Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

July through December 1947

NATIONAL APPLIED MATHEMATICS LABORATORIES
of the
NATIONAL BUREAU OF STANDARDS

PREFACE

This is a report on the projects and publications of Division 11 of the National Bureau of Standards for the period from July 1, 1947 to December 31, 1947. In the future it is planned to issue this report on a quarterly basis.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss, Chief
National Applied Mathematics
Laboratories

Approved:

E. U. Condon, Director
National Bureau of Standards
January 15, 1948

U. S. DEPARTMENT OF COMMERCE
W. Averell Harriman, Secretary

NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

Projects and Publications

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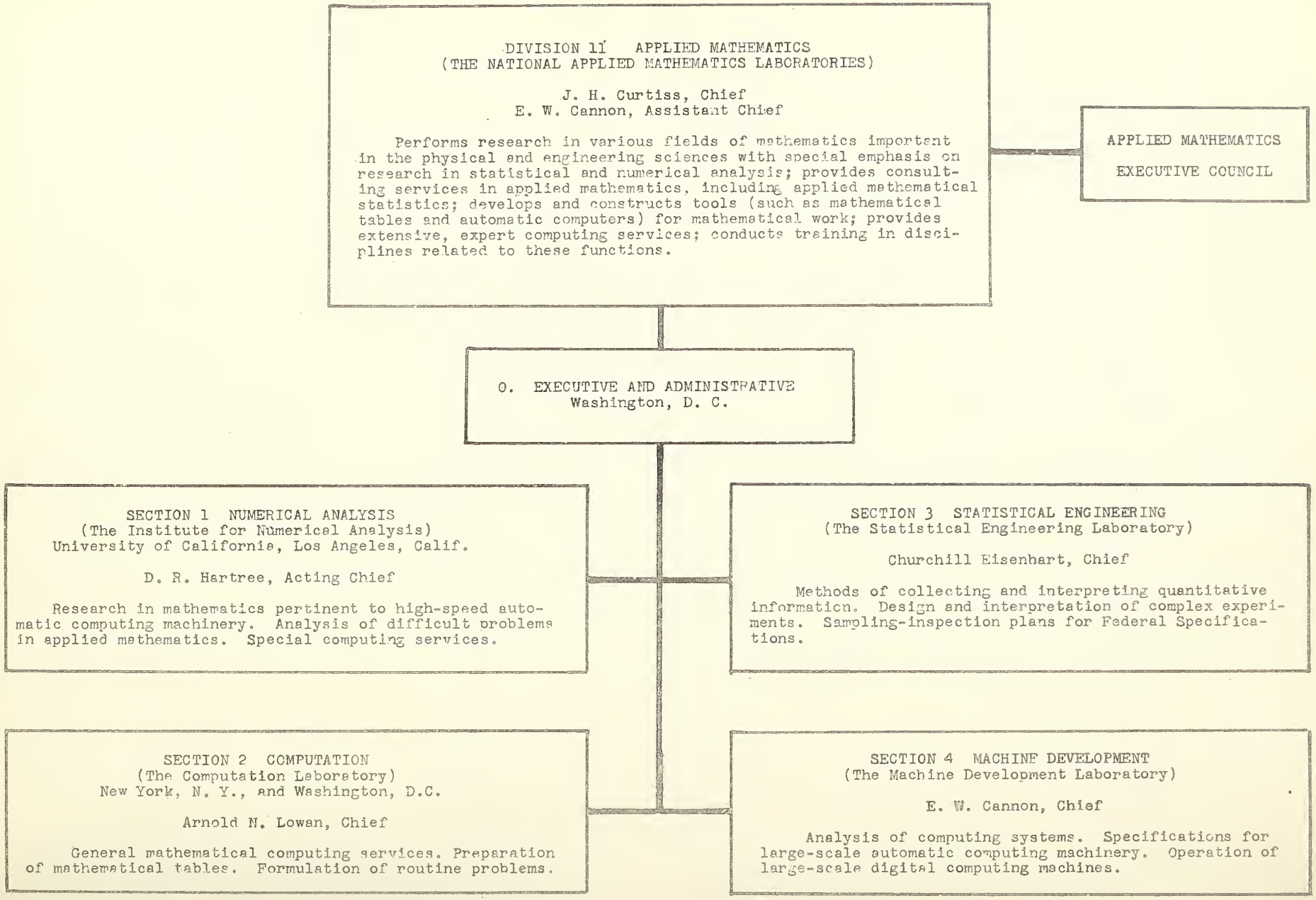
CONTENTS


DIVISION 11 APPLIED MATHEMATICS
(THE NATIONAL APPLIED MATHEMATICS LABORATORIES)
J. H. Curtiss, Chief
E. W. Cannon, Assistant Chief
Performs research in various fields of mathematics important in the physical and engineering sciences with special emphasis on research in statistical and numerical analysis; provides consulting services in applied mathematics, including applied mathematical statistics; develops and constructs tools (such as mathematical tables and automatic computers) for mathematical work; provides extensive, expert computing services; conducts training in disciplines related to these functions.
APPLIED MATHEMATICS EXECUTIVE COUNCIL
0. EXECUTIVE AND ADMINISTRATIVE
Washington, D. C.
SECTION 1 NUMERICAL ANALYSIS
(The Institute for Numerical Analysis)
University of California, Los Angeles, Calif.
D. R. Hartree, Acting Chief
Research in mathematics pertinent to high-speed automatic computing machinery. Analysis of difficult problems in applied mathematics. Special computing services.
SECTION 3 STATISTICAL ENGINEERING
(The Statistical Engineering Laboratory)
Churchill Eisenhart, Chief
Methods of collecting and interpreting quantitative information. Design and interpretation of complex experiments. Sampling-inspection plans for Federal Specifications.
SECTION 2 COMPUTATION
(The Computation Laboratory)
New York, N. Y., and Washington, D.C.
Arnold N. Lowan, Chief
General mathematical computing services. Preparation of mathematical tables. Formulation of routine problems.
SECTION 4 MACHINE DEVELOPMENT
(The Machine Development Laboratory)
E. W. Cannon, Chief
Analysis of computing systems. Specifications for large-scale automatic computing machinery. Operation of large-scale digital computing machines.

1. ADMINISTRATIVE ACTIVITIES

The National Applied Mathematics Laboratories were formally organized as Division 11 of the Bureau on July 1, 1947. The plan of organization is set forth in the chart on the opposite page. It follows closely the "Prospectus" issued in February, 1947 and approved by various officially interested agencies.

