NATIONAL BUREAU OF STANDARDS REPORT

2688

PROJECTS and PUBLICATIONS of the NATIONAL APPLIED MATHEMATICS LABORATORIES

A Quarterly Report

April through June 1953

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

U. S. DEPARTMENT OF COMMERCE

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The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

Electricity. Resistance Measurements. Inductance and Capacitance. Electrical Instruments. Magnetic Measurements. Applied Electricity. Electrochemistry.

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Electronics. Engineering Electronics. Electron Tubes. Electronic Computers. Electronic Instrumentation.

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These three divisions are engaged in a broad program of research and development in advanced ordnance. Activities include basic and applied research, engineering, pilot production, field testing, and evaluation of a wide variety of ordnance matériel. Special skills and facilities of other NBS divisions also contribute to this program. The activity is sponsored by the Department of Defense.

Missile Development. Missile research and development: engineering, dynamics, intelligence, instrumentation, evaluation. Combustion in jet engines. These activities are sponsored by the Department of Defense.

- Office of Basic Instrumentation
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PROJECTS and PUBLICATIONS
of the
NATIONAL APPLIED MATHEMATICS LABORATORIES

April through June 1953

NATIONAL APPLIED MATHEMATICS LABORATORIES

April 1 through June 30, 1953

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Contents

Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies, which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, Astronomy, and Automatic Translation

*New tasks

D. Mathematical Statistics

E. Mathematical Tables

*New tasks

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

June 30, 1953

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-10-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Managers: G. E. Forsythe, M. R. Hestenes, D. H. Lehmer
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. The SWAC experiments on the conjugate gradient method for solving a linear system AX=k have been written up in publication (4). These were performed before SWAC's magnetic drum became available, and hence were limited to matrices A of orders up to 12. The matrices A were treated as nonsymmetric. Let P=P(A*A) be the ratio of the largest to the smallest of the eigenvalues of A*A. When P is sufficiently near 1, the solution of Ax=k by the conjugate gradient method offers no difficulty and may not even require the use of floating-point operations. For P sufficiently large, it is probably impossible to solve the system Ax=k, even with floating-point operations. In the SWAC experiments the values of P ranged up to 10^8, for matrices of orders 5 to 8, and solutions were readily obtained with floating-point operations, using 28 significant bits. The method failed completely on one matrix of order 12 for which P is unknown, but apparently exceedingly large.

A SWAC code has been prepared by L. S. Wilson to solve Ax=k by elimination for a general order n up to 45, using fixed binary-point operations and a system of scaling.

Item (10) under Publications immediately below gives a mathematically elegant treatment of a number of numerical methods of solving linear systems and getting eigenvalues of finite matrices. There are many numerical examples worked in detail.

Publications: (1) "Tentative classification of methods and bibliography on solving systems of linear equations," by G. E. Forsythe; to appear in Simultaneous Linear Equations and the Determination of Eigenvalues, Proceedings of an NBS Symposium held in Los Angeles, August 1951, NBS Applied Mathematics Series 29, in press, U. S. Government Printing Office. (2) "On certain character matrices," by D. H. Lehmer; submitted to a technical journal. (3) "Punched-card experiments with accelerated gradient

methods for linear equations," by A. I. and G. E. Forsythe; to appear in Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues, NBS Applied Mathematics Series; (4) "Some numerical examples on solving systems of linear equations by the conjugate gradient method for nonsymmetric systems of equations," by M. R. Hestenes, U. Hochstrasser, and L. S. Wilson; IN MANUSCRIPT. (5) "On the linear iteration procedures for symmetric matrices," by A. M. Ostrowski; submitted to a technical journal. (6) "On over and under relaxation in the theory of the cyclic single step iteration," by A. M. Ostrowski; submitted to a technical journal. (7) "On the convergence of Gauss' alternating procedure in the method of the least squares, I," by A. M. Ostrowski; submitted to a technical journal. (8) "The solution of systems of linear equations by the conjugate gradient method for use on IBM equipment," by U. Hochstrasser; IN MANUSCRIPT. (9) "Solving linear algebraic equations can be interesting," by G. E. Forsythe; to appear in the Bulletin of the American Mathematical Society. (10) "Computational methods of linear algebra," by V. N. Faddeeva, translated from the Russian by C. D. Benster. (Chapter 1, edited by G. E. Forsythe, is an NBS report, now out of stock. Chapters 2 and 3 are IN MANUSCRIPT without displayed formulas.)

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS

Task 1101-10-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Managers: G. E. Forsythe and M. R. Hestenes
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. G. E. Forsythe has extended his results on the relation of the fundamental eigenvalue of the Laplace operator Δ to that of an approximating difference operator in two dimensions (see Jan-Mar 1953 issue, p. 2.) Let R be any convex domain in the plane, with boundary C. Let λ, $u(x,y)$ be the fundamental eigenvalue and corresponding eigenfunction for R; i.e., assume $\Delta u = u_{xx}+u_{yy} = -\lambda u$, in R, with $u=0$ on C. Let a square net with mesh-width h be constructed in the plane of R. Let R_h denote the nodes of the net which fall inside R. Over R_h one constructs a certain self-adjoint difference operator Δ_h which approximates Δ. Let λ_h be the fundamental eigenvalue of Δ_h. Then it is proved that as $h \longrightarrow 0$,

$$\frac{\lambda_h}{\lambda} \leq 1 - Ah^2 + \underline{o}(h^2),$$

where

$$A = \frac{\iint_R (u_{xx}^2+u_{yy}^2)\,dxdy + \int_C u_n^2 \sin^2 2\tau \; d\tau}{12 \iint_R (u_x^2 + u_y^2)\,dxdy}$$

Here u_n is the normal derivative of u, while τ is the angle between the tangent to C and the x-axis. As a consequence of (1), λ_h is a lower bound for λ for all $h \leq h_0$, for some sufficiently small h_0; no estimate of h_0 is

given. It is conjectured that $\lambda_h < \lambda$ for all h. Since good lower bounds are ordinarily very difficult to obtain, this result may have considerable practical importance.

In solving one problem for an aircraft company (see task 1101-50-5131/53-6, p. 19, the Institute for Numerical Analysis Mathematical Services Unit determined all 45 eigenvalues and eigenvectors of a symmetric matrix of order 45 on the SWAC. For k=0,1,2,...,44, the method was as follows: After the largest k eigenvalues and vectors were found, the gradient method of Hestenes and Karush (see "A method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; J. Res. NBS 47, 45-61 (July 1951)) was used to find the largest eigenvalue in the subspace S_k orthogonal to the k known eigenvectors. The maximizing vector was kept in S_k by successively orthogonalizing it against each known eigenvector. A combination of "optimal-α " and "fixed-α " procedures was followed. After the magnetic drum of SWAC became available, the elapsed time to get one eigenvalue and vector averaged approximately one hour for each of 25 eigenvectors. The method seems to have maintained great accuracy, as revealed by the smallness of the $\left| Au_i - \lambda_i u_i \right|^2$, and by the fact that

$$\frac{\sum_{i=1}^{45} \lambda_i - \text{trace } (A)}{\text{trace}(A)}$$

was approximately 10^{-11}.

Publication (10) under task 1101-10-5100/49-AE2, p. 1 also deals in part with eigenvalue problems.

Publications: (1) "Completely continuous normal operators with property L," by I. Kaplansky; submitted to a technical journal. (2) "Asymptotic lower bounds for the frequencies of polygonal membranes," by G. E. Forsythe; submitted to a technical journal. (3) "On the spectrum of a one parametric family of matrices," by A. M. Ostrowski; submitted to a technical journal.

STUDIES IN PURE MATHEMATICS
Task 1101-10-5100/50-4
(formerly 1101-11-5101/50-4)

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Managers: Various Staff Members
Full task description appears in July-Sept 1949 issue, see 11.1/1-50-4.

Status: CONTINUED. D. H. Lehmer and E. Lehmer have devised methods for investigating whether a given prime is irregular in the sense of Kummer. Results obtained by H. S. Vandiver over the past 20 years were rerun on the SWAC in two hours with nearly complete agreement. Three cases of irregular primes have been overlooked by Vandiver, the smallest being 389 which divides the 100th Bernoulli number. Thus the impossibility of

$$x^{389} + y^{389} = z^{389}$$

in nonzero integers remains unproved. An extended examination of primes between 617 and 1000, indicates that almost half of the primes are irregular.

A sieve process (see July-Sept 1952 issue, p. 16) for the solution of the diophantine equation

$$x^3 - y^2 = D$$

has been applied to the unsolved cases of D<100. A search up to $x=10^6$, and much beyond in some cases, failed to reveal any solution not previously known.

Publications: (1) "On a recursion formula and on some Tauberian theorems," by N. G. de Bruijn and P. Erdös; J. Res. NBS 50,161-164 (Mar. 1953). (2) "Boolean Geometry I," by L. M. Blumenthal; to appear in Rendiconti del Circolo Matematico di Palermo. (3) "On Polya frequency functions III: The positivity of translation determinants with an application to the interpolation problem by spline curves," by I. J. Schoenberg and A. Whitney; Trans. Am. Math. Soc. 74, 246-259(Mar. 1953). (4) "On smoothing operations and their generating functions," by I. J. Schoenberg; Bul. Am. Math. Soc. 59, 199-230(1953). (5) "An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space," by I. J. Schoenberg; submitted to a technical journal. (6) "On the distribution of totitives," by D. H. Lehmer; IN MANUSCRIPT.

COMPUTATION OF THE COMPLEX ZEROS OF THE RIEMANN-ZETA FUNCTION
Task 1101-10-5100/50-13
(formerly 1101-11-5101/50-13)

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Manager: D. H. Lehmer
Full task description appears in Apr-Jun 1950 issue, see 11.1/1-50-13.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-10-5100/51-1
(formerly 1101-11-5100/51-1)

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: W. Wasow and G. Blanch
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. G. Blanch has completed the experiments with the numerical method of Haskind-Reissner for the computation of lift and moment coefficients in subsonic (two-dimensional) compressible flow. Results obtained agree with Dietze's calculations considerably better than those heretofore obtained by other investigators.

G. Blanch and I. Rhodes completed the table of characteristic values of Mathieu's equations for large parameters for orders up to nine. Some further subtabulations are required in functions of higher order, but tables at intervals of 0.002 in t (where $t=1/\sqrt{s}$) are available, for orders up to fifteen. [See National Bureau of Standards "Tables relating to Mathieu functions," (Columbia University Press, New York, 1951) for definitions.] An examination of the entries shows that be$_r$(s) agre

$bo_{r+1}(s)$ to eight decimals for $t<.025$, even when the order r is as large as 14. In this region it should be possible to improve the known asymptotic expansion. Meixner communicated an expression for the difference $be_r - bo_{r+1}$. His formula was spot checked against the new table; and it appears that for low orders r and small t, the formula gives very good results. However, for orders greater than five and $t>.05$, it is not possible to obtain more than the order of magnitude of the functions in the region where the formula is applicable.

A report on mathematical aspects of the theory of relaxation oscillations with one or more degrees of freedom was prepared by W. Wasow (see publication (7)). It includes a simplified derivation of the first perturbation term for the period of oscillation in Van der Pol's case. In connection with this work the paper by A. A. Dorodnitsyn on "Asymptotic solution of Van der Pol's equation" was translated from the Russian. It contains a short account of a complete perturbation procedure.

The question as to when asymptotic series solutions in powers of a parameter of a linear differential equation are convergent (see Jan-March 1953 issue, p. 4.) has been further studied by W. Wasow in collaboration with R. M. Redheffer of the University of California at Los Angeles. Several sufficient conditions for convergence in special cases have been obtained. Other results shed some light on the nature of "inner friction layers" occurring in the theory of hydrodynamic stability. A joint manuscript is in preparation.

Publications: (1) "On the numerical solution of parabolic partial differential equations," by G. Blanch; accepted by the Journal of Research of the NBS. (2) "On mildly nonlinear partial difference equations of elliptic type," by L. Bers; accepted by the Journal of Research of the NBS. (3) "Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point," by W. Wasow; accepted by the Annals of Mathematics. (4) "An expansion method for parabolic partial differential equations," by J. W. Green; accepted by the Journal of Research of the NBS. (5) "On small disturbances of plane Couette flow," by W. Wasow; accepted by the Journal of Research of the NBS. (6) "Tables of lift and moment coefficients for oscillating airfoils in subsonic compressible flow," (formulation of Reissner and Haskind) by G. Blanch; an NBS report. (7) "Singular perturbation methods for nonlinear oscillations," by W. Wasow; IN MANUSCRIPT. (8) "Asymptotic solution of Van der Pol's equation," by A. A. Dorodnitsyn, translated from Russian by C. D. Benster, edited by W. Wasow; an NBS report.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-10-5100/51-2
(formerly 1101-11-5100/51-2)

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: W. Wasow
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. In connection with mathematical service work done for the USAF School of Aviation Medicine under the guidance of D. Teichroew, the transformation of an almost normal random variable t into a normal one x and vice versa, was studied theoretically by W. Wasow. It was shown that for a certain class of distributions the formal expansions in terms of a parameter obtained for these transformations are asymptotic. The coefficients of the expansions are polynomials in x or t, respectively. The class

of distributions considered includes Student's distribution and the chi-square distribution. A paper on this subject is being prepared.

P. Erdös and T. S. Motzkin solved a problem on frequencies left open by Dvoretzky and Motzkin. If $f(p,q,\alpha)$ is the probability that, in a sequence of p zeros and q ones, every initial sequence should have at least (or more than) α times as many zeros as ones, it is shown that for $p \longrightarrow \infty$, $q \longrightarrow \infty$, $p/q \longrightarrow \lambda$ the limit of f exists. The limit function has jumps exactly at all rational α. Recurrence relations for the probabilities are established which facilitate the computation of the limit. In the simplest non trivial cases, the value of the limit is the quotient of two hypergeometric functions with immediately available arguments.

Publications: (1) "Additive functionals of a Markoff process," by R. Fortet; submitted to a technical journal. (2) "Metodi probabilistici per la soluzione numerica di alcuni problemi di analisi," by W. Wasow; Rend. Mat. App. {V} XI, 336-346 (Roma 1952); also issued separately as Pubblicazioni dell'Istituto per le Applicazioni del Calcolo N. 354 (Roma 1953). (3) "Statistical estimation of matrix quantities by means of a class of discrete Markov chains," by H. P. Edmundson; IN MANUSCRIPT. (4) "Limits for permanent preponderance," by P. Erdös and T. S. Motzkin; IN MANUSCRIPT.

VARIATIONAL METHODS
Task 1101-10-5100/51-3

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

Publications: (1) "On methods for obtaining solutions of fixed end-point problems in the calculus of variations, " by M. L. Stein; J. Res. NBS 50, 277-297 (May 1953). (2) "Iterative methods of solving linear problems on Hilbert space," by R. M. Hayes; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

STUDIES IN APPLIED MATHEMATICS
Task 1101-10-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: C. Lanczos, G. E. Forsythe, and D. Teichroew
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. C. B. Tompkins, T. S. Motzkin, and G. E. Forsythe have organized a seminar on numerical analysis, a lecture and discussion group meeting twice weekly, in which the public is invited to participate. Eighteen meetings were held between April 20 and June 17. The central topic of the seminar has been the use of automatic digital computers (especially SWAC) for the solution of analytical or combinatorial problems. The computational problems discussed have dealt with: a military attrition function, linear inequalities, the assignment problem, vibrational frequencies of membranes , eigenvalues of a 45th order matrix, matrix inversion,

and the maximization of a function of many variables. (See Lectures and Symposia for a detailed list of the sessions.) The seminar has attracted participants from a variety of scientific organizations in the Los Angeles area.

Although divided differences are of basic importance in numerical analysis, they are harder to interpret than ordinary differences, partly because their properties differ radically from familiar ordinary differences. Thus, for sufficiently regular functions f(t) over a region R, the divided difference or order n can be expressed by $f^{(n)}(y)/n!$, while the nth ordinary difference of f(t), at uniform intervals h, has the form $h^n f^{(n)}(z)$, y and z in R. G. Blanch has shown that by introducing an average interval and modifying the definition of divided differences, the latter acquire many of the characteristics of ordinary differences. They can be used quite readily when functions are available only at unequal intervals of the argument, and an examination of successive differences for error-patterns resembles the corresponding use of ordinary differences. A paper on the subject is being prepared.

Of interest to numerical analysts is publication (7) described below. It makes a large body of material in English available for the first time to applied mathematicians. It gives lucid explanations, with many references to the world literature, of numerical and analytical methods for solving ordinary and partial differential equations and integral equations. The chapter titles are: 1) "Methods based on representation of the solution as an infinite series"; 2) "The approximate solution of the integral equations of Fredholm"; 3) "The method of nets"; 4)"Variational methods"; 5) "The conformal transformation of regions"; 6) "The principles of the application of conformal transformation to the solution of the fundamental problems for canonical regions"; and 7) "Schwarz's method".

