NATIONAL BUREAU OF STANDARDS REPORT

2916

PROJECTS and PUBLICATIONS

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A Quarterly Report

July through September 1953

FOR OFFICIAL USE

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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Electricity. Resistance Measurements. Inductance and Capacitance. Electrical Instruments. Magnetic Measurements. Applied Electricity. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Gage.

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Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion.

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Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Machine Development.

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Ordnance Development.
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These three divisions are engaged in a broad program of research and development in advanced ordnance. Activities include basic and applied research, engineering, pilot production, field testing, and evaluation of a wide variety of ordnance matériel. Special skills and facilities of other NBS divisions also contribute to this program. The activity is sponsored by the Department of Defense.

Missile Development. Missile research and development: engineering, dynamics, intelligence, instrumentation, evaluation. Combustion in jet engines. These activities are sponsored by the Department of Defense.

- Office of Basic Instrumentation
- Office of Weights and Measures.

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NATIONAL BUREAU OF STANDARDS

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Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies, which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, Astronomy, and Automatic Translation

*New task

D. Mathematical Statistics

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

September 30, 1953

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-10-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Managers: G.E. Forsythe, M. R. Hestenes, D. H. Lehmer
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. M. R. Hestenes has continued his theoretical investigation of the conjugate gradient method of solving linear systems. By extending the concept of gradient somewhat he shows that every n-step iterative procedure can be interpreted as a conjugate gradient method.

T. S. Motzkin has thought of an n-step process, believed new, for solving a system of linear equations. Let the system be regarded as the intersection of n hyperplanes $H_1,\ldots,H_n$. Select n+1 points $P_1,\ldots,P_{n+1}$ at random. Using one point, say P_{n+1}, as a center of projection, project the other points into H_1, to obtain the points $P_1^{(1)},\ldots,P_n^{(1)}$. Using $P_n^{(1)}$ as a center, project $P_1^{(1)},\ldots,P_{n-1}^{(1)}$ onto H_2, obtaining $P_1^{(2)},\ldots,P_{n-1}^{(2)}$. Continuing in this way, the point $P_1^{(n)}$ is at the intersection of all hyperplanes, and thus a solution to the system is found.

Publications: (1) "Tentative classification of methods and bibliography on solving systems of linear equations," by G. E. Forsythe. Simultaneous linear equations and the determination of eigenvalues, Proceedings of an NBS Symposium held in Los Angeles, August 1951; NBS Applied Mathematics Series 29, p. 1-28 (U. S. Government Printing Office, Washington, D. C.) (2) "On certain character matrices," by D. H. Lehmer; submitted to a technical journal. (3) "Punched-card experiments with accelerated gradient methods for linear equations," by A. I. and G. E. Forsythe; to appear in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series; in press, U. S. Government Printing Office. (4) "Some numerical examples on solving systems of linear equations by the conjugate gradient method for nonsymmetric systems of equations," by M. R. Hestenes, U. Hochstrasser, and L. S. Wilson; IN MANUSCRIPT. (5) "On the linear

iteration procedures for symmetric matrices," by A. M. Ostrowski; submitted to a technical journal. (6) "On over and under relaxation in the theory of the cyclic single step iteration," by A. M. Ostrowski; MTAC 43, 152-159 (July 1953). (7) "On the convergence of Gauss' alternating procedure in the method of the least squares, I," by A. M. Ostrowski; submitted to a technical journal. (8) "The solution of systems of linear equations by the conjugate-gradient method for use on IBM equipment," by U. Hochstrasser; IN MANUSCRIPT. (9) "Solving linear algebraic equations can be interesting," by G. E. Forsythe; Bul. Am. Math. Soc. 59, 299-329 (July 1953).

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-10-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Managers: G. E. Forsythe and M. R. Hestenes
Full task description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

Publications: (1) "Completely continuous normal operators with property L," by I. Kaplansky; submitted to a technical journal. (2) "Asymptotic lower bounds for the frequencies of polygonal membranes," by G. E. Forsythe; submitted to a technical journal. (3) "On the spectrum of a one parametric family of matrices," by A. M. Ostrowski; submitted to a technical journal.

STUDIES IN PURE MATHEMATICS
Task 1101-10-5100/50-4
(formerly 1101-11-5101/50-4)

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: C. B. Tompkins
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. The calculations for the determination of irregular primes were continued for all primes less than 2000. To do this it was necessary to recode the problem to make use of the magnetic drum memory. It was discovered that there are 118 irregular and 184 regular primes up to that limit. It has been proved by Kummer that Fermat's Last Theorem is true for all regular primes. Further tests were deemed necessary in order to complete the proof for irregular primes. Such a test - a modification of a Kummer criterion - was programmed for the SWAC. This test was found to hold for all irregular primes less than 2000 thus proving Fermat's Last Theorem up to that limit.

T. S. Motzkin, jointly with P. Erdös, E. G. Straus, and J. D. Swift, studied the representations of a positive integer n as sum of three integers $n=a_k+b_k+c_k$, $a_k \geq b_k \geq c_k \geq 1$, $k=1,2,\ldots,r$, such that $a_1 b_1 c_1 = \ldots = a_r b_r c_r$.

Besides asymptotic properties it was established that sets of distinct triples with arbitrarily large r exist, and that sets of two distinct triples exist for every $n>18$.

Background studies on embedding of surfaces in Euclidean space and in problems connected with minimizing Dirichlet's Integral under various conditions (the Problem of Plateau and problems of lesser difficulty) have been initiated by G. Blanch, M. R. Hestenes, C. B. Tompkins, and others. These studies will produce some partial differential equations to be attacked on SWAC.

Discrete variable problems were attacked by various workers including E. Kleinfeld, E. Lehmer, D. H. Lehmer, and C. B. Tompkins. (For discrete variable problems see also task 1101-10-5150/53-1, p. 10).

R. H. Bruck, M. R. Hestenes, H. W. Kuhn, T. S. Motzkin, C. B. Tompkins, J. L. Walsh, and W. Wasow participated in the American Mathematical Society's Sixth Symposium held in Santa Monica on August 26-28, 1953.

Publications: (1) "Boolean Geometry I," by L. M. Blumenthal; Rend. Circ. Mat. Palermo, {II} , 1, 1-18 (1952). (2) "An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space," by I. J. Schoenberg; submitted to a technical journal. (3) "On the distribution of totitives," by D. H. Lehmer; IN MANUSCRIPT.

COMPUTATION OF THE COMPLEX ZEROS OF THE RIEMANN-ZETA FUNCTION
Task 1101-10-5100/50-13
(formerly 1101-11-5101/50-13)

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Managers: D. H. Lehmer and C. B. Tompkins
Full task description appears in Apr-June 1950 issue.

Status: CONTINUED. Careful analysis of the zeta function at the 400 special Gram points is being carried out. (See Jan-Mar 1953 issue, p. 4.)

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-10-5100/51-1
(formerly 1101-11-5100/51-1)

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: G. Blanch and W. Wasow
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Much of the activity of the past quarter centered around the symposium on numerical analysis (see p. 62) and on the teaching of courses at UCLA during the summer session. In connection with the latter, W. Wasow is preparing a set of lecture notes on the asymptotic theory of linear differential equations.

The table of characteristic values of Mathieu's equation (see Apr-June 1953 issue, p. 4) by G. Blanch and I. Rhodes is being put in final form during spare hours. It is hoped that the table will shed more light on the asymptotic behavior of the characteristic values and will stimulate research which will result in better asymptotic approximations.

Status of Projects

Publications: (1) "On the numerical solution of parabolic partial differential equations," by G. Blanch; J. Res.NBS 50, 343-356 (June 1953). (2) "On mildly nonlinear partial difference equations of elliptic type," by L. Bers; accepted by the Journal of Research of the NBS. (3) "Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point," by W. Wasow; accepted by the Annals of Mathematics. (4) "An expansion method for parabolic partial differential equations," by J. W. Green; accepted by the Journal of Research of the NBS. (5) "On small disturbances of plane Couette flow," by W. Wasow; accepted by the Journal of Research of the NBS. (6) "Tables of lift and moment coefficients for oscillating airfoils in subsonic compressible flow," (formulation of Reissner and Haskind) by G. Blanch; an NBS report. (7) "Singular perturbation methods for nonlinear oscillations," by W. Wasow; IN MANUSCRIPT.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-10-5100/51-2
(formerly 1101-11-5100/51-2)

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: D. Teichroew and W. Wasow
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. D. Teichroew has continued his investigation of methods of generating random variates. A subroutine to compute values of random variates on the SWAC by the polynomial transformation method has been coded and checked. A routine to compute functions of sequences of random variates computed by this method has also been coded and checked. The functions are: (i) the first eight moments, (ii) the serial correlation, and (iii) the frequency distribution. These statistics are used to test the hypothesis that the random variates have the desired distributions. Experience up to the present indicates that the hypothesis will not have to be rejected.

An investigation of methods of computing probability distributions appropriate to high speed computers has been started by D. Teichroew. A code for computing the incomplete gamma function (for integral values of the parameter) has been checked and is being used to evaluate the accuracy of other methods. (It is hoped that an opportunity will arise to compute an extensive table of the incomplete gamma function). One method which, to our knowledge, has not yet been investigated is that based on the inversion of the characteristic function. For example,

$$F(x) = \int_0^x \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-t^2/2}\,\frac{\sin xt}{t}\,dt\ .$$

From preliminary calculations it appears that, for many distributions, numerical integration of an integral analogous to the integral on the right-hand side may be a reasonable way of computing the value of the distribution function for arbitrary values of the argument. Another method being investigated is that based on asymptotic expansions in terms of normal deviates. The expansion has been obtained for certain distributions that tend to the normal as a parameter tends to a limit. One advantage of this method is that it permits computations of variates associated with a given probability and therefore eliminates the need for inverse interpolation in tables of distribution functions. Routines have been coded and checked for computing t and chi-square variates from

normal deviates and vice versa, normal deviates from t and chi-square variates. In connection with the incomplete gamma function the introduction to "Tables for the computation of the incomplete gamma function and the probability function of χ^2," by E. E. Slutskii, has been translated from the Russian by D. Benster.

A report is being prepared which contains the results of some sampling experiments (carried out on the SWAC by D. Teichroew) to determine the power of certain nonparametric tests, (see publication (3) below).

The following related articles have been translated from the Russian by C. D. Benster: (1) "On the maximum discrepancy between two empirical distributions," by B. V. Gnedenko and V. S. Koroliuk, (2) "On a problem of the comparison of two empirical distributions," by B. V. Gnedenko and E. L. Rvacheva, (3) "Some results on the maximum discrepancy between two empirical distributions," by B. V. Gnedenko, (4) "On the empirical distribution function in the case of grouping of the data," by I. I. Gikhman, (5) "On the distribution of the number of excesses of one empirical distribution function over another," by B. V. Gnedenko and V. S. Mikhalevich, (6) "Two theorems on the behavior of empirical distribution functions," by B. V. Gnedenko and V. S. Mikhalevich, and (7) "On the mutual disposition of two empirical distribution functions," by V. S. Mikhalevich.

F. G. Foster, on leave from the London School of Economics and Political Science, continued his research on a record test for trend in a time series by using the SWAC to compute, by sampling, power points of the test against normal alternatives, (see publication (4) below).

Publications: (1) "Statistical estimation of matrix quantities by means of a class of discrete Markov chains," by H. P. Edmundson; IN MANUSCRIPT. (2) "Limits for permanent preponderance," by P. Erdös and T. S. Motzkin; IN MANUSCRIPT. (3) "Some sampling results on the power of nonparametric tests against normal alternatives," by W. J. Dixon and D. Teichroew; IN MANUSCRIPT. (4) "Power function of the record test for trend in a time series," by F. G. Foster and D. Teichroew; IN MANUSCRIPT.

VARIATIONAL METHODS
Task 1101-10-5100/51-3

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

Publications: "Iterative methods of solving linear problems on Hilbert space," by R. M. Hayes; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

STUDIES IN APPLIED MATHEMATICS
Task 1101-10-5100/51-4

Origin: NBS Authorized 9/1/50
Sponsor: Office of Naval Research, USN
Managers: G. E. Forsythe and T. S. Motzkin
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The seminar on numerical analysis continued to meet through July and early August. Under the influence of the SCAMP program in finite projective geometry, several talks were presented on computational aspects of discrete problems, and especially on the determination of perfect difference sets. There were also two talks on computational problems of theoretical physics. A detailed list of sessions will be found under Lectures and Symposia in the back of this issue. Summaries of the presentations have been prepared for the sessions from April 20 through May 13 by C. B. Tompkins and for the sessions from May 18 through June 8 by G. E. Forsythe.

T. S. Motzkin and J. L. Walsh continued their study of approximation by polynomials of a given degree on a real or complex finite point set, in particular of the minimization of $\Sigma\mu_i \mid f(z_i)\mid^p$, where the weight $\mu_i \geq 0$ and the exponent $p>0$, as well as the set $z_1,\ldots,z_m$ are given (for $p=\infty$ replace the sum by $\max \mu_i \mid f(z_i)\mid$). The set of all polynomials belonging, for given $p\geq 1$, to weights at least k of which are greater than 0, and the set of all infrapolynomials (that is, such that no other polynomial $g(z)$ has the property $g(z_i)= \varepsilon_i f(z_i)$, $|\varepsilon_i| <1$, $(i=1,\ldots,m)$ were determined and compared. Related investigations of real linear families of complex polynomials and of generalizations to higher-dimensional space were begun.

Publications: (1) "Numerical computation of low moments of order statistics from a normal population," by J. B. Rosser; submitted to a technical journal. (2) "Changes of sign of sums of random variables," by P. Erdös and G. Hunt; submitted to a technical journal. (3) "A numerical analyst's 15-foot shelf," by G. E. Forsythe; to appear in Mathematical Tables and Other Aids to Computation. (4) Seminar on numerical analysis -- summary of presentations between April 20 and May 13, 1953," by C. B. Tompkins, multilithed typescript, 10 p. (5) "Seminar on numerical analysis -- summary of presentations between May 18 and June 8, 1953," by G. E. Forsythe, multilithed typescript, 38 p.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-10-5100/51-5

Origin: Office of Naval Research, USN Authorized 9/1/50
Sponsor: " "
Managers: D. Saxon and C. B. Tompkins
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Item (5) under publications below deals with the application of variational methods to quantum mechanical scattering problems. A variational formulation for general acoustic scattering problems has also been developed (see publication (6) below).

Programming and preliminary numerical work is continuing on the calculation of the photo-disintegration of the deuteron (see Oct-Dec 1952 issue, p. 7). R. Woods is continuing a program of computation on

the elastic scattering of protons from various elements. A. Baños developed an extension of the saddlepoint method of integration valid when a pole exists in the vicinity of the saddlepoint.

Publications: (1) "Modes of vibrations of a suspended chain," by D. S. Saxon and A. S. Cahn; accepted by Quarterly Journal of Mechanics and Applied Mathematics (Oxford). (2) "A nonlinear model for the composite Pi-meson," by S. G. Gasiorowicz; submitted to a technical journal. (3) "Acoustic radiation pressure on a circular disk," by H. Levine; to appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society. (4) "A numerical solution of Schroedinger's equation in the continuum," by W. Futterman, E. Osborne, and D. S. Saxon; accepted by the Journal of Research of the NBS. (5) "Application of variational methods to intermediate and high energy scattering," by E. Gerjuoy and D. S. Saxon; IN MANUSCRIPT. (6) "Variational principles for the acoustic field," by E. Gerjuoy and D. S. Saxon; IN MANUSCRIPT.

STUDY OF RUSSIAN MATHEMATICAL PROGRESS
Task 1101-10-5100/52-1

Origin: NBS Authorized 3/15/52
Sponsor: Office of Naval Research, USN
Manager: G. E. Forsythe
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. The task manager is continuing to accumulate bibliographical cards on Russian mathematical monographs (see Jan-Mar 1952 issue, p. 11 and Oct-Dec 1952 issue, p. 8). The translations formerly reported under this task are now reported in connection with the pertinent tasks of the Institute for Numerical Analysis.

GENERALIZED RAYLEIGH-RITZ METHOD FOR EIGENVALUES OF A CLAMPED PLATE
Task 1101-10-5100/53-1

Origin: NBS Authorized 12/29/52
Sponsor: Office of Naval Research, USN
Manager: G. Blanch
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Some progress has been made in programming the computations for SWAC.

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-10-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF Authorized 6/15/50
Sponsor: " "
Manager: T. S. Motzkin
Full task description appears in Apr-June 1950 issue.

Status: CONTINUED. T. S. Motzkin investigated the assignment

problem from two different points-of-view. If the entries are sufficiently rough and the matrices sufficiently large, it turns out to be always feasible, even by slightly modified direct combinatorial trials, to obtain a permutation consisting of maxima in all rows and columns. The corresponding computational method, proposed by D. H. Lehmer and coded by B. Handy, was run for a 12x12 matrix and found to be too long, especially due to the large percentage of time consumed for access to the drum. On the other hand, in view of evaluation of application of the simplex method, the polyhedra corresponding to the assignment problem and some of its generalizations were studied and the faces of all dimensions determined.

