Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

April through June 1949

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES

April 1 through June 30, 1949

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Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

April through June 1949

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from April 1, 1949 through June 30, 1949.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss

Chief

E. U. Condon

Director
National Bureau of Standards
August 15, 1949

Status of Projects

June
~~September~~ 30, 1949

I. Institute for Numerical Analysis

(Section 11.1)

1. Research in Numerical Analysis

Key to letter symbols in project numbers:

A - Mathematical analysis
AE - Algebraic equations
AM - Analytical applied mathematics
CM - Numerical methods in conformal mapping
ODE - Ordinary differential equations
PM - Probabilistic methods in numerical analysis
SF - Studies of special functions

EXPLICIT FORMULAE FOR CALCULATION OF DEFINITE INTEGRALS
Project 11.1/1-49-A1

Origin: NBS — Authorized 4/1/49
Manager: A. M. Ostrowski — Completed 6/30/49

Objective: To derive rigorously various explicit formulae for evaluating definite integrals.

Background: The project was motivated in part by the need for extending the existing tables of definite integrals, and in part by a desire to supply rigorous proofs for certain classical formulae under carefully stated assumptions.

Status: Completed (New). The work on this project is set forth in the papers listed below under "Publications." The first three papers center about the Cauchy-Frullani integral formula,

$$\int_0^\infty \frac{f(at) - f(bt)}{t}\, dt = [f(\infty) - f(0)] \log \frac{a}{b} ,$$

where $a,b>0$, $f(0) = \lim_{x \downarrow 0} f(x)$, $f(\infty) = \lim_{x \to \infty} f(x)$, and $f(x)$ is integrable L over $0<A \leq x \leq B<\infty$. In the past, a number of authors have become interested in this surprising formula and have contributed discussions and generalizations. Typical results of the present series of papers are the following:

(1) Let $f(x)$ be absolutely continuous for $0<x<\infty$ and for some $\alpha>0$, and let

$$\int_0^\infty \frac{f'(x)}{x^\alpha} dx = \lim \int_\epsilon^A \frac{f'(x)}{x^\alpha} dx, \quad (\epsilon \downarrow 0, A \to \infty),$$

converge. Then $f(+0) = \lim_{x \downarrow 0} f(x)$ exists and

$$\int_0^\infty \frac{f'(x)}{x^\alpha} dx = \alpha \int_0^\infty \frac{f(x) - f(+0)}{x^{\alpha+1}} dx .$$

(2) Let $\int_p^x f(x)dx$ exist for $x \geq p > 0$, and let the integral

$$\int_p^\infty \frac{f(ax) - f(bx)}{x^{\alpha+1}} dx$$

converge, where $\alpha, a, b>0$. Then the integral

$$\int_p^\infty \frac{f(x)}{x^{\alpha+1}} dx$$

converges.

(3) If $\int_0^A \frac{f(t)}{t} dt$ exists for any $A>0$, and if

$$M(f) = \lim_{x \to \infty} \frac{1}{x} \int_0^x f(t)dt$$

exists, then for all $a,b>0$,

$$\int_0^\infty \frac{f(at) - f(bt)}{t} dt = M(f) \log \frac{a}{b} .$$

(4) If in addition to the assumptions in (3), it is assumed that

$$m(f) = \lim_{\epsilon \downarrow 0} \epsilon \int_\epsilon^1 \frac{f(t)}{t^2} dt$$

exists, then

$$\int_0^\infty \frac{f(at) - f(bt)}{t} dt = [M(f) - m(f)] \log \frac{a}{b}$$

(5) The result in (4) is generalized by replacing at and bt by more general functions of t.

The last paper of those listed below was written as a by-product of the first three papers. The result is as follows: Let $f(t,x)$ be a continuous function of the point (t,x) for $a<x<b$ and $t\geq T$, and suppose that $\lim_{t\to\infty} f(t,x) = f(x)$, where $f(x)$ is also continuous in (a,b); then for any $\epsilon > 0$ there exists a subinterval J of (a,b) and a number T_0 such that for x in J and $t\geq T_0$, $|f(t,x) - f(x)|<\epsilon$. (This theorem is the continuous analog of Osgood's famous theorem on uniform convergences of sequences of functions tending toward a continuous limit.)

Publications: (1) "Note on an infinite integral," by A. M. Ostrowski; submitted to the Duke Mathematical Journal. (2) "On some generalizations of the Cauchy-Frullani integral," by A. M. Ostrowski; submitted to the Proceedings of the National Academy of Science. (3) "On Cauchy-Frullani integrals," by A. M. Ostrowski; IN MANUSCRIPT, in possession of the author. (4) "Generalization of a theorem of Osgood to the case of continuous approximation," by A. M. Ostrowski; accepted for publication by the Bulletin of the American Mathematical Society.

DETERMINATION OF CHARACTERISTIC VALUES OF MATRICES
Project 11.1/1-49-AE1

Origin: NBS Manager: C. Lanczos. Authorized 2/15/49

Objective: To develop a practical and economical method of evaluating the characteristic values of arbitrary complex matrices.

Background: The calculation of characteristic values of matrices is of importance in many problems in physics, engineering, and applied mathematics. In particular, characteristic values play an important role in flutter and other vibration analysis problems, and it was the inadequacy of methods currently in use for such problems which furnished the motivation for the present project.

Comments: The principle underlying the present investigation consists in iterating a trial vector to set up a system of linear equations having a recurrent matrix, and then designing an algorithm for the solution of the recurrent system. The solution of the circular system yields the coefficients of the characteristic equation. This method is analogous to the method on which Project 11.1/1-49-ODE4 is based.

Status: Underway (Continuation). Manuscript in preparation.

Publication: It is expected that a detailed account will be submitted for publication in the NBS Journal of Research, and that a more condensed paper containing the essence of the theory will be submitted to one of the mathematical journals.

APPROXIMATE SOLUTION OF SETS OF ARBITRARY SIMULTANEOUS ALGEBRAIC EQUATIONS
Project 11.1/1-49-AE2

Origin: NBS Manager: C. Lanczos. Authorized 2/15/49

Objective: To develop a practical and economical method for solving simultaneous algebraic equations by matrix iteration.

Background: Such sets of equations arise repeatedly in all branches of applied mathematics.

Comments: Related to Project 11.1/1-49-AE1 in that the same technique of iterating a trial vector to obtain a set of linear equations having a recurrent matrix is used. An approximation is introduced by truncating the iteration of the trial vector and thus obtaining a recurrent system of lower order than the one employed in Project 11.1/1-49-AE1.

Status: Underway (Continuation). The following method for solving equations was investigated:

Consider the set of linear equations

$$Ay = b,$$

where A is a symmetrized matrix. Let us denote

$$b' = Ab,$$

and $$b_1 = b - \alpha b'.$$

Determine α by the condition that b_1^2 shall be made a minimum; this gives

$$\alpha = \frac{(b \cdot b')}{(b')^2}$$

Then $$y = \alpha b + y,$$

where $$Ay_1 = b_1 \quad . \qquad (*)$$

Now repeat the process for b_1, thus reducing (*) to

$$Ay_2 = b_2 \quad ,$$

where $b_2 = b_1 - \alpha_1 b_1'$. The process can be continued until b_n becomes negligibly small. In spite of the large number of steps occasionally demanded for good convergence, the process has the advantage of great simplicity. Moreover, the rounding errors do not accumulate, since essential digits are not lost as the process is continued. At present the investigation is directed to the possibility of increasing the convergence of the successive reductions, by replacing A by $\bar{A} = \gamma \cdot 1 + A$ with suitably chosen γ .

SOLUTION OF ALGEBRAIC EQUATIONS
Project 11.1/1-49-AE3

Origin: NBS Manager: A. M. Ostrowski. Authorized 4/1/49

Objective: To find new and better ways of dealing with algebraic equations in a single unknown.

Background: Such equations arise repeatedly in all branches of applied mathematics.

Status: Underway (New). A study was made of Vincent's method for separating the real roots of an algebraic equation. The basic theorem in Vincent's method is the following result:

If the equation $f(x) = A_0x^n + \ldots + A_n = 0$, $A \neq 0$, with real coefficients and without multiple roots, is transformed by successive transformations,

$$x = x_1 = a_1 + \frac{1}{x_2}, \quad x_2 = a_2 + \frac{1}{x_3}, \quad \ldots, \quad x_{m-1} = a_{m-1} + \frac{1}{x_m},$$

where $a_1, a_2, \ldots, a_{m-1}$ are not less than unity, into a new equation $f_m(x_m) = 0$, where $f_m(x_m)$ is a polynomial in x_m, then there is a value of m, say $m = m_0$, for which the polynomial $f_m(x_m)$ has at most one variation in signs.

Obviously it is of practical interest here to find how large m_0 must be. The project manager's main result consists of an improvement of an estimate for m_0 given by the late Professor Uspensky of Stanford University. The new estimate is this: If Δ is the smallest distance between two roots of $f(x)$, and if N_μ is the μ-th term of the Fibonacci series 1, 1, 2, 3, 5, 8,..., (which is determined by $N_\mu = N_{\mu-1} + N_{\mu-2}$), then m_0 can be taken as the first integer μ for which $N_\mu N_{\mu-1} > \sqrt{3}/\Delta$. The proof of the result is given in terms of several lemmas which are of interest in themselves. As of the end of the quarter this work had partially been written up, but had temporarily been suspended pending the availability at the INA of a copy of an old and rare algebra text-book, which was needed to insure historical authenticity in the exposition.

A paper mentioned below in Publication was prepared by the project manager, which deals theoretically with the possibility of improving or essentially modifying Horner's method of calculating the value of a polynomial $f_n(x)$ of degree n. More precisely, it is possible to calculate $f_n(x)$ with less than n additions or with less than n multiplications, and does any other rule for the calculation of $f_n(x)$ employing only n multiplications really reduce

to Horner's rule? It is shown in the paper that it is impossible to give a general rule for calculating $f_n(x)$ of a type which uses only addition and multiplication and which uses less than n additions.

The situation in the case of multiplications lies deeper. To formulate it properly, the project manager generalizes the problem by allowing an indefinite number of additions and multiplications by numerical constants. The problem then becomes one of considering a chain of moduli M_ν defined in such a way that each modulus $M_{\nu+1}$ is obtained from the preceding one M_ν by adjoining a product of two elements of M_ν. The questions stated above are then answered by the following two theorems about the index and structure of the first modulus M_ν containing $f_n(x)$, which are, however, proved only for small values of n:

(1) For no integer $n>1$ does there exist a chain of moduli M_ν such that $f_n(x)$ is contained in a modulus M_k with $k<n$. (Proved for $n\leq 4$. This theorem expresses the conjecture that the least number of multiplications necessary to calculate $f_n(x)$ by a rule involving only additions and multiplications by numerical constants is allowed.)

(2) If for a certain chain of moduli M_ν the expression $f_n(x)$, $n>1$, is contained in M_n, then for a suitable choice of A_ν,

$$M_{\nu+1} = M_\nu(A_{\nu+1}), \quad A_{\nu+1} = (\alpha_\nu x+\beta_\nu)A_\nu+m_\nu, \quad \nu = 0, 1, \ldots, n-1,$$

where each m_ν belongs to the corresponding modulus M_ν, and where $M_\nu(A_{\nu+1})$ is the new modulus formed by adjoining $A_{\nu+1}$ to M_ν. (Proved only for $n\leq 3$. If we allow an indefinite number of additions and multiplications by numerical constants, then Horner's rule is clearly not essentially the only way to calculate the value of $f_n(x)$ with exactly n multiplications, since a linear transformation of x should be carried out at each step. The theorem indicates the scope of essential modifications which are possible in

Horner's method when the question of the uniqueness of Horner's method is considered within the framework of the theory of moduli.)

In the same paper the extension of Horner's rule to polynomials in several variables is discussed. It is found that N-1 multiplications and N-1 additions must be carried out, where N is the number of terms in the corresponding "complete" polynomial.

Finally, work was also under way at the end of the quarter on an improvement of Newton's method for calculating the roots of a polynomial. A result was obtained whereby the amount of work necessary to achieve a given degree of precision is reduced by about 25%.

Publication: "On two problems in abstract algebra connected with Horner's rule," by A. Ostrowski; to be submitted to the American Mathematical Monthly.

SOLUTION OF THE TELEGRAPHER'S EQUATION FOR INITIAL CONDITIONS GIVEN ON ONLY ONE CHARACTERISTIC
Project 11.1/1-49-AM2

Origin: NBS — Authorized 11/1/48
Managers: G. E. Forsythe, Gertrude Blanch — Revised 3/31/49

Objective: (1) To solve the telegrapher's equation

$$\frac{\partial^2 v}{\partial x \partial t} + \frac{1}{4} v = 0$$

for t>o with the sole boundary condition that v is known for all x when t = 0. Here x is essentially terrestrial longitude and t is time. (2) To investigate the solution of a related difference equation. (3) To present the solution in such a form that meteorologists can see its consequences and try it out in forecasting.

Background: This boundary-value problem arises in Rossby's linearized treatment of the equation of non-divergent flow. The harmonic solutions have been discussed many times

and it is considered important to see what happens with arbitrary initial conditions. In general, the problem has no unique solution for $-\infty < x < \infty$. The fact that initial conditions are given on only one characteristic makes it a most unusual problem for a hyperbolic partial differential equation. In the present treatment the solution is rendered unique by using the roundness of the earth, i.e., by making the solution periodic in x.

Status: Inactive.

Publication: Paper entitled, "Solution of the telegrapher's equation with boundary conditions on only one characteristic," by G. E. Forsythe; accepted for publication by the NBS Journal of Research.

THEORETICAL AND PRACTICAL PROBLEMS IN THE APPLICATION OF A.D.C.M. TO THE DETERMINATION OF ORBITS
Project 11.1/1-49-AM3

Origin: NBS Authorized 12/1/48
Manager: S. Herrick

Objective: To find optimum procedures for determining two-body orbits, utilizing a.d.c.m. It is planned to re-examine existing methods and possible new ones in the light of new facilities that will almost certainly alter their present relative value, and to set up calculating machinery specifications (in terms of arithmetic speeds and memory size) to solve this problem, as well as actually program and code the problem for certain anticipated machines.

Background: The first spectacular success in the determination of orbits was the recovery of the lost minor planet Ceres by Gauss nearly a century and a half ago, though Olbers' cometary method and essays by Laplace and Lagrange antedate this time. The methods of Olbers, Gauss, and Laplace have been greatly improved in recent years; those of Lagrange and Willard Gibbs also will have to be taken into account in the study, and may prove to be even more applicable than the established ones.

Comments: The study is applicable to preliminary orbits and minor planets, and in part to the rocket problem.

Status: Underway (Continuation). An introduction was written to the set of tables prepared under project 11.1/2-48-3 entitled "Tables for rectilinear and nearly-rectilinear ('nearly parabolic') orbits," to be published in the NBS Applied Mathematics Series. In the course of writing the introduction, consideration was given to the possibility of applying these tables to parabolic as well as elliptic and hyperbolic orbits, with the result that the introduction constitutes an original paper introducing a new way for handling parabolas and "nearly parabolic" ellipses and hyperbolas, as well as rectilinear orbits. The introduction touches upon the fact that the tables should be investigated for other uses by mathematicians.

The previous status report for this project described progress on a manuscript entitled, "A modification and appraisal of Gibbs' orbit method," and mentioned that a trip to the Yale Library to consult certain recently discovered manuscripts of Willard Gibbs was being planned. The trip was made during the quarter covered by this report, and the papers were consulted. Evidence was found that Gibbs had considered the possibility of publishing tables such as the "Tables for rectilinear and nearly-rectilinear ('nearly parabolic') orbits," mentioned above, but nothing was found which anticipates the project manager's own paper on Gibbs. However, several promising leads for addenda to that paper were discovered.

Further attention was given to the problem of variation of parameters and perturbation theory, and correspondence with Brouwer, Clemence, and Herget laid the basis for another paper on the subject. This work will be included in a book on rocket navigation which the project manager is now writing.

Attention was also given to methods of orbit determination based upon the work of Lagrange. It was found that they tend toward those of Gibbs in certain phases, especially the differential correction. This work will also be included in the book on rocket navigation.

Publications: (1) Introduction to "Tables for rectilinear and nearly rectilinear ('nearly parabolic') orbits," to be published in the NBS Applied Mathematics Series. Also project 11.1/2-48-3. (2) "A modification and appraisal of Gibbs' orbit method" by S. Herrick; IN MANUSCRIPT, in possession of the author.

THEORETICAL AMPLITUDE DISTORTION IN A CONDENSER MICROPHONE CIRCUIT
Project 11.1/1-49-AM4

Origin: Department of Physics, UCLA Authorized 11/1/48
Manager: G. E. Forsythe

Objective: (1) To study mathematically the relation between the variable (input) capacitance of the condenser element and the variable (output) voltage across the resistance of an RC condenser microphone circuit. In particular, to find conditions under which the voltage faithfully reproduces the capacitance, as the resistance becomes infinite.
(2) To calculate the amplitudes of the various harmonics in the output voltage, corresponding to a pure sinusoidal input and a finite resistance.

Background: A standard reference on the condenser microphone, E. C. Wente, Physical Review 10, 39-63 (1917) makes some approximations which raised questions about the practical effect of dead capacitance in parallel with a condenser microphone. P. M. Morse, "Vibration and sound," gives a proof which is valid only for infinite resistance. It is believed that no one has presented an exact formula for the amplitudes of the various harmonics for finite R. All these matters are dealt with in the present project. Suggested by Professor R. W. Leonard of the Department of Physics, UCLA.

Comments: Professor Leonard plans an experiment to determine the amplitudes mentioned under (2) above. After his circuit constants are known, a calculation may be carried out to confirm his results. It would involve rapidly convergent series of Bessel functions.

Status: Inactive, pending an experiment to be performed by Professor Leonard.

NUMERICAL METHODS IN CONFORMAL MAPPING
Project 11.1/1-49-CM1

Origin: NBS Authorized 12/1/48
Managers: W. Seidel, E. F. Beckenbach, A. M. Ostrowski

Objective: To determine optimum methods of numerically constructing the univalent analytic function $w=f(z)$ which maps a given region in the complex z-plane on a given region (usually much simpler than the first: e.g., the unit circle) in the complex w-plane. The project will include the cases in which the boundary of the region in the z-plane is given analytically, graphically, and numerically.

Background: The method of conformal mapping is a powerful tool for solving boundary value problems for plane regions, and some use is made of it in nearly every branch of classical applied mathematics, as can readily be verified by referring to the standard texts. It is particularly applicable to potential problems, because Laplace's equation remains invariant under a conformal transformation and no new singularities are introduced. The method is particularly useful in fluid dynamics, and extensive applications have recently been made in the study of aerofoils.

Comments: A closely related problem is that of solving Laplace's equation with continuous boundary values given on a boundary of a region of the z-plane. A number of papers have appeared in the literature giving numerical procedures for this related problem as well as for the problem of conformal mapping. These have been largely aimed at hand methods of computing, whereas the present project is aimed at computation by automatic digital computing methods.

Status: Underway (Continuation). The major activity during the quarter consisted of organizing and conducting the Symposium on Conformal Mapping, held at the INA on June 22-25, 1949 (See section on Lectures and Symposia.) The groundwork for this Symposium was performed chiefly by Dr. Beckenbach, acting as chairman of the Committee on Arrangements.

Dr. Seidel's translation of the collection of Russian papers on numerical aspects of conformal mapping (see previous Status report in Projects and Publications Jan-Mar 1949) was polished and placed in the hands of a Reader, preparatory to submitting to the NBS Editorial Committee.

The bibliography on conformal mapping being prepared by Dr. Seidel was completed.

Dr. Ostrowski carried out a study of the convergence features of the method for conformal mapping developed by Theodorsen and Garrick. This method depends on the solution of a certain integral equation. Theodorsen and Garrick suggested a solution of the integral equation by successive approximations, the convergence of which was proved by Warschawski under certain conditions which included an assumption as to the existence of the required map. Dr. Ostrowski established the convergence under much less restrictive conditions; in particular, his proof does not include the assumption that the map exists. A valuable new feature of Dr. Ostrowski's approach is that the convergence is proved for an arbitrary initial approximation, which provides a sound basis for the use of a priori information to start with a good guess as to the final answer. The previous proof required a specific initial approximation.

Dr. Ostrowski also established the corresponding convergence and existence theorems for the more general integral equation:

$$\theta(\phi) - \gamma(\phi) = -\frac{1}{2\pi}\int_0^{\pi} P[\theta(\phi-t)] - P[\theta(\phi-t)]\,\mathrm{ctg}(t/2)\,dt.$$

Publication: The papers given at the Symposium will be submitted for publication by the NBS under the title, Proceedings of a Symposium on the Construction and Application of Conformal Maps, under the editorship of E. F. Beckenbach. The bibliography of Dr. Seidel and the paper of Dr. Ostrowski mentioned in the above status report will be published in this volume. It is expected that the translation by Dr. Seidel will be published in the NBS Applied Mathematics Series.

MONOGRAPH ON NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE1

Origin: NBS Authorized 11/26/48
Manager: W. E. Milne

Objective: To prepare a systematic exposition of the theory and techniques of integrating differential equations, with special emphasis on ordinary differential equations. The treatise will include chapters on the analysis of remainder

terms, the accumulation of errors, relaxation methods, and other topics pertinent to a.d.c.m.

Background: The literature of numerical methods in differential equations is at present scattered, incomplete, and often aimed at very special cases. A convenient, up-to-date treatise is very much needed. (See also project 11.1/1-49-ODE2.)

Comments: Project 11.1/1-49-ODE2 is intended to furnish some of the groundwork for this project.