Publications: (1) "Numerical computation of low moments of order statistics from a normal population," by J. B. Rosser; submitted to a technical journal. (2) "Sequential decision problems for processes with continuous time parameter. Testing hypotheses," by A. Dvoretzky, J. Kiefer, and J. Wolfowitz; Ann. Math. Stat. 24, No. 2, 254-264(1953). (3) "Changes of sign of sums of random variables," by P. Erdös and G. Hunt; submitted to a technical journal. (4)"A numerical analyst's 15-foot shelf," by G. E. Forsythe; submitted to a technical journal. (5) "Seminar on numerical analysis -- summary of presentations between April 20 and May 13, 1953," by C. B. Tompkins, multilithed typescript, 10 p. (6) "Seminar on numerical analysis -- summary of presentations between May 18 and June 8, 1953," by G. E. Forsythe, multilithed typescript, 38 p. (7) "Approximate methods of higher analysis," by L. V.Kantorovich and V. I. Krylov, translated by C. D. Benster; IN MANUSCRIPT. (This draft translation is typed without displayed formulas.) (8) "Translations of Russian articles on the Kolmogorov and Smirnov tests," by C. D. Benster, edited by D. Teichroew; IN MANUSCRIPT.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-10-5100/51-5

Origin: Office of Naval Research, USN Authorized 9/1/50
Sponsor: " "
Manager: D. Saxon
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. A manuscript is in preparation on the application of variational methods to quantum mechanical scattering problems (see Oct-Dec 1952 issue, p. 7). Programs have essentially been completed, and

numerical work is continuing on the calculation of the photo-disintegration of the deuteron (see Oct-Dec 1952 issue, p. 7). R. Woods has begun a program of computation on the elastic scattering of protons from various elements, under the guidance of D. Saxon. The main coding program has been completed and checked; some preliminary calculations have been carried out.

Publications: (1) "Modes of vibrations of a suspended chain," by D. S. Saxon and A. S. Cahn; accepted by Quarterly Journal of Mechanics and Applied Mathematics (Oxford). (2) "The torsion of anisotropic elastic cylinders by forces applied on the lateral surface," by H. Luxenberg; J. Res. NBS 50, 263-276 (May 1953). (3) "A nonlinear model for the composite Pimeson," by S. G. Gasiorowicz; submitted to a technical journal. (4) "Acoustic radiation pressure on a circular disk," by H. Levine; to appear in the Proceeedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society. (5) "A numerical solution of Schroedinger's equation in the continuum," by W. Futterman, E. Osborne, and D. S. Saxon; accepted by the Journal of Research of the NBS.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-10-5100/52-1

Origin: NBS Authorized 3/15/52
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. The task manager is continuing to accumulate bibliographical cards on Russian mathematical monographs (see Jan-Mar 1952 issue, p. 11 and Oct-Dec 1952 issue, p. 8). The translations formerly reported under this task are now reported in connection with the pertinent tasks of the Institute for Numerical Analysis.

GENERALIZED RAYLEIGH-RITZ METHOD FOR EIGENVALUES OF A CLAMPED PLATE
Task 1101-10-5100/53-1

Origin: NBS Authorized 12/29/52
Sponsor: Office of Naval Research, USN
Manager: G. Blanch
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. The programming of the computations for the SWAC is in progress.

SCAMP
Task 1101-10-5150/53-1

Origin: Office of Naval Research Authorized 6/10/53
Sponsor: " "
Manager: C. B. Tompkins

Objective: To develop numerical methods for discrete-variable problems arising in certain specialized quantitative aspects of military science,

with special emphasis on the application of automatic computers.

Background: This task is the result of a desire on the part of the sponsoring agency to bring together mathematicians for the purpose of contributing to the deep mathematical developments required.

Status: NEW. Preliminary work is now under way; full scale work starts July 1.

2. Applied Research

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-10-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF Authorized 6/15/50
Sponsor: " "
Managers: E. W. Barankin and T. S. Motzkin
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Investigation of the farthest projection method for solving linear inequalities was continued. T. S. Motzkin studied the problem of whether the solution necessarily becomes cyclic with respect to the order in which the projections are made on the hyperplanes bounding the regions defined by the inequalities. It is clear that the length of such a limiting cycle may be greater than the number of inequalities, and it is not clear that such a cycle need exist at all. One special result established was that the set of systems of three incompatible inequalities whose bounding lines form an acute-angled triangle and for which there are patterns of projections not cyclic after a sufficient number of projections has measure at most zero in a natural metric of triangles. The proof is through an argument facilitated by a choice of a special function on the boundary of the triangle. In terms of this function the projection operator is simple and an analysis (related to Borel's treatment of the measure of normal numbers) of the situations which may exist is possible.

The code of L. S. Joel and B. Handy for Gleyzal's method of solving the assignment problem was run on the SWAC with matrices up to order 8. The method is one of remarkable computational efficiency in which successive approximations to the desired permutation are made. Beginning with an arbitrary permutation, modifications are made by seeking closed circuits in the original matrix, each path in the matrix being either a vertical line across a row, and with the property that every second element of the circuit is an element which will be in the trace of the matrix modified by the approximating permutation. Such a circuit is acceptable as a modifying circuit if every element not on the trace of the matrix modified by the approximating permutation but lying on the circuit is larger than a corresponding element above the circuit on the trace. Careful and ingenious modification of the matrix in a way which leaves a solution to the problem invariant yields a code which takes the SWAC for problems of the size mentioned only a few seconds.

D. H. Lehmer proposed a direct combinatorial method of attacking the problem. It consists of an exhaustive search through permutations with an added feature that many permutations may be rejected in a block. With some simplications by T. S. Motzkin the method is being coded by B. Handy.

The cyclic projection and acceleration method of Kaczmarz and Tompkins was coded for the SWAC by R. B. Horgan and used in solving a large system of simple linear inequalities on the SWAC. The novel feature of this attack was the use of the collator input to present the inequalities systematically to the machine. Each inequality could be described with a small amount of data, and the machine could generate the coefficient of the inequality. Two inequalities can be described on a single IBM input card, and the machine can compute rapidly enough to permit rapid input of data, frequently with no interruption to the collator feed at all. The method seems to be working successfully, and there can be no question of the feasibility of this use of the collator as input.

E. W. Barankin has investigated convergence properties of the technique described in July-September 1952 issue, p. 12. A simplifying property of this iterative system has been discovered, and it is hoped that this will lend insight toward a convergence theorem. The computational work on this technique to date along with the intuitive motivation of the technique, gives reason to expect a very effective convergence theorem. Work on stochastic linear inequalities is continuing.

The stochastic search for the maximum of a non convex function has been studied by E. W. Barankin. Some results on this problem are already in hand, assuring that under moderate conditions a stochastic sequence of points in the domain of a function does bring with it stochastic convergence, to the maximum value of the function, of the largest values of the function on the successive sets of points turned up in the stochastic process.

As part of the program of computation for the purpose of a comparative study of various methods of solution of linear systems, with and without optimizers, E. E. Osborne has begun coding the traversal method of Brown and Koopmans.

Related to the task are investigations by C. V. Tompkins into the evaluation of a probabilistic game representing military attrition. A method of successive approximation has been developed and described in a working paper. Some computational experience in the use of this method has been obtained by T. H. Southard and P. L. Childress.

Publications: (1) "Basic solutions of the transportation problem," by T. S. Motzkin; IN MANUSCRIPT. (2) "The multi-index transporation problem," by T. S. Motzkin; IN MANUSCRIPT. (3) "Ray systems with maximum angle sum," by T. S. Motzkin; IN MANUSCRIPT. (4) "On Fejér sets in linear and spherical spaces," by T. S. Motzkin and I. J. Schoenberg; accepted by the Annals of Mathematics. (5) "On the relaxation method for linear inequalities," by T. S. Motzkin and I. J. Schoenberg; submitted to a technical journal. (6) "On the optimal character of the (s,S) policy in inventory theory," by A. Dvoretzky, J. Kiefer, and J. Wolfowitz; submitted to a technical journal. (7) "Least p-th power polynomials on a real finite point set," by T. S. Motzkin and J. L. Walsh; IN MANUSCRIPT. (8) "Least p-th power polynomials on a complex finite point set," by T. S. Motzkin and J. L. Walsh; IN MANUSCRIPT. (9) "A new type of existence theorem for systems of linear inequalities," by J. W. Gaddum; IN MANUSCRIPT. (10)"Boundedness of sequential projections," by T. S. Motzkin and C. B. Tompkins; IN MANUSCRIPT.

3. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(previously listed as Air Materiel Command Computing Machine)
Task 1101-20-5103/49-1
(formerly 1101-34-5103/49-1)

Origin: Aeronautical Research Laboratory Authorized 11/1/48
Wright Air Development Center, USAF
Sponsor: " " "
Managers: H. D. Huskey and R. Thorensen
Full task description appears in Apr-Jun 1949 issue, see 11.1/22-49-1.

Status: CONTINUED. During the last quarter the SWAC performed 875 hours of computation, working on 32 different problems. Of particular numerical interest has been the determination of the full set of eigenvalues and eigenvectors of a 45th order symmetric matrix.

A significant increase in the effectiveness of the computer was materialized during the latter part of the quarter when a magnetic drum memory of 4096 words was put into operation as an integral part of the machine. The drum memory has already been used for a variety of problems, such as in the solution of the matrix problem mentioned above, the solution of combinatorial and probability problems, and the computation of energy flow in a large electric power system. During this time the magnetic drum memory has performed with a very high degree of reliability. The only trouble encountered was traceable to defective read-write heads which since have been weeded out.

Other engineering changes of major importance include the installation in the SWAC of a new high level deflection system for the cathode ray tube memory and the start of construction of a new flexible breakpoint control. The high level deflection system has contributed significantly to the stability and reliability of the computer while the breakpoint control when completed will aid the operator both in code checking and in problem solution.

Publications: (1) "An improved cathode ray tube storage system," by R. Thorensen; to appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM, held in Los Angeles, Calif., February 4,5,6, 1953. (2) "The SWAC - design features and operating experience," by H. D. Huskey, R. Thorensen, B. F. Ambrosio, and E. C. Yowell; submitted to a technical journal.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-20-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS Authorized 2/15/49
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: H. D. Huskey
Full task description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Apr-Jun 1952 issue.

LANGUAGE TRANSLATION STUDY
Task 1101-20-5103/52-1
(formerly 1101-21-5104/52-1)

Origin: NBS — Authorized 8/15/51
Sponsor: The Rockefeller Foundation — Revised 10/15/51
Managers: H. D. Huskey and G. E. Forsythe
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. In the publication below V. A. Oswald and R. H. Lawson report on experiments in translating German texts on brain surgery with a limited glossary of 4328 entries. It is found that about ninety percent of the running text can be translated with this glossary. This concludes Prof. Oswald's research, sponsored by The Rockefeller Foundation and administered by the National Bureau of Standards, Los Angeles.

Other phases of the task are inactive.

Publication: "An idioglossary for mechanical translation," by V. A. Oswald, Jr., and R. H. Lawson; hectographed typescript, University of California at Los Angeles, Department of Germanic Languages, 16 p. (June 1953).

STUDIES IN THE THEORY OF DIGITAL COMPUTING MACHINES
Task 1101-20-5103/53-1

Origin: NBS — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: D. H. Lehmer
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. During the last quarter, a code was written to adapt SWAC to a three address, CPC-type operation. This code reads ten words from a card, obeys the ten instructions, and then calls for another card. Since all the orders are read from cards, logical instructions needed for controlling calculating loops are omitted. The arithmetic operations of addition, subtraction, multiplication, and division are included as well as square roots, sines, and cosines. Future plans call for the incorporation of other functions by making use of the magnetic drum.

This mode of operation, known as SWACPEC, performs 1,200 operations per minute. It then represents roughly a geometric mean between CPC speeds and SWAC speeds. It is felt now that the extra ease in coding for this mode of operation will counterbalance the increase in running time and will yield more efficient operations for small problems.

Publications: (1) "A general-purpose control panel for a model II CPC," by P. Bremer, D. Teichroew, and E. C. Yowell; to appear in the IBM Newsletter. (2) "SWAC coding guide," by R. Horgan; an NBS report. (3) "Distribution sampling with high-speed computers," by D. Teichroew; IN MANUSCRIPT. (4) "Systematic generation of permutations on an automatic computer and an application to a problem concerning finite groups," by L. J. Paige and C. B. Tompkins; IN MANUSCRIPT.

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-40-1111/49-1a
(formerly 1101-53-1101/49-1a and 1101-53-1100/49-1)

Origin: NBS Authorized 9/2/48
Sponsor: Office of Naval Research Revised 11/16/49
Managers: M. Howard, F. Hollander, P. Bremer
Full task description appears in July-Sept 1949 issue, see task 11.1/32-49-1.

Status: CONTINUED. The following problems were computed on IBM machines: (1) For D. Saxon (see task 1101-10-5100/51-5, p. 7): The integration of matrix elements involved in the photoelectric disintegration of the deuteron for a Yukawa potential. (2) For D. Teichroew: The key punching of the inverse table,

$$x(p), \text{ where } p = \tfrac{1}{2}[1 + \alpha(x)]$$

$$\text{and } \alpha(x) = \int_{-x}^{x} z(t)dt, \quad z(x) = \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}x^2),$$

for p = 0.5(.001)0.999; 10D.

Research staff problems involving SWAC were: (1) For D. H.Lehmer (see task 1101-10-5100/50-4, p. 3): All but nine numbers of the Mersenne problem have been computed twice. Those remaining will be checked as soon as time on the SWAC permits. (2) For S. Mayer (see task 1101-10-5100/51-5, p. 7): The calculations on electron density in crystal structure have been completed. (3) M. Hestenes (see task 1101-10-5100/49-AE2, p.1): A small amount of calculation was done on the 12x12 matrix supplied by the Douglas Aircraft Company. (4) For DeVogelaere: (Université Laval, Québec, Canada). Integration of a differential equation of the form $\ddot{x}=f(x)$. (5) For M. Muller: Some checking has been carried out on a code which will be used to compute, by sampling methods, the power function of sequential tests of the general linear hypothesis.

4. Mathematical Services

MATHIEU FUNCTIONS II
Task 1101-40-5131/45-1
(formerly 1101-53-1101/45-1)

Origin: Applied Mathematics Panel NDRC Authorized 7/1/47
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center,USAF
Manager: E. C. Yowell
Full task description appears in Apr-Jun 1949 issue, see 11.1/2-45-1.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

TABLES FOR ROCKET AND COMET ORBITS
Task 1101-40-5131/48-3
(formerly 1101-53-1101/48-3)

Origin: NBS
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: G. Blanch
Authorized 5/25/48
Completed 6/30/53

Objective: Computation of (a) V=1-cosx, S=sinx, and x as functions of U=x-sinx, and (b) V_h=(cosh x)-1, S_h=sinh x, and x as functions of U_h=(sinh x)-x for the range

$$\left.\begin{matrix} U \\ U_h \end{matrix}\right\} = 10^{-14}(10^{-15})10^{-13}(10^{-14})10^{-12}\ldots10^{-2}(10^{-3})10^{-1}(10^{-2})3.15.$$

Background: There are no similar tables now in existence. The tables will be of value in rapidly solving Kepler's equation. This was specifically requested by Dr. Samuel Herrick of the Astronomy Department of the University of California at Los Angeles.

Status: COMPLETED. The tables have been issued as an AMS publication.

Publication: "Tables for rocket and comet orbits," by S. Herrick, NBS Applied Mathematics Series 20; Available from U. S. Government Printing Office, Washington 25, D. C., $1.75.

PUNCHED CARD LIBRARY
Task 1101-40-5131/49-2
(formerly 1101-53-1101/49-2)

Origin: NBS
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: P. Bremer
Full task description appears in Apr-Jun 1949 issue, see 11.1/2-49-2.
Authorized 7/14/48

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles, 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following inverse table has been keypunched:

$$x(p), \text{ where } p= \tfrac{1}{2}[1+ \alpha(x)], \text{ and } z(x)= \frac{1}{\sqrt{2\pi}} \exp(- \tfrac{1}{2}x^2),$$

for p=0.5(.001)0.999; 10D. Here

$$\alpha(x) = \int_{-x}^{x} z(t)dt.$$

STATISTICAL SMOOTHING
Task 1101-40-5131/51-19
(formerly 1101-53-1101/51-19)

Origin: Stanford Research Institute, Stanford University Authorized 1/15/51
Sponsor: Office of Research Operations, U. S. Army
Manager: M. Howard
Full task description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Seven cases were completed this quarter.

TABLES OF THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-40-5131/51-32
(formerly 1101-53-1101/51-32)

Origin: Division 13, NBS Authorized 5/31/51
Sponsor: Office of Chief of Ordnance, U. S. Army
Manager: G. Blanch
Full task description appears in Apr-Jun 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

PRESSURE FIELDS OF POTENTIAL FLOW PAST A BODY OF REVOLUTION
Task 1101-40-5131/51-33
(formerly 1101-53-1101/51-33)

Origin: Naval Ordnance Test Station (Pasadena) Authorized 6/22/51
Sponsor: Bureau of Ordnance, USN Completed 6/30/53
Manager: R. R. Reynolds

Objective: To compute the potential, velocity, and pressure on the surface of a body of revolution.

Background: The Naval Ordnance Test Station needs these results in a research project to determine bodies of minimum resistance.

Comments: In task 1101-53-1101/50-13(see July-Sept 1950 issue, p. 30) an approximate procedure was used to determine a body for which a given velocity distribution was valid. Some of the results of that task are now being utilized to obtain the pressure distribution.

Status: COMPLETED. The results were sent to the originator.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-40-5131/51-34
(formerly 1101-53-1101/51-34)

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: E. C. Yowell
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. Two integrations were completed this quarter. The necessary coding was completed for transferring this problem from the CPC to SWAC.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-40-5131/51-36
(formerly 1101-53-1101/51-36)

Origin: University of Oregon — Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: A. D. Hestenes
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. This task has been reactivated and now will be computed with the aid of SWAC. Coding is well under way. It is expected that several important auxiliary tables will be available as a bi-product of these computations.

