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Methods of Estimating Loads in Plumbing Systems

by

ROY B. HUNTER

NATIONAL
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The National Bureau of Standards is a fact-finding organization; it does not "approve" any particular material or method of construction. The technical findings in this series of reports are to be construed accordingly.

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Foreword

For many years, differences in the plumbing regulations or plumbing-code requirements in different localities have been a source of annoyance to building owners, builders, and public health officials and have hindered the standardization, on a general or national scale, of plumbing materials and equipment, and of construction design. The principal reasons for the existence of plumbing codes are the protection of the health of the occupants of a building and of the inhabitants of the community or city in which the building is located and the protection of property from damage by water or sewage.

The results sought by plumbing requirements and health regulations applying to plumbing are or should be the same for low-cost or expensive and for small or large buildings, and may be covered by general regulations applying to all plumbing. Any simplified or standardized form of plumbing construction which gives these results—that is, which complies with necessary general regulations pertaining to sanitation or health—cannot be prohibited without violating the inherent rights of building owners and of the public.

Because of the above, this report has been written, as succeeding reports on plumbing in the Building Materials and Structures series will be written, first from the standpoint of essential general requirements and second from that of permissible construction within the general requirements—both general requirements and permissible construction being based on sound physical principles.

This report deals with one of the factors which must be considered in the selection of adequate yet economical sizes of pipes for plumbing systems—namely, the load to be expected from a given number and kind of plumbing fixtures. It is expected that it will be followed by other reports in the Building Materials and Structures series dealing with other aspects of plumbing problems.

Where the exercise of judgment is necessary in the choice of the numerical value of factors used in developing and illustrating the application of the method of estimating loads as described in this report, it will be understood that such numerical values, when not the actual results of Bureau tests or experiments, represent the author's judgment in regard to the most suitable factor to use in the application of the method, and that these are not to be regarded as standard values, unless later approved as such by a representative and authoritative body.

Lyman J. Briggs, *Director.*

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CONTENTS

ABSTRACT

This report describes a method of estimating the demand and sewage loads for which provision should be made in designing plumbing systems in order that the service may be satisfactory. The characteristics of flow through a plumbing system and of the operation of supply valves and plumbing fixtures are described, and their influence on the method of estimating the load to be expected is discussed. The relative load-producing values of different kinds of commonly used plumbing fixtures are analyzed, and a table is developed giving relative load weights in terms of a load factor called the "fixture unit." An estimate curve developed by means of the probability function is given, and its use in conjunction with the table of fixture units is illustrated.

I. INTRODUCTION

Aside from the choice of materials and equipment as regards quality, style, and quantity of fixtures, simplification and standardization of piping layouts offer the principal opportunities for reduction in cost of plumbing systems. Simplification and standardization in this respect must comply with accepted health regulations and minimum requirements for plumbing, which in turn should be based on scientific principles. This is true whether the building is small and "low-cost" or large and expensive. Few, if any, existing plumbing codes are based entirely on scientific principles; and they do not in general permit the simplification of plumbing piping layouts that could be accomplished well within the requirements for sanitation, and in many cases with more satisfactory operation than can be obtained by installations under existing codes.

This report deals with one of the factors on which minimum requirements should be based, the maximum load to be provided for in plumbing systems. Other reports are planned, dealing with the principles of water supply and water distribution in buildings, the principles of building drainage, and the principles of venting.

II. PURPOSE

The purpose of this series of papers is to collect in an organized form the mass of information obtained by the author over a number of years, beginning with the investigation in 1921 of plumbing of small dwellings, and including the current research (1937–40) on plumbing for low-cost housing, together with the results of intervening experiments related to plumbing requirements, and to interpret the results of these investigations in a form suitable for direct and practical application. It is hoped that this series of papers will supply the logical answer to many of the controversial questions pertaining to pipe sizes and design of plumbing construction.

III. DEFINITIONS AND SYMBOLS

A number of terms employed in the plumbing industry, and a few that are now introduced for the first time, are here defined in the sense in which they are to be used in this and later papers of this series. Insofar as they are adequate and applicable, the definitions from Recommended Minimum Requirements for Plumbing [1][1] will be utilized.

The *plumbing system* of a building includes the water-supply distributing pipes, the fixtures and fixture traps, the soil, waste, and vent pipes, the building drain and building sewer, and the storm-water drainage pipes, together with their devices, appurtenances, and connections within or adjacent to the building.

The *building main* is the pipe from the street water main or other source of supply to the building served.

The *water-distributing system* is the piping by which the water is conducted from the building main to its various places of use within and adjacent to the building, consisting of laterals, risers, and branches.

The *sanitary system* of a building is the piping system, including soil, waste, and vent pipes, that conducts waste water and water-borne wastes from the plumbing fixtures to the street sewer or other place of sewage disposal.

A *plumbing fixture* is any receptacle through which waste water or water-borne waste is discharged into the sanitary system.

A *drain* is any pipe that carries waste water or water-borne waste in either the storm or sanitary system.

A *fixture drain* is the single drain from a fixture-trap outlet to its junction with another drain.

A *waste pipe* is any drain that carries the waste water from one or more fixtures other than water closets.

[1] Figures in brackets indicate the literature references at the end of this report.

A *soil pipe* is any drain that carries the waste water from one or more water closets with or without the waste water from other fixtures.

A *stack* is a vertical section of pipe extending two or more stories with branches therefrom to serve the stories through which it passes, as waste stack, soil stack, or vent stack.

A *horizontal branch* is a drain, extending laterally from a soil or waste stack, with or without vertical sections, to which the fixture drains in the same story are connected.

A *building drain* is that part of a sanitary system which receives the sewage from soil and waste stacks and conducts it to the building sewer.

A *building sewer* is the main trunk of a sanitary, storm, or combined system from a point 5 feet outside the inner face of the building wall to the street sewer.

A *primary branch* is the single drain line leading from the base of soil or waste stack to its point of connection with the building drain or another branch thereof.

A *secondary branch* is any branch of the building drain other than a primary branch.

A *vent* or *vent pipe* is any pipe installed in a sanitary system for the purpose of permitting a circulation of air through or within the system.

A *vent stack* or *main vent* is a vent pipe paralleling a soil or waste stack to which the branch vents from the stack and its horizontal branches are connected.

Demand load is the volume rate of demand for water imposed on the water-distributing system or any branch thereof by the use of water, as by plumbing fixtures.

Sewage load is the load in volume or volume rate imposed on the sanitary system or any branch thereof by the use of plumbing fixtures.

Charging load is the volume inflow into the sanitary system from one or more plumbing fixtures, applicable when the capacity of the pipe receiving the load depends mainly on the internal volume (cross section times length) of the pipe.

Receiving capacity is the internal volume of a section (drain) of the sanitary system, applicable when the total sewage load is an additive function of the separate volumes but not of the separate rates.

Terminal velocity is the constant velocity of flow that exists under fixed conditions when equilibrium between the forces (gravitational) producing and the forces (frictional) opposing flow has been established.

Design factor m is the particular value of r out of n fixtures that will be found in operation a selected fraction of the time under the assumed conditions of use.

Fixture unit, or *load factor,* is a numerical factor which measures on some arbitrary scale the load-producing effect of a single plumbing fixture of a given kind. The use of the fixture unit makes it possible to reduce the load-producing characteristics to a common basis.

n=the total number of fixtures or supply openings of a given kind in the system.

r=the number of fixtures out of a total of n which at any given instant of observation are found operating to impose a demand load on the supply system, or a sewage load on the drainage system.

m=the design factor (see definition above).

q=the average volume rate of flow, in gallons per minute, to or from a plumbing fixture during actual operation.

Q=the total volume of water in gallons that flows to or is discharged by a fixture at each use.

t=the average duration of flow in seconds for a given kind of fixture for one use.

T=the average time in seconds between successive operations of any given fixture of a particular kind.

τ=a time interval in seconds such that the event in question (for example, exactly r fixtures will be found operating) will occur for an aggregate of 1 second.

C_r^n=the number of combinations of n things taken r at a time.

p_r^n=the probability of exactly r fixtures out of a total of n fixtures being found in operation at an arbitrary instant of observation.

$\sum_{r=m}^{r=n} p_r^n$=the probability that some number of fixtures between $r=m$ and $r=n$, inclusive, will be found operating at an arbitrary instant of observation.

IV. BASIS OF SELECTION OF PIPE SIZES

In order to select from a series of available or stock sizes the minimum size of pipe adequate to convey a given flow of a liquid, it is necessary to know accurately the capacities of the different sizes of pipe for the conditions under which the pipe is to be used and to know accurately the load the pipe will be called upon to carry, capacity and load being expressed in the same units or in convertible units.

Pipe formulas, given in terms of the velocity of flow and factors on which the velocity depends, are the usual means employed in estimating capacities of pipes. It is an easy step to convert a formula giving the velocity of flow in a pipe into one giving the volume rate of flow. As previously indicated, the applicability of pipe formulas to the conditions existing in plumbing systems in service will be treated in other papers. However, since it is necessary to express capacity and load in the same units in selecting the proper size of pipe for a given use, it becomes necessary to consider the units in which both the expected load and the capacity of the pipes involved can be expressed before choosing the units in which load estimates should be expressed. A pipe-flow formula is merely an equation of dynamic equilibrium and, as such, applies only to the particular conditions—namely, the condition of uniform continuous flow in the pipe. Hence the conventional pipe formulas apply to the irregular and intermittent flows that occur in plumbing systems only during that time (usually very short) and in that section of pipe in which the variable factors involved (velocity or volume rate of flow, pressure or hydraulic gradient, and hydraulic radius) are constant.