Preparation of an expository account of the theory of inequalities was begun by T. S. Motzkin and H. W. Kuhn. This presentation has been tentatively divided into four sections: (1) duality theorems, (2) rank theorems, (3) criteria for solvability, and (4) methods of solution; the latter will be worked out by A. J. Hoffman. Section (1) also contains systematic and elementary proofs of the incompatibility and implication theorems.

H. W. Kuhn investigated the feasibility of the face approach for the "traveling salesman" problem. It was found that, although there are but (n-1)! permutations to be considered, the convex hull has many more than n! faces in general. The exact number is known only for n=2,3,4; there are 450 distinct faces known for n=5. All of the conditions found thus far can be characterized in terms of a "blocking set", i.e., a minimal set of pairs (i,j) such that each admissible permutation takes some i into j. With the assistance of T. S. Motzkin, a finite zero-sum two-person game equivalent to the "traveling salesman" problem was constructed. The experience gained from this line of investigation seems to indicate that the problem is one of direct combinatorial enumeration.

A connection between convexity and contractibility was established by H. W. Kuhn who found that a contractible polyhedron in n-space with contractible faces must be convex. This result is related to recent econometric studies by Debreu and Arrow.

J. Marschak, in collaboration with R. Radner, studied optimal networks and rules for teams. The goal of a team is defined by the payoff function $u(a,x)$ where $a=\{a_i\}$ $(i=1,\ldots,n)$ and $x=\{x_j\}$ $(j=1,\ldots,m)$ are, respectively, the vectors of controlled (or action) variables and of non-controlled (or external) variables. Given the probability distribution $\Phi(x)$, the problem is to choose the best rule of actions, $\hat{\alpha}$, and the best communication network $\hat{N}$, defined as follows: To each communication network N corresponds a cost, C=C(N), and a matrix of "information structure," S=S(N), whose element $s_{ij}=1$ or 0 according to whether a_i does or does not depend on x_j. Let the vector $x_S^{(i)}$ consist of all x_j such that $s_{ij}=1$. Then the rule of actions is the vector $\alpha=\{\alpha_i\}$ of functions such that $a_i=\alpha_i(x_S^{(i)})$. The best expected gross payoff is

$$\hat{U}(S) = U(S(N),\hat{\alpha}) = \max_{\alpha} \int_x u[\alpha_1(x_S^{(1)}),\ldots,\alpha_n(x_S^{(2)}),x]\,d\Phi(x);$$

the best expected net payoff is

$$V(\hat{N},\hat{\alpha}) = \max_{N} [U(S(N),\hat{\alpha}) - C(N)].$$

The problem was studied for two cases: (I) u, a strictly concave function of a non-bounded vector a; (II) u, a linear function of a bounded vector a. Case (II) represents linear programming under uncertainty.

The economic example for case (I) was a firm in a competitive market, earning a profit $u=aQa'-xa'$, where a is the vector of inputs, x is the vector of their respective prices (the price of a unit of output being =1), and Q is a matrix characterizing the production technology.

The above profit function is a special case of a general quadratic function of a and x. Such a quadratic function can, without loss of generality, be assumed to be of the form

$$u(a,x) = aQa' - 2xRa',$$

where the expected value of x is zero. It is also assumed that for every x, u has a unique maximum in a, i.e., Q is negative definite.

For a given information structure S, the best rule of action $\hat{\alpha}$ is the solution of the system of integral equations

$$E(\alpha(x)Q - xR \mid x^{(i)}) = 0, \quad i = 1,\ldots,n.$$

If x has a normal distribution, then each $\hat{\alpha}_i(x^{(i)})$ is a homogeneous linear function of $x^{(i)}$. Formulas for the best rule of action and best expected gross payoff have been obtained under various special conditions on the distribution of x and on the information structures.

The economic example for case (II) was a firm engaged in speculative shipping of a commodity. If t_{ij} is the cost of transporting the commodity from city i to city j, if x is the vector of its prices in n cities, and if not more than one unit can be shipped from each city, the best expected gross payoff is

$$\hat{U}(S) = \sum_{i=1}^{n} E(\max_j E(x_j - x_i - t_{ij}) \mid x_S^{(i)}),$$

where the vector $x_S^{(i)}$ consists of prices known to the i-th branch, if the information structure is S. For n=2, with $t_{ij}=(1-\delta_{ij})t$, with c=cost of two-way communication between any two cities, and with $Ex_i=0$, $Ex_i^2=1$, and $Ex_1x_2=\rho$, the best networks were computed manually for given c, t, and ρ. A similar computation was made for n=3 based on the assumption of two alternative simple discrete distributions of x, one involving statistical dependence and the other free of it. Moreover, assume the latter distribution (with equal probabilities for each z_i to take the values 1 and -1) and permit n to change. Then, the optimal size of "subteams" (consisting of members who communicate with each other but not with members outside of the sub-team) turns out to be independent of n (it is equal to the largest integer that is smaller than or equal to $-\log_2 c$), provided the total number n of members is sufficiently large and t is sufficiently small.

For general normal distribution of x, with n=4 or larger, a SWAC procedure has been set up to compute optimal gross payoffs $\hat{U}(S)$ for all information matrices S. The procedure will use random samples of x, with population covariances $\sigma_{ij}=k^{|i-j|}\geq 0$, and alternatively $-1\leq\sigma_{ij}=1-k|i-j|\leq 1$.

With covariance matrices just mentioned and with n increasing indefinitely, it is also hoped to gain insight into the properties of a continuous version of the network problem.

Publications: (1) "Basic solutions of the transportation problem," by T. S. Motzkin; IN MANUSCRIPT. (2) "The multi-index transportation

problem," by T. S. Motzkin; IN MANUSCRIPT. (3) "Ray systems with maximum angle sum," by T. S. Motzkin; IN MANUSCRIPT. (4) "On Fejér sets in linear and spherical spaces," by T. S. Motzkin and I. J. Schoenberg; accepted by the Annals of Mathematics. (5) "On the relaxation method for linear inequalities," by T. S. Motzkin and I. J. Schoenberg; submitted to a technical journal. (6) "On the optimal character of the (s,S) policy in inventory theory," by A. Dvoretzky, J. Kiefer, and J. Wolfowitz; submitted to a technical journal. (7) "Least p-th power polynomials on a real finite point set," by T. S. Motzkin and J. L. Walsh; submitted to a technical journal. (8) "Least p-th power polynomials on a complex finite point set," by T. S. Motzkin and J. L. Walsh; IN MANUSCRIPT. (9) "A new type of existence theorem for systems of linear inequalities," by J. W. Gaddum; IN MANUSCRIPT. (10) "Boundedness of sequential projections," by T. S. Motzkin and C. B. Tompkins; IN MANUSCRIPT. (11) "Towards an economic theory of organization and information," by J. Marschak; to appear in a volume on Decision Processes edited by C. Coombs and R. Thrall. (The original manuscript written in December 1952, was substantially revised in August during the author's stay at the Institute.)

SCAMP
Task 1101-10-5150/53-1

Origin: Office of Naval Research Authorized 6/10/53
Sponsor: " "
Manager: C. B. Tompkins
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. A group of about 20 mathematicians participated in the most intensive part of the program, which lasted from 1 July through 28 August. A formal report has been submitted by the visiting chairman, Prof. S. S. Cairns, to the Department of Defense. Computations have been turned over to be carried out under task 1101-40-5131/54-1 (see p. 24).

2. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(previously listed as Air Materiel Command Computing Machine)
Task 1101-20-5103/49-1
(formerly 1101-34-5103/49-1)

Origin: Aeronautical Research Laboratory Authorized 11/1/48
Wright Air Development Center, ARDC, USAF
Sponsor: " " "
Managers: H. D. Huskey and R. Thorensen
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. During the last quarter the SWAC was used for 29 different problems for a computing time of 623 hours out of a scheduled time of 845 hours. A full set of eigenvalues and eigenvectors for a 32d order matrix was computed to 10 significant decimal digits. The complete solution was obtained in 18 hours of computing time, with the aid of the

magnetic drum memory.

Only minor engineering changes have been made on the computer during this past quarter. These changes have been limited to the completion of projects in progress and include such items as construction of a new breakpoint facility and installation of new power supplies and voltage regulators for an improved cathode ray tube deflection system. No new development projects have been initiated, and it is estimated that the SWAC may be kept in effective operation on a two shift basis with the services of only two maintenance engineers.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-20-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS Authorized 2/15/49
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: H. D. Huskey
Full task description appears in Apr-June 1949 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue.

LANGUAGE TRANSLATION STUDY
Task 1101-20-5103/52-1
(formerly 1101-21-5104/52-1)

Origin: NBS Authorized 8/15/51
Sponsor: The Rockefeller Foundation Revised 10/15/51
Managers: H. D. Huskey and G. E. Forsythe Terminated 9/30/53

Objective: This is a study of certain statistical and logical aspects of language translation, with a view to obtaining at least some partial answers to questions concerning the feasibility and practicality of performing language translations by automatic computing machines. Statistical aspects will include investigations of word frequencies and key-word positions in various types of vocabularies; logical aspects will include the study of the possibility of coding the elements of sentence structure in mathematical symbols understood by a computing machine so that the machine can make a beginning at constructing a real translation rather than doing just a word-substitution.

Background: This task originated from the interest of the Library of Congress and State Department in automatic language translation machines, as stated in a conference between the Librarian, Dr. Luther Evans, the Assistant Librarian, Dr. V. W. Clapp, and representatives of the State Department and of the National Bureau of Standards in January 1950.

Comments: This is a continuation of task 1101-21-1102/50-10 (see Apr-June 1951 issue, p. 11). The major portion of the work on this is being done by members of the language departments at UCLA.

Status: TERMINATED.

STUDIES IN THE THEORY OF DIGITAL COMPUTING MACHINES
Task 1101-20-5103/53-1

Origin: NBS Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: H. D. Huskey
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Methods of facilitating coding and new systems for using the drum in conjunction with the high-speed memory are being considered.

3. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-40-5130/54-1
(formerly 1101-40-1111/49-1a)

Origin: NBS Authorized 9/2/48
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF Revised 11/16/49
Managers: M. Howard, F. Holländer and P. Bremer
Full task description appears in July-Sept 1949 issue.

Status: CONTINUED. Research staff problems involving SWAC were: (1) For D. H. Lehmer: (a) The Mersenne problem has been completed. All numbers have now been computed twice as a check. (b) Turan's conjecture has been tested at 125,000 and checked by repeating the computation. (c) Certain prime numbers were tested by Alway's method. (d) Irregular primes have been tested in an investigation of Fermat's last theorem. (2) For M. Hestenes: Code checking was done on a conjugate gradient method of solving linear systems of equations. (3) For R. Woods: Code checking and some calculation was done on an optical model for calculating the scattering of protons by heavy nuclei. (4) For G. Blanch: An extension of the table of the bivariate normal distribution function was computed. (5) For E. C. Yowell: Several additional routines were checked out for the problem of the determination of periods of light variation of variable stars. (6) For D. Teichroew: SWAC has been used to compute a relatively large number of random variates by the polynomial transformation method, to check the efficiency of the method. Variates computed in this way have been used to compute power functions of certain nonparametric tests. (7) For the Geophysics Dept. of UCLA: Code checking was completed and calculation started on the solution, by the method of least squares, of the problem of high order tidal constituents for earth and ocean tides. (8) For the Chemistry Dept. of UCLA: Several routines concerned with the problem of crystal structure have been checked. Computations are continuing. (9) For the Physics Dept. of UCLA: Code checking is continuing on the routine for the solution of the molecular wave function for the Li_2 molecule. (10) For the Education Dept. of UCLA: Computations are being performed on item analysis of certain psychological instruments. (11) For the Business Administration Dept. of UCLA: A set of five linear equations in five unknowns was solved on SWAC.

MATHIEU FUNCTIONS II
Task 1101-40-5131/45-1
(formerly 1101-53-1101/45-1)

Origin: Applied Mathematics Panel, NDRC — Authorized 7/1/47
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Terminated 9/30/53
Manager: E. C. Yowell

Objective: To prepare a table of the periodic solutions:

$$Se_r(s,t) = \sum_{n=0}^{\infty} De_{2n+p} \cos(2n+p)t, \quad (p = 0,1)$$

$$So_r(s,t) = \sum_{n=1}^{\infty} Do_{2n-p} \sin(2n-p)t, \quad (p = 0,1)$$

for r=0(1)15, t=0(1°)90° over the range s=0 to s=100, of the Mathieu differential equation:

$$y'' + (b - s\cos^2 t)y = 0.$$

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press 1947). The existing tables by Ince and Goldstein are inadequate. The task was originally proposed by Dr. Philip Morse and Dr. James Wakelin.

Status: TERMINATED.

PUNCHED CARD LIBRARY
Task 1101-40-5131/49-2
(formerly 1101-53-1101/49-2)

Origin: NBS — Authorized 7/14/48
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: P. Bremer
Full task description appears in Apr-June 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles, 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following tables were added:
(1) Table 5020,

$$G(\delta;\alpha,\beta) = (\beta+1) \int_{-\infty}^{\infty} [F(x+\delta)]^{\alpha} [1-F(x)]^{\beta} f(x)dx,$$

where

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad \text{and} \quad F(x) = \int_{-\infty}^{x} f(t)dt$$

$$\alpha = 0(1)4; \ \beta = 1(1)9; \quad \delta = -3.20(.05)0(.01)6.4(.1)7.1;9D,$$

(2) Table 5021,

$$L(h,k,r) = \int_{h}^{\infty} dx \int_{k}^{\infty} dy \ \phi(x,y,r),$$

(3) Table 5022

$$V(h, \lambda h) = \int_{o}^{h} dx \int_{o}^{\lambda x} dy[z(x)z(y)]$$

where

$$\phi(x,y,r) = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left\{-\tfrac{1}{2}\left(\frac{x^2+y^2-2rxy}{1-r^2}\right)\right\}, \quad z(t) = \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}t^2).$$

[See tasks 1101-53-1101/51-32, Apr-June 1951, p. 25, and 1101-53-1101/52-50, Apr-June 1952, p. 28, for background.] For $L(h,k,r)$: $\pm$ r=0(.05)0.95(.01)1; 6D for positive values of r; 7D for negative values of r. h,k=0(.1)4; or up to point where L=0 to 7 decimals. For $V(h, \lambda h)$: h=0(.01)4(.02)4.6(.1)5.6 and ∞. For $V(\lambda h,h)$: h=0(.01)4(.02)5.6 and ∞. Both $V(h, \lambda h)$ and $V(\lambda h,h)$ are given for λ= .1(.1)1, 7D, with last place uncertain by two units.

STATISTICAL SMOOTHING
Task 1101-40-5131/51-19
(formerly 1101-53-1101/51-19)

Origin: Stanford Research Institute, Stanford University Authorized 1/15/51
Sponsor: Office of Research Operations, U.S. Army
Manager: M. Howard
Full task description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

TABLES OF THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-40-5131/51-32
(formerly 1101-53-1101/51-32)

Origin: Division 13, NBS Authorized 5/31/51
Sponsor: Office of Chief of Ordnance, U. S. Army
Manager: G. Blanch
Full task description appears in Apr-June 1951 issue.

Status: CONTINUED. A preliminary manuscript has been prepared with an introduction by the task manager concerning methods of interpolation and the method used to collate the tables. The collated volume also includes the function V(h,k), described in task 1101-40-5131/52-50, p. 16. The final manuscript will be prepared from the preliminary one on special forms. All values are now on punched cards.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-40-5131/51-34
(formerly 1101-53-1101/51-34)

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: E. C. Yowell
Full task description appears in Apr-June 1951 issue.

Status: CONTINUED. Integrations involving three sets of observed data were completed during this quarter.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-40-5131/51-36
(formerly 1101-53-1101/51-36)

Origin: University of Oregon Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: A. D. Hestenes
Full task description appears in Apr-June 1951 issue.

Status: CONTINUED. The Rosser method is being coded for SWAC solution. In order to achieve the desired results triple precision multiplication, addition, etc., is used. At this point values of

$$f(t) = e^{-\frac{t^2}{2}}/\sqrt{2\pi}$$

have been computed to 30 decimals at intervals of .02 units. The code for computing the integral of f(t) is well under way. Some work has been done on a few of the remaining routines.

SIERRA WAVE PROJECT
Task 1101-40-5131/52-36
(formerly 1101-53-1101/52-36)

Origin: Department of Meteorology, U.C.L.A. Authorized 4/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: T. H. Southard
Full task description appears in Jan-Mar 1952 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

LEGENDRE FUNCTIONS
Task 1101-50-5131/52-48

Origin: Hughes Aircraft Company — Authorized 6/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Terminated 9/30/53
Manager: E. C. Yowell

Objective: To determine the values of γ for which $P_\gamma^m(x)\Big|_{x=x_0} = 0$, where $P_\gamma^m(x)$ is the associated Legendre function. The first six values of γ for m=0(1)10 and for two values of x_0 are required. Also to determine

$$\int_{x_0}^{1} [P_\gamma^m(x)]^2 \, dx,$$

where γ is determined from the first problem.

