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Technical Note 100

REQUIRED SIGNAL-TO-NOISE RATIOS, RF SIGNAL POWER, AND BANDWIDTH FOR MULTICHANNEL RADIO COMMUNICATIONS SYSTEMS

E. F. FLORMAN AND J. J. TARY

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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CONTENTS

Page No.

Page No.

APPENDICES

ABSTRACT

A method is outlined for determining the relationships between the grade of performance, or message error rate, of a radio communication system and the system parameters. Results are presented in the form of design equations and system-design curves. The system parameters and variables considered are the following definable and measurable factors: signal-to-noise ratios, carrier-signal power level, message load, receiver noise figure, etc. These factors are used with the design curve scales as normalizing factors in order to yield quantitative results.

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by

E. F. Florman and J. J. Tary

1. INTRODUCTION

The objective of this work is to obtain the basic relationships between the grade of service, or the message error-rate, and the various system parameters, for radio communication systems. The results are presented in the form of design equations and sets of system-design curves, in which the system parameters are used as variables and as scale-normalizing factors. The design curves are basic and are general in form; however, they may easily be converted to families of curves which apply directly to specific cases, involving specified system parameters.

This paper outlines a method for obtaining design equations and sets of design curves for radio communication systems, based primarily on the combined performance characteristics of the radio receiver, the diversity combiner, and the message-signal decoder unit. Results are obtained in terms of the important specified, definable, and measurable system parameters, such as message error rates, modulating-signal power levels, modulating baseband signal bandwidth, signal-to-noise ratio at the radio receiver output, radio receiver noise figure, order of diversity, number of available message channels, etc. Through the application of these results it is possible to determine directly the optimum values for the important factors, such as the required radio-frequency signal power and the optimum bandwidth.

It is assumed that the grade of service for the system is directly associated with the radio receiver output signal-to-noise ratio; therefore, this ratio is used as a primary factor in determining the system performance.

In this work the radio receiver output message-channel signal-to-noise ratio, S_{oc}/N_{oc}, is determined in terms of:

(1) the radio-receiver input total RF-signal power and the total noise power levels,

(2) the pre-detection signal-to-noise ratio,

(3) the receiver noise figure,

(4) the relative power levels of the modulating signals at the radio transmitter,

(5) the power-spectrum bandwidth of the composite modulating signal,

(6) the power-spectrum bandwidth of the modulated radio-frequency signal,

(7) the modulation index, and

(8) position of the signal in the receiver-output baseband signal spectrum.

The effect of other factors on the receiver output signal-to-noise ratio may also be incorporated; these additional factors include:

(9) effects of bandwidth compression in the radio receiver IF, or at the pre-detection point,

(10) radio frequency path bandwidth capability, or effects of frequency selective fading on the radio-frequency signal, and

(11) intermodulation noise generated within the equipment.

The statistical characteristics of the signal-to-noise ratio at the output of the radio receiver depend upon the time-varying characteristics of (a) the received radio-frequency signal and (b) the total noise power, at the receiver input. The combined radio receiver output signal-to-noise characteristics also depend upon the order of diversity, type of diversity, type of combining, and the point within the receiver at which combining takes place, such as pre-detection and/or post-detection. For example, the distribution of the receiver-output signal-to-noise ratio, for a time-varying received RF signal, may be determined by combining the steady-signal characteristic of the radio receiver, with the combiner characteristics, for a particular comulative distribution of the time-varying received radio-frequency signal. This procedure is used here to determine the cumulative distribution of the combined receiver-output signal-to-noise ratio, for various orders of diversity; assuming that the received radio-frequency carrier signal is Rayleigh distributed. Other types of received RF signals may also be considered, provided that the statistical characteristics of these signals are known.

The cumulative distribution of the short-term message error rate is obtained by combining the steady-signal performance characteristic of the message-signal decoder unit with the cumulative distribution of the time-varying signal-to-noise ratio at the receiver output, or at the decoder-unit input. From the resultant cumulative distribution of the message error-rate we obtain the performance characteristics of the decoder unit for a time-varying signal-to-noise ratio at the decoder unit input. The above procedures are used for non, dual and quadruple diversity.

It should be noted in the above analysis that the performance of the radio receiver is considered in terms of the total RF-signal and total noise powers at the receiver input versus the receiver output signal-to-noise ratio. The performance of the decoder unit is taken to be in terms of the input signal-to-noise ratio versus the output-message error rate. The reason for the difference between the selected input-terminal factors for these equipment units is that the RF-signal power at the receiver input is a separable factor, which may be adjusted independently (within limits) while the signal-to-noise ratio at the receiver output or the decoder unit input, must be treated as a ratio because of the characteristics of the radio receiver.

2. RADIO COMMUNICATION SYSTEMS

From a technical viewpoint, a radio communication link should transfer information between two or more points at a specified rate, for a given amount of transmitted power, a required RF spectrum bandwidth, and with an acceptably small received-information error rate. An exchange between these major requirements can be effected; however, their mutual interdependence usually cannot be eliminated. Decisions involving the choice of the type of communication system to be used involve such factors as total initial cost, operational cost, time of delivery of equipment, etc.; most of these factors are obtainable from a technical evaluation of the various types of systems.

This paper deals with the comparative performance of point-to-point radio communication systems employing either SSBSC-AM (single sideband suppressed carrier, amplitude modulated) or FM (frequency modulated) types of modulation. Frequency division type of multiplexing is considered and the number of voice channels ranges from 1 to 120, or more. However, the subject is treated on the basis of message-error rates, required baseband frequency bandwidth, available received radio carrier power levels, signal-to-noise ratios, etc. Hence, the results are not restricted to frequency-division type of multiplexing nor to broadband systems, and are of general use in the field of electronic communications.

Results are presented in a form which should be useful to the system designer who has the responsibility of selecting the optimum combination of parameters; the results can also be used as a guide in setting up system performance tests.

The method of analyzing and evaluating a radio communication system, as outlined below, is to consider the performance characteristics of the major sections of the system and then optimize the performance of each of these sections. By this procedure, it is possible to compare directly the overall performance and relative merits of different types of equipment selected to perform the same function within a communication system. Referring to figure 2-1, the major sections comprising a radio communication system are seen to be:

(A) The encoding units which accept and encode the information into the form of an electronic message-signal, and a multiplexing unit which forms a composite baseband signal from the set of encoded information-bearing message signals.

(B) The modulator-radio transmitter transmitting-antenna section which generates, modulates and then directs a radio-wave signal to the radio receiver. This baseband-signal modulated radio-wave signal acts as a carrier to convey the baseband signal to the receiving end of the communication system.

(C) The radio-wave transmission paths having characteristics which are dependent upon such factors as antenna beam width, radio-carrier frequency, radio frequency spectrum, path length, etc.

(D) The receiving antenna, radio receiver, and diversity-combiner unit, considered as a section which accepts the modulated radio-frequency signals and delivers a replica of the transmitted composite baseband signal.

(E) The receiving multiplexing unit and decoding unit combined, which converts the received baseband signal to information in its original form.

2.1 Performance of System

Optimum performance of a radio communication system can be achieved only by proper considerations of the design parameters involved in the functioning of the various sections of the system. Referring to figure 2-1, the factors to be considered in this paper are:

BLOCK DIAGRAM OF BASIC UNITS
IN A MULTICHANNEL TROPOSPHERIC RADIO COMMUNICATION SYSTEM

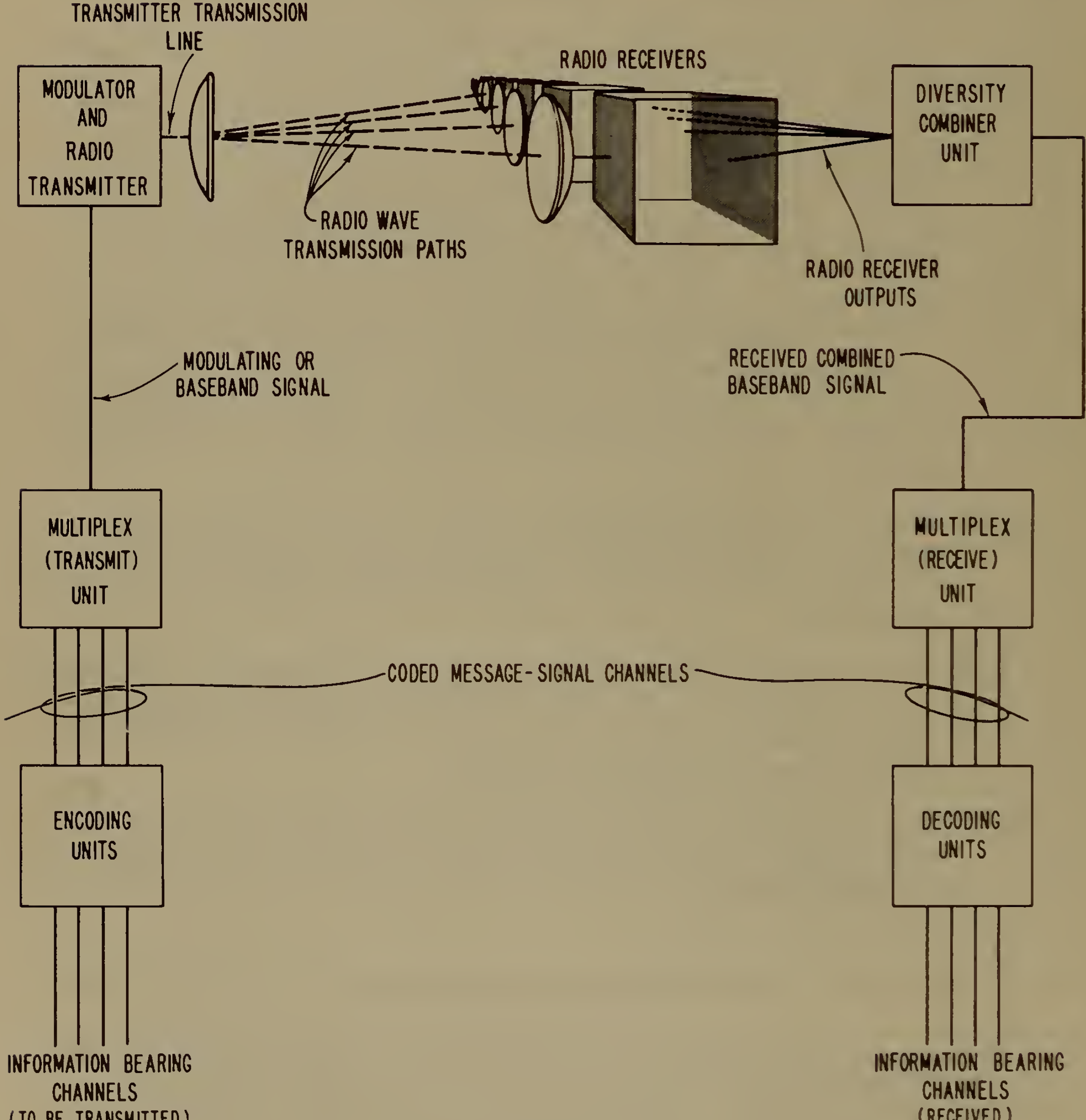


Figure 2-1

(1) Spectrum bandwidth required for the composite baseband signal, which contains the information to be transmitted.

(2) Cumulative amplitude distribution of the composite baseband or modulating signal; that is, the modulating-signal power levels exceeded for various percentages of the time.

(3) Required transmitted RF spectrum bandwidth.

(4) Signal-to-noise ratio in the radio receiver IF circuits at the demodulator input, and the statistical characteristics of this (received) signal-to-noise ratio.

(5) Received power level, P_r, at the radio receiver input and the characteristics of this received signal such as the cumulative amplitude distributions, fade rate, fading range and fade-duration distribution.

(6) Radio receiver noise level, combined with the receiving-antenna system noise power, the radio transmitter-output noise and the intermodulation noise, all referred to the radio receiver input terminals.

(7) Diversity gain, both theoretical and measured.

(8) Message-channel signal-to-noise power ratio, S_{oc}/N_{oc}, at the radio receiver output.

(9) The relationship between S_{oc}/N_{oc} at the radio receiver output and the message error rate of the received information.

From a study of the above definable and measurable parameters of a radio frequency communication system it is possible to determine the performance of various types of systems. Direct comparisons can then be made between the various systems at common points within the systems, in order to obtain relative figures of merit.

2.2 Design and Testing of Equipment and Systems

The principal requirement, which may be used as a guide when designing and testing radio communication systems, is that the average (received) message error rate should not exceed a specified value for specified message-load conditions on the system. <u>This allowable average message error rate determines a required time-average</u>

signal-to-noise ratio at the message-signal decoder-unit input. This required time-average signal-to-noise ratio at the message-signal decoder-unit input may be used as a starting point in the design of the radio frequency section of a communication system. It is understood that the signal level, at the input to the message-signal decoding unit, is required to be above a particular minimum level for proper operation of the decoding unit; proper amplification will maintain this signal level. However, the signal-to-noise ratio cannot be enhanced by amplification.

The design of each type of radio communication system can be carried out for various message-error rates and message loading; the different types of communication systems can then be compared for similar grades of service in terms of factors such as radio frequency power required at the transmitter output, required radio frequency spectrum, total cost of system, operational complexity, etc.

The performance and the cost of a radio communication system are governed largely by the radio-wave transmission loss, fading range, fading rate and the amount of frequency selective fading of the transmitted radio frequency signal. However, it is convenient to design the system for a non-fading or steady radio-frequency signal and then estimate the system performance with a fading type of radio frequency carrier signal; this is the procedure to be followed in this work. This procedure is feasible under conditions where the fade rate of the received carrier signal is low compared with either the duration of the binary pulse or the lowest frequency in the message signal. Methods of estimating the performance of radio communication systems, based on the above principles, are outlined in Sections 6 and 7 and Appendices B. C, and D.

3. MODULATOR-TRANSMITTER PERFORMANCE

The transmitter-output RF-signal characteristics, such as required spectrum bandwidth, B_{rf}, average power level, P_t, and time-amplitude distribution of the envelope, depend upon the type of modulation and the type of modulating signal. In order to obtain useful results it is necessary to derive the relationships between the average power level of the modulating signal, P_m, and the characteristics of the RF signal. For convenience, a sinusoidal modulating signal, with a power level P_{ms} at the Modulator input, is used as a "standard" of comparison. Proper or equivalent modulating-signal power levels, P_{mn}, for noise-type modulating signals are then obtainable relative to P_{ms} by means of the measurable or the specified characteristics of the Modulator. For amplitude modulation, the Modulator characteristics are in terms of P_{ms} versus the amplitude-modulation index, m_a; for frequency modulation, P_{ms} versus the carrier-frequency deviation, ΔF, would be required. The amplitude-modulation index, m_a, may vary from 0 to 100% without serious intermodulation problems; modulating-signal power levels greatly in excess of those required for 100% amplitude modulation will yield excessive intermodulation noise, and should not be used. For frequency modulation, the modulating-signal power level, P_m, determines the RF-signal spectrum bandwidth, B_{rf}, and hence the chosen level for P_m depends upon the choice of B_{rf} and the FM Modulator characteristics.

In this section quantitative relationships are derived between: (a) the modulating-signal average power level, P_m, (b) the RF signal bandwidth, B_{rf}, and (c) the distribution of the time-varying envelope of the transmitter-output RF signal. The results apply to both a sinusoidal modulating signal and a white-noise type of modulating signal. SSBSC-AM and FM systems are considered in this analysis.

3.1 Required Radio Frequency Signal Spectrum Bandwidth for Radio Communication Systems.

The required radio-frequency signal spectrum bandwidth depends upon the bandwidth of the modulating baseband signal and the type of modulation employed. In turn, the baseband-signal bandwidth depends upon the number of information or message channels, the bandwidth of each channel, and the type of multiplexing used to form the baseband-signal spectrum. The problem can be simplified somewhat by considering the radio-frequency signal bandwidth required in terms of a given baseband-signal bandwidth. This procedure makes possible a direct comparison of the radio-frequency signal bandwidth requirements for various types of radio communication systems, with the same baseband signal, or message load. The separate problem of minimizing the baseband signal bandwidth required, for a given number of information channels or message load, involves a consideration of the principles of encoding [Nyquist, 1924, 1928; Hartley, R. V.L., 1928] and multiplexing techniques [Landon, 1948] and is not considered in this paper.

3.1.1 Radio Frequency Bandwidth Required for SSBSC-AM Systems

In an SSBSC-AM (single sideband suppressed carrier, amplitude modulated) system the required radio frequency signal bandwidth is approximately equal to the bandwidth of the composite baseband or modulating signal.

3.1.2 Radio Frequency Bandwidth Required for FM Systems.

In an FM system the required radio frequency bandwidth depends upon the baseband or modulating-signal bandwidth, the modulation index, and also the amplitude distribution of the modulating signal, as will be shown later. The required radio-frequency signal bandwidth for FM systems is usually greater than the modulating-signal bandwidth, by a factor ranging from 2.5 to 25, or more.

3.1.2.1 Required FM Radio Frequency Signal Bandwidth for a Single Sinusoidal Modulating Signal.

When a single sinusoidal modulating signal, of frequency f_c, is employed to frequency-modulate a (radio-frequency) carrier signal, the resultant modulated-carrier power spectrum (theoretically) consists of an infinite number of components [Carson, 1922, 1929; Van der Pol, 1930; Hund, 1942] . For our purpose we restrict the required or transmitted radio-frequency signal spectrum bandwidth so as to include only the "significant" spectral components; that is, we arbitrarily include only those components having amplitudes equal to or greater than one percent of the amplitude of the unmodulated carrier-signal amplitude. Under these conditions the "required" radio-frequency signal bandwidth B_{rf} is given by

$$B_{rf} = f_c \times \phi\,(\Delta F/f_c) \qquad (3.1)$$

where,

f_c = Frequency of the sinusoidal modulating signal, in c/s.

ΔF = Peak deviation of the radio-frequency carrier signal from its un-modulated frequency, in c/s. This is the frequency deviation above and below the carrier-signal frequency.

$\phi\,(\Delta F/f_c)$ = Function of $\Delta F/f_c$, which gives the number of significant sidebands in the radio-frequency spectrum.

For a given value of ΔF, corresponding to a particular value of modulating-signal average-power level, P_{ms}, the radio-frequency bandwidth increases with increasing values of f_c. Hence, the maximum value of B_{rf} is determined for the case where the frequency

RELATIONSHIPS BETWEEN THE DEVIATION RATIO $\Delta F/f_m$ AND THE RF SPECTRUM BANDWIDTH B_{rf} WHICH INCLUDES 99.99 PERCENT OF THE RF SPECTRAL POWER

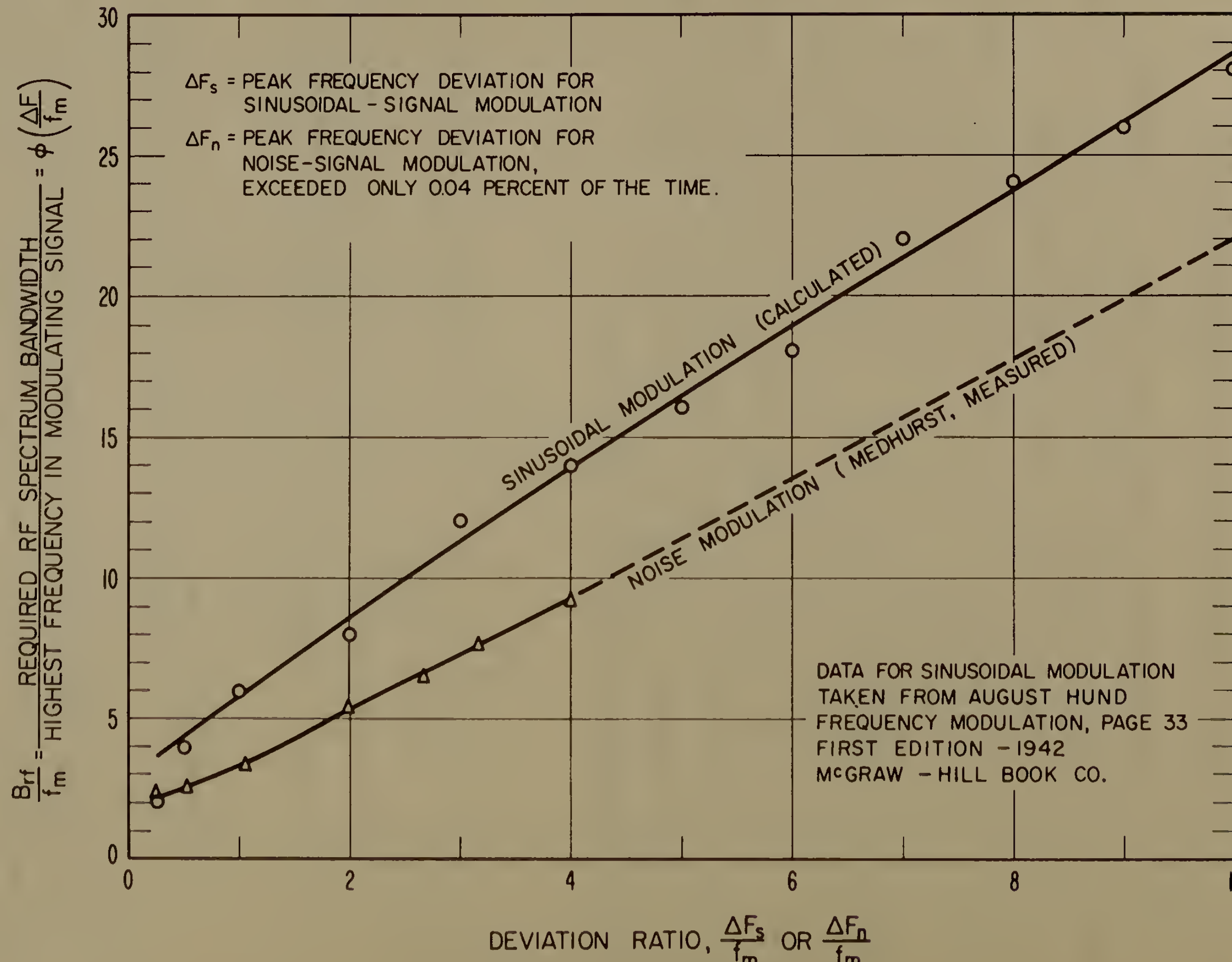


Figure 3-1

of the sinusoidal modulating signal is equal to f_m, the highest frequency in the modulating baseband signal. Therefore, throughout this report the required (maximum) radio-frequency signal spectrum bandwidth, B_{rf}, is derived from,

$$B_{rf} = f_m \times \phi (\Delta F/f_m) \tag{3.2}$$

where, f_m = Highest frequency in the modulating baseband signal, in c/s.

The function $\phi (\Delta F/f_m)$ is shown plotted in figure 3-1 for both a sinusoidal modulating signal and for a "noise-type" modulating signal; the latter case is discussed in Subsection 3.2.3. Note that a smoothed curve is drawn for the case of a sinusoidal modulating signal.

The decision to include only the RF spectral components having amplitudes greater than one percent of the unmodulated radio-frequency carrier signal, is based on measured results such as shown in figure 3-2. The curves in figure 3-2 indicate the effects of bandwidth clipping of the radio-frequency spectrum. These curves also show the performance of an FM receiver, in terms of the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, for a received radio-frequency signal at and above the "threshold level" of the receiver. At threshold (or below), the receiver IF bandwidth, B_{if}, should be somewhat greater than the radio-frequency spectrum bandwidth, B_{rf}; as given by (3.2), or the curve in figure 3-1. Above threshold, the receiver IF bandwidth may be somewhat less than B_{rf}, without seriously degrading the receiver output signal-to-noise ratio. Hence the use of (3.2) and figure 3-1 results in system performance estimates which are conservative above the receiver threshold level and are slightly optimistic below the receiver threshold level.

RECEIVER OUTPUT S_{oc}/N_{oc} RATIO VERSUS B_{rf}/B_{if} FOR VARIOUS RECEIVER-INPUT TOTAL RF SIGNAL POWER LEVELS

STEADY RECEIVER-INPUT SIGNAL

SINUSODIAL MODULATING SIGNAL

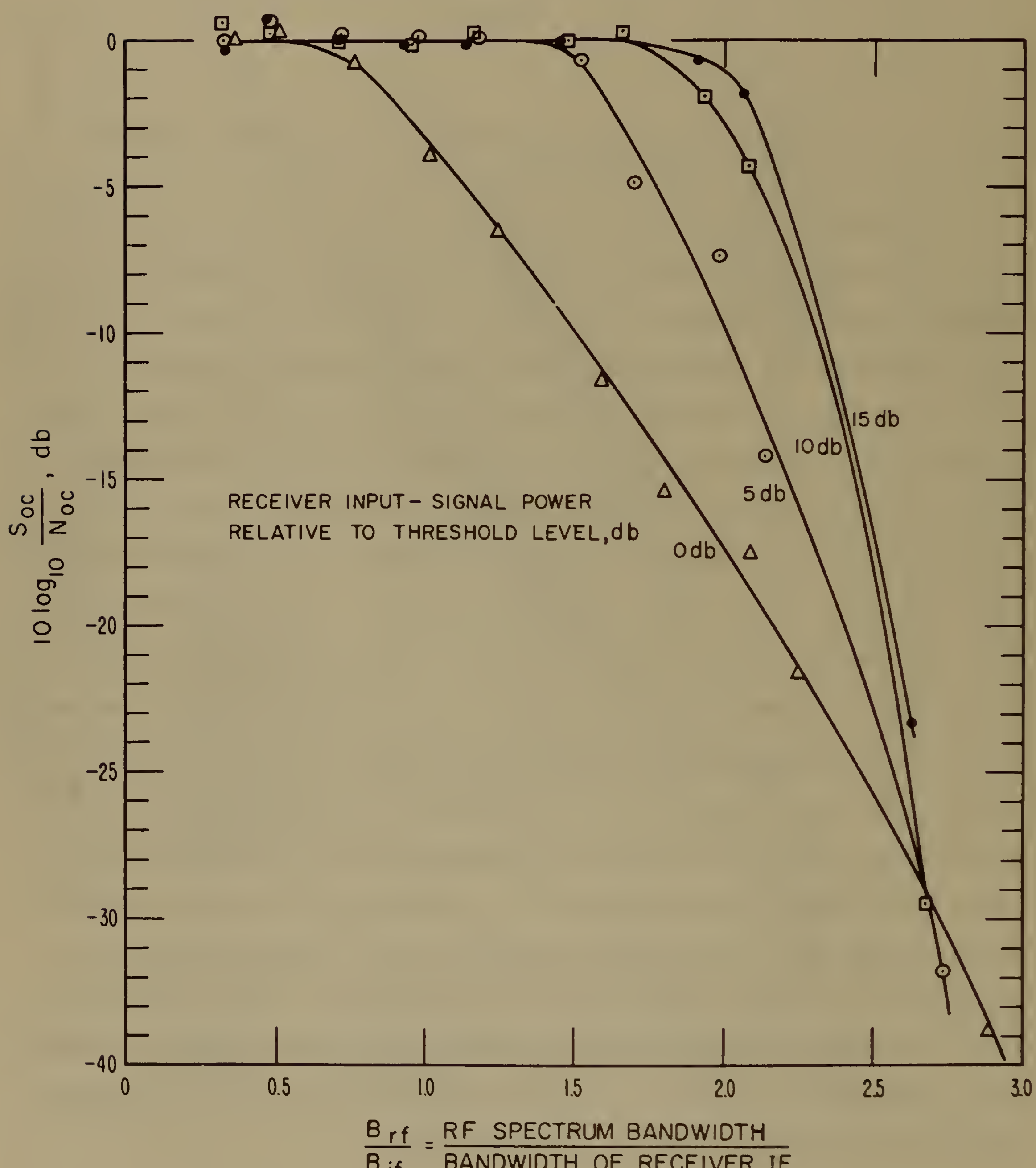


Figure 3-2

3.1.2.2 Required FM Radio Frequency Signal Bandwidth for Various Types of Modulating Signals.

In the preceding analysis the maximum value of the required radio-frequency signal bandwidth, B_{rf}, was derived on the assumption that the modulating signal was sinusoidal, with a frequency f_m, equal to the top frequency in the baseband, and that all of the modulating-signal power was concentrated in this one signal. Under these conditions the radio-frequency signal spectrum and the required bandwidth, B_{rf}, will be a maximum for a given set of system parameters and for a single sinusoidal modulating signal of frequency f_m.

We wish to determine the relationship between the required radio-frequency signal bandwidth, B_{rfs}, for a single sinusoidal modulating signal at the top baseband frequency, f_m, and the required radio-frequency signal bandwidth, B_{rfn}, for a composite modulating signal. The spectral components of the composite modulating signal are assumed to be distributed uniformly throughout the baseband modulating-signal spectrum and the cumulative amplitude distribution of this signal may be of various types. Radio-frequency signal bandwidth comparisons are made on the assumption that f_m is the highest frequency in the composite modulating signal.

The radio-frequency carrier-signal frequency deviation, ΔF, varies directly with the amplitude of the (voltage) envelope of the composite modulating signal, and hence the cumulative distribution of the envelope of the modulating signal is used to determine ΔF on a statistical basis. That is, the "short-term" values of ΔF can be determined for various percentages of time, including the maximum or peak value of ΔF which occurs during voltage or power peaks of the modulating signal. It should be noted that the short-term period is assumed to be short compared with the time variations of the envelope of the modulating signal.

The short-term value of the required radio-frequency signal spectrum bandwidth B_{rf} is determined by both the deviation ratio $\Delta F/f_c$ and f_c; where f_c is the short-term frequency of the modulating signal. The time included in the short-term period is assumed to be large compared with one cycle of the modulating frequency; during this period, ΔF and f_c are assumed to be constant. However, the values of ΔF and f_c are assumed to vary independently, with time.

The radio-frequency signal bandwidth, B_{rfn}, required for a composite-type of modulating signal, is defined here as the radio-frequency signal spectrum bandwidth containing 99.99 percent of the spectral energy of the modulated RF carrier signal. This definition of B_{rfn} is equivalent to the above definition of B_{rfs}, for the case where the modulating signal is a single sinusoid. However, this bandwidth is exceeded for a small percentage of the time, during which the (power) peaks in the composite modulating signal exceed the level required to generate a radio-frequency signal spectrum greater than B_{rfn}. Furthermore, these modulating-signal power-level peaks, which are required to generate radio-frequency signal spectrum bandwidths exceeding B_{rfn}, will also depend upon the simultaneous short-term frequency of the modulating signal.

From the above considerations it is evident that both ΔF_s and ΔF_n are the peak values of the short-term ΔF; while ΔF is seen to be proportional to the (voltage) envelope of the modulating signal.

3.1.2.3 Required FM Radio Frequency Signal Bandwidth for a White-Noise Modulating Signal.

Reliable information is scarce on the relationship between the required radio-frequency signal spectrum bandwidth, B_{rf}, and $\Delta F/f_m$, for modulating signals composed of groups of sinusoidal signals arranged in various frequency combinations in the modulating-signal baseband spectrum. However, the relationship between B_{rfn} and $\Delta F_n/f_m$ has been calculated and measured by Medhurst [1956] for the case where $2\pi(f_m - f_o)$ equal-amplitude, randomly-phased sinusoidal signals were assumed to simulate white-noise across the baseband; this artificial white-noise signal was used in his calculations as the composite modulating signal, where f_o was the lowest frequency and f_m was the highest frequency in the modulating signal. The results of his work are shown plotted in figure 3-1, see "noise modulation" curve; see also [Middleton, 1951; Stewart, 1954] .

In figure 3-1, ΔF is defined as the peak value of the frequency deviation of the carrier signal. When a sinusoidal modulating signal is used, ΔF_s is constant, and is proportional to the peak amplitude of the modulating-signal voltage. For noise-signal modulation, ΔF_n is proportional to the "peak amplitudes" of the voltage envelope of the noise-modulating signal; these peak amplitudes are exceeded for only .04 percent of the time, and are estimated to be 11 db above the average power value of the noise modulating signal. [Gladwin, 1947; Alversheim and Schafter, 1952; Medhurst, 1956] . ΔF_n should be considered as a peak deviation which is 11 db greater than the "rms ΔF_n" frequency deviation. The rms ΔF_n frequency deviation determines or generates the RF signal spectrum having a bandwidth, B_{rfn}. Both ΔF_n and rms ΔF_n are quantitatively defined (and measured) so as to make proper allowances, for the fact that the short-term deviation, ΔF, and the short-term modulating-signal frequency, f_c, vary with time for noise-signal frequency modulation.

The following derivations are for the purpose of obtaining the relationships between the frequency deviations, ΔF_s and ΔF_n, and the modulating-signal average-power levels, P_{ms} and P_{mn}; for the conditions:

$$B_{rf} = B_{rfs} = B_{rfn} \tag{3.3}$$

and,

$$f_m \text{ (for sinusoidal-signal mod.)} = f_m \text{ (for noise-signal mod.)} \tag{3.4}$$

The above factors are defined as follows:

ΔF_s = Peak deviation of the RF carrier signal for a sinusoidal modulating signal, in c/s.

ΔF_n = Peak deviation of the RF carrier signal exceeded for only .04 percent of the time; for a white-noise modulating signal, in c/s.

P_{ms} = Average long-term power level of the sinusoidal modulating signal; measured at the Modulator-input terminals, with a true rms reading meter.

P_{mn} = Average long-term power level of the noise modulating signal; measured at the Modulator-input terminals, with a true rms reading meter.

The advantage of referring our results to sinusoidal-signal modulation conditions is due to the fact that B_{rfs}, ΔF_s, and P_{ms} may either be calculated or may be measured conveniently with standard types of test equipment; these measured factors may then be used as reference levels. It is inconvenient to calculate or to measure B_{rfn}, ΔF_n, and P_{mn}, because ΔF_n can only be measured indirectly in terms of B_{rfn}.

Figure 3-3 shows the "relative levels", in db units, associated with B_{rf}, ΔF, and P_m, for sinusoidal-signal and for white-noise-signal frequency modulation. From figure 3-3,

$$10 \text{ Log}_{10} (\Delta F_n/\Delta F_s) = 10 \text{ Log}_{10} (P_{mn}/P_{ms}) + 8 \tag{3.5}$$

Values of the factor $10 \text{ Log}_{10} (\Delta F_n/\Delta F_s)$, for particular values of B_{rf}/f_m, were obtained from figure 3-1 and substituted in (3.5) to obtain corresponding values for $10 \text{ Log}_{10}(P_{ms}/P_{mn})$; the results of these calculations are shown in figure 3-4.

RELATIVE LEVELS OF ΔF AND P_m, SINUSOIDAL AND NOISE MODULATING SIGNALS AND FOR EQUAL RF SPECTRUM BANDWIDTHS

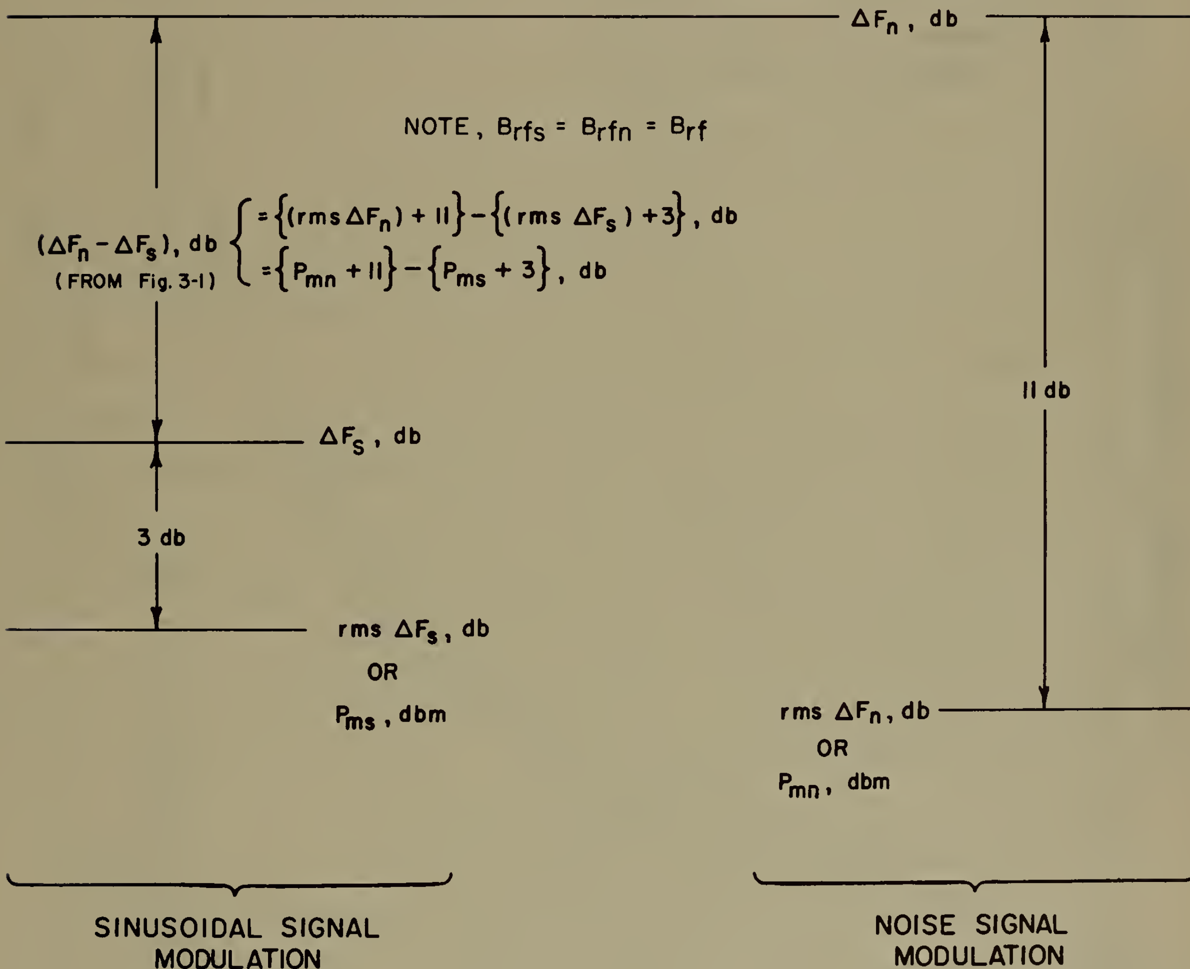


Figure 3-3

RF SPECTRUM BANDWIDTH, B_{rf},
VERSUS RELATIVE MODULATING-SIGNAL POWER LEVELS,
FOR FREQUENCY MODULATION

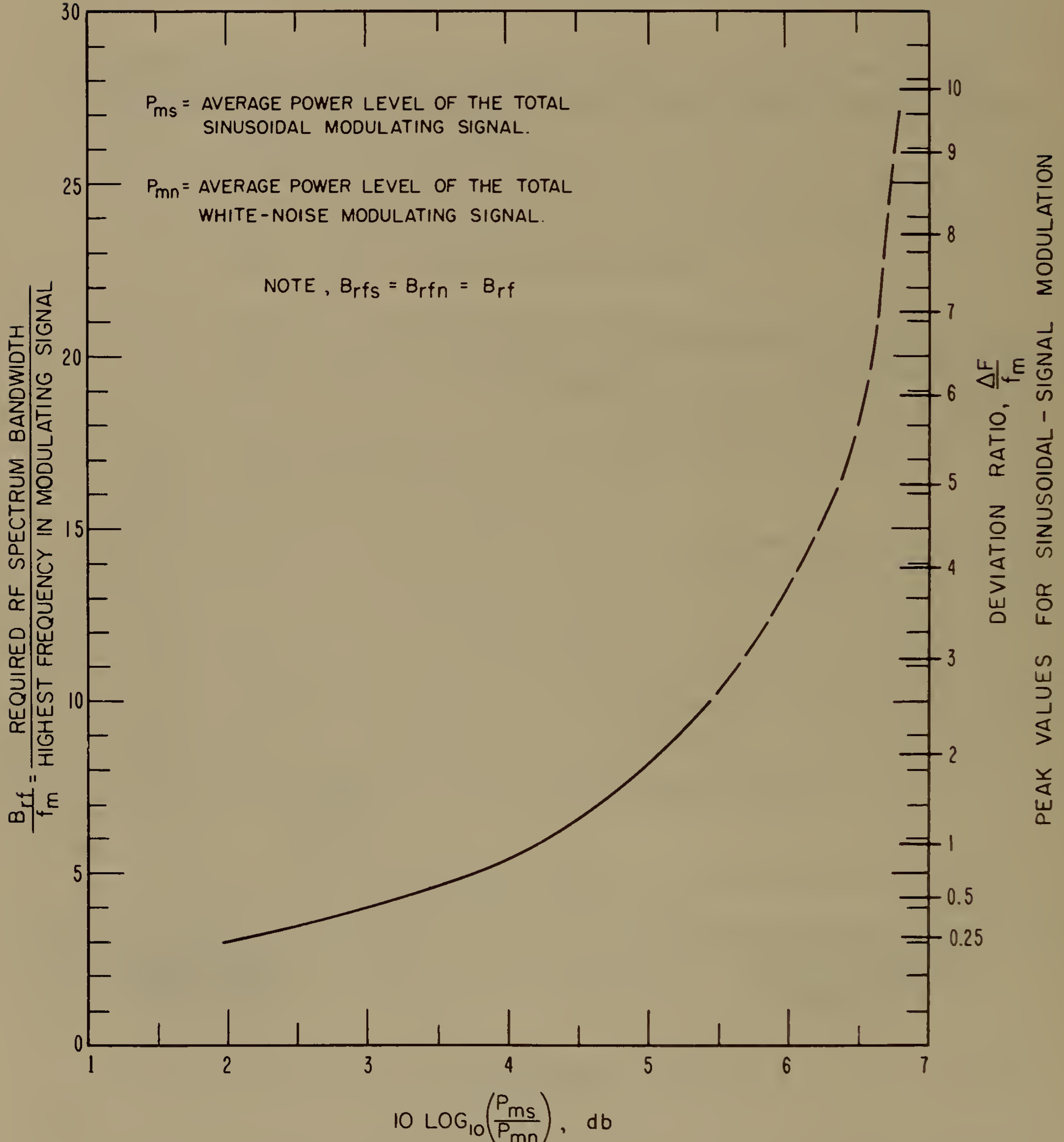


Figure 3-4

It should be noted that the RF spectrum bandwidth, B_{rf}, is either given or it may be chosen by the system designer. Also the highest modulating frequency in the modulating signal, f_m depends upon the number and the type of message channels and also the type of multiplexing equipment being used. Hence the value of B_{rf}/f_m is available for use in figures 3-1 and 3-4.