Appointments have now been made to the Applied Mathematics Executive Council, which will hold its first meeting early in February, 1948.

A formal project control system has been devised and installed; this system is reflected in the presentation appearing in the next section of this report.

Further administrative activities which are pertinent to individual Sections of the Division were as follows:

Section 11.1 The Institute for Numerical Analysis - The relationship of the Institute for Numerical Analysis to the University of California at Los Angeles has been studied at some length in cooperation with University authorities, with a view to tying in the Institute more closely with the Mathematics Department of the University. Two committees have been set up on the Campus in accordance with Bureau recommendation, one to review the qualifications of candidates for the P-4 to P-8 grades, and the other to perform a similar function for the lower professional and subprofessional grades. The chairman of the first committee is Professor E. F. Beckenbach and of the second committee is Professor Clifford Bell; both are members of the Mathematics Department of the University.

The bids for remodeling the building in which the Institute will be housed were opened on December 3, and the contract was let in the last week of December. The work will be completed by March 15, with occupancy possible not later than April 1, 1948. Purchase orders have been issued for office equipment and IBM equipment, requesting delivery by April first.

Section 11.2 The Computation Laboratory - A policy decision was made to move the Computation Laboratory from New York in the autumn of 1948, and by way of preparation, to activate at once a small problem computing laboratory in Washington. The delivery of an IBM computing installation to the new Wind Tunnel at the University of Maryland some months before the completion of the tunnel offered an opportunity to activate the local computing group without making immediate space demands at the Bureau. Steps have accordingly been taken to conclude an agreement with the University of Maryland whereby the Bureau would temporarily operate this IBM machinery at the wind tunnel, with the operation to start early in January, 1948.

Section 1. The Institute of Numerical Analysis

No formal projects have as yet been set up for this Section. Mr. and Mrs. John Todd and Mr. Albert Cahn were the scientists active in this Section during the period under review. Mr. and Mrs. Todd spent a considerable amount of time studying characteristics of automatic digital computers at the Bureau and elsewhere under development. Mrs. Todd also performed work on Project 47D4-3, and Mr. Todd initiated certain research and development activities which will be described in succeeding reports. (See also Sect. 3.2.3 of this Report.)

Section 2. The Computation Laboratory

Project: 43D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Bessel Functions $I_\nu(x)$; $\pm\nu$ = 1/3, 2/3, 1/4, 3/4.

Origin: NBS

Project Manager: Mr. M. Abramowitz

Objective: To provide tables of $I_\nu(x)$, $\pm\nu$ = 1/3, 2/3, 1/4, 3/4, for x = 0(.001).5(.01)25, 10D or 10S and $e^{-x}I_\nu(x)$ for x ranging from 25 to 30,000 at varying intervals to 10 decimal places. Central differences are also to be tabulated for purposes of interpolation.

Background: The Bessel Functions $J_\nu(x)$ and $I_\nu(x)$ arise in numerous problems in applied mathematics such as hydrodynamics, heat conduction, and elasticity. They are also of importance in the approximation to Bessel functions of large order, and in the approximation of solutions of differential equations of the form $y'' + p(x)y = 0$ in the Stokes transition region where p(x) changes sign. The need for such approximation arises for instance in the theory of wave propagation in stratified media with a constant gradient in the index of refraction; it also arises in many quantum mechanical problems. (See "Quantum Mechanics" by Condon and Morse.) The project was originally proposed by Dr. S. Schelkunoff of the Bell Telephone Laboratories.

Magnitude: Class III.

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.

Publication: IN MANUSCRIPT. To be submitted to Columbia University Press.

Project: 43D2-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Intensity Functions

Origin: National Defense Research Committee, Division X

Project Manager: Mr. W. Horenstein

Objective: The tabulation of functions which give the angular distribution of intensity and total light scattered by (1) transparent small spherical particles (such as a fog droplet) and (2) small spherical particles with small absorption coefficient k ($k < 0.01$) as a function of the parameter $\alpha = 2\pi r/\lambda$ when the particle radius r is roughly equal to the wave length λ of the incident light.

Background: Requested specifically by Professor V. K. LaMer of Columbia University. Intensity functions are used, for example,

1. To determine optimum particle size of DDT aeresols.
2. To determine optimum particle size of paint pigments with the view of obtaining maximum covering power of paint.
3. In connection with micro-wave radar studies.

Magnitude: Class III.

Date of Termination: 1/31/48

Comments: Part of the tabular material in the present volume, previously submitted to Professor LaMer, was included in OSRD report 1857 by V. K. LaMer and D. Sinclair. Earlier computations for very opaque particles (large values of k) were carried out for the Naval Research Laboratory.

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.
Publication: To be published in the NBS Applied Mathematics Series.

Project: 4302-3 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of $E_1(z)$, $(z = x + iy)$

Origin: Canadian National Research Council

Project Manager: Mr. A. Hillman

Objective: To prepare tables of the function $E_1(z) = \int_z^\infty (e^{-u})/u \, du$ and related functions for $z = x + iy$.

Table I: $E_1(z) + \log_e z$, $x = 0(.01)1$, $y = 0(.02)1$; 6D

Table II: $E_1(z)$, $x = 0(.02)4$, $y = 0(.02)3(.05)10$; 6D

Table III: $e^x E_1(z)$, $x = 0(.02)4$; $y = 0(.02)3(.05)10$; 6D

Background: The initial motivation for the preparation of a table of exponential integrals for complex arguments was a certain phase of the atomic bomb project which at present is still undisclosed. The table has however found applications in fluid mechanics. (See for instance, article by J. J. Stoker on "Surface Waves in Water of Variable Depth", in the April 1947 issue of the Quarterly of Applied Mathematics.)

Magnitude: Class IV
Date of Termination: 6/30/48

Status: UNDER WAY (CONTINUATION). Computations completed on IBM, and partly checked.
Publication: To be submitted for publication to the Columbia University Press.

Project: 4302-4 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Jacobi Elliptic Functions

Origin: NBS

Project Manager: Mr. W. Horenstein

Objective: (a) To prepare tables of the Jacobi elliptic functions: $sn(u,k) = \sin\varphi$, $cn(u,k) = \cos\varphi$, $dn(u,k) = \sqrt{1-k^2\sin^2\varphi}$ where φ is defined by $u = \int_0^\varphi (1-t^2)^{-1/2} (1-k^2t^2)^{-1/2} dt$. These functions are to be tabulated for $k^2 = 0(.01)1$ and for $u = pK$ with $p = 0(.01)1$ and $K = \int_0^{\pi/2} (1-k^2\sin^2\theta)^{-1/2} d\theta$.