V. CHARACTER OF FLOW IN PLUMBING SYSTEMS

Probably the best conception of the limitations of the applicability of a pipe formula for adequately estimating the capacities of plumbing pipes, and the best indication of the logical units to apply in expressing capacity and load in selecting the proper size of pipe to use in particular parts of the system, will be given by considering the changing character of flow as the water passes through the system from the source of supply to the building sewer. Water closets with flush-valve control of the supply are chosen for this illustration of varying flow characteristics, partly because critical loading conditions in the various parts of the plumbing system are usually produced by these fixtures and partly because the factors that determine the load for an individual fixture are more nearly constant and can be more readily evaluated than for other fixtures.

Assume that each water closet demands for each flush $Q=4$ gallons in the average time $t=9$ seconds, giving an average rate of supply to the closet of approximately $q=27$ gpm, and that each of these demands will occur on the average of once in $T=5$ minutes$=300$ seconds. Since the water-supply pipes are flowing full under pressure, there will be continuity of flow in both time and quantity in any part of the supply system through which water flows to the water closet or to a group of closets; and therefore the peak load at any time will be the sum of the separate demands, r, occurring at that time, or rq gpm, where q is the average flow per fixture. In this case a pipe formula is applicable for computing limiting capacities on the basis of diameters, lengths, roughness, and available head. The difficult part of the problem is to estimate the maximum r out of a total of n water closets supplied by the system for which provision should be made.

In the drainage system the flow from the water closet through the fixture drain (closet bend) into a horizontal branch will be at about the average rate of 27 gpm for a period of from 8 to 10 seconds. However, the flow will be less nearly uniform than the flow into the bowl, rising to a rate of from 30 to 40 gpm temporarily, as siphon or jet action comes into play in the bowl, and dropping immediately following the peak siphon action to a rate approximately equal to that of the supply at the time.

In the horizontal branch the temporary peak flow will flatten out, and the stream will tend to assume the terminal velocity (see definition) for the diameter and slope of the drain and the existing volume rate of flow. As the stream enters the vertical stack, the sudden change in direction piles the water up, tending to form a slug of water at the entrance. In fact, the surge from a single water closet entering a 3- or 4-inch vertical stack through a short horizontal branch of the same diameter will almost fill the stack at this point. Immediate acceleration takes place in the vertical stack and continues throughout the length of the stack or until the terminal velocity of the stack is reached for the volume rate of flow.

From the stack the stream will enter the sloping primary branch at a velocity much greater than the terminal velocity for that branch (shooting flow) [2] and will be decelerated until the terminal velocity for that volume rate of flow is approximately reached. If the volume rate of flow, the entrance velocity, and the slope, diameter, and roughness of the primary branch are in the proper relation, the transition from shooting flow at the entrance to gravity flow will occur in a hydraulic jump, a phenomenon which frequently occurs in the primary branches of building drains.

Throughout the drainage system in the direction of flow there is a continuous decrease in the volume rate of flow from a single closet or a group of closets flushing at the same time, a corresponding increase in the time of flow, and a continuous decrease in the peaks produced by siphon or jet action in the bowl or by the overlapping of the flows from different closets.

The conception of flow in plumbing drains as described has been verified by numerous experiments in the laboratory and by numerous observations and depth measurements made on the building drains of large systems in actual service. In plumbing drains the peak rate of flow from a water closet at the point of entrance to the horizontal branch is usually reduced to about one-half that rate at the point of transition from shooting flow to gravity flow in a primary branch, and the volume rate of flow continues to decrease from that point on.

If a single water closet flushes on the average at intervals of 5 minutes and delivers 4 gallons for each flush, the stream in the drain tends toward a continuous flow of $Q/T=4/5=0.8$ gpm as the distance from the water closet increases. Likewise for a very large number of water

[2] Shooting flow here refers to the high-velocity constantly decelerating flow that occurs in the primary branch of a building drain between its juncture with the soil stack and the hydraulic jump in the primary branch.

closets, n, each used at that average rate, and a very long building drain, the flow tends toward a uniform value 0.8 n gpm. If n is greater than T/t, it is obvious that flow in the main building drain will be continuous for the greater part of the time that each of the closets is being used at the average rate of once in T seconds, and that the tendency toward uniform continuous flow will increase as the size of the plumbing system increases—that is, as n and the length of the drains increase. Hence in a very large system in which all n closets are in continuous use, the outflow will be continuous and the rate will fluctuate around 0.8 n gpm.

Since the stream in the main building drain and building sewer of large buildings approaches uniform flow, and since these drains are ordinarily laid with a uniform slope, load and capacity can be most adequately expressed in volume rate of flow and a pipe formula for uniform flow may be used in estimating limits of capacity for building drains. However, it should be kept in mind that the volume rate of flow in any particular section of the drainage system is not an additive function of the separate volume rates of flow into the system.

There is another consideration that has a direct bearing on the method chosen for estimating the loads to be provided for in a building drainage system and on the choice of units such that load and capacity may be expressed in the same units. The horizontal branches, the entrance fittings to soil and waste stacks, and primary branches are parts of the drainage system in which critical loading or possible overloading are most likely to occur. In horizontal branches which lie near the fixtures, and to a lesser degree in the stack fittings and horizontal branches, the determinate factors in the selection of adequate drain pipes are the charging load and the receiving capacity. In estimating the charging load to be provided for, the problem is again to obtain a reasonable estimate of the value of r to be provided for in relation to n. The details of the method relating to receiving capacity will be completed in the papers dealing with capacities.

1. Cases of Loading

From the character of the flow in the water-supply system and in the drainage system as described, it is obvious that there are three distinct cases of loading to be considered: (1) One applying to the supply (the demand load) which will be measured by Σrq, where r is the number of separate like demands at the time for one kind of fixture and q the average volume rate of flow per demand, rq the total flow for r fixtures of one kind, and Σrq the sum or total flow for fixtures of different kinds; (2) one applying to horizontal branches (the charging load) which will be measured by ΣrQ, in which r is the number of like fixtures flowing at the time and Q is the average volume of sewage per fixture introduced into the drain within the time necessary to consider in relation to a particular location of the drain; and (3) a third case applying as a limit for building drains and building sewers which will be measured in terms of $\Sigma nQ/T$, in which n is the total number of any one kind of fixture installed in the system, Q the average volume used per fixture, and T the average time between uses. In cases 1 and 2, the load rq to be expected and provided for in the design of the plumbing system depends, with a few exceptions, on approximately the same time factors, t and T, and hence bears approximately the same relation to the number of fixtures, n. (See page 20.)

VI. STATEMENT OF THE PROBLEM

As stated in describing the conditions of flow in plumbing systems for which load estimates are sought, the most difficult part of the problem is to estimate the proportional part of the total possible load that should be provided for—that is, the number of demand or charging loads, r, bearing some logical relation to the total number of fixtures, n. In an earlier paper [1] the authors suggested the relation between r and n expressed by a probability function as a possible means of estimating loads to be expected in plumbing systems and developed the function applying to certain special cases.

It now appears that the possibilities of the practical application of this function and the limitations of the method under certain conditions have not been understood clearly. It seems advisable, therefore, to consider the probability function in its more general form, to explain more fully the suggested method of employing the function in making the estimates in question, and to point out certain limitations in its practical application. The problem may be stated as follows: Assuming that there are n fixtures in a system, each operated once in T seconds on the average, and that each operation is of t seconds' average duration, what is the probability that r fixtures will be found operating simultaneously at any arbitrarily chosen instant of observation?

It is necessary to define the expression, "operating simultaneously," in order to completely define a particular event of "r fixtures operating simultaneously." In the following development of the theory, this event will be considered as occurring when r, and only r, fixtures are found flowing at the instant of observation; and hence the r fixtures found flowing will include all those, and only those, which began their operation during the t-second interval immediately preceding the instant of observation.

VII. DEVELOPMENT OF THE PROBABILITY FUNCTION

By the generally accepted concept of probability, the probability that a particular fixture out of a number, n, will be found operating at any arbitrarily chosen instant of observation is t/T, where t has been defined as the duration of each operation and T as the time between operations of each fixture. In the same manner, the probability that the particular fixture will not be in operation at the instant of observation is $1-t/T$ or $(T-t)/T$.

A law of combinations that applies to the composite event of which the probability is sought in this problem may be stated as follows: The number of ways in which two or more independent events can occur together is the product of the ways each can occur separately. A similar law of probability may be stated as follows: The probability of two or more independent events occurring together, in this case at the same instant, is the product of the probabilities of their separate occurrence.

By the law of combinations, the probability that none of the remaining $n-1$ fixtures will be operating at the instant of observation is $\left(\frac{T-t}{T}\right)^{n-1}$. The probability that a particular one of n fixtures and none of the remaining $n-1$ fixtures will be operating at the instant is

$$(t/T)\left(\frac{T-t}{T}\right)^{n-1}.$$

But since there are n fixtures, there are n ways in which the event, one and only one fixture found flowing, can occur, and hence the probability of this event being found occurring at the instant of observation is

$$p_1^n=n\left(\frac{t}{T}\right)\left(\frac{T-t}{T}\right)^{n-1}. \tag{1}$$

This can be generalized for any group of r, and r only, constituting the particular event for which the probability is sought, by again applying the definitions and laws.

In any group of r fixtures the probability that any particular one will begin to operate within a given t seconds is t/T, and since there are r cases in the group, the probability, that all r fixtures will begin operation within the same t seconds is $(t/T)^r$. Also, of the n fixtures, any one of

$$\frac{n(n-1)(n-2) \text{ to } r \text{ factors}}{r!}=\frac{n!}{r!(n-r)!}=C_r^n$$

different groups is equally likely, and hence by the law of combinations the probability that some one of the equally likely groups of r fixtures will be in operation at the instant of observation will be $C_r^n\left(\frac{t}{T}\right)_r$. As before, the probability that the remaining $n-r$ flows will fall outside this particular t seconds is $\left(\frac{T-t}{T}\right)^{n-r}$.