Figure 3-1 may be used to determine values for $\Delta F_s/f_m$ and $\Delta F_n/f_m$; for the conditions given by (3.3) and (3.4). The ΔF_s vs $\sqrt{P_{ms}}$ characteristic of the (frequency) modulator should be linear for values of ΔF equal to ΔF_s, for sinusoidal-signal modulation; or equal to ΔF_n, for a white-noise modulating signal. If the modulator characteristic is non-linear for values of ΔF_n, obtained from figure 3-1, intermodulation "noise" will be added to the system noise, and the signal-to-noise ratio at the receiver output will be degraded.

Figure 3-4 may be used to obtain correct values for P_{ms}/P_{mn} which correspond to particular values of B_{rf}/f_m; assuming the conditions given by (3.3) and (3.4). The sinusoidal-signal modulating-power level, P_{ms}, which is required to produce a frequency deviation of the RF carrier signal, ΔF_s, and an RF spectrum bandwidth, B_{frs}, may be obtained from the measured ΔF_s vs $\sqrt{P_{ms}}$ characteristic of the frequency modulator. The required noise-signal modulating-power level, P_{mn}, may then be obtained from figure 3-4. If pre-emphasis is used, the pre-emphasis circuit must be inserted at a point in the system which follows the modulator-input terminals; note that P_{ms} and P_{mn} are measured at the modulator-input terminals.

It is common practice to estimate B_{rf} from,

$$B_{rf} = f_m \times 2\left(\frac{\Delta F}{f_m} + 1\right) \qquad (3.6)$$

The required bandwidth obtained from (3.6) will include only the RF spectral components having amplitudes which are approximately 10 percent of the unmodulated RF carrier-signal amplitude, or greater. More precisely, 99 percent of the RF-spectrum energy will be within the bandwidth determined from (3.6). Hence the use of (3.2) and figures 3-1 and 3-4 yield required-bandwidth values, B_{rf} (99.99%) which are somewhat greater that the B_{rf}(99 %) bandwidths, which would be obtained from (3.6). The following relationship between B_{rf}(99.99% bandwidth) and B_{rf}(99 % bandwidth) has been determined, using information similar to that used to obtain the curves in figure 3-1 (see Medhurst, 1956):

$$\left[\phi\left(\frac{\Delta F}{f_m}\right)\right]\left(99.99\%\ \text{bandwidth}\right) \approx 1.4\left[2\left(\frac{\Delta F}{f_m}+1\right)\right]\left(99\ \%\ \text{bandwidth}\right)$$

or,

$$B_{rf}\left(99.99\%\ \text{bandwidth}\right) \approx 1.4\ B_{rf}\left(99\%\ \text{bandwidth}\right) \qquad (3.7)$$

The use of the smaller RF spectrum bandwidth, obtained from (3.6), will result in a degradation of the receiver-output signal-to-noise ratio. The amount of this degradation is shown in figure 3-2.

3.2 Radio-Frequency Signal Time-Amplitude Characteristics for Radio Communication Systems

The time-amplitude characteristics, or the distribution of the envelope of the transmitter-output RF signal, depend upon the statistical characteristics of the modulating baseband signal and the type of modulation. Other considerations relating to the baseband signal are the same as discussed in section 3.1.

3.2.1 Radio-Frequency Signal Time-Amplitude Characteristics for SSBSC-AM Systems

An SSBSC-AM Modulator translates the baseband-signal spectrum to an RF-signal spectrum. In an ideal SSBSC-AM system, wherein there is no modulator distortion, the distribution of the RF signal is identical to the distribution of the baseband signal. Furthermore, the average transmitter-output power level, P_t, is related to the average power level of the modulating signal, P_m, in terms of the combined characteristics of the modulator and the transmitter. For maximum modulation efficiency and to limit over-modulation effects, the average power level of the modulating signal, P_m, should be adjusted so as not to exceed a short-term amplitude-modulation index, m_a, of 100% for more than a given percentage of the time.

3.2.1.1 Radio-Frequency Signal Time-Amplitude Characteristics of SSBSC-AM Systems for a Single Sinusoidal Modulating Signal

When a single sinusoidal modulating signal of frequency, f_c, and power level, P_{ms}, is employed in an SSBSC-AM system to amplitude-modulate a (radio-frequency) carrier signal, the resultant transmitter-output RF signal consists of a single (sinusoidal) RF signal.

The average power level of the sinusoidal modulating signal required for 100 percent amplitude modulation, P_{ms} (100%), may be obtained from the modulator characteristics, in terms of P_{ms} versus percent amplitude-modulation, m_a; the modulator characteristics may either be specified or measured. The above value of P_{ms} (100%), for 100 percent amplitude modulation, may be used as a "limiting reference level" for the average power of the modulating signal--for various types of modulating signals. From a practical viewpoint, it is convenient to estimate or to measure the performance of an amplitude modulator using a sinusoidal modulating signal.

3.2.1.2 Radio-Frequency Signal Time-Amplitude Characteristics of SSBSC-AM Systems for a White-Noise Type of Modulating Signal

In Section 3.2.1.1 it was noted that the maximum value of (sinusoidal) modulating-signal average power, P_{ms}, (100%), was obtainable from the SSBSC-AM modulator characteristics, on the assumption that the modulating signal was sinusoidal and that all of the modulating-signal power, P_{ms}, was concentrated in this one signal.

We wish to determine the average power level, P_{mn}, of a composite (noise-type) modulating signal which will insure that 100 percent amplitude modulation of the RF signal is exceeded for no more than a specified percentage of the time. The noise-type modulating-signal average power level, P_{mn}, is to be referred to the previously-determined sinusoidal modulating-signal power level, P_{mn} (100%).

The amplitude of the radio-frequency signal varies directly with the amplitude of the (voltage) envelope of the composite modulating noise-type signal, for values of modulating-signal power less than the "100 percent modulation" level, $P_m(100\%)$. Hence, for noise-signal modulation the "100 percent modulation peaks", P_{mnp}, correspond to particular "peak amplitudes" of the voltage envelope of the noise-modulating signal; these peak amplitude values, P_{mnp}, may be defined in terms of P_{mn} (100%) and also in terms of the average power level of the noise-type modulating signal, P_{mn}.

For a white-noise signal, the envelope of the signal will be 11 db or more above the average power level, P_{mn}, for .04 percent of the time [Gladwin, 1947; Alversheim and Schafter, 1952; Medhurst, 1956] .

Since the peak value of the sinusoidal modulating signal for 100 percent amplitude modulation, P_{msp} (100%) is 3 db above its average value, P_{ms} (100%), and P_{msp} (100%) has the same level as P_{mnp} (100%), it follows that the average sinusoidal modulating-signal power level, P_{ms} (100%), is 8 db higher than the average power level of the noise-modulating signal, P_{mn} --for the condition that the noise-modulating signal will exceed 100 percent amplitude for .04 percent of the time.

Following the above line of reasoning, combined with the distribution of a noise-type signal [Plush, R. W., et al, 1960] , the ratio, $P_{ms}(100\%)/P_{mn}$, was calculated for a range of percentage-of-time during which amplitude modulation exceeded 100 percent. The results of these calculations are shown in figure 3-5.

AMPLITUDE MODULATION EXCEEDING 100 PERCENT VERSUS RELATIVE MODULATING-SIGNAL AVERAGE POWER LEVELS, FOR SSBSC-AM

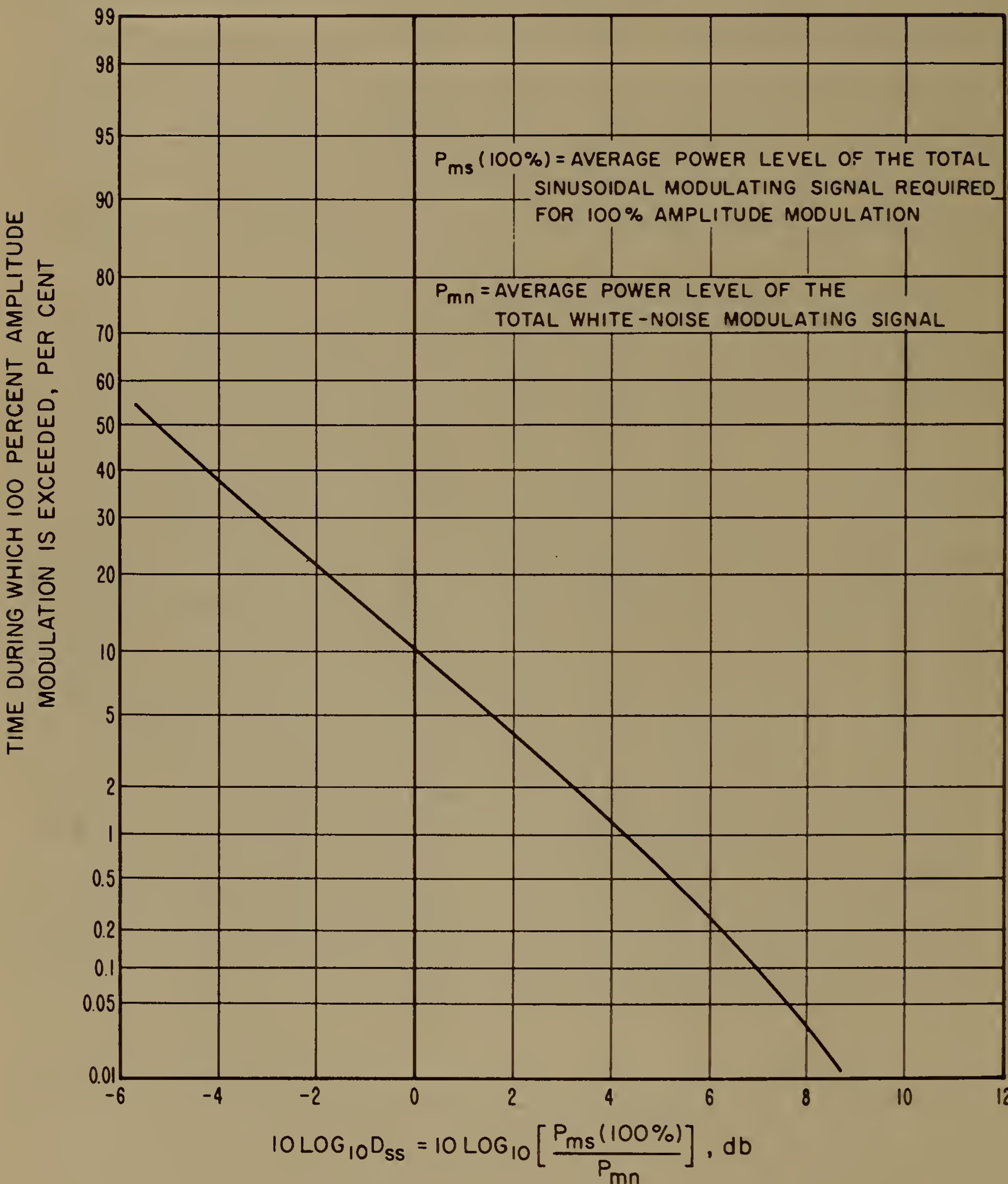


Figure 3-5

Figure 3-5 may be used as a guide in determining the (approximate) proper average power level, P_{mn}, for the noise-type modulating signal, in terms of the measurable average power level, P_{ms}, of a sinusoidal modulating signal, and the percentage of time that 100% amplitude-modulation conditions are exceeded. However, figure 3-5 does not yield accurate estimates of the intermodulation power generated within the Amplitude-Modulator, due to over-modulation; this work should be extended.

The factor, $D_{ss} = P_{ms}(100\%)/P_{mn}$, in the abscissa scale of figure 3-5, is required in the design or the performance-testing of SSBSC-AM systems, for proper adjustment of the modulating-signal power level, at the modulator input terminals.

3.2.2 Radio-Frequency Signal Time-Amplitude Characteristics for FM Systems.

In an FM system the amplitude of the transmitter-output RF signal is independent of the characteristics of the baseband modulating signal.

3.3 Peak Power Requirements for SSBSC-AM Systems

In AM (amplitude-modulated) systems, the transmitter-output short-term peak power levels, P_{tp}, should be related to the transmitter-output average power level, P_t, in precisely the same manner that the short-term peak power levels, P_{mnp}, of the composite modulating signal, are related to the average total power level of the modulating signal, P_{mn}. These relationships are required in the transmitter output in order to avoid excessive intermodulation distortion in the transmitter-output RF signal during periods of peak modulating power.

The above peak-power capability requirements for the transmitter depend directly upon the amplitude-time distribution of the envelope of the composite baseband modulating signal. A composite modulating signal which is composed of a number of independent voice-message signals combined with teletype-message and data-phone message signals, may be assumed to have a distribution which approximates the distribution of a white-noise signal. For a white-noise signal having a normal distribution, the peak power exceeded for one percent of the time is 7.3 db above the average power level of this signal. Other combinations of modulating signals, such as 8 or more equal-amplitude randomly-phased sinusoids, have distributions [Slack, 1946], [Plush, R. W., et al, 1960] in which the composite-signal peak-power-level amplitude, P_{mnp}, exceeds the average power level, P_m, by only 8 db for one percent of the time. Estimates of the percentage of time during which the transmitter-output short-term power exceeds its average power level, P_t, by 11 db, may be obtained from figure 3-5.

One method of qualifying the transmitter characteristics is to specify that the transmitter (power amplifier) must be linear to within a particular degree, usually one percent, for transmitter-output-power peaks, P_{tp}, approximately 10 db above the average power-output level, P_t, of the transmitter.

4. RADIO RECEIVER-COMBINER PERFORMANCE

This section deals with the performance of the radio receiver considered as a unit in the radio communication system. The method of analysis is general and may be applied to <u>any type of receiver</u>, provided that the steady received-signal performance characteristics of the receiver are available, either measured or calculated.

In the following work, the receiver performance characteristics are first determined for a steady received signal. The receiver performance, that is, the receiver output signal-to-noise ratio, is then calculated for a time-varying received signal by combining the steady-signal receiver characteristics with the cumulative distribution of the received radio-frequency signal. The effects of various orders of diversity on the resultant signal-to-noise ratio in the combined baseband signal are included in the analysis by combining the steady-signal characteristics of the radio receiver with the performance characteristics of the combiner; at this point, there may be a choice between predetection and post detection combining.

Radio receiver performance is defined here in terms of the relationship between the receiver-output signal-to-noise ratio, S_o/N_o, and the following factors:

(1) receiver-input radio-frequency signal total available power, P_r; see Appendix A for definition of available power,

(2) available receiver-input noise power, N_T; which is composed of the noise power received by the antenna plus the noise power which originates in the receiving-antenna system,

(3) noise power contributed by the radio receiver, $(F_r - 1)kT_oB_{if} = kT_{er}B_{if}$ and referred to the receiver input,

(4) the power level of the modulating signal, P_m,

(5) the signal-power loading ratio, P_m/p_{mi}, in the modulating baseband signal message channel.

(6) message-signal bandwidth, B_c, and

(7) the highest frequency in the modulating baseband signal, f_m.

The performance of a radio receiver, as defined above, depends upon the type of modulation being used and also depends upon the particular individual characteristics of the receiver. The required radio-frequency total signal power, P_r, at the receiver input, is directly related to the total noise power at the same point, (see Appendix A).

The nature of the radio receiver output signal-to-noise ratio is dependent upon the statistical characteristics and the median power level of the radio-frequency signal at the receiver input; for example, the characteristics of the output noise of a FM receiver, when operating in its threshold region, differ considerably from the output noise-power characteristics above the threshold region. The message error rate, for a given receiver output signal-to-noise ratio, will depend upon the characteristics of the noise at the receiver output, hence this factor is important when considering the overall performance of a radio communication system.

The above-listed factors (1) to (7) inclusive, are treated in Section 4. Detailed methods of applying the above analysis, to determine the radio receiver output signal-to-noise ratios, are given in Subsections 4.5 and 4.6; in which are developed quantitative performance characteristics in graphical form, for SSBSC-AM and FM receivers, respectively. This work includes steady received-signal and Rayleigh-fading received-signal performance curves for non-diversity, dual diversity, and quadruple diversity. Maximal-ratio type of combining is assumed.

4.1 Radio-Frequency Signal Characteristics and Power Levels in the Receiving System.

The radio frequency signal power level at the receiver input, P_r, is a key factor in the performance of a radio communication system; this condition applies for either a steady or a time-varying received signal. The power which the radio transmitter must provide is directly related to the power P_r required at the radio receiver input through the total transmission path loss. Furthermore, the performances of various types of radio receivers are best compared in terms of the receiver-input RF signal power levels required for the same value of receiver-output signal-to-noise ratio; this method of comparing receivers determines the receiver "sensitivity" and makes the proper allowance for the receiver noise figure and the receiver IF noise-power bandwidth. In practice, it is also found to be more convenient to measure the receiver-input RF signal power than to measure the pre-detection signal-to-noise ratio. This situation is particularly apparent to the system-testing engineer when measuring a time-varying received signal, and for cases where automatic gain control (agc) and/or limiting is used in the receiver.

The following characteristics of a time-varying received radio-frequency signal are of importance, insofar as they affect the receiver output signal-to-noise ratio:

(a) Median power level, for the sampling-time or sampling-period

(b) Cumulative distribution of the envelope of the received radio-frequency signal

(c) Fade rate at the median power level

(d) Time variability of the amplitude and phase cross-correlation, between the spectral components of the received radio frequency signal--due to "selective fading" effects and multipath radio-wave propagation conditions.

The effects of the above listed received-signal characteristics (a), (b), and (c) are considered later in this paper, to estimate radio receiver performance for a time-varying radio-frequency signal. For tropospheric-propagated radio-frequency signals, (a) may be calculated [Rice, Longley and Norton, 1959] , (b) has been extensively measured and has been found to closely approximate a Rayleigh distribution, and (c) has been found to vary between approximately 0.1 cps to 10 cps.

Additional experimental work is required to determine an accurate measure of the relationship between characteristic (d) and the radio receiver output signal-to-noise ratio; at present, experimental measurements of this type are very meager. In cases where relatively wide radio-frequency spectra are used, it is necessary to consider the non-correlated amplitude and phase variations between the signal components across the radio frequency and the baseband spectrums. These factors influence the receiver output signal-to-noise ratio and the message error rate and are usually involved in the transmission characteristics of the radio-wave path. The path-bandwidth cross-correlation or covariance factor is probably a function of the beam width of the antenna-pattern, length of path, and carrier frequency. Measurements of the tropospheric radio-path bandwidth capability have been made by Clutts, Kennedy and Trecker [1960] on a 185 mile path between Florida and Cuba.

From theoretical work by Staras [1955] we have the following relationship:

$$\beta = \frac{4}{d^3} \text{ Mc/s} \tag{4.1}$$

where d = radio wave transmission path length in hundreds of miles

β = correlation (radio frequency) bandwidth in Mc. By definition, the cross-correlation coefficient of the radio-frequency spectral components spaced β Mc/s is 0.5.

Equation 4.1 is probably conservative and hence can be used to obtain an estimate of the usable radio frequency bandwidth in cases where antenna beam widths are equal to or greater than the angle subtended by the effective "scatter volume". The usable radio frequency bandwidth is much greater for cases where the antenna beam width is considerably smaller than the angle subtended by the scatter volume. Where the antenna beams are of the order of 1 degree or less a somewhat larger value of usable radio frequency bandwidth could be assumed. Narrow-beam antennas reduce the multi-path effects and thereby result in an increase of usable radio frequency bandwidth; see section 4.3.

4.2 Pre-Detection Noise-Power Levels in the Receiving System

The noise performance of the receiving system depends upon the amount of noise power which is contributed by the various units in the receiving system, and which subsequently interferes with the pre-detection received signal, P_{if}. In this analysis an estimate is made of the total noise power contributed by the entire receiving system, from and including the receiving antenna to the pre-detection point in the radio receiver.

In order to make the results of this analysis convenient for use in system design work and performance tests, the total of the noise power contributed by the receiving system is referred to the radio receiver input, in terms of the receiving-system parameters. Using this procedure, the system designer can accurately estimate the level of the noise power at the receiver input; this same noise-power level can be conveniently measured, when performance tests are made, by using a calibrated (linear) receiver--where the receiver is calibrated in terms of receiver-input RF signal power versus receiver IF-output signal power or, the receiver agc voltage. Only amplitude-type of noise power is considered, as distinguished from phase or frequency-modulation noise; the latter is measurable only in the output of an FM receiver, and does not appear as amplitude noise in the predetection circuits of the receiver. Post-detection noise and its effects on the receiver-output signal-to-noise ratio is considered in section 4.3.

In this work all noise powers in the receiving system are added algebraically on an average power-level basis, as measured by a true rms-reading meter with an appropriate time constant. The noise power developed within the receiving-system units is assumed to be "thermal" type noise and to have a flat power spectrum over the bandwidth accepted by the receiver. Interference from other radio transmissions, which will introduce an effective noise component designated as N_{ai} in Appendix A, are excluded from the present analysis, since their inclusion would require a consideration of the statistical characteristics of the interfering signal. This task is beyond the scope of this report.

The transmitter-output "noise" power N_{at}, associated with the received RF signal, is composed of "transmitter amplitude noise" and "transmitter intermodulation noise."

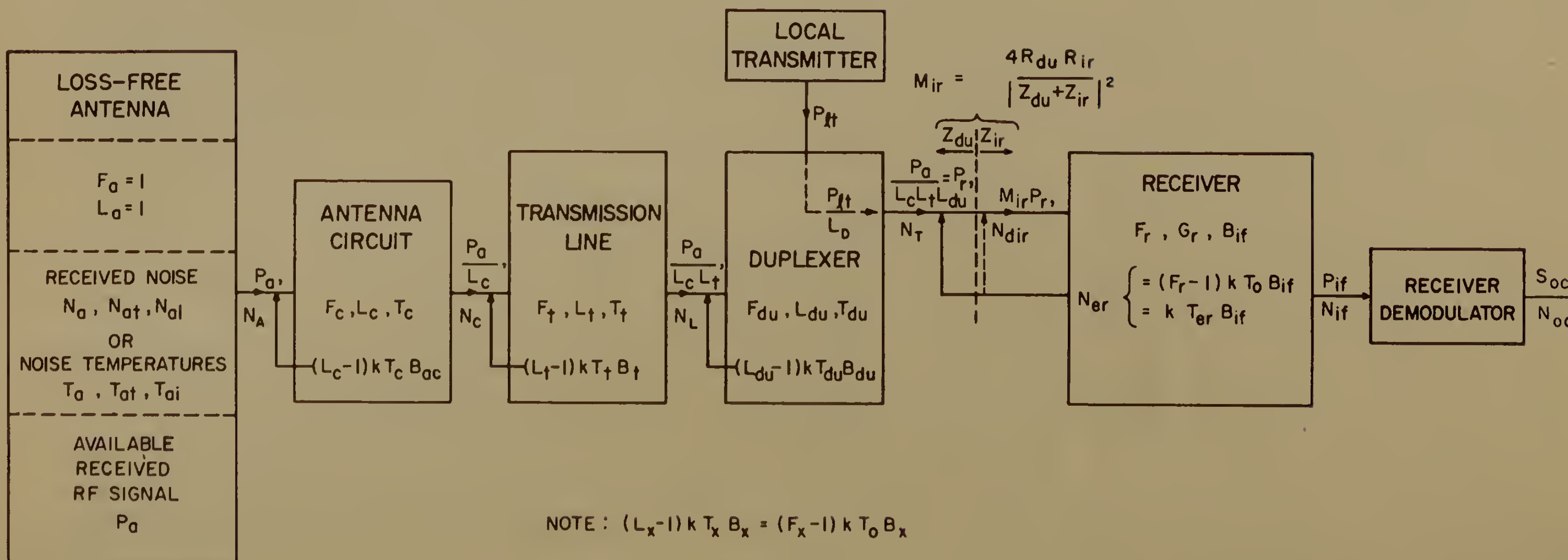
Transmitter amplitude noise usually originates in the transmitter output stages; this noise is present even though the transmitter is not being modulated, and may be measured, relative to the un-modulated RF carrier signal, at the transmitter output. It may be measured by connecting a transmitter and a receiver back-to-back through an attenuator, and measuring the receiver baseband-output noise power, under conditions of no modulation on the transmitter carrier signal. The receiver-output total noise, N_{oc}, (see figure 4-A) is measured for a range of receiver-input carrier-signal powers and the results plotted. The relative value of transmitter noise is indicated at that portion of the curve where the receiver-output noise does not decrease with further increase in the received carrier-signal power. The transmitter-output signal-to-noise ratio should be high enough to insure that the "leveling-off" portion of the curve will be above the operating or required RF signal-power levels of the receiver performance characteristic.

The net effect of transmitter-output noise is to place an upper limit on the receiver-output signal-to-noise ratio, and is similar to the effects of (a) the signal-to-noise ratio at the modulator input terminals, (b) equipment intermodulation noise, and (c) path-modulation effects. The latter subjects are discussed in section 4.3.

The conclusions to be drawn from the above discussion of the transmitter-output amplitude noise, N_{at}, are that proper modifications for the effects of this noise power can be made on the system design curves and also on system-performance test results, and that N_{at} need not be considered when calculating the noise levels or the performance of the receiving-antenna system. Consequently, N_{at} does not appear in the expression for the receiving-system available noise-power level, N_T, or effective noise temperature, T_A, at the receiver-input terminals; see figure 4-1 and Appendix A.

Following is an outline of the method used to estimate the total (average) available noise-power level, N_T, at the (radio frequency) receiver input. Referring to figure 4-1, the noise power, N_x, contributed by each unit in the receiving system, and subsequently referred to the receiver-input terminals, is estimated in terms of the parameters of each unit, such as its thermal temperature T_x, transmission loss L_x, noise figure F_x, etc. In the following derivations it is assumed that non-reflecting impedance-matched conditions exist at the junctions of the various units in the receiving system, except (possibly) at the receiver-input terminals, and that each unit is constructed with input and output positive-resistance terminations; hence each unit, preceding the receiver, delivers the "available" noise power, N_x, at its output terminals. See Appendix A for methods of estimating noise-power levels in the receiving system, including a discussion of the effects of impedance-mismatching at the receiver-input terminals.

BLOCK DIAGRAM OF RECEIVING SYSTEM



NOTE : $(L_x - 1) k T_x B_x = (F_x - 1) k T_0 B_x$

Fig. 4-1

Referring to figure 4-1, the total receiving-antenna system output noise power, N'_T, available at the receiver-input terminals and within the bandwidth $B_{rf}(=B_{if})$ "accepted" by the receiver, is given by (A.25) of Appendix A and is re-written here as (4.2)

$$N'_T = k T_A B_{if} \tag{4.2}$$

where,

k = Boltzman's constant = 1.3804×10^{-23} joules per degree Kelvin

T_A = Effective output noise temperature of the receiving-antenna system, degrees Kelvin

B_{if} = Effective receiver IF bandwidth, in c/s, see (A.18)

The effective output noise temperature of the receiving-antenna system, T_A, is defined by (4.2). T_A is a function of the receiving-antenna system parameters and is given by (A.30) of Appendix A, and reproduced here as (4.3),

$$T_A = \frac{T_a + (L_c - 1)T_c + L_c(L_t - 1)T_t + L_c L_t (L_{du} - 1) T_{du}}{L_c L_t L_{du}} + \frac{P_{\ell t}}{k B_{if} L_D} \tag{4.3}$$

The value of T_A, in degrees Kelvin, may be calculated by means of (4.3), by substituting values for the various receiving-system parameters; these parameters are completely described and defined in Appendix A.

If the thermal temperatures of the various units in the receiving-antenna system are equal to T_o, and if impedance-matching conditions exist throughout the receiving-antenna system, (4.3) becomes,

$$T_A = \frac{T_a - T_o}{L_c L_t L_{du}} + T_o + \frac{P_{\ell t}}{k B_{if} L_D} \tag{4.3a}$$

The portion of the noise power, N'_T, which is "delivered" to or absorbed by the receiver input circuit is,

$$N_{dT} = M_{ir}\, N'_T \tag{4.4}$$

where, M_{ir} = Impedance mismatch factor, at the receiver input terminals. This factor is a function of the impedance at the antenna-system output terminals, Z_{du}, and the receiver-input impedance, Z_{ir}, and is defined by (A.16) as,

$$M_{ir} = \frac{4 R_{du} R_{ir}}{|Z_{du} + Z_{ir}|^2} \tag{A.16}$$

where, R_{ir}, R_{du} = Resistive or "real" component of the complex impedances Z_{ir} and Z_{du}, respectively.

The noise power contributed by the linear section of the receiver, and delivered to the receiver-input terminals, is closely approximated by,

$$M_{ir} \{(F_r - 1) k T_o B_{if}\} = M_{ir} k T_{er} B_{if} \tag{4.5}$$

where, F_r = Receiver noise figure [IRE Comm. on noise, 1960] , as a ratio

T_o = Standard reference temperature, 290 degrees Kelvin [IRE Comm. on noise, 1960]

T_{er} = Effective receiver input noise temperature [IRE Comm. on noise, 1960] , degrees Kelvin

From (4.5) we obtain:

$$T_{er} = (F_r - 1) T_o \tag{4.6}$$

The total noise power delivered to the receiver-input terminals is equal to the sum of the noise powers given by (4.4) and (4.5) or,

$$N_{dT} + M_{ir} \{(F_r - 1) k T_o B_{if}\} = M_{ir} (T_A + T_{er}) k B_{if} \tag{4.7}$$

The dependence of the total noise power at the receiver input, relative to the noise temperatures T_A and T_{er}, is clearly evident from (4.7).

The total predetection noise power level is equal to the total noise power at the receiver input, $M_{ir}(T_A + T_{er}) k B_{if}$, multiplied by the receiver gain, G_r. The load-reflected [Siegman, 1961] noise power in the predetection circuit of the receiver is assumed to be negligible for communication-type receivers, and is not considered here; however, for extremely low-noise negative-resistance and/or bilateral types of receivers this noise power may be important. Hence, the total average noise power at the receiver-detector input point, N_{if}, is given by:

$$N_{if} = G_r M_{ir} [T_A + T_{er}] k B_{if} \tag{4.8}$$

where, G_r = Receiver power gain, as a ratio. This factor is defined as the ratio of the signal power delivered to the receiver input terminals to the available signal power in the receiver IF circuit.

When using (4.8) to estimate the predetection total noise power level, N_{if}, the first step is to determine the numerical value of the factor T_A. From (4.3) it is seen that the numerical value of T_A depends upon the receiving-system parameters. All of these parameters except T_a may be either measured or estimated with a high order of accuracy. T_a varies with time and antenna orientation and hence may (in some instances) be estimated only to a low order of accuracy.

The antenna noise temperature, T_a, or noise power, N_a, depends upon the RF carrier-signal frequency [CCIR Report 65, 1957], [Crichlow, Smith, Morton and Corliss, 1955], [Hogg and Mumford, 1960]. However, in the microwave frequency range it may be assumed [Grimm, 1959] that T_a varies between 50 to 300 degrees Kelvin, provided that the main lobe of the antenna pattern does not include the sun or an intense radio star.

The remaining factors in (4.8), G_r, T_{er}, and B_{if}, are obtainable from the receiver specifications; the substitution of these factors in (4.6) and (4.8) together with the previously calculated value of T_A, gives the predetection noise power level, N_{if}, and the receiver-input total delivered noise power level, $M_{ir}(T_A + T_{er}) k B_{if}$, for the estimated and the specified receiving-system parameters.

4.3 Post-Detection Noise Power Levels in the Receiving System

Post-detection noise, which appears at the receiver output, is generated at various points within the system and is due to various causes as listed below:

(1) Pre-detection (amplitude) noise, N_{if}, at the detector input.

(2) Noise power at the modulator input, N_m, relative to the total modulating-signal power, P_m.

(3) Intermodulation noise, generated within the equipment.

(4) Radio-wave path-modulation noise, imposed on the RF signal by transmission over the radio-wave path.

A complete analysis of post-detection noise requires a consideration of the statistical properties of each of the above noise powers in order to obtain the characteristics of the resultant total noise power per message-signal channel, N_{oc}, at the receiver output. The effects of N_{oc} on system performance may then be determined in terms of the resultant average signal power-to-average noise power ratio, S_{oc}/N_{oc}, at the receiver output.

Throughout this analysis it is understood that the post-detection noise is "available" at the point where it is being measured; usually, noise-power measurements are made for impedance-matched conditions and within a particular and carefully defined effective bandwidth. In FM systems, the post-detection noise level will depend upon its position in the baseband-signal spectrum, and will be a maximum in the top channels of the baseband signal, when the receiver-input signal is above the threshold point. If noise power levels are measured at a point beyond the receiver output, such as the output terminals of a multiplex unit, due allowances must be made for the noise properties of the intervening units and the power gains or losses in these units.

FM RECEIVER-OUTPUT SIGNAL AND NOISE IN THE TOP CHANNELS OF THE BASEBAND SIGNAL

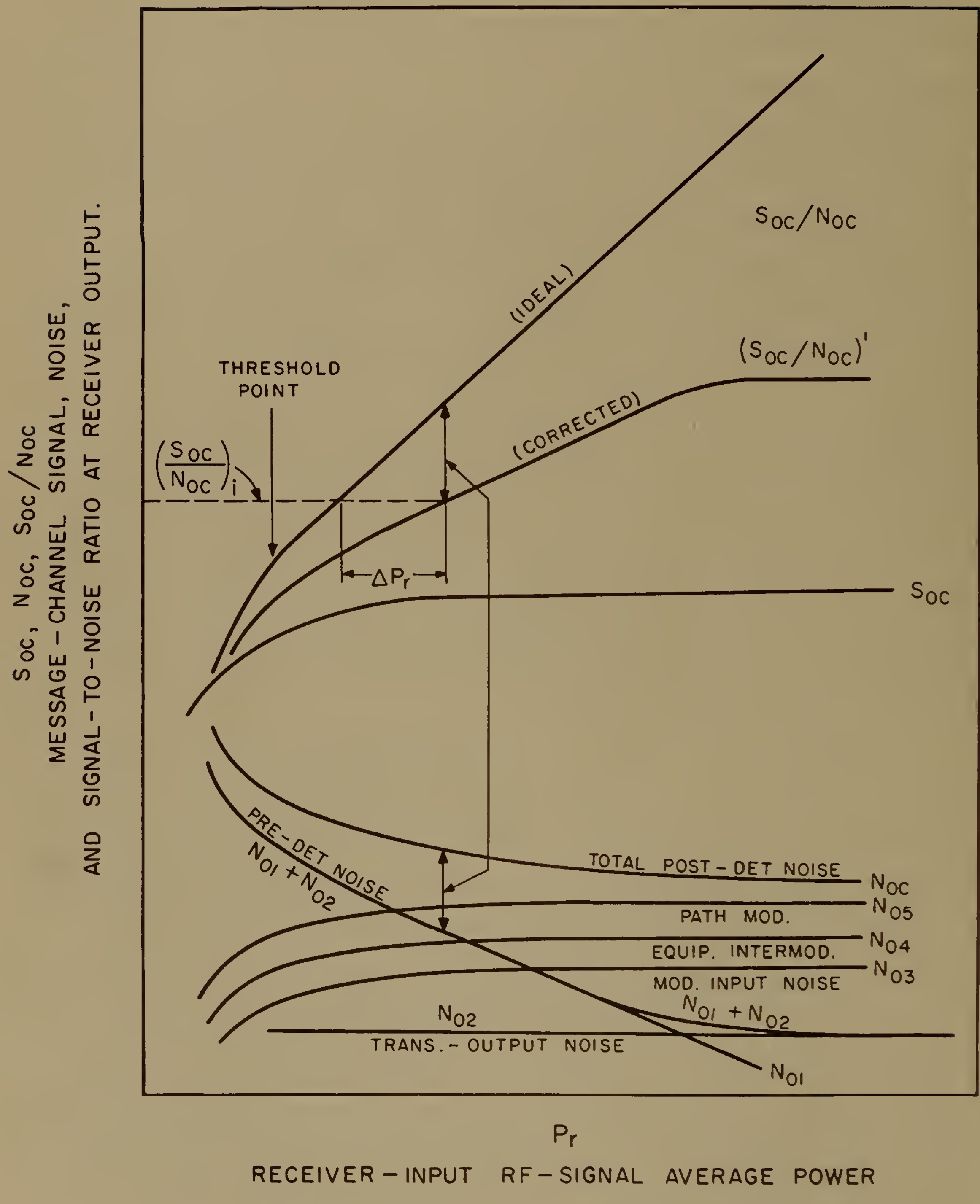


Figure 4-A

4.3.1 Post-Detection Noise Due to Pre-Detection Amplitude Noise

The effect of pre-detection noise, item (1) above, on the post-detection noise level, shown by the N_{01} curve in figure 4-A, is considered in detail in sections 4.5 and 4.6. The design equations and the design curves developed in this report are based primarily on the effects of the pre-detection noise with the understanding that the transmitter-output (amplitude) noise, N_{at}, was not included in the pre-detection noise. The reason for this omission may be seen from an inspection of figure 4-A, where the N_{01} curve gives the receiver-output noise, which is due to the pre-detection noise, N_{if}, excluding the transmitter-output noise. The receiver-output noise which is due to the transmitter-output noise is dependent upon the transmitter-output signal-to-noise ratio and is shown by the N_{02} curve. It is evident that the receiver-output noise, N_{02}, effectively limits the maximum value of the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}.

