50 percent or more for indicated reserves (mostly oil and gas resulting from further development of the reservoir) and amounting to perhaps several multiples for inferred reserves (oil and gas resulting from the discovery of additional reservoirs within the same field). Many U.S. professionals and Canadian government specialists would prefer, because of the uncertainty, to consider inferred oil and gas as not actually discovered.

To obtain more information on what lies beyond proved reserves in known fields involves a reservoir-by-reservoir examination of considerable magnitude. Considering the range of judgment involved and the unavoidable approximations, the added information obtained may not be worth the cost of acquiring it. In addition, there are problems of handling proprietary data, which, in any event, would be diverse, constantly changing, and of unknown quality and usefulness. It would require combining the personal judgment of the ultimate potential for field A made by a geologist from Company X with estimates for field B from Company Y's geologist, through all of the thousands of fields in the United States. In the final analysis, to know that indicated and inferred U.S. reserves are considered to be 2.4 times our proved reserves instead of 1.8 times has little significance in determining our policies with respect to oil and gas. The more important questions are found in the categories of subeconomic and undiscovered oil and gas resources. It is upon these quantities that our energy future depends most heavily in the medium term.

Estimation of Subeconomic Resources

Subeconomic resources include all of the oil and gas in known reservoirs that is not producible by present technology at present prices, but may become producible in the future with improved technology or higher prices. It should be noted that a simple downward movement in prices or other incentives can cause some reserves to be reclassified, at least temporarily, as subeconomic resources.

As a field is developed, following the drilling of a discovery well, the producer adopts a plan which he hopes will extract all of the oil from the reservoir that can be produced. He then hopes to sell the oil at an anticipated price that will return as much or more than the additional cost of producing it. He hopes that the aggregate return from all wells, over and above the direct costs of production, will pay him not only for production costs but also for the costs of exploration, dry holes, and abandoned wells. To minimize the duration of his exposure to uncertainty, he hopes the pace of production will permit a quick return of his initial investment.

The investment decision in oil production is a balancing of the total quantity to be recovered, the various costs, the rate at which the oil will be recovered, and the price at which it can be sold. Once the decision is made on how many wells to drill and what recovery technology to use, that is, natural flow, pumping, water flooding, injection of steam, or other methods, the amount of oil that will be recovered and the rate at which it will reach the surface are limited within a fairly narrow range. To change that plan, additional investments must be made in drilling additional wells or in altering the production methodology being used. Such a change in the production scheme will be adopted only if the faster production or greater recovery can justify the extra cost.

Thus, an increase in the price of oil or gas may not be adequate to change the plan for the operation of a field already being developed. The only effect of higher prices in that event may be to permit the reservoir to decline to a lower level of daily output per well before it is abandoned because of low oil flow or gas pressure. This additional quantity of oil and gas produced in the later life of a reservoir may only involve a 1 or 2 percent increase in the ultimate recovery because most of the oil or gas left behind is entrapped in the reservoir and could only be recovered by the use of a different technology. However, higher prices which occur before a production plan is fully implemented in a new field can lead not only to later abandonment but to higher recoveries because the prices can be reflected in a timely investment in a modified development scheme.

The appearance of significant improvements in production technology or markedly higher prices can justify modifying the way new reservoirs as well as fully developed fields are being operated. Even an abandoned reservoir can be reopened, although this is less likely because of the expense. It should be noted, however, that new methods of enhancing the recovery of oil and gas should not be viewed as applicable to all kinds of reservoirs. How successfully a new technology can be employed is determined by the kind of structure and natural energies in the reservoir, as well as the kind of rock that is found in it.

There has not been much experience in estimating the size of the national subeconomic resources of oil and gas because in the past the opportunity to discover new and plentiful oil and gas resources has always seemed more attractive to industry. Even for known fields, the estimation of subeconomic resources is complex. First, the estimator must face uncertainty about new and perhaps untested technology. Second, there is need to deal with the effect of price on production using established technology, as well as what price is required to make new technology commercially feasible. Third, there is a lack of information on exactly how much oil or gas is left in the reservoir to be recovered by new technology. Finally, if the data are to have meaning, there is need to deal with the problem of the time over which these prices and technology can be assumed to occur.

It is perhaps surprising that the amount of oil left behind in a reservoir is uncertain. Depending upon the kind of reservoir and the years during which it was exploited, oil recovery from the initial development plan used can vary anywhere from 10 percent to 80 percent of the oil estimated to have been in place originally.[4] In some cases, reservoirs have been reworked with a secondary production technology long after primary methods were begun. More recently, developed fields tend to be exploited by several integrated methods. Since oil in place, recoverable reserves, and a reservoir's recovery factor are all parts of the same equation, it is apparent that estimates for two of them allows the derivation of the third. Thus, if greater production from a reservoir is achieved than originally expected, one is never sure whether the cause is more oil in place, more reserves, or a higher recovery factor.

There is some indication that the overall recovery factor for oil in the United States did not improve very much during the 1960–75 period for several reasons. U.S. production shifted from regions with naturally higher recovery potential to areas with poorer recovery potential. Early estimates of the quantity of oil to be recovered by primary recovery techniques were probably overstated or, conversely, the oil in place may have been understated. Finally, there is a tendency to use a standard recovery factor in relating future production expectations to oil in the reservoir. Each of these tendencies could contribute to the assumption that the U.S. recovery factor has remained near 30 percent for many years. Realistically, it must be concluded that the true national recovery of oil is an unknown percentage.

The current interest in enhancing petroleum recovery by injecting heat, CO_2, or chemicals has led to more vigorous examination of subeconomic resources than ever before. Our major oil regions have been examined in terms of the amenability of the various kinds of reservoirs to newer methods for increasing recovery. Although some optimistic suggestions have been made that U.S. recovery could be increased from an assumed 32 percent to ultimately 60 percent, more modest near-term goals are now being set for the upgrading of some of our subeconomic resources to reserves. These suggest an overall increase of perhaps 5 to 8 percentage points in the U.S. recovery factor may be possible.

The uncertainty of how much subeconomic oil and gas may be produced is illustrated by the recent report of the National Petroleum Council (NPC). The additional oil from enhanced recovery, according to the NPC, could be as little as 7 billion barrels, under a price assumption of $10 per barrel (1976), but this would increase to 24 billion barrels at $25 per barrel. The effect on the rate of annual production would also vary. At the higher price level, U.S. oil production could be 3.5 million barrels per day greater in 1995. The uncertainty in the estimates is reflected in the judgment that the higher 24 billion barrel amount is merely the central value of an estimate ranging from as little as 12 or as much as 33 billion barrels. In contrast, another study viewed the outer limit of enhanced recovery at 76 billion barrels at $15 per barrel (1974). Despite the fact that enhanced recovery deals with "discovered" oil in known fields, this does not narrow the range of uncertainty. Technological and economic forecasting of recovery is a source of frustration equal to that of estimating the undiscovered (table 2).

Estimation of Undiscovered Resources

If the United States would suddenly cease drilling its customary 25 to 50 thousand new wells each year and would be content with merely producing what it could from existing wells, production would decline progressively by approximately 12 percent per year. After some forty years, production would fall to approximately 1 percent of present production. Since reservoirs do not cease production abruptly, some wells might still be producing a few barrels per day after one hundred years.

Unlike manufacturing, or some kinds of mining operations, the capacity to produce petroleum is not a constant. To avoid a decline in national production, there must be continuous drilling and development. The process of continual annual replacement of what we have produced is heavily dependent upon the magnitude of our undiscovered oil and gas resources.