INHERENT ERROR ANALYSIS FIRE CONTROL EVALUATION PROGRAM
Task 1101-40-5131/52-19
(formerly 1101-53-1101/52-19)

Origin: Naval Ordnance Test Station (Inyokern) — Authorized 4/1/52
Sponsor: Bureau of Ordnance, USN — Completed 6/30/53
Manager: R. R. Reynolds

Objective: This study is concerned with the inherent error analysis of the air to ground rocket fire control evaluation program of the client and consists of four phases as follows:

(A) Review of evaluation program and determination of the mathematical expressions for the inherent errors.

(B) Recommendation of statistical experiments by which NOTS can determine measurement errors, reading errors, etc., required for a numerical evaluation of the expressions obtained in (A).

(C) Study of the permissible measurement errors for a desired accuracy in the resulting data.

(D) Numerical evaluation of the mathematical expressions obtained in (A) using numerical values of errors obtained as a result of (B).

Background: This study is needed to determine the validity of the results of fire control evaluation programs.

Status: COMPLETED. Data supplied by the Test Station have been evaluated.

SYSTEMATIC AND RANDOM ERRORS
Task 1101-40-5131/52-29
(formerly 1101-53-1101/52-29)

Origin: North American Aviation Co. — Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF — Completed 6/30/53

Manager: A. D. Hestenes

Objective: To compute the salvo kill probability of a square target of side 2a as a function of the parameters: aiming (systematic) error, ammunition dispersion (random error), and salvo size. The expression to be evaluated is

$$P_{SK} = 2 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} Q(i,j,N)\, P_A(i,j)$$

where

$$Q(i,j,N) = 1 - [1 - P_k\, P_h(i,j)]^N$$

$$P_h(i,j) = \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 - \frac{i+\frac{1}{2}}{n}\right)\right) + \varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 + \frac{i+\frac{1}{2}}{n}\right)\right)\right]$$

$$\cdot \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 - \frac{j+\frac{1}{2}}{n}\right)\right) + \varphi\left(\frac{a\sqrt{2}}{\sigma_R}\left(1 + \frac{j+\frac{1}{2}}{n}\right)\right)\right]$$

$$P_A(i,j) = \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_A}\,\frac{i+1}{n}\right) - \varphi\left(\frac{a\sqrt{2}}{\sigma_A}\,\frac{i}{n}\right)\right]$$

$$\cdot \left[\varphi\left(\frac{a\sqrt{2}}{\sigma_A}\left(\frac{j+1}{n} - k\right)\right) - \varphi\left(\frac{a\sqrt{2}}{\sigma_A}\left(\frac{j}{n} - k\right)\right)\right]$$

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}}\, du.$$

The distributions are assumed to be circular normal distributions with standard deviations σ_R for the random error and σ_A for the aiming error. N is the salvo size and $y_0=ka$, $x_0=0$ represents the center of the aiming point distribution. P_k is a probability that a target will be destroyed if hit by a single shot independent of the effect of other shots. a/n represents the mesh size. n is to be chosen such as P_{SK} is correct to three decimals. P_{SK} is to be evaluated for the following values of the parameters P_k=0.2, 0.4, 0.7, and 1.0; N=1,5,10,25,50,100,150,200; y_0=0,a,2a,3a,5a,10a,15a,20a; σ_R=a,2a,3a,5a,10a,15a,20a; and σ_A=a,2a,3a, 5a,10a,15a,20a.

Background: These computations are needed to predict performance of new (as well as old) weapons.

Comments: Although much work of this nature has been done piecemeal in the past, the program outlined above goes beyond the previous work and represents a comprehensive program which will be useful to many organizations interested in defense programs.

Status: COMPLETED. The results have been submitted to the originator. A manuscript has been prepared for possible publication.

SIERRA WAVE PROJECT
Task 1101-40-5131/52-36
(formerly 1101-53-1101/52-36)

Origin: Department of Meteorology, U.C.L.A. Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: T. H. Southard
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. Theodolite data for several flights were reduced, and the results were submitted to the originator.

SUBSONIC COMPRESSIBLE FLOW PAST OSCILLATING AIRFOILS; REISSNER'S METHOD
Task 1101-40-5131/52-41
(formerly 1101-53-1101/52-41)

Origin: Air Materiel Command, Wright Field Authorized 6/23/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF Completed 6/30/53
Manager: G. Blanch

Objective: To test Reissner's method, outlined in NACA Technical Note 2363, by computing lift and moment coefficients for certain specified parameters.

Background: The problem of two-dimensional subsonic compressible flow was treated by Possio in 1938. In 1944 Dietze gave tables and curves for lift and moment coefficients. His computations are based on an iterative method, using the known results for the incompressible case as a starting point. H. E. Fettis, using a somewhat different technique, made some further computations, and his results agreed well with those of Dietze. In December, 1951, there appeared in the Journal of Aeronautical Sciences a paper by Timman, Van de Vooren, and Greidanus, with a treatment of the same problem, based on the use of Mathieu functions. Tables are given which agree with those of Dietze over a fairly wide range, but which disagree sharply with Dietze's in other regions. The authors could not account for the disagreement. In 1950, this laboratory made certain computations for Wright Field, using a method developed by Haskind, involving Mathieu functions. The results differ considerably (though not in order of magnitude) from those of Dietze. Reissner, in the note already cited, also gave a treatment of the same problem. This treatment is based on his earlier studies, and modified somewhat by taking account of Haskind's method. It is now the purpose to make some computations using Reissner's method, to see to what extent results agree with those of Dietze and Haskind. If possible, an attempt will be made to discover the reasons for differences in results based on the various methods. The work will be done in collaboration with H. E. Fettis of Wright Field, who has made extensive studies of the problem.

Status: COMPLETED. The results have been transmitted to the sponsor in the form of an NBS report (see publication).

Publication: "Tables of lift and moment coefficients for oscillating airfoils in subsonic compressible flow," by G. Blanch; an NBS report.

LEGENDRE FUNCTIONS
Task 1101-40-5131/52-48

Origin: Hughes Aircraft Company Authorized 6/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: E. C. Yowell
Full task description appears in July-Sept 1952 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

TABLES RELATING TO THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-40-5131/52-50
(formerly 1101-53-1101/52-50)

Origin: Division 13, NBS Authorized 6/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: G. Blanch
Full task description appears in Apr-Jun 1952 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

SEARCH FOR MAGIC SETS IN A LATIN SQUARE OF ORDER n
Task 1101-40-5131/53-3

Origin: Summer Symposium Group Authorized 7/28/52
Sponsor: Project SCAMP, ONR Completed 6/30/53
Manager: A. D. Hestenes

Objective: To determine the magic sets of a 10x10 latin square.

Background: A given n-th order latin square has a latin square orthogonal to it if, and only if, the given latin square has n nonoverlapping magic sets. A complete set of n-1 orthogonal latin squares determines a projective geometry of order n.

Status: COMPLETED. This task was initiated to study machine methods. As present work has grown beyond the scope of the task as written, the program will be continued under a future task which will represent general computing for the SCAMP Project.

EIGENVALUES
Task 1101-40-5131/53-6

Origin: Convair Authorized 9/26/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: E. C. Yowell
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. The eigenvalues and eigenvectors of this matrix

have been transmitted to the contractor. The determination of the influence coefficients has not been started.

METEOROLOGICAL MEANS
Task 1101-40-5131/53-10

Origin: Meteorology Department, UCLA — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: F. H. Hollander
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. The computation of $u_{i,j+\frac{1}{2}}$, and $v_{i,j}$ for all levels has been completed on the SWAC. Listings of this information have been made on the IBM tabulator and forwarded to the UCLA Meteorology Department. Coding has been completed on routines for time averages (based on daily readings for one month) and space averages (based on readings, for a given day, around an entire latitude circle) of the wind velocity, temperature, and height. For the sea level, pressure is computed instead of height.

Computing has been completed on the time averages, and has been begun, but not completed, on the space averages. Checking for these computations has been done by repeating all calculations. The results of the two runs, on punched cards, are compared in an IBM reproducer. Averages in space, of the time averages, and averages in time, of the space averages, have also been computed. This has been done on an IBM calculating punch, type 604, rather than on SWAC.. It is estimated that this problem is more than eighty percent completed. (See Jan-Mar 1950 issue, project 11.1/31-50-17, p. 15.)

PROLATE SPHEROIDAL WAVE FUNCTIONS
Task 1101-40-5131/53-11

Origin: Stanford Research Institute — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, USAF
Manager: R. R. Reynolds
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. For c=1.2, $\pi/2$, 2, $3\pi/4$, 2.5, 2.8, 3, π, 3.2, and n=9(2)19, the coefficients c_{2k}^{1n} and b_k^{1n} as well as $K_{1n}^{(1)}$ and Q_{1n} have been computed for k= μ(1) μ +5, where 2μ is the lower index of max c_{2k}^{1n}. These values are now being substituted in the general formulas for impedance.

NONLINEAR SERVO PROBLEM
Task 1101-40-5131/53-14

Origin: Douglas Aircraft, Santa Monica — Authorized 12/15/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Terminated 6/30/53

Manager: E. C. Yowell

Objective: To solve the set of differential equations

$$\ddot{\theta}_s = K_1\dot{\theta}_s + E_{\theta_s}K_2 + E_{\dot{\theta}_s}K_3 + K_4\,\theta_s + K_6\,\dot{\theta}_Q + K_7\,\theta_Q + K_8\,F_s$$

$$\ddot{\theta}_Q = K_{28}F_Z + K_{29}\,F + K_{30}\,\dot{\theta}_s + K_{31}\,\theta_s + K_{33}\,\dot{\theta}_Q + K_{34}\,\theta_Q + K_{35}\dot{\delta} + K_{36}\delta$$

$$\ddot{\delta} = K_{37}F_Z + K_{38}\,F + K_{39}\,\dot{\theta}_s + K_{40}\theta_s + K_{42}\dot{\theta}_Q + K_{44}\,\dot{\delta} + K_{45}\,\delta$$

where K_i are all constants, E_α are + 1 or -1, F_s is given as $15\sin 8\pi t$, F_Z and F are analytic functions of δ and θ_Q.

Background: This problem arises in the study of a servo control system. This solution is to provide a calibration check on a large set of differential analyser solutions of this problem.

Status: TERMINATED. At the contractor's request, this problem has been terminated. The work to date and the results obtained are summarized for possible future reference by the originator.

NONSYMMETRIC MATRIX
Task 1101-40-5131/53-15

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Authorized 12/15/52, Terminated 6/30/53
Sponsor: " "
Manager: M. W. Steinberg

Objective: Find the eigenvalues of an 18x18 nonsymmetrix matrix.

Background: This matrix originated at the Aeronautical Research Laboratory and was sent to us for the purpose of developing methods of handling problems of this nature.

Status: TERMINATED.

FLUTTER MATRICES
Task 1101-40-5131/53-17

Origin: Boeing Airplane Company — Authorized 12/15/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: E. E. Osborne
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. The eigenvalues and eigenvectors of the fifteen 6x6 matrices of complex elements have been found. The results have been sent to the requesting contractor. It is planned that a report describing the method used will be prepared and forwarded to the originator.

DISCRIMINANT FUNCTIONS
Task 1101-40-5131/53-18

Origin: Randolph Field, USAF — Authorized 12/15/52
Sponsor: "
Manager: D. Teichroew
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Coding of the problem for the SWAC is continuing

POWERS OF MATRICES
Task 1101-40-5131/53-19

Origin: Consolidated Vultee Aircraft Corporation (Convair) — Authorized 12/15/52, Terminated 6/30/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF
Manager: M. W. Steinberg

Objective: Find the last rows of the 1st, 2nd, 3rd, and 4th powers of the matrix

$$\begin{Bmatrix} \binom{0}{0}, 0, 0, \ldots & , 0 \\ \binom{1}{0}, P(1), \binom{1}{1}\tilde{P}(1), 0, \ldots & , 0 \\ \binom{2}{0}P^2(2), \binom{2}{1}P(2)\tilde{P}(2), \binom{2}{2}\tilde{P}^2(2), & , 0 \\ \vdots & \\ \binom{N}{0}P^N(N), \binom{N}{1}P^{N-1}(N)\tilde{P}(N), \binom{N}{2}P^{N-2}(N)\tilde{P}^2(N, \ldots \binom{N}{N}\tilde{P}^N(N) \end{Bmatrix}$$

with

$$\tilde{P}(j) = \left(1 - \frac{p}{j}\right)^n, \quad P(j) = 1 - \tilde{P}(j)$$

and for the following values of the parameters N=100; n=100,200,500,1000, 1200; and p=0.3,0.5,0.7,1.0.

Background: This problem is associated with the contractor's Air Force projects.

Status: TERMINATED.

LIGHT SCATTERING FUNCTIONS
Task 1101-50-5131/53-22

Origin: Naval Radiological Defense Laboratory, San Francisco — Authorized 12/15/52, Terminated 6/30/53
Sponsor: " "
Manager: T. H. Southard

Objective: A light beam is scattered by a spherical droplet. It is

desired to compute the intensities of light being transmitted in various directions, for various kinds and sizes of droplets.

Background: The problem arises in connection with the work of the originator. Intermediate results have been tabulated by Gumprecht and Sliepcevich in "Light scattering functions for spherical particles". In addition, the intensity functions have been tabulated for a different parameter field in "Tables of scattering functions for spherical particles," National Bureau of Standards Applied Mathematics Series 4(1948, U. S. Government Printing Office, Washington, D. C.)

Status: TERMINATED.

BIO-ASSAY PROBLEM
Task 1101-40-5131/53-24

Origin: Stanford University — Authorized 3/31/53
Sponsor: Office of Naval Research
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. A table for α=1(1)17; α=-5(.5)5; β=.5(.5)10 has been computed and is being checked.

SINGLE SHOT PROBABILITIES
Task 1101-40-5131/53-25

Origin: Naval Air Missile Test Center, Point Mugu — Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. A report on the theoretical investigation is being prepared.

DISCRETE MINIMAL SPACES
Task 1101-40-5131/53-26

Origin: Gilfillan Brothers, Inc. — Authorized 3/31/53
Sponsor: Evans Signal Laboratories, Army Signal Corps
Manager: A. Hestenes
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. This task was coded for SWAC by the originator. Several solutions have been made. These are being studied before the next set of solutions will be computed.

RAYLEIGH SCATTERING OF LIGHT IN THE ATMOSPHERE
Task 1101-40-5131/53-28

Origin: Naval Ordnance Test Station (Inyokern) — Authorized 3/31/53
Sponsor: Bureau of Ordnance, USN — Completed 6/30/53
Manager: G. Blanch

Objective: To produce tables similar to those computed in connection with task 1101-50-5131/51-25, "Computation in connection with a study of polarization of light," for five additional values of τ, namely τ=.0001, .001, .01, .02, .04. (These values may be replaced by slightly different ones after further consultation with Z. Sekera and sponsor.)

Background: The following report by Prof. Zdenek Sekera will be basic in this task: "Tables relating to Rayleigh scattering of light in the atmosphere," Scientific Report Number 3, UCLA Department of Meteorology, Contract AF19(122)-239,November 1952. The functions tabulated in this report depend on Chandrasekhar's "X" and "Y" functions, the computation of which involves the solution of nonlinear integral equations by lengthy iteration processes. It turns out that for small values of the parameter τ, it is possible to use a much simpler approximation, for the accuracy aimed at in the present task. Once these values of X and Y are available, there remain to be computed the various scattering functions and "intensities" (about 15 functions).

Status: COMPLETED. The contractor expects to issue a report.

LIGHT SCATTERING COMPUTATIONS
Task 1101-40-5131/53-30

Origin: University of California at Los Angeles — Authorized 3/31/53
Sponsor: Atomic Energy Commission — Completed 6/30/53
Manager: P. Bremer

Objective: To reduce data in accordance with the contractor's formulas. Approximately two sets of observations are processed on IBM machines every week.

Background: These computations arise from the work of the AEC Medical Group at the University of California at Los Angeles.

Status: COMPLETED. Additional data reduction was performed, and the results were fitted with straight lines by the method of least squares. (See Jan-Mar 1953 issue, p. 25.)

EXPERIMENT DESIGN
Task 1101-40-5131/53-31

Origin: Radioplane Corporation — Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Completed 6/30/53
Manager: D. Teichroew

Objective: To study data submitted by the originator and recommend a suitable design to determine a combination of parameters which will

maximize a desired function.

Background: The Parachute Testing Panel of the Committee of Aeronautic Equipment of the Research Development Board has suggested the parachute testing activities should make use of statistical techniques in its programs. This is an attempt to give assistance in that direction.

Status: COMPLETED. A report has been submitted to the contractor.

ROOTS OF DETERMINANTS
Task 1101-40-5131/53-32

Origin: North American Aviation — Authorized 3/31/53
Sponsor: Atomic Energy Commission
Manager: E. C. Yowell
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Eight cases were solved this quarter. Recoding for use with the magnetic drum memory of SWAC is now in progress.

PLANE SHOCK WAVES
Task 1101-40-5131/53-33

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Authorized 3/31/53
Sponsor: " " "
Manager: M. W. Steinberg
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

LINEAR EQUATIONS (CONVAIR)
Task 1101-40-5131/53-35

Origin: Consolidated Vultee Aircraft Corporation — Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: R. R. Reynolds
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. This task is held up temporarily pending the receipt of data.

