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Systems Analysis of Inland Consolidation Centers for Marine Cargo

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PREFACE

In July of 1967, the United States Maritime Administration asked the Technical Analysis Division, Institute for Applied Technology of the National Bureau of Standards, to develop analytical techniques to optimize the location and characteristics of inland centers to consolidate less-than-carload lots of cargo into "full" container loads for export and to unload and distribute containerized import cargo. It is presumed that such centers, if feasible, would encourage the use of containers and this, in turn, would promote the United States Merchant Marine. The study was undertaken by the National Bureau of Standards under Purchase Order No. P1-MA68-112, dated 1 August 1967.

The results of this study were communicated to the Maritime Administration in a report dated August 1968. In view of the importance and frequent occurrence of facility-location problems in governmental decision-making, the Bureau's Technical Analysis and Applied Mathematics Divisions found it appropriate to carry the work somewhat further than had been possible during the study for the Maritime Administration. These extensions included computer implementation and exercise of additional features of the mathematical model developed, design of improved summary-level output formats, and execution of more and better-based illustrative computations. The present document, which replaces the August 1968 report, covers these additions and also incorporates a few changes in the exposition of the material reported previously.

Appreciation is expressed to Mr. Paul Mentz of the Office of Research and Development of the Maritime Administration, Project Engineer, for his guidance and helpful suggestions in the conduct of the research, and to Mr. Maitland Pennington, Mr. Howard Marsden, Mr. Thomas Fay, and Mr. John Norris of the Office of Maritime Promotions, whose participation made possible a valuable understanding of the practical aspects of the problem.

Special acknowledgement is given to Mr. John Frazier, Mr. C. Nelson Bean, and Mr. Roupen Berberian of the Delaware River Port Authority for their wholehearted cooperation in making available unique data on shippers which had not been collected elsewhere, without which the success of the study would have been severely handicapped.

Appreciation is also expressed to Dr. George Suzuki for his technical editing of the report.

To Mrs. Verna Durkay goes the thanks of all participants for her part in arranging and preparing the many drafts and final version.

W. Edward Cushen
Chief, Technical Analysis Division

SYNOPSIS

This Technical Note documents a study, carried out for the U.S. Maritime Administration, to develop analytical techniques for use in optimizing the locations and characteristics of inland centers to facilitate the flow of containerizable marine cargo.* Such centers would perform the consolidation of small lots of break-bulk general cargo into container loads for export; for the reverse flow, they would carry out the handling and unloading of import containers for cargo distribution.** Performance of these functions inland, rather than exclusively at or near the ports involved, should result in savings to the shipping community from transporting cargo over land in full containers rather than as more costly less-than-carload (LCL) lots.

A mathematical model and associated solution technique have been developed, implemented in a digital computer program to a point compatible with the kinds of information available, and exercised using the body of data and background material accumulated during the fact-finding phases of the study. These sample calculations appear to establish the feasibility and value of our general approach and also aid in pinpointing the types of additional data needed for a more definitive analysis. The numerical results, although only tentative and illustrative, are highly encouraging as regards the economic value of the "inland center" concept.

Conclusions

The basic conclusions of this study can be summarized as follows:

1. The present line-haul container rates and consolidation costs at inland consolidation centers, when compared with cost of shipment in less-than-container load lots of break-bulk, appear to give a distinct monetary advantage to the shipping community from the use of inland consolidation centers. This conclusion remains intact under a variety of perturbations to our basic data set.

2. A computerized mathematical model to guide the selection of consolidation center locations is both feasible and useful; its use, level of detail, and the interpretations given its outputs must be duly sensitive to the quality of the data available. Relatively simple modifications could provide additional outputs bearing on the levels of service to particular shipper groups or ports.

3. For a definitive analysis to aid in realizing the full potential of a system of consolidation centers, as envisioned in this study, it is essential to have available pertinent data on which to base accurate, definitive, and sound judgments. For a port authority, a freight forwarder, an exporter or transportation company interested in containerization, or a government agency promoting such a program, it is essential to know points of origin, routes of transportation, times in transit, pertinent rates, volumes, seasonal variations, and points of destination of present and future flows of export and import cargo. These data are not presently available. The data are at least as important as the means by which they are manipulated.

4. The cost of acquiring land and operating a center, and thus the cost to a shipper of using that center, can vary appreciably depending on precise center location, but given reasonably acute acquisition choices and operating practices, this variation should not be so appreciable as to affect a shipper's choice of center. With this understood, it is not necessary for the mathematical model to pinpoint the exact geographical location of each center in order to indicate how to achieve near-minimum total costs.

The balance of this document, which reports fully the fact-finding and model-related work described just above, is relatively lengthy. It contains considerable technical detail, describes a number of data interpretations and modeling possibilities alternative to those actually adopted, and includes a good deal of material which (although relevant) proved peripheral to the main course of the study. This initial synopsis has therefore been

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*In view of the general importance and prevalence of facility-location decisions, some extensions to the work were carried out as an in-house project.

**See Chapter II for a fuller discussion of centers' functions.

inserted to provide the reader with a synoptic overview of the study's products and findings, as well as a guide to the body of the text. The object here is compactness, even at the cost of oversimplifying the description of the following chapters' contents.

Solution Concept

Each application of the model requires a specified value of

n = number of inland centers to be treated.

Thus applications would typically be arranged in sequence, with n varied systematically to cover some numerical range of interest.

The major outputs of the mathematical model are:

(a) For each shipper or aggregated class of shippers,* an assignment either to one of the n inland centers or to "his" port of use.

(b) For each of the n inland centers, a location.

(c) For each of the n centers, a measure of capacity ("sizing").

These outputs are required to satisfy (to within a specified tolerance) the following natural constraints:

(a') Each shipper should be assigned to that center most economical for him (or to his port of use if that choice is the most economical).

(b') Each center should be optimally located relative to the set of shippers it serves.

(c') Each center should be optimally sized relative to the volume of its patronage.

The terms "most economical" in (a'), and "optimally" in (b') and (c'), refer to a "generalized cost function" which will be described below. There are three other constraints which can be imposed if desired:

(d') Each center is required to achieve at least a certain minimum level of total patronage. (This serves as one control over the range of reasonable values for n.)

(e') A center is placed "off limits" to a particular category ("co-containerizable class") of cargo if its patronage within that category is insufficient to permit (on the average) reasonably rapid accumulation of reasonably full container-loads.

(f') A center may be constrained to be in a specified geographical subregion described as a convex polygon.

The satisfaction of these constraints provides the rationale for an iterative solution procedure which begins with a trial set of "guessed" locations for the centers. The procedure involves the alternation of two types of steps; the assignment (and sizing) step, and the location step.

In the assignment step, the locations and use-costs for the n centers are taken as "given"; that is, they are obtained from the previous location step.** In the simplest case, the assignment step simply consists of assigning the shippers to centers or ports so as to satisfy (a') above. If however the "patronage by category" constraint (e') is also to be enforced, then the sequence of shipper-by-shipper assignments is followed by a check for violations of this extra condition. For each cargo category, the center (if any) exhibiting the most severe violation is declared off-limits to that category, and the corresponding shippers are reassigned in accordance with (a') to the centers still open to them. This

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*We use "shipper" as a generic term for an exporter or importer of containerized cargo in LCL quantities.

**This is not strictly accurate; for details see p. 85 on "sub-substeps."

process continues until a pattern of assignments which satisfies (e') is achieved. (Data considerations did not permit bringing the logic to satisfy constraint (c')--i.e., the center-sizing technique described in Chapter VII--to a definite form incorporable in the computer program. How this logic would enter the assignment step is sketched on pp. 69-70.)

In the location step, the set of patrons for each of the n centers is taken as "given," i.e., it is obtainable from the previous assignment step. Using these assignments, the location for each center is determined so as to satisfy (b'); "geographical" constraints (f') can also be imposed if desired. (The logic for imposing (f') is worked out, but is not included in the present computer program.)

The alternation of assignment and sizing steps with location steps continues until all of (a'), (b'), and (c') are simultaneously satisfied within their tolerance limits. The nature of the process is such that (e') and (f'), if imposed, are also satisfied at this point. If the "total patronage" constraint (d') is to be enforced but in fact is violated, then new locations for the offending centers are chosen at random and the iterative process resumes. Should a prescribed number of such "restarts" occur without a successful termination, this is regarded as probable evidence that the stipulated number (n) of centers is incompatible with the minimum patronage level per center imposed in (d'), and the search for a solution is abandoned.

As noted earlier, this procedure has been implemented as a computer program (in FORTRAN), and applied to a set of illustrative data. (Because these data did not break down center-related costs into fixed and variable elements, the "optimal sizing" logic could not be exercised.) The illustrative calculations involved 3 ports (Baltimore, Philadelphia, and New York), 1400 aggregated shippers* in the East and Midwest, and up to 12 inland centers.** Running times on the Bureau's UNIVAC 1108 computer† were of the order of 1-3 minutes per passage through the alternating-step process to "convergence."

More than one such passage is generally advisable, because all the constraints together (except the geographical restrictions (f')) are typically not enough to single out a unique configuration of centers. The final configuration reached by the iterative process will in general depend on the initial "guesses" as to center locations; exploration of this point (pp. 94-98) indicates that these final configurations can be quite distinct in their geographical distributions, and can differ fairly significantly in desirability as measured by the "index of performance" described below. Thus several passages through the iterative process, using distinct starting guesses, are recommended. (Optional routines for generating such guesses are included in the program.) Even with this multiplicity, computer running times were found to be quite tolerable.

This capability for rapid solution, which permits examining a variety of parametrically defined cases with a reasonable outlay of resources, seems a vital feature in assuring the feasibility and usefulness of the entire approach. The price for this capability, of course, is that the model operates at a rather gross level of detail. There is no explicit representation of freight transport networks, of commodity-dependent freight rate structures in all their changeable complexity, of the schedules and itineraries of container-carrying ships. No account is taken in the formal model of an inland center's potential value as a possible reducer of pilferage, as a marshalling point (for full-container shipments) or as a convenient focus for information on shipment and ship movements. The pattern of demand for consolidation and break-bulk services is treated as "given" as regards both geographical distribution and volume; the very difficult area of demand forecasting, including the influence of the centers' existence and locations on the demand pattern, was beyond the scope of this study.††

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*The aggregation procedure is described on pp. 87-88.

**The present computer program can handle up to 3 ports, 3000 shippers, and 50 inland centers; tradeoffs among these limits can be easily calculated so that changes would require relatively little recoding.

†Conversion to other machines should not in principle be difficult; some of the necessary caveats are given on p. 82.

††Adaptation of the model to situations in which the demand for centers' services vary over the years, and new centers are gradually introduced, is sketched on pp. 72-73 but has not been programmed. This does not however account for the interactions between the demand and the supply of containerization services.

Most to the point, the center locations output by the computer must be regarded as specifying only in general terms where each center should be placed. Precise identification of the best site in each computer-specified locale would require a fine-grained analysis of that locale with respect to land-cost variations, access points to particularly attractive transportation facilities, and the like. Such "fine-tuning" appears unnecessary (see p. 20) and indeed inappropriate at the system-wide planning level, lying more in the province of the interests that undertake to initiate and operate a particular center in a particular locale. It appears possible (see p. 26) to place most centers on or near trunk-line railyards, contiguous to major highways; this should not be surprising in view of the natural tendency of centers of import-export activity to be located near major transportation facilities. Such locations should make quite feasible the use of unit trains to carry the containerized cargo between center and port.

Sample Computer Output

Now that the basic ideas of the solution process have been described, much of what follow can be conveniently based on references to a sample output sheet from the computer program. Such a sheet appears here as Figure A (and later as Figure 11 of the main text).

The first line shows that n=4 in the case under analysis. The fifth line shows that 5 sets of initial locations for the centers are to be tried; only the outcome from the first of these sets appears in the Figure. The third line declares that the "patronage by category" constraint is in force; the greatest allowable average delay to cargo while enough accumulates to yield an adequately full 40'x8'x8' container is an adjustable parameter whose nominal value (not shown in the Figure) was taken as 1.5 days, a figure compatible with the information gathered in the study's fact-finding phase.* If a center does not receive enough export cargo in a certain category (including U.S. port involved, and area of destination) to keep the average delay down to 1.5 days or less, the center is forbidden to serve that category.

The fourth line of the Figure declares that the "total patronage" constraint is in force; each center was required to handle on the average the equivalent of 10 90%-full containers per day. This lower limit of 10 was based on the impression gained from our fact-finding efforts concerning a reasonable cutoff point for economical operation.** Although up to 3 "restarts" would have been permitted in trying to satisfy this constraint, only one restart was in fact required. The printout shows that after the first termination of the alternating step process inland center No. 4 had only about 1/4 as much volume of business as the constraint demanded. Giving this one center a new (random) initial location and re-entering the iterative process led to a satisfactory configuration.

The printout's second line shows that initial locations for the centers were chosen at random in the region containing the shippers; these "first guesses" are specified in the first display on the output sheet.† The computer program permits as an alternative the initial placement of the n inland centers near n randomly selected shippers.†† Another option is to input initial trial locations for all centers. Some centers can if desired be held at fixed locations; they are then exempt both from random initialization and from the location-changing parts of the iterative process. This option permits savings in computer time when one is confident where certain centers should go.§

The second display in Figure A shows the final locations of the 4 inland centers, togethe with information on their respective hinterlands (the smallest rectangle with sides parallel to the coordinate axes containing all patrons), the average "payload-fullness" (load factors) of the containers they stuff, their average daily levels of activity and the division of that

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*Results of varying this value are reported in Chapter IX, p. 109.

**Chapter IX reports the results of varying this limit down to 8 containers a day and up to 12 containers a day.

†The (X,Y)-coordinate system used is specified on p. 87.

††Computational experiments on the relative merits of the two alternatives proved inconclusive; they are reported on pp. 94-98.

§Computational experiments based on this idea are reported on pp. 98-100.

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THIS PROGRAM COMPUTES OPTIMAL LOCATIONS FOR NUMBERS OF CENTERS FROM 4 TO 4 AT INTERVALS OF 1.
THE PROGRAM USES INITIALIZATION 2.
THE PROGRAM PROHIBITS CENTERS FROM ACCEPTING CARGO FOR A CO-CONTAINERIZABLE CLASS IF THE ACCUMULATION DELAY IS TOO GREAT.
CENTERS ARE REINITIALIZED 3 TIMES OR UNTIL EACH CENTER RECEIVES AT LEAST 432000.00 POUNDS OF CARGO.
5 INITIALIZATIONS ARE PERFORMED FOR EACH NUMBER OF CENTERS.
THE TRANSPORTATION RATE FOR CONTAINERIZED FREIGHT IS .0000135.
THE INVENTORY-TYPE CARRYING CHARGE IS .0004
THE PROJECTED EXPORT LOAD FACTOR IS .900.  THE MAXIMUM HOLDING TIME IS 4. DAYS.
THE AVERAGE IMPORT LOAD FACTOR IS .822.

INITIALIZATION FOR 4 CENTERS

CENTER     X          Y

   1    555.02     184.25
   2    339.09     454.70
   3    262.94     121.03
   4    135.04     179.92

CENTER 4 RECEIVING 119217.00 POUNDS AND LOCATED AT 169.750  208.930 REINITIALIZED  TO 741.921  420.389

CENTER     X        Y     MIN X OF  MAX X OF  MIN Y OF  MAX Y OF    POUNDS   CONTAINERS  LOAD   PER CENT  PER CENT  INDEX OF
                          PATRONS   PATRONS   PATRONS   PATRONS     INPUT     OUTPUT    FACTOR   EXPORT    IMPORT  PERFORMANCE

   3    355.21  246.79  253.75 -  874.12    141.57 -  484.07   1831374.20  43.37     .8798    76.00     24.00     .6471
   2    456.68  303.65  301.88 -  903.00    150.70 -  508.04   2234762.20  52.59     .8853    82.82     17.18     .5062
   1    591.09  189.97  469.88 -  886.37    100.47    485.21    941051.82  23.26     .8430    92.06      7.94     .4918
   4    770.00  333.37  607.25 - .952.87    139.28 -  509.18   1603106.30  36.95     .9039    82.76     17.24     .4037

  PLACE           X        Y        DELTA      NUMBER OF CENTERS
                                             WITHIN DELTA OF (X,Y)

PHILADELPHIA    108.50  203.22     50.              NONE
ALLENTOWN PA    135.62  247.74     50.              NONE
HARRISBURG      169.75  208.93     50.              NONE
SW. NEW YORK    249.49  355.66     30.              NONE
BUFFALO         298.37  393.80     30.              NONE
PITTSBURGH      367.50  236.33     50.               1
NE. OHIO        410.00  300.00     20.              NONE
CLEVELAND       456.75  303.68     30.               1
DETROIT         536.37  359.63     50.              NONE
SW. OHIO        592.37  188.38     50.               1
NW. OHIO        610.16  332.32     50.              NONE
S. MICHIGAN     650.00  350.00     50.              NONE
INDIANAPOLIS    690.37  189.52     50.              NONE
CHICAGO         770.00  333.37     50.               1
MILWAUKEE       784.87  412.14     50.              NONE
CENT. ILL.      837.37  195.23     50.              NONE

PORT       X        Y     MIN X OF  MAX X OF  MIN Y OF  MAX Y OF    POUNDS   CONTAINERS  LOAD   PER CENT  PER CENT  INDEX OF
                          PATRONS   PATRONS   PATRONS   PATRONS     INPUT     OUTPUT    FACTOR   EXPORT    IMPORT  PERFORMANCE

   1    112.80  200.80   87.50 -  310.38     93.62 -  371.04   906451.54    21.40    .8826    86.93     13.07      .8064
   2     52.50  255.00  ********       .00  ******** -     .00         .00      .00    .0000      .00    100.00      .0000
   3    188.00  156.20  157.50 -  784.87    102.75 -  484.07    99656.02     2.42    .8586    64.11     35.89      .8381

                    COST WITH CENTERS       COST WITHOUT CENTERS

VARIABLE COST           121277.23                252665.13

TOTAL COST              157432.18                288820.07

INDEX OF PERFORMANCE    .545087
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Figure A. Sample Output

activity between import and export trade. The last display gives similar information for the (fixed) centers regarded as associated with each of the three ports.*

The intermediate display, with first column headed "PLACE", was inserted to permit quick visualization of the final configuration of centers. The user can specify the names of several cities or subregions, a nominal location for each, and a tolerance distance (in miles) for each. The display will then describe how many of the inland centers lie within the tolerance distance of each of the specified locations, thus yielding a rough mental map of where the centers fall. (Actual map output, though a desirable adjunct, is not available from the present program.)

Index of Performance

We focus next on the last three lines of Figure A. They depend on the "generalized cost function," whose description we continue for the moment to defer.

"Total cost" refers to the sum over all shippers of the generalized costs associated with their shipments. "Variable cost" refers to those ingredients of "total cost" which depend on the locations of the centers (and on their capacities, should data suitable for exercising the model's "sizing logic" become available). In particular, total cost may contain summands, depending on the number (n) of inland centers, which are not included under variable cost; many but not all of these summands involve "fixed costs" associated with centers, which cannot be separated out using the presently available data.

Imagine now that "total cost" is evaluated according to each of two conceivable scenarios. In the first case, there are no inland consolidation centers at all; LCL cargo is containerized (if export) or broken down (if incoming) at a center located at the U.S. port of entry or egress. In the second case, n inland centers are available and are utilized by shippers according to the criteria described above; these n centers are located and sized (and the port-located centers are sized) in the way sketched previously.

The index of performance for a given value of n is taken to be the ratio of the numerical value of "total cost" under the second scenario, to its value under the first scenario. Thus a high value (close to 1) of this index indicates relatively little savings to the shipping community from consolidation centers, while a low value (close to 0) indicates a great savings. To the extent that the model reflects all relevant costs and benefits, the "best" value for the number n of inland centers would be that which minimizes this index of (mis)performance. As noted earlier, however, present data on center-associated costs are given on a "per container" basis and so do not separate out fixed costs; hence the present computer program lacks the essential ingredient needed to show how the index of performance begins to vary in the unfavorable direction as inappropriately large values of n are postulated.

Figure A indicates an index of performance of roughly 0.545 for 4 well-located inland centers.** In general, our illustrative computations yielded indices of performance between 0.51 and 0.63. These values are strikingly low, representing very substantial economic benefits accruing to the shipping community from the presence of the inland centers. That this savings results from the presence of inland centers rather than centers located in port areas is demonstrated in Chapter IX. The index of performance with only port centers available is about .95, indicating relatively little savings attributable to these centers.

Even taking into account the uncertainties in and incompleteness of the data employed in our illustrative work, plus the fact that the numerator and denominator of the present index of performance refer only to the U.S.-incurred portion of the generalized cost of origin-destination movement, these index values are so low as to create a strong presumption that the establishment of inland centers will provide worthwhile savings to the shipping community. Our assorted sensitivity analyses, noted in preceding and subsequent asides, maintain this picture. The question of how these savings might be divided among the elements of the community (shippers, center operators, transport operators, etc.) has been excluded from

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*A full list of presently and potentially available output information is given on pp.76-78.

**The importance of having n well-located centers, and not just n centers which "exist" but are randomly located, is investigated on pp. 93-94.

this study as far as possible*; our concern is with the size of the pie, not how it gets sliced up.

Number of Centers; Distribution of Demand

Because the "fixed-cost" element of center-associated cost was not available, the only factor limiting the number (n) of inland centers in our illustrative analyses was the total patronage constraint. As indicated in the discussion on pp. 101-102, this factor leads to the conclusion that as many as 7 or 8 inland centers might be justified for the particular region and demand pattern under scrutiny, a finding quite compatible with the "computerized map analysis" reported on pp. 92-93, and summarized in Table A (Table 18 of the main text).

This is in moderate contrast with the indication in Chapter VI, based on a hand analysis performed before our shipper-related data were assimilated into computer processable form, that 10 or more centers might well be justifiable. As might be expected, the contrast proves to be attributable to differences in underlying assumptions. The "map analysis" in Chapter VI treats all flow from the shippers in question as grist for the "consolidation center" mill. All later calculations, in comparison, worked only with estimates of the "LCL component" of that flow,** reducing the total volume by over 50%.

The line of approach leading through Chapter VI to Table A, and the accompanying Table B (Table 19 of the main text), provides useful initial impressions of the likely general locations for centers. The total average daily volume (of estimated LCL cargo) associated with all the shippers in our data base is about 225 90%-full containers. (The nominal weight of a full 40'x8'x8' container is taken as 48,000 pounds.) Of this total, about 67%, or 150 containers per day, are associated with the 8 areas marked in Table A.

There is already a center at the port in Philadelphia. Three of the remaining areas, namely Chicago, Cleveland, and Pittsburgh, would individually support small to medium-sized centers based on the threshold level of 10 containers/day. The other 4 areas are at best marginal *per se*, but centers located in them could expect to attract some patrons from neighboring subregions.

Of course this material, based solely on flow volumes at shipper locations, does not take into account the effects on center location of such factors as port usage patterns and freight rates for containerized vs. LCL cargo. Thus such initial impressions on likely center locations required (and received) corroboration and refinement from use of the full model. We emphasize again that our work involved "as is" use of our shipper-related data; no account is taken of volume growth since the year 1964, to which those data refer,† of future growth in LCL cargo stimulated by the existence of inland centers, or of potential use of the centers by full-container shippers in order to take advantage of possible special freight schedules or reduced rates as for unit trains. In particular, the numerical results must be regarded as *illustrative* of what the methodology can provide, not as definitive.

Generalized Cost

We turn at last to a description of the costing procedure used in assigning shippers to "best" centers, in finding "best" locations and sizes for centers, and in calculating the index of performance. The description will be primarily in terms of export cargo, with unobvious differences for the import flow noted as they arise.

The accompanying Figure B (Figure 10 of the main text) shows those portions of the overall movement of cargo which are "costed out." The second line in the Figure refers to cargo which is consolidated at its U.S. port of exit, the first line to cargo which is containerized at an inland center (or possibly some other port).

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*See however pp. 7-8 and p. 63 on questions of ownership.

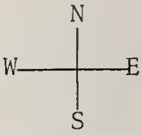
**The estimation step is described on pp. 87-88.

†Calculations with an assumed higher total demand level are reported on pp. 104-105. Additional calculations, with the levels of demand varied nonuniformly by commodity class, are reported on p. 108.

Table A. Geographical Distribution of Shipments*

.00	.49	.36	.00	.00	.00	.00	.00	.00	.00
.00	1.28	.00	.25	.76	.00	4.65	.00	.00	.02
.00	5.59 8	.00	1.06	.32 6	5.38	.00	2.59 2	4.47	1.07
.41	4.54	.00	1.01	2.49	.00	.00	1.61	2.72	.13
.15	.65	7 28.32	.60	3.78	21.81 5	2.17	.29	.37	.02
.60	.14	.09	1.12	1.57	8.68	8.45	3.16	4.73	.70
.00	.00	.48	.62	.77	3.93	7.82 3	7.74	.95	21.09 1
.01	.80	.39	1.41 4	5.11	1.77	1.97	.08	2.29	23.04
.01	.09	.06	3.58	4.77	1.59	.00	.00	.00	.94
.34	.00	.00	.00	.02	.00	.00	.00	.00	.30

1 Philadelphia-Allentown
2 Western New York
3 Pittsburgh-Johnstown
4 Southwest Ohio
5 Cleveland
6 Detroit
7 Chicago
8 Milwaukee



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*Each entry gives the average daily number of 90% full containers associated with the corresponding rectangle of the 10x10 grid overlaid on the Delaware Port Authority's hinterland.

Table B. Average Daily Number of 90%-Full Containers Imported or Exported for Each of the Areas Listed

Philadelphia	37.02	Detroit	11.04
Allentown, Pennsylvania	7.21	Southwestern Ohio	12.33
Harrisburg	3.46	Northwestern Ohio	3.31
Southwestern New York	2.52	Southern Michigan	.92
Buffalo	2.59	Indianapolis	1.83
Johnstown, Pennsylvania	6.82	Chicago	27.98
Pittsburgh	7.57	Milwaukee	7.03
Northeastern Ohio	.76	Central Illinois	.80
Cleveland	25.79	Other	62.41

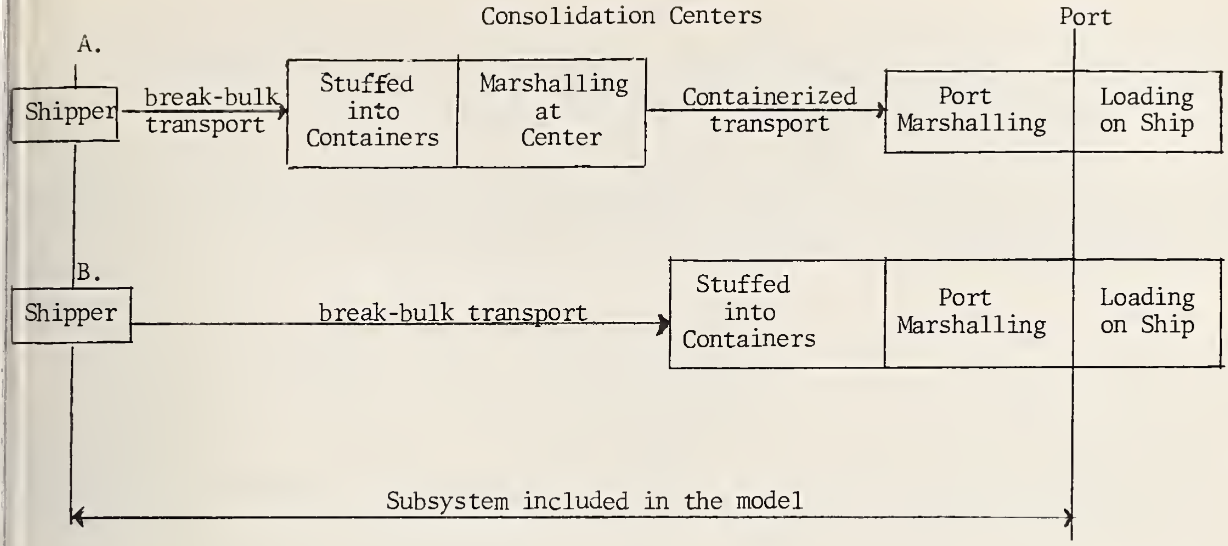

Figure B. Subsystem included in the model

The costs associated with a shipment naturally depend, in part, on what commodity is involved. The mathematical model per se is indifferent to what classification of commodities is employed, but a definite selection was required for use in the illustrative calculations. This selection, largely dictated by the need for a simple nominal set of freight rates for LCL cargo, led to a crude classification into 8 categories. Rail rates for container loads, and truck rates for noncontainer loads in each category were obtained for movement from one selected hinterland point (Cleveland) to 4 ports (New York, Philadelphia, Baltimore, Norfolk). These were converted into 4 "per mile" rates which were averaged to provide single per mile rates for each of the 8 categories, both containerized and LCL. The details of this process* and of the various assumptions involved are given in Chapter V; what is most noteworthy is that the rate for containerized cargo** (0.135¢/cwt/mi.) is so much less than those for LCL cargo (an average of 0.644¢/cwt/mi.), providing the basis for the low recorded values of the index of performance.†

The movement of export cargo can be broken into three stages, at each of which cost is incurred:

shipper-to-center movement (uncontainerized),

passage through the consolidation center,

center-to-port movement (containerized).

(If consolidation occurs at the center associated with the port of exit, the last stage is regarded as vacuous.) The costing of the second stage is somewhat complicated, and will be discussed last.

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*Some of our calculations assumed LCL (uncontainerized) movements to be by rail, others by truck. Since truck rates are typically the lower of the two for short trips (say less than 300 miles; see Ref. 15, p. 135) the former choice tends to overcharge LCL shipments to nearby centers but not the typically longer LCL movements to ports, thus raising the index of (mis)performance of the system of centers.

**Center locations proved insensitive to variations of ± 20% in the rate, see p. 103.

†Results of varying the LCL truck transportation rates for each commodity class by random values less than ± 10% are reported on p. 108.

The freight rates described above determine the monetary costs associated with the firs and third stages of export cargo movement. A circuity factor (we used the I.C.C. value of 1.13) enters in passing from Euclidean to line-haul distance. In exercising the model the same factor was used for both rail and truck line-haul distance. The costing of the in-container movement requires a little care, however; an item in a 90%-full container must for example be charged 10/9 the charge it would incur as part of a perfectly full container. Thus this aspect of the costing includes multiplication by an appropriate inflation ratio (essentially but not exactly a reciprocal average load factor) which depends on the patronag pattern and dispatch policy of the center -- i.e., on the typical "fullness" of the containe it sends out with material from the particular co-containerizable class in question.

In addition to the direct monetary costs associated with shipper-to-center and center-t port movement, there are "time costs" to be considered. (Their inclusion motivates the adjective in "generalized costs.") Representing these in a way commensurable with ordinary costs requires both a means of converting from time-units to dollars, and a calculable expression for time in transit. Our approach to both subjects is taken from Meyer et al.*

Conversion from time to dollars is accomplished in two steps. First, the shipment quantity involved is multiplied by an "average value per pound" factor to obtain a dollar value; for our illustrative calculations, values for these multipliers for each of our 8 commodity classes were found by averaging the $/lb. ratios for the shippers in each class.** Second, this dollar value is multiplied by a factor proportional to the amount of time in question; the factor of proportionality signifies a representative charge (per unit time) for interest risk, obsolescence, and the like. The value of 10% per year (0.004/day, based on a 150-day effective year) suggested by Meyer et al was used in our numerical work, though values specific to commodity classes might be developed.[†]

Meyer et al also provide a formula for converting from line-haul distance to time in transit. It takes into account not only average speed but also delay in terminals and (for appropriate modes) time lost at sidings, interchanges, and switching points. The parameter values given by this source lead for the rail mode to the formula,

$$t = 0.0059\ d + 2,$$

where t denotes travel time (in days) and d denotes line-haul distance (in statute miles). Further assumptions by the study team yielded the analogous equation

$$t = 0.0014\ d + 1$$

for the truck mode.[††]

This completes the account of the costing method applied to the shipper-to-center and center-to-port movement of cargo. Costing of the cargo's movement through a center is discussed next.

Costing of Center Use

The mathematical model does not contain a detailed simulation of the sequence of operations in a consolidation center. Instead, formulas are provided for three cost elements associated with passage of material through the center. These are

a. accumulation delay (waiting for enough co-containerizable material to accumulate to yield a sufficiently full container),

b. service delay (a general queuing-type concept), and

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*Ref. 15, p. 135.

**The values are given in Table 17 (p. 90). One of our sensitivity analyses is equivalent t varying these values.

[†] The results of varying this factor by ± 10% are recorded on pp. 109-110.

[††] Chapter IX reports the results of varying the two sets of coefficients by ± 10%.

c. monetary cost.

These will be taken up in turn. The conversion of the two delay elements to dollar terms is carried out by the same method applied above to time in transit from shipper to center or center to port.

Average accumulation delay for a shipment depends both on the hold-dispatch policy of the center, and on the time distribution of arrivals at the center of material co-containerizable with the given shipment. We have assumed a simple but plausible type of hold-dispatch policy describable by two parameters,

H = maximum holding time,

L = "target" or "projected" load factor.

The policy is that a shipment is held until either there is enough co-containerizable cargo to fill a fraction L or more of a container (which is then dispatched) or a time H has elapsed, whichever comes first. Our nominal values for these parameters, based on information gathered during the study, are those indicated in the eighth line of Figure A's sample output:

H = 4.0 days, L = 0.90.

In our illustrative calculations, sensitivity analyses (reported on pp. 102-103) indicate that reasonable variations in these parameters lead to changes of order 0.01 in the index of performance (which itself lies between 0.51 and 0.63). Sensitivity is greater to changes in L than to variations in H. Striving for higher load factors (thus decreasing center-to-port transport costs, at the expense of higher accumulation delays) appeared advantageous, but the differences involved remained small.

A full treatment of arrivals of LCL cargo at the consolidation center might require information on seasonal fluctuations, size variations among lots from the same shipper, etc. In fact, however, the only relevant data available for each shipper were total yearly tonnage and mean frequency of shipment. The present model employs perhaps the simplest nontrivial mathematical representation determined by such data: the shippers to the center, of material in each co-containerizable class, are regarded as an aggregate Poisson source of equal shipments. The Poisson assumption is especially important in simplifying the delay-related and inflation-ratio calculations which must be redone each time the assignment step changes a center's set of users.

With the formulations of hold-dispatch policy and shipment-arrival distribution just described, explicit formulas are obtainable for average accumulation delay (for a co-containerizable class at a center) and also for average load factor and average inflation ratio. This applies to exports; accumulation delay (in the U.S.) is of course not relevant for imports, and in our illustrative work a load-factor of 0.822 (the average load factor in the second half of 1967 for inbound containers in the North Atlantic trade) was applied to all imports.* One more special point: accumulation delay is taken as zero for outbound cargo containerized at its port of exit.

To discuss average service delays at a facility, one must consider both its capacity and its level of activity. Let A_u denote the mean arrival rate of LCL cargo at the center, measured in "containerfuls" (units of 48,000 lbs.); it is calculable from the frequencies and sizes of shipments by the center's users. Let A_c denote the mean outflow rate of containers, taking inflation ratios into account. The "sizing" of the center is represented by two design parameters K_u and K_c, representing the center's respective throughput capacities for (a) the processing stages before accumulation delay becomes meaningful, and (b) for those following the point at which the dispatching rules first permit formation and dispatch of a container; these parameters are assumed chosen in the same ratio as A_u/A_c. The relation between mean waiting time and excess capacity is taken for simplicity as that for a simple queue (Poisson arrivals, exponential service times). The result is that the average delay for cargo in its uncontainerized form is the reciprocal of $K_u - A_u$, the average post-containerization delay is the reciprocal of $K_c - A_c$, and the average overall service delay is taken as the sum of the two.

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*The model's sensitivity to this factor was tested, and the results are recorded on p. 109.

The monetary costs to users of a center are calculated by prorating the total cost of center operations (perhaps multiplied by a "profit factor") among its patrons in proportion to the volumes of center output attributable to them. The total cost is hypothesized to be expressible as a sum of three terms: a constant, a "fixed-cost" term depending only on capacity (say, proportional to it), and a "variable-cost" term depending only on operating level (say, proportional to it). Capacity might be represented by K_c and throughput perhaps by A_c alone. The "sizing optimization" would then consist of selecting K_c to minimize the sum of the just-mentioned fixed-cost element and the dollar equivalent of the service delays for all patrons.

As noted earlier, this treatment was not in fact employed in our illustrative calculations, except for the calculation of accumulation delays, load factors, and inflation ratios. The reason is that the careful analysis of center-related costs carried out prior to model development and reported in Chapter III, did not separate out fixed-cost and variable-cost elements in the required way; after model development, the time and resources remaining did not permit performance of this extra data-analysis step. Thus the "sizing" portion of the location-sizing step could not be programmed or applied. Instead, a nominal service delay of 0.125 days inferred from the material of Chapter III was applied across the board. As for the monetary cost associated with using a center, Chapter III develops the representative values of Table C (Table 3 of the main text), yielding a nominal cost of \$94.28 per container this too was applied uniformly.[*] (Chapter III gives no reason to anticipate significant variation of cost with center location -- assuming prudent site selection and design -- but as a precaution a means for treating such variations has been included in the mathematical model and computer program.)

Table C. Summary of Representative Costs Attributed to Major Fixed and Variable Expenses Contributing to the "User Fee" for Stuffing One 40'x8'x8' Container[†]

Building	\$7.00
Land	2.00
Site Preparation	3.00
Maintenance and Utilities	3.00
Equipment - Forklifts	1.38
Yard Mule	1.20
Administration	25.00
Direct Labor	51.70
Total	\$94.28 [††]

[†] It is to be emphasized that these are representative costs and subject to variations in locations of Centers, ownership, policies of operations, etc.

[††] In comparison with this estimate of \$94.00, a well-known steamship company advertises that it will stuff a 24-foot container for \$60, which when converted to a 40-foot container, is \$100.

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[*] Chapter IX reports the effects of varying these center processing times and costs. The model is relatively insensitive to quite large changes in these parameters.

Sequence of Center Operations

The preceding description treated a consolidation center as an object for mathematical modeling; its activities were necessarily represented in only an abstract and symbolic way. This synoptic chapter should not leave the reader without a more "physical" picture of such a center and its operations. The picture described below and extracted from Chapter I is the general sequence and means of handling cargo; other configurations see some use today and further modifications in the future cannot be ruled out:

The consolidation center shed is a structure some 120 feet wide, 16 feet high, and a length consistent with the desired freight handling capacity. (See sections on the shed, Chapter III.)

Gates for the unloading and loading of freight are nearly continuous along the length on either side. It is a clear span structure to permit free movement around the floor of forklifts without interference from vertical supporting beams.

At one end there is office space for clerical work and customs personnel. On at least two sides of the building there is parking space for trucks and containers awaiting loading and unloading or over-the-road haulage. There may be a railroad spur along one side of the building or extending part way into the shed from one end.

Cargo is generally brought to the center by trucks. These trucks are either parked briefly in the yard awaiting a gate or are backed up immediately to a gate for unloading.

The unloading is generally done by the trucker. The cargo is then sorted by destination and piled along the center line of the shed, one pile for each destination. This sorting and piling is done using forklifts for the movement.

When a container load for a specific destination is collected, a container is backed up to a gate (the containers are kept on wheels) using an over-the-road tractor or a yard tractor and the container is stuffed. It is then moved out on line-haul over the road to the port or to a railroad yard for loading on a flatcar for the line haul. The latter would probably be done by center tractors as described in the Chapter on costs or by a trucking firm under contract to the center.

If a rail spur is run to the center, cargo is unloaded from boxcars through a gate or directly onto the floor if the spur runs into the shed. It is handled from here as described under the truck operation. For inland consolidation centers it is not anticipated that receipt of less-than-carload lots will commonly be by rail; the great majority will be by truck. Boxcar loads do arrive today at port consolidation centers and the cargo is stuffed into containers as described under the truck operation. But with the growth of inland consolidation centers it is anticipated that this mode of operation will become much less common.

If a spur is constructed to the consolidation center (which can be done through negotiation with a railroad, particularly if the center has enough business in prospect), then the containers can be stuffed while on flatcars on the spur alongside the consolidation shed. This method of loading has been tried but is not a common practice and, according to conversations with operational personnel, has not been particularly successful.

It is also conceivable that containers on wheels can be end-stuffed at gates as described above for over-the-road line haul and then loaded on flatcars using ramps and yard mules (as done generally by the railroads at the railyards). However, railroads prefer that stuffed containers be hauled by tractor to the railyard. It seems most practical to build the center at the railyard and thus eliminate spur construction or haulage to the railyard. (See Chapter III for a more detailed discussion.)

For a diagram of the flow see Figure C.

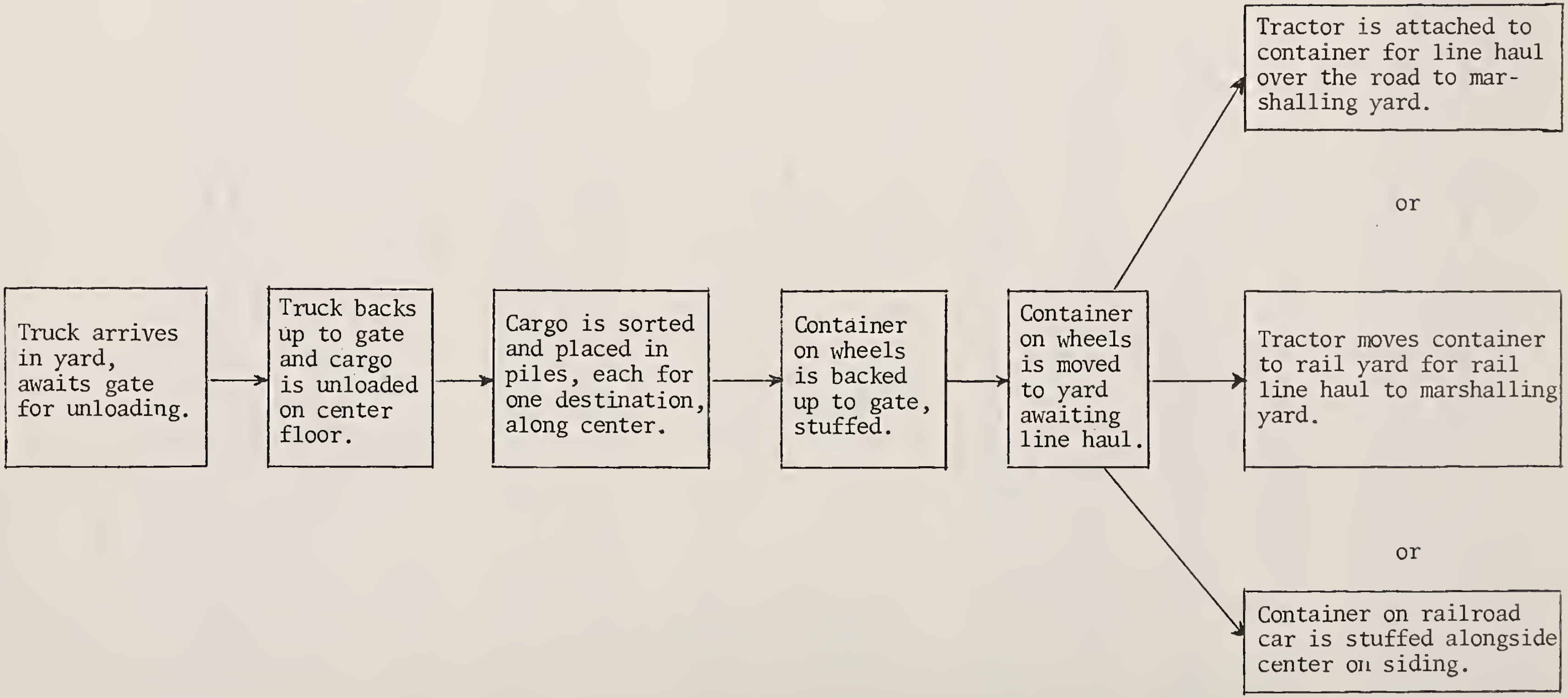

Figure C. Export Cargo Flow Through a Consolidation Center
(Import flow is the reverse of this flow)

Data Considerations

The previous material has alluded to or implied several areas in which better data would be valuable. Here we focus on additional major instances of data deficiencies, and note the makeshift devices used to bridge these gaps in our illustrative calculations. The principal related section of the main text is Chapter IV.

For comprehensive planning connected with international maritime commerce, it seems evident that regularly collected nationwide data are needed on goods movements with respect to ultimate source (domestic or foreign), mode(s) and route of transportation, destination (foreign or domestic), dollar value, commodity class, seasonal variations in flow, and the like.* No systematic studies have been conducted which incorporate all or even a significant part of these data. This situation seems to us a matter for serious concern.

It is perhaps symptomatic that the closest approximation to our needs was not a study conducted by a Federal agency, but rather the excellent survey of 1964 cargo flows conducted by the Delaware River Port Authority. This survey proved indispensable for our purposes; however, it lacked information on the foreign destinations of the cargo, and on seasonal or other variations over time in size and frequency of shipments. These omissions are of course not mentioned as criticisms of the survey, which was designed for a specific purpose other than the present study.

For studies specifically relating to containerization, still more information is required. Which commodity classes (or subclasses) are containerizable? Which combinations of individually containerizable cargo types can compatibly be shipped in a common container? What are the flow data when specialized to the LCL shipments which require the services of a consolidation center?

The Delaware River Port Authority survey data gave the commodity class (3-digit code) for each entry. Relying principally on the judgment of that Authority, we eliminated some commodity classes as representing mainly bulk commodities rather than general cargo.

Next came the elimination of those commodities considered unsuitable for containerizing. Although this elimination step was made on the basis of consultation with the Maritime Administration and use of Port of New York Authority studies, the degree of arbitrariness involved was distinctly disturbing.

The thinning-out of reported shipment flows, to reflect only the LCL component requiring consolidation services, was performed in the following crude but reasonable way. The average weight per shipment, for each shipper, could be calculated from the available data. If this average exceeded the nominal weight level (48,000 lbs.) corresponding to a full 40' container, the shipper was classified "non-LCL" and removed from the list of shippers of concern. All cargo from all of the remaining shippers is assumed to require consolidation at a center or at the port. This criterion errs in one direction by discarding the occasional LCL shipments of typically heavy shippers, but errs in the opposite direction by including in the centers' potential market all of the occasional full-container movements by "LCL on the average" shippers.**

The final "makeshift device" involves the question of which of a set of essentially co-located shippers could be regarded as producing "co-containerizable" LCL material, and hence could be aggregated. Although such information would surely be available in principle from an adequate data base for a consolidation-related study, it was not in fact deducible from the data at our disposal.

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*One would ideally also like a conceptual "explanation" of these data in terms of more fundamental variables, since mechanical extrapolation of observed trends is dangerous for planning purposes when underlying factors may change.

**It also ignores the possibility of some LCL cargo being noncontainerizable simply because its route lacks container-handling facilities.

If cargo from several shippers is to be co-containerizable, it must

(a) move in the same direction (into or out of the U.S.) through the same U.S. Port,

(b) be of physical natures permitting joint presence in a common container without damage, and

(c) involve the same overseas distribution/concentration point (or perhaps port-of-call area).

The available data permitted the straightforward application of criterion (a).

Regarding (b), there was the question of which combinations of commodity classes, each individually containerizable, would admit joint containerization. The variety within each commodity class precludes a really clearcut answer. Without a detailed study on this point, it seemed necessary to choose as working hypothesis one of the two logical extremes, either that any containerizable commodity class's material can be combined in a container with any other class's, or that no two commodity classes can share a container. The first of these alternatives was believed nearer the truth and was adopted. There is an obvious need for better data and further study to permit more satisfactory treatment of this point.

Since no information bearing on (c) was at hand, fictitious "data" of the appropriate kind were generated by the following process. It was arbitrarily assumed that each of the three U.S. ports in question served overseas points (capable of handling containerized cargo) which constituted up to at most 20 co-containerizable categories. The exporters using each port were assigned at random, one by one, to some one of 20 categories, and the same was done for the importers. (An alternate random assignment to 20 categories and a random assignment to 15 categories were tried and are recorded on pp. 111-112.)

* * * * * *

In this synopsis we have attempted to give the reader an overview of the study and its principal findings, as well as a guide to the main text. The presentation has been largely structured around the mathematical model, its computer-program embodiment, and the illustrative calculations performed with it; the dependence of these developments on the more empirically oriented material in Chapters I - VI has been traced out. No attempt has been made above to provide guidance to the technical appendices (A-G), since these are adequately introduced at the appropriate points in the main text.

TABLE OF CONTENTS

Table of Contents (Cont.)

LIST OF FIGURES

LIST OF TABLES

List of Tables (Cont.)