The potential size of these resources is usually evaluated in one of three ways. There is the geologic or volumetric approach, which attempts to make a direct estimate of the quantity of oil

[4] Natural gas, because of its physical characteristics, normally has a high recovery factor—on the order of 75 percent. Subeconomic resources and opportunities for enhanced recovery of gas are thus limited for the most part to the fracturing of dense, low-permeability reservoir rock.

and gas remaining to be discovered and recovered. No attempt is made to show when or if these resources will be produced. The second approach is that of the engineer–manager who makes projections of the drilling, discovery, and production process. These future production profiles implicitly suggest the amount of recoverable oil and gas that is left in the ground. The third methodology is that of the economist who uses the equations in his model to suggest what future supply can be achieved by the oil and gas producers as they respond to price changes. Like the engineer–manager, the econometrician may indicate the quantity of remaining oil and gas resources in his model implicitly rather than explicitly.

The volumetric approach. The geologist's volumetric estimate is essentially what the name suggests. The total volume of sedimentary rock suspected to contain petroleum and natural gas is calculated for the entire United States, region by region. Based upon past geological knowledge, an estimate is made as to the total oil and gas that may exist in these rock volumes. It is quite apparent that this is a subjective judgment linked to past experience. Underestimates are possible since past experience does not readily account for unknown types of occurrences or future improvement in the ability to detect and produce oil. In contrast, since there is evidence that better areas and larger pools are found first, unexplored regions may prove to be less prolific.

The volumetric determination of the oil and gas that exists in the ground may not be the only calculation. The quantity of oil and gas in place in the rock strata only has economic meaning in terms of the proportion that is both discoverable and producible. The quantity of oil and gas eventually captured depends upon future effort, the effectiveness of the search, the size and depth of the reservoirs, and the economic and technical capacity for producing it.

Considering the many judgmental elements, volumetric estimates, not unexpectedly, have varied widely over the years. Part of this variation reflects the fact that some estimates are the product of extensive study by large groups while others may be the work of a single expert using a relatively simplistic approach to obtain a quick approximation. Moreover, subjective judgments about unknown resources change as more geological information becomes available. These normal divergences are further accentuated by the fact that different analysts have used different assumptions and have estimated different resource elements.

Table 2.

NATIONAL PETROLEUM COUNCIL REPORT ON ESTIMATES OF U.S. ENHANCED OIL RECOVERY POTENTIAL

Source of estimate and price assumption	Potential recovery (billions of barrels)	Production in 1985 (millions of barrels/day)
National Petroleum Council Report		
$5 (1976 dollars)	2.2	0.3
$10 (1976 dollars)	7.2	0.4
$15 (1976 dollars)	13.2	0.9
$20 (1976 dollars)	20.5	1.5
$25 (1976 dollars)	24.0	1.7
GURC[a]		
$10 (1974 dollars)	18–36	1.1
$15 (1974 dollars)	51–76	—
FEA/PIR[b]		
business as usual, $11	—	1.8
accelerated development, $11	—	2.3
EPA[c]		
$ 8–12 (1975 dollars)	7	—
$12–16 (1975 dollars)	16	—
FEA/Energy Outlook[d]		
$12	—	0.9
FEA[e]		
$11.28 (1975 dollars)	15.6–30.5	1–2

[a] Gulf Universities Research Consortium Reports, Number 130, November 1973, and Number 148, February 28, 1976.

[b] *Project Independence Report*, Federal Energy Administration, November 1974.

[c] *The Estimated Recovery Potential of Conventional Source Domestic Crude Oil*, Mathematica, Inc., for the U.S. Environmental Protection Agency, May 1975.

[d] *1976 National Energy Outlook*, Federal Energy Administration.

[e] *The Potential and Economics of Enhanced Oil Recovery*, Lewin & Associates, Inc., for the Federal Energy Administration, April 1976.

A careful examination of past geologic estimates reveals that it is rare for the same type of resource concept to be involved. Total oil and gas *originally* in place, oil and gas *remaining* in place, *discoverable* oil and gas in place, undiscovered *commercial accumulations* of oil and gas, or *recoverable* oil and gas under given economic and technologic conditions are markedly different quantitative concepts. Unfortunately, the authors of petroleum resource reports all too frequently are obscure about what they have estimated, their methodology, and their assumptions. Yet all of these numbers are generally identified as estimates of "the oil and gas resources" of the United States. The unsuspecting recipients of this information must then puzzle over how one expert can say that the oil resources of the United States are 50 billion barrels and another, with seemingly equal confidence, provides an estimate of 1,000 billion barrels.

If one reduces all of these various estimates as best he can to a common base, such as the quantity of undiscovered oil that is discoverable and producible at prices as of a certain date with an assumed 30 percent recovery factor, then the wide differences begin to shrink drastically. An estimate that appeared to be twenty times as large as another sudenly is only twice as large. Once reduced to a common base, there remain understandable differences in judgment between two analysts who possess varying degrees of optimism about what is still to be discovered. But this kind of comparison is not available to the casual reader who cannot know that one geologist has estimated all of the oil in the ground, another has assumed an optimistic 60 percent recovery factor, and another uses the current 30 percent recovery factor.

Geological resource analysis took on a new dimension with the 1975 publication by the U.S. Geological Survey of Circular 725, *Geological Estimates of Undiscovered Recoverable Oil and Gas Resources of the United States.* This was a major scale-up in the Survey's effort and involved a whole team of geologists working for a number of months. It entailed not only the use of traditional volumetric information, but incorporated sophisticated statistical integration of subjective judgments about each of 102 oil and gas provinces. The end product was a probabilistic appraisal of undiscovered, recoverable oil and gas.

Experimentation with this type of delphic approach has been going on for a number of years. Companies and various research groups have searched for a way to combine the various judgments of experienced individuals into a numerical expression of the probability of finding oil and gas. Circular 725 was the first attempt by the federal government to try this approach (see figure 4). Single number estimates that suggested a precision that does not exist have been abandoned. The public and Congress may now have to become used to resource estimates that indicate there is a 95 percent chance there may be a minimal quantity of oil resources but also a 5 percent chance that there could be quite a bit more. For example, the Survey estimates that there is a 95 percent probability that the remaining undiscovered recoverable oil reserves will be at least 50 billion barrels, but only a 5 percent probability that they will be as large as 127 billion barrels. Outside of these ranges there still remain low-level possibilities that a new province may have no oil or gas at all, or that it may contain an undetected Middle East. Only the drill can tell. Some cautious individuals still prefer not to try to attach numbers to what they consider immeasurable quantities.

The Geological Survey recognizes that Circular 725, while a major advancement, was a first effort and must be used with considerable care. Only the portion of the report concerning "undiscovered, economic" resources of crude oil and natural gas are totally new estimates. All other numbers presented were based on other sources of statistics or were derived by simple ratios. Thus the measured and indicated additional reserves are taken from the reports of the American Petroleum Institute and the American Gas Association. The inferred reserves of oil and gas are based not on an evaluation of fields and basins but on the historical trend of extensions and revisions of proved reserves through time. The subeconomic resources are based on simple ratios using two assumptions—that the average U.S. crude oil recovery might reach 60 percent and natural gas 90 percent at some unspecified time in the future.

The additional data provided by Circular 725 has been useful, but it presents problems for many of its users. If the undiscovered oil and gas resources are shown as a probability range,[5] what does one use if one needs a single number? Unfortunately, many seemed to prefer to use the lower limit. In using these data, it has been common to overlook the fact that the subjective judgments behind the estimates were based upon price and technological conditions that prevailed prior to the Arab embargo and the quadrupling of the world price of oil. This leads to the question of how much current prices might alter the estimates. It is believed by many that a recalculation would not make a large difference because the estimates are dominated by large fields which were economic before 1973.

