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MEASUREMENT OF PITCH DIAMETER OF SCREW THREAD GAGES

CONTENTS

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CONTENTS (Continued)

1. INTRODUCTION

As the result of the extensive increase in the production of thread gages during the past few years, caused by the vast requirements of the war with Germany, much attention has been given to the development of methods of measuring such gages, and considerable information on the subject has been published. At this Bureau several thousand master thread gages for artillery ammunition, ordnance, and other munitions, were inspected during the two-year period beginning June 1917. In order to carry on this work, much effort was devoted to the design and construction of special measuring apparatus, and to the compiling of technical data and formulas.

It is the purpose of this circular to give a general résumé of the methods used for the measurement of pitch diameter and to present, particularly, those in use at this Bureau which are the result of the elimination of the less suitable apparatus and measuring devices available, and of the development of new instruments and methods.

The pitch diameter of a screw thread developed on a cylinder is the diameter of an imaginary cylinder which would pass through the threads at such points as to make the width of the threads and the width of the spaces equal. The pitch diameter of a symmetrical thread may also be defined as the length of a line perpendicular to the axis of the screw, and intercepted by the two helical surfaces of the screw. The term "effective diameter" has been commonly applied to this dimension; however, it should more properly refer to the abstract quantity obtained by adding to the measured pitch diameter an amount derived from the errors present in lead and angle, and which is herein referred to as the "effective size" of the thread. The pitch diameter and "effective diameter" are the same only when errors in lead and angle are not present. Thus, the effective size of a thread, or the quality of its fit with a perfect companion thread, is governed by three elements;- pitch diameter, lead, and angle of thread.

The measurement of the pitch diameter, as well as the major diameter (outside diameter), of a thread plug gage is accomplished by means of a micrometer caliper or other suitable apparatus used in connection with properly authorized standards. To measure the pitch diameter it is necessary to provide the micrometer or measuring machine with special contact points, or to apply the usual "wire" methods in which measurements are taken over small cylinders inserted in the thread groove. The cylinders or wires are allowed to assume an equilibrium position, in which they set themselves at an angle with the axis of the screw; this angle for each wire being the angle of the helix at the point of contact of the wire with the helical surfaces. The mathematical relations involved in the measurement of pitch diameter by the three-wire method are somewhat complex, and are considered in detail in Appendices 1 and 2, herein; and the formulas applied in making measurements are given below under the heading "Three-Wire Method".

Of the various methods which have been tried, the three-wire method has been found to be the most accurate and satisfactory when properly carried out. It has been in common use for nearly twenty years, and is the standard method used by the Gage Section of the Bureau of Standards.

2. THREE WIRE METHOD

A. Sizes of Wires

In the three-wire method of measuring screw threads, small, accurately ground cylinders, or wires, which have been lapped to correct size, are placed in the thread groove, two on one side of the screw and one on the opposite side as shown in Fig. 1. The contact over the two wires of the micrometer anvil or spindle must be sufficiently large in diameter to touch both wires; that is, it must be equal to, or greater than, the pitch of the thread. It is best, for reasons given later, to select wires of such a size that they touch the sides of the thread at the mid-slope. The size of wire which touches exactly at the mid-slope of a perfect thread, of a given pitch, is termed the "best-size" wire for that pitch. Any size, however, may be used which will permit the wires to rest on the sides of the thread and also project above the top of the thread.

The depth at which a wire of given diameter will rest in a thread groove depends primarily on the pitch and included angle of the thread; and secondarily, on the angle made by the helix, at the point of contact of the wire and the thread, with a plane perpendicular to the axis of the screw. Inasmuch as variation in the helix angle has a very small effect on the diameter of the wire which touches at the mid-slope of the thread, and as it is desirable to use one size of wire to measure all threads of a given pitch and included angle, the best-size wire is taken as that size which will touch at the mid-slope of a groove cut around a cylinder perpendicular to the axis of the cylinder, and of the same angle and depth as the thread of the given pitch. This is equivalent to a thread of zero helix-angle. The size of wire is given by the formula:

$$G = p/2 \sec a,$$

in which G = diameter of wire,
p = thread interval,
a = 1/2 included angle of thread.

This formula reduces to:

$$G = 0.5774 \times p \text{ for } 60^\circ \text{ threads},$$
$$G = 0.5637 \times p \text{ for } 55^\circ \text{ threads},$$
$$G = 0.5590 \times p \text{ for } 53^\circ 8' \text{ threads},$$
$$G = 0.5165 \times p \text{ for } 29^\circ \text{ threads}.$$

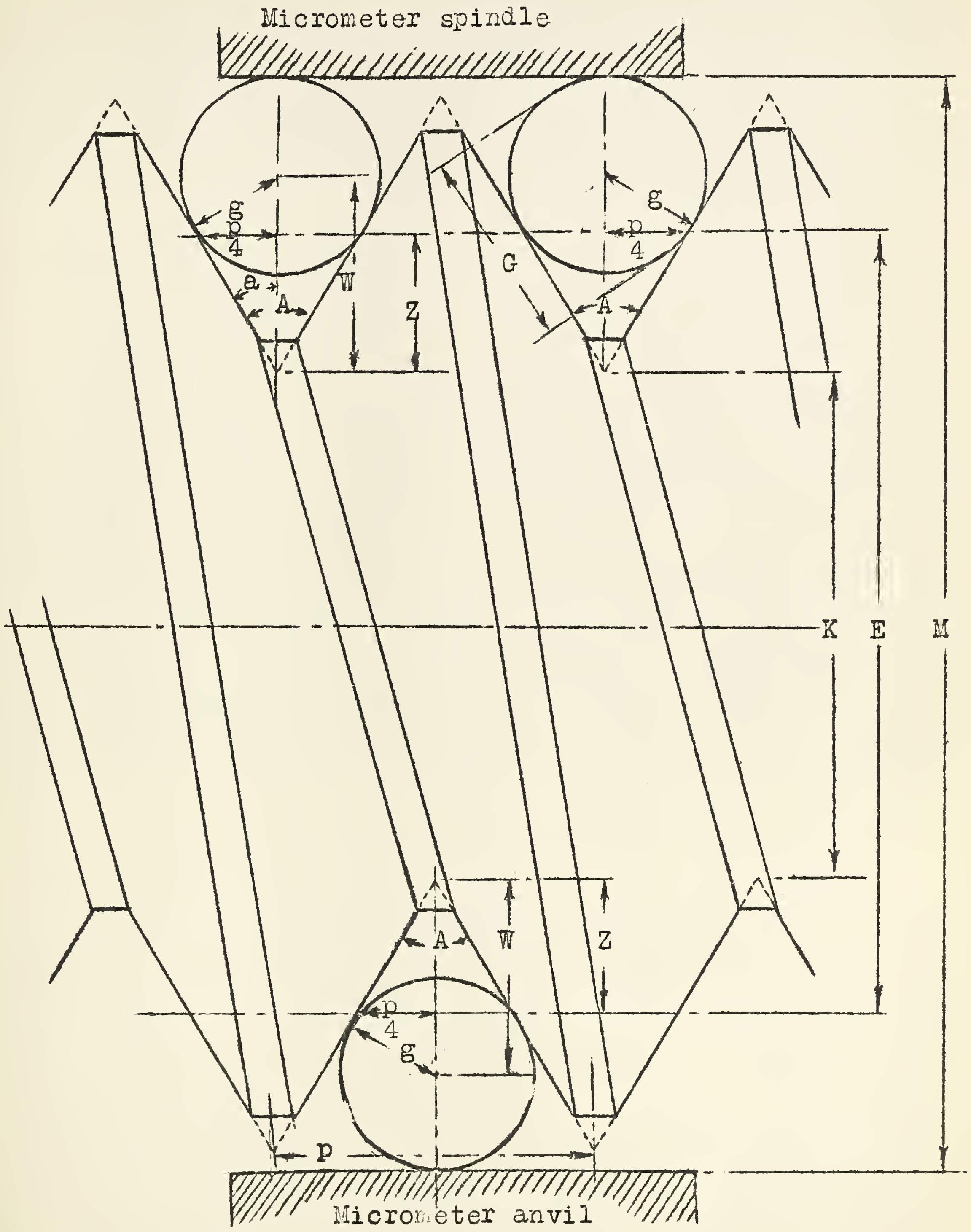

Fig. 1.- Three Wire Method of Measuring Pitch Diameter of Threaded Plug Gages

It is frequently desirable, as for example when a best-size wire is not available, to measure pitch diameter by means of wires of other than the best size. The minimum size which may be used is limited to the diameter which will permit the wire to project above the crest of the thread, and the maximum to the diameter which will not ride on the crest of the thread but will rest on the sides just below the crest. Tables 1 to 10, inclusive, which are appended, give the diameters of the best-size, maximum, and minimum wires for National (United States, A.S.M.E., and S.A.E. Standards), Whitworth, International Metric, Löwenherz, and Acme threads.

B. Specification for Wires

A suitable specification for wires is as follows:

1. The wires should be accurate cylinders of steel with working surfaces glass-hard and lapped to a high polish.
2. The working surface should be about one inch in length, and the wire should have a suitable handle which is provided at one end with an eye to receive a thread used to suspend the wire when taking measurements.
3. One side of the handle, which should be flattened, should be marked with the pitch for which the wire is the best size and with the diameter of the working part of the wire.
4. The wire should be round within 0.00002 in. and should be straight to 0.00002 in. over any quarter-inch interval.
5. One set of wires should consist of three wires which should have the same diameter within 0.00003 in. and this common diameter should be within 0.0001 in. of that corresponding to the best size for the pitch for which the wire is to be used.

C. Standardization of Wires

In order to measure the pitch diameter of a screw thread by means of wires, it is necessary to know the wire diameters accurately. The wires should be standardized by a method which approximates, as nearly as possible, the conditions under which they are used. This may be accomplished by placing the wire in contact with a hardened and lapped cylinder and measuring over the cylinder and wire with a micrometer caliper. The micrometer to be used for this purpose should be one which is graduated to ten thousandths of an inch and upon which hundred-thousandths of an inch can be estimated. Such micrometers are available in various forms of precision bench micrometers. Care should be taken to make sure that the measuring faces of the micrometer are flat and parallel to within 0.00001 inch. The object of measuring the wire in contact with a cylinder is to approximate the conditions of pressure and short line contact which exist when the wire is in contact with a thread. The variation in roundness and the taper are determined by measuring over the wire and cylinder in contact, and the deviation from straightness is determined by measurement between flat surfaces.

Since the wires, when in use, rest on the sides of the thread, a given pressure exerted on the top of the thread will have a magnified effect in distorting the wire and causing the measurement of the pitch diameter to be slightly less than it should be. For this reason a further modification in the method of standardization has been suggested, according to which the diameter of the wire may be determined under conditions duplicating those under which the wire is used. It consists in substituting a cylinder having a series of grooves of various depths, the diameters at the mid-slopes of which have been carefully determined, and taking measurements over two wires of equal size placed in the grooves. The diameter assigned to the two wires under test is such a value that when the pitch diameter of the groove in the cylinder is computed from the measurement over the wires and cylinder it is the same as the known pitch diameter of the groove. The diameter of the wires may be computed from the formula:

$$G = M - E - H \tan^2 a ,$$

in which
- G = diameter of wires
- M = measurement over wires and cylinder
- E = diameter at mid-slope of groove
- H = depth of groove
- a = 1/2 included angle of groove.

If the wire has been standardized by measurement over a plain cylinder and under light pressure and then is used with a heavier pressure, the diameter of wire which is substituted in the formula given below, for computing pitch diameter, will be larger than it should be. This difference is multiplied by the factor 3 in the formula and the tendency is to make the result small. The use of a grooved cylinder would obviate this difficulty to some extent, but it is not usually feasible, since a groove finished to the degree of accuracy required is very difficult to make.

D. Computation of Pitch Diameter of Symmetrical Threads

The general formula for finding the pitch diameter of any thread whose sides are symmetrical with respect to a line drawn through the vertex and perpendicular to the axis of the thread is:

$$E = M + \frac{\cot a}{2n} - G\left(1 + \operatorname{cosec} a + \frac{S^2}{2} \cos a \cot a\right),$$

in which
- E = pitch diameter
- M = measurement over wires
- a = 1/2 included angle of thread
- n = number of threads per inch
- G = diameter of wires
- S = tangent of the helix angle.

This formula differs from those given in most engineering handbooks in that the latter, as generally given, yield a result which should check with the major diameter of the screw measured, while the pitch diameter itself is not mentioned and no account is taken of the effects introduced by the helix angle.

The value of S, the tangent of the helix angle, is given by

$$S = \frac{P}{3.1416\ E},$$

in which P = lead
E = nominal pitch diameter.

In Table 12 are given the values of the term "$\frac{S^2}{2}$ cos a cot a"

for 60° and 55° threads for various values of the helix angle. It will be seen that this term, when multiplied by G, the diameter of the wires used, amounts to as much as 0.0001 in. only when the helix angle is large. For this reason this term is commonly neglected, and the above formula takes the form:

$$E = M + \frac{\cot a}{2n} - G\ (1 + \operatorname{cosec} a).$$

In order that the practice followed, in the measurement of thread gages, may be uniform, the Gage Section of this Bureau uses the latter formula for the threads used on all ordinary fastening screws having helix angles of less than 5°. For a 60° thread of correct angle and thread form this formula simplifies to

$$E = M + \frac{0.86603}{n} - 3G.$$

Similarly, for Whitworth 55° threads

$$E = M + \frac{0.96049}{n} - 3.16568G,$$

and for Löwenherz 53°8' threads

$$E = M + \frac{1.00000}{n} - 3.23594G\ .$$

For Acme threads, the general formula given above is used, since the helix angle is usually large.

For a given set of best-size wires

$$E = M + X,$$

where

$$X = \frac{\cot a}{2n} - G\ (1 + \operatorname{cosec} a).$$

The quantity X is a constant for a given angle, and, when the wires are used for measuring threads of the pitch and angle for

which they are the best size, the pitch diameter is obtained by the simple operation of subtracting this constant or factor from the measurement taken over the wires. In fact, when best-size wires are used this factor is changed very little by a change in the angle of the thread and it has, therefore, been the practice of this Bureau to tabulate and apply the factors for the various sets of wires in use, thus saving a considerable amount of time in the inspection of gages. However, when wires of other than the best sizes are used this factor changes quite appreciably with a variation in the angle of the thread. The following table shows the relative amount of change in the factor with changes in angle and size of wire for threads of different pitches.

Typical Changes in Quantity X with Changes in Thread Angle and Sizes of Wires

Wires for 60° threads	Dia. of Wire	Factor X for half angle		
		28°	30°	32°
4 threads per inch				
min. wire	0.12630	0.16023	0.16239	0.16461
best wire	.14434	.21670	.21651	.21669
max. wire	.25259	.55553	.54126	.52922
20 threads per inch				
min. wire	0.02526	0.03205	0.03248	0.03292
best wire	.02887	.04335	.04331	.04334
max. wire	.05052	.11111	.10826	.10585
50 threads per inch				
min. wire	0.01010	0.01281	0.01298	0.01316
best wire	.01155	.01734	.01733	.01735
max. wire	.02021	.04445	.04331	.04235

This table shows that, with the exception of coarse pitch screws, variation in angle from nominal value causes no appreciable change in the quantity X for best-size wires. On the other hand, when a wire near the maximum or minimum allowable size is used, a considerable change occurs, and the values of the cotangent and cosecant of the actual measured half-angle are to be used. It is apparent, therefore, that there is a great advantage in using wires very closely approximating the best size. It should be remembered, moreover, that the best size wire for a 60° thread, for example, will not be the best size of wire for a Lowenherz 53°8' thread.

For the sake of convenience in carrying out computations, the values of $\frac{\cot a}{2n}$ for the various pitches of the National Coarse (U. S. Standard), National Fine (S.A.E.), Whitworth, International Metric and Löwenherz systems have been tabulated and are given in Tables 1, 3, 6, and 8. In Table 11 the values of the cotangent and cosecant functions for angles varying by intervals of five minutes to one degree on either side of the standard half-angles are given.

E. Computation of Pitch Diameter of Unsymmetrical Threads

The approximate formula which gives the pitch diameter of any thread whose sides are not symmetrical with respect to a line drawn perpendicular to the axis of the screw, such as the Harvey Grip and modified buttress threads, is:

$$E = M + \frac{\cos a_1 \cos a_2}{n \sin A} - G\left(1 + \frac{\cos a_1 + \cos a_2}{\sin A}\right),$$

in which

E = pitch diameter
M = measurement over wires
a_1 and a_2 = angles which sides of thread make with line perpendicular to axis of screw
$A = a_1 + a_2$
p = pitch of thread
G = diameter of wires

This formula does not include the terms which involve the tangent of the helix angle, since they are not appreciable in amount, and the formula is cumbersome in practical use when they are included. The complete formula is equation (4) given in Appendix 1 under Derivation of the General Formula for Pitch Diameter.