Modifications of the receiver-performance characteristics for the effects of that portion of the post-detection noise which is due to items (2), (3), and (4), are considered below.

4.3.2 Post-Detection Noise Due to Noise at the Modulator Input

The design equations and curves in this report were based on the assumption that the signal-to-noise ratio in the modulating (baseband) signal, at the modulator input, was very much greater than the required (baseband) signal-to-noise ratio at the radio receiver output terminals. The net effect of noise at the modulator input is to place an upper limit to the receiver-output signal-to-noise ratio. In general, we cannot expect to obtain a signal-to-noise ratio at the receiver output higher than the signal-to-noise ratio at the modulator input. The post-detection or receiver-output noise-power level, due to noise N_m, at the modulator input, is shown by the N_{03} curve in figure 4-A.

Estimated and measured system-performance characteristics must involve the baseband signal-to-noise ratio at the modulator input, as a factor which limits the radio receiver-output signal-to-noise ratio. The actual value of the receiver-output baseband signal-to-noise ratio will be lower than the modulator-input baseband signal-to-noise ratio, and will depend upon the signal-to-noise "degradation factor" of the radio-frequency section of the communication system.

4.3.3 Equipment-Intermodulation Post-Detection Noise

Intermodulation noise is generated within the equipment whenever the limits of linear modulation or demodulation are exceeded and when the receiver IF effective bandwidth, B_{if}, is insufficient. For FM systems, the post-detection noise power level, due to equipment intermodulation noise, is shown in figure 4-A, by the N_{04} curve. Hence, the post-detection (equipment) intermodulation noise-power levels may be estimated from the characteristics of the modulator, the demodulator, and the passband characteristics of the receiver IF circuits, using the material in Section 3.

For amplitude modulation, the required modulator characteristic is in terms of the sinusoidal modulating-signal power level, P_{ms}, versus the percent modulation. The percent of time during which 100% amplitude modulation is exceeded, for particular power-levels of a noise-type modulating signal, P_{mn}, may be obtained from figure 3-5. The percent of time during which 100% amplitude modulation is exceeded may be used as an estimate of the percent of the modulating signal power which is converted to noise power. By arbitrarily assigning a limit to the percent of time during which 100% amplitude modulation may be exceeded, the value for D_{ss} may be obtained from figure 3-5. At the same time, the corresponding value for the average power level of the noise-type modulating signal, P_{mn}, may be obtained from figure 3-5. It is evident from figure 3-5 that intermodulation noise in the modulator is dependent upon the factor D_{ss}; low values of intermodulation noise require high values for D_{ss}, and vice versa.

If the amplitude demodulator is linear to 100% amplitude modulation of the RF signal, it will not add appreciable intermodulation noise power to the system.

For frequency modulation, the required modulator characteristic is in terms of the sinusoidal modulating-signal power level, P_{ms}, versus the peak frequency deviation, ΔF_s. The modulator characteristic should be linear for values of ΔF somewhat greater than ΔF_s in order to prevent excessive "distortion" of the RF spectrum during the time when the peaks of the noise-modulating signal occur. This distortion is comparable to that produced in an amplitude modulator when the modulating-signal power level exceeds P_{ms} (100%); the net effect results in distortion of the baseband signal at the demodulator output, and the production of receiver-output noise, shown by the N_{04} curve.

The relationship between P_{ms}/P_{mn}, for FM systems, and the required RF spectrum bandwidth, B_{rf}, is shown in figure 3-4. The effect of "bandwidth clipping" in the receiver, on the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, is shown in figure 3-2. By combining the modulator characteristic with the data in figures 3-2 and 3-4, it is possible to estimate the relationship between the increase of the post-detection noise level or the degradation of the receiver-output signal-to-noise ratio, and the modulating-signal power level, P_{mn}. The results will be relative to the FM receiver threshold point and the bandwidth ratio, B_{rf}/B_{if}.

If the FM demodulator characteristic is linear to $\pm \Delta F_n$ it will not add an appreciable amount of intermodulation noise; however, if this condition does not hold the demodulator characteristics should be included in the above analysis of intermodulation noise effects.

The portion of the receiver-output noise power, N_{04}, due to modulator-transmitter-receiver intermodulation effects, may also be measured by means of a transmitter-receiver back-to-back test. For this test, the receiver performance characteristic is first measured using a modulated RF signal having a negligible amount of intermodulation noise; this condition is attained by using a comparatively low value for the modulating-signal power level, P_m. For multichannel systems, the modulating signal should be white noise which is transmitted through a band-rejection slot filter; noise and signal measurements are made at the receiver output within the clear channels or spectrum "slots." Meaningful results will only be obtained if the entire system is properly adjusted for normal operation. The modulating-signal power level, P_m, is then increased in steps, and either or both the receiver-output increase in noise or the change in the signal-to-noise ratio are measured for each modulating-signal power level, P_{mi}, and for a range of receiver-input RF signal power levels, P_r. From the results of these tests it is possible to determine the relationship between the modulating-signal power level, P_m, and the amount of intermodulation-noise power; in terms of the received RF signal power, P_r, required to produce a given receiver-output signal-to-noise ratio--for particular values of the modulating-signal power,

P_m. For FM systems the results would be similar to those shown in figure 4-A.

Since the design curves in this report were measured and calculated under conditions of negligible intermodulation noise power; the above information may be used to make the proper corrections for the effects of equipment intermodulation noise in the design curves; see section 4.3.6. Alternatively, departures of system-performance measured results from design-curve values may be attributed in part to equipment intermodulation-noise effects, and corrections may be made from the above data, after due allowances have been made for all other factors.

The results of the above intermodulation-noise tests will include intermodulation effects in both the modulator-transmitter unit and the receiver. Furthermore, the relationship between the modulating-signal power level, P_m, and the system intermodulation noise power will be obtained; these equipment-performance data should be used as guides for setting up system-performance tests and for operation of the particular system under test. The above types of tests may also be conducted on a complete system, utilizing an over-the-path signal. In fact, this method of testing is necessary in order to determine the performance characteristics of the combiner and to obtain measurements of the path-modulation noise; see section 4.3.4. A complete-system test is not as convenient to conduct as an equipment back-to-back test, primarily because of the lack of control of the received RF signal, in the former case.

4.3.4 Post-Detection Path-Modulation Noise

Radio wave path-modulation effects on the transmitted RF signal are due to "frequency selective fading" of the RF-signal spectrum components, which results in distortion of the received-signal spectrum and consequent distortion of the baseband-signal components. Path modulation arises primarily because of the differences in the relative delays of the (received) RF-signal spectrum components due to multipath conditions. In a multichannel FM system, the effects of path modulation appear at the receiver output as interchannel modulation noise. In an AM system the effect of path modulation on the receiver-output noise is somewhat different from that in an FM system; however, the overall effects on system performance are similar for both types of systems.

The severity of selective fading increases with the RF-spectrum bandwidth and hence determines the "transmission bandwidth capability" of a radio-wave path. Selective fading, and therefore, transmission bandwidth capability, depend also on the type of RF signal, such as HF via ionospheric reflection, tropospheric, line of sight, etc.

A measure of the degree or the amount of selective fading may be obtained from the cross-correlation of the amplitudes of the spectral components of the received RF-signal. However, a more direct and useful measure of the transmission-bandwidth of a radio-wave path may be obtained by measuring the path-modulation noise in the baseband signal at the receiver output [Clutts, C. E., et al, 1960]. The power level of the path-modulation noise will depend upon the RF-signal spectrum bandwidth, B_{rf}, and the various system-path parameters, such as path length, antenna beamwidth, and to some extent, the order of diversity and the RF-signal frequency. This noise is time-varying and hence its study requires statistical treatment.

Path-modulation acts the same as signal modulation and sets a lower limit to the receiver-output noise and hence, determines an upper limit to the receiver-output signal-to-noise ratio, regardless of the power transmitted or received. For a given system, RF bandwidth, and type of RF signal, any apparent dependency of the median receiver-output path-modulation noise level on the median received RF signal level is due primarily to variations of the type of received RF signal. However, the path-modulation noise level at the receiver-output will also appear to be dependent upon the median received RF signal, for RF-signal levels below the receiver threshold point; see figure 4-A. Hence, the net effect of path-modulation noise is similar to the effects of either noise at the modulator input, or equipment intermodulation noise. Receiver-output noise power, due to path-modulation effects, is shown by the N_{05} curve in figure 4-A.

4.3.5 Total Post-Detection Noise

The total receiver-output noise-power per message-signal channel, relative to the received RF-signal, P_r, is shown by the N_{oc} curve in figure 4-A. The absolute levels of the noise-power curves, N_{01} to N_{05}, inclusive, are dependent upon the various factors described above. These curves are positioned as shown in order to indicate their relative effect on the level of the noise curve, N_{oc}, and the resultant effect on the receiver-output signal-to-noise ratio curve, S_{oc}/N_{oc}. The shapes of the curves are dependent upon the type of modulation-demodulation employed and include the effects of agc (automatic gain control) and limiting, in the receiver. For the case where system back-to-back and over-the-path tests are made, the curves in figure 4-A may be measured accurately. However, it should be understood that the absolute levels of the post-detection or receiver-output noise powers, N_{01} to N_{05}, inclusive, are dependent upon power gains (or power losses) within the receiver, therefore, these noise-power levels are meaningless unless they are referred to the post-detection or receiver-output signal-power level, S_{oc}. Hence, estimates and measurements of receiver-output noise power levels should only be made for conditions where signal-power levels are properly adjusted throughout the system.

4.3.6 Effect of Post-Detection Noise on Receiver Characteristics and Method of Correcting System-Design Curves

The actual performance characteristic of the receiver will be as indicated by the "corrected" receiver-output signal-to-noise ratio curve, $(S_{oc}/N_{oc})'$, in figure 4-A. The corrected curve is obtained from the "ideal" curve by reducing the ideal-curve values of S_{oc}/N_{oc} an amount equal to the noise-power-level increase (in db units) between the N_{01} curve and the N_{oc} curve; for particular values of P_r. These corrections also apply to the design curves in this report, which are identical to the ideal curve shown in figure 4-A. In practice, corrections to the design curves would be in terms of the additional required receiver-input RF-signal power, ΔP_r. This additional power would be required to attain a particular value of receiver-output signal-to-noise ratio, $(S_{oc}/N_{oc})_i$, or a particular grade of service. The actual value of ΔP_r would be approximately equal to the noise-power-level difference between the N_{01} curve and the N_{oc} curve. Note that this type of correction is limited by the "levelling-off" point of the corrected receiver-characteristic curve; beyond this point, no improvement in the receiver-output signal-to-noise ratio will be obtained by increasing the receiver-input RF signal power, P_r.

The net effect of post-detection noise on the performance of a radio communication system may be determined from the corrected receiver characteristic curve. The use of the corrected receiver characteristic curve results in an accurate determination of the distribution of the time-varying post-detection (baseband) signal, noise and signal-to-noise ratio; these distributions are very important factors in the performance of either single-link or tandem-link radio systems. The performance of single-link and tandem-link radio systems should be determined from the distribution of the time-varying post-detection (combined) baseband signal-to-noise ratio. Any attempt to estimate the performance of radio systems solely in terms of the pre-detection signal-to-noise ratio, is likely to lead to the erroneous conclusion that this type of solution is complete and general.

For single-link systems, the characteristics of the post-detection or receiver-output signal, noise, and the signal-to-noise ratio would be obtained by the methods outlined above.

In tandem-link systems, the signal and the noise at the intermediate points in the system may be "transferred" either at the pre-detection point or at the post-detection point. If the signal and the noise are transferred at the pre-detection point, signal de-modulation is not involved at the intermediate points in the system and the receivers are used as linear repeaters at these points. Hence, in this type of system the characteristics of the intermediate-point pre-detection combined signal-to-noise ratio will influence the transmitted and the received (RF) signal-to-noise ratio. However, in a linear-repeater type of system, the system-intermodulation noise which is developed within each of the tandem links, is transferred through the system and arrives at the output of the terminal demodulator. Therefore, in a tandem-link system which employs linear repeaters at the intermediate points, the total system-intermodulation noise at the terminal-demodulator output may be approximated by the sum of the system-intermodulation post-detection noise in each of the tandem links.

In tandem-link systems employing demodulation at the intermediate points in the system, the characteristics of the post-detection or receiver-output signal and noise at each intermediate point would be determined by the methods outlined above for single-link systems. The total receiver-output noise, N_{oc}, at an intermediate point in the system would be applied to the input terminals of a modulator, and hence this noise power (N_m, see section 4.3.2) would influence the system-intermodulation noise developed in the succeeding link of the system. Under these conditions, the performance of each succeeding link in a tandem-link system becomes progressively worse because of the progressive deterioration of the signal-to-noise ratio at the modulator input terminals.

Therefore, the characteristics of the post-detection noise at the output terminals of a tandem-link system will depend somewhat upon whether the intermediate links are operated as

(a) linear repeaters, which transfer the received signal and noise at each intermediate point in the system to another portion of the RF spectrum; or

(b) as receiver-demodulator units which produce the baseband noise (and signal) at each receiving-point in the system.

In either case, the characteristics of the post detection noise may be determined by the methods outlined in section 4.3.

The case of re-construction of the message signal at each intermediate point in a tandem-link system is equivalent to single-link operation, and the total message errors would be equal to the sum of the message errors in the individual links.

4.4 System Parameters Which Affect the Message-Channel Signal-to-Noise Ratio at the Radio Receiver Output

In the previous Subsections we discussed the effects of predetection and post-detection noise on the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}. In this Subsection we will consider the relationship between the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the radio receiver output, and the following factors: (1) the average total power of the composite modulating signal, P_m, (2) the individual message-channel average signal-power level, p_m, in the modulating baseband signal, relative to the total average power level of the modulating baseband signal, P_m, and (3) the message-signal spectrum bandwidth, B_c. It is shown later that these parameters determine the per message-channel signal-to-noise ratio at the radio receiver output, in terms of the total RF signal power-to-total noise power ratio and the RF spectrum bandwidth, B_{rf}, at the receiver input.

It should be noted that the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the message-signal decoder-unit input, is related to the radio receiver output total signal-to-noise ratio, S_o/N_o, through the characteristics of the receive-multiplex equipment. The net result of this situation is that for some types of message-signal multiplexing equipment [Landon, 1948] , the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, cannot be measured directly in the radio receiver output baseband signal. For SSB frequency-division multiplex, being considered in this paper, S_{oc}/N_{oc} may be measured directly in the receiver output baseband signal, in terms of the message-signal spectrum bandwidth and its position in the baseband signal. For example, in FM multiplex, S_{oc}/N_{oc} cannot be measured directly in the radio receiver-output baseband signal, and must be determined at the output terminals of the receive-multiplex equipment.

Hence the multiplex receiver characteristics must be included in the analysis, in terms of the total baseband signal-to-noise ratio, S_o/N_o, at the multiplex receiver input, and the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the multiplex receiver output. The above considerations make proper allowances for any change in the message-channel signal-to-noise ratio which might be attributed to the message-signal multiplexing system.

4.4.1 Dependence of Receiver Output Signal-to-Noise Ratio on the Modulating-Signal Power Level, P_m.

In SSBSC-AM systems the power level of the modulating signal, P_m, affects the receiver output signal-to-noise ratio only in terms of the intermodulation noise generated within the modulator. The dependence of the receiver output signal-to-noise ratio on the average power level, P_{mn}, of the noise-type modulating signal is referred to the power level, P_{ms} (100%) of a sinusoidal modulating signal; where P_{ms} (100%) is the modulating-signal power level required to provide 100% amplitude-modulation of the RF carrier signal. The factor, P_{mn}/P_{ms} (100%), may be obtained from figure 3-5.

Note that for a sinusoidal modulating signal, P_m or, P_{ms}(100%) for SSBSC-AM systems and P_{ms} for FM systems, would be obtained from the modulator characteristic. For a noise-type modulating signal, P_m or, P_{mn} would be obtained from figure 3-4 for FM systems, and from figure 3-5 for SSBSC-AM systems.

In FM systems, the modulating-signal power level, P_m, determines the required RF-signal spectrum bandwidth, B_{rf}. The relationship between the factor, P_{mn}/P_{ms}, and B_{rf} may be obtained from figure 3-4. Note that for the FM case, the reference power level is P_{ms} and is not a limiting factor as is P_{ms}(100%), which is used in the SSBSC-AM system.

A value for B_{rf} is required in order to establish the noise-power level at the radio receiver input, see Section 4.2. The radio receiver IF bandwidth, B_{if}, is associated with B_{rf}, and hence these factors influence the radio receiver output signal-to-noise ratio. The relationship between the receiver output signal-to-noise ratio, S_o/N_o, and B_{rf}, for FM systems, is derived fully in Sections 3.2, 4.6, and Appendix C. In SSBSC-AM systems, the radio-frequency signal spectrum bandwidth is independent of the modulation index, and is approximately equal to the highest frequency in the modulating baseband signal.

The portion of the radio-receiver output noise power which is related to the modulating signal power, P_m, is known as "intermodulation noise". This intermodulation noise is generated in a radio communication system whenever the limits of linear modulation or demodulation are exceeded, in either or both the radio transmitter and the radio receiver. It is difficult to estimate intermodulation noise power levels and accurate results must be based on the equipment performance; however, it is usually assumed that the intermodulation noise power level is equal to the thermal noise power level, for practical system design. See section 4.3 on equipment intermodulation effects.

4.4.2 Dependence of Receiver Output Message-Channel Signal-to-Noise Ratio, S_{oc}/N_{oc}, on the Message-Channel Signal-Power Loading Ratio, P_m/p_m, in the Modulating Baseband Signal.

The message-signal power, p_{mi}, which must be allocated to each individual message signal, in the modulating baseband signal, depends upon the required signal-to-noise ratio at the radio receiver output. This required radio-receiver output signal-to-noise ratio depends upon the specified grade of service or message error rate, for a particular message. The relationship between the radio receiver output message-channel signal-to-noise ratio and the message error rate is discussed in Section 5.

In this work it is assumed that the individual message-signal average power level in the modulating baseband signal, must be adjusted so as to give a specified or required average message-channel signal-to-noise ratio at the receiver output. Under these conditions (see Appendix B), the message-channel signal-power loading ratio in the modulating baseband signal is:

$$\frac{P_m}{p_{mi}} = \frac{M_A\left[B_c\left(\frac{S_{oc}}{N_{oc}}\right)\right]_A + \ldots + M_I\left[B_c\left(\frac{S_{oc}}{N_{oc}}\right)\right]_I + \ldots + M_N\left[B_c\left(\frac{S_{oc}}{N_{oc}}\right)\right]_N}{B_{ci}\left(\frac{S_{oc}}{N_{oc}}\right)_i} \tag{4.9}$$

where, p_{mi} = Average power level of a particular message signal in the modulating baseband signal.

P_m = Average total power level of the composite modulating baseband signal, under conditions when a specified number of simultaneously-active message channels are operating. For sinusoidal-signal modulation, $P_m = P_{ms}$, and for noise-signal modulation, $P_m = P_{mn}$.

B_{ci} = A particular message-signal channel bandwidth in c/s, at the receiver output. This channel bandwidth is assumed to be equal to the spectrum bandwidth of the message signal.

S_{oc} = A particular message-channel signal average power level, at the radio receiver output, or a related message-channel signal average power level at the decoder-unit input.

N_{oc} = Average noise-power level of a particular message-channel having a bandwidth B_c, at the radio receiver output or, a related noise-signal average power level at the decoder-unit input.

$\left(\frac{S_{oc}}{N_{oc}}\right)_i$ = Required radio receiver output signal-to-noise ratio, or required decoder-unit input signal-to-noise ratio plus a degradation factor for the multiplex receiver, in a particular message-signal channel, for a specified allowable average message error rate.

M_A = Number of simultaneously-active message signals each of which requires the <u>same</u> value for the product $\{B_c(S_{oc}/N_{oc})\}_A$.

M_I = Ditto, for the product $\{B_c(S_{oc}/N_{oc})\}_I$

M_N = Ditto, for the product $\{B_c(S_{oc}/N_{oc})\}_N$.

As noted in Subsection 4.3, for some types of message-multiplexing systems, it may be necessary to determine the message-channel signal-to-noise ratio at the output of the multiplex receiver because of the impossibility of measuring this ratio directly in the receiver-output baseband signal. Conversely, at the transmitting end of the system for some types of message-signal multiplexing, it may not be possible to measure the modulating signal power levels directly in the modulating baseband signal; under these conditions, p_m and P_m must be measured at the multiplex transmitter input. This situation will not arise where SSB frequency-division multiplex equipment is employed.

In order to use (4.9) it is necessary to determine the required message-channel signal-to-noise ratio, S_{oc}/N_{oc}, and the message-signal bandwidth B_c, at the radio receiver output. This required information would be obtained from measured or estimated data such as shown in figure 5-3 of Section 5.

The specified message error rate depends upon the type of message. Consequently, the required receiver-output signal-to-noise ratio will be different for each type of message. However, if different types of messages require the same value for the product, $\{B_c(S_{oc}/N_{oc})\}_I$, then these messages may be grouped in the same class, M_I.

Proper methods for adjusting the power levels of the individual message signals, at the terminals of the multiplexing equipment, are outlined in the manuals for this equipment; the instructions in these manuals should be followed carefully in order to avoid overloading of the multiplex circuits, and thereby prevent consequent intermodulation distortion. The power levels p_{mi} and P_m must be considered in terms of their effect at the radio-transmitter modulator input; if the multiplex baseband power level is inadequate, a power amplifier or an attenuator is required at this point in the system.

4.5 Performance Characteristics of SSBSC-AM Radio Receivers

In this section the performance of SSBSC-AM (single sideband suppressed carrier amplitude modulation) radio receivers is defined in terms of the signal-to-noise ratio per message channel at the receiver output, S_{oc}/N_{oc}, versus: (a) the total received RF signal power and (b) the total available noise power at the radio receiver "front-end" input. In order to obtain accurate estimates of the radio receiver performance it is necessary to consider the statistical characteristics of the received RF signal power and the noise power, at both the receiver input and at the receiver output. Methods for estimating the noise level and the characteristics of the noise at the receiver input are outlined in Section 4.2 and Appendix A. It is assumed here that the total noise-power at the receiver input approximates "white" noise and that the power level of this total noise is proportional to the RF spectrum bandwidth accepted by the receiver.

The effects of system-intermodulation noise are not included in the analysis of receiver performance, at this point, and hence, the results will be in terms of "ideal" receiver performance. Modifications required for the effects of system-intermodulation noise on the ideal receiver-performance characteristics, are outlined in section 4.3. These remarks apply also to FM-receiver performance, as developed in section 4.6.

Since SSBSC-AM receivers are linear, it follows that the characteristics of the receiver-output noise are similar to the characteristics of the noise at the receiver input, within the limits of receiver linearity.

The ideal SSBSC-AM receiver does not exhibit a "threshold effect", that is, there is a linear relationship between the receiver-input signal power level and the receiver-output signal-to-noise ratio. The following procedures for determining SSBSC-AM receiver performance are also applicable in case the receiver characteristic curve is non-linear, and is known.

In this work the relationship between the receiver output signal-to-noise ratio and the receiver input-signal power level was calculated for SSBSC receivers, assuming that the receiver-input RF signal was steady, that is non-varying with time. To obtain the receiver-performance characteristic for a time-varying receiver-input RF signal, the above steady receiver-input signal receiver-output signal-to-noise ratio characteristic was combined with the distribution of the time-varying receiver-input signal; see Appendix D.

4.5.1 Receiver-Input Radio-Frequency Signal Power Requirements for SSBSC-AM Systems Using Steady Received Signals.

For an ideal SSBSC-AM receiver we have,

$$\frac{S_o}{N_o} = \frac{P_{if}}{N_{if}} \tag{4.10}$$

where, P_{if} = Total of the average IF signal power at the receiver demodulator input, in watts.

N_{if} = Total of the average noise power at the receiver-demodulator input, for IF bandwidth B_{if}, in watts.

S_o = Total of the average baseband signal power at the demodulator output, in watts

N_o = Total of the average noise power at the demodulator output for baseband bandwidth $\approx B_{if}$.

On a per message-channel basis, with a message-channel signal-power loading ratio of P_m/p_m in the modulating baseband signal at the radio transmitter (see Section 4.4.2) we have,

$$\frac{S_{oc}}{N_{oc}} = \frac{S_o}{N_o} \times \frac{p_m}{P_m} \times \frac{B_{if}}{B_c} \tag{4.11}$$

or, from (4.10) and (4.11)

$$\frac{S_{oc}}{N_{oc}} = \frac{p_m}{P_m} \times \frac{B_{if}}{B_c} \times \frac{P_{if}}{N_{if}} \tag{4.12}$$

where, S_{oc} = Signal power in the receiver output, per message-signal channel, in watts.

N_{oc} = Noise power in the receiver output in a message-signal bandwidth, B_c, in watts.

p_m = Average power level of a message-signal in the modulating baseband signal, in watts.

P_m = Total average power level of the composite modulating baseband signal; determined under conditions when a specified number of simultaneously-active message signals are operating, in watts.

B_c = Message-signal bandwidth, in c/s.

$B_{if} = B_{rf}$ = Effective bandwidth at the radio receiver IF output, or demodulator input.

In an SSBSC-AM system the radio-frequency spectrum bandwidth B_{rf}, and also the receiver IF bandwidth, B_{if}, are approximately equal to the highest frequency, f_m, in the modulating baseband signal; hence,

$$B_{rf} = B_{if} \approx f_m \tag{4.13}$$

Combining (4. 12) and (4.13) and rearranging terms, we obtain,

$$\frac{S_{oc}}{N_{oc}} \times \frac{B_c}{f_m} \times \frac{P_m}{p_m} = \frac{P_{if}}{N_{if}} \tag{4.14}$$

The term, P_{if}/N_{if},* in (4.14) is the required predetection signal-to-noise ratio, associated with a specified value of S_{oc}/N_{oc} at the receiver output, for particular values of the parameters: f_m, B_c, and P_m/p_m. However, in radio communication system design and test work it is much more convenient to deal with the radio-frequency signal power and the noise power at the radio receiver input terminals than it is to measure P_{if}/N_{if} in the IF section of the receiver. Hence the form of (4.14) was modified in the following derivations to include the total received signal power, P_r, and total received noise power. First,

$$P_{if} = G_r M_{ir} P_r \tag{4.15}$$

*The symbol R is sometimes used to designate $\frac{P_{if}}{N_{if}}$ or $10 \operatorname{Log}_{10} \frac{P_{if}}{N_{if}}$.

and (see 4.8),

$$N_{if} = G_r M_{ir} \{T_A + T_{er}\} k B_{if}$$

or, since $B_{if} \approx f_m$ for the SSBSC-AM case,

$$N_{if} = G_r M_{ir} \{T_A + T_{er}\} k f_m \qquad (4.16)$$

therefore,

$$\frac{P_{if}}{N_{if}} = \frac{P_r}{k(T_A + T_{er}) f_m} \qquad (4.17)$$

where,

G_r = Power gain of the cascaded RF and IF linear sections of the SSBSC radio receiver

P_r = Total average radio-frequency signal power available at the receiver input, in watts

k = Boltzman's constant = 1.3804×10^{-23} joules per degree Kelvin

M_{ir} = Defined by (A.16); see Appendix A.

f_m = Highest frequency in the modulating signal

T_A = Effective receiving-antenna system output noise temperature, degrees Kelvin. See Section 4.2 and Appendix A on noise at receiver input.

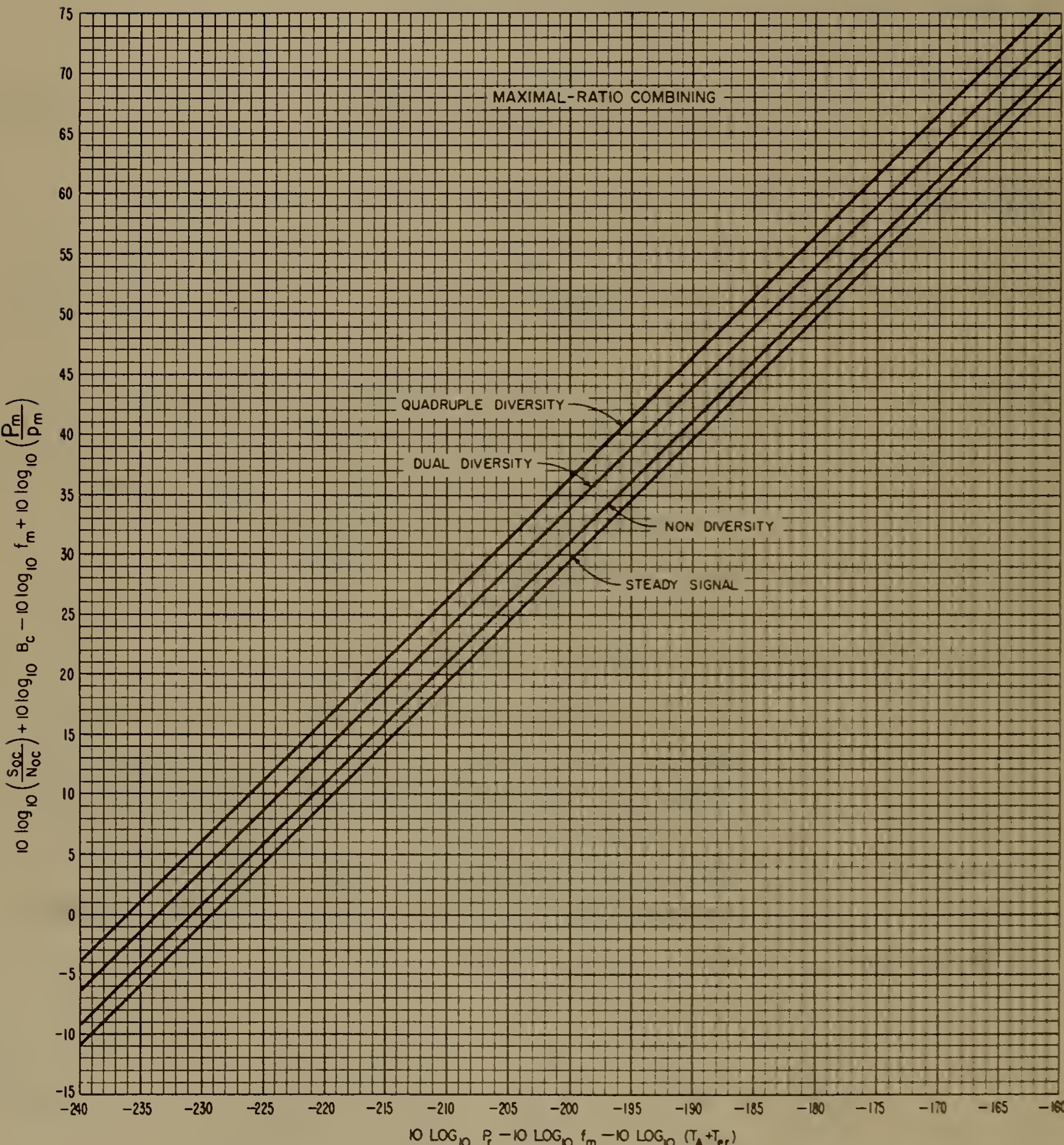
T_{er} = Effective receiver-input noise temperature, degrees Kelvin [IRE Comm. on Noise, 1960] .

Substituting (4.17) in (4.14) we obtain,

$$\frac{S_{oc}}{N_{oc}} \times \frac{B_c}{f_m} \times \frac{P_m}{p_m} = \frac{P_r}{k(T_A + T_{er}) f_m} \qquad (4.18)$$

The factor f_m was retained in both sides of (4.18) in order to obtain power-level values of P_r on the abscissa scale of the design curves shown in figure 4-2, relative to the total noise power, $k(T_A + T_{er}) f_m$, at the receiver input.

SSBSC-AM RADIO RECEIVER PERFORMANCE CURVES
STEADY AND RAYLEIGH-FADING TYPES OF RECEIVED SIGNAL



(STEADY SIGNAL) $\frac{P_r}{f_m(T_A+T_{er})} = \frac{\text{AVERAGE OF THE TOTAL SIGNAL POWER, WATTS}}{\text{AVERAGE OF THE TOTAL RELATIVE NOISE POWER}}$, AT RECEIVER INPUT

(TIME-VARYING SIGNAL) $\frac{P_r}{f_m(T_A+T_{er})} = \frac{\text{SAMPLING-PERIOD MEDIAN OF THE NON-DIVERSITY AVERAGE TOTAL SIGNAL POWER, WATTS}}{\text{SAMPLING-PERIOD AVERAGE OF THE TOTAL RELATIVE NOISE POWER}}$, AT RECEIVER INPUT

Figure 4-2

Equation (4.18) may be used to calculate the required radio-frequency signal power level, P_r, at the (SSBSC-AM) radio receiver input, to yield a specified receiver-output signal-to-noise ratio, S_{oc}/N_{oc} --for particular values of the parameters: f_m, P_m/p_m, B_c, T_A, and T_{er}. However, it is more convenient to use the information contained in this equation in a graphical form. Hence, (4.18) was plotted as shown in figure 4-2 for a time-invariant or steady RF signal at the receiver input. Scale values in figure 4-2 are normalized in terms of the parameters in (4.18).

To obtain quantitative values for P_r from figure 4-2, correct values must be used for S_{oc}/N_{oc}, f_m, and p_m/P_m. The required value of S_{oc}/N_{oc} is obtained from data similar to those shown in figure 5-3. f_m is taken to be equal to the highest frequency in the modulating baseband signal, in c/s. The message-channel signal-power loading ratio, P_m/p_m, may be calculated by means of (4.9). The factor T_A may be calculated from (4.3). The radio-receiver input noise temperature, T_{er}, depends largely upon the carrier (radio) frequency and the type of receiver; T_{er} may be calculated by means of (4.6). Procedures for using the above information to estimate the performance of SSBSC-AM systems are outlined in Section 6.

The predetection signal-to-noise ratio, $\frac{P_{if}}{N_{if}}$, which corresponds to a particular value of P_r, may be obtained from (4.17). It should be noted that (4.18) involves the modulation index, m_a, which is tacitly assumed to be unity, for SSBSC-AM systems, that is, $P_{ms}(100\%)$ is used as the reference level of the modulating-signal power. If the modulation index exceeds unity the resultant (intermodulation) distortion will appear as additional noise at the radio receiver input, and at the receiver output and will degrade the signal-to-noise ratio at the receiver output, for all values of receiver input power, P_r.

4.5.2 Receiver-Input Radio-Frequency Signal Power Requirements for SSBSC-AM Systems Using a Rayleigh-Fading Type of Received Signal.

The value of the required receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, will not be the same for time-varying received RF-signal amplitudes as for steady received signals for the same message error rate. Also, S_{oc}/N_{oc} will depend upon the order of diversity. The value of the ratio, S_{oc}/N_{oc}, to be inserted in (4.9) may be obtained from either steady-signal or varying-signal data similar to those shown in figure 5-3. However, each different type of message will require a different set of curves similar to those shown in figure 5-3.

The curves in figure 4-2 for non-diversity, dual diversity and quadruple diversity were obtained by combining the SSBSC-AM steady-signal characteristic curve (in figure 4-2) with the (maximal-ratio) combiner characteristics given in figure D-1b, Appendix D. Details of the method used to combine these characteristic curves are given in Appendix D. It should be noted that the abscissa-scale values in figure 4-2 for the fading-type signals refer to the median of the non-diversity received total RF signal average power, P_r, while the ordinate-scale values yield the average of the time-varying signal-to-noise ratio, S_{oc}/N_{oc}, at the receiver output; all other parameters are the same as for the "steady-signal" case. The 1.59 db displacement between the "steady-signal" curve and the "non-diversity" curve in figure 4-2 is due to the fact that the average value of the Rayleigh-distributed received RF signal power, P_r, is $(1.59 + 10 \log_{10} n)$db greater than the median value of P_r; where n is the order of diversity. Dual and quadruple-diversity curves are similarly displaced. Note that maximal-ratio combiner performance is assumed.

In the above derivations no allowance was made for degradation of the signal-to-noise ratio in the Multiplex unit; the effect of this degradation is to increase the required signal-to-noise ratio at the receiver output. In the absence of information on this degradation factor, it may be assumed to be approximately 1 to 2 db for SSB-SC frequency division multiplex equipment.

Procedures for using the information in figure 4-2, to estimate the performance of SSB systems, are outlined in Section 6 for a Rayleigh-fading type of received signal and for non-diversity, dual-diversity and quadruple-diversity.

4.6 Performance Characteristics of FM Radio Receivers

The performance of an FM (frequency modulated) radio receiver may be defined in terms of the signal-to-noise ratio per message channel at the receiver output versus: (a) the total available RF-signal power, (b) the total available noise power at the receiver front-end input, and (c) the radio-frequency signal spectrum bandwidth; other system parameters may be considered as normalizing factors.

A method for determining the above relationships for FM systems follows; this method is based on a combined mathematical and graphical analysis of typical characteristic curves of FM radio receivers. These curves consist of an upper linear portion, and a lower non-linear portion called the "threshold" region. In the work that follows, the slope and the position of the upper linear portion of the receiver curve were calculated and the curvature of the threshold region was obtained from measured results on several different types of FM radio receivers; however, the threshold region portion of the curve may also be calculated [Stumpers, 1948]. The position of the threshold-level point was assumed to correspond to a signal-to-noise power ratio (P_{if}/N_{if}) of 10 db, at the input to the receiver limiter-discriminator.

Accurate estimates of the FM radio receiver performance requires quantitative data on the statistical characteristics of the received RF signal and the noise power, at the receiver input, and corresponding data on the signal power and the noise power at the receiver output.

4.6.1 FM Radio Receiver Characteristic Curves

Throughout this work, covering the development and use of the FM radio receiver characteristic curves, the following ideas and assumptions are used as guides:

(a) The modulating signal is sinusoidal and its frequency is equal to the highest frequency in the modulating baseband signal. Furthermore, all of the power in the modulating baseband signal is contained in this single sinusoidal modulating signal. The work is then extended to cover receiver performance for modulating signals at lower frequencies. Receiver performance is shown to be improved, <u>above the threshold region</u>, directly proportional to the square of the ratio of the highest baseband-signal frequency to the frequency of the modulating signal being considered.

(b) The radio-frequency spectrum bandwidth required for a noise-type modulating signal is then calculated in terms of the results obtained for the conditions in (a).

(c) Receiver-input noise is unweighted, that is, its power spectrum is uniform or flat.

(d) Limiting in the radio receiver is used, preceding the frequency discriminator.

(e) The radio receiver IF bandwidth, B_{if}, is assumed to be only wide enough to pass the radio frequency spectrum, B_{rf}; that is, $B_{if} = B_{rf}$. If "bandwidth-compression" type of receivers are used, then $B_{if} < B_{rf}$. The effects of bandwidth-compression techniques on the system design curves are considered.

(f) The effects of pre-emphasis and de-emphasis techniques are considered.*

(g) Companders and limiters (for speech messages) are not considered.