Therefore, the probability that exactly r fixtures will be operating at a particular instant of observation in the manner defined is

$$p_r^n=C_r^n\left(\frac{t}{T}\right)^r\left(\frac{T-t}{T}\right)^{n-r}=C_r^n\frac{t^r(T-t)^{n-r}}{T^n} \quad . \ . \ . \ (2)$$

This is the general term of a series, expressing the probability that r out of n fixtures will be found flowing at any arbitrary instant of observation. This series may be represented as

$$\sum_{r=0}^{r=n} p_r^n = \sum_{r=0}^{r=n} C_r^n t^r \frac{(T-t)^{n-r}}{T^n} = 1 \quad . \quad . \quad . \qquad (3)$$

Equation 3 is equivalent to the binomial expansion of $\left(\frac{t}{T}+\frac{T-t}{T}\right)^n=1$ and is the conventional expression for certainty, since either no fixtures, $r=0$, or some number of fixtures from $r=1$ to $r=n$ must be operating at the instant of observation.[3]

VIII. INTERPRETATION OF THE PROBABILITY FUNCTION

As previously stated, the probability function, p_r^n, eq 2, gives the probability that exactly r fixtures out of a total of n will be found operating at an arbitrary instant of observation, provided that all n fixtures are in continuous use at the assumed rate. The probability, p_r^n, may also be interpreted as the percentage or fraction of the time in the long run that r flows will occur in the manner defined, since the fraction

$$\frac{\text{time of } r \text{ fixtures operating}}{\text{total time}}$$

is also the probability of the occurrence. Hence for any given values of n and r, if the probability function as developed is multiplied by a time τ in seconds such that

$$\tau p_r^n = \tau C_r^n \left(\frac{t}{T}\right)^r \left(\frac{T-t}{T}\right)^{n-r} = 1 \text{ second}, \quad . \quad . \quad . \qquad (4)$$

or

$$C_r^n \left(\frac{t}{T}\right)^r \left(\frac{T-t}{T}\right)^{n-r} = \frac{1}{\tau},$$

the equation signifies that r fixtures will be in simultaneous operation for an aggregate of 1 second out of every τ seconds that all n fixtures are in use at the assumed rate. Likewise, the condition that a chosen design factor, $r=m$, will not be exceeded more than a given fraction of the time $1/\tau$, is expressed by

$$\sum_{r=m+1}^{r=n} p_r^n = 1/\tau \quad . \quad . \quad . \qquad (5)$$

[3] Equations 2 and 3 may be developed by other methods and in different forms. However, the results will be identical or may be reduced to an identity, as evidenced by the function developed by Fry [2] for the analogous problem of busy lines in telephone systems (see eq 156, p. 335, of the reference), which differs from eq 2 only in the form to which it is reduced and in the notation employed.

Equations 4 and 5 are based on the assumption that all n fixtures will be in continuous use over the entire time τ at the average rate of once in T seconds. The time τ may be reduced to days or years on the basis of the daily period of peak use, by assuming or determining this period and computing on the basis of a day of that length.

IX. PROPOSED USES OF THE PROBABILITY FUNCTION FOR MAKING LOAD ESTIMATES

1. Pertinent Information Obtainable From the Probability Function

It may be helpful in judging the reasonableness of the proposed application of the probability function to summarize the information obtainable from the probability function before proceeding with the selection of the time factors applicable to practical cases.

The following pertinent information can be obtained from the equations developed:

(a) The probability that a given number of fixtures, m, out of a total of n, will be operating at an arbitrary instant of observation, determined by eq 2.

(b) The fraction of the time that m and only m fixtures will be operating at the same instant, a second interpretation of eq 2.

(c) The fraction of the total time that any number of fixtures greater than the design number, m, will be operating at the same instant, determined by eq 5.

(d) The ratio of any two successive terms in the series of eq 2; for example, the ratio of the fraction of the time $r+1$ to the fraction of the time r fixtures will be operating at the same instant, determined by

$$C_{r+1}^n \left(\frac{t}{T}\right)^{r+1} \left(\frac{T-t}{T}\right)^{n-r-1} \div C_r^n \left(\frac{t}{T}\right)^r \left(\frac{T-t}{T}\right)^{n-r}$$

$$= \frac{n-r}{r+1} \times \frac{t}{T-t}. \qquad (6)$$

There are certain characteristics of the probability series given in eq 2 which will be of material aid in determining the conditions on which the design factor m is based. In the series, eq 2, the value of successive terms increases as r increases up to the most probable value or values and then decreases. Also, the ratio of any term to the preceding term of the series decreases continuously as r increases from $r=1$ to $r=n$. These characteristics make it possible to determine the value m that will not be exceeded more than the fraction $1/\tau$ of the time without completing the entire summation from the $m+1$ to nth term.

If $p_m^r \leqq 1/\tau$, and if the ratio (eq 6)

$$\frac{n-m}{r+1} \times \frac{t}{T-t} \leqq \frac{1}{2},$$

the summation $\sum_{r=m+1}^{r=n} p_r^n$ must be less than $1/\tau$, because the finite series

$$\frac{1}{\tau}\left[\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2+\delta}\right)+\frac{1}{2}\left(\frac{1}{2+\delta}\right)\left(\frac{1}{2+\delta'}\right)+\ \ldots\ \right],$$

which results from the series

$$p_{m+1}^n + p_{m+2}^n + \ \ldots \ + p_n^n,$$

by multiplying and dividing this last series by p_m^n, substituting the value 1/2 for the ratio p_{m+1}^n/p_m^n, and replacing the factor p_m^n by $1/\tau$, is less, term by term, than the infinite series

$$\frac{1}{\tau}\left[\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ \ldots\ \right],$$

which is known to approach the value $1/\tau$ as the number of terms increases without limit. When the series satisfies this condition, it can be positively stated, without summation, that a design factor $r=m$ will be exceeded less than $1/\tau$ of the time.

If $\frac{n-r}{r+1} \times \frac{t}{T-t} > 1/2$, summation is necessary in order to determine a value of $r=m$ that will not be exceeded more than $1/\tau$ of the time, but it is unnecessary to carry the summation beyond the term where $\frac{n-r}{r+1} \times \frac{t}{T-t} \leqq 1/2$.

2. Values of t, T, τ, and q

In applying the probability function for estimating the design load mq, it is necessary to select values of t, T, and τ from which to compute the value of m and to select a value of q, the factors excepting τ pertaining to a particular kind of fixture and service. The actual values selected in any case are largely a matter of engineering judgment. In this connection, it is to be understood that in the following development and illustrative examples, the values selected represent the author's judgment in regard to the appropriate values for producing satisfactory service and are based on the author's interpretation of the information available.

For the purposes of this discussion *satisfactory service* is defined in a relative sense as that in which interruption in service because of controllable factors, such as the sizes and arrangement of pipes, is infrequent and is of sufficiently short duration to cause no inconvenience in the use of fixtures or any unsanitary condition in the plumbing system. Attainment of satisfactory service will depend on the selection of the design factor m, or more specifically on the value of τ from which the value of m is computed. The value of τ selected for illustrating the proposed application is 100 seconds, which provides for wholly satisfactory service 99 percent of the time and for reasonably satisfactory service all of the time if the design load mq gpm is not greatly exceeded. In this connection, it will be observed that if m is exceeded in actual service it is most likely to be exceeded by one fixture only and is progressively less likely to be exceeded by two, by three, or more.

Obviously t and q bear a direct relation to Q in respect to the values necessary to provide satisfactory service if m fixtures are in operation simultaneously. Since there is a considerable range in the values of t and Q on which the value of q depends for any particular fixture, it will be very helpful to the engineer in determining reasonable values to be used for a particular kind of fixture to consider the characteristics of operation of that kind of fixture.

It is a characteristic of water closets that they will operate more or less effectively under any average rate of supply from about 15 gpm up to rates of about 30 gpm or more delivered in any time ranging from about 6 seconds up. For each type and design of water-closet bowl there is an intermediate smaller range of average rate of supply within which there is no detectable difference in the effectiveness of the flush in emptying the bowl of its contents. There is likewise a range in time of flow within which the bowl will be effectively emptied of its contents with an average rate of supply anywhere within the smaller range referred to.

From the evidence of experiments reported in the earlier paper [1], the Subcommittee on Plumbing of the United States Department of Commerce Building Code Committee agreed upon an average rate of 30 gpm for 10 seconds as a reasonable and safe basis for estimating loads to be expected in building drainage systems. The experiments referred to were designed to obtain the maximum loads per closet that could be delivered to the drains within the operational range of water closets, and no attempt was made to determine either the most effective rate of supply for a particular type of bowl or an average rate that would produce a satisfactory flush in all types of closet bowls. It is to be expected, as has been the case, that overestimates would result when the maximum values of all load factors involved are employed in estimating. More recent experiments [3] by Camp give rates of supply for safe and economical flushes ranging from 20 gpm to 29 gpm for different types of closet bowls and times of flush ranging from 7.5 to 9 seconds, omitting data for one bowl with spiral flow action. The averages for six different bowls are 25.9 gpm and 8.2 seconds. Unpublished data from still more recent experiments [4] at the National Bureau of Standards indicate that the most effective removal of the contents of a closet bowl occurs with rates of supply ranging from about 20 to 24 gpm and that the complete contents of the bowl are removed, if removable by any rate of flow, in times ranging from about 6 to about 10 seconds for different types of bowls.

