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U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Sampling Techniques for Electric Power Measurement

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7. Appendix

SAMPLING TECHNIQUES FOR ELECTRIC POWER MEASUREMENT

R. S. Turgel

A system is described that determines average electric power by periodically sampling current and voltage waveforms and calculating the result from digitized values of measured instantaneous currents and voltages. System performance is modeled on a digital computer to investigate the effects of noise, harmonics and sampling time errors on the result of a simulated power measurement. With 15 bit analog-to-digital conversion and 512 measured sample points an accuracy of 0.01% can theoretically be obtained from dc up to 5 kHz.

Key Words: Analog-to-digital conversion; digital; electric power; electricity; measurement; sampling; simulation; wattmeter.

1. Introduction

This report contains a study of the feasibility of a new method for making precision measurements of audio frequency electric power. The study examines the limitations imposed by the nature of the measurement process itself as distinct from the additional uncertainties introduced by the departure from ideal characteristics of the actual hardware.

None of the currently used methods to measure low frequency ac power is an absolute method. As in the measurement of ac voltage and current some transfer device is used to relate the measurements to the corresponding dc units. The use of transfer devices is based on some assumptions regarding the relative response of the device to ac and dc quantities. Generally an upper limit of the ac/dc error can be deduced from theoretical considerations.

The definition of (average) ac power is given by

$$W = \frac{1}{T} \int_{0}^{T} ei\,dt, \qquad (1)$$

where e = instantaneous voltage,

i = instantaneous current,

t = time, and

T = some definite time interval, usually a period (or multiple thereof) of the fundamental frequency component.

Integration of equation (1) is performed within the measuring device, making use of mechanical or thermal inertia, by the first three of the following well-known methods of measuring electric power:

(a) Electrodynamometer
(b) Electrostatic
(c) Electrothermic (thermoelement, calorimeter)
(d) Hall effect

The Hall effect devices produce dc voltages proportional to the instantaneous product of e and i which are then usually averaged (integrated) by the readout system.

In all the above methods a physical transducer is used to obtain a measure of the product of current and voltage. Their relative advantages and disadvantages derive mainly from the nature of that transducer.

The method to be described in this report follows directly from the fundamental definition of ac power, equation (1). Instead of carrying out the necessary integration with an analog system, instantaneous values of current and voltage are measured and the integration is performed by numerical (digital) methods. The integration is therby replaced by a summation:

$$W \approx \frac{1}{N} \sum_{1}^{N} e_j i_j . \qquad (2)$$

The problem is therefore twofold. The number of samples of instantaneous voltage and current and the method of summation must be chosen so that the value of the integral can be approached within a given tolerance. Further, the measurement of current and voltage must be made with sufficient accuracy in a given time interval.

2. Sampling Method

To reconstruct the waveform from sampled data the minimum sampling rate must be at least twice the frequency of the highest sinusoidal frequency component contained in the signal according to the sampling theorem[1]. Since instantaneous power varies at twice the frequency of the current and voltage, the sampling theorem would require a minimum sampling rate of four times the highest sinusoidal frequency component contained in the current and voltage waveforms. However, we are interested in "average power", and as shown below the minimum sampling rate is thereby reduced. It should be remembered that conditions encountered in practice differ from the ideal assumed for theoretical derivations, and therefore the minimum sampling rate is seldom sufficient in actual cases.

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[1] Raised figures indicate literature references on page 17.

In the present application the average integral

$$\frac{1}{T} \int_o^T f(x)dt$$

must be approximated by summing a sufficient number of samples during one period T so that

$$\frac{1}{N} \sum_N f(x_j) \rightarrow \frac{1}{T} \int_o^T f(x)dt$$

The trapezoidal rule has been shown to converge at least as fast as $\frac{1}{N^2}$.[2] The last term of the summation represents the error of the approximation

$$\int_a^b f(x)dx = h[\frac{f(a)}{2} + f(a+h) + \ldots + f(a+(N-1)h) + \frac{f(b)}{2}] - \frac{(b-a)^3}{12N^2} f''(\xi)$$

where

$$a<\xi<b \text{ and } h = \frac{b-a}{N}$$

For sampling at random intervals the error term decreases at a rate proportional to $\frac{1}{N}$. It appears therefore that random sampling is not the right approach if high precision is desired.

For the special case of periodic functions, the trapezoidal rule is exact for the 2N periodic functions:[3]

$$1, \sin (x), \cos (x), \ldots, \sin (N-1)x, \cos (N-1)x, \sin (Nx)$$

Therefore, the integral representing the average power of sinusoidal current and voltage with phase angle Θ

$$\frac{1}{2\pi} \int_o^{2\pi} \sin (x) \sin (x+\Theta)dx = \frac{1}{2\pi} \int_o^{2\pi} [\cos (\Theta) - \cos (2x+\Theta)]dx$$

can be evaluated exactly with only three terms, (N=3), of the summation formula. This can be verified by direct substitution. Theoretically, therefore, the number of samples per cycle need be only 2K + 1, where K is the order of the highest harmonic present in the input signal. If harmonics up to the fourth are included at least 9 samples must be taken in each period. If the input waveform is no longer a combination of harmonically related pure sinusoids, but includes small random fluctuations (noise), or if the sampling time is subject to small fluctuations (jitters), then a higher sampling rate is necessary. In practice, therefore, the minimum sampling rate is never sufficient. To ease implementation of binary arithmetic, the number of samples, N, has always been chosen as an integral power of 2 in this study.

3. Description of Measuring System

The system is capable of measuring rms current, or voltage, and average power with either a real load or phantom load connection. For purposes of discussion it can be divided into four main functional blocks as shown in fig. 3.1.

3.1 Analog Signal Conditioning

Signal conditioning preamplifiers translate the input voltages and currents to the required 10 volt level at the input to the A/D converter. Figure 3.2 shows one such possible arrangement.

To preserve the accuracy of the overall system the closed loop gain-frequency characteristic of the amplifiers has to be constant within 50 ppm from dc to the upper frequency limit. This requires a loop gain of at least 80 db. The preamplifiers can be built from commercially available operational amplifiers that satisfy these requirements.

3.2 Analog to Digital Converter (A/D)

The A/D subsystem samples the input signals at predetermined intervals and converts them to two 15 bit binary numbers.

Two configurations are possible. (1) The signals are captured in two sample-and-hold amplifiers and then successively applied to the converter through an analog multiplexer, or (2) the sample-and-hold amplifiers feed two parallel converters. The second scheme has the advantage of eliminating additional errors due to the multiplexer; on the other hand, it requires equality of the A/D's or suitable corrections. Specifications of commercially available units indicate that an overall short term accuracy of 0.01% can be maintained under favorable circumstances.

3.3 Digital Computation

This subsystem has the following functions:

1. Temporarily stores pairs of successive numbers from the output of the A/D converters.
2. Forms the product of these numbers.
3. Adds the product to a total.

4. Divides the total by the number of products added to obtain an average.

5. Displays the average on an output device.

6. Applies numerical corrections where necessary.

There are two general approaches to the problem of handling the digital data from the A/D converter(s).

1. A general purpose minicomputer can be used to store sets of data during the measurement, and to perform the computation after the measurement is completed.

2. Special arithmetic hardware can be built to perform the necessary calculations in real time.

Method 1 is preferable during the development period, because it provides flexibility of operation and permits the introduction of diagnostic routines. On the other hand, when the immediacy of results is of prime importance (e.g. certain control applications), method 2 is more appropriate. Minicomputer data manipulation is generally too slow in this case, and hard-wired integrated circuits are required to perform the arithmetic operations in between sampling periods.

The final result can be displayed on electro-optical readouts, or, if a minicomputer is used, on a printer.

3.4 Timing and Control Circuits

The timing and control section is of vital importance to the correct performance of the whole system. The two main functions of the control circuit are:

1. to determine the proper sampling time,

2. to coordinate the various subsystems so that the signals and digital information are processed in the correct sequence.

Of these two functions the first is the more critical. The degree to which the result from the sampled data approximates the integral desired is directly dependent on the timing of the sampling interval. Although a certain amount of random fluctuation (jitter) of the precise time of the sampling command can be tolerated, any systematic error in the periodicity of the samples will produce a proportional error in the final result. It is therefore very important to synchronize the timing pulses with the fundamental frequency to be measured. At low frequencies all the necessary samples for one reading can be obtained during a single cycle of the fundamental of the singal frequency. For instance, with an input frequency of 60 Hz 512 samples can be taken per cycle (fig. 3.3). At higher frequencies, or when more samples are required, the measurement time has to be spread over several cycles.