MISCELLANEOUS CORONA COMPUTATIONS
Task 1101-40-5131/53-36

Origin: NBS, Div. 15 — Authorized 3/31/53
Sponsor: "

Manager: P. Bremer
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

ATTRITION FUNCTIONS
Task 1101-40-5131/53-37

Origin: Logistic Research — Authorized 3/31/53
Sponsor: Office of Naval Research, Logistics Branch — Completed 6/30/53
Manager: C. B. Tompkins

Objective: To solve some simple arithmetically described battlelike games with probabilistic attrition in order to create models of evaluation functions applicable to military or to other competitive situations; to present these to experienced students of military science (quantitatively inclined officers of the Armed Forces) in a small working group discussion in order to guide the sponsor and the workers in future studies of war games.

Background: Studies of this kind have been carried on with low priority for several years by several groups. This task is a natural application of the large computing instrument to these studies which have heretofore been hampered by inadequate computational facilities.

Status: COMPLETED.

FLUTTER PROBLEM
Task 1101-40-5131/53-38

Origin: Douglas Aircraft Company — Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Terminated 6/30/53
Manager: F. Hollander

Objective: To obtain the powers of a 9x9 matrix with complex elements, using double precision programming.

Background: The originator coded the problem for SWAC and supplied three matrices, one of which was chosen as a test problem. The results are to be compared with answers obtained by the originator on his IBM equipment.

Status: TERMINATED.

MONTE CARLO STUDIES
Task 1101-40-5131/53-39

Origin: RAND Corporation — Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF

Manager: A. D. Hestenes
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. This problem was coded for SWAC by RAND personnel. The INA has provided machine and operator time. Several cases have been computed, and an equal amount of computing is anticipated in the future.

RANKING PROBABILITIES
Task 1101-40-5131/53-40

Origin: Cornell University — Authorized 3/31/53
Sponsor: Office of Naval Research
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The problem has been coded for the SWAC, and the routine is being checked.

ACCEPTANCE TESTS
Task 1101-40-5131/53-41

Origin: Jet Propulsion Laboratory, California Institute of Technology — Authorized 6/29/53
Sponsor: Army Ordnance Corps, U. S. Army
Manager: E. C. Yowell

Objective: To design and administer the acceptance tests for the high speed digital computer being purchased by Jet Propulsion Laboratories.

Background: The amount of computation being performed at JPL has made it advisable to purchase a large scale automatic digital computer. Since JPL's past computing experience has been primarily analogue, it was felt that INA could be of assistance to them, particularly in evaluating a new machine.

Status: NEW.

B. P. A. Studies
Task 1101-40-5131/53-42

Origin: Bonneville Power Administration — Authorized 6/29/53
Sponsor: " "
Manager: E. C. Yowell

Objective: To evaluate the utility of digital computing machinery in solving power transmission problems.

Background: Many problems arising in the work of the BPA are being solved on analogue equipment. This investigation seeks to determine which of their problems can be more effectively solved on digital equipment. All coding is being done by BPA personnel.

Status: NEW.

Status of Projects

ROOTS OF A TRANSCENDENTAL EQUATION
Task 1101-40-5131/53-43

Origin: U. S. Naval Ordnance Test Station, Pasadena — Authorized 5/20/53
Sponsor: " " " — Completed 6/30/53
Manager: A. D. Hestenes

Objective: To obtain α (if such exists) satisfying

$$\rho(\alpha,\psi)\sin(\alpha-\phi) = \rho_0 \sin(\alpha_0 - \phi)$$

$$\alpha_0 < \alpha \leq A + 10^\circ$$

where $\rho(\alpha,\psi)$ is given empirically, and

$$\phi = \arctan\frac{-\sin\psi}{\lambda' - \cos\psi}, \quad -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$$

$$\lambda' = \frac{\lambda \sin A}{A}.$$

ψ is to be taken at 10° intervals in the range $-\pi \leq \psi \leq \pi$. The parameters have the following values: $\lambda = 1.5, 3$, $A = \pi/6, \pi/4, \pi/3$, $\rho_0 = 1500, 1250, 1000, 750, 500$, and $\alpha_0 = -15^\circ, -12\frac{1}{2}^\circ, \ldots, 12\frac{1}{2}^\circ, 15^\circ$.

Status: COMPLETED (NEW). The roots were obtained using a suitable SWAC code and have been submitted to the originator.

HINGE MOMENTS
Task 1101-40-5131/53-45

Origin: Aeronautical Research Laboratory, Wright Air Development Center, Air Research and Development Command, USAF — Authorized 6/30/53
Sponsor: " " "
Manager: G. Blanch

Objective: To compute the integrals $RP_h(c)$, $RT_h(c)$, $RP_\alpha(c)$, $RT_\alpha(c)$, for $c = -.9(.1)9$.

Background: In the process of computing lift and moment coefficients for task 1101-53-1101/52-41, there were computed certain constants A and B, in terms of which Reissner's $p_a^{(2)}$ for the circulatory flow can be determined. Once this is done, the following integrals are required:

$$RP_h(c) = 2\int_c^1 [p_a]_h dx; \quad RT_h(c) = 2\int_c^1 (x - c)[p_a]_h dx$$

$$RP_\alpha(c) = 2\int_c^1 [p_a]_\alpha dx; \quad RT_\alpha(c) = 2\int_c^1 (x - c)[p_a]_\alpha dx$$

where R is some constant.

Status: NEW.

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-10-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS Authorized 1/1/50
Managers: J. Todd, M. Abramowitz, and H. A. Antosiewicz
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. H. Antosiewicz investigated the differential equation $\ddot{x}+k[f(x)+g(x)\dot{x}]\dot{x}+h(x)=ke(t)$. A paper including his recent as well as all his previous results on this equation has been prepared (see publication (1) below).

M. Abramowitz has been engaged in a further study of the problem of heat convection in laminar flow of fluids inside a tube, an extension of his previous work in this field. A paper on this has been prepared (see publication (3) below).

He has also been investigating the numerical inversion of the Laplace transform.

Publications: (1) "On the differential equation $\ddot{x}+k[f(x)+g(x)\dot{x}]\dot{x}+h(x)=ke(t)$," by H. A. Antosiewicz; submitted to a technical journal. (2) "Some implications of Liapunov's conditions for stability," by H. A. Antosiewicz and P. Davis; submitted to a technical journal. (3) "On forced convection in laminar flow through a tube," by M. Abramowitz; IN MANUSCRIPT.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-10-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS Authorized 3/1/50
Manager: P. Davis
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1952 issue.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-10-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS Authorized 1/1/50
Managers: O. Taussky Todd, J. Todd, M. Abramowitz, and A. Hoffman
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The manuscript for a proposed volume in the Applied Mathematics Series which deals with numerical experiments in conformal mapping is in preparation. The volume will contain articles by A. M. Ostrowski, S. E. Warschawski, J. Todd, L. Ahlfors, and G. Blanch and L. K. Jackson.

In connection with this work elaborate experiments have been carried out on SEAC by I. Rhodes on the mapping of an ellipse with axis ratio 5:1 on a circle. Many interesting phenomena have turned up, one of which lead to the research on the eigenvalues of integral equations (mentioned by H. Wielandt in task 1102-10-1104/52-34, p.32). It has been found, for example, that although some fifty iterations would be necessary to obtain results correct to 9 decimal places directly, some 12 are sufficient when these are subjected to multiple accelerations of the Aitken form. In order to obtain 9 decimal places it is necessary to use a 1° interval in the quadrature, and some 25 minutes were required for each iteration. This indicates the economy effected by use of this process. Standard subroutines for the Aitken process have been prepared by P. Henrici.

Publication: "Linear functional equations and interpolation series," by P. Davis; submitted to a technical journal.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-10-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS Authorized 6/1/50
Managers: O. Taussky Todd and K. Goldberg
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. The table of least positive primitive roots is being extended when time is available on SEAC. The preparation of an index table is under consideration.

Publication: "A table of Wilson quotients and the third Wilson prime," by K. Goldberg; J. London Math. Soc., 28, Pt. 2, 252-256 (Apr. 1953). Reprints available.

ANALYSIS OF CRYSTAL STRUCTURE
Task 1102-10-1104/51-3

Origin: NBS
Manager: Ethel C. Marden

Authorized 9/28/50
Terminated 6/30/53

Objective: Calculation of electron density $\rho(x,z)$ corresponding to structure factors $F(h,0,\ell)$ satisfying the reciprocal relationships

$$\rho(x,z) = \sum_{h=-\infty}^{\infty} \sum_{h=-\infty}^{\infty} F(h,\ell) \cos 2\pi(hx - \ell z)$$

$$F(h,\ell) = \int_0^1 \int_0^1 \rho(x,z) \cos 2\pi(hx + \ell z)dx\, dz.$$

Background: This problem arises in the study of crystal structures. It is the two-dimensional special case of a more general three-dimensional problem in which ρ is a function of x,y,z and F depends on three parameters (Miller indices) h, k, ℓ. The function ρ measures the distribution of electrons in the unit cell of a crystal, treated as a continuous function, and F measures the intensity of light reflected from a crystal surface with Miller indices h,k, ℓ.

The approach is to start with a preliminary experimentally determined set of structure factors $F(h,0,\ell)$, and calculate the corresponding ρ's. Then the ρ's are modified so as to make the distribution more "acceptable," in particular so that ρ is non-negative and conforms to certain preconceived ideas about the shape of such distribution functions. Changes in ρ are to be accomplished in such a way as to keep F as close as possible to the observed values.

Comments: This work is part of an investigation of crystal structures being carried on by the Portland Cement Association Fellowship. The problem was proposed by Dr. F. Ordway, who suggested a method to be followed.

Status: TERMINATED.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-10-1104/51-6

Origin: NBS
Manager: M. Abramowitz
Full task description appears in July-Sept 1950 issue.

Authorized 9/28/50

Status: INACTIVE. For status to date see Apr-Jun 1952 issue.

THREE-BODY PROBLEM
Task 1102-10-1104/52-4

Origin: NBS Authorized 10/5/51
Manager: A. Goldstein
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-10-1104/52-8

Origin: NBS Authorized 8/10/51
Manager: C. J. Swift
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-10-1104/52-34

Origin: NBS Authorized 11/6/51
Manager: O. Taussky Todd
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. A. J. Hoffman and O. Taussky Todd continued their investigations concerning characterizations of normal matrices (see publication (1) below). T. S. Motzkin and Mrs. Todd continued their work on matrices with property L (see publication (2) below). They were also trying to find an alternative treatment of their problem by a function theoretic method. This has now been taken up successfully by H. Wielandt. O. Taussky Todd performed more work on commutators of matrices and permutations of products of matrices.

A new volume in the Applied Mathematics Series was edited by O. Taussky Todd (see publication (3) below), a sequel to AMS29, "Simultaneous linear equations and the determination of eigenvalues," and includes: (1) "Practical solution of linear equations and inversion of matrices," by L. Fox; (2) "Punched card experiments with accelerated gradient methods for linear equations," by A. I. Forsythe and G. E. Forsythe; (3) "Iterated methods of solving linear problems on Hilbert space," by R. M. Hayes; (4) "Tables of inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; (5) "The condition of the finite segments of the Hilbert matrix," by J. Todd; and (6) "Lower bounds for the rank and location of the eigenvalues of a matrix," by Ky Fan and A. J. Hoffman. Descriptions of all these papers have appeared earlier except that of J. Todd. This paper is an examination of the condition of the Hilbert matrix H_n and a study of its inversion on SEAC. The condition number of H_n is exponentially large; the average condition number of a matrix is $O(n)$. While it has been possible to invert matrices with condition number $O(n^2)$ of order 50 or more on SEAC, obtaining about seven significant figures, it is not possible to invert a 6x6 Hilbert matrix. The process used is a modification of the Gauss elimination process as presented by von Neumann and Goldstine which was coded for SEAC by M. Newman and S. L. Pollack.

H. Wielandt extended his earlier results concerning matrices

with property L (see publication (4) below). He found alternative proofs and generalizations of the the theorems in publication (2) below. These proofs are based on function-theoretical arguments, and the generalizations refer to the case of matrices whose elements are polynomials of arbitrary degree (not only linear) in a parameter z.

Dr. Wielandt found new inequalities for the eigenvalues γ_i, of the sums C=A+B of two Hermitian matrices with given eigenvalues α_i, β_i. These inequalities have the form

$$\gamma_{\nu_1} + \ldots \gamma_{\nu_s} \leqslant \alpha_{\eta_1} + \ldots + \alpha_{\eta_s} + \beta_{\rho_1} + \ldots + \beta_{\rho_s}$$

and include inequalities of H. Weyl and Lidskii. The proofs are based on a maximum-minimum characterization of $\alpha_{\eta_1} + \ldots + \alpha_{\eta_s}$, which includes the maximum characterization of $\alpha_1 + \ldots + \alpha_s$ given by Ky Fan. H. Wielandt determined those values γ which may occur as eigenvalues of A+B, where A and B run over all normal (not necessarily Hermitian) matrices with given eigenvalues. He found the following comparison theorem for the eigenvalues α_i, β_i of normal matrices A,B which are "near" in the sense that their difference A-B has a given rank $r < n$: Every circle containing b eigenvalues of B contains at least b-r eigenvalues of A. He proved a similar theorem for the eigenvalues of normal segments of normal matrices. He further determined the possible spectra of the segments of all Hermitian matrices with a given set of eigenvalues. Dr. Wielandt developed a method for estimating the error of the eigenvalues which occur if an Hermitian integral equation $\int K(x\xi)y(\xi)d\xi = ky(x)$ is solved approximately by use of any given formula for numerical quadrature. The method can be applied without any previous knowledge concerning the eigenfunctions or eigenvalues. It seems the first known method to determine in advance a number of interpolation points sufficient to guarantee a prescribed accuracy.

In connection with O. Taussky Todd's work on commutators of matrices, Ky Fan determined the commutators of the unitary group and the orthogonal group. Ky Fan continued his investigation on inequalities concerning the eigenvalues of Hermitian matrices and on unitary-invariant metric properties of the space of matrices.

Publications: (1) "A characterization of normal matrices," by A. J. Hoffman and O. Taussky; IN MANUSCRIPT. (2) "Pairs of matrices with property L, II," (summary) by T. S. Motzkin and O. Taussky; to appear in the Proceedings of the National Academy of Science. (3) "Contributions to the solution of systems of linear equations and the determination of eigenvalues," edited by O. Taussky; to appear in the NBS Applied Mathematics Series. (4) "Pairs of matrices with property L," by H. Wielandt; accepted for publication by the Journal of Research of the NBS.

Status of Projects

ROOTS OF POLYNOMIAL EQUATIONS
Task 1102-10-1104/52-51

Origin: NBS
Manager: D. I. Rubin

Authorized 2/1/52
Terminated 6/30/53

Objective: To produce a SEAC routine which will determine the roots, real and complex, of any given polynomial equation, provided its degree is not so large that computation time becomes excessive.

Background: Let C be a closed curve in the complex plane, and f(z) a function which is meromorphic within, and has no singularities on C. Then

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}\,dz = \sum_j r_j - \sum_k s_k$$

where $r_1, r_2, r_3, \ldots$ are the orders of the zeros and $s_1, s_2, s_3, \ldots$ the orders of the poles of f(z) within C. Since a polynomial has no poles, it can be determined whether the polynomial has zeros in a given region, and, if so, the region can be subdivided and the process repeated until the roots are known to any desired accuracy.

Status: TERMINATED. This task was terminated in favor of tasks of higher priority.

DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC LATTICES
Task 1102-10-1104/52-62

Origin: NBS
Sponsor: "
Manager: T. W. Ledley and F. J. Stockmal

Authorized 2/25/52
Completed 6/30/53

Objective: To compute the distribution of normal modes of vibration in cubic lattices, on which depend the vibrational contribution to the thermodynamic properties of polyatomic molecules and crystals.

Exact distribution functions have been found for two-dimensional lattices, but the amount of computation necessary in the three-dimensional case has hitherto been prohibitive. This program will investigate application of high-speed computing devices to the problem.

Background: The characteristic frequencies of normal modes of vibration of a cubic lattice are roots of cubic equations. Since the number of equations is of the order of the number of particles in the lattice ($O(10^{23})$), the time required for the calculation of the frequencies is tremendous. It is expected that by taking small crystals ($O(10^4)$ lattice points) one would obtain a considerable amount of information concerning the distribution.

Status: COMPLETED. The results were transmitted to the originator.

AN INTEGRAL ARISING IN THE THEORY OF COOPERATIVE PHENOMENA
Task 1102-10-1104/52-69

Origin: NBS — Authorized 5/9/52
Manager: J. Todd
Full task description appears in Apr-Jun 1952 issue.

Status: CONTINUED. It has not yet been possible to carry out experiments on handling this work by Monte Carlo methods. The work done on this task is described in the publication below.

Publication: "The tabulation of an integral arising in the theory of cooperative phenomena," by M. Tikson; J. Res. NBS 50, 177-178 (Mar. 1953).

ROOTS OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS
Task 1102-10-1104/52-76

Origin: NBS — Authorized 5/21/52
Manager: D. I. Rubin — Terminated 6/30/53
Full task description appears in Apr-Jun 1952 issue.

Status: TERMINATED. This task was terminated in favor of tasks of higher priority.

DIFFERENTIAL EQUATION FOR NERVE FIBER REACTION
Task 1102-10-1104/53-15

Origin: National Naval Medical Institute — Authorized 12/8/52
Sponsor: " "
Managers: H. A. Antosiewicz and P. Rabinowitz
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Further results were obtained for various parameter values t_0, I_0. Also a periodic solution V(t) was found by numerical integration of the system. These results were transmitted to the sponsor.