SYSTEMS ANALYSIS OF INLAND CONSOLIDATION CENTERS FOR MARINE CARGO*

R. H. Jordan, M. C. Stark, C. O. Bunn, J. L. Donaldson, W. J. Obright, H. R. Millie
J. Gilsinn, A. J. Goldman, W. A. Horn

This Technical Note documents a study, carried out for the U.S. Maritime Administration and completed by interested Bureau staff, to develop analytical techniques to optimize the locations and characteristics of inland consolidation centers for marine cargo. Such centers would consolidate less-than-container lots of cargo into "full" container loads for export, and would unload and distribute containerized import cargo.

After discussing the nature and scope of the study problem and outlining the functions and operations of the centers, the paper reports the fact-finding phase of the analysis. Successive chapters present and analyze data on: initial and operating costs for centers, current demand for their services as derived from a recent survey by the Delaware River Port Authority of its hinterland, and relevant (ground) transportation rates for containerized and uncontainerized material.

Reported next is the development of a mathematical model for estimating good locations and sizes for the consolidation centers. The selective implementation of this model as a computer program is described in sufficient detail to guide prospective users. A final chapter describes the illustrative application of this program to the data at hand, with results quite encouraging for the "inland center" concept.

Key words: Transportation; maritime, cargo; containerization; systems analysis; mathematical models; optimal locations.

Chapter I. INTRODUCTION

In order to exploit the full potential of future U.S. foreign trade and a compatible merchant marine industry, the Maritime Administration has undertaken a comprehensive research program. A part of this research pertains to the consolidation and containerization of small shipments of break-bulk general cargo into full loads, each load destined for a designated overseas distribution point, and the reverse, the receipt of full loads from overseas at domestic centers for unloading and distribution. On August 1, 1967, the National Bureau of Standards was requested by the Maritime Administration to develop analytical techniques to optimize the characteristics and location of such consolidation centers, first on a regional and then on a nationwide basis.

The study was conducted in two phases. The first consisted of a detailed definition of the problem and the development of an approach and analytical techniques to solve the problem. Phase II consisted of further development of Phase I techniques and their application in determining the most productive locations and characteristics of consolidation centers in a selected market region serving a specific port facility. A Phase III similar to Phase II was planned to cover the problem nationwide but because of lack of nationwide data the third phase was postponed.

The specifications of the study are outlined in the Maritime Administration Purchase Order, "Scope of Work, Systems Analysis of Inland Consolidation Centers," dated August 1, 1967.

The Problem and the Objective

It has been customary to transport general break-bulk cargo to ports in boxes and crates of different sizes and shapes depending on the nature of the unit objects being shipped. This procedure requires handling numerous small units and results in excessive time and labor costs to the shipper. These costs are further escalated because of pilferage and damage.

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*Sponsored by the United States Maritime Administration, Washington, D. C., 20235.

Recently there has been a strong and rapidly accelerating trend toward the containerization of break-bulk general cargo. That is, the individual units of cargo are loaded or "stuffed" into large containers, generally from 20 to 40 feet in length and 8 feet by 8 feet in breadth and height, each destined for a specific foreign distribution point, and the container is handled as one unit in transport over the land, loading onto ships, transport at sea, and unloading. This procedure can yield large savings in time and labor costs and presumably decrease losses from pilferage and damage.

The containers are commonly stuffed at a consolidation point in a seaport facility. It is believed by some segments of the transportation industry that consolidation and containerization at common centers, optimally sized and located in relation to the shippers and to the pertinent ports, would be less costly to those shipping in less than carload lots and would thus encourage foreign trade and foster growth of the United States Merchant Marine. If this consolidation and containerization is performed at an inland point, handling may be minimized and the economy of movement in full container loads can be exploited. The cargo of shippers with less-than-carload lots can be merged with others to make up full containers. The shipper who has sufficient cargo for a given foreign distribution point to fill a container might well bypass the inland consolidation centers but, on the other hand, he may benefit by taking advantage of lower cost rates to the port as in the case of the speed and lower cost of unit trains. The study includes the flow of both export and import cargo.

The goals of the study are to analyze the functions of consolidation centers and to develop methods for determining their size and location. The research includes means of estimating the cost of the use of centers in relation to the location of potential shippers. The results of this endeavor provide tools for comparing various proposed configurations of the fixed consolidation center concept and comparing these configurations with the present break-bulk system of inland distribution.

Also the techniques can be used with forecast data to establish guidelines for the determination of new environmental features, such as direct lines from inland centers to ports, improved feeder highways, and probable site locations of new centers.

General Approach

The approach has been the development of a general procedure for locating centers, utilizing costing techniques that involve transportation costs, center costs, and the location of exporters (and importers) and their volume of business. A symbolic representation of the system was developed and an algorithm for computer application was formulated and exercised. This development of the model was evolutionary. A preliminary model was constructed, its strengths and weaknesses considered, and improved versions developed. In this manner, a series of progressively improved models was evolved. During this process, the function of consolidation centers and the effects of various modes of operation and ownership have become increasingly clear. To develop the model, the problem was divided into three parts: the necessary handling and transportation of cargo from the shippers to the consolidation centers; the handling at the centers for the necessary consolidation; and transportation to the ports. The problem is one of providing a system which maximizes the effective movemen of shipments, determining locations of consolidation centers that are efficient with respect to time and overall costs, and reducing transportation costs through moving full loads insofar as possible over optimum routes.

In addition to the development of the general location model for solution by automatic data processing techniques, data on exporters and importers and their cargos were obtained for the area from Indiana throughout the Midwest to Philadelphia-Camden (see Chapter IV, Data: Source, Adequacy, and Preparation). These data were used in exercising the computer model for consolidation center location in relation to shippers and ports. As well as giving guidance for computer runs and for checking computer answers, these data were used manually for a graphic portrayal of export origins, indicating cargo tonnages, and generally, by inspection, probable location of consolidation centers. Although the "map analysis" lacks some of the rigor and refinements of the computer model, it has provided an intimate knowledg of the data and a direct feel for the problem not always attainable from sophisticated automated approaches.

Bounds of the Problem

The scope of the problem is defined as follows:

1. Only small shipments, less than container lots, of break-bulk containerizable cargo are considered. It is assumed that if a shipment to a specific overseas destination is large enough to fill a container, the container will be filled at the point of origin (the factory) and shipped by the most favorable route (perhaps through a consolidation center) to the port facility.

2. The ownership of consolidation centers was not originally considered a problem affecting the study nor a part of the research. However, the importance of the effects of different ownerships has considerably increased as the study has progressed. These effects in relation to the function of centers are discussed later, although no attempt is made to recommend any one type of ownership.

3. Furthermore, in exercising the model it is assumed that consolidation centers do not own or control the transportation systems draying cargo to them, the line haul systems to the domestic ports, or the containers. Ownership of these systems can make a marked difference in the location, size, and function of centers; but because the numerous possibilities are a study in themselves, they are not researched in depth but are discussed generally to emphasize their importance. No decisions are reached as to the ownership plans most benefiting the shipping community.

4. Political, sociological, and economic effects on a community are not treated.

5. Legal aspects such as responsibility for losses and damage to cargo, changes required in documentation, etc., are not considered.

6. Origin and destination data for break-bulk general cargo by commodity class, volume, and frequency required as inputs to this study were to be provided and, to the extent possible, were provided by Maritime Administration or sources designated by the Maritime Administration. Lack of adequate origin and destination data, particularly foreign destination data, has proved to be a major problem and necessitated substantial modification of the original plans for conducting the study.

7. All cargo considered in this study is assumed to be containerized by the time it is loaded on the ship.

8. Bulk cargo or cargo not subject to containerization is eliminated from consideration.

9. The advantages of reduction of packing costs and pilferage are very important arguments for containerization but they are not important factors in function, location, or size of consolidation centers and therefore are not discussed. The reduction in damage is a debatable point under present methods of stuffing (insurance rates have generally not been reduced). It is therefore omitted from consideration in this limited study.

Specific Considerations

In preparation for the study, a broad survey of background information was conducted. In this survey it became clearly apparent that there were many considerations bearing either directly or indirectly on the problems that influence the scope of the study, and to a certain extent, the mode of operation of the consolidation concept. Also, because of the exploratory nature of the study, it was realized that these considerations would come into sharper focus as the study unfolded; new ones would appear and some would not retain their original significance, assuming either greater or lesser importance. This has proved true. The most important of these considerations are: ownership of centers; length of time of holding cargo awaiting consolidation; the center as a shipping intelligence information point; cost of centers including cost of acquisition and operation as a basis for determining charges levied on the user; and rates and tariffs. These problems are discussed in the following chapters on "Function and Operation of Centers," "Cost of Consolidation Centers," and "Rates."

Other considerations are measures of effectiveness, symmetry of export and import flows, customs, ship scheduling, effects of centers on communities, and the marshalling function. These considerations are discussed in the following sections.

Cost - A Measure of Effectiveness

At the inception of the study it was believed that the best way to determine the number, size, and optimum location of centers, was through a comparison of respective total costs to the shipping community. These costs include the direct cost of transportation of cargo to the centers and from the centers to the port, and the cost of the centers including their operation which determines the charge to the user for the center services. The time required for shipment, while not always expressed in dollars, can in principle be converted to dollars.

It is still clear that these costs are significant determinants of the location of center

However, it soon became apparent that these direct costs are not by any means the only deciding factors. Others, such as traffic congestion in terminal areas, selection of shipping routes and selection of domestic ports sometimes are more important than direct costs incurre within the continental United States. Furthermore, as will be detailed in the chapter on "Rates," rates are difficult to obtain, change constantly, may be altered by the very construction and operation of a center, and, in regard to the use of containers, are in the midst of a very marked policy evolution. Therefore, while rates have been used in the exercise of the model and must be considered a very important factor, they have not been used in the detail expected at the outset of the study. Given the time and the money to determine more exact rates, they could be used more nearly as anticipated.

Symmetry

A consolidation center may also receive import cargo in containers from ships and reverse the functions previously described to forward cargo in the manner best suited to the shipper and the consignee. Except for the delay in the center awaiting a container load for export (there is no analogous wait in the domestic center concerning imports), this flow appears to be a "mirror image" of the flow to the ships insofar as modeling is concerned. However, an examination of the study data shows that optimum location of centers for imports is not necessarily best for exports because importers are not always exporters and vice versa. The model can be run to locate export centers or import centers alone, or, on the assumption that all centers will handle both exports and imports, it can be run to co-locate export-import centers simultaneously.

Customs

Space considerations for customs personnel and cargo handling are part of the costing in Chapter III. It is assumed here that customs inspection will be a part of all centers but will not affect location of centers.

Ship Scheduling

A knowledge of the departure and arrival of ships at pertinent ports is essential to the effective operation of a consolidation center. This knowledge is particularly important in relation to departures; it allows a scheduling of the arrival of export cargo at the center to avoid storage at the center and permits an optimum scheduling of the departure of containers from the center for most efficient marshalling and loading on the ship. More will be sai of the information on sailings in Chapter II, "Function and Operation of Consolidation Centers

Effect on Community

Containerization centers may well affect the communities in which they are built, particularly small communities. The study did not consider these effects. For example, it is well recognized that traffic congestion plays a definite part in the location of consolidatio centers. However important these effects are, they are specific to an area and each case must be considered separately. Therefore, they are recognized but omitted from the model.

Marshalling Function

Full containers, wherever loaded, must be held until time for them to be dispatched to the ship. This is a storage function, however temporary. Just prior to bringing containers to the crane that hoists them onto a ship, the containers will be retained at a place where they can be dispatched to the crane for a precise arrival time. This is a buffer function. In this report the storage function is considered to be performed at the consolidation center and land for this function is included in the land requirements of the center in the cost estimates of Chapter III. The buffer function must be performed at or near the docks for precision in timing. This function does not affect the location of the consolidation centers and is not included in the center cost estimates of Chapter III.

The Flow of Cargo Through the Center

By reading the various sections of this report, the flow of cargo through a consolidation center can be deduced. However, for convenience early in the report, a concise description in one place is advantageous. The picture described here is the general sequence and means of handling cargo although it would be erroneous to assume that other configurations are not used today or will not be devised in the future.

The consolidation center shed is a structure some 120 feet wide, 16 feet high, and a length consistent with the desired freight handling capacity. (See sections on the shed, Chapter III.) Gates for the unloading and loading of freight are nearly continuous along the length on either side. It is a clear span structure to permit free movement around the floor of forklifts without interference from vertical supporting beams. At one end there is office space for clerical work and customs personnel. On at least two sides of the building there is parking space for trucks and containers awaiting loading and unloading or over-the-road haulage. There may be a railroad spur along one side of the building or extending part way into the shed from one end.

Cargo is generally brought to the center by trucks. These trucks are either parked briefly in the yard awaiting a gate or are backed up immediately to a gate for unloading. The unloading is generally done by the trucker. The cargo is then sorted by destination and piled along the center line of the shed, one pile for each destination. This sorting and piling is done using forklifts for the movement.

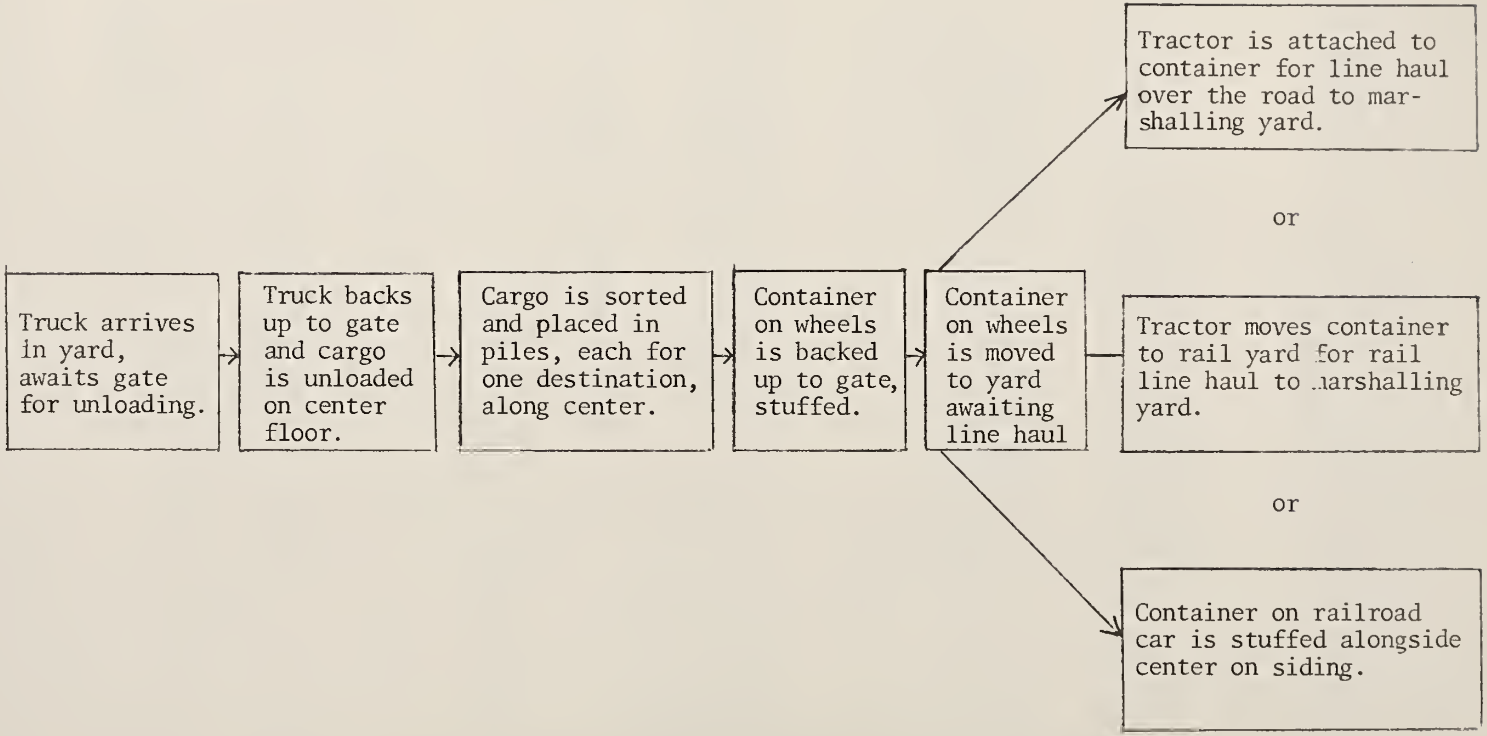
When a container load for a specific destination is collected, a container is backed up to a gate (the containers are kept on wheels) using an over-the-road tractor or a yard tractor and the container is stuffed. It is then moved out on line-haul over the road to the port or to a railroad yard for loading on a flatcar for the line haul. The latter would probably be done by center tractors as described in the Chapter on costs or by a trucking firm under contract to the center.

If a rail spur is run to the center, cargo is unloaded from boxcars through a gate or directly onto the floor if the spur runs into the shed. It is handled from here as described under the truck operation. For inland consolidation centers it is not anticipated that receipt of less-than-carload lots will commonly be by rail; the great majority will be by truck. Boxcar loads do arrive today at port consolidation centers and the cargo is stuffed into containers as described under the truck operation. But with the growth of inland consolidation centers it is anticipated that this mode of operation will become much less common.

If a spur is constructed to the consolidation center, which can be done through negotiation with a railroad, particularly if the center has enough business in prospect, then the containers will be stuffed while on flatcars on the spur alongside the consolidation shed. This method of loading has been tried but is not a common practice and, according to conversations with operational personnel, has not been particularly successful.

It is also conceivable that containers on wheels can be end-stuffed at gates as described above for over-the-road line haul and then loaded on flatcars using ramps and yard mules (as done generally by the railroads at the railyards). However, railroads prefer that stuffed containers be hauled by tractor to the railyard. It seems most practical to build the center at the railyard and thus eliminate spur construction or haulage to the railyard. (See Chapter III for a more detailed discussion.) For a diagram of the flow see Figure 1.

Figure 1. Export Cargo Flow Through a Consolidation Center (Import flow is the reverse of this flow)



Chapter II. FUNCTION AND OPERATION OF CONSOLIDATION CENTERS

One of the objectives of this study is the determination of the functions of consolidation centers. The basic definition is given by the Maritime Administration in the "Scope of Work" as, "An Inland Consolidation Center is a terminal for consolidation of containerizable cargo, located away from the containership berth, which performs these functions:

1. "Receives export cargo as presented by the shipper or forwarder.
2. "Processes, holds, and consolidates such cargo (to the extent practicable) into containers suitable for foreign trade, each of which contains cargo for only a single destination.
3. "Schedules and routes such containers to the ship berth in a manner which will minimize traffic congestion in the port.
4. "Receives import cargo in containers from the ship, and reverses the above functions to forward the cargo in the manner best suited to the shipper or his forwarder."

This definition is an accurate general statement, but a detailed study of consolidation indicates that certain functions and concepts of operations should be added. These functions are related to:

1. Ownership
2. Holding of cargo awaiting consolidation (storage)
3. Interchange of information (intelligence center).

There are at least three major plausible ownership arrangements for consolidation centers, and probably a number of combinations or variations of these three. One is ownership by a not-for-profit organization, perhaps a city, state, or regional port authority*; the second is by a private concern such as a freight forwarder; and finally by a cooperative composed of a number of shippers working together for their common benefit. Each of these types of ownership can have effects upon policy of operation and thus upon the functions conducted by the center.

It is not to be inferred here that the type of ownership be dictated by the government, nor is it within the scope of this study to decide which type of ownership would be of the greatest benefit to the general economy. Rather, the aim is to point out some possible influences of ownership upon functions so that comparisons for formulation of policy can be made.

The ownership concept in one of its simplest forms is by a not-for-profit organization, such as a port authority. The principal motivation would be to attract business to the area of the center. It is assumed that enough revenue would be received to cover operational expenses, retirement of mortgages and other expenses, with perhaps enough excess cash flow to cover expansion and emergencies. Also, although it need not be limited in this manner, it is assumed for this study that the organization would not own a fleet of trucks to collect less-than-container loads; instead cargo would be brought to the center by trucks owned by exporters or by local trucking companies. Finally, it is assumed that the not-for-profit center would neither own nor control, that is, keep track of and manage, the movement of the containers used in transportation nor would it own or manage the line haul equipment to the domestic ports. The center would charge a "user fee" for its services.

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*The term "port authority" here is not restricted to a marine or oceanic port but is used in a more general sense as, for example, an authority managing the airports of an area.

The shipper would gain through:

1. The opportunity to consolidate in full loads which he might not otherwise be able to accomplish (a number of long haul transportation companies are reluctant or refuse to handle less-than-carload lots).
2. Paying a lower rate on the line haul because of consolidated full loads.
3. Less pilferage.
4. Less breakage (hopefully).

This is the type of ownership assumed in this study for exercising the model, although with the proper reinterpretations the model can be used under any form of ownership.

A second means of ownership is by a private concern. This appears to be the direction in which the consolidation concept is moving. There are many variations of this type of ownership. Let us say, for example, that the owner and operator is a freight forwarder. He owns a fleet of trucks to collect cargo or he contracts with a transportation company to bring cargo to his center. (The latter can be done at lower than prevailing rates by a freight forwarder but not by certain other owners such as a trucking company, and is an incentive to attract customers.) He ships line haul to ports in full containers hopefully, owned by one or by various transportation companies. The forwarder is charged the container rate for the line haul, but he in turn charges the shipper the break-bulk less-than-container load rate. The difference is the basis for his profit. This is common practice today. In addition he may make a direct charge for stuffing containers.

The freight forwarder benefits by:

The difference between the full container line haul rates and the less-than-carload rates, and perhaps by charging for stuffing the containers.

The shipper benefits by:

1. Less pilferage,
2. Less breakage,
3. An opportunity to consolidate less-than-carload lots,
4. And perhaps by lower rates on the local short-haul to the center.

A third concept very similar in benefits to the first is a cooperative center owned and operated by the shippers themselves. The advantages to the shipper would be similar to those of the not-for-profit plan, although the shippers would probably have a greater voice in policies and management. Otherwise rates and user fees might well be almost the same.

<u>Holding of Cargo as a Function of Consolidation Centers</u>

The holding of cargo awaiting consolidation for specific overseas distribution points in full containers is a function of a consolidation center. How long it should be held is the question. The policy adopted has a direct and important bearing on the center capacity, number of shippers using the center, number of centers, and their operation.

On consideration of the problem certain facts are apparent:

1. Some cargo for out-of-the-way ports might have to be held for months awaiting enough to make up a full container load.
2. The larger the center's volume of business, generally speaking, the shorter the wait to fill a container.
3. The longer the cargo is held, the greater the need for storage space.

4. If cargo is stored in a building other than the consolidation center, there will be at least one extra handling and an extra transportation link. (If stored in the consolidation center building these two factors may or may not be true depending on such factors as labor arrangements and amount of cargo.)

5. Shippers cannot afford to have some commodities held because of competition, disadvantages of large inventories, perishability, etc.

During the study, discussions were held with consolidation companies concerning the holding of cargo to accumulate full loads, and their written opinions were examined, as in the hearings before the ICC for expansion of the New England Freight Forwarders, Inc.[1] There is an almost unanimous opinion among these companies that cargo cannot be held any appreciable time awaiting full loads. It appears that one week is close to the maximum and that two or three days or less is generally considered most desirable. (Although aircraft cargo is not part of this study, it was found that the aircraft consolidation centers visited hold cargo for no more than a matter of a few hours.)

One consolidation company stated emphatically that "if you hold cargo you are dead!"

Therefore, the present opinion among operators appears to be that consolidation centers are not storage centers, except for short periods for customs examination, actual stuffing operations, or cases where it is known that containers can be filled in a reasonable time. Storage is not considered a function of a center by consolidation operators.

On this basis the following guidelines were adopted; cargo is to be held no longer than one week and most cargo will be held no longer than three days; all centers will operate essentially in this manner; no storage capacity in the center shed will be included for purposes other than holding freight on the shed floor in piles for specified distribution points for a minimum time, although costs resulting from delay will be considered in the exercising of the model. A means of introducing them is detailed in the mathematical model in Chapter VII. (This guideline may appear from a theoretical sense to be an oversimplification, but it is a convincing one to a practical operator working against intense competition. If in the future it is found that this guideline is, indeed, an oversimplification, delay time factors can be implemented, given the necessary data and time for programming.)

The goal of minimum holding can be achieved in at least two ways, both of which affect the functions of the centers and are chiefly policy decisions:

1. A consolidation center can accept cargo only to those foreign distribution points to which the center can maintain a flow of sufficiently full containers. Consolidation centers generally follow this practice today by advertising shipments to certain ports only. Under special arrangements, less-than-carload lots could be accepted to other distribution points but no attempt would be made to consolidate by storing cargo beyond certain stated periods. The cargo would go in less-than-full containers perhaps under higher rates, or by ordinary LCL transportation.

   On the other hand, centers strategically located might accept cargo for consolidation for those distribution points not served by others. A center, say in Toledo, might serve certain European points but not South American points served by, say New Orleans, and vice versa. If a consolidating company should own a number of centers covering a large area of the country, it would probably send cargo to the center most likely to fill a container for the desired destination. One such company has stated this to be its future operating procedure.

2. A second means of minimizing the holding of cargo for consolidation is a part of a third possible function of a center and is treated in the following section.

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[1] Interstate Commerce Commission, New England Forwarding Company, Inc., Extension No. FF-96 (Sub-No. 2).

The Function of a Consolidation Center as an Intelligence Center

It appears that a consolidation center, regardless of ownership or other operational policy, should act as a shipping intelligence center if it is to operate efficiently. The center would hold on file all sailings from pertinent ports, departure dates, and overseas ports of call. These dates would be made available by the center to all exporters in the area served by the center and would be revised and updated as times for the actual sailings become more certain. The first forecasts might be weeks ahead of time and quite general as to the exact date, while the last would be several days prior to the actual sailing and quite precise. This procedure is used at present by steamship companies.

Similarly, shippers would notify the center of proposed shipments and would consult with the center concerning the probability of filling containers for specified overseas distribution points on chosen dates. The cargo would not be forwarded to the center (nor perhaps even manufactured) until requested, thereby reducing holding time and storage space at the center.

The shipper would decide whether to hold cargo at the factory until it could be consolidated; whether to ship direct in less than carload lots; whether to ship to another center or possibly ship in containers but in less-than-full containers. A decision of this kind is strictly a business decision predicated upon competition, type of commodity, prevailing local rate structures, cost of storage, space at the factory, inventory situations and many other factors specific to the individual shipper. It would be difficult if not impossible in a general model such as described in this paper to take all of these factors fully into account.

Chapter III. COST OF CONSOLIDATION CENTERS

It is assumed that consolidation centers must collect sufficient revenue to meet their capital and operating costs plus an appropriate level of profit. This operation assumes a "user fee" collected for the service rendered either directly or indirectly for stuffing the containers and associated services.

It is the purpose of this chapter to assess the cost of land acquisition, construction, operation, and maintenance of consolidation centers to determine the effects of these costs upon the location and use of centers.

This assessment of costs of building and operation is discussed under 7 major headings:

1. The building (the shed or shell)
2. The land
3. Site improvement, grading, etc.
4. Maintenance
5. Equipment
6. Administration
7. Labor

The total cost associated with each of these major factors is divided by the expected number of containers to be stuffed to determine the unit cost of stuffing and thus arrive at the user charge. In addition, the comparative effect of each is studied to determine the sensitivity of each contributing factor to the total cost per unit.

Prior to a discussion of these major cost factors, certain measures of performance and costing concepts pertinent to the problem are discussed.

Measure of Unit Production

Because the goal of the chapter is the determination of a user charge (average cost) for stuffing a container, it is logical and convenient to use the container as the unit of measure of production. However, because of the various sizes of containers, a decision must be made in selecting the unit. For the report, the 40' x 8' x 8' Group I container is selected as the unit of measure of production. (United States of America Standards Institute, MH5.1-1965 Specification Table 3.1.3.1.) A container stuffed to an average density of 24 short tons (48,000 lbs.) per container is the average density, although the referenced table cites a maximum gross weight of 33.6 tons for the container and contents.

No inference is intended nor is any recommendation to be assumed that this container is better than another type or that any particular size or standard is endorsed. The cost model is capable of adjustment and use with any other unit capacity if such becomes desirable.

In this study, the 40' x 8' x 8' container loaded to 48,000 pounds as a unit of measure is designated the Design Equivalent Container and is hereafter referred to as a D.E.C.

Center Throughput Capacity

The measure of throughput capacity for a center in this report will be the number of D.E.C.'s per week, each week considered as 5 days, one 8-hour shift per day. Annual or daily capacities could be used with equal facility, but for this report, all center capacities are per week. Hence a unit of capacity will be one container (D.E.C.) per week. The symbol K will be used to represent the specific capacity of a center.

Since the cost of providing consolidation center services depends significantly upon the utilization of facilities, it is assumed throughout this chapter that each center will operate at or near its designed capacity. That is to say, a center will be designed with a capacity near its expected operating level. The requirements for each proposed center will be estimated and the centers will be designed to operate at this level on the average.

Consolidation Service Charge

It is assumed that the consolidation center must obtain revenue to cover its costs and that this revenue will be derived by charging its customers a service fee. This service fee will be expressed in terms of dollars per D.E.C., and prorated among customers.

The general approach for estimating the service charge is to consider the fixed and variable costs of operating the center, including an appropriate return on investment. The pricing policy is simply to charge each container a fee derived from total cost for a year divided by the expected annual throughput of loaded containers, where total cost includes return on investment. Somewhat more specifically, if the annual throughput is X containers, the annual fixed cost is F dollars and the average direct cost of consolidating services per container is v dollars, then the service fee per container is

$$r = \frac{F}{X} + v \qquad [3.1]$$

F consists of return on capital investments, insurance, interest payments, maintenance of grounds, and administrative overhead. v is determined by direct labor and other direct operating costs per container. Equipment cost may or may not be variable depending on the manner in which the equipment is acquired and maintained, i.e., purchase or rental. It is considered a fixed cost in this study.

Since this study does not presume consolidation centers of fixed or predetermined sizes or location, it is desirable to develop the various cost elements on the basis of a unit of capacity. In this way a service charge can be estimated for consolidation centers of any particular size. To account for regional variations in costs, a regional cost factor is utilized.

In the work to follow, there is no suggested rate of return on investment. The proper choices for these rates are considered to be beyond the scope of this study. However, effects of several illustrative rates of return on that portion of service charge attributable to specific features of the consolidation center are given. It will be noted also that the cost components considered here are not exhaustive. The principal components are included, however.

Components of Cost

In order to analyze the service costs that would be incurred and to obtain better understanding of the basis for deciding upon the location and size of consolidation centers, it is useful to partition the per-container charge, r, into its principal components. For this we partition r into

$$r = r_B + r_L + r_P + r_M + r_E + r_A + v \qquad [3.2]$$

where r_B = service charge per container attributable to costs associated with the building,

r_L = service charge per container attributable to costs associated with land,

r_P = service charge per container attributable to costs associated with site preparation,

r_M = service charge per container attributable to costs associated with maintenance,

r_E = service charge per container attributable to costs associated with equipment,

r_A = service charge per container attributable to costs associated with administration,

v = service charge per container attributable to costs associated with labor and other direct operating costs as described above.

To illustrate the general approach, consider the land component. Suppose that in order to develop a consolidation center with a capacity of D containers per week, a land area of A acres is required (for building, parking, roadway, etc.). Then a = A/D denotes the land requirement per unit of capacity. If land costs C dollars per acre, then Ca is the investment in land per unit of capacity. Then r_L = I Ca/52 is the service charge per container for a rate of return on investment of I per year. The factor 52 is involved since I is an annual rate and capacity is related to one-week period.

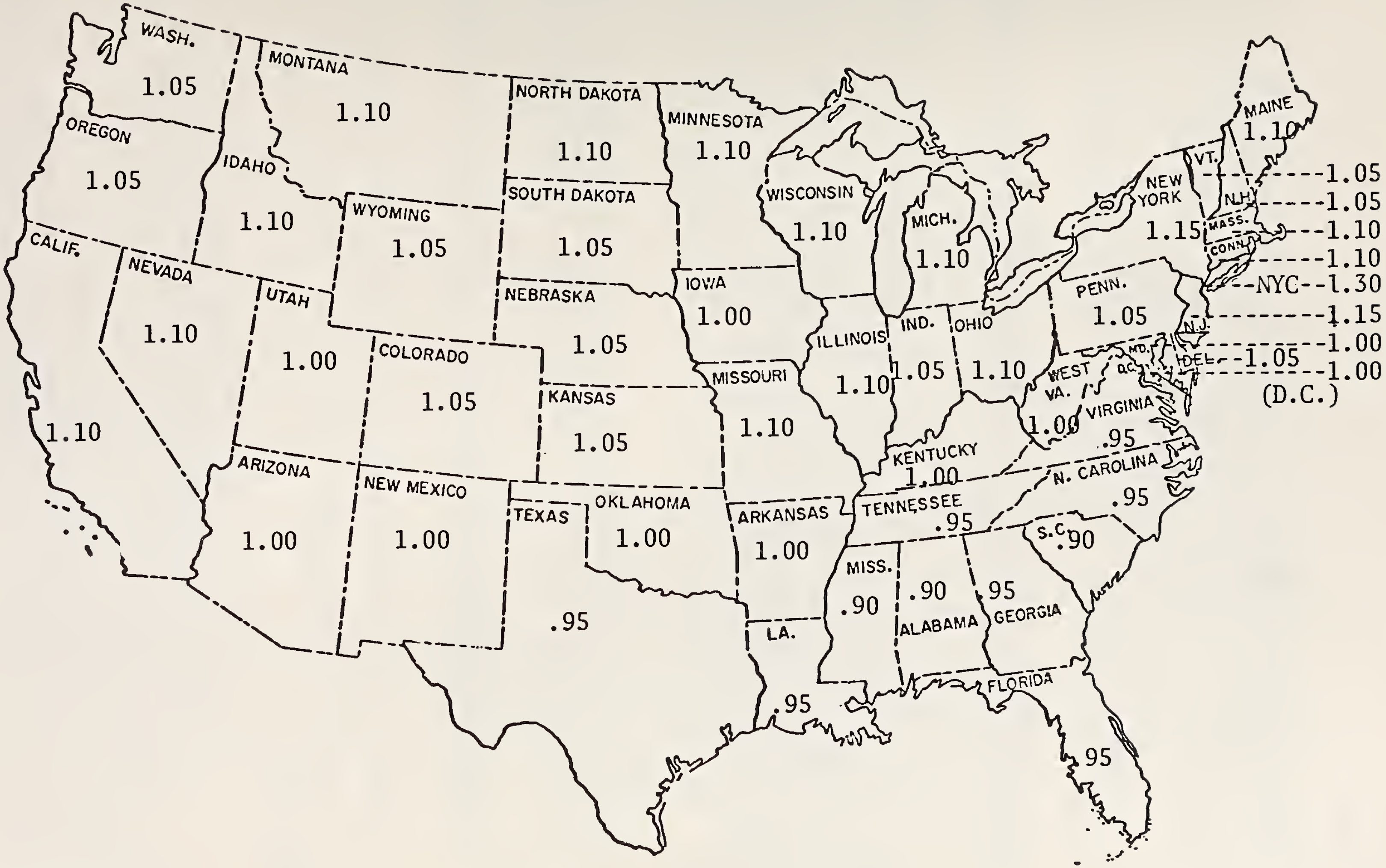
The symbols used in this chapter are listed in Table 1. Table 2 gives the specific values of the parameters used in this report. These values represent approximations of current practices and costs derived from observations at several operating enterprises, construction data from numerous sources, and on occasion, judgment of the project engineer. These values are specified as initial approximations to be used in testing the general methodology.

Table 1. Notations

a_L	Total land area required for a consolidation center per unit of capacity $a_L = a_B + a_L' + a_L''$
a_B	Land area required for building.
a_L'	Land area required for receiving and discharging noncontainerized freight including driveways and parking for trucks and railroad cars per unit of capacity.
a_L''	Land area for temporary parking, inbound and outbound, per unit of capacity.
C_L	Cost per acre of land.
y	Total cost of building shell only, exclusive of the double-high floor and mechanical-electrical construction costs.
C_F	Cost per unit area of double-high floor plus mechanical-electrical construction costs.
C_P'	Cost per acre of site preparation, including field surveys.
C_P''	Cost per acre for paving and storm drainage construction.
C_M'	Cost per acre of annual maintenance and repair of parking and roadway areas.
C_M^*	Cost per acre of annual maintenance and repair of freight shed facility.
C_M''	Cost per acre of annual maintenance and repair of electrical and other utility services.
C_M'''	Cost per acre of electrical power and service (not including power requirement within shed.)
C_ℓ	Total costs allocated to each average manhour of direct labor.
ε	Cost index factor which converts annual cost variants between any two given years and any two geographical areas.
α	Constant factor applied to type of shed construction.
I	Return on investment (land and site preparation).
I_B	Return on investment plus depreciation on building.
m	Number of manhours required to stuff a container and handle a DEC unit of freight.

Table 2. Suggested Values

a_L	0.455 acres
a_B	0.0098 acres = 427 square feet
a_L'	0.0082 acres
a_L''	0.0275 acres
b	0.0883
C_L	\$20,000 per acre
C_F	\$3.50 per square foot for the Washington, D. C., geographical area at a building cost index factor of unity
C_P'	\$11,000 per acre
C_P''	\$29,000 per acre for the Washington, D. C., area at a construction cost index factor of unity.
C_M'	\$125 per acre per year
C_M^*	\$0.198 per square foot per year. (\$8,625 per acre per year)
C_M''	\$141 per acre per year
C_M'''	\$1750 per acre per year, based upon a use factor of 1/3 (area illuminated during nonworking night periods) and \$0.03 per kwh utility rate.
C_ℓ	\$5.50 per manhour
ε	1.00 for March 1968, Washington, D. C., area
m	9 hours (m = 9) which is a most likely allowance of 3 hours for a 3-man team per DEC. Reported productivity varies from a low of 3 manhours per container (unloading tinplate from trucks) to a high of 25 manhours (unloading beer cases from railroad boxcars). Other classes of freight may be expected to fall within this range.



Construction Cost Factors
Figure 2

The Building

The costs of owning and operating a consolidation center vary from one place to another. Construction costs have been estimated to vary as much as 45% between some of the southern states and New York City. Figure 2 shows location factors assigned to various states which are adapted from Department of the Air Force criteria.[2] The location factors shown are multiples of 1.00 which has been assigned to Washington, D. C. In a countrywide study these factors should be taken into account. However, in the area under study in this report (see Chapter IV) they are much the same and are considered constant.

It is possible to estimate construction costs with more assurance than is possible with other elements of the cost model. The industrywide trend at the present time is toward a simple structure with many common characteristics:

(1) The building shell is a metal prefabricated structure about 16 feet high and with about 120-foot-wide clear span framing. The length varies dependent upon the freight handling capacity required of the facility.

(2) The building shell rests upon a "double high" foundation and floor about 4 ft. 2 in. above the finished roadway height for truck-trailer unloading. The floor is usually bituminous concrete suitable for forklift truck operation.

(3) There is no automated freight handling or sorting equipment. Some operators consider the possible use of an under-floor dragline or dolly towline, but the largest operation observed (1096 feet in length) was experiencing no difficulties with a forklift operation only and no future plans to convert to a dragline operation were under consideration, although there appear to be some possible advantages to limited automation in a freight shed which is very large and/or which requires a complex sorting of freight analogous to an airline freight operation. The approximations do not include any automatic freight handling or sorting equipment. Inclusion of a towline would not significantly affect the unit cost of the cost model for a large facility, however.

(4) There is a minimum of mechanical-electrical construction requirements. Freight sheds are either unheated or spot-heated in a minimal manner. Electrical construction is austere and minimum plumbing is provided. No air conditioning is included except in office areas.

(5) Overhead door openings are nearly continuous along the exterior walls. Access to rail cars are either from outside the building shell or from a freight car spur which extends into the building shell, being either a single or double track.

(6) There does not appear to be any reason why the building shell, mechanical facilities inside the building, plus the double-high floor, should cost more than $8 per square foot.[3]

(7) A large facility should probably include automatic data processing for office procedures as well as external information. The possible requirement is not included in the cost model because:

(a) Industry decisions (or trends toward an industry or government decision) are not clear with respect to ownership and management of containers (an automated data processing system seems mandatory if the centers control their containers and if the operation is of significant size, say handling 20,000 containers);

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[2] Military Construction Pricing Guide, AFP 88-16, March 1968.

[3] With respect to "General Purpose Warehouses" for Air Force construction, a Congressional cost limitation of $8 per square foot is currently applicable to facilities constructed in the United States less Alaska (including Hawaii) regardless of size of geographical cost index. AFP 88-16, March 1967, "Military Construction Pricing Guide," Department of the Air Force.

(b) Research on sizes, standards, and cost-benefit relationship regarding container ownership and fleet control are not a part of the study-mission;

(c) Some freight consolidation centers of medium and small freight-handling capabilities do function profitably without automatic data processing and information systems and there is no clear and readily available evidence for introduction of automatic data processing systems.

There are technical considerations which permit small economies of scale in the construction of a facility.[4] It has been recognized that many different classes of construction costs and equipment fabrication costs may be correlated by equations of the form:

$$y = \alpha A^{b} \qquad [3.3]$$

where y is the cost, α is a constant for the type of construction involved, A is the square footage of the unit, and b is also a constant, the factor of economics of scale. Engineering authors refer to the exponent b as the "slope," since in logarithmic form it is the slope of a linear function. Much research has been done to determine the value of b for various classes of construction.[5]

It has been pointed out by Chilton that although the median value of b for 36 curves was 0.66, several curves had slopes between 0.88 and 1.0. He explained that these curves represented plants which were composed predominantly of multiple units.[6] Based upon manufacturers' and contractors' data,[7] costed out for several sizes of consolidation center freight sheds, a value of b = 0.883 has been adopted here. This relates to the cost of building shell only, exclusive of the double high floor and mechanical-electrical construction. It can be noted that if the cost (y) is known for a facility with a square footage A and it is required to estimate the cost (y') of a similar facility with area A', then

$$y' = y\left(\frac{A'}{A}\right)^{b} \qquad [3.4]$$

The costs of the double high floor and mechanical-electrical construction can be influenced by local and regional cost variations, but do not appear to be significantly affected by economies of scale. A linear (direct) relation to area is assumed. The regional cost factors shown in Figure 2 may apply reasonably to both shed, flooring, and mechanical-electrical construction costs in order to adjust from one region to another. Local variations of these costs are not subject to precise analysis and a generalization is assumed.

The service charge attributable to the construction cost of the freight shed including shed and floor is:

$$r_B = (\alpha A_B^{b} + C_F A_B)\,\frac{\varepsilon I_B}{52K} \qquad [3.5]$$

or since

$$A_B = a_B K \qquad [3.6]$$

$$r_B = (\alpha a_B^{b}\,K^{b-1} + C_F a_B)\,\frac{\varepsilon I_B}{52} \qquad [3.7]$$

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[4] Cecil H. Chilton has presented a good description of the characteristics of the planning-estimating process, the problems involved, the meaning and use of the "six-tenths rule" and some of the caveats to be considered. The numerous representative curves for equipment items for chemical plants are shown. Some statements are made about William's six-tenths rule and its usefulness for entire plants. (Chemical Engineering, June 1949, pp 97-106).

[5] This relationship is discussed by Roger Williams, Jr., Chemical Engineering, June 1947, p 102.

[6] Chilton, Chemical Engineering, April 1950, pp 112-4.

[7] Numerous trade sources were used including major manufacturers who expressed reluctance to be specifically quoted due to competitive considerations.

where: r_B = Consolidation charge per container attributable to the building construction costs a and b as previously defined.

A_B = Total area covered by shed for shed with capacity K.

a_B = Area of shed required per unit of capacity (427 sq. ft. or .0098 acres).

K = Capacity in containers per week.

C_F = Cost per unit area for flooring and mechanical-electrical construction ($3.50 per square foot at Washington, D. C.).

ε = Regional construction cost factor (ε = 1 for Washington, D. C.)

I_B = Rate of return on investment (including depreciation).

α = Constant for shed type (α = $8.47)

The building area value, a_B = 427 square feet, represents an average of three field observations of operating freight consolidation sheds analogous to the considerations of this present study. There is no representation whatsoever that this value is an optimum for all specific local conditions. It is believed however that this number is a reasonable approximation of real-world operations today.

Figure 3 graphically portrays r_B through Equation 3.7) for a variety of Center capacities and for I_B = 10% and 20%. For Figure 3 , the values used were:

α = $8.47

a_B = 427 sq. ft.

b = .883

C_F = $3.50

ε = 1.00

In summary, it should be noted that economics of scale in building construction are relatively small and that the cost of stuffing a container is not particularly sensitive to the size of the building as long as it operates close to design capacity. A reasonable cost of stuffing a container attributable to building construction is about $7.00 ($I_B$ = .15).

Land

Land costs vary widely according to location. Since consolidation centers require relatively large amounts of land, these cost differences can affect significantly the choice of location centers. Land costs also can have an important effect upon the feasibility of a particular design and upon consolidation user charges.

The cost component associated with land (r_L) depends on the land area required, the cost per acre of land, and return on investment. Hence

$$r_L = \frac{C_L a_L I}{52} \qquad [3.8]$$

where a_L = Total land required per unit of capacity, in acres

C_L = Cost per acre of land

I = Return on investment

For this study, C_L = $20,000 and a_L = .0455 acres are assumed. a_L can be further partitioned into

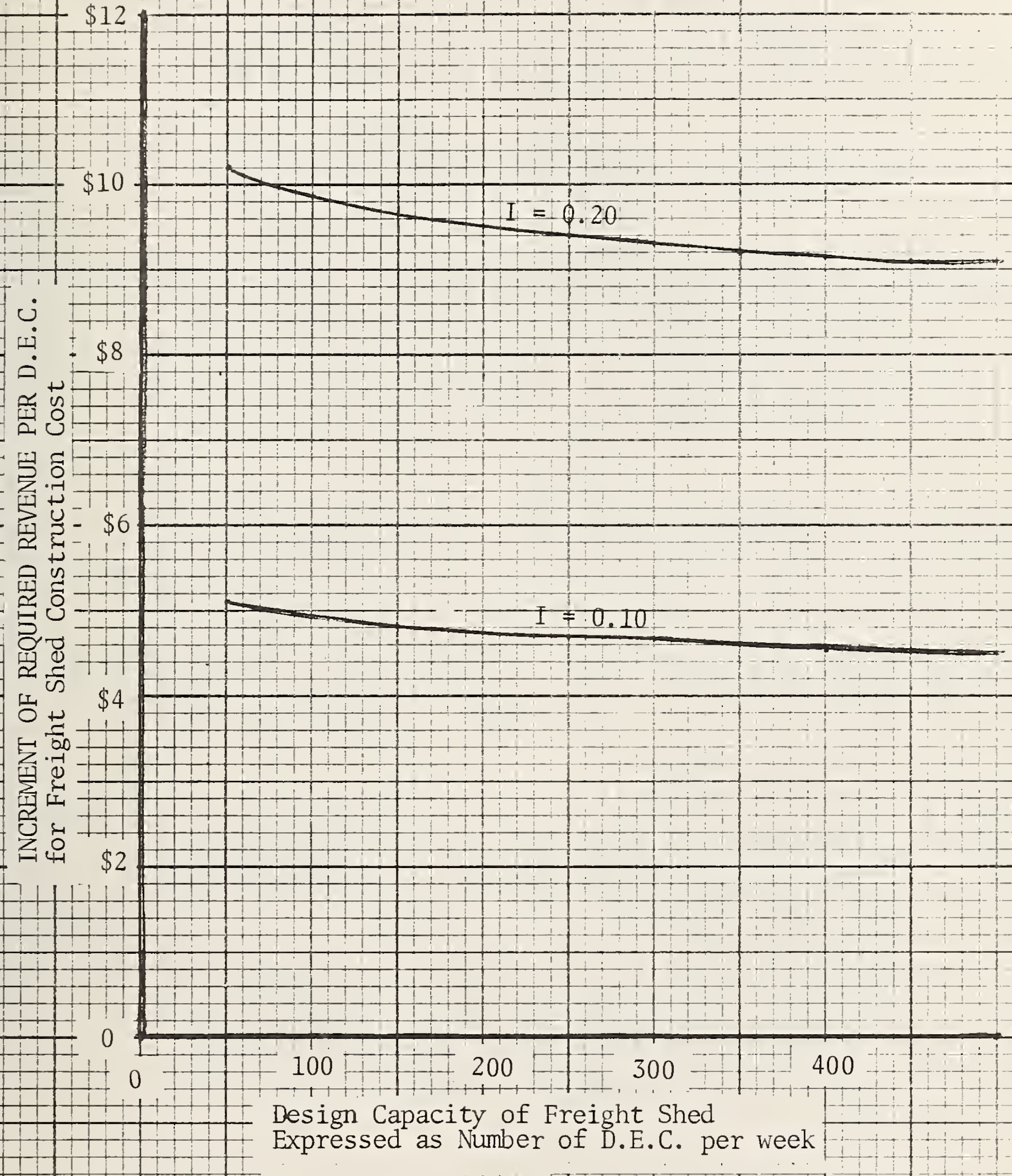

Figure 3. Shed

a_B = Land for building (.0098 acres)

a_L'= Land for receiving and discharging empty containers (.0082 acres)

a_L''= Land for temporary storage of inbound and outbound containers and freight (.0275 acres)

The cost relation [3.8] assumes that there is no economy of scale in the neighborhood of the capacity that might be built at any center.