The timing of the sampling period is related to the input signal frequency by means of a frequency divider and phase-lock circuitry. The sample command pulse enables the "hold" mode of sample-and-hold amplifiers and thereby starts the conversion. Delays to allow for settling time can be programmed independently or derived from the clock circuit of the A/D converter.

The time required to complete the analog-to-digital conversion will occupy the major portion of the period between samples. If calculations are performed by a minicomputer, enough additional time must be provided to transfer the results of the conversion into memory. A direct memory access (DMA) channel is useful for this purpose. If special hardware is used, computations can be carried out simultaneously with the conversion of the next set of sampled data. In fact, with a pipeline type arithmetic operation computation can be spread over several sampling periods if necessary, provided a sufficient number of registers for storage of intermediate results are available. The additional delay in obtaining the final result will still be only a small fraction of the total measurement period.

4. Mathematical Model of A/D Converter

In outlining the proposed system for measuring low frequency ac power a number of assumptions were made regarding accuracy and the number of samples required. Most of these assumptions were based on purely theoretical considerations. In an actual system disturbances due to noise, sampling time fluctuation, round off due to finite number of digits, harmonics in the signal, etc., contribute to the overall error. Because some of these are interrelated, estimation of the overall uncertainty is difficult. It seemed appropriate, therefore, to simulate the system operation to determine its characteristics.

The design of a mathematical model of the A/D conversion process is fairly simple. Into such a model the various error sources can be introduced by defining or specifying their bounds and using normalized random numbers to simulate perturbations. Separate calculations have to be performed for each set of conditions and at each sample point, but total computation time is relatively short.

4.1 Computer Program

The object of the simulation is the determination of the error limits of the measurement process. That is, the calculation of the difference between the "exact" integral (eq. 1.), which represents the analog input, and the response of the sampled data system (eq. 2.) under various conditions. For this purpose the voltage and current inputs are represented by

$$Y_1 = \sin(x) + A_{12} \sin(2x + \Theta_{12}) + A_{13} \text{Sin}(3x + \Theta_{13}) + A_{14} \sin(4x + \Theta_{14}) \quad (3)$$

$$Y_2 = \sin(x + \Theta_{21}) + A_{22} \sin(2x + \Theta_{22}) + A_{23} \sin(3x + \Theta_{23}) + A_{24} \sin(4x + \Theta_{14}) \qquad (4)$$

The amplitudes, A, of the 2nd, 3rd, and 4th harmonic and the respective phase angles, Θ, are assigned during execution of the program from input data. The sampling time x is expressed as a fraction of 2π depending on the number of samples, N, to be taken per cycle.

$$x = \frac{2\pi}{N} \left(n - \frac{1}{2}\right), \quad (n = 1 \ldots N). \qquad (5)$$

To each sampling time x, is added a "timing error" to simulate fluctuations occurring in a real system, and a term representing "noise" is included in the calculation of the instantaneous amplitude Y. Both of these values are obtained using normalized random numbers (range -1 to +1) multiplied by suitable parameters specified in the computer program input data. To obtain maximum resolution and to avoid cumulative rounding errors for the desired number of significant figures, the values of analog quantities are computed with "double precision."

Each Y value then represents the analog voltage or current applied to the input of the A/D converter. The converter, however, has a finite resolution of 14 bits (plus 1 sign bit). The remainder of the calculations are therefore carried out in the integer mode by multiplying Y by 2^{14} with appropriate rounding off by 1/2 bit. The number of bits retained in each subsequent step of the calculation closely parallels the real time calculations that would be performed by special hardware in an actual system following the A/D conversion. For convenience the final result was normalized and compared to the calculated "true value" of the integral.

The program has provisions for specifying the following parameters:

1. The amplitudes and phases of the harmonics (up to 4th order)
2. The phase angle between the fundamentals
3. The percentage "noise"
4. The timing error (in nanoseconds)
5. The frequency of the fundamental component
6. The number of samples per cycle for the initial run and whether the number of samples in the succeeding run is twice or ten times as large
7. The limit on the number of runs and/or the limit of error

of the summation approximation

8. Whether the error is to be expressed as percent of reading or percent of full scale

9. The percentage difference between the fundamental frequency and the appropriate submultiple of the sampling frequency (Sychronization error)

10. Ordered or random sample points.

5. Results of Simulation

The performance of the hypothetical system of power measurement is shown in computer print-out "A" in the appendix.

For this calculation a basic set of parameters was chosen to correspond to a typical situation. There "Reference" Conditions are listed in Table 1. Each parameter, one at a time, is then varied to test how the system behaves. The other parameters remain the same, except for the determination of timing error where the fundamental frequency is increased to 2000 Hz from 60 Hz in order to make the resulting changes more apparent. The computation is terminated for each set of conditions when the summation agrees with the integral within the specified tolerance (0.005%). The maximum number of sample points was also limited when the tolerance could not be reached.

Table 1

Reference Conditions

Phase Angle between Fundamentals	60°
2nd, 3rd, 4th Harmonic content	1.0%
Superimposed Noise	0.01%
Timing Error	100 ns
Frequency	60 Hz

The first four sets of data show the variation with phase angle. In each case no more than 32 sample points are required to reach an accuracy of 0.005 percent. For the 80 degree point and the 89.999 degree point the error was calculated as percent of full scale.

The next set demonstrates that when using random sample intervals 8192 samples are not enough to assure even 1 percent accuracy.

The following three sets of data show that increasing the harmonic amplitudes from 0 to 10 percent of the fundamental does not significantly affect the number of samples required.

In the next three sections the influence of noise in the input signals is demonstrated. For a noise amplitude of 0.01 percent or less not more than 32 samples are necessary. If the noise is increased by an order of magnitude 1024 samples are required to obtain the desired accuracy. Although this is a considerable increase in the number of samples, it does indicate that the method can produce valid results even when the noise level is high.

Similarly, the next nine sets of data show that the accuracy is relatively insensitive to random timing errors (in the sampling time), but very sensitive to systematic timing errors.

The last five sets of tests show the effect of signal frequency on the number of samples required. For a "clean" signal results can be obtained with 1024 samples even at 10,000 Hz.

5.1 Second Series of Tests

The results of the first series of tests suggested that 512 sample points would be a reasonable number. With this in mind, and considering that in practice an A/D converter has a limited throughput rate, the simulated mode of operation of the system was somewhat modified. To make efficient use of the A/D converter(s), it was decided to operate as close to the maximum throughput rate as practicable at all input frequencies. The number of sample points per cycle was therefore made dependent on the fundamental component of input frequency as shown in Table 2 which assumes a throughput rate of about 50 kHz.

Table 2

Input Frequency (Hz)	Number of sample points per cycle
50 - 100	512
100 - 200	256
200 - 400	128
400 - 800	64
800 - 1600	32
1600 - 3200	16
3200 - 6400	8
6400 - 12800	4

The computer program was modified accordingly, and tests were run to determine the attainable accuracy under the specified conditions. The total number of sample points for each run was varied from 512 to 2048.

The results are shown in the computer print-out "B" (Appendix), and are also summarized in Tables 3 and 4. Some rounding errors are evident (e.g. Table 4, column 4) which represent 1 in 2^{15}. Disregarding these, there appear to be no trends in the calculated errors as function of frequency, phase angle, noise content, or random timing fluctuations.

This indicates that 512 samples per measurement are sufficient and that the system can tolerate reasonable amounts of noise and timing errors and remain within the 0.01% accuracy limit. Because of occasional rounding errors it is advisable to make more than one measurement.

Table 3

Percentage error of the simulated power measurement as a function of phase angle, frequency, and the number of samples with a timing error of 50 ns and 0.01% noise amplitude.

Frequency (Hz)	Phase angle (degrees)			Total no. of sample points
	0	60	89.999	
60	0.009	0.007	*) 0.005	512
	0.009	0.007	0.005	1024
	0.009	0.007	0.005	2048
400	0.003	0.007	0.005	512
	0.003	0.007	0.005	1024
	0.003	0.007	0.005	2048
1000	0.009	0.007	0.005	512
	0.003	0.007	0.005	1024
	0.003	0.007	0.005	2048
2000	0.009	0.007	0.005	512
	0.003	0.007	0.001	1024
	0.003	0.007	0.005	2048
5000	0.012	0.005	0.01	512
	0.006	0.019	0.005	1024
	0.006	0.005	0.005	2048

---

*) percent of full scale

Table 4

Percentage error as a function of noise, timing error, and total number of sample points for 400 Hz and 2 kHz input with 60° phase angle.

