Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

July through September 1950

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES
July 1 through September 30, 1950

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NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

Projects and Publications
of the
NATIONAL APPLIED MATHEMATICS LABORATORIES

July through September 1950

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from July 1, 1950 through September 30, 1950.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J H Curtiss
Chief

E U Condon

Director
National Bureau of Standards
November 1, 1950

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

*New projects

E. Mathematical Tables

F. Manuals, Bibliographies, Indices and Technical Information

G. Computing Machine Development

*New projects

Index of Active Research and Development Projects

Announcement

THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)

The automatic digital computing machine which has been under construction at the Bureau's Institute for Numerical Analysis in Los Angeles, California, under task 1101-34-5103/49-1* was formally dedicated on August 17. This project was sponsored by the Office of Air Research of the Air Materiel Command, USAF. The official name of the machine is now the National Bureau of Standards Western Automatic Computer (SWAC)**.

The speakers at the dedication ceremony were Dr. E. U. Condon, director of the NBS; Colonel F. S. Seiler, chief of the Office of Air Research, Department of the Air Force; Dr. L. N. Ridenour, dean of the graduate School of the University of Illinois; Dr. J. H. Curtiss, chief of the National Applied Mathematics Laboratories; and Dr. H. D. Huskey, chief of the INA Machine Development Unit.

The dedication was followed by a symposium, on August 18, on applications of digital computing machines to scientific problems. The purpose of the symposium was to interchange information on various scientific problems which are being studied by West Coast laboratories and universities at the present time and to which high-speed automatic digital computing machinery may be applicable. The symposium was opened with introductory remarks by Dr. E. U. Condon, and by Dean L.M.K. Boelter of the College of Engineering, University of California at Los Angeles. Professor Boelter described the analogue computing machines operated by the Department of Engineering of UCLA. Dr. E. P. Little of the Office of Air Research was chairman of the morning session, and Dr. H. D. Huskey was chairman in the afternoon.

The following problems, of interest to their respective laboratories, were briefly outlined by the speakers indicated:

*See page
**See related tasks 1101-34-5103/50-1, 1101-34-5103/50-2, page 24 , and 1101-21-1102/50-10, page 22 .

Initiation of an airplane turn. Ellis Lapin, Douglas Aircraft Company, Inc.

Problems in water entry ballistics. E. P. Cooper, U. S. Naval Ordnance Test Station, Pasadena.

Reduction of measurements in free flight testing of missiles. Elmer Green, U.S. Naval Ordnance Test Station, Inyokern.

Solution of games by iterative processes. Paul Armer, Rand Corporation.

Nuclear reactor physics computations. Sidney H. Browne, North American Aviation, Inc.

The use of iterative processes in the solution of partial differential equations. Stanley Frankel, California Institute of Technology.

A problem of the naval air missile test center. L.H. Cherry, U.S. Naval Air Missile Test Center, Point Mugu.

Some problems in mathematical statistics. Jerzy Neyman, University of California, Berkeley.

An iterative construction of the optimum sequential decision procedure when the cost function is linear. Lincoln Moses, Stanford University.

Problems in pure mathematics. D. H. Lehmer, University of California, Berkeley.

On the Green's function of the clamped plate. Paul R.Garabedian, Stanford University.

Perturbations of a satellite rocket. Samuel Herrick, University of California, Los Angeles.

Physics research problems at Stanford susceptible to automatic computation. Paul H. Kirkpatrick, Stanford University.

An astronomical problem. Leland E. Cunningham, University of California, Berkeley.

Automatic computation in rocket engine research. H.L.Coplen, Aerojet Engineering Corporation.

The symposium was well attended, and considerable interest was displayed in the possibilities of machines like the SWAC.

The SWAC is an extremely fast automatically-sequenced general purpose electronic digital computer, operating in the parallel mode. Its development and construction was carried on under the direction of Dr. H.D.Huskey at the Institute for Numerical Analysis. The SWAC is to remain permanently at the Institute, where it is now being used to solve research problems of the INA staff, as well as problems of the United States Air Force and their contractors, and of other governmental agencies.

Commercially-produced cathode ray tubes are used as the storage element in the high-speed memory unit. This memory operates on a principle discovered by Professor F. C. Williams of Manchester University, England. The individual digits of the information are stored in the form of spots of charge, which are arranged in the form of a matrix on the face of the tube. Initially, 256 such spots are being stored on each tube, which makes the word length 37 binary digits. This memory unit has an extremely short access time, words being available from it in only 16/1,000,000 of a second.

The SWAC is capable of adding pairs of ten-digit numbers, in a binary representation, at the rate of 16,000 per second, and it can multiply such numbers at the rate of 2500 per second. These rates include the time it takes for the machine to receive the numbers from the memory, perform the required arithmetic operations upon them, return the result to the memory, and obtain the next instruction from it.

The SWAC performs the following operations through the insertion of a single command word: addition, subtraction, multiplication exact, multiplication rounded-off, comparison (both normal and absolute), extract, input, and output. There are also three special commands in addition, subtraction, and rounded-off multiplication which permit the transferring of the control to some point in the memory not in the regular sequence, thus taking the place of a transfer of control, or unconditional transfer, command. The comparison command determines the course of the computation depending upon the relative sizes of two numbers; thus, the machine may be said to possess a degree of choice. Extract divides numbers up into parts which the computer can then handle in different ways; it also provides for obtaining the logical product of two numbers and shifting the result an arbitrary amount.

Other arithmetic operations are accomplished by sequences of commands known as routines and subroutines. Routines and subroutines can be stored in the memory unit and are called from there into action by the use of special instructions. This procedure simplifies the process of preparing problems for the computer.

Problem preparation is further simplified by the storing of less frequently used routines on paper tape for insertion into the computer as needed. Thus, what might be thought of as a "library" of routines can be established for use by the machine whenever a problem needs to be solved in which one of these standard routines can be used.

A problem can be set up for solution on the SWAC simply by the insertion of a set of instructions (routines and subroutines) through the input-output unit. These instructions are stored in the high-speed memory unit along with the numbers.

Flexowriter units are used at present as the input and output devices. A flexowriter unit consists of electromatic typewriters, a teletype tape reader, and a teletype tape perforator. It is planned to supplement these units as soon as possible with a magnetic tape system.

All circuitry on the SWAC is on plug-in units, and there are spare plug-in units for about 80 percent of the chassis in the computer. This means that in the case of a failure of some component the faulty chassis can be removed and replaced by a spare one.

Components which are mass-produced commerically were used whenever possible in the SWAC, since they are expected to be more reliable, easily replaceable, and relatively economical.

The computer is compact, occupying about 50 square feet of floor space.

Final plans for the SWAC include an intermediate-speed auxiliary memory consisting of a magnetic drum, and a magnetic tape unit to serve as a slow-speed auxiliary memory. The magnetic drum for this purpose was constructed at the University of California at Berkeley under the direction of Professor Paul Morton. It has been delivered to the Institute, and at present the necessary circuitry and adjustments are being constructed to enable it to be incorporated into the SWAC memory system.

The drum will have the relatively large storage capacity of 8192 words, and an average access time of 8/1000 of a second. The slow-speed memory will have a capacity of about 100,000 words, and a word from it will be available on the average in $3\frac{1}{2}$ minutes. Delivery of the magnetic tape unit from the Raytheon Manufacturing Company has been promised for sometime in October. These three different types of memory units incorporated into one memory system will make possible the use of the SWAC for problems of great length and complexity.

Status of Projects

September 30, 1950

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-11-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

Publications: (1) "Matrix inversion by a Monte Carlo method," by G. E. Forsythe and R. A. Leibler; MTAC IV, No. 31, 127-129 (July 1950); reprints available. (2) "The extent of n random unit vectors," by G. E. Forsythe and J. W. Tukey; submitted to a technical journal for publication. (3) "A method of computing exact inverses of matrices with integer coefficients," by J. B. Rosser; accepted by NBS Journal of Research.

NUMERICAL METHODS IN CONFORMAL MAPPING
Task 1101-11-5100/49-CM1
(formerly 11.1/1-49-CM1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Managers: G. Blanch, L. K. Jackson
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The IBM calculations referred to in the Apr-June 1950 status report appeared to be consistent with the previous hand calculations, but did not seem to be converging to a reasonable answer. The results have been sent for analysis to Mr. Jackson (now with the University of Nebraska), with the hope that he will locate the trouble.

Publications: (1) "The construction and applications of conformal maps: Proceedings of a symposium," edited by E. F. Beckenbach; to be published in the NBS Applied Mathematics Series. The volume will include the following papers written in connection with this project: (i)"A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A. M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A. Weinstein. (2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W.Seidel. IN MANUSCRIPT at NAML; publication under consideration. (3) "Numerical methods in conformal mapping," by L. Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Task 1101-11-5100/49-ODE2
(formerly 11.1/1-49-ODE2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: W.E.Milne

Authorized 11/26/48
Terminated 9/30/50

Objective: In general, to develop and improve the techniques for the numerical integration of simple systems of ordinary differential equations. Specifically: (a) To investigate various devices to improve the accuracy of the step-by-step type of process; e.g., the use of quadrature formulas based on higher derivation of the unknown function in place of quadrature formulas using higher order differences or more lines in the table of the computation. (b) To determine optimum processes for presently proposed automatic digital computing machinery. (c) To obtain useful results concerning round-off and truncation error. (d) To develop improved methods for locating characteristic values.

Background: Ordinary differential equations are the mathematical model for problems in ballistics, celestial mechanics, control, circuit analysis, and many other important applied physical problems. A large number of formulas and procedures for numerical integration of such equations, all belonging to three or four main classes, have been developed in the past and an indefinite number of alternate formulas can be readily derived by similar methods. However, many problems involving choice and derivation of formulas and appraisal of error, which are essential to the application of a.d.c.m. to ordinary differential equations, still remain open.

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-1, "Studies in the numerical integration of differential equations," and is reported in its status.

STUDIES IN NUMERICAL INTEGRATION OF
PARTIAL DIFFERENTIAL EQUATIONS
Task 1101-11-5100/49-PD1
(formerly 11.1/1-49-PD1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: W. E. Milne

Authorized 3/1/49
Terminated 9/30/50

Objective: In general, to develop and improve the techniques for the direct numerical integration of partial differential equations. Specifically to investigate mathematical questions involved in the substitution of partial difference equations for partial differential equations, as for example, the determination of optimal forms of difference equations for a fixed number of terms, the development of best methods for handling curvilinear boundaries, and development and appraisal of remainder terms.

Background: Partial differential equations are the mathematical tools for attacking a wide variety of problems in physics and engineering. The literature pertinent to direct numerical integration of partial differential equations is at present incomplete and scattered. Much remains to be done to prepare this field for a.d.c.m.

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-1, "Studies in the numerical integration of differential equations," and is reported in its status.

Publications: See task 1101-11-5100/51-1.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL
EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Task 1101-11-5100/49-PM1
(formerly 11.1/1-49-PM1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Managers: R. P. Peterson, Jr. and J. H. Curtiss

Authorized 12/1/48
Terminated 9/30/50

Objective: To investigate methods of solving certain types of partial differential equations and integro-differential equations by sampling processes. A typical numerical technique consists in identifying the solution of the equation with the function specifying the distribution of distances traveled in a properly chosen statistical random walk or random flight problem and then computing this distribution function approximately by actually following through the life histories of a large number of random flights. The present investigation is aimed at determining the classes of problems and the types of a.d.c.m. for which such methods are more efficient than systematic quadrature methods, and answering related questions.

Background: The techniques referred to in the Objective are typical of those which are known as Monte Carlo methods and seem to have been suggested first by Ulam and von Neumann in an abstract in the Bulletin of the American Mathematical Society 53, 1120, (1947). They are natural ones to use in the study of diffusion problems, where the physical situation suggests directly the use of a random flight model.

Comments: As of the date of authorization, two computation projects involving the Monte Carlo method had already been worked on by the National Applied Mathematics Laboratories: project 11.2/33-48-17 (see Projects and Publications, Jan-Mar, Apr-Jun, July-Sept, Oct-Dec 1948), and project 11.1/31-49-10 (Projects and Publications Oct-Dec 1948).

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-2, "Probability methods and sampling techniques," and is reported under its status.

Publications: See task 1101-11-5100/51-2.

DETERMINATION OF EXTREMALS OF FUNCTIONALS
Task 1101-11-5100/50-1
(formerly 11.1/1-50-1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes

Authorized 7/1/49
Terminated 9/30/50

Objective: To study numerical and analytical methods of determining the maxima and minima of functionals; in particular, integrals of the types customarily studied in the Calculus of Variation.

Background: A substantial part of modern physical theory can be stated in terms of variational principles. In view of this fact, the Calculus of Variation is playing a continually expanding role in the solution of problems in applied physics. The existing literature on calculus of variation is extensive, but the numerical aspects have not kept pace with the theoretical ones.

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-3, "Variational methods," and is reported under its status.

Publications: See task 1101-11-5100/51-3.

DETERMINATION OF CRITICAL POINTS
Task 1101-11-5100/50-2
(formerly 11.1/1-50-2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes

Authorized 7/1/49
Terminated 9/30/50

Objective: To study numerical methods of determining critical points of functions of more than one variable, and in particular, to develop practical methods for locating maxima and minima of functions of many variables.

Background: Critical point determinations arise in the theory of aircraft design, the theory of games, and in many other physical and economic applications.

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-3, "Variational methods," and is reported under its status.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-11-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Work on sampling methods for obtaining eigenvalues is reported under task No. 1101-11-5100/51-2, "Probability methods and sampling techniques."

A report has been prepared on tests of methods for separating close eigenvalues of symmetric matrices. (see Projects and Publications Apr-Jun 1950). In addition to a detailed report on the methods of Lanczos and of Hestenes-Karush, there is a section describing how close eigenvalues may be separated by a modification of the classical power method. The report was being revised and typed as of the end of the quarter.

Prior machine and personnel commitments made it necessary to postpone further work on obtaining the eigenvalues of the 8 x 8 non-symmetric matrix with complex coefficients (see Apr-Jun 1950 status report).

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C. Lanczos; accepted by the NBS Journal of Research. (2) "Numerical determination of characteristic numbers," by W. E. Milne; NBS J. Res. 45, No. 3, 245-254 (Sept. 1950); reprints available. (3) "New matrix transformations for obtaining characteristic vectors," by W. Feller and G. E. Forsythe; accepted by the Quarterly of Applied Mathematics.

AN ITERATIVE SOLUTION OF FREDHOLM'S INTEGRAL EQUATION
Task 1101-11-5100/50-8
(formerly 11.1/1-50-8)

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: C. Lanczos

Authorized 11/1/49
Terminated 9/30/50

Objective: To develop a rapidly convergent approximation method for the solution of the general Fredholm type of integral equations.

Background: Fredholm's integral equation is fundamental for the general theory of ordinary differential equations. In spite of the very extensive literature on the analytical behavior of such equations, easily accessible methods of solution are not available. The field of non-symmetric kernels is particularly unexplored.

Comments: The present approach conceives the non-symmetry of the kernel function as the consequence of a skew-angular reference system in which the kernel is analyzed. The double set of biorthogonal functions associated with a non-symmetric kernel are then actually the covariant and contravariant representations of one single set of orthogonal functions. This viewpoint unifies the treatment of symmetric and non-symmetric kernels and the remarkable fact emerges that every non-symmetric kernel can be symmetrized in the proper reference system. In this system the matrix elements of the kernel are either real or purely imaginary numbers; they occupy the diagonal row plus one neighboring row on both sides. The transformation to the principal axes (if such axes exist) and the solution of the given integral equation is now easily accomplished.

Status: TERMINATED. Work performed this quarter has been integrated under the new task 1101-11-5100/51-4, "Studies in applied mathematics," and is reported under its status.