Figure 4 shows the effect upon consolidation charge of varying land costs and two different assumed values of I. The land area (a_L) which was used is .0455 acres as shown in Table 2.

The selection of the value .0455 acres per container as used in Figure 4 is the result of substantial field observations of operating facilities and consideration of the engineering design characteristics of a functional facility. No single freight consolidation operation was accepted as a typical prototype for the cost model. The values suggested for a_L and its components (also used in what follows) represent what are believed to be adequate but not excessive space provisions for a consolidation center operating at the capacity (K), which refers to outbound freight but which includes space for unloading inbound freight. A different treatment of variable costs is discussed later in this chapter.

It should be clearly understood that the land requirements for a consolidation center are necessarily determined from consideration and evaluation of numerous engineering and economic factors which influence and govern the functional characteristics of a facility. Each site is unique and will require an individual evaluation. However, it is also reasonable to assume that land at nominal cost can be found in most areas in which containerization centers, as envisioned in this study, are concerned and that excessive costs can be avoided.

It should be noted that this part of the cost of stuffing a container at the reasonable figure of $20,000 per acre will mean a charge of about $2.00 (I=.10) per container. This figure should not vary appreciably unless extremes in land costs are considered.

Site Construction and Preparation

There is no generally applicable procedure which will result in a thoroughly defensible estimate of site construction and preparation costs. Each site may be expected to present a unique situation. It is reasonable to presume that a consolidation center site will be at or closely adjacent to both rail and highway main transportation arteries. Under such conditions, demolition and removal cost might be substantial, or on vacant flat ground, would be negligible. The costs discussed here include all demolition and removal costs lumped as a generalized expected cost. In addition, field surveys, engineering layouts, clearing, fencing, rough-and-fine grading, paving, and electric lighting are all consolidated into one item (shown on Table 2 as $11,000 per acre suggested value). Experience from numerous sources entered into the selection of a specific figure which, as stated, may vary widely from one place to another because of different and unpredictable site conditions, but the figure is thought to be a fair estimate of expected costs.

$$r_p = \{C_p' a_L + C_p'' (a_L' + a_L'')\} \frac{\varepsilon I}{52} \qquad [3.9]$$

where r_p = The portion of consolidation charge per container attributable to site construction and preparation.

C_p'= Cost per acre of demolition, removal, field surveys, fencing, and other elements ($11,000 per acre).

C_p''= Paving cost per acre ($29,000 per acre).

I = Return on investment for site construction and preparation.

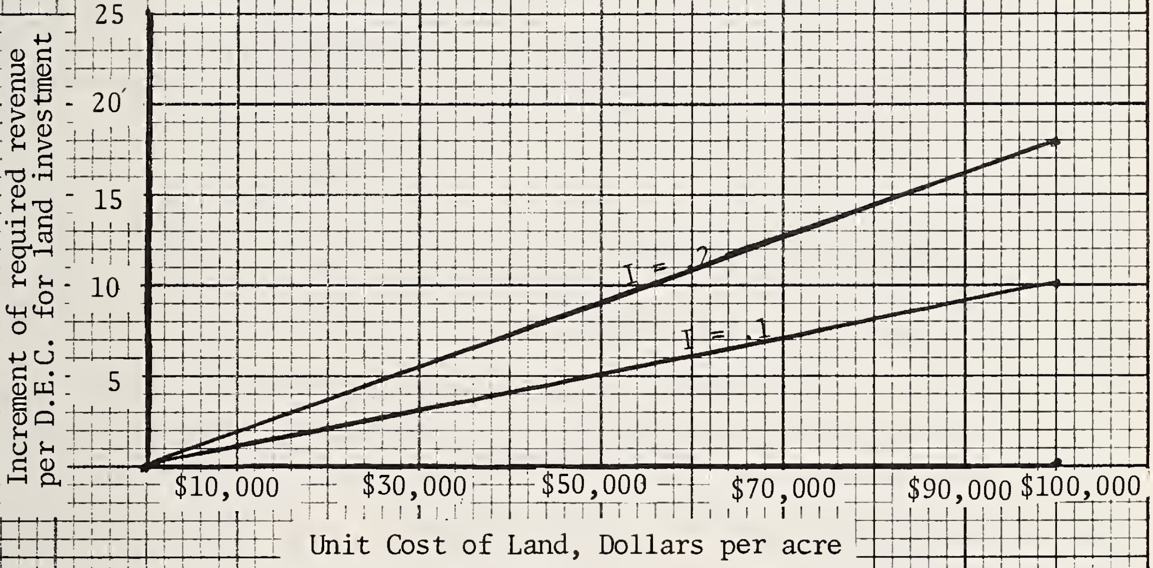

Figure 4. Cost of Land

The a's are acreage required per unit of capacity as defined previously. Spur trackage construction is not included because this is generally a negotiated affair with the railroad and the cost can vary widely; if operations are large it may cost the center nothing (see section on Equipment, this Chapter).

Figure 5 shows the effect upon service fees due to variations in the costs of site preparation and paving based upon the suggested values of Table 2. The cost of stuffing a container does not appear to be particularly sensitive to the costs involved in site preparatio A reasonable estimate for the cost of stuffing a container attributed to the cost of site preparation appears to be about \$3.00 (I = .10)

Maintenance, Repair, and Utilities Service

Operational costs of this nature are treated as fixed costs for purposes of our model. They may be expected to approximate the same annual total irrespective of the flow-through volume of freight handled. Paint will peel, concrete will spall, electric service lines wil break as the result of seasons and weather and not so much as the result of the usage of a facility. This might well not be true of a highly automated operation but for the type of operation described here it is a close approximation.

Some generalized statistical information is available for guidance with respect to such expected costs. Four major categories are recognized: paved areas and roadways, freight she maintenance and repair, electrical and utility services maintenance and repair, and electrical power costs and service. Utility costs for service for plumbing and heating other than electrical are not significant to the cost model. The service charge per container attribut able to maintenance, repair, and utility costs is:

$$r_M = \{(C_M' + C_M'' + C_M''')(a_L' + a_L'') + C_M^* \, a_B\}\frac{1}{52} \qquad [3.10]$$

where C_M' = Cost per acre for maintenance and repair services related to paved areas and roadways (\$125 per acre per year).

C_M'' = Cost per acre for maintenance and repair services related to electrical and utility services (\$141 per acre per year).

C_M''' = Cost per acre for electrical power (\$1750 per acre per year).

C_M^* = Cost per acre for maintenance and repair services related to freight shed (\$8625 per acre per year or \$0.198 per square foot per year).

One element of this cost equation is known to vary widely across the U. S. There is about 300 percent increase in the cost of electrical service when comparing Eugene, Oregon, with Belmont, Massachusetts.[9]

Figure 6 is plotted from Formula 3.10 to illustrate the effect of these costs upon consolidation center user fees. Parameter values used to develop Figure 6 are listed on Table 1 and identified in Table 2. A reasonable estimate of the cost ascribed to the stuffing of a container using the above suggested values is about \$3.00.

Equipment

There is a requirement for equipment to haul cargo in and adjacent to consolidation centers. This equipment is part of the overall cost. There are two categories, the forklifts used inside the centers, and yard equipment for moving containers within the confines of the parking lots for temporary storage awaiting line haulage.

Forklifts are almost universally used in the movement of cargo on pallets inside the centers. They are manufactured to carry various maximum loads, generally 2,000, 4,000, 6,000, or 8,000 pounds. One is used with each team of from one to three men loading a container. (Generally 2 laborers and one driver compose a team.)

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[8] Post Engineering Repairs and Utilities--Annual Summary of Operations, Office of the Chief of Engineers, Department of the Army, Fiscal Year 1966.

[9] Federal Power Commission, 1964. National Power Survey, Part 1, Table 10, p. 34.

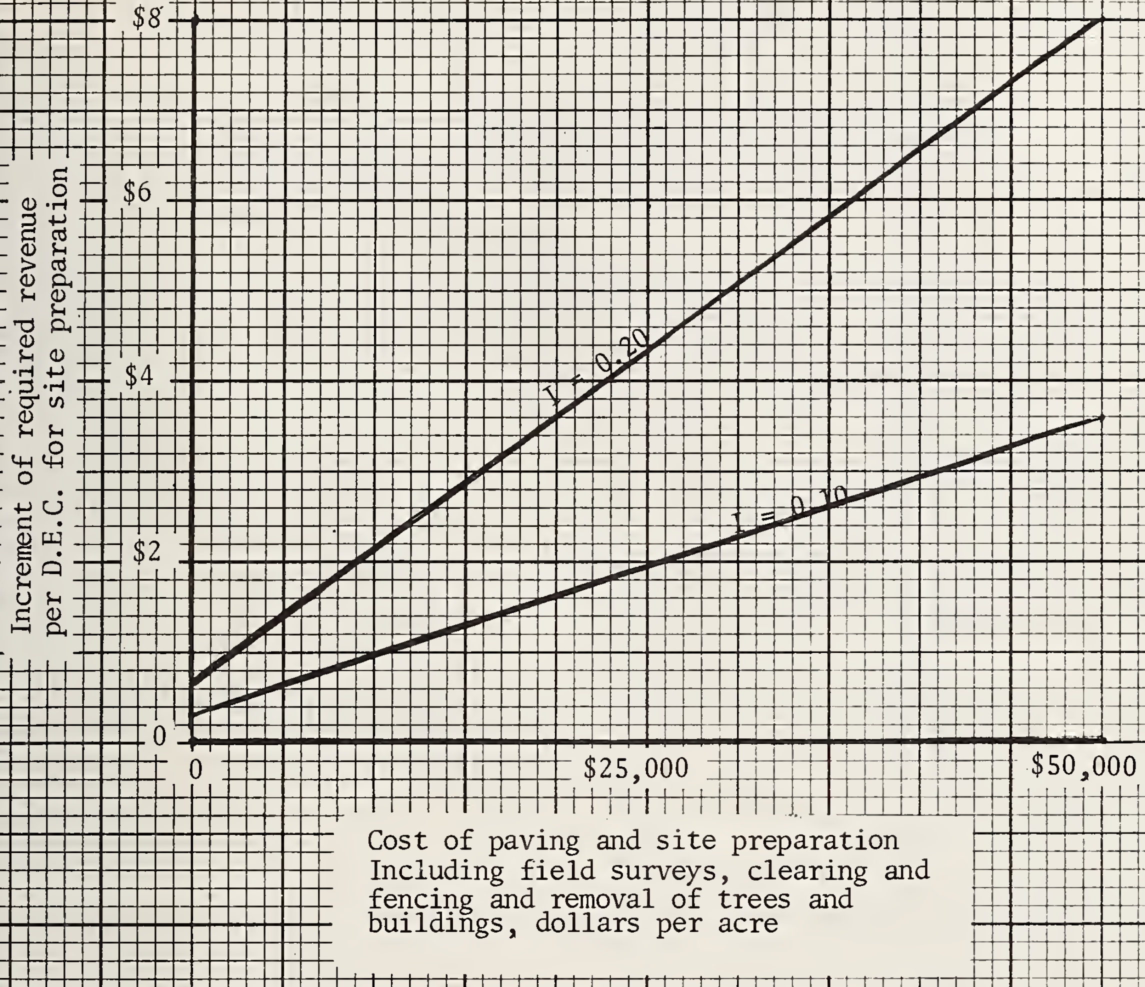

Figure 5. Site Preparation

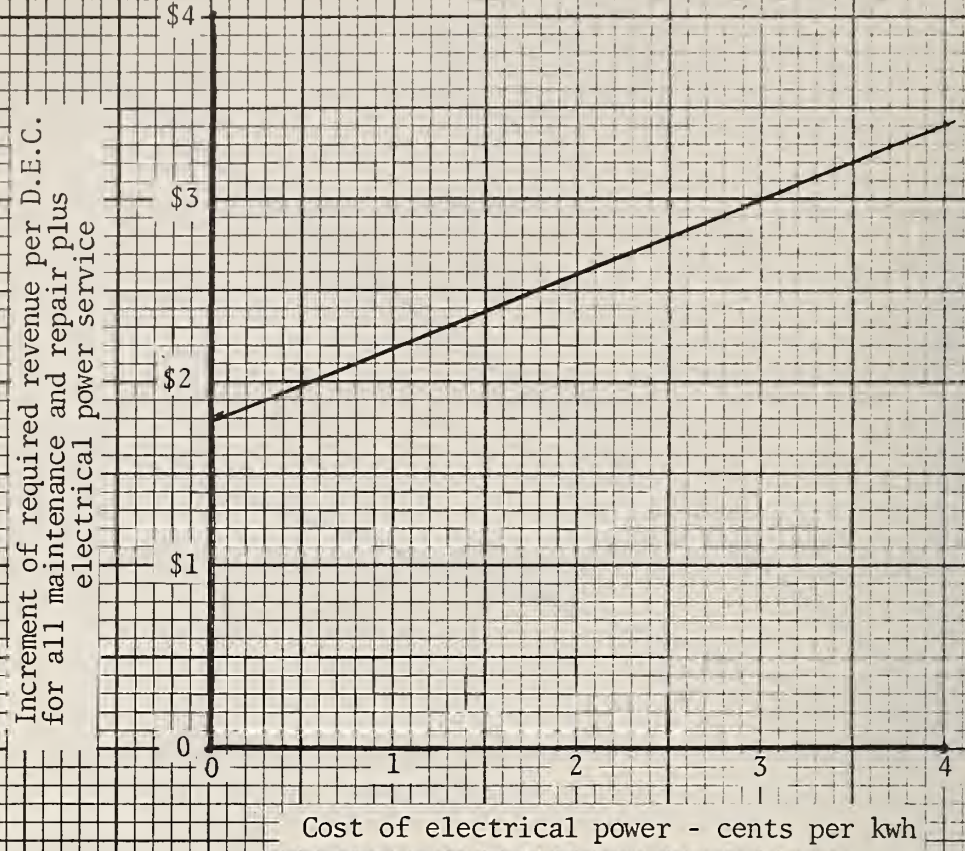

Figure 6. Maintenance Cost

Forklifts may be rented or purchased outright. They are considered to last about 5 years. If purchased they cost approximately:

2,000 lb. capacity	$5,300
4,000 " "	7,200
6,000	8,800
8,000	9,400

However, the trend appears to be toward leasing, and this study assumes leasing as a basis for cost estimates. The cost varies according to the environment in which they are used. If the environment is clean and floors are in good condition, they rent for about $100 a month for 170 hours running time, the equivalent of approximately one eight-hour shift per day for a month.

Thus in a clean operation in which 40 containers are stuffed per day (one shift), and the maximum use of forklifts are attained, ten gates are used and ten forklifts are required. This operation assumes an average of two hours for stuffing each container, or four containers per shift at each of the ten gates. One extra lift would be held in reserve for emergency. Thus monthly rental for the eleven lifts would be $1100.00 for stuffing 800 containers or $1.38 per container.

In contrast, under less favorable conditions at $147 per month, the cost would be $1617 for stuffing 800 containers or $2.02 per container. A suggested figure is $1.75 per container. (This figure does not include the operator whose pay is included under direct labor.)

In the containerization yard, containers are usually moved about on wheels using either over-the-road tractors or yard mules. In this study it is assumed that containers will be moved by a tractor or "yard mule", costing about $8,000, and that the containers will be kept on wheels.

It is also assumed here that two tractors can handle an operation of at least 40 containers per shift. This size operation is within bounds of a large operation, considered in this study to be about 40 containers per shift.

Tractors or yard mules cost about $3.00 an hour exclusive of the driver.[10] Again assuming close to full design operation, the cost per container in a 40-container-per-shift operation, would amount to $1.20 per container, plus $2.20 for the drivers.

Up to this point in this equipment discussion, it has been assumed that containers will be moved on the line-haul over roads by tractor-trailer. Many containers however will move by rail, particularly because of the very attractive rail rates.

There are five different container rates (plans) offered by the railroads. The selection of the rate to be used can affect the functions and the locations of the centers and the cost of stuffing containers as discussed below.

Railroads prefer not to handle containers beyond the rail yards.[11] They prefer that the shippers or freight forwarders assume the responsibility of hauling containers to and from the point of stuffing. However, they will do so, as in their plan II in which they offer a complete service between loading docks of shippers and receivers in separate terminal areas; but it is expensive and used generally for specialized operations.

The cheapest plans are Plan II½ and Plan III in which the costs are about the same. Both require the shipper to pick up the empty containers next to the unloading ramp at the railroad yard. The railroad loads and unloads from the flatcars without charge (today most railroads accept only containers on wheels because they do not have cranes for loading containers not on chasses. They move them up a ramp onto the flatcars using a yard mule.) These two plans are the most suitable to the consolidation concept developed in this study, and Plan II½ is selected for exercising the computer model. (See Chapter V on Rates.)

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[10] "Inland & Maritime Transportation of Unitized Cargo." NAS, NRC Publication 1135, p. 80.

[11] Conversations with railroad personnel.

However, these two plans, as stated, require that the shipper pick up the container from the railroad yard and transport it to the place of stuffing and then return it to the yard. Because a shipper bringing a less-than-carload lot to the center should not be responsible for hauling a container to the center nor for returning the container, stuffed with several shippers' goods, this function is a shuttle link that becomes the responsibility of the center.

There are alternative ways to handle this situation. First, the center could own, rent, or contract for tractors for the local hauling. At $3.00 an hour rental (the same as for the yard tractor costing about $8,000), an hour round trip to the railroad yard and a 40-containe per day operation would require 40 tractor hours or $120 per day or $3.00 per container, plus $5.50 for the driver.

Second, it is possible that a negotiated arrangement could be made with the railroad to run a spur to the center, particularly if the operation were of sufficient size, say, 40 twenty-foot containers (20 flatcars) per day. Under certain situations railroads would be willing to do this, but the possibility and cost depend on the particular situation. The few present similar operations end-stuff the containers on the cars through the regular shed doors, the track running parallel to the length of the shed. A few side loading containers are used and this may come to be a common technique as it appears highly practical for medium to large operations. Because of the greatly varying situations necessitating negotiated agreements and costs, no costing is given here, although it is probable in a large operation that it would be nominal.

A third solution, and probably the most practical and desirable, is the location of the center adjacent to the railroad yards. Railroads often have considerable land holdings adjacent to their yards and will at times sell or lease at a nominal price to a corporation, provided it appears that the business will be appreciable. There are many precedents for this type operation, for example, grain elevators. An operation of this kind would be highly appropriate for the unit train operation.

To summarize, the cost of stuffing a container attributed to forklifts varies according to the environment in which it is used but a representative figure is about $1.75 per container. A representative cost attributed to yard equipment for shuttling containers is about $1.20 per container plus $2.20 for the driver. If the center is located adjacent to a railroad yard, there should be no cost in moving containers to the flatcars, and if ordinary over-the-road tractor equipment is used on a one hour round trip to the yard from the center, a representative cost per container would be about $3.00 plus $5.50 for the drivers.

No costs are included for the case of the railroad spur to the center because of the negotiated costs which would vary widely according to the specific situation.

From a practical point of view, the location of consolidation centers at railroad yards, but also easily accessible to major highways, appears to be the most desirable arrangement and is recommended in this study.

Administration

Administrative overhead is considered a fixed cost dependent on the design capacity per week of the consolidation center. This cost does not vary, although the center may not be operating all times at its capacity. The cost of overhead, then, is dependent on the expected level of operations of the center. A sufficient number of clerks and administrators are employed to handle that amount of cargo.

It is most difficult to estimate this cost; it is a study in itself. It is complex because of the possible differences in the mode of operation of the center, for example, if the center owns its containers and must keep track and route them, the costs are greater. Furthermore, if this process is computerized, cost will be different. It is further complicated by determining the costs that must be allocated to central headquarters operation, if such centralization exists.

However, for planning, some estimates must be made. These were obtained by talking with administrative personnel in the business. They are admittedly rough because the administrators have difficulty themselves in breaking them down. They estimate that the cost is about $1300 per week for a small operation of say 10 containers per day (consisting of one administrator, one supervisor, two clerks, one secretary, and three checkers) or about $5000 for a large operation of 40 per day. This assumes that the costs are essentially linear with increasing capacity within a reasonable span of operation, say from 50 to 200 containers per week.

Then

$$r_A = \frac{C_A}{K} \qquad [3.11]$$

If C_A, cost of administration is $5000 and K, capacity per week, is 200, the r_A, a representative cost attributed per container, is $25.00.

Direct Labor

Reference is made to the second major design assumption of the cost model (page 12) where it was assumed that v (direct costs per container) is related to the number of containers stuffed during the operating year and included direct labor including benefits, payroll taxes, and insurance.

No attempt has been made to determine statistical averages which relate labor productivity to specific classes of freight. However, it takes much longer to load or unload certain commodities than others because of weight, packaging, and configuration. There is a wide range of variable costs dependent upon this factor. In one medium-sized operation observed during the study, 15,000 lbs. were loaded per manhour and in another only 2,000 per manhour.

Information of this nature can be obtained and used in the operation of the mathematical model at such time as the cost of statistical research into labor productivity is justified and commensurate with benefits to be derived from the more precise data.

However, in the representative operations observed, there were either two or three men working per container and it took from one to about six hours to load a container. In one case observed, for example, cartons of beer were being loaded by two men using a forklift in about three hours. Some companies work only an eight-hour day except during rush periods where they will work two eight-hour shifts; other companies work around the clock. The more hours worked, it is claimed, the less the requirement for container parking areas representing a saving which more than makes up for the overtime.

Because of the diverse nature, even within the same company, of the work and pay arrangements, it is difficult to pinpoint labor costs. Some companies work through a contracting company that not only hires but also furnishes forklifts and similar equipment and handles labor disputes. Other companies furnish their own equipment and do their own hiring. However, generally speaking, the costs are much as follows. ILA and other East Coast union labor cost about the same. The basic pay for loading is about $3.60 per hour with fringe benefits that increase it to approximately $5.50 per hour. The forklift operator receives about $.10 more in basic pay. Usually a team is composed of either one or two men plus a forklift operator. (A checker is also part of the team but is considered here under administration and supervision.)

Workers are generally paid whether they are loading or not, so in the calculations made here, as well as throughout the study, it is assumed that no more labor is hired than can be used and that operations are close to design capacity for an eight-hour shift. Furthermore, ILA or other union labor are assumed, although nonunion labor cost can be less, and can be employed in certain areas for certain kinds of work. (It should be noted that, according to companies visited during this study, where cheap labor is available, as in depressed areas, productivity is often low, to the detriment of the company and can more than offset the saving from the low cost labor.)

Under these conditions and assuming three hours to stuff a 40-foot container, the cost would be about $49.50 straight labor (see Figure 7).

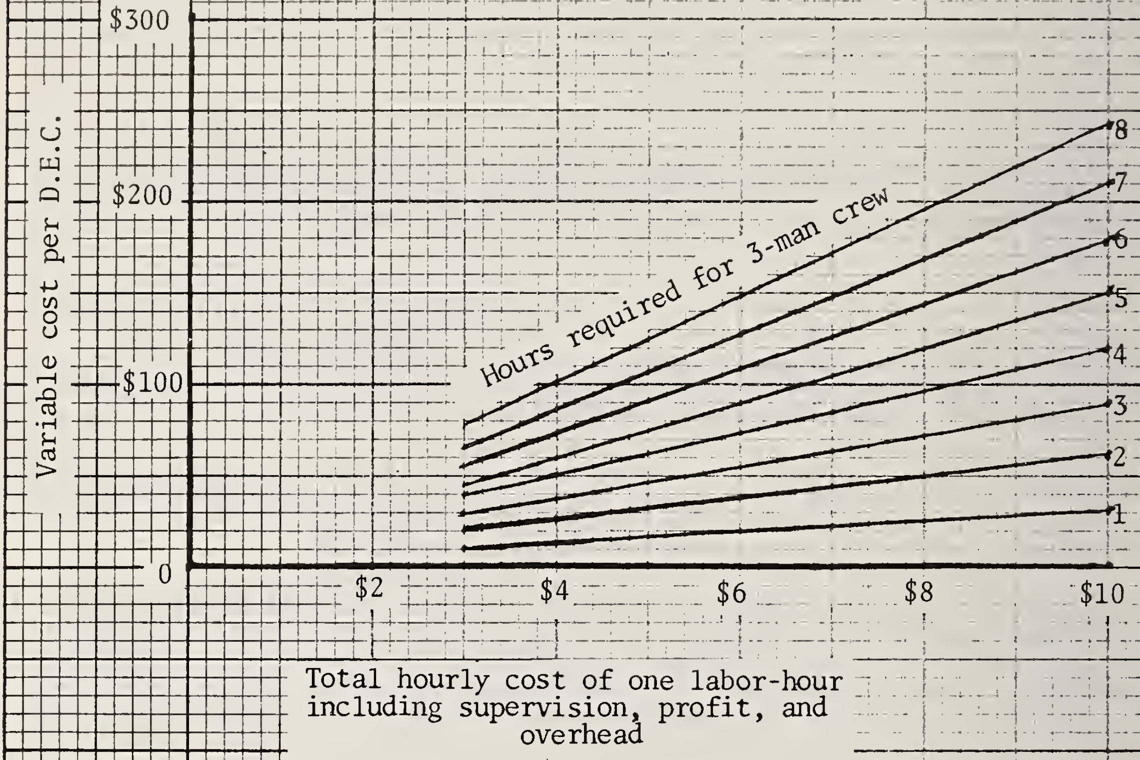

Figure 7. Labor Costs

The service charge in direct labor attributable to variable costs is expressed as:

$$v = mC_{\ell} \qquad [3.12]$$

where m = Number of manhours of direct labor required to stuff a container and handle the D.E.C. unit of freight (m = 9 hours).

C = Total costs allocated to each average manhour of labor (C = \$5.50).

It is obvious from Table 3 and from Figure 7 that the cost of labor and administration required to handle a D.E.C. unit of freight is the major contributing element of the revenue required for the use of the facility.

Table 3 summarizes representative costs for each of the seven major factors influencing the cost of stuffing containers and thus the determination of consolidation service fees.

Finally, it must be stressed again that the figures presented are approximations and can vary considerably from those actually found in any specific situation. They are, however, illustrative and are presented with this understanding.

Table 3. Summary of Representative Costs Attributed to Major Fixed and Variable Expenses Contributing to the "User Fee" for stuffing One 40'x8'x8' Container*

Building	$ 7.00	(I = .15)
Land	2.00	(I = .10)
Site Preparation	3.00	(I = .10)
Maintenance and Utilities	3.00	
Equipment - Forklifts	1.38	
Yard Mule	1.20	
Administration	25.00	
Direct Labor	51.70	
Total	$94.28**	

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*It is to be emphasized that these are representative costs and subject as noted in this chapter to variations in locations of centers, ownership, policies of operations, etc.

**In comparison with this estimate of $94.00, a well-known steamship company advertises that it will stuff a 24-foot container for $60, which when converted to a 40-foot container, is $100.

Although it has been recognized by those engaged in export-import transportation work, the study reemphasized the lack of origin-route-destination data on cargo. Quantitative solutions to important questions affecting our foreign trade cannot be determined without a concentrated effort to compile more exact and comprehensive knowledge of the subject.

This lack is felt by those who are planning and investing, not only in consolidation centers, but also in port authorities, new export businesses, transportation equipment, and by those government agencies engaged in the encouragement of these enterprises. Much of this planning is based on rules of thumb and inadequate data with important gaps. The available data inevitably lack the breadth to allow tracing goods from the point of manufacture to the ultimate destination. Bills of lading at a port show where goods were picked up, but the point of pickup may be a warehouse near the port, whereas the goods may have been manufactured in a city a thousand miles away.

The exporter, himself, while knowing the source, may not know the route or mode of transfer of his exports if, as is often the case, this part of the business is handled by a freight forwarder. If the routing is handled by a freight forwarder, he in turn may not know the ultimate source, nor the ultimate destination because the cargo may be handled overseas by a second freight forwarder. There is no central source or ready means of determining point of origin, routing, and destination.

Finally, in relation to lack of data on routing, the actual domestic port used must be a part of the data collected. The reason for a shipper or forwarder selecting a certain port in this country is often a matter of personal choice which is not subject to logical analysis. The shipper or freight forwarder may not choose the closest port or the least expensive route. The choice may be traditional or based on a personal relationship with the transporter, frequency of sailings, availability of special loading equipment, or anticipation of less congestion.

To attain systematic planning for efficient handling of cargo, as in this consolidation study, these factors should be known. The ultimate source, mode, and route of transportation, foreign destination, and seasonal variation in flow are all important factors in the location, size, numbers, function, and operation of consolidation centers.

No comprehensive, systematic nationwide studies have been conducted incorporating all or even a significant part of these data although a few limited surveys such as the Warrior[12] study have been made. The only study approaching the desired adequacy is the excellent survey conducted by the Delaware River Port Authority utilizing 1964 data, described below and used as the data source for the illustrative location of centers described in this paper.

The Delaware River Port Authority study consisted of 5100 interviews with exporters and/or importers tributary to the Philadelphia-Camden port. The interviewers were guided by a questionnaire completed during the interviews. About 3500 of the interviews gave pertinent data which were placed on magnetic tape.

The hinterland area covered by the study was all of Pennsylvania; western New York State; all of Delaware; eastern shore, Maryland; the West Virginia panhandle; all of Ohio; the northern two-thirds of Indiana and Illinois; southern Michigan; and southeast Wisconsin. See Figure 8.

For the southern New Jersey, Delaware, and Pennsylvania territory nominally within 100 miles of the port, all known shippers were surveyed. State lists of shippers were available for this area. Outside the immediate area peripheral to the port, sampling procedures were used which provided smaller samples as the distance from the port became greater.

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[12] The S.S. Warrior, An Analysis of an Export Transportation System from Shipper to Consignee (NAS-NRC Pub. 339, out of print).

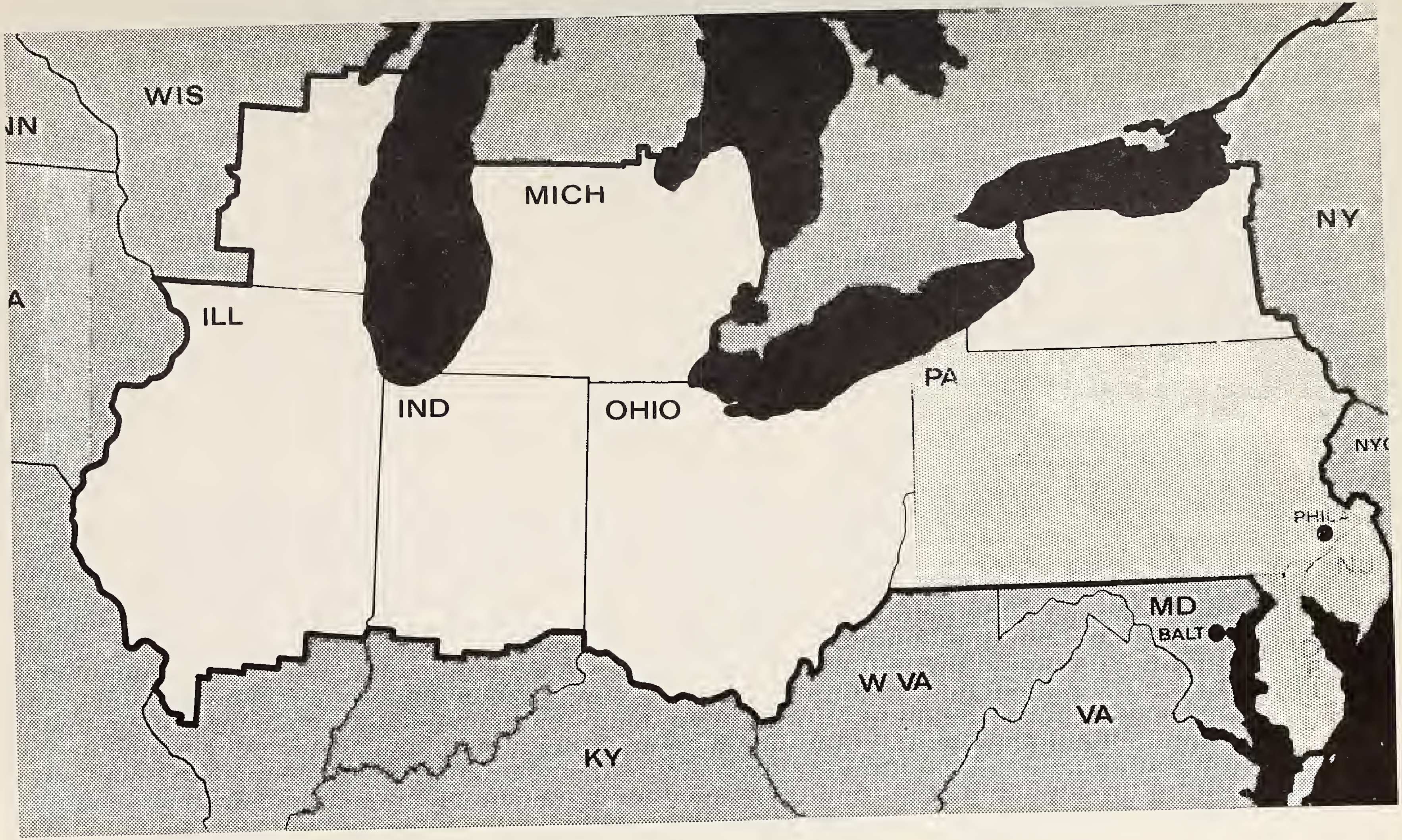

Figure 8. Area covered by study

In western Pennsylvania, the balance of Delaware, and Maryland, a complete census was taken of all firms that employed over 100 employees. In West Virginia, western New York, Ohio, Michigan, Indiana, Illinois, and Wisconsin, all firms with over 500 employees were interviewed.

With the smaller plants, i.e., with fewer employees, random samples were used. In western Pennsylvania, Delaware, and Maryland, 50% of those firms having 50 to 100 employees were sampled and 25% of those firms having up to 49 employees. In the other seven states, 25% of those firms having between 250-499 employees were sampled and 10% of those firms having 100 to 249 employees. In using the data, compensation was made for the sampling, i.e., from the sample design described above an item was counted 1, 2, 4, or 10 times.

An intermediate computer program was written to "clean up" the data from the Delaware River Port Authority study. All information irrelevant to the study was eliminated and miscellaneous editing was performed. Pertinent information was put onto a new tape in concise and orderly form to facilitate its use in later processing. Basically each "record" on the tape reported the commodity (by 3-digit code), location of the domestic shippers, the commodity classes shipped, the value of the commodity, the yearly tonnage shipped, the average frequency of shipment, domestic port through which it was exported, how it was packaged, the transportation mode used for shipping in this country and the sample factor, and the annual value of the shipments. Data for both exports and imports were processed.

The desired information not provided was the seasonal or other variations over time, and the foreign destination. Although the lack of this information was a distinct handicap, the study survey was by far the best that has been made in this field and cannot be criticized for these omissions; it was designed and used for specific purposes other than a consolidation study.

Certain information collected during the survey pertinent to this study was omitted during the editing, for example, that for shipments of bulk commodities. This elimination was based principally on the judgment of the Delaware River Port Authority. Also eliminated were those commodities considered unsuitable for containerizing. This elimination was made on the basis of consultation with the Maritime Administration, Port of New York Authority studies, and on clues in the source data such as the manner in which the goods were packaged. However, it must be understood in both of these elimination processes some arbitrary decisions were found necessary so that the work might proceed. A study of what is and what is not containerizable is a lengthy project in itself and beyond the bounds of this research. Furthermore, even after lengthy research, much is still left in doubt.

Also to simplify the process, the judgments were limited to "all" or "none" except in cases of considerable doubt when it was assumed that 50% could be containerized. See Tables 4 and 5 for a list of the commodities and how they were classed.

The location of each shipper was taken from the Port Authority data in latitude and longitude and then converted by a subroutine to corresponding X-Y coordinates. A straight line distance was then computed between the shipper location and the ports in question, and entered in the data bank. Actually, as a simplification, the distance-to-ports used were not those from the exact location of the shipper, but rather those from the center of the corresponding county. There were 409 counties involved. (In the Philadelphia area, because of the concentration of shippers, Philadelphia County was divided into 11 districts, each of which was considered as a county.) See Table 6 for a summary.

Data on costs of consolidation centers are treated in Chapter III, titled "Cost of Consolidation Centers" and data on rates and their application in Chapter V, titled "Rates."

Table 4
Philadelphia-Camden
Schedule S -- Exports 703
1962

005 Animals, edible
010 Meat, fresh frozen
013 Meat, & prods canned
017 Meat prod othrwis pre
*020 Animal oils fats, ed
033 Cond & evap milk
035 Dried milk
037 Cheese
039 Dairy products, nec
040 Fish, fresh frozen
043 Fish, canned
045 Fish prod othrwis pre
049 Shellfish & prods
050 Eggs & egg prods
055 Ed animal prods nec
060 Hides & skins, raw
065 Leather & mfgrs.
075 Furs & manufactures
080 Tallow, inedible
090 Animals, inedible
094 Shells, unmfgrd.
098 Animal prods ined nec
*100 Corn
*101 Rice
*102 Barley & rye
*103 Wheat
*104 Oats
107 Wheat flour & semol
108 Grain sorghums
*109 Flour & grains nec
*110 Animal feeds, nec
120 Veg, fresh, frozen
123 Vegetables, canned
127 Veg. preps, nec
130 Fruits, fresh froz.
132 Bananas
133 Fruits, dried evap.
135 Fruits, canned
136 Fruit juices
137 Fruits & preps
140 Nuts & preps
150 Veg oils fats, ed
160 Coffee
161 Cocoa beans shells
165 Tea
167 Table bev, mats nec
170 Spices
180 Sugar
185 Molasses ed. honey
190 Spirits, liquors wines
195 Bev, syrups, nec
200 Rubber, crude gum
201 Synthetic rubber
203 Rub scrap & rec.
205 Rubber tires & tubes
206 Aircraft tires tubes
207 Rubber mfgrs, nec
210 Naval store gum resin
220 Drugs, herbs, leaves, rts.
*231 Soybeans
232 Flaxseed
233 Copra
235 Oilseeds, nec
240 Veg. oils, ined. 50%
250 Veg. dyeing tan mats.
260 Seeds
280 Tobacco, unmfgrd.
285 Tobacco, mfgrd.
*290 Molasses, ined.
297 Veg. prod. ined. nec
300 Cotton, unmfgrd.
310 Cotton, semimfgrs.
320 Cotton mfgrs. cotton rags
324 Hemp, manila, abaca, unmfgrd.
326 Sisal, henequen, jute, unmfg,
328 Veg fibers, unmfgrd. nec
331 Burlap & jute baggings
335 Veg. fiber, semi & mfgr. nec
340 Wool, unmfgrd.
350 Wool, semi & mfgrd.
381 Man-made fibers & mfgrs.
390 Textile prod. nec
°400 Logs
°405 Posts, poles & piling
°408 Wood unmfgrd. nec
413 Lumber & shingles
416 Wood, cont. plywd. veneers
°417 Railroad ties
421 Wood mfgrs. nec
430 Cork & mfgrs.
°440 Pulpwood
441 Wood pulp
445 Paper base stocks, nec
450 Stand. newsprint paper
460 Paper board
475 Paper & prods. nec
*501 Anthracite coal
*502 Bituminous coal & lignite
*503 Coal & coke briquets
*504 Coke
*506 Motor fuel & gasoline
*508 Aviation motor fuels
*510 Gas oil distil fuel oil
*511 Petroleum, crude
*512 Jet fuels all types
*513 Kerosene

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*Bulk
°Not considered containerizable.
(% if given is % of commodity considered containerizable)

*514 Residual fuel oil bunker
516 Petroleum Asphalt
517 Lub oils & greases
518 Alphatic naptha
*520 Petroleum prods nec
521 Aviation lub oils
*522 Natural gasoline
523 Building cement 50 %
*526 Stone & mfgrs. nec
530 Glass & products
*540 Clays & earths
543 Brick & tile
547 Clay products, nec
*548 Gypsum or plaster rock
*550 Sulphur
*551 Limestone, crushed
553 Salt 50%
*554 Sand, gravel, crushed rock
*555 Nonmetal, min, mfgrs, nec
*600 Iron ore & concentrates
°601 Pig iron & spong iron
°602 Iron & steel scrap
603 Iron steel semifin. prods. 50%
°605 Iron steel castgs. forgs.
606 Tools & basic hardware
607 Hsld, kitchen, hosp, utens
608 Iron steel pipe tub tubing. 50%
609 Rld fin. steel mill prods 50%
°610 Bridges, prtble knckdwn nec
611 Metal mfgrs parts nec
*613 Manganese
*614 Chrome
615 Ores & metals, nec 50%
*617 Aluminum ores & scrap
618 Aluminum ore & semifab.
*620 Copper ore conc. scrap
622 Refined copper, crude
624 Copper, semifabricated
632 Copper base alloys sem.
*640 Lead ores & scrap
*642 Lead & allys, semifab.
*652 Nickel ore scrap semi.
*662 Tin ore, conc. scrap
*670 Zinc ore, conc, scrap
672 Zinc crd. semif. fms.
*682 Nonfer ore metls mfgrs.
690 Precious metls mfgrs.
701 Gen. elec. mach appar
705 Spec elec mach. appar
708 Radio comm trans receiv
710 Engines, turbines, nec 50%
°722 Constr. & mining mach.
731 Machine tools 50%
739 Ammo & rifle machines
740 Textile shoe machinery
742 Ind & off machines nec
°770 Agri machines & trac.
781 Auto, trk bus & trlrs.
°784 Military water craft
°785 Merchant vesls & pts
°786 Railway locos & pts
787 Auto, trk bus & trlrs pts
°788 Military auto trks bus
789 Mil auto trk bus trlr pts
°793 Aircraft comm & civ.
794 Aircraft & pts, nec 50%
796 Vehicles & parts, nec 50%
801 Coal-tar products
802 Benzol or benzene
806 Other coal-tar prods
807 Toluene or toluol
810 Med & phar preps
825 Sulphuric acid
826 Alcohol
827 Sodium hydroxide
828 Other indus chemicals
833 Military gases
837 Synthetic resins
844 Chemical specs, nec
845 Carbon black
847 Pigments, paints, varn
849 Ammonium sulfate
*851 Nitrogenous fert.
*852 Phosphate rock
854 Superphosphate
*855 Potash fert. matls.
859 Fertilizer & mats. 50%
862 Dynamite
863 Explosives
865 Soap & toilet preps
901 Gen. misc. commods. nec
903 Small arms
909 Spec. misc. commods. nec

Table 5
Philadelphia-Camden
Schedule T -- Imports 303
1964

005 Animals, edible
010 Meat prods., fresh
018 Meat prods., nec
020 Animal oils, fats, ed.
033 Cond. & evap. milk
035 Dried milk
037 Cheese
039 Dairy prods., nec
040 Fish, fresh, frozen
047 Fish, prods., nec
049 Shellfish & prods.
050 Eggs & egg prods.
055 Animal prods. ed., nec
060 Hides & skins, raw
065 Leather & mfgrs.
075 Furs & mfgrs.
090 Animals, ined.
094 Shells, unmfgrd.
095 Animal prods., ined. nec
*100 Corn
*101 Rice
*102 Barley & rye
*103 Wheat
*104 Oats
107 Wheat flour & semolina
*109 Flour grain preps. nec
*110 Animal feeds, nec
120 Veg. fresh, frozen
125 Veg. & preps., nec
130 Fruits & preps. fresh
132 Bananas, fresh
136 Fruit juices
138 Fruits & preps., nec
140 Nuts & preps.
150 Veg. oils, fats, ed.
160 Coffee, raw green
161 Cocoa beans & shells
165 Tea
167 Table bev. preps. nec
170 Spices
*180 Sugar
185 Syrups & prods., ed.
190 Spirits, liquors, wines
195 Bev., syrups, nec
200 Rubber, crude & gums
201 Synthetic rubbers
203 Rubber scrap
205 Rubber tires, tubes
207 Rubber mfgrs., nec
210 Naval stores, gums, resins
220 Drugs, herbs, leaves, roots, crude
*231 Soybeans
232 Flaxseed
233 Copra
234 Castor beans
236 Oil seeds, nec
240 Veg. oils, fats, waxes, inedible
250 Veg. dyeing, tanning matls.
260 Seeds except oilseeds
280 Tobacco, unmfgrd.
285 Tobacco, mfgrd.
*290 Molasses, ined.
297 Veg. prods., ined., nec
300 Cotton, unmfgrd.
310 Cotton, semimfgrd.
320 Cotton mfgrs. incl. rags
324 Hemp, manila, abaca, unmfgrd.
326 Sisal, henequen, jute, unmfgrd.
328 Veg. fibers, unmf., nec
331 Burlap & jute bagging
335 Veg. fibers, nec
340 Wool, unmfgrd.
350 Wool, nec
381 Man-made fibers & mfgrs.
390 Textile prods., nec
°400 Logs
°405 Posts, poles & piling
°408 Wood, unmfgrd., nec
413 Lumber & shingles
416 Wood contnrs., plywd., veneers
°417 Railroad ties
421 Wood mfgrs., nec
430 Cork & mfgrs.
°440 Pulpwood
441 Wood pulp
445 Paper base stock, nec
450 Std. newsprint paper
457 Paper prods. & mfgrs. nec
*501 Anthracite coal
*502 Bituminous coal, lignite
*503 Coal & coke briquets
*504 Coke incl. petroleum coke
*507 Gasoline & other motor fuels except jet fuel
*510 Gas, oil, distil. fuel oil
*511 Petroleum, crude
*512 Jet fuels, all types
*513 Kerosene
*514 Residual fuel oil
516 Petrl. asphalt & prods.
*519 Lub. oils & greases
*520 Petroleum prods., nec
523 Building cement
°526 Bldg., other stone, nec
530 Glass & glass prods.
*540 Clays & earths
543 Brick & Tile
547 Clay prods., nec
*548 Gypsum, plaster rock
*550 Sulphur

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*Bulk
°Not containerizable (or as noted 50% is estimated to be containerizable)

Table 5 Continued

*551 Limestone, crushed
°553 Salt 50%
*554 Sand, gravel, crushed rock
*555 Non-metallic minerals, mfgrs., nec
*600 Iron ore & concentrates
°601 Pig iron & sponge iron
°602 Iron & steel scrap
603 Iron & steel semifin. prods.
°605 Iron & steel castgs., forgs.
606 Tools & basic hardware
607 Hshld., kitchn., hosp. uten.
608 Iron & steel pipe tubing
609 Rld. fin. steel mill prods.
612 Metal mfgrs. & pts., nec
*613 Maganese
*614 Chrome
°615 Ferroalloys, nec 50%
*617 Bauxite crude alm. scrap
618 Alum. metals, semifab.
*620 Copper ore, conc. scrap
622 Ref. copper crude forms
624 Copper, semifab. forms
632 Copper base alloy semifab.
*640 Lead ores conc.
642 Lead alloys crude smfb.
*652 Nickel ore conc. scrap smfb.
*660 Tin ore conc. scrap
665 Tin metal crude smfb. fms.
*670 Zinc ore conc. scrap
672 Zinc crude semifab.
*682 Other nonferrous ores
690 Precious metals & mfgrs.
700 Elec. machy. & appar.
710 Engines, turbs., pts., nec 50%
730 Machine tools & parts
740 Textile, sewing, shoe machy., parts
745 Machinery & parts, nec
°770 Agricultural machy., pts.
°780 Autos, trucks, busses
782 Auto, truck, bus pts., acces.
°783 Merchant vessels & parts
°786 Railway locos. cars. parts
°790 Aircraft & parts 50%
°796 Vehicles & parts, nec 50%
802 Benzol or benzine
805 Coal-tar prods.
810 Med. & phar. preps.
825 Sulphuric acid
827 Sodium hydroxide
829 Industrial chemicals
848 Pigments, paints, varn.
849 Ammonium sulfate
*851 Nitrogenous fertilizers
*852 Phosphate rock
854 Superphosphate
*855 Potash fertilizer matls.
°859 Fertilizer matls., nec 50%
860 Misc. chemical prods.
900 Commods. misc., nec
920 Articles, U. S., returned

Table 6. Counties Used for Location of Shippers

Survey Area	Number of Counties or Districts Included and Assigned		Location
1	11	(7 districts in Phila. County)	Philadelphia
2	4	(all 67 counties in Pennsylvania)	Pennsylvania
3	8		Pennsylvania
4	9		Pennsylvania
5	10		Pennsylvania
6	9		Pennsylvania
7	10		Pennsylvania
8	12		Pennsylvania
9	8	(of 21 in New Jersey	New Jersey
10	3		Delaware
11	9	(of 24 in Maryland)	Maryland
12	17	(all 88 counties in Ohio)	Ohio, NE
13	45		Ohio, S
14	26		Ohio, NW
15	24	(of 62 in New York)	New York, W
16	4	(of 55 in West Virginia)	West Virginia, Panhandle
17	20	(of 83 in Michigan)	Michigan, SE
18	21		Michigan, SW
19	9	(of 92 in Indiana)	Indiana, SW
20	55		Indiana, Central
21	8	(of 102 in Illinois)	Illinois, Chicago
22	65		Illinois, Central
23	22	(of 71 in Wisconsin)	Wisconsin, SE
Total	409		

As previously outlined, a basic incentive for containerization is that commodities can be transported more cheaply when stuffed into "full" containers than when handled in small lots. According to where a shipper is located in relation to the port and an inland consolidation center, it may be economically advantageous for the shipper to use the consolidation center.