Status: Underway (Continuation). About 90% of the material has been assembled; about 20% of the final manuscript completed.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE2

Origin: NBS Authorized 11/26/48
Manager: W. E. Milne

Objective: In general, to develop and improve the techniques for the numerical integration of simple systems of ordinary differential equations. Specifically: (a) To investigate various devices to improve the accuracy of the step-by-step type of process; e.g., the use of quadrature formulas based on higher derivatives of the unknown function in place of quadrature formulas using higher order differences or more lines in the table of the computation. (b) To determine optimum processes for presently proposed automatic digital computing machinery. (c) To obtain useful results concerning round-off and truncation error. (d) To develop improved methods for locating characteristic values.

Background: Ordinary differential equations are the mathematical model for problems in ballistics, celestial mechanics, control, circuit analysis, and many other important applied physical problems. A large number of formulas and procedures for numerical integration of such equations, all belonging to three of four main classes, have been developed in the past, and an indefinite number of alternate formulas can be readily derived by similar methods. However, many

problems involving choice and derivation of formulas and appraisal of error, which are essential to the application of a.d.c.m. to ordinary differential equations, still remain open.

Comments: This project is intended to furnish some of the groundwork for project 11.1/1-49-ODE1.

Status: Underway (Continuation). The main results of the work on this project during the quarter are set forth in the three papers listed below in "Publications."

In the first paper, a method for the numerical integration of ordinary differential equations is presented which appears to possess rather outstanding advantages when applied to certain types of equations. The equations to which the method most readily applies are those for which it is possible to obtain in comparatively simple form expressions for two additional derivatives. That is, for an equation of n-th order.

$$y^{(n)} = f(x, y, y^1, \ldots, y^{n-1}),$$

we obtain by differentiation expressions for $y^{(n+1)}$ and $y^{(n+2)}$. If these expressions are not so involved as to make the labor of substitution prohibitive, then the method here proposed is applicable.

The advantages claimed for the process are: (1) The start of the integration is accomplished by the same formulas that are used in the regular routine of the process so that no special formulas or procedures are required in order to get the computation under way. (2) Each step of the integration makes use of two lines only of the computation, whereas a method employing differences and having a comparable degree of accuracy would require five lines in the computation. (3) A change in the length of step for step-by-step integration is often necessary as the integration proceeds. Such a change can be more readily made in this process than where five-line formulas of integration are employed. (4) The coefficients occurring in the formulas are simpler than those in comparable five-line quadrature formulas so that the machine calculation is not at all complicated.

The most obvious disadvantage of the process is that it requires the calculation of two additional derivatives at each step, and the labor of substitution in certain instances may be excessive. In such cases this method is not recommended. On the other hand, for equations of simple analytical form, and particularly for linear differential equations, it should prove valuable.

In the second paper, examples are exhibited in which the Runge-Kutta method is less satisfactory than a

method based on mechanical quadrature. This paper was being revised as of the end of the quarter to bring into consideration the appraisal of the error term in the Runge-Kutta method given in Bieberbach's "Theorie der Differentialgleichungen." (Note. The error term is given in Bieberbach's book without proof. At the request of Dr. Ostrowski, Dr. Bieberbach has submitted to the INA a proof which establishes an even sharper appraisal than that which appears in the book.)

In the third paper, the Lanczos method of obtaining eigenvalues of matrices and differential operators (see projects 11.1/1-49-ODE4 and 11.1/1-49-AE1) is presented in slightly modified form so as to provide a simple numerical routine for computation. A number of numerical examples are worked out to illustrate the procedure and to give an indication of the accuracy attained. The exposition is limited to differential equations of the second order, but the method can be extended to higher orders.

Publications: (1) "A note on the numerical integration of differential equations," by W. E. Milne; IN MANUSCRIPT (at INA), to be submitted to the NBS Journal of Research. (2) "Note on the Runge-Kutta method," by W.E.Milne; IN MANUSCRIPT (at Prof. Milne's residence in Corvallis, Oregon); further work on the paper now underway. (3) "Numerical determination of characteristic numbers," by W. E. Milne; IN MANUSCRIPT (at INA); to be submitted to the NBS Journal of Research, where it will follow a paper by Dr. Lanczos presenting the basic theory.

APPLICATION OF NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS TO MISCELLANEOUS PROBLEMS IN NUMERICAL ANALYSIS
Project 11.1/1-49-ODE3

Origin: NBS Authorized 11/26/48
Manager: W. E. Milne

Objective: To investigate the practicability of replacing certain problems in numerical analysis, such as the quadrature of difficult integrals and the solution of algebraic or transcendental equations, by equivalent problems involving the numerical solution of ordinary differential equations.

Background: In certain cases, irrationalities and

singularities present in the original formulation of a problem in numerical analysis can be avoided by setting up an equivalent differential equation, and the problem can then be solved quickly and efficiently by numerical integration of the differential equation. An example is the elliptic integral

$$y = \int [P(x)]^{-1/2} dx$$

where P(x) is a polynomial; the corresponding differential equation is

$$\left(\frac{dx}{dy}\right)^2 = P(x)$$

in which irrationalities are absent. One possible advantage of reducing various different problems to ordinary differential equations is that the library, external and built-in,of sub-routines for a given automatic computing machine might thereby be reduced.

Status: Terminated. The Oregon State College Ph.D. thesis of Mr. James Price, mentioned in the status report for January through March 1949 for this project, was completed under the direction of Professor A. T. Lonseth. The problem worked on by Mr. Price was one of replacing a system of irrational or transcendental ordinary differential equations by a system (usually of higher order) of rational differential equations.

APPROXIMATE DETERMINATION OF THE CHARACTERISTIC FUNCTIONS AND CHARACTERISTIC VALUES OF ARBITRARY LINEAR DIFFERENTIAL OPERATORS
Project 11.1/1-49-ODE4

Origin: NBS Authorized 2/15/49
Manager: C. Lanczos

Objective: To develop a practical and economical method of obtaining the characteristic values and functions of arbitrary linear differential operators.

Background: In many problems of physics and engineering, the determination of the characteristic solutions of a linear differential equation is of great importance. The

frequently employed Rayleigh-Ritz method is applicable only to self-adjoint systems, it requires integrations, and is basically restricted to the characteristic value which is least in absolute value. The present study is aimed at developing a method which will not have these limitations.

Comments: The principle underlying the present investigation consists in constructing a trial function which together with its iterations up to a certain order satisfies the given boundary conditions, and then determining the characteristic equation by a method entirely analogous to that used in project 11.1/1-49-AE1.

Status: Underway (Continuation). A number of consultations were held with W. E. Milne, which led to the experimental and expository work on his part referred to in the status report for 11.1/1-49-ODE2.

Also, consideration was given to a related problem involving the solution of integral equations of a certain type. Consider the integral equation

$$f(x) - \lambda K f(x) = g(x), \tag{1}$$

where K f(x) indicates the operation

$$\int_a^b K(x,\xi)\, f(\xi)\, d\xi. \tag{2}$$

The solution of this integral equation can be given by the Neumann series:

$$f(x) = g(x) + \lambda g_1(x) + \lambda^2 g_2(x) + \ldots \tag{3}$$

where

$$g_n(x) = K\, g_{n-1}(x) \ . \tag{4}$$

This series converges only if λ is smaller than the first eigenvalue λ of the integral operator. The present investigation obtains the solution of (1) for arbitrary λ by the following expansion:

$$f(x) = \lim_{n\to\infty} [\beta g(x) + \beta_1 g_1(x) + \ldots + \beta_n g_n(x)] \ , \tag{5}$$

where the coefficients β_i depend on n, but can be explicitly given on the basis of knowing the functions $g(x)$, $g_1(x)$, ..., $g_n(x)$. The convergence is rapid if the dispersion of the eigenvalues is of the nature usually encountered in the customary problem of physics and engineering.

STUDIES IN NUMERICAL INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PD1

Origin: NBS Authorized 3/1/49
Manager: W. E. Milne

Objective: In general, to develop and improve the techniques for the direct numerical integration of partial differential equations. Specifically to investigate mathematical questions involved in the substitution of partial difference equations for partial differential equations; as for example, the determination of optimal forms of difference equations for a fixed number of terms, the development of best methods for handling curvilinear boundaries, and the development and appraisal of remainder terns.

Background: Partial differential equations are the mathematical tools for attacking a wide variety of problems in physics and engineering. The literature pertinent to direct numerical integration of partial differential equations is at present incomplete and scattered. Much remains to be done to prepare this field for a.d.c.m.

Comments: This project is intended to furnish some of the groundwork for project 11.1/1-49-ODE1.

Status: Underway (Continuation). A rather extensive study of step-by-step methods of integration for parabolic and hyperbolic partial differential equations was made, as well as a study of successive approximation methods for the solution of elliptic differential equations. The application of the Liebmann method to the solution of the Dirichlet problem for a variety of boundaries and for several types of boundary conditions including mixed boundary conditions was investigated. Some of the results will be incorporated in a paper which is to be presented by the project manager at the dedication of the Harvard Mark III Computer in September 1949.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PM1

Origin: NBS — Authorized 12/1/48, Revised 3/31/49

Managers: R. P. Peterson, Jr. and J. H. Curtiss

Objective: To investigate methods of solving certain types of partial differential equations and integro-differential equations by sampling processes. A typical numerical technique consists in identifying the solution of the equation with the function specifying the distribution of distances traveled in a properly chosen statistical random walk or random flight problem and then computing this distribution function approximately by actually following through the life histories of a large number of random flights. The present investigation is aimed at determining the classes of problems and the types of a.d.c.m. for which such methods are more efficient than systematic quadrature methods, and answering related questions.

Background: The techniques referred to in the Objective are typical of those which are known as Monte Carlo methods and seem to have been suggested first by Ulam and von Neumann in an abstract in the Bulletin of the American Mathematical Society 53, 1120, (1947). They are natural ones to use in the study of diffusion problems, where the physical situation suggests directly the use of a random flight model.

Comments: As of the date of authorization, two computation projects involving the Monte Carlo method had already been worked on by the National Applied Mathematics Laboratories: project 11.2/33-48-17 (see Projects and Publications, Jan-Mar, Apr-June, July-Sept, Oct-Dec 1948), and project 11.1/31-49-10 (Projects and Publications Oct-Dec 1948).

Status: Underway (Continuation). The activities continued to fall in the background-research and administrative categories, although Dr. Forsythe gave some consideration to the problem of unbiased estimation of certain integrals by means of the average ordinate of a sample of N points in k-dimensional space.

The major activity consisted in organizing the Symposium on the Monte Carlo Method, which is described in the section of this report on Lectures and Symposia.

The joint RAND-INA Seminar on Stochastic Processes met regularly every Thursday throughout the quarter; reports were given by Forsythe, Peterson, Curtiss, Blackwell, Hoel, Germond, Bellman, Mood, and others.

GENERATION AND TESTING OF RANDOM DIGITS
Project 11.1/1-49-PM2

Origin: NBS Authorized 4/1/49
Managers: G. E. Forsythe, R. P. Peterson, Jr., and Everett Yowell, Jr.

Objective: (1) To devise machine methods of generating random digits by arithmetical processes. (2) To test whether the methods give random digits suitable for use in various Monte Carlo sampling process.

Background: Random digits (or random-sampling digits) are needed for many purposes in "experimental mathematics," applied statistics, and numerical analysis. (See for example, project 11.1/1-49-PM1.) Tippett and Kendal-Smith in England have published tables of random-sampling digits obtained by laborious hand work with tables and machines of the roulette-wheel type. The RAND Corporation has generated random digits from the thermionic noise. It is inherent in the nature of a.d.c.m., however, that neither precomputed digits now produced by a non-reproducible process are very well suited for calculations. It therefore seems essential to have techniques for internal generation of random digits (1) which will be repeatable, and (2) which will give digits which behave like the products of a truly random process.

Comments: Some of the Atomic Energy Commission laboratories have experimented with the arithmetic generation of sampling digits. However, the reports at the Symposium on the Monte Carlo Method held at the INA on June 29, 30, July 1, 1949, showed a need for more basic investigations.

Status: Underway (New). Several methods were considered for generation of digits on the IBM Electronic Calculating Punch Type 604. It was found for the simpler methods that it was possible to predict the average duration of runs before they terminated in zeros or other cycles of digits. This gives valuable information on the probable duration of the

more complex methods usable in practice.

One method was carried through to the generation of 100,000 individual digits. This consisted in the multiplication of a certain five digits of an 8-digit number a_0 by an 8-digit number a_1 to yield a 13-digit number. Then a certain set of eight digits of the product were retained and called a_2. Similarly a_{n+1} were used to get a_{n+2} ($n=1, 2, \ldots, 12498$). To these 100,000 digits were applied the four tests for randomness given by Kendall and Smith; see Pearson's Tracts for Computers, vol. 26 (1939). It was found that the method was almost certainly not quite random enough. The numbers would be suitable for some purposes, but a more refined method is needed. It is easy to devise more refined methods, but the testing for randomness is time-consuming, in spite of ingenious use of IBM equipment.

Publication: These results were presented in a talk to the Monte Carlo conference, June 30, 1949, at the INA (see section on Lectures and Symposia), and will probably be published in the Proceedings of that conference.

DETERMINANTS OF ORTHOGONAL POLYNOMIALS
Project 11.1/1-49-SF1

Origin: NBS Authorized 11/29/48
Managers: E. F. Beckenbach, W. Seidel, G. E. Forsythe

Objective: To derive identities, inequalities, and convexity properties of minors of recurrent determinants whose elements are orthogonal polynomials, e.g.,

$$\Delta(n, h, k; x) = \begin{vmatrix} P_n(x) & P_{n+h}(x) \\ P_{n+k}(x) & P_{n+h+k}(x) \end{vmatrix},$$

where $P_n(x)$ is the Legendre polynomial of degree n; and to develop applications of these results, especially to the problem of moments.

Background: This is primarily a long-range project aimed at extending general mathematical knowledge. However, orthogonal polynomials are of great importance in applied mathematics, and any further information concerning them has an appreciable probability of being immediately useful.

The present project was suggested by a certain inequality of Turan for $\Delta(n, 1, 1; x)$ published by Szego in the Bulletin of the American Mathematical Society 54, 401-105 (1948).

Comments: During his visit to the Institute for Numerical Analysis in the spring and summer of 1948, Dr. O. Szasz became interested in Turan's result and derived a considerable improvement of it, applicable to the ultraspherical polynomials as well as to the Legendre polynomials. He derived a similar inequality for Bessel functions (project 11.1/1-49-9 "Nonlinear inequalities concerning ultra-spherical polynomials and Bessel functions," described in Projects and Publications report for July-Sept 1948.)

The present project originated in discussions concerning his results held by Dr. Szasz with the staff of the Institute.

Status: Underway (Continuation). In the previous report for this project, mention is made of two papers by Beckenbach, Seidel, and Szasz (mentioned below in "Publications"). The first one has been completed, and work was underway on the second one.

A conference was held with the project managers, Professor O. Szasz, and Professor G. Szego of Stanford University during the Symposium on Conformal Mapping (see section Lectures and Symposia), during which Professor Szego proposed the problem stated below; this was later solved and will be incorporated in a future paper. The problem is to prove that

$$\frac{1}{2} + R \sum_{n=1}^{\infty} \frac{P_n^{(\lambda)}(x)}{P_n^{(\lambda)}(1)} z^n > 0$$

for $-1 \leqslant x \leqslant 1$, $|z| < 1$, $\lambda > 0$, where $P_n^{(\lambda)}(x)$ is the ultraspherical polynomial of degree n and order λ. (The case $\lambda = 1$ was proved by Szego. The case $\lambda = 0$ is the kernel of the Poisson Integral.)

Dr. Forsythe was shown for $n = 1, 2, 3, \ldots$ that $\Delta(n, 1, 2; x)$ and $\Delta(2n+1, 2, 2; x)$ are negative for $0 < x < 1$; Turan had shown the same for $\Delta(n, 1, 1; x)$. It is conjectured that no other $\Delta(n, h, k; x)$ have this property. Extensive numerical calculations of special cases verify this conjecture; also, analytical proofs have been obtained for approximately seven-eighths of the cases.

Publications: (1) "Second-order determinants of

Legendre polynomials," by G. E. Forsythe; to be submitted to the Duke Mathematical Journal. (2) "Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials," by E. F. Beckenbach, W. Seidel, and O. Szasz; submitted to the Duke Mathematical Journal.

2. Mathematical Tables

Note: Sponsorship for the preparation of the mathematical tables in this section has been assumed by the Office of Air Research, AMC, USAF.

MATHIEU FUNCTIONS II
Project 11.1/2-45-1

Origin: Applied Mathematics Panel, NDRC Authorized 7/1/47
Manager: Gertrude Blanch

Objective: To prepare a table of the periodic solutions:

$$Se_r(s,t) = \sum_{n=0}^{\infty} De_{2n+p}\cos(2n+p)t, \quad (p = 0, 1)$$

$$So_r(s,t) = \sum_{n=1}^{\infty} Do_{2n-p}\sin(2n-p)t, \quad (p = 0, 1)$$

for r=0(1)15, t=0(1°)90° over the range s=0 to s=100, of the Mathieu differential equation:

$$y'' + (b - s\cos^2 t)y = 0.$$

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press 1947). The existing tables by Ince and Goldstein are inadequate. The project was originally proposed by Dr. Philip Morse, and Dr. James Wakelin.

Comments: Related to project 11.2/2-46-2.

Status: Inactive. Computations about 45% completed.

DECOMPOSITIONS OF ARC TANGENTS
Project 11.1/2-48-1

Origin: NBS Authorized 5/25/48
Managers: Gertrude Blanch, J. Todd

Objective: To find the integral values of u_i and f_i satisfying

$$\arctan (a/b) = \sum_{1}^{k} f_i \arctan u_i$$

where $a/b<1$, and a and b are relatively prime integers for all positive $b<100$.

Background: This computation arose from project 11.1/1-48-4 (see Projects and Publications Oct-Dec 1948) and is being performed as part of the training program of the computation unit in section 11.1.

Comments: Mr. Todd computed a table of decompositions of arctan a/b, for b=100, and a prime to b. The laboratory is checking this table against a 15-place table of arc tangents which was computed by the New York unit of the NBS Computation Laboratory.

Status: Underway (Continuation). The table was being processed for photo offset reproduction.

Publication: The table will appear in the NBS Applied Mathematics Series. The table will include the decompositions for integral arguments $n \leq 2,100$. "A Problem of J. C. P. Miller on Arc Tangent Relations," by J. Todd has been accepted by the American Mathematical Monthly.

TABLES OF $E_1(z)$, SECOND QUADRANT
Project 11.1/2-49-1

Origin: NBS Authorized 7/14/48
Managers: Gertrude Blanch, Roselyn A. Siegel

Objective: To prepare tables of the function $E_1(z) = \int_z^\infty [e^{-u}/u]du$, where the path of integration is along a

line parallel to the real axis, and of related functions for z=x+iy:

Table I: $E_1(z) + \ln z$; [x = -1(.01)0, y = 0(.02)1; 6D]
Table II: $E_1(z)$; [x = -4(.02)0, y = 0(.02)3(.05)10; 6D]
Table III: $e^x E_1(z)$; [x = -4(.02)0, y = 0(.02)3(.05)10; 6D]

Background: A similar table has been prepared by section 11.2 for the first quadrant. It is thought that values for the second quadrant will make a useful companion table (see project 11.2/2-43-3).

Comments: This project is intended to be used as a fill-in problem when machines would otherwise be idle. Study may show more useful intervals and ranges.

Status: Underway (Continuation). Define $U_n(z) = U_n(x+iy) = z^n/n!$, $T_n(z) = U_n(z)/n$; then $E_1(z)$, Ei(z), Ci(z), and Si(z) involve certain combinations of $T_n(z)$ and, except for Si(z), a logarithmic term. Also sin z, cos z, sinh z, cosh z, e^z, and e^{-z} involve combinations of $U_n(z)$. Thus, ten important functions can be computed from a set of values of $U_n(z)$ and $T_n(z)$. It is enough to determine $U_n(z)$ and $T_n(z)$ in the first octant of the complex plane. The values of these functions in the remaining portion of the plane can be obtained from the values in the first octant. Values of $U_n(z)$ are being generated by IBM equipment during spare machine time, for x, y=.1(.1)3.1. These will be used as "KEY VALUES." It is now planned to secure values of $U_n(z)$ and $T_n(z)$ to 10D, and of the functions mentioned to 8D. Values of $U_n(z)$ through n=7 are already available.

PUNCHED CARD LIBRARY
Project 11.1/2-49-2

Origin: NBS Authorized 7/14/48
Manager: Roselyn A. Siegel

Objective: To compute from time to time, and to accumulate upon punched cards, useful and needed tables.

Background: Since IBM 602-Multipliers execute 10-digit multiplication, some 8-decimal-place punched card tables need to be calculated to 10 places. Also, extensive tables of often used functions are an essential part of a computing laboratory. It is contemplated that otherwise idle machine time will be used to compute such tables.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: The following tables were punched in the computation unit of the INA, but checking had not been completed: (1) Characteristic values b of Mathieu's differential equation $y'' + (b - s\cos^2 x)y = 0$, $s \leqslant 100$ (first 16 even and 15 odd orders). (2) Joining factors associated with solutions of Mathieu's equation, $s \leqslant 100$. (3) Bessel functions $Y_0(x)$, $Y_1(x)$, $x = 10(.01)25$ (based on B.A.A.S. tables).

In addition, the laboratory completed differencing the table of Arc sin x, punched by Consolidated Vultee Aircraft Corporation, for the range: $x = [0(.0001).989(.00001)1;$ 12D]. C.V.A. has also undertaken to punch the NBS Tables of Natural Logarithms, vol. III and IV (MT10 and 12), during spare time.

3. Development of Automatic Computing Machinery

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.1/22-49-1

Origin: Air Materiel Command, USAF Authorized 11/1/48
Manager: H. D. Huskey Anticipated termination January 1950

Objective: To develop and construct an experimental automatic-sequenced electronic digital computing machine suitable for general mathematical computation, using, if possible, an electrostatic high-speed memory device.