Timing error (ns)	Maximum noise amplitude (%)				Total number of samples
	0.0	0.01	0.03	0.1	
			400 Hz		
50	0.007	0.007	0.019	0.005	512
	0.007	0.007	0.007	0.005	1024
	0.007	0.007	0.007	0.019	2048
100	0.007	0.007	0.007	0.019	512
	0.007	0.007	0.007	0.019	1024
	0.007	0.007	0.007	0.007	2048
			2000 Hz		
50	0.007	0.007	-	0.007	512
	0.007	0.007		0.019	1024
	0.007	0.007		0.007	2048
100	0.019	0.007	-	0.007	512
	0.007	0.019		0.007	1024
	0.007	0.007		0.007	2048

6. Conclusions

A system has been described that will measure audio frequency power by computation from the numberical values of simultaneous samples of the instantaneous current and voltage. A mathematical model of the system has been investigated and has shown that the method is capable of an accuracy of 0.01 percent. If the maximum sampling rate is 50 kHz, 512 sampling points for each determination are adequate in most cases. In the calculation quantizing errors arising from finite resolution of the A/D converters were taken into account, but non-linearities and offsets were not considered. The results therefore correspond to the best that can be obtained by this sampling method with the assumption of a 15 bit conversion resolution under conditions where the hardware does not contribute additional errors.

A reprint of a paper (previously published)[4] describing the experimental verification of this method is included in the appendix. Measurement results show agreement with the NBS wattmeter generally within a ±0.02% error band for frequencies up to 5 kHz. Contributing to the experimintal uncertainty is the deviation from linearity of the A/D converters, the frequency response and voltage dependent offsets of the sample and hold circuitry. Improvements in the operating characteristics of the hardware could reduce the overall uncertainty by perhaps 50 ppm.

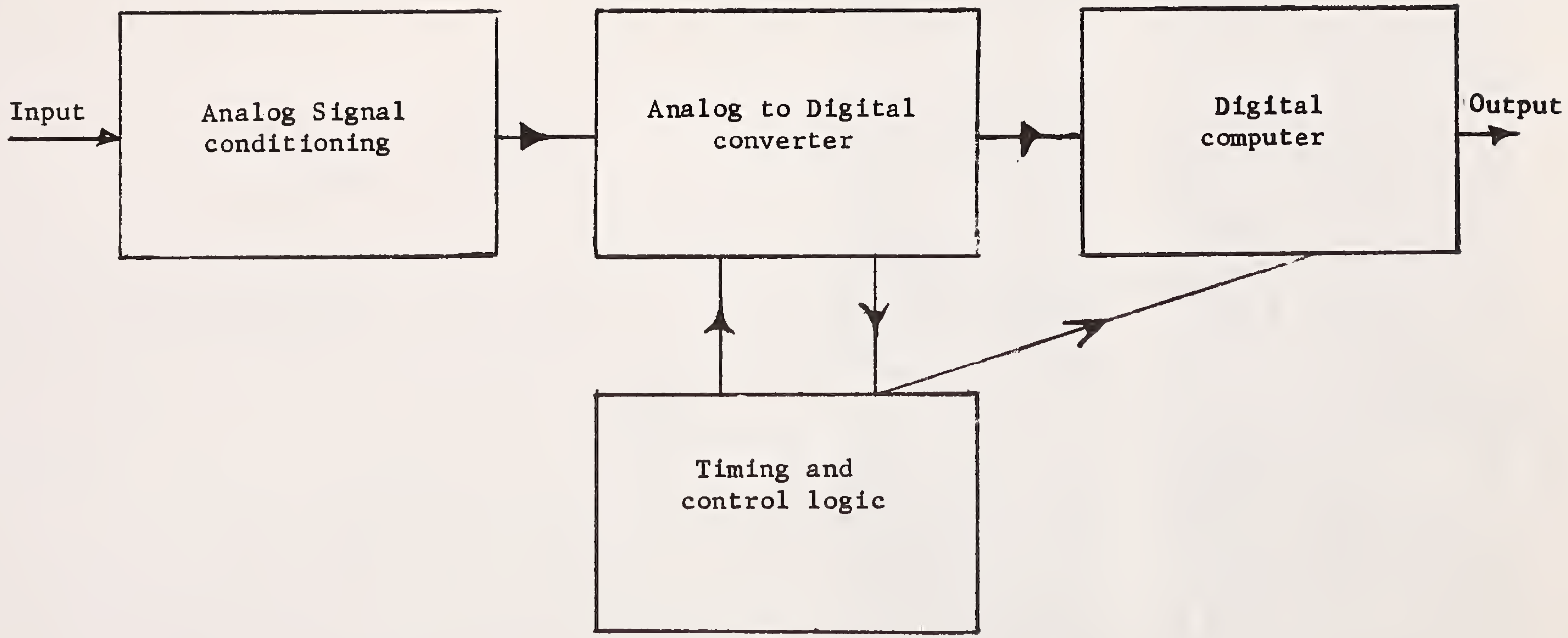
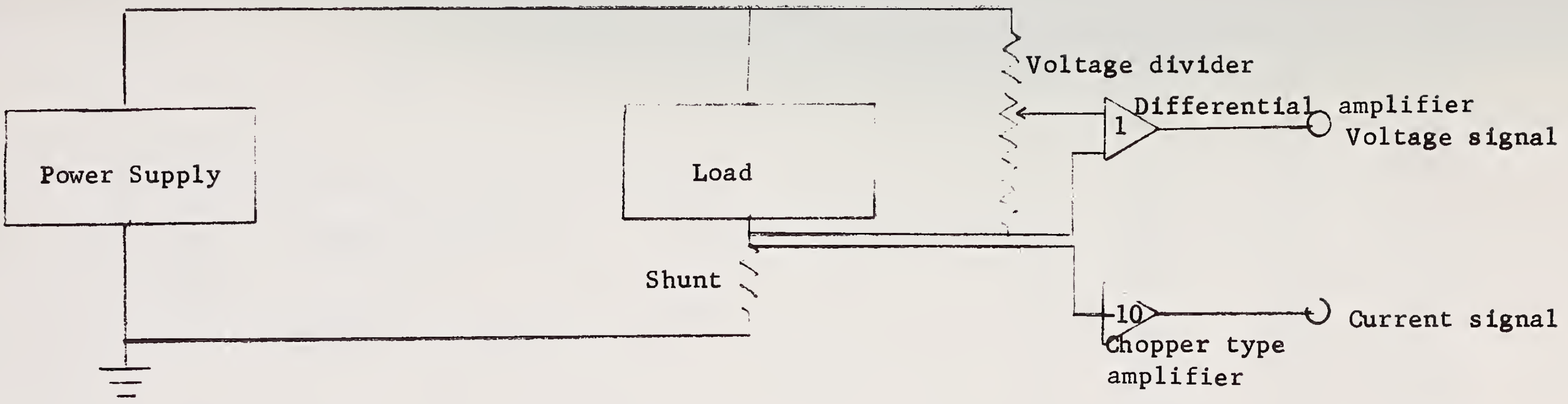
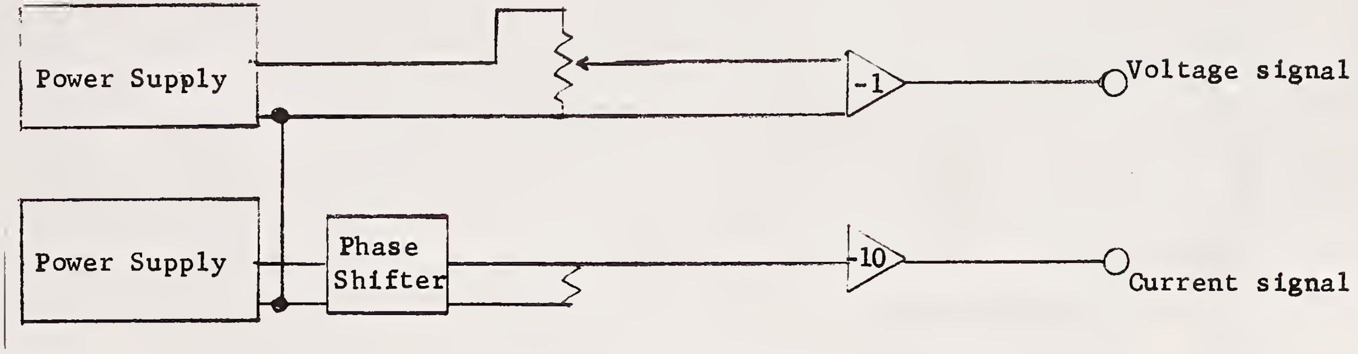