Background: These eigenvalues are necessary for the expansion in orthogonal functions of the radiation fields from certain antennae.

Status: TERMINATED.

TABLES RELATING TO THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-40-5131/52-50
(formerly 1101-53-1101/52-50)

Origin: Division 13, NBS — Authorized 6/1/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Completed 9/30/53
Manager: G. Blanch

Objective: To provide tables of V(h,k) and V(k,h), relating to the bivariate normal distribution function, where

$$V(h,k) = \frac{1}{2\pi} \int_0^h dx \int_0^{xk/h} \exp(-\tfrac{1}{2}x^2 - \tfrac{1}{2}y^2)dy.$$

Parameters: h=0(.01)4, and h=∞; k/h=0.1(.1)1.
Accuracy 6 decimals.

Background: A seven-place table of V(h,k), at intervals of 0.05 or 0.1 in h, was computed by the Mathematical Tables Project, under the auspices of NDRC, at the request of H. H. Germond. The latter subtabulated this table, and published an extensive table of V(h,k),but not of V(k,h) in a restricted NDRC report, which has since been declassified. There is an intimate relationship between the function V(h,k) and the bivariate normal distribution function L(h,k,r), where

$$L(h,k,r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^{\infty} dx \int_k^{\infty} \phi(x,y,r)dy$$

and

$$\phi(x,y,r) = \exp\left[-(x^2+y^2-2rxy)/2(1-r^2)\right].$$

It can be shown that

$$L(h,k,r) = V\left(h, \frac{k-rh}{\sqrt{(1-r^2)}}\right) + V\left(k, \frac{h-rk}{\sqrt{(1-r^2)}}\right) + \frac{1}{4}\left[1-\alpha(h)-\alpha(k)\right] + \frac{\text{arc sin } r}{2\pi}$$

where

$$\alpha(x) = \frac{1}{\sqrt{2\pi}} \int_{-x}^{x} \exp\left(-\tfrac{1}{2}t^2\right)dt.$$

For some purposes, the function V(h,k) is more convenient than L(h,k,r), and it is deemed desirable to include a table of both V(h,k) and V(k,h) in the volume of collated tables L(h,k,r), described under task 1101-53-1101/51-32.

Status: COMPLETED. The table is included in the collated tables of the Bivariate Normal Distribution Function. See task 1101-40-5131/51-32, p. 14.

EIGENVALUES
Task 1101-40-5131/53-6

Origin: Convair Authorized 9/26/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: E. C. Yowell
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. The first portion of the problem was completed by the determination of all eigenvalues and eigenvectors of the 45th order matrix, and these results have been transmitted to the proposer of the problem. The second portion, which involves the inversion of this matrix, has not yet been started.

METEOROLOGICAL MEANS
Task 1101-40-5131/53-10

Origin: Meteorology Department, UCLA Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. H. Hollander
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Coding has been completed for the 850 mb level calculations. The programming for the "Mountain Term" is complete, and

coding has begun. Time averages for two levels remain to be coded. The problem has been extended by the addition of data for the 100 mb level for January-February 1949 (see project 11.1/31-50-17, Jan-Mar 1950 issue, p. 15). The calculation of results for these data will use the codes already formed.

PROLATE SPHEROIDAL WAVE FUNCTIONS
Task 1101-40-5131/53-11

Origin: Stanford Research Institute — Authorized 9/30/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Terminated 9/30/53
Manager: R. R. Reynolds

Objective: To compute

$$Y = \frac{c^2}{30(\log \frac{b}{a})^2} \sum_{n=1,3,5,7} A_n[I_{2n} - B_n(I_{1n})^2]$$

and

$$Z = \frac{1}{Y} ,$$

for the 108 sets of values $\xi_0=1+10^{-m}$ (m=2(2)8); b/a=2,4,8; c=1.2,$\pi/2$,2, $3\pi/4$,2.5,2.8,3,π,3.2. A_n and B_n are tabulated complex functions of c and ξ_0,

$$I_{1n} = \int_{\xi_0}^{\xi_b} \xi(\xi^2 - 1)^{-\frac{1}{2}} R_{1n}^{(3)}(c,\xi)\, d\xi$$

$$I_{2n} = 2 \int_{\xi_0}^{\xi_b} \xi(\xi^2 - 1)^{-\frac{1}{2}} R_{1n}^{(3)}(0,\xi)\, d\xi \int_{\xi_0}^{\xi} \xi'(\xi'^2-1)^{-\frac{1}{2}} W_n(c,\xi')\, d\xi',$$

$$W_n(c,\xi') = R_{1n}^{(1)}(c,\xi')\, dR_{1n}^{(3)}(c,\xi_0)/d\xi_0 - R_{1n}^{(3)}(c,\xi')\, dR_{1n}^{(1)}(c,\xi_0)/d\xi_0,$$

$$\xi_b = [1 + (b/a)^2 (\xi_0^2 - 1)]^{\frac{1}{2}},$$

$$R_{1n}^{(3)}(c,\xi) = R_{1n}^{(1)}(c,\xi) + i\, R_{1n}^{(2)}(c,\xi), \quad i^2 = -1,$$

and $R_{1n}^{(1)}$ and $R_{1n}^{(2)}$ are the first and second solutions of

$$\left[\frac{d}{d\xi}\left\{(\xi^2 - 1)\frac{d}{d\xi}\right\} - \left\{\frac{1}{\xi^2-1} + \lambda_{1n} - c^2\xi^2\right\}\right] R_{1n}(c,\xi) = 0$$

with the expansions

$$R_{1n}^{(1)} = [K_{1n}^{(1)}]^{-1} (\xi^2 - 1)^{\frac{1}{2}} \sum_{k=0}^{\infty} c_{2k}^{1n} (-)^k (\xi^2 - 1)^k,$$

$$R_{1n}^{(2)} = \frac{1}{2} Q_{1n} \quad R_{1n}^{(1)} \log[(\xi + 1)/(\xi - 1)] + \xi(\xi^2-1)^{-\frac{1}{2}} \sum_{k=0}^{\infty} b_k^{1n} (\xi^2-1)^k.$$

[Q_{1n} and $\kappa^{(1)}_{1n}$ are tabulated real functions of c, and $1 \leq \xi < \infty$.]

Background: Y and Z are the input admittance and impedance of a monopole antenna in the form of half a prolate spheroid which is fed by a coaxial line whose outer conductor spreads out into a conducting plane.

Comments: The coefficients c^{1n}_{2k} and b^{1n}_{k} are tabulated as real functions of c for n=1(2)7, k=0(1)5 in "Prolate spheroidal wave functions," by C. Flammer, Technical Report 16, Stanford Research Institute, 1951.

Status: TERMINATED.

FLUTTER MATRICES
Task 1101-40-5131/53-17

Origin: Boeing Airplane Company — Authorized 12/15/52
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Completed 9/30/53
Manager: E. E. Osborne

Objective: To find the eigenvalues of each of fifteen 6x6 non-symmetric complex flutter matrices.

Background: The problem is associated with the contractor's work on flutter.

Status: COMPLETED. The final report is being reproduced.

DISCRIMINANT FUNCTIONS
Task 1101-40-5131/53-18

Origin: Randolph Field, USAF — Authorized 12/15/52
Sponsor: " "
Manager: D. Teichroew
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. SWAC codes have been prepared and checked. Preliminary results have been submitted to the contractor.

BIO-ASSAY PROBLEM
Task 1101-40-5131/53-24

Origin: Stanford University — Authorized 3/31/53
Sponsor: Office of Naval Research
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The table which was computed in the previous quarter has been submitted to the contractor.

SINGLE SHOT PROBABILITIES
Task 1101-40-5131/53-25

Origin: Naval Air Missile Test Center, Point Mugu Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Several additional problems connected with the estimation of probabilities were submitted and have been more rigorously formulated.

DISCRETE MINIMAL SPACES
Task 1101-40-5131/53-26

Origin: Gilfillan Brothers, Inc. Authorized 3/31/53
Sponsor: Evans Signal Laboratories, Army Signal Corps
Manager: A. Hestenes
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

ROOTS OF DETERMINANTS
Task 1101-40-5131/53-32

Origin: North American Aviation Authorized 3/31/53
Sponsor: Atomic Energy Commission
Manager: E. C. Yowell
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Eight cases were solved during this quarter.

PLANE SHOCK WAVES
Task 1101-40-5131/53-33

Origin: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF Authorized 3/31/53 Terminated 9/30/53
Sponsor: " " "
Manager: M. W. Steinberg

Objective: To solve numerically two nonlinear second-order differential equations arising in the problem of motion of a compressible fluid. These equations are

$$\ddot{\theta} = (8\theta)/(3w\varnothing)\,[w' - \varnothing^{-1}\,(w + \theta\, w^{-1} - 1)]$$

$$-2/3(3\theta+\beta)/(\theta+\beta)w'^{2} - 3/2(\theta+3\beta)/(\theta+\beta)\theta'^{2}\theta^{-1}$$

$$-2[(w'\theta)w^{-1}]';$$

$$\ddot{w}=4/(23\theta)\left\{280\theta/(45w\phi)\,[\theta'-5/7\phi^{-1}(\theta-\delta(1-w)^2+\alpha)]\right.$$

$$\left.-(115\theta+96\beta)/4(\theta+\beta)w'\theta'-2\theta w'^2w^{-1}\right\}$$

where α and β are given constants and $\phi=\theta^{3/2}/(\theta+\beta)$. The prime means differentiation with respect to x. There are four different cases with two-point boundary conditions; w and θ are given for $x=-\infty$ and $x=+\infty$.

Background: The theoretical discussion of these equations is found in L. H. Thomas' "Note on Becker's Theory of the shock front," J. Chem. Phys. 12, No. 11 (Nov. 1944). Numerical work and additional discussion is given in K. Zoller's "Zur Struktur des Verdichtungsstosses," Zeit. Phys. 130 (1951).

Status: TERMINATED.

LINEAR EQUATIONS (CONVAIR)
Task 1101-40-5131/53-35

Origin: Consolidated Vultee Aircraft Corporation Authorized 3/31/53
Sponsor: Bureau of Aeronautics, USN
Manager: R. R. Reynolds
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. As a single precision SWAC code was ineffectual, a double precision code was prepared. It is hoped that the system will be solved early next quarter.

MISCELLANEOUS CORONA COMPUTATIONS
Task 1101-40-5131/53-36

Origin: NBS, Div. 15 Authorized 3/31/53
Sponsor: " Terminated 9/30/53
Manager: P. Bremer

Objective: This task was established to perform small miscellaneous computations for the NBS Corona Laboratories.

Comments: From time to time other small tasks will be reported. The objective of these tasks will be included in the current issues of Projects and Publications.

Status: TERMINATED.

MONTE CARLO STUDIES
Task 1101-40-5131/53-39

Origin: RAND Corporation — Authorized 3/31/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: A. D. Hestenes
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

RANKING PROBABILITIES
Task 1101-40-5131/53-40

Origin: Cornell University — Authorized 3/31/53
Sponsor: Office of Naval Research
Manager: D. Teichroew
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. A table of $P(h/r,k)$ for $\sqrt{2h}=0(.01)6.4$ and the required values of r and k have been computed, checked, and submitted to the contractor.

ACCEPTANCE TESTS
Task 1101-40-5131/53-41

Origin: Jet Propulsion Laboratory, California Institute of Technology — Authorized 6/29/53
Sponsor: Army Ordnance Corps, U.S. Army
Manager: E. C. Yowell
Full task description appears in Apr-June 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

B. P. A. Studies
Task 1101-40-5131/53-42

Origin: Bonneville Power Administration — Authorized 6/29/53
Sponsor: " "
Manager: E. C. Yowell
Full task description appears in Apr-June 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

HELICOPTER STABILITY STUDIES
Task 1101-40-5131/53-44

Origin: J. B. Rea Company Authorized 6/30/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. H. Hollander

Objective: To determine the coefficients of the characteristic equations, the characteristic roots, and some of the eigenvectors of certain 16th-order matrices, and to obtain various transient responses by Fourier synthesis techniques, using these frequency solutions.

Background: Dynamic stability characteristics for various flight velocities of aircraft are needed. This can best be accomplished on high speed digital computing equipment such as SWAC. Forty-four sets of sixteen simultaneous linear equations in sixteen unknowns have been supplied, on which the above operations are to be performed.

Status: NEW. Programming for the calculation of the characteristic polynomial has been completed. Hand-punching of data has been done. The remaining parts of the problem have been partially programmed.

HINGE MOMENTS
Task 1101-40-5131/53-45

Origin: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF Authorized 6/30/53 Terminated 9/30/53
Sponsor: " "
Manager: G. Blanch

Objective: To compute the integrals $RP_h(c)$, $RT_h(c)$, $RP_\alpha(c)$, $RT_\alpha(c)$, for $c = -.9(.1)9$.

Background: In the process of computing lift and moment coefficients for task 1101-53-1101/52-41, there were computed certain constants A and B, in terms of which Reissner's $p_a^{(2)}$ for the circulatory flow can be determined. Once this is done, the following integrals are required:

$$RP_h(c) = 2\int_c^1 [p_a]_h dx; \quad RT_h(c) = 2\int_c^1 (x - c)\,[p_a]_h\, dx$$

$$RP_\alpha(c) = 2\int_c^1 [p_a]_\alpha\, dx; \quad RT_\alpha(c) = 2\int_c^1 (x - c)\,[p_a]_\alpha\, dx$$

where R is some constant.

Status: TERMINATED.

COMPUTATIONS FOR SCAMP
Task 1101-40-5131/54-1

Origin: Office of Naval Research — Authorized 9/29/53
Sponsor: " "
Manager: C. B. Tompkins

Objective: Calculations to support SCAMP (see task 1101-10-5150/53-1, p. 10).

Background: This task is the result of a desire on the part of the sponsoring agency to bring together mathematicians for the purpose of contributing to the deep mathematical developments required.

Status: NEW. Some calculations were completed during the quarter. Calculations involving exhaustive search were started and are continuing. About 375 sets of Steiner Triples of order 19 were calculated, an inconsiderable fraction of the ones which probably can be found. New calculations will be started.

EIGENVALUES AND EIGENVECTORS
Task 1101-40-5131/54-2

Origin: Consolidated Vultee Aircraft Corporation — Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF — Completed 9/30/53
Manager: A. D. Hestenes

Objective: Determine the eigenvalues and eigenvectors of a 30x30 symmetric matrix.

Status: COMPLETED (NEW). Routines coded for task 1101-40-5131/53-6, p. 17, were used. Results have been forwarded to the originator.

CRUISE CONTROLLER
Task 1101-40-5131/54-3

Origin: J. B. Rea Company — Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. H. Hollander

Objective: A cruise controller is an instrument which is designed to determine the appropriate throttle setting to obtain the maximum miles per pound. The associated SWAC problem is to determine the type of throttle input which results in the smallest hunting period.

Comments: The originator is analyzing this problem to determine its appropriate mathematical form.

Status: NEW.

RAM ROCKET TRAJECTORIES - PARAMETER STUDY
Task 1101-40-5131/54-4

Origin: Marquardt Aircraft Company Authorized 9/29/53
Sponsor: Aeronautical Research Laboratory, Wright Air Development Center, ARDC, USAF
Manager: F. H. Hollander

Objective: To determine engine characteristics for optimum performance over a wide range of trajectory variables. The analysis and coding is to be performed by the originator. INA will supply SWAC time and coding consultation as is necessary.

Status: NEW. Some routines have been prepared and are presently being checked on SWAC.

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-10-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS Authorized 1/1/50
Managers: J. Todd, M. Abramowitz, and H. A. Antosiewicz
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. H. A. Antosiewicz investigated the integral

$$\frac{1}{\pi}\int_0^{\infty} e^{-kx^2t}\,\frac{J_0(rx)Y_0(ax) - J_0(ax)Y_0(rx)}{J_0^2(ax) + Y_0^2(ax)}\,\frac{dx}{x} \quad \text{for } t\longrightarrow\infty$$

which arises in the solution of the equation of heat flow in a region internally bounded by a circular cylinder. This problem was suggested by B. A. Peavey of the NBS Building Technology Division (Div. 10).

M. Abramowitz has completed an investigation of forced heat convection in laminar flow through a tube. M. Abramowitz and W. Cahill are investigating the problem of vibrations of plates by the method of finite differences.

In continuation of earlier work of Gohner (Ing. Arch. 1, 619 (1930) and Freiberger (Australian J. Sc. Research {A} 2, 354 (1949), P. Henrici investigated the stress distribution in a helical spring of finite circular cross-section and developed the stress-concentration factor into a power series in terms of the spring index.