Background: The project was authorized by the Applied Mathematics Executive Council at its meeting on October 19, 1948.

with a view to (1) diversifying the types of machine development underway at the NBS, (2) making the most effective use of the abilities of the project manager, (3) providing a practical machine more quickly and inexpensively than then seemed possible under outside contract, and (4) promoting the early development of electrostatic memory tubes.

Comments: Close coordination between this project and the 11.4 series of machine development projects is being effected by various means.

Status: Underway (Continuation). Bids were obtained for the fabrication of 80 chassis for the arithmetic unit. It was anticipated that this contract would be awarded in August. The construction of a test unit design to test the arithmetic chassis was initiated.

The Williams' memory tube circuitry was nearly in final form. Two final DC deflection amplifiers are to be built, and the circuitry is to be housed in a relay cabinet to minimize radiation pick-up. A prototype cathode ray tube chassis was being designed in the same form in which it is planned to use it in the final computer. The present Williams' tube setup has operated as a memory, but not with the adequate safety factors necessary for its use in the computer.

Ten chassis of the control unit of the computer were in production, and the logical design had been frozen. The balance of the control chassis were about to be laid out.

Considerable coding has been done using the present logical design and the system is believed to be adequate.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Project 11.1/22-49-2

Origin: NBS Authorized 2/15/49
Managers: S. Herrick, H. D. Huskey

Objective: To establish a suitable and consistent terminology, notation, and symbolism for the automatic computing machine being constructed under project 11.1/22-49-1, with a view to developing thereby an acceptable proposal for a standard language for describing logical aspects of a.d.c.m. in general.

Background: Because of the diversity of groups working on computing machines a number of different sets of conventions regarding terminology and symbolism are currently in

use. Criticism has developed among the mathematicians of the Institute for Numerical Analysis regarding the consistency, compactness, and suitability of some of the systems of notation and symbolism now in use. It has seemed to be definitely worthwhile from the viewpoint of the research program of the Institute to develop a notation and symbolism for the Institute machine which will be acceptable to the research staff, and it is hoped that if this task is accomplished thoughtfully, a standard of wide applicability may result. Work on the project is being performed by a discussion group of which Dr. Herrick and Dr. Huskey are co-chairmen.

Comments: This project will not be primarily concerned with the aspects of a.d.c.m. development which lie purely in the field of electronics. The project is carried on in cooperation with the Machine Development Laboratory (NBS section 11.4).

Status: Underway (Continuation). The group working on this project have had several meetings at which the terminology and notation in use at present were discussed. Professor D. R. Hartree of Cambridge University, England, and Mrs. Ida Rhodes of the NBS Machine Development Laboratory (11.4) were present at one of the meetings. A number of worthwhile suggestions resulted and will be incorporated in the practice of the coding group at the Institute.

4. Computing Services

SEPARATION OF EXPONENTIALS
Project 11.1/31-48-2

Origin: NBS Authorized 5/25/48
Manager: Roselyn A. Siegel

Objective: To test a method of determining the unknown quantities, n, a_i, and τ_i from a known function F(t) and the relationship

$$F(t) = \sum_{1}^{n} a_i e^{-t/\tau_i}$$

Background: An important problem in physics involves the task of identifying the decay constants of the components present in a mixture of radio-active substances. The measured total activity is represented by the above equation. Mr. Albert Cahn of section 11.1, NBS, has worked out a method for finding the unknown quantities by using two theorems from the problem of moments. It was desired to compute a numerical example. The project is being performed as part of the training program of the computation unit in section 11.1.

Status: Underway (Continuation). Further work on several test problems indicated that methods so far considered are unstable. However, there seems to be some promise in varying the coefficients of the test equations. This approach will be studied.

TEST OF AN ADAPTATION OF THE MONTE CARLO METHOD
Project 11.1/31-49-10

Origin: RAND Corporation Authorized 11/16/48
Managers: L. L. Bailin, Roselyn A. Siegel

Objective: By use of an adaptation of the so-called "Monte Carlo" method, to obtain a statistical solution of the neutron transport equation

$$(1) \qquad \gamma \frac{\partial \psi}{\partial x} + \psi = \frac{\gamma}{2} \int_{-1}^{+1} \psi(x, \gamma') d\gamma',$$

and compare the solution with one obtained by analytic methods.

Background: One considerable disadvantage in using the Monte Carlo method to solve an equation such as (1), where the independent variable is greatly attenuated for certain physically interesting situations, is that often an impossibly large number of "particles" must be calculated in order to obtain a reasonably accurate statistical approximation to the answer. This project is to test a conjecture that this difficulty can be removed by substituting $\psi = \varphi e^{-\alpha x}$ and solving the resulting equation for φ. This corresponds to a physical situation where particles may multiply as well as be absorbed.

Status: Underway (Continuation). 98% completed.

Publication: Results are transmitted to the RAND

Corporation as they are obtained.

RADIATION CHARACTERISTICS OF A TURNSTILE ANTENNA SHIELDED BY A SECTION OF A METALLIC TUBE CLOSED AT ONE END
Project 11.1/31-49-12

Origin: North American Aviation Corporation Authorized 3/4/49
Manager: L. L. Bailin

Objective: To determine by theoretical methods the radiation characteristics of the antenna, i.e., the far field radiation pattern and polarization, the behavior of the propagating modes in the cylindrical shielding can, the impedence seen by the antenna, the field and power gain functions, subject to the following assumptions:

(i) The antenna consists of perpendicularly crossed wires, of equal length, excited by a sinusoidal current distribution which differs 90° in time phase between members of the cross.

(ii) The shield consists of a cylindrical can open at one end. The plane of the antenna is perpendicular to the axis of the cylinder; the center of the crossed wires is on the axis.

(iii) The shield is assumed to be a perfect conductor; the effect of the antenna supports is neglected.

Background: This problem was proposed by the Aerophysics Laboratory of the North American Aviation Corporation as a step in obtaining a satisfactory basis for efficient design of antennas of this type.

Comments: Professors A. Baños, Jr., and D. S. Saxon of the University of California at Los Angeles are acting as consultants on this problem.

Status: Underway (Continuation). 90% completed.

OPTIMUM STAR CONFIGURATIONS
Project 11.1/31-49-14

Origin: Hughes Aircraft Company, Inc. Authorized 5/25/49
Manager: E. Yowell

Objective: To find the parameters of certain types of star shaped figures which maximize the area between the star and an enclosing circle.

Background: This problem arises in the study of methods of packing solid rocket fuel in cylindrical rockets.

Status: Underway (New). 85% completed.

MINOR COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Project 11.1/32-49-1

Origin: NBS Authorized 9/2/48
Manager: Gertrude Blanch

Objective: To assist the research staff of the Institute for Numerical Analysis by performing small computational tasks.

Background: From time to time it is worthwhile to make small computations for the research staff. These are not properly listed as separate projects. The totality of this work is not trivial, however, and is so described as one project.

Status: Underway (Continuation). The following were among the larger tasks undertaken:

(1) Solution of a problem in forced vibrations for several parameters, suggested by D. R. Hartree:
Given $\ddot{x} + \frac{9}{4}\dot{x} + x(1 + \frac{x}{20}) = 5 \cos t$, with various initial conditions x_0 and $\dot{x}_0$; a fixed point was found (period 2π):
$x_0 = -7.83$, $\dot{x}_0 = 7.62$.

Given $\ddot{x} + \frac{8}{9}\dot{x} + x(1 + \frac{x}{20}) = 14 \cos t$; a subharmonic was found: $x_0 = -6.015$, $\dot{x}_0 = 12.12$.

(2) Studies in the generation of random digits: This was reported on at the Symposium on the Monte Carlo Method (see section on Lectures and Symposia) by G. E. Forsythe.

(3) Studies on relaxation methods in connection with 11.1/1-49-PD1. Part of this work was done with the aid of the 604 IBM multiplier. (For W. E. Milne.)

REDUCTION OF THEODOLITE DATA
Project 11.1/32-49-2

Origin: Naval Air Missile Test Center (Point Mugu)
Manager: E. Yowell Authorized 11/16/48

Objective: To determine from the theodolite descriptions of a missile flight the actual positions and velocities of that flight as a function of time.

Background: The Naval Air Missile Test Center periodically photographs missile flights through angle recording theodolites. It is necessary to calculate from the angles a description of such flights in terms of the position and velocity of the missile as a function of time. A method of performing this computation, using punched card machinery, has been worked out by Mr. Yowell.

Status: Underway (Continuation).

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Project 11.1/32-49-4

Origin: NBS Authorized 3/8/49
Manager: E. Yowell Anticipated termination 9/15/49

Objective: (1) The determination of the periods and amplitudes of the individual components contributing to the

observed light variations of δ Scuti and 12 Lacertae. (2) The determination of the best methods to be used with IBM machines to determine amplitudes and periods of periodic phenomena.

Background: The light variations in stars of the type of δ Scuti and 12 Lacertae is the result of a superposition of several components. The amplitudes of the two predominating components are much larger than the errors of observation. For the remaining components, the amplitude is of the order of the errors of observation. No method is in general use which will find the periods and amplitudes of these small variations. It is important to establish their presence and their amplitudes, for this information appears to offer one of the few observational checks on theoretical models for giant stars.

Status: Inactive.

CALCULATION OF DIFFUSION COEFFICIENTS
Project 11.1/32-49-5

Origin: Western Regional Research Laboratory,
U. S. Department of Agriculture
Manager: E. Yowell Authorized 3/23/49 Completed 6/30/49

Objective: Given several sets of S as a function of s, to obtain the following:

a) $$\frac{2\pi S_m^2}{\Sigma^2(S)}\left[\frac{\Sigma(s^2S)}{\Sigma(S)} - \frac{\Sigma^2(sS)}{\Sigma^2(S)}\right]$$

b) AC

c) $$\frac{\omega^2\Sigma^2(S)}{4\pi S_m^2}$$

where S_m is the maximum value of S for any given set, and ω^2 is an enlargement factor.

Background: These quantities enter into the determination of diffusion coefficients. These particular sets of data were measured for serum albumen.

Status: Completed.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Project 11.1/32-49-6

Origin: NBS Authorized 3/21/49
Manager: S. Herrick Anticipated termination September 1949

Objective: To compute orbit of Halley's comet from 1835 to 1910, and the orbits of the ninth satellite of Jupiter and certain minor planets.

Background: The orbit of Halley's comet from 1835 to 1910 as determined by Cowell and Cromellin in their well-known prediction of the last return, is being checked and experimentally paralleled by a new variation of parameters procedure (Herrick: Publications of the Astronomical Society of the Pacific, October 1948). The calculations are fundamental to a prediction of the next return and to the establishment of the relative merits of the old and new procedures for this prediction. The orbits of the ninth satellite of Jupiter and certain minor planets are to be undertaken in similar experimental calculations.

Status: Underway (Continuation). Computations relating to Halley's comet from 1835 to 1910 were about 50% completed.

HARMONIC ANALYSIS OF GIVEN DATA
Project 11.1/32-49-7

Origin: Ames Laboratory, N.A.C.A Authorized 5/25/49
Manager: E. Yowell Completed 6/1/49

Objective: To obtain the amplitude of the sine and cosine components of the first ten harmonics for several sets of data.

Background: This computation was requested by the Ames Laboratory as related to and needed in the course of their scientific investigations.

Status: Completed (New).

COMPUTATION IN CONNECTION WITH ACOUSTIC PROPAGATION STUDIES
Project 11.1/32-49-8

Origin: Engineering Department, U.C.L.A Authorized 5/25/49
Manager: L. L. Bailin Completed 6/1/49

Objective: To compute, using the John W. Miles method, an approximation to the length, d, which an inner guide must be extended to account for the effect of the discontinuity upon plane longitudinal waves in a bifurcated circular guide.

Background: The Miles method is described in the following reference: Journal of the Acoustical Society of America, 19, 4, 579-584, July 1947. These results will exhibit an approximate method of dealing with the scattering of sound in the case of a circular tube inserted into a larger tube in order to measure local fields. The validity of the approximation may be determined by comparison with exact results obtained by L. L. Bailin in his Thesis, U.C.L.A (1949), through the use of integral equations.

Status: Completed (New).

EVALUATION OF DETERMINANTS
Project 11.1/32-49-9

Origin: North American Aviation Corporation
Manager: E. Yowell Authorized 5/25/49 Completed 6/10/49

Objective: To evaluate a large number of four-rowed determinants.

Background: The need for this computation arose from work being performed by the North American Aviation Corporation under a contract with the Atomic Energy Commission.

Status: Completed (New).

TRAFFIC SURVEY COMPUTATION
Project 11.1/32-49-10

Origin: Insitute of Transportation and Traffic Engineering, U.C.L.A.
Managers: E. Yowell, Roselyn Siegel
Authorized 6/15/49 Completed 6/23/49

Objective: Given a number of sets of observations of F, L, and Y (representing counts of vehicles moving in three ways through an intersection), to fit by least squares a surface of the form

$$Y = a + bL + cL^2 + dF + eFL$$

and to calculate certain statistics used in significance tests.

Background: The Traffic Institute made a number of surveys of traffic conditions at various street intersections in Los Angeles. In this particular data, Y represents the number of cars in the left traffic lane going in one direction, L represents the number of cars turning left from that lane, and F represents the total number of cars going in the opposite direction in the left and center lanes of the other side of the street. The observed values of these variables are counts of cars passing through an intersection during one green light.

Status: Completed (New).

5. Training

NUMERICAL ANALYSIS COLLOQUIUM SERIES
Project 11.1/4-49-2

Origin: NBS Authorized 7/9/48
Manager: J. H. Curtiss Terminated 4/1/49

Objective: To present a series of lectures on the theory and practice of numerical analysis.

Background: The Colloquia are aimed at college graduates with the equivalent of a major in mathematics, who are

interested in the theory or the application of numerical analysis, or both. They are used as a means of broadening the horizon of the computers of the Institute for Numerical Analysis and other computing laboratories in the Southern California area, and are also intended to stimulate interest in the underlying theory.

Comments: The lectures of this series are normally held each Monday afternoon in the Institute Building at 4 p.m.

Status: Terminated (although the Colloquium Series is continued). Information previously given in status report of this project will henceforth be contained in a new section of Projects and Publications, on Lectures and Symposia.

COURSE ON A.D.C.M. FOR MATHEMATICIANS
Project 11.1/4-49-5

Origin: NBS — Authorized 12/1/48
Manager: H. D. Huskey — Completed 6/30/49

Objective: To acquaint the research staff of the Institute for Numerical Analysis with the fundamentals of programing and coding for a.d.c.m.

Comments: The plan of the course consists in presenting the commands, times for their performance, and other pertinent data (but no details of electronics engineering) of a given computing system (the EDVAC was chosen) and assigning numerous problems to be translated by the class into programs and coded instructions. The course is divided into three parts:

I. Lectures and problems pertinent to the EDVAC
II. Advanced and unsolved problems
III. Comparative study of other computing systems.

Status: Completed.

CHARACTERISTIC VALUES OF MATHIEU'S DIFFERENTIAL EQUATION
Project 11.1/4-49-6

Origin: NBS Authorized 7/6/49
Manager: Gertrude Blanch

Objective: To provide a training program for computers using desk calculators, and, at the same time, to furnish a very useful table directly related to work soon to be published by the N.B.S.

Background: The tables to be published by N.B.S. provide the characteristic values b (for solutions of period π and 2π) of

$$y'' + (b - s\cos^2 x)y = 0.$$

Corresponding to a given s, there exists a countably infinite set of <u>even</u> solutions y of period π or 2π; these values of b are denoted by $be_r(s)$, $r = 0,1,2,\ldots$. There also exists another countably infinite set of values b giving rise to <u>odd</u> periodic solutions; these are denoted by $bo_r(s)$, $r = 1,2,3,\ldots$. When r is odd, the solutions are of period 2π; when r is even, the solutions are of period π. The tables now in process of publication give $be_r(s)$ and $bo_r(s)$ for $r \leqslant 15$ and $0 \leqslant s \leqslant 100$, to 8 decimals.

At $s = 100$, the asymptotic expansions for $be_r(s)$ and $bo_r(s)$ yields five significant figures or better for $be_0(s)$ and $be_1(s)$, $bo_1(s)$, and $bo_2(s)$; for $be_r(s)$ only 4 significant figures are obtainable and for $be_4, \ldots, be_8$ barely one or two significant figures. It appears that the characteristic values could be computed for the argument $\frac{1}{\sqrt{s}} = t$, for $t = 0(.01)0.1$. Such a table, for $r \leqslant 15$, would therefore tie in with the existing table, and complete the required calculations for orders $r \leqslant 15$. It is proposed to tabulate

$$Z = \frac{b}{(2r+1)\sqrt{s}}, \text{ or else } W = b - (2r+1)\sqrt{s} + \frac{(2r+1)^2}{8}$$

Both Z and W are regular at $s = \infty$ $(t = 0)$; for $Z \to 0$; $W \to -1/8$, as $t \to 0$. It is expected that both Z and W will be equally satisfactory from the viewpoint of regularity and "smoothness"; and if so, W will be tabulated. Decision on this point can be made after some work has been done.

The method to be used is fully outlined in the paper by G. Blanch, "On the Computation of Mathieu Functions,"

XXV No. 1, 1-20, (Feb. 1946).

Status: Underway (New).

II. Computation Laboratory

(Section 11.2)

1. Mathematical Tables

Note: Sponsorship for the preparation of the mathematical tables in this section has been assumed by the Office of Air Research, AMC, USAF.

TABLES OF $E_1(z)$, $(z = x + iy)$
Project 11.2/2-43-3

Origin: Canadian National Research Council
Manager: Irene Stegun Authorized 7/1/47

Objective: To prepare tables of the function $E_1(z) = \int_z^\infty [e^{-u}/u]du$ and related functions for $z = x + iy$.

Table I: $E_1(z) + \log_e z$, [$x = 0(.02)1$, $y = 0(.02)1$; 6D]

Table II: $E_1(z)$, [$x = 0(.02)4$, $y = 0(.02)3(.05)10$; 6D]

Table III: $e^x E_1(z)$, [$x = 4(.1)10$, $y = 0(.02)3(.05)10$; 6D]

Background: The initial motivation for the preparation of a table of exponential integrals for complex arguments arose in connection with atomic research. However, the table has found applications in fluid mechanics. See for instance, J. J. Stoker, "Surface Waves in Water of Variable Depth," Quar. App. Math. (Apr. 1947).

Status: Underway (Continuation). Computations completed; final manuscript in preparation.

TABLE OF JACOBI ELLIPTIC FUNCTIONS
Project 11.2/2-43-4

Origin: NBS Manager: H. E. Salzer Authorized 7/1/47

Objective: (a) To prepare tables of the Jacobi

elliptic functions:

$sn(u,k) = \sin\varphi$, $cn(u,k) = \cos\varphi$, $dn(u,k) = \sqrt{1-k^2\sin^2\varphi}$ where φ is defined by $u = \int_0^{\varphi} (1-t^2)^{-1/2} (1-k^2t^2)^{-1/2}dt$. These functions are to be tabulated for $k^2 = 0(.01)1$ and for $u = pK$ with $p = 0(.01)1$ and $K = \int_0^{\pi/2}(1-k^2\sin^2\theta)^{-1/2}d\theta$.

(b) To prepare tables of $sn(iu,k') = i\, sn(u,k)/cn(u,k)$, $cn(iu,k') = 1/cn(u,k)$, $dn(iu,k') = dn(u,k)/cn(u,k)$, for same values of u and k as in (a), and $k^2 + k'^2 = 1$.

Background: Professor Milne-Thompson originally suggested the preparation of a table of Jacobi elliptic functions for complex arguments. Because of the magnitude of this task, it was deemed sufficient to undertake the computation of the functions in question for real and purely imaginary arguments. The known addition formulae would then enable the user to evaluate elliptic functions for complex arguments. The chief applications contemplated by Professor Milne-Thompson were in the field of hydrodynamics.

Subsequently there was extensive correspondence with members of the Mathematical Tables Committee of the BAAS regarding the scope of this table. The present specifications incorporate the suggestions contained in the correspondence.

Comments: The proposed tables are to be computed at equidistant intervals of k^2. A similar table of elliptic functions, Smithsonian Institution, Publication No. 3863 (1947), gives the tabulations at equidistant intervals of $\alpha = \arcsin k$.

Status: Terminated, in favor of projects of higher priority. Part (a) had been completed for $k^2 = 0(.01)1$ and $p = 0(.1)1.0$.

MATHIEU FUNCTIONS II
Project 11.2/2-45-1

To be continued at the Institute for Numerical Analysis as Project 11.1/2-45-1. See page 24.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Project 11.2/2-46-1

Origin: NBS Manager: H. E. Salzer Authorized 7/1/47

Objective: (a) To prepare a table of $\log_e \Gamma(z)$, $z = x + iy$ for $x = 0(.1)10$, $y = 0(.1)10$, 10D to 12D.
(b) A table of $\Gamma(z)$ for same arguments as (a).
(c) A table of $1/\Gamma(z)$ (near the origin) for $x = 0(.01)\ p$, $y = 0(.01)q$ where p and q will be at least equal to unity and probably somewhat larger.

Background: Gamma functions for complex arguments occur in numerous physical problems such as the attraction between two particles in a Coulomb field of force. The existing tables are entirely inadequate for the needs of modern nuclear problems. Originally suggested by Dr. R. D. Evans of the Massachusetts Institute of Technology.

Status: Inactive. Computations of $\log_e \Gamma(x + iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ completed. Computations for $x = 0(.1)9$, 25% completed.

MATHIEU FUNCTIONS I
Project 11.2/2-46-2

Origin: Applied Mathematics Panel, NDRC
Manager: Gertrude Blanch Authorized 7/1/47

Objective: An eight-place table of the first 15 odd and 16 even characteristic values b of Mathieu's differential equation

$$y'' + (b - s\cos^2 t)y = 0$$

for s ranging from 0 to 100 at various intervals, and the Fourier coefficients of the solutions corresponding to these characteristic values as well as certain related functions.