Input
Analog Signal conditioning
Analog to Digital converter
Digital computer
Output
Timing and control logic

Fig. 3.1

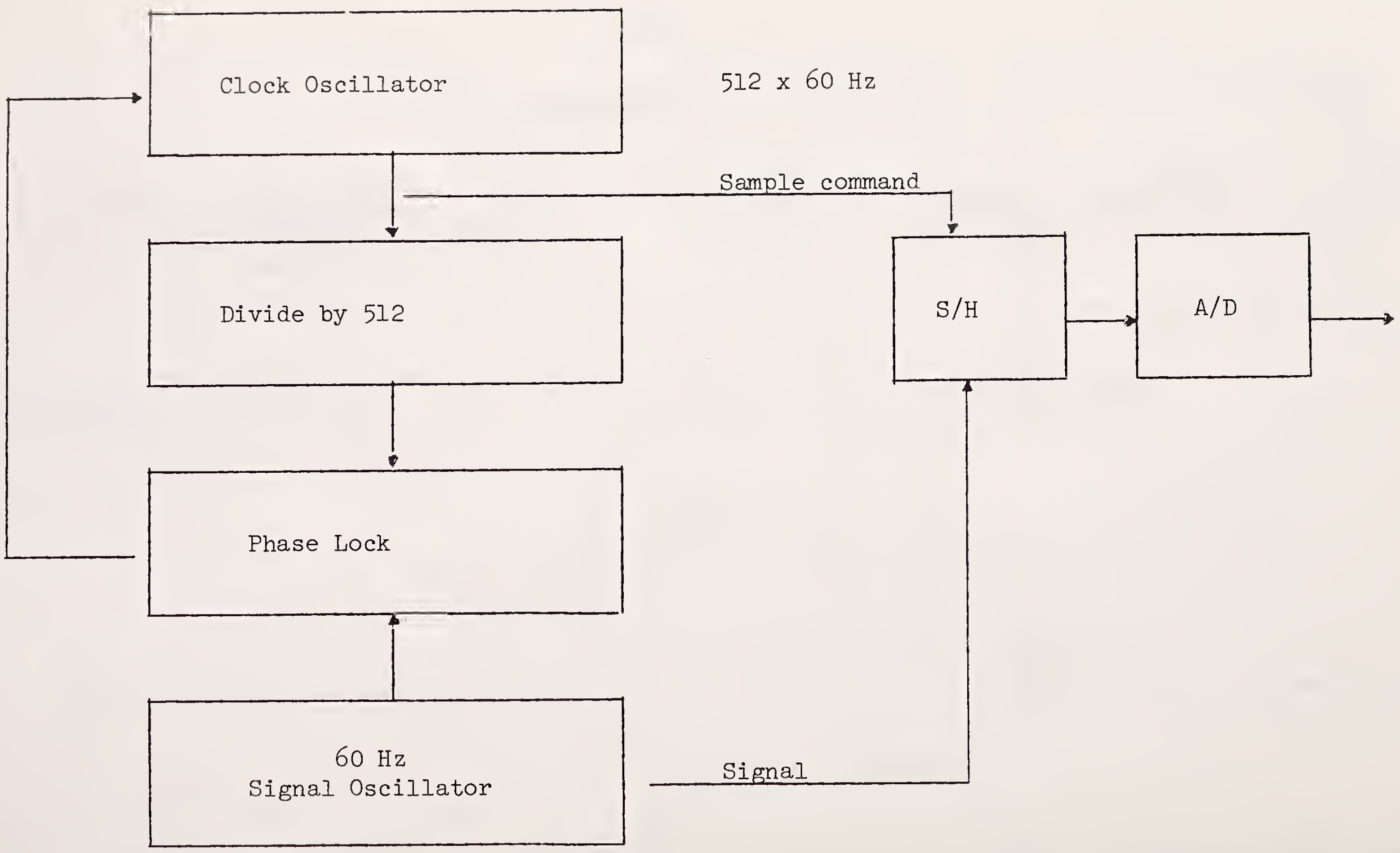


(a) Real Load



(b) Phantom Load

Fig. 3.2



Control circuit for 512 samples at 60 Hz signal frequency

Fig. 3.3

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DIGITAL POWER MEASUREMENT                                              PAGE  1 A

NO. OF         PHASE   INTEGRAL   SUM        PERCENT     PCNT.  PCNT.  TIMING  FREQUENCY
POINTS         ANGLE                         ERROR       HARM.  NOISE  ERROR     HZ

    4            .000   .500041   .508362   -1.664087     1.    .0100   100.       60.
    8            .000   .500041   .500092    -.010169     1.    .0100   100.       60.
   16            .000   .500041   .500000     .008141     1.    .0100   100.       60.
   32            .000   .500041   .500031     .002037     1.    .0100   100.       60.

    4          60.000   .250041   .254974   -1.973143     1.    .0100   100.       60.
    8          60.000   .250041   .250092    -.020336     1.    .0100   100.       60.
   16          60.000   .250041   .250031     .004074     1.    .0100   100.       60.

    4          80.000   .086865   .090088     .644620*    1.    .0100   100.       60.
    8          80.000   .086865   .086914     .009854*    1.    .0100   100.       60.
   16          80.000   .086865   .086823    -.008456*    1.    .0100   100.       60.
   32          80.000   .086865   .086853    -.002353*    1.    .0100   100.       60.

    4          89.999   .000049   .002472     .484497*    1.    .0100   100.       60.
    8          89.999   .000049   .000092     .008423*    1.    .0100   100.       60.
   16          89.999   .000049   .000031    -.003784*    1.    .0100   100.       60.

    4 RAND       .000   .500041   .720123  -44.012934     1.    .0100   100.       60.
    8 RAND       .000   .500041   .651794  -30.348276     1.    .0100   100.       60.
   16 RAND       .000   .500041   .535919   -7.175113     1.    .0100   100.       60.
   32 RAND       .000   .500041   .553162  -10.623318     1.    .0100   100.       60.
   64 RAND       .000   .500041   .440918   11.823586     1.    .0100   100.       60.
  128 RAND       .000   .500041   .458466    8.314349     1.    .0100   100.       60.
  256 RAND       .000   .500041   .513672   -2.726012     1.    .0100   100.       60.
  512 RAND       .000   .500041   .538971   -7.785414     1.    .0100   100.       60.
 1024 RAND       .000   .500041   .495331     .941902     1.    .0100   100.       60.
 2048 RAND       .000   .500041   .494537    1.100580     1.    .0100   100.       60.
 4096 RAND       .000   .500041   .496918     .624545     1.    .0100   100.       60.
 8192 RAND       .000   .500041   .508270   -1.645778     1.    .0100   100.       60.

    4          60.000   .250000   .250031    -.012207     0.    .0100   100.       60.
    8          60.000   .250000   .249969     .012207     0.    .0100   100.       60.
   16          60.000   .250000   .250000    -.000000     0.    .0100   100.       60.