The best-size wire for measuring an unsymmetrical thread may be computed by the following formula derived in Appendix 2:

$$G = \frac{p(\cos a_1 + \cos a_2) \cos a_1 \cos a_2}{(\cos a_1 + \cos a_2)^2 - \sin^2 A}.$$

F. Measuring Apparatus Used with Wire Methods

It has been common shop practice to hold the wires down into the thread, when making measurements, by means of elastic bands. This has a tendency to prevent the wires from adjusting themselves

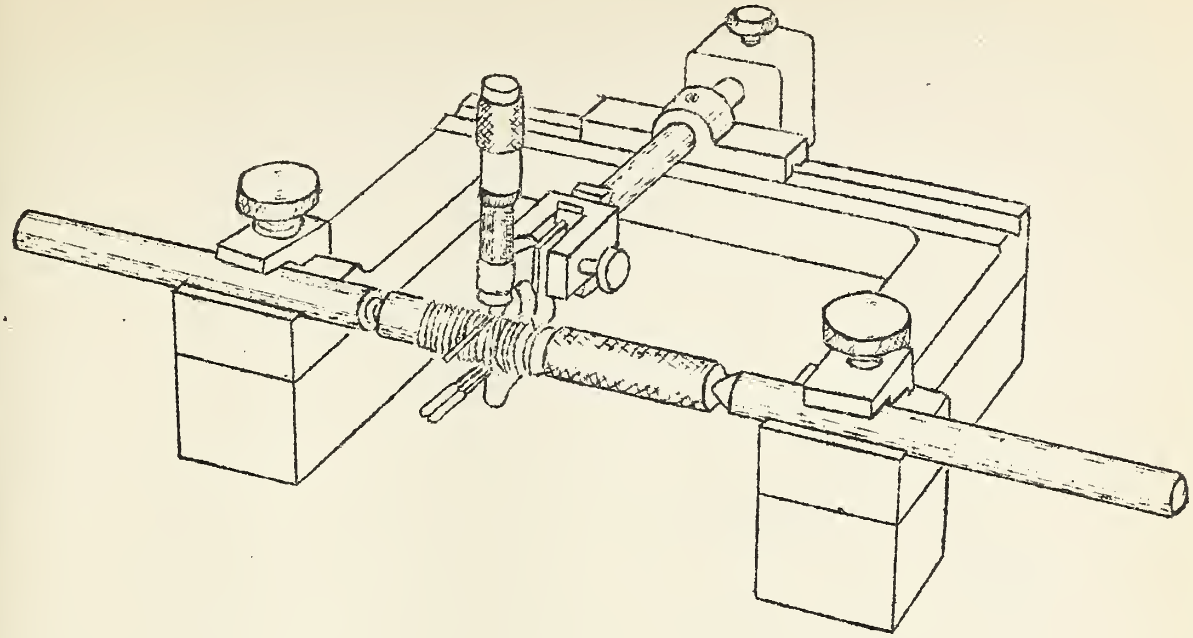

Fig. 2

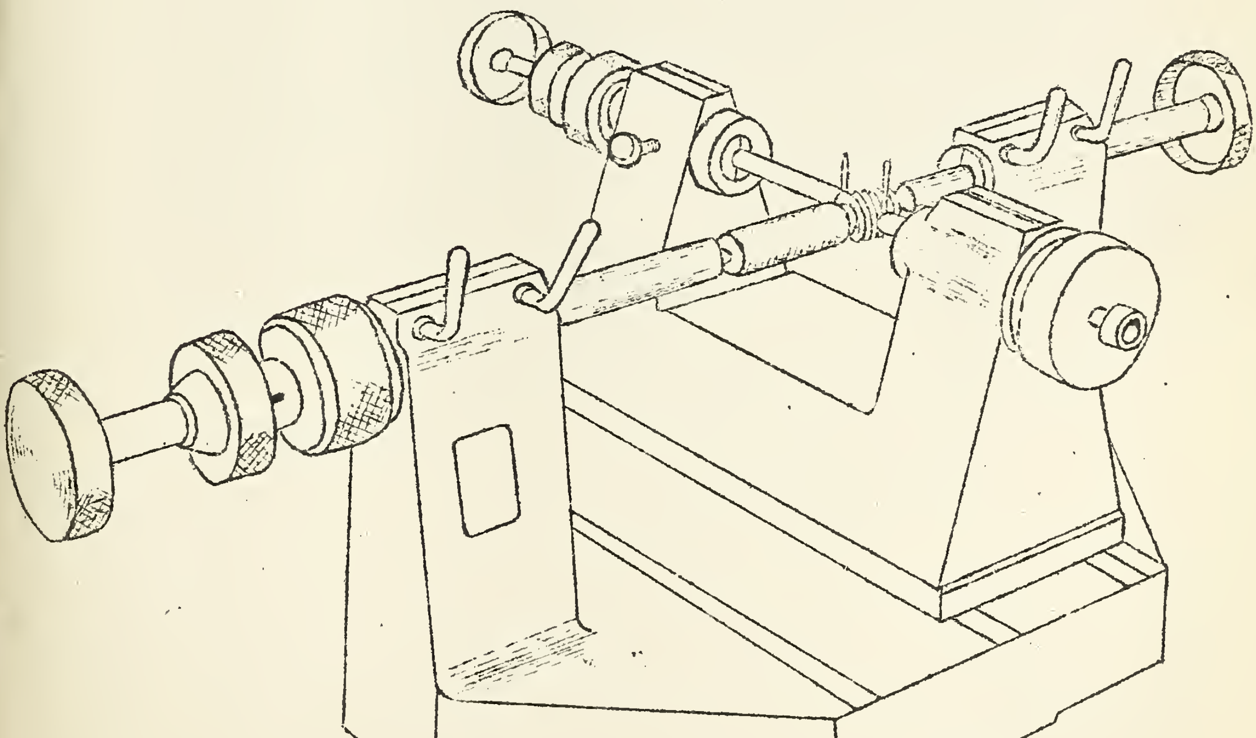

Fig. 3

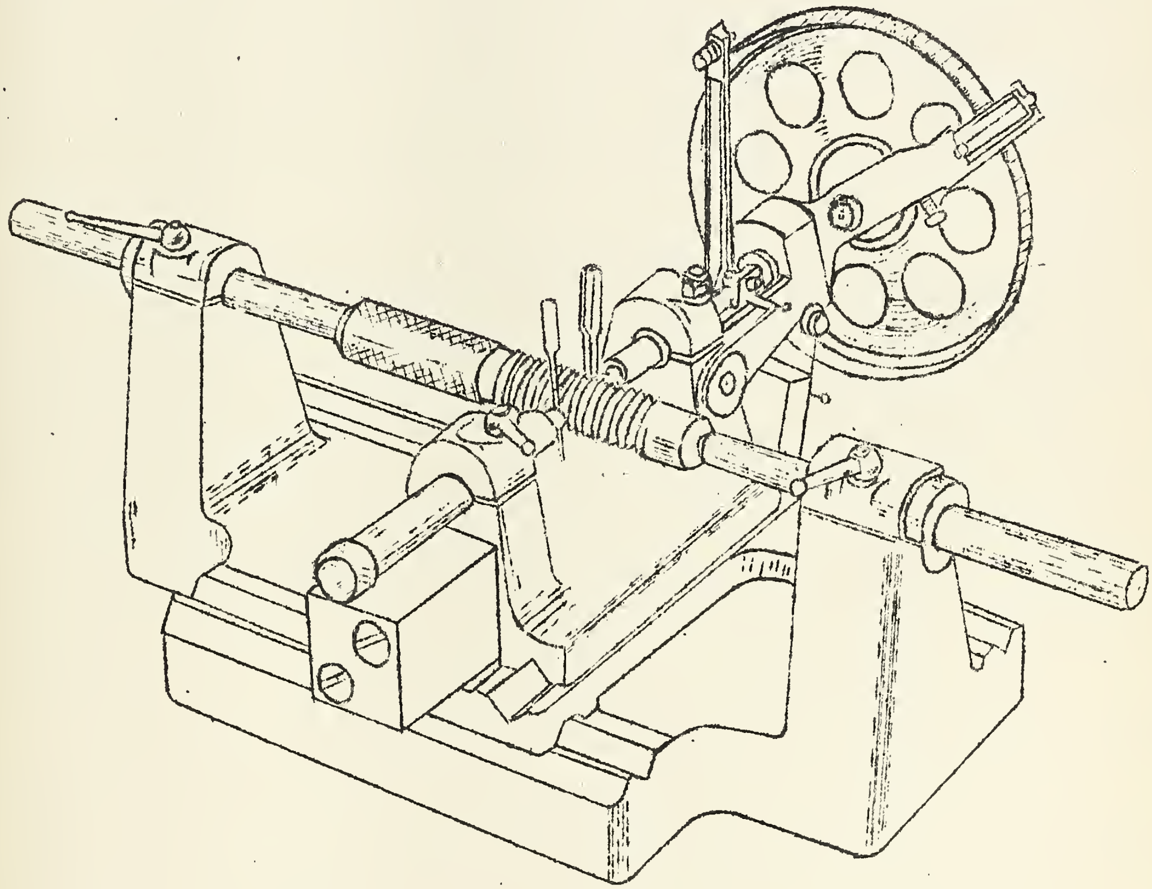

Fig. 4

to the proper position in the thread grooves; thus a false measurement is obtained. In some cases, it has also been the practice to support the screw being measured on two wires which are in turn supported on a horizontal surface and measuring from this surface to the top of a wire placed in a thread over the gage. If the screw is of large diameter, its weight causes a distortion of the wires and an inaccurate reading is obtained. For these reasons these practices should be avoided and subsidiary apparatus for supporting the wires and micrometer should be used. A convenient apparatus for this purpose, which is known as a balanced micrometer, is shown in Fig. 2. The screw is supported between centers and the micrometer is supported on a counterbalanced arm as shown. The micrometer clamp is pivoted on its supporting arm, thus allowing a slight movement of the micrometer, in the vertical plane which passes through the axis of the screw, and permitting the micrometer to adjust itself to contact on all wires. Two of the wires are supported on the anvil of the micrometer below the thread and one is supported over the thread. The proper "feel" is obtained by sliding the wires in the thread groove. This apparatus is very simple in construction and is recommended as being very convenient where a large number of gages are to be tested.

Another form of apparatus for making wire measurements, but with which it is better to use only one wire on each side of the screw, is shown in Fig. 3. This instrument was especially designed and constructed for the Bureau of Standards by the Mt. Wilson Solar Observatory and has been found useful for measuring threads of relatively large diameter. It embodies a precision bench micrometer, having a screw accurate in lead to within about two hundred thousandths of an inch, supported on a heavy base, and carried by steel balls. The position of the anvil is adjusted by means of precision gage blocks to provide a gap between the anvil and spindle to suit the diameter of the gage. The thread gage is carried on adjustable centers which are mounted in the supporting brackets on the ends of the platform. The machine is of massive construction, and its rigidity, which is not common to ordinary measuring instruments, permits, when precautions are taken to avoid temperature changes, measurements which are accurate to within a few hundred thousandths of an inch. In determining pitch diameter, using two wires with the micrometer constrained perpendicular to the axis of the screw, the computation is the same as that involved in the three-wire method. Care must be taken to insure that the gage is accurately centered, so that the axis of the thread is perpendicular to the axis of the micrometer spindle.

A similar machine of smaller construction, which provides for lateral motion of the carriage to bring the micrometer opposite any thread of the screw, was designed by the National Physical Laboratory of England, and is shown in Fig. 4. A number of these machines built by manufacturers in this country embody an electrical indicating device intended to assure a uniform pressure and eliminate the errors due to personal equation.

3. PITCH DIAMETERS OF THREAD RING GAGES

The measurement of threaded ring gages presents many problems, owing to the difficulties with which observations are made on the various elements. While there are various ways of obtaining the measurements desired, it is not possible to work to the same degree of accuracy that is obtainable in measuring threaded plug gages. In determining the pitch diameter of a threaded ring gage it has been the customary practice to fit it to a master threaded check plug having the standard thread form. It is usually considered that, when the ring fits snugly on the plug, the pitch diameter of the ring is the same as that of the plug. This assumption, however, is not strictly correct since there is practically always a variation between the lead of the plug and that of the ring, and, also, the thread angle of the plug and that of the ring. These variations make it necessary that the ring have a pitch diameter larger than that of the plug on which it fits snugly. In most cases the difference actually required for fit, due to differences in lead and angle, is appreciable, sometimes as much as several thousandths of an inch. The relations between this difference in pitch diameter of a screw and nut, which fit together, and the errors in lead and angle present are discussed in Appendix 3.

The major diameters of the master check plugs for both the "Go" and "Not Go" thread ring gages are made, according to usual commercial practice, to the maximum or "Go" dimension to insure clearance at the root of the thread in the ring. This practice involves the difficulty that, in case the pitch diameter of the ring is large and the thread in the ring is not clear at the root, the check plug will have contact with the ring at the top of the thread but not on the sides. Since the pitch diameter of a thread is the fundamental dimension, the check plug should gage this dimension. As the minor diameter and lead may be measured directly, and the clearance at the root of the thread and the angle may be determined by inspection of a cast of the thread in a projection lantern, it is only necessary that the check plug determine the pitch diameter. The check plug should, therefore, so far as possible, check the pitch diameter only, and its thread form should be modified to meet this condition. The thread form illustrated in Fig. 5 is recommended for this purpose. The crest of the thread is located at about one-fourth the depth of the sharp-V thread above the pitch diameter line and, similarly, at a distance of one-fourth the depth of the thread below the pitch diameter line the thread is cleared as shown. This form of thread is easy to make since the major and minor diameters need not be kept within close limits and therefore need not be finished by grinding and lapping after hardening. (For further details see Bureau of Standards Letter Circular LC 19).

Since the bearing surface of such a check plug is much less than that of a full form thread, the effect on the fit of the plug in the ring, due to difference of angle between the two threads, is considerably reduced and a more accurate determination of the

pitch diameter of the ring is obtained. The length of the check plug should be at least four threads, but to prevent errors in lead having an appreciable effect on the fit of the plug in the ring, the length of the plug should not greatly exceed this amount.

The ultimate advantage resulting when threaded components are gaged by "Go" and "Not Go" rings, which were checked by plugs of this recommended type, is that the clearance or neutral space between mating parts, and, thus, the quality of fit, is more consistently maintained.

If a projection lantern is not available, the clearance at the root of the thread in the ring should be tested by means of a threaded check plug having a major diameter of the "Go" or maximum size, and having the angle relieved so that it will not bear on the sides of the thread in the ring but at the major diameter only. This check plug can be used to inspect the clearance of both the "Go" and the "Not Go" thread ring gages if the angle is sufficiently relieved.

In order to measure the pitch diameter of a threaded ring gage when a check plug is not available, a method similar to the three-wire method is applied. In this method three steel balls of the same diameter as that of the best-size wire for the pitch of the ring are used. The ring is placed on a flat surface so that the axis of the thread will be vertical. Two balls are placed in the thread, a few threads apart, and the third is placed diametrically opposite. Against these balls are placed blocks having parallel faces and with corners chamfered, as shown in Fig. 6. In the intervening space a combination of precision gage blocks, together with a pair of tapered parallels, is inserted. (See Fig. 6). The balls are held in position in the threads by embedding them in vaseline or other light grease. The purpose of the tapered parallels is to secure the proper pressure on the balls; the pressure should be just enough to insure contact, since the balls are easily distorted under pressure. Difficulty in securing the proper pressure is the chief objection to this method. The slopes of the wedges of the parallels should be small, 0.001 inch in total adjustment being sufficient. A micrometer reading is taken over the entire combination and this measurement is analogous to the reading M taken over the wires when measuring a plug gage. In this case, however, the signs in the pitch diameter formula are changed and it has the following form:

$$E = M - \frac{\cot a}{2n} + G\,(1 + \operatorname{cosec} a),$$

in which E = pitch diameter
M = measurement between balls
n = threads per inch
G = diameter of balls
a = half of included angle of thread.