W. Cahill together with S. Levy of the NBS Engineering Mechanics Section are investigating the problem of modes of vibration of an airplane wing.

Publications: (1) "On a certain integral involving Bessel functions" by H. A. Antosiewicz; IN MANUSCRIPT. (2) "Forced heat convection in laminar flow through a tube" by M. Abramowitz; IN MANUSCRIPT. (3) "On helical springs of finite thickness by P. Henrici; IN MANUSCRIPT. (4) "Computation of vibration modes and frequencies on SEAC" by W. Cahill and S. Levy; IN MANUSCRIPT. (5) "On a problem in the theory of mechanical quadratures" by P. Davis; IN MANUSCRIPT. (6) "On the estimation of quadrature errors for analytic functions" by P. Davis and P. Rabinowitz; IN MANUSCRIPT.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-10-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS Authorized 3/1/50
Manager: P. Davis

Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1952 issue.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-10-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS Authorized 1/1/50
Managers: O. Taussky Todd, J. Todd, M. Abramowitz, and A. Hoffman
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. O. Taussky Todd made a survey of computational problems in algebraic number theory and gave a report on this subject to the Applied Mathematics Symposium of the American Mathematical Society which was held at Santa Monica, California, in September. Computational problems connected with the following topics were mentioned: (1) integral bases, (2) factorization of rational primes in number fields, (3) algebraic units (with special attention given to a conjecture of Ankeny, Artin, and Chowla concerning the quantity u in the fundamental unit $\frac{1}{2}(t + u\sqrt{p})$ in the field generated by $\sqrt{p}$, at present under examination on SEAC,) (4) ideal classes and class numbers, and (5) the determination of the ideal class of a field F with non-cyclic ℓ-class group which becomes the principal class in an unramified relatively cyclic extention field of F of relative degree ℓ.

A. Hoffman, M. Newman, and O. Taussky Todd, in collaboration with E. Straus of U.C.L.A., present in their paper (see item (1) under publications below) a new approach to a problem treated earlier by R. Baer and R. W. Ball. By a judicious numbering of the points and lines of a finite projective plane, the number of absolute points becomes trace A, where A is the incidence matrix of the finite projective plane, which is studied in terms of the relation between the characteristic polynomial of A and the transitivity properties of the collineation which is the square of the given correlation. This approach is capable of easy generalization to combinatorial configurations more complicated than projective planes and even in the case of planes obtains information apparently stronger than previously known.

K. Goldberg and M. Newman, in collaboration with E. Straus and D. Swift of U.C.L.A., have completed a paper (see item (2) under publications below) in which the integers represented by quotients of certain binary quadratic forms are studied.

The preparation of a code to determine a certain number-theoretic invariant associated with finite projective planes was begun by M. Newman in collaboration with G. Pall. The problem will be run on SEAC for the incidence matrices of finite projective planes of order nine (for which the incidence matrices are of order 91), both for Desarguian and nonDesarguian finite projective planes.

Publications: (1) "The number of absolute points of a correlation," by A. Hoffman, M. Newman, O. Taussky, and E. Straus; IN MANUSCRIPT. (2) "The representation of integers by binary quadratic rational forms," by K. Goldberg, M. Newman, E. Straus, and D. Swift; submitted to a technical journal. (3) "Ueber die Funktionen von Gegenbauer," by P. Henrici; IN MANUSCRIPT. (4) "On certain series expansions involving Whittaker functions and Jacobi polynomials," by P. Henrici; IN MANUSCRIPT. (5) "Linear functional equations and interpolation series," by P. Davis; submitted to a technical journal.

NUMBER THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-10-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS Authorized 6/1/50
Managers: O. Taussky Todd and K. Goldberg
Full task description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A routine to determine the primes $p \leq 100{,}000$ for which $2^{p-1}-1$ is divisible by p^2 has been prepared and checked.
A routine to determine the fundamental unit of a real quadratic field has been prepared and checked.
The table of least positive primitive roots is being extended as time is available.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-10-1104/51-6

Origin: NBS Authorized 9/28/50
Manager: M. Abramowitz
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Apr-Jun 1952 issue.

THREE-BODY PROBLEM
Task 1102-10-1104/52-4

Origin: NBS Authorized 10/5/51
Manager: A. Goldstein
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

ANALYSIS OF GEOMAGNETIC FIELD
Task 1102-10-1104/52-8

Origin: NBS Authorized 8/10/51
Manager: C. J. Swift
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1951 issue.

SPECIAL PROBLEMS IN FINITE MATRIX THEORY
Task 1102-10-1104/52-34

Origin: NBS Authorized 11/6/51
Manager: O. Taussky Todd
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. O. Taussky Todd continued work with T.S.Motzkin

on matrices with property L. In particular they extended theorem II in their report "Matrices with property L, II" to more general matrices. Ky Fan prepared a manuscript (see item (1) under publications) concerning his results on commutators of matrices. He generalized a result of O. Taussky Todd concerning a pair of matrices with equal determinant to two elements x,y in a group for which xy^{-1} is a commutator, and proved a corresponding and generalized result. It was also shown that any unimodular unitary matrix is a commutator in the unitary group, and that any unimodular normal matrix is a commutator of normal matrices.

Ky Fan developed in publication (2) some new inequalities for eigenvalues of Hermitian matrices. The first section presents inequalities for the successive eigenvalues of a Hermitian matrix $A=(a_{ij})$ in terms of the quantities a_{ii} and

$$\rho_i = \Big(\sum_{j>i} |a_{ij}|^2 \Big)^{\frac{1}{2}} .$$

A typical result is: if $c_1,\ldots,c_{n-1}$ and $d_1,\ldots,d_n$ are $2n-1$ nonnegative numbers such that

$$c_i > 1, \quad d_i \geq \frac{c_i^2}{c_i - 1} d_{i+1}, \quad |a_{ii}| + c_i \rho_i \leq d_i ,$$

then

$$-d_{n-i+1} \leq \lambda_i \leq d_i ,$$

where $\lambda_1 \geq \ldots \geq \lambda_n$ are the eigenvalues of A. The second section compares the eigenvalues of A with those of $B=(b_{ij})$, where B is Hermitian and consists of blocks on the diagonal agreeing with A and of zeros elsewhere. Let the eigenvalues of B be $K_1 \geq \ldots \geq K_n$. One of the results is that for any h, where $1 \leq h \leq n$, we have

$$\sum_{i=1}^{h} K_i \leq \sum_{i=1}^{h} \lambda_i .$$

Further, if A is positive definite, then

$$\prod_{i=1}^{h} \lambda_{n-i+1} \leq \prod_{i=1}^{h} K_{n-i+1} .$$

Ky Fan and A. Hoffman presented in publication (3) matricial analogues of some well-known inequalities for complex numbers. For example, let us consider the inequalities (i) x real, z complex which implies that

$$|z - x| \geq \left| z - \frac{z + \bar{z}}{2} \right|$$

and (ii) $z = \rho e^{i\theta}, \rho \geq 0, |w| = 1$ which imply that

$$|z + e^{i\theta}| \geq |z - w| \geq |z - e^{i\theta}| .$$

These inequalities are generalized by replacing a complex number by a matrix, a real number by a Hermitian matrix, the conjugate of a complex

number by the adjoint of a matrix, a nonnegative number by a positive-semi-definite matrix, a number of absolute value unity by a unitary matrix, and absolute value by any unitarily invariant norm.

A. Hoffman together with R. Bellman of the RAND Corporation, derived in publication (4), alternative methods of proving the theorem of A. M. Ostrowski and O. Taussky that $A=A^*$, $B=B^*$, $A \geq 0$, $C=A+iB$ imply $|\det C| \geq \det A$. This theorem and others are shown to be consequences of (1) the formula

$$\frac{(\sqrt{\pi})^n}{(\det C)^{\frac{1}{2}}} = \int_{E^n} e^{-(Cx,x)} dx ,$$

or (2) convexity arguments based on a geometric formulation of results of K. Fan.

J. Todd completed a study of the condition of the system of equations which approximates to the biharmonic equation. It was found that the P-condition number of this system is the square of that of the system corresponding to the harmonic equation. The order of the condition number, as a function of the reciprocal of the mesh length, is independent of the number of independent variables. Use was made of a theorem of Ledermann later generalized by H. Wielandt and K. Fan about the effect on the characteristic roots of finite changes in a small number of elements of a matrix. These theoretical results have been confirmed by extensive experiments on SEAC.

H. Wielandt continued his investigations concerning the numerical calculation of the eigenvalues of integral equations with real symmetric kernels; he finished his work concerning the method based on numerical quadrature (see publication (7)) and developed error estimates for the variational method which corresponds to the Rayleigh-Ritz method in differential equations. The latter estimates are consequences of new inequalities which connect the eigenvalues of a Hermitian (finite or infinite completely continuous) matrix with the eigenvalues of a segment of the matrix (see publication (8)).

Publications: (1) "Some remarks on commutators of matrices," by K. Fan; submitted to a technical journal. (2) "Inequalities for eigenvalues of Hermitian matrices," by K. Fan; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series. (3) "Some metric inequalities in the space of matrices," by K. Fan and A. Hoffman; IN MANUSCRIPT. (4) "On the theorem of Ostrowski and Taussky," by R. Bellman and A. Hoffman; IN MANUSCRIPT. (5) "Pairs of matrices with property L, II" by T. S. Motzkin and O. Taussky; accepted for publication in the Proceedings of the National Academy of Science. (6) "The condition of certain matrices,II" by J. Todd; IN MANUSCRIPT. (7) "Error bounds for eigenvalues of symmetric integral equations," by H. Wielandt; IN MANUSCRIPT. (8) "Einschliessung von Eigenwerten hermitescher Matrizen nach dem Abschnittsverfahren," by H. Wielandt; IN MANUSCRIPT. (9) "Characteristic roots of quaternion matrices," by O. Taussky; to appear in Archiv der Mathematik.

AN INTEGRAL ARISING IN THE THEORY OF COOPERATIVE PHENOMENA

Task 1102-10-1104/52-69

Origin: NBS
Manager: J. Todd

Authorized 5/9/52
Terminated 9/30/53

Objective: To tabulate

$$F(b) = \frac{1}{\pi^3} \int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \frac{dw_1\, dw_2\, dw_3}{3b - (\cos w_1 + \cos w_2 + \cos w_3)}$$

for $\frac{1}{b} = \mu = .01(.01)1$.

Background: Integrals of the form

$$\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} f(3b - \cos w_1 - \cos w_2 - \cos w_3)\, dw_1\, dw_2\, dw_3$$

occur in problems involving averages of the characteristic values, , of the difference equation $\Delta^2 \gamma_{\ell,m,n} = \lambda \gamma_{\ell,m,n}$ (where Δ^2 is the Laplacian second difference operator) with the periodic boundary condition

$$\gamma_{\ell,m,n} = \gamma_{\ell+N,m,n} = \gamma_{\ell,m+N,n} = \gamma_{\ell,m,N+n}.$$

Such averages arise in the theory of ferromagnetism, cooperative phenomena in solids, and critical fluctuations in liquids.

Status: TERMINATED. It is hoped to resume the study of this integral by Monte Carlo methods later.

DIFFERENTIAL EQUATION FOR NERVE FIBER REACTION
Task 1102-10-1104/53-15

Origin: National Naval Medical Institute Authorized 12/8/52
Sponsor: " "
Managers: H. A. Antosiewicz and P. Rabinowitz
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Computations are performed as requested by the sponsor.

BAKER-HAUSDORFF FORMULA
Task 1102-10-1104/53-16

Origin: NBS Authorized 12/8/52
Manager: O. Taussky Todd and K. Goldberg
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. The publication below describes a method for the calculation of the coefficients in the Baker-Hausdorff equation.

Publication: "The equation $e^z = e^x.e^y$ in a free associative ring," by K. Goldberg; IN MANUSCRIPT.

Status of Projects

POISSON-BOLTZMAN EQUATION
Task 1102-10-1104/53-31

Origin: NBS
Manager: M. Abramowitz

Authorized 1/23/53
Terminated 9/30/53

Objective: To determine the maximum values of δ for which the non-linear differential equation

$$\frac{d^2\theta}{dz^2} + \frac{k}{z}\frac{d\theta}{dz} = -\delta e^{\theta}$$

has a solution satisfying the conditions $\theta'=0$ for $z=0$, $\theta=0$ for $z=1$. The parameter k is a positive integer.

Background: The differential equation occurs in the theory of thermal explosion under the assumption that the critical condition for inflammability is reached when the amount of heat developed is equal to the heat lost to surroundings.

Status: TERMINATED.

HYPERGEOMETRIC FUNCTIONS
Task 1102-10-1104/53-35

Origin: NBS
Managers: P. Rabinowitz, W. Cahill
Full task description appears in Jan-Mar 1953 issue.

Authorized 3/25/53

Status: CONTINUED. The code for the hypergeometric function has been used to compute the complex error function Erf(z), $z=x+iy$ for x,y = 0(.2)1.2(.4)3.2.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin: Office of Scientific Research, ARDC, USAF and Office of the Air Comptroller, USAF
Sponsor: Office of Scientific Research, ARDC, USAF
Managers: J. Todd and A. Hoffman
Full task description appears in Jan-Mar 1950 issue, see 11.2/12-50-1.

Authorized 3/31/50

Status: CONTINUED. Applications in support of requirements of the Air Comptroller's Office, formerly reported under this task, will be reported under task 1102-10-5116/54-9, p. 35. The basic aspects of the program will continue to be reported under this heading.

Most of the machine work during the quarter was confined to the development and testing of a matrix inversion code using tapes for temporary storage. This code will be used in a revision of the simplex method to obtain greater efficiency for sparse matrices. In addition, some experimentation was undertaken, using the simplex method, in support of research of H. W. Kuhn on the faces of the convex hull of the cyclic permutation matrices.

A fully automatic machine procedure for solving the so-called two-parameter problem (i.e. for each λ, μ, minimize $(a+\lambda b+\mu c, x)$,

where a,b,c are given vectors and x ranges over a convex polyhedron) has been devised and awaits trial. Briefly, it consists of (i) noting, with each solution corresponding to a given λ, μ, the sides of the convex polygon in the (λ,μ) plane in which that solution minimizes, (ii) introducing into the basis all vectors which contribute sides of the polygon, and (iii) keeping track of past introductions so as to minimize superfluous changes of basis.

The following variation of the smoothing problem has been considered: $r_1, r_2, \ldots, r_n$ are given nonnegative constants, representing requirements in each month; and $x_1, \ldots, x_n$, $s_1, \ldots, s_n$ are given nonnegative variables denoting production and storage, respectively, for all values of the variables such that

$$x_1 + \ldots + x_h - s_h = r_1 + \ldots + r_h \quad (\text{where } h = 1, \ldots, n)$$

minimize, for each $\lambda \geq 0$, $\sum |s_h - s_{h-1}| + \lambda \sum (x_h - x_{h-1})_+$. In the case $r_h < r_{h+1}$ (where $h=1,\ldots,n-1$), the answer may be described as follows: for

$$\lambda \geq \frac{r_1 + \ldots + r_n}{n}, \text{ set } x_h = \frac{r_1 + \ldots + r_n}{n}$$

for $h=1,2,\ldots,n$. For smaller λ, if $0 \leq \lambda \leq 2m$ (where $m=0,1,2,\ldots$), set $x_n = r_{n-m}$, $x_{n-1} = r_{n-m}$, $\ldots$, $x_{n-k+1} = r_{n-m}$; $x_1 = r_1$, $x_2 = r_2$, $\ldots$, $x_{n-k-1} = r_{n-k-1}$.

$$x_{n-k} = -\sum_{i=1}^{n-k-1} x_i - \sum_{i=n-k+1}^{n} x_i + \sum_{i=1}^{n} r_i ,$$

where k is determined by the property

$$k\, r_{n-m} \leq \sum_{i=n-k+1}^{n} r_i , \quad (k+1) r_{n-m} > \sum_{i=n-k}^{n} r_i .$$

Publications: (1) "On 'overshoot' in the furthest hyperplane method," by R. Bryce; IN MANUSCRIPT. (2) "Lower bounds for the rank and location of the eigenvalues of a matrix," by K. Fan and A. J. Hoffman; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series. (3) "On the caterer problem," by J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky; IN MANUSCRIPT. (4) "Remarks on the smoothing problem," by A. J. Hoffman; IN MANUSCRIPT. (5) "Computational experience in solving linear programs," by A. J. Hoffman, M. Mannos, D. Sokolowsky, and N. A. Wiegmann; J. Soc. Ind. App. Math. I, 17-34 (Sept. 1953). (6) "A characterization of normal matrices," by A. J. Hoffman and O. Taussky; to appear in the Journal of Research of the NBS. (7) "On block relaxation," by L. S. Joel; IN MANUSCRIPT. (8) "Eigenvectors of matric polynomials," by M. Mannos; J. Res. NBS, 51, 33-36 (July 1953). (9) "Experimental results with the double description method, by S. Pollack; IN MANUSCRIPT. (10)"A method of solving the transportation problem," by A. Gleyzal; IN MANUSCRIPT.

