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Systematic Errors in Power Measurements Made With a Dual Six-Port ANA

Cletus A. Hoer

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The purpose of this report is to determine the systematic error in measuring power with a dual 6-port ANA. Most of the report concentrates on developing equations for estimating systematic errors due to imperfections in the test port connector, imperfections in the connector on the power standard, and imperfections in the impedance standards used to calibrate the 6-port for measuring reflection coefficient. These are the largest sources of error associated with the 6-port. For 7 mm connectors, all systematic errors which are associated with the 6-port add up to a worst-case uncertainty of ±0.00084 in measuring the ratio of the effective efficiency of a bolometric power sensor relative to that of a standard power sensor.

Key Words: bolometric detectors; connector model; effective efficiency; error analysis; microwave; power measurements; six-port.

1. Introduction and Review

Microwave power measurements at NIST are based on the dc substitution technique [1-5]. A bolometer unit, that is, a temperature-sensitive resistor mounted in a transmission line, is biased to a specific resistance with a dc current. The dc power to do this is measured by a bridge network [2] or a power meter [3]. Then, when rf power is applied to the input of the mount, the dc power is reduced by the bridge or power meter to maintain the resistance at its original value. The ratio of this change in the dc power P_{dc} to the net rf power P_{rf} applied to the input of the mount is defined as the effective efficiency η, where.

$$\eta = \frac{P_{dc}}{P_{rf}}. \tag{1.1}$$

If all of the microwave power applied to the mount were dissipated in the bolometer itself, the mount would be 100 percent efficient. However, because of losses within the mount, the effective efficiency is less than 1. In addition, there may be a small dc-rf substitution error due to the different paths taken by the dc and rf currents. All of these imperfections in the mount and the bolometer sensor are included in the measurement of η.

The effective efficiency of specially designed bolometric power detectors is measured calorimetrically in NIST microcalorimeters which are the national standards of microwave power. After calibration in a micro- calorimeter, these "transfer standards" are used to obtain the effective efficiency of other power detectors by means of a 6-port reflectometer which measures the ratio of two η's.

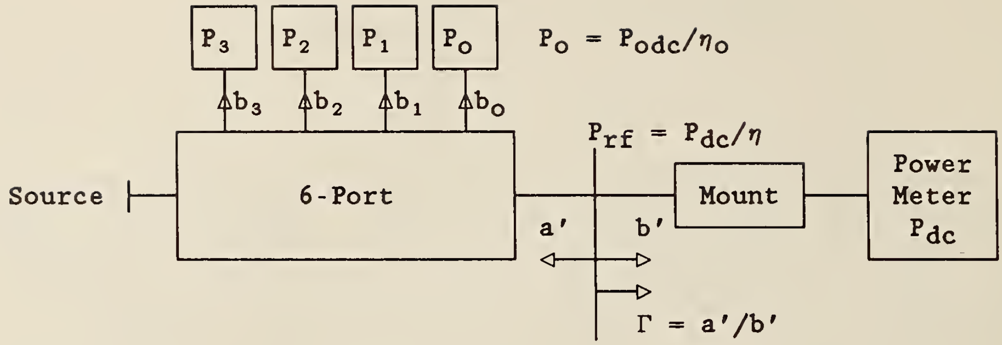


Figure 1. A 6-port reflectometer being used to measure the reflection coefficient Γ and the net rf power P_{rf} into a bolometer mount whose effective efficiency is η.

2. <u>Measuring Power with a Six-Port Reflectometer.</u>

When a termination or power detector with a reflection coefficient Γ is connected to the measurement port of a 6-port reflectometer as shown in figure 1, the net rf power into either device is given by[1] [6]

$$P_{rf} = \frac{P_o|1 - wC/A|^2}{K_p|1 - BC/A|^2}(1 - |\Gamma|^2), \qquad (2.1)$$

where P_o is the emerging rf power from the 6-port reference sidearm. This sidearm is designed to have an output b_o that is proportional mainly to b' at the measurement plane. The complex quantity w is the ratio of one of the remaining three sidearm emerging voltage waves (b_1, b_2, or b_3) to b_o and is calculated from P_o ... P_3. The "error-box" parameters A, B, C are constants that characterize the 6-port and are independent of the reflection coefficient of the generator and the load. These error-box parameters are determined by the Thru-Reflect-Line (TRL) or Line-Reflect-Line (LRL) calibration technique [6,7].

The parameter K_p in (2.1) is also a constant ("power constant") of the 6-port, and is determined by the effective efficiency η_s assigned to the transfer standard by the microcalorimeter. For the 6-port or any other reflectometer, Γ and the calibration parameters A, B, C and K_p all refer to the same reference plane, so the reference plane for power is the same as the reference plane for Γ.

The calibration parameters B and C/A appear in (2.1), but the parameters A and C do not appear alone. Since B and C/A can be determined without the "reflect" measurement in the TRL or LRL calibration, power as well as Γ

---

[1] K_p in (2.1) above is written $|r_{22}|^2$ in reference 6.

measurements can be made independent of the "reflect" if the power detector is measured on both 6-ports to determine Γ_A on one 6-port, and Γ_B on the other 6-port. Then Γ calculated from $\sqrt{\Gamma_A \Gamma_B}$ is independent of the "reflect" measurement as explained in Section 5.3 of Appendix G of reference [8].

For any reflectometer, w and Γ are related by

$$w = \frac{A\Gamma + B}{C\Gamma + 1} \quad \text{or} \quad \Gamma = \frac{w - B}{A(1 - wC/A)}. \tag{2.2}$$

Using this result, (2.1) can be put in a more familiar form by writing

$$1 - wC/A = 1 - \frac{C\Gamma + BC/A}{C\Gamma + 1} = \frac{1 - BC/A}{1 + C\Gamma}. \tag{2.3}$$

Then (2.1) becomes

$$P_{rf} = \frac{P_o}{K_p} \frac{1 - |\Gamma|^2}{|1 + C\Gamma|^2}. \tag{2.4}$$

The parameter C in (2.4) is sometimes written as $-\Gamma_g$ or $-S_{22}$ where Γ_g or S_{22} represents the reflection coefficient looking back toward the generator from the measurement port.

From (1.1) and (2.4) the effective efficiency η_u of a detector connected to the 6-port can be written

$$\eta_u = K_p N_u, \tag{2.5}$$

where N_u is defined as

$$N_u = \frac{P_{dcu}}{P_{ou}} \frac{|1 + C\Gamma_u|^2}{1 - |\Gamma_u|^2}. \tag{2.6}$$

A subscript u has been added to parameters that refer to the device under test.

The power constant K_p is determined by measuring a standard mount which has a known effective efficiency η_s. From (2.5),

$$K_p = \eta_s / N_s, \tag{2.7}$$

where

$$N_s = \frac{P_{dcs}}{P_{os}} \frac{|1 + C\Gamma_s|^2}{1 - |\Gamma_s|^2}. \tag{2.8}$$

A subscript s has been added to parameters that refer to the standard. Equations (2.5)-(2.8) combine to give

$$\frac{\eta_u}{\eta_s} = \frac{N_u}{N_s} = \frac{P_{dcu}}{P_{dcs}} \frac{P_{os}}{P_{ou}} \left| \frac{1 + C\Gamma_u}{1 + C\Gamma_s} \right|^2 \frac{1 - |\Gamma_s|^2}{1 - |\Gamma_u|^2} \tag{2.9}$$

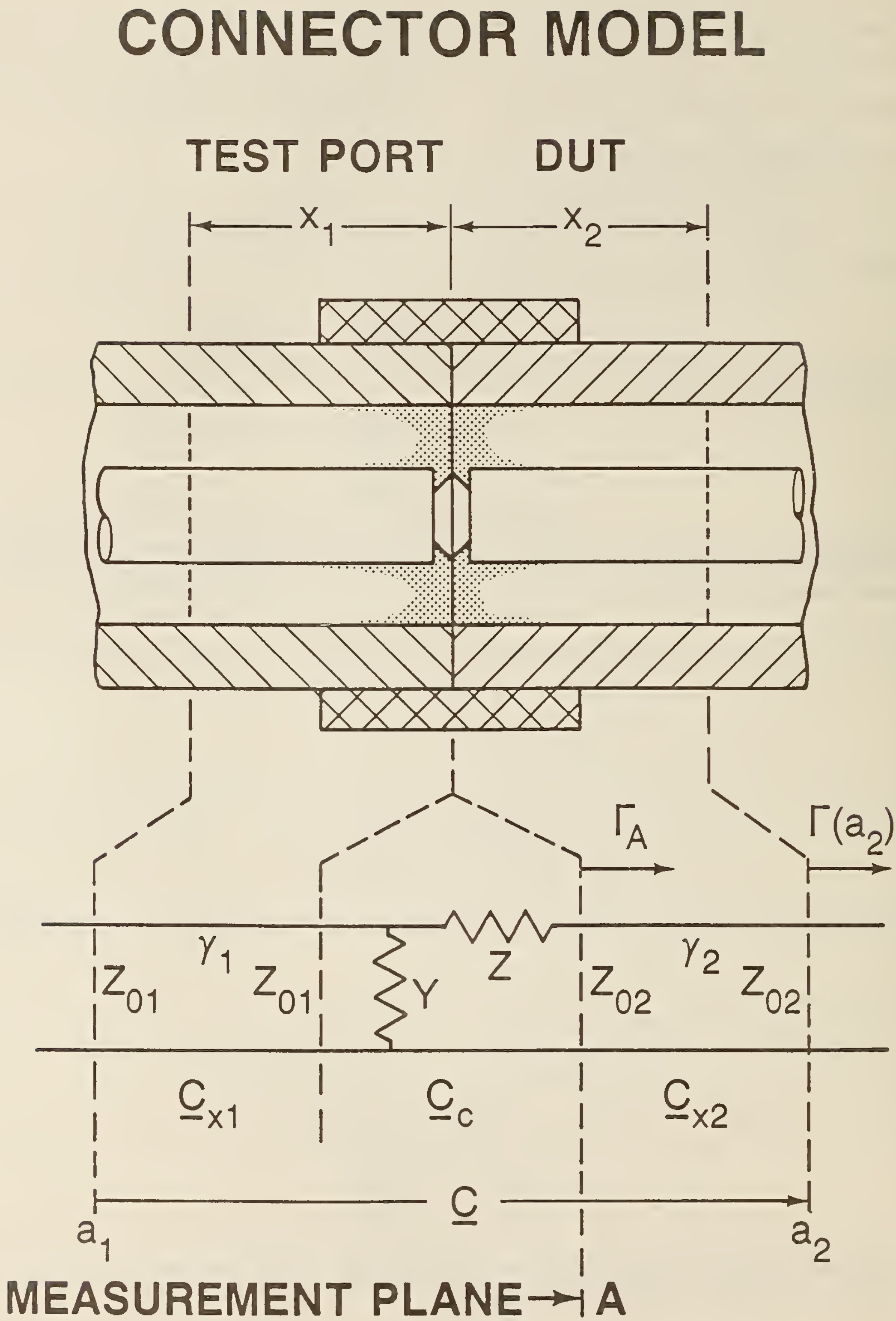


Figure 2. Equivalent circuit for a pair of imperfect connectors.

3. Connector Model and Reference Plane

When considering sources of systematic error it is necessary to take a careful look at the reference planes. The reference plane for power measurements in the microcalorimeter may not be the same as the reference plane for power in the 6-port. To investigate this possibility, we will first look at the reference plane in the microcalorimeter where η_s of a transfer standard is determined and then see how this η_s is used in calibrating the 6-port. The connector model developed by Hoer [11] which is described next will be used to analyze all connections.

A typical connection of two 7 mm connectors is shown in the top of figure 2, where the shaded area represents unwanted modes generated by the discontinuities present at the junction of the two connectors.

When more than one mode is present at the measurement plane, these modes depend on the properties of both connectors. For this reason it is better to define a model for a connector pair rather than for a single connector.

3.1. Connector Model

An exact equivalent circuit for the pair of connectors is shown in the bottom of figure 2. In the derivation of this equivalent circuit, the junction part of the connector is first represented by either a T-network or a π-network, either of which is sufficiently general to account for any discontinuity at the junction. Either representation reduces to one series and one shunt element as shown in figure 2 [11]. The T- or π-network is inserted between two lengths of uniform lossy transmission line which represent the rest of the connector. The lengths x_1 and x_2 of these two lines are chosen long enough that all unwanted modes generated near the mating plane are negligible by the time they reach the fictitious reference planes at a_1 and a_2. Since only one mode is present at these fictitious reference planes, everything between them can be, represented exactly by a set of network parameters for a 2-port. This 2-port is therefore an exact representation of a connector pair.

3.2. Cascading Matrix Representation

Let $\underline{C}$ be the cascading matrix of the 2-port which represents the connector pair. If $\underline{C}_{x1}$ and $\underline{C}_{x2}$ are the cascading matrices of the two lines of length x_1 and x_2, and $\underline{C}_c$ is the cascading matrix of the T-network, then

$$\underline{C} = \underline{C}_{x1}\underline{C}_c\underline{C}_{x2}. \tag{3.1}$$

The cascading matrices of the two lines are given by [9]

$$\underline{C}_{xi} = \begin{bmatrix} e^{-\gamma_i x_i} & 0 \\ 0 & e^{+\gamma_i x_i} \end{bmatrix}, \tag{3.2}$$

where γ_i is the complex propagation constant of line $i = 1,2$, whose characteristic impedance is Z_{oi}.

The matrix $\underline{C}_c$ can be obtained by combining the cascading matrices for the individual Z and Y elements of the T- or π-network [9]. The result is

$$\underline{C}_c = \begin{bmatrix} 1-y-z & z-y \\ y-z & 1+y+z \end{bmatrix} + \Delta z_o \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3.3}$$

where the dimensionless normalized admittance y and impedances z and Δz_o, are defined by

$$y = YZ_o/2, \tag{3.4}$$

$$z = Z/2Z_o, \tag{3.5}$$

$$\Delta z_o = (Z_{o1} - Z_{o2})/2Z_o. \tag{3.6}$$

The characteristic impedance Z_o in (3.4)-(3.6) is the nominal value for the transmission line. Z_{o1} and Z_{o2} are the characteristic impedances at the reference planes a_1 and a_2 of the 2-port representing the connector pair. If the junction is represented by a T-network, Z in (3.5) is the sum of the two series elements of the T-network. The same equations are obtained if the junction is represented by a π-network. In that case Y in (3.4) is the sum of the two shunt elements. In either case the results show that one series impedance and one shunt admittance are adequate to represent the imperfections at the junction. In deriving (3.3) we have assumed that

$$|2yz| \ll 1, \quad |2y\Delta z_o| \ll 1, \quad |2z\Delta z_o| \ll 1, \quad \text{and} \quad |2\Delta z_o^2| \ll 1, \tag{3.7}$$

which are true for all connectors in reasonably good condition.

3.3. Reference Plane

The choice of reference plane is not unique. For the application where a network analyzer is being calibrated with a length of line as the standard, the reference plane is chosen as shown at the bottom of figure 2. This lumps all of the discontinuity into the test port where it can be calibrated out [11].

After calibration, if a termination with a connector that is identical to the standard line connector is measured, the matrix $\underline{C}$ is the same as before and none of the discontinuity is attributed to the device under test (DUT). The discontinuity is calibrated out.

If a termination with a connector different from the line connector is measured, the matrix $\underline{C}$ changes to a different matrix $\underline{C}' = \underline{C} \cdot \underline{\Delta C}$. The matrix $\underline{\Delta C}$ contains this difference and appears as part of the DUT. The original discontinuity remains calibrated out as part of the test port and creates no error.

4. Microcalorimeter Reference Plane for Power

Let figures 3a,b,c represent the connection when the standard mount is being calibrated in the microcalorimeter (MC) to determine its effective efficiency η_s.