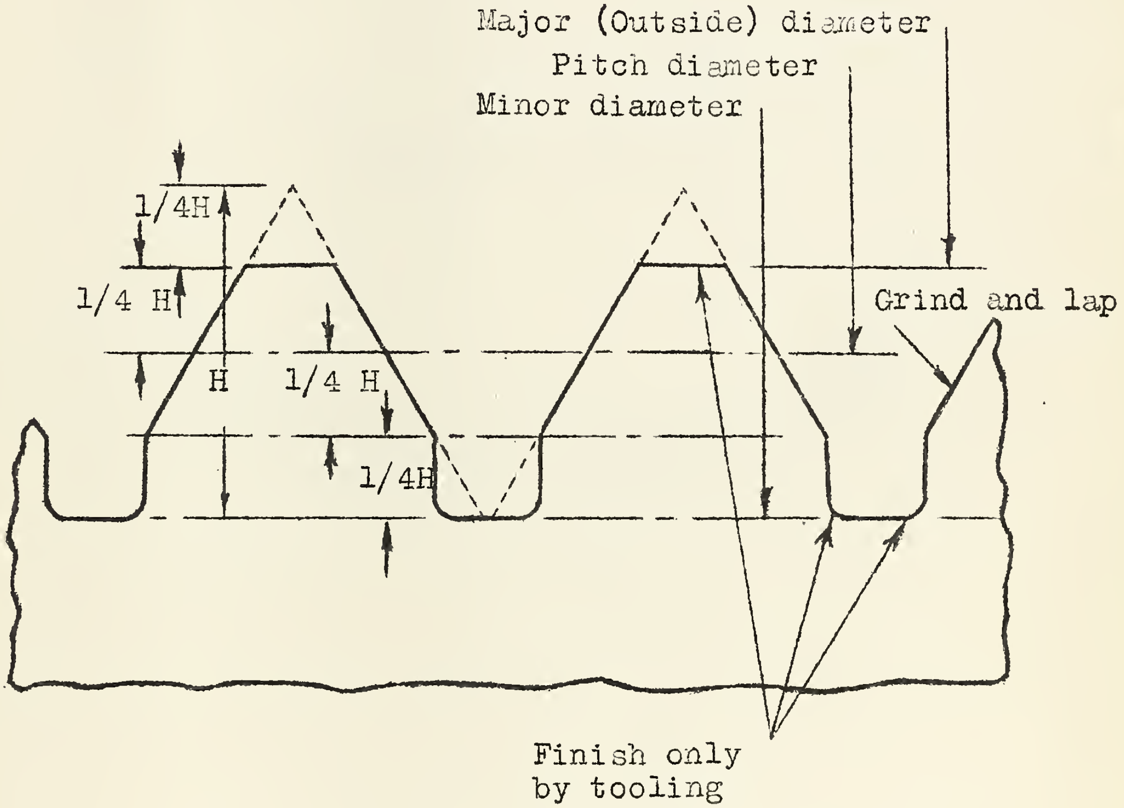

Fig. 5. - Recommended Thread Form of Check Plug for Checking Pitch Diameter of Threaded Ring Gage

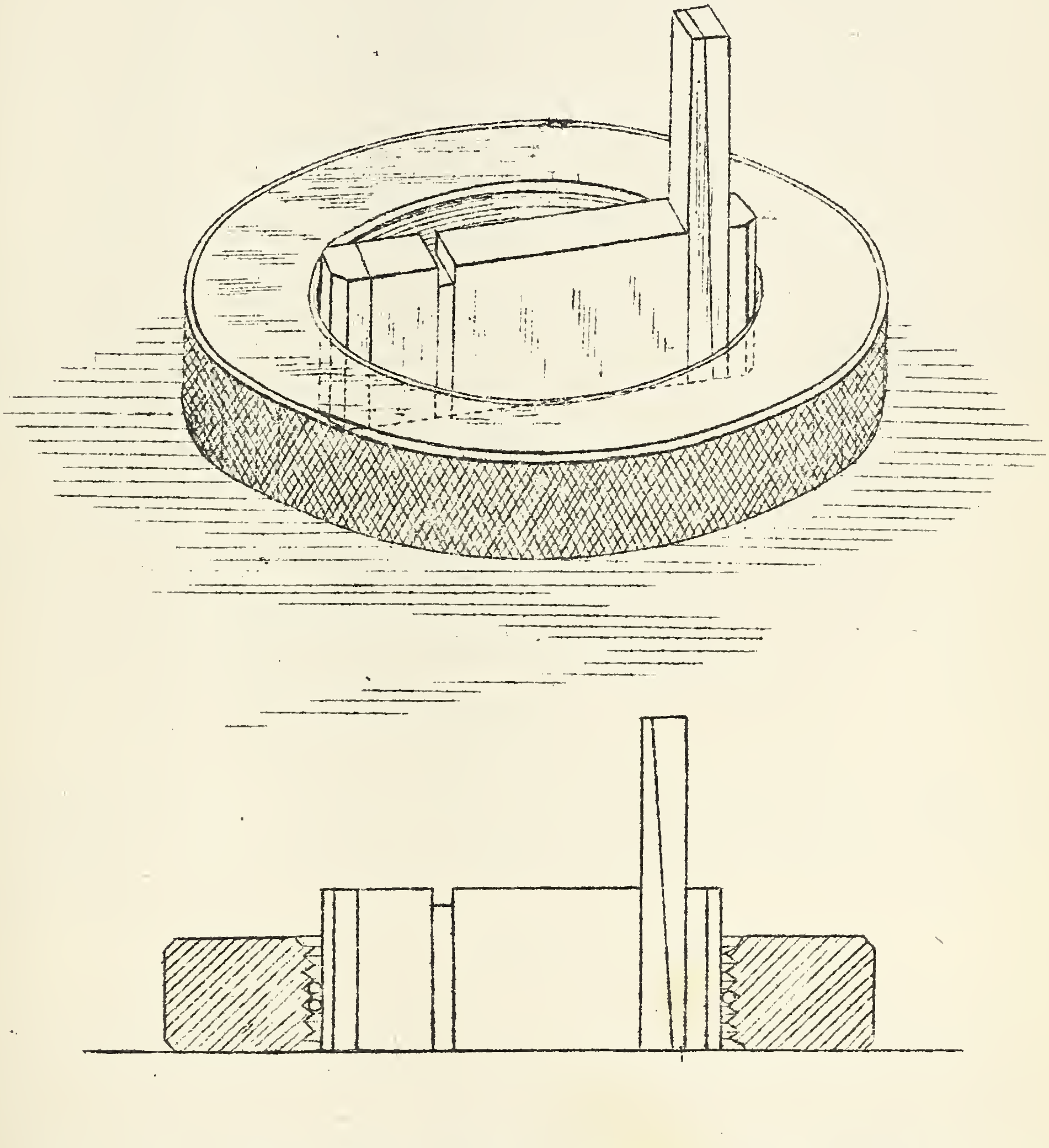

Fig. 6. - Three Ball Method of Measuring Threaded Ring Gages

4. PITCH DIAMETERS OF TAPERED THREAD GAGES

The pitch diameter of a tapered thread gage is measured by the three-wire method in very nearly the same manner as straight thread gages. A point at a known distance L from the end of the gage is located by means of a combination of precision gage blocks and the cone point furnished as an accessory with these blocks, as shown in Fig. 7. The gage is set vertically on a surface plate, the cone point is placed with its axis horizontal at the desired height, and the plug is turned until the point fits accurately into the thread. The position of this point is marked by placing a bit of Prussian Blue or wax immediately above it. The gage is placed between centers of the balanced micrometer and a single "best-size" wire is placed in the thread at this point and the other two wires are placed in the adjoining threads on the opposite side. Measurement is made over the wires in the usual manner but care must be taken that the gaging surfaces of the micrometer make contact with all three wires since the micrometer is not perpendicular to the axis of the screw when there is proper contact. (See Fig. 7). Owing to this inclination the measurement over the wires must be multiplied by the secant of the half-angle of the taper of the thread. The general formula for the pitch diameter of any tapered thread plug gage, the threads of which are symmetrical with respect to a line perpendicular to the axis, then has the form

$$E = M \sec y + \frac{\cot a}{2n} - G\left(1 + \operatorname{cosec} a + \frac{S^2}{2} \cos a \cot a\right),$$

in which
E = pitch diameter
M = measurement over wires
y = half-angle of taper of thread
n = number of threads per inch
a = half-angle of thread
G = diameter of wires
S = tangent of helix angle.

Neglecting the term involving the tangent of the helix angle, the pitch diameter of a National (Briggs') Standard Pipe Thread Gage, having correct angle (60°) and taper (3/4 in. per foot) is then given by the formula:

$$E = 1.00048\ M + 0.86603\ p - 3G.$$

To obtain the pitch diameter at any other point along the thread, multiply the distance, parallel to the axis of the thread, between this point and the point at which the measurement was taken by the taper per inch; then add the product to or subtract it from the measured pitch diameter, according to the direction in which the second point is located with respect to the first.

The following method, illustrated in Fig. 8, has a theoretical advantage over the first method in that it is independent of the taper of the thread and, therefore, requires less computation; or, if the taper is not measured but assumed to be correct, it is more

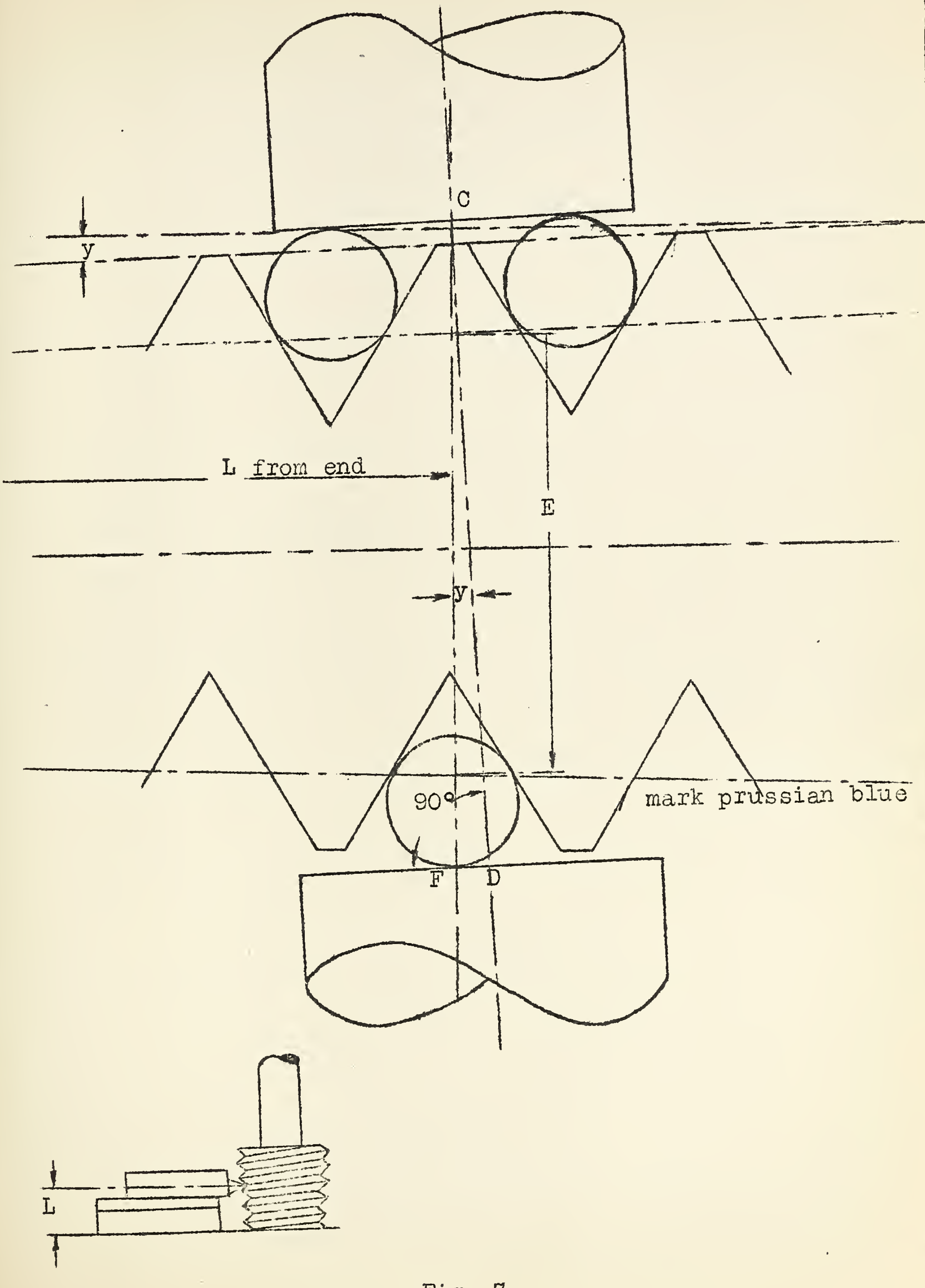

Fig. 7

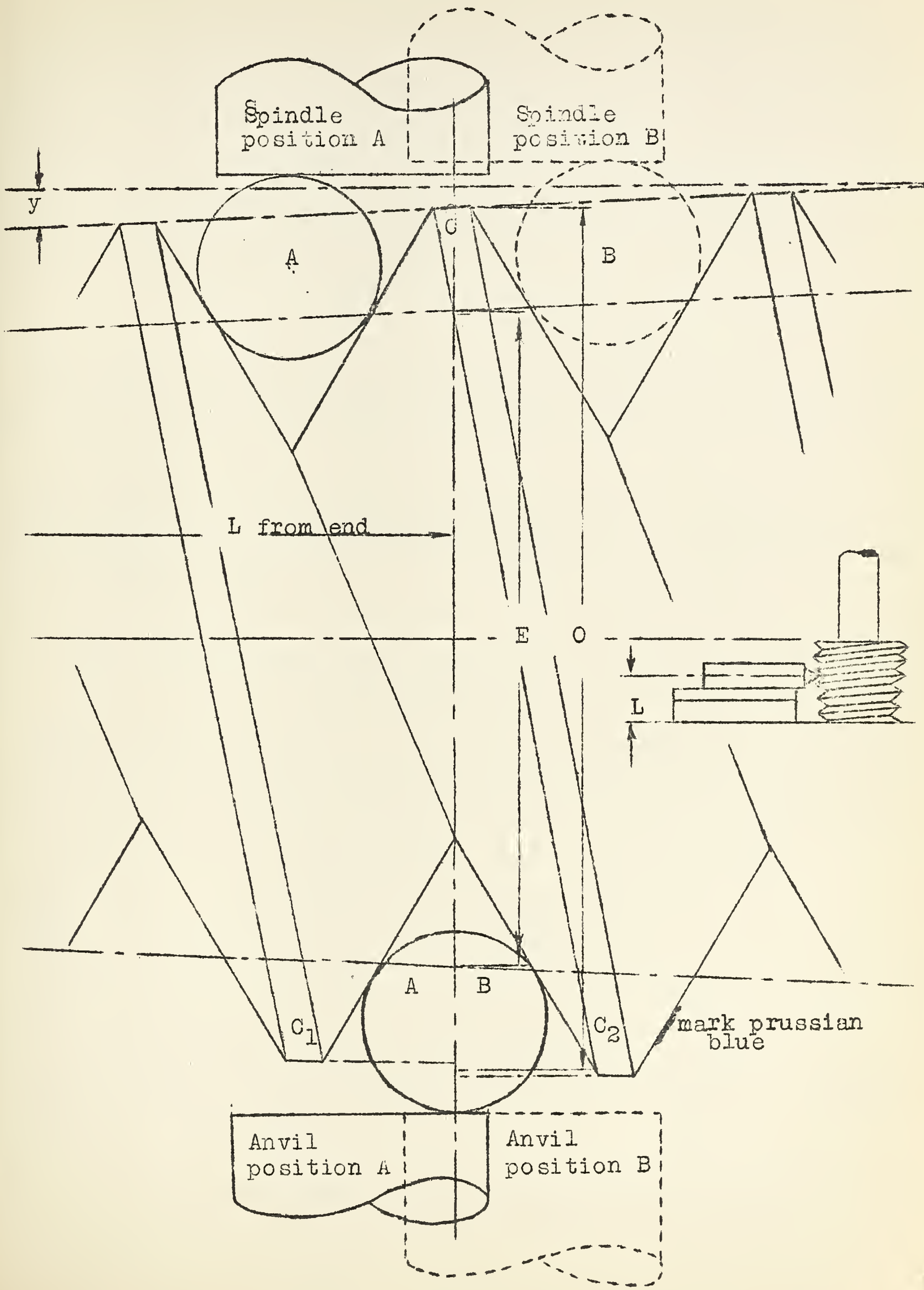
Spindle
position A
Spindle
position B
y
A
C
B
L from end
E
O
L
A
B
C1
C2
mark prussian
blue
Anvil
position A
Anvil
position B

Fig. 8

accurate. In this case the micrometer is constrained perpendicular to the axis of the screw, either by a solid arm substituted for the swivel arm in the balanced micrometer, or by placing the gage on a surface plate with its axis vertical and supporting the micrometer in a horizontal position with its anvil and spindle resting on two equal combinations of gage blocks. A single wire is inserted in the thread at the point located as in the previous method, and one other wire is placed in the upper thread on the opposite side. A measurement is taken over the two wires; the second wire is then moved to the thread immediately below, and a second reading is taken. The mean of these two readings is substituted as the value of M in the formula:

$$E = M + \frac{\cot a}{2n} - G\,(1 + \operatorname{cosec} a).$$

The taper can be readily computed by taking readings over the wires, first in any thread near the small end, and then in any thread near the large end, the exact number of threads between the two points being known.

Satisfactory methods for measuring the pitch diameter of tapered thread rings have not been worked out. Accordingly, the only procedure available is to determine the fit of the ring on a master thread plug.

5. CONCENTRICITY OF PITCH DIAMETER AND MAJOR OR MINOR DIAMETERS

When the major and pitch diameters of a thread plug gage, or the minor and pitch diameters of a thread ring gage, have been determined by readings taken at right angles to each other, and at different points along the thread, the concentricity of these diameters at a few places should be checked. This is important if these diameters were finished separately by using different laps or in different set-ups in grinding, since, in these cases, the diameters might be eccentric. The eccentricity may be readily determined in the case of a plug gage by measuring over one wire placed in the thread, with the anvil of the micrometer in contact with the wire and the spindle in contact with the crest of the thread. Observations are made on the variation in the readings obtained during one revolution of the gage, keeping on the same thread. Another method, whereby eccentricity may be detected, consists in rotating the gage in the projection lantern and observing the presence of any pronounced variations in the width of the flat at the crest of the thread.