In the original concept, the model was to utilize actual transportation rates to be applied to the commodity shipment data. This would involve rail and truck rates, container and noncontainer, by each class or commodity.

As the study progressed and the rate structure was examined in more detail, it became clear that the situation was enormously complicated, requiring the amassing of container rates, less-than-container rates and local drayage rates between all possible pairs of places. The problem becomes even more complicated when it is realized that initially it is not known where the consolidation centers will be. Thus there could be literally an infinite number of points and an infinite number of each kind of rate for each commodity.

Even with the most complete present rate data, the following problems would present themselves:

1. An existing commodity rate may be artificial in that it is geared to a special condition or is induced by a competitive situation.

2. An absence of established rates may exist between hinterland points.

3. An absence of container rates may exist where no container service is yet in operation.

4. The effect of future competition on rates cannot be accurately anticipated.

5. Probable, unforeseeable, major changes may take place in the overall rate structure as the technology and the use of containerization spreads.

6. The actual building of a center will undoubtedly influence, in itself, the rate structure to and from the center.

Despite these problems it was deemed highly desirable to use some kind of real rate data to exercise the model. This was in lieu of a completely arbitrary R1 and R2 (one rate for all noncontainer traffic and another rate for all container traffic), which had been used in the operation of the model in its preliminary runs. That is, to debug the computer program, artificial shipment data were used (simulated with a random number generator). It was actually the ratio of R1 to R2 that was instrumental in determining center locations as the result of the operation of the model.

Because neither the time nor the resources were available to develop the complete rate details deemed desirable, compromises were necessary. Rail rates were obtained from one selected hinterland point (Cleveland) for container and noncontainer loads to four selected ports (New York, Philadelphia, Baltimore, and Norfolk) for an arbitrary, small number of commodity groups. All commodities were forced into these limited groups. The rates were then converted to a basis of equivalent rate per pound per computed mile. The computed mile is in effect the air line distance as computed by the computer program from the latitude and longitude data furnished.* Thus the program is able to take a prospective consolidation center at any point (x,y), compute the distance to the port and to a shipper in question, and apply an equivalent rate per mile to arrive at a rate to be evaluated by the selection

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*An alternative computation leading to the same total transportation cost could be made by applying a circuity factor which would yield a somewhat greater number of route miles and result in a correspondingly lower rate per route mile. (See Chapter VII on the Mathematical Model.)

criteria in the optimization program. Comparison of rates will dictate use of the particular consolidation center or will deny it, as determined for each shipper. The noncontainer rates were furnished with various weight breaks and with a minimum charge per shipment. The information about shipments from shippers provided insufficient information for a determination of a distribution of shipment sizes to significant destinations. Available data represent, for each shipper, annual export tonnage to domestic ports.

Also, container rates for one group of commodities were different according to whether the shipment was insured fully by the carrier or only to a certain amount. The shipment data gave no clue on this distinction. Furthermore, an agreement for a lower rate if the particular commodity in question is less than fully insured is an arbitrary arrangement that could not be predicted for the 5 to 10 year future and beyond. Again, a simple average of the two furnished rates was used.

The container rates used are for the trailer-on-flatcar, the so-called piggy-back mode of operation. In brief, five basic plans for piggy-backing trailers or containers are currently in general use and are included in interstate rail and truck rate structures. (See Table 7.) These plans may be modified to fit particular situations.

A strong assumption exists that consolidation centers will be located near rail lines. The ideal site would apparently be adjacent to a siding at the railroad yard. This location would obviously offer the maximum advantages of time and economy for all parties concerned.

Conversations with representatives of the railroads indicate that the railroads greatly prefer the shippers to do their own handling of containers away from the terminal yard. However, because of the pressures of competition and expediency, they will collect and deliver containers locally, but with an additional charge. The various rail rates generally reflect the amount and nature of the services performed. The cheapest rail rates are without the local drayage service.

Plan II½, or some modification thereof, seems to be the most appropriate of the various plans for pricing the line haul transportation of containers from prospective consolidation centers to the vicinity of the port or marshalling area. Plan II½, often referred to as "ramp to ramp" service, is the simplest of the rail rate plans in concept and provides the most ready yardstick for measuring direct line-haul charges unencumbered by the inclusion or exclusion of other cost factors.

Under Plan II½, the carrier furnishes the flatcars. The containers are purchased and maintained by a transportation company but are under control of the railroad during transit. It is assumed in this exercise that they are not owned by the consolidation center. This assumption eliminates the need for analyzing container inventory and capital costs. In the overall system analysis, the costs of the containers themselves would have a very slight effect of favoring shorter line-haul distance, that is, pulling the consolidation center toward the port. However, when this factor is part of the rate for transportation service provided by the carrier, such considerations are automatically taken care of.

This treatment is entirely appropriate because physical containers are the same kind of item in the total transportation picture as are railroad flatcars, locomotives, roadbeds, and bridges. It was not contemplated that the present study should be involved in consideration of these rudiments of the transportation service.

The list of pertinent Schedule S export commodities used in this study included 201 commodities. Having in mind Cleveland exports, because the representative rates were based on Cleveland rates, the commodities were assigned to seven arbitrary groups which were deemed to include the great majority of Cleveland exports. All other commodities were arbitrarily assigned to the designated group 8 for which the rate was taken as an average of the rates of the other seven groups. Table 8 defines the rate groups and shows, for less-than-truckload shipments, the derived equivalent rate by motor carrier per ton-mile* for each group. As for container rates by rail (freight-all-kinds), a flat figure per ton-mile was applied to all groups.

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*Tons are 2000 lb short tons.
Miles are 5280 ft statute miles.

Table 7. Plans for Piggyback Trailers/Containers

	Plan I	Plan II	Plan III	Plan IV	Plan V
Who owns or leases container	TRUCKER	RAILROAD	SHIPPER-OWNED OR SHIPPER-LEASED	SHIPPER	RAILROAD OR TRUCKER
Who owns or leases the rail car	RAILROAD	RAILROAD	RAILROAD	SHIPPER	RAILROAD
Who hauls container to and from rail terminal	TRUCKER	RAILROAD*	SHIPPER OR SHIPPER'S AGENT	SHIPPER OR SHIPPER'S AGENT	RAILROAD OR TRUCKER
Who loads and unloads container on railroad flatcar	RAILROAD	RAILROAD	RAILROAD	SHIPPER OR RAILROAD (When railroad loads and/or unloads, charge is assessed for that service.)	RAILROAD
How shipping charges are assessed	Trucker pays fixed charge per container based on weight and distance, regardless of commodity.	Assessed by railroad under published tariffs on file with ICC.	Assessed by railroad under published tariffs on file with ICC.	Fixed charge per car (whether or not containers are loaded).	Assessed by railroad or motor carrier under published tariffs on file with ICC
Additional information	Railroad has no direct contact with shipper. Trucker solicits shipper and bills freight at truck rates	Railroad deals with shipper, furnishes all equipment and provides pickup and delivery between shipper's plant and rail terminal.*		Railroad performs only line haul movement.	This plan is a joint motor common carrier-railroad operation. Class and commodity rates are carried in both motor common carrier and railroad tariffs on file with ICC.

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*While Plan II generally includes complete transportation service from shipper's dock at origin to consignee's dock at destination, arrangements may be made for shipper and/or consignee to perform all or part of the terminal service at origin and/or destination. These arrangements, frequently called "Modified Plan II" or Plan II½", provide patrons with maximum flexibility in using railroad trailer service.

Table 8. Truck LCL and Rail Container Rates

Group		LCL Rate in ¢/cwt/mile
1	Automotive parts	.758
2	Chemicals, n.o.i.	.693
3	Iron or steel articles	.418
4	Foodstuffs	.540
5	Petroleum products	.581
6	Alcoholic beverages	.892
7	Rubber articles	.628
8	All other commodities assigned to average	.644 (average)

Sample reading: 0.644¢/cwt/mi = \$0.00644/cwt/mi = 12.88¢/Short Ton-mile

Container rate f.a.k. = .135¢/cwt/mi

f.a.k. = freight all kinds

A supporting tabulation, Table 9, shows the source data for the uncontainerized less-than-truckload rates. Rates for the seven groups are shown by weight breaks from Cleveland to each of the four ports (New York, Philadelphia, Baltimore, and Norfolk). The rates were converted to a per-cwt-mile basis with the assumption that the majority of shipments would be in the 5000 lb. or over category and thus eligible to receive the most favorable rate for comparative purposes. Then the per-ton-mile rates to the four ports were averaged to provide a single per-ton-mile rate for each commodity group to the average port, this figure to be used by the computer program for application to each shipper. Table 10 shows this development.

Similarly, full-container rates were obtained for transport on flatcars from Cleveland to the four ports named above. These source rates were in two categories, single-container and two-container rate. The two-container rate was used in view of the anticipated volume of consolidation center output. Table 11 shows these figures. Again, these rates were converted to a per-ton-mile basis and averaged over the four ports to provide a single per-ton-mile rate to be applied by the computer program in the optimization routine.

Table 9. Truck LCL Rates from Cleveland to 4 Ports in Cents per Hundredweight

	Weight Breaks	Automotive Parts	Chemicals, NOI		Iron/Steel art., viz. Plate, sheet, Bars, Rods, etc.	Foodstuffs, canned or processed (not baked goods)	Petroleum Products (Lubricants)	Alcoholic Bev., e.g. whisky, wine (not beer)	Rubber Art. e.g. tires tubes, etc.	Miles		
			w/o Rel.	Released to 50¢/1b						Truck	Rail	Air
New York		(A)	(A)	(A)	(B)	(A)	(A)	(A)	(A)	481	562	407
	>1 M	392	392	324	196	282	305	456	324			
	1 M - 2 M	358	358	296	187	257	277	416	296			
	2 M - 5 M	331	331	274	183	238	257	384	274			
	5 M & over	293	293	240	158	209	224	343	240			
Philadelphia		(C)	(C)	(C)	(D)	(C)	(C)	(C)	(C)	415	490	360
	>1 M	361	361	303	176	265	282	422	303			
	1 M - 2 M	330	330	276	172	243	257	385	276			
	2 M - 5 M	306	306	256	167	224	238	357	256			
	5 M & over	270	270	223	143	192	207	318	223			
Baltimore		(E)	(E)	(E)	(F)	(E)	(E)	(E)	(E)	344	444	309
	>1 M	348	348	292	177	254	272	404	292			
	1 M - 2 M	317	317	267	169	231	249	370	267			
	2 M - 5 M	293	293	247	165	214	230	341	247			
	5 M & over	258	258	215	141	183	198	303	215			
Norfolk		(G)	(G)	(G)	(H)	(G)	(G)	(G)	(G)	517	636	426
	>1 M	413	413	345	254	298	321	481	345			
	1 M - 2 M	377	377	315	231	272	293	439	315			
	2 M - 5 M	349	349	291	214	252	271	406	291			
	5 M & over	310	310	257	184	221	238	368	257			

NOTE A: Minimum charge per shipment $7.17
B: " " " " 6.79
C: " " " " 6.99
D: " " " " 6.66
E: " " " " 6.99
F: " " " " 6.66
G: " " " " 7.34
H; Same as (G)

M = 1000 1b.

Table 10. Truck LCL Rates from Cleveland to Four Ports in Cents per Hundredweight per Computed Mile

	Auto-motive Parts	Chemicals N.O.I.			Iron Steel Art.	Food-stuffs	Petro-leum Prod.	Alco-holic Bev.	Rubber Art.	Overall Avg.
		w/o Rel.	Released to 50¢/lb	Avg.						
New York	.720	.720	.589	.654	.388	.514	.550	.842	.589	.608
Philadelphia	.750	.750	.620	.685	.397	.533	.575	.883	.620	.635
Baltimore	.835	.835	.696	.766	.456	.592	.641	.981	.696	.710
Norfolk	.728	.728	.603	.666	.432	.519	.559	.864	.603	.624
Average	.758			.693	.418	.540	.581	.892	.628	.644

Sample Reading: 0.644¢/cwt/mi = $0.00644/cwt/mi = 12.88¢/Short Ton-mile

Table 11. Rail Container Rates (Plan II$\frac{1}{2}$), Cleveland to Four Ports

	1-Trailer Rate (Min.38500 lb)	2-Trailer Rate (Min.73500 lb)	Computed Miles	Equiv. Rate in cents/cwt per Computed Mile
New York	$232.78	$380.07	407	.127¢
Philadelphia	219.39	358.44	360	.135¢
Baltimore	206.00	336.81	309	.148¢
Norfolk	249.26	405.82	426	.130¢
Average				.135¢

Sample conversion:

$$\frac{\overset{\$/\text{load}}{380.07} \times \overset{\#/\text{cwt}}{100} \times \overset{¢/\$}{100}}{\underset{\#/\text{load}}{73500} \times \underset{\text{miles}}{407}} = .127\ ¢/\text{cwt/mi}$$

It is assumed that all containers will be filled to the minimum weight. (In some cases container might be only 2/3 full.) Excess weight above minimum is extension of same rate on per pound basis.

Chapter VI. MAP ANALYSIS OF DEMAND DATA

While the demand data described in Chapter IV were being converted from their original magnetic tape form to edited form, and the mathematical location model and its computer implementation were in progress, a careful manual map analysis of these data (export only) was also undertaken. Requiring about 5 man-weeks, this analysis proved quite worthwhile, providing insights and visual comprehensibility not so easily gained by the machine approach.

Such an exercise yields convenient representations of the amount of containerizable export cargo by day and by geographical distribution of its origins. It displays clustering around industrial areas, and location in relation to trunk transportation routes (which can be superimposed on the maps). Such information gives a rather reliable basis for inferring first impressions concerning number, size, and location of consolidation centers. These rough calculations have value in checking initial outputs of the computerized models for general reasonableness, and in estimating the number and kinds of machine runs to be made. They are particularly important in providing preliminary insight as to the number of consolidation centers appropriate for the export flow; earlier speculation on this point had ranged from some hundreds to ten or fifteen. Finally, such a manual-map operation provides a more intimate familiarity with the data than could be attained by the "impersonal" handling associated with the (computing) machine work.

To keep the different phases of the research in proper perspective, however, it must be remembered that inferences about the proper configuration of the consolidation center system, drawn from inspection of the results of the map analysis, involve the transport rate structure and user fees only in an intuitive, implicit way. Explicit quantitative consideration of these factors involves a mass of calculations calling for the electronic computer, and it is just such calculations which comprise the computerized model discussed in Chapter VII.

The actual conduct of the map exercise involved the following steps:

1. The total annual number of pounds of export commodities for each exporter in each city or town was totaled. (Only annual figures and only weight, not cubic, are available.)

2. The total annual weight in pounds was divided by 250 working days per year (one 8-hour shift per day) to determine the average pounds exported per export location, generally a town, per day.

3. For a measure of capacity more easily visualized, the average pounds were then divided by 51,000 lbs.,* a reasonable figure for a "fully" loaded 40' x 8' x 8' container to determine the average number of container loads stuffed per day in one 8-hour shift. (The figure 51,000 lbs. is now known to be higher than the average outbound load to Europe today. For six months (July-December, 1967) the average was 16,755 lbs. per 20-foot container or 33,510 lbs. for a 40' x 8' x 8' container.[13])

4. The number of container loads per day per shift were then compared with representative numbers stuffed by small, medium, and large consolidation centers visited during the study. A center stuffing 10 per day (one shift) for this study was arbitrarily considered a small operation, 20 per day a medium, and 40 per day a large operation. (Much larger operations apparently are feasible and three shifts rather than one are common.)

5. The average number of containers stuffed during one shift per day for the cities and towns involved were then plotted on a map.

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* It should be noted that 51,000 lbs. are used here, not the 48,000 lbs. in Chapter III. This discrepancy should cause no problems in that both are used only as measures of amount of cargo not the actual number of containers stuffed.

[13] "North Atlantic Container Statistics Report for Six Months Period Ending Dec. 31, 1967," O.M.P. MARAD.

The limitations of this procedure are readily apparent;

1. Each container was considered stuffed "full" regardless of the ultimate destination of the cargo. In other words, some containers were in effect loaded with cargo for more than one ultimate distribution point. (This same limitation is true of the machine solution as ultimate origin-destination figures are not available, nor are they known. See Chapter IV titled "Data: Source, Adequacy, and Preparation.")

2. The figures are averages computed from annual exports. They do not reflect daily or seasonal fluctuations.

3. The number of exporters that would actually use a consolidation center, particularly from towns averaging less than one container load per day, is not known because transportation rates to the centers and from the centers to the ports are not introduced in the manual computations. The time limitations imposed on the study precluded the very time consuming manual handling of rates.

The analysis was done by states, principally to test progressively the value of the work in relation to allotted time and the resume is by state (and then summed for a total picture). This procedure is not without other merit in that state borders can affect transportation rates and thus specific locations, for example, the Gary, Indiana-Chicago, Illinois complex.

After elimination of bulk commodities and those not adapted to containerization, and after application of the sample blow-up factor (see Chapter IV), the data yielded the results shown in tabular form (see tables 12, 13, 14, and 15) and are represented pictorially on a map (Figure 9).

Table 12

Illinois

Summary, Towns Shipping One or More Containers per Day Average, One 8-hour Shift

City	Average lbs.per day	Containers per Day (40x8x8 @ 51000 lbs)
Chicago	1,837,170	36.0
Granite City	80,000	1.6
Aurora	73,200	1.4
Rockford	72,400	1.4
Galesburg	64,400	1.3
Evanston	54,700	1.1
	2,181,870	42.8
All other towns 56 towns less than one container each per day.	921,180	18.1

Table 13

Indiana

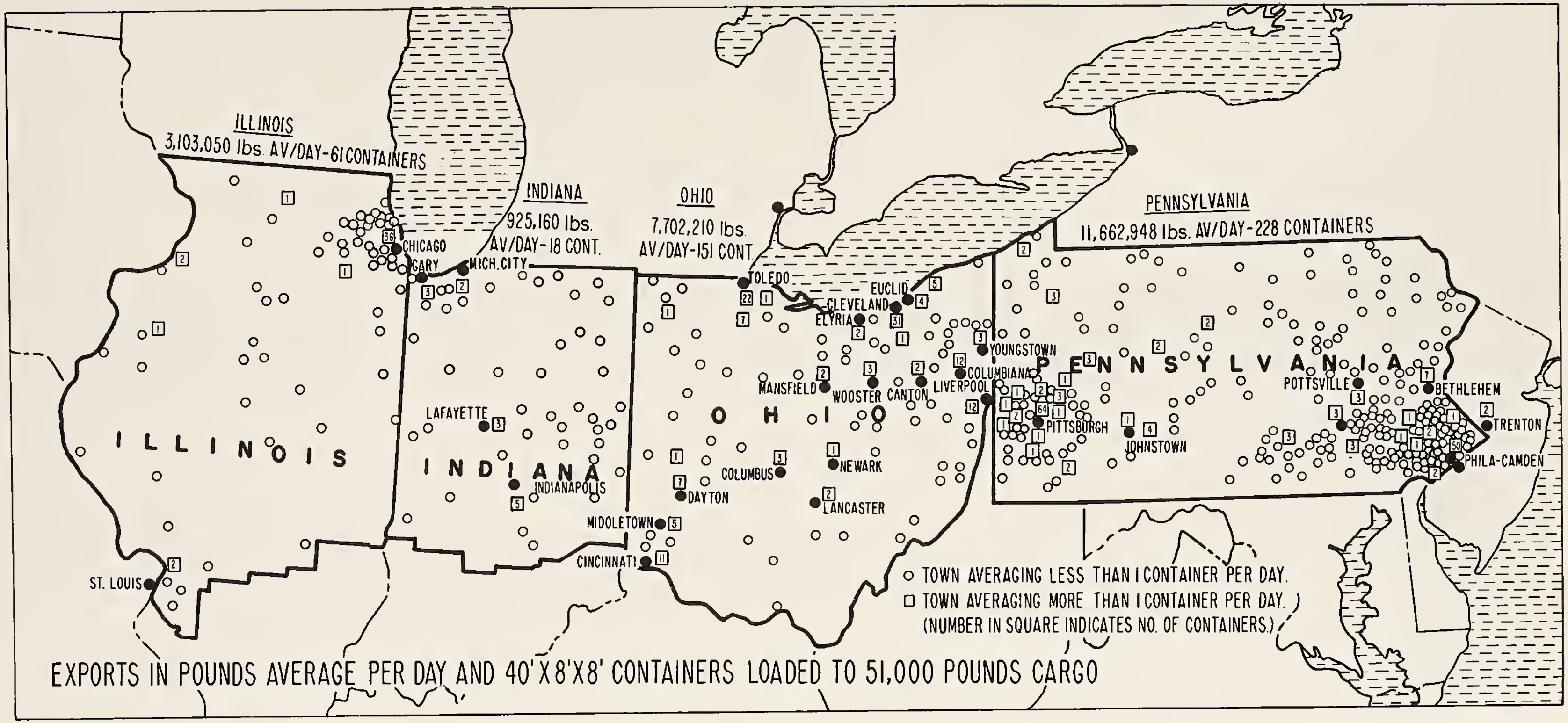
Summary, Towns Shipping one or more Containers per Day Average, one 8-hour Shift.

City	Average lbs.per day	Containers per Day (40x8x8 @ 51000 lbs)
Indianapolis	231,000	4.5
East Chicago	128,000	2.5
Lafayette	129,000	2.5
Michigan City	93,200	1.8
	581,200	11.3
All other towns (43 towns, less than one container each per day)	343,960	6.7

Table 14

Ohio (plus two towns in West Virginia)
Summary, Towns Shipping One or More Containers per Day Average, One 8-hour Shift

City	Average lbs.per day	Containers per Day (40x8x8 @ 51000 lbs)
Cleveland	1,562,336	31
Toledo	1,133,663	22
E. Liverpool	640,000	13
Columbiana	576,000	11
Cincinnati	548,007	11
Woodville.	400,000	8
Dayton	365,443	7
Middletown	220,000	4
Wickliffe	216,666	4
Painesville	191,428	4
Fairport	171,428	3
Youngstown	164,477	3
Columbus	151,770	3
Canton	115,827	2
Mansfield	110,881	2
Defiance	96,000	2
Salem	88,644	2
Lancaster	80,240	2
Newark	70,088	1
Williston	66,560	1
Akron	61,312	1
Troy	52,822	1
22 towns	7,083,592	138 containers
All other towns (71 towns, less than one container each per day)	618,618	10 containers



EXPORTS IN POUNDS AVERAGE PER DAY AND 40'X8'X8' CONTAINERS LOADED TO 51,000 POUNDS CARGO

FIGURE 9

Table 15
Pennsylvania
Summary, Towns Shipping One or More Containers per Day Average, One 8-hour Shift

City	Average lbs.per day	Containers per Day (40x8x8 @ 51000 lbs)
Pittsburgh	3,230,000	63.4
Philadelphia (not environs)	2,500,000	49.0
Johnstown	600,000	11.8
Bethelehem	371,000	7.3
Franklin	215,000	4.2
Lebanon	186,800	3.7
Steelton	180,000	3.5
Templeton	180,000	3.5
New Kensington	174,700	3.5
Cressona	174,500	3.4
Oil City	158,500	3.1
Norristown	136,400	2.7
Lockhaven	133,000	2.6
Connellsville	132,000	2.6
Ambridge	120,000	2.4
Fairless Hills	120,000	2.4
Lancaster	116,000	2.3
Erie	115,400	2.3
E. Pittsburgh	100,000	2.0
Ford City	95,200	1.9
Bridgeport	94,000	1.8
Hempfield Township	92,700	1.8
Borough Township	92,500	1.8
Clearfield	92,200	1.8
Beaver	92,000	1.8
Beaver Falls	80,600	1.6
Clairton	80,000	1.6
Pottstown	72,000	1.4
Glassport	66,800	1.3
York	60,600	1.2
Leetsdale	60,000	1.2
Morrisville	56,000	1.1
32 towns	9,891,900	196.0
All other towns	1,771,048	32.7

(296 towns, less than one container each per day)

Summary - Map Analysis

According to the Delaware River Port Authority data, a daily average of 23,393,368 pounds of cargo is exported from the four states of Illinois, Indiana, Ohio, and Pennsylvania amounting to 458 40-foot containers with a preponderance, 11,662,948 pounds, from Pennsylvania. As expected, the greatest amount is generated from the industrial areas and most of that originates near the coastal areas--in this case, Philadelphia-Camden. At the time of data collection, 1964, very little of this cargo was containerized.

The major exporting areas by states were:

Illinois

Chicago (very little originates in the rest of the state)

Indiana

Gary-Michigan City
Indianapolis
Lafayette
(Small shipments originate throughout the state)

Ohio

Toledo
Cleveland
Youngstown-Salem-E.Liverpool
Cincinnati-Middletown-Dayton
(Small amounts originate over the whole state)

Pennsylvania

Pittsburgh-Johnstown
Harrisburg-Lebanon-Pottsville
Philadelphia and environs, bounded by Lancaster, Trenton, Bethlehem
(Very little originates in Central Pennsylvania)

According to operational personnel interviewed, it appears that about ten 40'x8'x8' containers per eight-hour day must be stuffed to support a center, and under the constraints detailed above, it would appear from inspection on the basis of the distribution and volume of these data that a center would be located in each of the following areas.

1. Chicago-Gary
2. Indianapolis-Lafayette
3. Toledo-Gibsonburg
4. Cleveland-Elyria-Euclid-Fairport
5. Cincinnati-Middletown-Dayton
6. Columbus-Lancaster-Newark
7. Mansfield-Wooster-Canton
8. East Liverpool-Columbiana-Youngstown
9. Pittsburgh and environs
10. Johnstown
11. Harrisburg-Lebanon-Pottsville-Lancaster
12. Philadelphia-Camden-Trenton and environs

Trunk lines of major railroads interlace these states, and in many cases the cities of each area are located along a major trunk line. This is to be expected. It is assumed, and recommended, that the center or centers in each area be located at a railroad yard (also with major road access) to take advantage of the low rail container rates (See Chapter V on Rates, Chapter III, Cost of Consolidation Centers). A seaport area such as Philadelphia-Camden, in which a center would be located, would possibly be an exception. A fine-tuned study of each area would decide upon the exact location of each center.

A striking result of this graphical plotting exercise is the adaptability of distribution of exporters to unit train operation. Examples are the Penn Central (Northern route) from Chicago through Toledo, Cleveland, Buffalo, etc., to New York City and Boston; the Penn Central (Southern route) from St. Louis through Indianapolis, Dayton, Columbus, Pittsburgh, Harrisburg, and on to the Philadelphia area; the Baltimore & Ohio-Chesapeake & Ohio from St. Louis, Cincinnati, to Washington, Baltimore, and connecting to Philadelphia and New York; and Norfolk & Western from Cincinnati through to Norfolk. The distribution of export centers could hardly be more admirably suited to unit train operation. A study of train routes in relation to volume and domestic ports of embarkation should certainly be made to expedite this type of long haul transportation.

In this chapter and the next two, the following work will be described.

(a) The mathematical model and technique developed for determining good locations and capacities for cargo consolidation centers,

(b) The computer program written to implement a version of (a) appropriate to the data at hand, and

(c) The results of applying this program in illustrative exercises based on the data delineated in Chapters III, IV, and V.

Chapters VII and VIII present some technical mathematical and computer-oriented material which goes more naturally with the exposition here than it would elsewhere in the report, but which can be omitted without loss of management-level understanding. Additional technical material is presented in the Appendices.

A comment is in order at the outset, concerning the level of precision to be attributed to the model's outputs (results). This level must reflect both the accuracy of input data, and the level of aggregation of the model as a whole. Hence, the center locations obtained by the computer must be regarded as specifying only in general terms where each center is to be placed. Precise identification of the best site in each computer-specified locale would require a fine-grained study of that locale with regard to land cost variations, access points to particularly attractive transportation facilities, and the like. Such "fine tuning" appears unnecessary and indeed inappropriate at the system-wide planning level, lying more in the province of the interests that undertake the initiation and operation of a particular center in a particular locale.

Scope of Model

Any system is imbedded in one or more larger ones, so that isolating it as the focus of a study is almost certain to involve some distasteful excisions. To place the model to be described in better perspective, it may be useful to list some of the requirements for an analysis that would do full justice to the ramifications of, and influences on, consolidation center location and sizing:

(a) Models to predict U.S. exports and imports, year by year over the planning period, by season, (containerizable) commodity class, shipper location, U.S. port of departure, and overseas port of delivery.

(b) Predictions of which U.S. and overseas ports will develop capability for efficient handling of containerized cargo, and how this capability will develop over time.

(c) Models for the growth, deployment, and itineraries of the container-carrying vessel fleet.

(d) Computer representations of relevant U.S. transportation networks, and their evolution over the design period.

(e) Computer representation of (land) transportation rates for the various commodity classes, and for containers.

(f) A model describing redistribution of empty containers may prove necessary.

Clearly these items are not independent; one would need a "supermodel" to combine them and their interactions with models of shipper* behavior to determine volumes and frequencies of LCL shipments to centers, all as affected by the locations and sizing of those centers.

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*"Shipper and/or importer" is actually meant, but the terminology here will generally be export-oriented.

Some of the items concern policy questions not within the scope of this study. Some involve (and are currently involving) massive research projects of their own. Some involve masses of data too enormous to permit the rapid computer manipulations needed to accomplish the location of centers under a variety of scenarios. One might possibly formulate such a battery of models (though this may not even be reasonable as a goal), but securing the necessary information to validate them would be yet another monumental task.

Appreciation of this situation led to two basic modeling decisions. First, no attempt would be made to venture beyond the project's assigned scope into the area of demand forecasting. Therefore, the pattern over time of demand for containerization-consolidation services--by shipper location, commodity class, U.S. port of departure, and overseas port of delivery--would be regarded as an input to the model. The computer can be used to work out the consequences of many alternative assumptions about these critical inputs.

Second, it was judged essential to avoid developing and manipulating computer representations of the nation's complex and extensive freight transportation networks, or of its transportation rate structures (which have become notorious for their lack of description-easing regularities). As regards the first factor, the matter of centers' proximity to appropriate transportation facilities is left to the type of "fine-tuning" mentioned earlier. (A discussion of locating centers in a network is given in Appendix A.) As for transportation rates, they are treated on a "distance" basis in the manner described below. Such a basis of course represents a considerable simplification of reality, but one which appears reasonable in view of the apparent absence of alternatives permitting rapid calculations.

Decision Criteria

In determining "good" or "optimal" locations and capacities for centers on a generalized cost basis, it is necessary to try to be explicit as to how "goodness" is to be measured. The factors should be those which depend sensitively on location and sizing (the topics for decision), a test which appears to rule out such items as pilferage reduction. The obvious remaining indicators of system performance are the resources of time and money expended in the movement of the cargo from shipper to center to port of departure, so defined as to include the costs of initiating and operating the centers themselves. Our objective, therefore, is to minimize a total generalized cost which has both dollar and time components.

Even this, however, is not quite an accurate description of the situation. The difficulty is that "cost minimization" suggests the existence of a single decision-maker with the authority to determine which center each shipper should use so as to minimize the system-wide cost, even if some shippers are thereby not assigned to the centers they would most prefer. (Such models are briefly discussed in Appendix B.) It did not seem appropriate to base the model on such a concept. On the contrary, it seemed best to adhere as closely as possible to the following:

Assignment Principle: In an "optimal" solution, the assignment of shippers to centers should be compatible with the centers' locations in that, given these locations, no shipper should prefer a center other than the one to which the solution assigns him.

This desirable attribute of a "solution" has associated with it the following logical difficulty: whether or not a shipper prefers a particular center depends on the generalized cost of his using that center, which may in turn depend on the extent to which other shippers patronize it. (With insufficient patronage, processing costs may be excessively high, or there may be unacceptable delay awaiting arrival of sufficient additional cargo for the same destination to permit stuffing a reasonably full container, or perhaps excessive cost to the center--ultimately passed on to users--in sending out insufficiently full containers to avoid such delays.) (Such factors are discussed in Chapter II, "Function of Consolidation Centers.")

This point will be discussed again later. The next observation here is that the "solution" concept should reflect not only the assignment principle, but also the complementary

Location Principle: In an "optimal" solution, the location of each center C should be compatible with the locations and relative importance (as customers) of the shippers assigned to C by the solution, in the sense of minimizing the total cost associated with this patronage.

In applying this principle, the set of shippers assigned to C is regarded as temporarily "known." Thus the patronage pattern at C is known, and hence the total processing cost at C (assuming a design properly matched to the patronage pattern) is known.* Thus the principle calls for locating C so as to minimize the total transportation cost involved in the use of C by its (known) clientele. Note that this is to hold for each center.

Figure of Merit

A full evaluation of the costs and benefits stemming from any particular configuration of consolidation centers would of course be a complex matter, especially if it is to deal carefully with how these costs and benefits are distributed among the various interests involved (shippers, transport operators, center operators, etc.). For the present study it was considered important, despite the risk of over-simplification, to be able to attach a single numerical "rating" or "score" to each situation studied.

The total generalized cost to the shipping community,

$$\sum_S [T_{C(S)}(S) + P_{C(S)}(S)] \qquad [7.0]$$

in the later notation of [7.33], is a natural candidate for such a figure of merit (more precisely, "demerit"). Its appropriateness is evidenced by the fact that, very roughly speaking, the main thrust of the model's calculations is to reduce this quantity, taking into account the interactions among the independent decision-makers involved.

However, [7.0] as it stands has some deficiencies as a figure of merit. Its absolute magnitude will not be too reliable, in view of data uncertainties and the unlikeliness that all relevant cost elements can be incorporated in the numerical work. It does not directly make vivid the relative merits of different configurations of centers, or the effects of changes in the values of the model's parameters, or the general benefits from a system of inland centers versus not having such centers.

Accordingly, the figure of merit chosen is not the total generalized cost itself but rather a modification of it, "normalized" to facilitate comparative evaluations of the types mentioned above. This normalized version is obtained by dividing the total generalized cost, for the configuration of centers to be evaluated, by the corresponding cost for the case in which no inland consolidation centers are available. Thus the figure of merit, to be interpreted on a "high values bad, low values good" basis, will normally assume values between 0 and 1; the former extreme represents an unrealistic ideal state, while the latter extreme represents the (presumably unrealistic) possibility of no economic advantage from inland centers.

To convey a more accurate impression of the meaning of this scoring function, it may be useful to discuss the scope of the word "total" in the phrase "total generalized cost." Here one must distinguish between the conceptual mathematical model (the subject of the present chapter), and its present computer implementation as restricted by data considerations and other practical factors. What follows refers to the more comprehensive of the two, the mathematical model.

First, "total" shares with "generalized" the intent of reflecting "time costs" as well as "money costs." Second, it suggests a comprehensive aggregation over all relevant cargo, as indicated by the summation over all shippers S (exporters and importers) in [7.0]. This objective is itself less than "total," since direct and indirect effects on sectors of society outside the shipping community are not considered. Moreover even the limited objective is not fully attained in the present model concept, which focuses on LCL shipments to the exclusion of full-container shipments which do not require a center's consolidation services but might benefit from its marshalling activities and associated unit train rates to ports.

The term "total" also suggests a treatment of the total movement of cargo, from origin to destination. How far the present analysis falls short of this ideal is schematized in Figure 10 (drawn for simplicity for exports only). In evaluating the numerator of the

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* Possible effects of location (independent of patronage) upon processing costs, and a way to handle them within the approach described here, are discussed later.

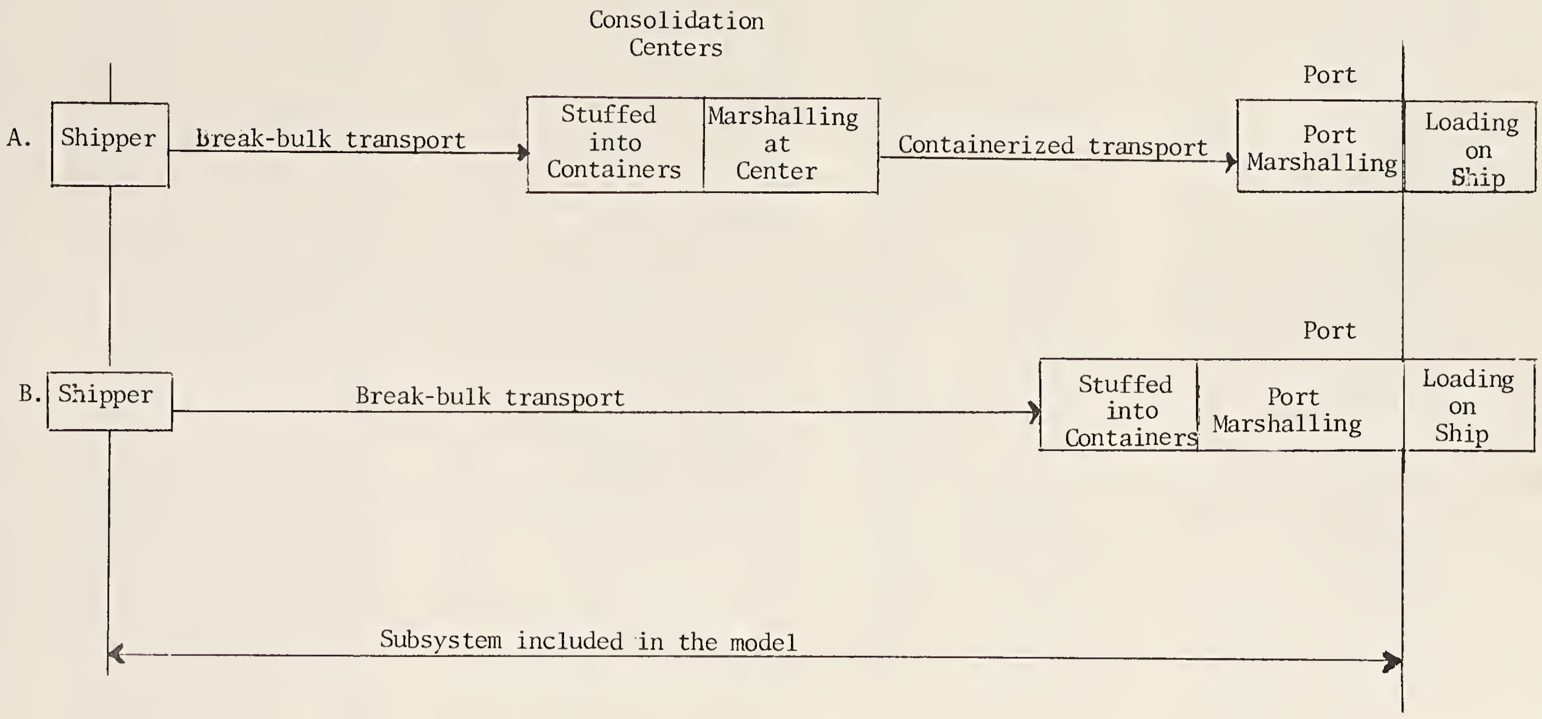

Figure 10. Subsystem included in the model

figure of merit, i.e., the generalized cost for a system of inland centers, alternatives A and B are both available to shippers, who choose between them (and for A, select a particular center) on a generalized-cost basis. In evaluating the denominator, only alternative B is regarded as available.

Two technical points regarding Figure 10 may be noted here. For alternative A, the cost associated with the marshalling function will not be separated out from the general center processing cost described in Chapter III; thus economies of scale from full-container shippers using the marshalling facilities are not explicitly considered, nor do such shippers (in the present model) influence the locations or the number of centers. And for alternative B, the delay to LCL cargo -- in waiting for the accumulation at the port of enough cargo to be merged with it -- is treated as negligible. To the extent that this assumption is optimistic, the present calculation of the figure of merit is biased against the use of inland centers.

Sketch of Solution Process

The two principles (Assignment and Location) correspond, respectively, to the two types of computational step whose alternation constitutes the solution technique. The process is to be applied for each of a number of values of

n = number of centers

in order to determine which number of centers is "best" in the sense of yielding lowest cost after optimization of center locations. More precisely, n is the number of centers to be located; there may also be centers at some or all ports whose presence there is taken as "given" by the model.

The solution method for a particular value of n begins with an initial or "trial" set of locations for the n centers. This may represent a "best guess" by the user of the model, or a set of random choices made by the computer, or perhaps a systematic choice such as placing the n centers initially at the n "heaviest" shippers.

With these locations regarded as fixed, an assignment step (whose nature is discussed later) is performed to produce an assignment of shippers to centers, which satisfies the assignment principle. Next, this just-produced assignment is regarded as fixed, i.e., the patrons of each center are regarded as known, and so a location step (whose nature is also discussed below) is performed to select, for each center, a new location which satisfies the location principle. These center locations are then taken as fixed, another assignment step is performed, and so on, with assignment steps and location steps alternating. The process is terminated when the center locations--and thus the assignments as well--have "settled down" (to within a prescribed tolerance level), indicating a situation in which (as desired) both principles are satisfied to a good approximation; no shift of any center's location will significantly reduce the cost to that center's users, and no shipper can achieve a non-negligible saving by transferring his patronage to some center other than the one to which he is assigned. The final assignment determines each center's patronage, and therefore its appropriate sizing as well.

The preceding description was oversimplified, for ease of initial exposition. First, assigning of shippers to centers was spoken of as if such an assignment were irrevocable. Such an approach might be plausible if we are optimizing for some single designated "target year," or if a shipper must sign some sort of long-term commitment to avail himself of a center's services. But it seems more likely that a shipper will be relatively free to shift his patronage from one center to another during the planning period, and may well have an incentive to do so (perhaps because the second center has just become operational, or because exports from its vicinity to certain destinations have grown great enough to permit rapid filling of containers). It turns out, however, that the difficulties raised by this time-varying ("dynamic") aspect of the problem are predominantly technical--i.e., neither practically nor conceptually of great significance. Therefore discussion of their treatment is deferred to the end of this chapter. Until then the language used will be as if the "single target year" objective were in force.

Second, the natural concept of a "shipper," as an institutional entity with a specific location, requires refinement. Such a "shipper" may have exports in several commodity classes, involving different freight rates, on their way to a center in uncontainerized form.

He may have exports to different foreign distribution points, or for the same foreign distribution point but via different U.S. ports of departure; such cargo categories are not co-containerizable,* can suffer different delays at a center in waiting for enough of a containerful to build up, and may in fact be sent to different centers. This makes it convenient to fractionate ordinary "shippers" into subentities, each characterized by

(a) a single location (essentially),

(b) a single commodity class, and

(c) a single combination of overseas distribution point and U.S. port-of-departure.

It is these subentities which will be referred to as "shippers" from now on. Shippers which agree in items (b) and (c), and have near y locations, can of course be aggregated if one is willing to accept the resulting loss in accuracy for the sake of the resulting reduction in the number of "shippers," and thus in the computational labor required. Note that while the above-mentioned fractionation is conceptually natural and analytically convenient, it requires more care in interpreting data and/or in designing questionnaires for the purpose of collecting data on the demand for consolidation services.

Possible Difficulties

The practical reader, observing the requirement that this alternating sequence of assignment steps and location steps "settle down," may well question whether the results of this process will in fact settle down, and (more important) whether they will do so in few enough steps to make the solution method realistically feasible. A mathematical analysis of such questions is often possible, but generally difficult. It is preferred here to adopt a "try it and see" attitude, referring the reader to our actual computational experience (Chapter IX) for affirmative evidence of computational feasibility. Such evidence does not provide the same certitude as would a theoretical proof, that rapid settling-down will occur for input-data combinations other than those specifically tested, but it does produce a rather powerful intuitive conviction that this will be the case for "reasonable" inputs to the model.

A potentially more dangerous difficulty arises from quite another source. It is a well-known property, of "successive improvement" solution methods for the optimal location of more than one facility, that the final pattern of locations can depend on the initial locations used. In particular, an unfortunate choice of initial guesses on suitable locations for centers might yield a final configuration which--though obeying both the assignment and location principles--corresponded to a total generalized cost distinctly greater than the true minimum possible. For proper assurance of achieving the true minimum (generalized cost) or close to it, it was considered essential to explore the possible seriousness of this problem. These explorations are reported later (Chapter IX), but a general recommendation is a common-sense precaution that the method be applied using several initial sets of trial locations (for each n).

Treatment of Transportation Costs

Our main aim in the balance of this chapter is to describe, in turn, the location step and the assignment step whose alternation constitutes the model's solution method. Recall that the location step is used in the following setting: For each center C, the set of shippers using C can for the moment be regarded as "given," and the problem is to locate C so as to minimize the total (generalized) transportation cost associated with this usage.

A description of the location step must therefore begin with a discussion of how transportation costs (in dollars and time) are represented in the model. This is the subject of the present section. Considerable use of mathematical notation becomes necessary at this point.

Consider any particular shipper S among the users of center C. Our previous "fractionation" of shippers ensures that S's usage of the center involves a definite commodity class, k_S, and a definite U.S. port of departure, P_S. S's generalized transportation costs can be broken up (at least in principle) into two parts, relating respectively to shipper-to-center movement, and to center-to-port movement. Symbolically,

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*They may in fact be co-containerizable, depending on break-bulk policies, if their ports of arrival are on a common sequence of ports-of-call from their (common) U.S. port of departure. What follows can readily be modified to reflect this possibility.

$$T_C(S) = T(S,C) + T(C,P_S), \qquad [7.1]$$

where

$T_C(S)$ = transportation cost to shipper S, using center C,

$T(S,C)$ = (generalized) cost of S-to-C movement,

$T(C,P_S)$ = (generalized) cost of C-to-P_S movement.

Note that $T(S,C)$ and $T(C,P_S)$ have both dollar and time components, and that each depends on the location of C, which is to be chosen in the location step to minimize

$$\textstyle\sum_S T_C(S),$$

the sum being taken over all users S of C.

Dollar Costs

We must now develop formulas for $T(S,C)$ and $T(C,P_S)$, beginning with the former. The dollar component of this generalized cost is taken as a product

$$M_m(S)\ d_m(S,C), \qquad [7.2]$$

where

$d_m(S,C)$ = distance (in miles) from S to C by mode m

$M_m(S)$ = money cost (per mile) for hauling S's material by mode m.

The underlying assumption that the cost of a movement should be proportional to distance moved is in line with general regulatory policy.

The second factor in [7.2] will now be decomposed further into

$$d_m(S,C) = c_m d(S,C) \qquad [7.3]$$

where

c_m = "circuity factor" for mode m,

$d(S,C)$ = short-line distance from S to C.

For example, ICC line-haul unit costs for rail have been based[14] on the value $c_m = 1.13$. In principle, c_m could be varied from territory to territory to reflect regional differences in the circuity of the transportation network. At any rate, combining [7.2] and [7.3] gives the formula

$$M_m(S)c_m d(S,C) \qquad [7.4]$$

for the dollar component of the generalized shipper-to-center transportation cost $T(S,C)$.

There is still the matter of how a numerical value for $M_m(S)$ is to be found. One possibility is

$$M_m(S) = R_m(k_S)A(S) \qquad [7.5]$$

where

$R_m(k)$ = rate of mode m, per commodity unit per mile, for moving commodity k,

$A(S)$ = amount shipped by S. [7.6]

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[14] Interstate Commerce Commission, Rail Carload Unit Costs by Territories for the Year 1963, Statement No. 5-65, March 1965.

However, the implied assumption of a constant rate $R_m(k)$, independent of shipment size, may not be tenable.* If the available data provide the distribution of S's shipments, among the size categories corresponding to different rate levels, then $M_m(S)$ can be found by expanding the right-hand side of [7.5] to a sum of products, one per size category. But if only the frequency of S's shipments is known, then about the only thing to do is to calculate S's average shipment size, and use the rate corresponding to that size in [7.5].

