NBS TECHNICAL NOTE 679

U.S. DEPARTMENT OF COMMERCE/ National Bureau of Standards

Frequency Domain Stability Measurements: A Tutorial Introduction

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FREQUENCY DOMAIN STABILITY MEASUREMENTS: A TUTORIAL INTRODUCTION

David A. Howe

This report introduces the concept of stability measurements of oscillators by spectral analysis. Development of topics does not rely heavily on mathematics. The equipment and set-up for stability measurements in the frequency domain are outlined. Examples and typical results are presented. Physical interpretations of common noise processes are discussed. The last section provides a table by which typical frequency domain stability characteristics may be translated to time domain stability characteristics.

Key Words: Fractional frequency fluctuations; frequency stability; phase fluctuations; power law noise processes; spectral density; spectrum analysis.

I. The Sine Function and Noise

A sine wave signal generator produces a voltage that changes in time in a sinusoidal way as shown in fig. 1. The signal is an oscillating signal because the sine wave repeats itself. A cycle of the oscillation is produced in one period "T". The phase is the angle "Φ" within a cycle corresponding to a particular time "t". Therefore, we have $\frac{\Phi(t)}{2\pi} = \frac{t}{T}$.

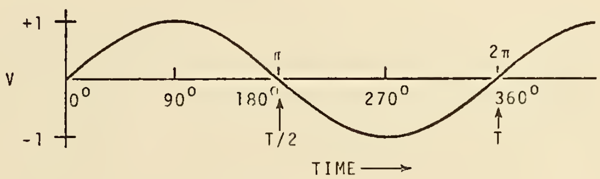


FIGURE 1

It is convenient for us to express angles in radians rather than in units of degrees, and positive zero-crossings will occur at even mulitiples of π-radians. The frequency "ν" is the number of cycles in one second, which is the reciprocal of period (seconds per cycle). The expression describing the voltage "V" out of a sine wave signal generator is given by $V(t) = V_p \sin[\Phi(t)]$ where V_p is the peak voltage amplitude. Equivalent expressions are

$$V(t) = V_p \sin\left(2\pi \frac{t}{T}\right)$$

and

$$V(t) = V_p \sin(2\pi\nu t). \tag{1}$$

Consider figure 2. Let's assume that the maximum value of "V" equals 1, hence "V_p" = 1. We say that the voltage "V(t)" is normalized to unity. If we know the frequency of a signal and the signal is a sine wave, then we can determine the incremental change in the period "T" (denoted by Δt) at a particular angle of phase.

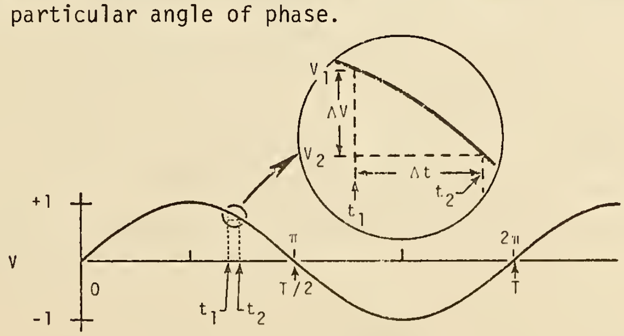


FIGURE 2

Note that no matter how big or small Δt may be, we can determine ΔV. Let us look at this from another point of view. Suppose we can measure ΔV for a particular Δt. From this there is a sine wave at a unique minimum frequency corresponding to the given ΔV and Δt. For small Δt, this frequency is called the <u>instantaneous frequency</u> at time t. The smaller the interval Δt, the better the approximation of instantaneous frequency at t.

When we speak of oscillators and the signals they produce, we recognize that an oscillator has some nominal frequency at which it operates. The "frequency stability" of an oscillator is a term used to characterize the frequency fluctuations of the oscillator signal. There is no formal definition for "frequency stability". However, one usually refers to frequency stability when comparing one oscillator with another. As we shall see later, we can define particular aspects of an oscillator's output then draw conclusions about its relative frestability stability. In general terms,

> "Frequency stability is the degree to which an oscillating signal produces the same value of frequency for any interval, Δt, throughout a specified period of time".

Let's examine the two waveforms shown in figure 3. Frequency stability depends on the amount of time involved in a measurement. Of the two oscillating signals, it is evident that "2" is more stable than "1" from time t_1 to t_3 assuming the horizontal scales are linear in time.

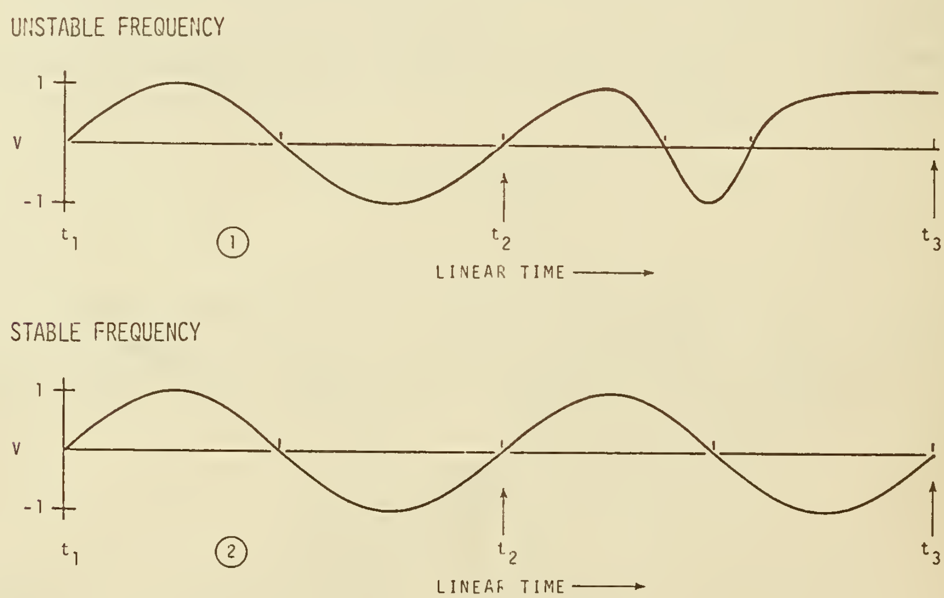


FIGURE 3

From time t_1 to t_2, there may be some question as to which of the two signals is more stable, but it's clear that from time t_2 to t_3, signal "1" is at a different frequency from that in interval t_1 to t_2.