6. MEASUREMENT OF PITCH DIAMETER BY MEANS OF THE SCREW THREAD MICROMETER

The screw thread micrometer, shown in Fig. 9, is one of the adaptations of the micrometer for measuring directly the pitch

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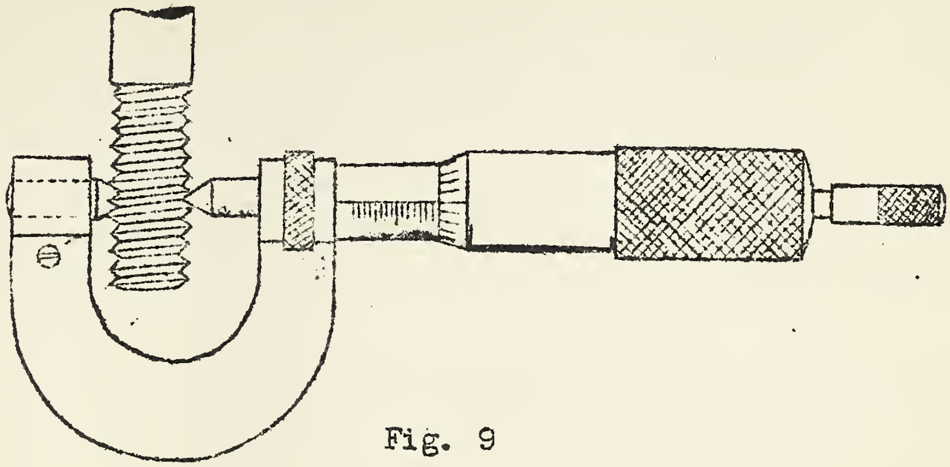


Fig. 9

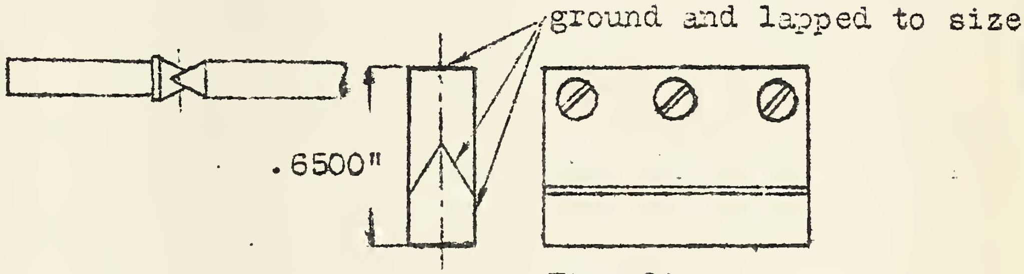


Fig. 9A

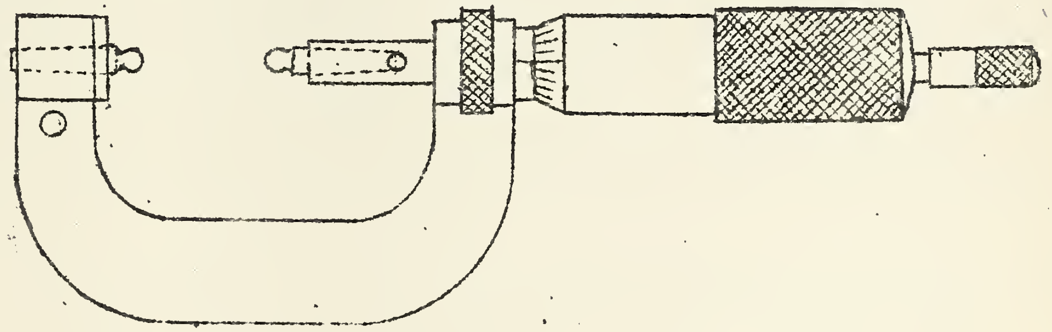
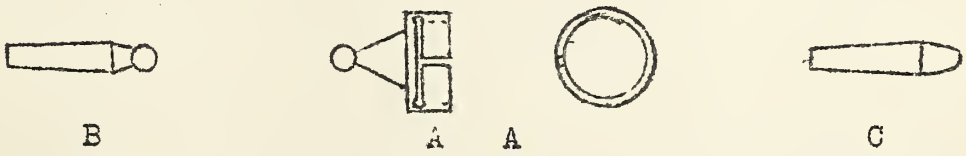


Fig. 10



diameter of a screw thread. It is a slight modification of a micrometer caliper which, although not practical in application, will theoretically measure directly the true pitch diameter. The anvil of a micrometer having this ideal form would consist of two cone points placed closely together, and the spindle of the micrometer would also have a cone point. The points of the cones would be truncated, in order that the cones might touch on the sides of the thread rather than on the bottom. To measure accurately, the axes of all three of the cones must lie in the same plane, the angles of all of the cones and of the screw being measured must be the same, and, when taking measurements, the axis of the screw must lie in the plane which contains the axes of the three cones.

Since it is not practicable to meet all of these conditions, the modified form of thread micrometer shown in Fig. 9 has two parallel wedges formed into one V-shaped piece, which is free to rotate, in the place of the two cones on the anvil. This instrument gives pitch diameter readings which are slightly large; however, this excess is usually not over 0.0002 in. provided that the thread angle is the same as the angle of the wedge and cone of the micrometer. The end of the cone point of the spindle is truncated, and the groove in the anvil is cleared at the bottom, thus allowing both the anvil and the spindle to make contact with only the sides of the thread. When the spindle and anvil are in contact, the zero line on the thimble represents the plane XY, Fig. 9. The anvil and spindle are limited in their capacity, and to cover all pitches it is necessary to provide different micrometers for various ranges of pitches. On account of the above limitations, and the fact that careful and frequent adjustment are required, this instrument is unsatisfactory for accurate measurement. If used at all in the measurement of thread gages, the thread micrometer should only serve as a means to obtain an approximate check on measurements made by the three-wire method. It is very useful, however, in transferring measurements from a standard gage to the work at hand.

A convenient check for a screw thread micrometer is shown in Fig. 9A. It consists of two pieces, one grooved to fit the spindle and one, which is wedge-shaped, to fit into the anvil. The faces opposite the wedge and groove are lapped flat. A micrometer is checked at various points by inserting precision gage blocks between the two flat faces of the check. The length of the check is determined by measuring over the flat surfaces, the check being assembled with the wedge and groove together as shown.

For the approximate measurement of screw plugs in the shop and commercial inspection laboratory a standard micrometer fitted with different types of points is commonly used. There are various types of points used; in Fig. 10 three are shown. The type shown at A is made to slip over both the anvil and spindle of the micrometer but unless very carefully made these often do not fit solidly over the measuring points of the micrometer even when

they are split, and for this reason may cause errors in measurement. At B a type of point is shown which can be used to measure threads of coarser pitch than sixteen threads per inch. For measuring threads of sixteen pitch or finer the point is formed as shown at C and this type of point can be successfully used to measure threads as fine as seventy-two threads per inch. These points are used only when measurements are referred to a standard gage.

7. MEASURING THREAD ANGLE BY TWO-SIZE WIRE METHOD

In case special facilities for measuring the included angle of a thread are not available, the angle may be determined approximately by means of two sets of wires of different diameters. Measurement is made over the wires, which are inserted in the thread, in the same manner as when the pitch diameter is measured. One measurement is taken over the minimum or the best-size wires and a second is taken over the maximum wires. The sizes of maximum and minimum wires which may be used with various pitches are given in Tables 1 to 7 inclusive. The angle may be computed from the measurements by applying the formula, derived in Appendix 4:

$$\text{Sin } a = \frac{(G_1 - G_2)\left(1 + \frac{S^2}{2}\right)}{(M_1 - M_2) - (G_1 - G_2)} ,$$

in which

G_1 = diameter of large set of wires
G_2 = diameter of small set of wires
M_1 = measurement over large wires
M_2 = measurement over small wires
a = half-angle of thread
S = tangent of helix angle of thread.

This method cannot be relied upon to give results as accurate as measurements made by means of an optical projection apparatus and shadow protractor. A variation of this method is to use a single wire of each size and make the measurement with the spindle of the micrometer in contact with the crest of the thread. In this case the formula has the form:

$$\text{Sin } a = \frac{(G_1 - G_2)\left(1 + \frac{S^2}{2}\right)}{2(M_1 - M_2) - (G_1 - G_2)} .$$

Values of S^2 for various helix angles are given in Table 12. Since the value of S^2 is small for small helix angles, the term $\left(1 + \frac{S^2}{2}\right)$ in the above formulas may be neglected when the helix angle is less than two degrees.

APPENDIX I

DERIVATION OF THE GENERAL FORMULA FOR PITCH DIAMETER MEASURED BY THE THREE-WIRE METHOD

The following is the derivation of the general formula for the determination of the pitch diameter of a screw thread by the three-wire method, when wires used are of any diameter within the maximum and minimum values, as given in Tables 1 to 8 inclusive. The formula is first derived with reference to an unsymmetrical thread whose sides make angles a_1 and a_2 with the perpendicular to the axis of the screw, and from this is deduced the formula with reference to a symmetrical thread.

It is assumed that:

(1). The wire is allowed to adjust itself to its natural or free position in the thread. In so doing the wire makes an angle with a plane perpendicular to the axis of the screw equal to the helix angle of the thread at the point y_o (Fig. 11). This point, y_o is the intersection of, (a), the plane perpendicular to the axis of the screw and passing through the center of the wire at the point which lies in the vertical plane passing through the axis of the screw, and of (b), the line passing through the points of contact of the wire with the sides of the thread.

(2). The horizontal wire touches both sides of the thread in a vertical plane containing the axis of the screw, the section of the wire cut by this vertical plane being an ellipse. This is not strictly true, since the wires, when in equilibrium, must make contact with the thread in a plane perpendicular to the axis of the wire. The error introduced by this assumption is negligible, as shown in "Notes on Screw Threads" by H. H. Jeffcott, published in Collected Researches, Vol. V, 1909, The National Physical Laboratory, England.

Let
$A = a_1 + a_2$ = total included angle of thread
p = pitch of thread
g = radius of wire
G = diameter of wire
M = measurement over wires
E = pitch diameter of thread
H = depth of sharp-V thread
s = helix angle
S = tangent of helix angle

$$= \frac{p}{3.1416\ E}$$

$$b = g\sqrt{1 + S^2}$$

K = minor (core) diameter

Referring to Fig. 11,

$$E = K + H,$$
$$K = M - 2g - 2RL.$$

(1) Therefore, $$E = M + H - 2g - 2RL.$$

$$RL = RT_1 \cot a_1,$$
$$RT_1 = CT_1 - CR,$$

(2) $$RL = (CT_1 - CR) \cot a_1.$$

The equations of the lines LT_1 and T_2L tangent to the ellipse, referring to Fig. 11 are:[1]

$$y = x \cot a_1 \pm \sqrt{b^2 \cot^2 a_1 + g^2},$$

and $$y = x \cot a_2 \pm \sqrt{b^2 \cot^2 a_2 + g^2}.$$

Taking $y = 0$, the intercepts on the X axis, $x = CT_1$ and $x = CT_2$ are given:

$$CT_1 = \frac{\sqrt{b^2 \cot^2 a_1 + g^2}}{\cot a_1},$$

$$CT_2 = \frac{\sqrt{b^2 \cot^2 a_2 + g^2}}{\cot a_2}.$$

Taking $x = o$, the intercepts on the Y axis, $y = Ct_1$ and $y = Ct_2$ are given:

$$Ct_1 = \sqrt{b^2 \cot^2 a_1 + g^2},$$
$$Ct_2 = \sqrt{b^2 \cot^2 a_2 + g^2}.$$
$$C't_2 = C'L \cot a_2,$$
$$t_1C' = C'L \cot a_1,$$
$$C't_2 + t_1C' = t_1t_2 = C'L (\cot a_1 + \cot a_2),$$

$$C'L = \frac{t_1t_2}{\cot a_1 + \cot a_2}.$$

$$CR = C'L = \frac{t_1t_2}{\cot a_1 + \cot a_2},$$

$$t_1t_2 = Ct_1 - Ct_2 = \sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}.$$

Then $$CR = \frac{\sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}}{\cot a_1 + \cot a_2}.$$

Substituting the above expressions for CR and CT_1 in equation (2):

$$RL = \cot a_1 \frac{\sqrt{b^2 \cot^2 a_1 + g^2}}{\cot a_1}$$
$$- \frac{\sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}}{\cot a_1 + \cot a_2}$$

$$= \sqrt{b^2 \cot^2 a_1 + g^2}$$
$$- \cot a_1 \frac{\sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}}{\cot a_1 + \cot a_2}.$$

To determine H:

$$m = H \tan a_1,$$

$$n = H \tan a_2,$$

$$m + n = p,$$

$$p = H(\tan a_1 + \tan a_2),$$

$$H = \frac{p}{\tan a_1 + \tan a_2}.$$

Formula (1) then becomes

(3) $$E = M + \frac{p}{\tan a_1 + \tan a_2} - 2g - 2\sqrt{b^2 \cot^2 a_1 + g^2} +$$
$$2 \cot a_1 \frac{\sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}}{\cot a_1 + \cot a_2}.$$

Remembering that

$$b = g \sec s = g\sqrt{1 + \tan^2 s} = g\sqrt{1 + S^2}\ ;$$

also

$$\tan a_1 + \tan a_2 = \frac{\sin (a_1 + a_2)}{\cos a_1 \cos a_2},$$

and $\cot a_1 + \cot a_2 = \dfrac{\sin (a_1 + a_2)}{\sin a_1 \sin a_2}$,

equation (3) reduces to

$$E = M + \frac{p \cos a_1 \cos a_2}{\sin A} - G\left\{1 + \sqrt{(1 + S^2) \cot^2 a_1 + 1} + \frac{\cos a_1 \sin a_2}{\sin A}\left[\sqrt{(1 + S^2) \cot^2 a_1 + 1} - \sqrt{(1 + S^2) \cot^2 a_2 + 1}\right]\right\}.$$

The quantities under the radicals may be expressed in terms of cosecant and cotangent functions, and the expression takes the form:

$$(4)\quad E = M + \frac{p \cos a_1 \cos a_2}{\sin A} - G\left\{1 + \sqrt{\operatorname{cosec}^2 a_1 + S^2 \cot^2 a_1} - \frac{\cos a_1 \sin a_2}{\sin A}\left[\sqrt{\operatorname{cosec}^2 a_1 + S^2 \cot^2 a_1} - \sqrt{\operatorname{cosec}^2 a_2 + S^2 \cot^2 a_2}\right]\right\}.$$

If the thread is symmetrical, that is, $a_1 = a_2 = a$, then equation (4) reduces to:

$$(5)\quad E = M + \frac{p}{2} \cot a - G\left[1 + \sqrt{\operatorname{cosec}^2 a + S^2 \cot^2 a}\right].$$

This formula may be simplified, by expanding the expression under the radical by the Binomial Theroem and neglecting all terms beyond the second, into the following very close approximation:

$$(6)\quad E = M + \frac{p}{2} \cot a - G\left(1 + \operatorname{cosec} a + \frac{S^2}{2} \cos a \cot a\right).$$

which is the formula given under "Computation of Pitch Diameter" (page 6).

Since the helix angle is usually small it may be assumed to be zero and equation (4) reduces to the form:

$$(7)\quad E = M + \frac{p \cos a_1 \cos a_2}{\sin A} - G\left(1 + \frac{\cos a_1 + \cos a_2}{\sin A}\right.$$

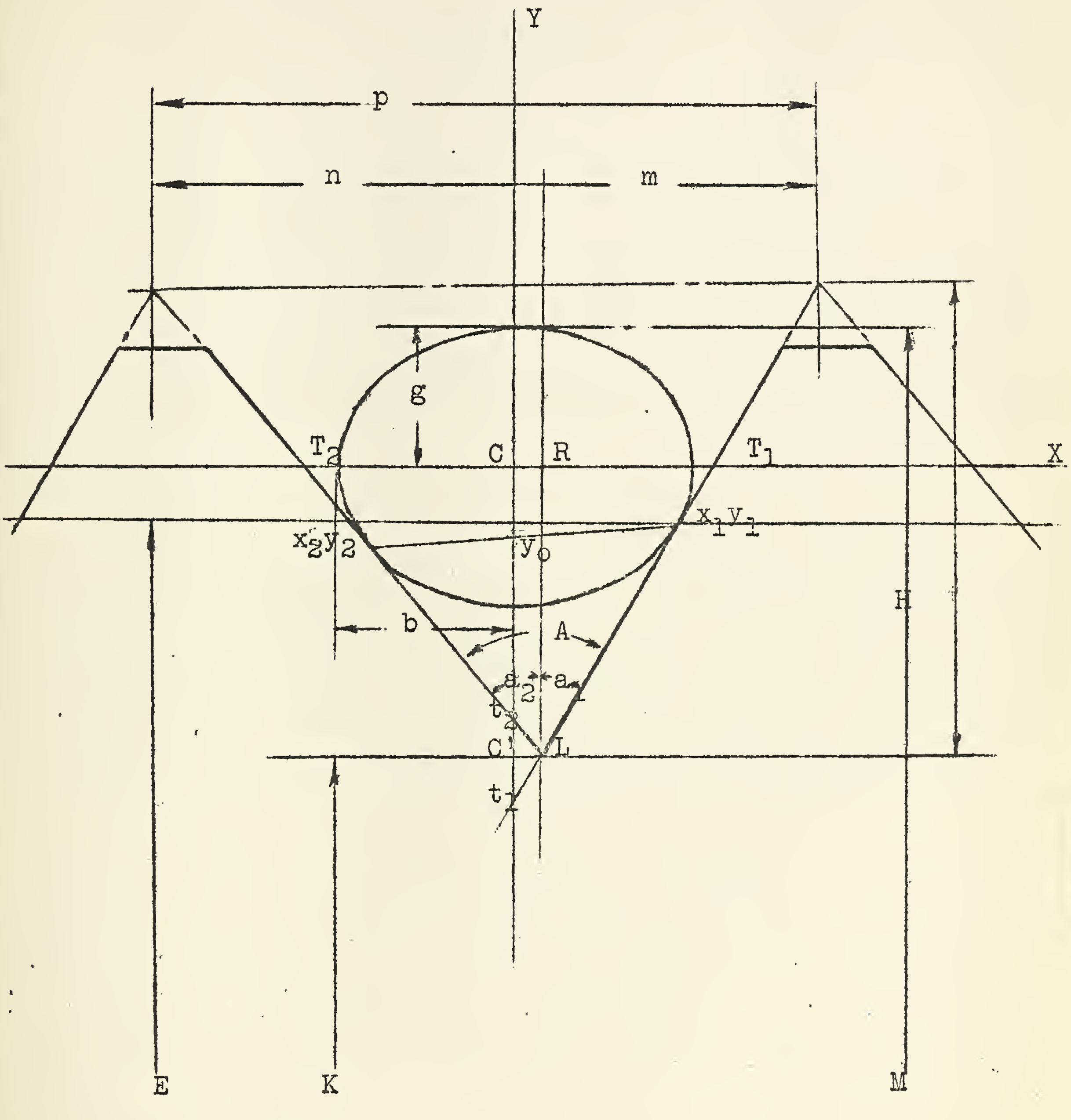


Fig. 11

which gives a result accurate within 0.0001 inch except when the helix angle is large. This formula (equation 7) should be used for most practical purposes in computing pitch diameter of threads of the buttress type.