BAKER-HAUSDORFF FORMULA
Task 1102-10-1104/53-16

Origin: NBS — Authorized 12/8/52
Manager: O. Taussky Todd and K. Goldberg
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. K. Goldberg has derived a formula for evaluating the coefficients of the powers of x and y in z, where $e^z = e^x . e^y$ and $xy \neq yx$.

POISSON-BOLTZMAN EQUATION
Task 1102-10-1104/53-31

Origin: NBS Authorized 1/23/53
Manager: M. Abramowitz
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

HYPERGEOMETRIC FUNCTIONS
Task 1102-10-1104/53-35

Origin: NBS Authorized 3/25/53
Manager: P. Rabinowitz, W. Cahill
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The code for the hypergeometric function with complex parameters and arguments has been modified to compute the confluent hypergeometric function. A report on the code for the general hypergeometric function is in preparation.

RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5115/50-2

Origin: Office of Scientific Research, and Authorized 3/31/50
Office of the Air Comptroller, USAF
Sponsor: " " "
Managers: J. Todd and A. Hoffman
Full task description appears in Jan-Mar 1950 issue, see 11.2/12-50-1.

Status: CONTINUED. The series of lectures on the "Theory of Games and Its Applications," jointly sponsored by the National Bureau of Standards and American University, which was reported in Jan-Mar 1953 issue, p. 37, continued into the current quarter. A list of the titles and speakers of the talks presented this quarter is given in Lectures and Symposia, p.68 .

The principal work of the quarter was the experimental investigation of various methods of solving linear programs. The "block relaxation" method is a variation of the furthest hyperplane scheme. Instead of moving to the furthest hyperplane, one moves to the intersection of the two furthest hyperplanes. The results of tests with this method are reported in publication (7) below. Further experiments with the furthest hyperplane procedure, in order to determine the optimal "overshoot", are reported in publication (1) below. The use of double description is described in publication (9) below.

A code for solving linear programs by a method proposed by Brown and Koopmans has been prepared by K. Christoph. New procedures for solving linear programs in which the objective function depends linearly on several parameters are under study.

Publications: (1) "On 'overshoot' in the furthest hyperplane method," by R. Bryce; IN MANUSCRIPT. (2) "Lower bounds for the rank and location of the eigenvalues of a matrix," by Ky Fan and A. J. Hoffman; to be included in "Contributions to the solution of systems of linear equations and the determination of eigenvalues," NBS Applied Mathematics Series.

(3) "On the caterer problem," by J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky; IN MANUSCRIPT. (4) "Remarks on the smoothing problem," by A. J. Hoffman; IN MANUSCRIPT. (5) "Computational experience in solving linear programs," by A. J. Hoffman, M. Mannos, D. Sokolowsky, and N. A. Wiegmann; accepted for publication in the Journal of the Society for Industrial and Applied Mathematics. (6) "A characterization of normal matrices," by A. J. Hoffman and O. Taussky; IN MANUSCRIPT. (7) "On block relaxation," by L. S. Joel; IN MANUSCRIPT. (8) "Eigenvectors of matrix polynomials" by M. Mannos; to appear in the Journal of Research of the NBS. (9) "Experimental results with the double description method," by S. Pollack; IN MANUSCRIPT.

SURVIVAL PROBABILITY IN PATTERN BOMBING
Task 1102-10-5115/53-13

Origin: Deputy Chief of Staff Operations, Directorate of Operations, Operations Analysis, USAF — Authorized 12/8/52, Completed 6/30/53
Sponsor: Office of the Air Comptroller, USAF
Managers: E. Marden and R. K. Anderson
Full task description appears in Oct-Dec 1952 issue.

Status: COMPLETED. Results were transmitted to the originator.

COMPRESSIBLE FLOW - METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS
Task 1102-10-5116/52-16

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF and Harvard University — Authorized 9/29/51
Sponsor: Aeronautical Research Laboratory, USAF
Managers: P. Davis and F. L. Alt
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computation of 12 particular solutions of the stream equation $\Delta\psi + 4F(\lambda)\psi = 0$ is proceeding. These solutions are given by

$$\psi = \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left\{ z^n - \int_{z_0}^{z}\int_{\bar{z}_0}^{\bar{z}} z^n f\left(\frac{z+\bar{z}}{2}\right) dz\, d\bar{z} \right\} \quad (n = 0,1,\ldots,5).$$

When reduced to real form, each of the above complex integrals is equivalent, for fixed n, to six real double integrals. Sixty of the seventy-two integrals have been computed. A code is being written to check these particular solutions in the differential equation itself.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-10-5116/52-33
(formerly 1102-21-5117/52-33)

Origin: NBS — Authorized 10/11/51
Manager: F. Oberhettinger
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The tables have been compiled and are now being edited.

INTEGRAL OPERATORS AND INTERPOLATION SERIES
Task 1102-10-5116/53-2

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF — Authorized 9/26/52, Revised 9/30/52
Sponsor: " "
Manager: P. Davis
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Theoretical investigation of Part II (see Jan-Mar 1953 issue, p. 38), continues.

WATER WAVES
Task 1102-10-5116/53-54

Origin: NBS, Division 6 — Authorized 6/1/53
Sponsor: Office of Naval Research
Manager: P. Davis

Objective: To study the partial differential equations governing the motion of water waves on a shallow beach and to formulate numerical methods for the solution of these equations on SEAC.

Background: The problem of the breaking of water waves requires the solution of the Laplace equation over a region bounded by a bottom of variable depth and a free surface whose location is governed by nonlinear boundary conditions. In their full generality, these equations cannot be solved in closed form, and it is therefore of great interest to obtain numerical solutions.

Status: NEW.

2. Applied Research: Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-10-1110/43-3
(formerly 1102-21-1110/43-3)

Origin: Canadian National Research Council Authorized 7/1/47
Manager: I. A. Stegun
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-43-3.

Status: CONTINUED. Revisions of the manuscript are in progress.

TABLE OF THE GAMMA FUNCTIONS FOR COMPLEX ARGUMENTS
Task 1102-10-1110/46-1
(formerly 1102-21-1110/46-1)

Origin: NBS Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-46-1.

Status: CONTINUED. This volume is now in press.

Publication: "Table of the Gamma function for complex arguments," NBS Applied Mathematics Series 34; in press, U. S. Government Printing Office.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-10-1110/47-2
(formerly 1102-21-1110/47-2)

Origin: NBS Authorized 7/1/47
Managers: M. Abramowitz and P. Rabinowitz
Full task description appears in Apr-Jun 1949 issue, see task 11.2/2-47-2.

Status: CONTINUED. The regular and irregular functions together with their derivatives have been computed on SEAC for ρ =1(1)10, η =1(1)10, L=0,5. These data are being processed.

TABLE OF ANTILOGARITHMS
Task 1102-10-1110/47-3
(formerly 1102-21-1110/47-3)

Origin: NBS Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-Jun 1949 issue, see 11.2/2-47-3.

Status: CONTINUED. This volume is now in press.

Publication: "Tables of 10^x," NBS Applied Mathematics Series 27; in press, U. S. Government Printing Office.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-10-1110/48-2
(formerly 1102/21/1110/48-2)

Origin: NBS — Authorized 5/25/48
Manager: J. Todd
Full task description appears in Apr-Jun 1949 issue, see 11.2/2-48-2.

Status: CONTINUED. This volume is in press.

Publication: "Tables of Lagrangian coefficients for sexagesimal interpolation," NBS Applied Mathematics Series 35; in press, U. S. Government Printing Office.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-10-1110/50-4a
(formerly 1102-21-1110/50-4a)

Origin: NBS, Section 11.3 — Authorized 12/31/49
Manager: J. Todd
Full task description appears in Oct-Dec 1949 issue, see 11.2/2-50-4.

Status: CONTINUED.

Publication: "Probability tables for analysis of extreme-value data," NBS Applied Mathematics Series 22; in press, U. S. Government Printing Office.

BIBLIOGRAPHY OF CODING PROCEDURES, MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-10-1110/50-5

Origin: NBS — Authorized 3/1/50
Managers: J. Todd, J. H. Wegstein, and P. Rabinowitz — Revised 1/9/53
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. Items for the coding library are being receiv from laboratories in this country and abroad. A comprehensive card cata is being maintained, and the more notable acquisitions are described in Mathematical Tables and Other Aids to Computation. The file of reviews of mathematical tables and papers in numerical analysis is being kept up to date and is being subdivided according to topic.

WAVE FUNCTION FOR LITHIUM
Task 1102-10-1110/50-7
(formerly 1102-21-1104/50-7)

Origin: NBS Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Managers: D. I. Rubin and W. H. Durfee
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Computation on SEAC continues as time is available.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-10-1110/51-4
(formerly 1102-21-1104/51-4)

Origin: NBS Authorized 9/28/50
Manager: J. Todd
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. This volume is in press.

Publication: "Tables of functions and of zeros of functions." Volume I of Collected Short Tables of the Computation Laboratory; NBS Applied Mathematics Series 37, in press.

REVISION OF MATHEMATICAL TABLES
Task 1102-10-1110/52-7

Origin: NBS Authorized 8/10/51
Managers: J. Todd, W. F. Cahill, and I. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The following action is being taken in connection with a mathematical table the sales stock of which has been exhausted:

Tables of circular and hyperbolic sines and cosines for radian arguments (a reissue of MT3) AMS36: revision completed, in press.

The following volume is now out of print:

Table of natural logarithms, $\log_e x$: x=5(.0001)9.9999, (1941). MT12; reissue is planned.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-10-1110/52-14
(formerly 1102-21-1110/52-14)

Origin: NBS Authorized 10/1/51
Manager: A. A. Goldstein
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The processing of punched cards continues.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-10-1110/52-18
(formerly 1102-21-1110/52-18)

Origin: NBS **Authorized** 9/17/51
Manager : W. F. Cahill
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computation on the SEAC is completed, and the results have been transferred to IBM cards.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-10-1110/52-23
(formerly 1102-21-1110/52-23)

Origin: NBS **Authorized** 10/4/51
Manager: P. Rabinowitz
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The introduction to the table has been revised and will include a derivation of the asymptotic formula for

$$\int_0^x A_0(x)dx.$$

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-10-1110/52-25
(formerly 1102-21-1110/52-25)

Origin: NBS **Authorized** 10/5/51
Managers: M. Abramowitz and F. J. Stockmal
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-10-1110/52-31
(formerly 1102-21-1110/52-31)

Origin: NBS **Authorized** 10/9/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-10-1110/52-37
(formerly 1102-21-1110/52-37)

Origin: NBS Authorized 11/28/51
Manager: T. Ledley
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Spheroidal wave functions have been computed for the following cases: (1) Prolate: $1/c=t=.01(.01)1$, $c^2=1(.01)0$, where $m=0$ and $\ell=0,1,2,3,4,5$; $m=1$ and $\ell=0,1,2,3$; $m=2$ and $\ell=0,1,2,3$; and $m=3$ and $\ell=0,1$. $1/c=t=.005(.005)1$, $c^2=1(.01)0$, where $m=1$ and $\ell=4,5$; $m=2$ and $\ell=4,5$; $m=3$ and $\ell=2.3$; $m=4$ and $\ell=0,1,2,3,4$; and $m=5$ and $\ell=0$. (2) Oblate: $1/c^2=t=.005(.005)1$, $c^2=1(.01)0$, where $m=0$ and $\ell=0,1,2,3$. More cases will be computed whenever machine time is available.

VAN DER POL EQUATION
Task 1102-10-1110/52-43

Origin: NBS Authorized 11/28/51
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see Apr-Jun 1952 issue.

RADIAL MATHIEU FUNCTIONS
Task 1102-10-1110/52-49

Origin: NBS Authorized 2/1/52
Managers: J. Todd, I. Rhodes, and G. Blanch

Status: CONTINUED. New tables which are suitable for differencing were computed, but some further subtabulation will be necessary.

SIEVERT'S INTEGRAL
Task 1102-10-1110/52-57

Origin: NBS Authorized 2/12/52
Managers: O. Steiner and R. B. Jasper
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. Computation of the tables on SEAC continues as time is available. Values in the range $0(.01)2(.02)5$ have been computed. Results are being transferred to punched cards for future printing. Seventy-five percent of the anticipated computation is finished.

SCATTERING FUNCTIONS
Task 1102-10-1110/52-63

Origin: NBS
Manager: A. Gleyzal
Authorized 3/10/52
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. Coding for the computation of the general solution of the wave equation is underway.

JACOBI ELLIPTIC FUNCTIONS
Task 1102-20-1110/52-74

Origin: NBS
Manager: T. Ledley
Authorized 7/1/52
Terminated 6/30/53

Objective: To tabulate the Jacobi elliptic functions $sn(u,k)=\sin \phi$, $cn(u,k)=\cos \phi$, $dn(u,k)=\sqrt{1-k^2\sin^2\phi}$, where

$$u = pK,$$

$$K = \int_0^{\frac{\pi}{2}} (1 - k^2 \sin^2 \theta)^{-\frac{1}{2}} \, d\theta,$$

ϕ is defined by $u = \int_0^{\phi} (1 - t^2)^{-\frac{1}{2}}(1 - k^2t^2)^{-\frac{1}{2}} dt,$

and $k^2=0(.01)1$, $p=0(.02)1$.

Background: A coarse tabulation of these functions (for p=0(.1)1) was undertaken under project 43D2-4 (see July-Dec 1947 issue, p. 4). The results have never been published. The purpose of this task is to produce a table, to a moderate number of decimals, probably 6 or 7, which is interpolable conveniently in the u direction. Every fifth value so obtained will be checked against the original hand-computations which were to about 16 decimals. Computations are to be done on SEAC.

Status: TERMINATED. sn(pK,k), cn(pK,k), and dn(pK,k) have been computed for $k^2=0(.01)1$, $p=0(.02)1$. The results are stored on punched paper tape from which Flexowriter copies can be made.

TABLE OF SECANTS AND COSECANTS
Task 1102-10-1110/52-81

Origin: NBS
Managers: K. C. Nelson and I. A. Stegun
Authorized 7/1/52
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. The editing of the introductory material was completed.

PAINLEVÉ EQUATION
Task 1102-10-1110/53-3

Origin: NBS Authorized 8/11/52
Managers: J. Todd and H. A. Antosiewicz
Full task description appears in July-Sept 1952 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-10-1110/53-52

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: C. J. Swift

Objective: To compute on the SEAC tables of internal conversion coefficients for the K and L shells of the atoms of atomic number 5(10)95.

Background: This is a continuation of task 1102-40-5126/51-19, p.47. For background, refer to that task.

Status: NEW. The code is in the final stage of checking, and a few preliminary answers have been obtained.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Managers: A. Hoffman and L. Joel
Full task description appears in Apr-Jun 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

3. Mathematical Services

LINEAR PROGRAMMING ON STANDARD PUNCHED CARD MACHINES
Task 1102-40-5126/49-3
(formerly 1102-53-1106/49-3)

Origin: Office of the Air Comptroller, USAF Authorized 3/21/49
Sponsor: " " Terminated 6/30/53
Manager: A. Hoffman
Full task description appears in Apr-Jun 1949 issue, see task 11.2/36-49-3.

Status: TERMINATED. The sponsoring agency, the Office of the Air Comptroller, will perform all future computations required in their program planning with their own computing facilities.

SHOCK WAVE PARAMETERS, I
Task 1102-40-5126/49-13
(formerly 1102-53-1106/49-13)

Origin: Bureau of Ordnance, Department of the Navy — Authorized 3/31/49
Sponsor: " " — Terminated 6/30/53
Manager: I. A. Stegun

Objective: To provide graphs and tables for the rapid determination of the parameters of spherical shock waves emitted by explosions. In particular, (a) to prepare basic tables of certain functions $I_n(x,c_v)$ [n=0,1,2,...,10, c_v=specific heat at constant volume] needed in the following parts of the problem, (b) to determine p_1 (the initial pressure at the shock front) in its dependence on p_e (the static pressure of the explosion gases) and a parameter K which measures the effect of density, temperature, etc.; (c) to compute tables of certain functions which facilitate the determination of K for a given explosive.

Background: OSRD Reports 1030, 2022, 3550, and 5649 contain tables describing the entire course of shock waves in air, fresh water, and salt water as a function of the initial conditions at the instant of explosion. The present task aims at facilitating the determination of these initial conditions from a knowledge of the chemical composition of the explosive, and thus to aid in studies of the effectiveness of explosives. The initial conditions, particularly pressure and velocity, are determined by the fact that they satisfy the equation of state of the explosion gas and the Hugoniot condition for the surrounding medium (water). The latter is tabulated in OSRD Reports 670 and 813; for the former the Wilson-Kistiakowsky equation may be used. Kirkwood and Montroll (OSRD Report 670) developed the theory which forms the basis of the present computations. They also introduced and tabulated the auxiliary functions $I_n(x,c_v)$. The range of their tables is, however, not quite sufficient, and subtabulation is needed for convenient use.

Comments: The problem was propsed by Dr. T. L. Brownyard of the Bureau of Ordnance and Dr. H. G. Snay of the Naval Research Laboratory. The latter developed the method used in this task.

Status: TERMINATED.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-40-5126/50-16
(formerly 1102-53-1106/50-16)

Origin: Naval Research Laboratory, USN — Authorized 3/31/50
Sponsor: " "
Manager: P. J. O'Hara
Full task description appears in Jan-Mar 1950 issue, see task 11.2/33-50-16.

Status: CONTINUED. Computations were performed as requested.