If we want an oscillator to produce a particular frequency ν_0, then we're correct in stating that if the oscillator signal frequency deviates from ν_0 over any interval, this is a result of something which is undesirable. In the design of an oscillator, it is important to consider the sources of mechanisms which degrade the oscillator's frequency stability. All undesirable mechanisms cause noise or a noise process to exist along with the sine wave signal of the oscillator. Simply stated, noise is anything which is undesirable. To account for the noise components at the output of a sine wave signal generator, we can modify equation (1) and express the output as

$$V(t) = [V_0 + \varepsilon(t)] \sin [2\pi\nu_0 t + \phi(t)], \tag{2}$$

where V_0 $\equiv$ nominal peak voltage amplitude,

$\varepsilon(t)$ $\equiv$ deviation of amplitude from nominal,

ν_0 $\equiv$ nominal fundamental frequency,

$\phi(t)$ $\equiv$ deviation of phase from nominal.

Ideally "ε" and "ϕ" should equal zero for all time. However, in the real world there are no perfect oscillators. To determine the extent of the noise components "ε" and "ϕ", we shall turn our attention to measurement techniques.

II. Phase Spectral Density

One method of characterizing noise is by means of spectrum analysis. Let's examine the waveform shown in figure 4.

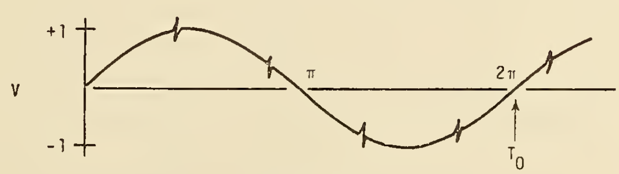


FIGURE 4

Here we have a sine wave which is perturbed for short instances by noise. Some loosely refer to these types of noises as "glitches". The waveform has a nominal frequency over one cycle which we'll call "ν_0"($\nu_0 = \frac{1}{T}$). At times, noise causes the instantaneous frequency to differ markedly from the nominal frequency. If a pure sine wave signal of frequency ν_0 is subtracted from this waveform, the remainder is the sum of the noise components. These components are of a variety of frequencies and the sum of their amplitudes is nearly zero except for the intervals during each glitch when their amplitudes momentarily reinforce each other. This is shown graphically in figure 5.

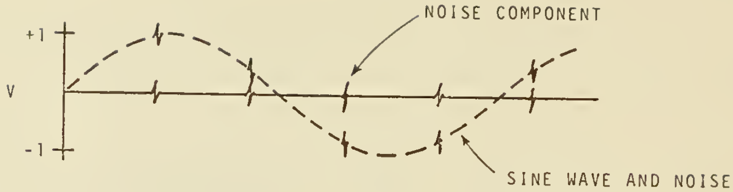


FIGURE 5

One can plot a graph showing rms power vs. frequency for a given signal. This kind of plot is called the power spectrum. For the waveform of figure 5, the power spectrum will have a high value at ν_0 and will have lower values for the signals produced by the glitches. Closer analysis reveals that there is a recognizable, somewhat constant repetition rate associated with the glitches. In fact, we can deduce that there is a significant amount of power in another signal whose period is the period of the glitches as shown in figure 5. Let's call the frequency of the glitches ν_s. Since this is the case, we will observe a noticeable amount of power in the spectrum at ν_s with an amplitude which is related to the characteristics of the glitches. The power spectrum shown in figure 6 has this feature. A predominant fundamental component has been depicted, and other harmonics also exist.

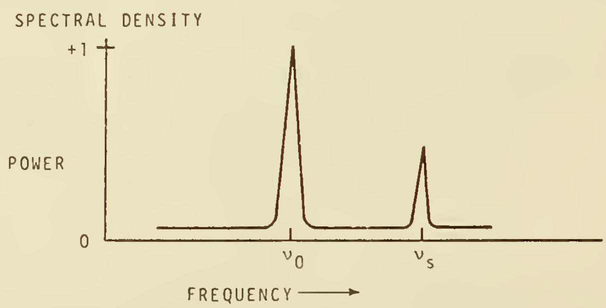


FIGURE 6

Some noise will cause the instantaneous frequency to "jitter" around ν_0, with probability of being higher or lower than ν_0. We thus usually find a "pedestal" associated with ν_0 as shown in figure 7.

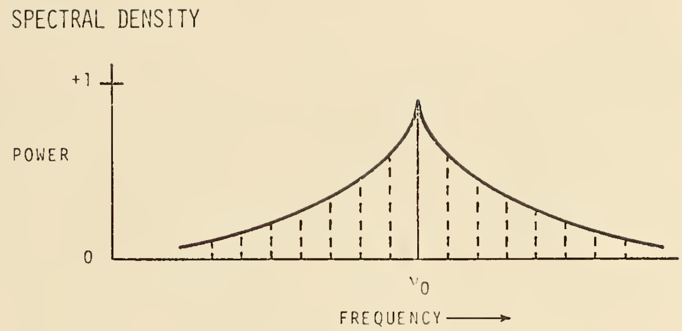


FIGURE 7

The process of breaking down a signal into all of its various components of frequency is called <u>Fourier expansion</u>. In other words, the addition of all the frequency components, called Fourier frequency components, produces the original signal. The value of a Fourier frequency is the difference between the frequency component and the fundamental frequency. The power spectrum can be normalized to unity such that the total area under the curve equals one. The power spectrum normalized in this way is the <u>power spectral density</u>.

The power spectrum, often called the RF spectrum, of V(t) is very useful in many applications. Unfortunately, if one is given the RF spectrum, it is impossible to determine whether the power at different Fourier frequencies is a result of amplitude fluctuations "$\varepsilon(t)$" or phase fluctuations "$\phi(t)$. The RF spectrum can be separated into two independent spectra, one being the <u>spectral density of "$\varepsilon(t)$"</u> often called the AM power spectral density and the other being the <u>spectral density of "$\phi(t)$"</u>.

For our purposes, the phase-fluctuation components are the ones of interest. The spectral density of phase fluctuations is denoted by $S_\phi(f)$ where "f" is Fourier frequency. For the frequently encountered case where the AM power spectral density is negligibly small and the total modulation of the phase fluctuations is small (mean-square value is much less than one rad^2), the RF spectrum has approximately the same shape as the phase spectral density. However, a main difference in the representation is that the RF spectrum includes the fundamental signal (carrier), and the phase spectral density does not. Another major difference is that the RF spectrum is a <u>power</u> spectral density and is measured in units of watts/hertz. The phase spectral density involves no "power" measurement of the electrical signal. The units are radians^2/hertz. It is tempting to think of $S_\phi(f)$ as a "power" spectral density because in practice it is measured by passing V(t) through a phase detector and measuring the detector's output power spectrum. The measurement technique makes use of the relation that for small deviations ($\delta\phi \ll 1$ radian),

$$S_\phi(f) = \left(\frac{V_{rms}(f)}{V_s}\right)^2 \qquad (3)$$

where V_{rms} (f) is the root-mean-square noise voltage per $\sqrt{Hz}$ at a Fourier frequency "f", and V_s is the sensitivity (volts per radian) at the phase quadrature output of a phase detector which is comparing the two oscillators. In the next section, we will look at a scheme for directly measuring $S_\phi(f)$.