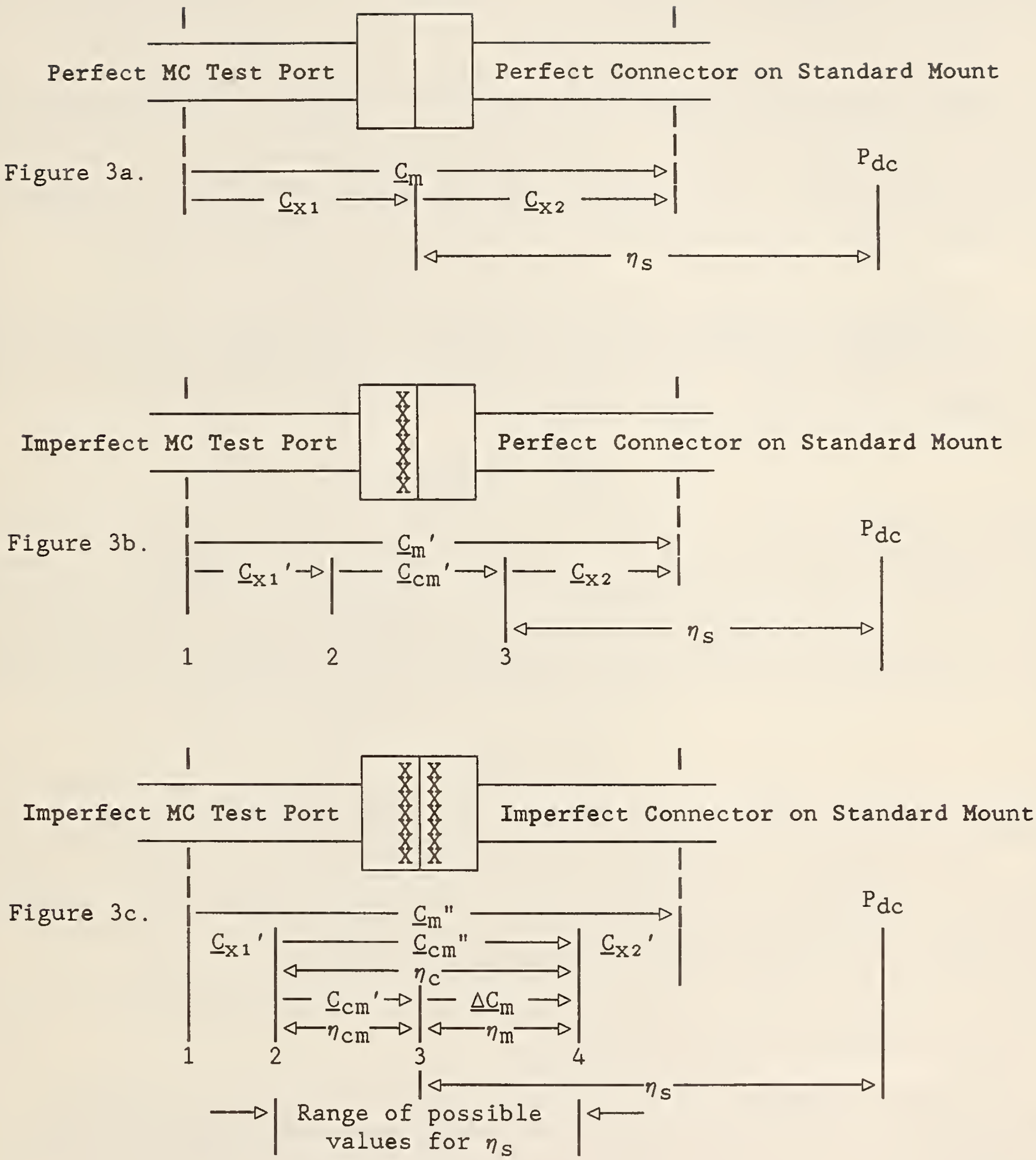


Figure 3. Equivalent circuit for the connector pair consisting of the microcalorimeter test port and the connector on the standard mount for different conditions.

In figure 3a, assume that the connector on the microcalorimeter test port and the connector on the power standard are both perfect and lossless. The cascading matrix $\underline{C}_m$ representing the connector pair is then simply $\underline{C}_{x1}$ times $\underline{C}_{x2}$, and the measurement plane is clearly between the 2-ports represented by these two matrices.

In figure 3b, assume that the connector on the microcalorimeter test port is imperfect but the connector on the power standard is still perfect. Let $\underline{C}_m'$ be the cascading matrix representing the connector pair, with component matrices $\underline{C}_{x1}'$, $\underline{C}_{cm}'$, and $\underline{C}_{x2}$. The matrix $\underline{C}_{cm}'$ represents all imperfections near the junction. The measurement plane is chosen as plane 3 between $\underline{C}_{cm}'$ and $\underline{C}_{x2}$, so all of the imperfections in the connector pair are lumped into the test port which, in this case, is the source of the imperfections.

In figure 3c, assume that the connector on the microcalorimeter test port and the connector on the power standard are both imperfect. Let $\underline{C}_m''$ be the cascading matrix representing the connector pair, with component matrices $\underline{C}_{x1}'$, $\underline{C}_{cm}''$, and $\underline{C}_{x2}'$. The matrix $\underline{C}_{cm}''$ includes the imperfections in both connectors near the junction. Setting

$$\underline{C}_{cm}'' = \underline{C}_{cm}'\underline{\Delta C}_m \tag{4.1}$$

makes reference plane 3 in figure 3c the same as reference plane 3 in figure 3b which corresponds to that for a mount with a perfect connector. Therefore, 3 is probably the best choice for the power reference plane because everything to the left of this reference plane is associated with only the imperfect test port. Let η_c be the efficiency of the 2-port connector junction represented by $\underline{C}_{cm}''$, and let η_{cm} and η_m be the efficiencies corresponding to $\underline{C}_{cm}'$ and $\underline{\Delta C}_m$, so

$$\eta_c = \eta_{cm}\eta_m. \tag{4.2}$$

Then η_{cm} becomes part of the test port, and η_m becomes part of the mount. However, it is usually not possible to separate η_c into the components η_{cm} and η_m. The efficiency η_c of the 2-port connector junction causes a systematic error in determining η_s because it is not clear how much of η_c should be assigned to the mount being calibrated. η_c then becomes a systematic error associated with the microcalorimeter measurement of η_s.

5. <u>Six-Port Reference Plane for Power.</u>

We next determine where the 6-port reference plane is located for power measurements. To do this, several different cases will be considered depending on the quality of the connectors on the line standard. In all of these cases, the connector on the test port of the 6-port and the connector on the power standard will be assumed to be imperfect.

Before the 6-port can be calibrated for power measurements, it must be calibrated for Γ measurements to obtain the "error box" parameters A,B,C.

5.1. Calibrating the Six-Port for Γ Measurements With a Perfect Line.

In figure 4a, assume that the connector on the line standard is perfect. This situation is similar to that shown in figure 3b. Let $\underline{C}_s$ be the cascading matrix representing the connector pair, with component matrices $\underline{C}_{x1}$, $\underline{C}_{cs}$, and $\underline{C}_{x2s}$. The matrix $\underline{C}_{cs}$ includes all imperfections near the junction. The measurement plane is chosen as plane 3 between $\underline{C}_{cs}$ and $\underline{C}_{x2s}$, so all of the imperfections in the connector pair are lumped into the test port which, in this case, is the source of the imperfections.

Since Γ and the error box parameters A, B, C refer to reference plane 3, the 6-port reference plane for power must also be at 3.

Figure 4a.

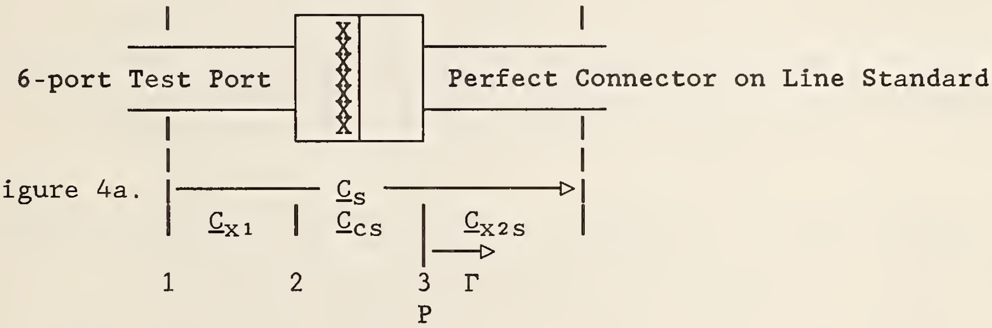


Figure 4b.

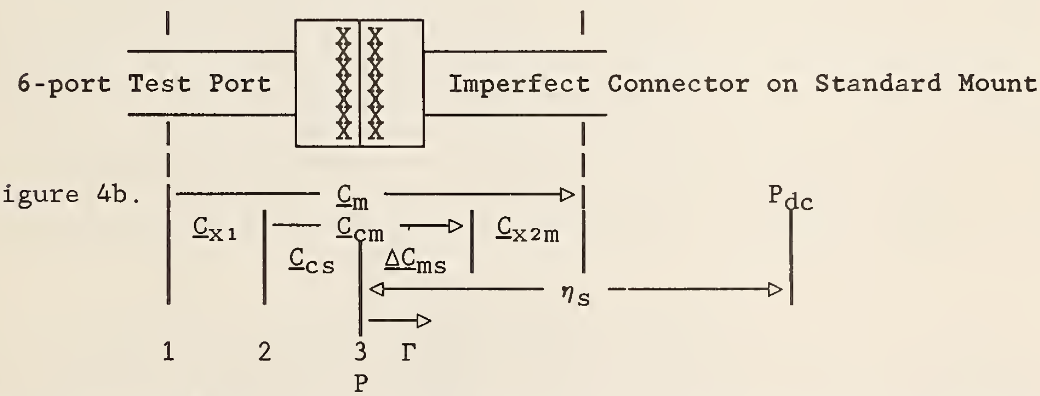


Figure 4. Equivalent circuit for the connector pair consisting of the 6-port test port and either the line connector or the connector on the standard mount.

5.2. Calibrating the Six-Port for Power Measurements Following Calibration for Γ With a Perfect Line Standard.

In figure 4b, assume that the connector on the power standard is imperfect. Let $\underline{C}_m$ be the cascading matrix representing the connector pair, with component matrices $\underline{C}_{x1}$, $\underline{C}_{cm}$, and $\underline{C}_{x2m}$. Setting

$$\underline{C}_{cm} = \underline{C}_{cs} \cdot \Delta\underline{C}_{ms} \tag{5.1}$$

makes it possible to create reference plane 3 in figure 4b so that everything to the left of 3 is the same as that in figure 4a. Since reference plane 3 is the correct reference plane for measuring P and Γ, the effective efficiency η_s of the transfer standard mount must be assigned to this reference plane. The effective efficiency η of other mounts will be measured at this reference plane relative to η_s.

The matrix $\underline{\Delta C}_{ms}$ represents the difference between the perfect connector on the line standard and the imperfect connector on the power standard. This difference becomes part of the Γ of the power standard.

5.3. Calibrating the Six-Port for Γ Measurements With an Imperfect Line.

In this Section we consider the general case shown in figure 5 where the connectors on the line standard and on the power standard are both imperfect. Let the cascading matrix $\underline{C}_\ell$ in figure 5a represent the connection when an imperfect line is used in the calibration, and let the components of this matrix be $\underline{C}_{x1}$, $\underline{C}_{c\ell}$, and $\underline{C}_{x2\ell}$.

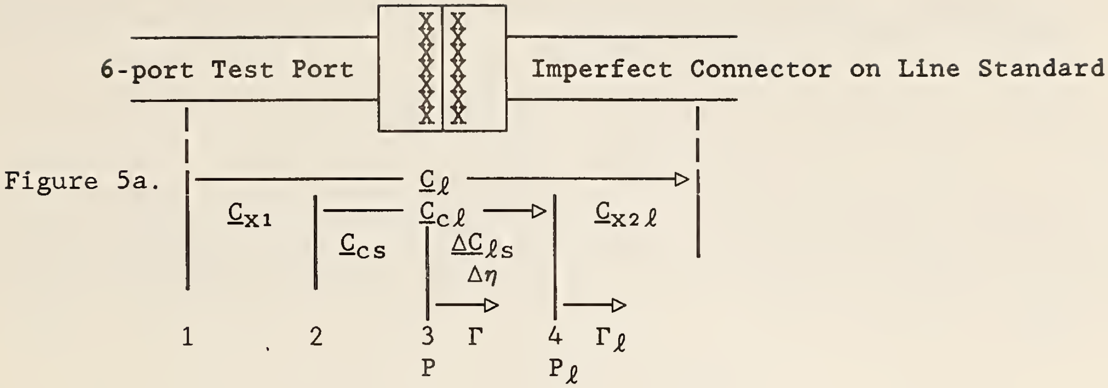
For a line standard with perfect connectors such as shown in figure 4a, the reference plane was chosen between the connector junction represented by matrix $\underline{C}_{cs}$ and the uniform line represented by the matrix $\underline{C}_{x2s}$. In the 6-port calibration, the line standard is always assumed to have perfect connectors, and the reference plane is always taken to be between the matrix representing the connector junction and the matrix representing the uniform line. This is reference plane 4 in figure 5a, between the matrices $\underline{C}_{c\ell}$ and $\underline{C}_{x2\ell}$. This choice for the reference plane lumps all of the imperfections associated with the test port and the line connector into the test port. The imperfections in the line connector create errors that we need to analyze and account for. Imperfections in the test port are still calibrated out.

Let P_ℓ and Γ_ℓ be the net power and reflection coefficient measured by the 6-port at reference plane 4. If we let

$$\underline{C}_{c\ell} = \underline{C}_{cs} \cdot \underline{\Delta C}_{\ell s}, \tag{5.2}$$

then everything to the left of reference plane 3 in figure 5a is the same as everything to the left of reference plane 3 in figure 4a. P and Γ at reference plane 3 are those values obtained after calibrating the 6-port with a line having perfect connectors. Therefore 3 is the correct reference plane in figure 5a for measuring power and reflection coefficient. This means that Γ_ℓ is in error by $\underline{\Delta C}_{\ell s}$, and P_ℓ is in error by $\Delta\eta$, where $\Delta\eta$ is the efficiency of the 2-port whose cascading matrix is $\underline{\Delta C}_{\ell s}$:

$$\Delta\eta = P_\ell / P. \tag{5.3}$$



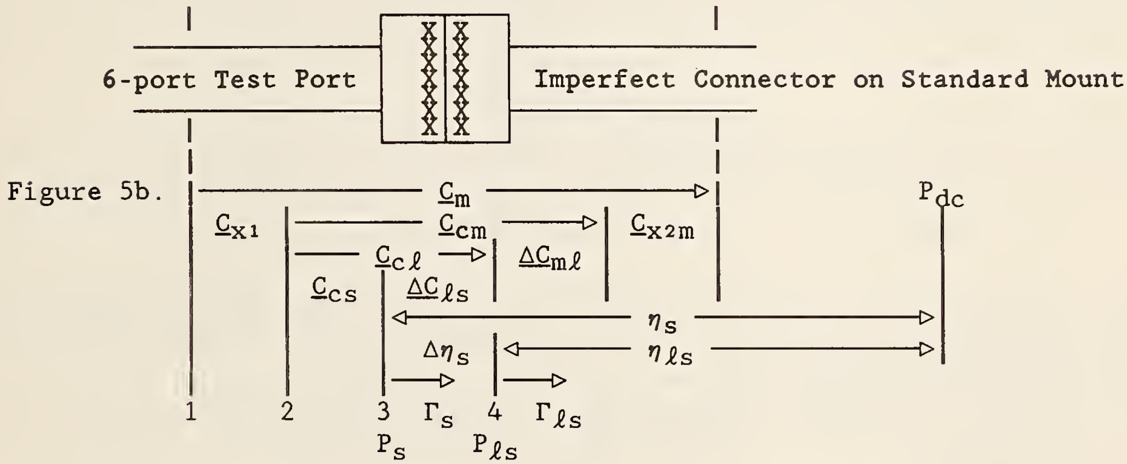


Figure 5. Equivalent circuit for the connector pair consisting of the 6-port test port and either the line connector or the connector on the standard mount.

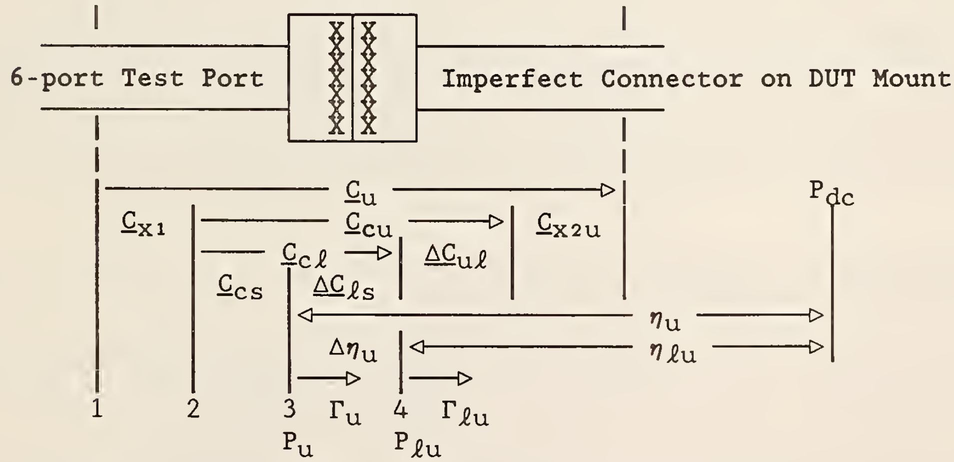


Figure 6. Equivalent circuit for the connector pair consisting of the 6-port test port and the connector on the DUT mount.

5.4. Calibrating the Six-Port for Power Measurements Following Calibration for Γ With an Imperfect Line Standard.

Figure 5b shows the situation when a power standard with an imperfect connector is connected to the test port. This situation is the same as that shown in figure 4b, where $\underline{C}_m$ is the cascading matrix representing the connector pair, with component matrices $\underline{C}_{x1}$, $\underline{C}_{cm}$, and $\underline{C}_{x2m}$. In figure 5b, $\underline{C}_{cm}$ is first set equal to $\underline{C}_{c\ell}\cdot\underline{\Delta C}_{m\ell}$, and then $\underline{C}_{c\ell}$ is set equal to $\underline{C}_{cs}\cdot\underline{\Delta C}_{\ell s}$ to make reference planes 3 and 4 in figure 5b the same as reference planes 3 and 4 in figure 5a. The η_s of the power standard still refers to reference plane 3 as in figure 4b, since that is the correct plane of measurement, and P at reference plane 3 is the correct power.