INTEGRAL TRANSFORMS AND NETWORK ANALYSIS
Task 1101-11-5100/50-9
(formerly 11.1/1-50-9)

Origin: NBS — Authorized 3/31/50
Sponsor: Office of Naval Research, USN — Revised 6/30/50
Manager: C. Lanzcos — Terminated 9/30/50

Objective: To replace the traditional expansion theorem of Heaviside, which requires the knowledge of the complex roots of an algebraic equation, by a simple algorithm which obviates the evaluation of the roots.

Background: A Laplace transform is usually inverted by integrating along the imaginary axis and then enclosing the path of integration through the negative infinite semi-circle. The integration is then reduced to loops around the singularities, which in the case of an electric network are simple poles. To obtain the position of these poles one must get the (generally complex) roots of an algebraic equation, which occasionally may be of rather high degree. This is frequently a cumbersome task. The present method uses conformal mapping by reciprocal radii, transforming the infinite imaginary axis into the unit circle. However, in contrast to the traditional procedure, it is the right half of the complex plane which is mapped inside the unit circle. The Laplace transform is regular in this region and allows expansion into a Taylor series around the center of the circle. Inverting this uniformly convergent series term by term we find that the original function f(x), which has the significance of the transient response (Green's function) of the network, is now expanded into a series of orthogonal Laguerre functions:

$$f(x) = \sum_{k=0}^{\infty} c_k \frac{L_k(2x)}{k!} e^{-x}$$

The coefficients c_k of this expansion are made readily available by a simple division algorithm. The convergence is such that 10 to 12 terms of the series will usually give a good overall representation of the transient response. The "noise" part of the response, which shows up in small humps of the curve and which is usually of minor physical significance, is automatically smoothed out in the representation. This method obviates the solution of an algebraic equation of high degree, and is particularly useful if we want to see how modification of a network parameter influences the transient response. Moreover, this method gives a new solution of the problem of network synthesis, when a network of given order is to be designed which shall approximate a given transient response as closely as possible.

Status: TERMINATED. Work performed this quarter has been integrated under task 1101-11-5100/51-4, "Studies in applied mathematics," and is reported under its status.

STUDIES IN MATHEMATICAL STATISTICS
Task 1101-11-5100/50-12
(formerly 11.1/1-50-12)

Origin: Office of Naval Research, USN — Authorized 6/1/50
Sponsor: " " — Terminated 9/30/50
Manager: A. Dvoretzky

Objective: To increase knowledge in those fields of mathematical statistics on which certain theories and practices in numerical analysis are based.

Background: The general expansion of scientific knowledge is the principal purpose of this project. There are, moreover, two active projects at the INA which will benefit especially from contributions to mathematical statistics, namely: project 11.1/1-49-PM1, and project 11.1/1-50-11.

Status: TERMINATED. Work performed this quarter has been integrated under task 1101-11-5100/51-2, "Probability methods and sampling techniques," and is reported there.

Publication: See task 1101-11-5100/51-2.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-11-5100/51-1

Origin: NBS — Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: F. John, W. E. Milne

Objective: To compare, develop, and improve the technique for numerical integration of differential equations.

Background: Differential equations are the mathematical tools for attacking a wide variety of problems in the physical sciences. The literature of numerical integration of differential equations is extensive, but particularly in the case of partial differential equations, it is still incomplete. Much remains to be done to prepare this field for automatic digital computing machinery.

Comments: This task is a continuation and generalization of tasks 1101-11-5100/49-ODE2 and 1101-11-5100/49-PD1.

Status: NEW. Upon his return to the Institute, Professor Milne resumed work on his monograph on the numerical solution of differential equations (see Jan-Mar 1949 issue of Projects and Publications). It is now planned that about one-third of the book will be devoted to the numerical solution of partial differential equations. The first draft is about 60 per cent completed.

Professor Milne has carried out a number of experimental numerical integrations of second-order differential equations with two-point boundary conditions.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of the Symposium held at Harvard Computation Laboratory, September 1949. (2) "Numerical solution of partial differential equation," by E. C. Yowell; to be published in the Proceedings, Scientific Computation Forum, held under the auspices of the I.B.M. Corporation in Endicott, N.Y., November 1949.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-11-5100/51-2

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: W. Wasow

Objective: To study and develop the use of statistical methods in numerical analysis; in particular, to investigate the use of random sampling methods ("Monte Carlo Methods") for solving equations involving linear operators, and for finding eigenvalues of linear operators.

Background: Numerical techniques based on statistical considerations are suggested naturally in certain cases by the underlying physical situation; for example, diffusion and transport problems. The techniques are interesting for themselves alone, as they are sometimes founded rather basically on the relationship between probability theory and functional equations. It is expected that in selected problems, these techniques may be particularly useful in connection with automatic digital computing machinery.

Comments: This task is a continuation and generalization of 1101-11-5100/49-PM1.

Status: NEW. Some methods for the solution of differential equations by random sampling were tested by Dr. Acton by applying them to the differential equation problem $y'' + 0.01\, xy = 0, y(0) = 0,\ y(10) = 1$. One of the questions under investigation was the efficiency of so-called "importance sampling". The work, carried out on IBM equipment, is near completion. A tentative conclusion is that for this type of problem importance sampling may not be a useful technique because the mean duration of the random walk may be increased to where it offsets the reduction in the number of samples.

Mr. Cutkosky developed a method for solving Fredholm integral equations by random sampling. The procedure is a modification of a sampling method for inverting matrices, described in a note by G.E.Forsythe and R.A.Leibler (MTAC, July 1950). Experiments are underway in which this method is applied to an integral equation equivalent to the differential equation problem of the preceding paragraph.

Under the direction of Dr. Kac the efficiency of the method of Kac and Donsker for the calculation of the lowest eigenvalue of the Schrodinger equation was explored. The calculations were carried out on IBM equipment by Dr. Yowell and Mr. M. Cohen. The results obtained for the harmonic oscillator and the hydrogen atom were very satisfactory, but for the helium atom the accuracy achievable on IBM equipment turned out to be insufficient for physical applications. The calculations were based on a sample of 900 random walks of 100 steps each in 6 dimensions. Normal random deviates were used.

Dr. Peterson gave a method for solving the initial value problem for the wave equation by random walks.

Dr. Wasow's investigations on the duration of random walks were continued. Asymptotic formulas for the moment generating function of the duration were derived which yield estimates for the relative error and for the probability of very long walks.

In publication (8) Drs. Dvoretzky and Erdos consider the symmetrical random walk in the lattice points of d-dimensional Euclidean space. Since such random walks are basic to Monte Carlo solutions of partial differential equations in d dimensions their investigation is essential ground work for numerical analysis. Two classes of problems are treated independently: (a) How many <u>different</u> lattice points does the random walk pass through in n steps? (b) How fast does a point walking randomly in d-space ($d \geqq 3$) escape to infinity? The answer to (a) is found to be approximately $\pi n/\log n$ when $d = 2$, and approximately $n \gamma_d$ when $d \geqq 3$, where $0 < \gamma_d < 1$. Even a strong law of large numbers is found to hold here. Concerning (b) a typical result is that in 3 dimensions for all $\epsilon > 0$, with probability 1, as $n \to \infty$,

$$(*) \qquad (\log^{-1-\epsilon} n)\sqrt{n} = \Theta(\| S(n) \|),$$

where S(n) is the distance of the point from the origin, while for $\epsilon = 0$ (*) is false.

In order to use the SWAC for Monte Carlo sampling processes (for example, in connection with task 1101-11-5100/49-PM1), it will probably be necessary to generate so-called random digits on the SWAC.

Being generated by and for binary (base 2) operations, these methods must be adapted to the binary system, whereas most previous efforts have dealt with the decimal system. Several processes have been proposed for the rapid generation of long sequences of 0s and 1's, of which the following are two examples: (a) One calculates the residues (modulo a) of the numbers $m, m^2, m^3, \ldots$, where m is a fixed integer and $a = 2^{36}$ or $2^{36} + 1$. For a favorable choice of m, this process has a period P of 2^{35} ($a = 2^{36}$) or 581,040 ($a = 2^{36} + 1$). A suitable selection of digits from each m^r ($r = 1, 2, \ldots, N$, where $N << P$) is proposed as a source of random digits. (This method was mentioned by D.H. Lehmer at Harvard in 1949.) (b) A second method is suggested by Dr. Rosser. Let $f_i(k)$ assume the values 0 or 1, where $k = 1,2,\ldots, n_i$, $i = 1,2,\ldots,r$. Assume $n_1,\ldots,n_r$ to be relatively prime. Let each $f_i(k)$ be extended <u>periodically</u> with period n_i over the domain $k = 1,2,3,\ldots$. Now let $F_k = \sum_{i=1}^{r} f_i(k)$ (modulo 2), for $k = 1,2,\ldots$. The sequence F_k is periodic with period $n_1.n_2\ldots n_r$. When $r = 4$, $n_1 = 35$, $n_2 = 34$, $n_3 = 33$, $n_4 = 31$, for example, the period is 1,217,370. If the $n_1 + \ldots + n_r$ values of the functions $f_1,\ldots,f_r$ are taken from a random digit table, and if one uses only a fraction of a full period of the F_k, the process may yield random digits. (c) It is also proposed to apply the method (b) to two or more sequences of digits arising from method (a).

1,217,370 digits have been computed on the SWAC by process (b). They are now being studied, to see how random the digits are. It is expected that other methods will be tested, and that a major test will be the use of digits in the estimation of the lowest eigenvalue of the Schrödinger equation for the harmonic oscillator,

$$\tfrac{1}{2} y'' - x^2 y = - \lambda y,$$

by Professor Kac's method.

Publications: (1)"Random walks and the eigenvalues of elliptic difference equations," by Wolfgang Wasow; accepted for publication in the NBS Journal of Research. (2) "Sampling methods applied to differential and difference equations, with special reference to equations of the elliptic type," by J. H. Curtiss; to appear in the Proceedings, Scientific Computation Forum, held under the auspices of the IBM Corporation in Endicott, N. Y., November 1949. (3) "The Monte Carlo Method: Proceedings of a symposium, held in Los Angeles, June 1949," to appear as NBS Applied Mathematics Series 12; in press. (4) "A Monte Carlo method for solving a class of integral equations," by R. E. Cutkosky; IN MANUSCRIPT. (5) "On some connections between probability theory and differential and integral equations," by Mark Kac; to be published in the Second Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, California. (6) "On the mean duration of random walks," by W. Wasow; accepted by the NBS Journal of Research. (7) "Elimination of randomization in certain statistical decision procedures and zero-sum two-person games," by A. Dvoretzky, A. Wald, and J. Wolfowitz; submitted to a technical journal for publication. (8) "Some problems on random walk in space," by A. Dvoretzky and Paul Erdos; to be published in the Second Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, California, 1950. (9) "Various techniques used in connection with random digits," by John von Neumann (summary written by G.E.Forsythe); to be included in item (3) above. (10) "Generation and testing of random digits at the National Bureau of Standards, Los Angeles," by G.E.Forsythe; to be included in item (3) above.

VARIATIONAL METHODS
Task 1101-11-5100/51-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes

Authorized 9/1/50

Objective: To develop the applications of variational methods in the numerical determination of eigenvalues, critical points of functions, extremals of functionals, and in other problems in numerical analysis.

Background: A substantial part of modern physical theory can be stated in terms of variational principles. The existing literature on variational methods is extensive, but the numerical methods have not kept pace with the theoretical ones.

Comments: This task replaces 1101-11-5100/50-1 and 1101-11-5100/50-2. Some of the work previously reported under 11.1/1-50-3 (now 1101-11-5100/50-3) will henceforth be described under this task.

Status: NEW. The Hestenes method of gradients for solving the finite matrix equation $Ax = \lambda x$ when $A = A^T$, originally described in Projects and Publications, Oct-Dec 1949, page 7, has been developed into a practical machine method. For an initial value of x_n one computes the Rayleigh quotient $\mu(x_n)$, where $\mu(x) = (x^TAx)/(x^Tx)$, and the gradient $\mathfrak{z}(x_n)$, where $\mathfrak{z}(x) = Ax - \mu(x)x$. When looking for the least eigenvalue of A, one next obtains an improved value x_{n+1} by the formula $x_{n+1} = x_n - \alpha_n \mathfrak{z}(x_n)$ where $\alpha_n > 0$ is a scalar. Since x_{n+1} is proportional to $(A - \beta_n I)x_n$, where β_n is a constant, the method is essentially a modification of the classical iteration $x_{n+1} = A^n x_1$. The art of the method lies in proper selection of the constants α_n; this is treated theoretically in publication (1). Dr. Karush and Mr. Hayes have acquired considerable practical experience with matrices of orders 6 to 12 on the IBM Card Programmed Calculator. Further eigenvalues are found by similar iterations, in which one intermittently subtracts the components in the directions of known eigenvectors. A code has been prepared for the SWAC (using only the high-speed memory) for matrices of order 12. Matrices of higher orders may be treated by minimization of $\mu(x)$ in 12 dimensions at a time.

In publication (2) Karush has extended the gradient method to formulas of the type

$$x_{n+1} = x_n + \alpha_1 A x_n + \alpha_2 A^2 x_n + \ldots + \alpha_j A^j x_n.$$

Such a method is useful for dealing with j nearly equal eigenvalues.

A related but more difficult problem is to solve the finite matrix equation $Ax = \lambda Bx$, where $A = A^T$ and where B is a positive definite matrix. The theory is treated in publication (4), in which the gradient method is generalized. Machine experiments in progress suggest that the method works, but with slow convergence.

The methods have been considered in the infinite case, where one wants the spectrum of the function equation $Ax = \lambda Bx$ in Hilbert space, where A is a positive-definite self-adjoint operator and B is a positive, completely continuous self-adjoint operator. Karush showed that the methods derived in publication (4) can be extended to obtain the extremities of the spectrum in the infinite case. The other result described in publications (1), (2), (4) have in the main been extended by Hestenes to the infinite case. A report is in preparation. These results are applicable to self-adjoint systems of ordinary and partial differential equations and to integral equations with a symmetric kernel. Numerical experiments are being planned.

Publications: (1) "The method of gradients for the calculation of roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; accepted by the NBS Journal of Research. (2) "An iterative method for finding the characteristic vectors of a symmetric matrix," by W. Karush; submitted to a technical journal for publication. (3) "Applications of the theory of quadratic forms in Hilbert space to the calculus of variations," by M. R. Hestenes; submitted to a technical journal for publication. (4) "The solutions of $Ax = \lambda Bx$," by M. R. Hestenes and W. Karush; IN MANUSCRIPT.

STUDIES IN APPLIED MATHEMATICS
Task 1101-11-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: C. Lanczos

Objective: To develop and improve the mathematical tools and models available to workers in physical sciences.

Background: Analytical solutions are very often more satisfying and more revealing than purely numerical solutions. A research organization devoted to the development of numerical methods, such as the Institute for Numerical Analysis, cannot afford to neglect possible analytical approaches. The purpose of this project is to take due cognizance of this fact by maintaining at the Institute a certain amount of research in classical analytical applied mathematics in parallel with the investigations into numerical methods. Furthermore, it is one of the stated functions of the Institute to provide consultation services in applied mathematics, and this project complements these services.

Comments: This task is a continuation and generalization of tasks 1101-11-5100/50-8 and 1101-11-5100/50-9.

Status: NEW. A brief table of the Laguerre functions was computed which will be sufficient for the average demands of electrical engineering. An investigation is in progress concerning the application of the method to the evaluation of the complex roots of an algebraic equation.