Time Costs

Now turn to the time component of the generalized cost T(S,C). It is represented mathematically in the form

$$V_{k(S)}\ I_{k(S)}\ A(S)t_m(S,C) \qquad [7.7]$$

where for typographical simplicity we have written k(S) for k_S, and where

V_k = average (per unit) value of commodity k,

I_k = inventory-type carrying cost factor for commodity k,

$t_m(S,C)$ = average time in transit from S to C by mode m,

while A(S) is as in [7.6]. Here I_k is intended as a representative charge (in percent per unit time) for interest, risk, obsolescence, and the like.

Next the factor $t_m(S,C)$ in [7.7] will be split further, in a way based on similar material in Meyer et al.[15] With $d_m(S,C)$ as in [7.2], the formula reads

$$t_m(S,C) = \frac{d_m(S,C)}{\sigma_m} + \alpha_m \frac{d_m(S,C)}{\sigma_m} + \beta_m \frac{d_m(S,C)}{\ell_m} + \gamma_m \frac{d_m(S,C)}{s_m} + \delta_m, \qquad [7.8]$$

where

σ_m = mean speed (velocity) of mode m,

α_m = fraction of "road" time spent on sidings,

β_m = mean time per interchange on mode m,

ℓ_m = mean distance between interchanges,

γ_m = mean time per switching operation for mode m (primarily for rail),

s_m = mean distance between switching points,

δ_m = mean delay in origin and destination terminals of mode m.

Note the assumption that delay in transit depends primarily on movement length (as an indicator of number of "opportunities" for delay) rather than shipment size. Note also that [7.8] can be rewritten as

$$t_m(S,C) = a_m d(S,C) + \delta_m, \qquad [7.9]$$

where $a_m = (1 + \alpha_m)/\sigma_m + \beta_m/\ell_m + \gamma_m/s_m$.

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*In such cases the fractionation of shippers may have to be modified. Customers might wish to take advantage of reduced "bulk rates" to ship cargo, for several destinations, to the center together. On the other hand, there would then be an extra operation, of sorting out the shipment by destination, to be performed at the center.

[15] J. R. Meyer et al, The Economics of Competition in the Transportation Industries, Harvard U. Press, 1964. See Chapter VII. Pp. 188-196 describe an application of this type of approach, including empirically based estimates of the parameters in [7.8] for rail and truck. On page 192, a value of 10% per year for I_k is suggested as reasonable.

Generalized Costs

We now combine [7.3], [7.4], [7.7], and [7.9] into a formula for the generalized transportation cost T(S,C). The result is a linear function of distance d(S,C),

$$T(S,C) = K_1(S)\ d(S,C) + K_2(S), \qquad [7.10]$$

where the coefficients K_1 and K_2 do not depend on distance and are given by

$$K_1(S) = c_m\{M_m(S) + V_{k(S)}\ A(S) I_{k(S)} a_m\} \qquad [7.11]$$

$$K_2(S) = V_{k(S)}\ I_{k(S)}\ A(S)\delta_m. \qquad [7.12]$$

If [7.5] can be used, then [7.11] can be converted to

$$K_1(S) = A(S) c_m\{R_m[k(S)] + V_{k(S)} I_{k(S)} a_m\}. \qquad [7.13]$$

The preceding paragraph does not quite tell the whole story. The coefficients K_1 and K_2 depend on the mode m, and the choice of mode may well depend on the distance d(S,C) to be covered. We should really have a mode-dependent notation $K_{1m}(S)$ and $K_{2m}(S)$, and should complicate [7.10] to indicate use of that mode which yields the lowest generalized cost:

$$T(S,C) = \min_m\{K_{1m}(S)\ d(S,C) + K_{2m}(S)\}. \qquad [7.14]$$

The method used to carry out the location step, however, depends critically on the linearity of [7.10]. This method is itself an iterative one, initialized with a starting guess as to a good location for C. It is suggested that this starting guess be used to determine a best mode m(S) for each user S of C, and that the location step be carried out using $K_{1m(S)}$ as coefficients in [7.10]. This yields a location for C which is optimal (cost-minimizing) if the shippers' "best-mode" choices for the starting-guess location remain valid for this "optimized" location. If such is not the case, take the new location as a new starting guess, altering the mode choices appropriately, and repeat the process. One might instead check for changes in "best modes" during the location step, interrupting the step to interject such changes as they occur.

(There is reason to expect that this problem will not prove too serious. For, changes in best mode will typically occur for S if two modes are nearly "tied" in attractiveness, but then it doesn't matter too much which of them is chosen. The computational experiments needed to back up this intuitive argument have not, however, been performed.)

We have now discussed the portion T(S,C) of shipper S's generalized transportation cost, which refers to shipper-to-center movement of goods. The total cost of such movements to C is

$$\begin{aligned} \sum_S T(S,C) &= \sum_S \{K_1(S)\ d(S,C) + K_2(S)\} \\ &= \sum_S K_1(S)\ d(S,C) + \sum_S K_2(S), \qquad [7.15] \end{aligned}$$

where the sums are over all users S of C. The second sum at the end of [7.15] is a constant (ignoring the mode-choice complication expressed in [7.14]), and so can be ignored in applying the location step to minimize total generalized transportation cost.

Center-to-Port Costs

It remains to consider the second transportation-cost portion, the one which refers to center-to-port movement. But this can be conceptually transformed to the type of situation already discussed, so that the previous material can be applied to it. That is, the movement from center to ports can be regarded as a ports-to-center movement in reverse, and the latter can be treated by the formulas developed above for shipper-to-center movement, but with "port" replacing "shipper." Of course, the transportation rates appropriate to containerized cargo must now be used.* Further, [7.7] must be changed to

$$[\sum_k V_k I_k A_k(C,P)]\, t_m(C,P), \qquad [7.16]$$

where the sum is over all commodity classes k, and

$A_k(C,P)$ = amount of commodity class k shipped through U.S. port P by users of C.

With these changes, one again ends up with a linear function of distance,

$$T(C,P) = K_1(P)\, d(C,P) + K_2(P). \qquad [7.17]$$

The resulting analog of [7.15] is

$$\sum_P T(C,P) = \sum_P K_1(P)\, d(C,P) + \sum_P K_2(P), \qquad [7.18]$$

and again the constant second sum in [7.18] can be ignored in the location step. The discussion previously centered on [7.14] applies here as well, but now concerning the choice of mode for shipment from center to ports.

Resumé

In summary, then, the location step applied to center C is aimed at finding a location for C to minimize a weighted sum of distances from C,

$$T(C) = \sum_S K_1(S)\, d(S,C) + \sum_P K_1(P)\, d(C,P), \qquad [7.19]$$

where the first sum is over the (temporarily fixed) set of shippers assumed to patronize C, and the second sum is over the set of U.S. ports serving these shippers.

From [7.19] we see that the shippers and the ports enter the problem in exactly the same way. It is convenient to have a notation which treats them uniformly. Suppose then that the shippers and ports involved with center C are located respectively at the (known) points $X_1, X_2, \ldots, X_{q(C)}$ (Note that these points, and the value of $q(C)$, can vary during the overall solution process as assignment steps change the patronage pattern of C. Note also that coincidence of two shippers, or of a shipper with a port, might lead to situations like $X_1 = X_2$.) Furthermore, we can define positive constants $w_1, w_2, \ldots w_{q(C)}$ by

$w_i = K_1(S)$ if X_i is the location of shipper S,

$w_i = K_1(P)$ if X_i is the location of port P.

Then the location step's purpose is to find a location for center C which minimizes the function** F(C) defined by

$$F(C) = w_1 d(X_1, C) + w_2 d(X_2, C) + \ldots + w_{q(C)} d(X_{q(C)}, C). \qquad [7.20]$$

---

*The reduction in rate may not be entirely passed along to the shipper, but the location step maximizes the total "transportation savings pie," without regard to how it might be sliced up among the various interested parties. Note that this criterion differs from that of finding the profit-maximizing location for a center, an objective which would normally involve attempts to gain additional users, contrary to the "known users" setting of the location step.

**In the following notation, as in the preceding, we will be somewhat loose in failing to distinguish between a center and its location.

The Location Step

The problem faced by the location step, and described in somewhat abstract form by [7.2 above, has a long history and (together with various special cases) has led to a number of publications in the mathematics and economics literature. These will not be be summarized. Instead, the interested reader is referred to the surveys by Witzgall[16] and Hargrave.[17] Rathe the concern is with finding an effective method for solving the problem.

The existing theory is adequate to show that there is exactly one optimal (= cost-minimizing) location for C. But no explicit formula for this location in terms of the proble data consisting of the w_i's and X_i's, has ever been found except for very special cases, and it is generally believed that no such formula exists. Thus the method sought must necessaril be some sort of iterative "successive approximations" technique.

Assume some (x,y)-coordinate system chosen to identify locations (of shippers, ports, and centers), and let

$$X_i = (x_i, y_i)$$

be the coordinates of X_i (shipper or port, as the case may be). Also let

$$C^{(t)} = (x^{(t)}, y^{(t)}) \qquad [7.21]$$

represent the estimated location for C at the t-th stage of the iterative process, which terminates when "settling down" is indicated by $d(C^{(t)}, C^{(t+1)})$ dropping below some prescribe threshold value. An iterative solution process will then give the coordinates $x^{(t+1)}$ and $y^{(t+}$ of a "new estimate" $C^{(t+1)}$ of the optimal location as computationally convenient expressions in the coordinates $x^{(t)}$ and $y^{(t)}$ of the "current" estimate $C^{(t)}$. Moreover, it will have the property that if $C^{(t+1)} = C^{(t)}$, i.e., if the locations obtained in two successive stages coincide, then this location is the optimal one.

Solution Method

The solution method is based on considering what mathematical conditions must hold if (and only if) some point C = (x,y), not at one of the points X_i, is to minimize F(C) as given by [7.20]. From the well known formula

$$d(X_i,C) = \sqrt{(x_i - x)^2 + (y_i - y)^2}, \qquad [7.22]$$

it can be shown that these conditions (obtained by equating the partial derivatives of F(C) to 0) are

$$\textstyle\sum_i w_i(x_i - x)/d(X_i,C) = 0, \qquad [7.23]$$

$$\textstyle\sum_i w_i(y - y_i)/d(X_i, C) = 0. \qquad [7.24]$$

These equations can be solved for the coordinates of C = (x,y), with the result

$$x = [\textstyle\sum_i w_i x_i/d(X_i,C)]/[\sum_i w_i/d(X_i,C)] \qquad [7.25]$$

$$y = [\textstyle\sum_i w_i y_i/d(X_i,C)]/[\sum_i w_i/d(X_i,C)] \qquad [7.26]$$

Of course these equations contain x and y in their right-hand sides as well as on the left, as coordinates of the "C" appearing there. However, they suggest basing an iterative solution method on the equations

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[16] C. Witzgall, Optimal Location of a Central Facility: Mathematical Models and Concepts, National Bureau of Standards Report 8388 (6/30/65).

[17] W. W. Hardgrave, Location-Allocation Problems: A Survey, Operations Research <u>16</u> (1968), Supplement 1, p. B-84(Abstract).

$$x^{(t+1)} = [\textstyle\sum_i w_i x_i/d(X_i,C^{(t)})]/[\sum_i w_i/d(X_i,C^{(t)})], \qquad [7.27]$$

$$y^{(t+1)} = [\textstyle\sum_i w_i y_i/d(X_i,C^{(t)})]/[\sum_i w_i/d(X_i,C^{(t)})]. \qquad [7.28]$$

This solution method, due to Kuhn and Kuenne,[18] is the one employed.

Some precautions are required, though. For, suppose that $C^{(t)}$, lies very close to some X_i, say X_1. Then the quantity $d(X_1,C^{(t)})$, which appears as a denominator in [7.27] and [7.28], is very small, and so the resulting quotients can be numbers too enormous to be handled properly by the computer. Now X_1 may in fact be the optimal position for C (i.e., the cost-minimizing location of a center may well be "at" one of the shippers or ports it serves), a possibility simply not provided for in the analysis starting with [7.25]. Fortunately, the theory provides a separate test for this possibility;* this test is a bit time-consuming, and so is applied only when "triggered" by the proximity of $C^{(t)}$ to X_1 (or some other X_i). If the test is not satisfied, i.e., if the sequence of tentative locations for C is only "passing by" X_1, then the numerical difficulty mentioned above is avoided by replacing [7.27] with the algebraically equivalent

$$\begin{aligned} x^{(t+1)} &= [w_1 x_1 + d(X_1,C^{(t)}) \textstyle\sum_{i>1} w_i x_i/d(X_i,C^{(t)})] \\ &\div [w_1 + d(X_1,C^{(t)}) \textstyle\sum_{i>1} w_i/d(X_i,C^{(t)})], \qquad [7.29] \end{aligned}$$

and similarly for [7.28].

Manhattan Metric

Satisfactory starting guesses, $C^{(0)} = (x^{(0)}, y^{(0)})$, can be obtained by solving a related problem, namely, replacing the straight-line distance formula [7.22] by

$$d^*(X_i,C) = |x_i - x| + |y_i - y| \qquad [7.30]$$

and choosing $C^{(0)}$ as a location for C which minimizes

$$f(C) = w_1 d^*(X_1,C) + w_2\, d^*(X_2,C) + \ldots + w_{q(C)}\, d^*(X_{q(C)},C)$$

with the d's given by [7.30]. This formula [7.30] is known as the "Manhattan metric," since it corresponds to travel distance in a network of streets (like much of Manhattan's) running in two perpendicular directions.

This procedure is useful because the cost minimization problem for the Manhattan metric does not require an iterative solution method, but instead can be solved in the following simple way.** Form the sum

$$W = w_1 + w_2 + \ldots. + w_{q(C)}. \qquad [7.31]$$

Renumber the points $X_i = (x_i,y_i)$ so that $x_1 \leq x_2 \leq \ldots \leq x_{q(C)}$. Now begin adding up $w_1, w_2 \ldots$, stopping as soon as the cumulative sum equals or exceeds W/2. If the sum exceeded W/2 when w_i was added (but was $\alpha W/2$ after the preceding addition of w_{i-1}), then the first coordinate $x^{(o)}$ of $C^{(o)}$ is given by $x^{(o)} = x_i$. If the sum equaled W/2 when w_i was added, then $x^{(o)}$ can be taken as any number between x_i and x_{i+1}, say their average. The second coordinate $y^{(o)}$ of $C^{(o)}$ is found analogously.

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[18] H. W. Kuhn and R. E. Kuenne, An Efficient Algorithm for the Numerical Solution of the Generalized Weber Problem in Spatial Economics, J. Regional Science 4 (1962), pp. 21-33.

*The test is described in the next chapter; its application requires reassembling the "fractionated" shippers associated with X_1.

**Witzgall, op cit, pp. 11-13. Again coincident shippers should be aggregated before applying this method.

Instead of using the Manhattan metric approach only to provide a starting guess for each center's location at the beginning of the location step, there is computational advantage in exploiting it more fully as follows: Preface the solution method as described so far with a preliminary phase, in which location steps and assignment steps still alternate, but where the location steps find center locations in terms of the Manhattan metric only. Switch over to the "real" straight-line distances only after this first phase settles down.

Restrictions on Locations

There is a further element of sophistication which might well prove desirable, but which has been carried only in part to completion. Namely, general policy decisions may place, on the locations of centers, additional conditions relating to governmental and/or geographical subdivisions. Such a condition might for example take a form like:

> There shall be at least one center in the northern part of (particular) state z, and at least one in its southern part.

Clearly this type of restriction must be accomodated in the location step rather than the assignment step. However, in the location step a center is "identified" by the set of shippers currently assumed to patronize it, rather than in any intrinsic way. So to implement a constraint reading "region R should contain at least one center," it is convenient to select some relatively prominent shipper S located fairly centrally* in R, and to rephrase the constraint as "the center currently assumed to serve S must be located in R." To avoid undue mathematical difficulty it must also be assumed that R has a simple geometrical shape --that of a convex polygon.** An approach to this "constrained" version of the location step is given in Appendix C.

Location-Dependent User Costs

In the preceding discussion of the location step, the assumed objective was to minimize the total generalized cost to the (known) users of the center being located. This ignores the possibility that the within-center cost, also, might depend directly on the center's location. Such dependence might enter the dollar component through variations (between locations) in land acquisition costs when constructing the center, or wage scales for the labor it employs. (See Chapter III, Cost of Consolidation Centers.) The time component might be affected by differences, among locations, in accessibility to transportation facilities.

As emphasized earlier, no attempt will be made to cope with locational effects at a fine-grained level. It is assumed that the choice of a specific site for a center, within a demarcated area, would be a judicious one. With this understanding, the influence of location on user fees can be treated in a rough way as follows:

The region of interest in locating a particular center C is assumed divided into subregions R_1, R_2, etc., such that (a) each of these subregions R_i is a convex polygon, and (b) labor costs and the costs of (careful) land acquisition can be regarded as sensibly constant for center locations in R_i, though these constants can, for instance, differ between R_1 and R_2. Thus the dollar component of processing cost (and hence, user fee) will be essentially constant within each R_i.

Now solve, for each R_i in turn the type of problem described in the last subsection: Find a location for C_i constrained to be within R_i, which minimizes the total generalized transportation cost to the center's users. Let T_i be the minimum-cost level corresponding to C_i, and let U_i be that level of total user costs associated with having C in R_i. Then choose the location of C to be a C_j for which

$$T_j + U_j = \min_i (T_i + U_i). \qquad [7.32]$$

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*If there is no such shipper, or analogous set of shippers assigned to a common center, then the justification for the constraint would be quite questionable.

**It appears that states and/or natural subdivisions of them can typically be rather well approximated by such shapes.

Treatment of Costs Incurred at Center

Now that the discussion of the location step has been completed, the assignment step must be described. Recall that, roughly speaking,* this step associates each shipper with the center he would most prefer. The locations of all centers are regarded as known.

It is assumed that the most preferred center, for shipper S, is the one whose use by S leads to the lowest sum of(a) the generalized cost of shipper-to-center and center-to-port transportation, and (b) the generalized cost incurred by S at the center. If, as before, the symbol C stands for an arbitrary center, and if

$P_C(S)$ = generalized "processing" cost incurred by S in using C,

then the center C(S) most preferred by S is characterized by the condition

$$T_{C(S)}(S) + P_{C(S)}(S) = \min_C[T_C(S) + P_C(S)]. \qquad [7.33]$$

The previous text included a mathematical formulation for the generalized transportation cost $T_C(S)$, defined in [7.1]. Before the assignment step can be described, a corresponding formulation of the generalized processing cost $P_C(S)$ must be developed. Like $T_C(S)$, it has both a monetary component and a time component. The conceptual analysis is much more complicated here, however, because the two components are strongly interrelated and because both of them are bound up with considerations of centers' sizing, pricing, and operating policies.

Consider just the matter of pricing, for example. A number of quite different scenarios are possible, at least in principle. One might imagine unified control (or regulation) of the entire system of centers as leading to the offering of a common schedule of prices by all centers. (This could certainly simplify the assignment step, since for each shipper S the monetary part of $P_C(S)$ could be dropped from the comparison of centers in [7.33].) Or, such unified control might lead to different prices at different centers, so set as to maximize the centers' total profit.

At the opposite extreme, one might conceive of individually operated centers, each striving to maximize its own profits. Here the appropriate mathematical model for pricing might well be that of an n-person non-cooperative game.[19] Unfortunately such "games" do not always have solutions[§] (in a sense meaningful for the present discussion), and even when solutions exist, they may not be unique.[†]

Intermediate between these two extremes (unified control vs. independent centers) is of course the case of a number of subsystems of centers, each with its own unified management.

This far-from-exhaustive treatment of pricing (and its dependence on assumptions concerning the ownership of centers) should really be expanded to include alternative possibilities as to price regulation. And much of the discussion would have to be repeated in describing alternative approaches to the analytical treatment of selecting sizes for centers, or to the mathematical representation of operating policies.

Investigating so great a variety of speculative possibilities, though desirable in the name of "comprehensiveness," is not really practical. Instead, a number of guidelines to be explained as they arise below were introduced in order to arrive at a definite model for analysis.

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*Complications have been alluded to earlier, and will be treated in detail shortly.

[19] See R. D. Luce and H. Raiffa, Games and Decisions, Wiley and Sons (1957), Chapters 4, 5, 7.

[§] For games with just 2 players, the chances that a solution will exist are quite good; see K. Goldberg, A. J. Goldman, and M. Newman, The Probability of an Equilibrium Point, Journal of Research NBS, 72B(1968), pp. 93-101. But the probability of existence decreases exponentially with the number of players (paper by M. Dresher of the RAND Corporation, presented at a 6/68 Conference on Combinatorial Mathematics, at Yale University).

[†] Suggestions to avoid this difficulty have been offered, but are not entirely convincing. See Luce and Raiffa, op cit, p. 173.

Time Costs

The time delays involved in the passage of goods through a center can be roughly classified into two categories: accumulation delays (waiting for enough co-containerizable material to arrive to yield a sufficiently full container), and service (or "queueing") delays. These categories will be treated in turn. Note that the time component of the generalized cost to shipper S incurred at center C is

$$V_{k(S)} I_{k(S)} A(S)\ [t_a(S,C) + t_s(S,C)], \qquad [7.34]$$

where the product of the first three factors (all defined earlier) yields a cost per shipment per unit of delay, and

$t_a(S,C)$ = average accumulation delay per unit,

$t_s(S,C)$ = average service delay per unit.

Thus we must develop formulas for t_a and t_s.

Dispatching Policies

Accumulation delays clearly depend on the center's operating policies concerning holding times and container load factors. If the latter are too high (too close to 1), there may be excessive delays to a user S while the center awaits enough cargo co-containerizable with his.* If the load factor is set too low (too close to 0), there will be excessive costs for the transportation of an inordinate number of containers, typically skimpy on payload.**

A plausible type of dispatching policy can be described by a pair of quantities,

L = "target" load factor,

H = maximum holding time.

The policy is that a shipment is held until either there is enough co-containerizable cargo to fill the fraction L of a container (which is then dispatched), or a time H has elapsed, whichever comes first. This is not fully realistic; for example if L = 0.75 and 3 pre-announced containerfuls of co-containerizable material arrived during the same morning, the center would presumably send out 3 full containers rather than 4 containers each 3/4 full of payload. Still, such (L,H)-type policies seem to be reasonable approximations of what one might expect.

Some policies might admit considerable sophistication. For example, one can conceive of the commodity classes being grouped into priority categories, each consisting of commodity classes with similar sensitivities to delay, i.e., $V_k I_k$-values. The p-th priority category would have a policy described by some (L_p, H_p), with both parameters presumably lower for high-priority categories. One interfering complication, however, is that co-containerizable cargo could contain material from several different priority categories, and it is not immediately clear what priority should be assigned to such a composite; giving it the priority of the most delay-sensitive category present might mean that in the long run too much low-priority material gets carried along on a high-priority basis. Alternatively, one might consider having a separate policy (L_c, H_c) for each c-th class of co-containerizable cargo. This too seems somewhat awkward, since such a class can contain cargo elements with quite different sensitivities to delay.§

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*And also excessive cost to the center (ultimately passed on to users) for storage of as yet insufficiently full containers or their contents.

**And there will be a similar inflation of cost of the overseas break-bulk operation, let alone the costs associated with acquisition and/or use of the containers.

§Partial exception to this statement can be taken, in that shippers with co-containerizable cargoes may be implicitly "synchronized" by trying to meet the same sailings.

For the present study, however, it seems best to avoid complications and to consider a single pair (L,H) for each center.* (It might vary from center to center.) Recall that L is a target load factor; because of the effect of H, the quantity

$\overline{L}_c$ = actual average load factor for c-th co-containerizable class

will in general be less than L.

The evaluation of $\overline{L}_c$, along with that of the average accumulation delay $t_a(S,C)$, will be described below. But first we note the important quantity

$$I_c = 1/\overline{L}_c \qquad [7.35]$$

which gives** the inflation ratio in converting from incoming quantities (of cargo in the c-th co-containerizable class, measured in containerfuls) to outgoing quantities (measured in containers). If c(S) is the unique co-containerizable class to which shipper S belongs (by virtue of the fractionation of shippers), and subscript "C" designates the center involved, then the term $T(C,P_S)$ in [7.1] should be multiplied by $I_{c(S)}$ before use in the location step and probably also when forming $T_C(S)$ for use in the assignment step via [7.33]. The situation is not quite so clear in the latter instance, because the increase in $T_C(S)$ is due to elements of the center's policy, and so might not be passed on (at least not entirely) to shippers. At present, though, it appears most consistent to perform this modification of $T_C(S)$ for the assignment step, as well as for the location step.

Evaluation of $\overline{L}_c$ and t_a

The evaluation of the (actual) average load factor $\overline{L}_c$, and average accumulation delay $t_a(S,C)$ will now be described. Since the delay depends on the co-containerizable class c(S) to which (fractionated) shipper S belongs, the notation will be changed from $t_a(S,C)$ to $t_a(c,C)$ with the understanding that

$$t_a(S,C) = t_a(c(S),C). \qquad [7.36]$$

Both quantities to be evaluated depend not only on the parameters (L,H) describing center C's dispatching policy, but also on the distribution (over times and shipment sizes) of new arrivals at C of class c cargo. This distribution is a composite of the analogous distributions for the individual shippers belonging to the c-th class. For general distributions, the evaluation of $t_a(c,C)$ and $\overline{L}_c$ would require a "Monte Carlo" simulation. But since the actual distributions are not presently available to us, it is reasonable to proceed under simplifying assumptions which have been observed to hold approximately in analogous real-life situations.

The mathematically simplest assumption, of course, is to treat the inflow of class c material to C as taking place continuously over time at a uniform rate. This however is too simple an assumption; under it the target load factor would either be consistently unattainable (so that dispatch times would be governed solely by the maximum holdover H), or consistently feasible (in which case the role of H would never come into play). Nor does such a formulation seem reasonable, to represent arrivals from a number of users whose shipping schedules are presumably independent.§

The type of formulation proposed for use here has become rather traditional, in the light both of practical experience and of theoretical development. It represents arrivals arising in such a way that the probability of at least one arrival, in any time interval of duration x, is

$$1 - \exp(-\lambda x) \qquad [7.37]$$

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*Another direction of possible sophistication, based on a suggestion by P. B. Mentz of the U. S. Maritime Administration, is described in Appendix E.

**Actually [7.35] is an approximation; the rigorous version is given in Appendix F.

§Support for this simplification comes from the observation that priority differences seem likely to involve distinctions between typical holding times of 1-2 days vs. 3-4 days. Cargo so delay-sensitive as to find such a distinction significant seems likely to follow the air route rather than marine routes.

Here λ is a positive parameter, with the interpretation that $1/\lambda$ is the average interval between successive arrivals.

There are (at least) two ways of applying this Poisson distribution of arrivals to our situation. One is to treat the arrivals at C, of shipments from each shipper, as Poisson distributed.* For each shipper S in class c, data like those of Chapter IV provide the tota volume and frequency of S's shipments; treating these shipments as uniform in time and size leads to a mean spacing between S's shipments, which can be equated to $1/\lambda_S$ and a mean shipment size $\bar{a}_S$. Suppose there are m_c shippers belonging to co-containerization class c. If all of them had the same mean shipment size $\bar{a}$, then their "resultant" would be a Poisso process with shipment size $\bar{a}$ and parameter $\lambda_c = \sum_S \lambda_S$, the sum being over all shippers S in the c-th class.

The evaluation of $\bar{L}_c$ is relatively direct in this case. Start counting time from the arrival of the first class c shipment (of size $\bar{a}$) following the dispatch of a class c container. Let m be the largest integer such that $m\bar{a}<L$. Then the maximum holding time (H) comes into effect if, and only if, fewer than m additional shipments arrive during the next time interval of duration H. By known properties of the Poisson distribution, the probabilit of this event is

$$p_{m-1}(\bar{H}) = \exp(-\bar{H}) \sum_{N=0}^{m-1} \bar{H}^N/N! \quad (\bar{H} = \lambda_c H), \qquad [7.38]$$

a function of m and $\bar{H}$ which has been extensively tabulated. In this situation the container' payload will be $(N + 1)\bar{a}$ for some N between 0 and m-1 inclusive, while in the complementary case it will be $(m + 1)\bar{a}$. Therefore,

$$\bar{L}_c = \bar{a}\,\{\exp(-\bar{H}) \sum_{N=0}^{m-1} (N + 1)\bar{H}^N/N! + [1 - p_{m-1}(\bar{H})](m+1)\}. \qquad [7.39]$$

This can be rewritten

$$\bar{L}_c = \bar{a}\,\{\bar{H}p_{m-2}(\bar{H}) + p_{m-1}(\bar{H}) + [1 - p_{m-1}(\bar{H})](m+1)\}. \qquad [7.40]$$

The evaluation of $t_a(c,C)$ is more complicated and is given in Appendix D; here only the result is given,

$$t_a(c,C) = (1/2\lambda_c)[(m-1)(1-p_m(\bar{H})) + \bar{H}p_{m-1}(\bar{H})]. \qquad [7.41]$$

The preceding formulas for $\bar{L}_c$, I_c, and t_a were based on the assumption that $(m+1)\bar{a} \leq 1$. In case $(m+1)\bar{a} > 1$, L is in effect replaced by $m\bar{a}$, and this can be accomplished by replacing m with m-1 in the formulas.

The corresponding formulas for $\bar{L}_c$ and t_a, when the users of C in the c-th co-containerization class have different average shipment sizes $\bar{a}_S$, can be written out explicitly and evaluated using a computer, but are very much more complicated. Until the quality of empirical data appears to warrant this extra substantial effort, it is suggested that a fictitious mean shipment size at C for the c-th class be formed as a weighted average

$$\bar{a}_c = \sum_S \lambda_S \bar{a}_S / \sum_S \lambda_S = (1/\lambda_c) \sum_S \lambda_S \bar{a}_S, \qquad [7.42]$$

where the sum is over all users of C belonging to the c-th class, and that [7.40] and [7.41] be used with $\bar{a}_c$ as $\bar{a}$. Still more complex models, which account explicitly for size fluctuations in shipments from individual users, can readily be devised (e.g., using "truncated stuttering Poisson distributions") but much less readily analyzed. This degree of sophistication will not be considered any further here. At the other extreme in model refinement, one might consider dropping the dependence of λ_c and $\bar{a}_c$ upon the center C, i.e., treating them as equal for all centers, with values calculated as averages over all shippers in the c-th class, rather than over all users of C in the c-th class.

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*Here the distribution of arrivals at C is identified with that of arrivals at the consolidation phase of processing.

A final note concerning accumulation delays and container load factors: In dealing with nports, accumulation delays at the U.S. centers should of course not be included in the eneralized costing. Impacts of overseas accumulation delays on import patronage at U.S. enters may, however, warrant inclusion in a more refined model. Average container load actors remain relevant, but would presumably have to be estimated directly from empirical ata rather than derived from a submodel of the sort described above for exports. This istinction is based on an impression that data on arrivals (frequencies and volumes) of LCL ipments at overseas consolidation-containerization centers will not be readily available.

Minimum-Patronage Restrictions

This is a convenient point at which to interrupt the discussion of (generalized) costs ncurred at a center, and to take up one of the difficulties described earlier when the ssignment principle was introduced. Consider a particular center C, a co-containerizable lass c, and a shipper S belonging to c. In considering a particular C for possible selection in the assignment step), one of the factors a shipper S must consider is the typical accumulation delays his shipments would encounter at C. But this depends on the degree of patronage f C by other members of c!

Of course excessive accumulation delays can be ruled out if the center sets low enough alues for one or both of the parameters characterizing its dispatching policy--namely the target" load factor L and the maximum holding time H. But then the average realized load actor $\overline{L}_c$ would be very low, resulting in high costs associated with an excessive number of ontainers. This is clearly just the same problem (insufficient patronage) from an alternaive viewpoint.

The recommended treatment of this problem involves setting a minimum acceptable level in one of the ways described below) on the patronage of C by shippers in the c-th class. f that level is not attained, it is assumed that C will not offer service to class c shipents.

In more detail: The assignment step for all centers is first performed without imposing e minimum patronage levels. Then, for each co-containerizable class c, the following rocess is carried out. A check is made on whether every center, to which shippers in the -th class have been assigned, has a patronage level which meets the "minimum acceptable" riterion. If this is not the case, the center at which the criterion is most severely iolated is declared "off limits" to class c shippers, and the former class c patrons of hat center are reassigned. This continues (with previous "off limits" restrictions remaing in force) until all class c shippers are assigned to centers with adequate class c atronage.

This approach seems both more manageable and more realistic than more esoteric alternaives, for example, the use of integer linear programming methods such as described in ppendix B. Note that the assignment step has now (like the location step) become an iteraive process.

It remains to describe how the "adequate patronage" criterion, for a specific center C d a co-containerizable class c, is to be established. The following three proposals are l essentially equivalent, so that the choice among them might be made on a basis of aturalness":

(a) Put an upper limit on p_{m-1} $(\lambda_c H)$, the probability of dispatching a container whose load factor is less than the target level.

(b) Put an upper limit on the average accumulation delay $t_a(c,C)$.

(c) Put a lower limit on the average realized load factor, $\overline{L}_c$.

A related alternative is based on the presumed saving in transportation cost, from ipping containerized rather than uncontainerized material over part of the trip to the S. port of departure. The idea is to choose the lower limit, in (c), as that level of $\overline{L}_c$ which the monetary effects of shipping containers largely consisting of "filler" in effect educe this saving to a specified fraction of its original value. Since the saving depends on a shipper's location, this criterion would most naturally apply to individual shippers rather than to a whole co-containerizable class. But perhaps this concept is somewhat too complex to be a likely basis for actual practice.

Evaluation of Service Delay

The next topic is the evaluation of $t_S = t_S(S,C)$, the average service delay per unit incurred by shipper S in the movement of his cargo through center C. In the absence of releva empirical data, the treatment of this point must necessarily be speculative, based on considerations of mathematical simplicity and of analogy to apparently similar situations which have been examined in some detail.

It seems natural to express t_s as a sum $t_s = t_{su} + t_{sc}$, and thus total time in the center as

$$t_a(S,C) + t_s(S,C) = t_a + t_s = t_{su} + t_a + t_{sc} \qquad [7.43]$$

where t_{su} refers to the time between (1) physical arrival at the center and (2) completion o processing to the point where accumulation delay (t_a) becomes meaningful. Similarly, t_{sc} refers to the period between (1) the time at which the dispatching rules first permit containerization, and (2) the actual departure of the container from the center.

For incoming cargo, there is a question of whether "excess capacity" should refer to al arriving material, or should be treated on a shipper-by-shipper basis or at least separately for each group of shippers which might utilize separate receiving and/or storage facilities-e.g., shippers using the same transportation mode to transfer their goods to the center. Th same question arises for outgoing material, where a category might consist of the users of a common U.S. port of departure, or (more grossly) of a common means of transit from center to port.

Pending the gathering of information on which to base a more refined treatment, we shal interpret "excess capacity" on a "total volume" basis for both inflow and outflow material. Thus t_{su} and t_{sc}, and hence their sum $t_s = t_s(S,C)$, will be treated as independent of the shipper-identity S. For the present, the relation between "excess capacity" (K_{ex}) and average wait (w) will be taken as that for a "simple" queuing process (Poisson distribution of arrivals, exponential distribution of "service" times):[20]

$$w = 1/K_{ex}. \qquad [7.44]$$

For the application of [7.41], set

$K_u(C)$ = mean handling capacity for uncontainerized material at center C,

$K_c(C)$ = analogous handling capacity, for outgoing (containerized) material,

$A(c,C) = \lambda(c,C)\bar{a}(c,C)$

= mean arrival rate at C of material in c-th co-containerizable class.

Then the over-all mean arrival rate at C is

$$A_u(C) = \textstyle\sum_c A(c,C), \qquad [7.45]$$

while the mean outflow of containers, taking into account the inflation factors $I_c = I(c,C)$, is

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* The point here is a well-known one: if the handling capacity of a facility is merely matched without "slack" to nominal input rate, then normal fluctuations in arrival patter can easily lead to lengthy queues and serious delays. "Capacity" here refers mean throug put capability per unit time.

[20] See for example, M. Sasieni, A. Yaspan, and L. Friedman, Operations Research - Methods an Problems, Wiley (1959), p. 133. In [7.44], w includes the duration of service time.

$$A_c(C) = \sum_c I(c,C)A(c,C). \qquad [7.46]$$

Application of [7.44]* gives

$$t_{su}(C) = [K_u(C) - A_u(C)]^{-1}, \qquad [7.47]$$

$$t_{sc}(C) = [K_c(C) - A_c(C)]^{-1}. \qquad [7.48]$$

An inflation ratio for center C can be defined by

$$I(C) = A_c(C)/A_u(C). \qquad [7.49]$$

It is reasonable that input and output capacities, $K_u(C)$ and $K_c(C)$, would be chosen in the ame ratio I(C) as the input and output volumes. Then combination of the preceding formulas eads to

$$t_s(C) = [1 + I(C)][K_c(C) - A_c(C)]^{-1}. \qquad [7.50]$$

Four brief comments are in order before leaving this topic. First, the preceding iscussion referred to exports, involving the center in the receipt of LCL cargo and the ispatch of containers. It seems likely that the queueing delays suffered by imports would e largely independent of those for exports, i.e., that (especially with proper peak-traffic anagement) the two flows would not interfere much with each other. In this case, a separate ormula like [7.50] would be required for imports.

Second, the capacity level $K_c(C)$ in [7.50] is a design parameter rather than an empiri-al datum; finding an appropriate value for it is one of the problems ("center sizing") to be reated in this study. Selecting such a value is a question that will be returned to later.

Third, both I(C) and $A_c(C)$ in [7.49] depend on which shippers patronize center C. But his information is the output of the assignment step, raising the question of how [7.50] can e used in the course of that step. This question, too, will be taken up shortly.

Fourth, it is interesting to observe a basic difference between the factors governing ccumulation delay t_a and service delay t_s. Roughly speaking, to minimize t_a a shipper would end to choose a heavily patronized center. But then there would be a tendency for t_s to be arge---or for there to be extra cost associated with additional capacity needed to rule out xcessive service delays. Of course this last effect might be more than counteracted by the eduction in unit cost due to spreading the fixed cost over numerous users.

Note that with eq. [7.41] for t_a and eq. [7.50] for t_s at hand, the time component [7.34] f the generalized cost to shipper S incurred at center C can be calculated. The monetary omponent will be treated next.

Monetary Processing Cost

The dollar component of the generalized processing cost $P_C(S)$ appearing above [7.33] will e denoted $M_C(S)$. It represents the user's fee that would be paid by shipper S for employing he services of center C. As part of the transportation system, one would expect consolida-ion centers to have price structures reflecting "cost of service" considerations more emphati-ally than a "value of service" concept. Thus it seems clear that user fees should, to a onsiderable extent, reflect the costs incurred by the center itself.

The area of pricing policy has already been mentioned, explicitly, as one in which sim-lifying assumptions (subject to later modification if required) must be introduced in order o get on with the analysis. The two assumptions made at the outset are (a) that a center's xpenses are regarded as allocated among its patrons in proportion to their volumes of usiness, and (b) that the fee charged a user is directly proportional to "his" share of the llocated cost; the constant π of proportionality (a "profit parameter" among the model nputs) being the same for all centers.

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*A fuller analysis would reflect the fact that [7.44] is not strictly applicable to yield 7.48], since the associated queue has non-Poisson inputs.

It should be noted that assumption (a) is not without plausible alternatives. For example, it might well be the case that some monetary cost elements are roughly proportional to the time periods required for the associated operations. This would lead to terms

$$\pi[k_u(S,C)t_{su}(S,C) + k_c t_{sc}(S,C) + k_a t_a(S,C)]$$

in $M_C(S)$, where the k's are appropriate constants. Such terms might be treated most conveniently in conjunction with the cost-of-delay portion of $P_C(S)$, and will not be discussed further here.

Another point concerning (a) is the question of whether the "user quantities" used for cost allocation should refer to incoming or outgoing material. In general different allocations would result from the two choices, because of the differences in inflation ratio among co-containerizable cargo classes. A suitably detailed costing model would permit the allocation to be based upon both quantities, involving separate allocations for the costs of the operations up through stuffing, and for the remaining operations. But since the cost information in Chapter III is developed on a "per container stuffed" basis, the allocation will be treated in terms of outgoing quantities. That is, if shipper S uses center C and is in the co-containerizable class c(S), he will be assigned a fraction

$$I(c(S),C)\ A(S)/A_c(C) \qquad [7.51]$$

of the center's cost, where $A_c(C)$ is defined by [7.46].

It remains to develop an expression for the costs incurred by the center. Since the letters "C" and "K" are already in use, the letter "E" (for "expense") will be used to designate this total cost. The approach chosen is a common one, in which E is expressed as the sum of a fixed cost F which depends on the throughput capacity of the center, and a variable (or operating) cost V, which depends on average realized throughput. Thus,*

$$E_C = F(K_c(C)) + V(A_c(C)). \qquad [7.52]$$

With further study, the description of center operations given in Chapter II could presumably be used to divide each of the cost elements developed in Chapter III into "fixed" and "variable" portions, associated with capacity and with operating level, respectively. However, available information suggests limiting the treatment to a linear cost equation,

$$E_C = b_o(C) + b_F(C)K_c(C) + b_V(C)A_c(C), \qquad [7.53]$$

where b_o, b_F, and b_V are constants which might in principle vary with center location. This would be valid for all values of $A_c(C)$ no less than some minimum level A_{min} (say, 50 containers /week) constituting a minimum overall patronage threshold for economical operation of a center.

In summary, then, the monetary component of the generalized cost to S of using C is giv by

$$M_C(S) = \pi E_C I(c(S),C)\ A(S)/A_c(C), \qquad [7.54]$$

where E_C is given by [7.52].

Sizing of Centers

The line of analysis developed above reduces the "sizing problem" to the selection of a appropriate level for the output capacity $K_c(C)$ of a center C, whose patronage pattern and mean output rate $A_c(C)$ are known.

A plausible objective, in selecting this level, is to minimize the total generalized cost associated with the center's use. Since accumulation delays do not depend on $K_c(C)$, only service delays and monetary costs need be considered. The expression to be minimized will be taken to include processing (dollar costs) rather than user fees; the transition to the latter, if preferred, would only require inserting a multiplier π at appropriate points.

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*A more detailed equation might involve additional fixed cost and variable cost terms, depending on $K_u(C)$ and $A_u(C)$, respectively.

To simplify the notation, references to the identity of the center (C) will be dropped in the remainder of this section. Moreover, let

$$V^* = \sum_S I_{k(S)} V_{k(S)} A(S),$$

where the sum is over all shippers using the center. Then K_C is to be chosen, subject to $K_C > A_C$, to minimize a function

$$G(K_C) = E + V^* t_s, \qquad [7.55]$$

which by [7.50] and [7.52] can be written

$$G(K_C) = F(K_C) + V(A_C) + V^*(1 + I)/(K_C - A_C). \qquad [7.56]$$

In general, the minimization can be carried out by equating to zero the partial derivative of G with respect to K_C. For example, in the linear case [7.53] the solution is

$$K_C = A_C + [V^*(1 + I)/b_F]^{\frac{1}{2}}. \qquad [7.57]$$

This minimizing K_C is then substituted into [7.52] for use in obtaining the user fee by [7.54], and is substituted into [7.50] to obtain the average service delay at C.

Enlargement of Assignment Step.

There is only one major point left to discuss in describing the main logic of the mathematical model and technique. It concerns a difficulty already alluded to above: the purpose of the assignment step is to assign each shipper S to that center whose use by S would involve the smallest generalized cost. Yet the generalized cost to S of using a particular center C involves quantities--mean accumulation delay $t_a(S,C)$, mean service delay $t_s(S,C)=t_s(C)$ and monetary service cost $M_C(S)$--which themselves depend both directly and indirectly (via the optimal sizing $K_C(C)$) on the patronage pattern of C. Specifically, they depend on the mean output rate $A_C(C)$ and overall inflation ratio I(C) of C, as well as (see [7.54]) the explicit inflation ratio I(c(S),C), and these in turn depend on the patronage pattern of C. Thus the information required by the assignment step appears to be that which is to be produced by the step!

The solution proposed for this difficulty will come as no surprise to the reader who has followed the unfolding of the model thus far. It involves increasing still further, and perhaps quite heavily, the iterative calculations to be performed within the assignment step.

Specifically, each pass through the assignment step (after completing a pass through the location step) begins with a set of patronage patterns "left over" from the last previous assignment step. For each shipper S, the total generalized cost of using each center C is calculated on the basis of this "left-over" pattern* of patronage for C. Once every shipper has been assigned, there is now an "observed" set of patronage patterns for the centers. These are used to determine new generalized costs which provide the decision basis for a new assignment of shippers to centers, and so on. The process terminates when the variable portion of the total cost for all shippers changes less than 0.5 percent from one run through the shippers to the next. The assignment corresponding to the cost is then the main output of that particular pass through the assignment step.** After the combined process (of location and assignment steps) has settled down to a solution, one would also want to record

(a) the optimized capacity $K_C(C)$ for each center C,

(b) the associated cost E_C for each center, and also

(c) the sum of the generalized costs to all users.

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* Alternatively, the first cycle of assignments within an assignment step (especially the first one) might be based solely on transportation costs. Including nominal values (the same for all centers) of other ingredients of generalized cost would of course be useless, since they would not affect choices among centers.

**Although there is nothing in the model to insure that the assignments settle down as the cost does, our experience with the model is that in general the assignments of less than one percent of the shippers (involving less than one percent of the total volume) change in two successive passes with a change of less than .5 percent in the variable cost.

This last quantity, in particular, should provide guidance as to the proper number of center

Limitations of time and data permitted only very limited implementation of these concept Specifically, our inability to separate a center's total monetary processing costs into fixed cost and variable-cost components, as in [7.52], precluded any numerical treatment of center sizing and capacity. This in turn ruled out treatment of the interactions, with center capaci and patronage, of user's service delay (cf. [7.50]) and user's processing cost (see [7.54]). Thus the present assignment step is "enlarged" only as regards the accumulation-delay element of generalized processing cost. Further gathering and/or interpretation of cost data, to permit explicit programming of and experience with the "center sizing procedure," would seem to have very high priority if development or application of the model is to be carried forwar

It should be noted that two other iterative processes, within the assignment step, must be appropriately meshed with the process introduced above. One is the procedure, already described, for placing some centers "off limits" to some co-containerizable cargo classes in order to insure enough patronage to prevent excessive average accumulation delays. The other insures that each center has at least the minimum total patronage A_{min} mentioned below [7.53] At present this condition is tested only when all other criteria for termination are satisfie Those centers (if any) which violate it have their locations re-initialized, and the computation starts afresh with a configuration consisting of the new locations for the "lean" center plus the "good" old locations for the remaining (nonviolating) centers. This procedure is repeated until there are no violations of the minimum total patronage restriction, or until a specified maximum number of repetitions is reached.

There is still another way in which the generalized cost to a shipper of using a center can depend on the patronage of C: the rates for center-to-port movement of containerized cargo may, in fact presently do, involve volume discounts. For example, a published schedule of rates* for transcontinental rail movement of nonrailroad-owned containers is:

Shipment Size	Rate per Carload**
1 - 10 cars	$1320
11 - 20 cars	1220
21 - 30 cars	1120
31 + cars	1020

This dependence of transport cost on LCL center patronage can in principle be accommodated in the present conceptual model, especially if one ignores tactical possibilities such as sending some containers to the port by an indirect route so that they can be combined with containers bound for another port, thus meriting a reduced rate for part of their trip. A more serious difficulty, however, concerns those shippers with full container-loads who utilize the marshalling capability of the center; their cargo is not only attracted to the center by the rate discounts for large shipments to the port, but also helps to determine what rates the (containerized) cargo from the LCL shippers will be subject to. The present model does not deal with the centers' marshalling activities, and considerably more analysis (and preferably, data) would be needed before attempting a treatment of those activities.

The Multi-Period Case.

Earlier in the present chapter, it was asserted that no basic difficulty not already present would arise in extending the solution process to a situation involving several time periods, and taking into account the possibility that a shipper might shift his patronage from one center to another as circumstances change. The reasoning on which this assertion was based will now be given.

For the situations considered up to now, a single "run" of the computer program would require as inputs the number (n) of centers to be considered, as well as demand, cost, and freight rate data for the single time period in question. In the multi-period case, data (on demand, cost, and rates) must of course be provided for each time period in the planning peric

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* Provided by the Project Engineer, P. B. Mentz of the Office of Research and Development, U.S. Maritime Administration.

**One carload (85 feet flatcar, 150 K-lb. capacity) = Two 40-foot containers.

If these data remain constant over time, or can be generated by the computer from initial values using simple projection formulas, the demands on computer memory and the associated programming effort could be drastically reduced.