Let η_ℓ be the effective efficiency measured by the 6-port at reference plane 4:

$$\eta_\ell = \frac{P_{dc}}{P_\ell}. \tag{5.4}$$

Then the correct effective efficiency η at reference plane 3 can be written

$$\eta = \frac{P_{dc}}{P} = \frac{P_{dc}}{P_\ell}\frac{P_\ell}{P} = \eta_\ell \Delta\eta. \tag{5.5}$$

This equation says that the correct effective efficiency is equal to the measured effective efficiency times an error term $\Delta\eta$. For a line standard with perfect connectors, $\Delta\eta = 1$.

Adding a subscript "s" in (5.5) to represent the power standard, we get the relationship

$$\eta_s = \eta_{\ell s}\Delta\eta_s \tag{5.6}$$

between the effective efficiency, η_s, of the power standard at reference plane 3 and the measured effective efficiency, $\eta_{\ell s}$, at reference plane 4.

Choosing reference plane 3 as the correct reference plane makes the effective efficiency at that plane independent of the test port on the 6-port, independent of the connector on the line standard, and independent of the connector on the power standard. The systematic error in the measured power P_ℓ and effective efficiency η_ℓ can be determined by estimating $\Delta\eta_s$ and $\Delta\eta_u$ as discussed later.

5.5. Measuring η_u of a DUT Power Detector

Let figure 6 represent the general situation when the effective efficiency η_u of a power detector is measured following a 6-port calibration with an imperfect line and an imperfect standard mount.

The situation is the same as in figure 5b except that the connector on the device under test is different from that on the standard mount, so subscripts "m" and "s" are changed to "u" to represent the DUT. Adding a subscript "u" in (5.5) to represent the device under test, we write

$$\eta_u = \eta_{\ell u}\Delta\eta_u. \tag{5.7}$$

From (5.6), (5.7), and (2.9), the ratio of η_u to η_s is

$$\frac{\eta_u}{\eta_s} = \frac{\eta_{\ell u}\Delta\eta_u}{\eta_{\ell s}\Delta\eta_s} = \frac{N_{\ell u}}{N_{\ell s}} \frac{\Delta\eta_u}{\Delta\eta_s}, \tag{5.8}$$

where $N_{\ell u}$ and $N_{\ell s}$ are obtained from (2.6) and (2.8). They are repeated below with the subscript ℓ added to emphasize that they refer to reference plane 4 and are calculated from actual measured values of Γ_ℓ at reference plane 4.

$$N_{\ell u} \equiv \frac{P_{dcu}}{P_{ou}} \frac{|1 + C_\ell \Gamma_{\ell u}|^2}{1 - |\Gamma_{\ell u}|^2}, \tag{5.9}$$

and

$$N_{\ell s} \equiv \frac{P_{dcs}}{P_{os}} \frac{|1 + C_\ell \Gamma_{\ell s}|^2}{1 - |\Gamma_{\ell s}|^2}. \tag{5.10}$$

We now need to determine $\Delta\eta_u/\Delta\eta_s$ which is the error in measuring η_u/η_s.

6. Efficiency of a 2-Port.

The error term $\Delta\eta$ is the efficiency of a 2-port whose cascading matrix is $\underline{\Delta C}_{\ell s}$. The efficiency of any 2-port whose reference planes are designated 1 and 2, when energy is fed into arm 1, is given by [9]

$$\eta_1 = \frac{P_L}{P_1} = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 - |S_{11} - (S_{11}S_{22} - S_{12}S_{21})\Gamma_L|^2}, \tag{6.1}$$

where Γ_L is the reflection coefficient of the termination on port 2, and the S_{ij} are the scattering parameters of the 2-port. In terms of the normalized cascading parameters, the efficiency of this 2-port is

$$\eta_1 = \frac{|d|^2(1 - |\Gamma_L|^2)}{|1 + c\Gamma_L|^2 - |b + a\Gamma_L|^2}, \tag{6.2}$$

where a, b, c, and d are the normalized cascading parameters defined by

$$\begin{aligned} a &\equiv r_{11}/r_{22} = -(S_{11}S_{22} - S_{12}S_{21}) = -\det(\underline{S}), \\ b &\equiv r_{12}/r_{22} = S_{11}, \\ c &\equiv r_{21}/r_{22} = -S_{22}, \\ d &\equiv 1/r_{22} = S_{21}, \end{aligned} \tag{6.3}$$

and the r_{ij} are the cascading parameters of the 2-port.

7. Efficiency of the Error 2-port.

The efficiency, $\Delta\eta$, of the "error" 2-port between reference planes 3 and 4 in figures 5 and 6 can be obtained from (6.2) using the cascading parameters in the matrix $\underline{\Delta C}_{\ell s}$ in place of the a,b,c,d. From (5.2),

$$\underline{\Delta C}_{\ell s} = \underline{C}_{cs}^{-1} \cdot \underline{C}_{c\ell}. \tag{7.1}$$

Both $\underline{C}_{cs}$ and $\underline{C}_{c\ell}$ can be represented by the matrix in (3.3) if subscript "s" or "ℓ" is added to y, z, and Δz_o. Since the assumptions in (3.7) are satisfied, substituting (3.3) into (7.1) leads to

$$\underline{\Delta C}_{c\ell s} = \begin{bmatrix} 1-\Delta y-\Delta z & \Delta z-\Delta y \\ \Delta y-\Delta z & 1+\Delta y+\Delta z \end{bmatrix} + \Delta z_o \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{7.2}$$

where

$$\Delta y \equiv y_\ell - y_s, \quad \Delta z \equiv z_\ell - z_s, \tag{7.3}$$

and

$$\Delta z_o \equiv \Delta z_{o\ell} - \Delta z_{os} = (Z_{os} - Z_{o\ell})/2Z_o. \tag{7.4}$$

In (7.4), Z_{os} is the characteristic impedance of a perfectly uniform line standard having diameters as specified in some standards document,[2] and $Z_{o\ell}$ is the characteristic impedance of the actual line used to calibrate the 6-port.

Comparing (7.2) with (6.3) defines the a,b,c,d parameters for the error 2-port. It will be more convenient to redefine the a,b,c,d parameters of the error 2-port as 1+Δa, Δb, Δc, 1+Δd. Then from (7.2) and (6.3),

$$\begin{aligned} a &\equiv 1+\Delta a = (1 - \Delta y - \Delta z + \Delta z_o)/(1 + \Delta y + \Delta z + \Delta z_o) \simeq 1 - 2\Delta y - 2\Delta z, \\ b &\equiv \Delta b = (\Delta z - \Delta y - \Delta z_o)/(1 + \Delta y + \Delta z + \Delta z_o) \simeq \Delta z - \Delta y - \Delta z_o, \\ c &\equiv \Delta c = (\Delta y - \Delta z - \Delta z_o)/(1 + \Delta y + \Delta z + \Delta z_o) \simeq \Delta y - \Delta z - \Delta z_o, \\ d &\equiv 1+\Delta d = 1/(1 + \Delta y + \Delta z + \Delta z_o) \simeq 1 - \Delta y - \Delta z - \Delta z_o. \end{aligned} \tag{7.5}$$

The approximations in (7.5) are valid when (3.7) is satisfied. Substituting these parameters into (6.2) gives the desired expression for $\Delta\eta$:

$$\Delta\eta = \frac{|1+\Delta d|^2 (1 - |\Gamma_\ell|^2)}{|1 + \Delta c \Gamma_\ell|^2 - |\Delta b + (1+\Delta a)\Gamma_\ell|^2}. \tag{7.6}$$

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[2]The outer and inner conductor diameters of a rigid precision 7 mm coaxial air line standard are specified to be 7.000 and 3.040 mm respectively. The conductivity of the material and the characteristic impedance of the line are not specified. See:

IEC Publication 457-2: Rigid precision coaxial lines and their associated precision connectors, 1974, pp. 5 and 7.

IEEE Standard for Precision Coaxial Connectors (IEEE Standards Publication No.287), IEEE Transactions on Instrumentation and Measurement, Vol IM-17, Sept 1968, pp. 204-222.

For the ideal case, $\Delta a = \Delta b = \Delta c = \Delta d = 0$ and $\Delta\eta = 1$. Typical values are given in the next Section.

If a second subscript s or u is added to designate the standard mount or a DUT mount, (7.6) gives

$$\frac{\Delta\eta_u}{\Delta\eta_s} = \frac{(1 - |\Gamma_{\ell u}|^2)(\,|1 + \Delta c\Gamma_{\ell s}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell s}|^2)}{(1 - |\Gamma_{\ell s}|^2)(\,|1 + \Delta c\Gamma_{\ell u}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell u}|^2)} \tag{7.7}$$

$$= 1 + \epsilon, \tag{7.8}$$

where ϵ is the error in the ratio of the effective efficiencies due to imperfect impedance standards and connectors. Equation (7.7) shows that $\epsilon = 0$ when $\Gamma_{\ell u} = \Gamma_{\ell s}$ or when $\Delta a = \Delta b = \Delta c = 0$. If Δa, Δb, and Δc are known, ϵ becomes a correction to the ratio of the measured effective efficiencies. However Δa, Δb, and Δc are usually unknown, and ϵ is used to estimate the systematic error in the ratio.

8. Approximation for ϵ.

Let the real and imaginary components of Δy, Δz, Δz_o, and Γ_ℓ be

$$\begin{aligned} \Delta y &\equiv \Delta G + j\Delta B \simeq j\Delta B, \\ \Delta z &\equiv \Delta r + j\Delta x, \\ \Delta z_o &\equiv \Delta r_o + j\Delta x_o, \\ \Gamma_\ell &\equiv \Gamma_r + j\Gamma_x. \end{aligned} \tag{8.1}$$

Then the real and imaginary components of Δa, Δb, Δc in (7.5) become

$$\begin{aligned} \Delta a &\equiv \Delta a_r + j\Delta a_x \simeq (-2\Delta r) + j(-2\Delta B - 2\Delta x), \\ \Delta b &\equiv \Delta b_r + j\Delta b_x \simeq (\Delta r - \Delta r_o) + j(\Delta x - \Delta B - \Delta x_o), \\ \Delta c &\equiv \Delta c_r + j\Delta c_x \simeq (-\Delta r - \Delta r_o) + j(\Delta B - \Delta x - \Delta x_o). \end{aligned} \tag{8.2}$$

Appendix A shows that substituting these expressions into (7.7) leads to the following approximate equation for ϵ:

$$\epsilon \simeq 4[\Delta\Gamma_r\Delta r - \Delta\Gamma_x\Delta x_o - \Delta|\Gamma|^2\Delta r], \tag{8.3}$$

where

$$\Delta\Gamma_r \equiv \Gamma_{\ell ur} - \Gamma_{\ell sr}, \tag{8.4}$$

$$\Delta\Gamma_x \equiv \Gamma_{\ell ux} - \Gamma_{\ell sx}, \tag{8.5}$$

$$\Delta|\Gamma|^2 \equiv |\Gamma_{\ell u}|^2 - |\Gamma_{\ell s}|^2. \tag{8.6}$$

Estimates of Δr and Δx_o for the NIST 7 mm lines are shown in figure 7. These estimates and the following expressions in Table 1 for Δy, Δz, and Δz_o are taken from Table 6 of Part 4 of reference 8. When the values of Δy, Δz, and Δz_o at 18 GHz from Table 1 are substituted into (8.2), they lead to the following worst-case errors for Δa, Δb, and Δc:

$$|\Delta a| \simeq 0.0037, \quad |\Delta b| \simeq 0.0021, \quad |\Delta c| \simeq 0.0021. \tag{8.10}$$

Table 1

			1 GHz	18 GHz	
$\Delta y \equiv \Delta G + j\Delta B;$	$\Delta G \simeq 0,$				
	$\Delta B \simeq$	$\pm 0.000032f.$	± 0.00003	± 0.00058	(8.7)
$\Delta z \equiv \Delta r + j\Delta x;$	$\Delta r \simeq -0,$	$+0.00007\sqrt{f}.$	0.00007	0.00030	(8.8)
	$\Delta x \simeq -0,$	$+0.00007f.$	0.00007	0.00126	
$\Delta z_o \equiv \Delta r_o + j\Delta x_o;$	$\Delta r_o \simeq$	$\pm 0.00061 \pm 0.0000\overline{4}/\sqrt{f}.$	± 0.00065	± 0.00062	(8.9)
	$\Delta x_o \simeq$	$\pm 0.00004/\sqrt{f}.$	± 0.00004	± 0.00001	

where f is the frequency in GHz.

9. Measured Power Constant

Substituting (7.8) into (5.8) leads to

$$\eta_u = \frac{\eta_s}{N_{\ell s}} N_{\ell u}(1 + \epsilon). \tag{9.1}$$

Comparing this with (2.5) and (2.7) suggests that we write (9.1) as

$$\eta_u = K_{\ell p} N_{\ell u}(1 + \epsilon), \tag{9.2}$$

where

$$K_{\ell p} \equiv \frac{\eta_s}{N_{\ell s}} \tag{9.3}$$

is the effective or measured power constant that refers to reference plane 4 in figure 6. $K_{\ell p}$ differs from K_p when $\Gamma_{\ell s} \neq \Gamma_s$.

10. Total Systematic Error in η_u

Equation (9.1) or (9.2) is used to calculate η_u. Differentiating (9.1) gives the following expression for the total systematic error $d\eta_u$ in η_u:

$$d\eta_u = \frac{\partial \eta_u}{\partial \eta_s} d\eta_s + \frac{\partial \eta_u}{\partial (N_{\ell u}/N_{\ell s})} d(N_{\ell u}/N_{\ell s}) + \frac{\partial \eta_u}{\partial \epsilon} d\epsilon, \tag{10.1}$$

which reduces to

$$\frac{d\eta_u}{\eta_u} = \frac{d\eta_s}{\eta_s} + \frac{d(N_{\ell u}/N_{\ell s})}{N_{\ell u}/N_{\ell s}} + \frac{d\epsilon}{1+\epsilon}. \tag{10.2}$$

The term $d\eta_s$ is the total systematic error in determining η_s in the microcalorimeter. For the second term on the right of (10.2) we need the ratio $N_{\ell u}/N_{\ell s}$ which from (5.9) and (5.10) is

$$\frac{N_{\ell u}}{N_{\ell s}} = \frac{P_{dcu}}{P_{dcs}} \frac{P_{os}}{P_{ou}} \frac{|1 + c\Gamma_{\ell u}|^2}{|1 + c\Gamma_{\ell s}|^2} \frac{1 - |\Gamma_{\ell s}|^2}{1 - |\Gamma_{\ell u}|^2}. \tag{10.3}$$

Differentiating this gives

$$\frac{d(N_{\ell u}/N_{\ell s})}{N_{\ell u}/N_{\ell s}} = \frac{d(P_{dcu}/P_{dcs})}{P_{dcu}/P_{dcs}} + \frac{d(P_{os}/P_{ou})}{P_{os}/P_{ou}} + \frac{d|G|^2}{|G|^2} + \frac{dM}{M}, \tag{10.4}$$

where

$$G \equiv \frac{1 + c\Gamma_u}{1 + c\Gamma_s}, \tag{10.5}$$

and

$$M \equiv \frac{1 - |\Gamma_s|^2}{1 - |\Gamma_u|^2}. \tag{10.6}$$

Each of the error terms in (10.2) and (10.4) will now be evaluated to obtain an estimate of the total systematic error in measuring power.

10.1. <u>Error in dc Power Ratio</u>

The error in the dc power ratio, the first term on the right in (10.4), is given by

$$\frac{d(P_{dcu}/P_{dcs})}{(P_{dcu}/P_{dcs})} = \frac{dP_{dcu}}{P_{dcu}} - \frac{dP_{dcs}}{P_{dcs}}. \tag{10.7}$$

For the NIST dual 6-port ANA, P_{dcu} and P_{dcs} are both measured by the same NIST type IV power meter which is connected through a scanner to a 5½ digit DVM. The power meter voltages can be read with or without a reference voltage generator (RVG) in series with the DVM as shown in figure 8. The RVG enables the DVM to operate on its optimum range for measuring small changes in voltage, reducing the effects of the percent-of-fullscale error in the DVM.

Figure 9 shows the relative error[3] dP_{dc}/P_{dc} expressed in percent when the power is measured with a NIST type IV power meter and a HP3457 DVM, with and without an RVG. These curves show that the error in P_{dc} due to the power meter is negligible, and that most of the error is due to the DVM. If two different DVMs are used to measure P_{dcu} and P_{dcs}, the two terms on the right of (10.7) can add, creating an error as large as 0.06% at 10 mW, and larger at lower power levels.

In the NIST system, P_{dcu} and P_{dcs} are read by the same power meter and DVM, and $P_{dcu} \approx P_{dcs}$. The systematic errors dP_{dcu} and dP_{dcs} are therefore approximately the same, and the terms on the right of (10.7) cancel. Setting

$$dP_{dcu} = dP_{dcs} \equiv dP_{dc} \tag{10.8}$$

reduces (10.7) to

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[3]Obtained from a computer program written by Neil Larsen, based on the error analysis developed in his paper "A New Self-Balancing DC-Substitution RF Power Meter," IEEE Transactions on Instrumentation and Measurement, vol. IM-25, Dec. 1976, pp. 343-347.