Solution of Hallen's integral equation: One of the most interesting mathematical problems of electrical engineering is the radiation of a cylindrical antenna. This problem is equivalent to the solution of an integral equation of finite domain and a kernel which is a function of $|x - \xi|$ only. The original kernel is not "smooth" and not well suited to analytic procedures. Two integrations smooth the kernel and make possible an effective approximation by exponential functions. After this approximation the problem is explicitly solvable in exponentials. The current distribution appears as a superposition of four waves with complex frequencies and amplitudes. Calculation of the radiation resistance and comparison with the experimental results of D. D. King and the theoretical results of R. King and S. A. Schelkunoff are in progress.

Equidistant curve fitting by Chebyshev polynomials: The project manager is developing a method of curve fitting which combines the advantages of the Chebyshev polynomials with an equidistant spacing of the data. Let $y = f(x)$ be a function whose second derivative is of bounded variation, considered in the range $0 \leq x \leq 1$. It is known from the nature of trigonometric interpolation that $f(x)$ can be approximated with ever increasing accuracy, as $n \to \infty$, by a polynomial of order n which interpolates $f(x)$ at the zeros of $T_{N+1}(x)$, where $T_k(x)$ are the Chebyshev polynomials, normalized to the range 0 to 1.

This method of curve fitting, characterized by very satisfactory convergence, has the disadvantage that the data $y_k = f(x_k)$ have to be given at the non-equidistant points

$$x_k = \frac{1 + \cos \frac{k\pi}{n}}{2}$$

On the other hand, Lagrangian interpolation at equidistant points has the disadvantage that the resulting expansion may not converge to $f(x)$, even if $f(x)$ is analytic in the entire range. An added difficulty arises if it is required to fit by the method of least squares an empirical function given by equidistant observations. We frequently do not know in advance how to choose the order of the least square polynomial which will not distort the true course of the function and yet will not yield too much to the scattering of the data caused by random errors.

The present method gives a possible answer to these problems, by introducing the trigonometric functions as an intermediary frame of reference. Let us replace $f(x)$ by

$$g(x) = f(x) - f(0) + [f(0) - f(1)]\, x \,, \qquad (1)$$

thus reducing the problem to the curve fitting of a function which vanishes at the two endpoints $x = 0$ and $x = 1$. We now expand $g(x)$ into a finite sine series

$$g_m^{(n)}(x) = \sum_{k=1}^{m} a_k^{(n)} \sin k\pi x, \quad (m \leq n - 1), \tag{2}$$

where the $a_k^{(n)}$ are the coefficients for interpolation of $g(x)$ by a sine polynomial of order $n - 1$:

$$a_k^{(n)} = \frac{2}{n} \sum_{\alpha=1}^{n-1} g\left(\frac{\alpha}{n}\right) \sin \frac{\alpha k}{n} \pi . \tag{3}$$

In the case of an empirical function the data $y_k = f(k/n)$ themselves decide how far to go in the sine series (2). For all sufficiently large n and $k << n$ the coefficients $a_k^{(n)}$ decrease with the speed k^{-3}, as far as the true course of the function is concerned. Actually in practice it will usually be observed that from a certain $k = m$ on, the a_k become stationary in order of magnitude, instead of decreasing still further. This is the point where the "noise" part of the measurements drowns out the true course of the function. Neglecting all the $a_k^{(n)}$ from here on automatically smoothes the function and estimates the inherent accuracy of the data.

We now approximate $g_m^{(n)}(x)$ by a least square Chebyshev series of order q:

$$\bar{g}_{m,q}^{(n)}(x) = \tfrac{1}{2} c_{m,o}^{(n)} + \sum_{k=1}^{q} c_{m,k}^{(n)} T_k(x) \tag{4}$$

where

$$c_{m,k}^{(n)} = \frac{2}{\pi} \int_0^{\pi} g_m^{(n)} \frac{1 + \cos\theta}{2} \cos k\theta \, d\theta . \tag{5}$$

From (5) we find that

$$c_{m,k}^{(n)} = \sum_{j=1}^{m} a_j^{(n)} \rho_{jk} \tag{6}$$

where

$$\rho_{jk} = 0, \qquad \text{for } j + k = 2p,$$

$$\rho_{jk} = 2(-1)^p J_k\left(j \frac{\pi}{2}\right), \quad \text{for } j + k = 2p + 1 . \tag{7}$$

$J_k(x)$ is the Bessel function of the order k.

The minimum order q of the approximating polynomial (4) is decided by forming the values $g_{m,q}^{(n)}(\alpha/n)$. If these values deviate from the given $g(\alpha/n)$ by less than a conveniently chosen statistical measure of the scatter of the data, we have obtained a polynomial which approximates $g(x)$--and finally $f(x)$--within the desired accuracy with a minimum number of terms.

The tabulation of the ρ_{jk} and application of the method to numerical examples is in progress.

Publications: (1) A manuscript on the antenna problem is in preparation for the Proceedings of the IRE.

(2) Dr. Lanczos has prepared an introduction on the applications of the tables of Chebyshev polynomials to be included in NBS

Applied Mathematics Series 9, "Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$;" now in press. See task 1102-21-1104/50-3a, page

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-11-5100/51-5

Origin: Office of Naval Research, USN — Authorized 9/1/50
Sponsor: " "
Manager: D. Saxon

Objective: To increase knowledge in theoretical physics, and particularly in those fields which furnish substantial problems for numerical analysis.

Background: The general expansion of scientific knowledge is a principal purpose of this project. In addition, a great many of the NAML mathematical research projects deal with computations which arise in theoretical physics, and it is considered desirable to conduct various researches into the bases of these projects.

Status: NEW.

STUDIES IN PURE MATHEMATICS
Task 1101-11-5101/50-4
(formerly 11.1/1-50-4)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In publication (10) Dr. Dvoretzky and his collaborators define $\{s\} = \mathcal{S}$ to be a Borel ring of subsets of $\{x\} = X$. Let $\mu_1, \ldots, \mu_r$ be atomless measures defined over $\mathcal{S}$, and let $\eta_1(x), \ldots, \eta_n(x)$ be non-negative, $\mathcal{S}$-measurable functions satisfying $\eta_1(x) + \ldots + \eta_n(x) = 1$. The author's main result asserts that X can be decomposed into disjoint sets $S_1, \ldots, S_n$ so that

$$\int_X \eta_j(x) \; d\mu_k(x) = \mu_k(S_j), \; (j = 1, \ldots, n; \; k = 1, \ldots, r) \; .$$

Applications to statistics and the theory of games are indicated, and are more fully treated in publication (7) under task 1101-11-5100/51-2.

In publication (11) Dr. Szasz considers

$$J_\alpha(t) = \left(\frac{t}{2}\right)^\alpha \sum_{n=0}^{\infty} \frac{(-1)^n \, t^{2n}}{4^n n! \, \Gamma(\alpha + n + 1)} ,$$

the Bessel function of order α ($\alpha > -1$). He puts

$$\Lambda_\alpha(t) = \left(\frac{2}{t}\right)^\alpha \Gamma(\alpha + 1) \; J_\alpha(t) \; ,$$

and denotes the relative maxima of $|\Lambda_\alpha|$ by $\mu_{r,\alpha}$. He then proves the following results:

(a) $\mu_{1,\alpha} > \mu_{2,\alpha} > \cdots$, for $\alpha > -\frac{1}{2}$;

(b) $\mu_{1,\alpha} < \mu_{2,\alpha} < \cdots$, for $-1 < \alpha < -\frac{1}{2}$;

(c) $\mu_{r,\alpha} > \mu_{r,\alpha+1} > \cdots$, for $\alpha > -1$, $r \geq 1$.

In publication (12) Dr. Szasz lets $\mu_{r,n}$ $(n \geq r \geq 0)$ be the absolute value of the $(r + 1)$-th relative extremum (as x decreases from $+\infty$ to 0) of the n-th Hermite orthogonal function $\phi_n(x)$. The author proves that $\mu_{r,n} > \mu_{r,n+1}$ for $n \geq r$. Analogous inequalities for Legendre, Laguerre, and ultraspherical polynomials have been previously proved by Szegö, J. Todd, and the author. As a by-product Dr. Szasz establishes a conjecture of G.E.H. Reuter.

In publication (13) Dr. Erdös deals with two sets of positive numbers c_k and f_k, where $\sum_{k=1}^{\infty} c_k = 1$. The numbers are connected by a recurrence formula,

$$f_1 = 1, \quad f_n = \sum_{k=1}^{n-1} c_k f_{n-k} ,$$

which has various applications in the theory of probability. Under certain assumptions it is proved that there exists an asymptotic relation between the partial sums $\sum_{k=1}^{n} c_k$ and $\sum_{k=1}^{n} f_k$, with an error term. The assumptions are of Tauberian type. The method is based on discussing the associated power series $\sum c_k x^k$ and $\sum f_k x^k$.

Publications: (1) "Gibbs phenomenon for Hausdorff means," by O. Szasz; accepted for publication in the Transactions of the American Mathematical Society. (2) "A generalization of S. Bernstein's polynomials to the infinite interval," by O. Szasz; accepted by the NBS Journal of Research. (3) "On some trigonometric transforms," by O. Szasz; for publication in the Pacific Journal of Mathematics. (4) "On positive harmonic functions and ultraspherical polynomials," by W. Seidel and O. Szasz; for publication in the Journal of the London Mathematical Society. (5) "On subharmonic and linear functions of two variables," by E. F. Beckenbach; submitted to Revista, Universidad Nacional de Tucuman (Argentina). (6) "Certain Fourier transforms of distributions," by E. Lukacs and O. Szasz; submitted to a technical journal for publication. (7) "On a Tauberian theorem for Abel summability," by O. Szasz; submitted to a technical journal for publication. (8) "On the relative extrema of ultraspherical polynomials," by O. Szasz; submitted to a technical journal for publication. (9) "Tauberian theorems for summability (R_1)," by O. Szasz; submitted to a technical journal. (10) "Relations among certain ranges of vector measures," by A.Dvoretzky; submitted to a technical journal. (11) "On relative extrema of Bessel functions," by O. Szasz; submitted to a technical journal. (12) "On the relative extrema of the Hermite orthogonal functions," by O. Szasz; submitted to a technical journal. (13) "On a recursion formula and on some Tauberian theorems," by Paul Erdös; accepted by the NBS Journal of Research.

COMPUTATION OF THE IMAGINARY ZEROS OF THE RIEMANN ZETA-FUNCTION
Task 1101-11-5101/50-13
(formerly 11.1/1-50-13)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. In preparation for the eventual computation of roots of the zeta function, two additional terms of a certain asymptotic series were determined, with an explicit upper bound for the error. The treatment was generalized so as to facilitate the determination of additional terms with an explicit bound for the error. The terms which we now have depend on a certain function and its derivatives, and for convenience in computation, these should be tabulated. A procedure for tabulating these was devised, involving determination of the coefficients and rate of convergence of certain power series. Reports on these matters will be forthcoming in time for use with the computations.

The coding of this problem for the SWAC is approximately 25 per cent complete.

2. Applied Research

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-21-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A seminar is being held on this subject at the INA. Efforts are being made to compile a library of pertinent literature in the field, and to contact persons from other organizations interested in the subject. Among the persons not on the INA staff who presented material on this subject or who were consulted during the quarter were: Dr. A. W. Tucker of Princeton and of Stanford Universities, Dr. Olga Todd of NBS, Dr. T. C. Koopmans of the Cowles Commission, Dr. E. W. Barankin of the University of California at Berkeley, Dr. M. Dresher and Dr. George Brown both of Rand Corporation. Dr. T. S. Motzkin joined the INA staff on September 12, and will devote full time to this project.

LANGUAGE TRANSLATION STUDY
Task 1101-21-1102/50-10
(formerly 11.1/1-50-10)

Origin: NBS Authorized 6/15/50
Manager: H. D. Huskey

Objective: To determine the feasibility and practicalness of doing substitution problems on automatic computing machines with a view to ascertaining the optimum type of automatic computer for this purpose. Particular emphasis will be given to the problem of the automatic translation of languages.

Background: Considerable thought has been given to this project by the project manager. Queries to various governmental agencies have confirmed the belief that there is a substantial need for faster and more economical methods of translation than are in use at present. Vast amounts of foreign data, which are not fully utilized by scientists and others due to the fact that no translations are available, have been continually accumulating. If it is shown that translation of this material by machines is practical then a satisfactory solution to the problem will have been found.

Status: CONTINUED. For the initial exploratory period of this project it was decided to attempt to determine the feasibility of the automatic substitution-translation of mathematical German by a computer such as the SWAC. Accordingly, Professor V. A. Oswald of the UCLA Department of German was engaged to work part-time on this subject during the summer months. Mr. S. Fletcher, a graduate student in the same department, was employed full time on the project during the summer as Professor Oswald's assistant. Professor Oswald and Mr. Fletcher made a detailed analysis of the problem connected with this phase of the project, attempting primarily to provide a rigidly mechanical solution of syntactical problems and of lexicographical problems which are inevitably bound up with syntax. The results of their studies were summarized in a report which is now being prepared for publication.

Formal work on the project by Professor Oswald and Mr.Fletcher terminated on September 15. However, due to their interest and belief in the possibilities of the project they are participating in a seminar on the subject conducted by the project manager. Other UCLA linguistic experts who are informally participating in the seminar are Professor W. E. Bull of the Spanish Department who has done considerable research on word counts of various languages, Professor H. Hoijer of the Department of Anthropology and Sociology, and Professor K. E. Harper of the Department of Slavic Languages who is an expert in the Russian language.

3. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(Previously listed as Air Materiel Command Computing Machine)
Task 1101-34-5103/49-1
(formerly 11.1/22-49-1)

Origin: Office of Air Research, AMC, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The fabrication and final assembly of the computer was completed at the beginning of the quarter. Formal dedication took place on August 17, followed by a symposium on the applications of digital computing machinery to scientific problems on August 18, and an open house at the Institute on August 19.

Testing of the various operations to be performed by the computer continued, interspersed with miscellaneous mathematical exercises and problems. In particular, work was done on a problem of Dr. Kac's; see task 1101-11-5100/51-2 (which continues 11.1/1-49-PM1), and 11.1/1-49-PM2 on page 6 of Oct-Dec 1949 issue.

Work continued on the design and construction of the chassis needed to integrate the magnetic drum and magnetic tape unit into the SWAC computer system. Arrangements were made to have the necessary reading heads for the drum fabricated at the University of California at Berkeley (where the magnetic drum was designed and constructed under the direction of Professor P. Morton). Delivery early next quarter has been promised on the magnetic tape unit ordered from the Raytheon Manufacturing Company to provide faster input-output for the SWAC.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-34-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Distribution of the preliminary lists of terminology and block diagram symbols to be used in connection with automatic computing machines continued. Comments on these lists are being continually received, and it is planned to reissue the lists taking into consideration all such comments, as soon as it seems feasible to do so. The preliminary lists are obtainable in manuscript form from the INA.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Task 1101-34-5103/50-1
(formerly 11.1/22-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

Publication: "Semi-automatic instruction on the Zephyr" by H. D. Huskey; to be published in the Proceedings of the Symposium held at the Harvard Computation Laboratory, September 1949.

PROGRAMING AND CODING OF PROBLEMS FOR SOLUTION ON THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER
Task 1101-34-5103/50-2
(formerly 11.1/22-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Many routines were coded for testing the various functions of the computer, singly and in various combinations. In addition, the following routines were prepared during the quarter:

(1) Several routines for generating random digits, their sums, and the distribution of their sums. The method of generating the digits is a new one proposed by Dr. J. B. Rosser. See task 1101-11-5100/51-2 (which is a continuation of 11.1/1-49PM1), and 11.1/1-49-PM2 on page 6 of Oct-Dec 1949 issue.

(2) Finding prime factors and testing integers for primeness.

(3) Solving any set of three linear simultaneous equations.

(4) The determination of eigenvectors of symmetric matrices.

(5) Table of sines and cosines.

(6) An improved formula for the computation of the imaginary zeros of the Riemann-zeta function (see task 1101-11-5101/50-13).

4. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-53-1100/49-1
(formerly 11.1/32-49-1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Among the problems involving the use of IBM equipment were the following:

(1) Computations relating to the eigenvalues of the helium atom, for M. Kac. (Task 1101-11-5100/51-2, which is a continuation of 11.1/1-49-PM1).

(2) Random walk problem, for R. Peterson. (Task 1101-11-5100/51-2, which is a continuation of 11.1/1-49-PM1).

(3) Problem in conformal mapping to test a method suggested by Ahlfors. (Task 1101-11-5100/49-CM1).

Shorter computations on desk calculators were made for various members of the staff. Among such calculations were the following:

For J. B. Rosser: (a) Extension of his work on the approximations of $\pi(x)$ and $\theta(x)$ for x beyond a million, in the range of Lehmer's table of primes; (b) Checking his equations and making computations relating to the roots of the zeta function on the line $x = \frac{1}{2}$.

For C. Lanczos: Computations relating to the theory of integral equations. (Task 1101-11-5100/51-4, which is a continuation of 11.1/1-50-8).

For H. Gruen: Computation of elements of a matrix. This work is needed in connection with a problem in finding the lowest eigenvalue, and corresponding eigenvector, of a matrix which arises from the Rayleigh Ritz variational principle applied to the Schrodinger equation, for a nuclear three-body problem with tensor forces. (Task 1101-11-5100/50-3).

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Task 1101-53-1100/49-4
(formerly 11.1/32-49-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

MATHIEU FUNCTIONS II
Task 1101-53-1101/45-1
(formerly 11.1/2-45-1)

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

SPECIAL TABLE OF BESSEL FUNCTIONS
Task 1101-53-1101/48-2
(formerly 11.1/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Computations completed. Final manuscript is to be made on card-controlled typewriter.

ROCKET NAVIGATION TABLES
Task 1101-53-1101/48-3
(formerly 11.1/2-48-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Computations completed. Final manuscript is to be made on card-controlled typewriter.

TABLES OF $E_1(z)$, SECOND QUADRANT
Task 1101-53-1101/49-1
(formerly 11.1/2-49-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Calculations are completed and checked. The final manuscript is to be made on a card-controlled typewriter.

PUNCHED CARD LIBRARY
Task 1101-53-1101/49-2
(formerly 11.1/2-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The tables of $E_1(z)$, $z = x + iy$, in the second quadrant of the complex plane were added to the library:

Table I. $$E_1(z) = \int_z^{\infty} \frac{e^{-u}}{u}\, du$$

Table II. $$E_1(z) + \log_e z$$

The path of integration is from z to $+\infty$, on a line parallel to the x axis. The branch cut has been taken on the negative real axis.

Range of Table I: $-x = 0(.1)3.1$, $y = 0(.1)3.1$, 10D

Range of Table II: $-x = 0(.1)1$, $y = 0(.1)1$, 10D.

REDUCTION OF RAYDIST DATA
Task 1101-53-1101/49-2a
(formerly 11.1/32-49-2)

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Two runs of Raydist data were calculated this quarter.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Task 1101-53-1101/49-6
(formerly 11.1/32-49-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. About 35% completed.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Task 1101-53-1101/50-2
(formerly 11.1/32-50-2)

Origin: Naval Air Missile Test Center, Point Mugu
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Test computation for one parameter was completed. Computations for a second parameter are in progress.

COMPUTATIONS ARISING IN THE THEORY OF HYPERSONIC FLIGHT
Task 1101-53-1101/50-4
(formerly 11.1/32-50-4)

Origin: Naval Ordnance Test Station, Pasadena Annex Authorized 11/1/49
Sponsor: Office of Naval Research, USN Completed 9/30/50
Manager: Gertrude Blanch

Objective: (1) To determine by a least squares fit the coefficients of the approximations

$$XY + A_1 - A_2Y=0$$

$$X=b_1 + b_2 Y$$

for certain tabulated data.

(2) To obtain the greatest root of the shock polar equation

$$Y_2 - \sqrt{}_o M_o^2 (B \cos^2\theta + 2 \sin^2\theta) Y + \sqrt{}_o^2 M_o^4 \sin^2\theta = 0$$

with $\sqrt{}_o = 1.405$; $B = .891, .833$; $M_o = 10, 12.5, 15, 17.5, 20, 22.5, 25$;

$\theta = 10^o, 20^o, 30^o, 45^o$; and to obtain the maximum value of θ for which the solutions will be real.

Background: These solutions were needed during the course of research by Dr. E. T. Benedikt of the U.C.L.A. Engineering Department, as a theoretical explanation for the results of experiments conducted by the Naval Ordnance Test Station.

Status: COMPLETED.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Task 1101-53-1101/50-7
(formerly 11.1/31-50-7)

Origin: Lockheed Aircraft Corporation
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE.

ANALYSIS OF RAM-JET DATA
Task 1101-53-1101/50-10
(formerly 11.1/31-50-10)

Origin: Marquardt Aircraft Company
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Two analyses were processed this quarter.

CONTOUR INTEGRAL
Task 1101-53-1101/50-10a
(formerly 11.1/32-50-10)

Origin: Naval Ordnance Test Station, Pasadena Authorized 6/1/50
Sponsor: " "
Manager: Gertrude Blanch

Objective: To evaluate the integrals

$$\int_C \left(y - s\frac{dy}{ds}\right) d\zeta$$

$$\int_C \left(x - s\frac{dx}{ds}\right) \frac{d\rho}{\rho}$$

where s is the arc-length $\rho^2 = (x - k_1)^2 + (y - k_2)^2$, $\zeta = \tan^{-1}\dfrac{y - k_2}{x - k_1}$

and the curves, C, are defined by various functions $y = f_j(x)$.

Background: The shape of the free water surface produced by a flat plate entering water obliquely has been deduced from experimental data obtained at Naval Ordnance Test Station. This curve contains one free parameter. It is planned to predict the value of this free parameter on the basis of the contour integrals proposed in this problem.

Comments: This is part of a general program on water entry problems being carried out at NOTS.

Status: COMPLETED.

"BOILING" COMPUTATIONS
Task 1101-53-1101/50-12
(formerly 11.1/32-50-12)

Origin: Engineering Dept., U.C.L.A.
Sponsor: Atomic Energy Commission
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Data reduction has continued throughout the quarter.

COMPUTATIONS OF POTENTIAL FLOW PAST A BODY OF REVOLUTION
Task 1101-53-1101/50-13
(formerly 11.1/32-50-13)

Origin: Naval Ordnance Test Station, Pasadena Annex Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN Completed 9/30/50
Manager: E. Yowell

Objective: To compute the potential flow past the surface of a body of revolution for various angles of attack.

Background: These results are needed by the Naval Ordnance Test Station in connection with research to determine bodies of minimum resistance. The formulae and procedures for making this calculation were specified by the Naval Ordnance Test Station. The theory of the computations is described in two British Air Ministry Reports and Memoranda, No. 2204, "A Family of Streamline Bodies of Revolution suitable for High-Speed or Low-drag Requirements," August 1945; and No. 2071, "A Simplified Theory for Streamline Bodies of Revolution, and its Application to the Development of High-speed Low-drag Shapes," July 1943.

Status: COMPLETED.

METEOROLOGICAL MEANS
Task 1101-53-1101/50-17
(formerly 11.1/31-50-17)

Origin: Department of Meteorology, U.C.L.A.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Originally, project was to reduce data for month of January 1950, where data was gathered at 4 pressure levels. This has been completed. The project has been extended to include data gathered for January 1950 and data for February 1950 at 8 additional levels. One of these 8 levels is 50 per cent complete; work will continue steadily on the processing of the data.

EARTH TIDES
Task 1101-53-1101/51-1

Origin: University of California, Los Angeles, Geophysics Department
Sponsor: Office of Naval Research, USN Authorized 9/28/50
Manager: E. C. Yowell

Objective: (1) To compute a table of $f(k, \cos x, \alpha)$ where

$$f(k,\cos x,\alpha) = k\left[\frac{\cos x}{\alpha} + \frac{1 - \frac{\cos x}{\alpha}}{(1+\alpha^2 - 2\alpha\cos x)^{3/2}}\right]$$

for $\alpha = .0000425$, $k = k_s = .02480289$; and $\alpha = .015(.01)(.018)$, $k = k_m = 12045.4$;

$\cos x = -1(.01)1$.

(2) To compute $\Delta g = \lambda[\tan^3 p\, f(k_m \cos z_m, \alpha_m) + D^{-3} f(k_s, \cos z_s \alpha_s)]$,
where z_m is zenith distance of moon,

α_m is ratio of distance between observing station and center of earth to distance between center of earth and center of moon,

z_s is zenith distance of sun,

α_s is ratio of distance between observing station and center of earth to distance between center of earth and center of moon,

p is moon horizontal parallax,

D is sun's distance in astronomical units,

λ is constant depending on observing station.

Δg is computed at 20-minute intervals for several stations and for times when gravimeter measurements were made.

Background: The Institute for Geophysics of U.C.L.A. desires to compare measured variations in gravity with that predicted by tidal theory treating the earth as a rigid body.

Comments: This work is being carried out by the Institute for Geophysics under a contract with the Office of Naval Research.

Status: NEW. The table of $f(k, \cos x, \alpha)$ has been completed.

FOURTH ORDER EQUATIONS
Task 1101-53-1101/51-2

Origin: Naval Ordnance Test Station (Pasadena) Authorized 9/28/50
Sponsor: Bureau of Ordnance, USN
Managers: G. Blanch, E. C. Yowell

Objective: To compute

$$\alpha_1 = 2\zeta_1\omega + \frac{\alpha_2 - (\omega^2 + \frac{1}{\omega^2})}{2\zeta_1\omega}$$

$$\alpha_3 = \frac{2\zeta_1}{\omega} + \frac{\alpha_2 - (\omega^2 + \frac{1}{\omega^2})}{\frac{2\zeta_1}{\omega}}$$

for $\alpha_2 = 3(1)6(2)20(5)30(10)50(50)100$,

$\zeta_1 = .02(.02).10(.10).80(.20)1.00(.50)3.00(1.00)6.00(2.00)10.00$,

$\omega = 1.0(.1)\sqrt{\alpha_2}$.

Background: These tables are useful in solving rapidly fourth degree algebraic equations of certain types.

Status: NEW. All computations have been completed and checked. Final listings are being prepared.

EVAPORATION COMPUTATIONS
Task 1101-53-1101/51-3

Origin: Naval Electronics Laboratory Authorized 9/28/50
Sponsor: Bureau of Reclamation, Department of the Interior
Manager: E. C. Yowell

Objective: To compute evaporation by means of eight different given equations which were derived from different theoretical assumptions in order to compare the computed quantities with measured evaporation.

Background: This computation is in connection with a study of evaporation from lakes being made jointly by the Naval Electronics Laboratory, the Bureau of Reclamation, and the Weather Bureau. Precise data is being taken of the meteorological elements at Lake Hefner, in Oklahoma City. The water budget of Lake Hefner is known accurately. The evaporation is calculated by each of the above mentioned eight equations. These answers are then compared with the actual evaporation obtained from the water budget. In this way it is hoped to find an equation which will accurately give the evaporation from a body of water if the pertinent meteorological data is known.

Status: NEW.

FLAT PLATE WATER ENTRY
Task 1101-53-1101/51-4

Origin: Naval Ordnance Test Station (Pasadena) Authorized 9/28/50
Sponsor: Bureau of Ordnance, USN
Managers: G. Blanch, E. C. Yowell

Objective: To compute the quantity $C_p(y_0)$ which is defined by the following equations:

$$B_1(y_0) = \int_C (x - s\frac{dx}{ds})\frac{d\rho}{\rho} + (y - s\frac{dy}{ds})\, d\zeta,$$

where $\tan\zeta = \frac{y - y_0}{x - 1}$, $\rho^2 = (x-1)^2 + (y - y_0)^2$, and C is a given curve in the (x,y) plane,

$$B_3 = \ln\left(\frac{2.3}{y_0} - 1\right),$$

$$F(y_0) = \frac{1}{\pi}(B_1 - B_3),$$

$$C_1(y_0) = \int_{y_0}^{2.3} F(t)\, dt,$$

$$C_p(y_0) = 2[y_0F(y_0) + C_1(y_0) - [F(y_0)]^2 - 2.645],$$

with C_p computed for the range $y_0 = .1(.05)1.1(.1)2.3$.

Background: This problem is another phase of the general problem of water entry which is being studied at Naval Ordnance Test Station. (See task 1101-53-1101/50-10.)

Status: COMPLETED.

BOUNDARY LAYER
Task 1101-53-1101/51-5

Origin: Northrop Aircraft Co., Inc. Authorized 9/28/50
Sponsor: Office of Air Research, USN
Manager: M. Howard, C. Lanczos

Objective: To solve, by finite differences, the simultaneous equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = UU' + \frac{\partial^2 u}{\partial y^2} ,$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 ,$$

subject to given necessary boundary conditions. U is a given function of x.

Background: This computation arises in connection with a study of the flow in the laminar boundary layer past an airfoil for the case when suction is applied. The method is based upon a translated report on the subject of Laminar Boundary Layers by H. Görtler. The report treats the case where there is no suction ($v = 0$ at $y = 0$). Adaption of the method for the case where suction is applied ($v = v_0(x)$ at $y = 0$) was made by Northrop Aircraft Co.

Status: NEW. The problem was coded for the I.B.M. Card Programmed Calculator.

II. Computation Laboratory

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-21-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. (1) The development of equally weighted n-point quadrature formulas for integrals of the types

$$\int_0^\infty e^{-x}f(x)dx \text{ and } \int_{-\infty}^\infty e^{-x^2}f(x)dx \,,$$

analogous to the Chebyshev quadrature formulas for finite intervals, was continued. The formulas investigated are of the form

$$\int_{-\infty}^\infty e^{-x^2} f(x)dx = \frac{1}{n} \sum_{i=1}^{n} f(x_i), \text{ and}$$

$$\int_{-\infty}^\infty e^{-x^2} f(x)dx = \frac{\sqrt{\pi}}{n} \sum_{i=1}^{n} f(x_i)$$

Efforts were concentrated on formulas which give an exact quadrature if $f(x)$ is any polynomial of degree n. In these cases the points $x_1, x_2, \ldots, x_n$ are the roots of certain polynomials. The coefficients of these polynomials had previously been determined for all $n \leqslant 10$, and the roots x_i (which are not necessarily real) are being calculated.

(2) "An elementary note on powers of quaternions" (cf. Apr-Jun 1950 issue, p. 28): A manuscript by this title was prepared and submitted to a technical journal for publication.

(3) "Radix table for obtaining trigonometric and inverse trigonometric functions to many places": A practical table for 18-place accuracy was calculated and a final version of the accompanying text with illustrations was prepared and submitted to a technical journal for publication.

(4) "Formulas for calculating the error function of a complex variable". Revised version of note (incorporating some suggestions of J.B. Rosser) was prepared.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS

Task 1102-21-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The inversion of matrices, which is required incidental to this project, was programmed for the IBM Card Programmed Calculator. The particular matrices arising in this problem have the property that their elements vary greatly in order of magnitude. Consequently, computations must be programmed for a "floating decimal point." The computation of the coefficients of power series representing the harmonic measures for the domains studied was begun.

Dr. Alt presented a paper, "Numerical methods for boundary value problems for the Laplace equation," at the Conference on Fluid Dynamics, Cambridge, Mass., August 29, 1950.