Also required, as an input, would be the time period in which each center is to begin operation. (For simplicity of discussion, it is assumed that no center is phased out during the planning period.) A more sophisticated version might involve constraints on how fast each center could expand to full capacity, but this refinement will not be considered here.

As before, the solution process involves an alternating sequence of assignment steps and location steps. An assignment step treats the locations of all centers as given, and produces an assignment of shippers to centers for each time period, in conformity with the assignment principle. A location step takes as given the patronage pattern over time for each center, and determines a location for each which minimizes the total (over time) location-dependent cost for its users. Once again, constraints on the locations of individual centers may be imposed.

There is nothing really new to be said about the location step. Each of the "weights" w_i in [7.20] would be a sum of terms, determined by the patronage patterns of center C in the various time periods. One might wish to apply discount factors to these time series.

The assignment step would still be an iterative process. It would involve a sequence of assignments of the type previously described, one for each time period in turn. Of course no shipper is to be assigned to a center during a time period before the latter goes into operation!

The only point appearing to offer difficulty is how to treat the evolution over time of the capacity of a center C. Suppose a value $K_c(C;t)$ has been determined for this capacity in time period t. If the optimized value for time period t+1 is less than $K_c(C;t)$, then it seems best to set

$$K_c(C;t+1) = K_c(C;t).$$

Because only data for a single time period were available, this concept (in particular, the initialization of iterative cycles) has not been thought through in full detail, but it appears feasible with suitable reprogramming. Some aggregation of time periods to reduce computer running time and memory storage requirements might prove advisable.

Management of Containers.

This final topic pertains to the likelihood of patronage patterns such that some centers will typically have more containers arriving than leaving, while the reverse will be true at other sites. Moreover, depending on such factors as balances of trade, the system of U.S. centers as a whole may display a net surplus or deficit of containers.

The treatment of this imbalance has not been emphasized because it seems unlikely to exert a serious influence on the location and sizing of consolidation centers. Suppose, however, that the center locations have been determined, and the patronage patterns of each predicted. Then the following submodel, though not suitable for the day-to-day management of the container supply, can provide planning-level guidance on the topic.

Since patronage patterns have been estimated, it is known which centers will over a time period be net sources of empty containers, and which will be net sinks; empty containers will be shipped from the former to the latter to redress the balance. Let

e_i = excess of containers at i-th source,

d_j = deficit of containers at j-th sink,

c_{ij} = cost of shipping an empty container from i-th source to j-th sink.

The quantities c_{ij} should be calculable from the center-to-center distances, and the applicable freight rates (or rates for whatever special arrangement might prove appropriate).

Suppose first that the container supply for the U.S. system of centers as a whole is in balance:

$$\sum_i e_i = \sum_j d_j. \qquad [7.58]$$

Consider an optimization problem involving the variables

x_{ij} = number of empty containers to be sent from i-th source to j-th sink,

which must obey the conditions

$$x_{ij} \geq 0,$$

$$\sum_j x_{ij} = e_i,$$

$$\sum_i x_{ij} = d_j.$$

The objective is to minimize the total cost of the balance restoring movements,

$$C = \sum_{ij} c_{ij} x_{ij}.$$

This problem falls under the heading of linear programming transportation problems; it has an extensive theory and powerful solution methods, embodied in readily available computer programs which can handle situations with rather large numbers of sources and/or sinks.

If [7.58] fails because there is an overall excess of containers, one introduces a fictitious extra sink whose "deficit" is just the right amount to restore the total balance. The "cost of shipping" from a source to this fictitious destination might represent the cost of storing an idle container, or might be a negative number standing for a fee paid the system by some external entity for a container or its use. Similarly, an overall deficit of containers can be accommodated in the submodel by introducing a fictitious source, "shipments from which represent acquisitions of additional containers. The variations on this basic theme, which can be handled by essentially the same techniques, are quite numerous.[21]

Although the above material referred only to the U.S. system of centers, the mathematica formulation also applies to the world-wide system, with excesses and deficits at centers relative to those containers of concern to the United States. Such a model would provide guidance for transoceanic shipment of empty containers between specific centers and/or ports, to restore total balance.

The minimized value of C represents an additional cost (possibly negative, i.e., a benefit) associated with the system of centers. Questions of ownership and policy, concerned wit how this cost's burden might be distributed, are beyond the scope of this discussion.

Combining these themes with that of the preceding section leads to "multi-period transportation problems," whose solution is not yet routine but seems quite feasible.[22]

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[21] L. R. Ford, Jr., and D. R. Fulkerson, Flows in Networks, Princeton U. Press (1962).

[22] D. Eklof, The Multi-Period Transportation Problem, Johns Hopkins U. doctoral thesis (1967). See also W. A. Horn, Determining Optimal Container Inventory and Routing, National Bureau of Standards Report 9936, 10/68.

Chapter VIII. THE COMPUTER PROGRAM

This chapter contains a description of the FORTRAN computer program written to implement a version, appropriate to the data at hand, of the mathematical model described in Chapter VII. Illustrative applications of this program are presented in the following Chapter IX while an annotated listing of the computer code itself is retained at the National Bureau of Standards and the Maritime Administration. Occasional references to Chapter VII are made below, but familiarity with that chapter, though helpful, is not essential in reading the present one.

The sequence of presentation for the following material is based on a desire to serve three classes of readers who require progressively finer levels of detail.

First are those readers seeking a management-level overview of the program. For this purpose, we have provided an informal specification of the kinds of input data required, the types of output information produced, and also the additional information of likely interest which is implicitly available from the program and could be explicitly output with a minimum of recoding.

Second are those who will be charged with the actual operation of the program "as is", i.e., setting up specific "runs", interpreting and reporting the results, and perhaps carrying out the (routine) conversion of the code for use on computers other than the one (UNIVAC 1108) for which it was written. The chapter goes on to provide information needed by such readers: the output formats, the allowable ranges of values for the various data inputs, and the specific forms of the necessary control cards and data decks. Additional information for associated off-line calculations is given early in the next chapter.

The text of the report includes a number of model refinements and features which have not been included in the present code. Moreover, this code has been written to fit within "internal memory" without any special use of memory-saving techniques. Those who might be called upon to incorporate such modifications--refined or additional model features and/or relaxation of limits on problem size--constitute the third class of readers we have in mind. For them, the balance of this chapter was to have contained a rather detailed account of the logic of the current code, its breakdown into subroutines, numerical problems encountered and how they are circumvented, etc. This goal of full detail has proven unattainable within the time available; instead, the program was provided with extensive "comment cards" which, together with the partial description written below, should provide sufficient detail for the experienced programmer.

General Program Description

The basic flow of the program involves iteration of the following two steps:

1. Assignment of shippers to centers, so as to minimize total generalized costs between each shipper and "his" port via the center.

2. For a given assignment, location of each center so as to minimize the costs to its users.

Step 1 is performed by subroutine ASSIGN, and step 2 by subroutine LOCATE. The process is started with an initial location of centers given by subroutine INITAL.

Subroutine ASSIGN tests each center in turn, as the center which shipper i should use in order to minimize the total generalized cost to his U.S. port of departure. The best of these is compared with the cost of direct shipment to that port. (Other ports are also tested as possible consolidation points.) The shipper is then assigned to that center or port which yields minimum generalized cost. This basic logic has been refined to insure satisfaction of two conditions relating to patronage at any center for any class of (co-containerizable) cargo: First, that the cost elements governing the assignment of shippers to the centers, insofar as they depend on the class's total patronage at the center, have numerical values consistent with the patronage pattern of the assignment itself. Second, t a center serve a cargo class only if there is sufficient patronage to provide (on the avera acceptably prompt filling of containers.

The generalized cost is made up of several factors: the dollar costs of transportation from shipper to center and center to port, the dollar cost of the time spent in shipper-to-center and center-to-port* transportation, and the cost of the accumulation delay** at the center. Center-to-port costs take into account the cargo inflation factor which results from a load factor <1 on outgoing containers. Processing-cost elements which depend on center location can be taken into account.

[Not represented in the present code are those generalized cost elements related to center-size-dependent service delays or processing costs at the center. The inclusion of these factors is discussed at the conceptual level in Chapter VII, and their future incorporation (if desired) seems quite feasible given the requisite data.]

The process used in subroutine LOCATE is the iterative one developed by Kuhn and Kuenn and independently by other authors,[24] to calculate the point C which minimizes a function o the form

$$f(C) = \sum_i w_i d(C,X_i),$$

where $d(C,X_i)$ is the Euclidean distance between points C and X_i, and w_i is a "weighting factor" associated with point X_i. In our problem, C represents a center's location, the points X_i are the locations of the shippers using that center (and the U.S. ports of entry or departure for those shippers' cargo), while w_i is the product of (a) the generalized shipping rate§ between C and X_i and (b) the quantity† shipped.

Iteration of the two steps--assignment of shippers to centers in subroutine ASSIGN, an relocation of centers in subroutine LOCATE--continues until the total generalized cost to a shippers changes less than 0.5% from one iteration to the next. The program then prints th desired information, and goes on to repeat the whole process for the next number of centers to be investigated.

<u>Input Data for Program</u>

The input information required by the program is listed below. Details, on the preparation of the data cards themselves, are given later in the chapter. Model modifications to accommodate more refined empirical data would of course involve corresponding refinements in this list.

1. The locations of the U.S. ports involved.

2. For each shipper: location, U.S. port used, class of commodity shipped, quantity (lbs.) shipped or received per year, reciprocal of average time (days) between successive shipments, co-containerization class.††

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*The present code reflects differences, among delay sensitivities of various commodity classes, in treating shipper-center movements but not center-port movements.

**Defined in the previous chapter.

[23] H. W. Kuhn and R. E. Kuenne, An Efficient Algorithm for the Numerical Solution of the Generalized Weber Problem in Spatial Economics, J. Reg. Sci. 4 (1962), pp. 21-33.

[24] L. Cooper, Location-Allocation Problems, Oper. Res. 11 (1963), pp. 331-343; W. Miehle, Link-Length Minimization in Networks, Oper. Res. 6 (1958), pp. 232-243; F. P. Palermo, A Network Minimization Problem, IBM J. Res. Dev. 5 (1961), pp. 335-337.

§"Generalized" in that it includes a term to represent the cost of time spent in transit.

†These quantities are measured in weight units throughout; some of the necessary conversion are given early in the next chapter.

††Shipments are "co-containerizable" if their natures and overseas distribution points permi their being consolidated together.

3. Number of U.S. ports involved, of exporters, of commodity classes, of co-containeriz- le cargo classes per port. Set of values of n (= number of inland centers) for which ogram is to be run, specified by starting level; final level, and increment.

4. The transportation rate per pound-mile for a full container; the LCL rate per pound- le for each commodity class.

5. The dollar value per pound for each commodity class, and the average value per pound r all cargo. Multiplicative factors (for rail and truck, respectively) for converting stance traveled into time consumed, and an inventory carrying factor which (together with e dollar values) is used to convert from time consumed to dollar costs.

6. The fraction of total nominal cargo to be regarded as LCL material requiring con- lidation. (In the present code this same fraction is applied to every shipper; varying it ovides a means of testing sensitivity to the level of demand for consolidation services.)

7. The value to be attributed to "work-days per year" at the centers.

8. The nominal weight (lbs.) of a full container.

9. The "target" load factor and maximum permitted holding time (in days) for outgoing ntainers, and the average load factor for incoming containers.

10. If center-related costs are assumed to vary with location among the "boxes" of a ctangular grid: the number of rows and columns of the grid, and the incremental unit cost plicable for locations in each box.

In addition to these basic data, there are several other input categories:

11. Maximum allowable average accumulation delay (if desired).

12. Minimum acceptable total patronage at a center in terms of containers filled to the rget load factor (if the minimum patronage restriction is to be applied).

13. A list of cities, their locations, and a tolerance distance in miles; for use in rinting out which cities have centers nearby, when this output is desired.

14. Average cost and average time to stuff a container at a center.

15. Specification of the desired version of the program and of the outputs to be rinted. (The available alternatives are described below.)

16. If random initial locations for the centers are to be used: the "seed" or "kick-off" etting for the pseudo-random number generator (so that the "random" factors can be duplicated other computer runs if desired, or duplication can be avoided if preferred).

tput Information

The user of the program can specify which of the following types of information are to printed out. Any subset of these categories can be chosen.

1. A repetition of the input data for each shipper. (Selection of this option of course eads to relatively voluminous output.)

2. For each number of centers considered, the initial trial locations for all centers.

3. For each center: its final (optimized) location, its zone of patronage (the smallest ectangle, with sides parallel to the coordinate axes, which contains all the center's users), he mean daily volume of business (lbs. of payload), daily output (of containers), average utgoing load factor, distribution of center input tonnage between import and export (per- entages), total generalized cost to users expressed as a fraction of the cost to them if hey had to ship LCL direct to the U.S. ports they use.

4. For each shipper: the center he uses, and the generalized cost incurred as a fraction of that which would be incurred in the absence of inland centers.

5. For each commodity class: the total generalized cost incurred in the processing and movement of that class's shipments, as a fraction of the cost in the absence of inland center

6. For each co-containerization class: the average load factors of containers, and total generalized cost as a fraction of that in the absence of inland centers.

7. The final random number generated, for use in continuing the pseudo-random sequence in later "runs." (The initial "seed" must be recorded separately by the user if it is to be employed again.)

8. Information as in 3 for each port center.

9. The number of centers within a specified distance of each of a list of cities.

Other Potential Outputs

A number of quantities of potential interest, in addition to those listed above, are explicitly or implicitly calculated during the program's operation and could be provided as outputs with relatively little reprogramming. While the present output repertoire was felt to be most suitable for the current exercising of the model and to illustrate adequately the types of information that could be obtained, a brief sketch of readily available further outputs may be of value here.

For each shipper, the model calculates and could print out the accumulation delay, time in transit to the center and from the center to port, dollar cost of transportation between shipper and center and between center and port, and total dollar cost. These outputs might be used to analyze which shippers derive the most benefit from the system of centers and how this benefit is divided between dollar and time components.

The distributions of benefits among commodity classes, among co-containerizable cargo classes, and among ports seem likely to be of interest. One might want to know which centers serve which ports (to various degrees), and which commodity classes would constitute the bulk of particular centers' business. As an aid to such analyses, printouts could be obtained--for each commodity class, or co-containerizable class, or port--of percent of cargo quantity associated with each center, as well as overall average time delays and dollar costs.

Program Options

Besides the choice of outputs described above, a number of additional options are available to the user of the program:

1. The model may be used to evaluate a specific set of locations for centers. In this case, of course, no attempt is made to optimize the center sites.

2. Four different procedures for choosing initial center locations have been incorporated*:

   (a) Initial locations can be specified on punched-card input.

   (b) They can be randomly chosen, within the rectangle bounded by the maximum and minimum x and y coordinates of the shippers' locations.

   (c) They can be chosen near shippers, selected at random from the shipper population.

   (d) They can be placed near the shippers with the greatest annual cargo flows (in weight).

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* Still other procedures are plausible, but these four seemed representative.

3. The program can be instructed to impose a restriction which prohibits a center from accepting cargo from a co-containerizable class if the center's patronage for that class is so low that mean accumulation delay (for an adequately full container-load) would exceed the maximum allowed.

4. To speed up the calculations, ordinary (Euclidean) distances may be replaced throughout by the "Manhattan metric" distances described in Chapter VII.

5. To take into account those elements of center-associated costs (e.g., pertaining to land acquisition and for labor) which might prove sensitive to rough geographical location, the region can be divided into rectangles each of which has a different associated additional cost per container. These cost increments are entered into the assignment-step and location-step calculations.

6. Since different initializations may lead to different locally optimal solutions, the program may be directed to try more than one set of initial locations for the same number of centers.

7. The program may start out using the Manhattan metric and only use the Euclidean metric after the center locations have settled down under the Manhattan metric.

8. The program may enforce a minimum total patronage restriction on each of the centers. This is accomplished by reinitializing centers which receive too little throughput, and then starting the assignment-location process again.

9. The program may be required to keep the locations of some centers fixed. The fixed-location centers affect the assignment and enter into the total cost figures, but are never allowed to be relocated.

General Remarks

In concluding this management-level overview of the program, it must be repeated that model outputs are only as good as the input data. Since actual locations of centers may depend on many factors not explicitly represented in the computer program, such as access to railroads or interstate highways, availability of land in the exact area chosen, and so forth, the program outputs can only be used as a guide to location. Also, since no work was done on demand forecasting, the model's results will reflect only the existing distribution of demand. Runs testing the sensitivity to demand can be made, but these do not include changing patterns of demand (resulting from the existence of centers, for instance). The next chapter includes a description of the data manipulations, nominal values of parameters, and the sources of the data. Further efforts to obtain more precise values for some of these data appear highly desirable.

Input Formats

Passing to a more detailed phase of the exposition, this section describes the input formats required in setting up a problem to run on the computer. Shipper-related information constitutes a tape input, while all other data are entered on punched cards.

The card formats will be described first. All numbers should be right justified in their fields. The number of decimal places for each floating-point input is given below.* Card input is from logical unit 5. Users should consult subroutine INPUT in the program listing for further information.

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* E.g., the phrase "3 dec. places" applied to a variable with a 6-digit field implies a number of decimal form (ab.cde). (The decimal point is counted as one field.)

Card No.	Columns	Information
1	1-10	Smallest number of centers considered.
	11-20	Largest number of centers considered (≤50).
	21-30	Increment in number of centers considered.
	31-40	Number of commodity classes (≤100).
	41-50	Number of co-containerizable classes per U.S. port (≤20).
	51-60	Number of exporters.
2	1-5	Successive entries of array IDO, specifying program options.
	6-10	See program listing's comments for instruction code.
	⋮	
	71-75	
3	1-5	
	6-10	Continuation of the entries in the array IDO.
	11-15	
4	1-10	Fraction of nominal cargo to be treated as LCL. (4 dec. places)
	11-20	Number of working days per year.
5	1-20	Transportation rate ($/lb-mi) for a full container (7 dec. places).
6	1-10	LCL transportation rate for each commodity class (7 dec. places).
	11-20	
	⋮	
	71-80	
7	1-10	Inventory type carrying charge (yr.$^{-1}$)(4 dec. places).
	11-20	Factor for converting distance to rail time (days/stat.mi.)(4 dec.places)
	21-30	Factor for converting distance to truck time (days/stat.mi.)(4 dec.places)
8	1-10	Average value/lb. ($/lb), all cargo (3 dec. places).
9	1-10	Average value ($/lb) for each commodity class (3 dec. places).
	11-20	
	⋮	
	71-80	
10	1-10	Circuity factor used to reflect the difference between straight line distance and the actual distance upon which rates are based (3 dec. places).

Card No.	Columns	Information
11	1-10	Target load factor for outgoing containers (3 dec. places).
	11-20	Maximum holding time (days) at all centers (3 dec. places).
	21-30	Nominal weight (lbs) in a full container.
12	1-10	Average load factor for incoming containers (3 dec. places).
13	1-10	Average cost to stuff a container at an inland center (3 dec. places).
	11-20	Average amount of time (in days) to stuff a container (3 dec. places).
	21-30	Number of days spent in rail terminals (3 dec. places).
	31-40	Number of days spent in truck terminals (3 dec. places).
14	1-10	x-coordinate* of first port (2 dec. places).
	11-20	y-coordinate* of each port (2 dec. places).
		NOTE: This is followed by a similar card for each of the other ports. (At present, number of ports ≤ 3.) These must be followed by a card with 7/8 in column 1, EØF in column 3.
15	1-24	Random number "seed". (Needed only if location initialization 2 or 3 is used or if the minimum total patronage restriction is imposed.)
16	1-10	Maximum allowable average delay at any center (3 dec. places). NOTE: This card is needed only if IDO(3) > 0.
17	1-10	Minimum allowable total patronage at an inland center, in containers per day (5 dec. places). NOTE: This card is needed only if IDO(17) > 0.
18	1-12	City name
	21-30	x-coordinate of city location (2 dec. places).
	31-40	y-coordinate of city location (2 dec. places).
	41-50	Tolerance distance (2 dec. places).
		NOTE: This is followed by similar cards for each other city for which it is desired to print out the number of centers within the specified distance of the city. The final card must have 7/8 in column 1, EØF starting in column 3. These cards are needed only if IDO(18) > 0.
19	1-10	Number of rows in grid.
	11-20	Number of columns in grid. NOTE: This card is needed only if IDO(12) > 0, i.e., location-dependent costs are considered. The number of boxes in the grid must be ≤ 100.
20	1-10 11-20 ⋮ 71-80	Cost increment (F/container) for successive boxes in first row of grid (2 dec. places). NOTE: Followed by similar cards for the remaining rows. Needed only if IDO(12) > 0.

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* The coordinate system is described early in the next chapter.

The shipper-related input information is on a tape to be mounted on logical unit 7. Exporter data appear in a block at the beginning of the tape, and importer data in a block at the end. At present the number of shippers must be ≤ 3000. There is one tape record per shipper, with the following format:

Columns	Information
1-11	x-coordinate of location (2 dec. places).
12-22	y-coordinate of location (2 dec. places).
24	U.S. port used.
26-28	Commodity class.
30-40	Quantity (lbs./yr.)
41-47	Number of shipments/yr. (0 dec. places).
48-50	Co-containerization class.

Conversion to Other Computers

The present program is written for the National Bureau of Standards' UNIVAC 1108 computer, under the EXEC II operating system. Its "language" is FORTRAN V, which contains the instructions of FORTRAN IV as a subset and has some additional features. If the program is to be run on some other computer (with at least FORTRAN IV capability), the following machine-dependent features may need alteration.

1. The UNIVAC 1108 at NBS has 65,000 36-bit words of storage, of which about 50,000 words are available under EXEC II. Reduction of array dimensions might be necessary or expansions might become possible for the core storage of some other machine.

2. On our machine, card input is from logical unit 5, and printer output is on unit 6. Unit 7 has been used in the program for tape input. These selections of logical input/output units might vary for another computer.

3. The present code contains comments written on program instruction cards following a 7/8 punch.

4. The present code includes FORTRAN "READ" statements of the form

READ(unit, format no., END=k).

Such a statement yields a transfer of control to statement number k when an end-of-file condition is encountered in the input.

5. The code employs a random number routine (RANDNO) written in 1108 assembly language. A similar routine would be needed on another machine.

6. The 1108 allows and correctly calculates expressions involving mixed-mode arithmetic. In view of possible conversion difficulties, an attempt has been made to replace such statements throughout the present code, but some may have survived.

7. The 1108 FORTRAN V allows the use of Hollerith characters appearing between quote marks instead of the nH... form. This avoids the necessity of counting characters to be printed.

Subroutines and their Functions

The final level of detail to be presented consists of brief descriptions of the program's various subroutines and their roles.

1. MARAD: This is the main routine, which controls the iterations through assignment and location steps alternately. It also calls the input routine and the routine which calculates initial center locations.

2. CDIST: This subroutine calculates an array DIST used for a rapidly calculable approximation to Euclidean distances. (Thus it is not performed if the Manhattan metric is to be used.)

3. $D(X_1, X_2, Y_1, Y_2)$: The function D calculates the distance between points (X_1,Y_1) and (X_2, Y_2). If the Manhattan metric is involved, the exact formula

$$D = | X_1 - X_2 | + |Y_1 - Y_2 |$$

is employed. If Euclidean distance is involved, the quantities

$$M = \max \{|X_1 - X_2|, |Y_1 - Y_2|\} ,$$

$$m = \min \{|X_1 - X_2|, |Y_1 - Y_2|\} ,$$

are first calculated. Then the exact result

$$D = M \{1 + (m/M)^2\}^{\frac{1}{2}}$$

is approximated by replacing the proper fraction m/M by the nearest fraction of the form I/1000, I an integer. The approximate square root is obtained from the above-mentioned array DIST, which contains the 1001 values

$$\{1 + (I/1000)^2\}^{\frac{1}{2}} , \qquad I = 0, 1,\ldots 1,000 .$$

4. INPUT: Card and tape program inputs are read in by this routine. Transportation rates (per quantity unit per mile) are incremented to include transit time factors. The annual nominal cargo levels are multiplied by the input parameter WLCL (to obtain the fraction attributed to LCL cargo requiring consolidation at a center or port), and then divided by the "working days per year" factor to obtain an average daily level. In addition, the reprinting of desired input data (to "label" the output fully) is carried out by this subroutine.

5. INITAL. This subroutine calculates initial locations for each number of centers. There are four initialization procedures; IDO(1) controls which one of these is used for a particular run. The first reads user-specified initial positions from punched cards. (WARNING: a center should never be initially located exactly at any shipper. The location subroutine is such that such a center will never move.) A second initialization option randomly locates centers in the area bounded by the minimum and maximum x and y coordinates of shippers. The third initialization procedure locates centers near randomly chosen shippers. The final method locates centers initially near the "heaviest" shippers. For the purposes of this last initialization, the tonnage shipped by all shippers at one point is aggregated to decide which locations ship the greatest amount of cargo.

6. LOCATE: This subroutine performs the location step; for a given assignment of shippers to centers it calculates, for each center in turn, the location of that center which is best for its users. Accumulation delay need not be taken into account, since it is constant (independent of center location) for a given assignment. However, inflation factors (describing container load factors less than 1) must be taken into account in costing cargo movement between center and port. LOCATE calls the location subroutines appropriate to the metric used.

7. KK: This subroutine involves the use, for each center C, of the Kuhn-Kuenne procedur (cited earlier) to choose the location of C so as to minimize a function of the form

$$F(C) = \sum_i w_i d(C,X_i).$$

This process is an iterative one; if (x_i,y_i) are the coordinates of X_i, and $(x^{(t)}, y^{(t)})$ are the coordinates of the t-th stage estimate $C^{(t)}$ of the optimal location, then the coordinates of the next estimate $C^{(t+1)}$ are given by

$$x^{(t+1)} = [\sum_i w_i x_i/d(X_i,C^{(t)})]/[\sum_i w_i/d(X_i,C^{(t)})],$$

$$y^{(t+1)} = [\sum_i w_i y_i/d(X_i,C^{(t)})]/[\sum_i w_i/d(X_i,C^{(t)})].$$

This process obviously will run into difficulty if at some (n-th) iteration, $C^{(t)}$ is very close to some point X_I among the X_i's, so that $d(C^{(t)}, X_I) \ll 1$. This will certainly occur, for example, if C is "homing in" on the optimal solution, which happens to be at the point X_I. It is therefore important to have a test (necessary and sufficient condition) for the optimal C to be at X_I. Kuhn and Kuenne (op.cit.) show that the following condition constitutes such a test:

$$(w_I)^2 \geq [\sum_{i \neq I} w_i(x_i - x_I)/d(X_i,X_I)]^2 + [\sum_{i \neq I} w_i(y_i - y_I)/d(X_i,X_I)]^2 .$$

In our implementation of this algorithm, whenever any $d(C^{(t)}, X_I)$ becomes less than 0.1, the preceding test is applied to the point X_I to see whether the center should be located at X_I. If the test fails, C is allowed to draw closer to X_I in its passage to the solution. (In this case, as an extra precaution, a program halt accompanied by an error message occurs if convergence is not achieved after another 35 iterations. This precaution has proven unnecessary so far.)

Termination of the iterations through subroutine KK occurs in one of two ways:

1. if the location $C^{(n+1)}$ differs very little from the point $C^{(n)}$ obtained in the previous iteration,

2. if the partial derivatives of F(C) evaluated at $C^{(t)}$, namely

$$\sum_i (w_i/d(C^{(t)},X_i))(x_i - x^{(t)}), \ \sum_i (w_i/d(C^{(t)},X_i))(y_i - y^{(t)}),$$

are both sufficiently close to zero.

The second section of KK is also employed for each center in turn if IDO(12)>0, i.e., when possible dependence of unit processing costs on center location is to be considered. Here the total shipping area is divided into a grid of rectangles. For each of these rectangles, the optimal location within that rectangle is determined by the method described in Appendix C, and the resulting minimized generalized cost for transportation is added to the location-dependent cost associated with the rectangle. The optimal location, in the rectangle for which the sum is least, is then selected.

8. SORT: This subroutine and the next are used when distances are to be measured by the Manhattan metric, rather than the Euclidean one. SORT is used first to sort the x-coordinates of the shippers and ports using a given center, and then to sort the corresponding y-coordinates.

9. MEDIAN: Finding the optimal center location under the Manhattan metric reduces to finding the x and y medians of the shippers and ports weighted by their annual quantities of LCL cargo. This subroutine calculates these x and y medians.

10. ASSIGN: In this subroutine, "current" locations for all centers are given. For each shipper, each center (as well as the U.S. port involved) is tested as a possible consolidation point, and the point yielding the smallest generalized cost is chosen.

For reasons explained in the last chapter, this basic logic requires considerable modification. Each assignment step (i.e., each passage through ASSIGN) consists of a number of substeps, each in turn consisting of a number of sub-substeps. These two levels of detail will be explained in sequence.

Each substep begins with a specification, for each co-containerizable cargo class, of which centers are "off limits" to that class (because of insufficient patronage to ensure a prompt or economical accumulation of material). These prohibitions are enforced throughout the substep, which ends with an assignment of all shippers to centers. The patronage pattern from this assignment may be such that additional centers have insufficient patronage from some of the co-containerizable classes they serve. For each such class, that center serving it which is the worst violator of the patronage adequacy criterion is now also declared unavailable to the class. These additional prohibitions, together with the previous ones, make up the specifications for the next substep. The assignment step terminates when no further prohibitions are required, i.e., when all centers serve only those shippers whose cargo they can handle expeditiously. Note that termination after some finite number of substeps must occur (if each substep terminates), unless some co-containerizable class has too little volume for efficient service even when concentrated at a single center.

The particular condition, which in the present code triggers prohibition of a center to a co-containerizable class, is that the mean delay in waiting for the "target" fraction (L) of a containerful to accumulate exceeds a specified limit. Different time limits at different centers could easily be inserted. The mean accumulation delay is calculated using the Poisson-arrivals model and the (L,H) dispatching policy described in Chapter VII. This concludes the description of the breakdown of an assignment step into substeps, except to remark that this portion of the logic is exercised only if IDO(3) >0; otherwise the assignment step consists of a single substep, and no service prohibitions due to inadequate patronage are imposed.

Each sub-substep begins with assumed values for mean accumulation delay and mean inflation ratio, for each co-containerizable cargo class at each center. These values are used in calculating the generalized costs to a shipper, of using the various centers available to him. On the basis of these costs, the shippers are assigned one by one to centers. When this is complete, the patronage pattern from the resulting assignments is used to calculate the accumulation delays and inflation ratios for the next sub-substep. The substep terminates when two consecutive sub-substeps yield total generalized costs (summed over all shippers) which differ by less than 0.5%. That is, the generalized costs leading to the final assignments and those derivable from these assignments match fairly closely, at least so far as their totals (over all shippers)--aggregate measures of system performance--are concerned.*

Both the initialization and the recursive aspects of this process must be specified further. As to the former: in the first sub-substep of the first substep of an assignment step, the accumulation delays and inflation ratios are set at 0 and 1 respectively, so that only generalized transportation costs enter this first set of assignments. As to the recursion: it would be simplest to take the accumulation delays and inflation ratios for a new sub-substep, to be those corresponding to the patronage pattern for the assignments found in the last sub-substep. This "total updating," however, was found to constitute an over-adjustment which failed to yield convergence. Instead, the accumulation delays and inflation ratios for a new step are calculated as a weighted average of (a) those assumed throughout the last set of assignments, and (b) those corresponding to the patronage pattern resulting from these assignments. The weight of (a) is designated WEIGHT in the program, and so that of (b) is 1-WEIGHT. Taking WEIGHT = 0.25 has proven satisfactory so far.

11. DELAY: This subroutine calculates the average accumulation delay and inflation factor resulting from an assignment for each co-containerization class at each center (see equations [7.40] and [7.41]).

12. OUTPUT: This subroutine prints the desired outputs.

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* Experimentation with alternative termination criteria might be desirable.

Chapter IX. ILLUSTRATIVE APPLICATION OF THE MODEL

This chapter describes the application of the computer program described in Chapter VIII to illustrative situations based on the data of Chapters III to V. The discussion of "results" is aimed at giving the reader a clearer picture of what kinds of qualitative and quantitative inferences can be drawn from the outputs of the model. It must be emphasized, however, that the particular results described below are illustrative (of what the methods developed in this study are capable of), but not definitive; they are based on input data with serious deficiencies (e.g., concerning the overseas destinations of exports), on a computer program which does not reflect the final phase of model development,* and on numerical explorations severely restricted (for practical reasons) to only a fraction of the cases that could and would be covered in a full-scale application effort.

Despite these limitations, it is felt that the usefulness and feasibility of the study's approach--and the desirability of more intensive data collection coupled with further model refinement and implementation--have in fact been established. The reader will of course wish to judge for himself whether this feeling is justified.

The first sections below deal with input data. They state what values were selected as "most reasonable" nominal ones for transportation rates, average dollar values for commodity classes, load factors for containers arriving from overseas, and other input parameters. Also included is a description of the manipulations performed to bridge the gaps in the empirical information available, and thus to arrive at a semblance of a suitable data base.

Taken up next are the results from applying the program, under the above-mentioned "nominal" values of all input parameters. These computer runs were aimed primarily at investigating the effects of altering the number (n) of inland centers. Another objective was to evaluate the practical seriousness of the theoretical danger that plausible initial guesses concerning center locations could lead to far-from-optimal final results. In addition, these runs established that the solution method--involving many iterative calculations whose good behavior could not be guaranteed in advance--really would converge to an answer with a tolerable expenditure of computer time. Specifically, the computer runs averaged about 1.5 minutes per case. This was roughly doubled when the minimum total patronage restriction was in force.

The results described next in the chapter pertain to "sensitivity analyses" carried out to study the effects of plausible perturbations of selected input parameters from their nominal values. Within the time available, it was possible to perform such analyses only on a quite limited scale.

Other computer runs illustrate the capability for and consequences of treating the possibility of location-dependent summands in centers' unit processing costs.

Throughout this chapter the index of performance by which runs of the model are compared is the ratio of the total cost when inland centers are available, to the total cost if there are no inland centers. The total cost, for shippers offered no advantage by inland centers, is the same whether there are such centers or not. For shippers who would avail themselves of the centers, the total cost of doing so consists of a generalized cost of transportation to the center (containing both a dollar and a time component), a cost of accumulation delay at the center, a generalized cost for processing at the center (containing both a time and a dollar component), and a generalized cost for transportation in a container to the port. The corresponding cost when there are no centers consists of a generalized transportation cost LCL to the port, and a generalized processing cost for stuffing the container at the port.** The index of performance is calculated as the sum over all shippers of their total cost when inland centers are available, divided by the corresponding sum when there are no inland centers. Note that good performance of the system of centers is indicated by low values of this index, poor performance by values near 1.

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* The computer program does not attempt to optimize center sizes, nor does it evaluate the effects of center size on processing cost and time.

** Similarly for imports.

Typically, for the computer runs made using the Delaware Port Authority hinterland data described in Chapter IV, the value of the index of performance was between .51 and .63. This low figure would represent a substantial savings to the shipping community from such centers. Even with all the disclaimers and cautions that must accompany numerical results from an illustrative example only partially based on empirical data, the impressive magnitude of these reductions in total generalized cost provides a <u>very encouraging indication</u> of the economic worth of the inland consolidation center concept. (It should be recalled that this study has refrained from stipulations as to how these savings might be distributed among the sectors of the shipping community, which would include the inland centers themselves.)

<u>Shipper Related Data</u>

The shipper-related data employed have been described in detail in Chapter IV. The present study was confined to those shippers sending or receiving containerizable commodities* through the ports of Baltimore, Philadelphia, and/or New York. For each of those shippers in the area surveyed by the Port of Delaware River Authority and for their usage of the through ports just mentioned,** the following pieces of information were extracted from data tapes supplied by the Authority:

Total yearly tonnage,
Total yearly dollar value,
Mean frequency of shipment.

The shippers in each survey area (these areas are specified in Chapter IV) were "moved" slightly so as to lie at a single centroid point chosen in this area. The centroids (hence, the approximate shipper locations) are initially identified by latitude and longitude, but these coordinates are then linearly converted to (x,y) coordinates. The conversions are given by:

1° latitude = 68.5 mi., 1° longitude = 52.5 mi.,

and the origin of the (x,y)-system is at

Latitude: 37° N., Longitude: 73° W.

At this point, the list of shippers numbered over 3,000 exporters and over 900 importers. Further simplification was required, if computer running times and memory storage requirements were to be kept within acceptable bounds. The simplification process will be described next.

The first step was based on the observation that many of the shippers' cargo movements involve full container loads, not requiring a center's consolidation services. Thus the reported flow of shipments had somehow to be thinned out to an estimate of its "LCL component," the portion constituting the demand for consolidation services. It is important to recognize that this reduction was carried out only after the map analysis reported in Chapter VI had been performed; thus the results reported below are not directly comparable with those from the map analysis, which is based on all reported (export) cargo movement rather than an estimated LCL component of it.

This thinning-out was performed in the following crude but reasonable way. The average weight per shipment, for each shipper, could be calculated from the available data. If this average exceeded the nominal weight level (48,000 lbs.) corresponding to a full 40' container, the shipper was classified "non-LCL" and removed from the list of shippers of concern. All cargo from all of the remaining shippers is assumed to require consolidation at a center or at the port. This criterion errs in one direction by discarding the occasional LCL shipments of typically heavy shippers, but errs in the opposite direction by including in the centers'

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* Recall from Chapter IV that the decisions as to what should be regarded as containerizable, were far from cut and dried.

**A potential bias against inland centers is introduced here, in that their value relative to usage of ports other than these three is not "scored."

potential market all of the occasional full-container movements by "LCL on the average" shippers.* Further work aimed at a more refined treatment of this step seems desirable.**

The second step in the simplification process involved aggregation rather than elimination of shippers. All shippers lying in the same survey area (hence, essentially co-located) and belonging to the same co-containerizable class were aggregated to form a single new shipper. The difficulty here is that information on common membership of shippers in a co-containerizable class, although surely available in principle from an adequate data base, was not in fact deducible from the data at our disposal. Thus some "creative" operations on the data were required. These are detailed next; at any rate, the joint effect of the elimination and aggregation steps was to reduce the numbers of exporters and importers to tolerable levels (roughly 1025 and 375, respectively).

If cargo of several shippers is to be co-containerizable, it must

(a) move in the same direction (into or out of the U.S.) through the same U.S. port,

(b) be of physical natures permitting joint presence in a common container without damage, and

(c) involve the same overseas distribution/concentration point (or perhaps port-of-call area).

The available data permitted the straightforward application of criterion (a).

Regarding (b), there was the question of which combinations of commodity classes, each individually containerizable, would admit joint containerization. The variety within each commodity class precludes a really clearcut answer. Without a detailed study on this point, it seemed necessary to choose as working hypothesis one of the two logical extremes; either that any containerizable commodity class's material can be combined in a container with any other class's, or that no two commodity classes can share a container. The first of these alternatives was believed nearer the truth and was adopted. There is an obvious need for better data and further study to permit more satisfactory treatment of this point.

Since no information bearing on (c) was at hand, fictitious "data" of the appropriate kind were generated by the following process; it was arbitrarily assumed that each of the three U.S. ports in question served overseas points (capable of handling containerized cargo) which constituted up to at most 20 co-containerizable categories. The exporters using each port were assigned at random, one by one, to some one of 20 categories, and the same was done for the importers. The results of varying the "20" as well as the particular assignment to categories are described later in this chapter.

In summary, the following steps were performed on the data from the Delaware River Port Authority:

1. The following information for each shipper was taken from the data tape:

   a. shipper location code
   b. total yearly tonnage
   c. total yearly dollar value
   d. mean frequency of shipment
   e. commodity sent
   f. U.S. port of debarkation or entry
   g. sample factor

2. Shipments of non-containerizable commodities were eliminated.

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*It also ignores the possibility of some LCL cargo being noncontainerizable simply because its route lacks container-handling facilities.

**The program provides an option whereby the demand of each shipper may be multiplied by a constant factor. A run using this option to increase demand by 25 percent is described later in this chapter.

3. Shipments not departing or arriving at New York, Philadelphia, or Baltimore were eliminated.

4. Each tonnage and frequency figure was multiplied by the appropriate sample factor.

5. The average shipment size (total yearly tonnage divided by mean frequency shipment)was calculated; those shippers sending more than 48,000 pounds on the average were eliminated.

6. The commodity class (see Table 8) for each shipper was found.

7. The average dollar value per pound for each commodity class was determined.

8. Each of the remaining shippers was located at the centroid of the study subarea containing it.

9. All shippers in the same subarea who ship the same commodity class through the same U.S. port were aggregated into one shipper whose total tonnage and frequency were the sums of the constituent shippers' tonnages and frequencies.

10. Each of the aggregate shippers was randomly assigned to one of 20 co-containerization categories.

The input tape to the model then contained for each (aggregate) shipper:

x-coordinate of location
y-coordinate of location
port of U.S. entry or debarkation
commodity class
total pounds shipped per year
number of shipments per year
co-containerization category.

Other Input Parameters

The computer runs were based on conventions of 250 work-days per year, and 48,000 lbs. as nominal weight for a full 40' container, consistent with Chapter III.

J. Norris of the Maritime Administration's Office of Maritime Promotions provided the study with a suitable set of rough transportation rates based on an 8-fold commodity classification. The rate for cargo in a full container is taken as

0.135 cents/hundredweight/mile,

while the rates for uncontainerized cargo are given in Table 16. This subject was discussed in Chapter V.

Equation [7.4] gives the dollar component of travel cost, as a transportation rate times the amount of cargo shipped times the Euclidean distance multiplied by a circuity factor. The value 1.13 discussed in Chapter VII is used for the circuity factor.

Equation [7.8] shows how time in transit can be estimated as a linear function of distance traveled. These times are required in calculating the generalized costs of the shipper-center and center-port movements. The values suggested by Meyers et al (op.cit., p. 192) lead to a conversion factor of .0059 days/mi. for the rail mode. By omitting those delays not involved in truck transport, the same equation yields .0014 days/mi. for truck. The value suggested by Meyers et al for the constant term δ in equation [7.8], for the train mode, is 2 days; a value $\delta = 1$ day was felt to be representative for the truck mode. Within the program, it was in effect assumed that all uncontainerized shipments use the truck mode, while containers move by rail. Thus material containerized or broken out at a center uses the truck mode for the uncontainerized portion of its trip and the rail mode for the containerized portion.

Meyers et al suggests that for trips less than 300 miles, the truck mode is usually cheaper than rail, and the reverse for trips of over 300 miles. Most of the shipper-to-center trips (which are LCL) are less than 300 miles (many within one city), so the truck mode is most appropriate for these trips. The rates quoted in Table 10 are based on an average of rates to the 4 ports of New York, Philadelphia, Baltimore, and Norfolk, all trips of more than 300 miles. Therefore the rates used in the model will overestimate the cost of short trips. The direction of this bias is further enhanced by the fact that all these rates are for interstate trips, which are more expensive than intrastate hauls. Thus our use of the truck LCL rates given in Table 10 biases to some extent against the use of inland centers and raises their index of performance.

Recall that inventory-type charges I_k are employed to convert from time (delays) to dollars. Meyers et al (p. 192) suggest a figure of 10%/year, which gives a factor of 0.0004/day.* Appearing as a multiplier is the unit value/unit quantity (V_k); numerical values for these were found by averaging the $/lb. ratios for shippers in each commodity class, and these values appear in Table 17.

Table 16. Transportation Rates (cents/hundredweight/mile)

Commodity Class[†]	Rate	Commodity Class[†]	Rate
1	0.758	5	0.581
2	0.693	6	0.892
3	0.418	7	0.628
4	0.540	8	0.644

Table 17. Average Unit Values of Cargo ($/lb.)

Commodity Class[†]	Value	Commodity Class[†]	Value
1	2.334	5	0.091
2	1.788	6	20.029
3	1.111	7	0.887
4	0.834	8	2.933

[†] These numbered classes are identified in the corresponding table of Chapter V.

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*The use here of a 250 day effective year can of course be questioned; sensitivity to alternate figures could of course be tested, but off-hand is not expected to be substantial. Of course factors dependent on commodity class would give a more realistic picture of the situation.

The nominal load factor for inbound containers of all co-containerizable classes was taken as the single figure given in the document

North Atlantic Container Statistics
Report for 6-month Period Ending 12/31/67
(Office of Maritime Promotions, MARAD)

Of course the extension to all trade areas of results for the North Atlantic trade is questionable, and no account is taken of the effects of "filler" in the containers observed. (In the model, "load factors" are really "payload factors.")

The average figure given in the document, with 20'-containers as nominal, was 8.81 long tons. Doubling for conversion for 40'-containers, and recalling the 48,000 lb. nominal weight assumed for such a container, leads to a load factor of

$$2 \times 8.81 \times 2{,}240/48{,}000 = 0.822.$$

The nominal values used for the parameters describing the centers' dispatching policy represented what appeared to be the central tendency of the information and impressions gained during the study. These values are

L = "target" load factor = 0.90,

H = maximum holding time = 4 days.

For all the runs which are discussed in this chapter, centers were required to process enough cargo in each co-containerization class so that the average accumulation delay experienced by each class is less than 1.5 days. Whenever a center violated this condition for some class, the center at which cargo of that class experienced the largest accumulation delay was prohibited from accepting cargo of that class.

Chapter III provides nominal values for the cost and time to process one container: $94.28 and .125 days, respectively. Since all cargo is assumed to be containerized before going on board ship, and since center processing costs and times are being treated as independent of the center at which they occur, the values of center processing cost and time are constant and do not enter into the optimizing process.* They do, however, contribute to the index of performance and are added in at the end when it is computed.

It is recognized that many of the decisions described in this section and the preceding one are somewhat arbitrary, and no doubt can be criticized on a variety of grounds. They were selected by the project team as the "most reasonable" choices within the time and information available, and do appear fairly adequate for the illustrative aims of the computer exercises reported here. Further sensitivity runs to probe the criticality of the various assumptions made would of course be desirable. At any rate, the necessity to work through the detailed specification of a complete set of input parameters, made particular needs for better empirical data much more vividly apparent.

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* There is a provision in the program to consider center processing costs which vary for different geographical areas as described in Chapter VII. A run of the computer model was made to illustrate the use of this provision, and the results are described later in this chapter.

Computer "Map Analysis"

As noted above, the manipulation of the shipper input data was performed only after the (manual) map analysis of Chapter VI had been carried out. In order to display the distributio of the shipments (both exports and imports) of the revised demand data, two computer runs were made. The first divided the rectangle bounding the shippers into a 10x10 grid and calculated the output, measured in 90%-full 48,000 pound containers of export or import cargo, associated with each grid element. The results of this run are included in Table 18. The second run calculated the amount of cargo originating or arriving near each of several cities. Its results are given in Table 19.

The information from these two runs can be used in two ways. First, it provided guesses as to good starting locations for centers. As can be seen in Table 18, there are approximatel 8 areas from which the bulk of cargo is exported or imported - Philadelphia, western New York, Pittsburgh, Cleveland, southwestern Ohio, Detroit, Chicago, and Milwaukee. The total amount of cargo associated with all shippers is about 225 90%-full containers per day. Of this total, about 67%, or 150 containers per day, originates in the 8 areas above when Harrisburg and Allentown are included with Philadelphia, the southwestern New York area and Buffalo are lumped together as "western New York", and Pittsburgh and Johnstown are coupled as the Pittsburgh area.

Second, this information also gave an indication what numbers of centers were reasonable candidates for analysis in computer runs of the mathematical model. There is already a center located at the port in Philadelphia. With the cargo originating in the Philadelphia area subtracted from the total, the other 7 of the 8 areas listed produce a daily average of 103 out of the remaining 177 containers. Three of these areas, namely, Chicago, Cleveland, and Pittsburgh, would individually support small to medium sized centers. The other four areas appear at best marginal. From an examination of the two tables it seems unlikely that the study area would support more than 7 or 8 inland centers if each is typically to process at least 10 90%-full containers of LCL cargo per day.

Table 18. Geographical Distribution of Shipments*

.00	.49	.36	.00	.00	.00	.00	.00	.00	.00
.00	1.28	.00	.25	.76	.00	4.65	.00	.00	.02
.00	5.59 8	.00	1.06	.32 6	5.38	.00	2.59 2	4.47	1.07
.41	4.54	.00	1.01	2.49	.00	.00	1.61	2.74	.13
.15	.65	7 28.32	.60	3.78	21.81 5	2.17	.29	.37	.02
.60	.14	.09	1.12	1.57	8.68	8.45	3.16	4.73	.70
.00	.00	.48	.62	.77	3.93	7.82 3	7.74	.95	21.09 1
.01	.80	.39	1.41 4	5.11	1.77	1.97	.08	2.29	23.04
.01	.09	.06	3.58	4.77	1.59	.00	.00	.00	.94
.34	.00	.00	.00	.02	.00	.00	.00	.00	.30

1 Philadelphia-Allentown	5 Cleveland
2 Western New York	6 Detroit
3 Pittsburgh-Johnstown	7 Chicago
4 Southwest Ohio	8 Milwaukee

---

*Each entry gives the average daily number of 90% full containers associated with the corresponding rectangle of the 10x10 grid overlaid on the Delaware Port Authority's hinterland.