(b) To prepare tables of $sn(iu,k') = i\, sn(u,k)/cn(u,k)$, $cn(iu,k') = 1/cn(u,k)$, $dn(iu,k') = dn(u,k)/cn(u,k)$, for same values of u and k as in (a).

Background: Professor Milne-Thompson originally suggested the preparation of a table of Jacobi elliptic functions for complex arguments. Because of the magnitude of this task, it was deemed sufficient to undertake the computation of the functions in question for real and purely imaginary arguments. The known addition formulae would then enable the user to evaluate elliptic functions for complex arguments. The chief applications contemplated by Professor Milne-Thompson were in the field of hydrodynamics.

Subesequently there was extensive correspondence with members of the Mathematical Tables Committee of the BAAS regarding the scope of this table.

The present specifications incorporate the suggestions contained in the correspondence.

Magnitude: Class III

Comments: The proposed tables are to be computed at equidistant intervals of k^2. A similar table is being published (9/30/47) by the Smithsonian Institution in which the tabulation is at equidistant intervals of $\alpha = \arcsin k$.

Status: INACTIVE. Computations 50% completed.

Project: 43D2-5 *Priority: 3* *Date Auth.* 7/1/47

Title: Tables of Bessel Functions $Y_0(z)$ and $Y_1(z)$

Origin: NBS

Project Manager: Mr. A. Hillman

Objective: To prepare tables of the Bessel functions of the second kind, $Y_0(z)$ and $Y_1(z)$, for $z = \rho e^{i\theta}$, $\rho = 0(.01)10$; $\theta = 0°(5°)90°$, 10D.

Background: Bessel functions of order zero and one arise in the theory of potential, heat conduction, and wave motion, for a domain bounded by a circle or circular cylinder. They occur also in the propagation of electromagnetic waves with a straight wire as a guide, the theory of the skin effect for poorly conducting wires and many other boundary value problems. The tabulation of these functions was originally suggested by Dr. S. Schelkunoff, Bell Telephone Laboratories.

Magnitude: Class III

Date of Termination: 6/30/48

Status: UNDER WAY (CONTINUATION). Final manuscript prepared; checking about 40% completed; introduction in process of preparation.

Publication: To be submitted to the Columbia University Press.

Project: 43D2-6 *Priority: 3* *Date Auth.* 7/1/47

Title: Table of Sines and Cosines to Hundredths of a Degree

Origin: NBS

Project Managers: Messrs. M. Abramowitz, J. Laderman, H. E. Salzer

Objective: To prepare a fifteen-place table of sines and cosines at intervals of .01 of a degree. Second central differences are also to be tabulated.

Background: A fifteen-place table of trigonometric functions at an interval of one hundredth of a degree was computed in 1633 by H. Briggs and H. Gellibrand under the title "Trigonometria Brittanica." This table is very scarce. For this reason and in order to meet the frequent demands for a very accurate table of trigonometric functions with decimal subdivision of the degree, the Mathematical Tables Committee of the British Association for the Advancement of Science has suggested the preparation of a fifteen-place table of all the six trigonometric functions at intervals of one thousandth of a degree, A first phase of this program is the preparation of a fifteen-place table of sines and cosines at intervals of one hundredth of a degree.

Magnitude: Class II

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.

Publication: IN MANUSCRIPT. NBS Applied Mathematics Series.

Project: 45D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Mathieu Functions II

Origin: Applied Mathematics Panel, NDRC

Project Managers: Dr. G. Blanch, Mr. M. Abramowitz

Objective: To prepare a table of the periodic solutions:

$$Se_r(s,t) = \sum_{n=0}^{\infty} De_{2n+p} \cos(2n+p)\,t, \quad (p = 0, 1)$$

$$So_r(s,t) = \sum_{n=1}^{\infty} Do_{2n-p} \sin(2n-p)\,t, \quad (p = 0, 1)$$

for $r = 0(1)\,15$, $t = 0^\circ(1^\circ)\,90^\circ$ over the range $s = 0$ to $s = 100$, of the Mathieu differential equation:

$$y'' + (b - s\cos^2 t)\,y = 0.$$

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu Functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press 1947). The existing tables by Ince and Goldstein are inadequate. The project was originally proposed by Dr. Philip Morse, now Director of the Brookhaven National Laboratory, and Dr. James Wakelin, formerly of the Office of Naval Research.

Magnitude: Class IV

Date of Termination: 12/31/48

Comments: Related to Project 46D2-2

Status: UNDER WAY (CONTINUATION). Computations about 40% completed.

Publication: To be published by the Columbia University Press.

Project: 46D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Gamma Functions for Complex Arguments

Origin: NBS

Project Manager: Mr H. E. Salzer

Objective:

a) To prepare a table of $\log_e \Gamma(z)$, $z = x + iy$ for $x = 0(.1)\,10$, $y = 0(.1)\,10$, 10D to 12D

b) A table of $\Gamma(z)$ for same arguments as (a).

c) A table of $1/\Gamma(z)$ (near the origin) for $x = 0(.01)\,p$, $y = 0(.01)\,q$ where p and q will be at least equal to unity and probably somewhat larger.

Background: Gamma functions for complex arguments occur in numerous physical problems such as the attraction between two particles in a Coulomb field of force. The existing tables are entirely inadequate for the needs of modern nuclear research problems. Originally suggested by Dr. R. D. Evans of the Massachusetts Institute of Technology.

Magnitude: Class IV

Date of Termination: a) 3/30/48, b) 7/30/48, c) 10/30/48

Status: UNDER WAY (CONTINUATION). Computation of $\log\Gamma(x + iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ about 80% completed.

Publication: To be published by the Columbia University Press.

Project: 46D2-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Mathieu Functions I

Origin: Applied Mathematics Panel, NDRC

Project Manager: Dr. G. Blanch

Objective: An eight-place table of the first 15 odd and 16 even characteristic values b of Mathieu's differential equation

$$y'' + (b - s\cos^2 t)y = 0$$

for s ranging from 0 to 100 at various intervals, and the Fourier coefficients of the solutions corresponding to these characteristic values as well as certain related functions.