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions"

by N. W. McLachlan (Oxford Press, 1947). The existing tables by Stratton-Morse-Chu-Hutner, Ince and Goldstein are inadequate. The project was proposed by Dr. Philip Morse and Dr. James Wakelin.

Comments: Related to Project 11.1/2-45-1.

Status: Underway (Continuation). Final manuscript in preparation for publication.

SPHEROIDAL WAVE FUNCTIONS
Project 11.2/2-47-1

Origin: NBS Manager: M. Abramowitz Authorized 7/1/47

Objective: (a) To prepare tables of the characteristic values of orders L = 0,1,...,10 for the differential equation

$$(1-x^2)w'' - (2m+1)xw' + (b-c^2x^2)w = 0$$

for m = 0(1)10 and c^2 ranging from 0 to about 1000 at various intervals.

(b) Tables of the solutions of the differential equation corresponding to the characteristic values under (a).

Background: Spheroidal wave functions are the solutions of the wave equation in prolate and oblate spheroidal coordinates. In his introduction of the "Elliptic Cylinder and Spheroidal Wave Functions," Professor Morse states "Solutions of problems involving the radiation and scattering of waves from strips of material, from wires of finite length and from discs of material, all require the knowledge of the mathematical properties and the numerical values of solutions of the wave equation for these coordinate systems. The solutions are likewise required for the study of the diffraction of waves through slits and circular openings, the absorption of sound by strips or by circular patches of material and the behavior of electrons in diatomic molecules." Originally proposed by Dr. Philip Morse.

Status: Terminated, in favor of projects of higher priority. Computations had been completed for m = 1,

L = 0,1,2,3,4, and c^2 ranging from 0 to 1000 at various intervals;

also for m = 0, L = 0, c^2 = 0(1)20(2)64(4)140(10)300(20)400,
L = 1, c^2 = 0(1)26(2)58(4)150(8)318(16)414,
L = 2, c^2 = 0(1)20,
L = 3, c^2 = 0(1)16(2)64(4)120(8)224,
L = 4, c^2 = 0(1)10(2)40(4)120(8)216;
and for m = 2, L = 0, c^2 = 0(1)10(2)30(4)54,
L = 1, c^2 = 0(1)10(2)32(4)100.

TABLES OF COULOMB WAVE FUNCTIONS
Project 11.2/2-47-2

Origin: NBS Authorized 7/1/47
Manager: M. Abramowitz Anticipated completion 12/31/49

Objective: Tabulation of the regular solution

$F_L(\rho,\eta) = C_L\rho^{L+1}\varphi_L(\rho,\eta)$ and its derivative $F'_L(\rho,\eta) = C_L\rho^L\varphi^*_L(\rho,\eta)$

and of the irregular solution $G_L(\rho,\eta)$ of the differential equation $y'' + \{1-\frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\}y = 0$ where

$$C_L^2 = \frac{2\pi\eta(1+\eta^2)\ (4+\eta^2)\ldots(L^2+\eta^2)2^{2L}}{(e^{2\pi\eta}-1)(2L+1)^2[(2L)!]^2}$$

Background: This equation arises in the quantum mechanical treatment of two particles moving in a Coulomb field of force; it arises in particular in the problems of proton-proton and proton -neutron interaction. The special case L = 0 occurs in a problem in classical hydrodynamics. Proposed by Professors Philip Morse of MIT, Gregory Breit of Yale University, and Herman Feshbach of MIT.

Status: Underway (Continuation). Computations completed; checking in process.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLE OF ANTILOGARITHMS
Project 11.2/2-47-3

Origin: NBS Authorized 7/1/47
Managers: M. Abramowitz, H. E. Salzer

Objective: To prepare a table of 10^x to ten decimal places for x = 0(.00001)1.

Background: The function 10^x is of basic importance. The best existing table is that of J. Dodson, "Antilogarithmic Canon," London (1742), which is out of print. The proposed publication, which was suggested by Mr. H. E. Salzer of the Computation Laboratory, will be an improvement over Dodson's table from the standpoint of accuracy and format.

Status: Underway (Continuation). Computations completed and 95% checked.

TABLES FOR THE OCCASIONAL COMPUTER
Project 11.2/2-47-4

Origin: NBS Authorized 7/1/47
Manager: Entire technical staff

Objective: To prepare an improved and amplified version of the Jahnke-Emde tables.

Background: The preparation of an improved version of the Jahnke-Emde tables had been originally suggested by Professor E. P. Wigner of Princeton University who submitted suggestions for the contents of the contemplated volume. This matter was discussed with Professor Tukey of Princeton, Professor J. Barkley Rosser of Cornell University and others.

Status: Inactive.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Project 11.2/2-48-2

Origin: NBS Authorized 5/25/48
Managers: M. Abramowitz, H. E. Salzer

Objective: Preparation of a volume of interpolation coefficients for the argument p at intervals of 1/3600 for 3-, 4-, 5-, 6-, 7-point interpolation and at intervals of 1/60 for 8-, 9-, 10-, 11-point interpolation.

Background: Functions tabulated with arguments in degrees occur in every branch of applied science. The present tables will simplify and reduce the labor of interpolation in such tables, whenever the functions are desired for minutes and seconds.

Status: Underway (Continuation). Computations completed for 3-, 4-, and 5-point interpolation.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN HERMITE POLYNOMIALS
Project 11.2/2-49-1

Origin: NBS Manager: H. E. Salzer Authorized 8/7/48

Objective: Preparation of a short table giving the zeros $x_i^{(n)}$, the Christoffel numbers $A_i^{(n)}$ and $e^{-x_i^{(n)2}} \times A_i^{(n)}$ to at least twelve significant figures, for the first sixteen Hermite polynomials.

Background: The Hermite polynomials and their zeros and corresponding Christoffel numbers play a fundamental role in the theory of orthogonal functions and in numerous applications in the fields of analysis and statistics. The

tables in existence give the zeros of only the first few polynomials to a number of places that is inadequate for the needs of a basic table. There are important applications, as in the numerical solution of integral equations, where the operations entail such a loss of significant figures that considerable more zeros, and Christoffel numbers to more places are needed.

Comments: This table will complete a series of three tables on zeros and Christoffel numbers of three fundamental and widely used sets of orthogonal polynomials - Legendre, Laguerre, and Hermite.

Status: Inactive. Computations 35% completed.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Project 11.2/2-49-2

Origin: NBS Manager: H.E. Salzer Authorized 3/14/49

Objective: To tabulate representations of numbers approximately between $1+10^{-20}$ and 10 in the form $\Pi_i p_i^{e_i}$ where p_i are prime numbers, e_i are positive or negative integers, and the product is taken over the first seven primes (i = 1, 2, ..., 7).

Background: For many years radix tables have been devised; they were tables of $\log(1 \pm n10^{-m})$, where n and m are integral, to aid in the calculation of logarithms to more than 20 places. Uhler recently produced such a table for calculating logarithms to about 130 places.

The present table in conjunction with values of the logarithms of seven basic primes will facilitate the computation of logarithms and antilogarithms to about 330 significant figures. The logarithms of the primes from 2 to 17 are known with the required accuracy, except for log 13 which will be calculated as part of this project.

This project was proposed by Dr. J. Barkley Rosser of Cornell University, who evolved the method of the present computation.

Status: Underway (Continuation). Computations 35% completed.

2. Computing Services

RANGE LAG OF BOMBS
Project 11.2/31-49-2

Origin: Air Materiel Command, USAF Authorized 6/25/49
Manager: F. L. Alt

Objective: To approximate the range lag of bombs as a simple analytic function of altitude and air-speed at time of release, and ballistic coefficient.

Background: The range lag, i.e., the difference between the actual range (horizontal travel) of a bomb and that range which would occur in a vacuum, is fitted by means of least squares by an expression of form $My + Ny^{3/2}$, where y is the altitude of release and where M and N are coefficients which in turn are to be fitted by simple functions of air-speed and ballistic coefficient. The reports are desired by contractors of the Air Materiel Command for the design of certain compensating equipment.

Status: Underway (New). Computations started.

HEAT CONDUCTION EQUATION
Project 11.2/33-46-1

Origin: Bureau of Ordnance, Department of the Navy
Manager: Ida Rhodes Authorized 7/1/47

Objective: To obtain numerical solutions of the non-linear partial differential equation

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2} + e^{-1/\theta} \quad (k = \text{constant})$$

satisfying the initial and boundary conditions; $\theta(x,0) = \theta_0$ and $\theta(0,t) = \theta_1$, for various values of θ_0 and θ_1.

Background: The problem had its origin in the investigation of the flow of heat from a hot thermostatic bath into an explosive substance in the form of a sphere immersed in the bath. For practical applications it is permissible to consider the explosive as a plane slab of infinite extent.

Status: Computations were completed for:

θ_o=.0600, θ_1=.1000; θ_o=.0600, θ_1=.141003; θ_o=.0600, θ_1=.2390.

Preliminary computations were carried out for other values of θ_o and θ_1.

Publication: Report to be transmitted to the Bureau of Ordnance.

COMPUTATIONS FOR METEOROLOGICAL PROJECT, N. Y. U.
Project 11.2/33-47-2

Origin: Office of Naval Research — Authorized 7/1/47
Manager: J. Laderman — Terminated 6/30/49

Objective: To perform computations required in objective analyses of meteorological elements. These computations include the determination of least squares solutions for divergence, wind velocity, barometric pressure, etc., based on data collected from weather stations throughout the eastern part of the U. S. Also the determination of large scale eddy stresses, stream lines, etc.

Background: The results obtained in addition to being of current interest in general circulation investigations are expected to establish the feasibility of constructing a machine to carry out these calculations. These computations were requested by Dr. H. Panofsky of New York University and Dr. J. von Neumann of the Institute for Advanced Study.

Status: Terminated. All problems submitted were completed by the end of the quarter.

Publication: Upon completion of each problem, the manuscript was transmitted to Dr. H. Panofsky, Meteorological Project, New York University.

PARTICLE DISTRIBUTION PROBLEM
Project 11.2/33-48-17

Origin: Oak Ridge National Laboratory Authorized 2/18/48
Manager: M. Karlin Terminated 6/30/49

Objective: To determine the frequency distribution of the distances traveled by particles (e.g., neutrons) passing through matter. The method consists in actually tracing the life history of many particles, determining distances between collisions and types of collisions by experimental sampling methods. Each particle is traced until it either disappears or its energy falls below a prescribed value.

Background: This problem is an example of the so-called "Monte Carlo" method which is now under consideration by nuclear scientists as a useful tool for solving a variety of diffusion problems. Such problems can generally also be formulated in terms of integro-differential equations, but these equations are sometimes intractable from the point of view of obtaining numerical results.

Comments: A basic reference for such problems is an abstract by Ulam and von Neumann in the Bull. Am. Math. Soc. 53, 1120 (1947).

Status: Terminated. In favor of projects of higher priority.

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Project 11.2/33-49-2

Origin: Naval Research Laboratory Authorized 10/1/48
Manager: Irene Stegun

Objective: To calculate ordinates of a function of the following type:

$$f(s) = \frac{a_1}{(1+m_1 s^2)^2} + \frac{a_2}{(1+m_2 s^2)^2} + \frac{a_3}{(1+m_3 s^2)^2},$$

for given values of a_1 and m_1 with s ranging between 0 and 12.566 at intervals of $\pi/10$. To evaluate

$$D(r) = \frac{2r}{\pi} \int_0^\infty s\, f(s) \sin rs\, ds$$

for r ranging from 0 to 1.5.

Background: The function f(s) represents the intensity of a beam of electrons at a given angle to the axis of the beam, and was fitted to experimental data. The goal is to compute the Fourier sine transform of f(s), and to correct this transform for deviation of f(s) from the observed data. The ordinates of f(s) calculated here are used in the correction process. The problem was proposed by Dr. J. Karle and Mr. H. Hauptman of the Naval Research Laboratory.

Status: Underway (Continuation). Computations for various values of the parameters a_1, m_1 were performed when requested.

SECOND AIR FORCE TEST PROBLEM
Project 11.2/33-49-4

Origin: Air Comptroller, USAF — Authorized 8/7/48
Manager: J. Laderman — Completed 6/30/49

Objective: To find a diet to be formed from 77 different foods with known nutritive values so that specified quantities of nine nutrients will be obtained.

Background: This is an experimental problem in simultaneous linear algebraic relationships, designed to study certain properties of such relationships. The setting up of a computational procedure and the comparison of this procedure with the one applied in the first Air Force Test Problem (Project 48S2-15) is the main objective, rather than the solution of this specific diet problem. The computational procedure for the present problem will follow the lines of a mathematical procedure prepared by Prof. J. von Neumann.

Comments: This project is a part of the mathematical work supporting Project 11.4/24-47-3.

Status: Completed. Report transmitted to the Office of the Air Comptroller.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Project 11.2/33-49-5

Origin: Section 3.2, NBS Authorized 11/29/48
Managers: J. H. Levin, Irene Stegun

Objective: To process a new critical compilation of tables giving thermodynamic properties of gases of technical importance to aeronautical engineers. This involves sub-tabulation by appropriate non-linear interpolation formulae; the sub-tabulation is to be carried to the extent where further linear interpolation will be valid. In addition, tables are converted from dimensional to non-dimensional form by appropriate conversion factors, and some are differenced.

Background: Existing tables are scattered and lacking in uniformity. Where temperature is the independent variable, some, like Gratch's "Table of Zero-Pressure Properties of Diatomic Nitrogen," use degrees Rankine in intervals of 4,8 or 16 degrees, while others (e.g., H. W. Woolley "Thermodynamic Functions for Molecular Oxygen in the Ideal Gas State," NBS J.Res. 40, No. 2, 163 (Feb. 1948)) use degrees Kelvin. Some refer to the most abundant isotope of a gas, others to the prevailing isotope mixture. Range of tables and units of measurement of tabulated functions also vary. The proposed tables are to form part of a larger collection of uniform tables, all expressed in dimensionless quantities and based on the most up-to-date measurements and calculations. This work is subsidiary to a project of Section 3.2, NBS, which is being sponsored by the National Advisory Committee on Aeronautics.

Status: Underway (Continuation). Tables of zero-pressure properties were completed for N_2 , O_2, A, air and normal H_2. Computations for real-gas properties were completed for dry air, normal H_2, and moist air. Tables of free-energy properties were in progress for H_2, O_2, and N_2.

ADMINISTRATIVE RECORDS ON PUNCHED CARDS
Project 11.2/33-49-8

Origin: Section M, NBS — Authorized 12/15/48
Manager: A. H. Rosenthal

Objective: To establish and maintain punched-card records of certain administrative items, such as stock records, travel data, etc. To prepare up-to-date printed listings of the records as required.

Background: Such records were formerly kept by manual methods. The punched card method will make it possible to compile necessary periodic reports in much less time and will make record data more easily and quickly available for use in statistical studies.

Status: Underway (Continuation). Records of administrative items were brought up-to-date as requested.

FERMI FUNCTION II
Project 11.2/33-49-10

Origin: Section 4.4, NBS — Authorized 12/30/48
Manager: Irene Stegun

Objective: To tabulate the Fermi Function

$$f(z,\eta) = \eta^{2+2s} e^{\pi\gamma\theta} \left| \Gamma(1+s+i\gamma\theta) \right|^2$$

where $\gamma = z/137$, $s = \sqrt{1-\gamma^2}-1$ and $\theta = \sqrt{1+\eta^2}/\eta$.

z is the atomic number of any chemical element, η the momentum of β-rays emitted. The function is to be tabulated for $z = 1(1)95$, $\eta = .05(.05)7.00$.

Background: The Fermi Function is used in the determination of the maximum momentum (or energy) of β-rays emitted from "synthetic" radio-active substances. (See I. Feister

and L. F. Curtiss, NBS J. Res. 40, 315 (1948), RP1877. See also projects 48S2-20 and 11.2/33-49-1. The computation was requested by Dr. L. F. Curtiss and Dr. I. Feister.

Status: Underway (Continuation). Computations were completed and checking was in progress.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Project 11.2/33-49-11

Origin: NACA, Lewis Flight Propulsion Laboratory, Cleveland, Ohio
Manager: Irene Stegun Authorized 2/1/49

Objective: To compute the proportions in which various gases occur in the exhaust after burning of hydrocarbon fuels. These proportions are to be determined in their dependence on temperature, pressure, fuel-air ratio and hydrogen-carbon ratio in the fuel, for a range of conditions occurring in practice.

Background: Computations of this kind are important in the design of combustion engines and jet engines and in the physical-chemical investigation of combustion in general. The particular set of computations to be performed here was proposed by L. R. Turner of the NACA. Similar computations on a very large scale were formerly carried out by S. R. Brinkley on the ENIAC; these, however, do not extend to the range of pressures and temperatures usually encountered in combustion gases.

Status: New.

SHOCK WAVE PARAMETERS
Project 11.2/33-49-13

Origin: Bureau of Ordnance, Department of the Navy
Manager: Irene Stegun Authorized 3/31/49

Objective: To provide graphs and tables for the rapid determination of the parameters of spherical shock

waves emitted by explosions. In particular, (a) to prepare basic tables of certain functions $I_n(x,c_v)$ [$n = 0,1,2,\ldots,10$, c_v = specific heat at constant volume] needed in the following parts of the problem, (b) to determine p_1 (the initial pressure at the shock front) in its dependence on p_e (the static pressure of the explosion gases) and a parameter K which measures the effect of density, temperature, etc.; (c) to compute tables of certain functions which facilitate the determination of K for a given explosive.

Background: OSRD Reports 1030, 2022, 3550, and 5649 contain tables describing the entire course of shock waves in air, fresh water, and salt water as a function of the initial conditions at the instant of explosion. The present project aims at facilitating the determination of these initial conditions from a knowledge of the chemical composition of the explosive, and thus to aid in studies of the effectiveness of explosives. The initial conditions, particularly pressure and velocity, are determined by the fact that they satisfy the equation of state of the explosion gas and the Hugoniot condition for the surrounding medium (water). The latter is tabulated in OSRD Reports 670 and 813; for the former the Wilson-Kistiakowsky equation may be used. Kirkwood and Montroll (OSRD Report 670) developed the theory which forms the basis of the present computations. They also introduced and tabulated the auxiliary functions $I_n(x,c_v)$. The range of their tables is, however, not quite sufficient, and subtabulation is needed for convenient use.

Comments: The problem was proposed by Dr. T.L. Brownyard of the Bureau of Ordnance and Dr. H. G. Snay of the Naval Research Laboratory. The latter developed the method used in this project.

Status: Underway (Continuation). (a) 90% completed; (b) 70% completed; (c) 25% completed.

BASIC IONOSPHERIC DATA
Project 11.2/33-49-14

Origin: Section 14.1, NBS Authorized 3/31/49
Manager: J. H. Levin*

Objective: To analyze multifrequency, vertical-incidence

observations of the ionosphere. The observations made at a number of observing stations are to be transcribed on punch cards. Various statistical analyses are to be performed and, in the case of current data, tables are to be prepared for publication.

Background: The observations consist of virtual heights and critical frequencies of the regularly ionized layers of the earth's upper atmosphere. These are measured hourly at 15 stations operated by or under agreement with the National Bureau of Standards. About 60 station-years of past data exist. Certain simple manipulations with this kind of data have been standard for many years, although the huge number of observations - some 6000 per station per month - has precluded much detailed investigation. Some analyses serve to extract typical ionospheric conditions to be applied in predictions of usable frequencies for long distance radio transmissions. Others will investigate statistically, changes in the characteristics of the ionospheric layers accompanying other geomagnetic phenomena, solar activity, etc., to increase knowledge on the origin and structure of the ionosphere and its effect on radio wave propagation.

Comment: *Dr. A. H. Shapley of Section 14.1 is directing this work.

Status: Underway (Continuation). Observation data was recorded and processed as received.

Publications: Current data are published in the monthly CRPL-F reports of the NBS Central Radio Propagation Laboratory; copy prepared from punched cards will replace hand-made copy as soon as procedures and schedules are worked out. The results of statistical computations will be included in scientific papers where appropriate.

SEQUENTIAL SAMPLING PLANS
Project 11.2/33-49-15

Origin: Hq. Quartermaster Corps Inspection Service, Department of the Army
Manager: Irene Stegun Authorized 4/14/49

Objective: To design truncated sequential sampling plans which have OC curves matching those of a given set of

single sampling plans at two points.

Background: This set of sequential sampling plans is to be used in the revision of JAN Standard 105.

Comments: Supervision is carried out in close contact with Mr. J. Cameron of the NBS Statistical Engineering Laboratory.

Status: Underway (New). Computations 75% completed.

STRAIN ROSETTE MEASUREMENTS
Project 11.2/33-49-16

Origin: David Taylor Model Basin, USN Authorized 4/14/49
Manager: Irene Stegun

Objective: To evaluate a series of strain rosette measurements taken at the David Taylor Model Basin.

Background: "Strain rosette measurements" consist in determining strains in three coplanar directions, two being perpendicular to each other and the third at a 45° angle. From these measurements it is desired to determine the magnitude and directions of the principal strains.

Status: Underway (New). Computations 90% completed.

RADIO-TELEGRAPH INTERFERENCE
Project 11.2/33-49-17

Origin: Section 14.4, NBS Authorized 4/14/49
Manager: A. H. Rosenthal Anticipated Completion 10/1/49

Objective: To determine percentage of intelligibility of telegraphic messages as a function of frequency separation and radio-intensity ratio. The observations, consisting of the scores on tests performed with a number of telegraph operators transcribing messages under controlled noise conditions, are to be recorded on punched cards, gross errors are

to be detected, and various statistical analyses are to be performed thereafter.