APPENDIX 2

DERIVATION OF WIRE SIZE FORMULAS

The formula for the size of wire which will make contact at any diameter E' on the side of an unsymmetrical thread, in which the helix angle is taken into account, is derived herein. From this formula are deduced the formulas for the size of wire touching at the pitch diameter of both symmetrical and unsymmetrical threads, the helix angle being taken into account; these formulas being required in the measurement of threads having small thread angles and large pitches, such as Acme threads. Finally, there is deduced the simple and commonly used formula for the "Best-Size" wire for ordinary screw threads, in which the helix angle is not taken into account.

As in Appendix 1, it is assumed that:

(1). The wire is allowed to adjust itself to its natural or free position in the thread. In so doing the wire makes an angle with a plane perpendicular to the axis of the screw equal to the helix angle of the thread at a point y_o (Fig. 12). This point, y_o is the intersection of, (a), the plane perpendicular to the axis of the screw and passing through the center of the wire at the point which lies in the vertical plane containing the axis of the screw, and of (b), the line passing through the points of contact of the wire with the sides of the thread. The diameter E' is taken as the diameter at the point y_o.

(2). The horizontal wire touches both sides of the thread in a vertical plane containing the axis of the screw, the section of the wire cut by this vertical plane being an ellipse. This is not strictly true, since the wires, when in equilibrium must make contact with the thread in a plane perpendicular to the axis of the wire. The error introduced by this assumption is negligible, as shown in "Notes on Screw Threads" by H. H. Jeffcott, published in Collected Researches, Vol. V, 1909, The National Physical Laboratory, England.

Referring to Fig. 12, let

$A = a_1 + a_2$ = total included angle of thread
p = pitch of thread
g = radius of wire = 1/2 minor axis of ellipse
G = diameter of wire
E' = diameter on side of thread at y_o
E = pitch diameter of thread

H = depth of sharp-V thread
s = helix angle
S = tan s
b = g sec s = $g\sqrt{1 + S^2}$ = 1/2 major axis of ellipse.

$(x_1, - y_1)$ and $(-x_2, - y_2)$ are points of contact of wire with sides of thread.

Equation of ellipse $g^2x^2 + b^2y^2 = g^2b^2$,

$$\text{or } y^2 = g^2 - \frac{g^2}{b^2}x^2. \tag{1}$$

Equations of tangents LB and CL are,

$$y = x \cot a_1 \pm \sqrt{b^2 \cot^2 a_1 + g^2}, \tag{2}$$

and

$$y = x \cot a_2 \pm \sqrt{b^2 \cot^2 a_2 + g^2}. \tag{3}$$

At the points of tangency,

$$y_1 = x_1 \cot a_1 - \sqrt{b^2 \cot^2 a_1 + g^2}.$$

Also, substituting in (1),

$$y_1^2 = g^2 - \frac{g^2}{b^2} x_1^2.$$

Then

$$\left[x_1 \cot a_1 - \sqrt{b^2 \cot^2 a_1 + g^2}\right]^2 = g^2 - \frac{g^2}{b^2} x_1^2.$$

Squaring left hand term and solving for x_1,

$$x_1 = \frac{b^2 \cot a_1}{\sqrt{b^2 \cot^2 a_1 + g^2}}.$$

Similarly

$$x_2 = -\frac{b^2 \cot a_2}{\sqrt{b^2 \cot^2 a_2 + g^2}}.$$

Substituting values of x_1 and x_2 in (2) and (3),

$$y_1 = -\frac{g^2}{\sqrt{b^2 \cot^2 a_1 + g^2}}$$

and

$$y_2 = -\frac{g^2}{\sqrt{b^2 \cot^2 a_2 + g^2}}$$

Equation of line through $(x_1, -y_1)$ and $(-x_2, -y_2)$ is

$$\frac{y - (-y_1)}{x - (+x_1)} = \frac{+(-y_2) - (-y_1)}{+(-x_2) - (+x_1)}$$

Substituting, and letting $x = o$, $y = y_o$,

$$\frac{y_o + \dfrac{g^2}{\sqrt{b^2 \cot^2 a_1 + g^2}}}{-\dfrac{b^2 \cot a_1}{\sqrt{b^2 \cot^2 a_1 + g^2}}} = -\frac{\dfrac{g^2}{\sqrt{b^2 \cot^2 a_1 + g^2}} - \dfrac{g^2}{\sqrt{b^2 \cot^2 a_2 + g^2}}}{\dfrac{b^2 \cot a_1}{\sqrt{b^2 \cot^2 a_1 + g^2}} + \dfrac{b^2 \cot a_2}{\sqrt{b^2 \cot^2 a_2 + g^2}}}$$

which reduces to

$$y_o = -\frac{g^2 (\cot a_2 + \cot a_1)}{\cot a_1 \sqrt{b^2 \cot^2 a_2 + g^2} + \cot a_2 \sqrt{b^2 \cot^2 a_1 + g^2}} . \qquad (4)$$

But $$y_o = \frac{E' - E}{2} + \frac{H}{2} - RL. \qquad (5)$$

Equating (4) and (5), and substituting values of $\frac{H}{2}$ and RL derived in Appendix 1,

$$\frac{E' - E}{2} + \frac{p}{2(\tan a_1 + \tan a_2)} - \sqrt{b^2 \cot^2 a_1 + g^2}$$

$$+ \frac{\cot a_1\left(\sqrt{b^2 \cot^2 a_1 + g^2} - \sqrt{b^2 \cot^2 a_2 + g^2}\right)}{\cot a_1 + \cot a_2}$$

$$= - \frac{g^2 (\cot a_2 + \cot a_1)}{\cot a_1\sqrt{b^2 \cot^2 a_2 + g^2} + \cot a_2\sqrt{b^2 \cot^2 a_1 + g^2}} .$$

Remembering that $\tan a_1 + \tan a_2 = \dfrac{\sin A}{\cos a_1 \cos a_2}$,

$$b = g \sec s = g\sqrt{1 + S^2},$$

and $\cot a_1 + \cot a_2 = \dfrac{\sin A}{\sin a_1 \sin a_2}$.

and substituting, we have

$$\frac{E' - E}{2} + \frac{p \cos a_1 \cos a_2}{2 \sin A} - g\sqrt{(1 + S^2) \cot^2 a_1 + 1}$$

$$+ \frac{g \cos a_1 \sin a_2}{\sin A}\left[\sqrt{(1 + S^2) \cot^2 a_1 + 1} - \sqrt{(1 + S^2) \cot^2 a_2 + 1}\right]$$

$$+ \frac{g^2(\cot a_2 + \cot a_1)}{g(\cot a_1 \sqrt{(1 + S^2) \cot^2 a_2 + 1} + \cot a_2 \sqrt{(1 + S^2)\cot^2 a_1 + 1}} = 0.$$

Substituting,

$G = 2g$, $\sqrt{(1 + S^2) \cot^2 a_1 + 1} = \sqrt{\csc^2 a_1 + S^2 \cot^2 a_1}$, and

$$\sqrt{(1 + S^2) \cot^2 a_2 + 1} = \sqrt{\csc^2 a_2 + S^2 \cot^2 a_2}$$

then,

$$G = \left[(E' - E) + \frac{p \cos a_1 \cos a_2}{\sin A}\right] \div \left[\sqrt{\csc^2 a_1 + S^2 \cot^2 a_1} + \frac{\cos a_1 \sin a_2}{\sin A}\left(\sqrt{\csc^2 a_1 + S^2 \cot^2 a_1} - \sqrt{\csc^2 a_2 + S^2 \cot^2 a_2}\right) - \frac{\cot a_2 + \cot a_1}{\cot a_1\sqrt{\csc^2 a_2 + S^2 \cot^2 a_2} + \cot a_2 \sqrt{\csc^2 a_1 + S^2 \cot^2 a_1}}\right] \quad (6)$$

When $S = \tan s = o$, equation (6) takes the form,

$$G = \frac{E' - E + \dfrac{p \cos a_1 \cos a_2}{\sin A}}{\csc a_1 \dfrac{\cos a_1 \sin a_2}{\sin A}(\csc a_1 - \csc a_2) - \dfrac{\cot a_1 + \cot a_2}{\cot a_1 \csc a_2 + \cot a_2 \csc a_1}}$$

which reduces to

$$G = \frac{\left[\sin A\,(E' - E) + p \cos a_1 \cos a_2\right]\left[\cos a_1 + \cos a_2\right]}{(\cos a_1 + \cos a_2)^2 - \sin^2 A} \quad (7)$$

When $a_1 = a_2 = a$, equation (6) takes the form

$$G = \frac{(E' - E) + \frac{p}{2} \cot a}{- \sqrt{\csc^2 a + S^2 \cot^2 a} - \dfrac{1}{\sqrt{\csc^2 a + S^2 \cot^2 a}}}$$

which reduces to,

$$G = \frac{2\,(E' - E) \tan a + p}{2\,(1 + S^2)} \sqrt{\sec^2 a + S^2}\,. \quad (8)$$

When $S = \tan s = o$, equation (8) reduces to:
Likewise, when $a_1 = a_2 = a$, equation (7) reduces to:

$$G = \sec a \left[(E' - E) \tan a + \frac{p}{2}\right]. \quad (9)$$

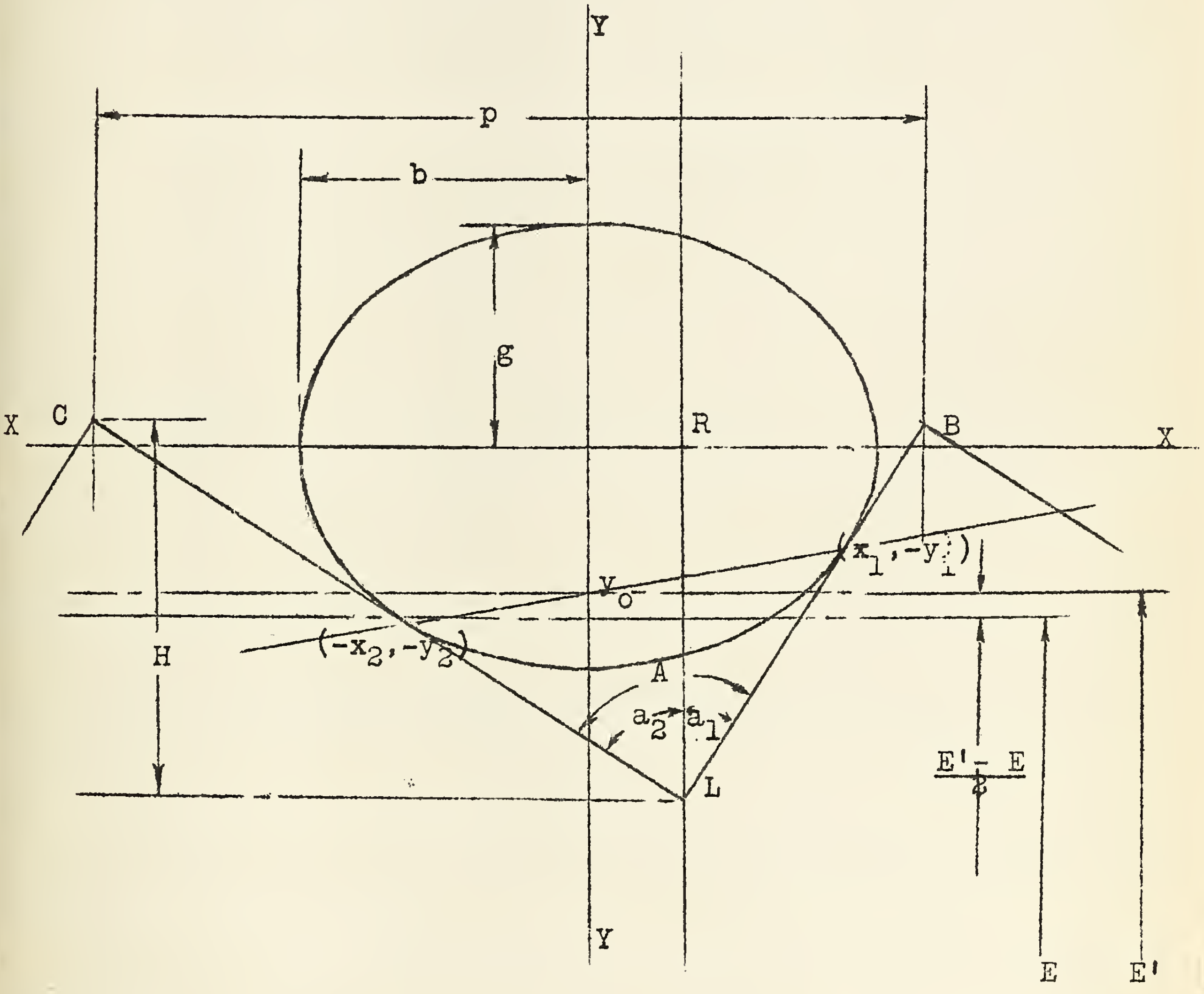


Fig. 12

When E' = E, equation (8) reduces to:

$$G = \frac{p\sqrt{\sec^2 a + S^2}}{2\,(1 + S^2)} \tag{10}$$

When E' = E, equation (7) reduces to:

$$G = \frac{p\,(\cos a_1 + \cos a_2)\cos a_1 \cos a_2}{(\cos a_1 + \cos a_2)^2 - \sin^2 A} \tag{11}$$

or in terms of half of the total angle and one of the angles,

$$G = \frac{p \cos^2 a_1 \left(2 \cos \frac{A}{2} + \sec \frac{A}{2} + 2 \sin \frac{A}{2} \tan a_1\right)}{2 \cos\left(\frac{A}{2} - a_1\right) - 2 \sin^2 \frac{A}{2} \sec\left(\frac{A}{2} - a_1\right)}.$$

when E' = E, equation (9) reduces to:
Likewise, when S = o, equation (10) reduces to:
And, when $a_1 = a_2 = a$, equation (11) reduces to: (12)

$$G = \frac{p}{2} \sec a.$$

Thus:

Equation (6) is the general formula for the size of wire touching at any given diameter, E' (see assumption (1)) of any unsymmetrical thread, in which the helix angle is taken into account.

Equation (7) is the formula for the size of wire touching at any given diameter, E', of any unsymmetrical thread (of small helix angle) in which the helix angle is not taken into account.

Equation (8) is the formula for the size of wire touching the sides of a symmetrical thread at any given diameter E', in which the helix angle is taken into account.

Equation (9) is the formula for the size of wire touching the sides of a symmetrical thread at any given diameter E', in which the helix angle is not taken into account.

Equation (10) is the formula for the size of wire touching at the pitch diameter of a symmetrical thread in which the helix angle is taken into account. This formula, as well as equation (9) is useful in the measurement of threads of coarse pitch such as Acme thread, and also in the measurement of multiple threads.

Equation (11) is the formula, in which the helix angle is not taken into account, for the size of wire touching the sides of an unsymmetrical thread at such points that the line connecting them intersects the pitch line in the plane passing through the center of the wire perpendicular to the axis of the screw. It may be considered the "best-size" of wire for an unsymmetrical thread of given angles (a_1 and a_2) and pitch.

Equation (12) is the formula for the size of wire touching at the pitch diameter of a symmetrical thread in which the helix angle is not taken into account. This size is commonly known as the "best-size" of wire.