In both cases the data were obtained with approximately uniform flow through the bowl, and that average rate of flow which maintained continuous siphon action through the bowl was taken as the criterion for the most effective flushing rate. Considering the wide ranges in rate and time of supply within which a closet bowl will operate effectively, Camp's and Golden's data may be regarded as in substantial agreement. However, none of the experiments cited closely simulates service conditions, for the rate of supply in actual service is not uniform, whether supplied by flush valve or flush tank: it rises to a maximum at or near the beginning and gradually decreases as the valve closes or as the tank empties. The characteristics of an effective flush may be summarized as follows:

A quick priming of the siphon; a continuous siphon action for sufficient time to clear the bowl of its contents and carry them through the trapway; and a breaking of the siphon action before the flow ceases, in order to refill the trap. Whether by design or chance, the characteristic operation of conventional supply devices (flush valves and flush tanks), if properly adjusted for volume and time, controls the rate of supply in a manner that meets these flushing requirements admirably. In figure 1, curve 1 is the reproduction from Camp's report of a typical rate curve for a flush valve, representing a flow of 3.8 gallons in 8.5 seconds giving a peak rate at the end of two seconds of approximately 37 gpm and an average rate of 26.9 gpm over the time the valve is in operation. Curve 2 is the rate curve for a flush tank plotted from Golden's data for a flow of 4 gallons in 9.1 seconds giving a peak rate about 31.5 gpm at the end of 1 second and an average rate of 26.4 gpm. In both cases the flow reaches a rate within the first second amply sufficient to prime the siphon of any standard type of closet bowl and thereafter is maintained for 6 or 7 seconds within the range required to produce continuous

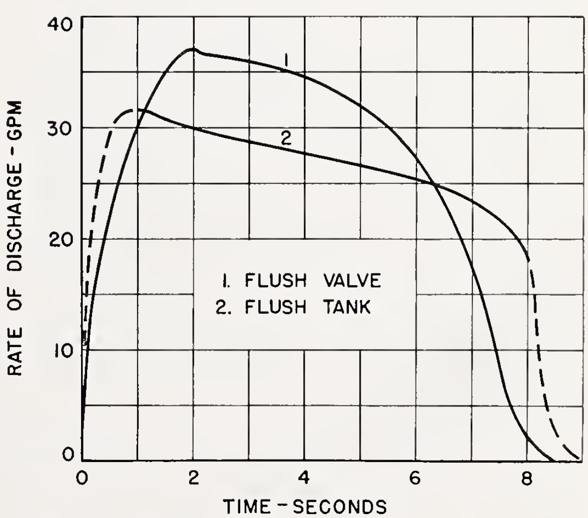

Figure 1.—*Typical time-supply curves for water-closet bowls.*

siphon action. The total volume, the average rate of flow, and the duration of the flush in these examples are all greater than necessary for an effective flush in the types of bowls most commonly used—washdown, reverse trap, and siphon-jet bowls. It is impractical to attempt to estimate individual variations either in rates of supply or in rates of use of fixtures. Hence, it is advisable to set the factors chosen for making estimates high enough to allow some leeway in adjusting down to the volume and rate of supply needed for satisfactory service in particular cases, since it is impossible to adjust for greater rates of supply than the supply pipes are capable of delivering.

Considering the problem of estimates from all angles, an average rate of supply of 27 gpm for a duration of 9 seconds, giving a volume of approximately 4 gallons per flush, appears to meet requirements generally as nearly as can be done in round numbers and will be employed for flush valves for water closets in evaluating the probability function in the further development of the methods of estimating demand loads.

The rate of supply to flush tanks for water closets is not related directly to the rate of supply required by the closet for effective operation. The only essential for operation of the fixture is that the tank refill in the interim between operations. A rate of 4 gpm is ordinarily considered sufficient, which for a volume of 4 gallons gives a value of 60 seconds for t. The value of T should obviously be the same for flush-valve and flush-tank supply.

In case of faucet-controlled supply for fixtures, it is not possible to base the time factors or the quantity of water used on the characteristic operation of the fixture as was done for flush valves for water closets, because the manner in which faucets are operated depends largely on personal habits or preferences. For such fixtures an arbitrary selection of values, made from a consideration of relative rates of supply and volumes used, appears to be the only recourse.

An average rate of supply of 8 gpm would permit the drawing of 8 gallons in 1 minute (60 seconds), 16 gallons in 2 minutes, 24 gallons in 3 minutes, etc. Bathtubs, depending on size and style, hold from 25 to 40 gallons when filled to the overflow. Ordinarily only a fraction, possible ⅓ to ½, of these volumes will be used for a bath. The time, T, between uses will include the time required to draw the water, the time taken for bathing, the time required to empty the tub, and any additional time consumed in the complete bathing operation. This total time, T, between operations of the fixture seems likely to range from about 15 minutes for hurried baths to about 30 minutes for baths taken more leisurely. It also seems likely that smaller quantities of water will be used under the former than under the latter conditions. Now, assume that 8 gpm is an ample average rate of supply for a bathtub. This is 60 percent higher than some [5] and 20 percent lower than other [6] estimates of a satisfactory average rate of supply for a bathtub. An average rate of supply of 8 gpm, an assumed average time of filling of $t=60$ seconds, and an assumed time between operations of $T=900$ seconds (15 minutes) would provide for an average of 8 gallons per bath. For an average volume of 16 gallons per bath and the same rate of supply, 8 gpm, 120 seconds (t) will be required to draw the bath. If the rate of operation (average time taken per bath) is once in 30 minutes, giving $T=1{,}800$ seconds, the ratio of t/T is the same in both cases; $60/900=120/1{,}800=1/15$. Since for a given value of n the value of p_r^n for any value of r is determined by the ratio t/T, the probability of a selected design load mq being exceeded will be exactly the same for the two cases cited or for any other case in which the time T is proportional to the volume used and the same basic rate q is employed. From these considerations, a design load mq, for bathtubs in congested service, based on an average rate of supply of 8 gpm, and a ratio $t/T=1/15$ appears to insure fairly satisfactory service and is used as the basis of the development and comparisons following.

The values selected for the three fixtures discussed are given in table 1.

TABLE 1.—*Values of t/T, q, and Q*

Kind of fixture	t/T	q in gpm	Q in gallons
Flush valves for water closets	9/300	27	4
Flush tanks for water closets	60/300	4	4
Bathtubs	1/15	8	8 to 16

3. Presentation of Probability Data

Equations 4 and 5 are not satisfied in general by integral values of r and n and a given value of $1/\tau$; that is, for occurrence exactly a given percentage of the time. However, by trial and error, a value of r for each given value of n can be computed for which the probability of occurrence is in excess of a given percentage, for example 1 percent of the time, by the smallest possible fraction. From this probability, p_r^n, a particular value m of r, for which the probability of occurrence of any number greater than m is not greater than a given percentage of the time, may be readily determined by an approximation of $\sum\limits_{r=m+1}^{r=n} p_r^n$.

If $p_{m+1}^n \leqq (1/2)p_m^n$, no further computation is necessary as pointed out on page 8. If $p_{m+1}^n > (1/2)p_m^n$, it will ordinarily be necessary to carry the summation to a few terms only to determine the value of m required.

Probability data for flush valves for water closets, flush tanks for water closets, and bathtubs are given in table 2, in which r is the greatest number of fixtures out of n that will be in operation 1 percent of the time and m is the number of fixtures that will not be exceeded more than 1 percent of the time.

Table 2.—*Probability data for three types of fixtures*

Flush valves, for $t/T=9/300=3/100$				Flush tanks, for $t/T=60/300=1/5$				Bathtubs, for $t/T=1/15$			
n	r	p_r^n	m	n	r	p_r^n	m	n	r	p_r^n	m
6	2	0.0119	2	3	2	0.0960	2	3	2	0.0124	2
16	3	.0102	3	5	3	.0512	3	8	3	.0118	3
30	4	.0101	4	7	4	.0281	4	15	4	.0126	4
47	5	.0104	5	9	5	.0165	5	22	5	.0107	5
66	6	.0106	6	15	7	.0138	7	31	6	.0118	6
85	7	.0100	7	25	10	.0118	10	40	7	.0112	7
107	8	.0105	8	40	14	.0115	14	59	9	.0104	9
129	9	.0104	9	64	20	.0112	20	113	14	.0102	14
151	10	.0101	10	132	36	.0104	37	172	19	.0103	19
199	12	.0102	12	240	60	.0101	67	221	23	.0102	23
299	16	.0101	16	305	74	.0102	78	309	30	.0100	31

It will be observed that for flush valves t/T is small, and $r=m$ for all values of n in the range given, and that the greater t/T the smaller is the range in which $r=m$.

Figure 2 shows the curves obtained by plotting m and n for the three fixtures, curve 1 being