The Survey is now searching out the answers to a number of questions. Can the reserves portion of the estimate, which received modest attention in the first effort, be improved? In estimating subeconomic resources, there is the important question concerning the realism of ever reaching an overall 60 percent recovery. This may now be better understood because of the extensive work just completed in examining enhanced recovery. Since the availability of actual experience by Survey geologists for every oil and gas "play" was naturally somewhat limited, there is an interest in how to tap a broader spectrum of judgment. There is a need to have future appraisals encompass more data on: size distribution of undiscovered fields, depths, reservoir types, and basins found in deep water offshore. These data may be the key to determining actually how "price sensitive" are oil and gas resource estimates. Until these answers are forthcoming, the user of the Survey's Circular 725 must remain fully aware that these are subjective views by a group of government geologists concerning the recoverable amount of pre-1973 "commercial" oil left in the unexplored portions of U.S. basins onshore or to a depth of 200 meters offshore. This recoverable amount is only a portion of the total physical quantity of oil and gas in place remaining beneath the surface in the United States.

Engineering projections. The fact that oil production is a process in which production declines and costs increase became apparent to engineer-managers soon after Colonel Drake, a retired railroad conductor, drilled the first well in Pennsylvania in 1859. Fortunately, many wells do better than Drake's, but a ruler placed on the graph of past production and the cost per barrel of any well or field always provides a dismal picture of a downward trend in the absence of new discoveries and technology. In contrast, projections made by individual companies or industry groups showing increasing future production are illustrations of how additional investment in exploration, drilling, or applications of new technology can cause the aggregate production of oil or gas to increase in the future despite the fact that the older wells are declining and future efforts will face

[5] The Survey does provide an average of the most likely and upper and lower estimates. However, the statistical usefulness of this average is uncertain and it only appears in the tables.

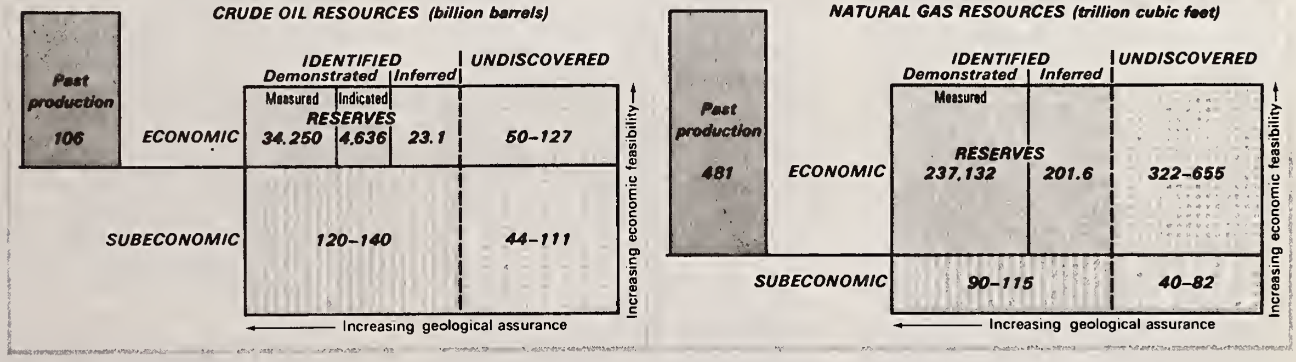

Figure 4. USGS Estimates of Crude Oil and Natural Gas Resources of the United States, December 31, 1974.

greater costs per barrel produced per foot drilled.

The best illustrations of this kind of analysis are found in the extensive series of reports by the National Petroleum Council (NPC) to the U.S. Department of the Interior. They contain many examples of how a given number of dollars invested, assuming a specified rate of return to the investor, could generate a required level of geological-geophysical work, leading to the drilling of wildcat wells, and finally the development effort needed to produce the new oil found. Figure 5 from the 1972 NPC study demonstrates how this can lead to various perceptions of the future.

By their very nature, these projections of future production, based on a specified amount of additional effort or investment, are expansive. These speculative futures may or may not incorporate a judgment as to whether the remaining oil and gas resources in the ground are sufficient to provide for these annual flows. In the NPC's extensive study of U.S. energy from 1970 to 1973, projections were coupled to a geologic study of the U.S. petroleum provinces. Since most of the NPC's projections were only to the year 1985, the possibility of a production decline after 1985 due to dwindling resources was not shown.

Many government or industry projections of future production are not primarily designed to deal with the ultimate size of our oil and gas resources. Nonetheless, they still may foster a public belief that resources are adequate in size to meet the projected goals. In addition, there may be only minimal attention paid to the price required to elicit the necessary investment. And whatever that price may be, the accompanying alterations in the demand for oil and gas, given that price, may not be addressed at all.

One specialized form of engineering projection that has been employed by many authors over the years has been to use a production-history profile that will follow the standard pattern observed when minerals or fuels are produced from a finite deposit. The classical configuration is a bell-shaped curve showing an upward sweep, a peaking of production, followed by a decline to final depletion. Not only a specific deposit, but a state, a region, or a nation, as an aggregate of many deposits, often appears to follow this pathway.

If one assumes that this is the general behavior of production, then it becomes possible to estimate when the peak will occur, the quantity that will be produced, and how it will be distributed over time. Customarily this is done by rather simple curve-fitting techniques

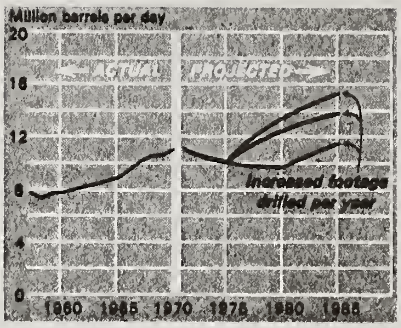

Figure 5. Estimated U.S. Production of Petroleum Liquids at Three Levels of Future Drilling Activity
(from National Petroleum Council, *U.S. Energy Outlook,* 1972)

using an appropriate mathematical formula. One of the most popular production-history curves has been the logistic growth curve, since it is both bell-shaped and symmetrical when fitted to annual data. It can also be used as a long attenuated "S" showing how cumulative production will approach asymptotically a line representing the maximum recoverable resources. In reviewing forecasts between 1948 and the mid-1950s, RFF's Sam Schurr and Bruce Netschert found at least six authors using this kind of approach who expected U.S. petroleum production to peak by 1970.

Among this group, perhaps the greatest amount of attention in recent years has been directed toward the work of M. King Hubbert. Relying heavily on the logistic curve and a family of various statistical series to track the behavior of U.S. oil production, his analysis is both extensive and detailed. Among the family of interrelated curves that he uses are: cumulative proved discoveries, cumulative production, proved reserves, annual production, annual increases and decreases in proved reserves, and discoveries of oil per foot drilled versus total footage drilled. The first three of these, with the appropriate fitted curves, are shown in figure 6.

The logistic curve provides a particularly good fit to any historical series that is approaching or has already reached a plateau. But one must be careful not to assume that the fitting of the curve to the several variations of the same historical series in some way confirms the validity of its use. Despite the vigor with which Hubbert examines past behavior and projects the various patterns of U.S. oil discovery and production into the future, this does not necessarily indicate that the logistic curve is a more reliable predictor of the future than any other curve that might have been chosen.

The use of mathematical formulas to project trends forward provides an aura of precision and objectivity. However, the process of fitting and projecting is a more subjective process than it might appear. The choice of the type of curve to be used preordains in a general way what the future will look like. Then the analyst must exercise further judgment as to the time period to be used and how the curve is fitted to the data. Judgments at this stage of the analysis are particularly critical, because the manner in which the final years are fitted affects the steepness of the expected production decline.

In addition to his decision to use the logistic curve rather than another, Hubbert's case for future decline is bolstered by his assumption that a declining amount of oil will be found per foot drilled. This is a persuasive position to take if one expects that the largest and near-surface fields have been found first. Hubbert supports this hypothesis by another projection involving again the choice of proper data and a mathematical formula[6] to project the trend of oil found per foot drilled. In this extrapolation he shows that despite extensive drilling the quantity of oil found in the future becomes relatively insignificant and his gloomy expectations for the future are further substantiated.