COMPRESSIBLE FLOW - METHOD OF ORTHOGONAL AND KERNEL FUNCTIONS
Task 1102-10-5116/52-16

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF, and Harvard University — Authorized 9/29/51
Sponsor: Aeronautical Research Laboratory, USAF
Managers: P. Davis and F. L. Alt
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The computation of the 12 particular solutions of the stream equation $\Delta\Psi + 4F(\lambda)\Psi = 0$ has been completed. These solutions are given by

$$\Psi = \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left\{z^n - \int_{z_0}^{z}\int_{\bar{z}_0}^{\bar{z}} z_n\, f\left(\frac{z+\bar{z}}{z}\right) dz d\bar{z} + \ldots\right\} \quad (n=0,1,\ldots,5).$$

The local accuracy of these solutions is now being checked.

TABLES OF INTEGRALS INVOLVING THE HIGHER TRANSCENDENTAL FUNCTIONS
Task 1102-10-5116/52-33
(formerly 1102-21-5117/52-33)

Origin: NBS — Authorized 10/11/51
Manager: F. Oberhettinger
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The table of integral transforms has been varityped. The rest of the table is being prepared for varityping. Dr. Oberhettinger has added a section on the Lebedev transform to the table of integral transforms.

INTEGRAL OPERATORS AND INTERPOLATION SERIES
Task 1102-20-5116/53-2

Origin: Flight Research Laboratory, Wright Air Development Center, USAF — Authorized 9/26/52
Sponsor: " " " — Revised 9/30/52
Managers: F. L. Alt, P. Davis — Terminated 9/30/53

Objective: To investigate the relationship between the method of integral operators of the Bergman type, the symbolic operator method of Cauchy-Heaviside, and the method of analytic functionals as developed in Italy by L. Fantappie. Also to investigate the generation of complete sets of solutions of differential equations by the method of orthogonal complements.

Background: The first of these investigations is to extend to the theory of ordinary linear differential equations with nonconstant coefficients as well as to the case of linear partial differential equations. In the case of ordinary equations, Dr. Davis has recently obtained a number of results, showing how the Bergman integral operator provides new representations for the Duhamel kernel of the equation. This work is related to some results of I. M. Schiffer (Tokyo Journal, 1932), and has potential application to the theory of variable networks and to the

statistical theory of noise in such networks. It is desired to press this investigation to the field of partial differential equations, and new representations for the solution of initial value problems are anticipated.

The method of orthogonal complements would be intimately related to the theory of kernel functions of Bergman type and to the theory of interpolation series for analytic functions, inasmuch as the kernel of the orthogonal complement of the subspace of solutions is to be obtained by the method of interpolation series. This investigation may be regarded as a continuation of some recent work in the theory of interpolation series by Walsh and Davis ("Interpolation and orthonormal systems," by J. L. Walsh and P. Davis; to appear in Journal D'Analyse Mathematique). It is to be carried out for differential equations of arbitrary order and for an arbitrary number of variables, but for analytic coefficients. By the introduction of kernel tensor fields, it will also be possible to treat systems of linear partial differential equations. Some theoretical work on this investigation has been done; in particular, it has been shown how, by the use of interpolation series, we may generate a kernel $K_s(Z,\bar{w})$ such that the transform $g(z)= \int K_s(z,\bar{w})f(w)dV$ maps the space of analytic functions of n complex variables onto the subspace of solutions.

Status: TERMINATED. The results obtained have been incorporated by P. Davis into a paper (see publication below).

Publication: "Linear functional equations and interpolation series," by P. Davis; submitted to a technical journal.

WATER WAVES
Task 1102-10-5116/53-54

Origin: NBS, Division 6 Authorized 6/1/53
Sponsor: Office of Naval Research
Manager: P. Davis
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. A joint paper by P. Davis and R. F. Dressler entitled "A computational scheme for a free boundary problem" is now being prepared. The possibility of using the formulation of the boundary value problem in Lagrangian coordinates as a basis for computation is also being studied.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Office of the Air Comptroller, USAF Authorized 9/25/53
Sponsor: Office of Scientific Research, ARDC, USAF
Managers: J. Todd and A. J. Hoffman

Objective: (a) The computation of solutions to specific programming problems originating in the Planning and Research Division, AFACO. (b) The development of improved computation schemes for solving general dynamic linear programs. (c) The development of computation schemes for solving special dynamic programs.

Background: (a) Based on past experience, the specific programming problems of the Planning and Research Division, AFACO, will frequently be in the nature of experimental computations to verify or deny intuition and preliminary heuristic analysis, or to test the validity of the mathematical formulation of a practical situation.

(b) The improvements in the computation schemes for solving general dynamic linear programs are expected to be modifications of the basic principles of the simplex algorithm, in order to facilitate processing of problems by high-speed computers. Where the problem deals with a single objective, we are now in possession of what appears to be sound ideas for taking advantage of the quasi-canonical format of dynamic programs, and these await the test of machine trial. Where the problem is concerned with a class of objectives, (e.g., where the objective cannot be precisely formulated), further study is necessary. The success already attained, however, for the case that the class of objectives is linear and one-dimensional, suggests the direction of research for the more general multi-parameter problem. One line of attack, recently devised for the two-parameter problem, deserves experimentation. The development of these new tools will substantially enlarge the domain of applicability of linear programming.

(c) Many important special dynamic problems occur so frequently in military and logistic planning that it is worthwhile, if possible, to find either a formula or an ingenious method for the solution.

Over the past few years, and especially during the last year, the Air Force and National Bureau of Standards personnel have, as a team, acquired "technique" in the formulation and solution of these special problems, and further co-operation along these lines should produce additional contributions to saving and efficiency in the military program.

Comments: This is a continuation of certain phases of former task 1102-10-5116/50-2 "Research in Linear Programming."

Status: NEW. One problem was attacked during this quarter, and its solution was completed shortly after the end of the quarter. This is a generalization of the so-called "Transportation Problem", which arises in the awarding of purchase contracts. The problem actually occurs in the operation of such agencies as the Textiles and Apparel Procurement Agency, which purchases uniforms and similar articles for the Army and Air Force. The problem is to select, from all the potential suppliers who submit bids on a given requirement, a number of suppliers in such a way as to minimize the total cost, including cost of shipment. The problem differs from the classical transportation problem in a number of special conditions such as upper and lower limits on bids, volume discounts, etc. Two amplifications of this problem have been considered but not satisfactorily solved. These refer to the conditions that the quota of each successful bidder be distributed over several different sizes (in the case of apparel) in fixed proportions, or be scheduled uniformly over a period of time. These problems lead to very difficult questions in the theory of inequalities. We are sure of one method of solution, namely, the general simplex method; this, however, is prohibitively long. We suspect at present that no shortcuts to the simplex method exist for these two problems. If this conjecture is confirmed, attention will be concentrated on methods for approximate solution.

These problems are closely related to earlier work done here for the Air Comptroller's Office and for the Logistics Research Project of George Washington University (see task 1102-40-5126/53-55, p. 51).

2. Mathematical Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$

Task 1102-10-1104/43-3
(formerly 1102-10-1110/43-3)

Origin: Canadian National Research Council Authorized 7/1/47
Manager: I. A. Stegun
Full task description appears in Apr-June 1949 issue, see task 11.2/2-43-3.

Status: CONTINUED. Revisions of the manuscript are in progress.

TABLE OF THE GAMMA FUNCTIONS FOR COMPLEX ARGUMENTS

Task 1102-10-1104/46-1
(formerly 1102-10-1110/46-1)

Origin: NBS Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, see task 11.2/2-46-1.

Status: CONTINUED. The page proof has been received for this volume.

Publication: "Table of the Gamma function for complex arguments," NBS Applied Mathematics Series 34; in press, U. S. Government Printing Office.

TABLES OF COULOMB WAVE FUNCTIONS

Task 1102-10-1104/47-2
(formerly 1102-10-1110/47-2)

Origin: NBS Authorized 7/1/47
Managers: M. Abramowitz and P. Rabinowitz
Full task description appears in Apr-June 1949 issue, see task 11.2/2-47-2.

Status: CONTINUED. The tables of the regular and irregular functions together with their derivatives for $\rho = 1(1)10$, $\eta = 1(1)10$, L=0,5, are being proofread, and an introduction is being prepared.

TABLE OF ANTILOGARITHMS

Task 1102-10-1104/47-3
(formerly 1102-10-1110/47-3)

Origin: NBS Authorized 7/1/47
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, see task 11.2/2-47-3.

Status: CONTINUED. This volume is now in press.

Publication: "Tables of 10^x," NBS Applied Mathematics Series 27; in press, U. S. Government Printing Office.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-10-1104/48-2
(formerly 1102-10-1110/48-2)

Origin: NBS Authorized 5/25/48
Manager: J. Todd
Full task description appears in Apr-June 1949 issue, see task 11.2/2-48-2.

Status: CONTINUED. The page proof has been received for this volume.

Publication: "Tables of Lagrangian coefficients for sexagesimal interpolation," NBS Applied Mathematics Series 35; in press, U. S. Government Printing Office.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-10-1104/50-4a
(formerly 1102-10-1110/50-4a)

Origin: NBS, Section 11.3 Authorized 12/31/49
Sponsor: Aeronautical Research Laboratory, Completed 9/30/53
Wright Air Development Center, ARDC, USAF
Manager: J. Todd

Objective: To prepare a set of short tables relating to the statistical theory of extreme values, including:
(1) The inverse of the (cumulative) distribution function of extremes, $y=\log(-\log x)$, x= .0001(.0001).005(.001).988(.0001).9999, 5D.
(2) The direct (cumulative) distribution of extremes, $x=\exp(-\exp(-y))$, y=-3(.1)-2.4(.05)0(.1)4(.2)8(.5)17, 7D.
(3) Probability density of extremes, $x'=e^{-y}e^{-e^{-y}}$, range of y same as (2), 7D.
(4) Probability density of extremes as a function of the probability, $x'=-x\log x$, x= .0001(.0001).01(.001).999, 5D.
(5) Probability points for n'th extremes.
(6) Distribution and density function for the range $\psi(R)$ and $\psi'(R)$, as defined by $\psi''+\psi'-e^{-R}\psi=0$; R between -4.6 and 20; 6 or 7D.

Background: The theory of extremes has been developed in recent years, primarily by Dr. E. J. Gumbel. At his suggestion this compilation of pertinent tables has been undertaken. The table listed as (1) above is an extension of the one computed by this Laboratory under project 11.2/33-49-7 (see Oct-Dec 1948 issue, p. 50). Tables (2) to (5) are taken from unpublished manuscripts of A. J. Greenwood, table (6) from an unpublished table computed at Dahlgren Proving Ground, with additions by A. J. Greenwood. All tables are to be checked and edited by this Laboratory. Dr. Gumbel participated in the detailed planning.

Status: COMPLETED.

Publication: "Probability tables for analysis of extreme-value data," NBS Applied Mathematics Series 22. Available from U. S. Government Printing Office, Washington, 25, D. C.

BIBLIOGRAPHY OF CODING PROCEDURES, MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-10-1104/50-5
(formerly 1102-10-1110/50-5)

Origin: NBS Authorized 3/1/50
Managers: J. Todd, J. H. Wegstein, and P. Rabinowitz Revised 1/9/53
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. Preparation of the bibliographies continues, and reports prepared by J. H. Wegstein on the bibliographies are issued periodically in Mathematical Tables and Other Aids to Computation.

WAVE FUNCTION FOR LITHIUM
Task 1102-10-1104/50-7
(formerly 1102-10-1110/50-7)

Origin: NBS Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Managers: D. I. Rubin and W. H. Durfee
Full task description appears in Apr-June 1950 issue.

Status: CONTINUED. Computation on SEAC continues as time is available.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-10-1104/51-4
(formerly 1102-10-1110/51-4)

Origin: NBS Authorized 9/28/50
Manager: J. Todd
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. The first volume of such tables is in press.

Publication: "Tables of functions and of zeros of functions," Volume I of Collected Short Tables of the Computation Laboratory; NBS Applied Mathematics Series 37, in press.

REVISION OF MATHEMATICAL TABLES
Task 1102-10-1104/52-7
(formerly 1102-10-1110/52-7)

Origin: NBS Authorized 8/10/51
Managers: J. Todd, W. F. Cahill and I. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The following action is being taken in connec-

tion with mathematical tables the sales stock of which has been exhausted:

Tables of normal probability functions, originally MT14 (Tables of probability functions, vol. II), reissued as AMS23: available from U. S. Government Printing Office, Washington, D. C.

Table of natural logarithms for arguments between zero and five to sixteen decimal places, originally MT10, to be reissued as AMS31: in press.

Table of sine and cosine integrals for arguments from 10 to 100, originally MT13, to be reissued as AMS32: in press.

Tables of circular and hyperbolic sines and cosines for radian arguments, originally MT3, to be reissued as AMS36: in press.

Table of probability functions, vol. I, originally MT8: revision completed for reissue in the AMS.

Tables of sines and cosines for radian arguments, MT4: revision in progress.

Table of natural logarithms, MT12: revision in progress.

The following two volumes are now out of print:

Tables of sine, cosine, and exponential integrals vol. I, MT5 (1940): reissue under consideration.

Tables of scattering functions for spherical particles, AMS4(1949): not to be reissued.

TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-14
(formerly 1102-10-1110/52-14)

Origin: NBS Authorized 10/1/51
Manager: A. A. Goldstein
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The information has been put on punched cards for checking and editing.

EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Task 1102-10-1104/52-18
(formerly 1102-10-1110/52-18)

Origin: NBS Authorized 9/17/51
Manager: W. F. Cahill
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The punched cards are being processed for typing.

TABLE OF THE MODIFIED AIRY INTEGRAL
Task 1102-10-1104/52-23
(formerly 1102-10-1110/52-23)

Origin: NBS Authorized 10/4/51
Manager: P. Rabinowitz
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The tabular manuscript is in process of preparation for publication. An introduction to accompany the table has been completed; it consists of two parts by E. E. Osborne and P. Rabinowitz.

TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-10-1104/52-25
(formerly 1102-10-1110/52-25)

Origin: NBS Authorized 10/5/51
Managers: M. Abramowitz and F. J. Stockmal
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. The table for $x,y=0(.1)2$ has been computed. Additional values were also obtained for $x,y=2(.4)3.2$.

EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR NEGATIVE ARGUMENTS
Task 1102-10-1104/52-7
(formerly 1102-10-1110/52-7)

Origin: NBS Authorized 10/9/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-10-1104/52-37
(formerly 1102-10-1110/52-37)

Origin: NBS Authorized 11/28/51
Manager: T. Ledley
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. The computation of the spheroidal wave functions for the prolate case for $m=0,1,2$, $\ell=0(1)10$; $m=3$, $\ell=0(1)6$; and $m=4,5$, $\ell=0(1)5$ has been completed. Also the computation for the oblate case for $m=0,1$, $\ell=0(1)10$; $m=2,3$, $\ell=0(1)6$; and $m=4,5$, $\ell=0(1)5$ has been completed.

VAN DER POL EQUATION
Task 1102-10-1104/52-43
(formerly 1102-10-1110/52-43)

Origin: NBS Authorized 11/28/51
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue.

RADIAL MATHIEU FUNCTIONS
Task 1102-10-1104/52-49
(formerly 1102-10-1110/52-49)

Origin: NBS Authorized 2/1/52
Managers: J. Todd, I. Rhodes, and G. Blanch

Status: CONTINUED. All the subtabulation has been completed. The results are being processed for punched cards. This work is being carried out under the supervision of I. Rhodes.

SIEVERT'S INTEGRAL
Task 1102-10-1104/52-57
(formerly 1102-10-1110/52-57)

Origin: NBS Authorized 2/12/52
Managers: O. Steiner and R. B. Jasper
Full task description appears in Jan-Mar 1952 issue.

Status: CONTINUED. Computation of the tables on SEAC has been completed, and the results have been transferred to punched cards for differencing and future printing.

SCATTERING FUNCTIONS
Task 1102-10-1104/52-63
(formerly 1102-10-1110/52-63)

Origin: NBS Authorized 3/10/52
Manager: A. Gleyzal
Full task description appears in Jan-Mar 1952 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

TABLE OF SECANTS AND COSECANTS
Task 1102-10-1104/52-81
(formerly 1102-10-1110/52-81)

Origin: NBS Authorized 7/1/52
Managers: K. C. Nelson and I. A. Stegun
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED.

Publication: "Table of secants and cosecants to nine significant figures at hundredths of a degree"; to appear in the National Applied Mathematics Series.