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press, 1947). The existing tables by Stratton-Morse-Chu-Hutner, Ince and Goldstein are inadequate. The project was proposed by Dr. Philip Morse, now Director of the Brookhaven National Laboratory, and Dr. James Wakelin, formerly of the Office of Naval Research.

Magnitude: Class III

Date of Termination: 12/31/47

Comments: Related to Project 45D2-1

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.

Publication: To be published by the Columbia University Press.

Project: 47D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Spheroidal Wave Functions

Origin: NBS

Project Manager: Mr. M. Abramowitz

Objective: a) To prepare tables of the characteristic values for the differential equation of orders $L = 0, 1, 2, \ldots, 10$

$$(1-x^2)w'' - (2m+1)xw' + (b-c^2x^2)w = 0$$

for $m = 0(1)10$ and c^2 ranging from 0 to about 1000 at various intervals.

b) Tables of the solutions of the differential equation corresponding to the characteristic values under (a).

Background: Spheroidal wave functions are the solutions of the wave equation in prolate and oblate spheroidal coordinates. In his introduction to the "Elliptic

Cylinder and Spheroidal Wave Functions," Professor Morse states "Solutions of problems involving the radiation and scattering of waves from strips of material, from wires of finite length and from discs of material, all require the knowledge of the mathematical properties and the numerical values of solutions of the wave equation for these coordinate systems. The solutions are likewise required for the study of the diffraction of waves through slits and circular openings, the absorption of sound by strips or by circular patches of material and the behavior of electrons in diatomic molecules." Originally proposed by Dr. Philip Morse of the Massachusetts Institute of Technology (now Director, Brookhaven National Laboratory.)

Magnitude: Class IV

Status: UNDER WAY (CONTINUATION). Computations completed for:

$m = 1,\ L = 0$ c^2 ranging from 0 to 100 at various intervals

$m = 1,\ L = 1$ c^2 ranging from 0 to 600 at various intervals

$m = 1,\ L = 2,3,4$ c^2 ranging from 0 to 60 at various intervals

Exploratory work has been performed for other values of m and L.

Publication: Tables to be prepared at the Government Printing Office.

Project: 4702-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Coulomb Wave Functions

Origin: NBS

Project Manager: Mr. M. Abramowitz

Objective: Tabulation of the regular solution $F_L(\rho,\eta) = C_L \rho^{L+1} \varphi_L(\rho,\eta)$ and its derivative $F'_L(\rho,\eta) = C_L \rho^L \varphi^*_L(\rho,\eta)$ and of the irregular solution $G_L(\rho,\eta)$ of the differential equation $y'' + \{1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\} y = 0$ where $C_L^2 = \frac{2\pi\eta\,(1+\eta^2)\,(4+\eta^2)\ldots(L^2+\eta^2)\,2^{2L}}{(e^{2\pi\eta}-1)\,(2L+1)^2[(2L)!]^2}$

Background: This equation arises in the quantum mechanical treatment of two particles moving in a Coulomb field of force; it arises in particular in the problems of proton-proton and proton-neutron interaction. The special case L = 0 occurs in a problem in classical hydrodynamics. Proposed by Professors Philip Morse of MIT, Gregory Breit of Yale University, and Herman Feshbach of MIT.

Magnitude: Class IV

Date of Termination: 12/31/49

Status: UNDER WAY (CONTINUATION). Computations completed:

Values of the normalization factor, C_0 for η ranging from 0 to 10 at various intervals. Values of $\rho\varphi_0$ for $\eta = 0(.1)10$; $\rho = 0(.05)1.5$; values of $\log\rho\varphi_0$ and $\varphi_0^*/\rho\varphi_0$ and their derivatives with respect to η for $\eta = 4$ with $\rho = 1(.05)2(.1)8$ and $\eta = 6, 8, 10$, with $\rho = 1(.05)2(.1)10$.

Computations partially completed:

Values of φ_0^* for $\eta = 0(.1)10$, $\rho = 0(.05)1.5$, about 80% completed; values of $\rho\varphi_0$ and φ_0^* and their derivatives with respect to η for $\eta = .5, 1.5, 2.5$ with $\rho = 1.5(.1)10$ and $\eta = 4$ with $\rho = 8(.1)10$ about 20% completed.

Computations (related to G_L) completed:

Values of $\sigma_0 = \arg\Gamma(1+i\eta)$ and $\Gamma'(-i\eta)/\Gamma(-i\eta)$ for η ranging from 0 to 110 at various intervals.

Publication: Tables to be prepared at the Government Printing Office.

Project: 4702-3 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Antilogarithms

Origin: NBS

Project Managers: Mr. M. Abramowitz, Mr. H. E. Salzer

Objective: To prepare a table of 10^x to ten decimal places for $x = 0(.00001)1$.

Background: The function 10^x is of basic importance. The best existing table is that of J. Dodson, Antilogarithmic Canon, London, 1742, which is out of print. The proposed publication, which was suggested by Mr. H. E. Salzer of the Computation Laboratory, will be an improvement over Dodson's table from the standpoint of accuracy and format.

Magnitude: Class III

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Final manuscript prepared and about 25% checked by differencing.

Publication: IN MANUSCRIPT To be published by Columbia University Press.

Project: 4702-4 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables for the Occasional Computer.

Origin: NBS

Project Manager: Entire technical staff.

Objective: To prepare an improved and amplified version of the Jahnke-Emde tables.

Background: The preparation of an improved version of the Jahnke-Emde tables had been originally suggested by Professor E. P. Wigner of Princeton University who submitted suggestions for the contents of the contemplated volume. This matter was discussed with Professor Tukey of Princeton, Professor Barkley J. Rosser of Cornell University and others.

Magnitude: Class IV.

Status: INACTIVE. Further suggestions have been submitted by Mr. John Todd.

Project: 4802-1 *Priority:* 3 *Date Auth.* 12/26/47

Title: Tables for X-Ray Diffraction Analysis

Origin: Section 9.7 NBS

Project Manager: Mr. M. Abramowitz

Objective: To prepare tables of $\frac{\lambda}{2}\csc\theta$ for $\theta = 0^\circ(.01)90^\circ$ for various values of λ.

Background: These tables are based on new experimental values of the wave lengths. They will supersede previously published tables using incorrect wave lengths.

Specifically requested by Mr. H. E. Swanson.

Magnitude: Class II
Date of Termination: 6/30/48
Status: NEW. Computational procedure being planned.
Publication: Tables to be published by Government Printing Office.