In mature, densely drilled areas, such as the onshore areas of the lower 48 states, one might be persuaded that it is unlikely that there are many "surprises" left. It does appear unlikely that there are a number of Prudhoe Bays cleverly hidden by nature along the Gulf Coast or in the Rocky Mountains. For these areas it seems to be a question not of new peaks in production but of the duration of the current level of production and the nature of the subsequent decline. But in those areas where drilling has been infrequent—natural gas at greater depths along the Gulf or oil and gas in the Arctic or farther offshore—the future is not as clearly defined. For these regions, most analysts, including Hubbert, do not trust unalterable formulas but turn back to more traditional geologic speculation about the quality of the targets that may be found in these unexplored volumes of rock.

Production-history profiles provide for many a sense of "rightness" because they follow the classical pattern of mineral production. Yet they leave a number of questions unanswered. Is it fair to assume that the historical interrelations between the many factors that affect oil exploration and production

[6] In this case a line representing a constant percentage of decrease each year is used.

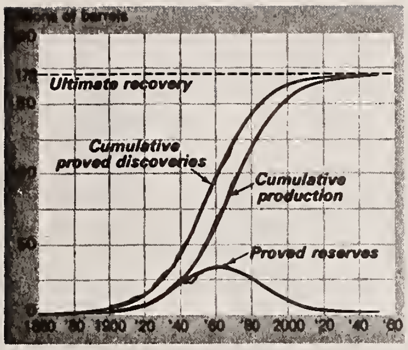

Figure 6. Application of Logistic Curve to U.S. Petroleum Data
(from M. King Hubbert; see References).

will continue unchanged into the future? If our production of oil and gas has peaked, has that fact been induced by external institutional factors rather than by a limitation in our hydrocarbon endowment? Might not our technical ingenuity or the new frontiers of exploration once again produce a major surprise? If secondary peaks have been observed in the past for states and nations, why not for the United States?

There are no certain answers to these questions. But for the United States, M. King Hubbert has reminded us of one unavoidable truth—we are not debating whether there will be decline, merely when and how. This does not deter the exploration optimist from reminding us how we drilled for many years in Texas, Saudi Arabia, and the Arctic with little or no success.

Econometric models. The third group to deal with the future supply of oil and gas has been the economists. By professional instinct and training, they initially turn to the marketplace as the starting point for their analysis. Their facility for portraying relationships by mathematical equations, combined with the ability of modern computers to provide rapid and complex calculations, has led to the use of econometric models. Normally found within these models are equations that relate the supply of oil and gas to exploratory and development efforts prompted by changes in price.

Perhaps over the decades more general attention has been directed toward oil rather than gas resources. But more recently, gas supply has demanded considerable attention from the econometrician. This reflects the fact that gas prices have been regulated and there are questions about what would happen if the regulated price were increased or if regulation were to be removed entirely. Interest has intensified with the recent, rapid decline in proved gas reserves. The occurrence of gas and the search for it are not necessarily linked to oil, so gas supply can be disengaged for separate study. The task is also simplified by the fact that gas supply models are essentially domestic, that is, they need not incorporate, as in the case of oil, the impact of large imports from abroad.

A number of models for gas supply have appeared over the last two decades. Some provide projections and forecasts and a number deal with the supply–price relationships. However, like the engineer–manager projections, the econometric models are not designed to provide estimates of the remaining oil and gas resources of the United States. Nonetheless, in addressing the supply–price question they unavoidably give an impression of resource availability.

An examination of the better-known gas supply models reveals that the inclusion of total U.S. gas resources, or any limit to discoverable and producible resources, tends to be implicit rather than explicit. This is intentional and is not a serious flaw for the intended use of the models. For the most part, econometric equations are not considered particularly reliable beyond short periods of time—of the order of five or ten years. Given these limits, to introduce total resource quantities is an unneeded refinement.

However, many models are designed to indicate that price will trigger an exploration response and the resulting greater production suggests that oil and gas resources are adequate to support higher levels of production than now prevail. The MacAvoy–Pindyck model, developed in the early 1970s to analyze the effects of deregulation policy, suggested the possibility that 34 trillion cubic feet of natural gas could be produced in 1980 at an average wholesale price of 88.3 cents per thousand cubic feet (MCF) with a newly-discovered-field price of 100.3 cents per MCF (see table 3). This result did not hinge upon the total quantity of undiscovered resources required for the United States to achieve that level of production. Nor was the model constructed to deal with the mechanics of annual investment, the number of wells to be drilled, or the physical ability of the productive system to achieve the required level of effort. The MacAvoy–Pindyck model shares with most of its econometric companions a necessity to simplify the national energy economy. It was designed to answer a specific question—in that process it ignored others.

Econometric models have their own special link to the past. The response, or elasticity, of oil and gas supply to price must be judged in large measure in terms of historical data, despite the realization that in each future year we will deal with a different segment of the original resource. Future resources may very well differ in character and, as a consequence, in cost from those discovered in the past. Economic behavior patterns of operations conducted on vast federal leases in 1,000 feet of water are not the same as those encountered in the private farmlands of Kansas. Nor will the response of gas supply to a doubling in price (in constant dollars) be the same when it starts at 10 cents per MCF as when it starts at $1. A reason to question further the future validity of past experience is to recall that much of the past was characterized by smaller movements in the price of oil and gas relative to other prices, and that for the most part this was downward not upward.

Current efforts. In making resource estimates geologists, engineers, and economists are all to some degree projecting past experience into the future. Insofar as the past does not adequately represent the future, their estimates are likely to be in error. In addition, each profession, starting from its own particular analytical framework, is the victim of a certain amount of tunnel vision. The geologist prefers to perform his task in a price-free, time-free fashion. The engineer may ignore resource constraints and economic reactions in his production model. The econometrician may demonstrate what market price is necessary to reach an equilibrium point but in so doing may violate the time sequence or engineering requirements needed for the process to be accomplished, given the magnitude of the remaining resources and national capabilities.

It is not suggested that the various analysts are totally unaware of the limits of their work. More often than not the problem is the difficulty of trying to link all dimensions of the resource system into one model or into one forecast. Moreover, the purposes being served may not demand a complexity that exceeds available time and financial resources.

The Federal Energy Administration (FEA), in projecting the needs of the nation by 1985 for Project Independence, initially employed the committee approach to the problem; so, too, did the National Petroleum Council. However, subsequent in-house work by the FEA staff on the *1976 National Energy Outlook* (NEO) led to the development of a complex computer model (PIES). This effort has been an excellent illustration of the long and difficult task of attempting to introduce the many dimensions of energy into one integrated analysis. The many scenarios developed for the *National Energy Outlook* required an analysis of demand, supply, finance, the environment, the national economy, and international aspects.

This should not be construed as suggesting that the ultimate model is now available to the new Department of Energy. A close examination of PIES reveals that the tie between energy and the national economy tends to be one directional. In the 1976 report, environmental and international aspects were not introduced as specifically as one might desire. The model reflects the many imperfections in our understanding of the behavior of energy demand in the marketplace. The resource component of the model is still the familiar 1973 data from the USGS Circular 725. Perhaps most important to the user is the fact that the PIES model does not generate a single forecast, but rather as many forecasts as there are policy combinations that an administration wishes to test (see table 4). It is easy to overlook, in the copious statistics and discussions of the model and its scenarios, that much reliance has been placed on a few key sources of data or relationships. Thus, to whatever extent Circular 725 is limited in its perspective of U.S. oil and gas resources, the *National Energy Outlook* series is equally limited.

Since so many analyses have come to depend upon it, the further work of the USGS has become extremely critical. Currently, the Survey is hoping to refine its presentation of probability data on undiscovered oil and gas resources so that the full range of potential resources within the hypothetical and speculative categories is more apparent. This will allow for an appreciation that beyond the 5 and 95 percent probability boundaries there still remain possibilities for either zero finds or major discoveries for which past experience has not prepared us. Recent interest on the part of the National Petroleum Council and other groups in enhanced recovery will now permit the Survey to be somewhat more specific about the magnitude of subeconomic, discovered resources. In addition, the presentation of data on indicated and inferred reserves (reserves beyond proved) in known fields may be expanded by the Survey.