APPENDIX 3

EFFECTIVE SIZE - PITCH DIAMETER PLUS INCREMENTS DUE TO LEAD AND ANGLE ERRORS

A. Pitch Diameter Increment Due to Lead Error:

As stated on page 8, when a threaded plug having a given pitch diameter and correct thread form, but having an error in lead, fits snugly in a nut having correct lead, angle, and thread form, the thread in the nut must have a pitch diameter which is larger than that of the plug. The fit of the nut on the screw depends entirely on the maximum lead error present within the length of engagement, regardless of the number of threads within the interval within which it occurs. A formula, which gives the amount of this difference, for straight threads, between the pitch diameters of the screw and nut due to the maximum lead error present between any two threads engaged, is as follows:

$$E' = (\pm p') \cot a$$

in which E' = pitch diameter increment due to lead error; or difference between pitch diameters of incorrect screw and perfect nut, or vice versa

p' = the maximum lead error between any two of the threads engaged

a = half-angle of thread

This formula is derived, with reference to Fig. 13, as follows:

Let P_1 = distance between the two threads on plug between which the maximum lead error occurs

P_2 = corresponding correct or nominal distance = distance between corresponding threads in correct component

$p' = P_1 - P_2$

a = half-angle of thread

K_1 = minor diameter (at sharp-V) of screw

K_2 = minor diameter (at sharp-V) of nut

E_1 = pitch diameter of screw measured by the three wire method and based on the nominal pitch of the thread. (It is not the actual average pitch diameter of the threads in which the lead error occurs).

E_2 = pitch diameter of nut

$E' = E_2 - E_1$

Referring to Fig. 13.

$$\frac{m}{n} = \frac{\frac{P_2 - P_1}{2}}{K_2 - K_1} = \frac{P_2 - P_1}{K_2 - K_1} = \tan a$$

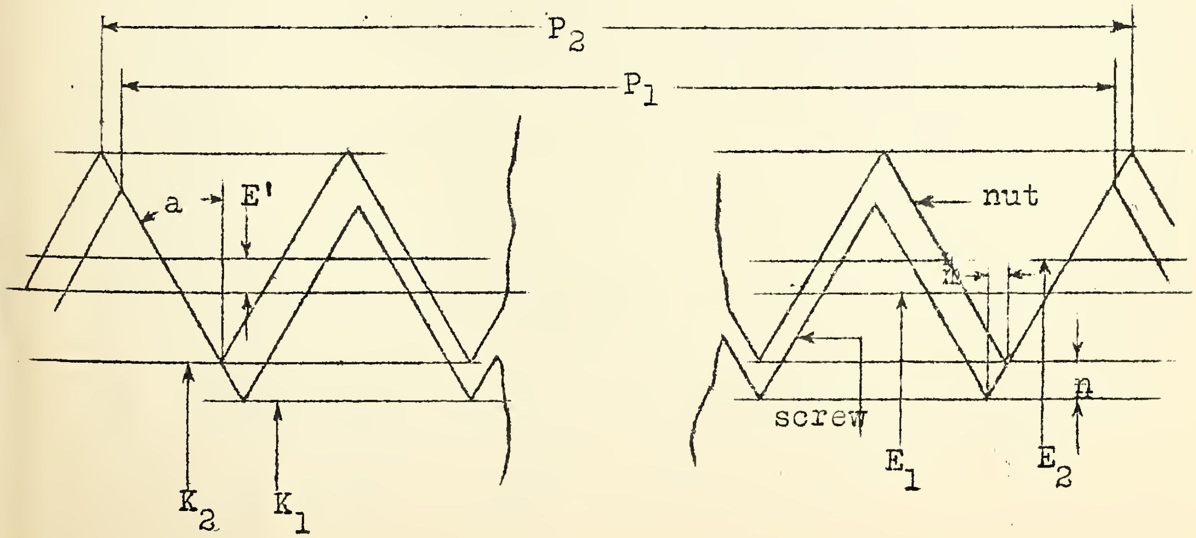
$$K_2 - K_1 = \frac{P_2 - P_1}{\tan a}$$

$$K_2 - K_1 = E_2 - E_1$$

$$E_2 - E_1 = (P_2 - P_1) \cot a$$

$$E' = (\pm p') \cot a$$

The quantity E' is always added to the measured pitch diameter in the case of an external thread, and it is always subtracted in the case of an internal thread, regardless of the sign introduced by the lead error p'.



B. Pitch Diameter Increment Due to Angle Error

The presence of an error in the included angle of a thread likewise necessitates a difference between the pitch diameters of a screw and nut which fit snugly together. The formulas which express the increment E" which, in order to compensate for an error in angle, is to be added to the measured pitch diameter of a screw and subtracted from the measured pitch diameter of a nut, are derived below, with reference to Figs.14 A, B, and C.

As the basis for their development, the side of the thread is conceived of as having been rotated from the correct position through a small angle about a point at the midslope of the thread as the center of rotation. The errors are assumed to be in the thread of the screw, and the latter fitted into a thread ring gage of basic dimensions having a truncated crest and cleared at the root of the thread, as shown in Figs, 14 A, B, and C. The major diameter of the screw is assumed to be basic, as would ordinarily be the case. The pitch diameter of the screw is less than basic by the increment E" to be derived.

If the errors were assumed to be in the nut and this were fitted to a thread plug gage of basic dimensions, the pitch diameter of the nut would be greater than basic by the increment E".

Formulas for three different cases are derived, namely:

First Case, - Angle errors on both sides of thread are plus,
Second Case, - Angle errors on both sides of thread are minus,
Third Case, - Angle error on one side of thread is plus and on the other side is minus.

In these formulas,

p = thread interval
F = width of basic flat
a = correct half-angle of thread
a_1 = angle error on one side of thread
a_2 = angle error on other side of thread

e'' = pitch radius increment due to angle error
E'' = pitch diameter increment due to angle error

1. First Case, - Plus Angle Errors on Both Sides of Thread

Referring to Fig. 14 A,

$$e'' = r - t + n - f$$

$$r = \frac{p}{4} \cot a$$

$$t = \frac{p}{2\,(\tan\,(a + a_1) + \tan\,(a + a_2))}$$

$$n = \frac{F}{\tan\,(a + a_1) + \tan\,(a + a_2)}$$

$$f = \frac{F}{2} \cot a$$

Then,

$$e'' = \frac{p}{4}\cot a - \frac{p}{2(\tan(a+a_1)+\tan(a+a_2))} + \frac{F}{\tan(a+a_1)+\tan(a+a_2)} - \frac{F}{2}\cot a$$

$$e'' = \left(\frac{p}{2} - F\right)\left(\frac{\cot a}{2} - \frac{1}{\tan(a+a_1) + \tan(a+a_2)}\right)$$

$$E'' = 2e'' = \left(\frac{p}{2} - F\right)\left(\cot a - \frac{2}{\tan(a+a_1) + \tan(a+a_2)}\right). \qquad (1)$$

If $+a_1 = +a_2 = a'$,

$$E'' = \left(\frac{p}{2} - F\right)\left(\cot a - \cot(a + a')\right). \qquad (2)$$

2. Second Case, - Minus Angle Errors on Both Sides of Thread

Referring to Fig. 14B,

$$e'' = t - r + f - n$$

$$t = \frac{p}{2(\tan(a-a_1) + \tan(a-a_2))}$$

$$r = \frac{p}{4}\cot a$$

$$f = \frac{F}{2}\cot a$$

$$n = \frac{F}{\tan(a-a_1) + \tan(a-a_2)}$$

Then,

$$e'' = \frac{p}{2(\tan(a-a_1) + \tan(a-a_2))} - \frac{p}{4}\cot a + \frac{F}{2}\cot a - \frac{F}{\tan(a-a_1) + \tan(a-a_2)}$$

$$e'' = \left(\frac{p}{2} - F\right)\left(\frac{1}{\tan(a-a_1) + \tan(a-a_2)} - \frac{\cot a}{2}\right)$$

$$E'' = 2e'' = \left(\frac{p}{2} - F\right)\left(\frac{2}{\tan(a-a_1) + \tan(a-a_2)} - \cot a\right). \qquad (3)$$

If $-a_1 = -a_2 = -a'$,

$$E'' = \left(\frac{p}{2} - F\right)\left(\cot(a - a') - \cot a\right). \qquad (4)$$

3. Third Case, - Plus Angle Error on One Side and Minus Angle Error on Other Side of Thread

Referring to Fig. 14C,

$$e'' = \frac{m + n}{2}\cot a$$

$$m = v(\tan(a+a_2) - \tan a)$$

$$n = u(\tan a - \tan(a-a_1))$$

$$v = \left(\frac{p}{4} - \frac{F}{2}\right)\cot a - e''$$

$$u = \left(\frac{p}{4} - \frac{F}{2}\right)\cot a + e''$$

Then,

$$e'' = \frac{\cot a}{2}\left[\left(\left(\frac{p}{4} - \frac{F}{2}\right)\cot a - e''\right)\left(\tan(a+a_2) - \tan a\right) + \left(\left(\frac{p}{4} + \frac{F}{2}\right)\cot a + e''\right)\left(\tan a - \tan(a-a_1)\right)\right]$$

$$e'' + e''\frac{\cot a}{2}\left(\tan(a+a_2) + \tan(a-a_1) - 2\tan a\right) = \frac{\cot a}{2}\left[\left(\frac{p}{4}-\frac{F}{2}\right)\cot a\left(\tan(a+a_2) - \tan(a-a_1)\right)\right]$$

$$e'' = \frac{\left(\frac{p}{4}-\frac{F}{2}\right)\cot a\left(\tan(a+a_2) - \tan(a-a_1)\right)}{\tan(a+a_2) + \tan(a-a_1)} \qquad (5)$$

$$E'' = 2e'' = \frac{\left(\frac{p}{2}-F\right)\cot a\left(\tan(a+a_2) - \tan(a-a_1)\right)}{\tan(a+a_2) + \tan(a-a_1)}. \qquad (6)$$

If $a_1 = a_2 = a'$,

$$E'' = \frac{\left(\frac{p}{2}-F\right)\cot a\left(\frac{\tan a + \tan a'}{1-\tan a\tan a'} - \frac{\tan a - \tan a'}{1+\tan a\tan a'}\right)}{\frac{\tan a + \tan a'}{1-\tan a\tan a'} + \frac{\tan a - \tan a'}{1+\tan a\tan a'}},$$

$$E'' = \frac{\left(\frac{p}{2}-F\right)\cot a\left(\frac{2\tan a'(1+\tan^2 a)}{1-\tan^2 a\tan^2 a'}\right)}{\frac{2\tan a(1+\tan^2 a')}{1-\tan^2 a\tan^2 a'}}$$

$$E'' = \left(\frac{p}{2}-F\right)\frac{\cot^2 a\ \tan a' + \tan a'}{1+\tan^2 a'}. \qquad (7)$$

Formula (7) does not apply when the angle errors are of such magnitude as to result in contact of the crest of the thread of the screw at a point nearer the pitch line of the ring gage than the position of the basic flat. As such angle errors would exceed ordinary commercial tolerances, this case need not be considered.

4. Reduction to apply to National Standard Threads

For the National Standard Thread form, $a = 30°$ and $F = \frac{p}{8}$.

Substituting in (2),

$$E'' = \frac{3}{8}p\left(\sqrt{3} - \frac{\sqrt{3}\cot a' - 1}{\sqrt{3}+\cot a'}\right),$$

which reduces to,

$$E'' = \frac{3p}{2(\sqrt{3}+\cot a')}. \qquad (8)$$

Similarly, substituting a = 30° and $F = \frac{p}{8}$ in (4)

$$E'' = \frac{3}{8}p \left(\frac{\sqrt{3}\cot a' + 1}{\cot a' - \sqrt{3}} - \sqrt{3} \right),$$

which reduces to,

$$E'' = \frac{3p}{2(\cot a' - \sqrt{3})} . \qquad (9)$$

Thus the formula for E", when the error a' is equal and of the same sign on both sides of the thread, can be written

$$E'' = \frac{3p}{2(\cot a' \pm \sqrt{3})} \qquad (10)$$

Substituting a = 30° and $F = \frac{p}{8}$ in (7), we have,

$$E'' = \left(\frac{3}{8}p\right)\frac{4 \tan a'}{1 + \tan^2 a'}$$

which reduces to,

$$E'' = \frac{3p}{2(\cot a' + \tan a')} , \qquad (11)$$

when the error is equal on both sides, but plus on one and minus on the other side.

As the $\sqrt{3}$ is small compared with the cotangent of a small angle, and the tangent of a small angle is small compared with its cotangent, equations (10) and (11) may be summarized into the following close approximation:

$$E'' = 1.5\ p \tan a', \qquad (12)$$

which gives values close to the mean of values given by equation (10), and is, therefore recommended for general use.

(c). Effective Size:

The equivalent pitch diameter E_q, or what may correctly be termed the "Effective Size", of a screw having errors in lead and angle is:

$$E_q = E + E' + E''$$

and of a nut having errors in lead and angle is:

$$E_q = E - E' - E''.$$

The summation of E' and E" in this way is permissible although not absolutely correct. It can be shown that the presence of an angle error has a slight effect on the value of E'; and similarly an error in lead has a slight effect on the value of E", but these are of secondary magnitude and may be disregarded.

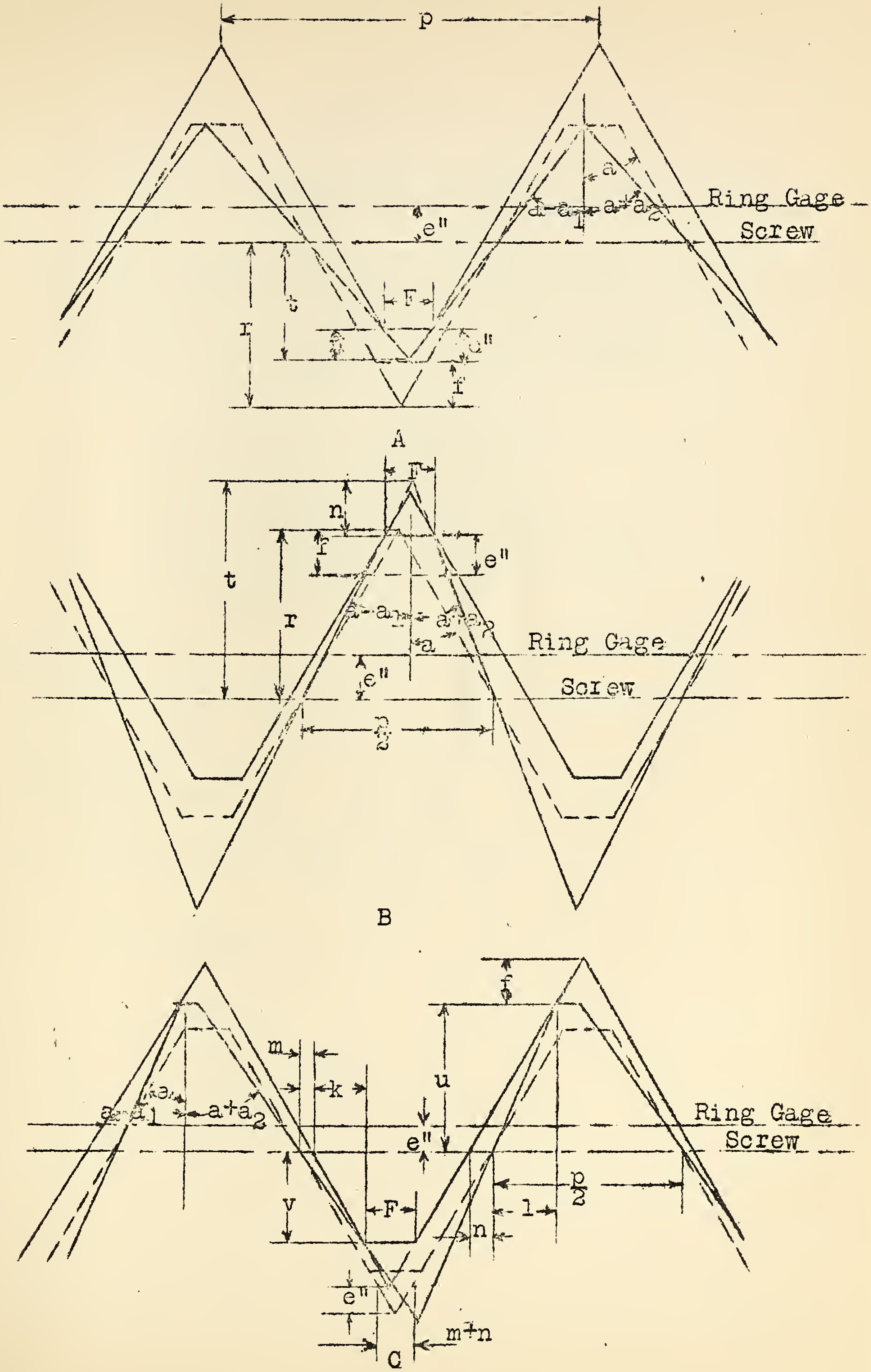


Fig. 14.- Pitch Diameter Increment Due to Angle Errors

APPENDIX 4

DERIVATION OF FORMULA FOR THREAD ANGLE MEASURED BY TWO-SIZE WIRE METHOD.

The following is a derivation of the formula for determining thread angle of a symmetrical thread by measuring over two sizes of wires, in which the helix angle is taken into account. The formula is a close approximation, in which the helix angle, s, is taken to be the helix angle at the pitch diameter.

Let
a = half-angle of thread
g_1 = radius of large wire
G_1 = diameter of large wire
g_2 = radius of small wire
G_2 = diameter of small wire
M_1 = measurement over large wires
M_2 = measurement over small wires
s = helix angle
S = tan s.