One question we might ask is, "How do frequency changes relate to phase fluctuations?" After all it's the frequency stability of an oscillator that is a major consideration in many applications. The frequency is equal to a rate of change in the phase of a sine wave. This tells us that fluctuations in an oscillator's output frequency are related to phase fluctuations since we must change the rate of "$\phi(t)$" to accomplish a shift in "$\nu(t)$", the frequency at time t. A rate of change of total "$\Phi_T(t)$" is denoted by "$\dot{\Phi}_T(t)$". We have then

$$2\pi\nu(t) = \dot{\Phi}_T(t) \tag{4}$$

The dot denotes the mathematical operation of <u>differentiation</u> on the function Φ_T with respect to its independent variable t.* From equation (4) and equation (2) we get

$$2\pi\nu(t) = \dot{\Phi}_T(t) = 2\pi\nu_0 + \dot{\phi}(t)$$

Rearranging, we have $$2\pi\nu(t) - 2\pi\nu_0 = \dot{\phi}(t)$$

or $$\nu(t) - \nu_0 = \frac{\dot{\phi}(t)}{2\pi} \tag{5}$$

The quantity $\nu(t) - \nu_0$ can be more conveniently denoted as $\delta\nu(t)$, a change in frequency at time t. Equation (5) tells us that if we differentiate the phase fluctuations $\phi(t)$ and divide by 2π, we will have calculated the frequency fluctuation $\delta\nu(t)$. Rather than specifying a frequency fluctuation in terms of a shift in frequency, it is useful to denote $\delta\nu(t)$ with respect to the nominal frequency ν_0. The quantity $\frac{\delta\nu(t)}{\nu_0}$ is called the <u>fractional frequency fluctuation</u>** at time t and is signified by the variable y(t). We have

$$y(t) = \frac{\delta\nu(t)}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} \tag{6}$$

---

* As an analogy, the same operation relates the velocity of an object with its acceleration. The acceleration is equal to the rate of change of velocity.

** Some international recommendations replace "fractional" by "normalized".

The fractional frequency fluctuation $y(t)$ is a dimensionless quantity. When talking about frequency stability, its appropriateness becomes clearer if we consider the following example. Suppose in two oscillators $\delta\nu(t)$ is consistently equal to + 1 Hz and we have sampled this value for many times t. Are the two oscillators equal in their ability to produce their desired output frequencies? Not if one oscillator is operating at 10 Hz and the other at 10 MHz. In one case, the average value of the fractional frequency fluctuation is 1/10, and in the second case is 1/10,000,000 or 1×10^{-7}. The 10 MHz oscillator is then more precise. If frequencies are multiplied or divided using ideal electronics, the fractional stability is not changed.

In the frequency domain, we can measure the spectrum of frequency fluctuations $y(t)$. The spectral density of frequency fluctuations is denoted by $S_y(f)$ and is obtained by passing the signal from an oscillator through an ideal FM detector and performing spectral analysis on the resultant output voltage. $S_y(f)$ has dimensions of (fractional frequency)^2/Hz or Hz^{-1}. Differentiation of $\phi(t)$ corresponds to multiplication by $\frac{f}{\nu_0}$ in terms of spectral densities. With further calculation, one can derive that

$$S_y(f) = \left(\frac{f}{\nu_0}\right)^2 S_\phi(f) \tag{7}$$

We will address ourselves primarily to $S_\phi(f)$, that is, the spectral density of phase fluctuations. For noise-measurement purposes, $S_\phi(f)$ can be measured with a straightforward, easily duplicated equipment set-up. Whether one measures phase or frequency spectral densities is of minor importance since they bear a direct relationship. It is important, however, to make the distinction and to use equation (7) if necessary.

III. The Phase-Locked Loop

Suppose we have a noisy oscillator. We wish to measure the oscillator's phase fluctuations relative to nominal phase. One can do this by phaselocking another oscillator (called the reference oscillator) to the test oscillator and mixing the two oscillator signals 90° out of phase (phase quadrature). This is shown schematically in figure 9. The two oscillators are at the same frequency in long term as guaranteed by the phase-lock loop (PLL). A low-pass filter is used after the mixer since the difference signal is the one of interest. By holding the two signals at a relative phase difference of 90°, short-term phase fluctuations between the test and reference oscillators will appear as voltage fluctuations out of the mixer.

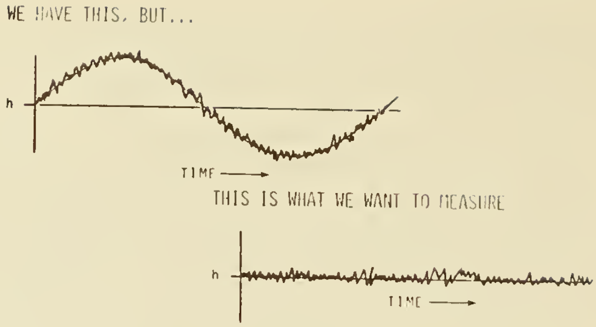


FIGURE 8

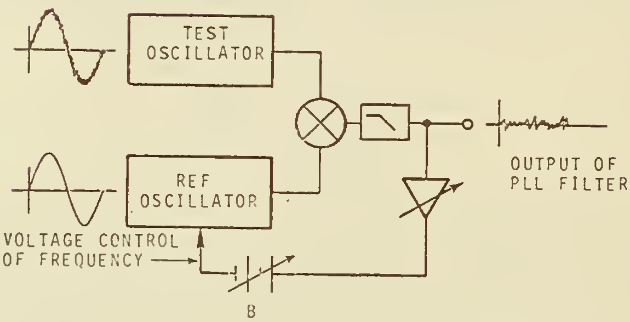


FIGURE 9

With a PLL, if we can make the servo time constant very long, then the PLL bandwidth as a filter will be small. This may be done by lowering the gain A_v of the loop amplifier. We want to translate the phase modulation spectrum to baseband spectrum so that it is easily measured on a low frequency spectrum analyzer. With a PLL filter, we must keep in mind that the reference oscillator should be as good or better than the test oscillator. This is because the output of the PLL represents the noise from both oscillators, and if not properly chosen, the reference can have noise masking the noise from the test oscillator. Often, the reference and test oscillators are of the same type and have, therefore, approximately the same noise. We can acquire a meaningful measurement by noting that the noise we measure is from two oscillators. Many times a good approximation is to assume that the noise power is twice that which is associated with one oscillator. $S_\phi(f)$ is general notation depicting spectral density on a per hertz basis. A PLL filter output necessarily yields noise from two oscillators.