Background: To assist the Provisional Frequency Board at Geneva, Switzerland, and other interested agencies in frequency allocation of radio-telegraph services.

Comments: Specifically requested by W. Q. Crichlow, 14.4, NBS.

Status: Underway (New). Recording of data was begun.

PERCENTAGE POINTS OF THE ARITHMETIC MEAN IN RANDOM SAMPLES FROM THE SECH AND SECH^2 DISTRIBUTIONS
Project 11.2/33-49-18

Origin: Section 11.3, NBS Authorized 5/3/49
Manager: Irene Stegun

Objective: To produce two tables, one relating to the sech distribution, and one to the sech^2 distribution, giving the values of $t_\epsilon = u_\epsilon/\sqrt{N}$, where u_ϵ is the solution of

$$\int_{-\infty}^{u_\epsilon} p_N(u)du = 1 - \epsilon ,$$

and for the sech distribution,

$$p_N(u) = \begin{cases} \frac{1}{2(N-1)!}\, u \operatorname{csch} \tfrac{1}{2}\pi u \cdot \prod_{r=1}^{n-1} [u^2 + (2r)^2], & \text{if } N = 2n \text{ is even} \\ \frac{1}{2(N-1)!} \operatorname{sech} \tfrac{1}{2}\pi u \cdot \prod_{r=0}^{n-1} [u^2 + (2r+1)^2], & \text{if } N = 2n+1 \text{ is odd;} \end{cases}$$

for the sech^2 distribution,

$$P_N(u) = \frac{2^N}{\pi\sqrt{3}} \int_0^\infty \left(\frac{w}{e^w - e^{-w}}\right)^N \cos \frac{uw}{\sqrt{3}}\, dw .$$

t_ϵ is to be tabulated to 6D for N = 1(1)15, and for (one-tail) probability levels ϵ = .001, .005, .010. .025, .050, .100, .200, .250.

Background: The quantity t_ϵ, called "percentage (or probability) point of the (standardized) mean" for given sample size N and probability ϵ, is related (by a simple transformation) to the value that will be exceeded with probability ϵ by the mean of random sample of N independent observations from a given population. Such percentage points are of fundamental importance in statistical tests of significance and related problems. Approximate values, based on approach to normality, have served in the past when dealing with large samples but have been found inadequate for relatively small samples. Exact percentage points of the mean have previously been computed (project 11.3/1-47-1) for samples from the normal, double-exponential and rectangular distributions. Computations are now needed for samples from the distribution sech,

$$p(x)dx = \frac{dx}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}, \quad (-\infty \leq x \leq \infty),$$

and sech^2

$$p(x)dx = \frac{\pi}{\sqrt{3}} \frac{dx}{\left(e^{\frac{\pi x}{\sqrt{12}}} + e^{\frac{-\pi x}{\sqrt{12}}}\right)^2}, \quad (-\infty \leq x \leq \infty),$$

to complete the last two papers mentioned in that project.

Status: New.

BODIES OF REVOLUTION WITH PRESCRIBED CHARACTERISTICS
Project 11.2/33-49-19

Origin: David Taylor Model Basin, U. S. Navy
Manager: S. Prusch Authorized 6/4/49 Completed 6/30/49

Objective: To compute the coefficients of polynomials representing the meridians of bodies of revolution when certain geometric parameters of these bodies are prescribed, and to compute offsets for such bodies for useful ranges of the parameters.

Background: Systematic families of streamlined bodies of revolution are to be tested to study the variation of hydrodynamic resistance with the geometric parameters. The prescribed parameters for defining a body are the nose and tail radii, the position of the maximum cross section, the volume of the circumscribed cylinder and the fineness ratio (ratio of length to maximum diameter). The method may readily be extended to include additional parameters, if desired.

Status: Completed (New). Results transmitted to David Taylor Model Basin.

COMPUTATION OF NORMAL MODES OF BOX BEAM
Project 11.2/33-49-20

Origin: Section 6.4, NBS Authorized 6/20/49
Manager: J. H. Levin

Objective: Computation of three lowest resonant frequencies and associated mode shapes for box beam. For purposes of simplifying the calculation of the mode shapes, the box beam is replaced by an idealized structure consisting of discrete mass points connected by weightless springs. The validity of such an approximation is to be tested in section 6.4 by comparison of observed resonant frequencies and mode shapes for the box beam with calculated frequencies and mode shapes for the substitute structure. The mathematical formulation of this problem leads to the calculation of the three highest characteristic roots and associated modes of a matrix of order 14. If the characteristic roots are λ_1^2, then the desired frequencies are $\omega_1 = 1/\lambda_1$.

Background: These studies are of importance in investigating dynamic properties of aircraft, -- in particular, the dynamic response of airplane wings to transient influences such as landing impacts, gusts, gun blast, and maneuvering.

Status: Underway (New). 50% completed.

TABLES OF CABLE FUNCTIONS
Project 11.2/33-49-21

Origin: David Taylor Model Basin, USN — Authorized 6/25/49
Manager: S. Prusch — Anticipated Completion 8/49

Objective: To evaluate the functions

$$\log_e \tau = \int_0^{\phi} \frac{P}{Q} d\phi \ , \qquad \sigma = \int_0^{\phi} \frac{\tau}{Q} d\phi$$

$$\eta = \int_0^{\phi} \frac{\tau \sin\phi}{Q} d\phi \ , \qquad \xi = \int_0^{\phi} \frac{\tau \cos\phi}{Q} d\phi$$

where $P = f\frac{\cos\phi}{|\cos\phi|} + w \sin\phi$, $Q = -|\sin\phi| \sin\phi + w \cos\phi$

and $w = \sec \phi_c - \cos \phi_c$, for $f = .01(.01).03$, $\phi_c = 0(5^\circ)90^\circ$;

$\phi = [\phi_c(1^\circ)\phi_c + 180^\circ]$.

Background: These functions facilitate the determination of the shape of and tensions in a flexible cable that is held in a uniform stream. The effects of the weight of the cable and frictional drag are included. The types of configurations that may be treated by means of these functions are most general.

Status: Underway (New). 50% completed.

STANDARD LORAN TABLES
Project 11.2/34-49-1: Atlantic Chain
Project 11.2/34-49-2: Aleutian Chain

Origin: U. S. Navy Hydrographic Office — Authorized 10/13/48
Manager: M. Abramowitz — Completed 6/30/49

Objective: Preparation of tables giving coordinates of hyperbolic lines of positions.

Background: Standard Loran Navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: Completed.

Publication: Tables to be published by U. S. Navy Hydrographic Office.

TRAINER SKY WAVE COMPUTATIONS
Project 11.2/34-49-4

Origin: U. S. Navy Hydrographic Office Authorized 4/14/49
Manager: M. Abramowitz Completed 6/1/49

Objective: To prepare a table of sky wave corrections for use with Loran Trainer.

Status: Completed (New).

EFFECT OF NUCLEAR RADIATIONS ON HUMAN BEINGS
Project 11.2/35-49-1

Origin: Operational Research Office, U. S. Army (Johns Hopkins University)
Manager: J. H. Levin Authorized 2/1/49

Objective: To systematize data related to the effect of nuclear radiations on the mortality and life expectancy of animals. An auxiliary task is the computation of the ionization produced in different tissues by incident mono-energetic radiations.

Background: The numerical work to be accomplished in this project will contribute toward a research task being conducted by Johns Hopkins University under contract with the Department of the Army, for the preparation of tables which show the effect of nuclear radiations on the mortality

and life expectancy of human beings. The greater part of the source material was derived from experiments on mammals.

Status: Underway (Continuation). Normalized life expectancies for various groups of experimental animals were completed. Computations of attenuation of radiation penetrating body tissues were completed for energies of .25 and .1 mev.

TABLES FOR COMBAT CREW PLANNING
Project 11.2/36-49-2

Origin: Air Comptroller's Office, USAF Authorized 3/21/49
Manager: J. H. Levin

Objective: These tables pertain to combat losses among air crew personnel under specified conditions. Corresponding to assumed loss rates and tours of duty, the tables will give: (1) chance of completing tour, (2) chance of not completing tour, (3) potential sorties per new crew, (4) average potential sorties per crew in combat in mature theater, and (5) effective loss rate per sortie (attrition plus retirement). "Tour of duty" is here defined to mean a given number of sorties for a crew to fly, at the completion of which it is automatically dropped from combat flying duty. "Mature theater" is a hypothetical theater in which the crew flow balances the losses and holds constant the proportion of crews at each experience level.

Background: These tables are an extension of the tables in AAF Manual 150-35-1. They are of importance, among other things, in the establishment of training programs. Thus, it is necessary to determine what attrition rates might be expected so that crews could be scheduled for completion of training in time to meet resulting replacement needs.

Status: Underway (Continuation). Computations had been completed and checked on punched cards. Preparation of final manuscript was under way.

A PROBLEM IN LINEAR PROGRAMING
Project 11.2/36-49-3

Origin: Air Comptroller's Office, USAF Authorized 3/21/49
Manager: F. L. Alt

Objective: To experiment with methods for the efficient large-scale solution of problems of the following type: Given n equipment items $E_1, E_2, \ldots, E_n$ (such as various types of airplanes, personnel, supplies) and n+1 activities $A_0, A_1, A_2, \ldots, A_n$, (such as combat, procurement, shipment, training) which consume certain equipment items and produce others. Given further the quantities $a_{ij}^{(t)}$ and $\bar{a}_{ij}^{(t)}$, where $a_{ij}^{(t)}$ is the amount of item E_i required per unit of activity A_j at the beginning of time period t, and $\bar{a}_{ij}^{(t)}$ is the amount of E_i produced per unit of activity A_j at the end of time period t. Given finally the amounts $A_0^{(t)}$ of activity A_0 at each of a number of successive time periods. Determine the levels of the other activities in each time period such that exactly the required amounts of each equipment item will be available at each time.

Background: The problem can easily be formulated as a system of n simultaneous linear difference equations. Special methods of solution are required only because (a) n is very large, (b) many of the coefficients $a_{ij}^{(t)}$ and $\bar{a}_{ij}^{(t)}$ are zero; in particular, the solution was greatly simplified because most of the coefficients above the principal diagonal $(j \geq i)$ are zero. There was also some simplification by the fact that $a_{ij}^{(t)}, \bar{a}_{ij}^{(t)}$ are independent of t for $j \geq 1$; only $a_{io}^{(t)}, \bar{a}_{io}^{(t)}$ depend on t. Since the solution of problems of this type contributes greatly to the efficient management of organizations such as the Armed Forces, it is desirable to develop special methods which take advantage of all these simplifications. The methods employed in the present project were developed by G. B. Dantzig, E. Schell and M. K. Wood.

Status: Underway (Continuation). Additional modifications of the methods were tried.

MAGNETIC MOMENT OF ELECTRONS
Project 11.2/37-49-1

Origin: Atomic Energy Commission Authorized 6/30/49
Manager: M. Oliphant

Objective: To evaluate analytically a certain fourfold integral of a rational function of four variables.

Background: The integral represents effects of fourth-order electromagnetic interactions on the magnetic moment of electrons.

Comments: The problem was proposed by Drs. R. Karplus and M. Kroll of the Institute for Advanced Study, Princeton.

Status: New.

3. Training

TRAINING OF PERSONNEL IN THE OPERATION OF NEW IBM MACHINES
Project 11.2/4-49-3

Origin: NBS Authorized 3/31/49
Manager: A. H. Rosenthal Anticipated termination 9/49

Objective: To instruct the personnel of the machine unit of the Computation Laboratory in the uses and applications of the various new IBM machines, especially the 602, 602A and 604 types.

Background: The idea for such instruction originated because of the need of trained personnel for the new machines, in order to insure the accuracy of the work and the efficient use of the equipment.

Comments: Conferences are held three times a week for one hour per session.

Status: Underway (Continuation).

III. Statistical Engineering Laboratory

(Section 11.3)

1. Research in Mathematical Statistics

THE MEAN DEVIATION, STANDARD DEVIATION, AND RANGE AS ESTIMATORS OF SCALE PARAMETERS (MEASURES OF DISPERSION) OF PROBABILITY DISTRIBUTIONS
Project 11.3/1-47-2

Origin: Section 11.3, NBS Authorized 7/1/47
Managers: Churchill Eisenhart, Lola S. Deming

Objective: To evaluate percentiles and other features of the distributions of these estimators in small random samples from normal (Gaussian) and various non-normal (see project 11.3/1-47-1) populations.

Background: Previous studies of the relative merits of the mean deviation, standard deviation, and range as estimators of scale parameters of probability distributions have, in the main, concentrated on (a) evaluation of adjustment factors for rendering them unbiased estimators of, say, the standard deviation of the population; and (b) comparison of their "efficiencies" (as measured by the ratios of their sampling variances when so adjusted). Since their distributions in small samples are non-normal and generally differ in form, comparisons of their "efficiencies" in small samples may not truly represent their relative merits with regard to accuracy and precision in such cases. The approach via percentiles and other features (e.g., the probability of underestimating the true value of the relevant scale parameter) is expected to yield important new information.

Status: Inactive. For present status see Projects and Publications Oct-Dec 1948.

Publication: A paper on "The relative frequencies with which certain estimators of the standard deviation of a normal population tend to underestimate its value," by Churchill Eisenhart and Celia S. Martin will be submitted to the Journal

of the American Statistical Association. A second paper, "Probability center lines for standard deviation and range charts," by Churchill Eisenhart will appear in Industrial Quality Control.

STATISTICAL TESTS OF SIGNIFICANCE FOR 2 x 2 TABLES WHEN THE NUMBER OF OBSERVATIONS IS SMALL
Project 11.3/1-47-3

Origin: Program Committee, Institute of Mathematical Statistics
Manager: Churchill Eisenhart
Authorized 7/1/47

Objective: To compare Fisher's "exact" test, Barnard's "C.S.M." test, and certain other statistical tests for data arranged in 2 x 2 tables with respect to (a) scope, i.e. conditions for which the respective tests are valid, and (b) operating characteristics, i.e. bias, power, etc., under the conditions for which they are jointly valid.

Background: The project was undertaken in connection with an invited address given at the Symposium on 2 x 2 Tables sponsored by the Institute of Mathematical Statistics at the New Haven, Connecticut meeting on September 2, 1947.

Status: Inactive. For present status see Projects and Publications Oct-Dec.1948.

Publication: The three-dimensional model showing the power surfaces of the exact and the C.S.M. tests for the case discussed above is available for examination in the Statistical Engineering Laboratory.

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Project 11.3/1-49-1

Origin: Section 11.3, NBS Authorized 9/21/48
Manager: J. Lieblein

Objective: To evaluate the statistical properties of three observations (1) when all are drawn at random from a

single universe and (2) when more than 1 universe is involved, with particular reference to the processing of experimental measurements and especially to the problem of anomalous readings.

Background: The wide prevalence of triplicate readings in chemistry, other physical sciences, and engineering testing invites thorough study of samples of this size under a variety of assumptions regarding the experimental conditions under which they are made. This study is of special importance since there is great interest among scientists in whether a sound statistical criterion for the rejection of one of three observations can be devised that makes use only of the observations themselves. This study was undertaken at the suggestion of Mr. Youden, as a follow-up to an empirical study that he had made of some published data.

Status: Underway (Continuation). About 80% completed. A comparative study of the results of this project with the related results of W. J. Dixon, F. E. Grubbs, K. R. Nair and others, has revealed certain qualitative differences between samples of 3 and samples of larger sizes. For information regarding theoretical and numerical results obtained to date in the Statistical Engineering Laboratory see Projects and Publications Oct-Dec 1948, Jan-Mar 1949.

Publications: The theoretical and numerical results, together with related results of other writers, are being incorporated in a paper on samples of 3, by Julius Lieblein, to be submitted to the NBS Journal of Research. Their practical implications are being discussed in detail by W, J. Youden in a paper on duplicate and triplicate determinations, to be submitted to the Scientific Monthly.

"The fallacy of the best two out of three", a summary of the results of this study and their practical implications, prepared for the July 1949 issue of the NBS Technical News Bulletin and released in advance to the press and to various technical and trade journals, has resulted in brief notices in the press (e.g. New York Times, and Science Service) and elsewhere.

STATISTICAL PROCEDURES FOR INTERPOLATED MEDIANS
Project 11.3/1-49-2

Origin: Institute for Numerical Analysis (Section 11.1, NBS)
Managers: Churchill Eisenhart and Julius Lieblein
Authorized 3/21/49

Objective: To provide practical statistical procedures for employing interpolated medians in estimation and tests of significance for parameters of grouped populations, together with a rigorous mathematical development of the underlying theory.

Background: In response to a request from Section 11.1 (see project 11.1/32-49-3) asymptotic standard errors and tests of significance were developed for interpolated medians in samples from grouped populations and form the basis of a note, "Standard errors and tests of significance for medians of grouped distributions," by Churchill Eisenhart and Mirian L. Yevick. It is the aim of the present project to provide a more complete mathematical treatment of the underlying theory, and a fuller treatment of the practical procedures.

Status: Underway (New). About 50% completed.

Publication: An abstract of the above note appears in the Annals of Mathematical Statistics, March 1949.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Project 11.3/1-49-3

Origin: Section 11.3 NBS Authorized 3/15/49
Managers: Henry B. Mann and Miriam L. Yevick

Objective: To prepare a monograph on stochastic processes from the viewpoint of mathematical statistics.

Background: Stochastic (i.e., random) processes are becoming increasingly important in many branches of science and technology, e. g., in physics, electrical engineering, ionospheric research, meteorology, epidemiology, actuarial science, economics, and biology. A fairly well advanced but widely scattered literature exists, in which the subject is approached from diverse points of view. The mathematical treatment has been largely from the viewpoints of either pure or of classical applied mathematics. It seems difficult to apply modern

principles and techniques of statistical inference to stochastic processes via the existing theory of the subject. It is the aim of the project to develop a theory of stochastic processes that will facilitate the application of modern principles and techniques of statistical inference to the study of these processes.

Status: About 70% completed. The monograph will consist of six chapters. The theorems and proofs comprising the first five chapters have been written up in preliminary form, without illustrative examples and other instructional aids. The sixth chapter has been completed in rough draft.

During the period June 13-17, 1949, Dr. Mann delivered a group of five lectures at the Bureau, "On the theory of stochastic processes from the viewpoint of mathematical statistics" as item VIII of the NBS Applied Mathematics Colloquium Series. The subjects of the lectures are given in the section on Lectures and Symposia in this report.

2. Manuals of Statistical Methods

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS
Project 11.3/2-47-2

Origin: Section 11.3, NBS Authorized 7/1/47
Manager: C. Eisenhart

Objective: To provide a useful collection of formulas for the operating characteristics and the number of observations needed for certain single-sample one-sided tests of statistical hypotheses, with instructions for their application.

Background: Procedures are given in the statistical literature (e.g., in textbooks, journal articles) for determining operating characteristics (discriminating power) and the number of observations needed (cost) for certain single-sample one-sided tests of statistical hypotheses, but they generally require the use of specialized probability tables. It does not appear to have been generally recognized that many problems of these types can be satisfactorily handled by means of relatively simple approximate formulas requiring for their use only certain readily accessible and easily remembered normal-probability deviates.

Status: Inactive. About 90% completed. For status to date, see Projects and Publications Jan-Mar 1949.

STANDARD SAMPLING-INSPECTION PROCEDURES
Project 11.3/2-48-1

Origin: Office of Naval Research, and Research and Development Division of the Department of the Army
Managers: Churchill Eisenhart, J. Lieblein
Authorized 7/15/47

Objective: To revise, expand, and extend the present Navy Department manual "Standard Sampling Inspection Procedures" (Part D, Chapter 4, of the Administration Manual of the Material Inspection Service, U.S.N.) so that it will be suitable for referencing in all types of Government specifications, particularly in Federal specifications.

Background: A task group of the Inspection Advisory Council of the War Department undertook during the fiscal year 1947 to effect a revision of the Navy "Procedures" to adapt them to War Department needs. Additional tables recommended by this task group were computed by Section 11.2 in that fiscal year. It was decided that further mathematical work would be needed on the entire task and that the task should be generalized to include all Federal procurement. Theoretical aspects have been assigned to Section 11.3 while responsibility for practical aspects remains in the Department of National Defense.

Comments: In order to foster a wider appreciation and understanding of some of the standard sampling-inspection systems now in use in the National Military Establishment and elsewhere, a one-term in-hours course in the basic statistical principles, devices, and techniques involved was offered at the Bureau this quarter (see Project 11.3/4-49-7).

Status: Underway (Continuation). About 10% completed. Work on this project is being coordinated with the activities of the National Military Establishment in sampling inspection and quality control.

A Quality and Sampling Committee was established in the Munitions Board Material Inspection Agency for the purpose of studying and developing standards and procedures of statistical quality control and quality assurance. This Committee has proposed (a) the establishment of a broad National

Military Establishment research and development project on the application of statistical method to quality assurance and quality control, to disclose and cope with such problems as were mentioned above; and (b) the development of an interim standard, utilizing the best knowledge and experience to date, for use pending the outcome of the more comprehensive study. In the meanwhile "General Specifications for Inspection of Material, Appendix X, Standard Sampling Inspection Tables for Inspection by Attributes", issued by the Navy Department, April 1, 1946, with the addition of an appropriate cover sheet and such changes in the general instructions as were necessary to convert it to a joint Army, Navy, Air Force document pending the promulgation of a revised standard, has been approved by the Departments of the Army, Navy,and the Air Force as JAN-STD-105, "Sampling Inspection Tables for Inspection by Attributes", and where applicable is mandatory for these departments effective 16 Aug. 1949.

A Subcommittee for Revision of JAN Standard 105, of which Dr. Eisenhart is a member, was active throughout the quarter, and a third draft of the "revision" was completed as the quarter ended. In this connection, NBS section 11.2 developed multiple-sampling plans to match certain additional single-sampling plans developed by the Quartermaster Corps (See project 11.2/33-49-15).