Referring to Fig. 15,

$$\sin a = \frac{BD}{AB} = \frac{CE}{AC}$$

$$\frac{M_1 - M_2}{2} = AC + g_1 - (AB + g_2)$$

$$M_1 - M_2 = 2(AC - AB) + (G_1 - G_2)$$

$$AB = \frac{BD}{\sin a}$$

$$AC = \frac{CE}{\sin a}$$

$$M_1 - M_2 = \frac{2\,(CE - BD)}{\sin a} + (G_1 - G_2)$$

$$\sin a = \frac{2\,(CE - BD)}{(M_1 - M_2) - (G_1 - G_2)}$$

Approximately, $CE = g_1 \sec s = g_1\sqrt{1 + \tan^2 s} = g_1\sqrt{1 + S^2} = g_1(1 + \frac{S^2}{2})$

Similarly, $BD = g_2 \sec s = g_2\,(1 + \frac{S^2}{2})$

Then
$$\sin a = \frac{2\,(g_1 - g_2)\,(1 + \frac{S^2}{2})}{(M_1 - M_2) - (G_1 - G_2)} \qquad (1)$$

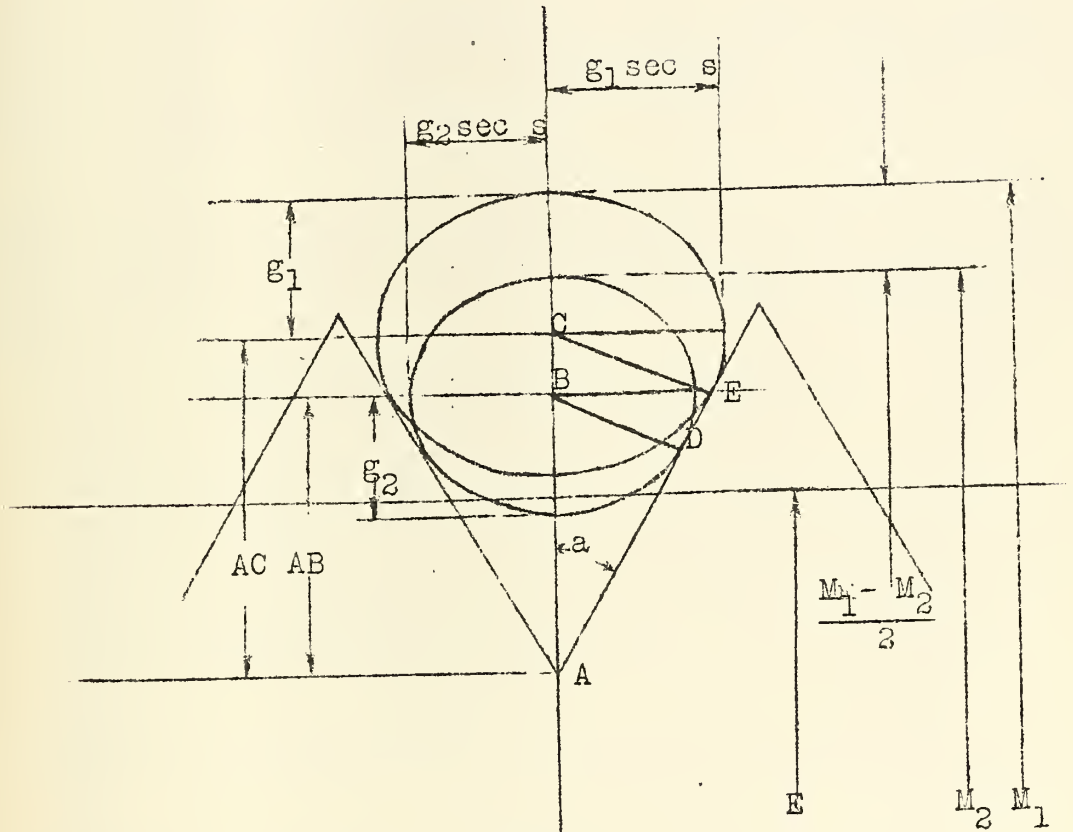


Fig. 15

$$\sin a = \frac{(G_1 - G_2)\,(1 + \frac{S^2}{2})}{(M_1 - M_2) - (G_1 - G_2)}.$$

For values of S^2 see Table 10.
If the helix angle is small it may be neglected, and (1) reduces to,

$$\sin a = \frac{(G_1 - G_2)}{(M_1 - M_2) - (G_1 - G_2)}.$$

APPENDIX 5

SYMBOLS FOR SCREW THREAD NOTATION AND FORMULAS

The following symbols are being used by the Gage Section, Bureau of Standards, in connection with the inspection of screw thread gages, for expressing relations of screw thread elements, for use on drawings, and other similar purposes. In determining the particular symbols given herein, consideration was given to the following:

Use of Symbols Found on Typewriter: For convenient reference in reports and letters, symbols were chosen that could be written on the typewriter. This made it necessary to abandon the very good practice of using Greek letters for angles, but this was thought to be justified.

Consistent Use of Large and Small Letters: Since it seemed desirable to provide symbols for both the diameters and radii of the various elements of the screw, it was decided that the use of the large and small le tters in a systematic manner would give the best results.

SYMBOLS FOR SCREW THREAD FORMULAS.

Major diameter (outside diameter)--------------D
Corresponding radius---------------------------d

Pitch diameter---------------------------------E
Corresponding radius---------------------------e

Minor diameter (core diameter)-----------------K
Corresponding radius---------------------------k

Angle of thread--------------------------------A
Half-angle of thread---------------------------a

Number of turns per inch-----------------------N
Number of threads per inch---------------------n

Lead---$P = \frac{1}{N}$

Pitch or thread interval-----------------------$p = \frac{1}{n}$

Helix angle------------------------------------s

Tangent of helix angle-------------------------$S = \frac{P}{3.14159}$

Width of basic flat at top, crest, or root-----------F
Depth of basic truncation---------------------------f

Depth of sharp V-thread-----------------------------H
Depth of National (U.S.S.) form of thread-----------h

Included angle of taper-----------------------------Y
One-half angle of taper-----------------------------y

Radius of curvature (Whitworth crest or root)--------c
Width across crest or root - Whitworth thread--------C

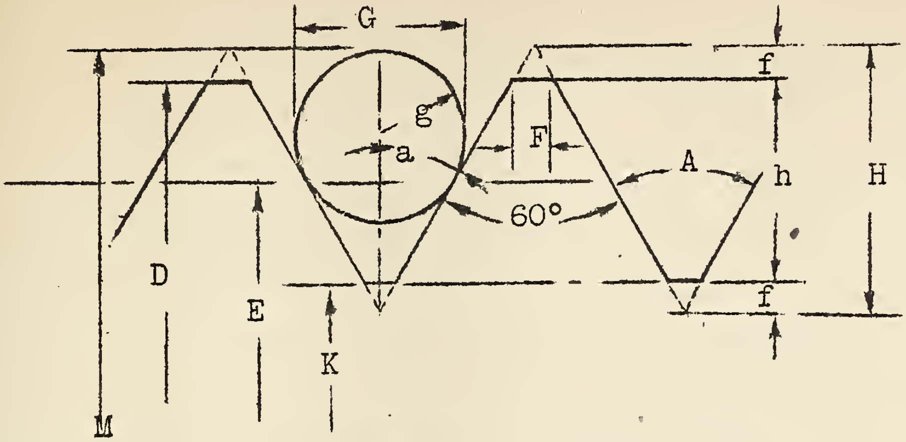
Symbols used with Wire Measurements

Measurement over wires------------------------------M

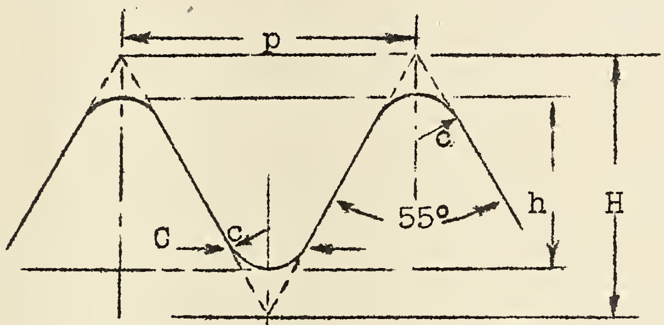
Diameter of wire------------------------------------G
Corresponding radius--------------------------------g

*Factor for wire measurement------------------------X

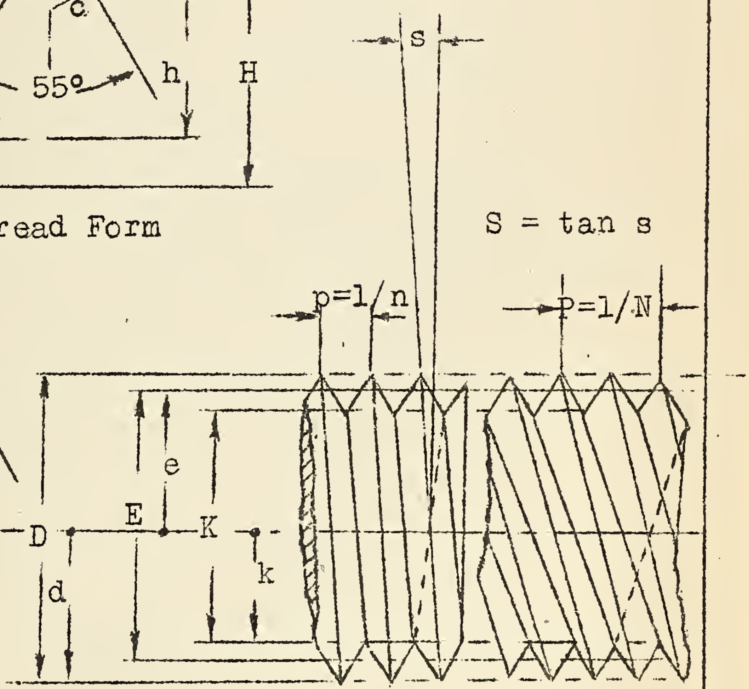
$$*X = \frac{\cot a}{2n} - G\,(1 + \operatorname{cosec} a).$$



National Thread Form (U. S. Standard)



Whitworth Thread Form



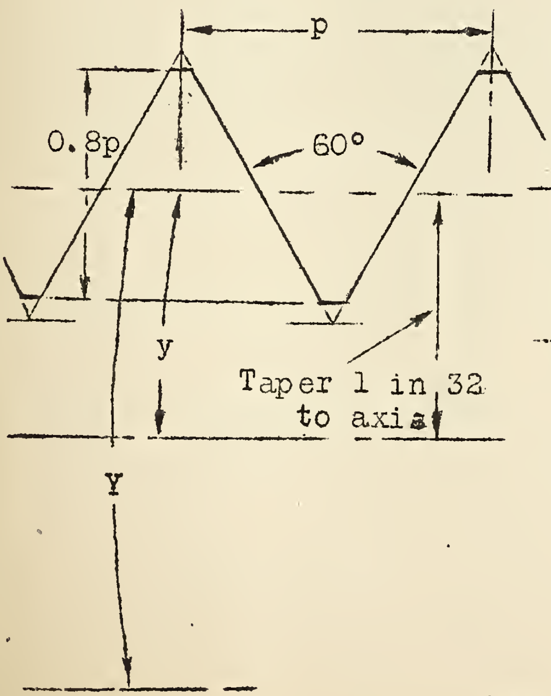


Fig. 16

Table 1.—Wire Sizes and Constants, National (U.S. Standard), and National (Brigg's) Pipe, Threads - 60°

1	2	3	4	5	6	7
Wire sizes*						
Best 0.577350p	Maximum 1.010363p	Minimum 0.505182p	Threads per inch n	Pitch $p = \frac{1}{n}$	Pitch $\frac{p}{2} = \frac{1}{2n}$	Depth of V-thread $\frac{\cot 30°}{2n}$
Inches	Inches	Inches		Inches	Inches	
0.00722	0.01263	0.00631	80	0.01250	0.00625	0.01083
.00802	.01403	.00702	72	.01389	.00694	.01203
.00902	.01579	.00789	64	.01562	.00781	.01353
.01031	.01804	.00902	56	.01786	.00893	.01547
.01203	.02105	.01052	48	.02083	.01042	.01804
.01312	.02296	.01148	44	.02273	.01136	.01968
.01443	.02526	.01263	40	.02500	.01250	.02165
.01604	.02807	.01403	36	.02778	.01389	.02406
.01804	.03157	.01579	32	.03125	.01562	.02706
.02062	.03608	.01804	28	.03571	.01786	.03093
.02138	.03742	.01871	27	.03704	.01852	.03208
.02406	.04210	.02105	24	.04167	.02083	.03608
.02887	.05052	.02526	20	.05000	.02500	.04330
.03208	.05613	.02807	18	.05556	.02778	.04811
.03608	.06315	.03157	16	.06250	.03125	.05413
.04124	.07217	.03608	14	.07143	.03571	.06186
.04441	.07772	.03886	13	.07692	.03846	.06662
.04811	.08420	.04210	12	.08333	.04167	.07217
.05020	.08786	.04393	11.5	.08696	.04348	.07531
.05249	.09185	.04593	11	.09091	.04545	.07873
.05773	.10104	.05052	10	.10000	.05000	.08660
.06415	.11226	.05613	9	.11111	.05556	.09623
.07217	.12630	.06315	8	.12500	.06250	.10825
.08248	.14434	.07217	7	.14286	.07143	.12372
.09623	.16839	.08420	6	.16667	.08333	.14434
.11547	.20207	.10104	5	.20000	.10000	.17321
.12830	.22453	.11226	4.5	.22222	.11111	.19245
.14434	.25259	.12630	4	.25000	.12500	.21651

*For zero helix angle.

n = threads per inch on single thread screws.

Table 3.—Relation of Best Wire Diameters and Pitches,—Wires for National (U.S.S., S.A.E., A.S.M.E. and Briggs') Screw Threads.

Best wire sizes	Threads per inch																											
Inches	80	72	64	56	48	44	40	36	32	28	27	24	20	18	16	14	13	12	11.5	11	10	9	8	7	6	5	4.5	4
0.00722	⊗	x																										
.00802	x	⊗	x																									
.00902	x	x	⊗																									
.01031	x	x	x	⊗																								
.01203	x	x	x	x	⊗	x																						
.01312		x	x	x	x	⊗	x																					
.01443			x	x	x	x	⊗	x																				
.01604				x	x	x	x	⊗	x																			
.01804					x	x	x	x	⊗																			
.02062					x	x	x	x	x	⊗	x																	
.02138						x	x	x	x	x	⊗	x																
.02406							x	x	x	x	x	⊗																
.02887									x	x	x	x	⊗	x														
.03208										x	x	x	x	⊗	x													
.03608											x	x	x	x	⊗													
.04124												x	x	x	x	⊗	x											
.04441													x	x	x	x	⊗	x	x									
.04811													x	x	x	x	x	⊗	x	x								
.05020													x	x	x	x	x	x	⊗	x								
.05249														x	x	x	x	x	x	⊗	x							
.05773															x	x	x	x	x	x	⊗	x						
.06415																x	x	x	x	x	x	⊗	x					
.07217																	x	x	x	x	x	x	⊗					
.08248																		x	x	x	x	x	x	⊗				
.09623																					x	x	x	x	⊗			
.11547																							x	x	x	⊗	x	
.12830																								x	x	x	⊗	x
.14434																									x	x	x	⊗

Note: The crosses (x) indicate those wire diameters which can be used for each pitch. An encircled (⊗) indicates the "best-wire" diameter for that pitch which heads the column.

Table 3.—Wire Sizes and Constants, British Standard Whitworth and British Standard Fine Threads - 55°

1	2	3	4	5	6	7
Wire sizes*			Threads per inch n	Pitch $p = \frac{1}{n}$	$\frac{\text{Pitch}}{2}$ $\frac{p}{2} = \frac{1}{2n}$	$\frac{\cot 27°30'}{2n}$
Best 0.563692p	Max. 0.852727p	Min. 0.505679p				
Inches	Inches	Inches		Inches	Inches	
0.01409	0.02132	0.01264	40	0.02500	0.01250	0.02401
.01566	.02369	.01405	36	.02778	.01389	.02668
.01762	.02665	.01580	32	.03125	.01562	.03002
.02013	.03045	.01806	28	.03571	.01786	.03430
.02168	.03280	.01945	26	.03846	.01923	.03694
.02349	.03553	.02107	24	.04167	.02083	.04002
.02563	.03876	.02299	22	.04545	.02273	.04366
.02818	.04264	.02528	20	.05000	.02500	.04802
.03132	.04737	.02809	18	.05556	.02778	.05336
.03523	.05330	.03160	16	.06250	.03125	.06003
.04026	.06091	.03612	14	.07143	.03571	.06861
.04697	.07106	.04214	12	.08333	.04167	.08004
.05124	.07752	.04597	11	.09091	.04545	.08732
.05637	.08527	.05057	10	.10000	.05000	.09605
.06263	.09475	.05619	9	.11111	.05556	.10672
.07046	.10659	.06321	8	.12500	.06250	.12006
.08053	.12182	.07224	7	.14286	.07143	.13721
.09395	.14212	.08428	6	.16667	.08333	.16008
.11274	.17054	.10114	5	.20000	.10000	.19210
.12526	.18949	.11237	4.5	.22222	.11111	.21344
.14092	.21318	.12642	4	.25000	.12500	.24012
.16105	.24364	.14448	3.5	.28571	.14286	.27443

*For zero helix angle.
n = threads per inch for single thread screws.