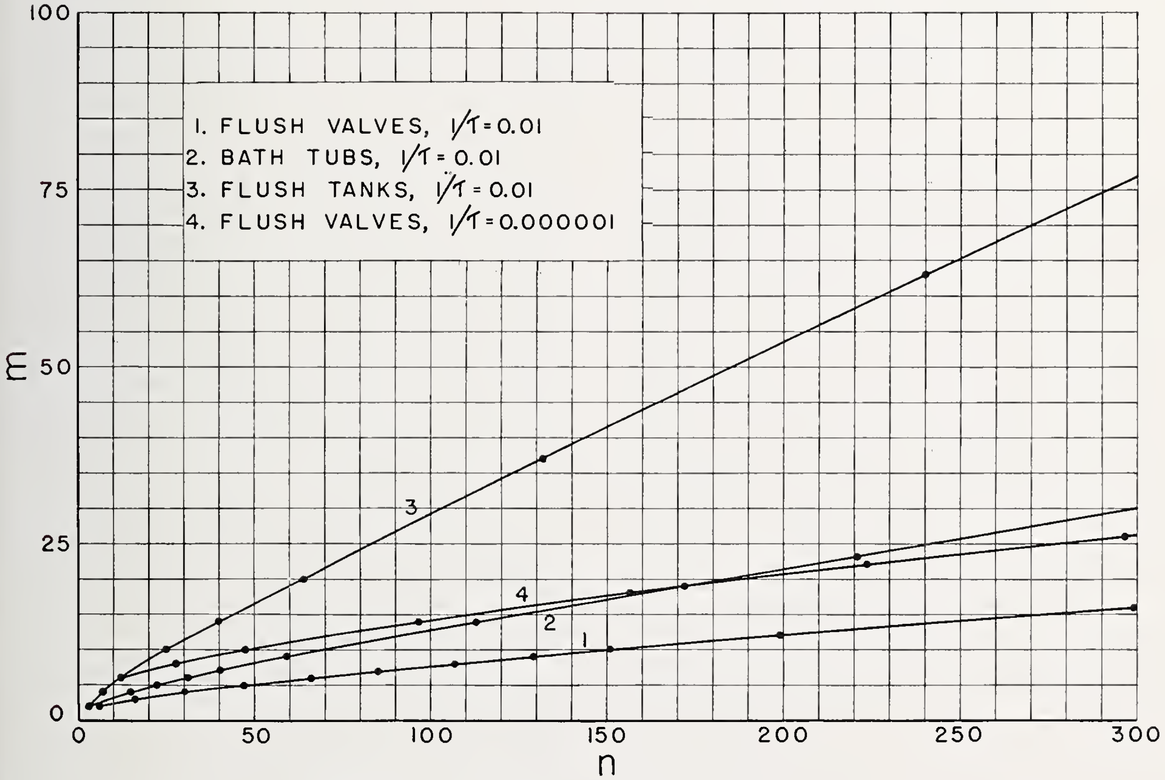

Figure 2.—*Probable relation of m to n from table 1.*

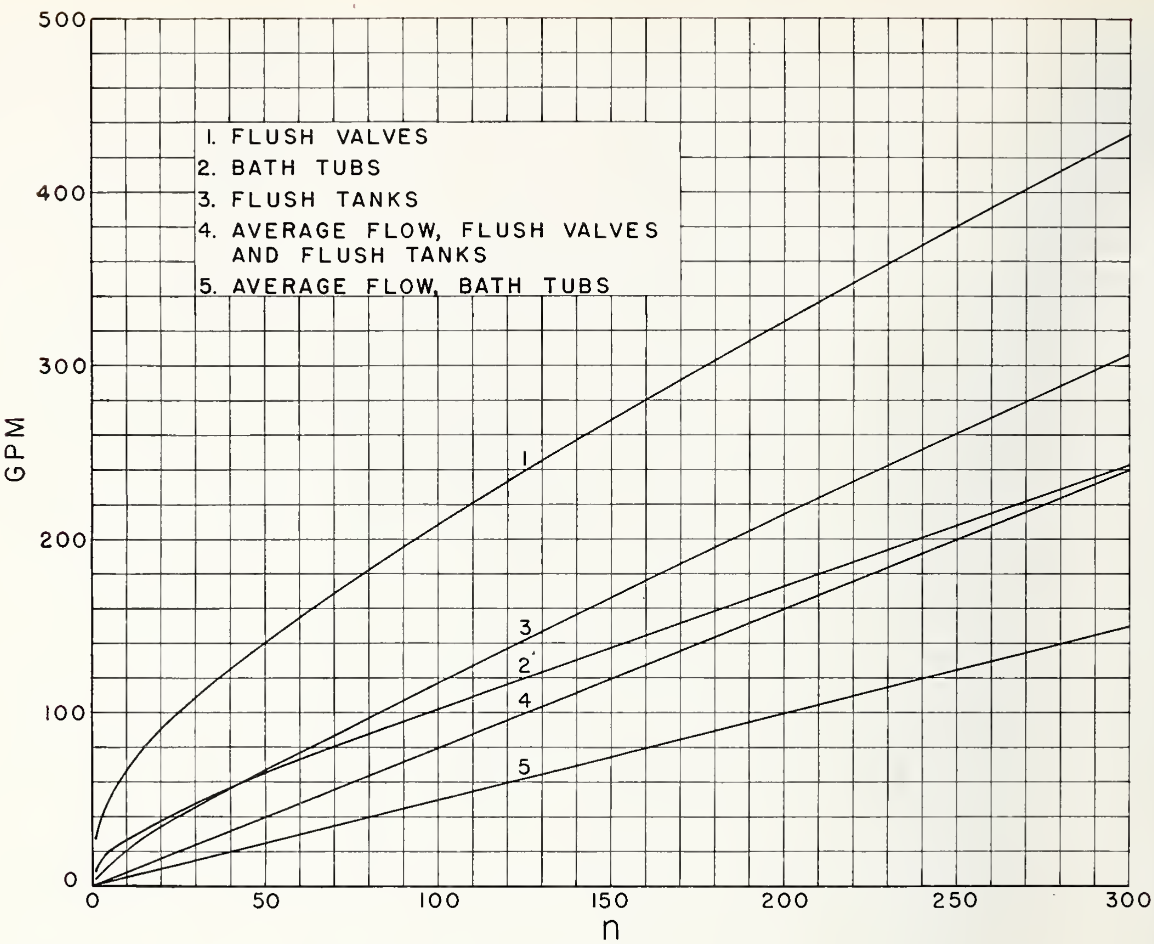

FIGURE 3.—*Probable flow in relation to n.*

for flush valves, curve 2 for bathtubs, and curve 3 for flush tanks. A value of m for flush valves that would not be exceeded for more than one-millionth of the time was also computed, and the results are represented in figure 2 by curve 4.

Estimates of pipe capacities will ordinarily be obtained in flow units, for example in gallons per minute. Hence it will be more convenient in practice to have the demand load expressed in the same units. This may be obtained by plotting mq and n as shown by figure 3, in which q is taken as 27 gpm for flush valves, 4 gpm for flush tanks, and 8 gpm for bathtubs. The average flow, nQ/T, during the time the fixtures are being used at the rate of once in the time T is also represented in figure 3, for flush valves and flush tanks by line 4 and for bathtubs by line 5.

Curves 1, 2, and 3, in figure 3, show the relation of demand loads to number of fixtures, based on estimated time factors representing congested conditions of service—that is, the maximum practical rate at which fixtures can be used continuously in actual service. Assuming the correctness of the factors employed in evaluating the probability functions, the curves may be used for estimating the demand loads for any particular number of fixtures of one given kind. However, the design load for all kinds of fixtures installed in one system should not be the sum of the design loads computed separately for each kind of fixture, even though the individual curves may be correct. Simultaneous operation of different kinds of fixtures is a chance occurrence which would have to be evaluated by another probability function. Although such an evaluation is possible on the basis of the assumed average time factors, the process is too complicated for convenient application in the field or office and, because the time factors for faucet-supplied fixtures on which the probability of simultaneous operation depends cannot be determined in general, would in the end lead to doubtful results. For these reasons it is believed that reasonably satisfactory results, which are much simpler to apply, may be obtained by weighting each kind of fixture and applying the weighted sum of the total numbers of fixtures of all kinds to a load curve for flush valves or a load curve for flush tanks, according to which type of supply is to be used.

4. Derivation of Fixture Weights (Fixture Units)

In Recommended Minimum Requirements for Plumbing [1], fixtures were weighted in the scale from 1 to 6, this scale being selected largely because, of the fixtures installed in the greatest numbers, the smallest load (lavatory) was estimated to be about one-sixth of the largest load (water closet). The scale chosen is purely arbitrary, and it is now suggested that a decimal scale, 1 to 10, would give a much more flexible system of estimating.

A comparison of the relative loading effects of the three kinds of fixtures may be made by means of the curves in figure 3. Obviously there is no common exact ratio of the loading effects of any two different kinds of fixtures, each kind using water at different volumes, Q, volume rates, q, and different time intervals, T, over any considerable range in number of fixtures, n. For example, for a demand of 50 gpm, the number of fixtures from which this load would be occurring not more than 1 percent of the time is approximately (fig. 3) as 5:34:33, respectively, for flush valves for water closets, flush tanks for water closets, and bathtub supply (total). The load-producing weights are inversely as the number of fixtures producing a given load; hence, on the basis of 10 for flush valves, the weights for a demand of 50 gpm are 10:1.47:1.52. Table 3 gives the relative weights computed in this manner from a comparison of the three curves for demand loads at increments of 50 gpm from 150 to 300 gpm, the range in which the proposed method of estimates will have its greatest application and usefulness.

Table 3.—*Relative demand weights of fixtures*

Demand, gpm	Flush valves		Flush tanks		Bathtubs	
	Number of fixtures n	Weight	Number of fixtures n	Relative weight	Number of fixtures n	Relative weight
150	56	10	133	4.20	167	3.35
200	93	10	185	5.03	238	3.90
250	133	10	238	5.59	312	4.30
300	176	10	292	6.03	398	4.42
Average weight		10		5.21		3.99

Neither the possible accuracy in estimating the demand to be expected in a plumbing system nor in estimating the capacity of pipes selected for supplying the estimated demand justifies the assignment of weights in the scale 1 to 10 closer than to the nearest integer. Accordingly, on the basis of average values, the relative weights of the three fixtures become 10 for flush valves for water closets, 4 for bathtubs (total hot- and cold-water supply), and 5 for flush tanks.

It will be observed that these fixture weights have the same significance as the fixture unit ratings assigned to the different kinds of fixtures in Recommended Minimum Requirements for Plumbing [1]. This unit in the weight scale is not a definite unit of flow but is simply a load factor to be applied through a relation between m and n such as developed in figures 2 and 3. However, since the term *fixture unit* in this significance has become fairly well established by usage during the past 15 years, the term will be retained here to designate load weights of fixtures.