4.6.1.1 Linear Region of FM Receiver Characteristic Curves

For the condition where the pre-detection signal-to-noise ratio, P_{if}/N_{if}, exceeds 10 db, it can be shown (see Appendix C, (C-15)) that,

$$\frac{S_{oc}}{N_{oc}} \times \frac{B_c}{f_m} \times \frac{P_m}{p_m} = \left(\frac{\Delta F}{f_m}\right)^2 \times \frac{P_r}{2k(T_A + T_{er})f_m} \qquad (4.19)^{**}$$

where,

S_{oc} = Average message-signal power level per message-channel, at the receiver output, in watts.

N_{oc} = Receiver output average noise power level, per message-signal spectrum bandwidth, in watts.

B_c = Message-signal spectrum bandwidth, in c/s.

f_m = Frequency of the modulating sinusoidal signal, in c/s. This is also the highest frequency in modulating baseband signal.

p_m = Average power level of an individual modulating signal in the baseband signal.

P_m = Total average power in the composite modulating baseband signal. This is the power level which exists at a time when a specified number of simultaneously-active message signals are being transmitted.

*Acceptable system performance may be obtained in the "threshold region" where pre-emphasis and de-emphasis do not improve system performance, see Section 6.

**For a discussion of the term $(T_A + T_{er})$ and the effects of impedance mismatching in the receiving system, see Appendix A.

ΔF = Peak radio-frequency carrier deviation, in c/s, for a single sinusoidal modulating signal, having an average power level P_m equal to the total average power level of the composite modulating baseband signal.

P_r = Total available carrier-signal power at the input terminals or "front-end" of the FM receiver, in watts.

k = Boltzman's constant = 1.3804×10^{-23} Joules per degrees, Kelvin.

T_A = Receiving antenna-system output noise temperature, degrees Kelvin; see Appendix A.

T_{er} = Radio receiver effective input noise temperature [IRE Comm. on Noise, 1960] , degrees Kelvin.

$\frac{\Delta F}{f_m}$ = Deviation ratio.

P_r and S_{oc}/N_{oc} are to be considered as the "variables" in (4.19); $\Delta F/f_m$ is the "variable-parameter" and the remaining terms in this equation are considered as "scale-normalizing factors".

Since (4.19) was developed for the purpose of calculating the performance of FM radio receivers, a discussion of the limitations and application of this equation is appropriate. It should be noted that S_{oc}/N_{oc} is the receiver output signal-to-noise ratio for a particular message signal having a power spectrum centered at a frequency of f_m cycles per second in the baseband. Furthermore, this equation is valid only for values of the predetection signal-to-noise ratio, P_{if}/N_{if}, greater than approximately 10 db. In other words, there is a linear relationship between the predetection signal-to-noise ratio in the FM receiver IF, and the signal-to-noise ratio at the receiver output, only for values of the predetection signal-to-noise ratio exceeding approximately 10 db. In this respect (4.19) can be deceptive because it does not include the non-linear threshold region. Hence, (4.19) should be used only to locate the position and the slope of the FM radio receiver characteristic curve above the threshold region.

The position of the "threshold-level" point of the FM radio receiver characteristic performance curve is determined from the relationship

$$\frac{P_{if}}{N_{if}} = \frac{P_r}{k(T_A + T_{er})\, B_{if}} = \frac{P_r}{k(T_A + T_{er})\, f_m \times \phi\left(\frac{\Delta F}{f_m}\right)} = 10 \qquad (4.20)$$

where,

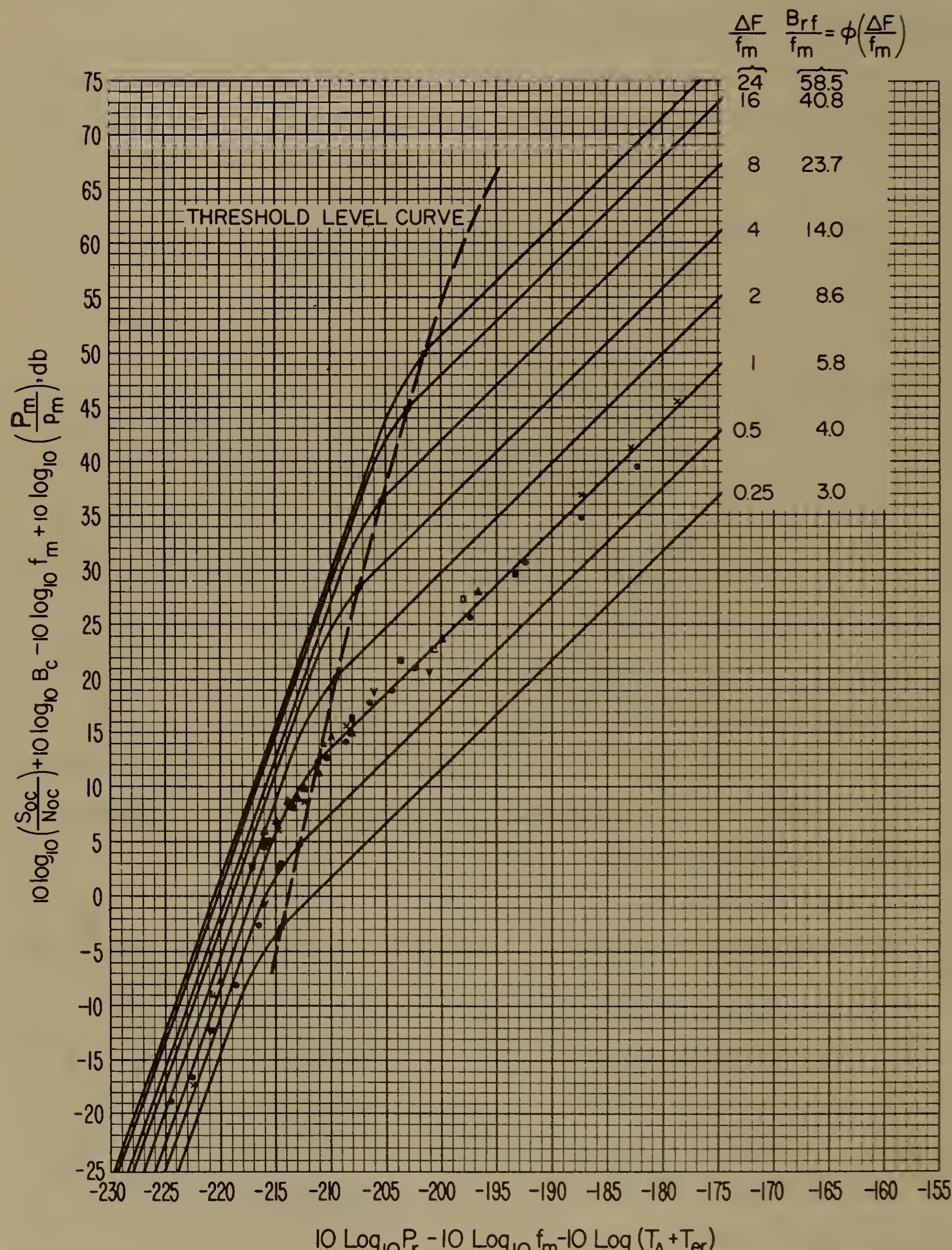
$\phi(\Delta F/f_m)$ = Function of $\Delta F/f_m$, calculated [Hund, 1942] and plotted in figure 3-1 for "sinusoidal-signal" FM modulation. This function gives the number of significant sidebands in the radio frequency carrier spectrum; assuming that the modulating signal is a single sinusoidal voltage wave at the highest frequency f_m, in the modulating baseband signal.

Equation 4.20 was used to determine the "threshold level" points on the curves in figure 4-3; these points are the limits at which the curves cease to be linear for decreasing value of P_r. The slope of the upper linear portion of the receiver characteristic curves, shown in figure 4-3, was determined from (4.19).

4.6.1.2 Threshold Region of FM Receiver Characteristic Curves

The shape or curvature of the non-linear "threshold region" of the radio receiver characteristic curve was obtained from averaged measured data on several different types of conventional FM radio receivers. Calculated threshold-level points are shown in figure 4-3 as the intersections of the threshold-level curve and the receiver-characteristic curves.

FM RADIO RECEIVER CHARACTERISTIC CURVES
STEADY RECEIVED SIGNAL



$$\frac{P_r}{f_m(T_A+T_{er})} = \frac{\text{AVERAGE OF THE TOTAL SIGNAL POWER, WATTS}}{\text{AVERAGE OF THE TOTAL RELATIVE NOISE POWER}}, \text{ AT RECEIVER INPUT}$$

Figure 4-3

From the above outline it is possible to determine the shape and the position of an FM radio-receiver characteristic curve, in terms of the receiver-input RF-signal power to the receiver-input relative noise power ratio, $\frac{P_r}{k(T_A + T_{er}) f_m}$; and the receiver output (message) channel-signal power to channel-noise power ratio, S_{oc}/N_{oc}; for a given set of system parameters.

4.6.2 FM Radio Receiver Input-Power and Bandwidth Requirements for Steady Received Signals.

Following is a description of the method used to construct the FM radio receiver curves shown in figure 4-3, using the scale normalizing parameters as noted in the abscissa and in the ordinate titles. The shape of these curves and their locations, relative to the absicssa and the ordinate scales and relative to each other, are important if they are to be used to obtain accurate determinations of the required bandwidth $B_{rf}(= B_{if})$ and the receiver input-power, P_r, required for particular grades of service, or for particular values of receiver-output signal-to-noise ratios, S_{oc}/N_{oc}.

The non-linear threshold-region portions of the curves were obtained from averaged measurements on a number of FM radio receivers of different types, as indicated by the measured points shown in figure 4-3. It was assumed that the threshold-region curvature of the curves did not vary appreciably with change in the deviation ratio $\Delta F/f_m$, hence all curves shown in figure 4-3 are identical in shape.

The positions and the slopes of the upper (linear) portions of the FM radio receiver curves in figure 4-3 were calculated, using (4.19) and (4.20); each of the scale-normalizing factors, B_c, f_m, and P_m/p_m were set equal to unity. T_A was set equal to the standard temperature, T_o (= 290 degrees Kelvin); this condition is equivalent to the assumption that the modulated RF signal was obtained from a signal generator with an output impedance temperature of 290 degrees Kelvin. T_{er} was assumed to be 0 degrees Kelvin; this assumption corresponds to a receiver noise figure of unity, or 0 db. Note that $\Delta F/f_m$ is considered here as the "variable" parameter and not as a scale-normalizing factor. The upper linear portions of the curves are displaced relative to each other on the ordinate scale an amount which is dependent upon the relative values of $(\Delta F/f_m)^2$.

The threshold-region section of the receiver curves are drawn tangent to the upper linear portions of the curves, at the points given by (4.20); these points correspond to an IF signal-to-noise ratio $P_{if}/N_{if} = 10$. This value of 10 db for P_{if}/N_{if}, applies only to the case where the received radio-frequency signal is non-varying or steady.

The curves in figure 4-3 were drawn for a value of f_m assumed to be equal to the highest frequency in the modulating composite baseband signal; therefore, the receiver output signal-to-noise ratio values, for the linear portion of these curves, would be increased by a factor, $(f_m/f_c)^2$, or $20 \, \text{Log}_{10} \frac{f_m}{f_c}$ db, for modulating signals of frequency f_c -where $f_c < f_m$. However, when minimum receiver-input power P_r is the criterion, the system parameters should be adjusted so that the receiver will be operated in its threshold region for a small percentage of the time; under these conditions, when P_r is above the threshold-level value, the receiver output signal-to-noise ratio exceeds the required S_{oc}/N_{oc} value and pre-emphasis techniques are of doubtful value. In the threshold region, the receiver output signal-to-noise ratio, S_{oc}/N_{oc}, approaches the same value for all frequencies in the modulating baseband signal; see figure 4-8 and Section 4.6.4. Hence, the curves in figure 4-3 will yield accurate results in the threshold region and conservative results above the threshold level.

The radio-frequency spectrum bandwidth, B_{rf}, associated with particular values of $\Delta F/f_m$ and f_m (see (C.8)) is given by,

$$B_{rf} = f_m \times \phi(\Delta F/f_m) \tag{4.21}$$

where,

B_{rf} = Radio-frequency spectrum bandwidth which includes all components or sidebands having amplitudes equal to or greater than one percent of the unmodulated carrier amplitude.

The relationship between $\Delta F/f_m$ and $\phi(\Delta F/f_m)$ was calculated [Hund, 1942] and is shown in figure 3-1, for sinusoidal-signal modulation. Values for B_{rf}/f_m, associated with particular values of $\Delta F/f_m$, were obtained by combining (4.21) and figure 3-1; these associated values of $\Delta F/f_m$, B_{rf}/f_m and $\phi(\Delta F/f_m)$ are shown in figure 4-3.

The receiver characteristic curves in figure 4-3 were re-plotted in a more convenient and useful form, as shown in figure 4-4.

The interrelationships between three important factors in an FM radio communication system are shown in figures 4-3 and 4-4. These factors are: (1) required bandwidth or radio frequency spectrum, B_{rf}, (2) grade of service or minimum permissible receiver output signal-to-noise ratio, S_{oc}/N_{oc}, and (3) required radio frequency carrier signal power, P_r, relative to the noise-power level, $k(T_A + T_{er})f_m$, at the receiver input.

The designer of a radio communication system is usually given the specified number of voice channels, that is, the baseband spectrum bandwidth, from which he can determine f_m; the grade of service or the maximum tolerable average message error rate is also specified, from which the system designer can establish the required receiver output signal-to-noise ratio, S_{oc}/N_{oc}. The initial problem, is to choose the required radio-frequency carrier spectrum bandwidth B_{rf} (where

FM RADIO RECEIVER PERFORMANCE CURVES
STEADY SIGNAL

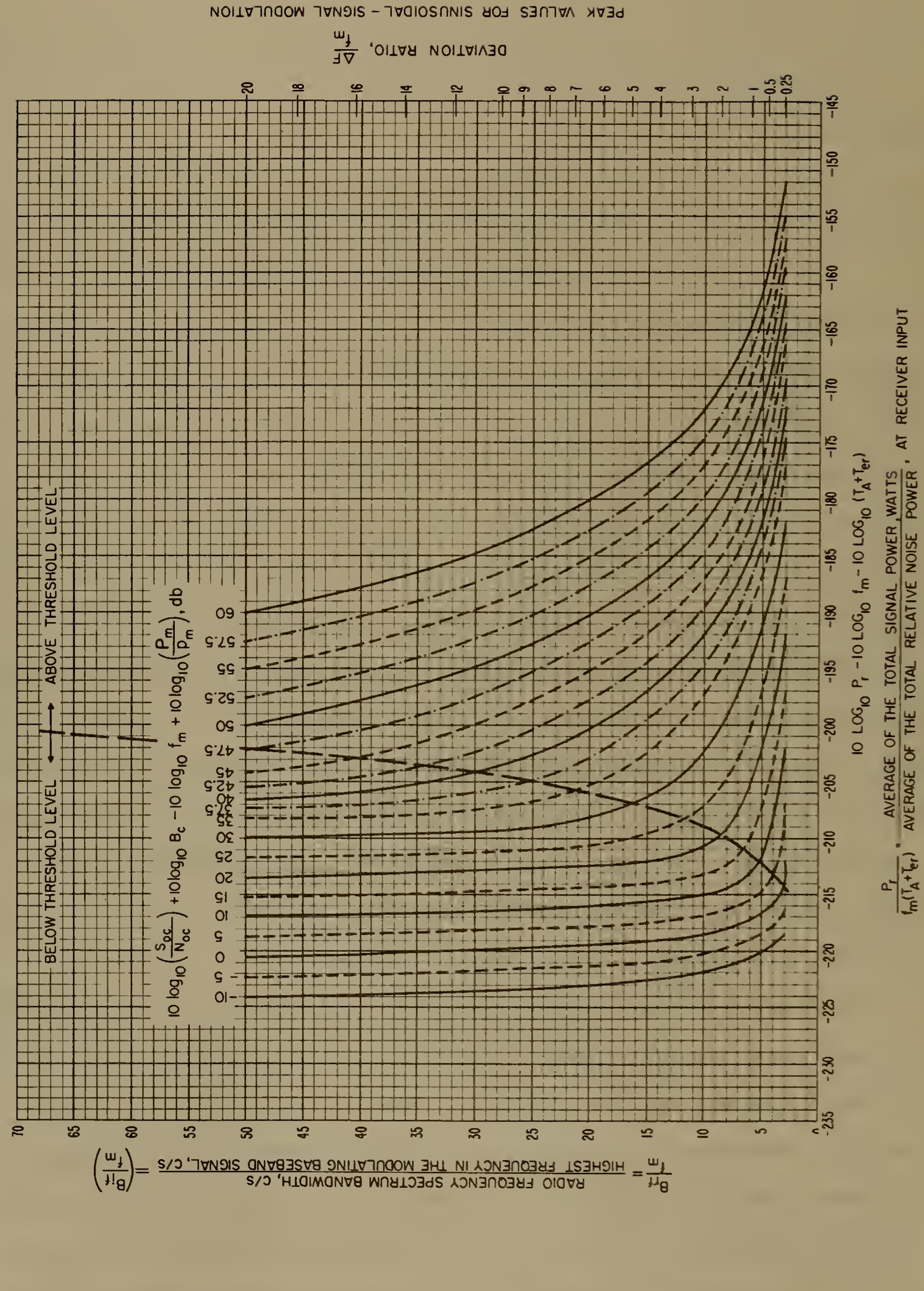


Figure 4-4

FM systems are involved), and the required radio frequency carrier-signal power level, P_r, at the receiver input. The curves in figures 4-3 and 4-4 make it possible to select required sets of values for B_{rf} and P_r, for a particular grade of service, or S_{oc}/N_{oc}, for specified types of messages. The curves in figure 4-4 can also be used as guides when it is desired to trade or interchange required P_r, for required bandwidth, B_{rf}.

The required deviation ratio, $\Delta F/f_m$, associated with the required bandwidth ratio, B_{rf}/f_m, and for a sinusoidal modulating signal, may be obtained from the right-hand scale in figure 4-4. Hence, the required deviation ΔF can be obtained for a given baseband bandwidth B_b or a particular value of f_m. It should be understood that the data in figures 4-3 and 4-4 applies only to the case of a steady or non-varying radio frequency received signal. Procedures for using the design curves in figure 4-4 are outlined in Section 6.

4.6.3 FM Radio Receiver Input-Power and Bandwidth Requirements for a Rayleigh-Fading Type of Received Signal.

FM receiver characteristic curves, were obtained for a Rayleigh-fading type of received signal and for non-diversity, dual diversity, and quadruple diversity by graphically combining the curves in figure 4-3 (for steady received signal) with the appropriate maximal-ratio Combiner-output distributions [Brennan, 1958] shown in figure D-1b; details of the method used to combine the FM receiver performance characteristic for steady received signal with the Combiner characteristics are given in Appendix D. The three sets of curves obtained by this procedure were then scaled and replotted as shown in figures 4-5, 4-6 and 4-7; for non-diversity, dual diversity, and quadruple diversity, respectively.

FM RADIO RECEIVER PERFORMANCE CURVES

RAYLEIGH-FADING SIGNAL, NON DIVERSITY

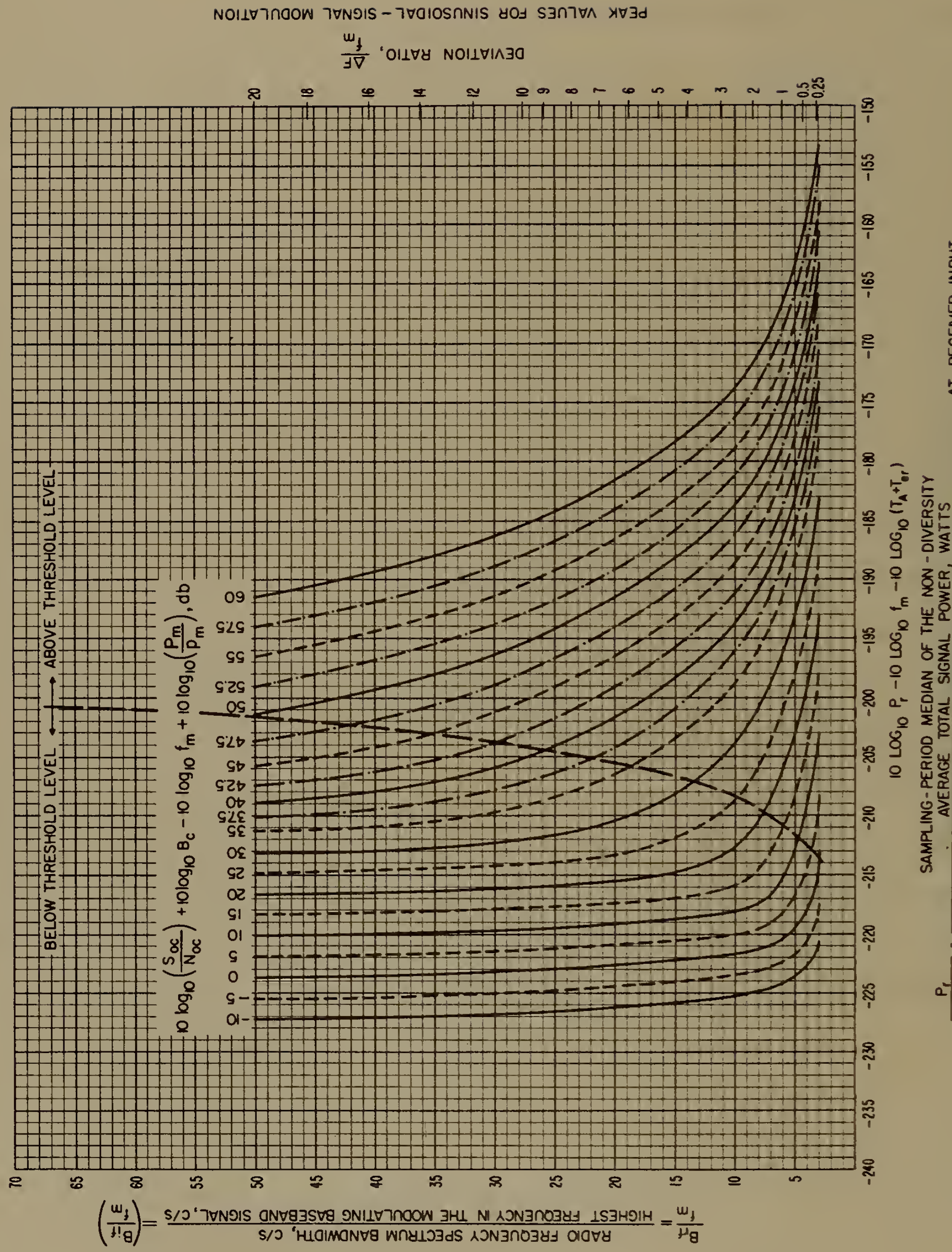
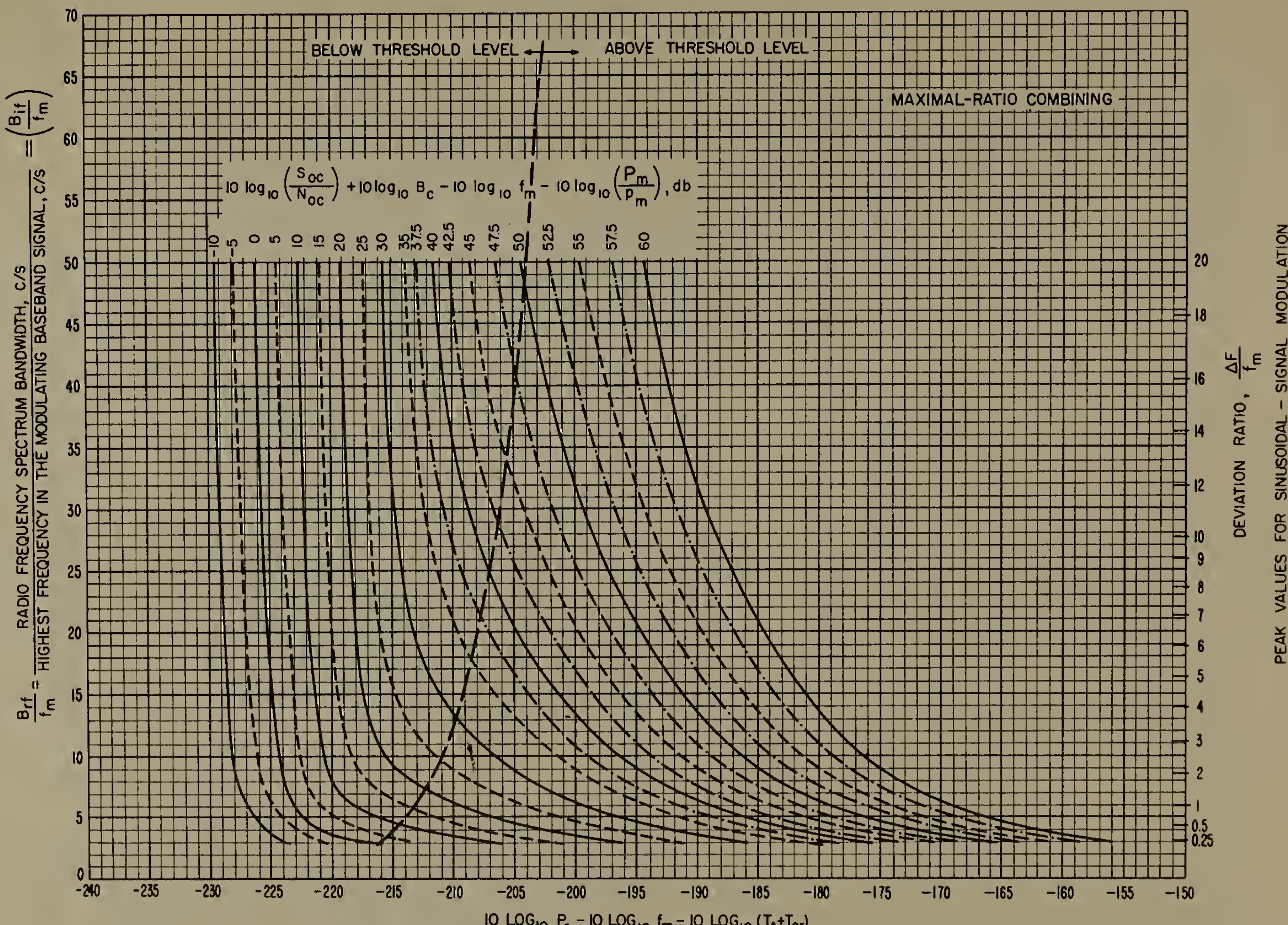


Figure 4-5

FM RADIO RECEIVER PERFORMANCE CURVES
RAYLEIGH – FADING SIGNAL, DUAL DIVERSITY



$\frac{P_r}{f_m(T_A+T_{er})} = \frac{\text{SAMPLING-PERIOD MEDIAN OF THE NON-DIVERSITY AVERAGE TOTAL SIGNAL POWER, WATTS}}{\text{SAMPLING-PERIOD AVERAGE OF THE TOTAL RELATIVE NOISE POWER}}$, AT RECEIVER INPUT

Figure 4-6

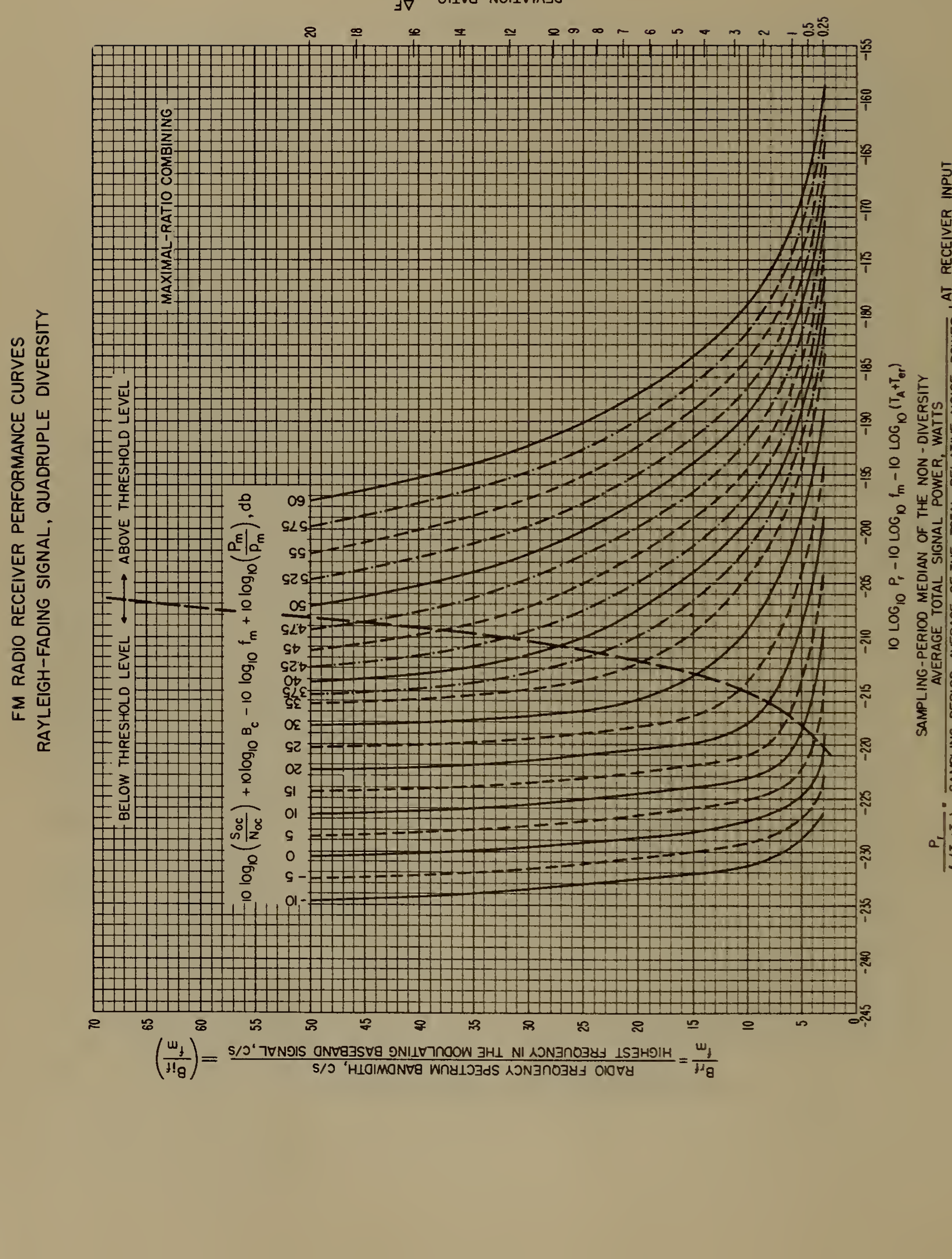

FM RADIO RECEIVER PERFORMANCE CURVES
RAYLEIGH-FADING SIGNAL, QUADRUPLE DIVERSITY
MAXIMAL-RATIO COMBINING
BELOW THRESHOLD LEVEL ⟷ ABOVE THRESHOLD LEVEL
10 log10 (Soc/Noc) + 10 log10 Bc − 10 log10 fm + 10 log10 (Pm/pm), db
PEAK VALUES FOR SINUSOIDAL SIGNAL MODULATION
DEVIATION RATIO, ΔF/fm
Brf/fm = RADIO FREQUENCY SPECTRUM BANDWIDTH, C/S / HIGHEST FREQUENCY IN THE MODULATING BASEBAND SIGNAL, C/S = (Bif/fm)
10 LOG10 Pr − 10 LOG10 fm − 10 LOG10 (TA+Ter)
Pr / fm(TA+Ter) = SAMPLING-PERIOD MEDIAN OF THE NON-DIVERSITY AVERAGE TOTAL SIGNAL POWER, WATTS / SAMPLING-PERIOD AVERAGE OF THE TOTAL RELATIVE NOISE POWER, AT RECEIVER INPUT

Figure 4-7

The major differences between the design curves in figures 4-4, 4-5, 4-6 and 4-7 are that the P_r values for the steady received signals are average values while the P_r values for the fading type of received signals are median values of the non-diversity average-power levels of the received signal. The positions of the curves, relative to the abscissa and the ordinate scales, are of course different for the different orders of diversity; furthermore, their relative positions are also dependent upon the type of combining and the actual performance of the Combiner. This latter point greatly influences the accuracy of system-performance estimates based on the design curves which are developed in this report, compared to actual system-performance measurement results. Procedures for using the design curves in figures 4-4, 4-5, 4-6, and 4-7 are outlined in Section 6.

4.6.4 Pre-emphasis-De-emphasis Techniques

In FM systems, the rms noise voltage in the receiver-output baseband signal is proportional to the frequency in the baseband-signal spectrum. If equal power levels are assigned to each spectral component in the modulating baseband signal at the modulator input, regardless of the positions of the modulating signals in the baseband signal, it will be found that the receiver-output signal-to-noise ratio will vary as shown in figure 4-8.

Pre-emphasis of the spectral components in the modulating baseband signal, proportional to the frequency of the component in the baseband, and a corresponding de-emphasis of the baseband components in the radio receiver, result in an equilization of the radio receiver output signal-to-noise ratio for all components in the baseband signal. However, the above conditions apply only when the radio receiver is operating above its "threshold level"; below the receiver threshold

FM RECEIVER CHARACTERISTIC CURVES, WITHOUT PRE-EMPHASIS

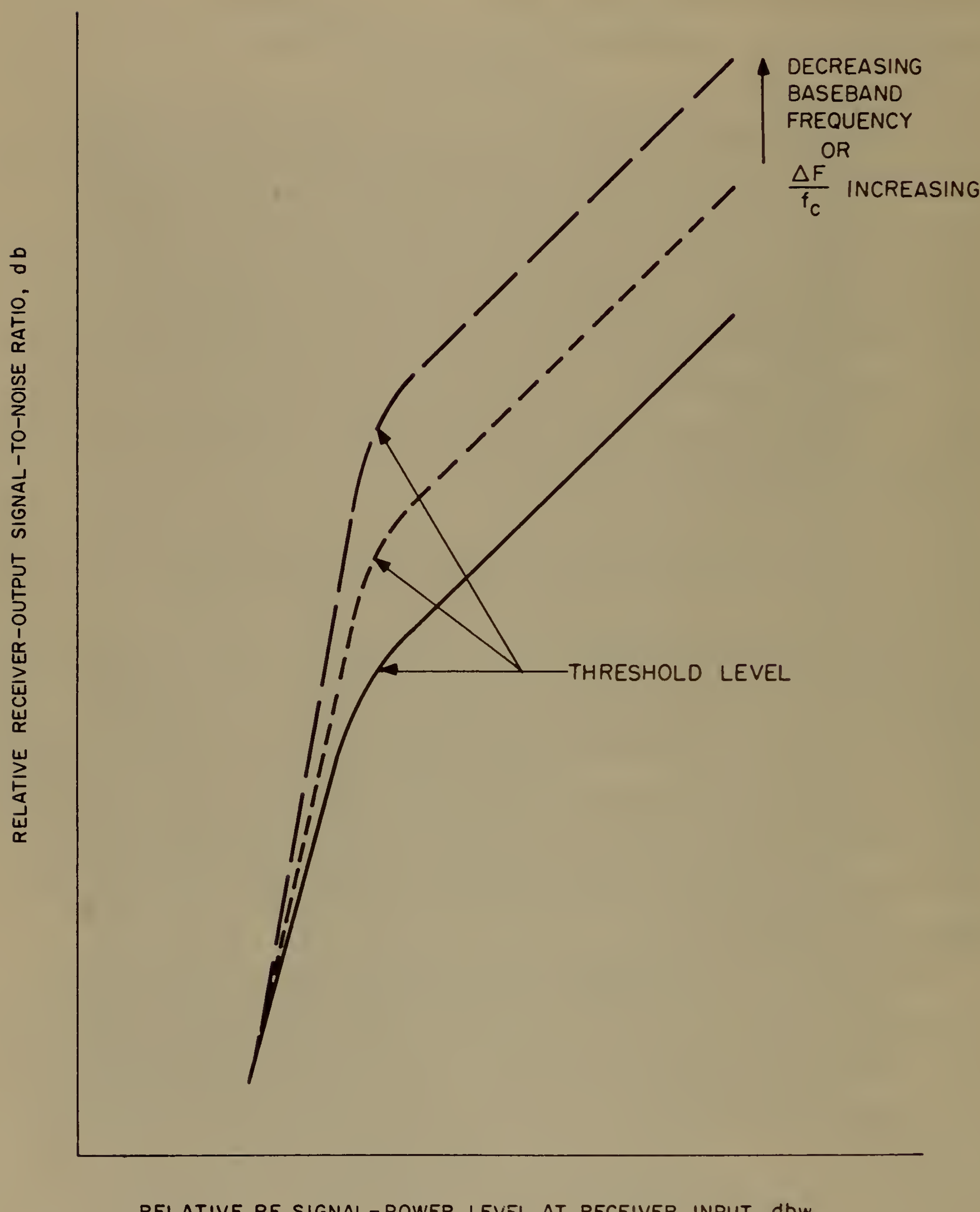


Fig. 4-8

level the use of pre-emphasis-de-emphasis techniques results in a degradation of receiver-output signal-to-noise ratio. This condition is evident from figure 4-9. Hence, pre-emphasis-de-emphasis techniques will only be effective when the FM receiver is operated above its threshold level.

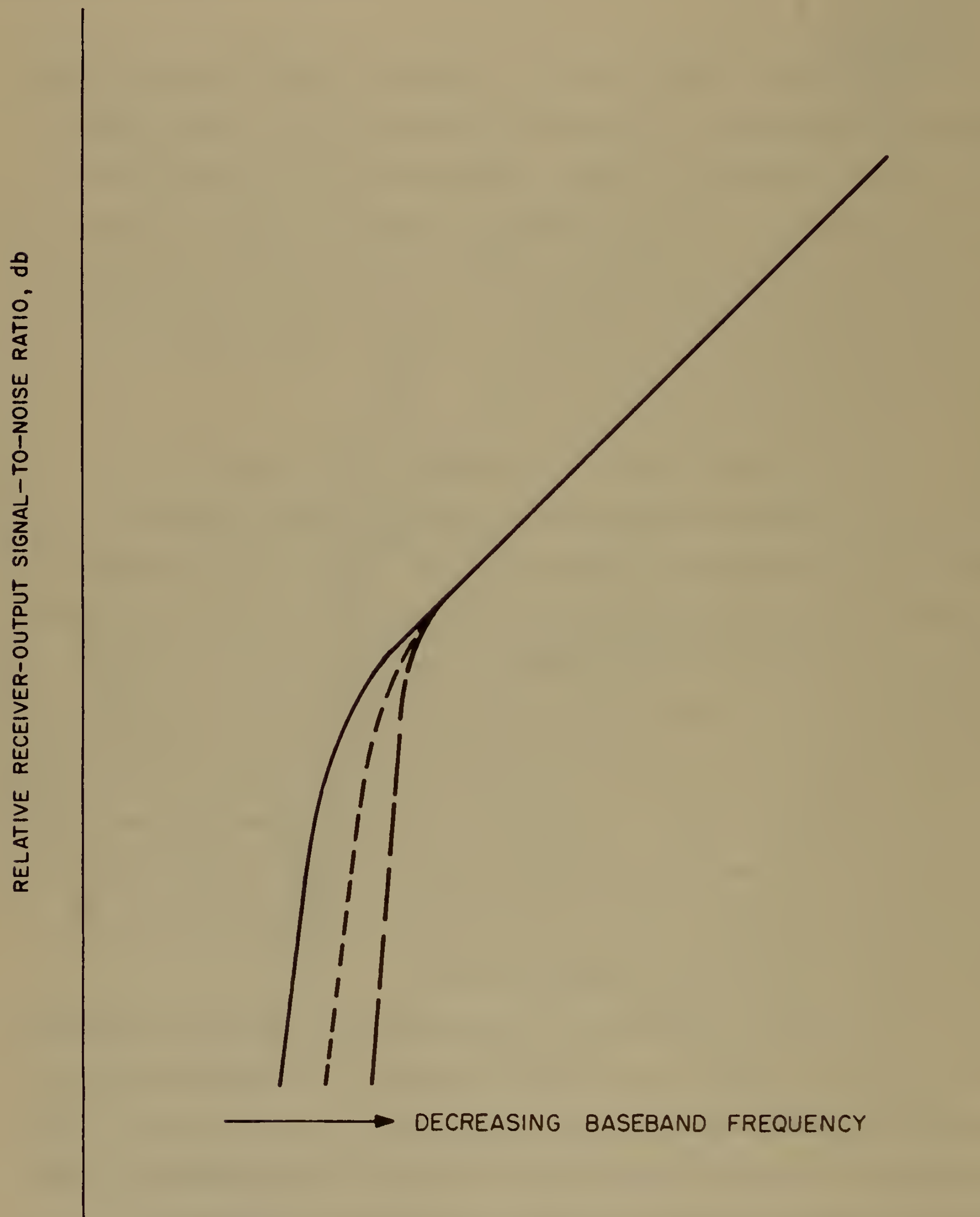
FM system performance may be estimated without pre-emphasis-de-emphasis techniques as outlined in this paper; the results will be accurate in the receiver threshold region and will be conservative above the threshold level. Final design estimates may be modified, in terms of the baseband-signal frequency, by the use of data similar to that shown in figure 4-8; however, there may be some question as to the practicability of this procedure.