PAINLEVÉ EQUATION
Task 1102-10-1104/53-3
(formerly 1102-10-1110/53-3)

Origin: NBS Authorized 8/11/52
Managers: J. Todd and H. A. Antosiewicz
Full task description appears in July-Sept 1952 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

L-SHELL CONVERSION COEFFICIENTS
Task 1102-10-1104/53-52
(formerly 1102-10-1110/53-52)

Origin: Oak Ridge National Laboratory Authorized 5/20/53
Manager: C. J. Swift
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The code has been completed and is being checked. More sample computations have been performed.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Managers: A. Hoffman and L. Joel
Full task description appears in Apr-June 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

3. Mathematical Services

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-40-5126/50-16
(formerly 1102-53-1106/50-16)

Origin: Naval Research Laboratory, USN Authorized 3/31/50
Sponsor: " "
Manager: P. J. O'Hara
Full task description appears in Jan-Mar 1950 issue, see task 11.2/33-50-16.

Status: CONTINUED. Computations were performed as requested.

Status of Projects

PROGRAM COMPUTATION ON THE SEAC
Task 1102-40-5126/51-7
(formerly 1102-53-1106/51-7)

Origin: Office of the Air Comptroller, USAF — Authorized 9/1/50
Sponsor: " " — Terminated 9/30/53
Manager: A. Hoffman

Objective: To compute Air Force programs on the SEAC by the "triangular" method.

Status: TERMINATED. It is expected that the computing equipment of the Air Comptroller's Office will be adequate to handle this task in the future.

FLOW IN SUPERSONIC NOZZLES
Task 1102-40-5126/51-13
(formerly 1102-53-1106/51-13)

Origin: Naval Ordnance Laboratory — Authorized 12/15/50
Sponsor: " " — Terminated 9/30/53
Manager: N. Levine

Objective: To calculate the variation of flow in a test section of a supersonic nozzle for a given range of Mach numbers.

Background: The contour of a supersonic nozzle can be determined, neglecting boundary layer effects, such that the flow in a test section of nozzle will be uniform for one assumed Mach number. If the ratio of the nozzle diameter at the throat to the nozzle diameter at the test section is varied, the Mach number of the flow will change and the flow will become nonuniform. The problem is to calculate the amount of this nonuniformity in the flow. Analytically, the problem is one in potential theory where it is necessary to integrate the following system of hyperbolic equations over the area of a meridional nozzle section:

$$\frac{\partial \theta}{\partial \phi} = \frac{1}{M\left(1 + \frac{M^2}{3}\right)^{3/2}} \frac{\partial M}{\partial \psi}$$

$$\frac{\partial M}{\partial \phi} = \frac{M}{(M^2-1)\left(1 + \frac{M^2}{5}\right)^{3/2}} \frac{\partial \theta}{\partial \psi}$$

where ϕ are the potential lines of the flow
ψ are the stream lines of the flow
M is the Mach number
θ is the angle of the stream line with respect to the axis of symmetry of the nozzle.

Status: TERMINATED.

LIQUID-VAPOR TRANSITION
Task 1102-40-5126/51-22
(formerly 1102-53-1106/51-22)

Origin: Naval Medical Research Institute — Authorized 2/1/51
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

MOLECULAR STRUCTURE, III
Task 1102-40-5126/51-37
(formerly 1102-53-1106/51-37)

Origin: Naval Research Laboratory, USN — Authorized 8/10/51
Managers: P. J. O'Hara and I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED. Computations were performed as requested.

SHOCK WAVE PARAMETERS, II
Task 1102-40-5126/51-38
(formerly 1102-53-1106/51-38)

Origin: Bureau of Ordnance, USN — Authorized 6/28/51
Sponsor: " "
Manager: I. A. Stegun
Full task description appears in Apr-June 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

PRESSURE DISTRIBUTION ON BODIES OF REVOLUTION
Task 1102-40-5126/52-3
(formerly 1102-53-1106/52-3)

Origin: David Taylor Model Basin, USN — Authorized 8/10/51
Sponsor: U. S. Navy — Terminated 9/30/53
Managers: A. Gleyzal and O. Steiner

Objective: To calculate the axially symmetric potential flow about elongated bodies of revolution for 30 different shapes. An iteration method is employed to solve a Fredholm integral equation of the first kind.

$$\int_0^P \frac{U(x)y^2(x)}{2r^3}\,ds = 1$$

where $U(x)$ is the unknown velocity function and the kernel is $y^2/2r^3$. The functions $y^2(x)$ are given polynomials of 6th degree which describe the shape of the body, $r^2=(x-t)^2+y^2$, and ds is arc length.

The iteration formula used is

$$U_{n+1}(t) = U_n(t) + \cos\gamma(t)\left[1 - \int_0^P \frac{y^2(x)}{2r^3} U_n(x)ds\right]$$

$$U_1(t) = 1 + k\cos\gamma\ ,\quad \gamma = \frac{dy}{ds}$$

where k is the longitudinal mass coefficient of an "equivalent" prolate spheroid.

Background: The origin of the problem and the method of solution are described in TMB Report No. 761, "Axially symmetric potential flow about elongated bodies of revolution," by L. Landweber.

Status: TERMINATED.

POWDER DIFFRACTION
Task 1102-40-5126/52-6
(formerly 1102-53-1106/52-6)

Origin: NBS, Section 9.7 Authorized 9/17/51
Manager: E. Marden
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 1102-40-5126/52-17
(formerly 1102-53-1106/52-17)

Origin: NBS, Division 2 Authorized 9/29/51
Sponsor: "
Manager: I. A. Stegun
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

SPHERICAL BLAST
Task 1102-40-5126/52-20
(formerly 1102-53-1106/52-20)

Origin: Naval Ordnance Laboratory Authorized 9/27/51
Sponsor: " "
Manager: D. H. Jirauch
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

MAGNETIC FIELD EXTRAPOLATION
Task 1102-40-5126/52-22
(formerly 1102-53-1106/52-22)

Origin: Naval Ordnance Laboratory — Authorized 10/1/51
Sponsor: " " " — Terminated 9/30/53
Manager: N. Levine

Objective: To evaluate the integral

$$H_z(x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{H_z(\xi,\eta,\rho)(z-\rho)\,d\xi\,d\eta}{\{(x-\xi)^2+(y-\eta)^2+(z-\rho)^2\}^{3/2}}$$

for several specified values of z and a sufficiently fine grid of pairs x,y. $H_z(x,y,z)$ is the unknown vertical component of the magnetic intensity at any point in the x,y plane below a given ξ, η plane in which the magnetic intensity $H_z(\xi,\eta,\rho)$ at every point is known.

Background: Formerly empirical means were used to measure the field generated by a given electromagnetic apparatus. This calculation, if feasible and sufficiently accurate, should replace costly measurement procedures.

Status: TERMINATED.

CALCULATIONS FOR d SPACINGS
Task 1102-40-5126/52-44
(formerly 1102-53-1106/52-44)

Origin: NBS, Div. 9 — Authorized 12/7/51
Sponsor: "
Manager: I. Stegun
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Computations were performed as requested.

GAS ADSORPTION BY HIGH POLYMERS
Task 1102-40-5126/52-70
(formerly 1102-53-1106/52-70)

Origin: Bethesda Naval Medical Center — Authorized 4/1/52
Sponsor: " "
Manager: I. Stegun
Full task description appears in Jan-Mar 1952 issue.

Status: INACTIVE. For status to date see Apr-June 1952 issue.

CHEMICAL TRANSITION PROBABILITIES
Task 1102-40-5126/52-82
(formerly 1102-53-1106/52-82)

Origin: NBS, Section 3.2 and Hydrocarbon Research Corp. Authorized 6/1/52
Sponsor: Office of Naval Research
Managers: A. Gleyzal and A. A. Goldstein
Full task description appears in Apr-June 1952 issue.

Status: CONTINUED. A code for the second part of the problem has been prepared and checked.

NEUTRON DIFFUSION, II
Task 1102-40-5126/53-4

Origin: Atomic Energy Commission, New York Office, (NDA) Authorized 9/30/52
Sponsor: " " "
Managers: O. Steiner and N. Levine
Full task description appears in July-Sept 1952 issue.

Status: CONTINUED. Computations are being performed as requested, and results are transmitted to the sponsor.

STRENGTH OF WING COMPONENTS
Task 1102-40-5126/53-11

Origin: National Advisory Committee for Aeronautics Langley Field, Va. Authorized 12/8/52
Sponsor: " " "
Manager: W. H. Durfee
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED.

RADIANT HEATING OF SOLIDS
Task 1102-40-5126/53-20

Origin: NBS, Section 10.2 Authorized 12/15/52
Sponsor: "
Manager: W. F. Cahill
Full task description appears in Oct-Dec 1952 issue.

Status: CONTINUED. Solutions are being computed for various values of the parameters and transmitted to the sponsor as completed.

COMPUTATION OF THERMODYNAMIC FUNCTIONS
Task 1102-40-5126/53-27

Origin: NBS, Division 5 Authorized 3/18/53
Sponsor: "
Manager: E. Marden
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Computation of the tables continues.

STUDY OF TRICALCIUM ALUMINATE
Task 1102-40-5126/53-28

Origin: NBS, Division 9 Authorized 3/30/53
Sponsor: "
Manager: R. Anderson
Full task description appears in Jan-Mar 1953 issue.

Status: INACTIVE. For status to date see Jan-Mar 1953 issue.

DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Task 1102-40-5126/53-29

Origin: NBS, Section 6.4 Authorized 1/23/53
Sponsor: "
Manager: I. Rhodes
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. Computations are being performed as requested, and results are being transmitted to the sponsor.

REDUCTION OF ECLIPSE DATA
Task 1102-40-5126/53-34

Origin: Air Photographing and Charting Services, USAF Authorized 3/30/53
Sponsor: " "
Manager: A. Goldstein
Full task description appears in Jan-Mar 1953 issue.

Status: CONTINUED. The following results have been obtained to diminish the required calculations: Defining the light function L as $L = \sum_{n=0}^{\infty} c_n L_n$, where

$$L_n = \int_{b-\Delta}^{a} \rho \, (a^2 - \rho^2)^{\frac{n}{2}} \arccos \frac{b^2 - \Delta^2 - \rho^2}{2\Delta\rho} \, d\rho ,$$

it is found that

$$\Delta L_\Delta + a L_a + b L_b = 2L + \sum_{n=0}^{\infty} n \, C_n L_n ,$$

and an expansion for L_n as an infinite series is given involving Γ-functions.

A least squares solution has been carried out for Bangui with differential corrections for the solar and lunar radii and the time of mid-totality. The L_n's are now being computed for Khartoum.

SPECTRAL ANALYSIS OF STATIONARY TIME SERIES
Task 1102-40-5126/53-37

Origin: Statistical Research Center, University of Chicago — Authorized 4/20/53
Sponsor: Office of Naval Research, USN
Manager: I. Stegun
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. A report is being prepared.

ACOUSTICAL IMPEDANCES
Task 1102-40-5126/53-39

Origin: NBS, Section 6.1 — Authorized 6/30/53
Sponsor: "
Managers: S. Prusch and K. Nelson
Full task description appears in Apr-June 1953 issue.

Status: INACTIVE. For status to date see Apr-June 1953 issue.

STANDARD LORAN TABLES - Rates 2H2, 2H3, 2H4
Task 1102-40-5126/53-40

Origin: Hydrographic Office, U. S. Navy — Authorized 4/20/53
Sponsor: " " — Terminated 9/30/53
Managers: W. H. Durfee, D. Jirauch, and K. Nelson

Objective: To prepare tables giving coordinates of hyperbolic lines of position.

Background: Standard Loran navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: TERMINATED. The results were transmitted to the sponsor.

LORAN UNIVAC CODE
Task 1102-40-5126/53-41

Origin: Hydrographic Office, U. S. Navy — Authorized 4/20/53
Sponsor: " "
Managers: W. H. Durfee, I. Rhodes, and D. Rubin
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The code has been completed and is almost completely checked.

Status of Projects

NOMOGRAMS FOR RF PERMEAMETER
Task 1102-40-5126/53-42

Origin: NBS, Division 14.8 — Authorized 6/29/53
Sponsor: " — Terminated 9/30/53
Manager: I. Stegun

Objective: To perform analysis as necessary to prepare nomograms.

Background: The problem arises in connection with the use of the RF permeameter.

Status: TERMINATED.

OPTIMUM SECTIONS FOR DELTA WINGS
Task 1102-40-5126/53-49

Origin: National Advisory Committee for Aeronautics, Langley Field, Va. — Authorized 4/29/53
Sponsor: " "
Manager: O. Steiner
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The computations desired were completed, and the results were transmitted to the sponsor.

NEUTRON DIFFUSION III
Task 1102-40-5126/53-51

Origin: NBS, Section 4.8 — Authorized 6/9/53
Sponsor: Armed Forces Special Weapons Project
Manager: F. Stockmal
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. The code for the first part of the problem has been completed and checked and a sample run of 50 histories has been made. Coding for the second part of the problem is underway.

TRANSPORTATION PROBLEM II
Task 1102-40-5126/53-55

Origin: Logistics Research Project, George Washington University — Authorized 6/9/53
Sponsor: Office of Naval Research
Managers: A. J. Hoffman and L. Gainen
Full task description appears in Apr-June 1953 issue.

Status: CONTINUED. All necessary codes for SEAC for the first part of the problem have been completed and checked.

STANDARD LORAN TABLES - Rates 1L4,1L5
Task 1102-40-5126/54-2

Origin: Hydrographic Office, U. S. Navy — Authorized 9/16/53
Sponsor: " "
Manager: W. H. Durfee

Objective: To prepare tables giving coordinates of hyperbolic lines of position.

Background: Standard Loran navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: NEW. Computations are in progress.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-40-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2 — Authorized 11/29/48
Sponsor: National Advisory Committee for Aeronautics
Manager: F. L. Alt
Full task description appears in Apr-June 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

BASIC IONOSPHERIC DATA
Task 1401-34-1473/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.3 — Authorized 3/31/49
Sponsor: " — Terminated 9/30/53
Manager: M. Stein

Objective: To analyze multifrequency, vertical-incidence observations of the ionosphere. The observations made at a number of observing stations are to be transcribed on punch cards. Various statistical analyses are to be performed and, in the case of current data, tables are to be prepared for publication.

Background: The observations consist of virtual heights and critical frequencies of the regularly ionized layers of the earth's upper atmosphere. These are measured hourly at 15 stations operated by or under agreement with the National Bureau of Standards. About 60 station-years of past data exist. Certain simple manipulations with this kind of data have been standard for many years, although the huge number of observations - some 6000 per station per month - has precluded much detailed investigation. Some analyses serve to extract typical ionospheric conditions to be applied in predictions of usable frequencies for long distance radio transmissions. Others will investigate statistically, changes in the characteristics of the ionospheric layers accompanying other geomagnetic phenomena, solar activity, etc., to increase knowledge on the origin and structure of the ionosphere and its effect on radio wave propagation.

Comment: Dr. A. H. Shapley of Section 14.1 is directing this work.

Status: TERMINATED.

RAY TRACING
Task 0202-10-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 2.2 Authorized 3/1/50
Sponsor: "
Managers: R. K. Anderson and D. Rubin
Full task description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations are being performed as requested.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS Authorized 1/9/49
Manager: L. S. Deming
Full task description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Abstracts taken from Mathematical Reviews and Zentralblatt fur Mathematik are put onto file cards to form a bibliography of literature dealing with probability, statistics, and their applications. Each card is classified by a code number according to its subject and arranged alphabetically by author for easy reference.

Abstracts from Mathematical Reviews cover the literature from 1939 to the present. Abstracts from Zentralblatt start with 1940 and work back through the years as far as there is activity in the statistical field. The file at present includes Zentralblatt abstracts back through 1935.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS Authorized 3/1/50
Manager: F. S. Acton
Full task description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1952 issue.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS Authorized 7/1/50
Managers: C. Eisenhart and L. S. Deming
Full task description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS Authorized 7/1/50
Manager: E. Lukacs
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. (1) J. Lieblein found that a method which he had used in an earlier paper (see "On the exact evaluation of the variances and covariances of order statistics in samples from the extreme value distribution," Ann. Math. Stat. 24, 282-287 (June 1953) for obtaining the exact covariances of order statistics for one type of extreme-value distribution can be applied to two other types as well. (2) J. Lieblein prepared two working papers, one dealing with a conjecture of E. J. Gumbel concerning the asymptotic relationship between the expected mean, expected variance, and expected largest value of an extreme variate; and the other treating a problem of rejection in connection with duplicate measurements. (3) M. Zelen found bounds on a cumulative distribution function, which are functions of the moments up to order four. Explicit expressions for the Tchebycheff-Markoff inequalities are given.