Project: 46S2-1 *Priority:* 2B *Date Auth.* 7/1/47

Title: Heat Conduction Equation

Origin: Bureau of Ordnance, Navy Department

Project Manager: Mrs. I. Rhodes

Objective: To obtain numerical solutions of the non-linear partial differential equation $\frac{\partial\theta}{\partial t} = k\frac{\partial^2\theta}{\partial x^2} + e^{-1/\theta}$ (k = constant) satisfying the initial and boundary conditions: $\theta(x,0) = \theta_0$; $\theta(0,t) = \theta_1$, for various values of θ_0 and θ_1.

Background: The problem had its origin in the investigation of the flow of heat from a hot thermostatic bath into an explosive substance in the form of a sphere immersed in the bath. For practical applications it is permissible to consider the explosive as a plane slab of infinite extent.

Magnitude: Class IV
Date of Termination: 12/31/47
Status: UNDER WAY (CONTINUATION). Computations completed for following cases: $\theta_0 = .0600$, $\theta_1 = .1000$; $\theta_0 = .0600$, $\theta_1 = .141003$; $\theta_0 = .0600$, $\theta_1 = .2390$. Preliminary computations carried out for other values of θ_0 and θ_1.
Publication: Report to be submitted to Bureau of Ordnance for photo-offset reproduction.

Project: 47S2-1 *Priority:* 2C *Date Auth.* 7/1/47

Title: Shock Wave Computations

Origin: Bureau of Ordnance, Navy Department

Project Manager: Mr. William Horenstein

Objective: The determination of a number of shock wave parameters corresponding to $\gamma = c_p/c_v = 1.1$ and 5/3 and the representation of the results in graphical form.

Background: These computations were originally requested by the Explosive Section of the Bureau of Ordnance, and are now sponsored by Dr. Raymond Seeger of the Naval Ordnance Laboratory. Earlier computations of a similar nature were used in the preparation of Explosives Research Report No. 13, "Regular Reflection of Shocks in Ideal Gases," and Explosives Research Report No. 14, "Interaction of Shock Waves in Water-Like Substances," as well as in checking experiments on three-shock solutions in air which were carried on at the Princeton Station of Division 2, NDRC.

Magnitude: Class IV
Date of Termination: 12/31/47
Status: UNDER WAY (CONTINUATION).
Computations corresponding to $\gamma = 1.1$ completed.
Computations corresponding to $\gamma = 5/3$ approximately 30% completed.
Publication: Manuscript to be submitted to Bureau of Ordnance, Navy Department.

Project: 47S2-2 *Priority:* 2C *Date Auth.* 7/1/47

Title: Computations for Meteorological Project, N. Y. U.

Origin: Office of Naval Research

Project Manager: Mr. J. Laderman

Objective: To perform computations required in objective analyses of meteorological elements. These computations include the determination of least squares solutions for divergence, wind velocity, barometric pressure, etc., based on data collected from weather stations throughout the eastern part of the U. S. Also the determination of large scale eddy stresses, stream lines, etc.

Background: The results obtained in addition to being of current interest in general circulation investigations are expected to establish the feasibility of constructing a machine to carry out these calculations. These computations were requested by Dr. H. Panofsky of New York University and Dr. J. Von Neumann of the Institute for Advanced Study.

Magnitude: Class III
Date of Termination: 12/31/48
Status: UNDER WAY (CONTINUATION) New problems are being processed as prior ones are completed.
Publication: Manuscript submitted to Dr. H. Panofsky of Meteorological Project, New York University.

Project: 48S2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: The "τ" Integral

Origin: Section 6.1, NBS

Project Manager: Mr. H. E. Salzer

Objective: To compute the value of

$$\tau = \frac{\mu}{x^2} \int_0^{x\sqrt{2/\mu}} \frac{w\,dw}{1 + w^2\left[\cos \mu w - \frac{w}{2} \sin \mu w\right]^2}$$

for $x = .2, .5, 1.0, 2.0, 5.0, 10, 20, 50, 100$

and $\mu = .001, .002, .004, .006, .008, .01, .02, .04, .06, .08, .1$

Background: The integral arises in the theory of the transmission of sound through a double partition with an enclosed air space. Requested by Dr. Richard K. Cook of the National Bureau of Standards.

Magnitude: Class III
Date of Termination: 9/30/47
Comments: The work was facilitated by an asymptotic expansion derived by Dr. G. Blanch with the assistance of Miss I. Stegun.
Status: COMPLETED.
Publication: Manuscript submitted to Sound Section, NBS.

Project: 48S2-2 *Priority: 2B* *Date Auth. 7/1/47*

Title: Problem in the Theory of Atomic Spectra I.
Origin: Section 4.1 NBS
Project Manager: Mr. A. Hillman
Objective: For each of nine given matrices (b_{kj})

a) To compute the solution of the secular equation for the roots $A^{(1)}$, $A^{(2)},\ldots,A^{(n)}$

b) For each root $A^{(i)}$ to compute the solution of the system of homogeneous equations $\sum_{j=1}^{n} b_{kj}\, y_j^{(i)} = A^{(i)}\, y_k^{(i)}$ subject to the normalization condition $\sum_{k=1}^{n} [y_k^{(i)}]^2 = 1$

c) To compute $g^{(i)} = \sum_{k=1}^{n} g_k [y_k^{(i)}]^2$ for given g_k.

Background: The matrices and equations arise in the theory of atomic spectra. The computations were requested specifically by Dr. George Shortley.
Magnitude: Class III
Date of Termination: 9/30/47
Status: COMPLETED.
Publication: Manuscript submitted to Dr. Shortley.

Project: 48S2-3 *Priority: 1C* *Date Auth. 7/1/47*

Title: Human Centrifuge Calculations
Origin: Special Devices Branch, Office of Naval Research
Project Manager: Mr. A. Hillman
Objective: To determine 14 curves w(t) satisfying the following differential inequalities:

$$|w| \leq 2.46; \qquad |.0426\ddot{w} + .808\dot{w} + w| \leq 6.35;$$

$$|.00852\dddot{w} + .1616\ddot{w} + 1.008\dot{w} + w| \leq 15.96;$$

$$|.000468\ddddot{w} + .01741\dddot{w} + .217\ddot{w} + 1.063\dot{w} + w| \leq 44.9.$$

w(t) must also satisfy certain prescribed boundary conditions, and minimize a given expression.