Table 19. Average Daily Number of 90%-Full Containers Imported or Exported for Each of the Areas Listed

Philadelphia	37.02	Detroit	11.04
Allentown, Pennsylvania	7.21	Southwestern Ohio	12.33
Harrisburg	3.46	Northwestern Ohio	3.31
Southwestern New York	2.52	Southern Michigan	.92
Buffalo	2.59	Indianapolis	1.83
Johnstown, Pennsylvania	6.82	Chicago	27.98
Pittsburgh	7.57	Milwaukee	7.03
Northeastern Ohio	.76	Central Illinois	.80
Cleveland	25.79	Other	62.41

When the results of the two computer map analysis runs are compared with the hand calculations in Chapter VI, the most striking observation is the great reduction in total demand (by a factor exceeding 2) introduced in the process of passing to the LCL component of the cargo. This occurs despite the use in the computer runs of a nominal figure of 43,200 pounds per container (90%-full 48,000 pound containers), as compared with the 51,000 pounds per container used in the hand analysis. As a result there are only two centers in Ohio, one medium and one small, after non-LCL shipments are deleted, whereas there are six listed in Chapter VI. Since the map analysis in Chapter VI was done only for Pennsylvania, Ohio, Indiana, and Illinois, it had no opportunity to exhibit centers in Milwaukee or western New York.

In summary, although the hand analysis in Chapter VI indicated that the study area could easily support 10 or more centers, passing to the LCL component of demand as is done here leads to a more conservative estimate: that present levels of demand probably justify at most 7 or 8 centers. This of course makes no allowance for subsequent growth of shipping volumes, especially for LCL cargo stimulated by the existence of the inland centers.

Effects of the Location Step and Minimum Patronage Constraint

A natural question is whether the optimization process embodied in the model's iterations through its "location step" is not more or less of a frill. That is, would not the mere availability of the inland centers, even if haphazardly located, secure essentially the same savings? Results bearing on this question are given in Table 20, which compares some values of the index of performance when centers were located at random with those obtained when the model was used to optimize starting with these locations.

The relatively low value of the index for the randomly located centers indicates that the very existence of centers provides a great savings, as much as one third. An additional reduction of about 1/10 in the index is provided by the optimizing procedure. The optimization procedure also provides a better chance that the centers will process at least the minimum amount of cargo--10 containers per day--indicated during the study as representing a profitable level of operation.

Table 21 shows the effects on the index of performance of requiring that centers meet the minimum total patronage constraint. This constraint reduces the index (for results from the optimization) by approximately an additional 5 percent. Although at first glance such a constraint might be expected to raise the index, it actually has the effect of lowering it for the following reason: In general, the index of performance is lower when more cargo is attracted to centers, and the minimum total patronage constraint has the effect of shifting "small" centers to points where they may attract more patronage.

Table 20. Effect on Index of Performance of Optimizing Center Locations*

No. of Centers	With Locations Random in Area	Optimized from Random Locations
4	.695	.579
5	.683	.577
6	.668	.571
7	.646	.562

---

* The random center configurations referred to in the second column were used as initializations for the optimizations to which corresponding figures in the third column refer. The figures are each averaged over 4 cases.

Table 21. Effect of the Minimum Total Patronage Constraint on the Index of Performance

No. of Centers	No Constraint†	Patronage Constraint††
4	.579	.553
5	.577	.550
6	.571	.530
7	.562	.523

---

† The figures for the case of no minimum total patronage constraint are averages over 4 cases.

†† The figures for the runs with the minimum total patronage constraint in effect are averages over 5 cases (each involving up to three initializations to secure satisfaction of the constraint).

Effects of Random Initial Locations

Recall that the solution obtained by the computer program can depend on the initial "guesses" as to center locations. Thus it is dangerous to examine only the results from a single initial guess since that guess might have been an unfortunate one leading to far from optimal results.

As noted earlier, several methods for generating initial guesses (if they are not supplied by the user) have been included in the program. Two are random in nature; one ("random at shippers") places the n centers near n randomly selected shippers, while the other ("random in area") places them at random in the smallest rectangle, with sides parallel to the axes of the (x,y)-coordinate system, which contains all the shippers.

Initial runs were performed, in part, to determine the effects of the random variations in initialization within each of the two random methods. While the resultant configurations of center locations were of course of great interest, it was felt even more important to get insight into the resultant fluctuations in the benefits provided by the system.

Table 22 presents the indices of performance found in these limited runs. The first thing to strike the eye is the low values--all between .5 and .63--recorded for the index of performance. These indicate very substantial economic benefits accruing from the presence of the inland centers. Such an impression is reinforced by the fact that the index of performance, as presently calculated, carries an anti-system bias in that accumulation delays for containerization at the ports are taken as zero.

Some immediate caveats are in order. There are of course all the previously-mentioned uncertainties and arbitrariness in the data and input parameters employed. Another point is that both numerator and denominator of the index of performance refer only to the U.S.-incurred portion of the generalized costs of origin-to-destination movement; one should consider how the results would be modified if the benefits of containerization for the over-water trip segment and overseas-land segment were taken into account.

Allowing for all these factors, the reported values still appear small enough to warrant the inference in the Summary that the establishment of inland centers promises wholesale advantages to the shipping community.

From an examination of the indices of performance in Table 22, there seems no reason to anticipate order-of-magnitude variations in the solutions obtained from different random initializations. The serious fluctuations are at about the 10% level, which seems significant enough to merit continuing the practice of multiple initializations for each case studied.

Table 22. Effects of Randomizations of Initial Locations on Index of Performance

Initialization Method	No. of Centers	Values of Index	Avg.	Min.
Random	4	.587,* .568, .600, .562	.579	.562
in	5	.626, .562, .544, .575	.577	.544
Area	6	.564, .551, .577, .589	.571	.551
	7	.571, .570, .565, .541	.562	.541
Random	4	.559, .616, .655	.610	.559
at	5	.545, .572, .578	.565	.545
Shipper	6	.550, .551, .566	.556	.550
	7	.566, .559, .617	.579	.559

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* Each value of the index refers to a separate initialization.

The previous material referred to the effects of random initialization on the index of system performance. It is also necessary to consider the effects on the locations of centers. For this purpose, a rough geographical classification of locations was developed, and is employed in the illustrative Tables 23-26. Examination of these tables reveals unanimity on some points, e.g., the need for one or more centers in the general areas of Chicago, Cleveland, and Pittsburgh. Other areas, in which a center was located starting from a substantial number of different initial configurations, are: southwestern Ohio, western New York, and Philadelphia. Milwaukee, although marginal, would probably also support a small center.

Perhaps the most striking difference between the location patterns obtained from the two random initialization methods is in their respective treatments of the eastern Pennsylvania area. Heavy concentration of centers here is especially odd in view of the proximity of the port's facility. For an explanation, recall from Chapter IV the sampling scheme used by the Delaware Port Authority. There are two factors in it which might bias the distribution of shippers in favor of Philadelphia. The first is the fact that Philadelphia is divided into 11 study subareas, whereas all other subareas are full counties. Since a "shipper", for the purpose of the model, is an aggregate of all shippers in a subarea who ship the same commodity through the same U.S. port, the definition of 11 subareas in Philadelphia would tend to produce as many as 11 times more shippers in the Philadelphia area as in an area of comparable size elsewhere. Secondly, the sampling scheme only samples larger (as measured by number of employees) firms in areas outside the immediate

Table 23. Variations in Rough Locations of Centers (4 Centers)

Area	Random in Area**				Random at Shipper**		
E. Pennsylvania (including Philadelphia)					1	1	2
Western New York	1	2	1			1	
Greater Pittsburgh				1			
N.E. Ohio					1	1	
Cleveland	1	1			1		1
Detroit							
S.W. Ohio			1				
N.W. Ohio						1	
South Central Michigan							
Central Indiana							
Chicago	1	1	1	1	1		
Milwaukee				1			
Other	1*		1	1		1	

Table 24. Variations in Rough Locations of Centers (5 Centers)

Area	Random in Area				Random at Shipper		
E. Pennsylvania (including Philadelphia)					1	1	2
Western New York	1		1		1	1	
Greater Pittsburgh			1			1	1
N.E. Ohio	1				1		
Cleveland		1	1	1		1	1
Detroit							
S.W. Ohio				1	1		
N.W. Ohio							
South Central Michigan	1						
Central Indiana							
Chicago		1	1	1	1	1	1
Milwaukee		1					
Other	1,1*	2	1	2		1*	

---

* Center received no cargo.
** Each column corresponds to a separate initialization.

Table 25. Variations in Rough Locations of Centers (6 Centers)

Area	Random in Area				Random at Shipper		
					1	2	3
E. Pennsylvania (including Philadelphia)	1						
Western New York		1		1	1	1	1
Greater Pittsburgh	1	1			1	1	
N.E. Ohio							
Cleveland	1	1	1	1	1	1	1
Detroit							
S.W. Ohio		1		1			
N.W. Ohio							
South Central Michigan							
Central Indiana							
Chicago	1	1	1	1	1	1	1
Milwaukee	1						
Other	1*	1	3,1*	2	1		

Table 26. Variations in Rough Locations of Centers (7 Centers)

Area	Random in Area				Random at Shipper		
					2	3	3
E. Pennsylvania (including Philadelphia)							
Western New York		1	1	1		1	1
Greater Pittsburgh				1	1		2
N.E. Ohio							
Cleveland	1	2	1	1		1	1
Detroit							1
S.W. Ohio				1			1
N.W. Ohio					1		
South Central Michigan							
Central Indiana	1						
Chicago	1	1	1	1	1	1	
Milwaukee	1	1	1				
Other	3	1,1*	3	2	2	1	

---

* Center received no cargo.

Philadelphia area. Such a sampling scheme may bias the distribution of commodities, since some commodities may be produced or imported mainly by smaller firms. Since shippers are aggregates handling the same commodity, the sampling procedure may again bias the distribution of shippers in favor of Philadelphia where every known exporting or importing firm was interrogated. Therefore, for both of these reasons, the model's input data may exaggerate the relative number of shippers in the Philadelphia area. Such a bias would lead the random-at-shippers method of getting an initial configuration of centers to start out, on the average, with unduly many center sites in the Philadelphia area, and thus to tend to end up with configurations suffering the same bias.

These results do not seem to permit a clear selection between the two random initialization methods. The values of the index of performance are approximately the same for the two methods. For the particular cases reported here the random-in-area method gives average values of the index which decrease with the number of centers. This would be expected on purely theoretical grounds since (a) the computer program does not incorporate any dependence of processing costs on center size, and (b) the minimum total patronage restriction was not in effect. The random-in-area method also avoids the bias towards Philadelphia mentioned above. However, the random-in-area method has a higher incidence of centers receiving no cargo. This may be explained by the fact that this method locates randomly in the smallest rectangle with sides parallel to the axes which contains the region bounding the shippers. If the region bounding the shippers were roughly oval in shape, then points near the corners of the rectangle would be apt to be bad points for locating centers.

From the results here, it is suggested that both methods be used to get an idea of possible combinations of initial locations to be tried. The next section will describe the choice and results of such combinations of locations.

Systematic Selection of Possible Initial Configurations

In testing the random initialization procedures, many different "good" final configurations were found from a variety of starting positions. Although in several cases the index of performance for the random start itself was quite good (low), improvement was still possible; the likelihood of happening by pure chance on an unimprovable configuration is clearly very small. However, it can be seen from the computerized map analysis and the results of the preceding section that centers should almost certainly be located in Chicago, Cleveland, and greater Pittsburgh. This suggested using a more systematic approach, which could take such guidance from previous experiments into account, and which should therefore give "good" final configurations with fewer initializations than would the purely random methods.

The computer runs described below all rely on taking the best locations among those encountered in placing n-1 centers, and then systematically choosing starting positions for the nth center . Three different ways of initializing the nth center were tried:

1. It was located near one of the n-1 positions already found.
2. It was placed at one of the cities which in the computerized map analysis was associated with at least 5 to 10 containers per day.
3. It was placed systematically in areas which did not yet have centers.

Although it is not logically necessary that the best locations for n-1 centers be (or lie close to) a subset of the best configuration of n centers, such a procedure seemed likely to provide a good starting approximation, and has in practice led to good final n center configurations involving only small adjustments in the initial n-1 locations.

The first method of initializing the extra center was to locate it near one of the original n-1. The results of such a procedure for 5 centers are given in Table 27. In general, this method ended up with two centers in one area. The one exception is Chicago, where the extra center ends up in Milwaukee. Cleveland and Pittsburgh are both associated with enough cargo to support two small centers; however, the southwestern Ohio area is not. This method does not seem too promising since it does not lead to significantly new configurations in most cases.

Table 27. Initializing by Splitting a Good Center into Two Located Near Each Other*

	5 Centers			
Western New York				
Greater Pittsburgh	2	1	1	1
Cleveland	1	2	1	1
Southwest Ohio	1	1	2	1
Chicago	1	1	1	1
Milwaukee				1
Index	.536	.537	.539	.536

---

* Based on a 4-center configuration with locations in Pittsburgh, Cleveland, southwest Ohio, and Chicago. The columns correspond respectively to initializations of the fifth center near each of the first 4, in turn.

An examination of the results of the computerized map analysis and of the runs with random initial locations, strongly indicates that Chicago, Cleveland, and Pittsburgh should have centers located at them. For this reason, the program includes an option to require some locations to have centers. These centers affect the assignment process but are never moved in the location step.

Table 28 gives the results of runs made to test initializing at some of the cities which are associated with at least between 5 and 10 containers per day in the map analysis (see Table 19). Line I of Table 28 gives the index of performance when the indicated centers are fixed; Line II gives the corresponding index when the line I entries are freed and all centers are allowed to move. The difference between the values of the indices in the two lines is quite small, and since the computer running time for fixed-center problems is only about half that of regular runs, this procedure provides a valuable tool for investigating starting configurations indicated by the distribution of shippers in the data. The best configuration for 5 centers was western New York, Pittsburgh, Cleveland, southwestern Ohio and Chicago; for 6 centers it was western New York, Pittsburgh, Cleveland, southwestern Ohio, Chicago and Milwaukee; and for 7 centers western New York, Pittsburgh, Cleveland, southwestern Ohio, Detroit, Chicago and Milwaukee. All of these centers process at least the desired minimum cargo per day (10 containers).

Two other investigations of possible initial center configuration were carried out using the procedure of fixing some of the centers. These involved fixing n-1 centers and investigating several points on a line as starting points for the nth center. Table 29 records the results of initializing centers at evenly spaced intervals on an east-west line segment west of Pittsburgh from central Ohio to central Illinois. In general, the resulting location of the last center is somewhat east of its initialization but in the same rough area. The indices are slightly higher than those in Table 28. Table 30 records the results of initializing at evenly spaced intervals on a line along the Pennsylvania-New York border east of Cleveland. All three different initializations produced essentially the same final configuration, with the free center ending in western New York state.

In conclusion, a systematic search for good initial locations lowered the final index of performance by about another 2 percent as compared with a purely random approach. For the present set of data, using the best known n-1 center locations as a start on the initial locations of n centers is quite useful. Simply splitting one of the n-1 centers in two does not greatly reduce the index. Trying a systematized search in areas where centers are not yet located seems to give the best results. When centers were initialized along the Pennsylvania-New York line east of Cleveland, there was a strong tendency to locate a center in western New York, and a corresponding lowering of the index from about .537 to about .523.

Table 28. Effects on Index and Location of Initializing Centers in Cities Chosen from the Map Analysis

	5 Centers			6 Centers		7 Centers
E. Pennsylvania (including Philadelphia)						
Western New York			1	1^F	1^F	1
Greater Pittsburgh	1^{F*}	1^F	1^F	1^F	1^F	1
N.E. Ohio						
Cleveland	1^F	1^F	1^F	1^F	1^F	1
Detroit		1			1	1
S.W. Ohio	1^F	1^F	1^F	1^F	1^F	1
N.W. Ohio						
South Central Michigan						
Central Indiana						
Chicago	1^F	1^F	1^F	1^F	1^F	1
Milwaukee	1			1		1
Other						
I. Index-some centers fixed	.540	.537	.523	.522	.522	
II. Index-no centers fixed	.533	.535	.522	.520	.522	.517

Table 29. "New" Center Initialized on Line West of Pittsburgh

	5 Centers					6 Centers					
Western New York						1^f	1^f	1^f	1^f	1^f	1^f
Greater Pittsburgh	1^{f**}	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f
N.W. Pennsylvania	1					1					
Cleveland	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f
Central Ohio		1					1				
S.W. Ohio	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f
N.W. Ohio			1					1			
North Central Indiana				1						1	
Chicago	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f	1^f
Milwaukee											1
Central Illinois					1						
Index	.536	.539	.538	.538	.539		.522	.523	.523	.523	.524

---

* The superscript F indicates those centers which were fixed for the runs of line I.

** The superscript f indicates those centers whose locations were fixed.

Table 30. "New" Center Initialized on
Line East of Cleveland

	5 Centers		
Western New York	1	1	1
Greater Pittsburgh	1^{F*}	1^{F}	1^{F}
Cleveland	1^{F}	1^{F}	1^{F}
Detroit			
S.W. Ohio	1^{F}	1^{F}	1^{F}
Chicago	1^{F}	1^{F}	1^{F}
Milwaukee			
Other			
Index	.523	.523	.526

* The subscript F indicates those centers whose location is fixed.

The Number of Centers

An obviously important question is that of the most appropriate value or range of values for the number (n) of inland centers to be established. As n increases, the transportation portion of total generalized cost should decrease. But for n past some critical value, the generalized processing-cost portion is expected to increase, since the large sets of centers will tend to contain some which are marginal with respect to total patronage, or to patronage by some of the co-containerizable cargo classes they serve. These two conflicting trends (decreasing generalized transportation costs and increasing generalized processing costs, as n increases) will typically interact to produce an n-value or range of n-values within which total generalized cost is minimized.

The mathematical model developed in Chapter VII appears adequate to grapple with the problem of optimizing n. However, this capability could be exercised only partially here, since the present stage of computer implementation does not include treatment of center-size-dependent processing costs and times. It does, however, contain two provisions for restrictions based on adequate patronage. The first prohibits centers from accepting cargo for a co-containerization class if the expected accumulation delay is too great. This prohibition was imposed in all the computer runs reported in this chapter. It should be noted here that the cost of accumulation delay (like other time components) is taken to be a linear function of the delay time for values less than the threshold, and (in effect) becomes infinite at the threshold level. For values of delay close to but less than the threshold, this linear approximation is probably too low. One would expect that the cost rate of time delay would increase as the delay approached the threshold. The effect of this underestimation of the cost of accumulation delay is to make a larger number of centers look more advantageous than it really is. The second prohibition method depends on total patronage at a center. Since any center processing fewer than 10 containers per day was to be regarded as uneconomical, whenever the final configuration of centers contained centers which processed too few containers, all such centers were randomly reinitialized. The model was then rerun with these new center locations plus the old "good" locations as initial configuration. This prohibition mechanism was used for the computer runs reported in Table 21, but was not used in all of our work because of the additional computer time required.

Although it is possible to have 7 or 8** carefully placed centers which meet the minimum total patronage restriction (the 7 final locations of Table 28 do), such a configuration is unlikely to be reached accidentally. Only one out of 5 of the configurations of 7 centers had all centers meeting the restriction after three reinitializations, and this configuration resulted in two centers in the Pittsburgh area. Two of the other cases had one center which received no cargo at all. Thus the computer runs tend to confirm the intuitive conclusion from the computer map analysis, that the given study area and demand pattern would support about 7 or 8 centers at most.

**A configuration of 8 centers would include one in the Philadelphia area.

Table 31 records the results of two runs made to test the 11 locations listed in Chapter VI. One run just evaluated the index of performance for these locations; the other used the locations as a starting configuration and optimized. The index of performance is lower, but not significantly lower than the smallest for 7 centers. On the other hand, only 5 centers in one run and 6 in the other processed the requisite minimum of 10 containers per day. From this it is clear that the operations performed on the Delaware Port Authority data (to isolate the LCL component) have significantly reduced the estimate of how many centers this area would support--on the basis of the cargo flow in the study year with no growth. Again it should be noted that the present model gives no "credit" for full container shippers utilizing the centers in order to take advantage of possible special reductions in transportation rates such as those for unit trains. The inclusion of such benefits would of course tend to increase the number of centers which the area could advantageously support.

Table 31. Computer Evaluation of the Locations of Centers Suggested by the Manual Analysis of Chapter VI

	No Optimization	Optimized Locations
E. Pennsylvania (including Philadelphia)	2	2
Central Pennsylvania	1	1
Western New York	1	1
Pittsburgh	1	1
Cleveland	1	1
Detroit	1	1
Southwest Ohio	1	1
Northwest Ohio	1	1
Indianapolis	1	1
Chicago	1	1
Index	.516	.514

Sensitivity to Dispatching Policy

The computer program was used to perform several illustrative sensitivity analyses. The first of these, pertaining to the centers' dispatching policy, is reported here.

Recall that in the mathematical model, a dispatching policy is described by the two parameters

L = "target" load factor (a dimensionless proper fraction),

H = maximum allowable holding time (days).

The nominal values selected for these parameters were

$L = 0.90$, $H = 4$ days.

For the sensitivity analysis, the "low" and "high" values of L were taken as $L = 0.85$ and $L = 0.95$ respectively. Lower values of L as "targets" are possible but seem somewhat unlikely, since rather full containers are desirable for payload-exploitation of the reduced rate for transporting containerized material. The low and high values of H were taken as 3 days and 5 days respectively. Beyond the 5 day limit, the disadvantages of extra delay might be accompanied by the cost of extra storage capabilities.

The results of this analysis are given in Table 32. The dispatching policy has an effect of 1.5 to 2.5% on the index of performance. The index seems much more sensitive to the target load factor than to the maximum holding time. Striving for high load factors (decreasing center-to-port costs, at the expense of higher accumulation delays) seems to be a good policy. Increasing the maximum holding time does not seem to make up, in extra cargo accumulated and the resultant lower containerized transportation cost, for the extra accumulation delay time involved. On the whole, the index of performance is less sensitive to dispatching policy than to different initial configurations.

Table 32. Variation of Index of Performance with Dispatching Policy

L	H	4 Centers	5 Centers	6 Centers	7 Centers
.95	4	.573*	.575	.567	.558
.95	5	.575	.575	.568	.558
.90	3	.577	.576	.571	.561
.90	4**	.579	.577	.571	.562
.90	5	.580	.579	.571	.563
.85	3	.583	.584	.577	.568
.85	4	.587	.583	.577	.569

---

* All the indices listed here are averages over 4 cases. The same set of initial locations was used for each of the 7 policies tested.

**Nominal case.

Sensitivity to Rate for Containers

The second sensitivity analysis was concerned with the (monetary) component of the transport rate for containerized material. This is an important parameter, since its difference from the corresponding rate for uncontainerized cargo provides the most obvious economic motivation for consolidation of LCL cargo in containers.

Table 33 records the indices of performance for variations of ± 10% and ± 20% in the value of the containerized freight rate. A change of 10% in the containerized freight rate brings about a 3% change in the index; a change of 20% in the freight rate causes about a 6% index change. Lower freight rates of course have correspondingly lower indices. The locations of centers are not substantially affected by lower freight rates and the lower rates (for this range of change at least) do not significantly alter the amount of cargo attracted to centers.

Table 33. Variation of the Index of Performance with Transportation Rate for Container Movement

Rates	4 Centers	5 Centers	6 Centers	7 Centers
.0000108	.542	.538	.534	.530
.0000122	.562	.562	.553	.545
.0000135†	.579	.577	.571	.562
.0000148	.597	.597	.587	.579
.0000162	.615	.615	.604	.599

---

† Nominal case. The value of the rate is in $/lb.

Sensitivity to Level of Total Demand

The third sensitivity analysis was concerned with the level of demand. Forecasting cargo demand for a future year was not within the scope of this project. However, it is clear that the proper number of centers and their index of performance depend on demand level. Also, the admittedly approximate operations which were performed on the Delaware Port Authority data have significantly affected the level of assumed LCL demand. Therefore, some analysis of the effects of alternative levels of demand is necessary to give insight into the consequences of both present data uncertainties and future growth.

Tables 34 and 35 record the results of increasing demand by 25% over the "nominal" level used in all other work. This increase is accomplished by increasing both the total yearly volume and the total yearly shipment frequency for each shipper by 25%. Thus the average shipment size remains the same; each shipper just ships more often. The same initial locations were used for both the nominal demand pattern and the "raised by 25% version." Both runs produced the same final configurations. The realized load factors remained substantially the same since the target load factor and average shipment sizes did not change. The number of containers processed by all centers increased by about 25% although the increase is not distributed evenly over the centers. (For instance, in Table 34, the center in Western New York gained only about 11%, while the one at Pittsburgh gained over 33%.) The center at Milwaukee, which was marginal at the nominal demand level, became much more robust once demand had been raised. Further investigations would be needed to evaluate whether the study area would support more than the seven inland centers or whether the seven centers would just each process more. The increase in demand level produced a small decrease in the index of performance, but this is not especially significant in comparing situations with different demand patterns.

Location-Dependent Processing Costs

Four computations are made to illustrate the treatment of monetary processing costs which depend on center location. The region of interest was divided into 4 rectangles by a 2x2 grid. It was first assumed that the processing cost per container would vary by the following additive quantities:

N.W. Rectangle = \$2.00
S.W. Rectangle = -6.50
N.E. Rectangle = -3.00
S.E. Rectangle = 5.00

These quantities were arbitrary, but (based on Chapter III and information gained during the study) are thought to be of reasonable order of magnitude.

The computation was made for 7 centers initialized as in Table 28. The index of performance rose, but only by about .02 percent which is probably less than the resolution capability of the model. No significant change in the locations of the seven centers was noted. The reduced cost in the southwest rectangle did not draw any other centers to join the one in southwest Ohio. The increased cost in the southeast rectangle did not drive out its single center at Pittsburgh, nor did the increased cost of the northwest rectangle drive out the centers of Detroit, Chicago, and Milwaukee. For this particular example the variation in location-dependent costs from -6.5 to +5.0 dollars per container did not produce a significant change in either locations of centers or the index of performance. This range of variation is about that to be expected from the material in Chapter III. The remaining calculations described below were made in order to test the logic of the model; the values of the incremental cost in the southwestern rectangle are an order of magnitude greater than ones which one might reasonably expect.

A second computation was made with the cost in the southwestern rectangle very large (\$9,999,999.99 per container) and the costs of the other rectangles as above. This drove the center originally in southwest Ohio out of business. A third computation with the incremental cost in this southwest rectangle at \$480 per container did not change its location but reduced the southwestern Ohio center's patronage to less than 4 containers per day. A fourth computation with the incremental cost in the southwest rectangle at \$800 per container drove all patronage except one shipper away from the southwestern Ohio center and moved its location to that shipper.

Table 34. Sensitivity to Total Demand
(6 centers)

	Nominal Demand		Demand Raised by 25%	
	Containers/day	Load Factor	Containers/day	Load Factor
Western New York	21.51	.8842	23.99	.8924
Pittsburgh	36.09	.8930	48.34	.8922
Cleveland	55.25	.8764	68.37	.8746
Southwest Ohio	17.40	.8851	23.58	.8552
Chicago	25.39	.9020	31.29	.9086
Milwaukee	12.35	.8669	14.67	.8838
Index	.520		.517	

Table 35. Sensitivity to Total Demand
(7 centers)

	Nominal Demand		Demand Raised by 25%	
	Containers/day	Load Factor	Containers/day	Load Factor
Western New York	20.41	.8928	24.00	.8922
Pittsburgh	39.77	.8896	51.74	.8873
Cleveland	42.36	.8492	47.56	.8519
Detroit	14.10	.8816	22.27	.8561
Southwest Ohio	16.29	.8746	21.82	.8574
Chicago	24.09	.9087	30.10	.9092
Milwaukee	11.22	.8875	14.38	.8893
Index	.517		.514	

Sample Program Output

Figure 11 contains part of a sample program output. The program contains options to print more information than that recorded here; this particular output contains only those types of information which were desired for the runs described in this chapter.

The upper section contains the run description, which is printed once at the beginning of a run. This particular run is going to perform 5 separate initializations of type 2 ("random in area") for 4 centers. Both types of patronage restrictions are in force. The dispatching policy and containerized freight rates are printed for use in the sensitivity analyses.

The next section of output contains the initial center locations. The coordinate system used in the output is the one described earlier in this chapter.

The next line is printed whenever a center violates the minimum total patronage restriction (if imposed), and is therefore reinitialized to a new location.

```
THIS PROGRAM COMPUTES OPTIMAL LOCATIONS FOR NUMBERS OF CENTERS FROM 4 TO 4 AT INTERVALS OF 1.
THE PROGRAM USES INITIALIZATION 2.
THE PROGRAM PROHIBITS CENTERS FROM ACCEPTING CARGO FOR A CO-CONTAINERIZABLE CLASS IF THE ACCUMULATION DELAY IS TOO GREAT.
CENTERS ARE REINITIALIZED 3 TIMES OR UNTIL EACH CENTER RECEIVES AT LEAST 432000.00 POUNDS OF CARGO.
5 INITIALIZATIONS ARE PERFORMED FOR EACH NUMBER OF CENTERS.
THE TRANSPORTATION RATE FOR CONTAINERIZED FREIGHT IS .0000135.
THE INVENTORY-TYPE CARRYING CHARGE IS .0004
THE PROJECTED EXPORT LOAD FACTOR IS .900.  THE MAXIMUM HOLDING TIME IS 4. DAYS.
THE AVERAGE IMPORT LOAD FACTOR IS .822.

INITIALIZATION FOR 4 CENTERS

CENTER      X          Y

   1     555.02     184.25
   2     339.09     454.70
   3     262.94     121.03
   4     135.04     179.92

CENTER 4 RECEIVING 119217.00 POUNDS AND LOCATED AT 169.750  208.930 REINITIALIZED  TO 741.921  420.389

CENTER     X        Y     MIN X OF  MAX X OF  MIN Y OF  MAX Y OF     POUNDS   CONTAINERS  LOAD   PER CENT  PER CENT  INDEX OF
                          PATRONS   PATRONS   PATRONS   PATRONS      INPUT      OUTPUT   FACTOR   EXPORT    IMPORT  PERFORMANCE

   3     355.21   246.79  253.75 -  874.12    141.57 -  484.07    1831374.20  43.37     .8798    76.00     24.00      .6471
   2     456.68   303.65  301.88 -  903.00    150.70 -  508.04    2234762.20  52.59     .8853    82.82     17.18      .5062
   1     591.09   189.97  469.88 -  886.37    100.47    485.21     941051.82  23.26     .8430    92.06      7.94      .4918
   4     770.00   333.37  607.25 - .952.87    139.28 -  509.18    1603106.30  36.95     .9039    82.76     17.24      .4037

  PLACE            X        Y       DELTA     NUMBER OF CENTERS
                                            WITHIN DELTA OF (X,Y)

PHILADELPHIA    108.50   203.22      50.            NONE
ALLENTOWN PA    135.62   247.74      50.            NONE
HARRISBURG      169.75   208.93      50.            NONE
SW. NEW YORK    249.49   355.66      30.            NONE
BUFFALO         298.37   393.80      30.            NONE
PITTSBURGH      367.50   236.33      50.              1
NE. OHIO        410.00   300.00      20.            NONE
CLEVELAND       456.75   303.68      30.              1
DETROIT         536.37   359.63      50.            NONE
SW. OHIO        592.37   188.38      50.              1
NW. OHIO        610.16   332.32      50.            NONE
S. MICHIGAN     650.00   350.00      50.            NONE
INDIANAPOLIS    690.37   189.52      50.            NONE
CHICAGO         770.00   333.37      50.              1
MILWAUKEE       784.87   412.14      50.            NONE
CENT. ILL.      837.37   195.23      50.            NONE

PORT       X        Y     MIN X OF  MAX X OF  MIN Y OF  MAX Y OF     POUNDS   CONTAINERS  LOAD   PER CENT  PER CENT  INDEX OF
                          PATRONS   PATRONS   PATRONS   PATRONS      INPUT      OUTPUT   FACTOR   EXPORT    IMPORT  PERFORMANCE

   1     112.80   200.80   87.50 -  310.38     93.62 -  371.04     906451.54   21.40    .8826    86.93     13.07      .8064
   2      52.50   255.00  ********       .00  ******** -      .00         .00     .00    .0000      .00    100.00      .0000
   3     188.00   156.20  157.50 -  784.87    102.75 -  484.07      99656.02    2.42    .8586    64.11     35.89      .8381

                        COST WITH CENTERS     COST WITHOUT CENTERS

VARIABLE COST              121277.23               252665.13

TOTAL COST                 157432.18               288820.07

INDEX OF PERFORMANCE      .545087
```

Figure 11. Sample Output

The next section contains the output information for each center: final location, zone of attraction, the number of pounds of cargo processed per day, the number of containers stuffed (or unstuffed) per day, the average load factor of these containers, the percentages of the cargo which are export and import, respectively, and the index of performance for the patrons of each center.

The next section gives a rough geographical description of the locations of the centers. Several cities (16 in this example) are chosen together with a tolerance distance, labeled DELTA. The number of centers which are "near" (within the tolerance distance of) each of the cities is tabulated.

The next section contains the same information for port centers as is printed for the inland centers. Here it should be noted that port center 2 received no patronage. The asterisks and 100.00 percent import result from this fact and should be ignored.

The final section contains information on the index of performance of the system and the breakdown of the costs which make up this index. The total variable cost is that cost which enters into the optimization phase--transportation costs (except the constant term of equation [7.8]), and center accumulation delay costs at inland centers.

Further Sensitivity Analyses

After the sensitivity analyses described above were performed and a preliminary report completed, occasion arose to make further improvements in the model as well as further sensitivity analyses.

The computer program was expanded to include two versions of the equation [7.8] which gives travel time as a function of line haul distance; one of these is applied to containerized freight (in effect assumed to go by rail), the other to uncontainerized (moved by truck). Therefore since two values of the δ_m of equation [7.8] were needed, one for rail and one for truck, this factor had to be included in the assignment step, whereas previously it only had to be added in at the end when evaluating the index of performance. The circuity factor (see [7.3]) is now used in calculating the travel time as well as travel cost. These changes affect the values of the index of performance, typically raising them, but they still remain generally between .55 and .63. The changes also decrease the differences among the index-values for different sets of initial locations. They do not, however, have any appreciable effect on the center locations determined.

The following sections describe the nine additional sensitivity analyses performed. Since the changes described above are included in the computer runs for these analyses, the indices of performance which follow are not properly comparable to those of the previous sections. For this set of runs the nominal case had the minimum total patronage constraint in operation. All other cases used the center locations determined for the nominal case as starting configuration, and the minimum total patronage constraint was not imposed except in the analysis summarized in Table 46.

The general findings are essentially the same as those noted earlier for the previous set of sensitivity analyses. Final center locations are most sensitive to the set of initial locations; relatively large variations in other parameters produce only small variations either in locations or in the index of performance. Throughout our tests, sensitivity of the index to parameter changes never significantly exceeded its sensitivity to the choice of initial locations and most often was significantly less. Because the tables which follow do not include the evidence for this assertion (specifically, the index-values arising from different location-initializations), we illustrate the point here with a fairly typical example: Three runs of the model with different sets of initial locations for five centers produced indices of performance of .5969, .5775, and .5901. A decrease in the import load factor by 10% produced indices of .5998, .5813, and .5906 for the same three sets of initial locations. The maximum raise in index is .0038, occurring for the second set of initial locations. The variation in index due to initial location set, however, is much larger: .0194 for the base set of runs and .0185 for the runs with lower import load factor.

Sensitivity to LCL Transportation Rates

The LCL transportation rates listed in Table 16 are averages over the commodities in each of the eight commodity classes. As described in Chapter V, transportation rates depend in reality on such factors as origin and destination, nature of the commodity, and the packaging of goods; Table 16's figures are representative averages over all these factors. The present analysis was performed to investigate sensitivity to deviations from these nominal rates. Each of the eight rates was given a random perturbation uniformly distributed between ±10% of the nominal value in Table 16; the eight perturbations were independent. Two sets of perturbed rates (8 rates each) were formed in this way.

The results are reported in Table 36. (The new transportation rates for the results are given in footnotes.) Both sets of variations in transportation rates happened to reduce the total cost without centers,of the set in line 1 by only 0.4% and that in line 3 by 4%. However, the commodity mix is such that the numerator of the index of performance for line 3 was reduced relatively less than was the denominator, causing higher values of the index. On the whole, the index appears less sensitive to changes or moderate errors in the LCL transportation rates than to different starting positions.

Table 36. Variation of Index of Performance with Random Perturbations of Uncontainerized Transportation Dollar Rates*

	4 Centers	5 Centers	6 Centers	7 Centers
1.**	.600	.591	.596	.580
2.†	.607	.588	.594	.577
3.††	.611	.602	.605	.590

---

* All entries are averages over 3 cases.

**The new transportation rates for this case are .696, .730, .450, .508, .566, .959, .671, and .615 cents per hundredweight mile for each of the eight commodity classes, respectively.

† Nominal case.

††The new transportation rates for this case are .801, .640, .380, .553, .548, .955, .604, and .586 cents per hundredweight mile for each of the eight commodity classes, respectively.

Sensitivity to Level of Demand by Commodity Class

An earlier sensitivity analysis varied the level of total demand while keeping its distribution constant. However, it is reasonable to suppose that the demands for different commodities have different growth characteristics. This sensitivity analysis, therefore, was concerned with variation of demand by commodity class. No detailed forecasting of demand level by commodity class was attempted, such an analysis requiring resources and data far greater than those available. Rather, a growth factor between -10% and +10% was chosen randomly for each of the eight commodity classes.

Table 37 records the results of two such runs. (The growth factors chosen are given in footnotes.) Both lines 1 and 3 have lower total cost without centers than the base run, line 1 by 5% and line 3 by only 0.4%. However, the commodity mix is such that line 3 did not have a commensurate reduction in the cost with centers, while line 1 did. (The measure of disbenefit is reduced only if the savings with centers available is a larger fraction of the cost with centers available, than the savings without centers available is of the total cost without centers.) The most significant inference which can be drawn from the results in Table 37 is that the index of performance is not very sensitive to random variations in the level of demand by commodity class.

Table 37. Variation of Index of Performance with Random Perturbations of Demand by Commodity Class*

	4 Centers	5 Centers	6 Centers	7 Centers
1.**	.595	.584	.589	.573
2.†	.607	.588	.594	.577
3.††	.604	.594	.599	.584

---

* All entries are averages over 3 cases.

** The growth factors used in this case are +.05731, -.07523, -.08977, +.02383, -.05636, +.07106, -.03835, and -.08930 for the eight commodity classes, respectively.

† Nominal case.

†† The growth factors used in this case are -.08188, +.05303, +.07668, -.05893, -.02567, +.07539, +.06786, and -.04501 for the eight commodity classes, respectively.

Sensitivity to Import Load Factors

As described earlier, our nominal value for the import load factor was taken from the North Atlantic Container Statistics (Report for 6-month period ending 12/31/67). This figure only represents container trade levels in the North Atlantic; although one might expect the bulk of container import cargo through the ports of New York, Philadelphia, and Baltimore to result from the North Atlantic trade, this load factor figure may be quite different for other trade areas. Also, in 1967 container trade was still somewhat of a fledgling industry, and we have no reason to expect that load factors will remain at the 1967 level as the industry grows. For these reasons, the model's sensitivity to the value of the import load factor was tested. The results of this analysis are summarized in Table 38. An increase of 10% in the nominal import load factor brought about approximately a 1% drop in the index of performance, while a decrease of 10% brought an even smaller increase in the index. For this analysis it seems clear that for a rather wide range of average import load factors (from 75% full to 90% full), the model remains relatively insensitive.

Sensitivity to Maximum Allowable Average Accumulation Delay

The dispatching policy used in the model involves holding cargo until either a full enough container accumulates at the center or cargo has been held for too long a time. In addition to this maximum holding time, if the average time cargo of one co-containerization class is held at a center would be greater than some threshold value, that center is prohibited from accepting cargo of that class. In this way, the number of centers available to a given cargo class may be reduced, so that the remaining centers have a better chance of accumulating full enough containers within the maximum allowable holding time.

The nominal value of the threshold for the average accumulation delay is 1.5 days. Table 39 gives the results of varying this value. As the threshold was decreased to 1 day, the value of the index was increased about 1%. A corresponding 1% decrease in the index was observed when the maximum allowable average accumulation delay was raised to 2 days. Thus, allowing some extra time for cargo to accumulate seems better than forcing cargo to be sent to a less convenient center where a higher load factor can be achieved. However, the variations in the index of performance with rather wide changes in the average accumulation delay threshold are less than those observed for different starting configurations.

Sensitivity to Inventory Carrying Charge

In converting various time components (e.g., travel times, processing times, and accumulation delay times) to dollar equivalents for inclusion in the generalized cost function, an inventory-type charge is used. This charge represents the fraction of the value of the cargo

Table 38. Sensitivity to Import Load Factor.*

Import Load Factor	4 Centers	5 Centers	6 Centers	7 Centers
.740	.602	.591	.595	.581
.822**	.607	.588	.594	.577
.904	.593	.582	.587	.572

---

* All entries listed here are averages over 3 cases.

** Nominal case.

Table 39. Sensitivity to Maximum Allowable Average Accumulation Delay (days)*

Delay	4 Centers	5 Centers	6 Centers	7 Centers
1.0	.606	.593	.601	.587
1.5**	.607	.588	.594	.577
2.0	.592	.581	.587	.571

---

* All entries are averages over 3 cases.

** Nominal case.

being handled which is to be charged for each unit of time delay. Variation of the inventory charge is equivalent to variation of the dollar values of all commodity classes by a constant factor, since these two quantities always appear within the model as a product.

Our nominal value of the inventory charge is 10% per year, i.e., 0.0004% per day for a 250-day year. Table 40 gives the results of varying this charge by ±10%. An increase of 10% in the inventory carrying charge brings a 1 to 2% increase in the index of performance. A decrease of 10% produces a decrease of more than 2% in the index. Again it should be noted that the model is not greatly sensitive to this parameter.

Using a center is time-consuming since it involves both a roundabout route to the port and the experiencing of accumulation delay. Thus an increase in the inventory carrying charge might be expected to cause not only an increase (as observed) in the (mis-)performance index, but also a diversion of traffic from inland centers to containerization at the ports. Sensitivity in this regard also proved quite slight, the patronage at centers remaining almost the same for corresponding runs of all three cases.

Table 40. Sensitivity to Inventory Carrying Charge Factor*

Value Fraction/day	4 Centers	5 Centers	6 Centers	7 Centers
.00036	.588	.574	.580	.564
.00040**	.607	.588	.594	.577
.00044	.608	.598	.600	.587

---

* All entries are averages over 3 cases.

** Nominal case.

Sensitivity to Parameters Converting Travel Distance to Travel Time

Equation [7.8] estimates time in transit as a linear function of distance. Meyer et al (p. 135, Ref. 15) provide values of the parameters for the rail mode. By omitting those delay factors inappropriate to the truck mode, and estimating appropriate terminal delays, corresponding parameters were constructed for the truck mode. Equation [7.8] can be rewritten (cf. [7.9])

$$t_m = a_m d_m + \delta_m \qquad [9.1]$$

where t_m is the time to go a distance d_m by mode m. The nominal values of a_m and δ_m for the rail mode are given in Meyer et al as .0059 days/mi. and 2 days, respectively. The nominal values chosen for truck were .0014 days/mi. and 1 day, respectively. Shippers are charged truck rates for uncontainerized shipments and rail rates for those in containers. The tradeoff is one of cost versus time, with truck faster and rail cheaper. The nominal values represent averages of values which may vary for different railroads and different times of the year.

Table 41 shows the variation of the index of performance with changes of ±10% in the parameters a_m and δ_m. The values of the index are almost identical to those produced by ±10% changes in the interest carrying charges, although these latter affect the cost of accumulation delays and center processing times, while the variations in a_m and δ_m do not. Again it should be noted that the variation in the index is not very large, i.e., the model is not very sensitive to the coefficients in the travel time equation.

Table 41. Sensitivity to Factors for Converting Travel Distance to Travel Time*

	4 Centers	5 Centers	6 Centers	7 Centers
Nominal Values - 10%	.585	.575	.576	.564
Nominal Values	.607	.588	.594	.577
Nominal Values + 10%	.608	.598	.602	.587

---

* All entries are averages over 3 cases.

Sensitivity to Co-containerization Classification

Within the model, the accumulation delay is calculated separately for each (co-containerization class, U.S. port) pair. Although there are some exceptions it was assumed in our illustrative calculations (see page 88) that most types of commodities can be loaded into the same container. On the other hand, shipments for different destinations (for our purposes, foreign decontainerization points) cannot be put into the same container if efficiency of unloading is desired. Therefore, for the most part, our co-containerization classification scheme is a surrogate for the foreign destination of each shipment. This information is unavailable from the Delaware Port Authority data; a hypothetical version was concocted by randomly assigning a class to each shipment.

The nominal number of such classes was 20. Table 42 gives the variation in the index of performance for (a) a different randomly assigned classification with 20 classes, and (b) also for one with 15 classes. The new set of 20 classes produces a rise of less than 1% in the index. The 15 classes reduce the index by 1 to 2%. (This is the appropriate direction of change; classifying the same amount of cargo into fewer co-containerization classes should reduce the accumulation delay since there should be more cargo available in any one class.) The model therefore appears not very sensitive either to the classification scheme or to the number of classes.

Table 42. Sensitivity to Co-Containerization Classification*

Classification	4 Centers	5 Centers	6 Centers	7 Centers
15 classes	.593	.580	.583	.567
Nominal (20 classes)	.607	.588	.594	.577
20 classes	.604	.594	.598	.583

---

* All entries are averages over 3 cases.

Sensitivity to Processing Times and Costs

Chapter III provides nominal values of .25 days and $94.28 to process a 40'x8'x8' container. Since all cargo entering the model is eventually containerized and these processing times and costs are taken independent of the center at which the cargo is containerized, these cost elements need not enter into the optimization process. However, they do affect the index of performance and are added into both numerator and denominator of that index at the end of the model run. The formula for the index of performance thus becomes

$$(T_1 + T_p)/(T_2 + T_p)$$

where T_1 is the total variable cost with centers available, T_2 is the corresponding cost when centers are not available and T_p is the processing (generalized) cost for stuffing containers.

Table 43 records the results of varying the cost to process a container by ±10% and increasing the time to process a container by a factor of 2 (from 3 hours to 6). The variation in the cost of processing a container affected the index more than the change in time did. On the other hand the total range of index values changes only about 1%, which indicates that the model is quite insensitive to these parameters over the range under consideration.

Table 43. Sensitivity to Processing Times (Days) and Costs ($)*

Time	Cost	4 Centers	5 Centers	6 Centers	7 Centers
.125	84.852	.604	.585	.591	.573
.250	84.852	.606	.587	.592	.575
.125**	94.280	.607	.588	.594	.577
.250	94.280	.609	.590	.595	.578
.125	103.708	.610	.591	.597	.580
.250	103.708	.612	.593	.599	.581

---

* All entries are averages over 3 cases.

** Nominal case.

Sensitivity to Total Patronage Constraint

Since the center costing analysis described in Chapter III does not separate out the fixed cost element from the variable cost element of centers, we were unable to exercise the center sizing portion of the model as described on pages 70 and 71. As a result, for each number n of centers there is a set of center locations which under our present computer program will reduce or at least not increase the index of performance beyond the best set of n-1 locations, so that we are presently unable to base the choice of the best number of centers on minimum cost alone. As a reasonable alternative, we can require each center to process at least a given minimum amount of cargo. The number of centers is thus limited by the number which can process this minimum cargo level.

Recall that the logic is as follows: centers are located by the alternation of assignment and location steps as described previously. When this process has settled down, the final set of centers is checked to see if each obeys the minimum patronage restriction. The locations of any centers which fail this test are randomly reinitialized, and the assignment-location process is started again. This is continued up to a specified number of tries or until all centers obey the restriction. Such a procedure may be described as testing several different initial configurations, tentatively retaining at each point those center locations from the last stage that worked out well.