Publications: (1) "On absolute measurements," by E. N. Dorsey and C. Eisenhart; Sci. Monthly 77, 103-109 (August 1953). (2) "On some procedures for rejection of suspected data," by E. P. King; J. Am. Stat. Assoc. 48, 531-533 (September 1953). (3) "A property of the normal distribution," by E. P. King and E. Lukacs; submitted to a technical journal. (4) "A new method of analyzing extreme-value data," by J. Lieblein; accepted for publication as an NACA Technical Note. (5) "On strongly continuous stochastic processes," by E. Lukacs; submitted to a technical journal. (6) "Certain Fourier transforms of distributions (II)," by E. Lukacs and O. Szász; accepted for publication in the Canadian Journal of Mathematics. (7) "Nonnegative trigonometric polynomials and certain rational characteristic functions," by E. Lukacs and O. Szász; accepted for publication in the Journal of Research of the NBS. (8) "A historical note on the application of the "weakest-link" idea to tensile strengths," by J. Lieblein; submitted to a technical journal. (9) "Tables of expected values of $1/X$ for positive Bernoulli and Poisson variables," by E. Grab and I. R. Savage; submitted to a technical journal. (10) "Tables of the inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; to be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series. (11) "On optimum grouping in one-criterion variance components analysis," by E. P. King; IN MANUSCRIPT. (12) "On the variances and covariances of order statistics from the Weibull distribution," by J. Lieblein; IN MANUSCRIPT.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS Authorized 6/23/51
Managers: C. Eisenhart and I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1951 issue.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS Authorized 9/17/51
Manager: I. R. Savage
Full task description appears in July-Sept 1951 issue.

Status: CONTINUED.

Publication: "Bibliography of non-parametric statistics and related topics," by I. R. Savage; accepted for publication by the Journal of the American Statistical Association.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description appears in the Oct-Dec 1952 issue.

Status: CONTINUED. (1) M. Zelen proved that a partially balanced incomplete block (PBIB) design with (m+1) associate classes can be obtained from a PBIB design with m associate classes by replacing each variety of the design by n new varieties. (See item (5) under publications below.) (2) M. Zelen derived the intra- and inter-block analysis of covariance for PBIB designs with two associate classes. (3) W. H. Clatworthy studied the class of PBIB designs with two associate classes, two plots per block, and ten or fewer replications of each variety. He either proved impossible or constructed all but four of the designs. (See item (2) under publications below.) (4) W. J. Youden discovered a new class of incomplete block designs in which each variety is replicated twice. (5) W. S. Connor and W. J. Youden prepared a paper which describes by example the use and methods of analysis of new experimental arrangements with two plots per block. The designs treated are the "two-group" and "rectangular" arrangements. (See item (7) under publications below.)

Publications: (1) "The chain block design," by W. J. Youden and W. S. Connor; Biometrics 9, 127-140 (June 1953). (2) "Partially balanced incomplete block designs with two associate classes and two treatments per block," by W. H. Clatworthy; submitted to a technical journal. (3) "Some theorems for partially balanced designs," by W. S. Connor and W. H. Clatworthy; accepted for publication in the Annals of Mathematical Statistics. (4) "An embedding theorem for balanced incomplete block designs," by M. Hall, Jr., (Ohio State University) and W. S. Connor; accepted for publication in the Canadian Journal of Mathematics. (5) "A note on partially balanced designs," by M. Zelen; submitted to a technical journal. (6) "Analysis for some incomplete block designs having a missing block," by M. Zelen; submitted to a technical journal. (7) "Designs for paired observations," by W. S. Connor and W. J. Youden; IN MANUSCRIPT.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3737-60-0002/51-1
(formerly 3011-60-0002/51-1)

Origin: NBS Authorized 7/1/50
Manager: W. J. Youden
Full task description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this task fell into two main categories:

A. Design of Experiments: A partially balanced incomplete block design involving two items per block was constructed for use in an experiment on spark plug performance. Two-cylinder engines were involved in the test, and a two way elimination (of cylinder position differences and of engine differences) was effected in a case where this elimination could not have been achieved in the corresponding completely balanced design.

B. Development or Selection of the Appropriate Method for Analysis and Interpretation of Data: (a) A nonorthogonal analysis of covariance was worked out for testing the statistical significance of the effects of heaters and the effect of temperature rate in the measurement of the thermal conductivity of aluminum, and (b) some investigations were made concerning the adequacy of certain statistical distributions such as the Weibull and log-normal in representing fatigue data.

Publications: (1) "Confidence and tolerance intervals for the normal distribution," by F. Proschan; J. Am. Stat. Assoc. 48, 550-564 (Sept. 1953). (2) "Estimating the standard deviation of a normal population," by E. P. King; Ind. Quality Control 10, 30-33 (Sept. 1953). (3) "Sets of three measurements," by W. J. Youden; Sci. Monthly 77, 143-147 (Sept. 1953). (4) "Units for measuring variations in measurements," by W. J. Youden; Metals Progress 64, 91-96 (Sept. 1953). (5) "Acceptance sampling of electroplated articles," by J. M. Cameron and F. Ogburn (5.6); accepted for publication in Plating. (6) "Probability limits for the average chart when process standards are unspecified," by E. P. King; submitted to a technical journal. (7) "Rejection of outlying observations," by F. Proschan; accepted for publication in the American Journal of Physics. (8) "The principles of experimental design," by W. J. Youden; accepted for publication in Selection, Training and Use of Personnel in Industrial Research. Proceedings of the Third Annual Conference on Industrial Research. (9) "Making one measurement do the work of two," by W. J. Youden and W. S. Connor; accepted for publication in Chemical Engineering Progress. (10) "Performance of inspectors and gasoline pumps, by W. J. Youden and M. W. Jensen; submitted to a technical journal.

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1
(formerly 3011-60-0002/52-1)

Origin: NBS Authorized 10/1/51
Manager: I. R. Savage
Full task description appears in Oct-Dec 1951 issue.

Status: INACTIVE. For status to date see July-Sept 1952 issue.

APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO THE STUDY OF TRAJECTORIES
Task 1103-21-5119/52-1

Origin: U. S. Naval Ordnance Test Station, Inyokern — Authorized 1/1/52
Sponsor: " " — Terminated 9/30/53
Manager: E. Lukacs

Objective: To determine whether mathematical-statistical tools associated with the theory of stochastic processes can be profitably applied to the analysis of trajectory data of the type gathered at the Naval Ordnance Test Station, Inyokern.

Background: The theory of stochastic processes deals with time-dependent phenomena in which there is a probability relationship between a state at a given instant and one or more states at preceding instants. Some of the difficulties encountered in analyzing ordnance data can be overcome by considering a trajectory to be a stochastic process, thus eliminating the difficulties created by the fact that only one observation is available for the position of the missile at each instant.

Status: TERMINATED.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-50-5105/52-1

Origin: Ship Structure Committee, NRC — Authorized 12/1/51
Sponsor: " "
Manager: W. J. Youden
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Data from an interlaboratory study in which thirteen laboratories participated are being analyzed. The data provide comparisons among the laboratories of their notching and testing techniques, and estimates of experimental error.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO PROBLEMS OF THE CHEMICAL CORPS
Task 1103-50-5118/52-1

Origin: Biological Laboratories, Chemical Corps, Dept. of the Army — Authorized 10/1/51
Sponsor: " " "
Manager: C. Eisenhart
Full task description appears in Oct-Dec 1951 issue.

Status: CONTINUED. Reports on two special problems were transmitted to the sponsor.

IV. Machine Development Laboratory

(Section 11.4)

in cooperation with

Electronic Computer Section

(Section 12.3)

1. Development: Design and Construction of Automatic Digital Computing Machines

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. The first UNIVAC installation has continued to be operated on an around-the-clock basis by the Bureau of the Census. The machine is maintained by a crew of one contract engineer plus the Census personnel. The National Bureau of Standards has continued its obligation of obtaining a stock pile of replacement parts under contract with Remington Rand Inc., Eckert-Mauchly Division. A complete set of acceptable maintenance manuals has not yet been furnished by the company for any of the UNIVAC installations procured under NBS contract.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF — Authorized 7/1/47
Sponsor: " "
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. The second UNIVAC System has continued to be operated and maintained by personnel of the Office of the Air Comptroller. However, a reduction in maintenance personnel as well as in workload has necessitated a reduction from an around-the-clock seven-day a-week schedule to a schedule consisting of three 40-hour shifts per week. A minor amount of specialized procurement of parts has been handled by the National Bureau of Standards during this quarter, now that channels for the procurement of UNIVAC parts have been firmly established at the Pentagon.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Origin: NBS Authorized 12/15/48
Sponsor: Office of the Air Comptroller, USAF
Full task description appears in Apr-June 1949 issue.

Status: CONTINUED. Operating efficiency for the quarter July through September 1953 continued at the relatively high level of 81% (which represents total productive time over total time scheduled). The electrostatic memory, now available for problems which require more than the regular 512 words of acoustic memory, was used on at least 45 different occasions, with error-free runs ranging from 10 minutes to nine hours in duration. The acoustic memory has been provided with an odd-even (parity) check for all words read from the memory, which has added to the efficiency of machine operation. An additional magnetic wire input-output dumper has been added to the installation. Using this dumper, a code has been devised which will check-sum all but eight words of the 1024-word memory, transfer the entire contents of the memory to a removable wire cartridge, check the complete recording for accuracy, and enable the machine automatically to resume computation, all within 90 seconds.

WRIGHT DEVELOPMENT CENTER COMPUTING MACHINE
(formerly AIR MATERIEL COMMAND COMPUTING MACHINE)
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Aeronautical Research Laboratory, Wright Air Development Center, USAF Authorized 3/21/49
Sponsor: " " "
Manager: E. W. Cannon

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for use by the Wright Air Development Center in connection with air research problems.

Background: The Wright Air Development Center requires a high-speed and flexible computing machine to perform the involved calculations arising in the multifold problems associated with the design and development of aeronautical equipment and accessories. The problems involved range from the computation and evaluation of voluminous test data to the solution of intricate problems related to the design and performance prediction of airfoils, propellers, propulsive equipment and both subsonic and supersonic missiles. The computer should have a large memory capacity and a precision sufficiently high to furnish significant answers to problems like those in flutter analysis, which involve the treatment of large linear systems.

Comments: This task is related to projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/24-47-3a (see Apr-June 1949 issue, p. 87, 88, and 90, respectively).

Status: TERMINATED. With the delivery of manuals and diagrams to Wright Air Development Center during this quarter, the contractual obligations of the General Electric Company to the National Bureau of Standards were completed and the contract terminated.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, USA — Authorized 12/15/48
Sponsor: " " — Terminated 9/30/53
Manager: E. W. Cannon

Objective: To develop specifications for and to construct an automatic-sequenced electronic digital computing machine suitable for use by the Army Map Service in cartography.

Background: In the development of maps, particularly where various maps based on different projection systems are combined into one large map based upon a single projection system suitable for military application, there is required mass production of detailed calculations on point coordinates. A suitable computer must be able to handle huge masses of raw data, perform a calculation on each item of data and print output results in usable tabular form with speed comparable to that at which data enters the machine.

Comments: This task and projects 11.4/21-47-1, 11.4/22-47-2 and 11.4/3-47-4 (see Apr-June 1949 issue, p. 87, 88 and 93, respectively), are interrelated. Project 11.4/3-47-4 serves to coordinate the mathematical direction of the three computing machine projects.

Status: TERMINATED.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS OF THE SOCIAL SECURITY AGENCY
Task 1104-53-5108/51-1

Origin: Social Security Agency — Authorized 12/31/50
Sponsor: " " — Terminated 9/30/53
Manager: I. Rhodes

Objective: The purpose of the survey will be to investigate the applicability of automatic digital electronic computing to the record keeping and statistical operation of the Division of Accounting Operations of the Social Security Agency.

Background: The Social Security Agency has requested this survey to be performed by staff members of the Machine Development Laboratory of the National Bureau of Standards, with the cooperation of procedural consultants of the Division of Accounting Operations. Equipment requirements are to be evaluated by the Electronics Division of the Bureau. In connection with the survey, representative problems of the Division are to be tried out on existing computing machines. At the conclusion of the survey, a report of findings and recommendations are to be submitted to the Division.

Status: TERMINATED. This task is being terminated with the submission of the final report which is in preparation.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

American Mathematical Society
Sixth Symposium in Applied Mathematics: Numerical Analysis

The National Bureau of Standards was a co-sponsor with the American Mathematical Society of a Symposium on Numerical Analysis held at Santa Monica City College, Santa Monica, California, August 26-28. The program was as follows:

August 26:

RIDDICK, Dr. Morford (Director of the General Division, Santa Monica City College). Greetings.

TOMPKINS, C. B. (National Bureau of Standards). Application of automatic digital computers to problems with discrete variables.

BRUCK, R. H. (University of Wisconsin). Computational aspects of certain combinatorial problems.

TAUSSKY, Olga (National Bureau of Standards). Some computational problems in algebraic number theory.

LEHMER, Emma (Berkeley, California). Number theory on the SWAC.

WIELANDT, H. (University of Tübingen). Error bounds for eigenvalues of symmetric integral equations.

GIVENS, W. (Oak Ridge National Laboratory). A method of error analysis.

YOUNG, D. M., Jr. (University of Maryland). On the solution of linear systems by iteration.

HESTENES, M. R. (University of California at Los Angeles and National Bureau of Standards). The conjugate gradient method for linear systems.

August 27:

BERGMAN, S. (Stanford University). Some methods for solutions of boundary value problems of linear partial differential equations.

MOTZKIN, T. S. (University of California at Los Angeles). The assignment problem.

BELLMAN, R. (The RAND Corporation). On some computational problems in the theory of dynamic programming.

KUHN, H. W. (Bryn Mawr College). The traveling salesman problem.

August 28:

WARSCHAWSKI, S. E. (University of Minnesota). Recent results in numerical methods of conformal mapping.

TAUB, A. H. (University of Illinois). Numerical solution of the equations of pseudo-stationary flow.

FISCHBACH, J. W. (Ballistic Research Laboratories). Some applications of gradient methods.

FRANKEL, S. P. (California Institute of Technology). Stability problems in partial differential equations.

WALSH, J. L. (Harvard University). Best-approximation polynomials of given degree.
HASTINGS, C., Jr. (The RAND Corporation). Approximations in numerical analysis.
WASOW, W. R. (National Bureau of Standards). On the asymptotic transformation of certain probability distributions.
ROSENBLOOM, P. C. (University of Minnesota). The method of steepest descent.
SARD, A. (Queens College). Function spaces and approximation.

Numerical Analysis Colloquium Series
(Los Angeles, California)

PALL, G. (Illinois Institute of Technology). Some properties of finite projective planes. July 13.

LEFSCHETZ, S. (Princeton University). Some recent contributions to non-linear differential equations. July 16.

MARSCHAK, J. and RADNER, R. (Cowles Commission for Research in Economics, University of Chicago). Optimal communication in teams. August 10.

BAÑOS, A., Jr. (University of California, Los Angeles). On Sommerfeld's electromagnetic surface waves. August 17.

FOSTER, F. G. (London School of Economics, University of London). A review of some recent work in the theory of queues. September 21.

Applied Mathematics Division Technical Meetings

At Los Angeles, California:

YOWELL, E. C. Coding for SWAC. July 7 and July 9.

BRUCK, R. H. (University of Wisconsin). Abelian difference sets. July 10.

ALBERT, A. A. (University of Chicago and University of California). On proper solutions of the incidence equation for finite projective planes. August 11.

ROSENBLOOM, P. (University of Minnesota). Methods of steepest descent applied to calculus of variations. September 9, 11, 14.

At Washington, D. C.:

FAN, K. A combinatorial lemma and antipodal point theorem. July 9.

CONNOR, W. A problem in the design of experiments. July 23.

DAVIS, P. Errors of numerical approximation for analytic functions. August 14.

WIELANDT, H. The converse of Abel's convergence theorem. August 19.

Seminar on Numerical and Computational Aspects of Linear Problems: Games, Linear Equalities, Linear Inequalities, Programming,...(cont'd.)
(Los Angeles, California)

SOUTHARD, T. H., Numerical aspects of projecting and war games. July 6.

LEHMER, D. H., and LEHMER, EMMA. Programming for discrete variable problems. July 8, 13.

HOFFMAN, A. J., NEWMAN, M., TAUSSKY, O., TODD, J. Recent work on SEAC. July 15.

PAIGE, L. J. A continuous method in a discrete problem. July 20.

SAXON, D. S. Problems for theoretical physics suitable for SWAC computation. July 22, 30.

ROBERTS, A. E. Projection methods in discrete problems. July 28.

TOMPKINS, C. B. A continuous manifold of quasi-solutions to a discrete problem. August 4.

BAÑOS, A., Jr. On the saddle-point method of integration. August 6.

MARSCHAK, J., and RADNER, R. Some problems in the mathematical theory of organization. August 13.

TOMPKINS, C. B. Numerical local embedding of a surface with Riemannian metric into Euclidean 3-space, I. Geometrical introduction and classical formulas. September 29.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

Papers presented at the joint meeting of the American Mathematical Society and the Institute of Mathematical Statistics, Kingston, Ontario, August 31 - September 5:

ANTOSIEWICZ, H., and DAVIS, P. Some implications of Liapunov's conditions for stability.