RESEARCH IN MODERN NUMERICAL ANALYSIS: CONDITION OF MATRICES

Task 1102-21-1104/50-3
(formerly 11.2/11-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS

Task 1102-21-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Dr. Taussky-Todd continued her work on matrix classes and ideal classes in algebraic number fields. She continued her study of the properties of the characteristic roots of polynomials in finite matrices, both from the algebraic and arithmetic standpoint. Results were obtained concerning a problem proposed by M. Kac to characterize forms of non-commutative matrices A,B for which $\alpha A + \beta B$ has as roots $\alpha_i + \beta\mu_i$ where λ_i are the characteristic roots of A and μ_i are the roots of B. Dr. Taussky-Todd presented the paper (1) below at the International Congress of Mathematicians, Cambridge, Mass.

Publications: (1) "Classes of matrices and quadratic fields," by Olga Taussky-Todd, submitted for publication in a technical journal. (2) "On the relative extrema of the Laguerre orthogonal functions," by John Todd, to appear in Atti della Reale Accademia delle Scienze di Torino. (3) "The convergence of the Cauchy-Riemann sums to the Cauchy-Riemann integral," by O. Szasz and J. Todd, accepted for publication in the NBS Journal of Research. (4) "Coulomb wave functions expressed in terms of Bessel-Clifford functions, and Bessel functions" by M. Abramowitz, accepted for publication in the Journal of Mathematics and Physics. (5) "Table of the function

$$\int_0^x e^{-u^3}\, du,"$$

by M. Abramowitz, accepted for publication in the Journal of Mathematics and Physics.

STUDIES IN METHODS OF IMPROVING THE CONVERGENCE OF SERIES
Task 1102-21-1104/50-6
(formerly 11.2/11-50-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A paper, "The determination of converging factors for the asymptotic expansions for the Weber parabolic cylinder function," was presented by Dr. J.C.P. Miller at the International Congress of Mathematicians, Cambridge, Mass., Aug. 30-Sep. 6, 1950. The work is being continued.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-21-1104/51-6

Origin: NBS Authorized 9/28/50
Manager: Viola D. Hovsepian

Objective: To investigate the Monte Carlo method as a means of solution of Laplace's equation

$$\sum_{\nu=1}^{n} \frac{\partial^2 V}{\partial x_\nu^2} = 0 .$$

"Cubical" regions and simple boundary conditions will be used.

Background: The Monte Carlo approach is particularly suitable for high-speed machine calculations, and is therefore a natural one to program for the SEAC. Compare task 1101-11-5100/44-PM1.

Status: NEW. Two variations of the method are being considered for solution on the SEAC. In the first method it is assumed that a point makes a series of random walks starting at a fixed interior point of a net spread over the region, and moves with equal probability positively or negatively in any of the coordinate directions. The walks are terminated when they first hit the boundary. The average of the values of the boundary function at these points gives an approximation to the value of V at the starting point of the walks. In the second method, we start with an approximation to the function V. In this method the walks proceed with weighted probabilities depending on the approximation, and this weighting is compensated by attaching an appropriate weight to the boundary value. It is to be expected that the convergence would be faster than in the first method; experiments will be made with various approximating functions. (See task 1101-11-5100/49-PD1).

At the meeting of the Association for Computing Machinery, Washington, D. C., September 9-11, 1950, Mr. Todd presented a paper on "Some recent experiments with the Monte Carlo method." Mr. Todd also presented a paper, "Application of high speed automatic computing machinery in problems of fluid dynamics," at the Conference on Fluid Dynamics, Cambridge, Mass., August 29, 1950; this paper includes application of the Laplace equation.

NUMBER-THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-21-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. (1) Further modifications of the basic routine have been developed, and a routine has been prepared for the identification of large primes. A table of factors of $n^2 + 1$ for n up to 20225, prepared by Dr. John W. Wrench, Jr., has been extended to 21500, and a factor table for $(1000n)^2 + 1$, for n = 1(1)200, of which about 22 factorizations were already known, has been completed; this serves as an extended sample of the fuller table. Results obtained from a study of these tables are being written up, in collaboration with Dr. Wrench, and it is hoped to publish them shortly.

(2) Further progress has been made, and routines prepared for finding Haupt-exponents of 3, and for searching for primes p with small exponents, for example, not exceeding 2000.

(3) Consecutive residues: Dr. Taussky-Todd and Dr. Miller have continued the programing and coding of the problem mentioned in the Apr-Jun report. A talk on the problems considered and the proposed methods of attack was given at the meeting of the Association for Computing Machinery in Washington on September 8 under the title, "Number-theoretical problems on the SEAC." A fuller account of the methods and techniques used has been prepared.

2. Applied Research: Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-21-1104/43-3
(formerly 11.2/2-43-3)

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Subtabulation in certain parts of the range will be carried out on the SEAC. The completion of this phase of the project has been delayed due to lack of machine time.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-21-1104/46-1
(formerly 11.2/2-46-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations of $\log \Gamma(x+iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ were completed. Extension of the table to include values for $x = 0(.1)9$ continued.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-21-1104/47-2
(formerly 11.2/2-47-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. These tables are to appear in two volumes. The manuscript for the first volume was completed and checked; for the second, computations have been completed except for a small amount of subtabulation.

TABLE OF ANTILOGARITHMS
Task 1102-21-1104/47-3
(formerly 11.2/2-47-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of the manuscript by differencing continued.

TABLES FOR THE OCCASIONAL COMPUTER
Task 1102-21-1104/47-4
(formerly 11.2/2-47-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1948 issue.

TABLE OF LAGRANGIAN COEFFICIENTS
FOR SEXAGESIMAL INTERPOLATION
Task 1102-21-1104/48-2
(formerly 11.2/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN
HERMITE POLYNOMIALS
Task 1102-21-1104/49-1
(formerly 11.2/2-49-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Roots, weight factors, and auxiliary coefficients for the first 16 polynomials were determined. Computations will be extended as far as the 20th polynomial.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Task 1102-21-1104/49-2
(formerly 11.2/2-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Revision of the preliminary manuscript pursuant to comments received from Dr. Rosser was underway.

FERMI FUNCTION, II
Task 1102-21-1104/49-10
(formerly 11.2/33-49-10)

Origin: NBS, Section 4.4
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office, and will be issued as "Table for the analysis of β-spectra," NBS Applied Mathematics Series 13.

TABLES TO FACILITATE SEQUENTIAL t-TESTS
Task 1102-21-1104/50-2a
(formerly 11.2/2-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as "Tables to facilitate sequential t-tests," NBS Applied Mathematics Series 7.

TABLE OF CHEBYSHEV POLYNOMIALS
Task 1102-21-1104/50-3a
(formerly 11.2/2-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office, and will be issued as NBS Applied Mathematics Series 9.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-21-1104/50-4a
(formerly 11.2/2-50-4)

Origin: NBS, Section 11.3
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. All tables are now in form suitable for publication. Computations of interpolation schedules in the few places where they would be useful have been completed. The final manuscript with accompanying text describing the method of computation and the interpolation was submitted to Dr. E. Gumbel, who originally suggested the compilation of these tables. Dr. Gumbel has prepared an introduction to this table.

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-21-1104/50-5
(formerly 11.2/2-50-5)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED.

TABLE OF n! and $\Gamma(n+\frac{1}{2})$
Task 1102-21-1104/50-6a
(formerly 11.2/2-50-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Complete manuscript copy, with introduction, was prepared for publication.

Publication: Accepted for publication in the NBS Applied Mathematics Series.

WAVE FUNCTION FOR LITHIUM
Task 1102-21-1104/50-7
(formerly 11.2/2-50-7)

Origin: NBS
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Study of suitable numerical methods is underway, preparatory to coding for the SEAC.

RELATIVE ABUNDANCE OF THE ELEMENTS
Task 1102-21-1104/51-1

Origin: Applied Physics Laboratory, Johns Hopkins University
Manager: J. H. Levin Authorized 9/28/50

Objective: To solve the following system of differential equations:

$$\frac{d\xi_n}{d\tau} = -\left(1 + \frac{3}{2}\right)\xi_n - P_1\xi_n\xi_1 - P_2\xi_n\xi_2$$

$$P_3\xi_n\xi_3 - P_4\xi_n\xi_4$$

$$\frac{d\xi_1}{d\tau} = \xi_n - P_1\xi_n\xi_1 - \frac{3}{2\tau}\xi_1$$

$$\frac{d\xi_i}{d\tau} = P_i\xi_n\xi_{i-1} - P_i\xi_n\xi_i - \frac{3}{2\tau}\xi_i, \quad (i = 2,3,\ldots,) .$$

In the above equations τ represents time, ξ_n is the relative abundance of neutrons, and ξ_i is the relative abundance of the element having atomic number i. P_i is essentially the effective neutron capture volume swept out per second by nuclei of element having atomic number i.

Background: The above equations arise out of a formulation of the theory of the "neutron capture process" for the formation of the elements, suggested by R. A. Alpher, H. Herman, and G. Gamow. This theory, in explaining the relative abundance of the elements, takes into account formation of elements by neutron capture, radioactive decay of neutrons, and dilution of matter resulting from expansion of the universe.

References:

Comments: /(1) A neutron-capture theory of the formation and relative abundance of the elements, R. A. Alpher, The Physical Review, vol. 74 (Dec. 1, 1948). (2) On the relative abundance of the elements, R. A. Alpher and H. Herman, The Physical Review, vol. 74, (Dec. 15,1948). (3) Remarks on the evolution of the expanding universe, R. A. Alpher and H. Herman, The Physical Review, vol. 75 (Apr. 1, 1949).

Status: NEW. Solutions were carried out on the SEAC for several trial sets of initial conditions. It is contemplated that further solutions will be required corresponding to 2 or 3 trial sets of initial conditions in order to arrive at those initial conditions which will be consistent with empirical data.

CRYSTAL STRUCTURE PROBLEM FOR POINT ATOMS
Task 1102-21-1104/51-2

Origin: Naval Research Laboratory, USN — Authorized 9/28/50
Manager: Rose Rowen

Objective: Corresponding to r sets of indices h, k, ℓ, and observed quantities $F_{hk\ell}{}^2$, to find the coordinates of points (x_i, y_i, z_i), $(i = 1, \ldots, m)$ satisfying the relationships

$$(1)\quad |F_{hk\ell}|^2 = \sum_{j=1}^{m} n_j^2 + 2\sum_{u<\gamma<1}^{m} n_\mu n_\gamma ck\ 2\pi\,[h(x_\mu - x_\gamma) + k(y_\mu - y_\gamma) + \ell(z_\mu - z_\gamma)]\,.$$

In general $r < m$. The technique used is therefore to choose points x_i, y_i, z_i so as to minimize the sum

$$\sum_{h,k,\ell}\left[\,|F_{hk\ell}\ (\text{computed})|^2 - |F_{hk\ell}\ (\text{observed})|^2\,\right]^2 .$$

Background: The expression above arises in the study of crystal structures by X-ray diffraction methods. The expression (1) describes the relationship between the intensity $F_{hk\ell}{}^2$ of the scattered beam and the coordinates (x_i, y_i, z_i) of the atoms in the crystal. The quantities n_i are proportional to the atomic numbers, and the "Miller indices" h, k, ℓ, are the reciprocals of the intersections of the crystal planes with the coordinate axes.

Status: NEW. Coding for the SEAC was completed but not checked.

ANALYSIS OF CRYSTAL STRUCTURE
Task 1102-21-1104/51-3

Origin: NBS Authorized 9/28/50
Manager: Ethel C. Marden

Objective: Calculation of electron density $\rho(x,z)$ corresponding to structure factors $F(h,0,\ell)$ satisfying the reciprocal relationships

$$\rho(x,z) = \sum_{h=-\infty}^{\infty} \sum_{h=-\infty}^{\infty} F(h,\ell) \cos 2\pi(hx + \ell z)$$

and

$$F(h,\ell) = \int_0^1 \int_0^1 \rho(x,z) \cos 2\pi(hx + \ell z)\,dx\,dz \;.$$

Background: This problem arises in the study of crystal structures. It is the two-dimensional special case of a more general three-dimensional problem in which ρ is a function of x,y,z and F depends on three parameters (Miller indices) h,k,ℓ. The function ρ measures the distribution of electrons in the unit cell of a crystal, treated as a continuous function, and F measures the intensity of light reflected from a crystal surface with Miller indices h,k,ℓ.

The approach is to start with a preliminary experimentally determined set of structure factors $F(h,0,\ell)$, and calculate the corresponding ρ's. Then the ρ's are modified so as to make the distribution more "acceptable," in particular so that ρ is non-negative and conforms to certain preconceived ideas about the shape of such distribution functions. Changes in ρ are to be accomplished in such a way as to keep F as close as possible to the observed values.

Comments: This work is part of an investigation of crystal structures being carried on by the Portland Cement Association Fellowship. The problem was proposed by Dr. F. Ordway, who suggested a method to be followed.

Status: NEW. Coding for the SEAC was completed but not checked.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-21-1104/51-4

Origin: NBS Authorized 9/28/50
Managers: A. N. Lowan and H. E. Salzer

Objective: To assemble for publication the short mathematical tables and articles by members of the Computation Laboratory in two volumes under the general title of "Short Mathematical Tables of the Computation Laboratory." Volume I probably will be entitled "Tables of Functions and of Zeros of Functions," with an introduction by A. N. Lowan, and Volume II, "Tables of Coefficients" by H. E. Salzer.

Background: All of the tables in this collection originated with members of the Computation Laboratory of the National Bureau of Standards, and most of them have been published in mathematical journals. It is considered useful to have them assembled.

Comments: The collection of these tables and articles was suggested by Dr. A. N. Lowan.

Status: NEW. Most of the tables which are to constitute the first volume were assembled and an introduction was drafted.

Publication: To be submitted to Columbia University Press.

X-RAY PENETRATION PROBLEM
Task 1102-21-1104/51-5

Origin: NBS Authorized 9/28/50
Manager: O. Steiner

Objective: To solve on the SEAC the following integral equations for $a_{\ell n}(\lambda)$:

$$\mu(\lambda)\, a_{\ell n}(\lambda) = \int_{\lambda_0}^{\lambda} k(\lambda, \lambda')\, P_\ell(1-\lambda+\lambda')\, a_{\ell n}(\lambda')\, d\lambda'$$

$$- \alpha \sum_{n'=0}^{n-1} \left[\frac{\ell+1}{2\ell+1}\, a_{\ell+1,n'}(\lambda') + \frac{\ell}{2\ell+1}\, a_{\ell-1,n'}(\lambda') \right]$$

$$+ \lambda_0 k(\lambda_0, \lambda) P_\ell(1-\lambda+\lambda_0) \frac{\alpha}{\mu_0} \left[1 - \frac{\alpha}{\mu_0} \right]^n ,$$

where $k(\lambda', \lambda)$ is the Klein-Nishina cross-section,

P_ℓ is the ℓ^{th} Legendre polynomial,

μ is total cross section (varying with material),

$\alpha = \mu(\text{minimum})$, and

$a_{\ell n}(\lambda)$ are used to solve the transport equation governing X-ray penetration.

Background: The transport equation governing X-ray penetration of materials has been reduced to the above chain of Volterra integral equations. It is desired to investigate the possibility of using the SEAC in the solution of such chains of integral equations. If this problem can be easily handled on the SEAC, then a program will be considered for determining X-ray penetrations for a wide variety of cases.

Comments: This project was proposed by Dr. L. Spencer, NBS 4.8.

Status: NEW. Coding for the SEAC was completed and partially checked.

3. Mathematical Services

HEAT CONDUCTION EQUATION
Task 1102-53-1106/46-1
(formerly 11.2/33-46-1)

Origin: Bureau of Ordnance, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. The computation of additional cases on the SEAC was delayed because of lack of machine time.

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Task 1102-53-1106/49-2
(formerly 11.2/33-49-2)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameters a_i, m_i, were performed when requested.