The output of the PLL filter at Fourier frequencies above the loop bandwidth is a voltage representing phase fluctuations between reference and test oscillator. It is necessary to make the time-constant of the loop long compared with the inverse of the lowest Fourier frequency we wish to measure. That is, $\tau_c > \frac{1}{2\pi\ f(\text{lowest})}$. This means that if we want to

measure $S_\phi(f)$ down to 1 Hz, the loop time-constant must be greater than $\frac{1}{2\pi}$ seconds. One can measure the time-constant by perturbing the loop (momentarily disconnecting the battery is convenient) and noting the time it takes for the control voltage to reach 70% of its final value. The signal from the mixer can then be inserted into a spectrum analyzer. A preamp may be necessary before the spectrum analyzer. The analyzer determines the mean square volts that pass through the analyzer's bandwidth centered around a pre-chosen Fourier frequency f. It is desirable to normalize results to a 1 Hz bandwidth.Assuming white phase noise (white ϕM), this can be done by dividing the mean square voltage by the analyzer bandwidth in Hz. One may have to approximate for other noise processes. (The phase noise sideband levels will usually be indicated in rms volts-per-root-Hertz on most analyzers.)

IV. Equipment for Frequency Domain Stability Measurements

(1) Low-noise mixer

This should be a high quality, double-balanced type, but single-ended types may be used. The oscillators should have well-buffered outputs to be able to isolate the coupling between the two input RF ports of the mixer. Results that are too good may be obtained if the two oscillators couple tightly via signal injection through the input ports. We want the PLL to control locking. One should read the specifications in order to prevent exceeding the maximum allowable input power to the mixer. It is best to operate near the maximum for best signal-to-noise out of the IF port of the mixer and, in some cases, it is possible to drive the mixer into saturation without burning out the device.

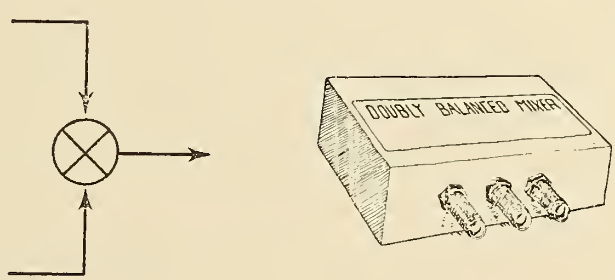


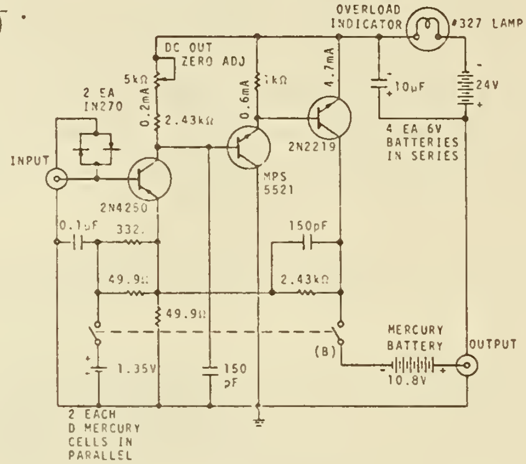
FIGURE 10

(2) Low-noise DC amplifier

The amount of gain A_V needed in the loop amplifier will depend on the amplitude of the mixer output and the degree of varactor control in the reference oscillator. We may need only a small amount of gain to acquire lock. On the other hand, it may be necessary to add as much as 80 dB of gain. Good low-noise DC amplifiers are available from a number of sources, and with cascading stages of amplification, each contributing noise, it will be the noise of the first stage which will add most significantly to the noise being measured. If a suitable low-noise first-stage amplifier is not readily available, a schematic of an amplifier with 40 dB of gain is shown

in figure 11 which will serve nicely for the first stage. The response of the amplifier should be flat from DC to the highest Fourier frequency one wishes to measure. The loop time-constant is inversely related to the gain A_v and the determination of A_v is best made by experimentation knowing that

$$\tau_c < \frac{1}{2\pi f\ (\text{lowest})} .$$



LOW NOISE AMPLIFIER

FIGURE 11

(3) Voltage-controlled reference quartz oscillator

This oscillator should be a good one with specifications available on its frequency domain stability. The reference should be no worse than the test oscillator. The varactor control should be sufficient to maintain phase-lock of the reference. In general, low quality test oscillators may have varactor control of as much as 1×10^{-6} fractional frequency change per volt. High quality test oscillators may have only 1×10^{-10} fractional frequency change per volt. Some provision should be available on the reference oscillator for tuning the mean frequency over a frequency range that will enable phase-lock. Many factors enter into the choice of the reference oscillator, and often it is convenient to simply use two test oscillators phase-locked together. In this way, one can assume that the noise out of the PLL filter is no worse than 3 dB greater then the noise from each oscillator. If it is uncertain that both oscillators are contributing approximately equal noise, then one should perform measurements on three oscillators taking two at a time. The noisier-than-average oscillator will reveal itself.

(4) Spectrum analyzer

The signal analyzer typically should be capable of measuring the noise in rms volts in a narrow bandwidth from near 1 Hz to the highest Fourier frequency of interest. This may be 50 kHz for carrier frequencies of 10 MHz or lower. For voltage measuring analyzers, it is typical to use units of "volts per Hz". The spectrum analyzer and any associated input amplifier will exhibit high-frequency rolloff. The Fourier frequency at which the voltage has dropped by 3 dB is the measurement system bandwidth f_h, or $\omega_h = 2\pi f_h$. This can be measured directly with a variable signal generator.

Rather than measure the spectral density of phase fluctuations between two oscillators, it is possible to measure the phase fluctuations introduced by a device such as an active filter or amplifier. Only a slight modification of the existing PLL filter equipment set up is needed. The scheme is shown in Figure 12.

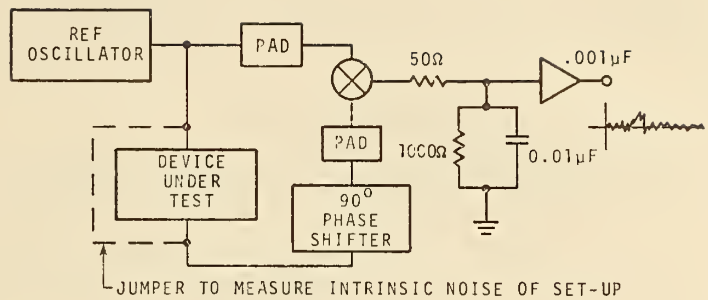


FIGURE 12

Figure 12 is a differential phase noise measurement set-up. The output of the reference oscillator is split so that part of the signal passes through the device under test. We want the two signals going to the mixer to be 90° out of phase, thus, phase fluctuations between the two input ports cause voltage fluctuations at the output. The voltage fluctuations then can be measured at various Fourier frequencies on a spectrum analyzer.