Background: The equation arises in a study of the effect of centrifugal force on the human body. The mathematical formulation of the problem is due to Professor F. J. Murray of Columbia University.

Magnitude: Class III

Date of Termination: 11/3/47

Status: COMPLETED.

Publication: Report to be submitted to Special Devices Branch.

Project: 48S2-4 *Priority:* 2B *Date Auth.* 8/4/47

Title: Calculation of Mutual Impedances, Part I

Origin: Engineer Research and Development Laboratory
Engineer Center, Ft. Belvoir, Va.

Project Manager: Mr. H. E. Salzer

Objective: To prepare a table of several thousand values of mutual impedances of metal detector head coils. These values were obtained from bridge arm data by means of the following formula:

$$R_n + jX_n = \frac{(1/R_B) + jwC_B}{(1/R_A) + jwC_A}$$

where $j = \sqrt{-1}$, $w = 2000\pi$, $R_B = 991.036\ R_{AR}$, $C_B = .001009\ C_{AR}$ and R_A, C_A, R_{AR}, C_{AR}, are given data.

Background: These impedance calculations are needed in an investigation of the characteristics of metal detector heads under way at the Engineer Research and Development Laboratory. The specific request was made by Mr. Chandler Stewart, Jr., of that laboratory.

Magnitude: Class III

Date of Termination: 10/24/47

Status: COMPLETED.

Publication: Manuscript submitted to Chandler Stewart, Jr.

Project: 48S2-5 *Priority:* 3 *Date Auth.* 7/1/47

Title: Computation of Lattice Sums

Origin: Section 7.7 NBS

Project Manager: Mr. J. Laderman

Objective: To compute the sum of the quantities

$$\frac{w}{K} = \frac{3(a^2 - b^2 + c^2)(a^2 + b^2 - c^2)(-a^2 + b^2 + c^2) + 8a^2 b^2 c^2}{8(a^2 b^2 c^2)^{5/2}}$$

over the face centered cubic and hexagonal lattices, where a^2, b^2, c^2, are given functions of n_1, n_2, n_3, and m_1, m_2, m_3, with n_1 and m_j taking on all possible integral values satisfying certain inequalities. A total of about 21000 terms (values of $\frac{w}{K}$) have to be computed and summed.

Background: The purpose of this project is to obtain the difference in the lattice sums of the third order Van der Waals interaction for the two closest packed lattices, the hexagonal and the face centered cubic. The problem was proposed by B. M. Axilrod; approved by Dr. Teller, University of Chicago, and Dr. J. Weyl, Office of Naval Research.

Magnitude: Class III

Date of Termination: 1/31/48

Status: UNDER WAY (CONTINUATION). Computations about 20% completed.

Publication: To be submitted to Mr. Axilrod of NBS.

Project: 48S2-6 *Priority:* 1B *Date Auth.* 9/1/47

Title: Air Forces Problem

Origin: Office Air Comptroller, War Department

Project Manager: Mr. J. Laderman

Objective: To solve the following system of linear equations (written in matrix notation) for $A^{(18)}$.

$$\bar{\alpha}_A{}^{(18)} + \bar{U}^{(18)} = \alpha_A{}^{(19)} + U^{(19)}; \qquad \rho\alpha_A{}^{(18)} + \rho U^{(18)} = \rho\bar{U}^{(17)}$$

$$\rho^{(1)}\alpha_A{}^{(18)} + \rho^{(1)}U^{(18)} = \rho^{(1)}\bar{U}^{(17)} - \sigma^{(1)}\rho(\bar{U}^{(16)} - U^{(17)})$$

$$\rho^{(2)}\alpha_A{}^{(18)} + \rho^{(2)}U^{(18)} = \rho^{(2)}\bar{U}^{(17)} - (\sigma^{(2)}\rho + \tau^{(2)}\rho^{(1)})(\bar{U}^{(16)} - U^{(17)}) + \tau^{(2)}\sigma^{(1)}\rho(\bar{U}^{(15)} - U^{(16)})$$

where all matrices in above equations are given except $A^{(18)}$. After $A^{(18)}$ has been found, it is required to find $A^{(17)}$ from a similar system obtained by reducing superscripts on $A^{(t)}$, $U^{(t)}$, and $\bar{U}^{(t)}$ by 1. Then similarly to solve in order for $A^{(16)}$, $A^{(15)}$,..., $A^{(1)}$.

Background: The solutions of these systems of linear equations were specifically requested by Dr. G. Danzig and Mr. Marshall Wood of the Air Comptroller's Office. The problem was proposed as a test problem to evaluate certain general techniques for solving linear systems involving perhaps as many as 500 unknowns being developed by personnel of the Air Comptroller's Office, with mathematical assistance of the Bureau. The solution of such large linear systems is required in the determination of Air Force Combat programs, and is the source of the need of the Air Comptroller's Office for an automatic sequenced electronic digital computing machine.

Magnitude: Class III

Date of Termination: 12/15/47

Comments: This is a test problem originating in the mathematical work involved in Project 47D4-3.

Status: COMPLETED.

Publication: To be submitted to Office Air Comptroller War Department

Project 48S2-7 *Priority:* 2B *Date Auth.* 8/28/47

Title: Problem in the Theory of Atomic Spectra II

Origin: Section 4.1, NBS

Project Manager: Mr. A. Hillman

Objective: Tabulation of the radial integrals F^k and G^k for specified values of n, L, n', L', where

$$F^k(n,L,n',L') = \int_0^\infty r_1^k R_1^2(n,L)\, dr_1 \int_{r_1}^\infty \frac{R_2^2(n',L')}{r_2^{k+1}}\, dr_2$$

$$+ \int_0^\infty \frac{R_1^2(n,L)}{r_1^{k+1}}\, dr_1 \int_0^{r_1} r_2^k\, R_2^2(n',L')\, dr_2$$

$$G^k(n,L,n',L') = 2\int_0^\infty r_1^k R_1(n,L)\, R_1(n',L')\, dr_1 \int_{r_1}^\infty \frac{R_2(n,L)\, R_2(n',L')}{r_2^{k+1}} dr_2$$

where

$$R_1(n,L) = \sqrt{\frac{(n-L-1)!}{n^2[(n+L)!]^3}}\, e^{-r_1/n} \left(\frac{2r_1}{n}\right)^{L+1} L_{n+L}^{2L+1}\left(\frac{2r_1}{n}\right) \text{ and}$$

L_{n+L}^{2L+1} is the associated Laguerre Polynomial.