PROGRAM COMPUTATION ON THE SEAC
Task 1102-40-5126/51-7
(formerly 1102-53-1106/51-7)

Origin: Office of the Air Comptroller, USAF — Authorized 9/1/50
Sponsor: " "
Manager: A. Hoffman
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

FLOW IN SUPERSONIC NOZZLES
Task 1102-40-5126/51-13
(formerly 1102-53-1106/51-13)

Origin: Naval Ordnance Laboratory — Authorized 12/15/50
Sponsor: "
Manager: N. Levine
Full task description appears in Oct-Dec 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1952 issue.

INTERNAL CONVERSION COEFFICIENTS FOR L-SHELL
Task 1102-40-5110/51-19
(formerly 1102-50-5126/51-19 and 1102-53-1106/51-19)

Origin: Atomic Energy Commission, Oak Ridge National Laboratories — Authorized 3/30/51, Terminated 6/30/53
Sponsor: " " "
Manager: C. J. Swift

Objective: To compute on the SEAC tables of internal conversion coefficients for the K and L shells of atoms of atomic number 5(10)95.

Background: A radioactive nucleus may make a transition in which the energy is carried off either as a photon (gamma ray) or by one of the extra-nuclear electrons. The ratio of the number of electrons to photons observed is known as the internal conversion coefficient. A comparison of calculated and observed coefficients is a tool used in classifying the radioactive transition and in understanding the decay scheme.

Since extra-nuclear (atomic) electrons are present in K,L,M... shells one may consider conversion coefficients for these various shells. Due to the difference in energy of converted K,L,M electrons, they are experimentally distinguishable. In fact, the ratio of K to L conversion is easily measurable, and corresponding calculations are desirable.

At present, some accurate K-shell calculations exist. However only a few scattered calculations on the L-shell exist in addition to some rough approximate formulae. The present program is intended to obtain a wide range of L-shell internal conversion values accurately calculated with due attention to all important physical effects. In addition, the gaps in the present K-shell tables should be filled in.

Thus, at the end of the present calculations there will exist tables of K-shell and L-shell internal conversion coefficients in sufficient completeness that the experimenter will have a powerful tool at

his disposal to aid in classification of nuclear states and decay schemes.

Status: TERMINATED. See task 1102-10-1110/53-52. p.45.

LIQUID-VAPOR TRANSITION
Task 1102-40-5126/51-22
(formerly 1102-53-1106/51-22)

Origin: Naval Medical Research Institute — Authorized 2/1/51
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

MOLECULAR STRUCTURE, III
Task 1102-40-5126/51-37
(formerly 1102-53-1106/51-37)

Origin: Naval Research Laboratory, USN — Authorized 8/10/51
Managers: P. J. O'Hara and I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computations were performed as requested.

SHOCK WAVE PARAMETERS, II
Task 1102-40-5126/51-38
(formerly 1102-53-1106/51-38)

Origin: Bureau of Ordnance, USN — Authorized 6/28/51
Sponsor: " "
Manager: I. A. Stegun
Full task description appears in Apr-Jun 1951 issue.

Status: CONTINUED. Exploratory and test cases have been completed for several explosives.

PRESSURE DISTRIBUTION ON BODIES OF REVOLUTION
Task 1102-40-5126/52-3
(formerly 1102-53-1106/52-3)

Origin: David Taylor Model Basin, USN — Authorized 8/10/51
Managers: A. Gleyzal and O. Steiner
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

POWDER DIFFRACTION
Task 1102-40-5126/52-6
(formerly 1102-53-1106/52-6)

Origin: NBS, Section 9.7 Authorized 9/17/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

NEUTRON DIFFUSION
Task 1102-40-5111/52-15
(formerly 1102-50-5126/52-15 and 1102-53-1106/52-15)

Origin: Rand Corporation Authorized 9/17/51
Sponsor: Air Materiel Command, USAF Terminated 6/30/53
Manager: C. J. Swift

Objective: To determine depth of penetration and other theoretical data for gamma radiation entering a semi-infinite medium, using the Monte Carlo technique.

Background: Some information has been obtained from ENIAC, but SEAC is a better machine to give the extensive information wanted.

Status: TERMINATED.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-40-5126/52-17
(formerly 1102-53-1106/52-17)

Origin: NBS, Division 2 Authorized 9/29/51
Sponsor: "
Manager: I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computations were performed as requested.

SPHERICAL BLAST
Task 1102-40-5126/52-20
(formerly 1102-53-1106/52-20)

Origin: Naval Ordnance Laboratory Authorized 9/27/51
Sponsor: " "
Manager: D. H. Jirauch
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

MAGNETIC FIELD EXTRAPOLATION
Task 1102-40-5126/52-22
(formerly 1102-53-1106/52-22)

Origin: Naval Ordnance Laboratory, USN — Authorized 10/1/51
Sponsor: " "
Manager: N. Levine
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Apr-Jun 1952 issue.

CALCULATIONS FOR d SPACINGS
Task 1102-40-5126/52-44
(formerly 1102-53-1106/52-44)

Origin: NBS, Div. 9 — Authorized 12/7/51
Sponsor: "
Manager: I. Stegun
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Computations were performed as requested.

INTENSITIES OF SPECTRAL LINES II
Task 1102-40-5126/52-46
(formerly 1102-53-1106/52-46)

Origin: NBS, Div. 14 — Authorized 12/10/51
Sponsor: " — Terminated 6/30/53
Manager: R. Zucker

Objective: To perform additional calculations on the theoretical intensities of the rotational spectral lines of water vapor.

Background: This task is an outgrowth of task 1409-31-1457/51-5, Jan-Mar 1951 issue, p. 48.

Status: TERMINATED.

MULTIPLE COMPTON SCATTERING OF LOW ENERGY GAMMA RADIATION
Task 1102-40-5126/52-65
(formerly 1102-53-1106/52-65)

Origin: Naval Research Laboratory — Authorized 4/1/52
Sponsor: " " — Terminated 6/30/53
Manager: I. Stegun

Objective: To evaluate Fourier integrals for a selected set of parameters.

Background: The integrals arise in the theoretical studies being carried out at the Naval Research Laboratory. These computations were specifically requested by Mr. O'Rourke.

Status: TERMINATED.

LONG PATH USABLE FREQUENCY PREDICTIONS
Task 1102-40-5126/52-66
(formerly 1102-53-5126/52-66)

Origin: NBS, Section 14.4 Authorized 4/1/52
Sponsor: " Completed 6/30/53
Manager: M. Stein

Objective: To analyze available radio traffic and field strength data to determine observed maximum usable frequencies over long paths. To develop empirical methods of calculating long path maximum usable frequencies to bring them into agreement with observations.

Background: Experience has shown that the maximum usable frequencies predicated by methods currently in use at the Central Radio Propagation Laboratory are discrepant with and in general lower than those actually observed. It is believed that empirical methods can be devised to improve this situation.

Status: COMPLETED. The results have been transmitted to the sponsor.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-40-5126/52-70
(formerly 1102-53-1106/52-70)

Origin: Bethesda Naval Medical Center Authorized 4/1/52
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1952 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue.

STANDARD LORAN TABLES
Task 1102-40-5126/52-77
(formerly 1102-53-1106/52-77)

Origin: U. S. Navy Hydrographic Office Authorized 5/15/52
Sponsor: " " Completed 6/30/53
Manager: M. Abramowitz

Objective: Preparation of tables giving coordinates of hyperbolic lines of positions.

Background: Standard Loran Navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: COMPLETED. The results were transmitted to the sponsor.

CHEMICAL TRANSITION PROBABILITIES
Task 1102-40-5126/52-82
(formerly 1102-53-1106/52-82)

Origin: NBS, Section 3.2 and Hydrocarbon Research Corp. Authorized 6/1/52
Sponsor: Office of Naval Research
Managers: A. Gleyzal and A. A. Goldstein
Full task description appears in Apr-Jun 1952 issue.

Status: CONTINUED. Code checking of the first portion of the problem has been completed. Code checking of the second portion, using dummy routines for certain spherical Bessel functions, has also been completed.

NEUTRON DIFFUSION, II
Task 1102-40-5126/53-4

Origin: Atomic Energy Commission, New York Office, (NDA) Authorized 9/30/52
Sponsor: " " "
Managers: O. Steiner and N. Levine
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Ninety percent of the original program has been completed, and the results have been transmitted to the sponsor.

STRENGTH OF WING COMPONENTS
Task 1102-40-5126/53-11

Origin: National Advisory Committee for Aeronautics Langley Field, Va. Authorized 12/8/52
Sponsor: " " "
Manager: W. H. Durfee
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED.

RADIANT HEATING OF SOLIDS
Task 1102-40-5126/53-20

Origin: NBS, Section 10.2 Authorized 12/15/52
Sponsor: "
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Solutions are being computed for various values of the parameters and transmitted to the sponsor as completed.

STANDARD LORAN TABLES - EXTENSION OF RATES 1L4, 1L5, 1L6
Task 1102-40-5126/53-26

Origin: Hydrographic Office, U. S. Navy — Authorized 1/23/53
Sponsor: " " — Terminated 6/30/53
Manager: W. H. Durfee

Objective: To prepare tables giving coordinates of hyperbolic lines of position.

Background: Standard Loran navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: TERMINATED. The results were transmitted to the sponsor.

COMPUTATION OF THERMODYNAMIC FUNCTIONS
Task 1102-40-5126/53-27

Origin: NBS, Division 5 — Authorized 3/18/53
Sponsor: "
Manager: E. Marden
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Tables of thermodynamic functions and their differences were computed for additional molecules. Modification of the code to take into account additional equations is in process.

STUDY OF TRICALCIUM ALUMINATE
Task 1102-40-5126/53-28

Origin: NBS, Division 9 — Authorized 3/30/53
Sponsor: "
Manager: R. Anderson
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Task 1102-40-5126/53-29

Origin: NBS, Section 6.4 — Authorized 1/23/53
Sponsor: "
Manager: I. Rhodes
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Computations were performed as requested and transmitted to the sponsor.

SKYWAVE TRAINER DELAY CURVES
Task 1102-40-5126/53-32

Origin: U. S. Navy Hydrographic Office — Authorized 3/30/53
Sponsor: " " — Terminated 6/30/53
Manager: W. H. Durfee

Objective: To prepare tables of skywave corrections for certain existing standard Loran tables giving coordinates of hyperbolic lines of positions.

Background: These tables will be used in the training of military personnel in the use of Loran equipment.

Status: TERMINATED. The results were transmitted to the sponsor.

REDUCTION OF ECLIPSE DATA
Task 1102-40-5126/53-34

Origin: Air Photographing and Charting Services, USAF — Authorized 3/30/53
Sponsor: " "
Manager: A. Goldstein
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. To start the program it was necessary to solve an integral equation of the type of Volterra's integral equation of the first kind. A method was developed and, using the SEAC, results were obtained. In addition, investigations were carried out leading to recommendations for the observations of the next eclipse. An interim report has been issued giving details of the accomplishments to date.

SPECTRAL ANALYSIS OF STATIONARY TIME SERIES
Task 1102-40-5126/53-37

Origin: Statistical Research Center, University of Chicago — Authorized 4/20/53
Sponsor: Office of Naval Research, USN
Manager: I. Stegun

Objective: To compute the serial products

$$c_\nu = \sum_{n=1}^{N-|\nu|} x_n x_{n+\nu} \quad \text{and} \quad \gamma_s = \sum_{n=1}^{N-|s|} x_n^2 x_{n+|s|}^2$$

which are used in estimating the spectral distribution function and its kurtosis, respectively.

Background: The present statistical theory of analysis of stationary time series has assumed complete knowledge of the covariance sequence or, equivalently, of the spectrum of the process. It is, therefore, important to be able to estimate one of these. Knowledge of the spectrum seems to yield greater immediate insight into the structure of the process.

Comments: This problem was proposed by M. Rosenblatt and U. Grenander, SRC, University of Chicago.

Status: NEW. Computations have been completed, and checking is in progress.

ACOUSTICAL IMPEDANCES
Task 1102-40-5126/53-39

Origin: NBS, Section 6.1 Authorized 6/30/53
Sponsor: "
Managers: S. Prusch and K. Nelson

Objective: To solve the following simultaneous equations for Z_s (internal impedance of the source) and Z_X (the internal impedance of the ear),

$$E_1 = \frac{E_B Z_1}{Z_1+Z_s} , \quad E_2 = \frac{E_B Z_2}{Z_2+Z_s} , \quad E_X = \frac{E_B Z_X}{Z_X+Z_s} ,$$

where E_1 and E_2 are the probe output voltages observed for lengths ℓ_1 and ℓ_2 of a tube, respectively, E_x is the complex voltage put out by the microphone, and Z_1 and Z_2 are the corresponding impedances calculated for the tube. Tne impedance of the tube for a length ℓ is given by $z=-i\, S \rho\, c \cot k(\ell\, d)$, where ρ is the density of air, S is the cross-section area of the tube, c is the velocity of sound and k = (2π/wavelength).

Background: The problem arises in the present work of the Sound Laboratory which is directed at obtaining an artificial ear that is more nearly representative of the human ear, at devising measuring equipment that will minimize the effects of human variations, and, ultimately, to set up instrumentation for a more precise determination of the physical aspects of normal hearing.

Status: NEW. Solutions of equations for various parameters have been sent to the Sound Laboratory for further study and investigation.

STANDARD LORAN TABLES - Rates 2H2, 2H3, 2H4
Task 1102-40-5126/53-40

Origin: Hydrographic Office, U. S. Navy Authorized 4/20/53
Sponsor: " "
Managers: W. H. Durfee, D. Jirauch, and K. Nelson

Objective: Toprepare tables giving coordinates of hyperbolic lines of position.

Background: Standard Loran navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: NEW. The computations were performed as requested, and charting coordinates were given to the sponsor. The tables are now being edited.

LORAN UNIVAC CODE
Task 1102-40-5126/53-41

Origin: Hydrographic Office, U. S. Navy — Authorized 4/20/53
Sponsor: " "
Managers: W. H. Durfee, I. Rhodes, and D. Rubin

Objective: To prepare a code for computing on the UNIVAC standard Loran tables with sky wave corrections.

Background: A code for performing these computations on the SEAC already exists. In view of the importance of this work it was deemed advisable to be able to prepare Loran tables, if necessary, on another computer.

Status: NEW. The code is being prepared, and parts of it have been checked in.

NOMOGRAMS FOR RF PERMEAMETER
Task 1102-40-5126/53-42

Origin: NBS, Division 14.8 — Authorized 6/29/53
Sponsor: "
Manager: I. Stegun

Objective: To perform analysis as necessary to prepare nomograms.

Background: The problem arises in connection with the use of the RF permeameter.

Status: NEW. Several nomograms have been set up, and analysis is being performed for other combinations of nomograms.

OPTIMUM SECTIONS FOR DELTA WINGS
Task 1102-40-5126/53-49

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. — Authorized 4/29/53
Sponsor: " "
Manager: O. Steiner

Objective: To perform computations to determine optimum sections for delta wings at supersonic speeds.

Status: NEW. All the codes necessary to run this problem were completed and checked. The pilot computations were performed, and the results were turned over to the sponsor, who will determine what further computations are needed.

WEATHER INFORMATION
Task 1102-40-5126/53-50

Origin: Weather Bureau — Authorized 5/20/53
Sponsor: Office of Naval Research — Completed 6/30/53
Manager: P. J.O'Hara

Objective: To compute a number of sea level pressure indices for each day of the winter seasons from 1920 - 1940. These indices are based on weather data collected from the entire northern hemisphere.

Background: Pressure data obtained from the northern hemisphere daily weather map can be summarized in a way to provide various indices of the general circulation of the atmosphere. These indices can then be used to study the reason for weather changes and should lead to improved forecasts. Original data obtained from the Weather Bureau list daily pressure observations from approximately one thousand stations.

Status: COMPLETED. (NEW). The results obtained have been submitted to the sponsor.

NEUTRON DIFFUSION III
Task 1102-40-5126/53-51

Origin: NBS, Section 4.8 — Authorized 6/9/53
Sponsor: Armed Forces Special Weapons Project
Manager: F. Stockmal

Objective: To solve boundary-value problems in gamma-ray diffusion by means of the Monte Carlo method.

Background: This problem arises in connection with theoretical work on gamma-ray propagation performed by the Nuclear Physics Section of the Radiation Laboratory. The nature of this problem is such that while a purely analytical treatment is extremely difficult, accurate results can be obtained with relative ease by means of random sampling techniques.

Status: NEW. A code has been completed for the first phase of this problem, the computation of energy and angular histories. Results have been verified by hand computation. Phase two will be concerned with the spatial histories corresponding to the results of phase one, with imposition of various boundary conditions.

TRANSPORTATION PROBLEM II
Task 1102-40-5126/53-55

Origin: Logistics Research Project, George Washington University — Authorized 6/9/53
Sponsor: Office of Naval Research
Managers: A. J. Hoffman and L. Gainen

Objective: To solve systems of equations arising in the problem of allocating shipments from bidders to depots minimizing the total cost of the operation. To investigate problems arising from conditions imposed by bidders by time phasing of requirements, etc., and to formulate these conditions for computation.

Background: The technique of G. B. Dantzig in applying the simplex method in finding solutions of the transportation problem of Hitchcock-Koopmans is used in solving standard allocation problems.

Status: NEW.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-40-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2 Authorized 11/29/48
Sponsor: National Advisory Committee for Aeronautics
Manager: F. L. Alt
Full task description appears in Apr-Jan 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

BASIC IONOSPHERIC DATA
Task 1401-34-1473/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.3 Authorized 3/31/49
Sponsor: "
Manager: M. Stein
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The results are being transmitted to the sponsor as completed.