A further comparison and an illustration of the results of applying the average weights of different kinds of fixtures to a single basic probability curve may be made by reducing the number of fixtures to fixture units (n times weight) and replotting, as in figure 4. It will be observed that the curves for flush tanks and bathtubs (total supply) lie very close together, indicating that the relative weights, 5 to 4, are approximately correct throughout the range illustrated for the individual volumes and rates allowed. It will also be observed that the two curves both intersect the flush-valve curve between 193 gpm 880 fixture units and 210 gpm 1,040 fixture units, indicating again that the relative weights 10:5:4 are approximately correct in that range. For 300 flush tanks, 1,500

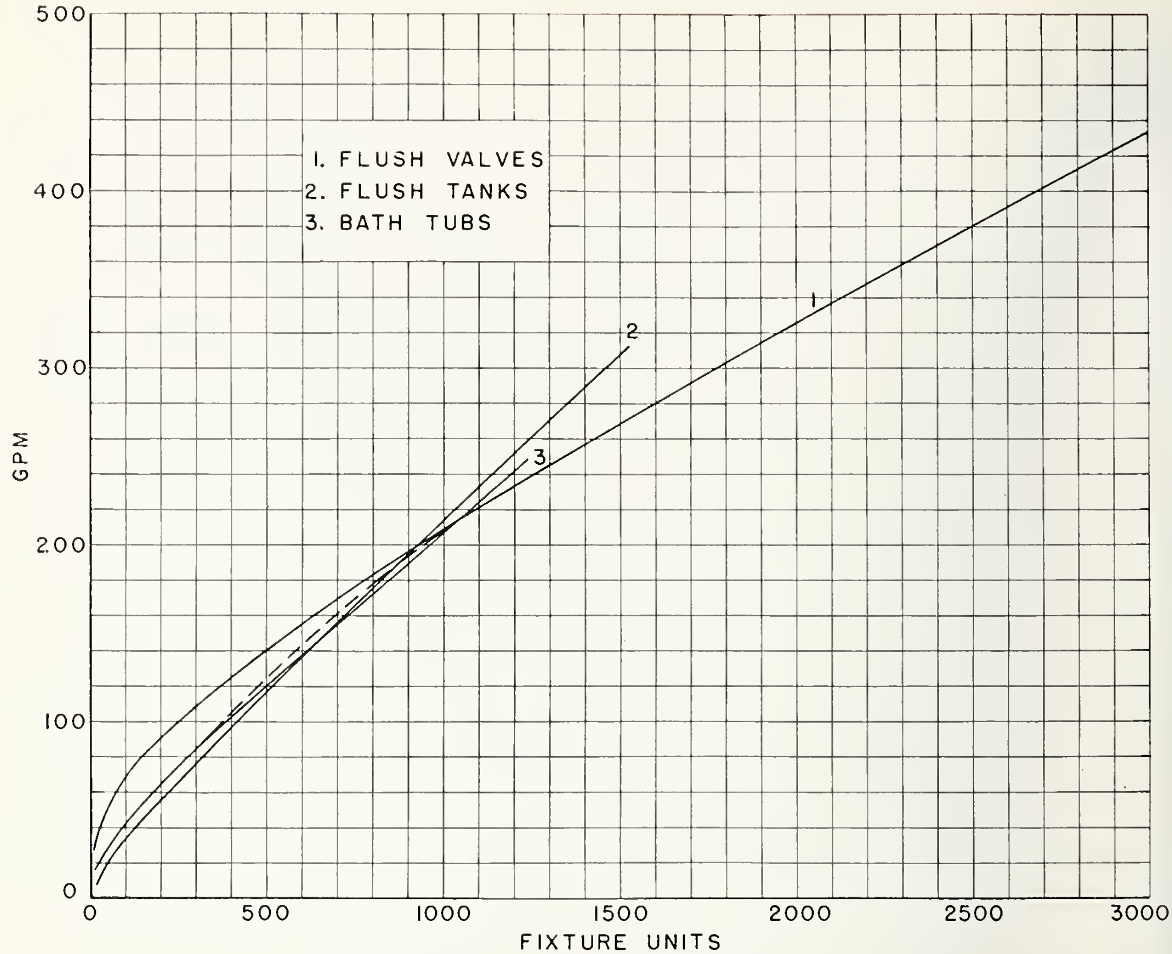

FIGURE 4.—*Relation of demand to fixture units.*

fixture units, referred separately to the flush-valve curve for an estimate, the demand estimate would be about 15 percent lower than the corresponding estimate made directly from the flush-tank curve. The corresponding error in the estimate for 300 bathtubs made in the same manner would be about 3½ percent. These errors are immaterial, for the only result, in case the design load was exceeded in service by that amount, would be an increase in the time required to refill the fixtures by 15 percent and 3½ percent, respectively, or, in case the same time is occupied in refilling, a reduction by these percentages in the volume of water used. Below the point of intersection, referring flush tanks and bathtubs separately to the flush-valve curve would result in overestimating the demand by amounts varying from very small percentages for 880 to 1,040 fixture units to about 94 and 23 percent, respectively, for 100 fixture units. However, the error in an estimate made from curve 1 for the total demand load for flush valves for water closets and for bathtubs will be less than the error indicated by an estimate made separately for the bathtubs from the same curve in all cases in which the flush valves predominate, on the basis of total fixture units of the two kinds of fixtures. In cases where flush tanks for water closets are used exclusively or predominantly in the system, a closer estimate could be made on the basis of total weights, by using curve 2 and the total fixture units for all kinds of fixtures involved. Obviously the error made by using curve 2 for both flush tanks and bathtubs for any number of either up to 300 would be small. Also, the demand load relative to the number of fixture units may be approximately repre-

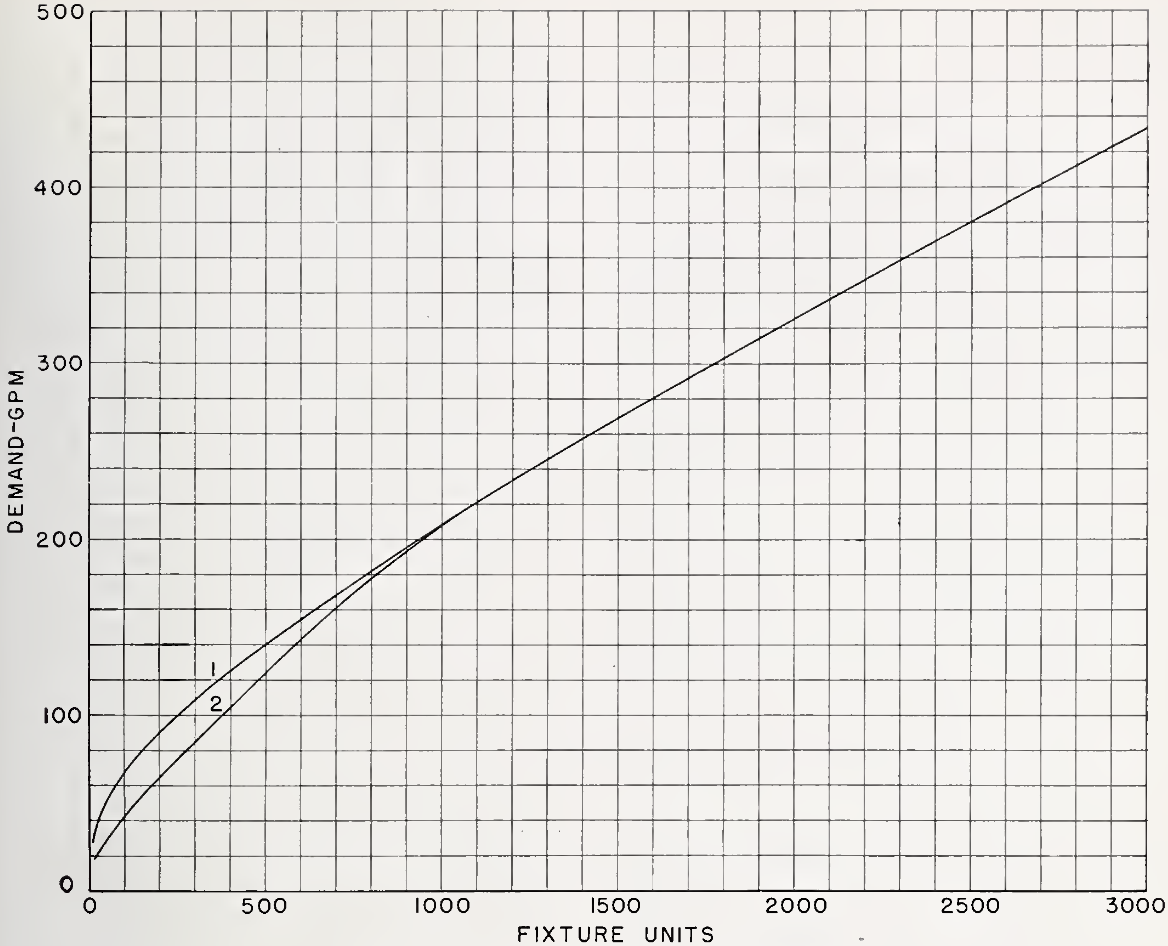

FIGURE 5.—*Estimate curve for design purposes.*

sented in this range by a smooth curve drawn above the two probability curves and merged with curve 1 as shown by the broken line in figure 4, thus giving estimates slightly in excess of the peak demands indicated by the separate curves for flush tanks and bathtubs. The broken line in figure 4 is reproduced in figure 5, together with curve 1. The curves in this figure are proposed for estimating design loads for water-supply lines in general, curve 1 to be used when flush valves predominate in the system and curve 2 to be used when flush tanks predominate, the common curve above the branch to be used for all weighted fixtures.

Of the fixtures commonly installed in the greatest numbers, these three produce the major part of the peak demand. Lavatories are installed in most systems in as great numbers, and frequently in greater numbers, but obviously have a much smaller load-producing weight, and are frequently ignored in estimating demand and sewage loads. Because, as pointed out in discussing supply demands for bathtubs, it is impossible to estimate the values of t and T reliably for faucet-supplied fixtures, and because of the relatively small effect on the total demand, it is suggested that satisfactory fixture-unit ratings may be assigned to irregularly used faucet-supplied fixtures independently of the probability function from a consideration of the sizes of the supply outlets and the relative quantities of water used. On the basis of this reasoning, the relative weights suggested for the kinds of fixtures most commonly subjected to congested service are: 10:5:4:2 for flush valves for water closets, flush tanks for water closets, bath tubs, and lavatories, respectively.