FORSYTHE, G. E. Arbitrarily close lower bounds for the fundamental frequency of a convex membrane.

LIEBLEIN, J. Estimation of extremal parameters by use of order statistics.

Papers presented at the Summer Statistical Seminar, University of Connecticut, Storrs, Conn., August 12, 1953:

EISENHART, C. Statistical method in experimentation: Individual experiments.

KING, E. P. Statistical resources for physicists.

Papers presented at the meeting of the Association for Computing Machinery, held at the Massachusetts Institute of Technology, Cambridge, Mass.,

September 9-11:

ALT, F. L. Numerical solution of some nonlinear heat transfer problems.

FORSYTHE, G. E., and HANDY, B. F. Finite-difference approximations to the fundamental frequency of a vibrating membrane.

Papers presented elsewhere:

CAMERON, J. M., and CONNOR, W. S., Statistical engineering at the National Bureau of Standards. Presented at a Seminar on Mathematical Statistics held at the Bureau of Ordnance, Department of the Navy, Washington, D. C., July 16.

CONNOR, W. S. Some new experimental designs. (1) Presented at the Gordon Research Conference, New Hampton, N. H., July 27-31. (2) Also presented at a meeting of the American Statistical Association, Chicago, Ill., September 7.

FORSYTHE, G. E. Solving linear algebraic equations can be interesting. Presented to the Department of Mathematics, University of Michigan, Ann Arbor, Mich., September 15.

LUKACS, E. Remarks on characteristic functions. Presented at a meeting of the Mathematical Society, University of Basel, Basel, Switzerland, July 6.

TEICHROEW, D. Use of continued fractions in high speed computing. Presented at a meeting of the Digital Computers Association, Los Angeles, Cal., July 24.

THORENSEN, R. (1) Memory systems: Introduction and survey; (2) The magnetic drum memory; (3) Selection and switching; (4) Magnetic drum memory system design; (5) Magnetic tape memories; (6) Megabit magnetic memories; (7) Electrostatic memories; (8) The Williams system; (9) Limitation of the Williams tube memory system and discussion of current research; (10) Survey of various types of memories. Lectures given at Summer Course in Computer Applications and Components, held at Wayne University Computation Laboratory, Detroit, Michigan, August 10-21.

WIELANDT, H. Error bounds for the eigenvalues of integral equations. Presented at the Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Md., September 25.

YOUDEN, W. J. Statistical units of measurement. Presented at a Symposium on Modern Statistical Methods held by the Charleston, West Virginia Section of the American Institute of Chemical Engineers, at Charleston, W. Va., September 24.

YOWELL, E. C. (1) Problem formulation; (2) Linear problems: Matrix inversion; (3) Linear problems: Characteristic values; (4) Approximation problems; (5) Monte Carlo techniques; (6) Subroutines; (7) Relative and abstract coding; (8) Checking procedures; (9) Machine design; (10) Operating efficiency. Lectures given at Summer Course in Computer Applications and Components, held at Wayne University Computation Laboratory, Detroit, Michigan, August 10-21.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Probability tables for analysis of extreme-value data. NBS Applied Mathematics Series 22. Available from U. S. Government Printing Office, Washington 25, D. C., 25 cents.

(2) Tables of normal probability functions. NBS Applied Mathematics Series 23. (Supersedes NBS Mathematical Table MT14, Tables of probability functions, Volume II.) Available from U. S. Government Printing Office, Washington 25, D. C., $2.75 (blue buckram).

(3) Tables of coefficients for the numerical calculation of Laplace transforms. H. E. Salzer. NBS Applied Mathematics Series 30. Available from U. S. Government Printing Office, Washington 25, D. C., 25 cents.

1.3 Technical Papers

(1) On the numerical solution of parabolic partial differential equations. G. Blanch. J. Res. NBS 50, 343-356 (June 1953); RP2424.

(2) Boolean geometry I. L. M. Blumenthal. Rend. Circ. Mat. Palermo II , 1, 1-18 (1952). Reprints available.

(3) On absolute measurement. N. E. Dorsey and C. Eisenhart. Scientific Monthly 77, 103-109 (Aug. 1953). Reprints available.

(4) Solving linear algebraic equations can be interesting. G. E. Forsythe. Bul. Am. Math. Soc. 59, 299-329 (July 1953). Reprints available.

(5) Tentative classification of methods and bibliography on solving systems of linear equations. G. E. Forsythe. Simultaneous linear equations and the determination of eigenvalues, Proceedings of an NBS Symposium held in Los Angeles, August 1951; NBS Applied Mathematics Series 29, pp. 1-28.(Available from U. S. Government Printing Office, Washington, D. C., $1.50).

(6) A non-harmonic Fourier series. J. M. Hammersley. Acta Math. 89, 243-260 (1953). Reprints available.

(7) Computational experience in solving linear programs. A. J. Hoffman, M. Mannos, D. Sokolowsky, and N. Weigmann. J. Soc. Ind. App. Math. I, 17-34 (Sept. 1953).

(8) Estimating the standard deviation of a normal population. E. P. King. Ind. Quality Control 10, 30-33 (Sept. 1953). Reprints available.

(9) On some procedures for the rejection of suspected data. E. P. King. J. Am. Stat. Assoc. 48, 531-533 (Sept. 1953). Reprints available.

(10) Eigenvectors of matric polynomials. M. Mannos. J. Res. NBS 51, 33-36 (July 1953); RP2429.

(11) The coefficients of certain infinite products. M. Newman. Proc. Am. Math. Soc. 4, 435-439 (June 1953). Reprints available.

(12) On over and under relaxation in the theory of the cyclic single step iteration. A. Ostrowski. MTAC 43, 152-159 (July 1953).

(13) Confidence and tolerance intervals for the normal distribution. F. Proschan. J. Am. Stat. Assoc. 48, 550-564 (Sept. 1953).

(14) Sur les problèmes de perturbation singuliers dans la théorie des vibrations non linéaires (On singular perturbation problems in the theory of non-linear vibrations). W. Wasow. Appeared in Actes du Colloque International des vibrations non linéaires, Ile de Porquerolles, France, 1951 (Publications Scientifiques et Techniques du Ministère de l'Air, No. 281).

(15) Pairs of normal matrices with property L. H. Wielandt. J. Res. NBS 51, 89-90 (Aug. 1953).

(16) Sets of three measurements. W. J. Youden. Sci. Monthly 77, 143-147 (Sept. 1953). Reprints available.

(17) Units for measuring variations in measurements. W. J. Youden. Metals Progress 64, 91-96 (Sept. 1953).

(18) The chain block design. W. J. Youden and W. S. Connor. Biometrics 9, 127-140 (June 1953). Reprints available.

1.5 Miscellaneous Publications

(1) Simultaneous linear equations and the determination of eigenvalues. Proceedings of an NBS Symposium held in Los Angeles, August 1951. NBS Applied Mathematics Series 29. Available from U. S. Government Printing Office, Washington 25, D. C., $1.50.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION September 30, 1953.

2.1 Mathematical Tables

(1) Tables of 10^x. NBS Applied Mathematics Series 27. In press, Government Printing Office.

(2) Table of natural logarithms for arguments between zero and five to sixteen decimal places. (A reissue of NBS Mathematical Table MT10, Table of natural logarithms, vol. III.) NBS Applied Mathematics Series 31. In press, Government Printing Office.

(3) Table of sine and cosine integrals for arguments from 10 to 100. (A reissue of NBS Mathematical Table MT13.) NBS Applied Mathematics Series 32. In press, Government Printing Office.

(4) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. In press, Government Printing Office.

(5) Tables of Lagrangian coefficients for sexigesimal interpolation. NBS Applied Mathematics Series 35. In press, Government Printing Office.

(6) Tables of circular and hyperbolic sines and cosines for radian arguments. (A reissue of NBS Mathematical Table MT3.) NBS Applied Mathematics Series 36. In press,Government Printing Office.

(7) Tables of functions and of zeros of functions. Volume I of Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(8) Table of secants and cosecants to nine significant figures at hundredths of a degree. To be issued in the NBS Applied Mathematics Series.

(9) Tables of expected values of 1/X for positive Bernoulli and Poisson variables. E. Grab and I. R. Savage. Submitted to a technical journal.

(10) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

2.2 Manuals, Bibliographies, Indices

(1) The statistical theory of extreme values and some practical applications. A series of lectures by E. J. Gumbel. To appear in the NBS Applied Mathematics Series.

(2) Bibliography of nonparametric statistics and related topics. I. R. Savage. Accepted for publication in the Journal of the American Statistical Association.

2.3 Technical Papers

(1) Evaluation of the integral $\int_0^\infty e^{-u^2-(x/u)}du$. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) On the solution of the differential equation occurring in the problem of heat convection in laminar flow through a tube. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(3) Regular and irregular Coulomb wave functions expressed in terms of Bessel-Clifford functions. M. Abramowitz. Accepted for publication in Journal of Mathematics and Physics.

(4) Approximate method for rapid Loran computation. M. Abramowitz, D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(5) The relaxation method for linear inequalities. S. Agmon. Submitted to a technical journal.

(6) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$. H. A. Antosiewicz. Submitted to a technical journal.

(7) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Submitted to a technical journal.

(8) The convergence of numerical iteration. H. A. Antosiewicz and J. M. Hammersley. Accepted for publication in the American Mathematical Monthly.

(9) On mildly nonlinear partial difference equations of elliptic type. L. Bers. Accepted for publication in the Journal of Research of the NBS.

(10) A general-purpose control panel for a model II CPC. P. B. Bremer, D. Teichroew, and E. C. Yowell. To be published in the IBM Newsletter.

(11) Programs for computing the hypergeometric series. W. F. Cahill. Submitted to a technical journal.

(12) Mathematical services useful in industry. A. S. Cahn, Jr. To appear in the Proceedings of a Symposium on Industrial Applications of Automatic Computing Equipment held by the Midwest Research Institute, Kansas City, Mo., January 8, 1953.

(13) Acceptance sampling of electroplated articles. J. M. Cameron and F. Ogburn. Accepted for publication in Plating.

(14) Partially balanced incomplete block designs with two associate classes and two treatments per block. W. H. Clatworthy. Submitted to a technical journal.

(15) Some theorems for partially balanced designs. W. S. Connor and W. H. Clatworthy. Accepted for publication in the Annals of Mathematical Statistics.

(16) Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains. D. van Dantzig. Submitted to a technical journal.

(17) Linear functional equations and interpolation series. P. Davis. Submitted to a technical journal.

(18) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack. Submitted to a technical journal.

(19) On representations and extensions of bounded linear functionals defined on classes of analytic functions. P. Davis and J. L. Walsh. Submitted to a technical journal.

(20) On the optimal character of the (s,S) policy in inventory theory. A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a technical journal.

(21) A test for statistical control applicable to a short series of observations. C. Eisenhart and E. P. King. Submitted to a technical journal.

(22) Changes of sign of sums of random variables. P. Erdös and G. A. Hunt. Submitted to a technical journal.

Publication Activities

(23) Inequalities for eigenvalues of Hermitian matrices. K. Fan. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(24) Some remarks on commutators of matrices. K. Fan. Submitted to a technical journal.

(25) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(26) A numerical analyst's fifteen-foot shelf. G. E. Forsythe. Accepted for publication in Mathematical Tables and Other Aids to Computation.

(27) Asymptotic lower bounds for the frequencies of polygonal membranes. G. E. Forsythe. Submitted to a technical journal.

(28) Punched-card experiments with accelerated gradient methods for linear equations. A. I. and G. E. Forsythe. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series. In press, Government Printing Office.

(29) Practical solution of linear equations and inversion of matrices. L. Fox. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series.

(30) A numerical solution of Schroedinger's equation in the continuum. W. Futterman, E. Osborne, and D. S. Saxon. Accepted for publication in the Journal of Research of the NBS.

(31) A nonlinear model for the composite pi-meson. S. G. Gasiorowicz. Submitted to a technical journal.

(32) Equations of physics in general Newtonian space-time. A. N. Gleyzal. Submitted to a technical journal.

(33) A 2-basic set of density zero. K. Goldberg. Submitted to a technical journal.

(34) The representation of integers by binary quadratic rational forms. K. Goldberg, M. Newman, E. G. Strauss, and J. D. Swift. Submitted to a technical journal.

(35) An expansion method for parabolic partial differential equations. J. W. Green. Accepted for publication in the Journal of Research of the NBS.

(36) The embedding theorem for balanced incomplete block designs. M. Hall, Jr., and W. S. Connor. Accepted for publication in the Canadian Journal of Mathematics.

(37) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(38) On a combinatorial theorem. A. J. Hoffman. Submitted to a technical journal.

(39) A characterization of normal matrices. A. J. Hoffman and O. Taussky. Accepted for publication in the Journal of Research of the NBS.

(40) The SWAC --design features and operating experience. H. Huskey, R. Thorensen, B. F. Ambrosio, and E. C. Yowell. To appear in the Proceedings of the Institute of Radio Engineers.

(41) Completely continuous normal operators with property L. I. Kaplansky. Submitted to a technical journal.

(42) Probability limits for the average chart when process standards are unspecified. E. P. King. Submitted to a technical journal.

(43) A property of the normal distribution. E. P. King and E. Lukacs. Submitted to a technical journal.

(44) On certain character matrices. D. H. Lehmer. Submitted to a technical journal.

(45) Acoustic radiation pressure on a circular disk. H. Levine. To appear in the Proceedings of the Fifth Symposium on Applied Mathematics of the American Mathematical Society.

(46) On strongly continuous stochastic processes. E. Lukacs. Submitted to a technical journal.

(47) Certain Fourier transforms of distributions (II). E. Lukacs and O. Szász. Accepted for publication in the Canadian Journal of Mathematics.

(48) Nonnegative trigonometric polynomials and certain rational characteristic functions. E. Lukacs and O. Szász. Accepted for publication in the Journal of Research of the NBS.

(49) On Fejer sets in linear and spherical spaces. T. S. Motzkin and I. J. Schoenberg. Accepted for publication in Annals of Mathematics.

(50) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Submitted to a technical journal.

(51) Pairs of matrices with property L, II (summary). T. S. Motzkin and O. Taussky. Accepted for publication in the Proceedings of the National Academy of Science.

(52) Least p-th power polynomials on a real finite point set. T. S. Motzkin and J. L. Walsh. Submitted to a technical journal.

(53) The arithmetic structure of certain modular subgroups. M. Newman. Submitted to a technical journal.

(54) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(55) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(56) On nearly triangular matrices. A. Ostrowski. Submitted to a technical journal.

(57) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(58) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(59) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Submitted to a technical journal.

(60) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(61) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(62) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(63) Rejection of outlying observations. F. Proschan. Accepted for publication in the American Journal of Physics.

(64) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(65) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(66) Modes of vibration of a suspended chain. D. S. Saxon and A. S. Cahn. Accepted for publication in the Quarterly Journal of Mechanics and Applied Mathematics.

(67) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Submitted to a technical journal.

(68) Characteristic roots of quaternion matrices. O. Taussky. Accepted for publication in Archiv der Mathematik.

(69) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(70) An improved cathode ray tube storage system. R. Thorensen. To appear in the Proceedings of the Western Computer Conference of the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6, 1953.

(71) The condition of the finite segments of the Hilbert matrix. J. Todd. To be included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series.

(72) On the accuracy of the numerical solution of the Dirichlet problem by finite differences. J. L. Walsh and D. Young. Accepted for publication in the Journal of Research of the NBS.

(73) Asymptotic solution of the differential equation of hydrodynamic stability in a domain containing a transition point. W. Wasow. Accepted for publication in the Annals of Mathematics.

(74) On small disturbances of plane Couette flow. W. Wasow. Accepted for publication in the NBS Journal of Research.

(75) Singular perturbation methods for nonlinear oscillations. W. Wasow. To appear in the Proceedings of a Symposium on Nonlinear Circuit Analysis, held by the Polytechnic Institute of Brooklyn, N. Y.

(76) On the eigenvalues of A+B and AB. H. Wielandt. Submitted to a technical journal.

(77) The principles of experimental design. W. J. Youden. Accepted for publication in Selection, Training, and Use of Personnel in Industrial Research. Proceedings of the Third Annual Conference on Industrial Research.

(78) Making one measurement do the work of two. W. J. Youden and W. S. Connor. Accepted for publication in Chemical Engineering Progress.

(79) Performance of inspectors and gasoline pumps. W. J. Youden and M. W. Jensen. Submitted to a technical journal.

(80) A note on partially balanced designs. M. Zelen. Submitted to a technical journal.

(81) Analysis for some incomplete block designs having a missing block. M. Zelen. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) Contributions to the solution of systems of linear equations and the determination of eigenvalues. To be issued in the NBS Applied Mathematics Series.

(2) A historical note on the application of the "weakest-link" idea to tensile strengths. J. Lieblein. Submitted to a technical journal.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.


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