Background: The integrals arise in the theory of atomic spectra. Proposed by Dr. George Shortley.

Magnitude: Class IV

Status: INACTIVE. Negotiations are under way with the proposer to reduce the scope of the project.

Publication: Manuscript to be submitted to Dr. Shortley.

Project: 48S2-8 *Priority:* 2B *Date* 7/18/47

Title: Computation of the function E(u,q)

Origin: Naval Research Laboratory

Project Manager: Mr. H. E. Salzer

Objective: Computation of the function

$$E(u,q) = \left| \int_{t_0}^{t_1} R^{\frac{1}{2}} I\, e^{-135.33 j R [1+\cos(t-u)]} dt \right|^2$$

where

$$R = \csc^2 t[2\sqrt{1+a \sin t \cos(t-q)} - a \sin t \cos q - 2\cos t]$$

$$a = 2\cos q - \sin q; \quad \tan t_0 = -a/2; \quad t_1 = t_0 + 90^\circ; \quad j = \sqrt{-1}$$

$$I = e^{-2.613(t-t_0-45^\circ)^2}$$

E(u,q) is to be tabulated for $q = 35^\circ(3^\circ)80^\circ$ and for a 4° range of u centered near the root of $\frac{d}{dt}\{R[1+\cos(t-u)]\} = 0$ for $t = t_0 + 45^\circ$ and for 20 values of u and 16 values of q.

Background: The integral arose in the course of an antenna theory investigation.

Magnitude: Class IV

Date of Termination: 6/30/48

Status: UNDER WAY (CONTINUATION). Location of the required arguments, u, order of magnitude of desired functions, and required interval of integration, determined from preliminary computations. The present formulation of this rather special problem would entail such expensive calculations that negotiations have been initiated with the proposers to reconsider the scope of the project.

Publication: Manuscript to be submitted to Naval Research Laboratory.

Project: 48S2-9 *Priority:* 3 *Date Auth.* 8/6/47

Title: Guided Missile Computations

Origin: Section 13.7 NBS

Project Manager: Dr. G. Blanch

Objective: Tabulation and graphs of Z, $\frac{dZ}{du}$, $\frac{d^2Z}{du^2}$, where

$$\frac{d^3Z}{du^3} - \frac{1}{f}\frac{d^2Z}{du^2} + \frac{1}{u}\frac{dZ}{du} - \frac{Z}{u^2} = 0$$

for f = 1, 2, 4, ∞, and various initial conditions.

Background: The differential equation arose in the theory of the trajectory of a homing guided missile. Requested by Dr. H. K. Skramstad of the National Bureau of Standards.

Magnitude: Class III

Date of Termination: 2/28/48

Status: UNDER WAY (CONTINUATION). 58 of 60 required tables completed.

Publication: Manuscript to be submitted to Section 13.7 NBS.

Project: 48S2-10 *Priority:* 2B *Date Auth.* 8/5/47

Title: Subsonic Compressible Flow Calculations

Origin: Office of Naval Research

Project Manager: Mr. W. Horenstein

Objective: To find particular solutions of the differential equation:

$$\frac{1-M^2}{\rho^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{w}{\rho}\frac{\partial}{\partial w}\left(\frac{w}{\rho}\frac{\partial\psi}{\partial w}\right) = 0,$$

for the stream function, ψ, and the analogous differential equation for the velocity potential, φ, suitable for the calculation of vortex patterns, where w and θ are hodograph coordinates, $M = \frac{w}{a}$ = local Mach number, a = acoustic velocity, and ρ = density of the fluid.

Background: The differential equation arose in connection with a problem of turbine design under study at the General Electric Company, Schenectady, N. Y.

The mathematical formulation of the problem is due to Professor Bergman and Mr. Hans Kraft of the Turbine Generator Engineering Division of the GE Company. The equations are of general interest in that their solution will furnish a basis for a direct numerical construction of any practically occurring subsonic compressible flow whose description in the hodograph plane is one-valued.

Magnitude: Class II

Status: UNDER WAY (CONTINUATION). Exploratory work in progress. A new method has been developed for solving the equivalent partial differential equation

$$\frac{\partial^2 \psi^*}{\partial Z \partial \bar{Z}} + F\psi^* = 0.$$

Publication: Manuscript to be submitted to ONR.

Project: 48S2-11 *Priority:* 2B *Date Auth.* 10/1/47

Title: Properties of Magnetic Electron Lenses

Origin: Section 4.2b, NBS

Project Manager: Mr. M. Abramowitz

Objective: To calculate tables of the values of the function

$$F = \frac{d}{\sqrt{\mu(2^{1/\mu} - 1)}} \cdot \frac{1}{\sin\{\pi/\sqrt{1 + A(\mu, d, V, H)}\}}$$

for various given values of d, μ, V, H.

Background: The experimental data for magnetic electron lenses is not completely covered by theory as yet. The present calculations will define theoretical characteristics of magnetic electron lenses which will later be compared with experimental values with the aim of obtaining an improved theory. The computations were requested by Dr. L. L. Marton of the National Bureau of Standards.

Magnitude: Class II.

Date of Termination: 11/31/47

Status: COMPLETED.

Publications: Report submitted to Section 4.2b, NBS.

Project: 48S2-12 *Priority:* 2C *Date Auth.* 9/19/47

Title: Loran Log Project

Origin: Section 14.1, NBS

Project Managers: Mr. J. Laderman, Mrs. I. Rhodes

Objective: To analyze observations on the "Musk-Calf" low frequency Loran system. The observations are to be recorded on punched cards, gross errors are to be detected and various statistical analyses are thereafter to be performed.

Background: The analysis was undertaken primarily to ascertain the cause of the discrepancies in the determination of locations by means of the low frequency Loran system of navigation. The results will also be of value in studies of the ionosphere and on anomalous propagation phenomena.

Magnitude: Class III
Date of Termination: 12/31/48
Status: UNDER WAY (NEW).