4.6.5 Bandwidth Compression in FM Receivers.

All of the evidence to date shows that the various proposed receiver bandwidth-compression schemes have the effect of lowering the "threshold-level" of the FM receiver, relative to the received carrier-signal power and the total noise power "accepted" by the receiver. The accepted noise power is that portion of the total receiver-input noise power which appears at the input to the FM receiver limiter-discriminator, or demodulator unit. It should be noted that the position of the threshold level also depends upon the characteristics of the limiter and hence the threshold level may, in some cases, possibly be lowered by improvements in the limiter characteristics.

The overall effect attained by compressing the receiver bandwidth is shown in figure 4-10. It is evident that there is no enhancement of the bandwidth-compressed receiver output signal-to-noise ratio above the threshold level point; this situation is due to the fact that the pre-detection frequency deviation, ΔF, is reduced when the bandwidth is compressed, and hence the net effect is that there is no improvement in the receiver performance above the threshold level.

EFFECT OF PRE-EMPHASIS ON FM RECEIVER CHARACTERISTIC CURVES



RELATIVE RF SIGNAL-POWER LEVEL AT RECEIVER INPUT, dbw

Fig. 4-9

EFFECT OF BANDWIDTH COMPRESSION ON FM RECEIVER CHARACTERISTICS

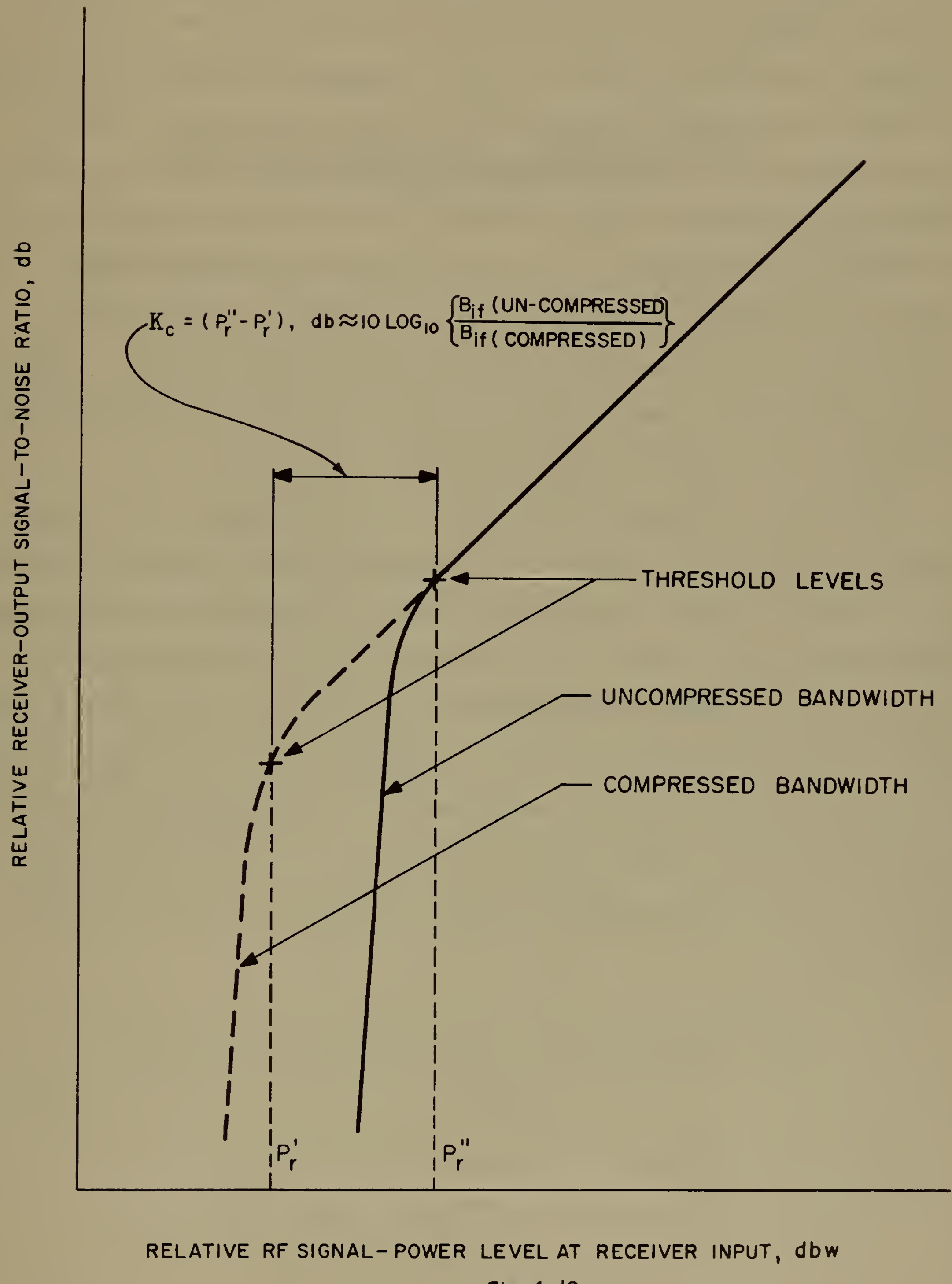


Fig. 4-10

The FM-receiver design-curve data presented in this report may be modified to include receiver bandwidth-compression or modified-limiter effects by incorporating the data from figure 4-10. The data shown in figure 4-10 may be obtained by either (1) performing the indicated comparative measurements on the FM receivers, for the conditions shown in the figure, and for identical frequency-modulated RF signals at the input terminals of the radio receivers; or (2) by estimating the value of the bandwidth compression factor, K_c, and modifying the design curves as shown in figure 4-10 or figure 4-11. The factor K_c is defined in terms of the compressed-bandwidth receiver threshold-point power level, P'_r relative to the similar power level, P''_r, for an un-compressed-bandwidth receiver. The value for K_c may be obtained from the measured characteristics of the receivers, as shown in figure 4-10.

The modified design curves, well below threshold and for high values of B_{rf}/f_m, will parallel the original curves and will be shifted to lower values on the relative P_r scale; this decrease in scale values will be equal to K_c, in db. The modified curves will converge with and will join the original design curves at the original threshold-level points. Above the threshold-level points, the modified design curves are identical to the original design curves--which apply to FM receivers without bandwidth compression. The effect of receiver-bandwidth compression on the shape of the FM system-design curves, is shown in figure 4-11.

EFFECT OF BANDWIDTH COMPRESSION ON FM SYSTEM DESIGN CURVES

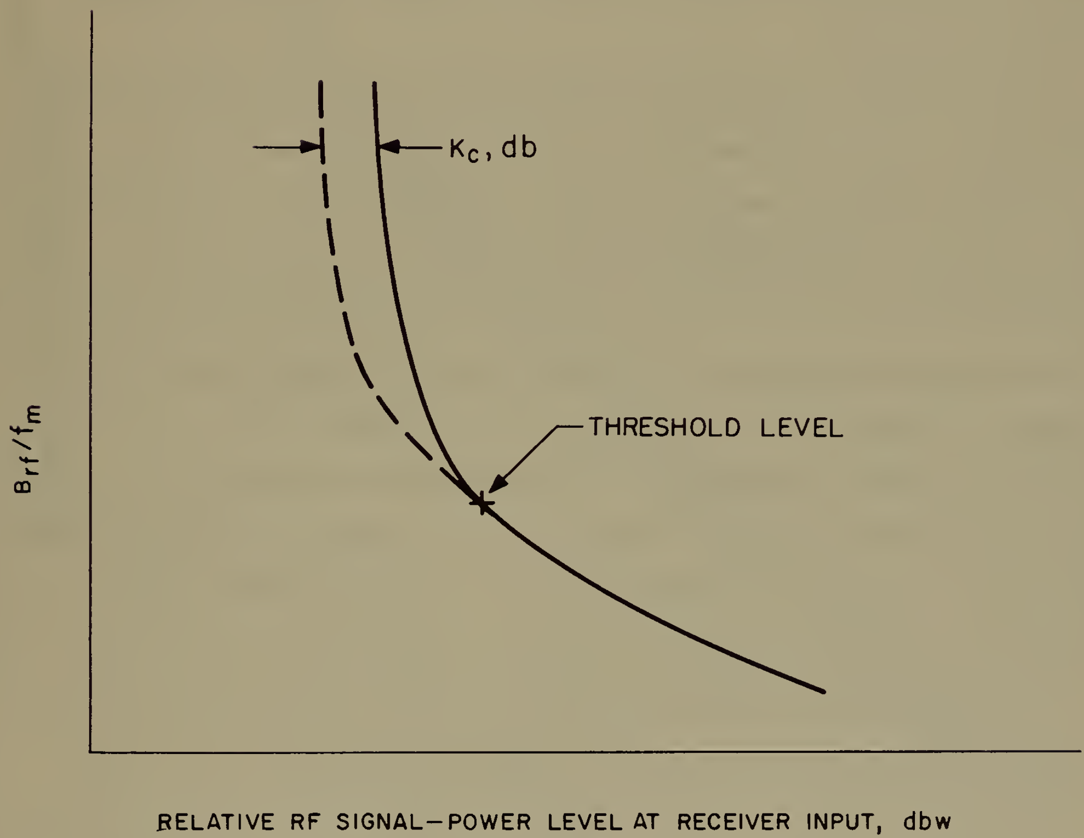


Fig. 4-11

5. PERFORMANCE CHARACTERISTICS OF MESSAGE-SIGNAL DECODER UNIT

The performance of the message-signal decoder unit is defined here in terms of the relationship between the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the decoder-unit input, and the error rate in the message at the decoder-unit output. This definition of the message-signal decoder unit performance is general and may be applied to any type of decoder, provided that proper consideration is given to the statistical characteristics of both the message-channel signal and the message-channel noise at the decoder-unit input, and the resultant message error rate.

At the decoder-unit input, both the short-term message-channel signal power, S_{oc}, and the short-term message-channel noise power, N_{oc}, will vary independently with time; furthermore, the distributions of S_{oc} and N_{oc} are dissimilar. Hence, in order to obtain measurable and definable results it is necessary to analyze the performance of the message-signal decoder unit in terms of the <u>average</u> signal power-to <u>average</u> noise power ratio, S_{oc}/N_{oc}, at the decoder input. The problem is further complicated by the fact that the ratio S_{oc}/N_{oc} varies with time and hence its distribution must be considered; finally, the distribution of the noise power affects the message error rate, for particular values of S_{oc}/N_{oc}, as shown in figures 5-1 and 5-2.

In this paper the decoder-unit performance is determined, for the above conditions at the decoder input, as follows:

(1) The decoder-unit performance is either estimated or measured for a range of values of the average signal power-to-average noise power ratio, S_{oc}/N_{oc}, at the decoder input. S_{oc}/N_{oc} is assumed to be non-time varying or steady, and the decoder-input noise, N_{oc}, is assumed to be "white". The

CUMULATIVE AMPLITUDE DISTRIBUTION OF NOISE AT RECEIVER OUTPUT FOR RECEIVED CARRIER POWER LEVELS ABOVE AND BELOW THE FM RADIO RECEIVER THRESHOLD LEVEL

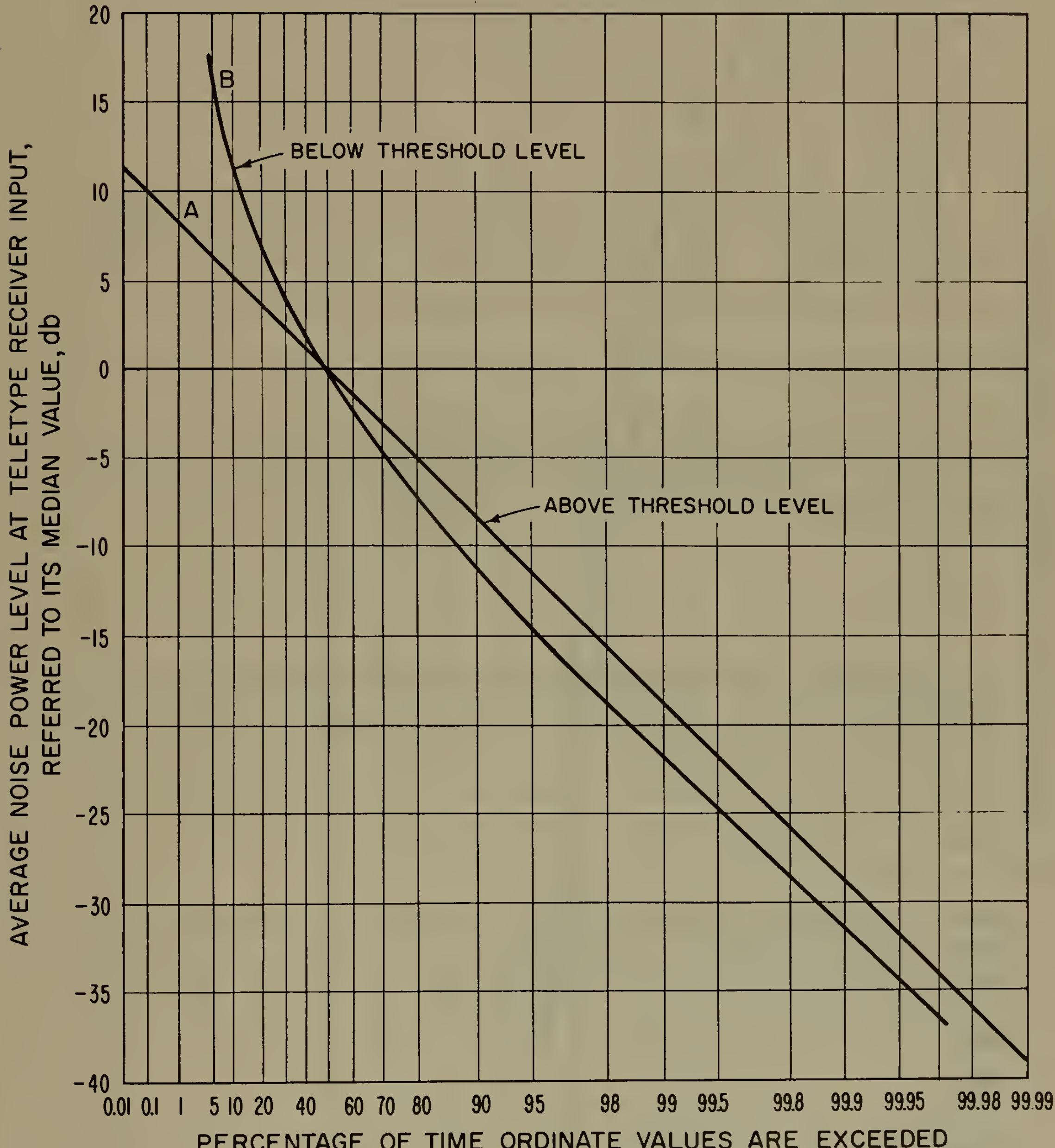
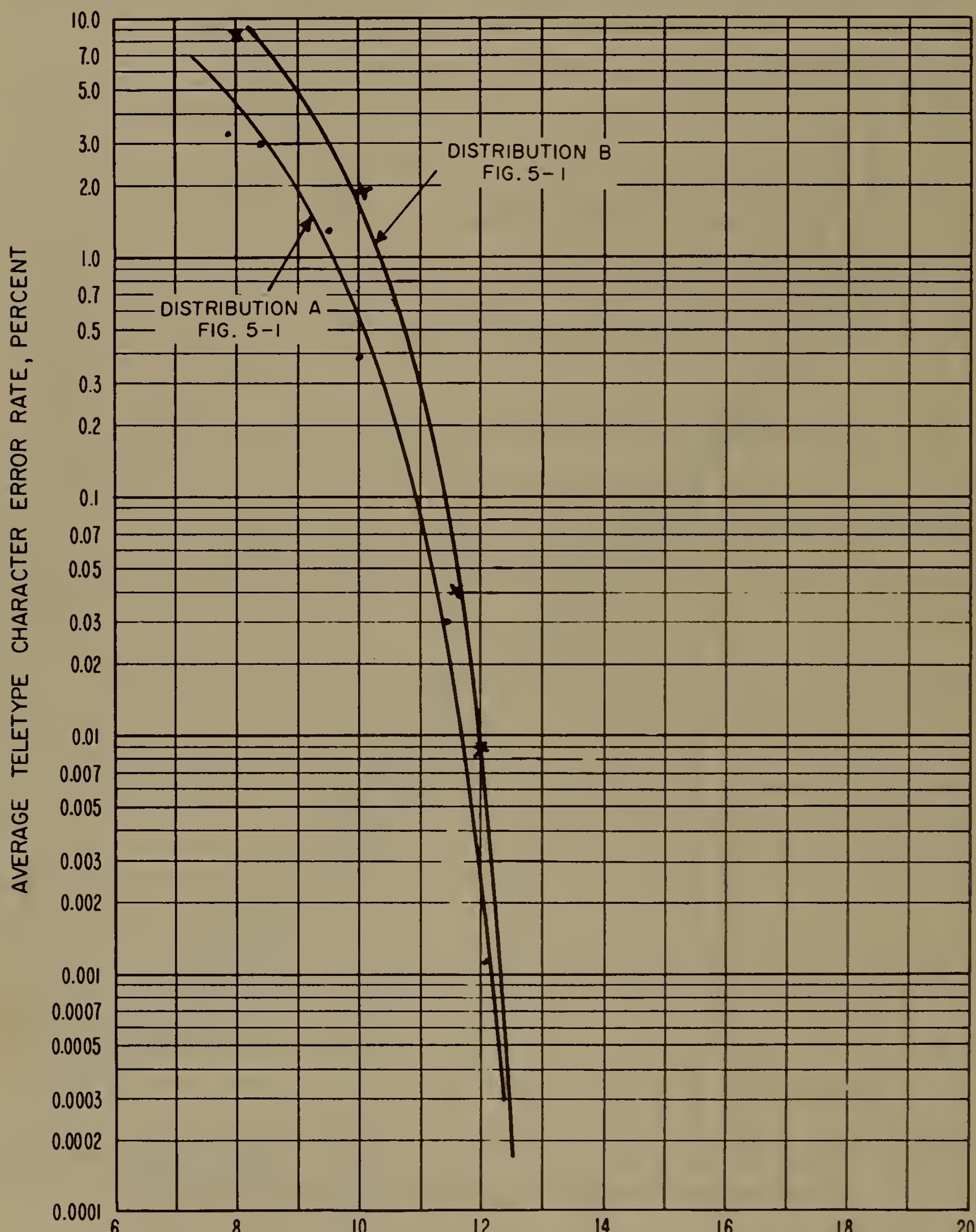


Figure 5-1

MEASURED RELATIONSHIP BETWEEN THE STEADY-SIGNAL POWER TO MEAN NOISE POWER RATIO, AT THE TELETYPE RECEIVER INPUT TERMINALS, AND THE AVERAGE TELETYPE CHARACTER ERROR RATE



$10 \log_{10} \frac{S_{oc}}{N_{oc}} = \frac{\text{TELETYPE MESSAGE-SIGNAL AVERAGE POWER LEVEL PER CHANNEL}}{\text{AVERAGE NOISE POWER LEVEL PER TELETYPE MESSAGE CHANNEL } B_c}$, db

MEASURED AT TELETYPE RECEIVER (DECODER-UNIT) INPUT

Fig. 5-2

"measured steady signal" curve shown in figure 5-3 is typical for narrow-band FSK teletype decoder-unit performance.

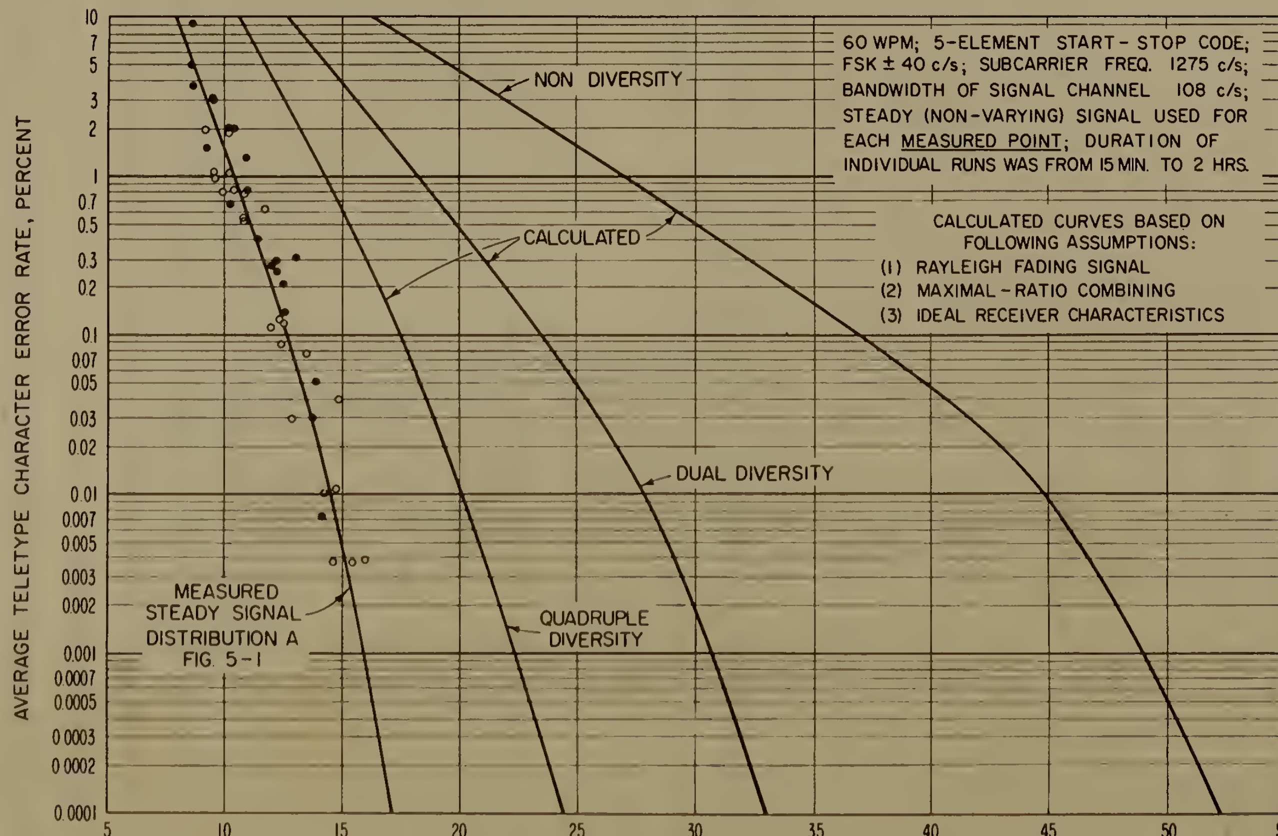
(2) The performance of the decoder unit, for time-varying (decoder-input) signal-to-noise ratios, is then estimated by combining the above steady-signal decoder-unit characteristic with the distribution of the time-varying decoder-unit input signal-to-noise ratio. Details of this procedure are given in Appendix D. 2; results are shown in figure 5-3 for non, dual, and quadruple diversity, assuming maximal-ratio combining.

(3) Under conditions where "impulse-type" of noise appears at the decoder-unit input, the procedures outlined in (1) and (2) above would be followed; the resultant decoder-performance curves would be similar to those shown in figure 5-3 (for white noise at the decoder input) but they would be shifted to higher values of S_{oc}/N_{oc}, for given values of message error rates. See figures 5-1 and 5-2.

The shapes and the positions of the curves in figure 5-3 are determined by the distribution of the signal-to-noise ratio, S_{oc}/N_{oc}, at the decoder-unit input; in turn, this distribution (for our purpose) is dependent upon: (a) the distribution of the received RF signal at the radio receiver input, (b) the radio-frequency receiver characteristics, (c) the order of diversity, and (d) the combiner characteristics. Combiner performance was assumed to be as shown in figure D-1b, for a Rayleigh-fading type of radio-frequency carrier signal, using maximal-ratio combining, and for SSBSC and FM radio receiver characteristics as given in Sections 4.5 and 4.6.

The principal requirement, which may be used as a guide when designing and testing radio communication systems, is that the (received) average message error rate should not exceed a specified value for

MEASURED AND CALCULATED RELATIONSHIP BETWEEN THE SIGNAL POWER TO MEAN NOISE POWER RATIO, AT THE TELETYPE RECEIVER INPUT TERMINALS, AND THE AVERAGE CHARACTER-ERROR RATE AT THE TELETYPE PRINTER OUTPUT



$$10 \log_{10} \frac{S_{OC}}{N_{OC}} = \frac{\text{TELETYPE MESSAGE-SIGNAL AVERAGE POWER LEVEL PER CHANNEL}}{\text{AVERAGE NOISE POWER LEVEL, PER TELETYPE-MESSAGE CHANNEL } B_c}, \text{db}$$

MEASURED AT TELETYPE RECEIVER (DECODER UNIT) INPUT

Figure 5-3

particular message-load conditions on the system. Message errors usually occur in bursts or groups, and are associated either with signal fades or noise peaks, or both. Hence, the "short-term message error rate" will vary from zero to some comparatively high value during the sampling period; therefore, the term "average message error rate" is used throughout this paper, in order to differentiate between the short-term message error rate and the message error rate which is obtained by averaging the short-term message error rates over the entire sampling period. For example, the average teletype character error rate is defined here as:

$$\frac{\text{Total number of teletype character errors occurring during sampling period}}{\text{Total number of teletype characters transmitted during the sampling period}}$$

The interval of time included in the sampling period is subject to the mutual agreement of all parties involved in the design and acceptance-tests of the radio communication system.

Narrow band FSK (frequency-shift-keyed) teletype equipment employing a frequency shift of approximately ± 40 cps is generally available in multichannel tropospheric radio communications systems, and considerable data are available on the correlation between the teletype character error rate and the signal-to-noise ratio at the teletype-receiver input or in the received baseband signal. Hence, teletype character error rates may be used conveniently as the criteron in designing and testing radio communication systems. Other types of messages, such as voice, digital data, or facsimile, could be used of course, provided that the relationship between the message error rate, i.e., grade of service, and the signal-to-noise ratio at the decoder-unit input is known.

For a specified allowable teletype-message error rate, from the data shown in figure 5-3, we can determine the required message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the radio receiver output; proper corrections are required for the effect of the message-signal multiplexing equipment on this signal-to-noise ratio. Similar information is required for the other types of messages being transmitted. These required radio-receiver output signal-to-noise ratios are then used in (4.9), Section 4, and with the proper set of system-design curves in figures 4-2, 4-4, 4-5, 4-6 or 4-7, to determine correct values for P_r, B_{rf}, ΔF, etc. The method of applying the above procedures is outlined in Sections 4.5 and 4.6. A typical design problem for SSBSC-AM and FM radio communications systems is solved in detail in Section 6.

From figures 5-1 and 5-2, it is evident that the performance of the FSK Teletype Decoder is not influenced, to a large degree, by the statistical characteristics of the noise at the decoder input terminals. This situation is due primarily to the effectiveness of the decoder limiter in removing the "amplitude" noise, which is associated with the signal. FM decoders for other types of messages would be expected to give results similar to the results obtained above for FSK teletype messages.

There is a need for up-dating voice-message performance data, similar to that given in figure 5-3 for teletype message errors vs. signal-to-noise ratios. In the system-design example worked out in section 6, the required voice message signal-to-noise ratio was obtained from the work of French and Steinberg, 1947; Eagan, 1944; and Beranek, 1947. An alternative solution to this problem is to accept the required voice message signal-to-noise ratio specified by the user of the system.

6. RADIO COMMUNICATION SYSTEM DESIGN AND TEST PROCEDURES

A typical system-performance problem is solved in this Section, in order to indicate the procedures to be followed when using the system design curves to estimate the following major factors: (A) average-power level of the total modulating signal, P_m, (B) required peak frequency deviation, ΔF (for FM systems), (C) required radio-frequency spectrum bandwidth, B_{rf}, and (D) required receiver-input RF-signal power level, P_r. The "required" values of these factors are associated with a specified maximum message error rate or grade of service. For the condition where certain parameters and factors such as B_{rf}, transmitter power, P_t, antenna gain, G_p, message load, etc. are given, the design curves may also be used to estimate the grade of service which will be provided by the system. Calculations are carried out for SSBSC-AM and for FM systems using dual diversity and maximal-ratio combining, and assuming a Rayleigh-fading type of received RF signal; for other orders of diversity the analysis is identical using the appropriate design curves.

It is assumed that the following system specifications are given:

(1) Number of available message (voice) channels.

(2) Specified number of simultaneously-active message channels.

(3) Number of messages of each type, M_I, and bandwidth, B_c, required for each message signal.

(4) Maximum allowable average message error rate.

(5) Type of Multiplex equipment to be used, and the baseband spectrum bandwidth required to accomodate the specified number of available message channels.

(6) Transmission losses in the transmitting and receiving antenna systems.

(7) Antenna noise-power temperature, T_a.

(8) Receiver noise figure, F_r, or receiver-input temperature, T_{er}.

The procedures for using the design curves in this report are as follows:

(a) Determine the required minimum value of the receiver output average signal power-to-average noise power ratio, S_{oc}/N_{oc}, associated with the specified maximum allowable average message-error rate, for a specified order of diversity. For teletype messages this information may be obtained from figure 5-3. For other types of messages, data similar to that given by figure 5-3 would be required, and could be developed using methods outlined in Appendix D.

(b) Determine P_m/p_m, the message-signal power-loading ratio in the modulating baseband signal, using (4.9) and associated information. Where p_m is the individual message-signal average modulating power and P_m is the total average message-signal modulating power in the modulating baseband signal, measured at a time when the specified number of simultaneously-active channels are operating. The maximum number of simultaneously-active channels should be specified, otherwise the designer is compelled to choose a value for this important factor.

(c) Determine the highest modulating frequency, f_m, from the specified number of available voice channels, or from the modulating baseband-signal bandwidth. The value of f_m will depend upon the type of multiplex system being used; f_m is always equal to the highest frequency in the modulating baseband signal.

(d) Calculate the antenna-system temperature, T_A, and the effective receiver-input temperature, T_{er}, from specified receiving-system parameters: $L_c L_t L_{du}$, T_a, T_c, T_t, T_{du}, and F_r. See Appedix A.

The above information on the system parameters is combined with the data in the appropriate equipment-performance design curves, in order to determine the required RF bandwidth, B_{rf}, and the required receiver-input RF signal power level, P_r; for the specified grade of service or permissible message error rate.

6.1 Determination of the Basic System Parameters: S_{oc}/N_{oc}, B_c, P_m/p_m, f_m, T_A, and T_{er} for Specified Average Message Error Rates, Number of Available Message Channels, and Specified Receiving-System Parameters.

This Subsection outlines the method of calculating the information required in items (a), (b), (c), and (d) in Section 6, from the system specifications which are outlined in Items (1) through (5) in Section 6. The parameters S_{oc}/N_{oc}, B_c, P_m/p_m, and f_m obtained from these calculations apply to both SSBSC-AM systems and FM systems. A specific example is considered in order to indicate more clearly the design procedures, and the calculated values of the above parameters are then used in Subsections 6.2 and 6.3.

Radio Communication System Specifications

Number of available voice channels	36
Maximum (specified) number of simultaneously-active channels	9
M_A = Maximum number of teletype message channels simultaneously-active (16 teletype messages per voice channel)	32

M_B = Maximum number of simultaneously-active voice-message channels (continuous talkers)	7
Bandwidth B_c required for each teletype signal	110 c/s
Bandwidth required for each voice signal	3200 c/s
Maximum allowable average teletype character error rate	0.01%
Type of Multiplex	SSBSC, Frequency div.
Multiplex baseband bandwidth, 4 x 36 =	144 kc
Type and order of Diversity	Space, Dual
Transmission-line and receiving-antenna loss factor, $L_c L_t L_{du}$	1.26(= 1db)
Antenna noise temperature, T_a	300 deg. K
Receiver noise figure, F_r	5 db

Calculations

From figure 5-3, the required signal-to-noise ratio at the teletype-message decoder unit input, for dual diversity operation, is 28 db; and allowing 2 db degradation of the signal-to-noise ratio through the Multiplex unit, we have

$$10 \text{ Log}_{10} (S_{oc}/N_{oc})\begin{pmatrix}\text{teletype}\\\text{message}\end{pmatrix} = 30 \text{ db}$$

and,

$$S_{oc}/N_{oc}\begin{pmatrix}\text{teletype}\\\text{message}\end{pmatrix} = 1000.$$

From the system specifications listed above we have,

$$B_c \text{ (teletype)} = 110. \text{ c/s}$$

$$B_c \text{ (voice)} = 3200 \text{ c/s}$$

The message-signal loading ratio in the modulating baseband signal, P_m/p_{mi}, is determined from (4.9), see p. 56. This equation shows that P_m/p_{mi} for the i^{th} message signal depends upon the required receiver-output signal-to-noise ratios, S_{oc}/N_{oc}, for each of the message signals in the modulating baseband signal. The required decoder-input signal-to-noise ratio for teletype message signals was obtained from figure 5-3, and was found to be 28 db. The required decoder-input signal-to-noise ratio for voice message signals (dual diversity) is assumed to be 33 db [French and Steinberg, 1947; Eagan, 1944; Beranek, 1947]. The estimated message intelligibility for voice messages, corresponding to a signal-to-noise ratio of 33 db, is 90 percent. Therefore, in the receiver baseband-signal at the receiver output, we have,

$$\frac{S_{oc}}{N_{oc}}\left(\begin{matrix}\text{voice}\\ \text{message}\end{matrix}\right) = 33 + 2 = 35 \text{ db} = 3160.$$

Substituting the above information in (4.9) p. 56, we obtain,

$$\frac{P_m}{p_{mi}}\left(\begin{matrix}\text{per teletype}\\ \text{message}\end{matrix}\right) = \frac{(32 \times 110 \times 1000) + (7 \times 3200 \times 3160)}{110 \times 1000} = 677.$$

and,

$$10 \text{ Log}_{10} \left|\frac{P_m}{p_m}\right| \left(\begin{matrix}\text{per teletype}\\ \text{message}\end{matrix}\right) = 10 \text{ Log}_{10} (677.) = 28.3 \text{ db}$$

In systems which require the transmission of pilot tones, it is necessary to include the pilot-tone modulating signal power levels, p_m, and the number of pilot tone signals, M, in the above calculations. The net effect of the inclusion of these pilot tone-signals will be to require an increase in the transmitter-output power; in order to transmit the pilot-tone signals without altering the grade of service of the "useful" message load.

The value of f_m is equal to the highest frequency in the modulating baseband signal and is,

$$f_m = 12 + (36 \times 4) = 156.\ \text{kc}$$

or,

$$10 \text{ Log}_{10} f_m = 52.\ \text{db}.$$

Note that the lowest-frequency in the multiplex is 12 kc and that each voice channel occupies a bandwidth of 4 kc in the modulating baseband signal including the guard bands; this simple relationship applies only to SSB frequency division type of Multiplex systems.

The value for T_A is obtained from either (4.3) or (4.3a), p. 37. Using (4.3a), and assuming $P_{\ell t}/k B_{if} L_D$ to be 200 degrees Kelvin* and T_a = 300 degrees Kelvin

$$T_A = \frac{T_a - T_o}{L_c L_t L_{du}} + T_o + P_{\ell t}/k B_{if} L_D$$

$$= \frac{300 - 290}{1.26} + 290 + 200 = 498 \text{ deg K}$$

The receiver noise figure F_r is assumed to be 5 db or, $F_r = 3.16$; therefore, from (4.6), p.38.

$$T_{er} = (3.16 - 1)\, 290 = 625 \text{ degrees Kelvin}$$

*Note that the required value of the Duplexer isolation factor, L_D, depends upon $P_{\ell t}$, k, B_{if} and the allowable level of the Local-Transmitter "noise temperature", at the Duplexer output.

6.2 Equipment Design Procedures, SSBSC-AM Systems, For a Rayleigh-Fading Type of Received Signal, Dual Diversity, and Maximal-Ratio Combining

It is required to determine P_r from the SSBSC-AM radio-receiver performance curves in figure 4-2; using the above listed specification items (6), (7), and (8), and the values of the parameters which were derived in Subsection 6.1.

For SSBSC-AM systems, the modulator-demodulator conversion factor, D_{ss}, may be obtained from figure 3-5. Assuming D_{ss} = 4 db, the (noise-type) modulating-signal power level, P_{mn}, (= P_m) required so as to insure that 100% amplitude-modulation conditions will be exceeded for only 1.2 percent of the time, will be 4 db below the power level, P_{ms} (100%). The value for P_{ms} (100%), corresponding to 100% amplitude modulation, may be obtained from the modulator characteristic, which is in terms of sinusoidal modulating-signal power, P_{ms}, versus percent amplitude modulation, m_a.

The power level, p_{mi}, of the individual message-signal, at the modulator input, may now be obtained from the previously calculated value for P_m/p_{mi} (= P_{mn}/p_{mi}). Note that this calculated value for p_{mi} applies only to the case where the maximum number of simultaneously-active message channels is one-fourth of the available channels, as specified in this example. Other specified message-loading conditions will require different values for p_{mi}.

From the above derivations we have

$$10 \, \mathrm{Log}_{10} \, D_{ss} = 4 \text{ db}$$

$$10 \, \mathrm{Log}_{10} \, P_{mn} = 10 \, \mathrm{Log}_{10} \, P_{ms} \, (100\%) - 4 \text{ db}$$

$$10 \, \mathrm{Log}_{10} \, p_{mi} = 10 \, \mathrm{Log}_{10} \, P_{mn} - 28.3 \text{ db}$$

Note that both P_{mn} and p_{mi} are dependent upon the measurable or estimated value of P_{ms} (100%), obtainable from the modulator-performance characteristics.