LINEAR PROGRAMING ON STANDARD PUNCHED CARD MACHINES
Task 1102-53-1106/49-3
(formerly 11.2/36-49-3)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Programs were being calculated as requested by the originators.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Task 1102-53-1106/49-11
(formerly 11.2/33-49-11)

Origin: NACA, Lewis Flight Propulsion Laboratory — Authorized 2/1/49
Sponsor: " " " — Completed 9/30/50
Manager: J. H. Levin

Objective: To compute the proportions in which various gases occur in the exhaust after burning of hydrocarbon fuels. These proportions are to be determined in their dependence on temperature, pressure, fuel-air ratio and hydrogen-carbon ratio in the fuel, for a range of conditions occurring in practice.

Background: Computations of this kind are important in the design of combustion engines and jet engines and in the physical-chemical investigation of combustion in general. The particular set of computations to be performed here was proposed by L. R. Turner of the NACA. Similar computations on a very large scale were formerly carried out by S. R. Brinkley on the ENIAC; these, however, do not extend to the range of pressures and temperatures usually encountered in combustion gases.

Status: COMPLETED. Results were transmitted to the originator. The composition of combustion gases was computed for the following sets of conditions: (a) Fuel of composition C_nH_n in stoichiometric mixture with air (i.e., fuel-air ratio F = 1), temperature T = 2800°K, pressures p of 1,4,16,64 atm. (b) Fuel C_nH_{2n}, other conditions as in (a). (c) Fuel C_nH_{3n}, F = 1.1, other conditions as in (a). (d) Fuel C_nH_{3n}, F = .8(.1)1.2, T = 2800°K, p = 1 atm. (e) Fuel C_nH_{3n}, F = 1, T = 2600°K, p = 1. For each of the cases enumerated, and for each of the 11 components of the system (N_2,O_2, etc.) the composition of the combustion gases, characterized by the partial pressures n_ν were determined, as well as the derivatives of these partial pressures taken in seven different directions.

The following derivatives were computed: (1) Partial derivative with respect to pressure at constant temperature, (2)-(5) Partial derivatives with respect to the amounts of nitrogen, hydrogen, oxygen, and carbon,-all taken at constant pressure and temperature, (6) Partial derivative with respect to the logarithm of temperature at constant pressure, (7) Partial derivative with respect to the logarithm of temperature at constant entropy.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/49-13
(formerly 11.2/33-49-13)

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

GUST ATTACKS ON DELTA WING
Task 1102-53-1106/50-1
(formerly 11.2/31-50-1)

Origin: Aircraft Laboratory, AMC, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Work on all phases originally requested was completed. Computations were started on certain additional cases and modifications subsequently requested by the originating agency.

STANDARD LORAN TABLES

Task 1102-53-1106/50-1a (formerly 11.2/34-50-1): Gulf Coast Chain
Task 1102-53-1106/50-5 (formerly 11.2/34-50-5): Hawaiian Islands Chain
Task 1102-53-1106/51-5: Marshall Islands Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Calculations were completed and results were submitted to the U. S. Hydrographic Office. Preparation of a final manuscript was underway.

RESEARCH IN LINEAR PROGRAMING

Task 1102-53-1106/50-2
(formerly 11.2/12-50-1)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Dr. Taussky-Todd completed a paper which includes pertinent results about the location of characteristic roots of matrices with positive elements, a class which is of importance in mathematical economics (see Publication item below). Also she presented a paper on linear programing at the Institute for Numerical Analysis seminar.

The programing of the problem of finding "feasible solutions" by means of triangular or almost triangular coefficient matrices for the IBM Card Programmed Calculator was completed. Time studies show that on this particular problem the use of standard IBM machines (type 604) is somewhat more efficient than the CPC. The use of the latter machine for this problem was therefore abandoned.

The large-scale use of the SEAC for these "triangular" problems was begun. A separate project (Task 1102-53-1106/51-7) has been set up for this phase of the work.

Research on "minimization" methods and efforts at coding "minimization" problems for the SEAC continue. Special emphasis was placed on the "transportation problem," which is a special case of the general minimization problem. This phase is being studied by Mrs. I. Rhodes and Mr. D. O. Larson. In addition, Dr. N. Wiegman concentrated on certain degenerate cases in which previously developed methods seem to fall.

Publication: "Bounds of characteristic roots of matrices, II," by O. Taussky-Todd, accepted by the NBS Journal of Research.

A PROBLEM IN MOLECULAR STRUCTURE, I
Task 1102-53-1106/50-3
(formerly 11.2/33-50-3)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations for various values of the parameters were performed as requested.

ADAPTATION OF LORAN CALCULATIONS TO CARD PROGRAMMED CALCULATOR AND NBS AUTOMATIC COMPUTER
Task 1102-53-1106/50-3a
(formerly 11.2/34-50-3)

Origin: Hydrographic Office, USN
Sponsor: " "
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A program for the Card Programmed Calculator has been worked out and is being tried. Programing for the SEAC is in progress.

WAVE RESISTANCE OF SHIPS, III
Task 1102-53-1106/50-11
(formerly 11.2/33-50-11)

Origin: David Taylor Model Basin, USN
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Computations were completed for certain specified values of the parameters and the results transmitted to the David Taylor Model Basin. Computations for a more general class of cases are in progress.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-53-1106/50-16
(formerly 11.2/33-50-16)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations were performed as requested.

MOMENTS OF THE DISTRIBUTION OF RANKED EXTREME VALUES
Task 1102-53-1106/50-18
(formerly 11.2/33-50-18)

Origin: N. Y. State Public Service Commission
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Computations were underway.

DETERMINATION OF OPTIMUM VALUES FOR SUBSAMPLING DESIGN
Task 1102-53-1106/51-1

Origin: Bureau of the Census — Authorized 9/28/50
Sponsor: " — Completed 9/30/50
Manager: Viola D. Hovsepian

Objective: To find the minimum of the function

$$\phi(q,m,\overline{N}) = \frac{\delta}{q} + \frac{(1-\delta)(1+\hat{\delta}_w\overline{N})}{q\,m\,\overline{N}}$$

subject to the following condition:

$$C = C_0 q\cdot\sqrt{m} + c_1 qm + C_2 qm\overline{N} + C_3 q + C_4\sqrt{q}$$

where C, $C_0,\ldots,C_4$ are constants. Also, to determine the range of values of q, m, $\overline{N}$ for which ϕ lies within 10% of the minimum. These problems are to be solved for about 10,000 combinations of parameters C, C_i.

Background: The problem arises in the design of statistical sampling plans. The function ϕ represents the variance in a stratified sample of size $qm\overline{N}$ which is divided into q primary units each of which is subdivided into m subsampling units of average size $\overline{N}$. The expression for C is an approximation to the cost of carrying out such a sampling survey. The solution of this problem will thus indicate how a sampling plan should be designed so as to give minimum variance at given cost. From the Lagrange equations of the original problem the following iterative expressions which converge to optimum values of $q,m,\overline{N}$, were derived:

$$\overline{N}_{i+1} = \frac{\dfrac{C_0}{2\sqrt{m_i}} + C_1}{\hat{\delta}_w}$$

$$\sqrt{q_{i+1}} = \frac{-C_4 + \sqrt{C_4^2 + 4C(C_0\sqrt{m_i} + C_1 m_i + m_i\overline{N}_{i+1} + C_3)}}{2(C_0\sqrt{m_i} + C_1 m_i + m_i\overline{N}_{i+1} + C_3)}$$

$$m_{i+1} = \frac{1}{\overline{N}_{i+1}} \frac{1 - \delta}{\delta} \left(\frac{C_0 \sqrt{m_i}}{2} + \frac{C_4}{2 \sqrt{q_{i+1}}} + C_3 \right).$$

Because of the magnitude of the problem it has not previously been attempted in its entirety. It is now being programmed for solution on the SEAC.

Status: COMPLETED (NEW). Results were transmitted to the originator.

A NON-LINEAR PROBLEM IN LEAST-SQUARE FITTING
Task 1102-53-1106/51-2

Origin: Operations Research Office, U.S.A. Authorized 9/28/50
Sponsor: " " Completed 9/30/50
Manager: M. Oliphant

Objective: To fit

$$P = \frac{c_r^2 \, c_z}{(\sigma_r^2 + c_r^2)(\sigma_z^2 + c_z^2)^{\frac{1}{2}}}$$

to given data Q, by choosing c_r and c_z to make $\sum(P - Q)^2$ a minimum.

Background: Requested by Mr. B. E. Phillips, ORO.

Status: COMPLETED (NEW). Results were transmitted to the originator.

ANALYSIS OF UNDERWATER SOUND MEASUREMENTS
Task 1102-53-1106/51-6

Origin: Underwater Sound Laboratory, USN Authorized 9/28/50
Sponsor: " "
Manager: B. Heindish

Objective: To make a time series study of radar and periscope bearing errors.

Background: This data was obtained from bearing measurements on a moving target by radar-type devices and periscopes. A time series study is necessary to establish standards on the expected accuracy of bearing measurements.

Status: NEW.

PROGRAM COMPUTATION ON THE SEAC
Task 1102-53-1106/51-7

Origin: Office of the Air Comptroller, USAF Authorized 9/1/50
Sponsor: " "
Manager: M. Montalbano

Objective: To compute Air Force programs on the SEAC by the "triangular" method.

Background: This project continues one phase of task 1102-53-1106/50-2. It is being set up as a separate project because the use of this method has become sufficiently routinized so that in the future the emphasis will be on large-scale production rather than on the development of new methods. Research on new or improved methods will continue to be reported under task 1102-53-1106/50-2.

Status: NEW. Two programs were completed, using different trial routines, and a third is in progress.

SCATTERING FUNCTIONS FOR SPHERICAL PARTICLES
Task 1102-53-1106/51-9.

Origin: Instrument Research Division, NACA, (Langley Field, Va.)
Sponsor: " " Authorized 9/1/50
Manager: Irene Stegun Completed 9/30/50

Objective: To extend the NBS tables of scattering functions for spherical particles by computing i_1, i_2 and K for m = 1,2, $0 < \alpha < 1.2$ and $\gamma = 30^\circ - 50^\circ$, $140^\circ - 160^\circ$.

Background: This is an extension of the tables published in NBS Applied Mathematics Series 4, "Tables of Scattering Functions for Spherical Particles," U. S. Government Printing Office, Washington 25, D.C.

Status: COMPLETED (NEW). Results were transmitted to the originator.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-51-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Tables of entropy, enthalpy, and specific heat of CO have been prepared for pressures ranging between .01 and 100 and temperatures between 200° and 280°.

BASIC IONOSPHERIC DATA
Task 1401-34-1412/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Apr-Jun issue.

Status: CONTINUED. Observational data were recorded and processed as received.

RADIO-TELEGRAPH INTERFERENCE
Task 1404-34-1423/49-17
(formerly 11.2/33-49-17)

Origin: NBS, Section 14.4
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

RATING OF WATER CURRENT METERS
Task 0605-41-0621/50-2
(formerly 11.2/33-50-2)

Origin: NBS, Section 6.5
Sponsor: "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were performed as requested.

IONOSPHERIC WINDS
Task 1401-11-1401/50-7
(formerly 11.2/33-50-7)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Data were processed as requested.

CRYSTAL STRUCTURES OF CEMENT COMPOUNDS
Task 9099-00-9009/50-9
(formerly 11.2/33-50-9)

Origin: NBS, Division 9, Portland Cement Research Associate Project
Sponsor: Portland Cement Association
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. Specification for parameters is pending.

RAY TRACING

Task 0202-21-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 1.6
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Skew rays have been traced and third order aberrations computed for optical systems as requested. The calculations were done on the IBM Card Programmed Calculator.

WAVE PROPAGATION IN THE IONOSPHERE

Task 1401-11-1400/50-14
(formerly 11.2/33-50-14)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Exploratory work was underway.

ELECTRON TRAJECTORIES

Task 1202-34-6354/51-1

Origin: NBS, Section 12.2 Authorized 9/28/50
Manager: M. M. Andrew

Objective: To calculate solutions on the SEAC of the non-linear differential equation

$$\frac{d^2\psi}{d\theta^2} = \eta_0 \Omega_0 \cos\psi \sin(\theta + \delta)$$

for $\psi(0) = 0$ and $\frac{d\psi(0)}{d\theta} = \Omega_0$. The exit velocity Ω_f of the electron from the electromagnetic field is to be calculated for $\eta_0 = .025(.05)0.475$, $\Omega_0 = .025(.025)0.35$ and $\delta = 0(\frac{\pi}{4})\frac{7\pi}{4}$. Ω_f is defined as $\frac{d\psi(\theta_f)}{d\theta}$ with $\psi(\theta_f) = \pi$. A total of 1120 Ω_f are to be computed, which will require about $5\frac{1}{2}$ hours time on the SEAC.

Background: This differential equation, which represents a one-dimensional electron trajectory in an electromagnetic field, arose from a study of Dr. R. T. Young of the NBS Tube Laboratory, in considering the efficiency of operation of tubes of the klystron type.

Status: NEW. The SEAC code is complete and about 40% of the computation has been performed.

Publication: Manuscript to be submitted to Section 12.2, NBS.

NUMERICAL SOLUTION OF A SET OF DIFFERENTIAL EQUATIONS CHARACTERIZING A DEPOLYMERIZING SYSTEM
Task 0700-12-0700/51-2

Origin: NBS, 7.0 Authorized 9/1/50
Manager: J. Blum

Objective: To obtain numerical solutions for the system of differential equations:

$$\frac{dQ_{N-1}}{d\tau} = -(1+\sigma)(N-4)Q_{N-1} ,$$

$$\frac{dQ_n}{d\tau} = -(1+\sigma)(n-3)Q_n + 2\left(\frac{K_0}{k_1} - \frac{1}{4}\sigma\right)Q_{n+2} + 2\sum_{i=1}^{\frac{N-n-3}{2}} \frac{K_i}{k_1} Q_{n+2i+2}, \quad \left\{ \begin{array}{l} 4 \leqslant n \leqslant N-3 \\ n \text{ even} \end{array} \right.$$

with these boundary conditions at $\tau = 0$:

$$\left\{ \begin{array}{ll} Q_{N-1} = 1 & \\ Q_n = 0 ; & 4 \leqslant n \leqslant N-3 . \end{array} \right.$$

The quantities K_i are defined in terms of the parameters σ and ϵ by the equation:

$$\frac{K_i}{k_1} = 2(1+\tfrac{1}{2}\sigma) \left\{ 1 - \frac{1}{4} \quad 1+3(1-\epsilon) \quad (1-\epsilon)^i \right\} + \sigma$$

The value proposed for N-1 is 1000; the following are the values for the parameters: $\sigma = 0.2,\ 1, 4, 10$: $\epsilon = 5\text{x}10^{-3},\ 10^{-2},\ 5\text{x}10^{-1}$.

Background: This system arises in the theory of chain reactions which describes the depolymerization of long polymer molecules as developed by R. Simha, L. A. Wall, and P. J. Blatz; J. Polymer Sci., in press (1950). (N-1) represents the initial number of units in a chain, Q_n the fraction of molecules containing n units and τ is a dimensionless variable related to time. $1/\epsilon$ is the average kinetic chain length of the reaction and σ the ratio between the probabilities of transfer and initiation of a free radical chain. For the limits of very small and very large values of ϵN, the solution has been evaluated. The case of intermediate values and of varying magnitudes of σ forms the object of this investigation.

Comments: Requested by Dr. R. Simha, 7.0.

Status: NEW.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES
FOR CERTAIN STATISTICAL TESTS
Task 1103-11-1107/47-2
(formerly 11.3/2-47-2)

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Word was received from Professor Chand to the effect that his work on the final manuscript was nearing completion.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Task 1103-11-1107/48-3
(formerly 11.3/2-48-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date, see Apr-Jun 1950 issue.