5. Relative Load Weights for Different Conditions of Service

Up to this point the discussion has been confined to four kinds of fixtures under congested conditions of service. There are several considerations that should be taken into account in determining load ratings for other fixtures and for other conditions of service, among which the following are especially important:

(1) Fixtures that are relatively few in number and unlikely to be used when the predominant kinds are being used most frequently will add very little to the demand or to the peak sewage load and hence may be ignored, except in regard to the branch supply lines and branch drain pipes of these fixtures. Slop or service sinks in office buildings, which are in use to any considerable extent only before or after office hours, add a negligible load to the peak loads of the day. Kitchen sinks and laundry trays in dwellings and residential apartments may also be placed in this category.

(2) Fixtures so installed that they cannot in general be subject to congested conditions of service in the same sense as fixtures installed in public comfort stations, general toilet rooms in office buildings, and other buildings in which each fixture is open and accessible for use at all times, should be given a rating in accordance with the possible extent or frequency of their use. Bathrooms in private dwellings, residential apartments, and private bathrooms in hotels may be considered in this class, and can be rated advantageously as a group.

(3) Water services that demand a continuous flow, such as lawn sprinklers, air-conditioning equipment, gang showers in factories, and athletic dressing rooms, present no element of chance in regard to overlapping and are not susceptible of a logical weighting in relation to water closets and other fixtures that use water at high rates for comparatively short periods of time. Hence the demand for this type of supply should be considered separately and estimated separately. If the use of these continuous water supplies is such that they overlap the rush period of the day for the weighted fixtures in the system, the separate estimates for the two classes of supply should be added to obtain the estimate for total demand on any supply pipe common to both services. If the two types of demand do not come at the same time of day, the greater demand of the two may be taken as the peak demand, or design estimate.

A bathroom group of fixtures can probably be more closely weighted to the selected scale of 1 to 10 by taking the three common bathroom fixtures—water closet, lavatory, and bathtub or shower—as a unit than by weighting each fixture separately. In hotels and apartment houses the bathrooms are not all in use at one time, and the loading weight of any fixture of the group—for example, a water closet—will be much lower than for the same fixture in congested service, for which it has been assumed that all fixtures will be in continuous use during the period of congestion. One peak period occurs in hotels and apartments in the morning during the rising hour and another in the afternoon during the preparations for dinner. Probably the best indication of the distribution of the use of the bathrooms in hotels and apartment buildings obtainable will be given by records of the activity during the hours when the peak use of the plumbing fixtures occurs. Table 4 is a summary of recorded morning (rising) calls from the switchboard for two large hotels in New York City for 7 consecutive days in one case and 9 days in the other. Hotel 1 is a large hotel in which the guests were principally transient. Hotel 2 is a large apartment hotel in which the greater part of the guests were resident but which also received and accommodated transients.

Table 4.—*Distribution of morning calls in hotels*

Place	Time of call									Total calls
	Before 6:30	6:30	6:45	7:00	7:15	7:30	7:45	8:00	After 8:00	
Hotel 1	174	228	98	486	146	700	185	463	382	2,862
Hotel 2	198	212	133	599	176	780	176	442	341	3,057
Total calls	372	440	231	1,085	322	1,480	361	905	723	5,919

It will be observed that the time of rising in the two hotels is distributed in approximately the same manner, the greatest number of calls in any 15-minute period being for that following 7:30 a. m., and that slightly more than one-half of all calls fall within the hour from 7:00 to 8:00 a. m. This is significant, and it may be assumed that the time of rising of guests leaving no morning call will be similarly distributed in time.

TABLE 5.—*Distribution of morning calls in apartments for 9 days*

Period	Time of call									Total calls
	Before 6:30	6:30	6:45	7:00	7:15	7:30	7:45	8:00	After 8:00	
1st day	3	2	7	11	3	6	3	1	1	37
2d day	8	4	8	12	2	9	3	1	2	49
3d day	6	2	9	12	3	9	3	2	2	48
4th day	4	3	9	12	3	8	3	1	1	44
5th day	3	2	9	12	2	8	3	1	1	41
6th day	5	4	7	11	2	8	2	1	1	41
7th day	8	4	7	13	3	11	4	2	1	53
8th day	4	4	9	11	4	11	2	2	3	50
9th day	4	3	10	12	4	10	3	1	1	48
Total calls	45	28	75	106	26	80	26	12	13	411

TABLE 6.—*Distribution of morning exits in apartments for 2 days*

Period	Interval of time									Total exits
	6:45 to 7:00	7:00 to 7:15	7:15 to 7:30	7:30 to 7:45	7:45 to 8:00	8:00 to 8:15	8:15 to 8:30	8:30 to 8:45	8:45 to 9:00	
1st day	11	4	18	31	47	70	41	31	26	279
2d day	6	8	15	27	49	59	54	23	21	262
Total exits	17	12	33	58	96	129	95	54	47	541

Table 5 gives the summarized record of morning calls for a large apartment house in Washington, D. C., for 9 different days other than Sundays during a period of 22 days. Table 6 gives the summarized record of morning exits from the same apartment house. Comparing tables 4 and 5, it will be observed that the number of morning calls in 15-minute periods in hotels and apartments fluctuates in about the same manner, the principal difference being that the peak number of calls in any 15-minute period is about 10 to 15 percent of the total number higher for the apartment house than for either the transient or apartment hotel. Also comparing tables 5 and 6, it will be observed that the peak of exits from the apartment house, indicating the beginning of the business or social activity of the day, occurs approximately 1 hour later than the peak of the rising activity. This indicates that extensive use of plumbing fixtures in any single apartment or in any group of apartments in which the occupants rise at the same hour will last for about an hour. The maximum number of calls within any single hour ranges from 69 to 75 percent for different days. The corresponding percentages for the hotels range from 47 to 62. It therefore seems reasonable to assume that not more than 75 percent of the bathrooms in an apartment house or in any hotel will be occupied and in use at one time. There may be exceptional apartment houses in which the occupants are all engaged in the same occupation that would show a higher concentration in rising time. However, it will be the exception rather than the rule and should be treated as an exception in estimating water-supply demands and sewage loads.

There is another consideration that is important in estimates of loads for bathrooms. In a leisurely use of a bathroom, the fixtures will ordinarily be used one at a time. In cases of hurry or congestion, two fixtures may be in use at the same time in a bathroom, in the sense that water is being drawn for or is being discharged from both at the same time. Hence it seems reasonable to assume that, on the average, not more than two-thirds of the total number of fixtures in all occupied bathrooms will be in use at the same time. The total number in use at one time on this basis, will not exceed two-thirds of 75 percent, or 50 percent, of the total number of fixtures in bathrooms in the building. Accordingly, it is suggested that a bathroom group in a residence or apartment, or a private bathroom in a hotel be given one-half the total weight for the same fixtures in public or congested service in estimating supply demands.

The term "bathroom group" is here defined as consisting of a water closet, a lavatory, and a bathtub with or without a shower head, or of a water closet, a lavatory, and a shower stall. These suggested weights become $(10+4+2)\div 2=8$, and $(5+4+2)\div 2=5.5$, respectively, for installations using flush valves and flush tanks. Again, in accordance with the convention previously used in selecting fixture weights, the nearest higher integral value will be assigned, giving for a bathroom group using a flush tank the weight of 6.

In suggesting this rating for a bathroom group, it is recognized that the condition of service for which the rating is made does not completely block out of use one-half of each kind of fixture, as would be required for the result obtained to be exact. What actually occurs in the installations to which the proposed rating would apply is that one-fourth or more of the fixtures of each kind are not in service during the time the other three-fourths or less are in actual use, and that the ratio of t/T for each kind of fixture is greatly decreased below the ratio applying to congested conditions of service. For example, if each of the water-closet flush valves is operated twice during the hour in which it was assumed that two-thirds of the bathrooms were in use, $t/T=9/1800=1/200$; if used four times, $t/T=9/900=1/100$, as compared to $t/T=9/300=3/100$ for congested service. A similar relation will exist between the ratios of t/T for flush tanks and bathtubs under the conditions described and under congested conditions. It may be noted in this connection that assigning a bathroom group one-half the rating that is given the same fixtures in congested service would give an estimate of the demand greater than one-half of that for congested service. For example, 10 flush valves for water closets, 10 bathtubs, and 10 lavatories in congested service total 160 fixture units, which by curve 1 in figure 5 gives an estimated demand of 81 gpm. The proposed rating for 10 bathroom groups with flush valves, 80 fixture units, gives an estimated demand of 64 gpm.

Table 7 gives the fixture weights suggested in accordance with the use to which the fixtures are subjected and the manner in which they are installed. The term "public" refers to fixtures which are individually open for use at all times when the building is open, as in public toilets or general toilets in office buildings. "Private" refers to fixtures installed in groups in such a manner that the entire group may be and generally is confined to the use of one person at a time, as in residences or private baths of hotels. "Total" refers to hot and cold supply combined. "Hot or cold" refers to hot or cold water supply only.