RAY TRACING
Task 0202-10-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 2.2 Authorized 3/1/50
Sponsor: "
Managers: R. K. Anderson and D. Rubin
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. A code for SEAC has been completed and several runs have been made. Additional runs will be made when new data are obtained.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE

Task 1103-10-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS — Authorized 1/9/49
Manager: L. S. Deming
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The card file consisting of abstracts of statistical literature is steadily growing as each issue of Mathematical Reviews appears and as additional pertinent selections from back issues of Zentralblatt are being photostated and prepared for inclusion in it.

MANUAL ON FITTING STRAIGHT LINES

Task 1103-10-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS — Authorized 3/1/50
Manager: F. S. Acton
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

Publication: "Analyzing straight line data," by F. S. Acton; J. Chem. Ed. 30, 128-133 (Mar. 1953).

TABLE TO FACILITATE DRAWING RANDOM SAMPLES

Task 1103-10-1107/51-1

Origin: NBS — Authorized 7/1/50
Managers: C. Eisenhart and L. S. Deming
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

Status of Projects

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: E. Lukacs
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. (1) E. Lukacs and E. P. King continued their studies of the independence of linear statistics. They proved the following theorem:

Let $X_1, X_2, \ldots, X_n$ be n independently but not necessarily identically distributed variables and assume that the nth moment of each $X_s (s=1,2,\ldots,n)$ exists. The necessary and sufficient conditions for the existence of two statistically independent linear forms

$$Y_1 = \sum_{s=1}^{n} a_s X_s \quad \text{and} \quad Y_2 = \sum_{s=1}^{n} b_s X_s$$

are: (A) Each random variable which has a nonzero coefficient in both forms is normally distributed; and

$$\text{(B)} \qquad \sum_{s=1}^{n} a_s b_s \sigma_s^2 = 0.$$

Here σ_s^2 is the variance of $X_s (s=1,\ldots,n)$. For n=2 this reduces to a theorem of Serge Bernstein [see "Généralisations de la loi de probabilité de Laplace," by M. Fréchet; Annales de l'Institut Henri Poincaré, XII (1951)]. (2) E. P. King continued his study of grouping control chart data. Monte Carlo experiments were initiated to check some of the results obtained by an approximate analytic method. (3) M. Zelen considered various extensions of the Tchebycheff inequalities which are based on second order moments. Bounds have been obtained for a univariate distribution function when a point of the distribution function is known, and for a bivariate distribution.

Publications: (1) "Contribution to the theory of Markov chains," by Kai Lai Chung; J. Res. NBS 50, 203-208 (Apr. 1953). (2) "On the exact evaluation of the variances and covariances of order statistics in samples from the extreme-value distribution," by J. Lieblein; Ann. Math. Stat. 24, 282-287 (June 1953). (3) "On some procedures for the rejection of suspected data," by E. P. King; accepted by the Journal of the American Statistical Association. (4) "A property of the normal distribution related to a theorem of S. Bernstein," by E. P. King and E. Lukacs; submitted to a technical journal. (5) "Certain Fourier transforms of distribution, II," by E. Lukacs and O. Szász; submitted to a technical journal. (6) "Nonnegative trigonometric polynomials and certain rational characteristic functions," by E. Lukacs and O. Szász; accepted by the Journal of Research of the NBS. (7) "On absolute measurement," by N. E. Dorsey and C. Eisenhart; accepted by the Scientific Monthly. (8) "A historical note on the application of the 'weakest link' idea to tensile strengths," by J. Lieblein; submitted to a technical journal.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS Authorized 6/23/51
Managers: C. Eisenhart and I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED.

Publication: "Bibliography of non-parametric statistics and related topics," by I. R. Savage; IN MANUSCRIPT.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. (1) W. S. Connor and W. M. Clatworthy continued the development of necessary conditions for the existence of partially balanced incomplete block (P.B. I. B.) designs, a study first referred to in the Oct-Dec 1952 issue, p. 60. The conditions arise from consideration of $|NN'|$, where N is the incidence matrix of a P.B.I.B. design. This determinant has been evaluated for two, three, and four associate classes, and detailed necessary conditions have been obtained for two associate classes. Bounds on b, the number of blocks in the design, have been obtained for all P.B.I.B. designs. The manuscript, "Necessary conditions for the existence of partially balanced incomplete block designs with two associate classes," by W. S. Connor and W. H. Clatworthy, which was submitted to a technical journal (see Jan-Mar 1953 issue, p. 76), has been withdrawn in order to incorporate the new results. (2) M. Zelen has prepared a manuscript (see publication (3) below) that describes the analysis which is appropriate when a block is lost from a design or from certain P.B.I.B. designs.

Publications: (1) "An embedding theorem for balanced incomplete block designs," by W. S. Connor and M. Hall, Jr.; accepted by the Canadian Journal of Mathematics. (2) "The chain block design," by W. J. Youden and W. S. Connor; accepted by Biometrics. (3) "Analysis for some incomplete block designs having a missing block," by M. Zelen; IN MANUSCRIPT.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1
(formerly 3011-60-0002/51-1)

Origin: NBS Authorized 7/1/50
Manager: W. J. Youden
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this task fell into two main categories:

A. Design of Experiments: For example a 1/9 fractional replication design was constructed using the general theory of a 2-way elimination for a $3x3x^{4}x27$ factorial design to be used in tests of metal insulator laminates.

B. Development or Selection of the Appropriate Method for Analysis and Interpretation of Data: For example: (a) A method of sampling of cement was developed in which the number of samples tested for each property measured depends on the closeness of the values to the specification limit. (b) Welch-Pitman randomization procedures were used for the statistical evaluation of results from studies on battery additives.

Publications: (1) "Control charts may be all right - but...," by F. Proschan; Industrial Quality Control IX, 56-58 (May 1953). (2) "Acceptance sampling of electroplated articles," by J. M. Cameron and F. Ogburn; submitted to a technical journal. (3) "A test for statistical control applicable to a short series of observations," by C. Eisenhart and E. P. King; submitted to a technical journal. (4) "Estimating the standard deviation of a normal distribution," by E. P. King; accepted by Industrial Quality Control. (5) "Confidence and tolerance intervals for the normal distribution," by F. Proschan; submitted to a technical journal. (6) "Rejection of outlying observations," by F. Proschan; accepted by the American Journal of Physics. (7) "The principles of experimental design," by W. J. Youden; accepted for publication in Selection, Training and Use of Personnel in Industrial Research, Proceedings of the Third Annual Conference on Industrial Research. (8) "Statistical units of measurement," by W. J. Youden; accepted by Metals Progress. (9) "Making one measurement do the work of two," by W. J. Youden and W. S. Connor; accepted by Chemical Engineering Progress. (10) "Performance of inspectors and gasoline pumps," by W. J. Youden and M. Jensen; submitted to a technical journal.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1
(formerly 3011-60-0002/52-1)

Origin: NBS Authorized 10/1/51
Manager: I. R. Savage
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES
Task 1103-20-5119/52-1

Origin: U. S. Naval Ordnance Test Station, Inyokern Authorized 1/1/52
Sponsor: " " "
Manager: E. Lukacs
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. It was decided to construct an "artificial trajectory" to be used to demonstrate by means of an example how the parameters of a Wiener process could be estimated from a single sample curve. The polynomial of degree four

$$f(t)=3400 + 310t - 2.7t^2 + 43(10^{-3})t^3 - 2.6(10^{-4})t^4$$

was chosen to represent the mean value curve of the fictitious process. The values of f(n) were computed for n=0(1)100. These values would correspond to the "true positions" at the 101 equidistant time points at which observations are made. In order to obtain the simulated observations one has to add random numbers as "errors" to the values f(n). These random numbers were obtained from H. Wold's tables entitled Random Normal Deviates (Cambridge University Press, 1948). These tables contain random numbers representing a normal population of zero mean and unit variance. They are arranged in columns of 50 and the sums $\Sigma(x)$ of the columns are also given. In view of the arrangement of Wold's tables it was decided to choose c=50/3 as the "true value" of the variance constant. Random numbers from a normal population with zero mean and variance c=50/3 were obtained by dividing the values $\Sigma(x)$ in Wold's table by $\sqrt{3}$. A set of 100 random numbers w_i (i=1,...,100) was derived in this manner. The "simulated observations" were finally obtained as

$$g(n) = f(n) + \sum_{i=0}^{n} w_i .$$

The parameters of the Wiener process with mean value curves were extended on the basis of the observations. Comparisons of estimated mean value curves of various degrees were made as well as comparisons with least square estimates. This work is reported in publication (4) below.

Publications: (1) "On strongly continuous stochastic processes," by E. Lukacs; submitted to a technical journal. (2) "Tables useful in estimating the mean value function of a fundamental random process," by E. Lukacs and I. R. Savage; IN MANUSCRIPT. (3) "Tables of inverses of

finite segments of the Hilbert Matrix," by I. R. Savage and E. Lukacs; to be included in Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues, NBS Applied Mathematics Series.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-50-5105/52-1

Origin: Ship Structure Committee, NRC — Authorized 12/1/51
Sponsor: " "
Manager: W. J. Youden
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The possibility of reducing failures of ship steels by a reduction in the specified transition temperature was investigated. It was necessary to construct the distribution of transition temperatures from the available plates which all came from ships with fractures.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-50-5118/52-1

Origin: Biological Laboratories, Chemical Corps, Dept. of the Army — Authorized 10/1/51
Sponsor: " " "
Manager: C. Eisenhart
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Final reports on two special problems were being prepared for the sponsor.

IV. Machine Development Laboratory

(Section 11.4)

in cooperation with

Electronic Computer Section

(Section 12.3)

1. Development: Design and Construction of Automatic Digital Computing Machines

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The installation of the No. 1 UNIVAC System at its permanent site in Federal Office Building No. 3, Suitland, Maryland, was completed during this period, and the machine is again in useful service. Maintenance is being provided primarily by engineers from the ERA Division of Remington Rand provided under extension of the installation contract between Remington Rand, Inc., and the Bureau of the Census. NBS personnel assisted in debugging the system following the move and were also made available to assist with two special problems that arose and have been solved during this period. One problem involved air flow through the cooling system, and the other was a problem of vibration of a heavy blower.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The second UNIVAC System has continued to be operated and maintained by personnel of the Office of the Air Comptroller on around-the-clock seven-days-a-week schedule. The NBS has as well as the individual owner-agencies concerned continued its cooperative endeavor to procure parts for the first three UNIVAC Systems. The stock piling of replacement parts continues to be a slow process due to other production priorities.

Status of Projects

WRIGHT DEVELOPMENT CENTER COMPUTING MACHINE
(formerly AIR MATERIEL COMMAND COMPUTING MACHINE)
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF — Authorized 3/21/49
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The OARAC (Office of Air Research Automatic Computer) was moved to its permanent site at Wright Air Development Center, Wright-Patterson Air Force Base and is now in operation by their Aeronautical Research Laboratory. Only a few small items such as manuals and diagrams remain to be completed prior to final termination of the contract between the National Bureau of Standards and the General Electric Company.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Origin: NBS — Authorized 12/15/48
Sponsor: Office of the Air Comptroller, USAF
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During the quarter from March through June 1953 approximately 90 hours a week have been used for computation. Operating efficiency for the period, i.e., total productive time versus total time scheduled, was 81 percent. Installation of the regulated d-c power supplies and a-c voltage stabilizers has now been completed. Engineering work on and modification of the experimental Williams type memory are nearing completion, and an extended problem which utilizes both the acoustic and electrostatic memories of SEAC has been prepared and scheduled. Mechanical tests of the cavity-type wire drive unit are in progress before its installation in SEAC for evaluation as part of the input-output system, and circuitry for adapting it to the SEAC has been assembled.

Additional technical memoranda issued are as follows:

Technical Memorandum
- No. 42: Four subroutines S30, S31, S32, and S33 for the step-by-step integration of the first order differential equation $y'=f(x,y)$.
- No. 43: Subroutine for 16-point Gaussian quadrature.
- No. 47: Subroutine for conversion of decimal degrees to binary radians and vice-versa.
- No. 50: Automonitor processing routine (designated as BOIE 0 BOIE).
- No. 51: Matrix inversion routine: von Neumann-Goldstine method (modified).
- No. 52: Subroutine for Log N; single precision, fixed binary point.
- No. 53: Matrix inversion routine: Lanczos-Hestenes method (for symmetric matrices).
- No. 54: Matrix multiplication routine.
- No. 55: Subroutine for arcsin x and arccos x, single precision; fixed binary point.

These memoranda are collected and issued as NBS reports under the title "SEAC operating and programming notes." Two additional reports in this series have been issued which contain Technical Memoranda Nos. 32-36 and 37-41.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, USA — **Authorized** 12/15/48
Sponsor: " "
Full task description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This period saw the completion of the cooperative effort on the part of NBS to procure replacement parts for the third UNIVAC System which has been in regular operation at its Army Map Service site. Maintenance has been provided under contractual agreement with the ERA Division of Remington Rand, Inc. Because operating efficiency for the quarter has been very low, an experimental program of large-scale tube replacement has been initiated by the Army Map Service technician.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS OF THE SOCIAL SECURITY AGENCY
Task 1104-53-5108/51-1

Origin: Social Security Agency — **Authorized** 12/31/50
Sponsor: "
Full task description appears in Oct-Dec 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Numerical Analysis Colloquium Series
(Los Angeles, California)

MINORSKY, N. (Stanford University) On the stroboscopic method. April 13.

DE VOGELAERE, R. (Université Laval, Québec, Canada) On nonlinear differential equations connected with conservative dynamical problems of two degrees of freedom. May 25.

BOCHNER, S. (Princeton University and University of California) Structure of infinitely subdivisible stochastic processes. June 22.

Seminar on Numerical and Computational Aspects of Linear Problems: Games, Linear Equalities, Linear Inequalities, Programming, ...
(Los Angeles, California)

TOMPKINS, C.B. A description of a two-person zero-sum game resembling a military battle with probabilistic attrition and a preliminary discussion of computational problems involved in its solution. April 20, 22, 24.

HORGAN, R. B. SWAC coding of a search for a solution to a large set (128) of simple linear inequalities in many (80) variables. May 5.

MOTZKIN, T. S. Elements of a theory of sequential projection. May 7 and 13.

HANDY, B. (1) Coding the assignment problem. May 11. (2) Frequencies of vibrating membranes, II. May 20.

FORSYTHE, G. E. (1) Frequencies of vibrating membranes--comparison between lowest eigenvalues of a differential equation and approximating difference equations, I. May 18. (2) Matrix eigenvalues--Chebyshev polynomials or Liebmann methods. June 1.

WEBER, M. SWAC experience on eigenvalues of a matrix of 45 rows and columns, I. May 25.

YOWELL, E. C. SWAC experience on eigenvalues of a matrix of 45 rows and columns, II. May 27.

HESTENES, M. R. Matrix inversion by the conjugate gradient method, I. Background and development of a program. June 3.

WILSON, L. Matrix inversion by the conjugate gradient method, II. Coding for SWAC and experience with computations. June 8.

TEICHROEW, D. A game theory approach to the search for a global maximum of a function. June 15.

BARANKIN, E. W. Stochastic research for the global maximum of a function. June 17.

Game Theory and Its Applications: A Series of Weekly Lectures Sponsored by The American University in Cooperation with the National Bureau of Standards

HOFFMAN, A. J. (National Bureau of Standards) Computational methods for solving games. April 7.

BROWN, G. W. (International Telemeter Corporation) Modelling of applications. April 14.

HAYWOOD, Colonel O. G. (Air Research and Development Command, USAF) The applications of game theory to the military doctrine of decision. April 21.

Statistical Engineering Seminar

SAVAGE, I. R. Nonparametric methods in the analysis of variance. April 17.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

Papers presented at the meeting of the American Mathematical Society, New York, N. Y. April 23-25:

ABRAMOWITZ, M. (1) On regular and irregular Coulomb wave functions in terms of Bessel-Clifford functions. (2) On the differential equation in a problem of heat convection of laminar flow through a tube. Both presented by title.

ANTOSIEWICZ, H. A. A boundedness theorem for a nonlinear differential equation.

LUKACS, E. On strongly continuous stochastic processes.

TODD, J. The condition of the finite segments of the Hilbert matrix. Presented by title.

Papers presented at a joint meeting of the Institute of Mathematical Statistics and the Biometric Society, Eastern North American Region, Washington, D. C., April 29 - May 1:

CAMERON, J. M. Control and measurement of experimental error.

CLATWORTHY, W. H. Necessary conditions for the existence of partially balanced incomplete block designs with two associate classes.

CONNOR, W. S. The correspondence between two classes of balanced incomplete block designs.

KING, E. P. A property of the normal distribution related to a theorem of S. Bernstein.

ZELEN, M. An analysis of some incomplete block designs with a missing block.

Talks presented to the Applied Mathematics Division, Naval Ordnance Laboratory, White Oak, Md.

DAVIS, P. Linear functional equations and interpolation series. June 4.

HOFFMAN, A. J. Bounds for the rank and location of eigenvalues of a matrix. May 14.

WEGSTEIN, J. H. Automatic coding techniques. April 30.

Papers presented elsewhere:

DAVIS, P. Linear functional equations and interpolation series. Presented at the Ballistics Research Laboratories, Aberdeen, Md., May 7.

EISENHART, C. Why mathematical statistics. Presented to the staff of the Logistics Research Project, George Washington University, Washington, D. C., June 17.

FORSYTHE, G. E. Arbitrarily close bounds for the fundamental frequency of certain vibrating membranes. Presented at a meeting of the American Mathematical Society, Palo Alto, Cal., May 2.