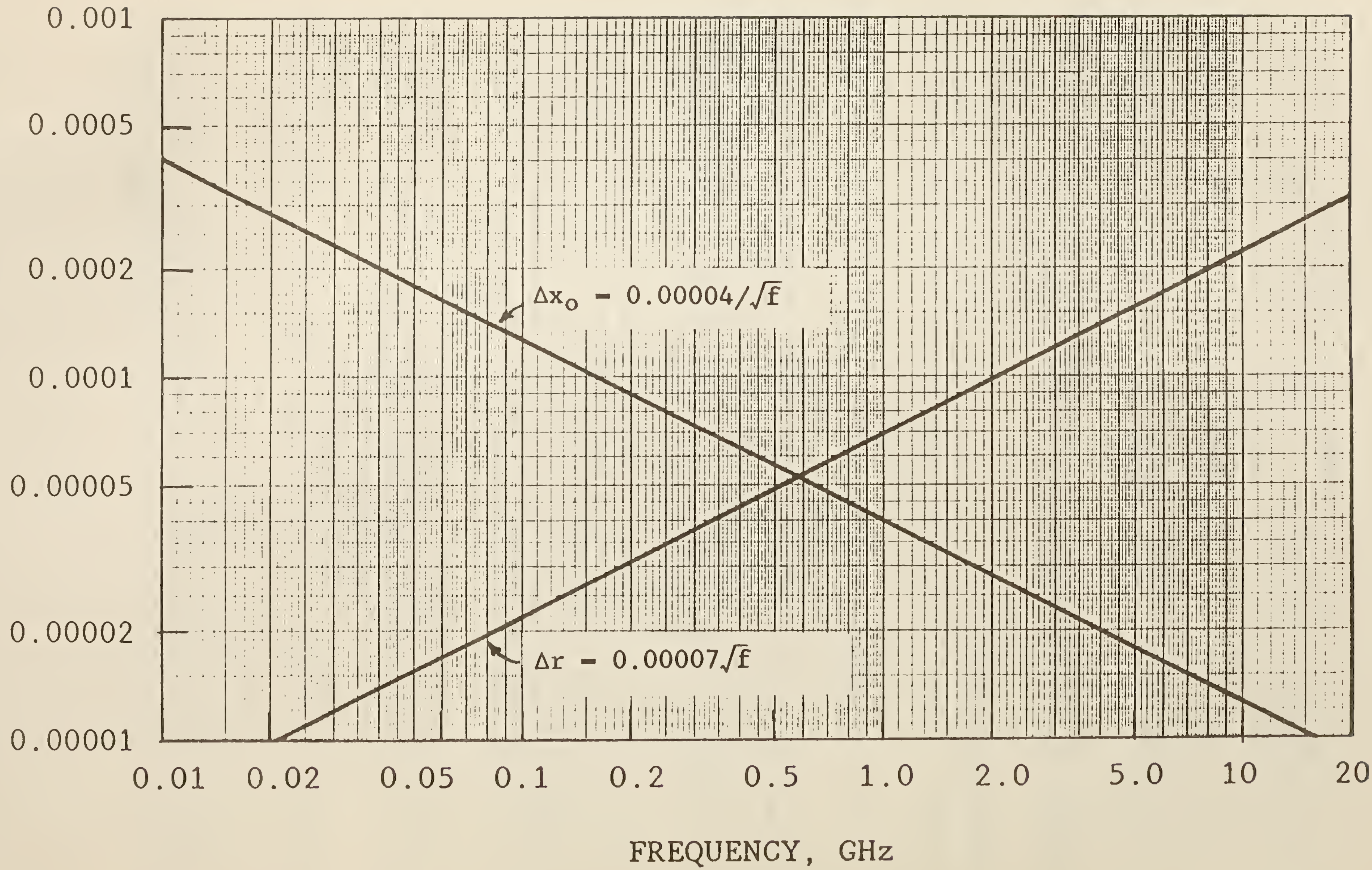

Figure 7. Average values of Δr and Δx_o for NBS 7 mm lines, for use in calculating error in effective efficiency from equation (8.3).

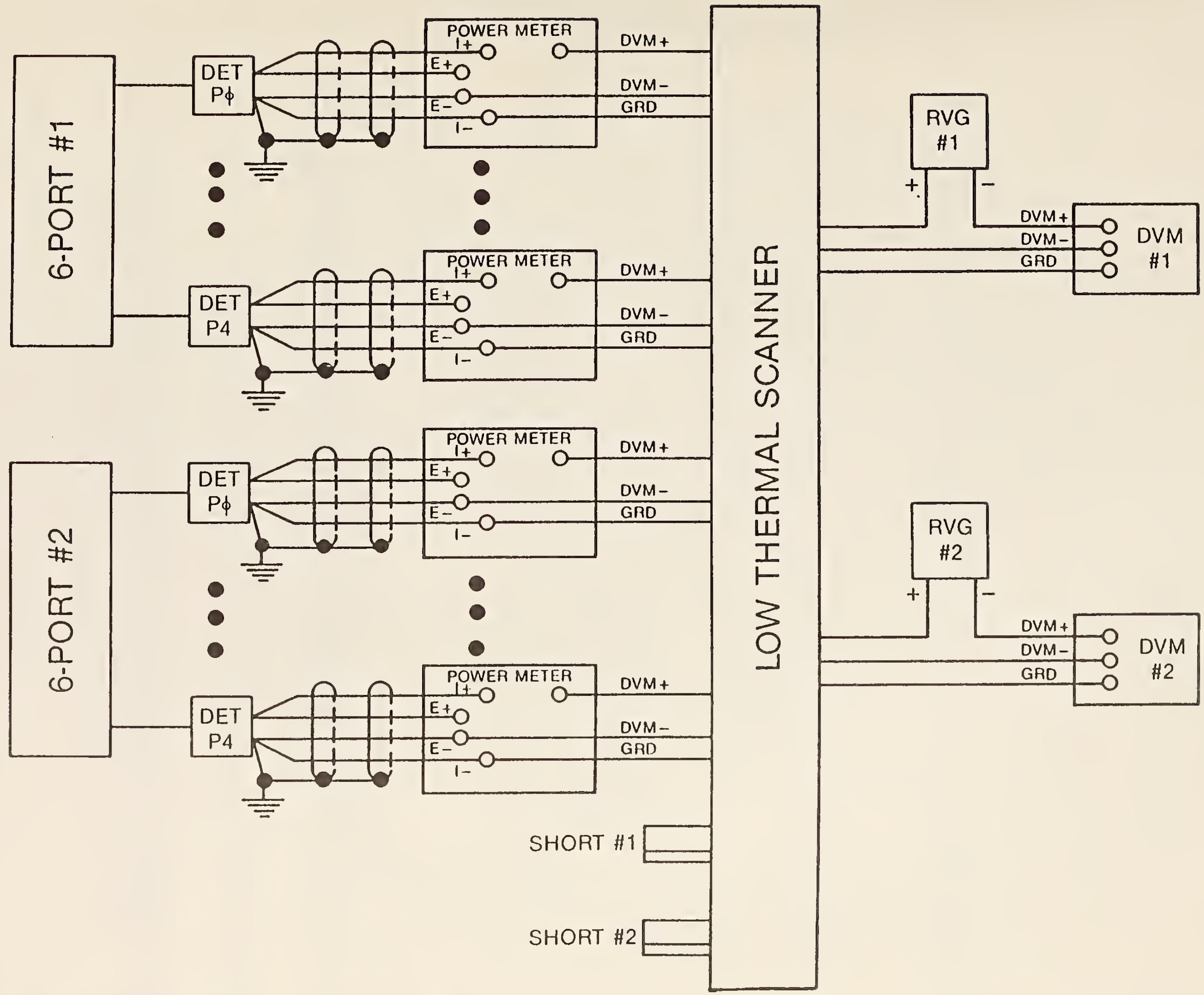

Figure 8. Block diagram showing how an RVG (Reference Voltage Generator) can be used in series with the DVM to enable the DVM to measure the voltages on its most sensitive range, increasing the resolution and decreasing the error in measuring power.

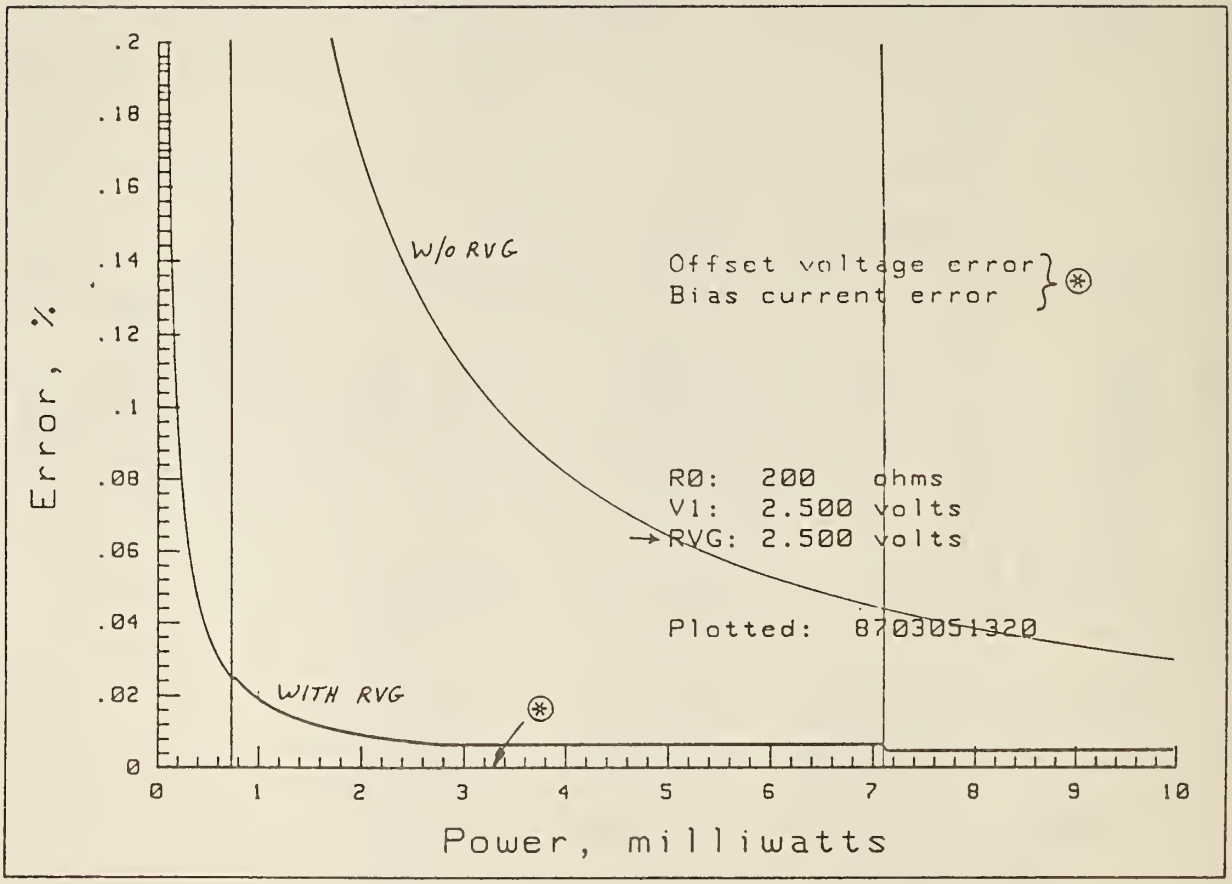

Figure 9. Relative error dP_{dc}/P_{dc} expressed in percent when the dc power is measured with an NIST type IV power meter and a 5½ digit DVM, with or without a Reference Voltage Generator (RVG).

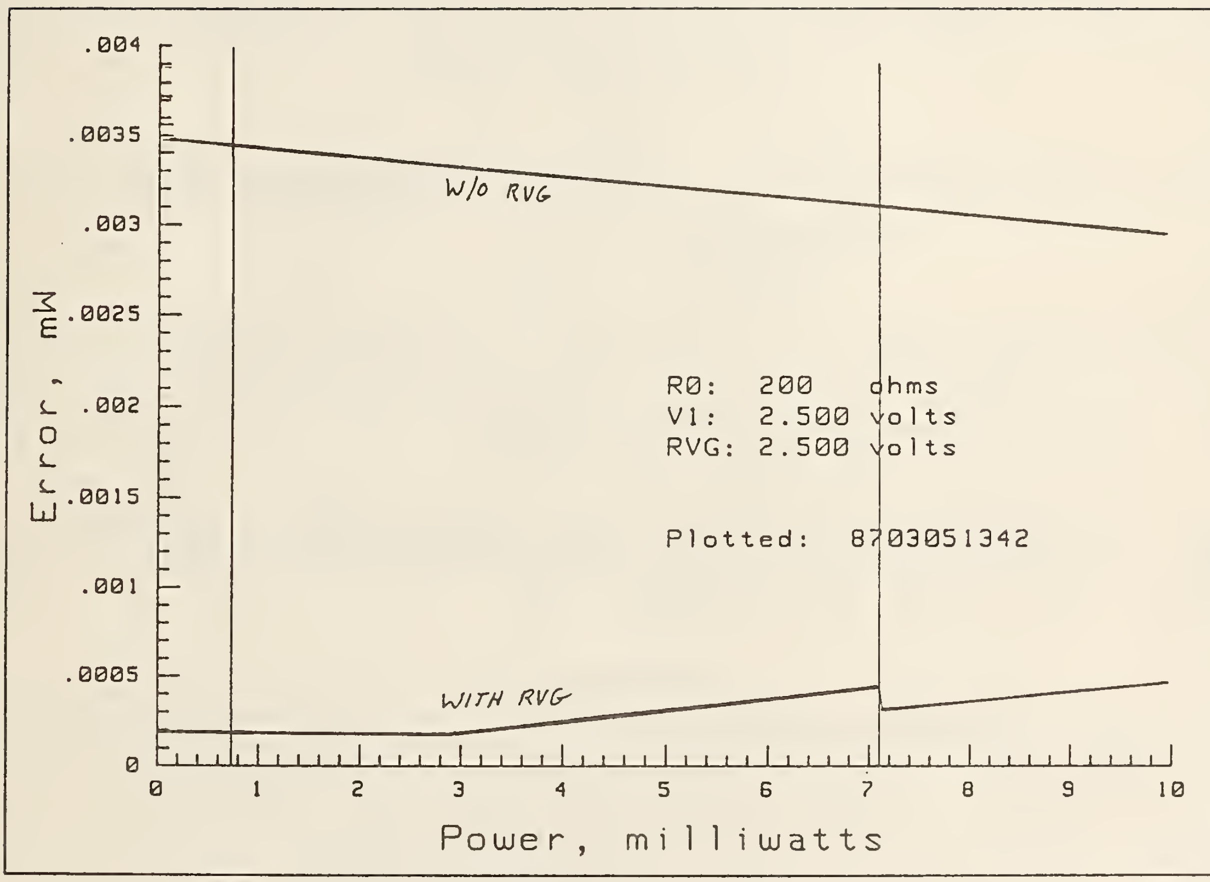

Figure 10. Uncertainty dP_{dc} in the dc power measured with an NIST type IV power meter and a 5½ digit DVM, with or without a Reference Voltage Generator (RVG).

$$\frac{d(P_{dcu}/P_{dcs})}{(P_{dcu}/P_{dcs})} = \frac{(P_{dcs} - P_{dcu})dP_{dc}}{P_{dcu}P_{dcs}}. \qquad (10.9)$$

The uncertainty dP_{dc} in the dc power is shown in figure 10. The error without an RVG is given by

$$dP_{dc} \simeq \pm(0.0035 - 0.00005P_{dc}), \text{ mW}. \qquad (10.10)$$

where dP_{dc} and P_{dc} are in milliwatts. This uncertainty assumes that the power-off voltage V_1 is $\simeq$ 2.5 V. As V_1 varies between 2.4 V and 2.6 V, the first constant in (10.10) varies from 0.00321 to 0.00375. Typically P_{dcs} and P_{dcu} differ by no more than 0.5 mW in the NIST system. Assuming that

$$P_{dcu} \simeq P_{dcs} \simeq 10 \text{ mW}, \qquad (10.11)$$

$$|P_{dcs} - P_{dcu}| < 0.5 \text{ mW}, \qquad (10.12)$$

and that dP_{dc} is given by (10.10), (10.9) gives a worst-case uncertainty of

$$\frac{d(P_{dcu}/P_{dcs})}{(P_{dcu}/P_{dcs})} = \pm 0.00002. \qquad (10.13)$$

This uncertainty is reduced by a factor of 6 to ±0.000003 if an RVG is used.