Table 7.—*Demand weights of plumbing fixtures*

Fixture or group	Occupancy	Type of supply	Weight per fixture or group in fixture units
Water closet	Public	Flush valve	10
Do	do	Flush tank	5
Pedestal urinal	do	Flush valve	10
Stall or wall urinal	do	do	5
Do	do	Flush tank	3
Lavatory	do	Total	2
Do	do	Hot or cold	1.5
Bathtubs	do	Total	4
Do	do	Hot or cold	3
Shower head	do	Total	4
Do	do	Hot or cold	3
Bathroom group	Private	Flush valve (total)	8
Do	do	Flush valve (cold only)	6
Do	do	Flush tank (total)	6
Do	do	Flush tank (cold only)	4
Do	do	Hot water only	3
Bathroom group with separate shower	do	Add to corresponding group above for total, 2; for cold or hot	1.5

6. Application of Load Chart and Weight Table

In estimating the demand load for a supply pipe in any building, the total number of each kind of weighted fixtures or weighted groups of fixtures will be multiplied by the weight of that fixture or group (table 7) and the products added to obtain the total number of fixture units. The demand load is given by the corresponding ordinate of the appropriate curve in figure 5. For example, assume that an apartment house or a hotel has 100 bathrooms with flush-tank supply for all water closets and that any other fixtures in the building are negligible [4] in relation to peak demand. The total number of fixture units will be 100×6, or 600. The ordinate on curve 2 of figure 5 corresponding to this abscissa is 147 gpm. This estimate, of course, applies only to the main supply or service pipe for the building and does not include any continuous demand, such as that for lawn sprinklers or air conditioning, etc. Again, assume that an office building has 100 water closets with flush-valve supply, 25 stall urinals with flush-valve supply, and 100 lavatories, and that the demand of other plumbing fixtures is negligible in relation to these, but that the building has an air-cooling system which demands a maximum rate of 225 gpm. By referring to table

[4] See discussion of negligible fixtures on page 16.

5 it is found that the total number of fixture units is $100\times10+25\times5+100\times2$, or 1,325. The corresponding estimated demand load is 247 gpm, to which 225 gpm must be added, giving 472 gpm for the total estimated demand on the service pipe of the building. The method of estimating for branch risers and other distributing pipes is similar, the estimate being based on the number of fixtures or groups supplied through the branch, and the weights being selected according to whether the branch supply pipe carried the total supply, the cold water only, or the hot water only.

X. ADEQUACY OF THE METHOD

The proof of the adequacy of the proposed method of estimating the demand loads to be expected in building-water-supply systems will, in the end, depend on its success in actual trial over a period of years. Fortunately, in this case we have a means of comparison with a similar method that has been given a trial.

Table 8 is an abridgement of a table from the construction manual [5] used by the mechanical engineering section of one of the Federal departments, based on the method of estimate by means of the probability function as originally proposed [1] in 1923. This table has been used for a number of years by that office with reasonably successful results. The only complaint against the results of using this table and method that has come to the author's attention is that it tends to give larger estimates than have been found necessary for satisfactory service. The principal reason for this tendency to oversize supply pipes does not lie in any inherent fault in the probability function, but in the application of the method by means of a table which does not provide for the probability, or rather the improbability, of overlapping between or among two or more groups of different kinds.

TABLE 8.—*Demand estimates in gallons per minute from Federal table*

Kind of fixture	Number of fixtures												
	2	4	8	12	16	24	40	50	80	100	150	200	300
Water-closet flush tanks	8	12	16	20	30	38	50	58	80	95	140	215	290
Water-closet flush valves	36	46	61	72	83	100	128	143	180	200	250	295	375
Urinal flush valves	18	23	30	36	42	50	64	72	90	100	125	148	188
Lavatories	6	10	15	18	20	25	30	36	51	60	75	85	115

The inherent fault of applying the probability function by means of a table is illustrated in the comparisons which follow. Table 9 gives estimated demand loads for flush valves for water closets, flush valves for urinals, flush tanks, and lavatory faucets, taken from the curves of figure 4 over the range in number of fixtures included in table 8. The values for flush valves for urinals were taken from the curve for flush valves for water closets on the basis of five fixture units per fixture, and the values for lavatories from the curve for bathtubs on the basis of two fixture units per fixture (table 7).

TABLE 9.—*Demand estimates in gallons per minute from figure 4*

Kind of fixture	Number of fixtures												
	2	4	8	12	16	24	40	50	80	100	150	200	300
Water-closet flush tanks	8	12	18	24	30	39	56	66	96	117	167	216	310
Water-closet flush valves	36	47	64	70	82	98	125	140	182	205	269	327	435
Urinal flush valves	27	36	47	54	61	71	89	98	125	140	175	205	269
Lavatories	8	12	16	20	23	29	39	43	58	65	84	102	137

There are obvious irregularities in the estimates given by table 8, especially in the estimates for flush tanks and lavatory faucets. This may be seen by plotting these values or by comparing them by steps as is done in table 10.

TABLE 10.—*Demand estimates in gallons per minute per fixture by table 8*

Kind of fixture	Number of fixtures				
	First 50	Second 50	Third 50	Fourth 50	Last 100
Flush tanks	1.16	0.74	0.90	1.50	0.75
Lavatories	0.72	.48	.30	0.20	.60

This makes the irregularities in table 7 obvious. To be consistent, the increment in the estimate for a given increment in number of fixtures should gradually approach a constant minimum as the total number increases.

Table 11 shows the regular decrease in demand per fixture as given in table 9.

TABLE 11.—*Demand estimates in gallons per minute per fixture by table 9*

Kind of fixture	Number of fixtures				
	first 50	second 50	third 50	fourth 50	last 100
Flush tanks	1.32	1.02	1.60	0.98	0.94
Lavatories	0.86	0.54	0.38	.36	.35

The regularly decreasing allowances per fixture in this table are proper and permissible. However, although the demand per fixture decreases with increasing number of fixtures, tending toward the average demands for entire systems, the estimate per fixture for satisfactory service of the entire group of fixtures cannot fall to that average, approximately 0.8 gpm per water closet and 0.16 gpm per lavatory, if the supply pipes are to be capable of supplying the peak demands.

There are two particular requirements that any practical method of estimating demand loads should meet. First, the method should give estimates greater than the average demand for all fixtures in the system during the periods of heaviest use; otherwise complete interruption of service will occur. Second, the method should give satisfactory and economical estimates for groups of fixtures all of the same kind and for groups composed of any number of different kinds of fixtures. The inherent defects of a table in the form of tables 8 or 9 in meeting the second requirement will become apparent from the comparisons given in table 12.

Table 12.—*Comparison of demand estimates for plumbing systems*

Number and kind of fixtures	Estimates by: Table 7 and figure 5 (fixture units)	Table 7 and figure 5 (gpm)	Table 9 (gpm)	Table 8 (gpm)
12 flush valves (W. C.) [a]	120		70	72
8 flush valves (Ur.) [b]	40		47	30
12 lavatories	24		20	18
Total	184	86	137	120
50 flush valves (W. C.)	500		140	143
24 flush valves (Ur.)	120		71	50
50 lavatories	100		43	36
Total	720	171	254	229
200 flush valves (W. C.)	2,000		327	295
100 flush valves (Ur.)	500		140	100
200 lavatories	400		102	85
Total	2,900	432	569	480

[a] Water closet.
[b] Urinal.

A comparison of these estimates shows clearly the tendency to overestimate when the separate estimates for the different kinds of fixtures are added directly to obtain the estimate for total demand for all fixtures. Although the manual from which the estimate values in table 8 were taken does not specifically state that the sum of the separate estimates for each kind of fixture is to be taken as the total estimate, it does not provide any other means of obtaining the total, and presumably this is the method that has been used in applying the table. If this is true, it accounts to a large extent for the overestimates complained of and previously referred to. Furthermore, no provision is made in the manual for the difference in demand for the same fixtures in public (congested) service and in private (noncongested) service, an omission that would still further increase the overestimates for the latter.

In addition to the fact that the method of weighting fixtures as to their demands (table 7) and applying the weighted total to probable use curves (fig. 5) will at least partially correct the errors inherent in the use of tables as illustrated, once the weighted method is understood thoroughly, it will be simpler to use. Tables in an abbreviated form, as illustrated by tables 8 and 9, require interpolation for values not appearing in the table, whereas interpolation is automatically taken care of in using the curve.

XI. ESTIMATES OF SEWAGE LOADS

It has already been pointed out on pages 4 and 5 that the probability function has only a limited application to the drainage systems of buildings. Insofar as the probability function is applicable to the estimation of peak sewage loads to be provided for, the same function as applied to the peak water demand (fig. 5) will apply also, with certain modifications, to the drainage system. In the main, the same table of fixture weights (fixture units) will also apply to both the supply and drainage systems. The principal exception is the water closet, which for the drainage system will in general have the same weight for a given congestion of service whether the water-supply control is by flush valve or flush tank. In certain other cases, it may become advisable to assign different weights to the same fixture, depending on the sizes of the outlet from the fixture and of the fixture drain. Because the details of the modifications indicated are closely related to the capacities of drains, the further development of the method of estimating sewage load will be treated in the paper dealing with capacities of building drains. Also, before the method can be successfully applied in practice, the abbreviated table of fixture weights for demand loads will have to be expanded and modified. These modifications will also be developed in the later paper.

XII. DISCUSSION

The principal purpose of this paper is to present the fundamental principles and data on which the proposed method of estimating loads to be provided for in plumbing systems is based. It has been pointed out that the details of application of any method in practice must be guided to a large extent by engineering judgment in order that it may lead to satisfactory results. The choice of values employed in evaluating the probability function and in converting estimates in number of fixtures flowing to estimates in gallons per minute represents the author's judgment.

It has also been pointed out that it is impossible to determine or to estimate closely either the maximum demand load or the maximum sewage load that will occur in service. It is only possible to estimate the loads having a certain probability of being exceeded in a given system. In this respect, it is believed that table 7 and figure 5 will enable the engineer to obtain that estimate as closely as the varying conditions encountered in plumbing systems permit, and that the results of applying the data presented in that form will in general lead to the selection of adequate sizes of supply pipes and as economical sizes as are consistent with safety and satisfactory operation, provided the proper judgment is exercised in estimating the capacity of the pipes under the particular conditions of installation.

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