HOFFMAN, A. J. Some problems in the foundations of geometry. Presented to the Columbian Mathematics Club, George Washington University, Washington, D. C., June 1.

LEHMER, D. H. Number theoretic studies with a high speed computer. Presented at a Peripatetic Seminar, California Institute of Technology, held on the UCLA campus, Los Angeles, Calif., May 4.

LIEBLEIN, J. New statistical methods for the study of wind data. Presented at a Conference of Utility Commission Engineers, St. Paul, Minn., May 28.

LUKACS, E. On some stochastic processes. Presented at the Admiralty Research Laboratory, Teddington, England, June 25.

MOTZKIN, T. S. and C. B. TOMPKINS. Boundedness of sequential projections. Presented at a meeting of the American Mathematical Society, Palo Alto, Cal., May 2.

TEICHROEW, D. Distribution sampling with high-speed computers. Presented at a Seminar of the Institute of Statistics, North Carolina State College, Raleigh, N. C., May 8.

THORENSEN, R. Electrostatic storage systems. Presented at a meeting of the IRE Professional Group on Electronic Computers, Detroit, Mich., April 24.

TODD, J. (1) The condition of the finite segments of the Hilbert matrix. Presented at a meeting of the Mathematical Association of America, Dahlgren, Va., May 2. (2) Linear equations and computing machines. Presented at a National Science Foundation Conference in Collegiate Mathematics, University of Colorado, Boulder, Colo., June 22.

WASOW, W. Singular perturbation methods. Presented at a Symposium on Nonlinear Circuit Analysis, Microwave Research Institute, Polytechnic Institute of Brooklyn, N. Y., April 23, 24.

YOUDEN, W. J. (1) Design of experiments. Presented at Johns Hopkins University, Baltimore, Md., April 1. Also presented to the technical staff of Rath and Strong, Inc., Pittsfield, Mass., April 22. (2) New experimental designs for research and development. Presented to the Hampton Roads Section of the American Chemical Society, Norfolk, Va., May 23.

WIELANDT, H. (1) Inclusion of eigenvalues. Presented to the Department of Mathematics, University of Pennsylvania, Philadelphia, Pa., April 16. (2) Proofs of impossibility. Presented to the Mathematics Club, Wilson College, Chambersburg, Pa., April 17. (3) On eigenvalues of matrices. Presented to the Department of Mathematics, University of North Carolina, Chapel Hill, N. C., May 6. (4) On eigenvalues of matrices arising in the numerical solution of integral equations. Presented at the NBS, June 12.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables for rocket and comet orbits. S. Herrick, NBS Applied Mathematics Series 20. Available from U. S. Government Printing Office, Washington 25, D. C., $1.75.

(2) Tables of Arctan x. NBS Applied Mathematics Series 26. (Supersedes NBS Mathematical Table MT16). Available from U. S. Government Printing Office, Washington 25, D. C., $1.75.

(3) A table of Wilson quotients and the third Wilson prime. K. Goldberg. J. London Math. Soc., 28, Pt. 2, 252-256 (Apr. 1953). Reprints available.

1.2 Manuals, bibliographies, indices

(1) Addendum to a guide to tables on punched cards. G. Blanch and E. C. Yowell. MTAC VII, 1-6(Jan. 1953). Reprints available.

1.3 Technical Papers

(1) Analyzing straight line data. F. S. Acton. J. Chem. Ed. 30, 128-133 (Mar. 1953). Reprints available.

(2) On a recursion formula and on some Tauberian theorems. N. G. de Bruijn and P. Erdös. J. Res. NBS 50, 161-164. (Mar. 1953).

(3) Contribution to the theory of Markov chains. Kai Lai Chung. J. Res. NBS 50, 203-208 (Apr. 1953).

(4) Elements of a mathematical theory of probability. J. H. Curtiss. Math. Mag. 26, 233-254 (May-June 1953).

(5) Sequential decision problems for processes with continuous time parameter. Testing hypotheses. A. Dvoretsky, J. Kiefer, and J. Wolfowitz. Ann. Math. Stat. 24, No. 2, 254-264 (1953). Reprints available.

(6) A Neumann series for the product of two Whittaker functions. P. Henrici. Proc. Am. Math. Soc. 4, 331-334 (Apr. 1953). Reprints available.

(7) On the exact evaluation of the variances and covariances of order statistics in samples from the extreme-value distribution. J. Lieblein. Ann. Math. Stat. 24, 282-287 (June 1953). Reprints available.

(8) The torsion of anisotropic elastic cylinders by forces applied on the lateral surface. H. Luxenberg. J. Res. NBS 50, 263-276 (May 1953).

(9) The double description method. T. S. Motzkin, H. Raiffa, G. L. Thompson, R. M. Thrall. Contributions to the theory of games II, edited by H. W. Kuhn and A. W. Tucker, Annals of Mathematics Study 28, chapter 3 (Princeton, 1953). No reprints.

(10) On the derivative of a polynomial and Chebyshev approximation. T. S. Motzkin and J. L. Walsh. Proc. Am. Math. Soc. 4, 76-87 (Feb. 1953). Reprints available.

(11) Two explicit formulae for the distribution function of the sums of n uniformly distributed variables. A. M. Ostrowski. Archiv Math. III, 3-11 (1952). Reprints available.

(12) On smoothing operations and their generating functions. I. J. Schoenberg. Bul. Am. Math. Soc. 59, 199-230 (1953). Reprints available.

(13) On Polya frequency functions III: The positivity of translation determinants with an application to the interpolation problem by spline curves. I. J. Schoenberg and A. Whitney. Trans. Am. Math. Soc. 74, 246-259 (Mar. 1953). Reprints available.

(14) On methods for obtaining solutions of fixed end-point problems in the calculus of variations. M. L. Stein. J. Res. NBS 50, 277-297 (May 1953).

(15) On the Gibbs phenomenon for a class of linear transforms. O. Szász. Acad. Serbe Sci. Publ. Inst. Math. 4, 135-144 (1952). Reprints available.

(16) Systems of equations, matrices and determinants. O. Taussky and J. Todd. Math. Mag. 26, 9-20 (Sept-Oct 1952); 71-88 (Nov-Dec. 1952). Reprints available.

(17) The tabulation of an integral arising in the theory of cooperative phenomena. M. Tikson. J. Res. NBS 50, 177-178 (Mar. 1953).

(18) Metodi probabilistici per la soluzione numerica di alcuni problemi di analisi. W. Wasow. Rend. Mat. App. {V} , XI, 336-346 (Roma 1952); also issued separately as Pubblicazioni dell'Istituto per le Applicazioni del Calcolo N. 354 (Roma 1953).

1.5 Miscellaneous Publications

(1) Scientific teamwork in a Computation Laboratory. J. H. Curtiss. Teamwork in research, edited by G. P. Bush and L. H. Hattery, chap. 17 (American University Press, 1953). No reprints.

(2) Control charts may be all right, but--. F. Proschan. Industrial Quality Control IX, 56-58 (May 1953).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1953.

2.1 Mathematical Tables

(1) Probability tables for analysis of extreme-value data. NBS Applied Mathematics Series 22. In press, Government Printing Office.

(2) Tables of normal probability functions. NBS Applied Mathematics Series 23. (Supersedes NBS Mathematical Table MT14, Tables of

probability functions, Volume II.) In press, Government Printing Office.

(3) Tables of 10^x. NBS Applied Mathematics Series 27. In press, Government Printing Office.

(4) Tables of coefficients for the numerical calculation of Laplace transforms. H. E. Salzer. NBS Applied Mathematics Series 30. In press, Government Printing Office.

(5) Table of natural logarithms for decimal numbers from 0.0001 to 5.0000. (A reissue of NBS Mathematical Table MT10, Table of natural logarithms, vol. III.) NBS Applied Mathematics Series 31. In press, Government Printing Office.

(6) Table of sine and cosine integrals for arguments from 10 to 100. (A reissue of NBS Mathematical Table MT13.) NBS Applied Mathematics Series 32. In press, Government Printing Office.

(7) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. In press, Government Printing Office.

(8) Tables of Lagrangian coefficients for sexagesimal interpolation. NBS Applied Mathematics Series 35. In press, Government Printing Office.

(9) Tables of circular and hyperbolic sines and cosines for radian arguments. (A reissue of NBS Mathematical Table MT3.) NBS Applied Mathematics Series 36. In press, Government Printing Office.

(10) Tables of functions and of zeros of functions. Volume I of Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(11) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(12) Tables of expected values of 1/X for positive Bernoulli and Poisson variables. I. R. Savage and E. Grab. Submitted to a technical journal.

2.2 Manuals, Bibliographies, Indices

(1) The statistical theory of extreme values and some practical applications. A series of lectures by E. J. Gumbel. To appear in the NBS Applied Mathematics Series.

2.3 Technical Papers

(1) Evaluation of the integral $\int_0^\infty e^{-u^2-(x/u)}du$. M. Abramowitz.

Accepted for publication in the Journal of Mathematics and Physics.

(2) On the solution of the differential equation occurring in the problem of heat convection in laminar flow through a tube. M. Abramowitz. Submitted to a technical journal.

(3) Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions. M. Abramowitz. Accepted for publication in Journal of Mathematics and Physics.

(4) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(5) Note on the simulation of autoregressive series. F. L. Alt. Submitted to a technical journal.

(6) The relaxation method for linear inequalities. S. Agmon. Submitted to a technical journal.

(7) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(8) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Submitted to a technical journal.

(9) The convergence of numerical iteration. H. A. Antosiewicz and J. M. Hammersley. Submitted to a technical journal.

(10) On mildly nonlinear partial difference equations of elliptic type. L. Bers. Accepted for publication in the Journal of Research of the NBS.

(11) On the numerical solution of parabolic partial differential equations. G. Blanch. Accepted for publication in the Journal of Research of the NBS.

(12) Boolean geometry I. L. M. Blumenthal. To appear in Rendiconti del Circolo Matematico di Palermo.

(13) A general-purpose control panel for a model II CPC. P. B. Bremer, D. Teichroew,and E. C. Yowell. To be published in the IBM Newsletter.

(14) Programs for computing the hypergeometric series. W. F. Cahill. Submitted to a technical journal.

(15) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn. Submitted to a technical journal.

(16) The embedding theorem for balanced incomplete block designs. W. S. Connor and M. Hall, Jr. Accepted for publication in the Canadian Journal of Mathematics.

(17) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Submitted to a technical journal.

(18) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack. Submitted to a technical journal.

(19) Linear functional equations and interpolation series. P. Davis. Submitted to a technical journal.

(20) On representations and extensions of bounded linear functionals defined on classes of analytic functions. P. Davis and J. L. Walsh. Submitted to a technical journal.

(21) On absolute measurement. N. E. Dorsey and C. Eisenhart. Accepted for publication in Scientific Monthly.

(22) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(23) A test for statistical control applicable to a short series of observations. C. Eisenhart and E. P. King. Submitted to a technical journal.

(24) Changes of sign of sums of random variables. P. Erdös and G. A. Hunt. Submitted to a technical journal.

(25) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(26) A numerical analyst's fifteen-foot shelf. G. E. Forsythe. Submitted to a technical journal.

(27) Asymptotic lower bounds for the frequencies of polygonal membranes. G. E. Forsythe. Submitted to a technical journal.

(28) Solving linear algebraic equations can be interesting. G. E. Forsythe. Accepted for publication in Bulletin of the American Mathematical Society.

(29) Tentative classification of methods and bibliography on solving systems of linear equations. G. E. Forsythe. To appear in Simultaneous linear equations and the determination of eigenvalues. Proceedings of an NBS Symposium held in Los Angeles, August 1951. NBS Applied Mathematics Series 29. In press, Government Printing Office.

(30) Punched-card experiments with accelerated gradient methods for linear equations. A. I. and G. E. Forsythe. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series.

(31) Additive functionals of a Markoff process. R. Fortet. Submitted to a technical journal.

(32) Practical solution of linear equations and inversion of matrices. L. Fox. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series.

(33) A numerical solution of Schroedinger's equation in the continuum. W. Futterman, E. Osborne, and D. S. Saxon. Accepted for publication in the Journal of Research of the NBS.

(34) A nonlinear model for the composite pi-meson. S. G. Gasiorowicz. Submitted to a technical journal.

(35) A 2-basic set of density zero. K. Goldberg. Submitted to a technical journal.

(36) An expansion method for parabolic partial differential equations. J. W. Green. Accepted for publication in the Journal of Research of the NBS.

(37) A non-harmonic Fourier series. J. M. Hammersley. Accepted for publication in Acta Matematica.

(38) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(39) On a combinatorial theorem. A. J. Hoffman. Submitted to a technical journal.

(40) Computational experience in solving linear programs. A. J. Hoffman, M. Mannos, D. Sokolowsky, and N. Weigmann. Accepted for publication in the Journal of the Society for Industrial and Applied Mathematics.

(41) The SWAC--design features and operating experience. H. Huskey, R. Thorensen, B. F. Ambrosio, and E. C. Yowell. Submitted to a technical journal.

(42) Completely continuous normal operators with property L. I. Kaplansky. Submitted to a technical journal.

(43) Estimating the standard deviation of a normal population. E. P. King. Accepted for publication in Industrial Quality Control.

(44) On some procedures for the rejection of suspected data. E. P. King. Accepted for publication in the Journal of the American Statistical Association.

(45) Probability limits for the average chart when process standards are unspecified. E. P. King. Submitted to a technical journal.

(46) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(47) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society.

(48) On strongly continuous stochastic processes. E. Lukacs. Submitted to a technical journal.

(49) A property of the normal distribution related to a theorem of S. Bernstein. E. Lukacs and E. P. King. Submitted to a technical journal.

(50) Certain Fourier transforms of distribution (II). E. Lukacs and O. Szász. Submitted to a technical journal.

(51) Nonnegative trigonometric polynomials and certain rational characteristic functions. E. Lukacs and O. Szász. Accepted for publication in the Journal of Research of the NBS.

(52) Eigenvectors of matric polynomials. M. Mannos. Accepted for publication in the Journal of Research of the NBS.

(53) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Submitted to a technical journal.

(54) On Fejér sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(55) Pairs of matrices with property L, II, (summary) by T. S. Motzkin and O. Taussky. Accepted for publication in the Proceedings of the National Academy of Science.

(56) The arithmetic structure of certain modular subgroups. M. Newman. Submitted to a technical journal.

(57) The coefficients of certain infinite products. M. Newman. Accepted for publication in Proceedings of the American Mathematical Society.

(58) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(59) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(60) On nearly triangular matrices. A. Ostrowski. Submitted to a technical journal.

(61) On over and under relaxation in the theory of the cyclic single step iteration. A. Ostrowski. Submitted to a technical journal.

(62) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(63) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(64) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Submitted to a technical journal.

(65) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(66) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(67) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(68) Confidence and tolerance intervals for the normal distribution. F. Proschan. Submitted to a technical journal.

(69) Rejection of outlying observations. F. Proschan. Accepted for publication in the American Journal of Physics.

(70) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(71) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(72) Modes of vibration of a suspended chain. D. S. Saxon and A.S. Cahn. Accepted for publication in the Quarterly Journal of Mechanics and Applied Mathematics.

(73) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Submitted to a technical journal.

(74) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(75) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6, 1953.

(76) The condition of the finite segments of the Hilbert matrix. J. Todd. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(77) On the accuracy of the numerical solution of the Dirichlet problem by finite differences. J. L. Walsh and D. Young. Accepted for publication in the Journal of Research of the NBS.

(78) On singular perturbation problems in the theory of nonlinear vibrations. W. Wasow. To be published in the Proceedings of the Symposium on Nonlinear Vibrations, held at Isle de Porquerolles, France, September 18-22, 1951.

(79) Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point. W. Wasow. Accepted for publication in the Annals of Mathematics.

(80) On small disturbances of plane Couette flow. W. Wasow. Accepted for publication in the NBS Journal of Research.

(81) Singular perturbation methods for nonlinear oscillations. W. Wasow. To appear in the Proceedings of a Symposium on Nonlinear Circuit Analysis, held by the Polytechnic Institute of Brooklyn, N. Y.

(82) On the eigenvalues of A+B and AB. H. Wielandt. Submitted to a technical journal.

(83) Pairs of normal matrices with property L. H. Wielandt. Accepted for publication in the Journal of Research of the NBS.

(84) Statistical units of measurement. W. J. Youden. Accepted for publication in Metals Progress.

(85) The principles of experimental design. W. J. Youden. Accepted for publication in Selection, Training, and Use of Personnel in Industrial Research. Proceedings of the Third Annual Conference on Industrial Research.

(86) The chain block design. W. J. Youden and W. S. Connor. Accepted for publication in Biometrics.

(87) Making one measurement do the work of two. W. J. Youden and W. S. Connor. Accepted for publication in Chemical Engineering Progress.

(88) Performance of inspectors and gasoline pumps. W. J. Youden and M. W. Jensen. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Simultaneous linear equations and the determination of eigenvalues. Proceedings of an NBS Symposium held in Los Angeles, August 1951. NBS Applied Mathematics Series 29. In press, Government Printing Office.

(2) Mathematical services useful in industry. A. S. Cahn, Jr. To appear in the Proceedings of a Symposium on Industrial Applications of Automatic Computing Equipment held by the Midwest Research Institute, Kansas City, Mo., January 8, 1953.

(3) A historical note on the application of the "weakest-link" idea to tensile strengths. J. Lieblein. Submitted to a technical journal.

(4) Contributions to the solution of systems of linear equations and the determination of eigenvalues. To be issued in the NBS Applied Mathematics Series.

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