TIMING ERROR IN NANOSECONDS
* = ERROR CALCULATED AS PERCENT OF FULL SCALE
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COMPUTER PRINTOUT "A"

```
DIGITAL POWER MEASUREMENT                                             PAGE  2 A

NO. OF      PHASE   INTEGRAL   SUM        PERCENT    PCNT. PCNT.  TIMING FREQUENCY
POINTS      ANGLE                         ERROR      HARM. NOISE  ERROR    HZ

   4        60.000  .250041   .255035   -1.997553    1.   .0100  100.     60.
   8        60.000  .250041   .250061    -.008132    1.   .0100  100.     60.
  16        60.000  .250041   .250031     .004074    1.   .0100  100.     60.

   4        60.000  .254071   .301117  -18.516986   10.   .0100  100.     60.
   8        60.000  .254071   .257935   -1.520784   10.   .0100  100.     60.
  16        60.000  .254071   .254059     .004670   10.   .0100  100.     60.

   4        60.000  .250041   .254974   -1.973143    1.   .0001  100.     60.
   8        60.000  .250041   .250031     .004074    1.   .0001  100.     60.

   4        60.000  .250041   .255005   -1.985349    1.   .0010  100.     60.
   8        60.000  .250041   .250061    -.008132    1.   .0010  100.     60.
  16        60.000  .250041   .250031     .004074    1.   .0010  100.     60.

   4        60.000  .250041   .254974   -1.973143    1.   .0100  100.     60.
   8        60.000  .250041   .250061    -.008132    1.   .0100  100.     60.
  16        60.000  .250041   .250031     .004074    1.   .0100  100.     60.

   4        60.000  .250041   .254791   -1.899913    1.   .1000  100.     60.
   8        60.000  .250041   .250214    -.069156    1.   .1000  100.     60.
  16        60.000  .250041   .249939     .040689    1.   .1000  100.     60.
  32        60.000  .250041   .250061    -.008132    1.   .1000  100.     60.
  64        60.000  .250041   .250092    -.020336    1.   .1000  100.     60.
 128        60.000  .250041   .250061    -.008132    1.   .1000  100.     60.
 256        60.000  .250041   .250000     .016280    1.   .1000  100.     60.
 512        60.000  .250041   .250000     .016280    1.   .1000  100.     60.
1024        60.000  .250041   .250031     .004074    1.   .1000  100.     60.

   4        60.000  .250041   .255035   -1.997553    1.   .0100    0.   2000.
   8        60.000  .250041   .250061    -.008132    1.   .0100    0.   2000.
  16        60.000  .250041   .250000     .016280    1.   .0100    0.   2000.
  32        60.000  .250041   .250031     .004074    1.   .0100    0.   2000.

TIMING ERROR IN NANOSECONDS
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 DIGITAL POWER MEASUREMENT                                                  PAGE   3 A

 NO. OF        PHASE   INTEGRAL   SUM        PERCENT    PCNT. PCNT. TIMING FREQUENCY
 POINTS        ANGLE                         ERROR      HARM. NOISE ERROR    HZ


     4         60.000  .250041  .255005  -1.985349    1.   .0100    50.   2000.
     8         60.000  .250041  .250000    .016280    1.   .0100    50.   2000.
    16         60.000  .250041  .250031    .004074    1.   .0100    50.   2000.


     4         60.000  .250041  .254915  -1.948734    1.   .0100   100.   2000.
     8         60.000  .250041  .249847    .077304    1.   .0100   100.   2000.
    16         60.000  .250041  .249817    .089510    1.   .0100   100.   2000.
    32         60.000  .250041  .250061   -.008132    1.   .0100   100.   2000.
    64         60.000  .250041  .249939    .040689    1.   .0100   100.   2000.
   128         60.000  .250041  .250061   -.008132    1.   .0100   100.   2000.
   256         60.000  .250041  .250031    .004074    1.   .0100   100.   2000.


     4         60.000  .250041  .255463  -2.168424    1.   .0100   200.   2000.
     8         60.000  .250041  .250458   -.166796    1.   .0100   200.   2000.
    16         60.000  .250041  .250244   -.081362    1.   .0100   200.   2000.
    32         60.000  .250041  .250122   -.032541    1.   .0100   200.   2000.
    64         60.000  .250041  .250061   -.008132    1.   .0100   200.   2000.
   128         60.000  .250041  .249908    .052895    1.   .0100   200.   2000.
   256         60.000  .250041  .250031    .004074    1.   .0100   200.   2000.


     4         60.000  .250041  .255005  -1.985349    1.   .0100   100.      60.
     8         60.000  .250041  .250031    .004074    1.   .0100   100.      60.


     4         60.000  .250041  .254974  -1.973143    1.   .0100   100.$     60.
     8         60.000  .250041  .250031    .004074    1.   .0100   100.$     60.
 $=TIMING PULSES BASED ON PERIOD   .010 PERCENT LONGER
   THAN PERIOD OF FUNDAMENTAL SIGNAL FREQUENCY


     4         60.000  .250041  .254547  -1.802273    1.   .0100   100.$     60.
     8         60.000  .250041  .249847    .077304    1.   .0100   100.$     60.
    16         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
    32         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
    64         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
   128         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
   256         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
   512         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
  1024         60.000  .250041  .249786    .101714    1.   .0100   100.$     60.
 $=TIMING PULSES BASED ON PERIOD   .100 PERCENT LONGER
   THAN PERIOD OF FUNDAMENTAL SIGNAL FREQUENCY

 TIMING ERROR IN NANOSECONDS
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DIGITAL POWER MEASUREMENT                                                  PAGE  4 A

NO. OF       PHASE   INTEGRAL   SUM        PERCENT    PCNT.  PCNT.  TIMING  FREQUENCY
POINTS       ANGLE                         ERROR      HARM.  NOISE  ERROR     HZ

   4         60.000  .250041   .251190   -.459717    1.    .0100  100.$     60.
   8         60.000  .250041   .247742    .919452    1.    .0100  100.$     60.
  16         60.000  .250041   .247894    .856428    1.    .0100  100.$     60.
  32         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
  64         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
 128         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
 256         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
 512         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
1024         60.000  .250041   .247955    .834017    1.    .0100  100.$     60.
$=TIMING PULSES BASED ON PERIOD  1.000 PERCENT LONGER
  THAN PERIOD OF FUNDAMENTAL SIGNAL FREQUENCY


   4         60.000  .250041   .256165  -2.449140    1.    .0100  100.$     60.
   8         60.000  .250041   .258728  -3.474364    1.    .0100  100.$     60.
  16         60.000  .250041   .257172  -2.851906    1.    .0100  100.$     60.
  32         60.000  .250041   .256897  -2.742061    1.    .0100  100.$     60.
  64         60.000  .250041   .256836  -2.717651    1.    .0100  100.$     60.
 128         60.000  .250041   .256836  -2.717651    1.    .0100  100.$     60.
 256         60.000  .250041   .256836  -2.717651    1.    .0100  100.$     60.
 512         60.000  .250041   .256805  -2.705446    1.    .0100  100.$     60.
1024         60.000  .250041   .256805  -2.705446    1.    .0100  100.$     60.
$=TIMING PULSES BASED ON PERIOD 10.000 PERCENT LONGER
  THAN PERIOD OF FUNDAMENTAL SIGNAL FREQUENCY


   4         60.000  .250041   .255005  -1.985349    1.    .0100  100.       60.
   8         60.000  .250041   .250061   -.008132    1.    .0100  100.       60.
  16         60.000  .250041   .250031    .004074    1.    .0100  100.       60.


   4         60.000  .250041   .255005  -1.985349    1.    .0100  100.      400.
   8         60.000  .250041   .250031    .004074    1.    .0100  100.      400.


   4         60.000  .250041   .255035  -1.997553    1.    .0100  100.     1000.
   8         60.000  .250041   .250183   -.056951    1.    .0100  100.     1000.
  16         60.000  .250041   .250031    .004074    1.    .0100  100.     1000.


   4         60.000  .250041   .255157  -2.046373    1.    .0100  100.     2000.
   8         60.000  .250041   .250061   -.008132    1.    .0100  100.     2000.
  16         60.000  .250041   .249786    .101714    1.    .0100  100.     2000.
  32         60.000  .250041   .249939    .040689    1.    .0100  100.     2000.
  64         60.000  .250041   .249969    .028484    1.    .0100  100.     2000.
 128         60.000  .250041   .250061   -.008132    1.    .0100  100.     2000.
 256         60.000  .250041   .250000    .016280    1.    .0100  100.     2000.
 512         60.000  .250041   .250031    .004074    1.    .0100  100.     2000.

TIMING ERROR IN NANOSECONDS
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DIGITAL POWER MEASUREMENT                                                PAGE  5 A

NO. OF       PHASE   INTEGRAL  SUM       PERCENT    PCNT. PCNT. TIMING FREQUENCY
POINTS       ANGLE                       ERROR      HARM. NOISE ERROR    HZ


   4         60.000   .250041  .253937  -1.558171    1.   .0100  100.  10000.
   8         60.000   .250041  .248688    .541095    1.   .0100  100.  10000.
  16         60.000   .250041  .249847    .077304    1.   .0100  100.  10000.
  32         60.000   .250041  .250122   -.032541    1.   .0100  100.  10000.
  64         60.000   .250041  .249756    .113919    1.   .0100  100.  10000.
 128         60.000   .250041  .250092   -.020336    1.   .0100  100.  10000.
 256         60.000   .250041  .250092   -.020336    1.   .0100  100.  10000.
 512         60.000   .250041  .250092   -.020336    1.   .0100  100.  10000.
1024         60.000   .250041  .250031    .004074    1.   .0100  100.  10000.

TIMING ERROR IN NANOSECONDS
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DIGITAL POWER MEASUREMENT                                        PAGE  1 B

NO. OF       PHASE   INTEGRAL   SUM       PERCENT   PCNT. PCNT. TIMING FREQUENCY
POINTS       ANGLE                        ERROR     HARM. NOISE ERROR    HZ


 512           .000   .500139   .500092    .009498    3.   .0100   50.     60.
1024           .000   .500139   .500122    .003396    3.   .0100   50.     60.
2048           .000   .500139   .500122    .003396    3.   .0100   50.     60.


 512           .000   .500139   .500122    .003396    3.   .0100   50.    400.
1024           .000   .500139   .500122    .003396    3.   .0100   50.    400.
2048           .000   .500139   .500122    .003396    3.   .0100   50.    400.


 512           .000   .500139   .500092    .009498    3.   .0100   50.   1000.
1024           .000   .500139   .500122    .003396    3.   .0100   50.   1000.
2048           .000   .500139   .500122    .003396    3.   .0100   50.   1000.


 512           .000   .500139   .500092    .009498    3.   .0100   50.   2000.
1024           .000   .500139   .500122    .003396    3.   .0100   50.   2000.
2048           .000   .500139   .500122    .003396    3.   .0100   50.   2000.


 512           .000   .500000   .499939    .012207    0.   .0100   50.   5000.
1024           .000   .500000   .499969    .006104    0.   .0100   50.   5000.
2048           .000   .500000   .499969    .006104    0.   .0100   50.   5000.


 512         60.000   .250139   .250122    .006789    3.   .0100   50.     60.
1024         60.000   .250139   .250122    .006789    3.   .0100   50.     60.
2048         60.000   .250139   .250122    .006789    3.   .0100   50.     60.


 512         60.000   .250139   .250122    .006789    3.   .0100   50.    400.
1024         60.000   .250139   .250122    .006789    3.   .0100   50.    400.
2048         60.000   .250139   .250122    .006789    3.   .0100   50.    400.


 512         60.000   .250139   .250122    .006789    3.   .0100   50.   1000.
1024         60.000   .250139   .250122    .006789    3.   .0100   50.   1000.
2048         60.000   .250139   .250122    .006789    3.   .0100   50.   1000.

TIMING ERROR IN NANOSECONDS
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COMPUTER PRINTOUT "B"

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 DIGITAL POWER MEASUREMENT                                            PAGE  2B

NO. OF          PHASE  INTEGRAL  SUM        PERCENT    PCNT. PCNT. TIMING FREQUENCY
POINTS          ANGLE                       ERROR      HARM. NOISE ERROR    HZ


 512            60.000  .250139  .250122     .006789    3.   .0100    50.  2000.
1024            60.000  .250139  .250122     .006789    3.   .0100    50.  2000.
2048            60.000  .250139  .250122     .006789    3.   .0100    50.  2000.


 512            60.000  .250049  .250061   -.004919     1.   .0100    50.  5000.
1024            60.000  .250049  .250000    .019491     1.   .0100    50.  5000.
2048            60.000  .250049  .250061   -.004919     1.   .0100    50.  5000.


 512            89.999  .000148  .000122   -.005142*    3.   .0100    50.    60.
1024            89.999  .000148  .000122   -.005142*    3.   .0100    50.    60.
2048            89.999  .000148  .000122   -.005142*    3.   .0100    50.    60.


 512            89.999  .000148  .000122   -.005142*    3.   .0100    50.   400.
1024            89.999  .000148  .000122   -.005142*    3.   .0100    50.   400.
2048            89.999  .000148  .000122   -.005142*    3.   .0100    50.   400.


 512            89.999  .000148  .000122   -.005142*    3.   .0100    50.  1000.
1024            89.999  .000148  .000122   -.005142*    3.   .0100    50.  1000.
2048            89.999  .000148  .000122   -.005142*    3.   .0100    50.  1000.


 512            89.999  .000148  .000122   -.005142*    3.   .0100    50.  2000.
1024            89.999  .000148  .000153    .000961*    3.   .0100    50.  2000.
2048            89.999  .000148  .000122   -.005142*    3.   .0100    50.  2000.


 512            89.999  .000057  .000000   -.011492*    1.   .0100    50.  5000.
1024            89.999  .000057  .000031   -.005388*    1.   .0100    50.  5000.
2048            89.999  .000057  .000031   -.005388*    1.   .0100    50.  5000.


 512            60.000  .250139  .250122     .006789    3.   .0000    50.   400.
1024            60.000  .250139  .250122     .006789    3.   .0000    50.   400.
2048            60.000  .250139  .250122     .006789    3.   .0000    50.   400.

TIMING ERROR IN NANOSECONDS
* = ERROR CALCULATED AS PERCENT OF FULL SCALE
```