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Task 1103-11-1107/49-1
(formerly 11.3/1-49-1)

Origin: NBS — Authorized 9/21/48
Manager: J. Lieblein — Completed 9/30/50

Objective: To evaluate the statistical properties of three observations (1) when all are drawn at random from a single universe and (2) when more than 1 universe is involved, with particular reference to the processing of experimental measurements and especially to the problem of anomalous readings.

Background: The wide prevalence of triplicate readings in chemistry other physical sciences, and engineering testing invites thorough study of samples of this size under a variety of assumptions regarding the experimental conditions under which they are made. This study is of special importance since there is great interest among scientists in whether a sound statistical criterion for the rejection of one of these observations can be devised that makes use only of the observations themselves. This study was undertaken at the suggestion of Dr. W.J.Youden as a follow-up to an empirical study that he had made of some published data.

Status: COMPLETED. Manuscript of final paper, "Properties of statistics involving the closest pair in a sample of three observations," submitted to NBS Editorial Committee for approval for publication in a technical journal.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-11-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The preparation of abstract cards continued.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Task 1103-11-1107/49-3
(formerly 11.3/1-49-3)

Origin: NBS
Manager: Henry B. Mann. (Dr. Mann, who is responsible for technical aspects of this project, undertook the project during a temporary employment by the Bureau from March to June 1949, and is expected to return for brief periods until the work is finished. During his absence Dr. Eisenhart handles inquiries and other matters related to the project.)
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Dr. Mann made slight revisions in his manuscript, "The estimation of parameters in certain stochastic processes," in accordance with comments received, and the paper was released by the Bureau for publication in a technical journal.

Work is progressing on revision of Professor Mann's monograph on the theory of stochastic processes. The object of this revision is to add more detail wherever it is necessary to facilitate reading the monograph, to add references and to eliminate disturbing typographical errors in the manuscript. Modifications of some of the proofs are being suggested. This work has been completed on the first chapter of the monograph; work on the second chapter is in progress.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Task 1103-11-1107/49-3a
(formerly 11.3/2-49-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Task 1103-11-1107/50-1
(formerly 11.3/1-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE.

EXTREME-VALUE THEORY AND APPLICATIONS
Task 1103-11-1107/50-1a
(formerly 11.3/2-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Collaboration was continued with Dr. Gumbel to complete several remaining technical matters in connection with the brochure on extreme-value theory and applications. Also covered in these discussions was Dr. Gumbel's manuscript for the introduction to the probability tables for extreme values (being prepared under task 1102-21-1104/50-4a, formerly 11.2/2-50-4). The introduction has now been completed and is in process of final typing prior to editorial review.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-11-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Background research on the subject continued.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-11-1107/51-1

Origin: NBS Authorized 7/1/50
Manager: Lola S. Deming

Objective: To tabulate the values of x defined by

$$\int_{x_P}^{\infty} f(t)\,dt = P$$

for

$$P = .0001(.0002).0019,\ .0022(.0004).0098,\ .0110(.0020).0390,\ .0420(.0040).0980,\ .1050(.0100).4950,$$

for the cases where f(x) is the probability-density function of the normal, sech^2, sech, Cauchy, Laplace, and rectangular ($-w/2 \le x \le w/2$) distributions respectively.

Background: In April 1947 Professor J. W. Tukey suggested the preparation of a matched set of tables of inverse probability transformations such that a sequence of random samples from the rectangular distribution (0.1) in the form of 4-digit random numbers could be "pumped through" these inverse probability transformations to yield matched sequences of random samples from the distributions (i.e., "populations") concerned. It was thought that, when studying the distributions of particular test functions and estimators by sampling methods, the use of such matched random samples from various alternative populations might help to distinguish between samples that were <u>intrinsically anomalous</u> and samples that were peculiar only when viewed through the "spectacles" of a particular population.

Status: NEW. In order to render the distributions comparable, the location and scale parameters of these distributions (with the exception of the Cauchy) were adjusted so that all have zero means and unit variances. To bring the Cauchy into the family we proceeded as follows: First, the values x_P for the 100 values of P were determined from

$$\frac{1}{\pi}\int_{x_P}^{\infty}\frac{dx}{1+x^2} = \frac{1}{2} - \frac{\arctan x_P}{\pi} = P,$$

that is,

from $$x_P = \tan \pi \left(\tfrac{1}{2} - P\right) ;$$

second, the variance of the discrete probability distribution so formed, that is, "σ_x^2" $= 2 \{(.0002)(x_{.0001})^2 + (.0002)(x_{.0003})^2 + \ldots + (.0004)(x_{.0022})^2 + \text{etc.}\}$ was calculated; and third, the values of x_P were then divided by "σ_x". The resulting tabular "x_P" values, by construction, thus correspond to a (discrete) distribution with zero mean and unit variance that retains some of the peculiarities of the Cauchy distribution for practical purposes.

The proposed table has been computed with the entries x_P given to 3D, except in the case of the rectangular distribution where 4D are given. The table is now being checked.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-11-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: E. Lukacs

Objective: To conduct research on problems in probability and mathematical statistics in connection with the work of the Statistical Engineering Laboratory.

Background: Problems in probability and mathematical statistics may originate from other tasks of the Statistical Engineering Laboratory or they may arise in connection with studies undertaken by members of the staff. These problems have often independent interest and warrant investigation.

Status: NEW. Dr. Lukacs completed a paper "Maximum likelihood estimate of position derived from measurements performed by hyperbolic instruments," that covers some work done by the author while employed at the Naval Ordnance Test Station, Inyokern, California. This paper deals with the problem of estimating position obtained from hyperbolic measurements when more than the minimum number of three observations are taken.

Dr. Lukacs also completed a paper "On the stochastic independence of symmetric and homogeneous linear and quadratic statistics", in which all univariate distributions are determined such that some homogeneous and symmetric quadratic statistic is stochastically independent of the mean. The discussion is restricted to distributions having moments of first and second order. A report on this paper was given by its author at the West Coast Meeting of the Institute of Mathematical Statistics held in Berkeley, California, on August 5, in conjunction with the Second Berkeley Symposium on Probability and Mathematical Statistics.

Work progressed on a joint paper of E. Lukacs and O. Szasz entitled "Some non-negative trigonometric polynomials connected with a problem in probability". The results were reported in Projects and Publications, Apr-Jun 1950, p. 8. Criteria for the non-negativity of two more classes of trigonometric polynomials were developed and work on a third class of such polynomials was started. A preliminary draft of a manuscript is in preparation. Dr. Lukacs gave a paper on this subject at the International Congress of Mathematicians.

Mrs. Deming prepared a set of tables collectively entitled "Some percentage points of the χ^2-distribution", based upon the following source tables: (1) R. A. Fisher, <u>Statistical Methods for Research Workers</u>, Table III, Oliver and Boyd, Edinburgh; (2) Catherine M. Thompson, "Table of percentage points of the χ^2-distribution", Biometrika, vol. XXXII, Part II, October 1941; (3) George A. Campbell, "Probability curves showing Poisson's exponential summation", Table II. The Bell System Technical Journal, January 1923. Her first table gives the values of $\chi^2_P(\nu)$ defined by

$$\frac{1}{2\Gamma(\frac{\nu}{2})} \int_{\chi^2_P}^{\infty} \frac{\chi^2}{2}^{\frac{\nu-2}{2}} \, d\chi^2 = P$$

to 3D for P, 1-P = .005, .01, .02, .025, .05, .10, .20, .25, .30, .50 when $\nu = 1(1)100$; the second, $\chi^2_P(\nu)$ to 3D for P, 1-P = .01, .10, .25, .50 when $\nu = 102(2)200$; and the third, $\chi^2_P(\nu)$ to 3D for P, 1-P = .0001, to 2D for P, 1-P = .000001, when $\nu = 2(2)200$. A limited number of copies were prepared by multilith process for use within the Bureau. However, since errors of 1 or 2 units in the last digit given in the present tables must be expected in some of the values obtained by interpolation in (2) or by transformation from (3), wide distribution of the tables is not contemplated at this time.

Dr. Eisenhart and Mrs. Deming prepared a brief table of the values of the quantity $P_\gamma(n,k)$ defined by

$$\text{Prob}\,\{P(n,k) \geq P_\gamma(n,k)\} = \gamma, \quad 0 < \gamma < 1,$$

where

$$P(n,k) = \frac{1}{\sigma\sqrt{2\pi}} \int_{\bar{x}-ks}^{\bar{x}+ks} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \, dy$$

and $\bar{x} = \left(\sum_{i=1}^{n} x_i\right)/n$ and $s = \left[\sum (x_i-\bar{x})^2/(n-1)\right]^{\frac{1}{2}}$ refer to an arbitrary sample of n observations $x_1, x_2, \ldots, x_n$ drawn independently at random from a normal population having mean μ and standard deviation σ. The table gives the values of $P_\gamma(n,k)$ that corresponds to $k_1=1.96$, $k_2=t_{.05}(n-1)$, and $k_3=k_2\sqrt{(n+1)/n}$ for $\gamma = 0.75, 0.90, 0.99$, for n=10, 50--except for the case $k=k_1$, $\gamma = 0.99$, n=10--where $t_{.05}(\nu)$

denotes the two-tail .05-probability level of Student's t for n-1 degrees of freedom. A note explaining the practical significance of these results is in preparation.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3000-21-0002/51-1

Origin: NBS Authorized 7/1/50
Manager: W. J. Youden

Objective: To furnish statistical advice and assistance to members of the NBS scientific staff on the planning, conduct and interpretation of experiments and tests.

Background: The advances in modern statistics, particularly in the design and arrangement of experiments, have for some years been employed in agricultural experimentation. The introduction of modern statistical method in the physical sciences and engineering has been slow, even though remarkable examples of their effectiveness have been recorded. These modern statistical methods in planning experiments should in time become as much a part of the experimenters "tool kit" as other forms of applied mathematics.

Status: NEW. Activity under this project fell into two main categories:

A. The Design of Experiments. Typical examples were: (1) A Youden square design was applied to the testing of seven textile abradants on four machines. (2) An allocation of specimens was constructed for testing the notch sensitivity of alloys so that comparisons of different curvatures of notching would not be biased by any systematic variation in the physical properties of the specimens.

B. The Development or Selection of the Appropriate Statistical Methods for Analysis and Interpretation of Data. For example a recurring problem is the fitting of straight lines when both variates are subject to error. The usual least squares (minimizing squares of vertical deviations) gives biased answers in this case and the appropriate method of fitting is the one developed by Wald. Another example occurred in the analysis of stress-fatigue data on alloys. It was suggested that a logarithmic transformation of the data would have the effect of stabilizing the variance and in certain cases permit the use of analysis of variance technique since the effect of varying different factors would be additive in the transformed scale.

RESEARCH ON APPLICATION OF THEORY OF EXTREME
VALUES TO GUST LOAD PROBLEMS
Task 1103-21-5106/51-1

Origin: NACA, Dynamic Loads Division Authorized 9/28/50
Sponsor: " "
Manager: J. Lieblein

Objective: Improved application of the statistical theory of extreme values to gust-load problems.

Background: Certain phases of existing methods in the statistical theory of extreme values have been applied to several types of gust-load problems by the NACA Langley Aeronautical Laboratory. During FY 1949 and FY 1950 the Statistical Engineering Laboratory, NBS, at the request of NACA informally furnished advice and assistance on certain statistical phases of this work. The NACA has now requested the SEL to undertake research directed toward more effective use of extreme-value theory and methods in the treatment of gust-load problems.

Status: NEW. During the quarter, Dr. Eisenhart and Mr. Lieblein spent three days at the NACA Langley Aeronautical Laboratory, Virginia, in a visit arranged to initiate collaboration between SEL, NBS, and the Gust Loads Branch, NACA. The discussions with NACA staff provided valuable information needed in connection with the project, including the nature of some of the statistical problems that had arisen and results that had previously been obtained. The visit also afforded opportunity to borrow several key records and reports for study. It culminated in agreement on the direction in which the initial phases of the work was to proceed, namely, with considerable attention to possible mathematical models for gust load distributions with application to actual data.

The balance of the work on the project during the quarter consisted in preparing a detailed account of the data and other information that had been obtained and in securing and studying some of the literature bearing on certain of the mathematical aspects.

IV. Machine Development Laboratory

(Section 11.4)

1. Development: Design and Construction of Automatic Digital Computing Machines

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Eckert-Mauchly Computer Corporation continued working on the UNIVAC. At the end of the quarter, completion of the first UNIVAC system by December 1950 appeared to be probable.

THE NAVY COMPUTING MACHINE
Task 1104-34-5107/47-2
(formerly 11.4/22-47-2)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Raytheon Manufacturing Company, contractor, made substantial progress on the two computers which it is constructing under another Government contract, and which will precede in delivery the computer being constructed under contract with the Bureau. The contractor estimated that on its first computer the engineering development was 90% completed and the actual construction 40% completed.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation, under a supplement to the Census UNIVAC contract. The design status is the same as that of task

1104-34-5107/47-1. Delivery of the computer by the summer of 1951 is expected.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Note: This computer has been previously referred to as the NBS Interim Computer and as the NBS Automatic Computer I.

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Some engineering testing continued on the computer during the quarter. In addition, work progressed on the 512-word Williams tube memory to be added, and on the development of faster input-output equipment. Use of the machine for computing continued, but on a limited scale because of the engineering testing.

AIR MATERIEL COMMAND COMPUTING MACHINE
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Work continued on this computer by the General Electric Company, at its Electronic Park Laboratories, in Syracuse, New York.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, U.S.A.
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation, under a contract in the form of a supplement to the Census UNIVAC contract. The design status is the same as that of task 1104-34-5107/47-1. Delivery of this computer, dependent upon the Eckert-Mauchly production facilities, is expected to follow closely that of the Air Comptroller's UNIVAC.

2. General Services

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Task 1104-53-1108/47-4
5108/
(formerly 11.4/3-47-4)

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The following routines were completed and run successfully on the SEAC.

1. A classified problem for the RDB.
2. Linear programming for the OAC (Dept. of the Air Force).
3. A diagnostic test for a 512-word memory.
4. Electron trajectories.
5. Partial differential equations for heat conduction for the Weather Bureau.
6. Jacobi Elliptic Function routine.

The following problems have been completed and are being checked.

1. A problem of X-ray penetration.
2. Calculation of Loran tables.
3. Analysis of crystal structure.
4. A routine for checking subroutines using a floating binary point.
5. Computation of the incomplete Gamma function.

CODING RELATED TO THE UNIVAC SYSTEM
Task 1104-53-5108/49-2
(formerly 11.4/3-49-2)

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

CODING RELATED TO THE RAYTHEON COMPUTER
Task 1104-53-5108/49-3
(formerly 11.4/3-49-3)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

THE MTAC SECTION
Task 1104-51-1109/47-1
(formerly 11.4/4-47-1)

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for inclusion in the January 1951 issue of Mathematical Tables and Other Aids to Computation was assembled, edited and forwarded to Professor D.H. Lehmer, editor. In addition, galley and page proofs for the October issue were edited.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Task 1104-51-1109/49-2
(formerly 11.4/42-49-2)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

Applied Mathematics Division Technical Meeting

CHERNOFF, H. (University of Illinois). Gradient methods of maximization. August 9, 1950.

Numerical Analysis Colloquium Series
(Los Angeles, California)

ALBERT, A. A. (University of Chicago and University of Southern California). Non-associative algebras. July 17, 1950.

WEINSTEIN, A. (Naval Ordnance Laboratory, White Oak, Md.) Approximation of eigenvalues. July 19, 20, 21, 1950.

ERDOS, P. (NBS and University of Aberdeen, Scotland). (1) Some Tauberian theorems. July 18, 1950. (2) On product of consecutive integers. July 24, 1950. (3) Probability and number theory. July 31, 1950. (4) Interpolation and distribution of roots of polynomials. August 8, 1950. (5) Combinatorial problems. August 15, 1950.