To estimate the noise inherent in the test set-up, one can in principle bypass the device under test and compensate for any change in amplitude and phase at the mixer. The PLL filter technique must be converted to a differential phase noise technique in order to measure inherent test equipment noise. It is a good practice to measure the system noise before proceeding to measurement of device noise.

A frequency domain measurement set-up is shown schematically in figure 13. The component values for the low-pass filter out of the mixer are suitable for oscillators operating at around 5 MHz.

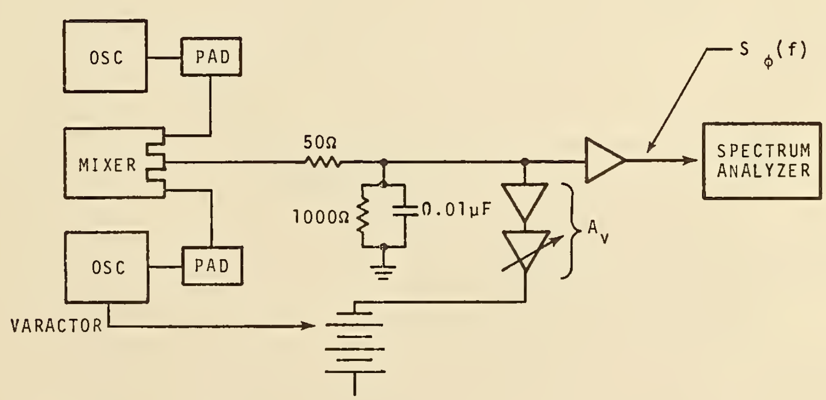


FIGURE 13

The active gain element (A_v) of the loop is a DC amplifier with flat frequency response. One may replace this element by an integrator to achieve high gain near DC and hence, maintain better lock of the reference oscillator in long term. Otherwise long-term drift between the reference and test oscillators might require manual re-adjustment of the frequency of one or the other oscillator.

V. Procedure and Example

At the input to the spectrum analyzer, the voltage varies as the phase fluctuations in short-term

$$S_\phi(f) = \left(\frac{V_{rms}\ (f)}{V_s}\right)^2$$

V_s is the phase sensitivity of the mixer in volts per radian. Using the previously described equipment set-up, V_s can be measured by disconnecting the feedback loop to the varactor of the reference oscillator. The peak voltage swing is equal to V_s in units of volts/rad if the resultant beat note is a sine wave.*

The value for the measured $S_\phi(f)$ in decibels is given by:

$$S_\phi(f) = 20 \log \frac{V_{rms} \text{ Voltage at } f}{V_s \text{ full-scale } \phi\text{-detector voltage*}}$$

Example: *Given a PLL with two oscillators such that, at the mixer output:*

$V_s = 1$ *volt/rad*

$V_{rms}(45\ Hz) = 100$ *nV per root hertz*

Solve for $S_\phi(45\ Hz)$.

$$S_\phi(45\ Hz) = \left[\frac{100\ nV\ (Hz^{-\frac{1}{2}})}{1\ V/rad}\right]^2 = \left[\frac{10^{-7}}{1}\right]^2 rad^2\ Hz^{-1} = 10^{-14}\ \frac{rad^2}{Hz}$$

In decibels,

$$S_\phi(45\ Hz) = 20 \log \frac{100\ nV}{1\ V} = 20 \log \frac{10^{-7}}{10^0}$$

$$= 20\ (-7) = -140\ dB \text{ at } 45\ Hz$$

*This may not be the case for state-of-the-art $S_\phi(f)$ measurements where one must drive the mixer very hard to achieve low mixer noise levels. Hence, the output will not be a sine wave.

VI. Power-Law Noise Processes

Power-law noise processes are models of precision oscillator noise that produce a particular slope on the spectral density plot. We often classify these noise processes into one of five categories. For plots of $S_\phi(f)$, they are:

1. Random walk FM (random walk of frequency), S_ϕ plot goes down as $1/f^4$.
2. Flicker FM (flicker of frequency), S_ϕ plot goes down as $1/f^3$.
3. White FM (white of frequency), S_ϕ plot goes down as $1/f^2$.
4. Flicker ϕM (flicker of phase), S_ϕ plot goes down as $1/f$.
5. White ϕM (white of phase), S_ϕ plot is flat.

Power law noise processes are characterized by their functional dependence on Fourier frequency.

The spectral density plot of a typical oscillator's output usually is a combination of different power-law noise processes. It is very useful and meaningful to categorize the noise processes. The first job in evaluating a spectral density plot is to determine which type of noise exists for a particular range of Fourier frequencies. It is possible to have all five noise processes being generated from a single oscillator, but, in general, only two or three noise processes are dominant. Figure 14 is a graph of $S_\phi(f)$ showing the five noise processes on a log-log scale. Figure 15 shows the spectral density of phase fluctuations for a typical high-quality oscillator.