Table 4.—Relation of Best Wire Diameters and Pitches —
Wires for Whitworth Threads

Best wire sizes	Threads per inch																					
Inches	40	36	32	28	26	24	22	20	18	16	14	12	11	10	9	8	7	6	5	4.5	4	3.5
0.01409	⊗	x																				
.01566	x	⊗																				
.01762	x	x	⊗																			
.02013	x	x	x	⊗	x																	
.02168		x	x	x	⊗																	
.02349		x	x	x	x	⊗	x															
.02563			x	x	x	x	⊗	x														
.02818				x	x	x	x	⊗	x													
.03132					x	x	x	x	⊗													
.03523						x	x	x	x	⊗												
.04026								x	x	x	⊗											
.04697									x	x	x	⊗	x									
.05124										x	x	x	⊗	x								
.05637											x	x	x	⊗	x							
.06263												x	x	x	⊗							
.07046												x	x	x	x	⊗						
.08053														x	x	x	⊗					
.09395															x	x	x	⊗	x			
.11274																	x	x	⊗	x		
.12526																		x	x	⊗		
.14092																		x	x	x	⊗	
.16105																			x	x	x	⊗

Note: The crosses (x) indicate those wire diameters which can be used for each pitch. An encircled cross (⊗) indicates the "best-wire" diameter for that pitch which heads the column.

Table 5.—Relation of Best Wire Diameters and Pitches,— National Wires for Whitworth Threads

National Threads		Whitworth Threads per Inch																					
Threads per inch	Best wire sizes Inches	40	36	32	28	26	24	22	20	18	16	14	12	11	10	9	8	7	6	5	4.5	4	3.5
44	0.01312	x																					
40	.01443	$\underline{x}$	x																				
36	.01604	x	$\underline{x}$	x																			
32	.01804	x	x	$\underline{x}$																			
28	.02062	x	x	x	$\underline{x}$	x																	
27	.02138		x	x	x	$\underline{x}$	x																
24	.02406			x	x	x	$\underline{x}$	$\underline{x}$															
20	.02887				x	x	x	x	$\underline{x}$	x													
18	.03208					x	x	x	x	$\underline{x}$	x												
16	.03608							x	x	x	$\underline{x}$												
14	.04124								x	x	x	$\underline{x}$											
13	.04441									x	x	x	x										
12	.04811										x	x	$\underline{x}$	x									
11.5	.05020										x	x	x	$\underline{x}$									
11	.05249										x	x	x	x	x								
10	.05773											x	x	x	$\underline{x}$	x							
9	.06415												x	x	x	$\underline{x}$	x						
8	.07217													x	x	x	$\underline{x}$						
7	.08248														x	x	x	$\underline{x}$					
6	.09623																x	x	$\underline{x}$				
5	.11547																	x	x	$\underline{x}$	x		
4.5	.12830																		x	x	$\underline{x}$	x	
4	.14434																			x	x	$\underline{x}$	

Note: The crosses (x) indicate those wire diameters which can be used for each pitch. An underlined cross ($\underline{x}$) indicates the nearest "best-wire" diameter for that pitch which heads the column.

Table 6.—Wire Sizes and Constants, — International Metric Screw Thread System and Extension - 60°

1	2	3	4	5	6	7	8
Wire Sizes*			Approx. Threads per inch	Pitch	Pitch	Pitch / 2	$\frac{p}{2}$ cot 30°
Best 0.577350p	Max. 1.010363p	Min. 0.505182p	n	p	p	$\frac{p}{2}$	
Inches	Inches	Inches		mm	Inches	Inches	
0.00546	0.00955	0.00477	105.8	0.24	0.00945	0.00472	0.00818
.00614	.01074	.00537	94.1	.27	.01063	.00531	.00921
.00682	.01193	.00597	84.7	.30	.01181	.00591	.01023
.00750	.01313	.00656	77.0	.33	.01299	.00650	.01125
.00818	.01432	.00716	70.6	.36	.01417	.00709	.01227
.00886	.01551	.00776	65.2	.39	.01535	.00768	.01330
.00955	.01671	.00835	60.5	.42	.01654	.00827	.01432
.01023	.01790	.00895	56.4	.45	.01772	.00886	.01534
.01364	.02387	.01193	42.3	.60	.02362	.01181	.02046
.01705	.02983	.01492	33.9	.75	.02953	.01476	.02557
.02046	.03580	.01790	28.2	.90	.03543	.01772	.03069
.02273	.03978	.01989	25.4	1.00	.03937	.01968	.03410
.02841	.04972	.02486	20.3	1.25	.04921	.02461	.04262
.03410	.05967	.02983	16.9	1.50	.05906	.02953	.05114
.03978	.06961	.03481	14.5	1.75	.06890	.03445	.05967
.04546	.07956	.03978	12.7	2.00	.07874	.03937	.06819
.05683	.09944	.04972	10.2	2.50	.09842	.04921	.08524
.06819	.11933	.05967	8.5	3.00	.11811	.05906	.10229
.07956	.13922	.06961	7.3	3.50	.13780	.06890	.11933
.09092	.15911	.07956	6.4	4.00	.15748	.07874	.13638
.10229	.17900	.08950	5.6	4.50	.17716	.08858	.15343
.11365	.19889	.09945	5.1	5.00	.19685	.09842	.17048
.12502	.21878	.10939	4.6	5.50	.21654	.10827	.18752
.13638	.23867	.11933	4.2	6.00	.23622	.11811	.20457
.14775	.25856	.12928	3.9	6.50	.25590	.12795	.22162
.15911	.27845	.13922	3.6	7.00	.27559	.13780	.23867
.17048	.29833	.14917	3.4	7.50	.29528	.14764	.25572
.18184	.31822	.15911	3.2	8.00	.31496	.15748	.27276

*For zero helix angle

n = threads per inch for single thread screws.

[illegible]

Table 7.—Relation of Best Wire Diameters and Pitches, — National Wires for International Metric Threads

National threads		International thread pitches in millimeters																											
Threads per inch	Best wire sizes Inches	0.24	0.27	0.30	0.33	0.36	0.39	0.42	0.45	0.60	0.75	0.90	1.00	1.25	1.50	1.75	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00
80	0.00722	x	x	x	$\underline{x}$	x																							
72	.00802	x	x	x	x	$\underline{x}$	x																						
64	.00902	x	x	x	x	x	$\underline{x}$	$\underline{x}$	x																				
56	.01031		x	x	x	x	x	x	$\underline{x}$																				
48	.01203				x	x	x	x	x	x																			
44	.01312					x	x	x	x	$\underline{x}$																			
40	.01443						x	x	x	x																			
36	.01604							x	x	x	$\underline{x}$																		
32	.01804									x	x	$\underline{x}$																	
28	.02062									x	x	x	$\underline{x}$																
27	.02138									x	x	x	x																
24	.02406										x	x	x																
20	.02887										x	x	x	$\underline{x}$															
18	.03208											x	x	x	x														
16	.03608												x	x	$\underline{x}$	x													
14	.04124													x	x	$\underline{x}$	x												
13	.04441													x	x	x	$\underline{x}$												
12	.04811													x	x	x	x												
11.5	.05020														x	x	x	x											
11	.05249														x	x	x	x											
10	.05773														x	x	x	$\underline{x}$											
9	.06415															x	x	x	x										
8	.07217																x	x	$\underline{x}$	x									
7	.08248																	x	x	$\underline{x}$	x								
6	.09623																	x	x	x	$\underline{x}$	$\underline{x}$							
5	.11547																		x	x	x	x	$\underline{x}$	x					
4.5	.12830																			x	x	x	x	$\underline{x}$	$\underline{x}$				
4	.14434																				x	x	x	x	x	$\underline{x}$	$\underline{x}$		

Note: The crosses (x) indicate those wires which can be used for each pitch. An underlined cross ($\underline{x}$) indicates the nearest "best-wire" diameter for that pitch which heads the column.

Table 8.—Wire Sizes and Constants, Lowenherz Standard Threads — 53°8'

1	2	3	4	5	6	7	8
Wire sizes*			Approx. threads per inch			Pitch/2	$\frac{p}{2}$ cot 26°34'
Best 0.559025p	Max. 0.978284p	Min. 0.540758p	n	Pitch p	Pitch p	$\frac{p}{2}$	
Inches	Inches	Inches		mm	Inches	Inches	
0.00550	0.00963	0.00532	101.6	0.25	0.00984	0.00492	0.00984
.00660	.01155	.00639	84.7	.30	.01181	.00591	.01181
.00770	.01348	.00745	72.6	.35	.01378	.00689	.01378
.00880	.01541	.00852	63.5	.40	.01575	.00787	.01575
.00990	.01733	.00958	56.4	.45	.01772	.00886	.01772
.01100	.01926	.01064	50.8	.50	.01968	.00984	.01968
.01321	.02311	.01277	42.3	.60	.02362	.01181	.02362
.01541	.02696	.01490	36.3	.70	.02756	.01378	.02756
.01651	.02889	.01597	33.9	.75	.02953	.01476	.02953
.01761	.03081	.01703	31.7	.80	.03150	.01575	.03150
.01981	.03466	.01916	28.2	.90	.03543	.01772	.03543
.02201	.03852	.02129	25.4	1.00	.03937	.01968	.03937
.02421	.04237	.02342	23.1	1.10	.04331	.02165	.04331
.02641	.04622	.02555	21.2	1.20	.04724	.02362	.04724
.02861	.05007	.02768	19.5	1.30	.05118	.02559	.05118
.03081	.05392	.02981	18.1	1.40	.05512	.02756	.05512
.03521	.06162	.03406	15.9	1.60	.06299	.03150	.06299
.03962	.06933	.03832	14.1	1.80	.07087	.03543	.07087
.04402	.07703	.04258	12.7	2.00	.07874	.03937	.07874
.04842	.08473	.04684	11.5	2.20	.08661	.04330	.08661
.05282	.09244	.05110	10.6	2.40	.09449	.04724	.09449
.06162	.10784	.05961	9.1	2.80	.11024	.05512	.11024
.07043	.12325	.06813	7.9	3.20	.12598	.06299	.12598
.07923	.13865	.07664	7.1	3.60	.14173	.07087	.14173
.08804	.15406	.08516	6.4	4.00	.15748	.07874	.15748
.09684	.16947	.09367	5.8	4.40	.17323	.08661	.17323

*For zero helix angle
n = threads per inch for single thread screws.

Table 9.—Relation of Best-Wire Diameters and Pitches,— National Wires for Löwenherz Threads.

National threads: Threads per inch	National threads: Best wire sizes, Inches	Lowenherz pitches in millimeters: 0.25	0.30	0.35	0.40	0.45	0.50	0.60	0.70	0.75	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.60	1.80	2.00	2.20	2.40	2.80	3.20	3.60	4.00	4.40
80	0.00722	x	x																								
72	.00802	x	x	x̲																							
64	.00902	x	x	x	x̲																						
56	.01031		x	x	x	x̲																					
48	.01203			x	x	x	x̲																				
44	.01312			x	x	x	x	x̲																			
40	.01443				x	x	x	x																			
36	.01604					x	x	x	x̲	x̲																	
32	.01804						x	x	x	x	x̲																
28	.02062							x	x	x	x	x̲															
27	.02138							x	x	x	x	x	x̲														
24	.02406								x	x	x	x	x	x̲													
20	.02887									x	x	x	x	x	x̲	x̲											
18	.03208											x	x	x	x	x	x̲										
16	.03608												x	x	x	x	x	x̲									
14	.04124													x	x	x	x	x	x̲								
13	.04441														x	x	x	x	x	x̲							
12	.04811															x	x	x	x	x	x						
11.5	.05020																x	x	x	x	x̲						
11	.05249																x	x	x	x	x	x̲					
10	.05773																	x	x	x	x	x					
9	.06415																		x	x	x	x	x̲				
8	.07217																			x	x	x	x	x̲			
7	.08248																				x	x	x	x	x̲		
6	.09623																						x	x	x	x̲	
5	.11547																							x	x	x	x
4.5	.12830																								x	x	x
4	.14434																									x	x

Note:—The crosses (x) indicate those wires which can be used for each pitch. An underlined cross indicates the nearest "best-wire" diameter for that pitch which heads the column.

Table 10. — Wire Sizes for Acme Threads — 29°.

No. of thds. per inch	Diameter of best wire 0.516450 p	Diameter of maximum wire 0.650013 p
	Inches	Inches
1	0.51645	0.65001
1-1/4	.41316	.52001
1-1/2	.34430	.43334
2	.25822	.32501
2-1/2	.20658	.26001
3	.17215	.21667
3-1/2	.14756	.18572
4	.12911	.16250
5	.10329	.13000
6	.08608	.10834
7	.07378	.09286
8	.06456	.08125
9	.05738	.07222
10	.05164	.06500
12	.04304	.05417

Table 11.—Cotangent and Cosecant Functions of Thread Angles

Angle a Deg.	Min.	Cotangent	Cosecant	Angle a Deg.	Min.	Cotangent	Cosecant
25	30	2.09654	2.32282	28	25	1.84818	2.10137
	35	2.08872	2.31576		30	1.84177	2.09574
	40	2.08094	2.30875		35	1.83540	2.09014
	45	2.07321	2.30179		40	1.82906	2.08458
	50	2.06553	2.29487		45	1.82276	2.07905
	55	2.05790	2.28800		50	1.81649	2.07356
26	0	2.05030	2.28117		55	1.81025	2.06809
	5	2.04276	2.27439	29	0	1.80405	2.06267
	10	2.03526	2.26766		5	1.79788	2.05727
	15	2.02780	2.26097		10	1.79174	2.05191
	20	2.02039	2.25432		15	1.78563	2.04657
	25	2.01302	2.24772		20	1.77955	2.04128
	30	2.00569	2.24116		25	1.77351	2.03601
	34	1.99986	2.23594		30	1.76749	2.03077
	35	1.99841	2.23464		35	1.76151	2.02557
	40	1.99116	2.22817		40	1.75556	2.02039
	45	1.98396	2.22174		45	1.74964	2.01525
	50	1.97681	2.21535		50	1.74375	2.01014
	55	1.96969	2.20900		55	1.73788	2.00505
27	0	1.96261	2.20269	30	0	1.73205	2.00000
	5	1.95557	2.19642		5	1.72625	1.99498
	10	1.94858	2.19019		10	1.72047	1.98998
	15	1.94162	2.18401		15	1.71473	1.98502
	20	1.93470	2.17786		20	1.70901	1.98008
	25	1.92782	2.17175		25	1.70332	1.97517
	30	1.92098	2.16568		30	1.69766	1.97029
	35	1.91418	2.15965		35	1.69203	1.96544
	40	1.90741	2.15366		40	1.68643	1.96062
	45	1.90069	2.14770		45	1.68085	1.95583
	50	1.89400	2.14178		50	1.67530	1.95106
	55	1.88734	2.13590		55	1.66978	1.94632
28	0	1.88073	2.13005	31	0	1.66428	1.94160
	5	1.87415	2.12425		5	1.65881	1.93692
	10	1.86760	2.11847		10	1.65337	1.93226
	15	1.86109	2.11274		15	1.64795	1.92762
	20	1.85462	2.10704		20	1.64256	1.92302

Table 12.—Values of Term in Pitch Diameter Formula Involving Function of Helix Angle

The values listed in columns 4 and 5 are to be multiplied by the diameter G of the wires used, in order to obtain the corrections to be subtracted from the values of E which are obtained by the formula:

$$E = M + \frac{p}{2} \cot a - G\,(1 + \operatorname{cosec} a).$$

1		2	3	4	5	6
Helix angle s				(National form)	(Whitworth form)	(Acme form)
Deg.	Min.	Tan s	s^2	$\frac{s^2}{2}$ cos 30° cot 30°	$\frac{s^2}{2}$ cos 27° 30' cot 27° 30'	$\frac{s^2}{2}$ cos 14° 30' cot 14° 30'
0	10	0.00291	0.0000085	0.000006	0.000007	0.000016
	20	.00582	.0000339	.000025	.000029	.000063
	30	.00873	.0000762	.000057	.000065	.000143
	40	.01164	.0001355	.000102	.000115	.000254
	50	.01455	.0002117	.000159	.000180	.000396
1	0	.01746	.0003049	.000229	.000259	.000571
	10	.02036	.0004145	.000311	.000353	.000776
	20	.02328	.0005420	.000407	.000461	.001015
	30	.02619	.0006859	.000514	.000584	.001284
	40	.02910	.0008468	.000635	.000721	.001585
	50	.03201	.0010246	.000768	.000873	.001918
2	0	.03492	.0012194	.000915	.001039	.002282
	10	.03783	.0014311	.001073	.001219	.002679
	20	.04075	.0016606	.001245	.001415	.003108
	30	.04366	.0019062	.001430	.001624	.003568
	40	.04658	.0021697	.001627	.001849	.004061
	50	.04949	.0024493	.001837	.002087	.004585
3	0	.05241	.0027468	.002060	.002340	.005141
	10	.05533	.0030614	.002296	.002608	.005730
	20	.05824	.0033919	.002544	.002890	.006349
	30	.06116	.0037405	.002805	.003187	.007001
	40	.06408	.0041062	.003080	.003498	.007686
	50	.06700	.0044890	.003367	.003824	.008402
4	0	.06993	.0048902	.003668	.004166	.009153
	10	.07285	.0053071	.003980	.004521	.009934
	20	.07578	.0057426	.004307	.004893	.010749
	30	.07870	.0061937	.004645	.005277	.011593
	40	.08163	.0066635	.004998	.005677	.012473
	50	.08456	.0071504	.005363	.006092	.013384