BOULANGER, G. R. (Polytechnic Faculty of Mons and University of Brussels). Nomographic computing techniques in physics and engineering. August 7, 1950.

MILLER, J.C.P. (NBS and Scientific Computing Service, London). Computing and computing machines. August 14, 1950.

CHARNEY, J. (Institute for Advanced Study). Some mathematical problems of numerical weather prediction. August 22, 1950.

FOX, L. (National Physical Laboratory, England). (1) Solution of some integral equations. September 18, 1950. (2) Determination of latent roots and latent vectors of matrices, with special reference to "escalator methods." September 21, 1950.

Statistical Engineering Seminar

DVORETZKY, A. (Hebrew University, Jerusalem). (1) Introduction to statistical decision theory. September 22, 1950. (2) Non-randomization of statistical decision rules. September 29, 1950.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

(A) Papers presented at the Second Symposium on Mathematical Statistics and Probability, held at Berkeley, California, August 7-11, 1950:

DVORETZKY, A. Some problems on the stability of sequences of random variables.

ERDOS, P. Some recursion formulas ir the theory of probability.

FEYNMAN, R. P. The concept of probability in quantum mechanics.

KAC, M. Some connections between probability theory and differential equations.

LUKACS, E. The stochastic independence of stochastic symmetric linear and quadratic statistics.

(B) Papers presented at the International Congress of Mathematicians, Cambridge, Mass., August 30-September 6, 1950.

ALT, F. L. Almost-triangular matrices.

DVORETZKY, A. On Hausdorff measures.

FORSYTHE, G. E., and W. FELLER. New matrix transformation of obtaining characteristic vectors.

FORTET, R. Elements aleatoires de nature quelconque.

FOX, L. The numerical solution of ordinary differential equations.

HESTENES, M. R. Iterative methods of obtaining solutions of boundary value problems.

HESTENES, M. R. and W. KARUSH. A method of gradients for the calculation of the characteristic roots and vectors of a symmetric matrix.

JACKSON, L. K. and E. F. BECKENBACH. Subfunctions and elliptic partial differential equations.

JOHN, F. On the fundamental solution of linear elliptic partial differential equations with analytic coefficients.

LUKACS, E. and O. SZASZ. Some non-negative trigonometric polynomials connected with a problem in probability.

MILLER, J.C.P. The determination of converging factors for the asymptotic expansions for the Weber parabolic cylinder function.

MOTZKIN, T.H. Duality and neighbor points.

ROSSER, J. B. Transfinite cardinal arithmetic in Quine's new foundations.

SZASZ, O. (1) Tauberian theorems for summability (R_1). (2) Representation of an analytic function by general Laguerre series (Preliminary Report). Presented jointly with N. Yeardley (Purdue University).

TAUSSKY-TODD, O. Classes of matrices and quadratic fields.

WASOW, W. On the mean duration of random walks in n dimensions.

(C) Papers presented at the Conference on Automatic Computation, held by the Association for Computing Machinery, Washington, D. C., September 7-9, 1950:

CURTISS, J. H. The role of a central computing laboratory.

MILLER, J.C.P. Number-theoretical problems on the SEAC.

TODD, J. Some recent experiments with the Monte Carlo method.

(D) Papers presented at other meetings:

ALT, F. (1) Numerical methods for boundary value problems for the Laplace equation. Presented at the Conference on Fluid Dynamics, Cambridge, Mass., August 29, 1950. (2) Numerical methods in matrix problems. Presented to a Study Group of mathematicians at the Pentagon, July 10, 1950.

HUSKEY, H. D. Automatic digital computing machines and their various applications. Presented at a meeting of the Optimist Club of Burbank California, September 21, 1950.

KAC, M. Some models of phase transition. Presented at the Physics Seminar, University of California at Los Angeles, July 12, 1950.

RHODES, I. Preparation of mathematical problems for electronic computations. Presented at the Tenth Annual Mathematics Institute, Duke University, North Carolina, August 14, 1950.

SZASZ, O. Tauberian theorems for summability (R_1). Presented at the Conference at Stanford held by the Office of Naval Research and the Department of Mathematics of Stanford University, Palo Alto, California, August 11, 1950.

TAUSSKY-TODD, O. Computing and computing machines. Presented to the Mathematics Department, Northwestern University, Evanston, Illinois, July 24, 1950.

TODD, J. Application of high speed automatic computing machinery in problems of fluid dynamics. Presented at the Conference on Fluid Dynamics, Cambridge, Mass., August 29, 1950.

YOUDEN, W. J. Statistical methods in analysis. Presented at the Gordon Research Conferences on Current Trends in Analytical Chemistry (AAAS), New Hampton, N. H., July 12, 1950.

Training Activities

A condensed course on programing and coding for the SWAC (Standards Western Automatic Computer) was given by Dr. H. D. Huskey at the beginning of the quarter in order to assist in orienting the summer mathematical research program at the Institute for Numerical Analysis.

The out-of-hours course in coding for the SWAC, which has been conducted by Mrs. Roselyn Lipkis, was discontinued in July for the summer. It will be resumed in October for orientation of new personnel at the Institute and for more advanced instruction of those who attended the previous course.

The training program for INA computers using desk calculators continued with further computations of the characteristic values of Mathieu's differential equation. Test computations for be_{14} were partially completed.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Table of coefficients for obtaining the first derivative without differences. H.E. Salzer. NBS Applied Mathematics Series 2. Reprinted. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 15 cents.

(2) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. Available from Columbia University Press, New York 27, N.Y. $7.50.

(3) Tables of integrals of Struve functions. M. Abramowitz. J. Math. Phys. 29, No. 1, 49-51 (Apr. 1950). Reprints available.

1.3 Technical Papers

(1) Matrix inversion by a Monte Carlo method. G. E. Forsythe and R. A. Leibler (Naval Communications Station, Washington, D. C.). MTAC IV, No. 31, 127-129 (July 1950). Reprints available.

(2) Numerical determination of characteristic numbers. W. E. Milne. NBS J. Res. 45, No. 3, 245-254 (Sept. 1950). Reprints available.

(3) Coefficients for polar complex interpolation. H. E Salzer. J. Math. Phys. XXIV, No. 2, 96-104 (July 1950). Reprints available.

(4) Use of statistics to determine the precision of test methods. W. J. Youden and J. M. Cameron. A.S.T.M. Special Publication No. 103, 27-34 (1950). Reprints available.

(5) The incorporation of subroutines into a complete problem on the National Bureau of Standards Automatic Computer I. The staff of the Machine Development Laboratory. MTAC IV, No. 31 164-168 (July 1950). Reprints available.

1.5 Miscellaneous Publications

The NBS Technical News Bulletin, September 1950, contains the following items relative to the NBS computer program:
(1) SEAC--the National Bureau of Standards Eastern Automatic Computer.
(2) Solution of skew-ray problem.
(3) SEAC's solution of a heat-flow problem.
(4) The NBS computer program.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION SEPTEMBER 30, 1950.

2.1 Mathematical Tables

(1) Tables of the exponential function e^x. Formerly NBS Mathematical Table MT2. Third edition to be issued as NBS Applied Mathematics Series 14. In press, Government Printing Office.

(2) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. In press, Government Printing Office.

(3) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(4) Tables for conversion of X-ray diffraction angles to interplanar spacing. H. Swanson. NBS Applied Mathematics Series 10. In press, Government Printing Office.

(5) Table of Arctangents of rational numbers. J. Todd. NBS Applied Mathematics Series 11. In press, Government Printing Office.

(6) Table for the analysis of β spectra. NBS Applied Mathematics Series 13. In press, Government Printing Office.

(7) Tables of n! and $\Gamma(n+\frac{1}{2})$ for the first thousand values of n. H. E. Salzer. Accepted for publication in the NBS Applied Mathematics Series.

(8) The hypergeometric and Legendre functions with applications to integral equations of potential theory. C. Snow. Revised version approved for publication in the NBS Applied Mathematics Series. (Formerly NBS MT15, out of print).

(9) Tables relating to the Mathieu functions. In press, Columbia University Press.

(10) Table of the integral $\int_0^x e^{-u^3}du$. M. Abramowitz. Accepted by the Journal of Mathematics and Physics.

2.3 Technical Papers

(1) Coulomb wave functions expressed in terms of Bessel-Clifford functions and Bessel functions. M. Abramowitz. Accepted by the Journal of Mathematics and Physics.

(2) Machine methods for finding the characteristic roots of a matrix. F. L. Alt. To appear in the Proceedings of a Scientific Computation Forum,held by the International Business Machines Corporation, Endicott, N. Y., November 1949.

(3) A problem in precision cam design. J. Blum. Accepted by the NBS Journal of Research.

(4) An analysis of the effect of the discontinuity in a bifurcated circular guide upon plane longitudinal waves. L. L. Bailin. Accepted for publication in the NBS Journal of Research.

(5) On subharmonic, harmonic and linear functions of two variables. E. F. Beckenbach. To appear in Revista, Universidad Nacional de Tucuman (Argentina).

(6) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. To appear in "The construction and applications of conformal maps: Proceedings of a symposium." to be published by the National Bureau of Standards.

(7) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Accepted by the Duke Mathematical Journal.

(8) The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation. J. M. Cameron and W. J. Youden. Submitted to a technical journal.

(9) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Accepted by the Duke Mathematical Journal.

(10) Forced oscillations in non-linear systems. M. L. Cartwright. Accepted for publication in the NBS Journal of Research.

(11) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J.H.Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(12) Sampling methods applied to differential and difference equations with special reference to equations of the elliptic type. J.H. Curtiss. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, New York, November 1949.

(13) Some problems on random walk in space. A. Dvoretzky and P. Erdos. To be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950.

(14) Relations among certain ranges of vector measures. A. Dvoretzky. Submitted to a technical journal.

(15) Elimination of randomization in certain statistical decision procedures and zero-sum two-person games. A. Dvoretzky, A. Wald, and J. Wolfowitz. Submitted to a technical journal.

(16) On a recursion formula and on some Tauberian theorems. P. Erdos. Accepted by the NBS Journal of Research.

(17) Generation and testing of random digits at the National Bureau of Standards, Los Angeles. G. E. Forsythe. To appear in "The Monte Carlo method: Proceedings of a symposium held June 29, 30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office.

(18) New matrix transformations for obtaining characteristic vectors. W. Feller and G. E. Forsythe. Accepted for publication in the Quarterly of Applied Mathematics.

(19) Second order determinants of Legendre polynomials. G. E. Forsythe. Accepted by the Duke Mathematical Journal.

(20) The extent of n random unit vectors. G. E. Forsythe and J.W.Tukey. (Princeton University). Submitted to a technical journal.

(21) Quadratic forms in Hilbert space, with applications in the calculus of variations. M. Hestenes. Accepted for publication in the American Journal of Mathematics.

(22) Applications of the theory of quadratic forms in Hilbert space to the calculus of variations. M. R. Hestenes. Submitted to a technical journal.

(23) The method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix. M. R. Hestenes. and W. Karush. Accepted for publication in the NBS Journal of Research.

(24) Semi-automatic instruction on the Zephyr. H. D. Huskey. To appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory, September 1949.

(25) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Accepted by the Bulletin of the American Mathematical Society.

(26) An iterative method for finding characteristic vectors of a symmetric matrix. W. Karush. Submitted to a technical journal.

(27) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. C. Lanczos. Accepted for publication in the NBS Journal of Research.

(28) Certain Fourier transforms of distributions. E. Lukacs and O. Szasz. Submitted to a technical journal.

(29) Numerical methods associated with Laplace's equation. W.E.Milne. To appear in the Proceedings of a Symposium held at Harvard Computation Laboratory, September 1949.

(30) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Accepted by the Bulletin of the American Mathematical Society.

(31) Note on an infinite integral. A. M. Ostrowski. Submitted to a technical journal.

(32) Note on Vincent's theorem. A. M. Ostrowski. Accepted for publication in Annals of Mathematics.

(33) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(34) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(35) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(36) Transformations to speed the convergence of series. J. B. Rosser. Accepted for publication in the NBS Journal of Research.

(37) Real roots of real Dirichlet L-series. J. B. Rosser. Accepted for publication in the NBS Journal of Research.

(38) A method of computing exact inverses of matrices with integer coefficients. J. B. Rosser. Accepted by the NBS Journal of Research.

(39) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Philosophical Magazine.

(40) Formulas for numerical integration of first and second order differential equations in the complex plane. H. E Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(41) Checking and interpolation of functions tabulated at certain irregular logarithmic intervals. H. E. Salzer. Accepted for publication in the NBS Journal of Research.

(42) An elementary note on powers of quaternions. H. E. Salzer. Submitted to a technical journal.

(43) Radix table for obtaining trigonometric and inverse trigonometric functions to many places. H. E. Salzer. Submitted to a technical journal.

(44) A bibliography of numerical methods in conformal mapping. W.Seidel. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards Applied Mathematics Series.

(45) On positive harmonic functions and ultraspherical polynomials. W. Seidel and O. Szasz. Accepted for publication in the Journal of the London Mathematical Society.

(46) A generalization of S. Bernstein's polynomials to the infinite interval. O. Szasz. Accepted for publication in the NBS Journal of Research.

(47) Gibbs' phenomenon for Hausdorff means. O. Szasz. Accepted for publication in the Transactions of the American Mathematical Society.

(48) On a Tauberian theorem for Abel summability. O. Szasz. Submitted to a scientific journal.

(49) Tauberian theorems for summability (R_1). O. Szasz. Submitted to a technical journal.

(50) On some trigonometric transforms. O. Szasz. Accepted by the Pacific Journal of Mathematics.

(51) On the relative extrema of ultraspherical polynomials. O. Szasz. Submitted to a technical journal.

(52) On the relative extrema of the Hermite orthogonal functions. O. Szasz. Submitted to a technical journal.

(53) On the relative extrema of Bessel functions. O. Szasz. Submitted to a technical journal.

(54) The convergence of Cauchy-Riemann sums to Cauchy-Riemann integrals. O. Szasz and J. Todd. Accepted by the NBS Journal of Research.

(55) Classes of matrices and quadratic fields. O. Taussky-Todd. Submitted to a technical journal.

(56) Bounds for characteristic roots of matrices, II. O.Taussky-Todd. Accepted by the NBS Journal of Research.

(57) On the relative extrema of the Laguerre orthogonal functions. John Todd. Submitted to a technical journal.

(58) On conformal mapping of variable regions. S. E. Warschawski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(59) Random walks and the eigenvalues of elliptic difference equations. W. Wasow. Accepted for publication in the NBS Journal of Research.

(60) On the mean duration of random walks. W. Wasow. Accepted by the NBS Journal of Research.

(61) A note on the four by four Latin squares. W. J. Youden. Accepted by Biometrics.

(62) Numerical solution of partial differential equations. E.C.Yowell. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., November 1949.

(63) A Monte Carlo method for solving Laplace's equation. E.C.Yowell. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., November 1949.

2.5 Miscellaneous Publications

(1) Problems for numerical analysis of the future. (Four papers presented at the Symposia on Numerical Analysis and Automatic Calculating Machinery, held at the NBS Institute for Numerical Analysis, Los Angeles, California, June 1948). To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(2) The Monte Carlo method; Proceedings of a symposium held on June 29, 30, July 1, 1949, in Los Angeles, California, under the sponsorship of the Rand Corporation, and the NBS, with the cooperation of the Oak Ridge National Laboratory. NBS Applied Mathematics Series 12. In press, Government Printing Office.

(3) The construction and applications of conformal maps: Proceedings of a symposium held at the NBS Institute for Numerical Analysis, Los Angeles, Calif., June 1949. To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(4) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

(5) High-speed computing and accounting. H. D. Huskey and V.R. Huskey. To appear in the Journal of Accountancy.