10.2. Error in rf Power Ratio at Six-Port Sidearm

The second term on the right side of (10.4) is the error in the ratio of the rf power measured at the 6-port reference sidearm. Each sidearm of the 6-port is terminated with a thermistor mount which is used to measure the rf power in terms of the dc substituted power. The net rf power P_o at the reference port sidearm can be written

$$P_o = P_{odc}/\eta_o, \qquad (10.14)$$

where P_{odc} is the dc substituted power, and η_o is the effective efficiency of the sidearm mount. Normally, η_o does not need to be known since it cancels in the power ratios. For the NIST dual 6-port ANA, P_{odc} is measured by a NIST type IV power meter which is connected through a scanner to the DVM. From (10.14),

$$\frac{P_{os}}{P_{ou}} = \frac{\eta_{ou} P_{odcs}}{\eta_{os} P_{odcu}}. \qquad (10.15)$$

A subscript has been added to η_o to allow for nonlinearity in the sidearm power sensor. Nonlinearity in a dual element thermistor mount can be caused by differences in the two thermistor elements [10]. Taking derivatives of (10.15) leads to

$$\frac{d(P_{os}/P_{ou})}{P_{os}/P_{ou}} = \frac{d\eta_{ou}}{\eta_{ou}} - \frac{d\eta_{os}}{\eta_{os}} + \frac{dP_{odcs}}{P_{odcs}} - \frac{P_{odcu}}{P_{odcu}}. \qquad (10.16)$$

Since $P_{os} \simeq P_{ou}$, the last two terms in (10.16) can be combined in the same way as the last two terms in (10.7) were combined to give

$$\frac{dP_{odcs}}{P_{odcs}} - \frac{P_{odcu}}{P_{odcu}} = \frac{(P_{odcu} - P_{odcs})}{P_{odcs}P_{odcu}} dP_{dc}, \tag{10.17}$$

where P_{dc} is given by (10.10). In the NIST dual 6-port design, P_o is about half the power at the test port. With 10 mW at the test port, P_{os} and P_{ou} are both about 5 mW. Assuming that (10.12) is true we will have

$$|P_{odcu} - P_{odcs}| < 0.25 \text{ mW}. \tag{10.18}$$

Then (10.17) gives

$$\frac{dP_{odcs}}{P_{odcs}} - \frac{P_{odcu}}{P_{odcu}} = \pm 0.00004. \tag{10.19}$$

10.2.1. Error Due to Detector Nonlinearity.

The net rf power P_{rf} into a bolometric type power sensor can be written

$$P_{rf} = bP_{dc} + \Delta P = P_{dc}/\eta, \tag{10.20}$$

where P_{dc} is the dc substituted power, ΔP is the dc-rf substitution error at P_{rf}, $1/b$ is the effective efficiency of the mount at low rf power levels where ΔP approaches zero, and η is the effective efficiency at the rf power level P_{rf}.

Experimental data reported by Engen [10] indicate that the dual element dc-rf substitution error has the form

$$\frac{\Delta P}{P_{rf}} = K_\epsilon P_{rf}^\alpha, \tag{10.21}$$

where in the region from 5 to 10 mW where the 6-port sidearm thermistor mounts are operated the exponent α is not constant but increases from 1 to almost 2.

$$1 < \alpha < 2. \tag{10.22}$$

At an rf power level of 10 mW, Engen's tests on 10 pairs of elements in three different mounts indicated a worst-case error of 1%, and a typical error of 0.2%. More recent tests by Neil Larsen[4] at NIST on mounts similar to those used on the NIST dual 6-port indicated a dual element error of only 0.02% or less. Inconsistencies in the sidearm power readings of the NIST dual 6-port

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[4]Neil Larsen compared the output of a coaxial dual element mount with the output from a single element waveguide mount, where both mounts were connected to the arms of an X-band waveguide magic-T. Measurements were made from 1 to 10 mW at 10 GHz. Since the waveguide mount has no dual element error, the difference in the responses was taken to be the dual element error of the coaxial mount. Neil Larsen, NIST, 723.01, Boulder, Colo. 80303.

ANA's indicate that the total nonlinearity in these power readings is approximately only 0.01%. These percent errors give the following values of K_ϵ at 10 mW.

$$\begin{aligned} \text{Engen worst case: } K_\epsilon &= 1\text{x}10^{-4} \\ \text{Engen typical case: } K_\epsilon &= 2\text{x}10^{-5} \\ \text{Larsen: } K_\epsilon &= 2\text{x}10^{-6} \\ \text{6-port inconsistencies: } K_\epsilon &= 1\text{x}10^{-6} \end{aligned} \tag{10.23}$$

A conservative estimate of K_ϵ for the 6-port systems would be double the value obtained by Larsen, or $K_\epsilon = 4\text{x}10^{-6}$.

Using (10.21) with $\alpha = 2$ for the worst case, the η terms in (10.16) become

$$\frac{d\eta_{ou}}{\eta_{ou}} - \frac{d\eta_{os}}{\eta_{os}} = K_\epsilon(P_{os}^2 - P_{ou}^2) = K_\epsilon(P_{os} + P_{ou})(P_{os} - P_{ou}). \tag{10.24}$$

When P_{odcs} and P_{odcu} are about 5 mW and are within 0.25 mW of each other, and if we choose for K_ϵ the value of $4\text{x}10^{-6}$, (10.24) gives

$$\frac{d\eta_{ou}}{\eta_{ou}} - \frac{d\eta_{os}}{\eta_{os}} = \pm 0.00001. \tag{10.25}$$

As (10.16) indicates, the total uncertainty in the rf power ratio at the 6-port reference sidearm is the sum of (10.25) and (10.19).

$$\frac{d(P_{os}/P_{ou})}{P_{os}/P_{ou}} = \pm 0.00005. \tag{10.26}$$

10.3. Error in η_s Due to Nonlinearity in Power Standard.

The value of η_s that is used in (9.1) is obtained from measurements made in the microcalorimeter at a specific rf power level, say P_{mc}, which is usually near 10 mW. If the power level P_s into the standard mount when it is connected to the 6-port is different from P_{mc}, the value of η_s at P_s could differ from the value of η_s at P_{mc} because of nonlinearity in the standard mount. Equations for calculating this error are the same as those derived in the previous Section. Let

η_{mc} = effective efficiency of the standard mount as measured in the microcalorimeter at power level P_{mc}, and let

η_{6p} = effective efficiency of this same standard mount at a power level P_s which is the net power at into this mount when connected to the 6-port.

The effective efficiency η_{mc} which is measured in the microcalorimeter at a power level P_{mc} includes any nonlinearity such as the dual element substitution error. When the standard mount is used at a different power level such as P_s, it is only the change in the effective efficiency due to the different power level that causes an error. An expression for this change in effective efficiency will be derived next.

Replacing the subscript rf in (10.20) with mc to indicate power levels used in the microcalorimeter, and with 6p to indicate power levels at the 6-port test port, (10.20) gives

$$\frac{1}{\eta_{mc}} = b + \frac{\Delta P_{mc}}{P_{dcmc}}, \tag{10.27}$$

$$\frac{1}{\eta_{6p}} = b + \frac{\Delta P_{6p}}{P_{dc6p}}. \tag{10.28}$$

The difference is

$$\frac{1}{\eta_{mc}} - \frac{1}{\eta_{6p}} = \frac{\Delta P_{mc}}{P_{dcmc}} - \frac{\Delta P_{6p}}{P_{dc6p}}. \tag{10.29}$$

This can be written

$$\frac{\eta_{6p} - \eta_{mc}}{\eta_{mc} \cdot \eta_{6p}} = \frac{P_{mc}\ \Delta P_{mc}}{P_{dcmc}P_{mc}} - \frac{P_{6p}\ \Delta P_{6p}}{P_{dc6p}P_{6p}},$$

$$= \frac{1}{\eta_{mc}} \cdot \frac{\Delta P_{mc}}{P_{mc}} - \frac{1}{\eta_{6p}} \cdot \frac{\Delta P_{6p}}{P_{6p}},$$

$$\frac{\eta_{6p} - \eta_{mc}}{\eta_{mc}} = \frac{\eta_{6p}}{\eta_{mc}} \cdot \frac{\Delta P_{mc}}{P_{mc}} - \frac{\Delta P_{6p}}{P_{6p}}. \tag{10.30}$$

When $P_{mc} \simeq P_{6p}$, then $\eta_{mc} \simeq \eta_{6p}$, and (10.30) gives the following desired expression for the dual element dc-rf substitution error:

$$\frac{\Delta\eta_{mc}}{\eta_{mc}} = \frac{\Delta P_{mc}}{P_{mc}} - \frac{\Delta P_{6p}}{P_{6p}}. \tag{10.31}$$

Substituting (10.21) into (10.31) with $\alpha = 2$ gives

$$\frac{\Delta\eta_{mc}}{\eta_{mc}} = K_\epsilon(P_{mc}^2 - P_{6p}^2) = K_\epsilon(P_{mc} + P_{6p})(P_{mc} - P_{6p}). \tag{10.32}$$

For the conditions stated in (10.11)-(10.12),

$$P_{mc} \simeq P_{6p} \simeq 10 \text{ mW}, \quad |P_{mc} - P_{6p}| < 0.5 \text{ mW}. \tag{10.33}$$

If $K_\epsilon = 4\text{x}10^{-6}$, these values in (10.32) give

$$\frac{\Delta\eta_{mc}}{\eta_{mc}} = \pm 0.00004. \tag{10.34}$$

10.4. Error in $|G|$

Equations for calculating $d|G|^2/|G|^2$ are derived in Appendix B. From (B37) of Appendix B,

$$\frac{d|G|^2}{|G|^2} \simeq \frac{2\Delta\Gamma_r dC_r - 2\Delta\Gamma_x dC_x.}{1 + 2(\Delta\Gamma_r C_r - \Delta\Gamma_x C_x)}. \tag{10.35}$$

To evaluate this expression we need to estimate $d\Gamma_r$, $d\Gamma_x$, dC_r, and dC_x. These errors are due to imperfections in the 6-port and not in the standards used to calibrate the 6-port. They can be obtained from the inconsistencies in the 6-port sidearm power readings [8]. Typical values are

$$|d\Gamma_r| \simeq |d\Gamma_x| \simeq |dC_r| \simeq |dC_x| \simeq 10^{-4}. \qquad (10.36)$$

If we assume that

$$|\Gamma_u| \le 0.15, \qquad |\Gamma_s| \le 0.15, \qquad |C| \le 0.3, \qquad (10.37)$$

the inequalities in (B24) give

$$|\Delta\Gamma_r| \le 0.3, \qquad |\Delta\Gamma_x| \le 0.3. \qquad (10.38)$$

For these values, (10.35) gives a worst-case error of

$$d|G|^2/|G|^2 \simeq \pm 0.00015. \qquad (10.39)$$

10.5. Error in M

Equations for calculating dM/M are derived in Appendix C. From (C14) of Appendix C,

$$dM/M \simeq 2(\Delta\Gamma_r d\Gamma_r + \Delta\Gamma_x d\Gamma_x). \qquad (10.40)$$

For the values assumed in (10.36)-(10.38), (10.40) gives as a worst-case error

$$dM/M \simeq \pm 0.00012. \qquad (10.41)$$

10.6. Error Due to Imperfect Impedance Standards and Connectors

The last term on the right of (10.2) is obtained by differentiating (8.3) to get

$$d\epsilon \simeq \pm 4[|\Delta\Gamma_r d(\Delta r)| + |\Delta\Gamma_x d(\Delta x_o)| + |\Delta|\Gamma|^2 d(\Delta r)|] \qquad (10.42)$$

The $d(\Delta r)$ and $d(\Delta x_o)$ in (10.42) are the uncertainties in estimating Δr and Δx_o. If Δr and Δx_o were known, ϵ is a correction and not an error. Then $d(\Delta r)$ and $d(\Delta x_o)$ would be zero, and so would $d\epsilon$ be 0. If Δr and Δx_o are error limits, then $d(\Delta r)$ and $d(\Delta x_o)$ in (10.42) are simply Δr and Δx_o.

For 7 mm connectors, Δr and Δx_o are given in figure 7. Over the 1 to 18 GHz frequency range this figure gives the limits

$$|\Delta r| \le 0.0003, \quad \text{and} \quad |\Delta x_o| \le 0.00004. \qquad (10.43)$$

Then for the values assumed in (10.36)-(10.38), (10.42) gives as a worst-case error

$$\frac{d\epsilon}{1+\epsilon} \simeq d\epsilon \simeq \pm 4[0.9 + 0.12 + 0.135]\text{x}10^{-4} = \pm 0.00046. \qquad (10.44)$$

This is the largest of all the errors in measuring ratios of effective efficiency.

Table 2

Summary of systematic errors in dual six-port power measurements, 1-18 GHz.

Section	Source of error	Parameter	Uncertainty $d\eta_u/\eta_u$ Worst case	Equations
10.1	dc power ratio measurement	$\frac{d(P_{dcu}/P_{dcs})}{P_{dcu}/P_{dcs}} = \Delta_{dc}$	$= \pm 0.00002$	(10.9)
10.2	rf power ratio at 6-port sidearm	$\frac{d(P_{os}/P_{ou})}{P_{os}/P_{ou}} = \Delta_{rf}$	=	(10.16)
	• Sidearm detector nonlinearity		± 0.00001	(10.24)
	• Power meter and DVM errors		± 0.00004	(10.17)
10.3	Power Standard nonlinearity	$\Delta\eta_{mc}/\eta_{mc} = \Delta_{mc}$	$= \pm 0.00004$	(10.34)
10.4	Uncertainty in G due to $d\Gamma_r$, Γ_x, and dc.	$d\|G\|^2/\|G\|^2 = \Delta_G$	$= \pm 0.00015$	(10.35)
10.5	Uncertainty in M due to $d\Gamma_r$ and $d\Gamma_x$	$dM/M = \Delta_M$	$= \pm 0.00012$	(10.40)
10.6	Imperfect 7 mm impedance standards and connectors	$\frac{d(\Delta\eta_u/\Delta\eta_s)}{\Delta\eta_u/\Delta\eta_s} = \Delta_\epsilon$	$= \pm 0.00046$	(10.42)
	Total uncertainty in 6-port measurement of η_u/η_s	$\frac{d(\eta_u/\eta_s)}{\eta_u/\eta_s} = \Delta_{sp}$	$= \pm 0.00084$	
	Uncertainty in power standard	$d\eta_s/\eta_s = \Delta_s$	$= \pm(0.00865 + 0.00105\sqrt{f})$	
	Total uncertainty in measurement of η_u	$d\eta_u/\eta_u = \Delta$	$= \pm(0.00949 + 0.00105\sqrt{f})$	

Assumptions

Power at the measurement port: $P_{rfu} \simeq P_{rfs} \simeq 10$ mW, $|P_{rfs} - P_{rfu}| < 0.5$ mW.

Power at the 6-port reference port: $P_{ou} \simeq P_{os} \simeq 5$ mW, $|P_{os} - P_{ou}| < 0.25$ mW,

The uncertainty dP_{dc} in the dc power measured without an RVG is given by $dP_{dc} \simeq \pm(0.0035 - 0.00005P_{dc})$, mW.

Uncertainty in Γ and c excluding imperfections in the standards and connectors: $|d\Gamma_r| \simeq |d\Gamma_x| \simeq |dC_r| \simeq |dC_x| \simeq 10^{-4}$.

Γ of the power sensors: $|\Gamma_u| \leq 0.15$, $|\Gamma_s| \leq 0.15$. Γ_g of test port: $|C| \leq 0.3$,

Normalized resistance due to the gap between actual line and perfect line: $|\Delta r| \leq 0.00007\sqrt{f}$ for gap ≃ 0.02 mm. Δr is the real part of $\Delta z \equiv (Z_\ell - Z_s)/2Z_o = \Delta r + j\Delta x$, where Z_s is the series impedance of a perfect line with the test port, and Z_ℓ is the series impedance of the actual line with the test port.

Normalized difference in reactive part of Z_o of a perfect line and the actual line: $|\Delta x_o| \leq 0.00004/\sqrt{f}$. Δx_o is the reactive part of $\Delta z_o \equiv (Z_{os} - Z_{o\ell})/2Z_o = \Delta r_o + j\Delta x_o$, where Z_{os} is Z_o of the perfect line, and $Z_{o\ell}$ is Z_o of the actual line. Z_o is the nominal value of the characteristic impedance of the connectors.

11. Total Systematic Error in η_u for 7 mm Connectors

Table 2 summarizes all of the components of error for power measurements in 7 mm connectors. All of the components of error in section 10 have been substituted in (10.2) to obtain this result. All of these errors are independent of the type of connector used except $d\epsilon$ in (10.42) which is due to imperfect impedance standards and connectors.

12. Outline of Measurements and Calculations

This outline assumes that there is only one standard mount used in the calibration of the 6-ports.

For the standard mount,

- Measure $\Gamma_{\ell s}$, P_{dcs}, & P_{os} for k connections on each test port, where $k \geq 3$ connections.
  $\Gamma_{\ell s}$ = Γ of the standard mount as measured by the 6-port.
  P_{dcs} = change in the dc power applied to the standard bolometer mount when rf power is applied to the input of the mount.
  P_{os} = rf power at the 6-port reference sidearm when rf power is applied to the input of the standard mount. For thermistor detectors we set $P_{os} = KP_{odcs}$, where K is a constant and P_{odcs} is the change in the dc power applied to the reference sidearm mount when rf power is applied to the input of the standard mount. When K is independent of power level, which we assume is true for the 6-port sidearm detectors, K drops out of the following calculations and can be set equal to 1. Then
  $P_{os} = P_{odcs}$.
- Calculate $N_{\ell s}$ from (5.10), and $K_{\ell p}$ from (9.3) for each connection.

$$N_{\ell s} \equiv \frac{P_{dcs}}{P_{os}} \frac{|1 + C_\ell \Gamma_{\ell s}|^2}{1 - |\Gamma_{\ell s}|^2}, \quad K_{\ell p} = \frac{\eta_s}{N_{\ell s}}. \qquad (5.10),(9.3)$$

  where η_s is the effective efficiency of the standard mount.
- Calculate $\overline{K}_{\ell p}$, the average $K_{\ell p}$ for each test port from the k values of $K_{\ell p}$ for that test port.
- Calculate and save the average $\Gamma_{\ell s}$, P_{dcs}, & P_{os} for 6-port #1, and the corresponding averages for 6-port #2 for use in calculating systematic errors.