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DIGITAL POWER MEASUREMENT                                              PAGE  3 B

NO. OF        PHASE   INTEGRAL   SUM         PERCENT   PCNT.  PCNT.  TIMING FREQUENCY
POINTS        ANGLE                          ERROR     HARM.  NOISE  ERROR    HZ


 512          60.000   .250139   .250092      .018989   3.    .0300    50.     400.
1024          60.000   .250139   .250122      .006789   3.    .0300    50.     400.
2048          60.000   .250139   .250122      .006789   3.    .0300    50.     400.


 512          60.000   .250139   .250153     -.005411   3.    .1000    50.     400.
1024          60.000   .250139   .250153     -.005411   3.    .1000    50.     400.
2048          60.000   .250139   .250092      .018989   3.    .1000    50.     400.


 512          60.000   .250139   .250122      .006789   3.    .0000   100.     400.
1024          60.000   .250139   .250122      .006789   3.    .0000   100.     400.
2048          60.000   .250139   .250122      .006789   3.    .0000   100.     400.


 512          60.000   .250139   .250122      .006789   3.    .0100   100.     400.
1024          60.000   .250139   .250122      .006789   3.    .0100   100.     400.
2048          60.000   .250139   .250122      .006789   3.    .0100   100.     400.


 512          60.000   .250139   .250122      .006789   3.    .0300   100.     400.
1024          60.000   .250139   .250122      .006789   3.    .0300   100.     400.
2048          60.000   .250139   .250122      .006789   3.    .0300   100.     400.


 512          60.000   .250139   .250092      .018989   3.    .1000   100.     400.
1024          60.000   .250139   .250092      .018989   3.    .1000   100.     400.
2048          60.000   .250139   .250122      .006789   3.    .1000   100.     400.


 512          60.000   .250139   .250122      .006789   3.    .0000    50.    2000.
1024          60.000   .250139   .250122      .006789   3.    .0000    50.    2000.
2048          60.000   .250139   .250122      .006789   3.    .0000    50.    2000.


 512          60.000   .250139   .250122      .006789   3.    .1000    50.    2000.
1024          60.000   .250139   .250092      .018989   3.    .1000    50.    2000.
2048          60.000   .250139   .250122      .006789   3.    .1000    50.    2000.

TIMING ERROR IN NANOSECONDS
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 DIGITAL POWER MEASUREMENT                                             PAGE  4B

NO. OF       PHASE   INTEGRAL   SUM       PERCENT    PCNT. PCNT. TIMING FREQUENCY
POINTS       ANGLE                        ERROR      HARM. NOISE ERROR    HZ


 512         60.000  .250139   .250092     .018989    3.   .0000  100.   2000.
1024         60.000  .250139   .250122     .006789    3.   .0000  100.   2000.
2048         60.000  .250139   .250122     .006789    3.   .0000  100.   2000.

 512         60.000  .250139   .250122     .006789    3.   .0100  100.   2000.
1024         60.000  .250139   .250092     .018989    3.   .0100  100.   2000.
2048         60.000  .250139   .250122     .006789    3.   .0100  100.   2000.

 512         60.000  .250139   .250122     .006789    3.   .1000  100.   2000.
1024         60.000  .250139   .250122     .006789    3.   .1000  100.   2000.
2048         60.000  .250139   .250122     .006789    3.   .1000  100.   2000.

TIMING ERROR IN NANOSECONDS
ND
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Digital Wattmeter Using a Sampling Method

RAYMOND S. TURGEL, MEMBER, IEEE

Abstract—**Average electric power can be measured by a system that samples voltages and currents at predetermined intervals. The sampled signals are digitized and the result is computed by numerical integration. The response of the system agrees with that of a standard electrodynamic wattmeter within 0.02 percent from dc to 1 kHz, with the possible exception of zero power factor measurements. Measurements up to 5 kHz can be made with somewhat greater uncertainties.**

I. INTRODUCTION

BECAUSE BASIC electrical standards are defined in terms of dc quantities, the measurement of ac quantities requires an ac–dc transfer process. Underlying all such transfer processes are certain assumptions about the limits of uncertainty associated with the measurement that cannot be determined precisely. Corroborative evidence from other independent methods of measurement is therefore the only means to increase the confidence in the method used and in the assumptions made.