A number of agencies have joined forces with the Survey in this effort to determine how additional information in economics and technology could be combined with the essentially geological data from the Survey's Resource Appraisal Group. This Inter-Agency Study Group on Oil and Gas Supply is examining not only the amount of the oil and gas resources present but their distribution with respect to size, geography, and depth. An attempt will then be made to define the level of exploratory drilling activity required to find these deposits. This can then be followed with studies that deal with drilling, production, and finding costs, which incorporate considerations of reservoir depth, water depth, and other geological characteristics.

Currently the group is in a testing phase, using three pilot areas to determine the feasibility of performing these various tasks. It is not expected that the current effort will attempt to deal immediately with the time distribution of future oil and gas supplies. But it is hoped that a better appreciation of the magnitude of oil and gas resources at various costs will be obtained, as well as an understanding of the exploratory effort that will be involved.

Table 3.

ECONOMETRIC SIMULATIONS OF PHASED DEREGULATION OF NATURAL GAS

Year	Total reserves (trillion cubic ft)	Production (trillion cubic ft)	Demand (trillion cubic ft)	New contract field price (cents per thousand cubic ft)
1972	233.4	23.3	23.5	31.7
1973	227.8	23.6	24.3	34.7
1974	222.9	24.3	26.3	39.7
1975	222.3	26.4	28.7	64.7
1976	226.1	27.6	30.4	71.7
1977	233.9	28.6	31.9	78.8
1978	245.8	30.2	32.9	85.9
1979	258.6	32.1	33.7	93.1
1980	271.2	34.1	34.2	100.3

From MacAvoy and Pindyck, *Price Controls and the Natural Gas Shortage,* 1975

Table 4.

PIES MODEL NATURAL GAS PRODUCTION REFERENCE SCENARIO

Assumed world oil prices $/bbl	1985 domestic production (trillion cubic ft)			1985 average city gate price (dollars per thousand cubic ft)
	Nonassoc. gas	Assoc. gas	Total	
8	16.3	4.1	20.4	1.79
13	17.4	4.9	22.3	2.03
16	17.4	5.1	22.5	2.07

Note: The Reference Scenario is a market clearing price at which supply and demand are at equilibrium in an uncontrolled market or deregulated condition.

From FEA's *National Energy Outlook, 1976.*

Other Occurrences

Other occurrences are frequently the source of possible deception about the size of the nation's usable oil and gas supplies. Billions of barrels of oil in low-quality shale, gas locked in impervious shales and sandstones, methane found in coal beds or dissolved in brines under great pressure at depths of 15,000 feet are all a part of our physical resource base. They can and should be accounted for in any total resource inventory, but they cannot and should not be considered comparable to reserves or subeconomic resources. The likelihood of their soon becoming producible under present or near-future prices and technology is small enough that their importance for present generations is uncertain. Thus considerable caution must be taken to avoid giving them too much leverage in current decisions. After fifty years of effort and anticipation, the first commercial barrel of U.S. shale oil has not yet been produced. To be deceived by a too hasty reliance on methane dissolved in the waters of the Gulf of Mexico would be foolish indeed.

Although the oil shales of the West have become the classic example of a "just-around-the-corner" resource, we must somehow account for such a vast quantity of hydrocarbons. Many oil and gas resource appraisals do not include the oil shales because they are restricted to conventional crude oil, natural gas, and natural gas liquids produced from wells. Other analysts do not include them because they are not economically producible at the present time. If, however, a complete accounting is desired, then it is appropriate to at least identify these as other occurrences or noneconomic resources which are currently not produced and are likely to be significantly more costly than other forms of energy now being used. Whether quantification is attempted depends upon the purposes of the inventory.

Hydrocarbons occur in many forms in nature. Just as there are many types of coal (anthracite, bituminous, or lignite) there are heavy oils, tar sands, and kerogens which will not flow to drilled wells. This requires the extraction of the material either through the use of heat and chemicals or physically mining the rock so that it can be processed above ground. Since these are sedimentary deposits, they can be vast in extent but highly variable in recoverable energy content. In effect, they are low-grade deposits requiring expensive processing. As such they must be considered as either subeconomic or probably nonexploitable in any period of time that is of significance to present generations.

A number of largely unexploited sources of methane, the most abundant of the natural gases, have also attracted considerable attention. Among these are natural gas in dense sandstones of the West and the Devonian shales of the East where the rock is relatively impermeable and does not allow the gas to flow freely to a well. As a consequence, the drilling of a well in these formations is not often rewarded with a great quantity of producible gas or a high daily rate of production. Methane in coal is well known as a hazard to mining and is actually recoverable by drilling holes in the coal bed in advance of mining. Another recent discovery has been of the presence of methane in underground salt water found at considerable depth in the Gulf Coast area. The gas is held in the water by the great pressures that exist at the depth.

Relatively simple calculations of the volume of oil shale in the Piceance basin, tarlike substances in Utah, and methane in coal beds or other geological settings yield vast quantities of energy that physically exist. However, like exotic rocks on the moon, the fact of their existence should not be confused with economic and technological accessibility.

In the other occurrence classification, there is also that portion of conventional oil and gas that we do not expect to recover. Similar to low-grade oil shale, it would be physically possible to produce this oil and gas at great cost. One could literally mine an oil reservoir and produce all of the oil, or let a gas well produce until there was no more pressure left. Obviously, long before this, it would be far more sensible to use some other source of energy. Thus, those portions of our oil and gas resources unlikely to ever be recoverable can be accounted for among the other occurrences.

Productive Capacity

It is virtually impossible to determine how much oil and gas can be produced in any given year solely on the basis of knowing the quantity of proved reserves of oil and gas. If the nation finds itself lining up at gas stations or shutting down factories because adequate pressure cannot be maintained in all the utility mains during cold weather, the immediate supply problem is the productive capability of the delivery system, not proved reserves or undiscovered resources.[7]

Over the years little study has been directed toward understanding the limits of the delivery system upon which we depend to move energy from the well to the burner tip. For the fossil fuel group, we have only the American Petroleum Institute's (API) estimate of productive capacity. This is the maximum daily rate of production which could be attained under specific condition on March 31 of any given year. It would require ninety days to reach, starting January 1, and is based upon existing wells, well equipment, and surface facilities. The estimate provides for no reduction in ultimate recovery, and environmental damage or other hazards are not accepted.

Obviously, it is useful to have such a measure of our capability. It is important, however, to be aware that the API definition of national productive capacity does not imply anything about the sustainability of this capacity over any specified period of time beyond the ninety days. Beyond March 31 of the year of estimate, the productive capability would begin to decline. This particular measure of productive capacity does not encompass our capability, or lack of it, for storage, transportation, and processing facilities to handle the oil once produced.

The gas shortage of the winter of 1976–77 was a combination of system limits and the declining deliverability of gas from existing wells. Through emergency measures, such as denying some customers gas and shifting gas between systems on an ad hoc basis, the nation coped with the situation. This does not alter the fact that the gas deliverability from wells in 1977–78 will be different from what it was in the past winter. It is not generally appreciated that ad hoc plans that worked once may not work as well in another heating season. The fact remains that our oil and gas systems involve thousands of enterprises delivering fuel to millions of households and commercial customers. Exactly how that system works and what its capability might be, given the declining production curves of oil and gas, is only partially understood. As of midwinter, the weather and the amount of natural gas in storage did provide some optimism for the 1977–78 heating season.

The inability to discriminate between reserves and deliverability is the source of considerable confusion in the reporting on the oil and gas situation in the United States. References to the estimated total reserves in a field or resources in a new region are equated with annual production or requirements. Since only approximately 10 to 15 percent of the reserves of a field can be produced in a single year, if connected to a delivery system, billions of barrels of oil reserves or trillions of cubic feet of gas translate into a much smaller amount of oil and gas available even in the early peak years.