For each mount under test,

- Measure $\Gamma_{\ell u}$, P_{dcu}, & P_{ou} for n connections on each test port, where $\Gamma_{\ell u}$ = Γ of the mount under test measured by the 6-port.
  P_{dcu} = change in the dc power applied to the mount under test when rf power is applied to the input of the mount.
  P_{ou} = rf power at the 6-port reference sidearm when rf power is applied to the input of the mount under test. As with P_{os}, set $P_{ou} = P_{odcu}$, where P_{odcu} is the change in the dc power applied to the reference sidearm mount when rf power is applied to the input of the mount under test.

- For each connection calculate $N_{\ell u}$ from (5.9), and η_u from (9.2) using $\bar{K}_{\ell p}$, the average value of $K_{\ell p}$ for that test port.

$$N_{\ell u} \equiv \frac{P_{dcu}}{P_{ou}} \frac{|1 + C_\ell \Gamma_{\ell u}|^2}{1 - |\Gamma_{\ell u}|^2}, \qquad \eta_u = \bar{K}_{\ell p} N_{\ell u}. \qquad (5.9),(9.2)$$

- Calculate $\bar{\eta}_{u1}$, the average η_u from the n values of η_u on 6-port #1, and $\bar{\eta}_{u2}$, the corresponding average on 6-port #2.
- Calculate $\bar{\eta}_u = (\bar{\eta}_{u1} + \bar{\eta}_{u2})/2$.
- Calculate the long term random error s_b ($=S_{NBS}$), the short term random error s_c ($=S_c$), and the total random error $s_{\bar{y}}$ in $\bar{\eta}_u$ following a procedure similar to that described in Part 5 of reference [8].
- Calculate total systematic error Δ_1 in $\bar{\eta}_{u1}$ from (10.2) and related equations listed in the right column of table 2 using average values of $\Gamma_{\ell s}$, P_{dcs}, P_{os}, $\Gamma_{\ell u}$, P_{dcu}, & P_{ou} for 6-port #1. Calculate corresponding total systematic error Δ_2 in $\bar{\eta}_{u2}$.
- Calculate total systematic error $\Delta = (\Delta_1 + \Delta_2)/2$.
- Calculate total uncertainty U_T in $\bar{\eta}_u$. $U_T = \Delta + 3s_{\bar{y}}$.

The equations for calculating the components of the systematic error are collected below. When (10.4) is substituted into (10.2), there are 6 components of the total systematic error which we will write as

$$\Delta = \Delta_s + \Delta_{dc} + \Delta_{rf} + \Delta_G + \Delta_M + \Delta_\epsilon \qquad (12.1)$$

where

$\Delta_s \equiv d\eta_s/\eta_s$ is the uncertainty in the effective efficiency of the power standard, presently given as

$$\pm\Delta_s = 0.865 + 0.105\sqrt{f}, \qquad (12.2)$$

where f is in gigahertz.

Δ_{dc} = uncertainty in measuring the dc power ratio,

$$\pm\Delta_{dc} = \frac{P_{dcs} - P_{dcu}}{P_{dcu} P_{dcs}} dP_{dc} \qquad (10.9)$$

where

$$dP_{dc} \simeq \pm(0.0035 - 0.00005P_{dc}), \qquad (10.10)$$

P_{dc} is the average value of P_{dcs} and P_{dcu}, and power is in milliwatts.
Δ_{rf} = uncertainty in measuring the rf power ratio at the 6-port sidearm,

$$\pm\Delta_{rf} = 2\text{x}10^{-6}(P_{os} + P_{ou})(P_{os} - P_{ou}) \qquad (10.24)$$

$$+ \frac{P_{odcu} - P_{odcs}}{P_{odcs} P_{odcu}} dP_{dc}. \qquad (10.17)$$

Δ_G = uncertainty in G due to $d\Gamma_r$, $d\Gamma_x$, and dC which are obtained by propagating the "power meter residuals" into Γ and C [8]. Since this propagation is not presently being done, we will set each of these errors equal to the power meter residual itself. This usually results in values of $d\Gamma_r \simeq d\Gamma_x \simeq dC_r \simeq dC_x \simeq 10^{-4}$.

$$\pm\Delta_G \simeq 2\{|C|^2\Delta\Gamma_r d\Gamma_r + |C|^2\Delta\Gamma_x d\Gamma_x + \Delta\Gamma_r dC_r - \Delta\Gamma_x dC_x\} \quad (10.35)$$

where in this equation we will use the absolute values of the differences

$$\Delta\Gamma_r = |\Gamma_{\ell sr} - \Gamma_{\ell ur}|, \quad (8.4)$$

$$\Delta\Gamma_x = |\Gamma_{\ell sx} - \Gamma_{\ell ux}|. \quad (8.5)$$

Δ_M = uncertainty in M due to $d\Gamma_r$ and $d\Gamma_x$,

$$\pm\Delta_M \simeq -2(\Delta\Gamma_r d\Gamma_r + \Delta\Gamma_x d\Gamma_x). \quad (10.40)$$

Δ_ϵ = worst-case uncertainty in the position of the reference plane due to imperfect standards and connectors,

$$\Delta_\epsilon = d\epsilon \simeq \pm 4[|\Delta\Gamma_r d(\Delta r)| + |\Delta\Gamma_x d(\Delta x_o)| + |\Delta|\Gamma|^2 d(\Delta r)|] \quad (10.42)$$

where

$$\Delta|\Gamma|^2 = |\Gamma_{\ell s}|^2 - |\Gamma_{\ell u}|^2, \quad (8.6)$$

$$d(\Delta x_o) = \Delta x_o \simeq 0.00004/\sqrt{f}, \quad \text{(figure 7)}$$

$$d(\Delta r) = \Delta r \simeq 0.00007\sqrt{f}, \quad \text{(figure 7)}$$

and where f is in gigahertz.

The total systematic error excluding that due to the power standard is Δ_{sp} which is the 6-port error in measuring the ratio of two effective efficiencies.

$$\Delta_{sp} = \Delta_{dc} + \Delta_{rf} + \Delta_G + \Delta_M + \Delta_\epsilon. \quad (12.3)$$

13. Conclusions and Observations.

Equations have been derived for calculating estimates of the systematic error in measuring ratios of effective efficiency of bolometric power sensors on a dual 6-port ANA. The largest source of error is imperfections in the transmission line impedance standards used to calibrate the dual 6-port for making Γ measurements. This error, given by ε in (8.3), is more than half the total systematic error in measuring ratios of effective efficiency. This error is based on a model that accounts for connector imperfections as well as imperfections in the line standard itself.

One important observation is that the error ε is usually considerably less when calculated from (7.8) or (8.3) than it is when calculated directly from (2.9) if worst-case errors are chosen for Γ and C. The reason for this is because errors in Γ and C are correlated, and cannot be chosen independent of one another. The correlation between errors in Γ and C is derived and explained in Appendix D. For realistic situations, the error in C is nearly equal to the error in Γ, and these errors then nearly cancel in (2.9).

For bolometric power sensors with 7 mm connectors, the total systematic error in measuring ratios of effective efficiency on the NIST 1-18 GHz dual 6-port ANA is estimated to be 0.00084 which occurs at 18 GHz. To this must be added the systematic error in η_s of the power standard to get the total systematic error in η_u of the DUT power sensor. The systematic error in η_s of the power standard is usually considerably larger than the 0.00084 contributed by the 6-port.

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Appendix A. Approximate Expression for $\Delta\eta_u/\Delta\eta_s$

The purpose of this Appendix is to derive a simple approximate expression for $\Delta\eta_u/\Delta\eta_s$ for use in estimating errors due to imperfections in the impedance standard. From (7.7) the exact expression for $\Delta\eta_u/\Delta\eta_s$ is

$$\frac{\Delta\eta_u}{\Delta\eta_s} = \frac{(1 - |\Gamma_{\ell u}|^2)(|1 + \Delta c\Gamma_{\ell s}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell s}|^2)}{(1 - |\Gamma_{\ell s}|^2)(|1 + \Delta c\Gamma_{\ell u}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell u}|^2)} \quad \text{(A1)}$$

$$\equiv 1 + \epsilon. \quad \text{(A2)}$$

Our purpose here is to get a simple expression for ϵ which is zero for perfect connectors and line standards. Let the real and imaginary components of the Δ terms be

$$\begin{aligned} \Delta a &\equiv \Delta a_r + j\Delta a_x, \\ \Delta b &\equiv \Delta b_r + j\Delta b_x, \\ \Delta c &\equiv \Delta c_r + j\Delta c_x. \end{aligned} \quad \text{(A3)}$$

Expanding the terms in (A1) with those in (A3), the terms

$$|1 + \Delta c\Gamma_{\ell s}|^2 \quad \text{and} \quad |1 + \Delta c\Gamma_{\ell u}|^2$$

become .

$$\begin{aligned} |1 + \Delta c\Gamma|^2 &= |1 + [\Delta c_r + j\Delta c_x](\Gamma_r + j\Gamma_x)|^2 \\ &= |1 + \Delta c_r\Gamma_r - \Delta c_x\Gamma_x + j[\Delta c_r\Gamma_x + \Delta c_x\Gamma_r]|^2 \\ &= 1 + 2[\Delta c_r\Gamma_r - \Delta c_x\Gamma_x] \\ &\quad + [\Delta c_r\Gamma_r - \Delta c_x\Gamma_x]^2 \\ &\quad + [\Delta c_r\Gamma_x + \Delta c_x\Gamma_r]^2 \\ &= 1 + 2[\Delta c_r\Gamma_r - \Delta c_x\Gamma_x] \\ &\quad + \Delta c_r^2\Gamma_r^2 - 2\Delta c_r\Gamma_r\Delta c_x\Gamma_x + \Delta c_x^2\Gamma_x^2 \\ &\quad + \Delta c_r^2\Gamma_x^2 + 2\Delta c_r\Gamma_x\Delta c_x\Gamma_r + \Delta c_x^2\Gamma_r^2 \\ &= 1 + 2[\Delta c_r\Gamma_r - \Delta c_x\Gamma_x] + \Delta c_r^2|\Gamma|^2 + \Delta c_x^2|\Gamma|^2 \\ &= 1 + 2[\Delta c_r\Gamma_r - \Delta c_x\Gamma_x] + |\Delta c|^2|\Gamma|^2, \end{aligned} \quad \text{(A4)}$$

where $\Gamma = \Gamma_r + j\Gamma_x$ represents either $\Gamma_{\ell s}$ or $\Gamma_{\ell u}$. The terms

$$|\Delta b + (1+\Delta a)\Gamma_{\ell s}|^2 \quad \text{and} \quad |\Delta b + (1+\Delta a)\Gamma_{\ell u}|^2$$

can be expanded to give

$$\begin{aligned} |\Delta b + (1+\Delta a)\Gamma|^2 &= |\Delta b_r + j\Delta b_x + (1+\Delta a_r + j\Delta a_x)(\Gamma_r + j\Gamma_x)|^2 \\ &= |\Delta b_r + (1+\Delta a_r)\Gamma_r - \Delta a_x\Gamma_x + j[\Delta b_x + (1+\Delta a_r)\Gamma_x + \Delta a_x\Gamma_r]|^2 \\ &= |\Gamma_r + (\Delta b_r + \Delta a_r\Gamma_r - \Delta a_x\Gamma_x) + j[\Gamma_x + (\Delta b_x + \Delta a_r\Gamma_x + \Delta a_x\Gamma_r)]|^2 \end{aligned}$$

$$
\begin{aligned}
&= \Gamma_r^2 + 2\Gamma_r[\Delta b_r + \Delta a_r\Gamma_r - \Delta a_x\Gamma_x] \\
&\qquad + [\Delta b_r + \Delta a_r\Gamma_r - \Delta a_x\Gamma_x]^2 \\
&\quad +\Gamma_x^2 + 2\Gamma_x[\Delta b_x + \Delta a_r\Gamma_x + \Delta a_x\Gamma_r] \\
&\qquad + [\Delta b_x + \Delta a_r\Gamma_x + \Delta a_x\Gamma_r]^2
\end{aligned}
$$

$$
\begin{aligned}
&= |\Gamma|^2 + 2\Gamma_r\Delta b_r + 2\Gamma_x\Delta b_x + 2\Delta a_r|\Gamma|^2 \\
&\quad + \Delta b_r^2 + \Delta a_r^2\Gamma_r^2 + \Delta a_x^2\Gamma_x^2 \\
&\quad + 2\Delta b_r\Delta a_r\Gamma_r - 2\Delta b_r\Delta a_x\Gamma_x - 2\Delta a_r\Gamma_r\Delta a_x\Gamma_x \\
&\quad + \Delta b_x^2 + \Delta a_r^2\Gamma_x^2 + \Delta a_x^2\Gamma_r^2 \\
&\quad + 2\Delta b_x\Delta a_r\Gamma_x + 2\Delta b_x\Delta a_x\Gamma_r + 2\Delta a_r\Gamma_x\Delta a_x\Gamma_r
\end{aligned}
$$

$$
\begin{aligned}
&= |\Gamma|^2 + 2\Gamma_r\Delta b_r + 2\Gamma_x\Delta b_x + 2\Delta a_r|\Gamma|^2 \\
&\quad + |\Delta b|^2 + |\Delta a|^2|\Gamma|^2 \\
&\quad + 2\Gamma_r(\Delta b_r\Delta a_r + \Delta b_x\Delta a_x) \\
&\quad + 2\Gamma_x(\Delta b_x\Delta a_r - \Delta b_r\Delta a_x),
\end{aligned}
\tag{A5}
$$

where again $\Gamma = \Gamma_r + \Gamma_x$ represents either $\Gamma_{\ell s}$ or $\Gamma_{\ell u}$. The first line of (A5) contains the most significant error terms. Lines 2, 3, and 4 of (A5) contain second order error terms (products or squares of Δ error terms).

Next, write (29) as

$$
\frac{|1+\Delta d|^2}{\Delta\eta} = \frac{|1 + \Delta c\Gamma_\ell|^2 - |\Delta b + (1+\Delta a)\Gamma_\ell|^2}{(1 - |\Gamma_\ell|^2)}. \tag{A6}
$$

Substituting (A4) and (A5) into (A6) gives

$$
\begin{aligned}
|1+\Delta d|^2/\Delta\eta = \{&1 - |\Gamma|^2 \\
&+ 2[\Delta c_r\Gamma_r - \Delta c_x\Gamma_x] + |\Delta c|^2|\Gamma|^2 \\
&- 2\Gamma_r\Delta b_r - 2\Gamma_x\Delta b_x - 2\Delta a_r|\Gamma|^2 \\
&- |\Delta b|^2 - |\Delta a|^2|\Gamma|^2 \\
&- 2\Gamma_r(\Delta b_r\Delta a_r + \Delta b_x\Delta a_x) \\
&- 2\Gamma_x(\Delta b_x\Delta a_r - \Delta b_r\Delta a_x)\ \}/(1 - |\Gamma|^2)
\end{aligned}
$$

$$
\begin{aligned}
= 1 + \{&2[\Gamma_r(\Delta c_r - \Delta b_r) - \Gamma_x(\Delta c_x + \Delta b_x) - \Delta a_r|\Gamma|^2] \\
&- |\Delta b|^2 - |\Delta a|^2|\Gamma|^2 + |\Delta c|^2|\Gamma|^2 \\
&- 2\Gamma_r(\Delta b_r\Delta a_r + \Delta b_x\Delta a_x) \\
&- 2\Gamma_x(\Delta b_x\Delta a_r - \Delta b_r\Delta a_x)\}/(1 - |\Gamma|^2).
\end{aligned}
\tag{A7}
$$

Neglecting terms that contain squares or products of Δ error terms,

$$
|1+\Delta d|^2/\Delta\eta \simeq 1 + 2[\Gamma_r(\Delta c_r - \Delta b_r) - \Gamma_x(\Delta c_x + \Delta b_x) - \Delta a_r|\Gamma|^2]. \tag{A8}
$$

The Δ terms are given in (8.2) as the following functions of the equivalent circuit parameters.

$$
\begin{aligned}
\Delta a &\equiv \Delta a_r + j\Delta a_x \simeq (-2\Delta r) + j(-2\Delta B-2\Delta x) \\
\Delta b &\equiv \Delta b_r + j\Delta b_x \simeq (\Delta r-\Delta r_o) + j(\Delta x-\Delta B-\Delta x_o), \\
\Delta c &\equiv \Delta c_r + j\Delta c_x \simeq (-\Delta r-\Delta r_o) + j(\Delta B-\Delta x-\Delta x_o).
\end{aligned}
\tag{8.2}
$$

Expanding the Δ terms in (A8) with these definitions reduces (A8) to

$$|1+\Delta d|^2/\Delta\eta \simeq 1 + 2[\Gamma_r(-2\Delta r) - \Gamma_x(-2\Delta x_o) - (-2\Delta r)|\Gamma|^2]$$

$$= 1 - 4(\Gamma_r\Delta r - \Gamma_x\Delta x_o - \Delta r|\Gamma|^2). \quad (A9)$$

Substituting (A9) in (A1) and adding appropriate subscripts gives

$$\frac{\Delta\eta_u}{\Delta\eta_s} \simeq \frac{1 - 4[\Gamma_{\ell sr}\Delta r - \Gamma_{\ell sx}\Delta x_o - \Delta r|\Gamma_{\ell s}|^2]}{1 - 4[\Gamma_{\ell ur}\Delta r - \Gamma_{\ell ux}\Delta x_o - \Delta r|\Gamma_{\ell u}|^2]}. \quad (A10)$$

This reduces further to

$$\frac{\Delta\eta_u}{\Delta\eta_s} \simeq \{1 - 4[\Gamma_{\ell sr}\Delta r-\Gamma_{\ell sx}\Delta x_o-\Delta r|\Gamma_{\ell s}|^2]\}\{1 + 4[\Gamma_{\ell ur}\Delta r-\Gamma_{\ell ux}\Delta x_o-\Delta r|\Gamma_{\ell u}|^2] + \ldots\},$$

where ... represents other terms in the series expansion of 1/(1 - 4[ ]). Discarding products or squares of error terms leads to

$$\frac{\Delta\eta_u}{\Delta\eta_s} \simeq 1 - 4[(\Gamma_{\ell sr}-\Gamma_{\ell ur})\Delta r - (\Gamma_{\ell sx}-\Gamma_{\ell ux})\Delta x_o - (|\Gamma_{\ell s}|^2-|\Gamma_{\ell u}|^2)\Delta r] \quad (A11)$$

so that the error in the ratio of the efficiencies is approximately

$$\epsilon \simeq 4[\Delta\Gamma_r\Delta r - \Delta\Gamma_x\Delta x_o - \Delta|\Gamma|^2\Delta r] \quad (A12)$$

where

$$\Delta\Gamma_r \equiv \Gamma_{\ell ur} - \Gamma_{\ell sr}\ , \quad (A13)$$
$$\Delta\Gamma_x \equiv \Gamma_{\ell ux} - \Gamma_{\ell sx}\ , \quad (A14)$$
$$\Delta|\Gamma|^2 \equiv |\Gamma_{\ell u}|^2 - |\Gamma_{\ell s}|^2. \quad (A15)$$

The approximate ϵ from (A12) agrees with ϵ calculated from (A1) to about 10% for $\Delta r = 0.01$, $\Delta x_o = 0.004$, and $|\Gamma| \leq 0.3$. The approximation is more accurate for smaller values of Δr, Δx_o, and $|\Gamma|$. Values of Δr and Δx_o for the NIST 7 mm lines are shown in figure 7.