The digital wattmeter described here is based on a method that differs from the electrodynamic, electrostatic, thermal, and Hall effect wattmeters, all of which make use of analog multiplication of current and voltage performed by a physical process in the device. In the digital instrument the multiplication involves discrete numbers and in itself introduces no experimental errors (other than predictable rounding errors).

Additionally, the digital method opens up possibilities of performing power measurements very rapidly, which may be of value in power control applications, and also of measuring nonsinusoidal waveforms with greater accuracy. Both of these advantages, however, have so far not been fully exploited.

II. EXPERIMENTAL METHOD

The fundamental equation for calculating the average electric power is

$$P = (1/T)\int_0^T ei\,dt. \qquad (1)$$

For sinusoidal quantities, T is chosen to be a multiple of the period of e or i,

$$T = n/f \qquad (2)$$

where n is a positive integer and f is the frequency of the sine wave. The present method is closely related to the fundamental formula (1). Instantaneous values of voltage and current are measured simultaneously, and from sets of these measurements the average power is computed using digital methods. Because the number of measurements is necessarily finite, the integral of (1) is replaced by a summation and the time inteval T by the number of measured points N,

$$P = (1/N)\sum e_j i_j, \qquad j = 1,\cdots,N. \qquad (3)$$

For dc power, where e and i are constant, the summation will obviously lead to the correct result. For sinusoidal quantities, however, we need to define conditions further. In (2) the time over which the integration extends is related to the period of the voltage or current waveforms. Similarly, the time taken for N samples must be such that the interval between samples is an integral fraction of the period, and all such intervals must be equal to one another. It can be shown that under these conditions, with at least 3 samples per cycle, the summation will theoretically be exact [1]. In practice, of course, more samples are needed to reduce the effects of random noise and fluctuations in sample timing. The important point is that samples are taken at definite intervals and not at random. Random sampling does not produce rapid convergence in the numerical integration process and therefore is not suited for high accuracy measurements.

To determine how many samples would be needed for a power measurement, a computer simulation of the measurement process was carried out. The study took into account the effects of input noise, timing uncertainty, presence of harmonics, and errors due to finite resolution. The known value of the integral given by (1) and (2) was compared with the computed summation of (3). The acceptable tolerance limit was set at 100 ppm and the presence of random noise with a peak value of 100 ppm was postulated. From this study, which is being published as a separate National Bureau of Standards (NBS) report, the following conclusions can be drawn.

1) Not more than 512 sample points are required.

2) A timing uncertainty of 50 ns is tolerable.

3) The percentage error due to systematic timing errors is proportional to the percentage deviation of the timing base period from the period of the input waveform.

4) A resolution corresponding to 15 bits for the measured values of current and voltage is adequate.

5) The sampling rate must be at least 40 kHz to resolve frequency components up to 10 kHz.

The measurement process can now be summarized as follows. Simultaneous samples of the current and voltage waveforms are taken at predetermined intervals. The sampling interval is an integral fraction of the period

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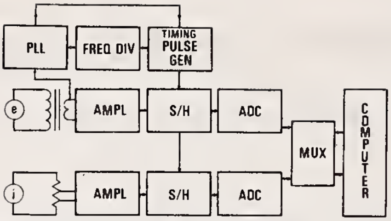

Fig. 1. Functional block diagram of digital wattmeter.

of the waveform and the time at which sampling occurs is defined with an uncertainty less than 50 ns. The sampling process is implemented by sample-and-hold amplifiers controlled by a special timing pulse generator. The values of the instantaneous signal voltage "captured" by the S/H circuits are digitized using analog-to-digital converters (ADC), with a resolution of 1 in 16384. The resulting digital output is then used in the numerical calculation of the power.

III. HARDWARE

A. *Signal Conditioning*

The functional blocks of the system are shown in Fig. 1. The voltage and current waveforms can be sampled either in a circuit with a real load, or with phantom loading.

The "current" signal is obtained from a specially constructed bifilar shunt with a time constant of 2 ns.[1] At 5 A, the voltage drop across the shunt is 0.5 V, thus keeping the dissipation to 2.5 W. Forced air cooling is used to minimize the temperature rise of the resistive element.

The "voltage" signal is obtained from a two-stage step-down transformer [2] with a 200-to-1 ratio which has negligible ratio and phase corrections. Two similar transformers are used, one for the frequency range of 50 to 400 Hz, the other for the range of 400 to 2000 Hz. The secondary voltage is 0.5 V. This value was chosen so that the current and voltage signals are comparable in magnitude and so that input channels can easily be interchanged.

For dc measurements a precision volt-box accurate to within 2 ppm is used instead of a transformer.

B. *Preamplifiers*

The two input signals are fed into high-impedance instrumentations amplifiers[2] which permit the shunt output and the transformer secondary to work essentially on open circuit. The amplifier gain is fixed at a nominal value of 13.5 and is flat within 50 ppm from dc to 5 kHz. The gain raises the 0.5-V inputs to the level required by the sample-and-hold circuitry, that is, ±10-V peak. An important consideration in power measurement is that the phase shift be the same in both channels. At 10 kHz the difference is phase shift of the two amplifiers was less than 600 μrad. Moreover, the effects of small remaining differences in phase shift can be canceled by averaging measurements with the two channels interchanged [3].

C. *Timing Pulse Generator*

A critical part of the measurement process is the timing of the sampling and conversion command. The timing pulse changes the S/H amplifiers from the "sample" to the "hold" state in which they retain at their outputs the values of the input voltages present at the time of the transition. The timing pulse also initiates the analog-to-digital conversion, which begins after a short delay to allow for settling of the S/H amplifiers.

The timing of the command pulse is brought into a definite relationship with the input signal by means of a phase-lock circuit.[3] The phase-locked loop (PLL) operates in a conventional manner with only minor modifications.

Part of the input voltage signal is fed to a high-impedance instrumentation amplifier with good common mode rejection which acts as a buffer to isolate the analog measuring circuit from the digital circuitry. The amplifier gain is chosen so that the phase lock will work reliably over an input voltage range from 0.1 to 1.0 V.

The frequency of the voltage signal appearing at the output of the amplifier is then compared to a subharmonic of the pulse repetition frequency in a phase sensitive detector which in turn controls the frequency of the pulse oscillator.

The subharmonic is obtained by means of a binary frequency divider which has 9 manually selectable ratios. These range from a ratio of 4 to a ratio of 512. Therefore, since the oscillator operates over a band from 20 kHz to 40 kHz the subharmonic generated by the highest ratio setting (512) ranges from 40 to 80 Hz. Similarly, for the lowest ratio setting (4) the subharmonic varies from 5 kHz to 10 kHz. Thus there is a definite ratio setting for every analog input frequency. It follows that, for instance, for an input of 60 Hz there are 512 sampling points per cycle, while for an input of 6 kHz there are only 4 samples per cycle. The ADC's however, always operate at the

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[1] Designed by T. M. Souders, Electricity Division, National Bureau of Standards, Washington, D. C.

[2] Designed by L. Marzetta, Measurement Engineering Division, National Bureau of Standards, Washington, D. C.

[3] Designed by O. Laug, Measurement Engineering Division, National Bureau of Standards, Washington, D. C.

maximum possible rate consistent with the phase-lock requirements from 20 000 to 40 000 conversions per second.

There is a tradeoff between capture range and stability in phase-lock circuits. Since in this application stability is of overriding importance, the loop filter circuits have been selected to provide high stability. As a consequence the capture range is fairly narrow and a manual frequency adjustment of the oscillator is required to bring it into lock. A 10-turn variable resistor serves as the tuning element. To determine the correct adjustment point, an indicator light signals the out-of-lock condition and a meter connected to the output of the phase comparator provides a reading for fine adjustment. An alternate method to determine proper adjustment uses the digitized data and a special software routine in the computer which will be described later.

D. Analog-to-Digital Converters (ADC)

The outputs of the preamplifiers are fed to two parallel S/H circuits that are physically part of the converter unit. The sampled "instantaneous" voltages are retained at the outputs of the S/H amplifiers within the specified accuracy limits long enough for the analog-to-digital conversion to be completed. The two resulting digital numbers are then multiplexed to a common 15-bit parallel output.

The configuration described is preferable to the perhaps more usual one where an analog multiplexer feeds signals to a single channel ADC. It avoids the additional errors introduced by analog multiplex switching circuits and, at least in this case, at no additional cost. The output digital multiplexer, which is not subject to disturbance by noise and offset voltages, does not contribute to the error of the measurement.