To understand oil and gas supply requires more than a realization that estimating reserves is an inexact process; it also involves an appreciation of the limited capability of a well to produce oil and gas upon demand. The time required to explore, find, drill development wells, lay pipe, and provide process facilities is a further restraint on translating reserves into production. To this year's energy consumer the only supply that counts is deliverable oil and gas, not reserves or resources. If that flow is inadequate, periods of five to ten years or more and considerable investment will be required to alter it in any significant way. Considerably more attention to the limits of this process, and perhaps less to reserves, seems warranted.

It is understandable that the productive capacity of the vast oil and gas producing industry and its downstream facilities presents problems in terms of measurement. One would expect, in contrast, that the capability of a known producing reservoir would be a reasonably precise number. This has taken on a new importance in recent years, as

[7] Productive capacity is the more common expression in the oil industry. The gas utility industry's immediate capability is referred to as the "deliverability" of the gas.

questions have been asked about whether or not producers holding federal leases have been producing oil and gas as diligently as they might if the price for oil and gas were higher.

This particular question has revived interest in a measurement that appeared during World War II called maximum efficient rate (MER). In concept, MER can mean the theoretical physical capability of a reservoir to produce oil and gas over time as a hydraulic unit. To exceed this rate may reduce the amount of oil and gas recovered. In practice, MER has come to mean the maximum rate, in terms of barrels or cubic feet per day, a reservoir can produce "efficiently" and "economically" from a fixed number of wells under actual operating and market conditions. In effect, this is the design capacity of the reservoir development plan, and reflects what the operator feels is economically justifiable. Producing the existing wells too rapidly could cause oil and gas to be lost in the reservoirs. Drilling additional wells could increase the flow from a reservoir without harm and might even increase the amount ultimately produced, but the decision then rests on whether the extra expense of the well can be justified by the economic advantage to be gained by producing more oil or gas or producing it faster. That particular judgment may depend on whether you are the producer or the federal government leaseholder.

Even the term efficiency provides its own share of problems. To the economist, efficiency will tend to be interpreted as determining how recovery should be distributed over time to arrive at a maximum value for the oil and gas produced regardless of the physical recovery. The reservoir engineer's technical efficiency will be to achieve maximum physical recovery at the lowest cost under current market conditions. The government administrator may be interested in a production rate that provides a maximum quantity of oil and gas to the public when it needs it, commensurate with a reasonable return to the producer and acceptable payments to the Treasury of bonuses and royalties. For a given reservoir, the annual production under these three criteria will not necessarily be the same.

A difficult aspect of using MER is that it is not a constant. Since production of a well or reservoir is from a declining reserve, the producible quantity actually decreases from day to day. In practice, MER is determined periodically for most federal leases. It should be remembered that a reservoir may include a large number of wells, and that the maximum efficient rate of production applies to the whole reservoir and not to any individual well. In the past, MER was considered a conservation technique to prevent wasting the natural energies of the reservoir by moving the various fluids so rapidly that oil or gas is left behind entrapped in the reservoir. Even so, in emergencies such as World War II, it is sometimes considered in the nation's best interest to produce oil or gas at a rate that actually causes some loss in ultimate production.

Offshore operations from costly platforms have introduced new dimensions to production and transport that have increased the difficulty of determining what is both physically and economically possible to accomplish. As a result, MER, which in the past has been primarily a state conservation regulatory tool, creates a number of problems when it is used as a measure of diligence in exploiting federal leases. The federal government continues to look for a means whereby it can properly monitor operator performance in terms of the national interest without harming the entitlement of the leaseholder to serve and to protect his own interests as a private enterprise.

Conclusions

The RFF staff has now engaged in over two years of studying, discussing and explaining the uncertainties of oil and resource estimation. That has led not to better numbers but to perhaps a better understanding of what the existing data can or cannot do for us. By and large, we find that most examinations of oil and gas resources reflect in part the professional background of the estimator but most importantly the purpose for which the work is designed. Many estimates that have been published provide total future quantities of oil and gas that may be produced rather than supply in the economic sense or rates of production over future time. All too frequently, these totals may be translated into years of remaining oil and gas by dividing them by the current or some other assumed rate of production. This leads to the too-simple conclusion that we may be out of oil and gas at the end of that number of years.

Published estimates of total future producible hydrocarbon fluids provide the public with a narrow view of future oil and gas supply. This is compounded by the fact that the public does not know how to interpret the figures. As one RFF workshop participant noted, "the difficulty [in publishing estimates] was the problem and confusion in the public's mind of what all these numbers mean. It has just been an absolute mess. They [the public] have taken undiscovered resources and related them to reserves, and this was not really our intent at all. Suddenly, we find ourselves quoted in the most peculiar ways. And much to our embarrassment."

To the economist, it is important to know how much and for what period of time oil and gas production rates might be increased by a change in the economic structure of the industry or in the cost–price ratios. Or how sensitive future oil and gas production rates are to changes in technology. Answers to these kinds of questions are not contained in the usual published estimates of future oil and gas reserves. As a participant in one RFF workshop said, "Not a single technique, approach, publication, or anything, has yet adequately dealt with what the economists would call the supply schedule. Somehow we have got to get some indication of what different levels of future supply would be available at different cost–price relationships."

In any given year the production of U.S. crude oil encompasses production from a reservoir that has been newly discovered, along with the production from reservoirs ten to fifty years of age. The important point to note is that the future rate of supply will be a composite of the rates from both old and new reservoirs.

The experts who have appeared in the RFF workshops are agreed that the rates of oil and gas production cannot be increased indefinitely. At some point, the rates must inevitably peak and then continue to decline until the hydrocarbon resources of the earth are exhausted. What the experts cannot agree upon is whether the annual rates of oil and gas supply can still be increased, and over what period of time, before the final decline begins. The extremes are illustrated by proposing different extensions into the future of the past oil production rate curve (see figure 7). The conservative view states that most of our giant fields have been found; the maximum annual rate has already been reached; and that, henceforth, there will be nothing but decline (see Curve A). An optimistic view might be that the annual oil production rate may still rise in response to exploration and new technology to some future maximum from whence it would decline until all reservoirs were exhausted (see Curve B). A moderate view would be a future annual production rate curve somewhat between these two extremes (see Curve C).

Whatever extension is predicted for the future rate of oil supply, the area under the resulting profile of the future can be no greater than the total amount of oil which one estimates can ultimately be recovered. Thus, the estimator of the conservative situation (Curve A) not only envisions a declining rate of production but also a limited amount of total production yet to be achieved. The optimistic estimator (Curve B) sees not only an increased annual rate of supply but also a larger volume of oil yet to be produced. Whether the estimator approaches the problem in terms of rates or total future production, the results of the estimate must be internally consistent with respect to the relationship between rates and cumulative production.

Many of the published estimates of future oil and gas supplies have provided a value for the total supply through use of the traditional "volumetric" method. No matter how polished and sophisticated the details may be, the volumetric method still contains two basic perceptions: (1) that the occurrence of hydrocarbons in an unexplored geological region and the parameters that are associated with its occurrence will probably bear a relationship to previously explored regions, and (2) that the searching efficiency for finding oil in new areas will likely resemble what it has been in the past in older regions.

Any resources estimate for future oil and gas production in an unknown region may convey a misleading impression to the nonprofessional. The only thing an estimator can say with absolute accuracy is that he does not know whether there is oil or gas in a given region until wells are drilled to find out. The history of past estimates is rife with situations in which either little oil was found in areas where there was a high expectancy or much oil was found in areas where there was a low expectancy. Thus the U.S. Geological Survey's probabilistic 2 to 4 billion barrels of undiscovered recoverable oil off the eastern U.S. Atlantic shore may be used by many as a "guaranteed" number. It may become the basis of a political decision either to explore or ignore the area. An individual particularly concerned about environmental protection might be inclined to conclude that this amount of oil does not justify taking the risk of polluting the environment to go after it. The fact is that there may be no oil at all off the Atlantic shore or there may be many times the estimated USGS figure.