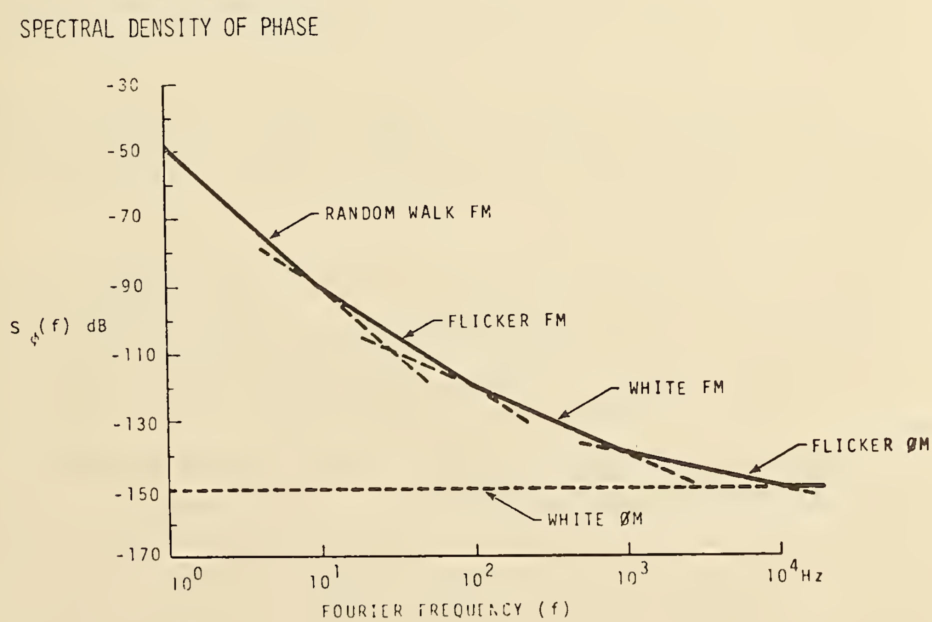


FIGURE 14

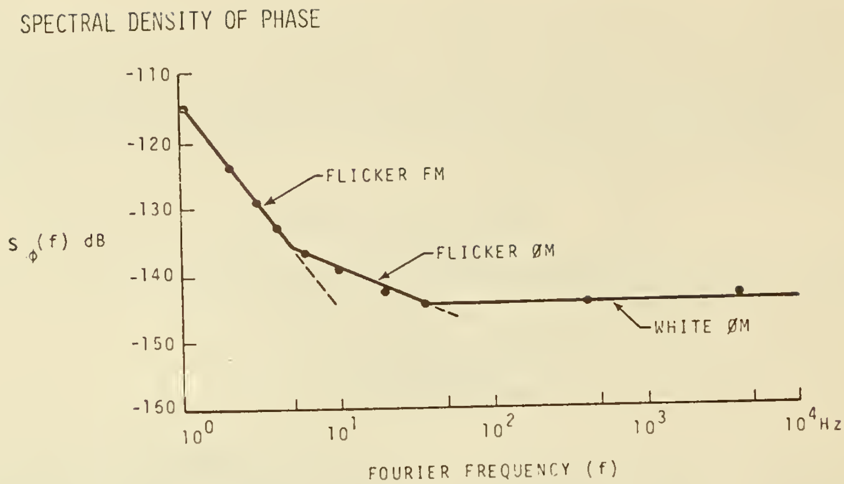


FIGURE 15

We can make the following general remarks about power-law noise processes:

1. Random walk FM ($1/f^4$) noise is difficult to measure since it is usually very close to the carrier. Random walk FM usually relates to the *oscillator's physical environment.* If random walk FM is a predominant feature of the spectral density plot then *mechanical shock, vibration, temperature,* or other environmental effects may be causing "random" shifts in the carrier frequency.

2. Flicker FM ($1/f^3$) is a noise whose physical cause is usually not fully understood but may typically be related to the *physical resonance mechanism of an active oscillator* or the *design or choice of parts used for the electronics, or even environmental properties.* Flicker FM is common in high-quality oscillators, but may be masked by white FM ($1/f^2$) or flicker ΦM ($1/f$) in lower-quality oscillators.

3. White FM ($1/f^2$) noise is a common type found in *passive-resonator frequency standards.* These contain a slave oscillator, often quartz, which is locked to a resonance feature of another device which behaves much like a high-Q filter. Cesium and rubidium standards have white FM noise characteristics.

4. Flicker ΦM ($1/f$) noise may relate to a physical resonance mechanism in an oscillator, but it usually is added by *noisy electronics.* This type of noise is common, even in the highest quality oscillators, because in order to bring the signal amplitude up to a usable level, amplifiers are used after the signal source. Flicker ΦM noise may be introduced in these stages. It may also be introduced in a frequency multiplier. Flicker ϕM can be reduced with good low-noise amplifier design (e.g., using rf negative feedback) and hand-selecting transistors and other electronic components.

5. White ΦM (f^0) noise is broadband phase noise and has little to do with the resonance mechanism. It is probably produced by similar phenomena as flicker ΦM ($1/f$) noise. *Stages of amplification* are usually responsible for white ΦM noise. This noise can be kept at a very low value with good amplifier design, hand-selected components, the addition of narrowband filtering at the output, or increasing, if feasible, the power of the primary frequency source.

VII. Translation from Frequency Domain Stability Measurement to Time Domain Stability Measurement

This section assumes the reader has some knowledge of the measurement of time domain stability. Our attention will be on how to translate a plot of $S_\phi(f)$ to $\sigma_y^2(\tau)$, the pair variance*. Elaboration on time domain stability measurements is beyond the intent of this Technical Note. A suggested reference is NBS Technical Note #669, "The Measurement of Frequency and Frequency Stability of Precision Oscillators," by D.W. Allan. Other references are listed in the Bibliography.

From the plot of phase spectral density, it is possible to make general comments about the time domain stability of an oscillator. Remarks are presented in the previous section. Recall that inferences of time domain stability are made based on specific kinds of power-law noise processes over a range of Fourier frequencies. More specifically, when dealing with noise processes, a power-law in the frequency domain corresponds to a particular power-law in the time domain. A convenient measure of frequency stability in the time domain is the pair variance, $\sigma_y^2(\tau)$. We have

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 \tag{8}$$

where M is the number of data values and $\bar{y}_k$ is the average over time τ of the kth data point (similarly, $\bar{y}_{k+1}$ corresponds to kth + 1 point.)

Example: Find the pair variance, $\sigma_y^2(\tau)$, of the following sequence of fractional frequency fluctuation values $\bar{y}_k$, each value averaged over one second.

$\bar{y}_1 = 4.36 \times 10^{-5}$	$\bar{y}_5 = 4.47 \times 10^{-5}$
$\bar{y}_2 = 4.61 \times 10^{-5}$	$\bar{y}_6 = 3.96 \times 10^{-5}$
$\bar{y}_3 = 3.19 \times 10^{-5}$	$\bar{y}_7 = 4.10 \times 10^{-5}$
$\bar{y}_4 = 4.21 \times 10^{-5}$	$\bar{y}_8 = 3.08 \times 10^{-5}$

(Assume no dead-time in measurement of averages)

Since each average of the fractional frequency fluctuation values is for one second, then the first pair variance calculation will be at τ = 1s. We are given M = 8 (eight values);

*Often called the Allan variance.

therefore, the number of pairs in sequence is M-1 = 7. We have:

Data values $\bar{y}_k$ ($\times 10^{-5}$)	First differences $(\bar{y}_{k+1} - \bar{y}_k)$ ($\times 10^{-5}$)	First differences squared $(\bar{y}_{k+1} - \bar{y}_k)^2$ ($\times 10^{-10}$)
4.36	--	--
4.61	0.25	0.06
3.19	-1.42	2.02
4.21	1.02	1.04
4.47	0.26	0.07
3.96	-0.51	0.26
4.10	0.14	0.02
3.08	-1.02	1.04

$$\sum_{k=1}^{M-1} (\bar{y}_{k+1} - \bar{y}_k)^2 = 4.51 \times 10^{-10}$$

Therefore,

$$\sigma_y^2(1s) = \frac{4.51 \times 10^{-10}}{2(7)} = 3.2 \times 10^{-11}$$

and

$$[\sigma_y^2(1s)]^{1/2} = \sqrt{3.2 \times 10^{-11}} = 5.6 \times 10^{-6}$$

Using the same data, one can calculate the pair variance for $\tau = 2s$ by averaging pairs of adjacent values and using these new averages as data values for the same procedure as above. For three second averages ($\tau = 3s$) take adjacent threesomes and find their averages and proceed in a similar manner. More data must be acquired for longer averaging times.