Each of the ADC's converts the analog input by the method of successive approximations to a 15-bit (14 bits plus sign) two's complement binary number. The actual conversion time is about 15 μs. Taking into account various settling times and the time required to transfer the output data, a minimum of 25 μs is required between conversion cycles. The maximum throughput rate is therefore 40 kHz. Binary signals control the output-channel selection, output-inhibit function, and indicate when the conversion is completed.

E. Computing Circuitry

At the time the project was planned, intergrated circuits for high-speed digital arithmetic were not as easily available as they are now. Building of special circuits for real time computation, therefore, did not appear advisable. The selection of a minicomputer instead of special hardware, in retrospect, certainly seems to have been the better choice, even though calculations cannot be done in real time. The flexibility afforded by a general purpose computer makes possible not only a relatively easy change of experimental procedures, but also provides the opportunity for using diagnostic routines to detect malfunction or input errors.

The computer chosen had a 16-bit word and originally had 4096 (4K) words of memory. The memory was later expanded to 12K to permit the use of a "basic" (programing language) interpreter.

In order to link the computer with the ADC's, a special interface is required which must generate a signal to interrogate the two ADC channels alternately. The time between samples is insufficient for the normal input–output (I/O) channels of the computer and the ADC, data are therefore transferred through a direct memory access (DMA) channel. Since the ADC data occupy only 15 bits, the sixteenth bit of the computer word is used as a channel indicator. Channel identification is helpful in verifying the proper functioning of the interface and in correlating corresponding ADC outputs.

With the 4K memory, 1024 data words (corresponding to 512 sample points), as well as the machine language program, could be accommodated, however, with little room to spare. On the other hand, in the 12K environment, the "basic" interpreter together with additional special routines and data storage do not fill all of the available memory.

Printed output from the computer is obtained through a conventional teleprinter.

F. Input Circuit Switching

Input connections to the two preamplifiers are made through a patchboard using British Post Office type coaxial connectors. These provide very reliable low-resistance connections and show little signs of wear. The choice of a patchboard type input was a matter of expediency. In the long run, however, a system of computer controlled switches would be more useful. In the present arrangement each of the two channels can be connected, singly or together, to the following "sources":

1) "zero" (100-Ω internal resistance);
2) calibration voltage;
3) shunt output voltage;
4) transformer output voltage; and
5) volt-box output (for dc voltage).

All of these are required for the measurement procedure.

IV. SOFTWARE

With the limited amount of memory available initially, all routines were written in assembly language using integer arithmetic. Because the ADC output has only 15 significant bits, a 16-bit computer word is adequate for the calculation of the results. Later, after the memory was expanded, some of the integer computations were incorporated as special subroutines into the "basic" system in the interest of speed. As an example, the computation time required for a typical power measurement using unmodified "basic" is roughly 55 s. With integer subroutines the time taken is reduced to less than 1 s.

It is usually desirable to provide at least some internal checks to guard against malfunction or improper inputs.

TABLE I

Frequency	Power Factor	Difference from Comparison Standard (percent of full scale)	Comparison Standard	Uncertainty of Standard (percent of full scale)
dc		−0.004	dc potentiometer and shunt	0.002
50	1	0.014	wattmeter	0.01
		0.007	DTVC[c]	0.005
		−0.004	rms[d]	0.005
	0.5 lead	0.018	wattmeter	0.02
	0.5 lag	0.014	wattmeter	0.02
60	1	0.008	wattmeter	0.01
		−0.018	rms[d]	0.005
	0.5 lead	0.018	wattmeter	0.02
	0.5 lag	0.010	wattmeter	0.02
400	1	0.026	wattmeter	0.02
		0.016	DTVC	0.005
		−0.018	rms[d]	0.005
	0.5 lead	0.015	wattmeter	0.03
	0.5 lag	0.008	wattmeter	0.03
	0 lead	0.035	wattmeter	0.03
1000	1	−0.009	wattmeter	0.03
		−0.017	DTVC	0.005
		−0.019	rms[d]	0.005
	0.5 lead	0.012	wattmeter	0.06
	0.5 lag	0.024	wattmeter	0.06
	0 lead	−0.046	wattmeter	0.06
	0 lag	−0.018	wattmeter	0.06
2000	1	−0.038[a]	wattmeter	0.04
		−0.016	DTVC	0.005
	0.5 lead	0.083	wattmeter	0.11
	0.5 lag	0.013	wattmeter	0.11
5000		0.016[b]	DTVC	0.005

[a] Amplifier response correction 0.141 ± 0.01 percent.

[b] Amplifier response correction 1.14 ± 0.1 percent.

[c] DTVC indicates differential thermocouple voltage converter.

[d] See text.

Diagnostic routines check the ADC channel number, the sequence in which data is stored in memory, and also whether the ADC output exceeded maximum positive or negative values. One indication of faulty connections in the analog signal path is the noise superimposed on dc measurements. Such noise can be detected from the standard deviation of a set of measurements performed at the 40-kHz sampling rate.

Certain test instruments, such as oscilloscopes, when attached at critical points modify the system response because of loading, ground loops, or pickup. To prevent such interference it is possible to use the computer itself to test the system by means of special software routines. For instance, offset adjustment of the data amplifiers can be checked by displaying the ADC output on the computer front panel indicator lights. Similarly, such a display can be used to locate the transition points when calibrating the ADC.

Under some conditions the sensitivity of the phase-lock null meter is insufficient for optimum adjustment of the oscillator. Finer adjustment can be obtained with the help of a routine that calculates the differences between successive peak-to-valley amplitudes of the input waveform as measured by the sampling circuit and the ADC. The phase lock can then be optimized by adjusting the oscillator so that the differences, which are displayed on the panel, are minimized. This method of adjustment presupposes that the ac source has sufficient cycle-to-cycle stability.

V. EXPERIMENTAL PROCEDURE

Wherever possible, measurement procedures relied on external standards in order to be independent of such circuit parameters as drift, offset, gain, and internal reference source stability. Each measurement sequence starts with a "zero reading" which is then applied as a correction to subsequent data. Next is the "calibration voltage" obtained from a Zener reference source stable to within a few parts per million per month. These measurements are the average of 512 readings on each channel. Both positive and negative calibration voltages are used to compensate for a slight asymmetry in the response of the S/H-ADC combination that varied slowly with time. During the computation of the power, the appropriate pair of calibration constants is used depending on the sign of the measured value and the measurement channel. Thus, if it can be assumed that in the brief interval between calibration and measurement the circuit parameters change only by negligible amounts, the computed value will be independent of offsets and the value of the gain in amplifiers and ADC's.

To investigate the performance of the system two types of measurements were made: 1) comparison with an electrodynamic wattmeter (Yokogawa APR–2); 2) comparison with a differential thermal voltage converter [4]. The results of these measurements are summarized in Table I, which shows the percentage differences between the readings from the digital system and those from the comparison standard. The intercomparison with the wattmeter was carried out at 5 frequencies at unity and at half-power factors, and in two cases at zero power factor. Power measurements were made at a nominal input of 100 V and 5 A.

The results using the differential thermal voltage converter with an input of 100 V to both channels were calculated either as a simulated power measurement at unity power factor, or by computing the rms value for each channel (indicated by "rms" in the table). In the first case the calculations were carried out exactly as in the wattmeter comparison using (3) with $N = 512$. In the second, the rms value was computed separately for each channel before forming the product. At 2 kHz and 5 kHz a correction was applied for the drop in the response of the S/H amplifier. This increased the uncertainty by 0.01 percent and 0.1 percent, respectively. Short term precision of 0.006 percent could be realized, and overall repeatability was estimated by combining all 400-Hz data, regardless of how they were obtained. This yielded a standard deviation of 0.02 percent of reading and 0.004 percent of the mean.

VI. CONCLUSIONS

A method was described of measuring average electric power by sampling instantaneous voltages and currents from which the result was computed by numerical integration. Intercomparisons with an electrodynamic wattmeter and a differential thermal voltage converter agreed within 0.02 percent from dc to 1 kHz, with the possible exception of zero power factor measurements and with somewhat larger uncertainties up to 5 kHz.

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A system is described that determines average electric power by periodically sampling current and voltage waveforms and calculating the result from digitized values of measured instantaneous currents and voltages. System performance is modeled on a digital computer to investigate the effects of noise, harmonics and sampling time errors on the result of a simulated power measurement. With 15 bit analog-to-digital conversion and 512 measured sample points an accuracy of 0.01% can theoretically be obtained from dc up to 5 kHz.

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