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Project 11.3/2-48-3

Origin: Section 11.0, NBS Authorized 1/23/48
Managers: Celia S. Martin and Irma S. Wachtel

Objective: To prepare a glossary of the statistical terminology associated with acceptance sampling and process control, statistical analysis and interpretation of experimental and test data, and statistical design of experiments and tests.

Background: The application of statistical concepts and techniques to acceptance sampling and process control has given rise to new terms, and many everyday terms are used with very specific connotations. The relatively new art of the statistical design of experiments and tests also has a special vocabulary. Finally, the concepts, principles, and techniques of statistical inference as applied to the analysis and interpretation of experimental and test data have been

revised and expanded considerably during the past two decades with consequent changes in the meanings of terms and the introduction of new terms.

It is highly desirable, therefore, that a glossary of statistical engineering terminology be prepared to eliminate some of the present confusion in this field and to facilitate wider understanding of the subject.

Comments: Related to, providing vocabulary for, but having a somewhat broader base than project 11.3/2-48-2.

Status: Underway (Continuation). About 60% completed. Work proceeded on the assembling, developing, and revising of definitions of terms.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Project 11.3/2-49-1

Origin: Section 11.3, NBS Authorized 1/9/49
Manager: Lola S. Deming

Objective: To prepare and maintain a card catalog of recent publications relating to statistical theory and methodology, containing a technical abstract for each item included, filed by (first) author, and cross-indexed by subject-matter content.

Background: The rate of development of statistical theory and methodology in recent years is alone such as to make it difficult to keep informed of developments in even a small area of the subject. Acquisition of more than a superficial acquaintance with current statistical developments is rendered enormously more difficult by the fact that articles containing statistical theory and methodology are published in a myriad of journals. Many of these, e.g. various journals in the fields of biology, education, medicine, psychology, and the social sciences are not readily accessible to the staff of SEL.

It is expected that this card catalog and associated index will not only facilitate and increase the effectiveness of SEL's operations, but also will be useful to others in the Washington area who are making an extensive or intensive study of some phase of statistical theory or methodology.

Comments: The general plan is to include in the card catalog only items of theoretical or methodological interests;

applications and illustrative material will be included only when of special interest from a theoretical or methodological point of view. Nonrecent and classical items the contents of which are superseded by later work will be included only when of special interest for historical or other reasons.

Status: Inactive. For present status see Projects and Publications Jan-Mar 1949.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Project 11.3/2-49-3

Origin: Section 11.3, NBS Authorized 3/23/49
Managers: Celia S. Martin and J. Lieblein

Objective: To ready for publication a Guide to Tables of the Normal Probability Integral consisting of: Part I, Direct Tables; Part II, Inverse Tables, Having the Integral as Argument; Part III, On Interpolation; Part IV, Comments (e.g. relations to other tabulated functions); and Part V. Bibliography.

Background: Care is required in using tables of the normal probability integral since two major types of table (direct and inverse) are common, and several different methods of tabulation are used for each. Accordingly, Parts I, II, and V were initially prepared over a year ago by Celia S. Martin of the Statistical Engineering Laboratory (SEL) for use in connection with a Bureau in-hours course in Techniques of Statistical Inference. The format and much of the content of Parts I and II were based upon the relevant sections of Chapter 15 of Fletcher, Miller, and Rosenhead, An Index of Mathematical Tables (McGraw-Hill, New York, 1946), with three important changes: the diagrams and the "key" tables were added by Miss Martin, and certain British tables (books) not readily available in U.S.A. were replaced by their American equivalents, with some additions. Corresponding changes were made in the Bibliography. Parts III and IV were prepared quite recently by Julius Lieblein of SEL.

Status: Underway (Continuation). About 90% completed. The text was being revised in the light of comments received, and the notes on interpolation were being rewritten to incorporate suggestions communicated by Dr. H. O. Hartley.

3. Statistical Services

STATISTICAL STUDIES OF CLINICAL THERMOMETER TESTING
Project 11.3/31-47-3

Origin: Section 3.1, NBS Authorized 7/1/47
Manager: J. M. Cameron

Objective: To determine whether, and in what form, acceptance sampling is feasible in inspection and testing of clinical thermometers.

Background: Various Government agencies (e.g., Veterans Administration) that purchase clinical thermometers under contracts referencing Federal Specification GG-T-311; Thermometers, Clinical, use the Bureau as the inspection and testing agency for these thermometers. Although the aforementioned specification includes a provision (Par. F-3) for acceptance or rejection of an entire delivery on the basis of the characteristics and performance of a sample of thermometers from the delivery, many contracts reference the paragraph (F-2) calling for complete inspection and testing of 100% of the thermometers of a delivery. At times backlogs of serious proportions have resulted from this practice. The present project is intended to explore ways and means of reducing the volume of inspection and testing without significant loss of protection to the purchasers.

Comments: Work on an additional report, "Analysis of the reproducibility of the tests for accuracy and consistency of readings," initiated under this project, was discontinued and will be renewed under another project, Project 11.3/31-49-1, under which more suitable data recently became available.

Status: Underway (Continuation). About 95% completed. The previous results of this project (see Projects and Publications Oct-Dec 1948) were reviewed during the quarter by personnel of the Thermometry Laboratory (NBS section 3.1) and by SEL personnel in connection with proposals for revision of Commercial Standard CS1-42, Clinical Thermometers.

On 3 June 1949, Dr. R. E. Wilson of the Thermometry Laboratory and Dr. Churchill Eisenhart of SEL conferred in Baltimore with Dr. Charles P. Winsor, Department of Biostatistics, the John Hopkins School of Hygiene and Public Health,

with Dr. A. McGehee Harvey, Physician-in-chief, the John Hopkins Hospital, and with others on (a) the accuracy and reproducibility required of clinical temperature measurements, (b) methods of determining the average length of life of clinical thermometers under hospital service conditions, and (c) the design of two cooperative experiments to be carried out at the John Hopkins Hospital to determine the reproducibility of clinical temperature readings under customary hospital conditions and procedures, and the correlation of the NBS mechanical hard-shaker tests with manual shaking results of doctors and nurses. Further details of the designs of these experiments were worked out by SEL staff the following week, and the experiments themselves were carried out at the John Hopkins Hospital on 13-14 June under the direction of Dr. R. E. Wilson and Mrs. Josephine S. Timko of the NBS Thermometry Laboratory, Mr. Joseph M. Cameron of SEL, and Dr. Roland V. Rider, Mr. Colin W. Churchill, and Miss McGee, R.N., of the John Hopkins Hospital. The data from these experiments were being analyzed statistically as the quarter closed.

WOOL CONTENT OF BLANKETS
Project 11.3/31-47-6

Origin: Section 7.5, NBS; Division of Statistical Standards, Bureau of the Budget.
Manager: Churchill Eisenhart
Authorized 7/1/47

Objective: To develop a procedure for sampling a lot of part-wool blankets and for taking one or more specimens from each of the sample blankets in order to determine the wool content and weight of the blankets with reasonable assurance.

Background: The present Federal Specifications for blankets gives no procedure, and the A.S.T.M. Standard, a very inadequate procedure, for sampling a lot of blankets. Neither specification gives any instructions for taking one or more specimens from each of the sample blankets for tests and analysis. The scanty instructions given in general specifications on the number of specimens to be subjected to any particular test apply only to the verification of the precision of the test procedures and leave variability of product out of consideration.

The omission of a proper sampling plan for blankets is serious as the specifications require a minimum wool content and a minimum weight without specifying whether these minima apply to the lot, sample, blankets,or specimen. Therefore the manufacturer will generally either have to furnish a large excess of wool content and weight over the requirement or else take a big chance that his blankets will be rejected. The purchaser has the same risk of obtaining a large quantity of material that is deficient in wool content and weight by accepting material on the basis of results obtained on a piece or pieces that are not representative of the material.

It is the aim of this project to determine the variation in wool content and weight from point to point within a blanket and from blanket to blanket within a lot. This will furnish a basis for a rational sampling procedure as well as for giving an operational meaning to the terms minimum "percent wool" and "weight" as commonly applied to blankets.

Comments: This project stems from a project completed in the fiscal year 1947 dealing with the components of the variance of a wool content determination based on a small piece taken at random from a part-wool blanket.

Status: Inactive. About 15% completed. For present status see Projects and Publications Oct-Dec 1948.

EFFECT OF GASOLINE AND OIL ADDITIVES ON CARBON AND GUM FORMATION
Project 11.3/31-47-8

Origin: Division 3 Authorized 7/1/47
Managers: J. M. Cameron, W. J. Youden

Objective: To advise and assist personnel of Section 3.5NBS, with the statistical aspects of the planning and conduct of experiments to determine the effect of gasoline and oil additives on carbon and gum formation of engines.

Background: By utilization of recent advances in the techniques of statistical inference and the principles of experimental design, it is expected that economy and increased efficiency will be effected in this research and testing program. The experiment involves the testing of over 20 additives in combination with a "control" gasoline and oil mixture on 80 similar engines, the problem being to design the

most efficient experiment to determine the performance of the various additives.

Status: Inactive. For present status see Projects and Publications Jan-Mar 1949.

STATISTICAL STUDY OF THE FLOW OF CERTAIN STOCKROOM ITEMS
Project 11.3/31-48-4

Origin: Section M.4, NBS Authorized 5/25/48
Manager: J. Lieblein

Objective: To ascertain for which (if any) of a selected group of stockroom items the pattern of demand is in a state of statistical control on the basis of a Poisson (or other) distribution.

Background: Mr. R. H. Wilson of the Bell Telephone Laboratories has formulated a set of procedures and tables for determining ordering points and related aspects of stockroom activities on the assumption that the demand for items may sometimes be approximated satisfactorily by a Poisson distribution. These procedures have been successfully used by the Bell Telephone Laboratories and others. The Budget and Management Division, NBS, is currently considering their use in stockrooms of the Bureau. The present study aims to provide a basis for evaluating their applicability in NBS stockrooms.

Status: Inactive. For present status see Projects and Publications Jan-Mar 1949.

STATISTICAL ANALYSIS OF THERMOMETRIC MEASUREMENTS
Project 11.3/31-49-4

Origin: Section 3.1, NBS Authorized 12/31/48
Manager: W. J. Youden

Objective: To cooperate with section 3.1 in the statistical analysis of data obtained in testing the efficiency of a new method of aging thermometers.

Background: Thermometers are aged by the manufacturer for a period of months before calibration to allow for volume changes after construction. Since this requires a large stock of thermometers in idle storage, a method for the rapid aging of thermometers has been devised and over a thousand thermometers so prepared have been submitted to Section 3.1 to determine whether the thermometers have been brought to a stable state. The study involves the determination of the precision of the tests and the detection of changes in the thermometers over a period of time. The results of this study will have application in the testing of thermometers generally.

Comments: The data from these tests afford an unusually good opportunity to demonstrate how the analysis of variance may be fitted to the special requirements of precision physical measurements.

Status: Underway (Continuation). About 75% completed. The 6 contemplated tests have been carried out by the Temperature Measurements Section (3.1) and the resulting data for 5 of these have been analyzed by SEL staff.

On 23 May 1949, Dr. R. E. Wilson of section 3.1, and W. J. Youden of SEL were the speakers at an NBS Thermodynamics Colloquium on "Some problems of measurement in testing clinical thermometers"; about 50 members of the Bureau's staff attended.

WOMEN'S BODY-MEASUREMENT STUDY
Project 11.3/31-49-5

Origin: Section 12.2, NBS Authorized 5/13/49
Manager: Lola S. Deming, Celia S. Martin

Objective: Reduction and analysis of certain body-measurement data for women in order to establish standard size designations for patterns, model forms, and wearing apparel. The analysis is intended to indicate which of the basic body measurements are the best predictors of all other measurements and from which representative areas may be chosen to yield sizes for garments, always cognizant of the practicability of fitting body types as well as the statistical efficiency in regard to "coverage" of data.

Background: Various consumer-, distributor-, and producer-groups have urged cooperation from the NBS in overhauling the present very unsatisfactory sizing system for women's apparel.

Some years ago 59 body measurements were taken on approximately 14,700 women living in the U. S. by carefully trained anthropometrists of the Textiles and Clothing Division, Bureau of Home Economics, USDA. The Department of Agriculture has made these data available and is cooperating with the NBS in their analysis.

Comments: This project is an extension, in a sense, of project 11.3/31-47-9, Teen-age girls' body measurement study (completed; see Projects and Publications July-Sept 1948) but studies a different set of data by methods somewhat similar but differing in detail where age demands the use of other basic body measurements.

Status: Underway (New). About 5% completed. The Subcommittee for Wearing Apparel, Sizes, and Measurements of the Mail Order Association of American met in Washington, D. C. on May 25-27, 1949. Representatives of this Laboratory took part in the meetings to assist in the planning of a work program for the analysis of women's body-measurement data upon which to be able to base a sensible and standardized sizing system for apparel. After a general outline of and familiarity with the particular set of data that we have available for this study, particular attention was given to the proper selection of age groups to best represent body types. The tentative age groups selected are 18 through 29 years, 30 through 40 years, 41 through 60 years. The distribution by age as it occurs in the actual population has been obtained from the Census Bureau and will be used to adjust the sample. Machine work is now in progress for obtaining frequency distributionsby single year of age for the 18 measurements considered to be the most basic for further study.

SOLID PROPELLANT DEVELOPMENT TESTS
Project 11.3/32-49-2

Origin: ORDTU, Research and Development Division, Ordnance Dept., U. S. Army
Manager: W. J. Youden
Authorized 7/16/48

Objective: To advise and assist personnel of ORDTU and its contractor on statistical aspects of formulating an ex-

perimental program for determining the significant variables affecting the ballistic properties and reproducibility of certain propellants.

Background: By utilization of recent advances in the techniques of statistical inference and the principles and devices of statistical design of experiments, it is expected that economy and increased efficiency can be effected in this research and development program.

Comments: This is the first of a series of projects sponsored by the Research and Development Group, Logistics Division, General Staff, U. S. Army.

Status: Underway (Continuation). For previous status see Projects and Publications Oct-Dec 1948. Dr. Henry Nocke of the Thiokol Corporation spent May 16, 1949 conferring with Dr. W. J. Youden on some lengthy experimental records and indicating the nature of the problem. He left these records for analysis and interpretation.

PROBABILITY STUDIES OF MISSILE EFFECTIVENESS
Project 11.3/33-49-1

Origin: Bureau of Ordnance, Navy Department, via Electronics Division, NBS
Manager: Gerald J. Lieberman
Authorized 6-13-49

Objective: To develop appropriate three-dimensional multi-parameter probability models to depict the performance of certain missiles, and to study the effects of varying the parameters involved in the probability of 'success'

Background: To aid in proper selection of parameter values in missile design.

Comments: Technical liaison with the Electronics Division (Div. 13) will be provided by Dr. H. K. Skramstad, Chief of the Missile Dynamics Section (13.9).

Status: New

4. Training

STATISTICAL RESEARCH COLLOQUIA
Project 11.3/4-49-4

Origin: Section 11.3, NBS — Authorized 1/18/49
Manager: Churchill Eisenhart — Terminated 6/30/49

Objective: To provide an exchange of information among the staff of the Statistical Engineering Laboratory concerning each member's research and development activity, and to aid in keeping the staff as a whole aware of new developments in statistical theory and methodology.

Background: In view of the extent and rate of development of statistical theory and techniques, individual members of the staff have undertaken to keep informed on developments in particular areas of special interest to them or related to their current tasks. It is intended that these Colloquia will serve to keep the staff as a whole aware of important advances in both statistical theory and technique, and fully informed regarding current research and development activities of SEL. The plan is to hold one-hour sessions once a week.

Status: Terminated (Inactive). Note: In the future, lectures of this type will be listed in the new section of Projects and Publications, entitled Lectures and Symposia.

STATISTICAL ENGINEERING COLLOQUIA
Project 11.3/4-49-5

Origin: Section 11.3, NBS — Authorized 1/18/49
Manager: W. J. Youden — Terminated 6/30/49

Objective: To provide for an exchange of information between personnel of the Statistical Engineering Laboratory

and the Bureau at large on applications of statistical method in physical-science research and engineering testing.

Background: The statistical approach utilized in the solution of a problem brought to the SEL is often applicable to a broad class of problems of which the problem in hand is but a particular example. Groups outside the Bureau are continually asking SEL staff to present expositions of such problems as worked examples of the statistical methods employed in their solution. It is intended that these Colloquia will serve as a proving ground for such expository talks, disseminating information on statistical methods among the Bureau's staff and gaining for SEL staff the benefit of suggestions and criticisms offered. It is anticipated that an hour's talk will be scheduled about once a month.

Status: Terminated (Inactive). Note: In the future, lectures of this type will be listed in the new section of Projects and Publications, entitled Lectures and Symposia.

STATISTICAL DESIGN OF SAMPLING-INSPECTION PROCEDURES
Project 11.3/4-49-7

Origin: NBS Educational Committee — Authorized 3/21/49
Manager: Churchill Eisenhart — Completed 6/7/49

Objective: To offer a one-term in-hours course in the statistical principles and devices appropriate to the construction, selection, and operation of sampling-inspection systems.

Background: There exists today a rapidly developing science of statistically-based sampling-inspection procedures and systems, which originated in the early twenties at the Bell Telephone Laboratories Inc., and first came to public attention with the publication in 1929 of H. F. Dodge and H. G. Romig's "Method of Sampling Inspection." Great strides in the development and application of statistically-based sampling-inspection systems were made during the recent war by the Army Ordnance Department, the Army Quartermaster Corps, and the Army Signal Corps; and by the Navy Bureau of Ordnance and the Navy Bureau of Ships. These advances have had wide effect throughout American industry. The aim of the present course is to expound and illustrate some of the basic statistical principles, devices, and techniques involved, in the hope that these may become everyday tools of those who are

engaged in the preparation or review of purchase specifications.

Comments: This course is listed, with the title "Statistical Design of Experiments (new material)" and designated A12.3 on p. 6 of the Announcement of Courses for 1948-1949 of the Graduate School of the National Bureau of Standards. The idea of having such material presented in a course at the Bureau originated with the Director; the impetus for the present course, with the Organic and Fibrous Materials Division (Division 7).

Status: Completed. Twenty-two persons (21 NBS, 1 outside) enrolled for this course, which began on March 29 and ended on June 7. The final examination consisted of a term paper by each student on an application of the subject matter of the course in his own work to be turned in on 8 July 1949.

IV. Machine Development Laboratory

(Section 11.4)

1. Design and Construction of Automatic Digital Computing Machines

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Project 11.4/21-47-1

Origin: The Bureau of the Census — Authorized 7/1/47
Manager: E. W. Cannon — Termination 2/1/50

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for the preparation of census reports.

Background: As a result of wartime work on electronic computing machines and related developments, it has appeared for some time that a revolution is imminent in methods of compilation and tabulation of statistical data. The construction and successful operation of an electronic digital computing machine, the ENIAC, and other developments in the electronic computing machine field clearly pointed out the possibility of constructing electronic digital equipment to carry out the types of manipulation of data involved in both the regular census compilations and in the newer sampling techniques. It is expected that the proposed electronic equipment will perform at increases in speed over existing equipment of a factor of 10 to 20 in some operations, and up to 100 or 200 or even more for other operations.

Comments: This project is related to Project 11.4/22-47-2, in that contractors for each project were made aware of the performance specifications for both projects, and were informed that a single model might finally be selected for both projects. Project 11.4/22-47-2 involves mathematical work related to the present project as well as to other projects.

Status: Underway (Continuation). The Eckert-Mauchly Computer Corporation tentatively approved the performance test for the BINAC submitted by the Bureau. This performance test was devised to serve as an acceptance test for those design features of the BINAC which will be incorporated in the UNIVAC System the Corporation is constructing to meet the needs of the Census Bureau. An evaluation of the test performance of the BINAC was under way by the Company. Also, the company demon-

strated to the Bureau a tape handling device, the UNISERVO, in a form approaching suitability for submission as a checkpoint under the UNIVAC construction contract.

THE NAVY COMPUTING MACHINE
Project 11.4/22-47-2

Origin: Mathematics Branch, Office of Naval Research
Manager: E. W. Cannon
Authorized 7/1/47 Termination 12/1/50

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for general mathematical computation.

Background: The project was undertaken to meet the need, recognized by the Mathematics Section of ONR, for faster and more efficient computing machinery than that now existing. Included among the problems at which the machine is aimed are the following: (a) Problems involving the systematic handling of large linear arrays (e.g., determination of the characteristic roots of matrices arising in vibration theory and quantum mechanics; solutions of systems of linear equations such as those which arise in vibration problems, metallurgical problems, weather problems, multivariate statistical analysis); (b) problems involving the solution of linear and non-linear partial differential equations, such as those which arise in the study of supersonic phenomena, turbulent flow, flow of viscous fluids, weather problems, servo-mechanisms, non-linear electrical oscillations and so on.

Comments: See Comments for 11.4/21-47-1.

Status: Underway (Continuation). Construction of a mercury delay line electronic computer to meet the needs of the Navy, which was contracted to the Raytheon Manufacturing Corporation, was progressing. Layout of the logical design and construction of the memory line component were begun. This computer will be similar in design to the general-purpose electronic digital computer that Raytheon is constructing in connection with the development of an integrated test data reduction system that it has under way for the Office of Special Devices of the Navy Department.

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.4/23-49-1

Origin: Air Material Command, USAF — Authorized 3/21/49
Manager: E. W. Cannon — Termination 6/1/50

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for use by the Air Materiel Command in connection with air research problems.

Background: The AMC requires a high-speed and flexible computing machine to perform the involved calculations arising in the multifold problems associated with the design and development of aeronautical equipment and accessories. The problems involved range from the computation and evaluation of voluminous test data to the solution of intricate problems related to the design and performance prediction of airfoils, propellers, propulsive equipment and both subsonic and supersonic missiles. The computer should have a large memory capacity and a precision sufficiently high to furnish significant answers to problems like those in flutter analysis, which involve the treatment of large linear systems.