One sees that with large numbers of data values, it is helpful to use a computer or programmable calculator. The confidence of the estimate of the pair variance improves nominally as the square root of the number of data values used. In this example, M=8 and the confidence can be expressed as being no better than $1/\sqrt{8} \times 100\% = 35\%$. This then is the allowable error in our estimate for the $\tau = 1s$ average.

Knowing how to measure $S_\phi(f)$ for a pair of oscillators, let us see how to translate the power-law noise process to a plot of $\sigma_y^2(\tau)$. First, consider $S_y(f)$, the spectral density of frequency fluctuations. There are two quantities which completely specify $S_y(f)$ for a particular power-law noise process: (1) the slope on a log-log plot for a given range of f and (2) the amplitude. The slope we shall denote by "α"; therefore f^α is the straight line (on log-log scale) which relates $S_y(f)$ to f. The amplitude will be denoted "h_α"; it is simply the coefficient of f for a range of f. When we examine a plot of spectral density of frequency fluctuations, we are looking at a representation of the addition of all the power-law processes. We have

$$S_y(f) = \sum_{\alpha = -\infty}^{\infty} h_\alpha f^\alpha \tag{9}$$

In the preceeding section, five power-law noise processes were outlined with respect to $S_\phi(f)$. These five are the common ones encountered with precision oscillators. Equation (7) relates these noise processes to $S_y(f)$. One obtains

1.	Random Walk FM	(f^{-2})	. . .	$\alpha = -2$
2.	Flicker FM	(f^{-1})	. . .	$\alpha = -1$
3.	White FM	(1)	. . .	$\alpha = 0$
4.	Flicker ϕM	(f)	. . .	$\alpha = 1$
5.	White ϕM	(f^2)	. . .	$\alpha = 2$
	with respect to $S_y(f)$			slope

Table 1 is a list of coefficients for translation from $\sigma_y^2(\tau)$ to $S_y(f)$ and from $S_\phi(f)$ to $\sigma_y^2(\tau)$. The left column is the designator for the power-law process. Using the middle column, we can solve for the value of $S_y(f)$ by computing the coefficient "a" and using the measured time domain data $\sigma_y^2(\tau)$. The rightmost column yields a solution for $\sigma_y^2(\tau)$ given frequency domain data $S_\phi(f)$ and a calculation of the appropriate "b" coefficient.

$S_y(f) = h_\alpha f^\alpha$ $\alpha =$	$S_y(f) = a\,\sigma_y^2(\tau)$ $a =$	$\sigma_y^2(\tau) = b\,S_\phi(f)$ $b =$
2 (white phase)	$\frac{(2\pi)^2\,\tau^2\,f^2}{3\,f_h}$	$\frac{3\,f_h}{(2\pi)^2\,\tau^2\,\nu_o^2}$
1 (flicker phase)	$\frac{(2\pi)^2\,\tau^2\,f}{3.81 + 3\,\ln(\omega_h\tau)}$	$\frac{[3.81 + 3\,\ln(\omega_h\tau)]f}{(2\pi)^2\,\tau^2\,\nu_o^2}$
0 (white frequency)	$2\,\tau$	$\frac{f^2}{2\,\tau\,\nu_o^2}$
-1 (flicker frequency)	$\frac{1}{2\,\ln(2)\cdot f}$	$\frac{2\,\ln(2)\cdot f^3}{\nu_o^2}$
-2 (random walk frequency)	$\frac{6}{(2\pi)^2\,\tau\,f^2}$	$\frac{(2\pi)^2\,\tau\,f^4}{6\,\nu_o^2}$

TABLE 1. Conversion table from time-domain to frequency-domain and from frequency-domain to time-domain for common kinds of integer power law spectral densities; $f_h (= \omega_h/2\pi)$ is the measurement system bandwidth. Measurement response should be within 3 dB from D.C. to f_h (3dB down high-frequency cutoff is at f_h).

$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f)$$

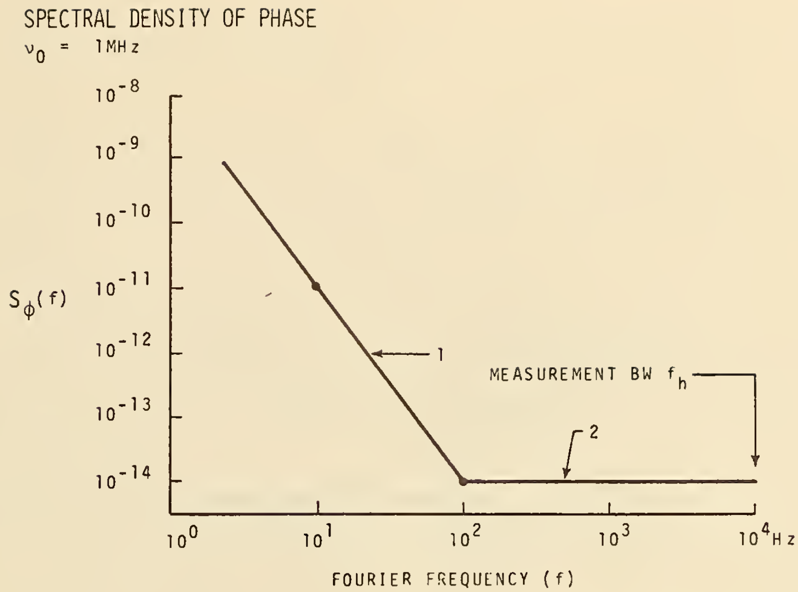


FIGURE 16

In the phase spectral density plot of figure 16, there are two power-law noise processes. For region 1, we see than when f increases by one decade (that is, from 10 Hz to 100 Hz), $S_\phi(f)$ goes down by three decades (that is, from 10^{-11} to 10^{-14}). Thus, $S_\phi(f)$ goes as $1/f^3 = f^{-3}$. For region 1, we identify this noise process as flicker FM. The rightmost column of Table 1 relates $\sigma_y^2(\tau)$ to $S_\phi(f)$. The row designating flicker frequency noise yields:

$$\sigma_y^2(\tau) = \frac{2\ln(2) \cdot f^3}{\nu_0^2} S_\phi(f)$$

One can now pick (arbitrarily) a convenient Fourier frequency f and determine the corresponding values of $S_\phi(f)$ given by the plot of figure 18. Say, f = 10, thus $S_\phi(10) = 10^{-11}$. Solving for $\sigma_y^2(\tau)$, given ν_0 = 1 MHz, we obtain:

$$\sigma_y^2(\tau) = 1.39 \times 10^{-20}$$

therefore, $\sigma_y(\tau) = 1.18 \times 10^{-10}$. For region 2, we have white ϕM. The relationship between $\sigma_y^2(\tau)$ and $S_\phi(f)$ for white ϕM is:

$$\sigma_y^2(\tau) = \frac{3f_h}{(2\pi)^2\,\tau^2\,\nu_0{}^2} S_\phi(f)$$

Again, we choose a Fourier frequency, say f = 100, and see that $S_\phi(100) = 10^{-14}$. Assuming $f_h = 10^4$ Hz, we thus obtain:

$$\sigma_y^2(\tau) = 7.59 \times 10^{-24} \cdot \frac{1}{\tau^2}$$

therefore,

$$\sigma_y(\tau) = 2.76 \times 10^{-12}\ \frac{1}{\tau}.$$

The resultant time domain characterization is shown in figure 17.

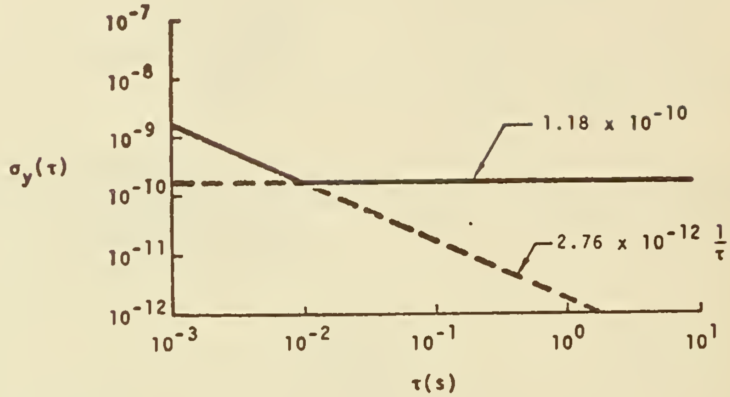


FIGURE 17. The translation of $S_\phi(f)$ of figure 16 yields this $\sigma_y(\tau)$ plot.

Figures 18 and 19 show a plot of time-domain stability and a translation to frequency domain. The procedure is left as an exercise for the reader.

VIII. Conclusion

Starting with the basic concepts associated with the sine-wave signal generator, this writing attempts to present fundamental aspects of spectral analysis with only light mathematical treatment. The RF spectrum is not a useful means of inferring the frequency stability of an oscillator, so a discussion of $S_\phi(f)$ and $S_y(f)$ is put forth as well as the technique for measurement. It was felt that there was a need for a tutorial writing that explains frequency-domain measurements without elaborating on existing time-domain techniques. A table for translating data from one domain to the other has been included, and the use of this table requires knowledge of the Allan Variance. The author has tried to be general in the treatment of topics, and a bibliography is attached for readers who would like details about specific items.

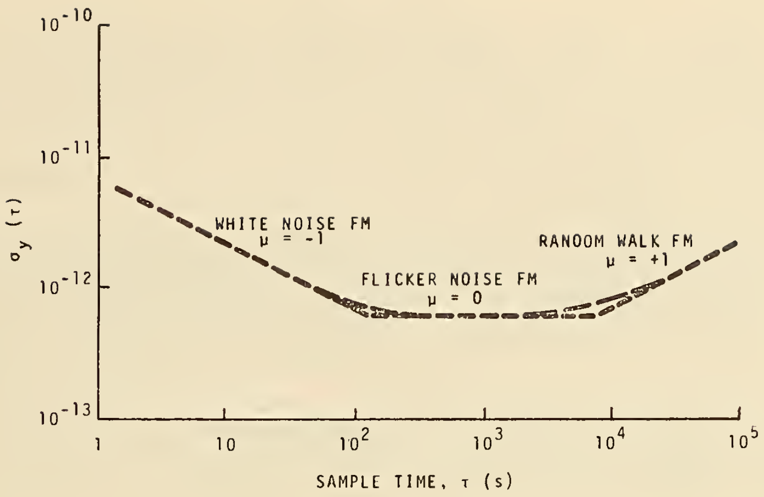


FIGURE 18. A $\sigma_y(\tau)$ versus τ plot of some actual oscillator data. Notice that if $\sigma_y^2(\tau) \approx \tau^{\mu}$, then $\sigma_y(\tau) \simeq \tau^{\mu/2}$ hence the $\tau^{-1/2}$ slope for $\mu = -1$, etc.

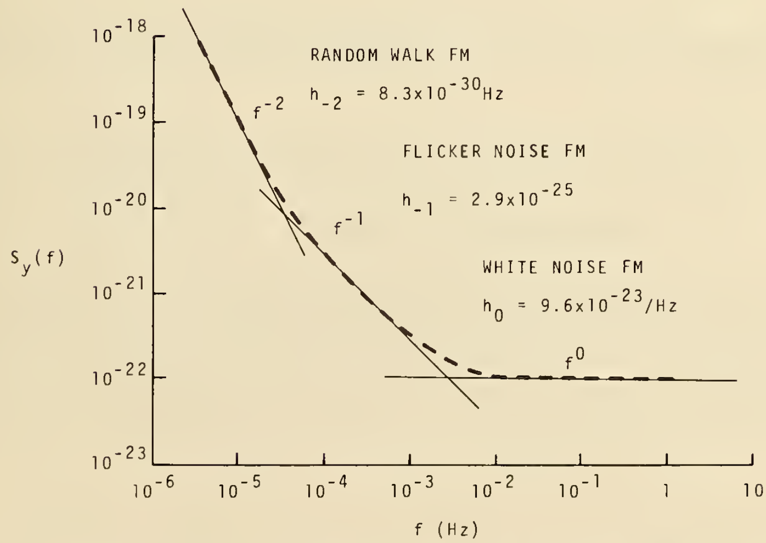


FIGURE 19. A plot of $S_y(f)$ as transformed from the time-domain data plotted in Figure 18.

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This report introduces the concept of stability measurements of oscillators by spectral analysis. Development of topics does not rely heavily on mathematics. The equipment and set-up for stability measurements in the frequency domain are outlined. Examples and typical results are presented. Physical interpretations of common noise processes are discussed. The last section provides a table by which typical frequency domain stability characteristics may be translated to time domain stability characteristics.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Fractional frequency fluctuations; frequency stability; phase fluctuations; power law noise processes; spectral density; spectrum analysis.

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