Appendix B. Systematic Error in η_u due to Systematic Error in C and Γ.

The purpose of this Appendix is to derive expressions for the error in G due to errors in Γ and C, or due to errors in w and C/A.

1. Exact solution.

Comparing (10.5) with (2.3) shows that G can be written as either

$$G = \frac{1 + \Gamma_u C}{1 + \Gamma_s C}, \quad \text{or} \quad G = \frac{1 - w_s C/A}{1 - w_u C/A}. \tag{B1}$$

We will derive an expression for the error in G due to errors in C and Γ. The resulting expression can also be used to calculate the error in G due to errors in w and C/A simply by replacing

Γ_u with w_s, Γ_s with w_u, and C with -C/A.

Let

$$\Gamma_u \equiv \Gamma_{ur} + j\Gamma_{ux}, \quad \Gamma_s \equiv \Gamma_{sr} + j\Gamma_{sx}, \quad \text{and} \quad C \equiv C_r + jC_x. \tag{B2}$$

Then

$$G = \frac{1 + (\Gamma_{ur} + j\Gamma_{ux})(C_r + jC_x)}{1 + (\Gamma_{sr} + j\Gamma_{sx})(C_r + jC_x)}$$

$$= \frac{1 + (\Gamma_{ur}C_r - \Gamma_{ux}C_x) + j(\Gamma_{ur}C_x + \Gamma_{ux}C_r)}{1 + (\Gamma_{sr}C_r - \Gamma_{sx}C_x) + j(\Gamma_{sr}C_x + \Gamma_{sx}C_r)} \tag{B3}$$

and

$$|G|^2 = \frac{[1 + (\Gamma_{ur}C_r - \Gamma_{ux}C_x)]^2 + (\Gamma_{ur}C_x + \Gamma_{ux}C_r)^2}{[1 + (\Gamma_{sr}C_r - \Gamma_{sx}C_x)]^2 + (\Gamma_{sr}C_x + \Gamma_{sx}C_r)^2}$$

$$= \frac{1 + 2(\Gamma_{ur}C_r - \Gamma_{ux}C_x) + (\Gamma_{ur}C_r - \Gamma_{ux}C_x)^2 + (\Gamma_{ur}C_x + \Gamma_{ux}C_r)^2}{1 + 2(\Gamma_{sr}C_r - \Gamma_{sx}C_x) + (\Gamma_{sr}C_r - \Gamma_{sx}C_x)^2 + (\Gamma_{sr}C_x + \Gamma_{sx}C_r)^2}$$

$$= \frac{1 + 2(\Gamma_{ur}C_r - \Gamma_{ux}C_x) + \Gamma_{ur}^2(C_r^2 + C_x^2) + \Gamma_{ux}^2(C_r^2 + C_x^2)}{1 + 2(\Gamma_{sr}C_r - \Gamma_{sx}C_x) + \Gamma_{sr}^2(C_r^2 + C_x^2) + \Gamma_{sx}^2(C_r^2 + C_x^2)} \tag{B4}$$

$$= \frac{1 + 2(\Gamma_{ur}C_r - \Gamma_{ux}C_x) + |\Gamma_u|^2|C|^2}{1 + 2(\Gamma_{sr}C_r - \Gamma_{sx}C_x) + |\Gamma_s|^2|C|^2}. \tag{B5}$$

Define the numerator of $|G|^2$ as N and the denominator as D so that

$$|G|^2 \equiv \frac{N}{D}. \tag{B6}$$

The systematic error in $|G|^2$ can be written

$$d|G|^2 = \frac{\partial|G|^2}{\partial\Gamma_{ur}}d\Gamma_{ur} + \frac{\partial|G|^2}{\partial\Gamma_{ux}}d\Gamma_{ux} + \frac{\partial|G|^2}{\partial\Gamma_{sr}}d\Gamma_{sr} + \frac{\partial|G|^2}{\partial\Gamma_{sx}}d\Gamma_{sx}$$
$$+ \frac{\partial|G|^2}{\partial C_r}dC_r + \frac{\partial|G|^2}{\partial C_x}dC_x \tag{B7}$$

where from (B5) and (B6),

$$\frac{\partial|G|^2}{\partial\Gamma_{ur}} = \frac{2C_r + 2\Gamma_{ur}|C|^2}{D} = |G|^2\,\frac{2C_r + 2\Gamma_{ur}|C|^2}{N} \tag{B8}$$

$$\frac{\partial|G|^2}{\partial\Gamma_{ux}} = \frac{-2C_x + 2\Gamma_{ux}|C|^2}{D} = |G|^2\,\frac{-2C_x + 2\Gamma_{ux}|C|^2}{N} \tag{B9}$$

$$\frac{\partial|G|^2}{\partial\Gamma_{sr}} = -\frac{N}{D^2}\,\frac{2C_r + 2\Gamma_{sr}|C|^2}{} = |G|^2\,\frac{-2C_r - 2\Gamma_{sr}|C|^2}{D} \tag{B10}$$

$$\frac{\partial|G|^2}{\partial\Gamma_{sx}} = -\frac{N}{D^2}\,\frac{-2C_x + 2\Gamma_{sx}|C|^2}{} = |G|^2\,\frac{2C_x - 2\Gamma_{sx}|C|^2}{D}. \tag{B11}$$

Also from (B5) we have

$$\frac{\partial|G|^2}{\partial C_r} = \frac{D(2\Gamma_{ur} + 2|\Gamma_u|^2C_r) - N(2\Gamma_{sr} + 2|\Gamma_s|^2C_r)}{D^2}$$

$$= 2|G|^2\left(\frac{(\Gamma_{ur} + |\Gamma_u|^2C_r)}{N} - \frac{(\Gamma_{sr} + |\Gamma_s|^2C_r)}{D}\right) \tag{B12}$$

$$\frac{\partial|G|^2}{\partial C_x} = \frac{D(-2\Gamma_{ux} + 2|\Gamma_u|^2C_x) - N(-2\Gamma_{sx} + 2|\Gamma_s|^2C_x)}{D^2}$$

$$= 2|G|^2\left(\frac{(-\Gamma_{ux} + |\Gamma_u|^2C_x)}{N} - \frac{(-\Gamma_{sx} + |\Gamma_s|^2C_x)}{D}\right). \tag{B13}$$

Substituting the partials from (B8)-(B13) into (B7) gives an exact equation for the systematic error in $|G|^2$.

2. Approximate Solution.

An approximation for $d|G|^2$ can be obtained by assuming that the systematic error in measuring Γ_u of the DUT mount is the same as the systematic error in measuring Γ_s of the standard mount. That is,

$$d\Gamma_{ur} = d\Gamma_{sr} \equiv d\Gamma_r, \tag{B14}$$

and

$$d\Gamma_{ux} = d\Gamma_{sx} \equiv d\Gamma_x. \tag{B15}$$

Then (B7)-(B13) combine to give

$$\frac{d|G|^2}{2|G|^2} = \frac{(D-N)C_r + |C|^2(D\Gamma_{ur} - N\Gamma_{sr})}{DN}d\Gamma_r + \frac{(N-D)C_x + |C|^2(D\Gamma_{ux} - N\Gamma_{sx})}{DN}d\Gamma_x$$

$$+ \left(\frac{(D\Gamma_{ur} - N\Gamma_{sr}) + (D|\Gamma_u|^2 - N|\Gamma_s|^2)C_r}{DN}\right)dC_r$$

$$+ \left(\frac{(D|\Gamma_u|^2 - N|\Gamma_s|^2)C_x - (D\Gamma_{ux} - N\Gamma_{sx})}{DN}\right)dC_x. \tag{B16}$$

When

$$|\Gamma_u C| << 1, \quad \text{and} \quad |\Gamma_s C| << 1, \qquad \text{(B17)}$$

both D and N in (B16) can be set equal to 1 except in the difference D-N where second order terms must be retained. This reduces (B16) to

$$\frac{d|G|^2}{2|G|^2} \simeq [|C|^2(\Gamma_{ur} - \Gamma_{sr}) + (D\text{-}N)C_r]d\Gamma_r$$
$$+ [|C|^2(\Gamma_{ux} - \Gamma_{sx}) - (D\text{-}N)C_x]d\Gamma_x$$
$$+ [(|\Gamma_u|^2 - |\Gamma_s|^2)C_r + (\Gamma_{ur} - \Gamma_{sr})]dC_r$$
$$+ [(|\Gamma_u|^2 - |\Gamma_s|^2)C_x - (\Gamma_{ux} - \Gamma_{sx})]dC_x. \qquad \text{(B18)}$$

From (B6) we have

$$D\text{-}N = 2[C_x(\Gamma_{ux} - \Gamma_{sx}) - C_r(\Gamma_{ur} - \Gamma_{sr})] - |C|^2(|\Gamma_u|^2 - |\Gamma_s|^2).$$
$$= 2[C_x\Delta\Gamma_x - C_r\Delta\Gamma_r] - |C|^2\Delta|\Gamma|^2 \qquad \text{(B19)}$$

where

$$\Delta\Gamma_r \equiv \Gamma_{ur} - \Gamma_{sr}, \qquad \text{(B20a)}$$
$$\Delta\Gamma_x \equiv \Gamma_{ux} - \Gamma_{sx}, \qquad \text{(B20b)}$$
$$\Delta|\Gamma|^2 \equiv |\Gamma_u|^2 - |\Gamma_s|^2. \qquad \text{(B20c)}$$

Substituting (B19) and (B20) into (B18) gives

$$\frac{d|G|^2}{2|G|^2} = \{|C|^2\Delta\Gamma_r + 2[C_rC_x\Delta\Gamma_x - C_r^2\Delta\Gamma_r] - |C|^2\Delta|\Gamma|^2C_r\}d\Gamma_r$$
$$+ \{|C|^2\Delta\Gamma_x - 2[C_x^2\Delta\Gamma_x - C_rC_x\Delta\Gamma_r] + |C|^2\Delta|\Gamma|^2C_x\}d\Gamma_x$$
$$+ [\Delta|\Gamma|^2C_r + \Delta\Gamma_r]dC_r + [\Delta|\Gamma|^2C_x - \Delta\Gamma_x]dC_x,$$

$$= [2C_rC_x\Delta\Gamma_x + (C_x^2\text{-}C_r^2)\Delta\Gamma_r - |C|^2\Delta|\Gamma|^2C_r]d\Gamma_r$$
$$+ [2C_rC_x\Delta\Gamma_r - (C_x^2\text{-}C_r^2)\Delta\Gamma_x + |C|^2\Delta|\Gamma|^2C_x]d\Gamma_x$$
$$+ [\Delta|\Gamma|^2C_r + \Delta\Gamma_r]dC_r + [\Delta|\Gamma|^2C_x - \Delta\Gamma_x]dC_x. \qquad \text{(B21)}$$

If we keep only the most significant terms, (B21) reduces to the following approximate expression for the systematic error in $|G|^2$.

$$\frac{d|G|^2}{2|G|^2} \simeq [2C_rC_x\Delta\Gamma_x + (C_x^2\text{-}C_r^2)\Delta\Gamma_r]d\Gamma_r$$
$$+ [2C_rC_x\Delta\Gamma_r - (C_x^2\text{-}C_r^2)\Delta\Gamma_x]d\Gamma_x$$
$$+ \Delta\Gamma_r dC_r - \Delta\Gamma_x dC_x. \qquad \text{(B22)}$$

3. Approximate Upper Bound

An upper bound to (B22) can be obtained from the inequalities

$$|\Delta\Gamma_r| \leq |\Gamma_u| + |\Gamma_s|, \quad |\Delta\Gamma_x| \leq |\Gamma_u| + |\Gamma_s|, \tag{B24}$$

and

$$|2C_rC_x \pm (C_x^2 - C_r^2)| \leq \sqrt{2}|C|^2. \tag{B25}$$

Then (B22) can be written

$$\frac{d|G|^2}{2|G|^2} \leq [\sqrt{2}|C|^2(|d\Gamma_r| + |d\Gamma_x|) + |dC_r| + |dC_x|](|\Gamma_u| + |\Gamma_s|). \tag{B26}$$

4. Typical Values

Typical maximum values for the parameters are

$$|\Gamma_u| \leq 0.15, \quad |\Gamma_s| \leq 0.15, \quad |C| \leq 0.3. \tag{B28}$$

Typical systematic errors in Γ_r, Γ_x, C_r, and C_x due to sources other than the impedance standards and the connectors are

$$|d\Gamma_r| \simeq |d\Gamma_x| \simeq |dC_r| \simeq |dC_x| \simeq 0.0001. \tag{B29}$$

Subststuting (B28) and (B29) into (B26) gives

$$\frac{d|G|^2}{|G|^2} \leq 2[0.000025 + 0.00020](0.3) = 0.000135. \tag{B30}$$

5. Alternate Approximate Solution.

For the typical maximum values of Γ_u, Γ_s and C given in (B28) we have

$$\max|\Gamma_u C|^2 = \max|\Gamma_s C|^2 = 0.002. \tag{B31}$$

We can therefore make the assumption in (B5) that

$$|\Gamma_u C|^2 << 1, \quad \text{and} \quad |\Gamma_s C|^2 << 1. \tag{B32}$$

This assumption reduces (B5) to

$$|G|^2 \simeq 1 + 2[(\Gamma_{ur}-\Gamma_{sr})C_r - (\Gamma_{ux}-\Gamma_{sx})C_x], \tag{B33}$$

or

$$|G|^2 \simeq 1 + 2(\Delta\Gamma_r C_r - \Delta\Gamma_x C_x). \tag{B34}$$

Using this expression to obtain the partials in (B7), (B7) becomes

$$d|G|^2 = 2C_r d\Gamma_{ur} - 2C_x d\Gamma_{ux} - 2C_r d\Gamma_{sr} + 2C_x d\Gamma_{sx} + 2\Delta\Gamma_r dC_r - 2\Delta\Gamma_x dC_x. \tag{B35}$$

When we can assume that the systematic error in measuring Γ_u of the DUT mount is the same as the systematic error in measuring Γ_s of the standard mount, this equation reduces to

$$d|G|^2 \simeq 2\Delta\Gamma_r dC_r - 2\Delta\Gamma_x dC_x. \qquad \text{(B36)}$$

Then from (B36) and (B34),

$$\frac{d|G|^2}{|G|^2} \simeq \frac{2\Delta\Gamma_r dC_r - 2\Delta\Gamma_x dC_x.}{1 + 2(\Delta\Gamma_r C_r - \Delta\Gamma_x C_x)}. \qquad \text{(B37)}$$

For the values given in (B28) and (B29) this expression gives a worst case error of 0.000146 which agrees fairly well with the value of 0.000135 obtained from (B26).

Appendix C. Systematic Error in η_u Due to Systematic Error in $|\Gamma|$.

The purpose of this Appendix is to derive expressions for the error in M due to errors in $|\Gamma|$.