Comments: This project is related to projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/24-47-3a.

Status: Underway (Continuation). Negotiations were under way with the General Electric Company and with Engineering Research Associates for an automatically-sequenced digital computer suitable for the needs of the AMC. Liaison with ERA regarding computational requirements of the AMC was strengthened by the assistance of the Harvard Computation Laboratory. A member of the staff of the ERA joined the Harvard Computation Laboratory for a month of orientation duty. It was planned that at the conclusion of this period the ERA proposals for magnetic-drum computers will be reevaluated at a joint conference of representatives of the Company, the Harvard Computation Laboratory and the Bureau.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Project 11.4/24-47-3

Origin: Office of the Air Comptroller, USAF. — Authorized 7/1/47
Manager: E. W. Cannon — Termination 1/1/50

Objective: To develop specifications for, and to construct an automatic-sequenced electronic digital computing machine suitable for use by the U. S. Air Force in program planning and control.

Background: The Air Comptroller's Office requires a high-speed and flexible computing machine to calculate detailed programs consistent with general policy decisions, and to facilitate rapid recomputation of programs to meet budgetary and other limitations. The problems involved are of wide applicability, and a part of the present project consists in formulating them mathematically. It is expected that the primary computation problem to be solved by the machine will consist of finding rapidly the solutions of large systems of simultaneous equations containing up to 1000 unknowns. The computer must be able to store and classify large quantities of data, and to refer rapidly for needed items to huge tables of organization, equipment, supply and other similar data. These tables will contain millions of items. It is required, in addition, that the printing devices associated with the computer will be capable of extremely high-speed printing of the complete details of the Air Force's programs that have been computed.

Comments: This project and projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/3-47-4 are interrelated. Project 11.4/3-47-4 serves to coordinate the mathematical direction of the three computing machine projects.

Status: Underway (Continuation). A contract had been signed with the Eckert-Mauchly Computer Corporation for the construction of a UNIVAC System for the Air Comptroller's Office. During this quarter work under this project was concerned with "proving in" of the UNIVAC basic design (by testing of the BINAC) and of engineering development related to the tape handling device, UNISERVO. See status of project 11.4/21-47-1.

THE NBS INTERIM COMPUTER
Project 11.4/24-49-1

Origin: NBS — Authorized 12/15/48
Manager: S. Lubkin — Termination 10/1/49

Objective: To develop specifications for and to construct a modest-scale electronic digital computing machine.

Background: Early use of even a limited capacity electronic digital computer system would help offset the disadvantage of the late delivery dates for the large-scale electronic computing equipment under construction by commercial suppliers. It is contemplated that the Interim Computer will be in operation by the fall of 1949, and that it will be of great value both as a problem-solving and a test device.

Comments: This project, supported by the Air Comptroller's Office, U. S. Air Force, and by the Department of the Army, has been undertaken by Divisions 11 and 13 of the National Bureau of Standards. The layout of the system is the responsibility of Division 11; Division 13 is directing the engineering design and construction. Dr. Lubkin serves as consultant to the Computer Section of Division 13 on certain phases of the circuitry design and breadboarding.

Status: Underway (Continuation). Modifications of the original logical design suggested by engineering considerations were evaluated from the standpoint of effect upon performance characteristics, cost in additional equipment and circuitry, and relation to the construction schedule. The logical design has been integrated with the actual circuitry and is now regarded as fixed. A contract for a 512-word mercury line memory unit for the computer was negotiated with the Technitrol Company, Philadelphia, Pa. Fabrication of the arithmetic and control units was under way in the laboratory and shop of the Electronic Computers Section.

ARMY MAP SERVICE COMPUTING MACHINE
Project 11.4/25-49-1

Origin: Army Map Service, U.S.A. Authorized 12/15/48
Manager: E. W. Cannon Termination 6/1/50

Objective: To develop specifications for and to construct an automatic-sequenced electronic digital computing machine suitable for use by the Army Map Service in cartography.

Background: In the development of maps, particularly where various maps based on different projection systems are combined into one large map based upon a single projection system suitable

for military application, there is required mass production of detailed calculations on point coordinates. A suitable computer must be able to handle huge masses of raw data, perform a calculation on each item of data and print output results in usable tabular form with speed comparable to that at which data enters the machine.

Comments: This project and projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/3-47-4 are interrelated. Project 11.4/3-47-4 serves to coordinate the mathematical direction of the three computing machine projects.

Status: Underway (Continuation). Progress on the construction of a UNIVAC system for the Army Map Service by the Eckert-Mauchly Corporation was the same as that indicated above for the UNIVAC for the Air Comptroller's Office, under 11.4/24-47-3.

DEPARTMENT OF THE ARMY COMPUTER DESIGN
Project 11.4/26/49-1

Origin: Department of the Army Authorized 12/15/48
Manager: S. Lubkin

Objective: To design a large-scale electronic digital computer which will be suited to specified needs of the Department of the Army.

Background: It was considered by the group of the Department of the Army dealing with the Bureau that an electronic computer of the EDVAC type would serve their needs along the lines of high-speed computing. Because of Dr. Lubkin's comprehensive knowledge of design features of the EDVAC-type computer, from both the electronics and logical systems viewpoint, the requested design was considered to be a feasible undertaking for the Bureau.

Comments: This project concerns the design of a system and the determination of circuit constants for a large-scale electronic computer of the EDVAC type. The work under this project and under project 11.4/24-47-3 is being conducted by the Bureau as far as possible. Division 11 is responsible for the mathematical sufficiency of the design, and Division 13 for its engineering soundness.

Status: Underway (Continuation). The logical design of a large-scale electronic digital computer of the EDVAC type to meet certain special needs of the Department of the Army was completed during the previous quarter. Preparation of the circuitry layout was started during this quarter by the Electronic Computers Section of the Electronics Division.

2. Programing Studies

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Project 11.4/3-47-4

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Manager: Ida Rhodes
Authorized 7/1/47

Objective: To program certain general types of mathematical and statistical routines such as sorting, collating, and the solutions of larger problems proposed by the Bureau of the Census, the Navy, the Air Force, and the Army for solution on automatic digital computing machines. Thereby to detect deficiencies in, and effect improvements in, the design of proposed machines; also to establish a library of routines for the above-mentioned types of problems and thus eliminate the necessity for the programmer to repeat the construction of a program whenever he is confronted with certain problems.

Background: The project was initially undertaken to insure proper coordination of projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/24-47-3. A further justification lies in the fact that when automatic computing machinery becomes generally available it will be necessary to have collections of programs for the routine mathematical operations, so that problem preparation can be expedited as much as possible.

Comments: This project serves as the foundation of the mathematical direction of all the projects of Section 11.4

relating primarily to the development of electronic digital computing machinery. The performance characteristics of proposed automatically-sequenced electronic digital computing machines are carefully analyzed, and the performance on selected classes of problems of the various machines under consideration is evaluated. In addition, the project concerns the development of mathematical techniques adapted to the newer electronic digital computing machines under design and construction. The project is expected to serve as a groundwork for the preparation of manuals of operation for the automatic computing machines constructed under the supervision of the Bureau, and for investigation by Section 11.4 of the solution by use of these machines of important and difficult problems now beyond the computational horizon.

Status: Underway (Continuation). The codes for the NBS Interim Computer were amended to provide greater flexibility and efficiency in the programing of problems. The binary point was moved two positions to the right; division and absolute value comparison orders were added. Subroutines were in process of being rewritten to reflect the above changes, and new routines were being prepared for application on this computer.

A study of the relative efficiency of the three-address versus the four-address instruction code was conducted. The elements considered were: 1) the number of memory positions required to achieve the solution of a problem, 2) machine time required to execute the problem, and 3) coding time required in preparing the problem for the computer. For subroutines, an average saving of approximately 20% in items (1) and (2) above was effected by the use of the four-address notation. The time required for the preparation of the problem was about the same under both systems. Although an insufficient number of complete problems were prepared for final evaluation of the codes, the analysis seemed to indicate a superiority of the four-address code in respect to 1) and 2) by a factor of 10-15 per cent.

CODING ON THE E. R. A. COMPUTER
Project 11.4/3-49-1

Origin: NBS Authorized 12/1/48
Manager: Ida Rhodes

Objective: To evaluate the performance characteristics of the Electronics Research Associates design for an electronic computer having a magnetic-drum internal memory.

Background: In order to check its appraisal of the suitability of magnetic-drum type computers for Office of Naval Research applications, the Bureau entered into a contract with the E.R.A.,for the preparation of designs of such computers and the evaluation of their mathematical capacity. Thus the Bureau's efforts to appraise fairly magnetic-drum computing machines were supplemented by intensive technical and mathematical work by E.R.A.

Comments: This project was essentially a development of Project 11.4/22-47-2.

Status: Underway (Continuation). At the request of the Air Materiel Command, the ERA computer design was submitted to the Harvard Computation Laboratory for evaluation. It was agreed by the Bureau and the Harvard Computation Laboratory that modifications of the design were necessary if the computer were to meet the needs of the AMC. Also it was agreed that further conformance with ERA concerning design proposals for rotating drum automatic computers suitable for the purposes of the AMC be deferred to provide an orientation opportunity for ERA. Dr. Aiken consented to act as consultant to the AMC and the Bureau in the further evaluation of design proposals of automatic computers for use by the Air Materiel Command.

CODING RELATED TO THE UNIVAC SYSTEM
Project 11.4/3-49-2

Origin: The Bureau of the Census Authorized 12/1/48
Manager: Florence Koons

Objective: To determine the performance to be expected of the UNIVAC SYSTEM on various problems of importance to the Bureau of the Census.

Background: A useful service of the Machine Development Laboratory to groups planning to use automatically-sequenced electronic digital computing machines is the preparation of instruction codes for the solution of important problems by these computing machines. The potential mathematical utility of the electronic computers for which this coding is per-

formed can be closely evaluated.

Status: Underway (Continuation). Extensive coding for the UNIVAC was discontinued temporarily because of the probability of modifications in the instruction code. A preliminary analysis of the changes in the performance of the input-output equipment was undertaken. This analysis indicates that although the machine time for the execution of certain problems may be slightly increased, the overall efficiency of the instruction code will not be seriously impaired.

CODING RELATED TO THE RAYTHEON COMPUTER
Project 11.4/3-49-3

Origin: Mathematics Branch, Office of Naval Research
Manager: Ida Rhodes
Authorized 12/1/48

Objective: To evaluate the mathematical sufficiency of the proposed Raytheon electronic computer.

Background: The Raytheon Company has submitted to the Bureau a series of design modifications intended to increase the flexibility and power of the proposed computer. By coding basic mathematical routines for solution on the Raytheon computer, in its various stages of design, the Bureau has evaluated successive designs.

Status: Inactive, pending further advancement in the design and construction of the proposed computer. See project 11.4/22-47-2.

PERFORMANCE TEST FOR THE BINAC
Project 11.4/3-49-4

Origin: NBS Authorized 12/1/48
Manager: Ida Rhodes

Objective: To devise an instruction code for a rigorous test problem on the BINAC.

Background: The BINAC (designed and under construction by the Eckert-Mauchly Computer Corporation) is an automatically-sequenced electronic digital computer embodying many of the important principles of engineering design of the larger and more powerful ENIAC. Satisfactory performance of the BINAC on an extensive test problem has therefore been stipulated by the Bureau as a prerequisite to consideration of the engineering design of the UNIVAC for approval.

Comments: This project is essentially an offshoot of project 11.4/21-47-1.

Status: Underway (Continuation). A problem for solution on the BINAC was submitted by the Air Comptroller's Office. This problem entailed the computation of requirements for the Air Force, and is designed to demonstrate the applicability as well as the limitations of an electronic computer with a small memory and manual input-output requirements. Programming of this problem for solution on the BINAC was under way.

3. Technical Reports on Computing Machinery

THE MTAC SECTION
Project 11.4/4-47-1

Origin: Committee on High-Speed Computing of the National Research Council
Manager: Edith T. Norris Authorized 7-1-47

Objective: To assemble and edit material for a Section entitled "Automatic Computing Machinery" in the quarterly Mathematical Tables and Other Aids to Computation (MTAC), published by the National Research Council.

Background: This Section is to serve as a news letter, a medium for exchange of opinions, and a vehicle for the publication of shorter technical papers, in the field of automatic high-speed calculating machinery. The need for such a service has been pointed out repeatedly by groups in-

terested in such machinery. The decision to sponsor this task and to assign the basic editorial work to the NAML was made at a joint meeting of the Committee on High-Speed Computing and on Mathematical Tables and Other Aids to Computation in New York in April 1947.

Status: Underway (Continuation). The July 1949 issue of Mathematical Tables and Other Aids to Computation contained a technical paper entitled, "The Solution of Simultaneous Linear Equations with the Aid of the 602 Calculating Punch," by Frank M. Verzuh of the Massachusetts Institute of Technology. The method of solving simultaneous equations described here is the well-known basic elimination method with the usual "back-substitution" procedure eliminated to allow repetition throughout of the same basic elimination procedure. The paper presents an interesting application of the IBM 602 Calculating Punch. Also included in this issue was a discussion article, "A New General Method for Finding Roots of Polynomial Equations," by Edward F. Moore. To alleviate the necessity for human intervention or for elaborate programs inherent in most machine methods for calculating roots of polynomial equations, an iterative method based on the Graeffs root-squaring process is proposed to work for all polynomial equations in a fixed number of iterations regardless of degree, location of roots, or their multiplicities. The method can be used without change on equations having complex coefficients covering a wide range of magnitudes.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Project 11.4/42-49-2

Origin: Section 11.4, NBS Authorized 5/5/49
Manager: Edith Norris

Objective: To prepare for publication and maintain a bibliography of recent important publications relating to high-speed automatic computing machinery.

Background: Recent major developments in the field of high-speed computing machinery, with particular reference to high-speed electronic computers, have necessitated the compilation and maintenance of a bibliography covering the electronic and mathematical phases of computer development. This specific project has been undertaken in response to numerous requests for such a bibliography and upon the informal recom-

mendation of an R.D.B. working group. It is hoped that this bibliography will not only prove valuable to Machine Development Laboratory personnel but also to interested persons in other Government agencies and in private industry.

Comments: It is expected that regular supplements to this bibliography will be prepared for distribution. The assistance of many of the large computer projects both in this country and abroad is hereby acknowledged. As most of the important work in this field has taken place during the past ten years, non-recent items will be included only when of special historical interest. Not all material published on the subject during this period has been listed; rather, an attempt has been made to include only the more significant of the more accessible writings.

Status: Underway (New). About 80% completed.

Lectures and Symposia

Symposia at the Institute for Numerical Analysis

A SYMPOSIUM ON THE CONSTRUCTION AND APPLICATIONS OF CONFORMAL MAPS, and a SYMPOSIUM ON THE MONTE CARLO METHOD were held in Los Angeles at the Institute for Numerical Analysis at the end of June. Sandwiched between these two symposia was a two-day condensed course in automatic computation open to the scientific public and given under the direction of Harry D. Huskey, head of the Machine Development Unit of the Institute. The symposia and the condensed course formed an integrated series of lectures and discussions on topics pertinent to the work of the Institute which lasted for some ten days, from June 22 to July 1 inclusive. The combined registration for all the events, as recorded at the Institute, totaled about 350 persons. The meetings were held in various buildings on the campus of the University of California at Los Angeles, where the Institute for Numerical Analysis is located.

The purposes of the SYMPOSIUM ON THE CONSTRUCTION AND APPLICATIONS OF CONFORMAL MAPS were to consider physical applications of conformal maps and their generalizations, and to study the construction of conformal maps with a view to determining the possible applicability of high-speed electronic digital computing machines in this direction. This symposium began on June 22 with a one-day course on programming for automatic computers given by the staff of the National Applied Mathematics Laboratories. This course was designed to acquaint those attending the later meetings of the symposium with the preparation of problems for automatic digital computing machines, and was especially arranged to appeal to mature mathematical analysts and applied mathematicians. This symposium was arranged by a committee of the Institute consisting of Edwin F. Beckenbach, chairman, Cornelius Lanczos, Alexander Ostrowski, and Wladimir Seidel. The speakers and topics of the sessions were as follows:

Wednesday, June 22 - Programming on Automatic Computers.

9.30 A. M. Lecture I: Definition of an automatic digital computing machine. Discussion of the transition from mathematical language to machine language. Hierarchies of routines. Habit-forming in machines. H. D. HUSKEY. National Bureau of Standards.

10.15 A. M. Lecture II: Description of a specific automatic computer. Discussion of coding and programming. Programming of simple operations. Use of flow diagrams. Floating operations. ROSELYN SIEGEL, National Bureau of Standards.

11.00 A. M. Lecture III: Programming the solution of n simultaneous linear equations. The use of parameters. Iterations. Positioning for size. Checking the computations. H.D. HUSKEY.

2.00 P. M. Laboratory. Participants urged to work out detailed routines for causing the computers to perform such operations as division, floating addition, solving a simple differential equation, and possibly, solving a set of simultaneous linear equations. MEMBERS of the STAFF of the National Applied Mathematics Laboratories.

Thursday, June 23 - General Session. J. H. CURTISS, National Bureau of Standards, Chairman.

9.45 A. M. On network methods in conformal mapping computation. R. VON MISES, Graduate School of Engineering, Harvard University.

11.00 A. M. Conformal mapping of domains of higher topological structure illustrated by flow patterns. R. COURANT, Institute for Mathematics and Mechanics, New York University.

Thursday, June 23 - Session on Physical and Industrial Applications. D. V. WIDDER, Harvard University, Chairman.

2.00 P. M. Applications of conformal mapping to torsional rigidity, principal frequency, and electrostatic capacity. G. SZEGO, Stanford University.

3.00 P. M. Some industrial applications of conformal mapping. H. PORITSKY, General Electric Company.

4.00 P. M. Conformal maps involving multiply connected regions and their technical applications. G. STEIN, Westinghouse Electric Corporation.

By Title. On the use of conformal mapping in problems of two dimensional elasticity. I.S. SOKOLNIKOFF, University of California, Los Angeles.

Friday, June 24 - Session on Fluid Dynamics. J.L. BARNES, University of California, Los Angeles, and North American Aviation, Inc., Chairman.

9.00 A. M. On the Helmholtz problem of conformal representation. ALEXANDER WEINSTEIN, U. S. Naval Ordance Laboratory and the University of Maryland.

10.00 A. M. Aspects of conformal mapping in aerodynamics. I. E. GARRICK, Langely Aeronautical Laboratory, N. A. C. A.

10.30 A. M. On Theodorsen's method of conformal mapping. ALEXANDER OSTROWSKI, University of Basle, and National Bureau of Standards.

11.00 A. M. The use of conformal mapping in the study of flow phenomena at the free surface of an infinite sea. EUGENE P. COOPER, U. S. Naval Ordnance Test Station, Pasadena, California.

11.30 A. M. Fluid dynamics, conformal mapping and numerical methods. ANDREW VAZSONYI, U. S. Naval Ordnance Test Station, Pasadena, California.

Friday, June 24 - Session on the Theory of Conformal Maps. W. T. MARTIN, Massachusetts Institute of Technology and University of California, Los Angeles (Summer Session). Chairman.

2.00 P. M. Some remarks on variational methods applicable to multiply connected domains. D. C. SPENCER, Stanford University.

2.45 P. M. A variational method for simply connected domains. A. C. SCHAEFFER, Purdue University.

3.15 P. M. Kernel functions and conformal mapping. STEFAN BERGMAN, Harvard University, and MENAHEM SCHIFFER, Stanford University.

4.00 P. M. The kernel function and canonical conformal maps. ZEEV NEHARI, Washington University.

4.30 P. M. A new proof of the Riemann mapping theorem. P. R. GARABEDIAN, Stanford University.

Saturday, June 25 - General Session. J. W. GREEN, University of California, Los Angeles, Chairman.

9.00 A. M. Conformal mapping applied to electromagnetic field problems. ERNST WEBER, Polytechnic Institute of Brooklyn.

9.30 A. M. Conformal invariants. L. V. AHLFORS, Harvard University.

10.00 A. M. Some generalizations of conformal mappings occurring in gas dynamics. LIPMAN BERS, Syracuse University.

10.30 A. M. On conformal mapping of variable regions. S. E. WARSCHAWSKI, University of Minnesota and National Bureau of Standards.

10.45 A. M. Recent contributions of the Hungarian school to conformal mapping. G. SZEGO, Stanford University.

11.00 A. M. On the effective determination of conformal maps. MAX SHIFFMAN, Stanford University.

11.30 A. M. Monodiffric functions. R. P. ISAACS, RAND Corporation.

12.00 N. The difference equation method for solving the Dirichlet problem. P. C. ROSENBLUM, Syracuse University.

Saturday, June 25 - Session on Numerical Methods. H. F. BOHNENBLUST, California Institute of Technology, Chairman.

2.00 P. M. Relaxation methods as ancillary techniques. SIR RICHARD SOUTHWELL, late Rector of the Imperial College of Science and Technology, London, England.

3.00 P. M. The use of conformal mapping to compute flows with free streamline. D. M. YOUNG, Harvard University.

3.30 P. M. (1) An approximation method for conformal maps. (2) Note on the preparation of a dictionary of conformal maps. LEE H. SWINFORD, University of California, Berkeley.

4.00 P. M. An application of conformal mapping to problems in conical supersonic flow. R.C.F. BARTELS and O. LAPORTE, University of Michigan.