The next step in the design procedure is to determine the operating value, on the normalized ordinate scale in figure 4-2. This scale value is obtained by substituting the above information on S_{oc}/N_{oc}, B_c, P_m/p_{mi}, and f_m in the "title" of the ordinate scale, which is,

$$10\mathrm{Log}_{10}\left(\frac{S_{oc}}{N_{oc}}\right) + 10\mathrm{Log}_{10} B_c - 10\mathrm{Log}_{10} f_m + 10\mathrm{Log}_{10}\left(\frac{P_m}{p_{mi}}\right)$$

or,

$$10\mathrm{Log}_{10}\,(1000)\left(\begin{matrix}\text{teletype}\\ \text{message}\end{matrix}\right) + 10\mathrm{Log}_{10}\,(110) - 10\mathrm{Log}_{10}\,(156000)$$

$$+ 10\mathrm{Log}_{10}(677.) = 30 + 20.4 - 52. + 28.3 = 26.7 \text{ db}$$

From figure 4-2, using the above calculated ordinate-scale value of 26.7 db and the dual-diversity curve, we obtain a value of -207.5 on the normalized abscissa-scale. Therefore,

$$10\ \mathrm{Log}_{10}\,P_r - 10\ \mathrm{Log}_{10}\,f_m - 10\ \mathrm{Log}_{10}\,(T_A + T_{er}) = -207.5$$

Substituting the values of T_A and T_{er} (and f_m = 52 db) in the above expression for the abscissa-scale value, we obtain for the SSBSC-AM system,

$$10\ \mathrm{Log}_{10}\,P_r = 52 + 10\ \mathrm{Log}_{10}\,(498 + 625) - 207.5 = -125.1 \text{ dbw}$$

However, this value of P_r (-125.1 dbw) must be corrected for the effects of system intermodulation noise; see section 4.3.6. Assuming that the receiver-output system intermodulation noise is equal to the receiver-output noise due to the pre-detection amplitude noise, the required correction to the above calculated value of P_r is 3 db. Hence,

$$10 \, Log_{10} P_r = -125.1 + 3 = -122.1 \text{ dbw (SSBSC-AM)}$$

The value of P_r (-122.1 dbw) is the median non-diversity received (total) RF signal-power level at the SSB receiver input, required to transmit the specified message load, with an average teletype-message error rate of .01 percent and a predicted voice-message intelligibility of 90 percent. This value of P_r is also the value to be substituted in (7.1) of Section 7, together with appropriate values of the other factors in (7.1), to obtain the required transmitter-output average power level, P_t.

In a SSBSC-AM system the radio-frequency spectrum bandwidth is approximately equal to the highest frequency in the modulating base-band signal; therefore,

$$B_{if} = B_{rf} = f_m = 156 \text{ kc.}$$

6.3 Equipment Design Procedures, FM Systems, for a Rayleigh-Fading Type of Received Signal, Dual Diversity, and Maximal-Ratio Combining

It is required to determine B_{rf} and P_r from the performance curves for FM systems using the system specifications and the values of the derived parameters in Subsection 6.1. Note that for the case of an FM receiver which does not use bandwidth compression, the "bandwidth compression factor" K_c (figure 4-10) is equal to 1 and, $B_{rf} = B_{if}$.

The first step in the design procedure is to ascertain which one of the design-performance curves in figure 4-6 p. 79 (dual diversity), is to be used. The particular design-performance curve to be used is found by substituting the above calculated values for S_{oc}/N_{oc}, B_c, f_m, and P_m/p_{mi} in:

$$10 \, Log_{10} (S_{oc}/N_{oc}) + 10 \, Log_{10} B_c - 10 \, Log_{10} f_m + 10 \, Log_{10}(P_m/p_m)$$

This procedure is identical to that used in the case of SSBSC-AM design calculations, except that the factor D_{ss} (= $P_{ms}(100\%)/P_{mn}$) is not involved for the FM case. A corresponding factor, P_{ms}/P_{mn}, is derived at the point in the FM-system design where the bandwidth ratio, B_{rf}/f_m, is chosen. Therefore, making the proper substitutions in the above expression we obtain,

$$10\text{Log}_{10}(1000)\left(\frac{\text{teletype}}{\text{message}}\right) + 10\text{Log}_{10}(110) - 10\text{Log}_{10}(152000) + 10\text{Log}_{10}(677.)$$

$$= 30 + 20.4 - 52 + 28.3 = 26.7 \text{ db}$$

The particular design-performance curve in figure 4-6, which has a "scale-value" of 26.7 db, is now used to determine B_{rf} and P_r. <u>Any point on this curve</u> yields a combination of values of B_{rf}/f_m and $P_r/f_m\ (T_A + T_{er})$ such that the average teletype character error rate is .01 percent, for the above values of f_m and P_m/p_m. The system designer may now decide to what extent he should interchange or trade bandwidth, B_{rf} or B_{if}, for required receiver-input power, P_r, that is, he may choose <u>any</u> point on the "26.7 db" design curve in figure 4-6 as the operating point.

For operation at the receiver threshold-level point on the "26.7 db" design curve in figure 4-6, we have,

$$B_{rf}/f_m = 10.5$$

or

$$B_{rf} = 10.5 \times 156 = 1638 \text{ kc}$$

and

$$\Delta F/f_m = 2.8$$

or

$$\Delta F = \Delta F_s = 2.8 \times 156 = 436.8 \text{ kc}$$

The required total modulating-signal average power level, P_{ms}, may be determined from the above required value of ΔF_s(= 436.8 kc) and the measured or the estimated modulator characteristics, ΔF_s versus P_{ms}, where the modulating-signal is a single sinusoidal signal. The required modulating-signal power level, P_{mn}, for a white-noise modulating signal, may be obtained from figure 3-4, relative to P_{ms}, using B_{rf}/f_m = 10.5; that is,

$$10 \text{ Log}_{10} P_{mn} = 10 \text{ Log}_{10} P_{ms} - 5.5 \text{ db}$$

The correct power level, p_{mi}, of the individual message-signal, at the modulator input may now be obtained from the previously-calculated value of P_m/p_{mi} (= 28.3 db) or,

$$10 \text{ Log}_{10} p_{mi} = 10 \text{ Log}_{10} P_{mn} - 28.3 \text{ db}$$

Note that this calculated value for p_{mi} applies to this specific example, where it was specified that the maximum number of simultaneously-active channels is one-fourth of the available channels. Other message-loading ratios will require different values for p_{mi}. Also, P_{mn} and p_{mi} are in terms of the measurable sinusoidal modulating-signal power level, P_{ms}.

The above calculated value of the radio-frequency signal bandwidth, B_{rf}, is the required bandwidth which includes 99.99 percent of the spectral energy in the FM radio-frequency spectrum. B_{rf} is the same for either a sinusoidal modulating signal or a noise-modulating signal, provided that the proper values are used for P_{ms} and P_{mn}.

The selected value for B_{rf}/f_m (teletype-voice) = 10.5, may now be used in figure 4-6 to determine the required median of the non-diversity receiver-input RF signal-power level, P_r, as follows: at the intersection of the B_{rf}/f_m = 10.5 level on the ordinate scale, and the "design curve" for 26.7 db, we have,

$$10 \, \text{Log}_{10} P_r - 10 \, \text{Log}_{10} f_m - 10 \, \text{Log}_{10} (T_A + T_{er}) = -210.5$$

or, for the FM system

$$10 \, \text{Log}_{10} P_r = 52. + 30.4 - 210.5 = -128.1 \text{ dbw.}$$

This value of P_r (-128.1 dbw) must be corrected for the effects of system intermodulation noise; see section 4.3.6 and figure 4-A. Assuming that the receiver-output system intermodulation noise is equal to the receiver-output noise due to the pre-detection amplitude noise, the required correction to the above calculated value of P_r is 3 db. Hence,

$$10 \, \text{Log}_{10} P_r = -128.1 + 3. = -125.1 \text{ dbw (FM)}$$

The above calculated value of P_r(-125.1 dbw) is the median non-diversity received (total) RF signal-power level at the FM receiver input, required to transmit the specified message load, with an average teletype-message error rate of .01 percent and a predicted voice-message intelligibility of 90 percent. This value of P_r is also the value to be substituted in (7.1) of section 7, together with appropriate values of the other factors in (7.1), to obtain the required transmitter-output <u>average</u> power level, P_t.

Note--The assumed value of 3 db for FM system intermodulation noise may be too low in a system which employs an RF spectrum band-width of several Mc/s. A more realistic value might be 3 db for equipment intermodulation noise and 2 db (B_{rf} = 1.64 Mc/s) for path-modulation noise. With the additional correction of 2 db for path modulation effects, the required value for median P_r (FM) = - 123.1 dbw.

The above design procedures apply to the condition where particular system specifications and a grade of service are given and it is required to determine the total average modulating-signal power level, P_{mn}, the individual message-channel average modulating-signal power level, p_{mi}, the RF signal spectrum bandwidth, B_{rf}, or B_{if}, and the required receiver-input RF signal-power level, P_r. The system design curves may also be used to estimate system performance when either or both B_{rf} and P_r are specified. Furthermore, system performance may be estimated for a <u>time-varying median-value</u> of P_r; this time-varying median P_r is obtainable by methods outlined in section 7.

6.4 Test Procedures for SSBSC-AM and FM Systems

Acceptance tests on radio communication systems involve the measurement of the system performance, in terms of the message error rate for particular message loads and transmitter output power. Test programs should be based on acceptable message-error rates, for a specified number of simultaneously-active message channels, with a given radio-frequency spectrum bandwidth, and for a particular value of transmitter-output power. The information contained in the system design curves of this report may be used as a guide for determining proper test procedures, and also for estimating equipment test-level values for the system variables and parameters, as applied to SSBSC-AM and FM multi-channel radio communication systems. Acceptance-test programs may be arranged conveniently by following the system design procedures outlined in Sections 6.1, 6.2, and 6.3. The major differences between design procedures and test procedures are due to the fact that in the former case certain parameters are to be estimated while in the latter case these same parameters are specified and the system performance is to be measured. In either case the design equations and the design curves in this report may be used to obtain quantitative estimates of the performance of the radio communication system.

6.5 System Performance Estimates for SSBSC-AM and FM Systems.

Using the procedures outlined in Sections 6.1, 6.2, 6.3, and the appropriate design curves and equations given in this report, the performance of a specific radio communication system may be estimated in terms of the received RF-signal power, P_r, and the corresponding message error rate. One form of presentation of these results is shown in figure 6-1. Note that the curves in this figure are only presented as being typical and are not accurate, hence they do not apply to a particular system; the same is true of figures 6-2 and 6-3.

A typical measured or estimated distribution [Rice, Longley, Norton; 1959] of a received tropospheric radio signal is shown in figure 6-2. The "sampling periods" should be restricted to intervals during which the distribution of the received RF signal approximates a Rayleigh distribution. The sampling period may also be defined in terms of an interval of time which is very large compared with the duration of an information bit; this definition permits the transmission of a "usable" sample of the message during the sampling period. The sampling-period median received-power level, P_r, is sometimes referred to as the "hourly median", with the (implied) understanding that it is not necessarily the median for an hour but that the period of time might only be a fraction of an hour, or it might be several hours; the above definitions of the sampling period may be used to clarify the meaning of the hourly median, of the received RF signal-power level. The total time involved in the percentage-of-sampling periods scale must be very much greater than the sampling-period time.

Combining the data in figure 6-1 with that in figure 6-2, using the method outlined in Appendix D, we obtain the distribution of the message error rate as shown in figure 6-3. Note that the positions of these curves, relative to the message error-rate scale, depend directly upon the median power level, P_r, of the received RF-signal, as given in figure 6-1. The total time to be considered in figure 6-3 is identical to the total time indicated in figure 6-2; that is, if figure 6-2 represents the distribution of the received RF-signal power levels for one day, then figure 6-3 will show the distribution of the message error rate for the same period of time, that is, for the same day; similar results may be obtained for a portion of a day, for one week, one month or for any specified total time.

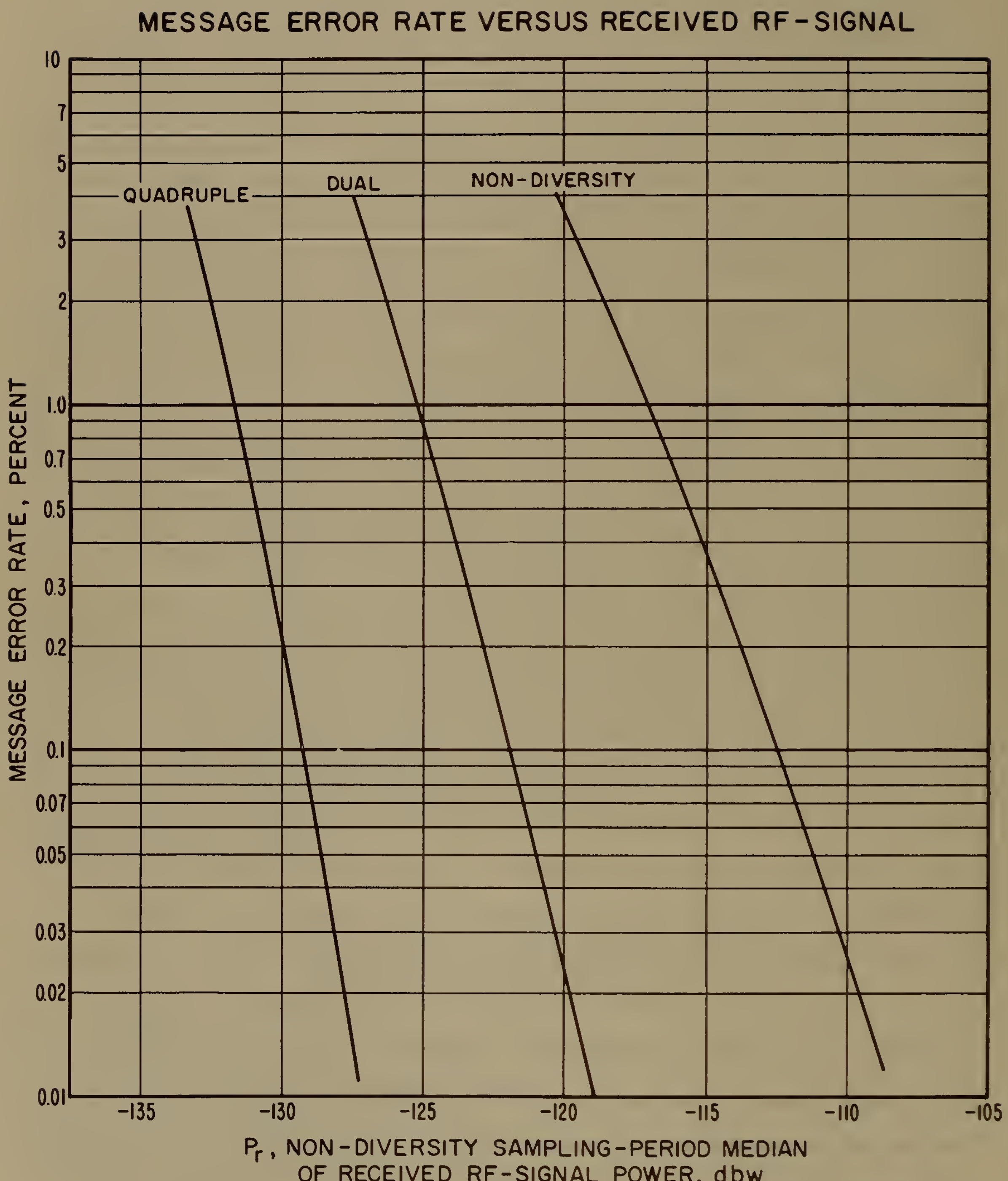

MESSAGE ERROR RATE VERSUS RECEIVED RF-SIGNAL
MESSAGE ERROR RATE, PERCENT
10
7
5
3
2
1.0
0.7
0.5
0.3
0.2
0.1
0.07
0.05
0.03
0.02
0.01
QUADRUPLE
DUAL
NON-DIVERSITY
-135
-130
-125
-120
-115
-110
-105
P_r, NON-DIVERSITY SAMPLING-PERIOD MEDIAN
OF RECEIVED RF-SIGNAL POWER, dbw

Figure 6-1

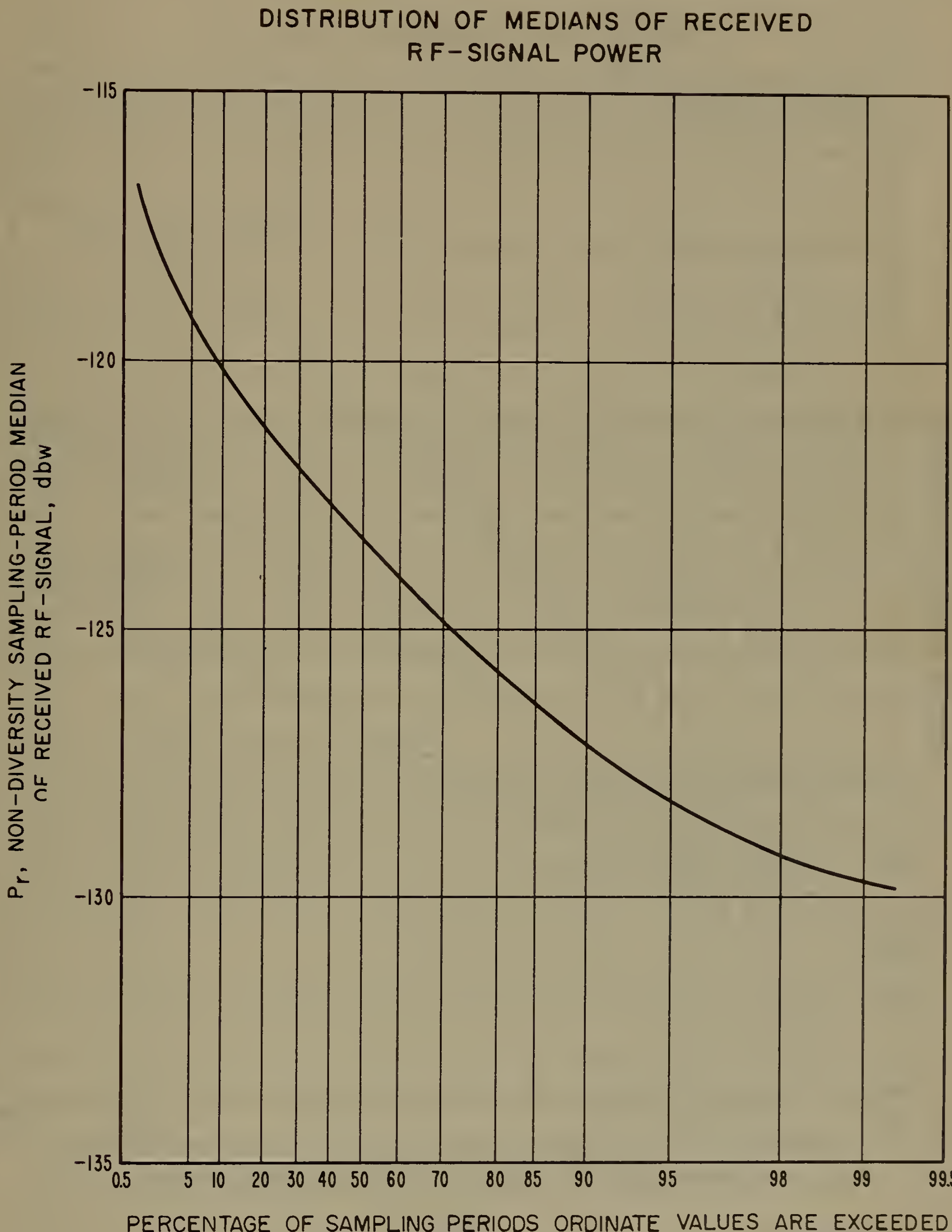

DISTRIBUTION OF MEDIANS OF RECEIVED
RF-SIGNAL POWER
-115
-120
-125
-130
-135
0.5
5
10
20
30
40
50
60
70
80
85
90
95
98
99
99.5
Pr, NON-DIVERSITY SAMPLING-PERIOD MEDIAN
OF RECEIVED RF-SIGNAL, dbw
PERCENTAGE OF SAMPLING PERIODS ORDINATE VALUES ARE EXCEEDED

Figure 6-2

DISTRIBUTION OF MESSAGE ERROR RATE

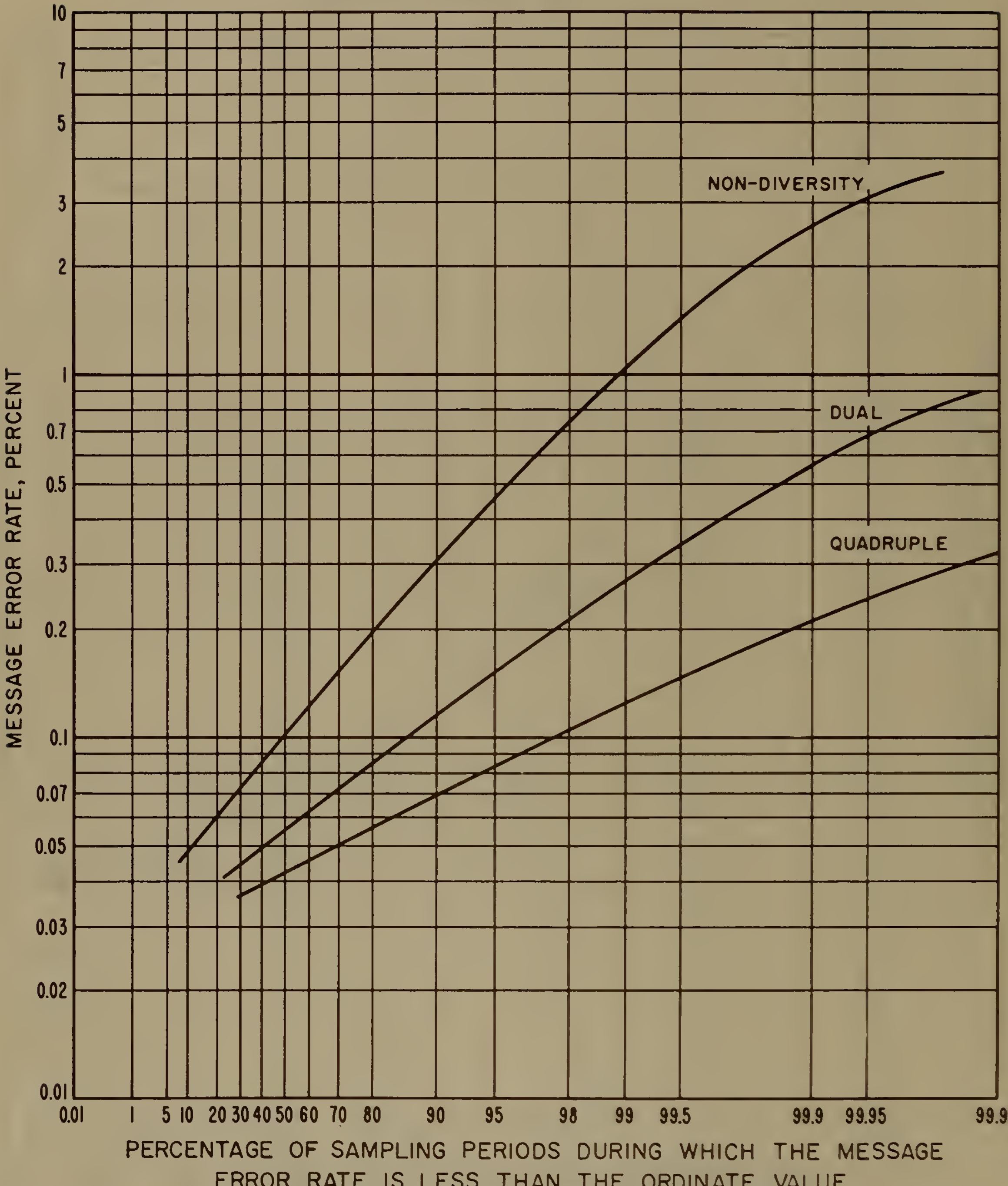


Figure 6-3

The information in figure 6-3 is presented in a form which may be easily and unambiguously interpreted. The statistical nature of the message error rate is clearly indicated by these curves. From data of the type shown in figure 6-3 it is possible to predict the system performance for various periods of time such as, all-year, worst-day, etc.

The data in figure 6-1 would apply to a particular message load on the specified system. The material in this report may also be used [Barsis, et al., 1961] to estimate radio communication system performance for various assumed system-parameter values, and the results may then be presented in terms of the percent message error and the various parameters such as, the RF-signal spectrum bandwidth, B_{rf}, number of message channels, N, percent message load, etc. This type of presentation of system performance estimates may be applied to specific cases, and it is sometimes more convenient to use this form of presentation than to use the general form of design curves shown in this report. However, these design curves are basic and of course are not restricted to specific cases.

Test procedures involve the calibration of the receiver so that it may be used to indicate the power level of the received RF signal. The receiver-input signal power, P_r, is available at the receiver-input terminals. Hence, when calibrating a receiver with a signal generator, using impedance mismatch conditions at the receiver input, in order to simulate operating conditions, care must be exercised to make certain that the values of P_r obtained from the signal generator output-power scale are properly corrected. These corrections would be in terms of the output impedance of the signal generator and the characteristics of the impedance-changing unit, which may be required between the generator output and the receiver input. Proper corrections (see Appendix A) will yield the available RF signal power at the receiver input terminal.

7. REQUIRED TRANSMITTER-OUTPUT POWER

Under conditions where the signal power is transmitted over a radio path, the required transmitter-output RF-signal power, P_t, may be calculated from the following (equivalent) equations:

$$P_t = (P_r L_T L_{bm} L_c L_t L_{du})/G_p \tag{7.1}$$

or,

$$\begin{aligned} 10\text{Log}_{10} P_t = & \ 10\text{Log}_{10} P_r + 10\text{Log}_{10} L_T + 10\text{Log}_{10} L_{bm} \\ & - 10\text{Log}_{10} G_p + 10\text{Log}_{10} (L_c L_t L_{du}), \text{ dbw} \end{aligned} \tag{7.1a}$$

where,

P_t = Transmitter output average power, watts.

P_r = Required available receiver-input median of the non-diversity signal power level, watts. Median values of P_r are used primarily because of the fact that calculations of the path transmission loss, L_{bm}, yield results in terms of median values for L_{bm}. Furthermore, it is easier to obtain measured values of median P_r (or L_{bm}) from their (measured) distributions than it is to obtain average values for these factors.

L_T = Total antenna-system transmission loss factor at the transmitter, as a ratio.

L_{bm} = Median of the basic transmission loss. This factor is defined as the ratio of the power input to the terminals of a loss-free isotropic transmitting antenna to the power available from the terminals of a loss-free isotropic receiving antenna. The median basic transmission loss, L_{bm}, and the effective path gain, G_p, may be calculated by methods outlined in NBS Technical Note #15. [Rice, Norton, Longley, 1959] and NBS Technical Note #101 [Rice, et al., 1962] .

L_c, L_t, L_{du}* = Transmission loss factors for the (receiving) antenna circuit, transmission line, and the duplexer network, respectively, as a ratio. These factors are defined as the ratio of the power delivered to the input of a network to that portion of the input power which is available at the network output. These factors are conveniently measured for non-reflecting impedance-matched conditions at the input and at the output of the network.

G_p = Effective path gain [Hartman, Wilkenson, 1959; JTAC, 1960] of the transmitting and receiving antennas combined, relative to an isotropic antenna, as a ratio.

Eq. (7.1) may also be written [Norton, 1956] as:

$$P_t = (R\, B_{if}\, F\, L_T\, L_{bm}\, k\, T_o)/G_p \qquad (7.2)$$

or,

$$10\text{Log}_{10}P_t = 10\text{Log}_{10}R + 10\text{Log}_{10}B_{if} + 10\text{Log}_{10}F + 10\text{Log}_{10}L_T + 10\text{Log}_{10}L_{bm} - 10\text{Log}_{10}G_p - 204, \text{ dbw} \qquad (7.2a)$$

The factor F in (7.2) is the "effective noise figure," [Barsis, Norton, Rice, Elder, 1961, Appendix III] . For a receiver with no spurious responses,

$$F = \frac{T_a + (L_c - 1)\, T_c + L_c(L_t - 1)\, T_t + L_c L_t(F_r - 1)}{T_o} \qquad (7.3)$$

The factors in (7.3) are identical to the factors used in (4.3) and (4.3a), provided that impedance-matched conditions exist in the receiving antenna system; these factors are described in Appendix A.

*These factors apply only to the case where non-reflecting impedance-matching conditions prevail in the transmitting and receiving antenna systems, except at the receiver input terminals.

The factor R in (7.2) is the predetection signal-to-noise ratio, P_{if}/N_{if}. The relationship between R and P_r may be obtained by combining (7.1) and (7.2), and is

$$P_r = \frac{R\, B_{if}\, F\, k\, T_o}{L_c\, L_t\, L_{du}} \qquad (7.4)$$

Multiplying both sides of (7.4) by $1/f_m (T_A + T_{er})$, we obtain

$$\frac{P_r}{f_m (T_A + T_{er})} = \frac{R\, B_{if}\, F\, k\, T_o}{L_c\, L_t\, L_{du}\, f_m (T_A + T_{er})} \qquad (7.5)$$

From (4.3) and (7.3), and neglecting the term $P_{\ell t}/k\, B_{if}\, L_D$, we have

$$F = \frac{L_c\, L_t\, L_{du} (T_A + T_{er})}{T_o} \qquad (7.6)$$

Combining (7.5) and (7.6) we obtain

$$\frac{P_r}{f_m (T_A + T_{er})} = \frac{R\, B_{if}\, k}{f_m} = \frac{R\, B_{rf}\, k}{f_m} \qquad (7.7)$$

Equation (7.7) is useful when it is desired to obtain values of the predetection signal-to-noise ratio, R, from the system design curves; this equation gives the appropriate substitution of abscissa-title factors in the design curves in figures 4-2, 4-4, 4-5, 4-6, and 4-7.

The required transmitter-output power, P_t, may be determined in terms of either:

(1) the required receiver-input power level, P_r, which is obtainable directly from the system design curves and then substituted in (7.1) or (7.1a), or

(2) the required predetection signal-to-noise ratio, R and the required bandwidth, B_{rf} (= B_{if}); values for these factors are inserted in (7.2) or (7.2a).

The choice between the use of either of the above procedures, (1) or (2), depends entirely upon the designer's preference; however, where performance-test measurements are made it is very much more convenient to measure P_r than to measure R. Furthermore, (7.1) and (7.1a) contain only power levels, loss factors and gain factors and hence these equations are easily understood. Equations (7.2) and (7.2a) involve the additional factors, F and R, each as a ratio; this situation apparently is confusing to many engineers.

The following relationships between P_{if}/N_{if} and the various system parameters have been found to be useful, and are included here for convenient reference:

$$R = \frac{P_{if}}{N_{if}} = \frac{P_r}{k(T_A + T_{er})\, B_{if}} \tag{7.8}$$

$$= \frac{P_a}{L_c L_t L_{du}\, k(T_A + T_{er})\, B_{if}} \tag{7.9}$$

$$= \frac{P_t G_p}{L_T L_{bm} L_c L_t L_{du}\, k(T_A + T_{er})\, B_{if}} \tag{7.10}$$

where,

k = Boltzman's constant = 1.3804×10^{-23}

T_A = Effective receiving-antenna system output noise-power temperature, in degrees Kelvin; see Appendix A, pp. 157-161.

T_{er} = Effective receiver-input temperature, in degrees Kelvin. This factor is given by $T_{er} = (F_r - 1) \times 290$.

P_a = Received RF-signal, available at the terminals of the "loss-free" antenna.

From (7.8), (7.9), and (7.10) we have,

$$P_r = \frac{P_a}{L_c L_t L_{du}} \tag{7.11}$$

$$P_r = \frac{P_t G_p}{L_T L_{bm} L_c L_t L_{du}} \tag{7.12}$$

CONCLUSIONS

The equipment-performance design curves presented in this report are based on the performance of "ideal" radio receivers and maximal-ratio type of combiners and hence these curves will yield optimistic results. However, for FM systems, placing the single sinusoidal modulating signal in the highest (baseband) frequency partially compensates for the optimistic assumptions regarding receiver (ideal) performance and combiner operation. Furthermore, proper allowance may be made for the effect of system intermodulation noise using the methods outlined in subsection 4.3.6 and following the examples worked out in section 6.

In practice, the degree of conformity between calculated and measured system performance will depend largely upon the performance of the combiner, compared to the (ideal) performance of the maximal-ratio combiner. In fact, a comparison between the calculated and the measured performance of a system constitutes a reliable check on the combiner performance, after due allowance is made for all other factors.

The method used to obtain the equipment-performance curves implies that pre-detection diversity combining is used; the use of these results, where post-detection combining is employed, might be questioned. However, if the combiner operates so as to improve the distribution of the signal-to-noise ratio of the (combiner) input signals, then our methods are justified; since the overall results should be similar whether the combiner precedes or follows the receiver demodulator.

The results of this work are presented in a form such as to be directly usable in determining system performance when combined with effective antenna gain and radio-path transmission-loss estimates.

The design curves shown are basic and may be used to determine sets of curves showing the system-performance characteristics for various combinations of the system parameters. These design curves involve the fundamental parameters of the system and hence may be used to establish equipment-performance standards on a quantitative basis.

It should be understood that the technical analysis of FM and SSBSC radio communication systems carried out in this paper does not involve economic factors or operational complexities. However, this type of technical analysis is required as a starting point for the determination of a complete analysis and comparison of the different types of systems.

In conclusion, a method has been outlined in this report, and a set of system-design curves have been developed based on the combined performance characteristics of the receiver, diversity combiner, modulator, and the message-signal decoder unit. The net results of this work are a set of curves in which all of the important system parameters are used as scale factors. The technique employed in this work essentially provides a link between information theory results and the application of these results.

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LIST OF SYMBOLS

B_b	Composite baseband signal spectrum bandwidth, c/s.
B_c	Message-signal spectrum bandwidth in c/s. In a single-sideband frequency-division multiplex system B_c is also the bandwidth occupied by the message signal in the composite modulating baseband signal.
B_{if}	Effective (noise-power) bandwidth of the receiver IF circuit, at the receiver demodulator input.
B_n	Effective bandwidth of a network, in c/s.
B_{rf}	Radio-frequency signal power-spectrum bandwidth of the total radio-frequency signal, in c/s.
B_{rfn}	Radio-frequency signal FM spectrum bandwidth for a white-noise type of modulating signal.
B_{rfs}	Radio-frequency signal FM spectrum bandwidth for a sinusoidal modulating signal.
D_{ss}	Modulation-demodulation conversion factor. This factor is defined as, $P_{ms}(100\%)/P_{mn}$, and applies only to amplitude modulation.
F	Effective noise figure (Barsis, et al, Appendix III, 1960).
f_a, f_b, f_c	Frequency in c/s of particular signals in the modulating composite baseband signals.
f_m	Maximum value of the modulating-signal frequency in c/s. Also, the highest frequency in a composite baseband modulating signal.
F_n	Noise figure of a network, as a ratio; see figure A-1.
F_r	Noise figure of (radio) receiver, or amplifier, as a ratio, measured according to IRE Standards (IRE Comm. 1953, 1960).

G_r — Power gain of the linear section of the (radio) receiver, or amplifier. This factor is defined as the ratio of the available output power to the power delivered to the receiver input. This is <u>not</u> the available gain.

k — Boltzman's constant = 1.38042×10^{-23} joules per degree Kelvin.

K_c — Bandwidth-compression factor, applied to the receiver. This factor indicates the degree of IF bandwidth compression in the receiver; note that $0 \leq K_c \leq 1$.

L_c — Transmission loss factor for the antenna network, due to the lossy elements in the receiving antenna. This factor is equal to the ratio of the non-reflecting impedance-matched delivered power at the network input to the non-reflecting impedance-matched power at the output of the network.

L_{du} — Ditto, for the duplexer.

L_t — Ditto, for the antenna-system transmission line.

L_n — Transmission loss factor of a network; defined as the ratio of the non-reflecting impedance-matched input power to the non-reflecting matched-impedance output power.

L_D — Isolation factor for the duplexer. This factor is equal to the ratio $P_{\ell t}/N_{\ell t}$, see figure 4-1.

L_T — Total transmitting antenna system transmission loss, as a ratio.

m_a — Amplitude-modulation index, in percentage.

M_{in} — Impedance mismatch factor, at network input.

M_{ir} — Impedance mismatch factor at the receiver input terminals.

M_A — Number of simultaneously-active message signals each of which requires the same value for the product $(B_c(S_{oc}/N_{oc}))_A$.

M_I — Ditto, for the product $(B_c\,(S_{oc}/N_{oc}))_I$.

M_N Ditto, for the product $(B_c (S_{oc}/N_{oc}))_N$.

N_a Receiving-antenna environmental noise power, in watts.

N_{ai} Ditto, for interfering signals.

N_{ao} Available noise power at the output terminals of a network, in watts; see figure A-1.

N_{at} Receiving antenna noise power, transmitted with the RF signal, in watts.

N_{au} Available noise power at receiving-antenna system output, due to antenna temperature, T_a, plus noise generated within the antenna-system units.

N_{ei} Noise power generated within and contributed by a network, in watts. This noise power is referred to the input terminals of the network, see figure A-1.

N_{if} Total of the average noise power at the IF output, or at the demodulator input, in watts.

N_o Receiver output average total noise power, in watts.

N_{oc} Average noise-power level of a message-signal channel having an effective bandwidth of B_c, at the receiver output; or, a relative average noise-power level per bandwidth, B_c, at decoder-unit input terminals.

N_T Total available noise power from the receiving-antenna system

p_m Average power level of a message-signal in the modulating baseband signal, in watts.

p_{mi} Average power level of the i^{th} message signal in the modulating baseband signal, in watts.

P_{if} Receiver IF-output total average signal power, or, average signal power at the demodulator input, in watts.

$P_{\ell t}$ Power output of the local transmitter, in watts.

P_m	Total average power level of the baseband modulating signal. It is understood that this power level is determined under conditions when a specified number of simultaneously-active message signals are in operation.
$\frac{P_m}{p_m}$	Message loading ratio, being the ratio of the total average power level of the modulating signal to the average power level of a message signal.
P_{mn}	Average power level of a "white-noise" modulating signal, in watts; where the modulating signal is distributed uniformly across a specified portion of the modulating baseband.
P_{mp}	Peak-power level in the composite modulating signal, exceeded for only a specified percentage of the time.
P_{ms}	Average power level of a sinusoidal modulating (system test) signal, in watts; under conditions where only one modulating signal is used. For FM systems the frequency of this modulating signal is f_m --the highest frequency in the baseband signal.
$P_{ms}(100\%)$	Sinusoidal-signal power level required to produce 100% amplitude modulation.
P_r	Total average radio-frequency signal power available at the receiver-input terminals, in watts. For a time-varying RF signal, the median value of P_r is used in this report.
P_t	Transmitter average total power output, in watts.
P_{tp}	Peak-power level at the transmitter output, exceeded for only a specified percentage of the time.
$R \equiv \frac{P_{if}}{N_{if}}$	Predetection total signal-to-total noise ratio. For the case of a varying received signal, R will be in terms of its median value.
S_o	Receiver output (baseband) average total signal power, in watts.

S_{oc} — Average message-signal power in the receiver output, or a relative message-channel signal average power level at the decoder-unit input terminals.

T_a — Receiving-antenna noise (power) temperature, associated with the sky noise and the environmental noise, in degrees Kelvin. This noise temperature, or its equivalent noise power N_a, is available at the output terminals of the loss-free antenna.

T_{ai} — Ditto, for interfering-signal power.

T_{at} — Ditto, for the transmitter-output noise power transmitted with the carrier signal.

T_c — Thermal temperature of the lossy elements in the (receiving) antenna-circuit network; see figures A-1, 4-1, and Appendix A.

T_{du} — Ditto, for the duplexer in receiving-antenna system.

T_{ei} — Effective input noise (power) temperature of a network, degrees Kelvin; see figure A-1.

T_{eo} — Effective output noise-power temperature of a network, degrees Kelvin; see figure A-1.