1. Exact solution.

From (10.6), M is defined as the following function of $|\Gamma|^2$.

$$M \equiv \frac{1 - |\Gamma_{\ell s}|^2}{1 - |\Gamma_{\ell u}|^2}. \tag{C1}$$

Let

$$\Gamma_{\ell u} \equiv \Gamma_{ur} + j\Gamma_{ux}, \quad \Gamma_{\ell s} \equiv \Gamma_{sr} + j\Gamma_{sx}. \tag{C2}$$

Then

$$M = \frac{1 - \Gamma_{sr}^2 - \Gamma_{sx}^2}{1 - \Gamma_{ur}^2 - \Gamma_{ux}^2}. \tag{C3}$$

Define the numerator of M as N and the denominator as D so that

$$M \equiv \frac{N}{D}. \tag{C4}$$

The systematic error in M can be written

$$dM = \frac{\partial M}{\partial \Gamma_{ur}} d\Gamma_{ur} + \frac{\partial M}{\partial \Gamma_{ux}} d\Gamma_{ux} + \frac{\partial M}{\partial \Gamma_{sr}} d\Gamma_{sr} + \frac{\partial M}{\partial \Gamma_{sx}} d\Gamma_{sx} \tag{C5}$$

where from (C3),

$$\frac{\partial M}{\partial \Gamma_{sr}} = -\frac{2\Gamma_{sr}}{D} = -M\frac{2\Gamma_{sr}}{N}, \tag{C6}$$

$$\frac{\partial M}{\partial \Gamma_{sx}} = -\frac{2\Gamma_{sx}}{D} = -M\frac{2\Gamma_{sx}}{N}, \tag{C7}$$

$$\frac{\partial M}{\partial \Gamma_{ur}} = \frac{N}{D}\frac{2\Gamma_{ur}}{D} = M\frac{2\Gamma_{ur}}{D}, \tag{C8}$$

$$\frac{\partial M}{\partial \Gamma_{ux}} = \frac{N}{D}\frac{2\Gamma_{ux}}{D} = M\frac{2\Gamma_{ux}}{D}. \tag{C9}$$

Substituting the partials from (C6)-(C9) into (C5) gives an exact expression for the systematic error in M.

2. Approximate Solution.

An approximation for dM can be obtained by assuming that the systematic error in measuring Γ_u of the DUT mount is the same as the systematic error in measuring Γ_s of the standard mount. That is,

$$d\Gamma_{ur} = d\Gamma_{sr} \equiv d\Gamma_r, \tag{C10}$$

and

$$d\Gamma_{ux} = d\Gamma_{sx} \equiv d\Gamma_x. \tag{C11}$$

Then (C6)-(C9) combine to give

$$\frac{dM}{2M} = \frac{-D\Gamma_{sr} + N\Gamma_{ur}}{DN} d\Gamma_r + \frac{-D\Gamma_{sx} + N\Gamma_{ux}}{DN} d\Gamma_x. \tag{C12}$$

When

$$|\Gamma_u|^2 \ll 1, \quad \text{and} \quad |\Gamma_s|^2 \ll 1, \tag{C13}$$

D and N defined by (C4) both reduce to approximately 1, and (C12) reduces to

$$\frac{dM}{M} = 2(\Delta\Gamma_r d\Gamma_r + \Delta\Gamma_x d\Gamma_x). \tag{C14}$$

where

$$\Delta\Gamma_r \equiv \Gamma_{\ell ur} - \Gamma_{\ell sr}, \tag{C15}$$

$$\Delta\Gamma_x \equiv \Gamma_{\ell ux} - \Gamma_{\ell sx}. \tag{C16}$$

Appendix D. Comparing Errors in Γ to Errors in C (or $-\Gamma_g$)

D1. Introduction.

The purpose of this Appendix is to show that when there are imperfections in the line standards used to calibrate the 6-port, these imperfections create errors in both the error-box parameter C (sometimes written $-\Gamma_g$) and also in the measured values of Γ of the DUT connected to the test port. The errors in C and Γ are therefore correlated, and this correlation must be taken into account when estimating errors in η_u/η_s.

This correlation creates a problem if we attempt to calculate the error in η_u due to errors on C and Γ from equation (2.9). It is possible to obtain unrealistically large estimated errors for η_u/η_s from (2.9) by choosing errors for C and Γ that give a worst case situation. The worst case usually comes about when the error in C has the opposite sign of the error in Γ, whereas in real situations the error in C has the same sign as the error in Γ, and these errors tend to cancel.

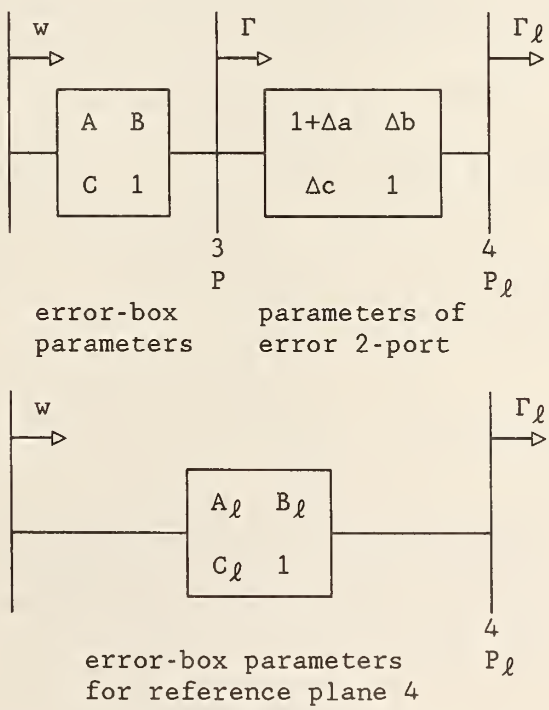

Figure D1. Equivalent circuit for the 6-port showing how the error box parameters A,B,C, which refer to reference plane 3 of the 6-port, and the parameters of the error 2-port combine to give a new set of error-box parameters A_ℓ, B_ℓ, C_ℓ that contain the imperfections of the line standard and its connectors, and refer to reference plane 4.

D2. Error in Γ.

As shown in figure D1, Γ and Γ_ℓ are related by the parameters of the error 2-port which in turn are functions of the imperfections in the line standard and its connectors.

$$\Gamma = \frac{(1+\Delta a)\Gamma_\ell + \Delta b}{\Delta c\Gamma_\ell + 1}. \tag{D1}$$

When $|\Delta c\Gamma_\ell|^2 << 1$ this becomes

$$\Gamma = [(1+\Delta a)\Gamma_\ell + \Delta b][1 - \Delta c\Gamma_\ell + \ldots]. \tag{D2}$$

If we discard terms that contain products or squares of Δ terms, (D2) becomes

$$\Gamma \simeq \Gamma_\ell + \Delta a\Gamma_\ell + \Delta b - \Delta c\Gamma_\ell^2. \tag{D3}$$

From (D3), the error in Γ due to imperfections in the connectors and the line is

$$\Gamma - \Gamma_\ell \simeq \Delta a\Gamma_\ell + \Delta b - \Delta c\Gamma_\ell^2. \tag{D4}$$

For the NIST 7 mm air line standards, the estimated maximum values of Δa, Δb, Δc at 18 GHz are given in (8.10) and are

$$|\Delta a| \simeq 0.0037, \quad |\Delta b| \simeq 0.0021, \quad |\Delta c| \simeq 0.0021. \tag{D5}$$

Typical maximum values for the Γ are

$$|\Gamma_u| \leq 0.15, \quad |\Gamma_s| \leq 0.15. \tag{D6}$$

Substituting the maximum values from (D5)-(D6) into (D4) shows that the most significant term in (D4) is Δb. Therefore

$$\Gamma - \Gamma_\ell \simeq \Delta b. \tag{D7}$$

D3. Error in C.

We next want to get an expression similar to (D4) or (D7) for the error in C. Returning to figure D1, note that the error-box parameters A, B, C of the 6-port and the parameters of the error 2-port combine to give a new set of error-box parameters A_ℓ, B_ℓ, C_ℓ that refer to reference plane 4. This new set of error-box parameters contains all the imperfections in the connectors and the line and is given by

$$\begin{bmatrix} A_\ell & B_\ell \\ C_\ell & 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & 1 \end{bmatrix}\begin{bmatrix} 1+\Delta a & \Delta b \\ \Delta c & 1 \end{bmatrix} \tag{D8a}$$

$$= \begin{bmatrix} A(1+\Delta a) + B\Delta c & A\Delta b + B \\ C(1+\Delta a) + \Delta c & C\Delta b + 1 \end{bmatrix}$$

$$= (1+C\Delta b)\begin{pmatrix} \dfrac{A(1+\Delta a) + B\Delta c}{1+C\Delta b} & \dfrac{A\Delta b + B}{1+C\Delta b} \\ \dfrac{C(1+\Delta a) + \Delta c}{1+C\Delta b} & 1 \end{pmatrix} \tag{D8b}$$

Comparing (D8a) and (D8b) shows that

$$C_\ell = \frac{C(1+\Delta a) + \Delta c}{1+C\Delta b}. \tag{D9}$$

When $|C\Delta b|^2 << 1$ this becomes

$$C_\ell = [C(1+\Delta a) + \Delta c][1 - C\Delta b + \ldots]. \tag{D10}$$

If we again discard terms that contain products or squares of Δ terms, (D10) becomes

$$C_\ell \simeq C + C\Delta a + \Delta c - C^2\Delta b. \tag{D11}$$

From (D11), the error in C due to imperfections in the connectors and the line is

$$C - C_\ell \simeq -(C\Delta a + \Delta c - C^2\Delta b). \tag{D12}$$

The maximum value for C is typically

$$|C| \leq 0.30$$

Substituting this maximum value into (D12) shows that the most significant term in (D12) is Δc. Therefore

$$C - C_\ell \simeq -\Delta c \tag{D13}$$

D4. Comparing Error in C to Error in Γ.

To compare the error in Γ to the error in C for a given imperfect line standard, expand Δb in (D7) and Δc in (D13) with their components from (7.5) to get

$$\Gamma - \Gamma_\ell \simeq \Delta b \simeq \Delta z - \Delta y - \Delta z_o \tag{D14}$$

and

$$C - C_\ell \simeq -\Delta c \simeq \Delta z - \Delta y + \Delta z_o. \tag{D15}$$

These equations show that the error in Γ and the error in C are not independent but are related approximately by

$$C - C_\ell \simeq \Gamma - \Gamma_\ell + 2\Delta z_o. \tag{D16}$$

Equation (D16) must be kept in mind when comparing the error in η_u/η_s calculated from (7.7) or (8.3) with that calculated directly from (2.9). From (7.7) we have

$$\frac{\Delta\eta_u}{\Delta\eta_s} = \frac{(1 - |\Gamma_{\ell u}|^2)(|1 + \Delta c\Gamma_{\ell s}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell s}|^2)}{(1 - |\Gamma_{\ell s}|^2)(|1 + \Delta c\Gamma_{\ell u}|^2 - |\Delta b + (1+\Delta a)\Gamma_{\ell u}|^2)} \qquad (7.7),(D17)$$

$$= 1 + \epsilon. \qquad (7.8),(D18)$$

Whereas from (5.8), (2.9), and (5.9)-(5.10) we have

$$\frac{\Delta\eta_u}{\Delta\eta_s} = \frac{\eta_u/\eta_s}{\eta_{\ell u}/\eta_{\ell s}} = \frac{N_u/N_s}{N_{\ell u}/N_{\ell s}} = \frac{\left|\frac{1 + C\Gamma_u}{1 + C\Gamma_s}\right|^2 \frac{1 - |\Gamma_s|^2}{1 - |\Gamma_u|^2}}{\left|\frac{1 + C_\ell\Gamma_{\ell u}}{1 + C_\ell\Gamma_{\ell s}}\right|^2 \frac{1 - |\Gamma_{\ell s}|^2}{1 - |\Gamma_{\ell u}|^2}}. \qquad (D19)$$

D5. An Example.

The problem is best illustrated by an example. Suppose we know that we have errors in $|\Gamma|$ and $|C|$ of about 0.005. If we were to use (D19) to calculate the error $\Delta\eta_u/\Delta\eta_s$ when

$$|\Gamma_u| \simeq |\Gamma_s| \simeq 0.2 \text{ and } |C| \simeq 0.1, \qquad (D20)$$

we would find that $\Delta\eta_u/\Delta\eta_s$ is a maximum of 0.0082 when the phase angles are such that

$$\Gamma_u = -j0.2, \quad \Gamma_s = +j0.2, \quad C = -0.1, \qquad (D21)$$

and

$$\Gamma_u - \Gamma_{\ell u} = \Gamma_s - \Gamma_{\ell s} = j0.005, \quad \text{and} \quad C - C_\ell = -j0.005. \qquad (D22)$$

If the error in η_u/η_s is calculated from (8.3) which is an approximation of (D17) we get

$$\epsilon = 4[\Delta\Gamma_r\Delta r - \Delta\Gamma_x\Delta x_o - \Delta|\Gamma|^2\Delta r] = 4\Delta\Gamma_x\Delta x_o = 4(0.4)\Delta x_o = 1.6\Delta x_o. \qquad (D23)$$

There is only one term in ϵ because both $\Delta\Gamma_r$ and $\Delta|\Gamma|^2$ given by (8.4) and (8.6) are 0 for the values assumed in (D21). To satisfy (D16) and (D22) requires that

$$\Delta z = \Delta y = 0, \quad \text{and} \quad \Delta z_o = j\Delta x_o = -j0.005, \qquad (D24)$$

so that $x_o = -0.005$. Substituting this value for x_o into (D23) gives ϵ=0.0080 which agrees with the worst case value of 0.0082 calculated from (D19). However, from (D14) and (D15) we can see that the errors in (D22) do not represent a realistic situation. A more likely situation is where x_o is negligible compared to Δz and Δy, and where Δz and Δy are mostly reactive. Then the errors in Γ are nearly equal to the error in C and these errors nearly cancel in (D19), which is what (7.7) or (8.3) also show.

To continue this example where the error in $|\Gamma|$ is known to be about 0.005, a more realistic approach would be to assume that the error in Γ is due primarily to a gap or discontinuities at the connector junction. We might assume that the normalized impedance Δz and admittance Δy are

$$\Delta z = \Delta r + j\Delta x = 0.001 + j0.006, \quad \Delta y = j0.001. \tag{D25}$$

Then from (D14), $|\Delta b|$ and the error in $|\Gamma|$ are 0.005, satisfying the original observation. However, the error in measuring ratios of effective efficiency as calculated from (8.3) is now <u>zero</u> for the values of Γ in (D21). If instead of (D21) we assume $\Gamma_u = -0.2$, and $\Gamma_s = +0.2$, ϵ is a maximum of 0.0016. This should be compared with the maximum worst case value of 0.0082 originally obtained from unrealistic values of Δz, Δy and Δz_o.

D6. Another Example.

Equation (8.3) indicates that ϵ can be 0.0080 also for the case when

$$\Gamma_u = -0.2, \quad \Gamma_s = +0.2, \quad C = \text{any value}$$

and

$$\Gamma_u - \Gamma_{\ell u} = \Gamma_s - \Gamma_{\ell s} = C - C_\ell = 0.005.$$

From (D14)-(D16), this can happen when $\Delta z = \Delta r = 0.005$, and $\Delta y = \Delta z_o = 0$. Equation (8.3) then becomes

$$\epsilon \simeq 4[\Delta\Gamma_r\Delta r - \Delta\Gamma_x\Delta x_o - \Delta|\Gamma|^2\Delta r] = 4\Delta\Gamma_r\Delta r = 1.6\Delta r = 0.0080.$$

However 0.005 is probably an unrealistically high value for Δr. From figure 7, Δr is probably less than 0.0003. Then $\epsilon = 0.00048$.

D7. Conclusion

If the error in measuring $|\Gamma|$ is some known value, say 0.005, then rather than calculate the maximum possible (but unrealisltic) worst case error in measuring ratios of effective efficiency from (2.9) or (D19), it is better to choose realistic values of Δz, Δy, and Δz_o that make Δb given by (D14) equal to 0.005. The error in measuring $|\Gamma|$ will then be 0.005, but the error in measuring ratios of effective efficiency as calculated from (8.3) will be considerably less than the maximum possible worst case error.

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4. TITLE AND SUBTITLE

Systematic Errors in Power Measurements Made With a Dual Six-Port ANA.

5. AUTHOR(S)

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6. PERFORMING ORGANIZATION (If joint or other than NBS, see instructions)

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
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9. SPONSORING ORGANIZATION NAME AND COMPLETE ADDRESS (Street, City, State, ZIP)

10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

The purpose of this report is to determine the systematic error in measuring power with a dual 6-port ANA. Most of the report concentrates on developing equations for estimating systematic errors due to imperfections in the test port connector, imperfections in the connector on the power standard, and imperfections in the impedance standards used to calibrate the 6-port for measuring reflection coefficient. These are the largest sources of error associated with the 6-port. For 7 mm connectors, all systematic errors which are associated with the 6-port add up to a worst-case uncertainty of ± 0.00084 in measuring the ratio of the effective efficiency of a bolometric power sensor relative to that of a standard power sensor.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

bolometric detectors; connector model; effective efficiency; error analysis; microwave; power measurements; six-port.

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