4.30 P. M. Asymptotic developments at the confluence of boundary conditions. H. LEWY, University of California.

The SYMPOSIUM ON THE MONTE CARLO METHOD was jointly arranged and sponsored by the RAND Corporation of Santa Monica and the Institute for Numerical Analysis, with the cooperation of the Oak Ridge National Laboratory. The committee on arrangements consisted of J. H. Curtiss, National Bureau of Standards, Hallett H. Germond, RAND Corporation, Alston S. Householder, Oak Ridge National Laboratory, Cuthbert C. Hurd, International Business Machines Corporation, and Raymond P. Peterson, National Bureau of Standards. The Symposium was held on June 29, 30 and July 1.

The Monte Carlo Method can be described quite generally as the representation of a physical or mathematical system by a sampling operation satisfying the same probability laws. Thus for example, the numerical integration of partial differential equations of a certain type can be accomplished by building up a large sample of trials of certain stochastic processes whose probability functions asymptotically satisfy the partial differential equations. In certain physical situations formerly represented by such equations, the physicist may prefer to place primary emphasis on the stochastic processes and

the associated sampling operations, which he will then regard as new mathematical models to be used in place of the continuous models of classical applied mathematics.

The purposes of this symposium were to interchange information concerning useful applications of the method, to stimulate discussion relative to its limitations and to indicate directions in which further theoretical research is needed. The speakers and topics of the sessions were as follows:

Wednesday, June 29 - 10.00 A. M. Orientation Session.
J. H. CURTISS, National Bureau of Standards, Chairman.

I. The Monte Carlo Method. S. ULAM, Los Alamos Scientific Laboratory.

II. Discussion. J. VON NEUMANN, Institute for Advanced Study.

Wednesday, June 29 - 2.00 P. M. Session on Physical Applications of Stochastic Methods. J. VON NEUMANN, Institute for Advanced Study, Chairman.

1. Introductory remarks by the Chairman.
2. Showers produced by low energy electrons and photons. ROBERT R. WILSON, Cornell University.
3. Nomograms for Monte Carlo Solution of the Milne Problem. B. I. SPINRAD, Argonne National Laboratory and G. H. GOERTZEL, New York University.
4. Neutron age calculations in water, graphite, and tissue. A. S. HOUSEHOLDER, Oak Ridge National Laboratory.
5. Methods of probabilities in chains applied to particle transmission through matter. LEWIS NELSON, Oak Ridge National Laboratory and WENDELL DEMARCUS, Nepa Project, Fairchild Engine and Airplane Corporation.
6. Multiply-scattered gamma rays. UGO FANO, National Bureau of Standards.
7. Stochastic methods in statistical mechanics. GILBERT W. KING, Arthur D. Little Company.

Thursday, June 30 - 10.00 A. M. Session on Physical Applications of Stochastic Methods (continued). FRANK C. HOYT, Argonne National Laboratory, Chairman.

1. Calculations on a water shield for fast neutrons, I. MARIA MAYER, Argonne National Laboratory.

2. Calculations on a water shield for fast neutrons, II. PRESTON HAMMER, Los Alamos Scientific Laboratory.
3. A Monte Carlo technique for estimating particle attenuation in bulk matter. B.A. SHOOR, Northrop Aircraft Incorporated, LEWIS NELSON, Oak Ridge National Laboratory, WENDELL DeMARCUS, and ROBERT L. ECHOLS, Nepa Project, Fairchild Engine and Airplane Corporation.
4. Neutron transmission through thick slabs. WILLISTON SHOR, U. S. Navy.
5. Estimation of particle transmission by random sampling. HERMAN KAHN and T. E. HARRIS, RAND Corporation.

Thursday, June 30, 2.00 P. M. - Session on Random Digits. JERZY NEYMAN, University of California, Chairman.

I. Generation and Testing.
1. GEORGE W. BROWN, RAND Corporation.
2. NICHOLAS METROPOLIS, Los Alamos Scientific Laboratory.
3. GEORGE E. FORSYTHE, National Bureau of Standards.

II. Various Techniques Used in Connection with Random Digits. JOHN VON NEUMANN, Institute for Advanced Study.

III. General Discussion From the Floor.

Friday, July 1, 10.00 A. M. - Mathematical Session, JOHN W. TUKEY, Princeton University, Chairman.

I. The Connection Between Stochastic Processes and Partial Differential Equations.
1. WILLIAM FELLER, Cornell University and the National Bureau of Standards.
2. MARK KAC, Cornell University and the National Bureau of Standards.
3. J. L. DOOB, University of Illinois.

II. Discussion From the Floor.

Friday, July 1, 2.00 P. M. - Round Table Discussion. Critique of Applications and Discussion of Possible New Directions for Research. JOHN W. TUKEY, Princeton University, Discussion Leader.

Panel J. NEYMAN, University of California, Berkeley,
M. A. GIRSCHICK, Stanford University,
W. A. WALLIS, University of Chicago,
D. H. BLACKWELL, Howard University
T. W. ANDERSON, Columbia University

Panel (Continued)

L. J. SAVAGE, University of Chicago, T. Harris, RAND Corporation, H. KAHN, RAND Corporation, and J. Wishart, University of California.

A BRIEF COURSE ON AUTOMATIC COMPUTATION was held between the two symposia, on July 27-28, under the direction of Harry D. Huskey. The purpose of this course was to introduce interested persons to the logical theory and performance characteristics of automatic digital calculators being developed at the present time. The course was very condensed and relatively elementary. Preparation of problems for such calculators was emphasized. The topics and speakers were as follows:

Monday, June 27 - Lecture Session

9.00 A. M. Introduction. Description of automatic computers. What speeds are possible? Why must they be automatic? Comparison with pre-electronic computing systems. E. W. CANNON, National Bureau of Standards.

9.45 A. M. Types of automatic computing machines. Parallel versus serial. Decimal versus binary. Machine language--ideal versus practical. Reliability versus speed. IDA RHODES, National Bureau of Standards.

10.30 A. M. The logical design of automatic computers. Coincidence, buffing, memories. The algebra of such circuitry. R. J. SLUTZ, National Bureau of Standards.

11.15 A. M. Programming. Flow diagrams. Division Floating operations. Double length operations. Library of routines. Generalized routines. G. E. FORSYTHE, National Bureau of Standards.

Monday, June 27 - 2.00 P. M. Laboratory Session

Tuesday, June 28 - Lecture Session

9.00 A. M. Hierarchies in coding. Abbreviated code instructions. Iterations. The use of parameters. H. D. HUSKEY, National Bureau of Standards.

9.45 A. M. Discussion of automatic digital electronic computers now under development. Status of various calculators, special characteristics and instruction codes. (EDVAC, EDSAC, UNIVAC, Raytheon, Institute for Advanced Study, Whirlwind, ERA, Mark III, N.B.S. Interim, Institute for Numerical Analysis Computer.) E. W. CANNON, H.D.HUSKEY, I. RHODES, R. J. SLUTZ, all of National Bureau of Standards, and P. MORTON, University of California.

Tuesday, June 28 - 2.00 P. M. Laboratory Session.

NBS Applied Mathematics Colloquium Series

Item VIII. Five lectures on "Stochastic Processes", presented by H. B. MANN of the National Bureau of Standards on June 13-17:

(1) "A Birdseye View of the Theory of Stochastic Processes."

(2) "Fundamental Concepts and Operations." - Random variables, definition of a stochastic process, Markoff processes with continuous and independent increments, differentiation and integration.

(3) "Theory of Statistical Inference Applied to Markoff Processes." - Modification of Markoff processes with continuous increments, Ornstein-Uhlenbeck process, estimation of parameters, stochastic differential equations.

(4) "Further Applications of Statistical Inference." - Markoff processes with discontinuous increments, general representation of Markoff processes.

(5) "Stochastic Aspects of Counting Devices." - Markoff processes modified by recording instruments, probability problems in counter theory.

Item IX. "Statistical Design of Experiments," presented by H. O. HARTLEY of Princeton University and the University of London on June 21, 1949.

Numerical Analysis Colloquium Series (Los Angeles, California)

LANCZOS, C. "An iterative method for the solution of the eigen-value problem for ordinary linear differential operators." April 4, 1949.

SIEGEL, ROSELYN A. "Coding for an electronic calculating machine." April 11, 1949.

BESICOVITCH, A. S. (University of Cambridge, England). (1) "Parametric surfaces and the problems of plateau." April 18, 1949. (2) "An extremal problem for convex curves." April 19, 1949.

GILVARY, J. J. (RAND Corporation). "Blind navigation by dynamical measurements." April 25, 1949.

DAVIS, C. F. (North American Aviation Inc.) "Some experiments in solving linear equations in many unknowns." May 9, 1949.

HASTINGS, C., Jr., (RAND Corporation). "Rational approximations in high-speed computing." May 16, 1949.

BROWN, C. (RAND Corporation). "An iterative method for solving discrete games." May 23, 1949.

CAHN, A. S., Jr. "Separation of exponentials." June 6, 1949.

OSTROWSKI, A. M. (University of Basle, Switzerland, and the National Bureau of Standards). "On the types of convergence and accuracy of approximation." June 13, 1949.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ABRAMOWITZ, M. "Laguerre functions expressed in terms of Bessel-Clifford functions." Presented by title at the meeting of the American Mathematical Society in Philadelphia, Pa., April 29-30, 1949.

ALT, F. L. "Digital Computation of Trajectories," a lecture on methods of digital computation applicable to trajectory problems, and on automatic equipment available for such purposes. Presented at a symposium on automatic computation arranged by the Research Development Board for a group of officers of the Services, April 25, 1949.

BLANCH, G. "On the Inversion of Matrices." Presented at the meeting of the Association for Computing Machinery held at Oak Ridge, Tennessee, April 18, 1949.

CANNON, E. W. (1) "The Bureau of Standards Digital Computer Program", presented to the Ad Hoc Committee on Scientific Analysis of the Research and Development Board, Washington, D. C., May 9, 1949. (2) "Idiosynchrasies of Automatic Electronic Digital Computers," presented before the Mathematics Club of the University of Delaware, Newark, Del., May 24, 1949. (3) "Limitations of Electronic Digital Computing Machines," given at the meeting of the Association for Computing Machinery, held at Oak Ridge, Tenn., May 30, 1949.

FORSYTHE, G. E. "Use of Computing Machines in Meteorology." Presented at the Los Angeles Seminar of the American Meteorology Society at U.C.L.A., April 20, 1949.

HERRICK, S. (1) "Rocket Navigation." Presented at the Institute of Navigation held at the Naval Academy, Annapolis, Maryland, June 9, 1949. (2) "Tables for Rectilinear and Nearly Rectilinear ("Nearly Parabolic") Orbits." Presented at the Meeting of the American Astronomical Society held at Ottawa, Canada. June 20, 1949.

HUSKEY, H. D. (1) "Statistical Theory of Round-Off Errors." Presented at the meeting of the Association for Computing Machinery held at Oak Ridge, Tennessee, April 19, 1949. (2) "Automatic Digital Computing Today." Presented at the Southwestern Section Meeting of the Mathematical Association of America held at the New Mexico College of Agriculture and Mechanical Arts, State College, New Mexico on April 29, 1949. (3) "Electronic Digital Computing in the United States." Paper read for Dr. Huskey at the Conference on High Speed Automatic Calculating Machines held at Cambridge University, England. June 22-25, 1949.

LANCZOS, C. (1) "An Iteration Method for the Solution of Eigen Value Problem of Ordinary Linear Differential Operators." Presented at the meeting of the American Mathematical Society held at Stanford University on April 30, 1949.

(2) A Logical Extension of the Field Equation of General Relativity." Presented at the American Physical Society Meeting held at the University of Washington, Seattle, June 29, 1949.

LUBKIN, S. "The NBS Interim Computer", given at the meeting of the Association for Computing Machinery held at Oak Ridge, Tennessee, May 30, 1949.

OSTROWSKI, A. M. (1) "On Two Problems in Abstract Algebra Connected with Horner's Rule." Presented at the Meeting of the American Mathematical Society at Stanford University, April 30, 1949. (2) "On the Continuity of Roots of Algebraic Equations." Presented at the Mathematic Department Seminar at the University of California at Los Angeles on May 24, 1949. (3) "On the Generalization of the Cauchy-Frullani Integral." Presented at the Peripathetic Seminar at the University of Southern California on June 6, 1949. (4) "On Descarte's Rule of Signs." Presented at the Rand Corporation Seminar on April 20, 1949.

RHODES, IDA. (1) "Computation and Its Tools," presented at the meeting of the Baltimore section of the National Council of Teachers of Mathematics, April 2, 1949. (2) "Modern Computing Machines," presented to the Association of Teachers of Mathematics of Philadelphia and vicinity, April 28, 1949. (3) "Computational Machines--from ADAM to MANIAC," presented to the Third Form Cadets at West Point Military Academy, May 31, 1949.

SEIDEL, W. "On Derivatives of Functions Analytic in the Unit Circle." Presented at the Peripathetic Seminar at the California Institute of Technology on April 4, 1949.

YOUDEN, W. J. A review of some studies conducted on clinical thermometer testing was presented at an NBS Thermodynamics Colloquium, "Some problems of measurement in testing clinical thermometers," on May 23, 1949.

Publication Activities

3.1 Publications which appeared during the quarter

3.1.1 Mathematical Tables

(1) Table of sines and cosines to 15 decimal places at hundredths of a degree. NBS Applied Mathematics Series 5. Available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 40 cents.

(2) On the function $H(m,a,x) = \exp(-ix)F(m+1-ia, 2m+2, 2ix)$. NBS Mathematical Tables MT19. Reissued. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 10 cents. (Originally printed in J. Math. Phys. XXI, No. 4, Dec. 1942).

(3) Seven-point Lagrangian Integration formulas. NBS Mathematical Table MT25. Reissued. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 10 cents. (Originally printed in J. Math Phys. XXII, No. 4, Dec. 1943).

(4) Table of coefficients for inverse interpolation with central differences. NBS Mathematical Table MT27. Reissued. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 25 cents. (Originally printed in J. Math. Phys. XXII, No. 4, Dec. 1943).

(5) Tables of circular and hyperbolic sines and cosines for radian arguments. NBS Mathematical Table MT3. Second edition 1949. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price $2.50.

3.1.2 Manuals, Bibliographies and Indices

(None)

3.1.3 Technical Papers

(1) On the backflow of a viscous fluid in a diverging channel. M. Abramowitz. J. Math. Phys. XXVIII, No. 1, 1-21 (Apr. 1949). Reprints available.

(2) On the reality of zeros of Bessel functions. A. Hillman. Bull. Math. Soc. 55, No. 2, 198-200 (Feb. 1949). Reprints available.

(3) Note on the expansion of confluent hypergeometric functions in terms of Bessel functions of integral order. M. Karlin. J. Math. Phys. XXVIII, No. 1, 43-44 (Apr. 1949). Reprints available.

(4) Conversion of numbers from decimal to binary form in the EDVAC. S. Lubkin and Florence Koons. MTAC III, No. 26, 427-431 (Apr. 1949). Reprints available.

(5) A new general method for finding roots of polynomial equations. E. F. Moore. MTAC III, No. 27, 486-488 (July 1949). Reprints available.

(6) Coefficients for repeated integration with central differences. H. E. Salzer. J. Math. Phys. XXVIII, No. 1, 54-61 (Apr. 1949). Reprints available.

(7) On a theorem by Latimer and MacDuffee. O. Taussky-Todd. Canadian J. Math. 1, No. 3, 300-302 (1949). Reprints available.

(8) The density of reducible integers. J. Todd and S. D. Chowla (Institute for Advanced Study). Canadian J. Math. 1, No. 3, 297-299 (1949). No reprints available.

3.1.4 Reviews and Notes

(1) Cybernetics: A new discipline. Review by Churchill Eisenhart of N. Wiener's "Cybernetics or control and communication in the animal and the machine." Science 109, No. 2834, 397-399 (Apr. 22, 1949).

3.1.5 Miscellaneous Publications

(1) A new era in calculating machines. H. D. Huskey and V. R. Huskey. Office Appliance Trade Journal, Jan. 1949.

(2) The Computation Laboratory of the National Bureau of Standards. A. N. Lowan. Scripta Mathematica XV, No. 1, 33-63 (Mar. 1949). Reprints available.

3.2 Manuscripts in the Process of Publication - June 30, 1949

3.2.1 Mathematical Tables

(1) Tables of the binomial probability distribution. NBS Applied Mathematics Series 6. Being printed by the Government Printing Office.

(2) Tables related to the inverse hypergeometric function. NBS Applied Mathematics Series 7. Being printed by the Government Printing Office.

(3) Tables of powers of complex numbers. NBS Applied Mathematics Series 8. Being printed by the Government Printing Office. - Exact values of z^n for $z = x + iy$, $x, y = 0(1)10$, $n = 0(1)25$; exact values of x^n for $x = 0(1)10$, $n = 0(1)25$.

(4) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. Being printed by the Government Printing Office.

(5) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. Being printed by Columbia University Press.

3.2.3 Technical Papers

(1) Tables of integrals of Struve functions. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) A class of mean value functions. E. F. Beckenbach. Submitted to the American Mathematical Monthly.

(3) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. Submitted to the Bulletin of the American Mathematical Society.

(4) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Submitted to the Duke Mathematical Journal.

(5) Table of modified Bernoulli polynomials. G. Blanch and R. Siegel. Accepted for publication in the NBS Journal of Research.

(6) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Submitted to the Duke Mathematical Journal.

(7) Formulas for the percentage points of the distribution of the arithmetic mean in random samples from certain symmetrical universes. Uttam Chand. Accepted for publication in the NBS Journal of Research.

(8) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(9) Acceptance sampling by variables, with special reference to the case in which quality is measured by average or dispersion. J. H. Curtiss. To appear in a special supplement to the Journal of the American Statistical Association.

(10) Probability center lines for standard deviation and range charts. C. Eisenhart. Accepted for publication in Industrial Quality Control.

(11) Exact particle trajectories for nonviscous flow in a plane with a constant coriolis parameter. G. E. Forsythe. Accepted for publication in the Journal of Meteorology. (Originally submitted under the title, Some exact solutions of the equations of non-viscous flow in a plane.)

(12) Solution of the telegrapher's equation with boundary conditions on only one characteristic. G.E. Forsythe. Accepted for publication in the NBS Journal of Research.

(13) Numerical integration for linear sums of exponential functions. R. E. Greenwood. Submitted to the Quarterly of Applied Mathematics.

(14) 1949 Ephemeris of Jupiter's ninth satellite. S. Herrick. To be published as a leaflet by the Astronomical Society of the Pacific.

(15) Some elementary problems in the calculus of variations. M. R. Hestenes. Submitted to the Mathematics Magazine.

(16) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Submitted to Bulletin of the American Mathematical Society.

(17) The remainder in linear methods of approximation. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(18) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Submitted to the Bulletin of the American Mathematical Society.

(19) Formulas for complex Cartesian interpolation of higher degree. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(20) Polynomials for best approximation over semi-infinite and infinite intervals. H. E. Salzer. Accepted for publication in Mathematics Magazine.

(21) Coefficients for polar complex interpolation. H. E. Salzer. Accepted for publication in Journal of Mathematics and Physics.

(22) Checking and interpolation of functions tabulated at certain irregular logarithmic intervals. H. E. Salzer. Submitted to the Journal of the American Statistical Association.

(23) Tables of zeros and weight factors of the first 15 Laguerre polynomials. H. E. Salzer and Ruth Zucker. Accepted for publication in the Bulletin of the American Mathematical Society.

(24) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Philosophical Magazine.

(25) Formulas for numerical integration of first and second order differential equations in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(26) Inequalities concerning ultraspherical polynomials and Bessel functions. O. Szasz. Submitted to Bulletin American Mathematical Society.

(27) Summation of slowly convergent series with positive terms. O. Szasz. Submitted to the Journal of Mathematics and Physics.

(28) On the Gibbs phenomenon for Euler means. O. Szasz. To be published in the 12th Anniversary volume of Acta Scientiarum Mathematicarum.

(29) A remark concerning the characteristic roots of the finite segments of the Hilbert matrix. O. Taussky-Todd. Accepted for publication in the Oxford Quarterly Journal.

(30) A recurring theorem on determinants. O. Taussky-Todd. Accepted for publication in the American Mathematical Monthly.

(31) A problem on arc tangent relations. J. Todd. Accepted for publication in the American Mathematical Monthly.

(32) Statistics in analytical chemistry. W. J. Youden. Accepted for publication in Transactions of the New York Academy of Sciences.

3.2.4 Reviews and Notes

At the request of the following journals, various staff members prepared abstract reviews of technical books and articles, as follows:

Mathematical Reviews

E. F. Beckenbach - 2
M. Kac - 1
W. Seidel - 6

Mathematical Tables and Other Aids to Computation

G. Blanch - 2
A. Lowan - 1
H. Salzer - 1

U. S. Quarterly Book List

E. W. Cannon - 2
C. Eisenhart - 1

Journal of Chemical Education

G. E. Forsythe - 1

Transactions of the American Mathematical Society

W. Seidel - 1

3.2.5 Miscellaneous Publications

(1) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

EXPLANATION OF PROJECT DESCRIPTIONS

The project descriptions appearing in this report are reproduced from the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With a view toward making this report more useful, an explanation of certain terms used in the Project Forms is given here.

Date of Authorization. This is the date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Status. Here is given the narrative of the progress to date on the project. Certain descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to which the entry applies. These terms, with their explanations, are as follows:

"New" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, but no work of any consequence has been performed as the date of the report.

"Underway (New)" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, and that work was in progress during that period.

"Underway (Continuation)" means that the work was initiated more than 3 months preceding the date of the report and was in progress during the 3 months preceding the date of the report.

"Inactive" means that the laboratories made a commitment more than 3 months preceding the date of the report, to work on the project, but no work of any consequence was performed on the project during the last 3 months.

"Completed" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof are to undergo extensive mathematical checks, the designation "Completed" is employed only after these checks have been performed.

"Terminated" means that, although all aspects of the objective had not been achieved, it was necessary to terminate the project due to circumstances beyond the control of the Laboratories.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "In Manuscript" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "Completed" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under *Status* or *Publication* to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.