T_{er} — Effective receiver-input temperature, degrees Kelvin. (IRE Comm. on Noise, 1960).

T_g — Output noise-power temperature of source or signal generator, degrees Kelvin; see figure A-1.

T_t — Thermal temperature of the receiving-antenna transmission line, degrees Kelvin.

T_n — Thermal temperature of the lossy elements in a passive network, degrees Kelvin; see figure A-1.

T_o — Standard reference temperature, assumed to be 290. degrees Kelvin. (Ire Comm. on Noise, 1960).

T_A — Effective output noise (power) temperature of the receiving-antenna system, degrees Kelvin. T_A is defined by: $N'_T = k T_A B_{if}$, see Appendix A.

Z_{du}	Characteristic output impedance of the duplexer. In general this is the characteristic output impedance of the network which precedes the receiver.
Z_{in}	Input impedance of network or amplifier, in ohms.
Z_{ir}	Input impedance of the radio receiver.
Z_{og}	Output impedance of source or signal generator, in ohms.
Z_{on}	Output impedance of network or amplifier, in ohms.
Z_{Ii}	Image or characteristic input impedance of a network or transducer.
Z_{Io}	Image or characteristic output impedance of a network or transducer.
ΔF	Peak radio-frequency carrier deviation above and below the un-modulated carrier frequency, in c/s; for a single sinusoidal modulating signal having an average power level P_{ms}, equal to the total average power level of the composite modulating signal.
ΔF_c	Peak radio-frequency carrier deviation in c/s; due to the message-signal modulating power, p_m.
$\frac{\Delta F}{f}$	Deviation ratio, in radians. This ratio is equal to the "phase deviation" of the radio-frequency carrier, that is, the extreme angular displacement of the carrier from its "average" position.
β	Correlation (radiofrequency) bandwidth in Mc/s. This bandwidth is defined as the frequency separation in c/s of spectral components whose cross-correlation coefficient has an average value of 0.5.
$\phi(\Delta F/f)$	Calculated ratio of the radio-frequency spectrum bandwidth B_{rf}, to the deviating frequency f; applicable only to the case where a single sinusoidal modulating signal is used--see figure 3-1.

APPENDIX A

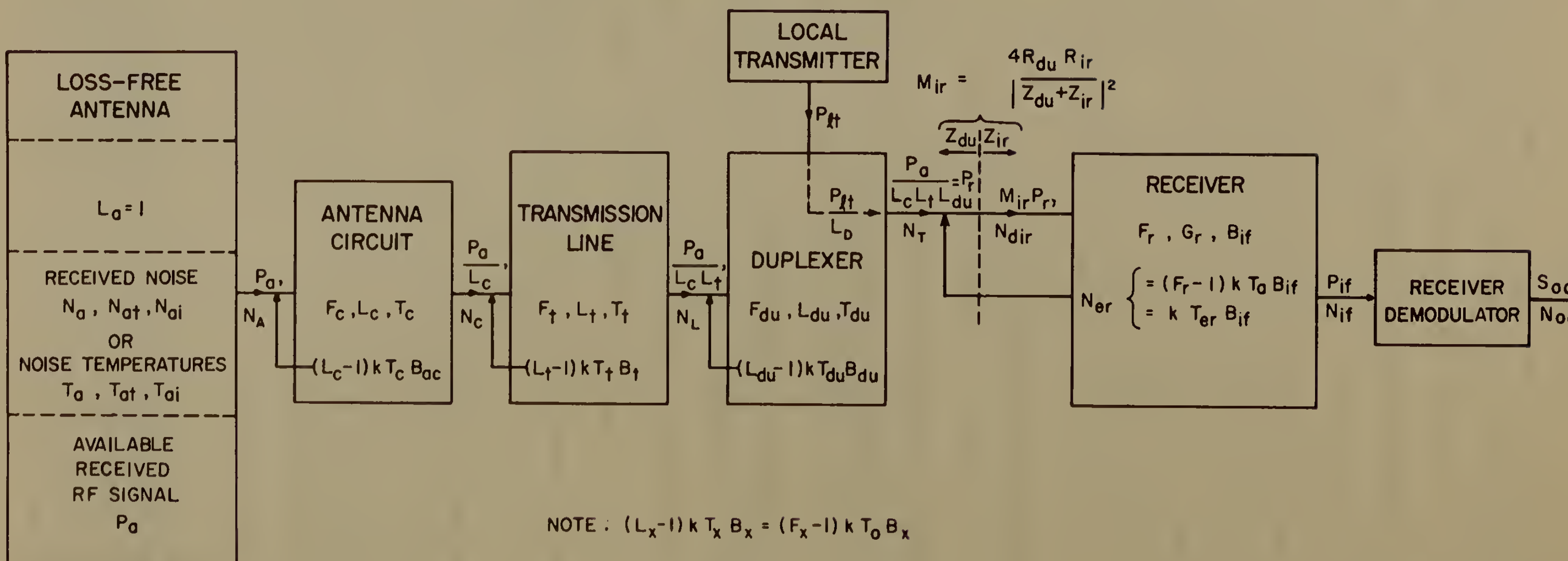
Pre-Detection Noise-Power Levels in the Receiving System.

The objective is to obtain the functional relationship between the predetection noise-power level, N_{if} in figure 4-1, and the various receiving-system parameters: F_x, L_x, G_x, T_x, etc., as indicated in this figure. The method used to determine the total pre-detection noise-power level in the receiver consists of properly adding the noise-power outputs of the receiving-system (linear) twoport network units in cascade [Ewen, 1959; Hansen, 1959]. Basically, the noise performance of each network is considered in terms of the amount of noise contributed by the network, and the resultant effect of this noise on the network-output signal-to-noise ratio.

In this analysis, the noise performance of each twoport network unit in the receiving system is determined in terms of the total noise power delivered* to and absorbed at the input of the unit (including the noise power originating in the unit) and the resultant available* noise power at the output of the unit. The use of delivered noise powers at the input of the units results in noise-performance factors which depend primarily upon the internal characteristics of the network units, such as power gain, effective bandwidth, image or characteristic impedance, and transmission loss. Furthermore, network noise figures, and noise performance characteristics, may be defined in terms of delivered (signal and noise) powers instead of available powers at the network input, without any loss in generality. The use of the delivered-power concept has the additional advantage of

*If impedance mismatch conditions exist at the input terminals of the network, the delivered power is dependent upon the input impedance of the network and the output impedance of the preceding network. The available power is the maximum power which may be obtained from the network, at its output terminals; see following subsection.

BLOCK DIAGRAM OF RECEIVING SYSTEM



NOTE : $(L_x - 1)\, k\, T_x\, B_x = (F_x - 1)\, k\, T_o\, B_x$

Fig. 4-1

making it possible to define and to calculate the impedance mismatch factors in terms of the various system impedances, independently of the power gain or the dissipative transmission loss, and the effective bandwidth of the networks.

The available-power concept, as applied to the noise power (and also the signal power) at the network output terminals, is valid because we are restricting this analysis to receiving antenna-system units having positive input and output resistances. In other words, the "exchangeable-power" concept is not required in this case [Haus, Adler, 1959] .

The various factors discussed above such as delivered power, available power, transmission loss, etc. are defined and discussed in the following subsections.

Effects of Impedance Matching on Delivered, Matched-Impedance, and Available Powers.

In this appendix the effects of impedance mismatching is treated in a general manner, for the purpose of indicating how the impedance mismatch factors affect the noise-power level, or the noise temperature, at the receiving-antenna system output terminals. No attempt is made here to derive the mismatch factors as explicit functions of the antenna system impedances.

The noise performance properties of a linear lossy passive bilateral twoport network, of the type generally used in a receiving-antenna system, are dependent upon impedance-matching conditions at the network input and output terminals. The effects of impedance matching on network-input and output powers are determined from the following considerations, based on information in the Radio Engineers Handbook [Terman, 1943] and the ITT Handbook [pp. 549-616] .

The power delivered to and absorbed by a load having an input impedance Z_ℓ ($= R_\ell + jX_\ell$), when connected to the output terminals of a source or generator having an internal impedance Z_g ($= R_g + jX_g$), and an internal (generated) voltage, E_g, is

$$P_{del} = |I|^2 R_\ell = \frac{|E_g|^2}{|Z_g + Z_\ell|^2} \times R_\ell \qquad (A.1)$$

where, I = Current flowing in the load.

If the generator internal (or output) impedance is non-reflecting matched to the load impedance, that is, $Z_g = Z_\ell$, the power in the load becomes

$$P_{mat} = \frac{|E_g|^2}{4\,|Z_g|^2} \times R_g \qquad (A.2)$$

Note that the impedance-matched power, P_{mat}, is absorbed in the load under non-reflecting impedance-matched conditions at the input terminals of the load.

The maximum power will be obtained from the generator when the generator impedance is conjugate to the load impedance; that is, $Z_g = Z_\ell^*$, and hence $R_g = R_\ell$, and $X_g = -X_\ell$. The available power is defined as the maximum power which may be obtained from a given source, and is

$$P_{av} \equiv \frac{|E_g|^2}{4R_g^2} \times R_\ell = \frac{|E_g|^2}{4R_g} \qquad (A.3)$$

A study of (A.1), (A.2), and (A.3) shows that:

(a) the available power closely approximates the non-reflecting matched-impedance power only when the ratio X_g/R_g is very much less than unity; that is, $X_g/R_g << 1$, and

(b) conjugate-impedance matching is a special case of impedance (mis)matching and hence its use results in power reflections at the conjugate-impedance-matched point.

The impedance mismatch factor is defined as the ratio of the power delivered to the load without matching to the power delivered with either type of impedance-matched conditions. Thus, the impedance mismatch factor may be defined in terms of either the non-reflecting matched-impedance power or the conjugate-impedance matched (available) power. In terms of the non-reflecting matched-impedance power, the impedance-mismatch factor is defined as

$$M_{mat} \equiv \frac{P_{del}}{P_{mat}} = \frac{4\,|Z_g|^2 R_\ell}{|Z_g + Z_\ell|^2 R_g} = \frac{4\,(R_g^2 + X_g^2)\,R_\ell}{|Z_g + Z_\ell|^2 R_g} \qquad (A.4)$$

In terms of conjugate-impedance matched or available power, the impedance-mismatch factor is defined as

$$M_{av} \equiv \frac{P_{del}}{P_{av}} = \frac{4\,R_g R_\ell}{|Z_g + Z_\ell|^2} \qquad (A.5)$$

From (A.4) and (A.5) it is evident that the impedance-mismatch factors for matched-impedance power and for available power are identical <u>only</u> when the ratio X_g/R_g is equal to zero. Hence, these powers are approximately equal when the ratio X_g/R_g is very much less than 1. In other words, maximum power transfer and non-reflecting impedance-matching can be attained simultaneously only when $X_g/R_g << 1$.

When a network is inserted between the generator and the load, the impedance-mismatch factors at the network input and output terminals may be determined in terms of the net or effective input and output impedances, Z_{in} and Z_{on}. In turn, Z_{in} and Z_{on} may be calculated from the network input and output image impedances, Z_{Ii} and Z_{Io}, and the generator and load impedances, Z_g and Z_ℓ [Terman, 1943]. The input image impedance of a network is defined as the impedance looking into the input terminals of the network with the output terminals terminated by the output image impedance; note that the input and output terminals may be interchanged in this definition. The image impedance of a network may be determined from

$$Z_I = \sqrt{Z_{oc} Z_{sc}} = R_I + jX_I \tag{A.6}$$

where Z_{oc} = Network input impedance with the network output terminals open-circuited

Z_{sc} = Network input impedance with the network output terminals short-circuited.

In a symmetrical network such as a transmission line, the input and output image impedances are equal to each other and also equal to the characteristic impedance of the line.

The image impedances of a network may also be calculated in terms of the equivalent T or π network impedances [Terman, 1943, pp. 208-210]. Conversely, the equivalent T or π network impedances may be calculated for known or assumed values of network image impedances. These latter relationships are useful where it is desired to calculate the net or effective input and output impedances, Z_{in} and Z_{on}, of a network in a tandem-network system in terms of network impedances

In a system of tandem networks, the impedance mismatch factors, M_{in}, at the input terminals of the networks are functions of the various network image impedances, or the equivalent T or π network impedances, and the generator and the load impedances. Each point or junction involving impedance mismatching may be treated as being equivalent to the simple case of impedance mismatch between a source (or generator) and a load; Z_g being replaced by Z_{on} and Z_ℓ being replaced by Z_{in}. The network-input impedance mismatch factors may be calculated by means of either (A.4) or (A.5); however, as stated previously, no attempt is made in this Appendix to derive the network-input mismatch factors explicitly in terms of the various impedances of the system.

The results of the above derivations are to be applied to the analysis of the noise performance of linear lossy passive bilateral networks in the antenna system; see pp. 147-151.

Note that if non-reflecting (image or characteristic) impedance matching conditions are assumed to exist at all points in the system except at the receiver input terminals, and if the ratios, X_x/R_x, are assumed to be very much less than 1, throughout the antenna system, under these conditions the available power will be approximately equal to the matched-impedance power. Furthermore, at all of the impedance-matched impedance points the available power will be approximately equal to the delivered power. However, in the general case, where impedance mismatched conditions exist at more than one point in the system, and the ratios, X_x/R_x, are not each very much less than 1, it is possible to define and use mismatch factors (to estimate delivered power) in terms of either the non-reflecting matched-impedance power or the available power.

Noise Performance of a Transducer Network

Each lossy (bilateral) transducer unit, or network, in a receiving-antenna system contributes (average) available noise power which is equal to $(F_n-1)kT_oB_n$ [Friis, 1944]; where F_n is the noise figure of the transducer unit, k is Boltzman's constant, and T_o is the "standard reference temperature" of the generator impedance used to measure F_n and is assumed to be 290 degrees Kelvin [IRE Comm. on Noise, 1960]. If the generator or source-impedance temperature, T_g, is not T_o degrees, the correct of "standard temperature" value of F_n is $F_n = (1 - \frac{T_g}{T_o}) + \frac{T_g}{T_o} F'_n$; where F'_n is the measured value of the noise figure for the condition that T_g is not equal to T_o [IRE Committee on Noise, 1960, p. 64, eq. (6.a)] .

The available noise power contributed by the network, N_{ei}, (see figure A-1) is referred to the input of the unit, and is added to the noise power available from the preceding unit, N_{ag}, to give a total available noise power, $N_{ei} + N_{ag}$. The "available-power transmission loss factor" is defined here as

$$L_{an} \equiv \frac{N_{ei} + N_{ag}}{N_{ao}} \qquad \text{(A.6a)}$$

where, N_{ao} is the available noise power at the network output terminals.

In terms of delivered noise power at the network input and available noise power at the network output, the "delivered power transmission loss factor" is defined here as,

$$L_{dn} \equiv \frac{\left(M_{in}\right)_{av}(N_{ei} + N_{ag})}{N_{ao}} \qquad \text{(A.6b)}$$

where $\left(M_{in}\right)_{av}$ is the network-input impedance-mismatch factor, in terms of available powers, and may be obtained from (A.5).

From (A.6a) and (A.6b) we obtain,

$$L_{an} = \frac{L_{dn}}{\left(M_{in}\right)_{av}} \qquad \text{(A.6c)}$$

For Impedance-matched conditions at the network input, $M_{in} = 1$ and $X_g = 0$ we have,

$$L_{an} = L_n \qquad \text{(A.6d)}$$

Hence, the transmission-loss factor, L_n, may be considered as a special value of L_{an} or L_{dn}, and is seen to be an internal characteristic of the network.

Note that in practice L_n may be measured conveniently by using non-reflecting impedance-matched conditions at the input and output terminals of the network with the (test) signal-power-level, P_{del}, much greater than the transducer noise-power level, $(F_n - 1)kT_oB_n$. Note also, that L_n is the "insertion loss" factor as defined by IRE Standards [59 IRE 2.S1] .

The transmission loss factor, L_n, is dependent upon the "real" part of the complex network "image transfer constant", θ, and is a measure of the dissipative loss of power in the network [Terman, 1943] . The image transfer constant, θ is given by

$$\tanh \theta = \sqrt{\frac{Z_{sc}}{Z_{oc}}} \qquad \text{(A.7)}$$

Where transmission lines are involved, L_n, in decibels is proportional to the product of the length of the transmission line, ℓ, and the "real" part (α) of the complex propagation constant, γ; where $\gamma = \alpha + j\beta$. That is,

$$L_n \propto \epsilon^{2\alpha\ell} \qquad \text{(A.7a)}$$

Network Bandwidth and Temperature Effects on Noise Power

The noise power level at the output of a network depends upon the effective bandwidth of the network, B_n. The power accepted by and processed in the network is proportional to B_n. Hence in a cascaded system of twoport networks, each of which may have different effective bandwidths, the network with the narrowest bandwidth will be the power-level and bandwidth-controlling unit.

It can be shown [Siegman, 1961a] that the exact expression for the (maximum or available) noise power originating in a lossy passive network having a thermal temperature of T degrees and an effective bandwidth, B_n, is given as,

$$N_n = \frac{h f B_n}{\exp(hf/kT)-1} \qquad (A.7b)$$

where, h = Planck's constant

f = Frequency

since $\frac{hf}{kT} << 1$ even at microwave frequencies and normal temperatures, $\exp(hf/kT) \approx 1 + \frac{hf}{kT}$ and (A.7b) becomes

$$N_n = k T B_n \qquad (A.7c)$$

and is valid for the radio frequencies and ambient temperatures used in radio communication systems.

The effective bandwidth of the network, in c/s, is defined as:

$$B_n \equiv L'_n \int \frac{1}{L(f)} df \qquad (A.7e)$$

where L'_n is the effective network transmission-loss factor; see discussion of the various types of L'_n above, and L(f) is the network transmission-loss factor, as a function of the frequency, f.

Noise-Power Levels in a Network

Noise power levels, and "effective noise temperature" [Strum, 1956, 1958], [Grimm, 1959, 1960], [Siegman, 1961a, 1961b], [IRE Committee on Noise, 1960], at the input and output terminals of a linear passive lossy bilateral twoport network unit are shown diagramatically in figure A-1. Figure A-1 applies to the case of impedance-mismatching conditions at the network input and output terminals. The interrelationships between: noise figures, F_n, the respective effective input noise powers of the network and receiver, N_{ei} and N_{er}, and the corresponding effective input noise temperatures, T_{ei} and T_{er}, are derived below.

The objective is to determine the available noise-power output from the network, N_{ao}, in figure A-1, in terms of the various system parameters.

The (fictitious) noise power N_{ei}, is contributed by the network and is referred to the network input terminals. This noise power is applied through the mismatch plane, M_{in}, [Rothe and Dalke, 1956; Haus and Adler, 1958; IRE Subcommittee 7.9 on Noise, 1960; Bell, 1951; Becking, et al., 1955] ; this condition is shown diagramatically in figure A-1. This concept of network noise power or current flowing through the generator impedance and hence, through the mismatch plane at the network input, is derived from the form of the equivalent "noise-source circuit" which is used in the above references.

The level of the noise power, N_{ei}, depends upon the network noise figure, F_n [Friis, 1944] , and is given by

$$N_{ei} = (F_n - 1) k T_o B_n \tag{A.8}$$

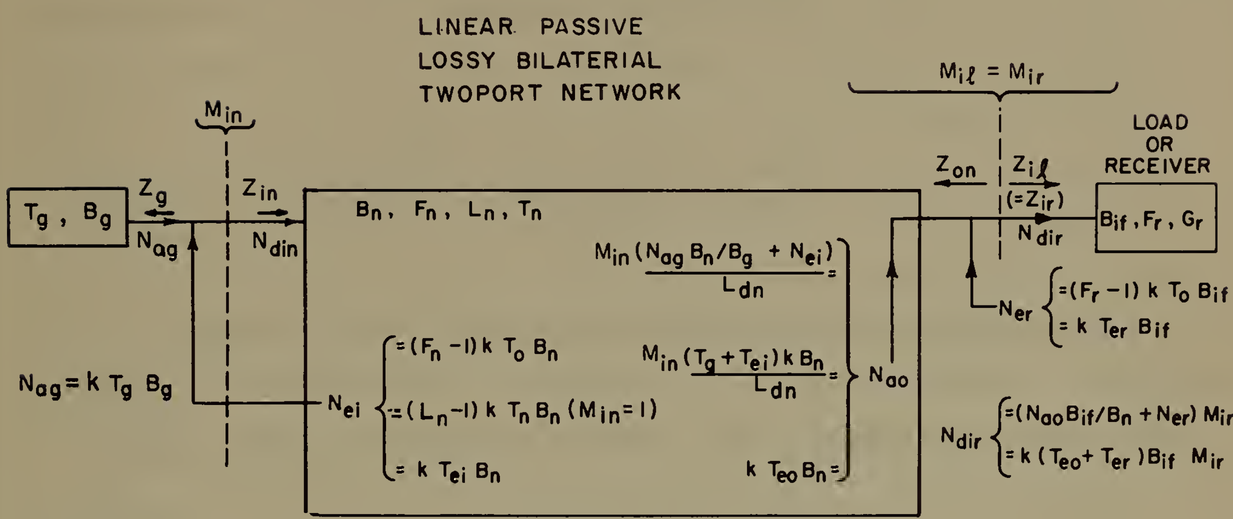
The available noise power from the source or generator is

$$N_{ag} = k T_g B_g \tag{A.9}$$

where,
- k = Boltzman's constant = 1.3804×10^{-23}
- T_g = Generator output-impedance (noise-power) temperature
- B_g = Generator effective bandwidth

For the general case, involving impedance mismatching conditions, and when $B_n \leq B_g$, the available noise power at the network output, N_{ao}, is given by

NETWORK NOISE POWERS AND NOISE TEMPERATURES



$T_{ei} = (F_n - 1) T_o$

$\quad = (L_n - 1) T_n, (M_{in} = 1)$

$$T_{eo} = \frac{(T_g + T_{ei}) M_{in}}{L_{dn}}$$

$T_{er} = (F_r - 1)\ T_o$

NOTE, $B_g > B_n > B_{if}$

$$L_{dn} \equiv \frac{N_{din}}{N_{ao}}$$

$L_{dn} = L_n$ (FOR $M_{in} = 1$)

B_x = EFFECTIVE PASSBAND, c/s

F = NOISE FACTOR

k = BOLTZMAN'S CONSTANT

L_n = TRANSMISSION LOSS FACTOR, AS A RATIO

N_a = AVAILABLE NOISE POWER

N_d = DELIVERED NOISE POWER

T_{ei}, T_{er} = EFFECTIVE INPUT NOISE TEMPERATURE, DEGREES KELVIN

T_g, T_{eo} = EFFECTIVE OUTPUT NOISE TEMPERATURE, DEGREES KELVIN

T_n = THERMAL TEMPERATURE, DEGREES KELVIN

T_o = STANDARD REFERENCE TEMPERATURE

Z_{in} = INPUT IMPEDANCE AT NETWORK INPUT TERMINALS

Z_{on} = OUTPUT IMPEDANCE AT NETWORK OUTPUT TERMINALS

M_i = IMPEDANCE MISMATCH FACTOR

Fig. A-1

$$N_{ao} = \frac{N_{din}}{L_{dn}} \tag{A. 9a}$$

$$= \frac{\left((N_{ag} B_n/B_g) + N_{ei}\right)}{L_{an}} \tag{A. 9b}$$

$$= \frac{\left((N_{ag} B_n/B_g) + (F_n - 1)kT_o B_n\right)}{L_{an}} \tag{A. 9c}$$

For the case of a transmission line or lossy passive twoport network, impedance-matched at its input and output.[Siegman, 1961a, p. 86], see also [Strum, 1956, 1958; Grimm, 1959, 1960]; we have

$$N_{ei} = (L_n - 1)kT_n B_n \tag{A. 10}$$

where,

T_n = Thermal temperature of the lossy elements of the network.

L_n = Dissipative transmission loss factor, defined above.

From (A.8) and (A.10) it is evident that

$$F_n = 1 + (L_n - 1)(T_n/T_o) \tag{A. 10a}$$

Effects of Impedance Mismatching In the Receiving System

Non-reflecting impedance-matched conditions are necessary throughout a receiving-antenna system, in order to avoid multiple reflections and consequent delayed or ghost signals at the receiver input terminals. However, an impedance mismatch at the receiver input is sometimes beneficial, provided that proper impedance mismatching at this point results in an improvement of the receiver noise figure. When the receiver-input impedance is not matched to the output impedance of a (matched-impedance) antenna system, the power which is reflected from the receiver-input terminals will be transmitted back through the antenna system where it will be absorbed in the transmission line and also re-radiated from the antenna, and multiple reflections will not exist in the receiving system. Consequently, if impedance-mismatch conditions exist only at the receiver input, interfering or ghost signals will not be developed at the receiver-input terminals.

From the above discussion it is evident that impedance mismatch conditions, at more than one point in the receiving-antenna system, cannot be tolerated in a multichannel or broadband system. Hence, in the following analysis it will be assumed that non-reflecting impedance-matched conditions exist throughout the receiving-antenna system, except that impedance-mismatch conditions may (or may not) exist at the receiver input. Under these conditions, the "load" in figure A-1 is replaced by the radio receiver, and the impedance mismatch factor, M_{ir} ($= M_{i\ell}$), will be equal to or less than unity.

Furthermore, in practically all radio receiving systems, the characteristic impedance of each network unit is essentially "real", hence the following conditions apply:

$$X_n/R_n << 1 \tag{A.11}$$

From (A.11), (A.6d), and the fact that the system is always matched at every point looking toward the antenna

$$L_{an} = L_n \tag{A.12}$$

From the previous subsection and for the conditions given by (A.11), and (A.12), and also referring to figure A-1, we have

$$N_{ao} = \frac{\left((N_{ag} B_n/B_g) + N_{ei}\right)}{L_n} \qquad \text{(A.14)}$$

or (see (A.10)),

$$N_{ao} = \frac{\left(N_{ag} B_n/B_g + (L_n - 1)kT_n B_n\right)}{L_n} \qquad \text{(A.15)}$$

Equation (A.15) may be used to determine the available noise-power output from a system of linear lossy twoport networks in tandem, under conditions of impedance matching at the input terminals of the networks, looking toward the antenna.

Noise Levels in the Receiving-Antenna System Networks

Referring now to figure 4-1, the "loss-free" antenna unit by definition does not contribute any noise; however, this unit receives and has available at its output the noise power $N_A = N_{at} + N_{ai} + N_a$ from the following sources:

(a) radio-transmitter output amplitude noise, N_{at}, associated with the received radio-frequency carrier signal;

(b) interfering radio-frequency signals, N_{ai};

(c) the antenna-received environmental noise power plus atmospheric and galactic noise, all of which add up to give N_a.

The effects of the received noise power, N_{at}, are discussed in Sections 4.2 and 4.3; where it is shown that proper allowances may be made for the effects of this noise power by using the results of equipment back-to-back tests to modify the (ideal) system-design curves. Therefore, N_{at}, need not be included at this point in the analysis of noise powers in the receiving system.

Also, the case of interference (noise) signals is not considered in this report. The effects of interfering-signals are dependent upon a variety of factors, including the "capture-effect" in FM systems; with the result that the interfering-signal noise power, N_{ai}, is not simply additive. Hence, for our purpose the available noise power at the output terminals of the loss-free antenna is

$$N_A = kT_aB_a \tag{A.16}$$

where B_a is the effective bandwidth of the receiving antenna, and T_a is the antenna received-noise temperature.

Each of the passive lossy units, in the receiving-antenna system, contributes noise power, and a portion of each of these noise powers is subsequently available at the duplexer output (receiver input) terminals. The total available noise power at the duplexer output, due to the noise contributed by each lossy antenna unit plus the antenna-temperature noise, is designated here as N_{au} and is assumed to be uniformly distributed over the effective bandwidth of the duplexer, B_{du}.

Interfering-signal noise power is also added to the receiving system from the local transmitter through the duplexer. The level of this noise power at the duplexer output (receiver input) terminals, is dependent upon the power output from the local transmitter, $P_{\ell t}$, and the "isolation factor" of the duplexer, L_D. The available local-transmitter noise power level at the duplexer output (receiver input) terminals is

$$N_{\ell t} = \frac{P_{\ell t}}{L_D} \qquad (A.17)$$

where, $P_{\ell t}$ = Power output of the local transmitter, in watts.

L_D = Transfer isolation factor of the duplexer. This factor is defined as the ratio of $P_{\ell t}$ to that portion of $P_{\ell t}$ which is available at the duplexer output (receiver input) terminals, and is within the receiver RF spectrum effective bandwidth $B_{rf}(= B_{if})$.

The position of the main portion of the RF spectrum of $P_{\ell t}$ must be well outside of the RF spectrum included by B_{rf} and the receiver "image-frequency" spectrum. Note that in the above definitions the available noise power, N_{au}, is distributed uniformly over the effective bandwidth of the duplexer, B_{du}, while the available noise power $N_{\ell t}$ is (assumed) to be distributed uniformly over the effective bandwidth of the receiver, B_{rf} or B_{if}. The selectivity characteristics of the duplexer and the receiver-input circuits are relied upon to exclude the local-transmitter power (outside the bandwidth B_{rf}) from the receiver IF circuits.

The sum of the noise powers in the receiving-antenna system at the duplexer output terminals is $N_T = N_{au} + N_{\ell t}$. This noise power, N_T, is available at the duplexer output, that is, at the receiver input terminals.

Because of the manner in which L_D has been defined, $N_{\ell t}$ is the noise power from the local transmitter as measured by the receiver having an effective bandwidth B_{if}. The value of $N_{\ell t}$ will depend upon the position of the effective bandwidth of the power-measuring device, (in this case the receiver) relative to the position and the shape of the power spectrum of $P_{\ell t}$.

Effective bandwidths of the cascaded units in the receiving-antenna system are assumed to be greater than the effective bandwidth of the receiver, B_{if}. Hence, as noted previously, the noise power "accepted" at the receiver-input terminals will be proportional to the receiver effective bandwidth, B_{if}, and to the noise-power densities, N_{au}/B_{du}, and $N_{\ell t}/B_{if}$. Furthermore, if impedance-mismatch techniques are employed at the receiver-input terminals, in order to minimize the receiver noise figure F_r, then the portion of N_T which is delivered to and absorbed at the receiver input terminals is dependent upon the value of the impedance-mismatch factor, M_{ir}. Therefore, the total receiving-antenna system noise power delivered to the receiver-input terminals, N_{Td}, and subsequently processed in the receiver and transferred to the receiver IF circuits, is

$$N_{Td} = M_{ir}\left(\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}}\right)B_{if} \qquad (A.18)$$

In (A.18), M_{ir} is the impedance mismatch factor at the receiver input.

In terms of available power at the duplexer output, M_{ir} is

$$(M_{ir})_{av} = \frac{4\,R_{du}\,R_{ir}}{|Z_{du} + Z_{ir}|^2} \qquad (A.19)$$

In terms of non-reflecting matched-impedance power at the duplexer output, M_{ir} is

$$(M_{ir})_{mat} = \frac{4(R_{du}^2 + X_{du}^2)\,R_{ir}}{|Z_{du} + Z_{ir}|^2\,R_{du}} \qquad (A.20)$$

where,

R_{du} = Resistive component of the output impedance of the duplexer.

Z_{du} = Output impedance of the duplexer. In general, this is the output impedance of the network which precedes the receiver.

Z_{ir} = Input impedance of the receiver.

In order to carry out the above procedures, and obtain an estimate of N_{au}, the noise power output of a linear passive lossy bilateral network, N_{ao}, is determined in terms of the noise power input to the unit, N_{ag}, the thermal temperature, T_n, transmission loss factor, L_n, and the noise figure F_n of the unit; see figure A-1 and (A.15).

Referring to figure 4-1, the output noise powers from each of the tandem units may be obtained by substituting the proper factors in (A.15). This procedure will be used in a later section of this Appendix to calculate the value for N_T, the total noise power at the duplexer output terminals, preceding the receiver input terminals.

Receiver Noise

The noise power contributed by the receiver [Bell, R. L., 1951] [Becking, et al., 1955] , [Rothe and Dalke, 1956] , [IRE Subcommittee 7.9 on Noise, 1960] and delivered to the receiver-input terminals is very closely approximated by,

$$M_{ir} N_{er} = M_{ir} (F_r - 1) k T_o B_{if} \tag{A.21}$$

where,

M_{ir} = Impedance mismatch factor at receiver-input terminals, as given by (A.16)

F_r = Noise figure of the receiver [IRE Standards, Proc. IRE, Jan., 1960] , as a ratio.

The receiver IF effective bandwidth, B_{if}, is defined by:

$$B_{if} = \frac{1}{G_r} \int G(f)\, df \qquad (A.22)$$

where,

G_r = Effective receiver power gain, as a ratio; this factor is defined as the ratio of the signal power delivered to the receiver input terminals to the available output signal power in the receiver pre-detection or IF circuit at an arbitrarily specified frequency. This frequency may correspond to that of maximum gain.

$G(f)$ = Receiver power gain as a function of the frequency, f.

Since our results apply to radio communication type of receivers, B_{if}, is assumed to be the effective single-response IF channel bandwidth.

The receiver noise figure, F_r, as used above must be measured using a "source" or generator output impedance which is equal to Z_{du}. In practice, the ratios X_{du}/R_{du} and X_{ir}/R_{ir} are each very much less than unity, and hence conjugate-impedance matching and non-reflecting impedance matching are equivalent. Furthermore, the signal generator or source impedance, Z_g, may be varied in order to obtain the optimum or minimum value for the receiver noise figure, F_o [Bell, 1951] , [Becking, et al. 1955] , [Rothe and Dalke, 1956] , and [IRE Subcommittee 7.9 on Noise, Jan., 1960] ; the particular value of Z_g which gives F_o would establish the correct value for Z_{du} required to insure optimum impedance-mismatch operating conditions at the receiver input.

Pre-Detection Noise Power, N_{if}

The receiver-contributed noise power, $M_{ir} N_{er}$, is added to the delivered noise power, N_{Td} (see (A.18)), from the receiving system which precedes the receiver, and this total noise power level is multiplied by the receiver power gain, G_r, to obtain the predetection noise power, N_{if}, that is,

$$N_{if} = (N_{Td} + M_{ir} N_{er}) G_r \qquad (A.23)$$

The pre-detection noise power, N_{if}, obtained from (A.23) applies only to a linear receiver having sufficient selectivity, in the input section, to eliminate "image-frequency" or "spurious-response" (signal and noise) power in the receiver IF circuits. The total IF noise power in multiple-response receivers depends upon the number of channels accepted by the receiver; this total IF noise power is equal to the sum of the noise power contributed by all of the channels. In radio communication systems only the "principal-response" channel is used. In receivers employing either or both agc and limiting, the receiver power gain, G_r, will be a function of the power level of the receiver-input RF signal, P_r.

The above ideas are now used as a guide in determining the functional relationship between the pre-detection noise power level, N_{if}, and the receiving-system parameters. The predetection load-reflected noise power [Siegman, 1961b] referred to in Section 4.2 is assumed to be negligible, and is not included in the following analysis. However, this factor would be important for very-low-noise bilateral receivers with an impedance-mismatch load connected to the receiver IF output terminals. Referring to figure 4-1, and substituting (A.18) and (A.21) in (A.23) we obtain the following relationship for N_{if}, the available pre-detection noise power,

$$N_{if} = G_r \left[M_{ir} \left(\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}} \right) B_{if} + M_{ir} (F_r - 1) k T_o B_{if} \right] \quad (A.24)$$

Each of the two terms inside the overall brackets on the right-hand side of (A.24) represents components of the <u>total</u> noise power delivered to the receiver-input terminals. The noise power contributed by the receiver, and delivered to the receiver input [IRE Subcommittee 7.9 on Noise, 1960] is given by , $M_{ir} (F_r - 1) k T_o B_{if}$; see (A.21). The noise power, $N'_T = (N_{au}/B_{du} + N_{\ell t}/B_{if}) B_{if}$, is composed of the noise power received by the antenna, N_A, plus the noise power generated within the lossy passive networks in the receiving-antenna system; including the noise power $N_{\ell t}$, which is contributed by the local transmitter; N'_T is available within the RF spectrum bandwidth B_{rf} (or B_{if}) at the receiver input terminals. This noise power, N'_T, is available at the output of the network which precedes the receiver, within the RF spectrum bandwidth B_{rf}(or B_{if}) accepted by the receiver, and is given by

$$N'_T = \left[\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}} \right] B_{if} \quad (A.25)$$

Note that $N'_T = N_T \, B_{if}/B_{du}$ only when the various noise powers in the receiving-antenna system are uniformly distributed over the effective bandwidth of the duplexer, B_{du}.

The factors within the brackets of (A.25) may now be determined in terms of the receiving antenna-system parameters, using the ideas outlined above to estimate the total noise power at the output of the tandem units in the antenna system, as follows:

From (A.15), (A.17), and figure 4-1 we obtain

$$\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}} = \left(\frac{N_L B_{du}/B_t}{L_{du}} + \frac{(L_{du}-1)kT_{du}B_{du}}{L_{du}} \right) \frac{1}{B_{du}} + \frac{P_{\ell t}}{B_{if} L_D}$$

$$= \left[\left(\frac{N_c B_t/B_{ac} + (L_t-1)kT_t B_t}{L_t L_{du}} \right) \frac{B_{du}}{B_t} + \frac{(L_{du}-1)kT_{du}B_{du}}{L_{du}} \right] \frac{1}{B_{du}}$$

$$+ \frac{P_{\ell t}}{B_{if} L_D}$$

$$= \left[\left(\frac{(N_A B_{ac}/B_a) + (L_c - 1)kT_c B_{ac}}{L_c L_t L_{du}} \frac{B_t}{B_{ac}} + \frac{(L_t - 1)kT_t B_t}{L_t L_{du}} \right) \frac{B_{du}}{B_t} \right.$$

$$\left. + \frac{(L_{du} - 1)kT_{du}B_{du}}{L_{du}} \right] \frac{1}{B_{du}} + \frac{P_{\ell t}}{B_{if} L_D} \qquad \text{(A.26)}$$

Since $N_A = kT_a B_a$, see (A.16), (A.26) may be written as

$$\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}} = \left[\frac{T_a + (L_c - 1)T_c}{L_c L_t L_{du}} + \frac{(L_t - 1)T_t}{L_t L_{du}} + \frac{(L_{du} - 1)T_{du}}{L_{du}} \right] k$$

$$+ \frac{P_{\ell t}}{B_{if} L_D} \qquad \text{(A.27)}$$

where,

L_c^*, L_t, L_{du} = Transmission (dissipative) loss factors for the antenna circuit, transmission line, and the duplexer network, respectively; as a ratio; see p. 143

*Antenna-earth "proximity effects" on the antenna circuit losses [Wait, J. R., 1959; Vogler, L. E., 1962], are included in the transmission loss factor, L_c.

The noise power available to the receiver within the receiver IF bandwidth, B_{if}, is given above by (A.25) as:

$$N'_T = \left[\frac{N_{au}}{B_{du}} + \frac{N_{\ell t}}{B_{if}} \right] B_{if} \tag{A.25}$$

Re-arranging terms in (A.27) and substituting in (A.25), we obtain

$$N'_T = \left[\frac{T_a + (L_c - 1) T_c + L_c (L_t - 1) T_t + L_c L_t (L_{du} - 1) T_{du}}{L_c L_t L_{du}} + \frac{P_{\ell t}}{B_{if} L_D} \right] k B_{if} \tag{A.28}$$

Equation (A.28) may be written as

$$N'_T = k T_A B_{if} \tag{A.29}$$

From (A.28) and (A.29) we obtain:

$$T_A = \left[\frac{T_a + (L_c - 1) T_c + L_c (L_t - 1) T_t + L_c L_t (L_{du} - 1) T_{du}}{L_c L_t L_{du}} + \frac{P_{\ell t}}{k B_{if} L_D} \right] \tag{A.30}$$

The factor, T_A, is the "effective receiving antenna-system (output) noise-power temperature", and is completely defined by (A.29). Because of its fundamental nature and simplicity of application, the concept of effective noise temperature is in general use in the fields of radio communication and radio astronomy, and is referred to throughout the literature in these fields.