If one prefers to approach the future by considering the production rate curve rather than total future production, then the immediate problem is the lack of an established theoretical basis for predicting the future shape of that curve. It appears to be somewhat unwarranted to assume that the curve would be symmetrical. On the contrary, there is reason to suggest that the rising leg of the curve is dominated by one physical and economic process, that is, the discovery of new reservoirs, while the decline will be dominated by a different physical and economic process, that is, the depletion of discovered reservoirs.

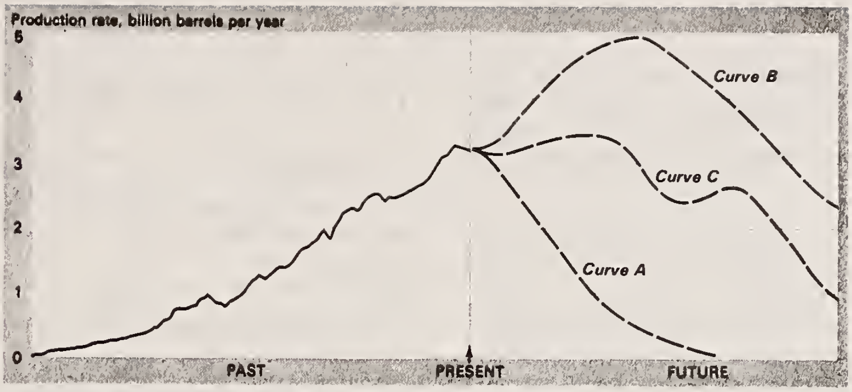

Figure 7. Alternative Futures for U.S. Oil Production

The nature of oil and gas reservoirs is such that the highest rates of production occur in the early life of the reservoir. The flow factors of a reservoir taken together generally mean that more than half the production from a particular reservoir will occur after the maximum rate of production is reached. If it is possible to supplement the natural producing energy of the reservoir or to apply technology that will make more of the reservoir oil accessible to production, the history of the reservoir may show additional production-rate peaks after the first one has been reached. This was the case in Pennsylvania and in Illinois.

The best current estimate is that, with present technology and prices, an average of between 30 and 40 percent of the oil known to exist in discovered reservoirs will have been produced at the time the reservoir is considered commercially exhausted. This is an average to be interpreted as we understand expectancy, that is, some people die younger and some live longer. The amount that can be taken from a particular reservoir is dependent upon the nature of the oil itself and the nature of the reservoir. There are known oil reservoirs from which the ultimate production will be as much as 80 percent of the oil contained in the reservoir. In other instances, the amount of oil that can be produced with present technology and prices may be as low as 10 or 15 percent of the oil in the reservoir. If technology improvement or a price rise permitted an abrupt change in recovery factors, a late production rate peak might show in the oil supply curve.

Another approach to examining the future of oil and gas is an examination of the rate at which exploratory drilling finds new oil and gas reservoirs. This approach does not necessarily depend upon prior estimate of whether oil or gas is present. It assumes that, if oil and gas are present, they will be found. The approach requires, however, an estimate of the efficiency for finding in the future. The published graphs which show the manner in which the amount of new oil found in the past related to the total exploratory footage drilled indicate that the finding rate has been decreasing. The reasons generally given for why less oil is being found per foot of exploratory hole drilled include:

1. We have already found the big reservoirs and those near the surface. Future reservoirs will be found at deeper horizons.
2. The most desirable geological regions have been explored and drilled.
3. The geological areas remaining to be drilled are more inaccessible and more expensive; for example, on the continental shelf.

Whatever the reasons may be for an expected decline in search efficiency, it is relatively easy to see that if one extrapolates this decline into the future, the contribution to production rates due to finding new reservoirs will diminish. Consequently, one would conclude that the maximum production rate probably has been reached, but not all the experts agree that the search efficiency must decline. Both technology and economics could affect the trend.

Disagreement resulting from various methods of estimating future oil and gas supply revolves around whether the volume of oil and gas remaining to be found, or our productive capability, is the primary limiting factor on the domestic production rate in the immediate years ahead. The National Petroleum Council concluded from its inquiry that at least until 1985 the amount remaining to be found is not the limiting factor. NPC visualizes that annual production can increase with appropriate attention to the drilling rates, finding rates, improvements in recovery factor, and economic adjustments.

The straightforward production-history approach of M. King Hubbert and others, which is appealing to many, does appear to be very useful in telling us what is likely to happen in the near future, if we continue doing things more or less the way we have been. It implies that production, drilling, and so on are insensitive to economics and policies. Barring an almost total interruption in exploration, such curves do seem to provide us with what should be our minimum expectation for future U.S. oil and gas output. However, this is not to suggest that any of the other kinds of appraisals are totally free of ties to past reserve and in-place figures and historical, economic, and technological factors.

There is greater satisfaction with recent estimates of future oil and gas supplies because the numbers appear to be converging. Instead of difference in orders of magnitude, two estimates may be within 10 or 25 percent of one another. In part, this merely reflects a greater consistency in methodology and assumptions than previously. Any comfort derived from this apparent consensus can be false. Although two estimators may now agree, even if they have used different methods, this does not necessarily mean that they are both right.

Finally, it is important to emphasize that all oil and gas resource estimates by the many analyses both public and private are dependent upon the same sets of numbers as starting points. Beyond this, there is no right methodology, and estimates are sophisticated guesses at best. All experts are agreed that the usable oil and gas hydrocarbon resources are probably sufficiently limited that the maximum annual rate of production and the decline until reserves are exhausted are events that will fall within a few decades, not much beyond that. The peak in the United States may have already been reached. Yet one must not minimize the importance of capturing the remaining one-half or one-third of our oil and gas.

So basically we are dealing with forecasts of annual production rates for two or three decades, and we must get an idea of the impacts of economic and technological changes on these rates. Cost data should be assembled so that it will be possible to analyze better the responses to economic change. Attention must be directed to the effect of technological change on increasing the recovery factor of existing reservoirs and the lead times needed to accomplish this.

This work will be aided if we can eliminate some of the past disagreements of estimators that stemmed from a lack of consistency in defining recovery factors and other concepts employed. If estimators are agreed on anything, it is that the definitions of terms must be examined closely and more standard definitions accepted for future resolution, if not of the supply question, then at least of why the estimators disagree in fact. Such a resolution would be an important step toward substantive agreement.

This is not as easy as it may seem. Even a seemingly simple term "total oil and gas in place" changes in meaning due to changes in information and the economic or technological perception of the analyst. Gas in tight sands or heavy oils would not have been encompassed within the definition of that term a few years ago. Yet some output from these sources has now joined the production stream.

The realization that there are no measurements in oil and gas resource appraisals is important to impress upon everyone. Even in discovered reservoirs we do not measure the oil in place—it is estimated. Reserves are an estimated value derived from a prior estimate of the oil in place, taking into account economics and technology. If the oil-in-place estimate changes, so will that of the reserves. Reserves and resources are equal to the estimated oil in place multiplied by an assumed recovery factor, substantially less than 100 percent for oil, less the amount of cumulative production. Nothing could be simpler yet so uncertain. There are no hidden formulas for predicting the end of the finite supply of oil and gas in the United States or the world. The definitive study of future oil and gas supply and how it may be altered by economic and technological parameters that have not yet emerged still remains to be done. It is of little comfort that the final, reliable, appraisal of the oil and gas resources of the United States will prove to be historic rather than predictive.