Tables 44 and 45 record the center locations resulting from runs changing the nominal constraint of 10 containers/day to 8 and 12, respectively. It should be noted that only one of the six-center configurations and none of the seven-center ones processed enough cargo at all centers, and this single six-center configuration worked for a constraint of only 8 containers/day. However, further inspection of the two tables suggests the best configurations for 6 and 7 centers respectively as: Cleveland, Chicago, Southwestern Ohio, Greater Pittsburgh, Western New York, and Detroit for 6 centers, with Milwaukee added when 7 centers are being located.

Table 44. Variations in Rough Locations of Centers (6 Centers) with Minimum Total Patronage Constraint

Area	Minimum Patronage = 8			Minimum Patronage = 10*			Minimum Patronage = 12		
E. Pennsylvania (incl. Philadelphia)						1**			
Western New York	1	1			1		1	1	
Greater Pittsburgh	1	1		1			1	1	1
N.E. Ohio									
Cleveland	1	1	1	1	1	1	1	1	1
Detroit			1			1			1
S.W. Ohio	1	1	1	1	1	1	1**	1	1
N.W. Ohio									
South Central Michigan									
Central Indiana								1**	
Chicago	1	1	1	1	1	1	1	1	1
Milwaukee		1	1**						
Other	1**		1**	2**	2**	1	1**		1**

* Nominal case
**Not enough total patronage

Table 45. Variations in Rough Locations of Centers (7 Centers) with Minimum Total Patronage Constraint

Area	Minimum Patronage = 8			Minimum Patronage = 10*			Minimum Patronage = 12		
E. Pennsylvania (incl. Philadelphia)									
Western New York		1	1		1	1		1	1
Greater Pittsburgh	1	1	1	1	1	1	1		1
N.E. Ohio									
Cleveland	1	1	1	1	1	1	1	1	1
Detroit	1	1		1				1	
S.W. Ohio	1	1		1	1	1	1	1	1
N.W. Ohio							1		
South Central Michigan									
Central Indiana		1	1	1			1		
Chicago	1		1	1	1	1	1	1	1
Milwaukee	1		1	1		1	1		
Other	1	1	1		1,1	1		2	2

---

* Nominal case.

Table 46 gives the indices of performance for this set of runs. No real conclusion as to dependence of index on level of patronage constraint can be drawn from this table. Although the original initial locations of centers are the same for each of the three rows, the process of reinitializing "bad" centers in effect tries several initial configurations. As noted earlier, the model is more sensitive to initial configuration than to any other single factor. The variations listed in Table 46 are as much a reflection of different "initial" sets of center locations as they are of the patronage constraint. Most locations which fail to attract enough patronage process only about 2 or 3 containers a day. In only a very few cases does one location satisfy a constraint of 8 containers but not of 12. This is a consequence of the particular shipment distribution with which we work.

Table 46. Sensitivity to Minimum Total Patronage Constraint*

Constraint	6 Centers	7 Centers
8 containers/day	.586	.584
10 containers/day**	.594	.577
12 containers/day	.582	.583

---

* All entries are averages over 3 cases.

** Nominal case.

Only Port Centers

In addition to the sensitivity runs just discussed, one run was made allowing centers only in the port areas. In this run, shippers have a choice between shipping directly to their own port, or consolidating at one of the other port centers and only then shipping on to their final U.S. port of debarkation. This run was made to investigate the value of inland versus port centers. The index of performance was .951, which when compared with the values of less than .63 for inland centers, indicates that the greater savings come from the inland centers, rather than just from freedom to consolidate at ports other than that of egress from or entry to the United States.

Concluding Remarks

The exercises with the model which were described above, serve to illustrate the ways in which it can be used. They provide only a sample of the types of analyses which should be done in using the model described here. Clearly more initial configurations should be tested, since the model seems most sensitive to this factor. All work should be done with the minimum total patronage restriction in force. For the Delaware River Port Authority data, the process described in Appendix G should be used to provide origin-destination input (U.S. shipper linked with specific foreign port). Sensitivity to changes in combinations of factors could also be checked in a more complete analysis.

Present program output could be expanded to include any time or cost components for any shipper or for groups of shippers, aggregated by port, center, or commodity class shipped. These items are all computed within the model and it could be reprogrammed with a minimum of effort to print them out. In addition, the inclusion in the model of a capability to output a map showing center locations would facilitate quick analysis and encourage more general usage.

The exercises performed so far have focused attention on some of the items which should be included in future data collection efforts and which are lacking in the Delaware River Port Authority data. The need for knowledge of the origin and destination of each shipment has been stressed above. The Delaware data contain only a general yearly description of shipments. More specific information on the distribution over the year of shipments (including such things as seasonal variations), and more information on the variation of shipment size, are also needed to permit realistic calculation of actual accumulation delays. Further work is necessary on the definition of which commodities are co-containerizable.

For a more realistic evaluation of the benefits of inland consolidation centers, and a better estimate as to their appropriate size and number, it would be necessary to predict future foreign cargo demands. In order to do this, data on past export and import demand trends are needed. Moreover, the present model only considers LCL shipments. It is necessary to separate out the costs and benefits of the marshalling function before full container-load shipments can be included in the analysis. Clearly such shipments will affect the size and possibly the location of centers. It is also necessary to separate out the variable and fixed cost portions of center processing costs, as given in equation [7.52], so that the processing cost (including cost of processing time) is not just a constant rate per pound of cargo processed but depends on the total amount being processed. This would make possible exercising the logic described in Chapter VII for optimal sizing of centers.

With these developmental features and studies added to the basic methodology developed in this report, the model should provide a powerful tool for guiding and executing analytical investigations and data-collection efforts aimed at determining favorable configurations for a system of inland consolidation centers.

APPENDIX A. OPTIMUM LOCATION OF CENTERS IN A NETWORK

The problem to be considered here is that of locating centers in a network so as to minimize the sum of the associated transportation costs to users. It is assumed that the center location-dependent portion of each user's cost (combining dollar and time components), which is the only portion that figures in this problem, can be expressed--as in the main text --as a multiple of user-center distance plus a multiple of center-port distance.

We begin with the case of a single center (n=1), the situation of concern in the location step of the mathematical model described in Chapter VII. Here the problem is that of choosing the center location C so as to minimize a function of the form

$$f(C) = \sum_i w_i d(X_i, C), \qquad [A.1]$$

where the X_i are the locations in the network of shippers (or importers) and ports, and the w_i are appropriate positive numerical "weights" reflecting shipment volumes and freight rates. The main novelty is that "d" now represents distance via a shortest path in the network.

If some X_i is an interior point of a network link with endpoints e' and e", we can conceptually split the link into two links, one with e' and X_i as endpoints and the other with X_i and e" as endpoints. Since this process can be repeated so long as there are any X_i's which are interior points (rather than endpoints) of links, it can for convenience be assumed in advance that every X_i is a <u>vertex</u> (node) of the network.

The main known theoretical result on this optimal location problem is due to Hakimi,[25] who showed that the search for optimal locations for C can be confined to the vertices of the network. (The same conclusion persists for cost functions more general than [A-1],[26] namely, those such that cost per unit distance along each link is a non-increasing function of distance traveled.)

This result justifies the following 3-step method for finding the optimal location C. The method is unpleasantly brute-force, but appears computationally feasible for problems of reasonable size.

STEP 1. For each vertex v_j of the network, set up a "location" in computer memory and initialize the value u_j of that location's contents at zero.

STEP 2. For each X_i in turn: Determine $d(X_i, v_j)$ for all vertices v_j. (This is not done vertex-by-vertex, but efficiently at once by a "labeling" shortest path algorithm. For a network with about 1000 nodes and twice as many links, this would take 1-2 seconds per X_i on a fast machine.) Add $w_i d(X_i, v_j)$ to the current value of u_j.

STEP 3. When Step 2 is complete, find the smallest u_j. Choose C to be the corresponding v_j.

The proof of Hakimi's result is brief enough to warrant presentation here. The generic symbol for a vertex of the network will be "v", and [A.1] is rewritten as

$$f(C) = \sum w_v d(v, C) \qquad [A.2]$$

where $w_v = 0$ if vertex v is not an X_i.

Consider any point x of the network. It lies on some link, with certain vertices v_r and v_s as endpoints, so that

$$d(v_r, x) + d(x, v_s) = d(v_r, v_s). \qquad [A.3]$$

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[25] S. L. Hakimi, Optimum Location of Switching Centers and the Absolute Centers and Medians of a Graph, Operations Research, <u>12</u> (1964), pp. 450-459.

[26] A. J. Goldman and P. R. Meyers, A Domination Theorem for Optimal Locations, Operations Research <u>13</u> (1965), p. B-147 (Abstract).

[27] C. Witzgall, On Labelling Algorithms for Determining Shortest Paths in Networks, NBS Report 9840 (5/68).

The vertices v of the network can be divided into 2 classes; the set V(r) of vertices v such that a shortest path between x and v passes through v_r, and those (not in V(r)) such that a shortest path between x and v passes through v_s. It follows that

$$d(v,x) = d(v,v_r) + d(v_r,x) \qquad (v \text{ in } V(r)),$$

$$d(v,x) = d(v,v_s) + d(v_s,x) \qquad (v \text{ in } V(s))$$

Hence

$$\begin{aligned} f(x) &= \textstyle\sum_v w_v d(v,x) \\ &= \textstyle\sum_{V(r)} w_v[d(v,v_r) + d(v_r,x)] + \sum_{V(s)} w_v[d(v,v_s) + d(v_s,x)]. \end{aligned}$$

If [A.3] is used to substitute for $d(v_s,x)$, the result is

$$f(x) = \textstyle\sum_{V(r)} w_v[d(v,v_r) + d(v_r,x)] + \sum_{V(s)} w_v[d(v,v_s) + d(v_s,v_r) - d(v_r,x)].$$

It follows from the definition of "d" as distance <u>via</u> a shortest path, that

$$d(v,v_s) + d(v_s,v_r) \geq d(v,v_r).$$

Thus the last formula yields

$$\begin{aligned} f(x) &\geq \textstyle\sum_{V(r)} w_v[d(v,v_r) + d(v_r,x)] + \sum_{V(s)} w_v[d(v,v_r) - d(v_r,x)] \\ &= \textstyle\sum_{V(r)} w_v d(v,v_r) + \sum_{V(s)} w_v\, d(v,v_r) + [\sum_{V(r)} w_v - \sum_{V(s)} w_v]\, d(v_r,x) \\ &= f(v_r) + [\textstyle\sum_{V(r)} w_v - \sum_{V(s)} w_v]\, d(v_r,x). \end{aligned}$$

It can be assumed without loss of generality that

$$\textstyle\sum_{V(r)} w_v \geq \sum_{V(s)} w_v.$$

This and the preceding inequality together imply $f(x) \geq f(v_r)$. That is, there is a vertex (v_r) which is at least as good a location for the center as x. But x was an arbitrary point of the network. So the search for optimal locations can be confined to vertices, and the proof is complete.

Because shipment volumes (and/or freight rates) are not perfectly known or predictable, the generalized problem in which the w_v's are random variables is of interest. Frank[28] treated the case in which they are independent random variables. This, however, is not the case for our situation; the w_v corresponding to a particular U.S. port of departure is determined by the w_v's corresponding to those shippers using that port. In a later paper[29] Frank considered the case in which the w_v's are correlated multinormally-distributed random variables, showing in particular how to reduce, to the numerical evaluation of a multiple integral of multivariate normal type, the determination of a point C in the network which for a fixed number R maximizes

$$\text{Prob}\,\{f(C) \leq R\}.$$

(In general such a C cannot be taken to be a vertex.) It is not clear how much error is introduced because the normality assumption permits negative values for the w_v's.

Next consider the problem of locating n centers, where $n > 1$. The natural generalization of [A.2] in this direction is

$$f(C_1, \ldots, C_n) = \textstyle\sum w_v \min_j d(v,C_j), \qquad \text{[A.4]}$$

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[28] H. Frank, Optimum Locations on a Graph with Probabilistic Demands, Operations Research <u>14</u> (1966), pp. 409-421.

[29] H. Frank, Optimum Locations on Graphs with Correlated Normal Demands, Operations Research <u>14</u> (1966), pp. 552-557.

corresponding to the idea that each vertex v is assigned to (or chooses) a center nearest him. Hakimi[30] has shown that his theorem for $n = 1$ generalizes to this case, i.e., in minimizing f it can be assumed that each C_j is at a vertex. Computational methods have been explored by Singer[31] (only the abstract of this paper has been published).

The proof that attention can be confined to vertex locations runs as follows. Consider any points $x_1, \ldots, x_n$ of the network. Let V be the set of all vertices, and V(1) the set of all vertices v for which

$$d(v,x_1) = \min_j d(v, x_j).$$

It follows, from [A.4], that

$$\begin{aligned} f(x_1, \ldots, x_n) &= \textstyle\sum_{V(1)} w_v d(v,x_1) + \sum_{V-V(1)} w_v \min_{j>1} d(v,x_j) \\ &= f'(x_1) + \textstyle\sum_{V-V(1)} w_v \min_{j>1} d(v,x_j), \end{aligned}$$

where f' is the objective function for the 1-center problem with weights

$$w'_v = w_v \qquad \text{if } v \text{ is in } V(1),$$

$$w'_v = 0 \qquad \text{otherwise.}$$

By the previous result for the 1-center case, there is a vertex v_1 such that $f'(v_1) \leq f'(x_1)$, so that

$$\begin{aligned} f(v_1, x_2, \ldots, x_n) &= \textstyle\sum_{V(1)} w_v \min [d(v,v_1), \min_{j>1} d(v, x_j)] \\ &\quad + \textstyle\sum_{V - V(1)} w_v \min [d(v,v_1), \min_{j>1} d(v, x_j)] \\ &\leq f'(v_1) + \textstyle\sum_{V-V(1)} w_v \min_{j>1} d(v,x_j) \\ &\leq f(x_1, \ldots, x_n). \end{aligned}$$

That is, x_1 can be replaced in $(x_1, \ldots, x_n)$ by some vertex v_1. In the same way it can be shown that x_2 in $(v_1, x_2, \ldots, x_n)$ can be replaced by some vertex v_2 and so on, completing the proof.

Unfortunately, the applications arising in the present study involve a more complicated function than [A.4]. This is because a vertex might represent a U.S. port of departure, or the locus of several shippers perhaps using different U.S. ports, or possibly even both. To represent this, replace the simple "weights" w_v by weights

$w(v,p)$ = "weight" associated with shipment from v due to leave CONUS* via port p,

$w^*(v,p)$ = "weight" associated with same shipment in its center-to-port movement.

Then [A.4] is replaced by

$$f(C_1, \ldots, C_n) = \textstyle\sum_{v,p} \min_j [w(v,p)\, d(v,C_j) + w^*(v,p)\, d(C_j,p)]. \qquad [A.5]$$

It will now be shown that Hakimi's result for [A.4] remains true for [A.5], i.e., that in minimizing f as given by [A.5], attention can be confined to vertex locations for centers. As above, it suffices to show that for any points $x_1, \ldots, x_n$ of the network, there is a vertex v_1 such that

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[30] S. L. Hakimi, Optimal Distribution of Switching Centers in a Communication Network and Some Related Graph Theoretic Problems, Operations Research 13 (1965), pp. 462-475.

[31] S. Singer, Multi-Centers and Multi-Medians of a Graph, with an Application to Optimal Warehouse Location, Operations Research 16 (1968), p. B-87 (Abstract).

* Continental U. S.

$$f(v_1, x_2, \ldots, x_n) \le f(x_1, \ldots, x_n).$$

Let V now denote the set of all ordered pairs (v,p), where v ranges over all vertices and p ranges over all vertices representing ports. Let V(1) now denote the subset of V consisting of those pairs (v,p) for which

$$w(v,p)\, d(v,x_1) + w^*(v,p)\, d(x_1,p) = \min_j [w(v,p)d(v,x_j) + w^*(v,p)d(x_j,p)].$$

It follows that

$$f(x_1, \ldots, x_n) = \textstyle\sum_{V(1)} [w(v,p)d(v,x_1) + w^*(v,p)d(x_1,p)]$$
$$+ \textstyle\sum_{V-V(1)} \min_{j>1} [w(v,p)d(v,x_j) + w^*(v,p)d(x_j,p)]. \qquad [A.6]$$

As in the previous proof, it suffices to show that the first sum in [A.6] can be written $f'(x_1)$, where

$$f'(x) = \textstyle\sum_V w'_v\, d(v,x)$$

is the objective function of a 1-center problem with suitable weights w'_v. This however is readily done by setting*

$$w'_v = \textstyle\sum\{w(v,p)\colon (v,p) \text{ in } V(1)\} + \sum\{w^*(u,v)\colon (u,v) \text{ in } V(1)\}.$$

It would be of evident interest to find an effective computational method for determining vertex locations to minimize the function f of [A.5]. The priorities of the present study, however, ruled out undertaking a research effort in this direction.

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*A sum over the empty set is taken to be zero.

APPENDIX B. MINIMIZATION of TOTAL COST

This Appendix contains a brief discussion of the situation in which some single decision-maker, based on cost minimization, could impose an assignment of shippers to centers.

There is a fairly extensive literature dealing with a simpler situation involving not three types of points--origins (shippers), intermediate points (centers), and U.S. ports (destinations)--but only two, namely, supply points (origins) and markets (destinations). Each destination has a prescribed demand, and the problem is that of locating the supply points, thought of as production facilities ("plants"), so as to meet the demands at minimum cost. Both production and transportation costs are involved, but since supply points (centers) are assumed to be known in advance, this situation (Hitchcock-Koopman's model) does not apply to the problem on hand.

The paper most akin to the approach in the body of the present report is that of Cooper,[32] who also employs an alternating sequence of assignment and location steps, with the Kuhn-Kuenne iterative method used for the latter. (Hartley[33] has proposed a similar approach to center location.) Balinski[34] gives a mixed-integer linear programming formulation:

$$\text{Minimize } \sum_{ij} c_{ij} x_{ij} + \sum_i f_i y_i$$

$$\text{subject to } \sum_i x_{ij} = 1 \qquad \text{(all } j\text{)},$$

$$0 \leq x_{ij} \leq y_i \leq 1 \qquad \text{(all } i,j\text{)},$$

$$y_{ij} = 0 \text{ or } 1.$$

Here f_i is the fixed cost associated with using plant location i, variable x_{ij} is the fraction of market j's demand (D_j) which is to be supplied from the i-th location, while the c_{ij} represent the appropriate transportation and (variable) production costs. While Balinski proposes a "partitioning" mixed-integer algorithm for this problem, Efroymson and Ray[35] develop what appears to be a more promising approach from a computational viewpoint, using a "branch and bound" technique together with several ingenious simplifications. The paper of Sharp et al[36] illustrates recent work featuring nonlinear production costs rather than the sharp discontinuity of a fixed cost. Further references can be found in the bibliographies of the cited papers.

One might hope to adapt the "suppliers and markets" model to the problem of locating consolidation centers. There are three difficulties, however. First, the model above presupposes knowledge of a finite and manageably small set of "allowed" locations for plants. This is not the way locations for centers are treated in the present study, though a change on this point would seem reasonable if a decisively better solution method would result. Second, the number of shippers and importers in the present study puts the problem well beyond the size range of the cases treated in the papers cited. Third, there is the matter of twisting the models around to make them represent the presence of two classes of customers (shippers/importers, and ports), with each member of the first class associated to a definite member of the second. This however turns out to give no real trouble, as will be seen below.

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[32] L. Cooper, Location-Allocation Problems, Operations Research 11 (1963), pp. 331-343. Also Cooper, Heuristic Methods for Location-Allocation Problems, SIAM Review 6 (1964), pp.37-53; Solutions of Generalized Locational Equilibrium Models, J.Reg.Sci. 7 (1967), pp. 1-18.

[33] H. O. Hartley, Optimisation of the Location of Serving Centers, 7/68, manuscript communicated by the author.

[34] M. Balinski, Integer Programming: Methods, Uses, Computations, Management Sci. 12 (1965), pp. 253-313.

[35] M. A. Efroymson and T. L. Ray, A Branch-Bound Algorithm for Plant Location, Operations Research 14 (1966), 361-368.

[36] J. F. Sharp, J. C. Snyder, and J. H. Green, An Algorithm for Solving the Multi-facility Production-Transportation Problem with Nonlinear Production Costs, Operations Research 16 (1968), p. B-87 (Abstract).

We conclude by presenting a mixed-integer linear programming formulation, patterned after that of Balinski, but including all three types of points (shippers/importers, ports, and centers) and explicitly including the problem of sizing the centers. It assumes given a manageably small set of possible center locations, and the possible sizes (levels of capacity) for each.

The model's data are as follows:

b_{jk} = fixed charge for operating capacity increment k of center at location j,

v_{jk} = unit variable cost for such operation,

t_{ij} = unit transport cost for use by shipper i of center at location j (including center-to-port cost),

a_{jk} = size of k-th capacity increment at j,

s_i = "supply" total to be sent by the i-th shipper.

The model's discrete variables are the

d_{jk} = 1 if k-th increment at j is operated, 0 otherwise,

while its continuous non-negative variables are the

x_{ijk} = amount sent by i-th shipper to center at j-th location, attributed to k-th increment.

The objective function to be minimized is then

$$\sum_{jk} b_{jk} d_{jk} + \sum_{ijk} (v_{jk} + t_{ij})\, x_{ijk}.$$

The supply balance equations read

$$\sum_{jk} x_{ijk} = s_i \qquad \text{(all i)},$$

while the capacity constraints can be expressed as

$$\sum_i x_{ijk} \leq a_{jk} d_{jk} \qquad \text{(all j,k)}.$$

The requirement that $d_{j,k+1} = 0$ unless the k-th increment at j is used to capacity, can be written as a linear constraint

$$a_{jk} d_{j,k+1} \leq \sum_i x_{ijk} \qquad \text{(all j,k)}.$$

What still would require investigation is the computational feasibility of this model, in view of the large number of shippers involved.

APPENDIX C: THE CONSTRAINED LOCATION STEP

The main text gave an iterative method, specified by formulas [7.27] and [7.28], for finding the location C of a center so as to minimize a total (variable) transportation cost given by

$$f(C) = \sum w_i d(X_i, C). \qquad [C.1]$$

Here the X_i are the locations of shippers (or importers) and ports, "d" represents Euclidean distance, and the w_i are appropriate positive numerical "weights" reflecting shipment volumes and freight rates.

The text also noted the desirability of being able to solve a constrained version of this problem, in which C is restricted to be within a prescribed convex polygon R. This is the problem to be treated here. The solution method will be informally described for the case of a general polygon R, to the point where its computational feasibility should be apparent. More detail will be given for the special case--R a rectangle with sides parallel to the axes--for which computer implementation has actually been carried out.

Let $V_1, \ldots, V_k$ be the vertices of R, in (say) counter-clockwise order around its perimeter. The polygon R is assumed to be "given" by listing (in order) the coordinates of these vertices V_j. Let S_j denote the side joining V_j and V_{j+1} (where V_{k+1} signifies V_1). The equation of the line carrying S_j can be found, say in the form

$$a_j x + b_j y = c_j$$

by standard analytic-geometry techniques. Substitute the coordinates of any vertex other than V_j and V_{j+1} into $a_j x + b_j y$; if the result is $>c_j$, replace (a_j, b_j, c_j) by their negatives. With this accomplished, R itself can be characterized as the set of points (x,y) for which

$$a_j x + b_j y \leq c_j \qquad (j= 1, 2, \ldots, k). \qquad [C.2]$$

The process for solving the constrained location problem begins by using the method, given in the main text, for solving the corresponding unconstrained problem. Let $C^*=(x^*,y^*)$ be the resultant location. If (x^*,y^*) satisfies the k conditions [C.2], then C^* lies in R and the constrained problem is also solved.

Suppose this is not the case. Then those j's, for which [C.2] are violated, correspond to those sides S_j of R which are "visible" (in an obvious sense) from C^*. And it is known[37] that the optimal location within R must occur on one of these sides. Thus it suffices to find the f-minimizing location C_j along each of the sides S_j visible from C^*, and to choose as C that C_j for which $f(C_j)$ is smallest.*

The solution of each of the one-dimensional optimization problems, i.e., the minimization of f(C) along a side S_j of R, can be carried out (for example) as follows. Perform a transformation of coordinates to make S_j lie along the x-axis; this will yield new coordinates for the X_i's. Now use only the first equation [7.27] of the iterative process, with $y^{(t)}=0$, to minimize f(C) along the (new) x-axis. If the minimizing point is in the segment S_j, choose it as C_j; if it lies outside S_j, choose the endpoint of S_j closer to it as C_j. Then reverse the coordinate transformation to find the "true" (i.e., original) coordinates of C_j.

Assume now, in particular, that R is the set of points (x,y) satisfying

$$x^- \leq x \leq x^+ , \quad y^- \leq y \leq y^+ ,$$

i.e., a rectangle with sides parallel to the axes. Then the characterization [C.2] of R can be written out explicitly as a set of four conditions:

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[37] A. J. Goldman, A Theorem on Convex Programming, paper delivered to the Mathematical Association of America, 1963 Annapolis Meeting.

* More elegant methods may be possible, but this one will surely do.

$$x \le x^+ ,$$

$$y \le y^+ ,$$

$$-x \le -x^- ,$$

$$-y \le -y^- .$$

If the solution $C^* = (x^*,y^*)$ of the unconstrained problem lies outside R, then it can violate at most two of the above conditions (e.g., either the first or the third must hold). Thus at most two sides of R, involving at most three vertices, can be visible from C^*. No transformation of coordinates is required; the minimization of f(C) along a horizontal side of R can be carried out using [7.27]--with $y^{(t)}$ set at whichever of y^- or y^+ applies--while the minimization along a vertical side can be done using [7.28]. If desired the appropriate partial derivative could first be evaluated at the two endpoints of a side, so as to identify an endpoint optimum at once.

APPENDIX D: EVALUATION OF MEAN ACCUMULATION DELAY

This appendix contains an evaluation of the mean accumulation delay, t_a, under a dispatching policy of the type (L,H) formulated in Chapter VII. As in the main text's discussion prior to [7.38], the actual situation is approximated by Poisson arrivals (with parameter $\lambda = \lambda_c$) of shipments of uniform size $\bar{a}$. Moreover, m denotes the largest integer such that $m\bar{a} < L$.

Let $\nu+1$ (a random variable) denote the number of shipments in a dispatched container, and $\bar{w}$ the average waiting time of those shipments. Then the quantity to be evaluated is

$$t_a = E(\bar{w}) = \sum_{N=0}^{m} E(\bar{w} | \nu = N) \text{ Prob } (\nu = N). \qquad [D.1]$$

Suppose first that N = m (i.e., the container has a "normal" payload $(m+1)\bar{a}$). Let z_i be the period between the arrival of the i-th shipment entering the container, and the next arrival after that, for $1 \le i \le m$. Since the (m+1)-st shipment suffers no accumulation delay, the total accumulation delay for all shipments in the container is $\sum_1^m iz_i$, so that the mean delay per shipment is

$$\bar{w} = (\sum_1^m iz_i)/(m+1). \qquad [D.2]$$

Thus the summand in [D.1] corresponding to N=m is given by the multiple integral

$$(m+1)^{-1} \int (\sum_1^m iz_i) \exp(-\lambda \sum_1^m z_i) \lambda^m \, dz_1 \ldots dz_m \qquad [D.3]$$

where the integration is over the region in $(z_1, \ldots, z_m)$ - space defined by

$$\text{all } z_i \ge 0 \ , \quad \sum_1^m z_i \le H. \qquad [D.4]$$

The change of variable $y_i = \lambda z_i$ converts this to

$$\lambda^{-1}(m+1)^{-1} \int (\sum_1^m iy_i) \exp(-\sum_1^m y_i) \, dy_1 \ldots dy_m, \qquad [D.5]$$

with the integration over

$$\text{all } y_i \ge 0 \quad , \qquad \sum_1^m y_i \le \bar{H} = H\lambda. \qquad [D.6]$$

Before proceeding with the main derivation, it will be useful to evaluate certain special integrals over the m-dimensional region described by [D.6]. In the first of these, the integrand is 1, so that the integral is just the (hyper-) volume of the region. Multi-dimensional analytical geometry gives

$$V_m = \int 1 \, dy_1 \ldots dy_m = \bar{H}^m/m! \qquad [D.7]$$

as the value of this volume.

The second special integral is

$$E_m = \int \exp(-\sum_1^m y_i) \, dy_1 \ldots dy_m. \qquad [D.8]$$

To evaluate it, note first that

$$E_1 = \int_0^{\overline{H}} \exp(-y)\ dy = 1 - \exp(-\overline{H}). \qquad [D.9]$$

Next, for m>1 perform the y_m-integration in [D.8] first, obtaining

$$E_m = \int\{1 - \exp\ (-H + \sum_1^{m-1} y_i)\}\ \exp\ (- \sum_1^{m-1} y_i) dy_1 \ldots dy_{m-1},$$

where the integration is over the (m-1)-dimensional analog of [D.6]. This yields

$$E_m = E_{m-1} - \exp(-\overline{H})V_{m-1}.$$

From this recursion (in m), the initial condition [D.9], and the evaluation [D.7] of the V integrals, we obtain

$$E_m = 1 - p_{m-1}(\overline{H}), \qquad [D.10]$$

where $p_{m-1}(\overline{H})$ is as in [7.38] of the main text.

The third special integral is

$$L_m = \int y_m\ dy_1 \ldots dy_m. \qquad [D.11]$$

To evaluate it, perform the y_m-integration first. The result is

$$L_m = (1/2) \int(\overline{H} - \sum_1^{m-1} y_i)^2\ dy_1 \ldots dy_{m-1},$$

where the region of integration is the (m-1)-dimensional analog of [D.6]. Now perform the y_{m-1}-integration; its result is

$$L_m = (1/6)\int(\overline{H} - \sum_1^{m-2} y_i)^3\ dy_1 \ldots dy_{m-2}\ .$$

where the integration is over the (m-2)-dimensional analog of [D.6]. From these two steps, the pattern of results from the remaining successive integrations is evident; the final step will be

$$\begin{aligned} L_m &= (1/m!)\int_0^{\overline{H}} (\overline{H}-y_1)^m dy_1 \\ &= \overline{H}^{\,m+1}/(m+1)!. \end{aligned} \qquad [D.12]$$

We return now to the evaluation of [D.5], the term of [D.1] corresponding to N-m. From [D.5], this term is

$$\lambda^{-1}(m+1)^{-1} \sum_1^m i \int y_i \exp\ (- \sum_1^m y_i)dy_1 \ldots dy_m. \qquad [D.13]$$

The integral in [D.13] is independent of i, so [D.13] becomes

$$\tfrac{1}{2}\lambda^{-1}m \int y_m\ \exp\ (-\sum_1^m y_i)\ dy_1 \ldots dy_m, \qquad [D.14]$$

and performance of the y_m-integration yields (if m>1)

$$\tfrac{1}{2}\lambda^{-1}m\int[1-(\exp(-\bar{H}+\sum_1^{m-1} y_i))(1+\bar{H}-\sum_1^{m-1} y_i)]\exp(-\sum_1^{m-1} y_i)\,dy_1\ldots dy_{m-1},$$

where the integration is over the (m-1)-dimensional analog of [D.6]. Simplifying, we obtain (for m>1)

$$\tfrac{1}{2}\lambda^{-1}m[E_{m-1}-\exp(-\bar{H})\cdot(1+\bar{H})V_{m-1}+\exp(-\bar{H})\int(\sum_1^{m-1} y_i)dy_1\ldots dy_{m-1}]. \qquad [D.15]$$

The integral in [D.15] is in turn reducible to $(m-1)L_{m-1}$, and so the preceding evaluations for the E, V and L integrals yield

$$\tfrac{1}{2}\lambda^{-1}m[1-p_{m-2}(\bar{H})-\exp(-\bar{H})\,[(1+\bar{H})\bar{H}^{m-1}/(m-1)!-(m-1)\bar{H}^m/m!]]$$

$$=\tfrac{1}{2}\lambda^{-1}m[1-p_{m-1}(\bar{H})-\exp(-\bar{H})\bar{H}^m/m!]$$

$$=\tfrac{1}{2}\lambda^{-1}m[1-p_m(\bar{H})]. \qquad [D.16]$$

This formula can also be verified (starting at [D.14]) to hold when m=1.

Next consider the N-th summand in [D.1] for some N<m, corresponding to a dispatched container containing N+1 shipments. For $1\le i\le N$, let z_i be the period between the arrival of the i-th and (i+1)-st of these shipments, and let z_{N+1} be the period between the arrival of the last of these shipments and the first shipment-arrival after the dispatch. The total accumulation delay for all shipments in the dispatch is

$$\sum_1^N iz_i+(N+1)\ (H-\sum_1^N z_i),$$

so that the mean delay per shipment is

$$\bar{w}=(N+1)^{-1}\sum_1^N iz_i+H-\sum_1^N z_i.$$

Thus the N-th summand in [D.1] is

$$\int[(N+1)^{-1}\sum_1^N iz_i+H-\sum_1^N z_i]\ \lambda^{N+1}\exp(-\lambda\sum_1^{N+1} z_i)\ dz_1\ldots dz_{N+1}\ ,$$

where the integration is over the region in $(z_1, \ldots, z_{N+1})$ - space defined by

$$\text{all } z_i\ge 0,\ \sum_1^N z_i\le H<\sum_1^{N+1} z_i.$$

The change of variable $y_i = \lambda x_i$ converts this to

$$\lambda^{-1}\int[(N+1)^{-1}\sum_1^N iy_i+\bar{H}-\sum_1^N y_i]\ \exp\ (-\sum_1^{N+1} y_i)\ dy_1\ldots dy_{N+1}.$$

As before, the terms $\sum_1^N iy_i$ and $\sum_1^N y_i$ can be replaced, respectively, by $N(N+1)y_N/2$ and Ny_N, yielding

$$\lambda^{-1}\int(\bar{H}-Ny_N/2)\ \exp\ (-\sum_1^{N+1} y_i)\ dy_1\ldots dy_{N+1}.$$

First perform the y_{N+1}-integration, which has range

$$y_{N+1}>\bar{H}-\sum_1^N y_i.$$

The result is

$$\lambda^{-1} \int (\bar{H} - Ny_N/2) \exp (-\bar{H} + \sum_1^N y_i) \exp (- \sum_1^N y_i) \, dy_1 \ldots dy_N ,$$

where the integral is over the N-dimensional analog of [D.6]. This immediately simplifies to

$$\lambda^{-1}\exp(-\bar{H}) \int (\bar{H} - Ny_N/2) \, dy_1 \ldots dy_N = \lambda^{-1}\exp(-\bar{H})[\bar{H}V_N - NL_N/2]$$

$$= \lambda^{-1}\exp(-\bar{H}) [\bar{H}^{N+1}/N! - (N/2)\bar{H}^{N+1}/(N+1)!]$$

$$= \tfrac{1}{2}\lambda^{-1}\exp(-\bar{H})[\bar{H}^{N+1}/N! + \bar{H}^{N+1}/(N+1)!]. \qquad [D.17]$$

From [D.16] and [D.17],

$$t_a = \tfrac{1}{2}\lambda^{-1}m[1-p_m(\bar{H})] + \tfrac{1}{2}\lambda^{-1}\exp(-\bar{H})[\bar{H} \sum_0^{m-1} \bar{H}^N/N! + \sum_{N+1=1}^{m} \bar{H}^{N+1}/(N+1)!]$$

$$= \tfrac{1}{2}\lambda^{-1}[m(1-p_m(\bar{H})) + \bar{H}p_{m-1}(\bar{H}) + p_m(\bar{H}) - 1],$$

or finally

$$t_a = \tfrac{1}{2}\lambda^{-1}[(m-1)(1-p_m(\bar{H})) + \bar{H}p_{m-1}(\bar{H})]. \qquad [D.18].$$

APPENDIX E: ALTERNATIVE DISPATCHING POLICIES

The main text proposed a particular type of dispatching policy, called an (L,H) policy, as a reasonable and tractable approximation to what might be expected. Here a container is dispatched whenever its load factor has reached the target level L, or some of its cargo has waited for the maximum holdover time H, whichever comes first.

Suppose however that very soon after the arrival of the first increment for some container, the target level L has already been reached. There would clearly be good economic reason to retain the container in hopes of soon filling it further, rather than dispatching it immediately. How might the (L,H) type of dispatching policy be altered to include such considerations?

Perhaps the simplest approach is to adjoin a minimum holding time h, with $h<H$, to the parameters describing the policy: The container would not be sent out until its first-arrived increment had suffered a wait of at least h, unless of course an as-full-as-possible container-load were achieved prior to that. If M is the largest integer for which $M\bar{a}<1$ (so that $M \geq m$, with equality only if $(m+1)\bar{a}>1$), then a container "as full as possible" of payload is of course one loaded with M of the (idealized) shipments of uniform size $\bar{a}$. A value h=(1 day) does not seem inconsistent with the information gathered during the Project, though a larger value than this would appear questionable.

The derivations, for an (L,H) policy, to obtain the average realized load factor $\bar{L}_c$ (in Chapter VII, see [7.3 9]). the inflation ratio I_c (in Appendix F), and the average accumulation delay t_a (in Appendix D), can be adapted to yield the same quantities for an (L,H,h) policy. One critical change is that the number $\nu+1$ of size $\bar{a}$ shipments in a container-load, which previously could be at most $m+1$(m, if $(m+1)\bar{a}>1$), can now be as much as M. For $m<N<M-1$ (where $(m+1)\bar{a}\leq 1$), we have the additional non-zero probabilities

$$\text{Prob}\{\nu = N\} = \exp(-\bar{h})\bar{h}^N/N! \qquad (\bar{h} = \lambda_c h),$$

as well as

$$\text{Prob}\ \{\nu = M-1\} = 1-p_{M-2}(\bar{h}).$$

This yields (for $(m+1)\bar{a}\leq 1$)

$$\begin{aligned}
\bar{L}_c &= \bar{a}\textstyle\sum_N (N+1)\ \text{Prob}\ \{\nu=N\} \\
&= \bar{a}\Big\{\exp(-\bar{H}) \sum_{0}^{m-1} (N+1)\bar{H}^N/N! + (m+1)\ \exp\ (-\bar{h})\Big[\bar{h}^m/m! + \sum_{0}^{m-1} (\bar{h}^N/N!)[1-p_{m-1-N}(\bar{H}-\bar{h})]\Big] \\
&\quad + \exp(-\bar{h})\sum_{m+1}^{M-2} (N+1)\bar{h}^N/N! + M[1-p_{m-2}(\bar{h})]\Big\},
\end{aligned}$$

$$\begin{aligned}
I_c &= (1/\bar{a})\textstyle\sum_N (N+1)^{-1}\ \text{Prob}\ \{\nu=N\} \\
&= (1/\bar{a})\ \Big\{\exp(-\bar{H}) \sum_{0}^{m-1} (N+1)^{-1}\ \bar{H}^N/N! \\
&\quad +(m+1)^{-1} \exp(-\bar{h})\Big[\bar{h}^m/m! + \sum_{0}^{m-1} (\bar{h}^N/N!)[1-p_{m-1-N}(\bar{H}-\bar{h})]\Big] \\
&\quad + \exp(-\bar{h}) \sum_{m+1}^{M-2} (N+1)^{-1}\bar{h}^N/N! + M^{-1}[1-p_{m-2}(\bar{h})]\Big\}.
\end{aligned}$$

These expressions can readily be simplified as for the (L,H) policy; for example

$$\begin{aligned}
\bar{L}_c &= \bar{a}\{(1+\bar{H})p_{m-2}(\bar{H}) + M[1-p_{m-2}(\bar{h})] \\
&\quad + \bar{h}\ [p_{M-3}(\bar{h}) - p_{m-2}(\bar{h})] + p_{m-2}(\bar{h}) + m\ p_{m-1}(\bar{h})\}.
\end{aligned}$$

A generalization is a policy represented by a decreasing function of the waiting time w such that a container is dispatched as soon as it is filled to a fraction f(w) after a wait of w.

A more sophisticated approach might be based upon the following concept. Let IV denote the cost of delay (in dollars per unit quantity per unit time) for the material already accumulated for a container, and let A be the amount (a fraction of a containerful) of this material. The cost of waiting an additional time t would be IVAt. If N additional shipments for the container arrive in that time--an event with probability $\exp(-\lambda_c t)(\lambda_c t)^N/N!$--and $A + N\bar{a} \leq 1$, then the center-to-port transportation cost for the original material is reduced from R (the rate for a container*) to $RA/(A + N\bar{a})$. Thus the expected saving for the original cargo is

$$\sum_N \{R - RA/(A + N\bar{a})\} \exp(-\lambda_c t)(\lambda_c t)^N/N! - IVAt$$

$$= R\bar{a} \ \exp(-\lambda_c t) \sum_N [N/(A + N\bar{a})] (\lambda_c t)^N/N! - IVAt.$$

The general idea is to choose t so as to maximize this quantity.

This last notion has been recorded for completeness of documentation, but has not been examined to the point where the advisability of ultimately including some version of it in the computer program can be regarded as established.

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* A more careful treatment would take the LCL container rate schedule into account.

APPENDIX F: RIGOROUS TREATMENT OF INFLATION RATIO

In [7.35] of the main text, the equation

$$I_c = 1/\overline{L}_c \qquad [F.1]$$

was proposed for the inflation ratio to be used in converting from incoming quantities (of cargo in the c-th co-containerizable class, measured in containerfuls) to outgoing quantities (measured in containers). Here $\overline{L}_c$ is the mean load factor for class s.

This is not quite correct; I_c should be the mean value of the reciprocal of the random variable L_c (the load factor) whose mean is $\overline{L}_c$. The discrepancy arises from the fact that the reciprocal of a mean (here, $1/\overline{L}_c$) is only by accident equal to the mean of the corresponding reciprocal (here, of $1/L_c$), otherwise underestimating the latter.*

As in the material around [7.38] and [7.39] in the main text, the notations

$\overline{a}$ = shipment size

m = largest integer with $m\overline{a}<L$

will be employed. The number $\nu+1$, of shipments which make up the payload of an outgoing container, is then a random variable with values between 1 and m+1 inclusive. If $\overline{a}$ is measured in containerfuls, then

$$L_c = (\nu+1)\overline{a} ,$$

and so

$$1/L_c = (1/\overline{a})(\nu+1)^{-1}.$$

Thus the rigorous expression for the inflation ratio is

$$I_c = (1/\overline{a}) \sum_{N=0}^{m} (N+1)^{-1} \mathrm{Prob}\{\nu=N\}. \qquad [F.2]$$

As noted in the main text, the probability of a "normal" container-load ($\nu=m$) is $1-p_{m-1}(\overline{H})$, while for $1 \le N<m$ we have

$$\mathrm{Prob}\{\nu=N\} = \exp(-\overline{H})\, \overline{H}^N/N!.$$

It follows that

$$I_c = (1/\overline{a})\{(m+1)^{-1}(1-p_{m-1}(\overline{H})) + \exp(-\overline{H}) \sum_{0}^{m-1} (N+1)^{-1}\, \overline{H}^N/N!\}. \qquad [F.3]$$

Since

$$(N+1)^{-1}\, \overline{H}^N/N! = \overline{H}^{-1}\, \overline{H}^{N+1}/(N+1)!,$$

the summation in [F.3] can be rewritten

$$\overline{H}^{-1} \sum_{0}^{m-1} \overline{H}^{N+1}/(N+1)! = \overline{H}^{-1} \{p_m(\overline{H}) - \exp(-\overline{H})\} \exp(\overline{H}).$$

Thus [F.3] yields

$$I_c = (1/\overline{a}) \{(m+1)^{-1} (1 - p_{m-1}(\overline{H})) + \overline{H}^{-1} [p_m(\overline{H}) - \exp(-\overline{H})]\}. \qquad [F.4]$$

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* That the direction of error is under-estimation follows from the Cauchy-Schwartz Inequality.

APPENDIX G: APPLICABILITY OF TRIP-END DISTRIBUTION MODELS

As noted in Chapter III, the available demand data included the U.S. member of each origin-destination pair (origin for exports, destinations for imports), but not the overseas member. Thus some arbitrary assumptions, detailed in Chapter IX, were required in order to exercise the model.

Quite late in the study, it was found that for each U.S. port, data on the total amount shipped to each overseas port might well have been available. From the study data, the amount shipped by each shipper to each of the three U.S. ports is known. The available "data" would then have included "trip end totals" at each of a number of U.S. shippers (referred to here as origins although for import cargo they are actually destinations) and foreign ports (referred to here as destinations). The remaining problem would be that of somehow using these totals to attribute a numerical value to the flow volume between each individual origin-destination pair.

The very same problem arises in what has become a somewhat stylized approach to the planning of urban transport systems.[38] The mathematical models developed for treating it are called trip-end distribution models, and include among their input data either observed or estimated values of the quantities

r_i = volume of flow from i-th origin,

c_j = volume of flow to j-th destination,

satisfying the obvious balance condition

$$\sum_i r_i = \sum_j c_j. \qquad [G.0]$$

The outputs of these models are proposed values for the quantities

x_{ij} = flow volume between i-th origin and j-th destination,

which of course must satisfy the conditions

$$x_{ij} \geqq 0, \qquad [G.1]$$

$$\sum_j x_{ij} = r_i, \qquad [G.2]$$

$$\sum_i x_{ij} = c_j. \qquad [G.3]$$

The overwhelming majority of trip-end distribution models have been of the type known as gravity models.[39] Here the model inputs include quantities

K_{ij} = "conductance" between i-th origin and j-th destination.

The basic idea is to mimic Newton's Law of Gravitation by setting

$$x_{ij} = r_i c_j K_{ij}.$$

However, this equation is in general inconsistent with [G.1] through [G.3], and so is replaced by

$$x_{ij} = r_i^* \, c_j^* \, K_{ij}, \qquad [G.4]$$

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[38] B. M. Levin and R. E. Schofer, The Urban Transportation Planning Process, Socio-Econ. Plan. Sci. Vol. 1 (1967), pp. 185-197.

[39] New work along this line, plus a useful bibliography, is given by P. S. Loubal, A Mathematical Model for Traffic Forecasting, 5/68, Berkeley doctoral thesis.

where r_i^* and c_j^* are quantities to be determined so as to make [G.4] consistent with [G.1] through [G.3].

The system of equations [G.1] through [G.4] is nonlinear. It is solved by an iterative process in which one starts with $r_i^* = r_i$ and $c_j^* = c_j$, computes the x_{ij} by [G.4], finds r_i^*'s so that (with the "current" c_j^*-values) the results of [G.4] will satisfy [G.2], finds c_j^*'s so that (with the "current" r_i^*-values) the results of [G.4] will satisfy [G.3], and so on, alternately imposing [G.2] and [G.3]. The convergence of this process, and the existence and uniqueness of x_{ij}'s satisfying [G.1] through [G.4], have only recently been put on a firm basis. Gravity models differ among themselves predominantly in how values for the conductances K_{ij} are constructed from the distance, travel time and travel cost associated with the (i,j)-th origin-destination pair.

The availability of data on overseas trip-end totals became known too late to warrant a literature search as to whether these concepts have previously been applied in the context of export-import trade. Though the techniques are presumably "better than nothing," their transferability from an urban transportation setting to overseas movements of cargo obviously cannot be asserted with any great confidence, pending actual empirical testing.

It is natural to ask how one might proceed to attribute values to the flows x_{ij}, when not even the data needed to estimate the K_{ij}'s are at hand.[40] With such a dearth of information, the most that can be hoped for is a method which is systematic and reproducible. Clearly the array X of x_{ij}'s must satisfy conditions [G.1] through [G.3]. Let d(X,Y) denote some measure of the "distance" between two such arrays, $X = (x_{ij})$ and $Y = (y_{ij})$; here d(X,Y) might be interpretable as indicating the seriousness of assuming the flow pattern to be X when in fact it is Y. Then the worst possible error that could result, from assuming the flow pattern to be X, is

$$F(X) = \max_Y d(X,Y),$$

and a conservative policy would be to choose X so as to minimize this worst possible error, i.e. to reduce it to

$$\min_X \max_Y d(X,Y).$$

The tractability of the mathematical problem of determining the minimizing X, of course, depends on the formula chosen for d(X,Y). For the not unreasonable choice

$$d(X,Y) = \max_{ij} | x_{ij} - y_{ij} | ,$$

this problem has been proved equivalent to the solution (which can be carried out by standard methods) of the following linear program in the x_{ij}'s and one extra variable z:

Minimize z

subject to [G.1] through [G.3] as well as the constraints

$$z \geq M_{ij} - x_{ij},$$

$$z \geq x_{ij} - m_{ij} ,$$

where the constants M_{ij} and m_{ij} are given by

$$M_{ij} = \min (r_i, c_j),$$

$$m_{ij} = \max (0, r_i + c_j - S),$$

and S is the common value of the two sides of [G.0].

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[40] The following material is adapted from A. J. Goldman and P. R. Meyers, Minimax Error Selection of a Bivariate Distribution with Given Marginals, manuscript in progress.

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