From (A.29) and (A.30) it is seen that T_A is a measure of the total noise power received by the antenna, plus the noise power originating within the receiving antenna system. The noise power, N'_T is "available" at the duplexer output terminals. The portion of N'_T that lies within the bandwidth B_{if} is processed by the receiver and is available at the "pre-detection" point within the receiver.

The factors * L_c, L_t, L_{du}, and L_D in (A.30) have been defined previously. T_A will be in degrees Kelvin if T_c, T_t, and T_{du} are in degrees Kelvin, and also if $P_{\ell t}$ is in watts and B_{if} is in c/s of bandwidth.

The effective antenna-system output noise temperature, T_A, as determined from (A.30) is seen to depend upon: (1) received noise-power temperature of the antenna, T_a, (2) thermal temperatures of the units in the receiving-antenna system, T_c, T_t, and T_{du}, (3) receiving antenna-system transmission loss factors, L_c, L_t, and L_{du}, and (4) other factors such as, the receiver IF bandwidth B_{if}, the power output of the local transmitter, $P_{\ell t}$, and the isolation factor of the duplexer, L_D.

It should be noted that (A.30) determines the value of T_A for <u>unrestricted</u> values of the thermal temperatures of the antenna-system networks, T_c, T_t, and T_{du}. Hence, this equation is applicable to the case where the networks in the receiving-antenna system are at a low thermal temperature; such as might be achieved with appropriate refrigeration, or perhaps in a "space" receiving system.

It should also be noted that (A.30) is restricted to the case where non-reflecting impedance matched conditions exist throughout the receiving-antenna system, when looking towards the antenna. The receiver input, may or may not be matched; (A.30) is further restricted to the case where, $X_n/R_n << 1$.

* See discussion of L_n on p. 142.

The effect of the loss factors, L_c and L_t, on the antenna-system temperature, T_A, may be seen by applying (A.30) to a typical example.* The receiving antenna system is assumed to consist of only an antenna and a transmission line, with the following paramaters:

$$T_a = 300 \text{ degrees Kelvin}$$
$$T_c = T_t = 290 \text{ degrees Kelvin}$$
$$L_c = 1.12 \ (= 0.5 \text{ db})$$
$$L_t = 1.58 \ (= 2. \text{ db})$$

From the above specified parameters and (A.30), we obtain

$$T_A = \frac{300 + (1.12 - 1)\,290 + 1.12\,(1.58 - 1)\,290}{1.12 \times 1.58}$$

$$= 170 + 19.6 + 106 = 295.6 \text{ deg. Kelvin}$$

The noise-temperature contribution from each unit in the antenna system is evident from these calculations. The calculated noise temperature, T_A (= 295.6° K) is seen to be only slightly different from the assumed antenna temperature, T_a (= 300° K); hence, the loss factors, L_c and L_t, have the dual role of effectively reducing the antenna temperature, T_a, and also contributing to the resultant noise temperature, T_A, at the receiver input terminals.

However, the receiver-input signal, P_r, will also be reduced by the product of the loss factors, $L_c L_t$, (= 1/1.12 x 158 = 0.56), and the net result will be a degradation of the predetection signal-to-noise ratio, P_{if}/N_{if}.

Equation (A.30) may be simplified if the thermal temperatures of the receiving-antenna passive-network units are each at the same (standard) temperature, T_o, and if impedance-matching conditions looking toward the antenna, exist throughout the receiving-antenna system; that is:

$$T_c = T_t = T_{du} = T_o \qquad \text{(A.31)}$$

* In this example a duplexer was not used.

Substituting conditions (A. 31) in (A. 30) and combining terms, we obtain:

$$T_A = \frac{T_a - T_o}{L_c L_t L_{du}} + T_o + \frac{P_{\ell t}}{k B_{if} L_D} \tag{A. 32}$$

If (a) the thermal temperature of each of the receiving-antenna elements is T_o, (b) $X_n/R_n << 1$, and (c) non-reflecting impedance-matching conditions exist throughout the receiving-antenna system looking toward the antenna, the effective antenna-system output temperature, T_A, may be calculated from (A. 32).

Equation (A. 2 3) may now be written as,

$$N_{if} = G_r M_{ir} [T_A + (F_r - 1) T_o] k B_{if} \tag{A. 33}$$

where F_r is the receiver noise figure, (see p. 153), and M_{ir} is defined by (A. 16), for "available" noise power at the duplexer output.

The factor, $(F_r - 1)T_o$, in (A. 33) is the effective receiver (input) noise temperature, T_{er}, [IRE Comm. on Noise, 1960] . Hence (A. 33) may be written as,

$$N_{if} = G_r M_{ir} [T_A + T_{er}] k B_{if} \tag{A. 34}$$

Equation (A. 34) is used throughout this paper to estimate the receiver IF noise power level, N_{if}.

The performance of a radio receiver depends upon the receiver IF total signal power-to-total noise power ratio, P_{if}/N_{if}. An inspection of (A. 34) shows that the receiver IF noise power level, N_{if}, depends upon both the receiving antenna-system noise power level, $k T_A B_{if}$, and the receiver-contributed noise power level, $k T_{er} B_{if}$.

The noise power, $kT_{er}B_{if}$, may be assumed to be thermal noise with a steady average power level. In the frequency range above (approximately) 100 Mc, the received noise power which is associated with the antenna noise temperature, T_a, may also be assumed to be thermal noise. Therefore, for radio communication systems operating with RF signals above 100 Mc, the receiver IF noise, N_{if}, may be assumed to be "white", and its average total power level may be estimated from (A.34). In the frequency range well below 100 Mc, the noise signal, kT_aB_{if}, may have the characteristics of atmospheric noise. Under these conditions, N_{if}, may be estimated from (A.34) provided that kT_aB_{if} is assumed to be an "average" value; obtained by averaging over a period of time which is long compared with the average period of time between noise peaks.

APPENDIX B

Derivation of the Message-Channel Signal-Power Loading Ratio P_m/p_m, in the Modulating Baseband Signal

Assuming that the message-channel signal power S_{oc}, at the radio receiver output, is proportional to the modulating signal-power loading ratio, p_m/P_m, we have:

$$S_{oc} = K_1 \times G_r \times \frac{p_m}{P_m} \tag{B.1}$$

where,

- S_{oc} = a particular message-channel average signal power level, at the radio receiver output, or decoder input.
- K_1 = constant of proportionality, dependent upon the radio system parameters.
- G_r = gain of the linear section of the radio receiver.
- p_m = average power level of a particular message signal in the composite modulating baseband signal.
- P_m = average of the total power level of the composite modulating baseband signal.

Note that for SSBSC time-division message-signal multiplexing systems which are being considered in this paper, p_m will be confined within a definite bandwidth in the baseband-signal spectrum and may be measured directly in the baseband. For other types of multiplexing systems (see Subsection 4.4) it may not be possible to measure or adjust p_m directly in the modulating baseband; hence, modifying factors might be required in (B.1).

The message-channel average noise-power level at the radio receiver output is,

$$N_{oc} = K_2 \times G_r \times B_c \tag{B.2}$$

where, K_2 = constant of proportionality, dependent upon the radio-system parameters.

B_c = bandwidth of a particular message-signal channel, at the receiver output, in c/s. This bandwidth is assumed to be equal to the spectrum of the message signal.

Since the radio receiver gain, G_r, is dependent upon the receiver-input radio-frequency signal power level, it is inconvenient to use either (B.1) or (B.2) separately. Hence, by combining (B.1) and (B.2) we obtain the message-channel signal-to-noise ratio at the radio receiver output, S_{oc}/N_{oc}, and also eliminate the variable radio-receiver gain factor, G_r. Combining (B.1) and (B.2), we obtain,

$$B_c \times \frac{S_{oc}}{N_{oc}} = K \times \frac{p_m}{P_m} \qquad (B.3)$$

The "constant" K in (B.3) is a function of the radio system parameters; its functional form and its value depend upon the type of radio system--see Sections 4.5 and 4.6.

From (B.3) we obtain,

$$\frac{p_{m1}}{p_{m2}} = \frac{B_{c1}(S_{oc}/N_{oc})_1}{B_{c2}(S_{oc}/N_{oc})_2} \qquad (B.4)$$

or, in general;

$$\frac{p_{m1}}{p_{mi}} = \frac{B_{c1}(S_{oc}/N_{oc})_1}{B_{ci}(S_{oc}/N_{oc})_i} \qquad (B.5)$$

and,

$$\frac{p_{mi}}{p_{mn}} = \frac{B_{ci}(S_{oc}/N_{oc})_i}{B_{cn}(S_{oc}/N_{oc})_n} \qquad (B.6)$$

From (B.4) to (B.6) we obtain:

$$p_{m1} = p_{mn} \frac{B_{c1}(S_{oc}/N_{oc})_1}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.7}$$

$$p_{m2} = p_{mn} \frac{B_{c2}(S_{oc}/N_{oc})_2}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.8}$$

$$p_{mi} = p_{mn} \frac{B_{ci}(S_{oc}/N_{oc})_i}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.9}$$

or, in general:

$$p_{mA} = p_{mn} \frac{[B_c(S_{oc}/N_{oc})]_A}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.10}$$

The subscript A denotes a "class" of message signals, each class requires the same value of the product $[B_c(S_{oc}/N_{oc})]$, at the radio receiver output. Continuing, we obtain:

$$p_{mB} = p_{mn} \frac{[B_c(S_{oc}/N_{oc})]_B}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.11}$$

$$p_{mN} = p_{mn} \frac{[B_c(S_{oc}/N_{oc})]_N}{B_{cn}(S_{oc}/N_{oc})_n} \tag{B.12}$$

The total power level, P_m, in the modulating baseband signal is,

$$P_m = M_A\, p_{mA} + M_B\, p_{mB} + \ldots + M_N\, p_{mn} \tag{B.13}$$

where, M_A = number of simultaneously-active message signals each of which requires the same value for the product $[B_c(S_{oc}/N_{oc})]_A$.

M_B = Ditto, for the product $[B_c(S_{oc}/N_{oc})]_B$.

M_N = Ditto, for the product $[B_c(S_{oc}/N_{oc})]_N$.

Combining (B.10) to (B.12) and (B.13) we obtain,

$$P_m = \frac{p_{mn}}{B_{cn}(S_{oc}/N_{oc})_n}\left(M_A[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_A + M_B[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_B + \ldots\right.$$

$$\left. + M_I[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_I + \ldots + M_N[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_N\right) \qquad \text{(B.14)}$$

Combining (B.6) and (B.14) we obtain:

$$\frac{P_m}{p_{mi}} = \frac{M_A[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_A + \ldots + M_I[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_I + \ldots + M_N[B_c\left(\frac{S_{oc}}{N_{oc}}\right)]_N}{B_{ci}(S_{oc}/N_{oc})_i} \qquad \text{(B.15)}$$

Equation (B.15) is reproduced as (4.9) in Section 4.3.2.

APPENDIX C

Derivation of FM Receiver Performance Characteristics

For the condition where the total signal power-to-total noise power ratio, P_{if}/N_{if}, at the IF output, that is, at the input to the limiter of a "conventional" type FM radio receiver, exceeds (approximately) 10 db, it is possible to calculate the relationship between the signal-to-noise ratio, S_o/N_o, at the receiver output, and the ratio of the radio-frequency signal power level, P_r, to the total noise power at the receiver input, $k(T_A + T_{er})B_{if}$. These calculated results apply only to the upper linear portion of the FM receiver characteristic curve, above the threshold level. The following deriviations indicate the method used to calculate the upper linear portion of the FM radio receiver characteristic curve, and the parameters to be considered.

The average total noise power in a one-ohm circuit at the output of an FM discriminator [Black, 1953] through a low pass filter having a cutoff frequency of f_k c/s is:

$$N_o = \frac{4\pi^2}{3} \times \frac{D_d^2}{P_{if}} \times f_k^3 \times \frac{N_{if}}{B_{if}} \qquad \text{(C.1)}$$

where,

D_d = demodulator conversion factor.

P_{if} = total average IF signal power at the receiver limiter input, in watts.

N_{if} = total average IF noise power at the receiver limiter input, in watts.

B_{if} = effective bandwidth of the receiver IF circuits.

Equation (C.1) indicates that the discriminator-output noise voltage spectrum is triangular, increasing with frequency up to the cutoff frequency f_k. This equation also shows that the total discriminator-output noise power is inversely proportional to the IF signal power P_{if}, and directly proportional to the noise power density N_{if}/B_{if} in the IF passband.

From (C.1) the total average noise power out of an FM discriminator in a message-channel passband from f_a to f_b is:

$$N_{oc} = \frac{4\pi^2}{3} \times \frac{D_d^2}{P_{if}} \times (f_b^3 - f_a^3) \times \frac{N_{if}}{B_{if}} \qquad (C.2)$$

Equation (C.2) gives the difference in discriminator-output noise power levels between the triangular-shaped output-noise voltage spectra having cutoff frequencies f_a and f_b.

The average signal power in a one-ohm load at an FM discriminator output, associated with a sinusoidal modulating signal of frequency f_c is [Black, 1953] :

$$S_{oc} = 2\,\pi^2 D_d^2 \Delta F_c^2 \qquad (C.3)$$

where, ΔF_c = peak frequency deviation of the radio frequency carrier, in cycles per second, due to the sinusoidal modulating signal of frequency f_c.

Combining (C.2) and (C.3) we obtain,

$$\frac{S_{oc}}{N_{oc}} = \frac{3}{2} \times \frac{\Delta F_c^2 B_{if}}{(f_b^3 - f_a^3)} \times \frac{P_{if}}{N_{if}} \qquad (C.4)$$

The value of ΔF_c is determined by the power level, p_m, of the modulating sinusoidal signal at the radio-transmitter frequency-modulator input and the modulator characteristics. This modulating power level may be adjusted, relative to the total baseband-signal power level so that:

$$\Delta F_c = (p_m/P_m)^{1/2} \Delta F \qquad (C.5)$$

where,

p_m = average power level of a particular sinusoidal modulating signal in the baseband; the frequency of the modulating signal being f_c c/s.

P_m = total average modulating-signal power level in the baseband signal.

ΔF = peak radio-frequency carrier deviation in c/s for a single sinusoidal modulating baseband signal, having an average power level equal to the total average power level of the total baseband signal, P_m.

The message-channel signal-power loading ratio, p_m/P_m, is determined by methods outlined in Section 4.4.2 and Appendix B.

We are concerned with a comparatively narrow portion of the baseband spectrum and hence $f_a \approx f_b$. For this condition it can be shown that,

$$f_b^3 - f_a^3 \approx 3(f_b - f_a)\left(\frac{f_a + f_b}{2}\right)^2 \qquad (C.6)$$

The portion of the baseband-signal spectrum which is given by $(f_b - f_a)$, is related to the message-signal bandwidth B_c, as follows,

$$f_b - f_a = K_M B_c \qquad (C.6a)$$

where,

K_M = factor which depends upon the type of multiplex system.

B_c = message-signal channel bandwidth, in c/s.

For SSBSC frequency-division multiplex, being considered in this paper, $K_M = 1$ therefore,

$$f_b - f_a = B_c \tag{C.6b}$$

The term $(f_a + f_b)/2$ in (C.6) is the mid-frequency, f_c, between f_a and f_b, hence

$$\frac{f_a + f_b}{2} = f_c \tag{C.6c}$$

Substituting (C.6b) and (C.6c) in (C.6) we obtain,

$$f_b^3 - f_a^3 = 3\ B_c\ f_c^2 \tag{C.6d}$$

where,

f_c = Center frequency of the message channel in the baseband signal spectrum, in c/s.

The radio-frequency spectrum bandwidth $(B_{rf})_c$, associated with particular values of $\Delta F_c/f_c$ and f_c, is given by:

$$(B_{rf})_c = f_c \times \phi(\Delta F_c/f_c) \tag{C.7}$$

However, the value of B_{rf}, is determined under conditions where all of the modulating-signal power is in a single sinusoidal modulating signal having a frequency = f_m; that is, $p_m/P_m = 1$, $f_c = f_m$, and $\Delta F_c = \Delta F$; hence, B_{rf} is given by

$$B_{rf} = f_m \times \phi(\Delta F/f_m) \tag{C.8}$$

where,

B_{rf} = Radio-frequency spectrum bandwidth which includes all components or sidebands having amplitudes equal to or greater than one percent of the unmodulated carrier amplitude.

$\phi(\Delta F/f_m)$ = function of $\Delta F/f_m$. This function gives the number of significant sidebands in the radio-frequency spectrum, assuming that the modulating signal is a single sinusoidal voltage wave having a frequency f_m, in c/s.

The function $\phi(\Delta F/f_m)$ in (C. 8) has been calculated [Hund, 1942] and is shown in figure 3-1.

Assuming that the FM receiver IF bandwidth, B_{if}, is adjusted to be only wide enough to pass the significant sideband components in the radio-frequency spectrum, we have,

$$B_{rf} = B_{if} \tag{C. 9}$$

Therefore,

$$B_{if} = f_m \times \phi(\Delta F/f_m) \tag{C.10}$$

Substituting (C.5), (C.6d) and (C.10) in (C.4) and re-arranging terms we obtain:

$$\frac{S_{oc}}{N_{oc}} \times \frac{B_c}{f_m} \times \frac{P_m}{p_m} = \frac{1}{2} \times \phi\left(\frac{\Delta F}{f_m}\right) \times \left(\frac{\Delta F}{f_c}\right)^2 \times \frac{P_{if}}{N_{if}} \tag{C.11}$$

The term P_{if}/N_{if} in (C.11) is the required predetection total signal power-to-total noise power ratio associated with a specified value of S_{oc}/N_{oc} at the receiver output, and for particular values of the parameters: f_m, B_c, p_m/P_m, $\phi(\Delta F/f_m)$ and $(\Delta F/f_c)$. However, in radio communication system design work and system performance testing it is usually more convenient to deal with the total radio-frequency signal power and the total noise power at the receiver input. Hence, the form of (C. 11) was modified as follows:

$$P_{if} = G_r M_{ir} P_r \tag{C.12}$$

From (A.30) in Appendix A and (C. 10) above, we have,

$$N_{if} = G_r M_{ir} [T_A + T_{er}] k f_m \times \phi\left(\frac{\Delta F}{f_m}\right) \tag{C.13}$$

therefore,

$$\frac{P_{if}}{N_{if}} = \frac{P_r}{[T_A + T_{er}] k f_m \times \phi(\Delta F/f_m)} \tag{C.14}$$

where,

G_r = power gain of the cascaded RF and IF linear sections of the FM receiver.

P_r = total average radio-frequency signal power at the receiver input, in watts.

k = Boltzman's constant = 1.3804×10^{-23} joules per degree Kelvin.

f_m = highest frequency in the modulating baseband signal.

T_A = receiving antenna-system output noise temperature, degrees Kelvin; see Appendix A.

T_{er} = radio receiver-input effective noise temperature [Proc. IRE Comm. on Noise, 1960], degrees Kelvin.

Substituting (C.14) in (C.11) we obtain,

$$\frac{S_{oc}}{N_{oc}} \times \frac{B_c}{f_m} \times \frac{P_m}{p_m} = \left(\frac{\Delta F}{f_c}\right)^2 \times \frac{P_r}{2k(T_A + T_{er}) f_m} \tag{C.15}$$

Note that the left-hand side of (C.15) coincides with that of (C.11); this similarity was preserved in order to compare the predetection total signal-to-noise ratio, P_{if}/N_{if}, with the RF signal-to-relative noise power ratio, $P_r/k(T_A + T_{er})f_m$, at the receiver-input terminals.

It is evident from (C.15) that the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, above the threshold region, is inversely proportional to the square of the modulating frequency f_c, for a given set of system-parameter values. This situation does not apply to the receiver performance in the threshold region; in this region the receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, approaches the same value for all frequencies in the modulating baseband signal. Hence f_c in (C. 15) was made equal to f_m, the maximum frequency in the modulating signal, in order to obtain a design equation which is conservative above the threshold region. Equation (C. 15) is reproduced as (4.19) in Section 4.6.1.1, with $f_c = f_m$.

APPENDIX D

Method of Estimating the Performance Characteristics of Radio Receivers and Message-Signal Decoding Units, for a Time-Varying Signal.

Estimated equipment performance, for a time-varying signal, is outlined by a method of graphically combining the steady-signal performance characteristic of the equipment with either the distribution of the time-varying signal power or the distribution of the time-varying signal-to-noise ratio at the equipment input.

The analysis of radio receiver performance in this Appendix does not include system intermodulation noise; see Section 4.3. Hence, the estimated performance will apply to the "ideal" receiver characteristic, as shown in figure 4-A. Proper modifications of the estimated-performance results may be made for the effects of system-intermodulation noise, using the procedures outlined in Section 4.3.6.

D.1 Estimated Performance Characteristics of Radio Receivers.

The total post-detection signal-to-noise ratio, S_o/N_o, at the output of an "ideal" radio receiver depends upon the total received radio-frequency signal power, P_r, and the total noise power, $N'_T + N_{er}$, available at the receiver input. Where N'_T is the antenna-system noise power which is available at the receiver input terminals, and lies within the RF signal spectrum bandwidth $B_{rf}(= B_{if})$. N_{er} is the noise contributed by the receiver, referred to the receiver input. See Appendix A. $N'_T + N_{er}$ may be estimated by adding the antenna-system noise and the receiver noise on a power basis and the resultant (for the case of radio systems operating at and above VHF) is assumed to have the characteristics of "white noise", with a steady average value. In the following derivations the total signal-to-noise ratio, S_o/N_o, is converted to the message-channel signal-to-noise ratio, S_{oc}/N_{oc} (see Section 4), in order to conform to symbols used in Sections D.1 and D.2.

In general, the statistical characteristics of both P_r and N_T must be considered. The characteristics of P_r to be considered are (1) the cumulative (time) distribution and (2) the fade rate. The characteristics of N_T to be considered are (1) the cumulative (time) distribution, including average and median power levels, and (2) the spectral density in the frequency range of the spectrum included in B_{rf}.

In this work the received RF signal, P_r, is assumed to be of the Rayleigh-fading type. Other types of received signals may be considered, provided that their distributions are known and also provided that the performance characteristics of the combiner are known for these other distributions. The fade rate of P_r at its median level is assumed to be lower than the lowest binary-bit rate in the modulating message signal. In other words, it is assumed that there is no appreciable change in the value of P_r during the duration of a message-signal binary pulse, or during the period of time required for one cycle of the lowest frequency in the modulating (message) signal.

At this point it should be noted that the characteristics of the receiver-output noise, N_{oc}, for an FM receiver, will depend upon the level of operation of the receiver with reference to the "threshold region". Hence, this situation must be considered when determining the performance characteristic of FM receivers.

The method of determining the receiver performance characteristic for a time-varying received RF signal, P_r, is based on the idea of combining the receiver-input steady RF signal performance characteristic with the cumulative power-level distribution of the received RF signal. Following is the detailed procedure for applying this method to determine the performance characteristic of an FM receiver for a Rayleigh-fading type of received RF signal.

Figure D-1a shows the measured steady-signal performance characteristic for an FM receiver, covering a range of receiver-input values of $P_r/f_m(T_A + T_{er})$ and receiver output message-channel signal-to-noise ratios, S_{oc}/N_{oc}. Normalized abscissa and ordinate scales are used to conform to other work in this report; see Section 4.6, figure 4-3. The normalizing factors are: f_m, T_A, T_{er}, B_c, and P_m/p_m. Note that $\Delta F/f_m = 1$ for this curve and hence curves derived from this characteristic, for a time-varying receiver-input relative signal-to-noise ratio, $P_r/f_m(T_A + T_{er})$, apply only to $\frac{\Delta F}{f_m} = 1$. P_r is the power level of the total RF signal, available at the receiver-input terminals. The total receiver-input noise consists of thermal noise from the signal generator and from the receiver; hence $T_A = T_g$ (generator output noise-power temperature) and $T_{er} = (F_r - 1)T_o$. $N'_T + N_{er}$ may be obtained from: $N'_T + M_{er} = k(T_A + T_{er})B_{if}$. Note that the relative value of $N'_T + N_{er}$ $(= k(T_A + T_{er})f_m)$ is used in the abscissa titles in figure D-1.

Curve A-A in figure D-1b gives the cumulative distribution of the Rayleigh-fading received RF signal, P_r. Any median value may be selected for P_r by proper adjustment of the ordinate scale values.

The method of obtaining the estimated FM receiver performance characteristic for a time-varying received RF signal is as follows: In figure D-1b, an abscissa value of percentage of time t_a is selected, and the corresponding normalized ordinate-scale value P'_r, is noted. The value of P'_r from figure D-1b was used in figure D-1a to determine an ordinate-scale value on the normalized scale; this normalized signal-to-noise ratio is denoted here as $(S_{oc}/N_{oc})'$. This value of $(S_{oc}/N_{oc})'$ was then plotted versus t_a as shown in figure D-1c. The above procedure was repeated to obtain additional points for the curve in figure D-1c.

The average value of the distribution of the relative signal-to-noise ratios was obtained by numerical integration of the curve and is shown in figure D-1c as $(S_{oc}/N_{oc})_{ave}$. Note that the integration must be

GRAPHICAL METHOD
FOR DETERMINING FM RECEIVER PERFORMANCE

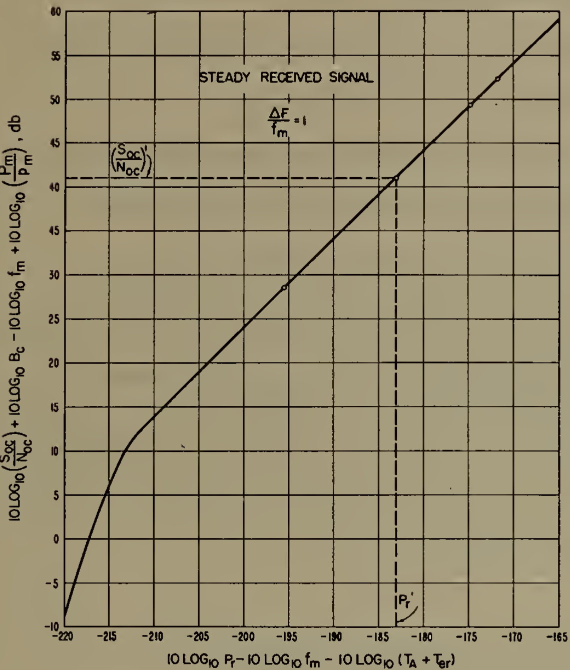


Fig. D-1a

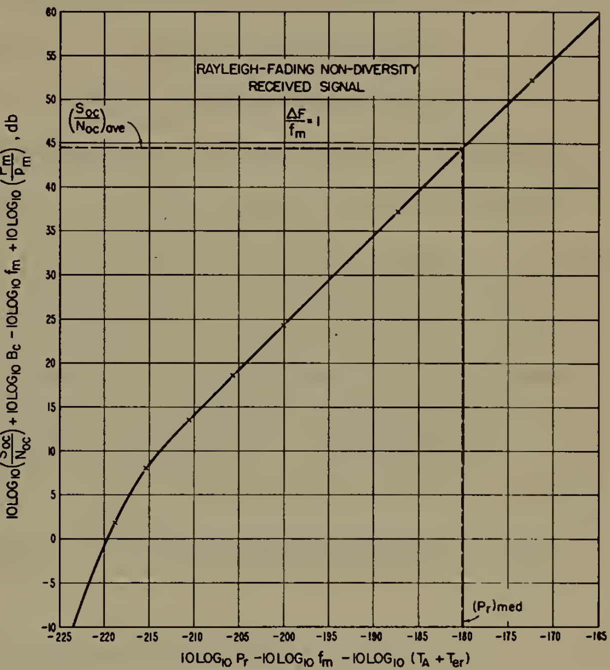


Fig. D-1d

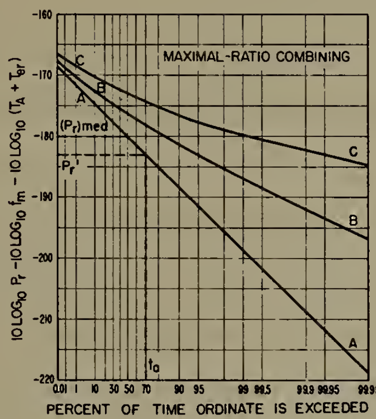


Fig. D-1b

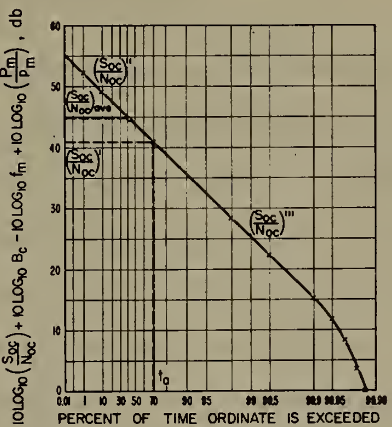


Fig. D-1c

carried out using ordinate-scale values as numerical ratios instead of decibel units. This value of $(S_{oc}/N_{oc})_{ave.}$, together with the value of P_r(med.) from figure D-1b, were plotted as shown in figure D-1d. Additional points for the FM receiver performance characteristic curve in figure D-1d were obtained from similar distributions (curve A-A) in figure D-1b but with different P_r(med.) values. The resultant FM receiver-performance characteristic curve in figure D-1d is in terms of the median of the time-varying received RF signal, P_r, and the average of the distribution of the time-varying receiver-output relative signal-to-noise ratio, S_{oc}/N_{oc}. The shape and the position of the curve in figure D-1d is related to the shape and the position of the curve in figure D-1a in terms of their respective abscissa and ordinate scales; hence, the accuracy of the estimated receiver-performance curve in figure D-1d depends directly upon the accuracy of the measured receiver-performance curve in figure D-1a and the accuracy of the combiner-performance curves in figure D-1b.

D.1.1 Estimated Performance Characteristics of Radio Receivers-With Diversity

The statistical characteristics of the "combined" receiver-output signal-to-noise ratio, S_{oc}/N_{oc}, depend upon both the receiver characteristics and the combiner characteristics.

The curves in figure D-1b are essentially the combiner-performance characteristics and apply to maximal-ratio type of combining; these curves were obtained from Brennan's work [1958]. Curve A-A is the distribution of the signal at the combiner input, curves B-B and C-C, are the corresponding distributions at the combiner output, for dual and quadruple diversity, respectively. Curve A-A may be considered as the distribution of the signal-to-noise ratio at the combiner input, instead of the distribution of the signal at this point. Proper phasing of the combiner-input signals is required in order to obtain accurate measurements of the radio receiver output signal-to-noise ratio.

The system design curves presented in this report were developed, for non, dual, and quadruple diversity, using the curves A-A, B-B, and C-C in figure D-1b, by the method outlined in Section D. 1.

D.2 Estimated Performance Characteristics of the Decoder Unit, for a Time-Varying Input Signal-to-Noise Ratio.

The number of message errors, and hence the average message error rate occurring during a sampling period, depends upon the message-channel signal-to-noise ratio, S_{oc}/N_{oc}, at the decoder-unit input. More in detail, we need to know the relationship between the average message-error rate and,

(1) the value of the average signal-to-average noise power ratio, S_{oc}/N_{oc}, and

(2) the statistical characteristics of:

(a) the message-channel signal power, S_{oc},

(b) the message-channel noise power, N_{oc},

(c) the message-channel signal-to-noise ratio, S_{oc}/N_{oc}.

In this work it is assumed that the performance characteristic of the decoder unit for a time-varying (input) average signal-to-average noise power ratio, S_{oc}/N_{oc}, may be determined by combining the decoder-unit performance characteristic for the non-time-varying S_{oc}/N_{oc} condition, with the cumulative distribution of the time-varying S_{oc}/N_{oc}. It is assumed that the time interval required for an appreciable change in S_{oc}/N_{oc} is large compared with the duration of a message-signal binary pulse, or the period of time required for one cycle of the lowest frequency in the modulating message signal. The results obtained by the above procedure will also depend upon the statistical properties of the noise, N_{oc}, at the decoder-unit input; each "type" of noise will have a different effect on the slope and the position of the decoder-unit performance-characteristic curve.

Measurements of the average message-channel signal-power level, S_{oc}, and the average message-channel noise-power level, N_{oc}, may easily be made with the proper type of power-measuring meters having appropriate time-constants. However, if either or both of the average power levels, S_{oc} or N_{oc}, vary (independently) with time, their separate measurements and also the measurement of the ratio, S_{oc}/N_{oc}, require special equipment capable of measuring the cumulative distribution of these variables.

Following is an outline of the method used to calculate the performance of a decoder unit for a time-varying signal-to-noise ratio, S_{oc}/N_{oc}, at the decoder-unit input. In this case, the decoder unit consisted of a teletype receiver and a teletype printer, combined.

Figure D-2a shows the measured performance characteristic for the teletype-message decoder, for a range of values of steady signal-to-noise ratios, S_{oc}/N_{oc}, and assuming that N_{oc} was thermal noise.

Figure D-2b shows the estimated distribution of the time-varying signal-to-noise at the output of an FM radio receiver, for the non-diversity case, and assuming a Rayleigh-fading type of signal at the radio receiver input. The method of obtaining this distribution, and its average value, $\left(\frac{S_{oc}}{N_{oc}}\right)_{ave.}$, is shown in Section D. 1. Note that figure D-2b is identical to figure D-1c, except that the ordinate scale values in figure D-2b are decreased over those in figure D-1c by the factor $10\ \mathrm{Log}_{10} B_c$ (= 20.4 db). This scale modification is required so that

GRAPHICAL METHOD FOR DETERMINING MESSAGE-DECODER PERFORMANCE

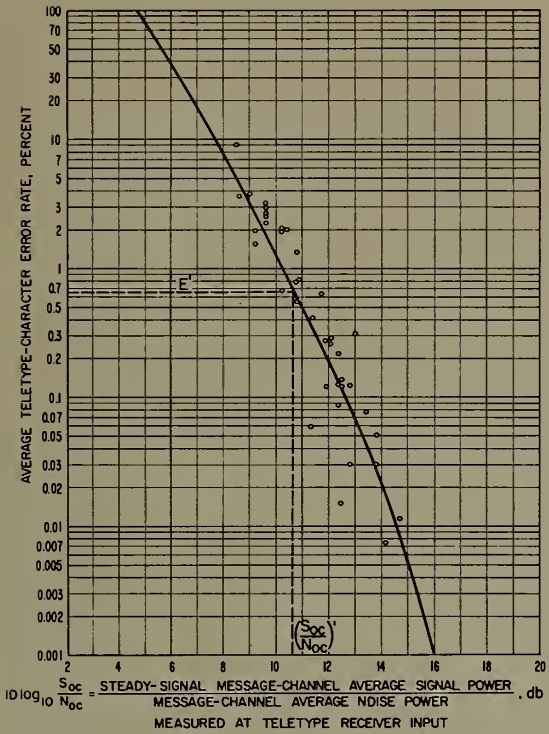


Fig. D-2a

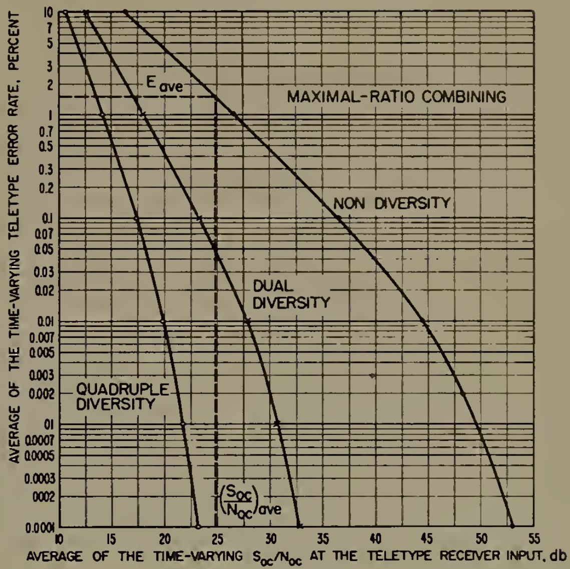


Fig. D-2d

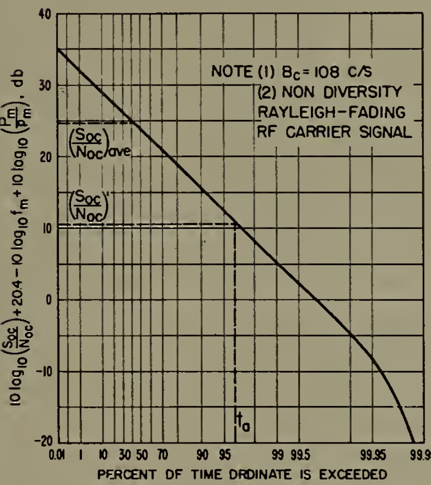


Fig. D-2b

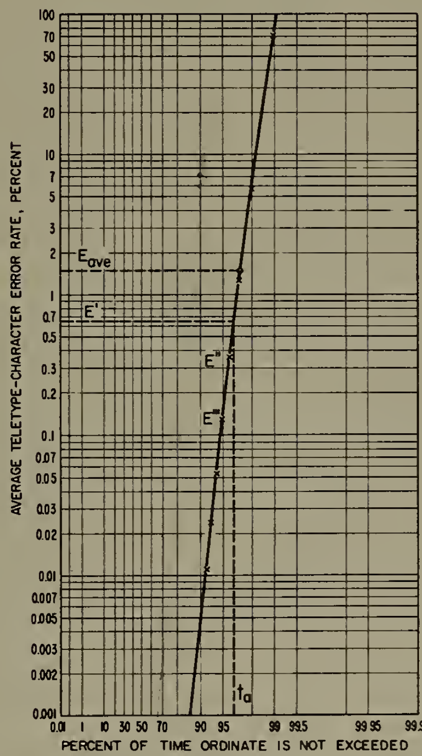


Fig. D-2c

the ordinate-scale values in figure D-2b correspond to the abscissa-scale values in figure D-2a. In other words, the procedures outlined in figure D-2 apply to a particular type of message-signal decoder unit.

The distribution of the teletype-character error rate, shown in figure D-2c, was obtained as follows: In figure D-2b, an abscissa value of the percentage of time, t_a, was selected and the corresponding normalized value of (S_{oc}/N_{oc}) was noted. The value of $(S_{oc}/N_{oc})'$, from figure D-2b, was used in figure D-2a to determine E'. This value of E' was then plotted versus t_a as shown in figure D-2c. The above procedure was repeated to obtain other points, E'', E''', etc., for the curve shown in figure D-2c.

The average value of the distribution shown in figure D-2c, $E_{ave.}$, was obtained by numerical integration of the curve. This value of $E_{ave.}$, together with the value of $(S_{oc}/N_{oc})_{ave.}$ from figure D-2b, were plotted as shown in figure D-2d. Additional points for the non-diversity curve in figure D-2d were obtained using the above procedures and the non-diversity distribution in figure D-2b, but with different $\left(\frac{S_{oc}}{N_{oc}}\right)_{ave.}$ values for this distribution. Various values may be obtained for $(S_{oc}/N_{oc})_{ave.}$ in figure D-2b by proper adjustment of the ordinate scale values.

Teletype decoder-unit performance curves for dual and quadruple diversity, shown in figure D-2d, were obtained using the same procedure as outlined above and using S_{oc}/N_{oc} distributions, in figure D-2b, for dual and quadruple diversity, from figure D-1c.

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