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RFF WORKSHOP PARTICIPANTS

Resources for the Future wishes to acknowledge the contributions made by participants in the series of workshops held in recent years. These were: the Workshop on Oil and Gas Resources chaired by John C. Calhoun, Texas A&M, and Earl Cook, Texas A&M, rapporteur; the Workshop on Maximum Efficient Rate chaired by John J. Schanz, Jr., Resources for the Future, and Hans H. Landsberg, Resources for the Future, rapporteur; and the Workshop which reviewed the FEA's *1976 National Energy Outlook,* chaired by William A. Vogely, Penn State, and Helmut Frank, University of Arizona, rapporteur. The participants, some of whom participated in more than one workshop, are listed alphabetically below with their affiliation at the time of the workshop.

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SYMPOSIUM ON

OIL AND GAS SUPPLY MODELING

- - - - -

Sponsored by
The Department of Energy and The National Bureau of Standards

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June 18, 19, 20, 1980

- - - - -

Main Auditorium
Department of Commerce
14th & E Streets, NW
Washington, DC

- - - - -

Wednesday, June 18, 1980

9:00 a.m. - 9:20 a.m.:

Welcome....................................Saul I. Gass, U. of MD/NBS
Symposium Chairman

Lincoln E. Moses
Administrator, EIA/DOE

H. William Menard
Director, USGS

9:20 a.m. - 9:30 a.m.:

Symposium Objectives..........................Frederic H. Murphy
DOE/EIA

9:30 a.m. - 10:15 a.m.:

Oil and Gas Supply: Public Perception, Modelers'
Abstraction, and Geologic Reality..........John J. Schanz, Jr.
CRS/LOC

10:15 a.m. - 10:30 a.m.: Coffee

10:30 a.m. - 11:15 a.m.:

Techniques of Prediction as Applied to the
Production of Oil and Gas..................M. King Hubbert

11:15 a.m. - 12:00 p.m.:

Current Problems in Oil and Gas Modeling..............William Stitt
ICF

12:00 p.m. - 1:30 p.m.: Lunch

Session Chairman: Frederic H. Murphy, DOE

1:30 p.m. - 2:00 p.m.:

The Evolution in the Development of Petroleum Resource Appraisal Procedures in the U. S. Geological Survey..................................Betty Miller
USGS

2:00 p.m. - 2:30 p.m.:

Review and Recommendations Concerning Statistical Procedures in Oil and Gas Resource Forecasting..John W. Harbaugh
Stanford

2:30 p.m. - 3:00 p.m.:

Probabilistic Approaches to Projecting Oil and Gas Supply In Our Time........................Gordon M. Kaufman
MIT

3:00 p.m. - 3:15 p.m.: Coffee

3:15 p.m. - 3:45 p.m.:

Analysis of Production and Investment Strategies for Petroleum Reserves.............................James W. McFarland
U. of Houston

3:45 p.m. - 4:15 p.m.:

A Methodology for Estimating Oil and Gas Production Schedules for Undiscovered Fields..........John H. Wood
DOE

4:15 p.m. - 4:45 p.m.:

Some Modern Notions on Oil and Gas Reservoir Production Regulation..............................John Lohrenz and
Ellis A. Monash
USGS

Thursday, June 19, 1980

Session Chairman: Wallace O. Keene, DOE

9:00 a.m. - 9:30 a.m.:

Historical Growth of Estimates of Oil and Gas Field Size....................................David Root
USGS

9:30 a.m. - 10:00 a.m.:

Results in Successive Sampling in Oil and Gas Exploration Models..............................Louis Gordon
DOE

10:00 a.m. - 10:30 a.m.:

Technology Specification and Economic Accounts for Resource Exploration and Production...........David Nissen
Chase Manhattan

10:30 a.m. - 10:45 a.m.: Coffee

10:45 a.m. - 11:15 a.m.:

Gulf Coast Resource Model Data Collection Process..Richard Zaffarano
DOE

11:15 a.m. - 11:45 a.m.:

A Methodology for Estimating Cost of Finding, Developing, and Producing Undiscovered Resources...Thomas Garland and
John H. Wood
DOE

11:45 a.m. - 1:00 p.m.: Lunch

Session Chairman: David S. Hirshfeld, DSH Assoc.

1:00 p.m. - 1:30 p.m.:

Petroleum Industry Exploration and Production Decision Methodologies..............................Ted Eck
Standard Oil of IN

1:30 p.m. - 2:00 p.m.:

The Economics of Exploration: Some Further Results in Empirical Implications.................James B. Ramsey
NYU

2:00 p.m. - 2:30 p.m.:

The Regulatory Framework in Oil and Gas Supply Modeling....................................Stephen L. McDonald
U. of TX at Austin

2:30 p.m. - 3:00 p.m.:

Firm Size and Performance in the Search for Petroleum....................................Lawrence J. Drew
USGS

3:00 p.m. - 3:15 p.m.: Coffee

3:15 p.m. - 3:45 p.m.:

Sensitivity and Statistical Analysis of Oil and Gas Supply Models..............................Carl M. Harris
Ctr. for Mgmt. and Policy Research

3:45 p.m. - 4:15 p.m.:

Natural Resource Exploration, Extraction, and Pricing Under Uncertainty.....................Sudhakar D. Deshmukh
Northwestern U.

4:15 p.m. - 4:45 p.m.:

The Depletion of U. S. Petroleum Reserves: Econometric Evidence...............................Dennis Epple and
Lars Hansen
Carnegie-Mellon

Friday, June 20, 1980

Session Chairman: Charles Everett, DOE

9:00 a.m. - 9:30 a.m.:

Finding Rates as a Factor in Oil and Gas Economic Projections.............................William K. Fisher
U. of TX at Austin

9:30 a.m. - 10:00 a.m.:

Modeling Future Onshore Domestic Production........Steve Muzzo, ICF and
Richard O'Neill, DOE

10:00 a.m. - 10:30 a.m.:

Long-Range Forecasting Methodologies for Conventional Oil and Gas........................Ellen Cherniavsky
Brookhaven

10:30 a.m. - 10:45 a.m.: Coffee

10:45 a.m. - 11:15 a.m.:

An Integrated Evaluation Model of Domestic Crude Oil and Natural Gas Supply................Robert Ciliano
Mathtech

11:15 a.m. - 11:45 a.m.:

Modeling Alaska Oil and Gas Supply.................Frederic H. Murphy and
William Trapmann
DOE

11:45 a.m. - 12:15 p.m.:

Oil and Gas Production Potential from the Lower 48 Outer Continental Shelf............Jerry Brashear and
Frank Morra
Lewin & Assoc.

12:15 p.m. - 1:30 p.m.: Lunch

1:30 p.m. - 3:30 p.m.: PANEL

Suggestions for Future Directions in Oil and Gas Supply Data Collection and Model Development....Frederic H. Murphy and
Richard O'Neill
DOE

The panel will give the speakers and audience an opportunity to present their views on oil and gas supply model research, development, and related needs.

Note: There is no registration fee. Coffee is available in the main cafeteria situated one floor below the auditorium. The cafeteria is open from 7:30 a.m. to 3:30 p.m. Conference Room 1851 (behind the front of the auditorium) is available each day for informal discussions.

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11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

The symposium on Oil and Gas Supply Modeling, held at the Department of Commerce, Washington, DC (June 18-20, 1980), was funded by the Energy Information Administration of the Department of Energy and co-sponsored by the National Bureau of Standards' Operations Research Division. The symposium was organized to be a forum in which the theoretical and applied state-of-the-art of oil and gas supply models could be presented and discussed. Speakers addressed the following areas: the realities of oil and gas supply, prediction of oil and gas production, problems in oil and gas modeling, resource appraisal procedures, forecasting field size and production, investment and production strategies, estimating cost and production schedules for undiscovered fields, production regulations, resource data, sensitivity analysis of forecasts, econometric analysis of resource depletion, oil and gas finding rates, and various models of oil and gas supply. This volume documents the proceedings (papers and discussion) of the symposium.

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