Project: 48S2-13 *Priority: 2B* *Date Auth. 11/13/47*

Title: Electron Ejection Problem
Origin: Clinton National Laboratories, Atomic Energy Commission
Project Manager: Mr. A. Hillman
Objective: Tables of the internal conversion coefficient $\beta = \frac{2\pi\alpha kI}{(I+1)(2I+1)}S$ where $\alpha = 1/137.03$ = fine structure, $I = 1(1)5$, and S is a complicated expression involving Gamma functions for complex arguments and hypergeometric functions for complex values of the parameters and the argument. All parameters and arguments are functions of the atomic number Z in the range from 20 to 90 and the energy of radiation k ranging from 0 to 5.
Background: The above calculations arise in the problem of electron ejection from atomic shells by nuclear gamma rays, and are expected to contribute considerably to the understanding of nuclear structure with particular emphasis on the determination of nuclear energy levels.
Magnitude: Class IV
Date of Termination: 12/31/48
Comments: Exact calculations have been done heretofore for one nucleus only (Z = 84) whereas present day experiments require a knowledge of internal conversion coefficients throughout the periodic tables.
Status: UNDER WAY (CONTINUATION) Computational procedure prepared. Computations postponed pending final decision by Clinton National Laboratories as to the scope of the problem.
Publication: Report to be submitted to Clinton National Laboratories, Atomic Energy Commission.

Project: 48S2-14 *Priority: 2A* *Date Auth. 12/9/47*

Title: L. F. Loran Stations
Origin: U. S. Navy Hydrographic Office
Project Manager: Mr. M. Abramowitz
Objective: Preparation of tables giving coordinates of hyperbolic lines of positions for three stations.
Background: Low Frequency or L. F. Loran Navigation Tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.
Magnitude: Class III
Date of Termination: 2/1/48
Status: UNDER WAY (NEW). Computations about 15% completed.
Publication: Tables to be published by U. S. Navy Hydrographic Office.

Section 3. The Statistical Engineering Laboratory

Project: 47R3-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: The Arithmetic Mean and the Median as Estimators of Location Parameters of Probability Distributions.

Origin: Section 11.3, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: To evaluate percentiles and other features of the distributions of the arithmetic mean, the median, and other estimators of parameters of location, in random samples from normal (Gaussian), Cauchy, Laplace (double-exponential), rectangular, sech, sech^2 (derivative of the "logistic"), and "contaminated normal", populations.

Background: This project stems from a study, undertaken in connection with another Project of the S series, of various procedures that have been advocated, over the years, for rejecting, or giving less weight to, anomalous or extreme observations. The motivating question was as follows: If the practice of reporting the medians of sets of measurements, instead of their arithmetic means, is adopted as a way of reducing the effect of occasional anomalous observations (due, perhaps, to faulty measurement, but possibly to more chance fluctuations) on the reported "averages", then what losses, if any, in accuracy and precision are to be expected when the measuring process is actually in control.

Previous studies of the relative merits of the arithmetic mean and the median have, almost without exception, concentrated on comparing the moments of their distributions in large samples, since in large samples the distributions of both the mean and the median tend to normality. Comparison of the arithmetic mean and the median in small samples (e.g., of 3, 5 or 7 observations) has been virtually neglected. The present approach via percentiles promises to give valuable new information.

Magnitude: Class II

Status: INACTIVE. About 70% completed.

Dittoed copies have been prepared of tables giving to 3D (i.e., to 3 decimal places) the .005, .01, .025, .05, .10, .25, and .50 probability points of the exact distributions of the median in random samples of N = 3(2)9, 15(10)95 independent observations from (a) Gaussian (normal), (b) Cauchy, and (c) Laplace distributions, and of a table giving to 3D the above probability points of the exact distributions of the mid-range in random samples of the same sizes from the rectangular distribution. These tables may be obtained from the Statistical Engineering Laboratory until the supply is exhausted.

Manuscript copies have also been prepared of tables giving the aforementioned probability points of the exact distributions of the median in random samples of the aforementioned sizes from (d) rectangular, (e) sech, and (f) sech^2 (derivative of the "logistic") distributions; also, the aforementioned probability points of the exact distributions of the arithmetic mean in random samples of N = 3(2)9, 15(10)95 from a normal distribution, in samples of N = 2(1)10 from a rectangular distribution, and in samples of N = 3, 5(1)11 from a Laplace distribution.

A paper entitled "Formulae for the Percentage Points of the Distributions of the Arithmetic Mean in Random Samples from Certain Symmetrical Universes" prepared by Uttam Chand, guest worker from India, may be consulted IN MANUSCRIPT at the Statistical Engineering Laboratory. The symmetrical distributions considered in detail in this paper are the rectangular, Laplace, sech, and $sech^2$ distributions.

Project: 47R3-2 *Priority: 3* *Date Auth. 7/1/47*

Title: The Mean Deviation, Standard Deviation, and Range as Estimators of Scale Parameters (Measures of Dispersion) of Probability Distributions.

Origin: Section 11.3, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: To evaluate percentiles and other features of the distributions of these estimators in small random samples from normal (Gaussian) and various non-normal (see Project 47R3-1) populations.

Background: Previous studies of the relative merits of the mean deviation, standard deviation, and range as estimators of scale parameters of probability distributions have, in the main, concentrated on (a) evaluation of adjustment factors for rendering them unbiased estimators of, say, the standard deviation of the population; and (b), comparison of their "efficiencies" (as measured by the ratios of their sampling variances when so adjusted). Since their distributions in small samples are non-normal and generally differ in form, comparisons of their "efficiencies" in small samples may not truly represent their relative merits with regard to accuracy and precision in such cases. The approach via percentiles and other features (e.g., the probability of underestimating the true value of the relevant scale parameter) is expected to yield important new information.

Magnitude: Class II

Status: INACTIVE. About 10% completed.

Tables have been prepared that show for sample sizes N = 2, 4, 5, 10, 20, and 50 the probabilities of under-estimating the true population standard deviation when various commonly-used multiples of the sample mean deviation, sample standard deviation and sample range are employed as estimators of the standard deviation of a normal population.

Project: 47R3-3 *Priority: 3* *Date Auth. 7/1/47*

Title: Statistical Tests of Significance for 2 x 2 Tables When the Number of Observations is Small.

Origin: Program Committee, Institute of Mathematical Statistics.

Project Manager: Dr. Churchill Eisenhart

Objective: To compare Fisher's "exact" test, Barnard's "C.S.M." test, and certain other statistical tests for data arranged in 2 x 2 tables with respect to (a) scope, i.e., conditions for which the respective tests are valid, and (b) operating characteristics (i.e., bias, power, etc.) under the conditions for which they are jointly valid.

Background: The project was undertaken in connection with an invited address given at the Symposium on 2 x 2 Tables sponsored by the Institute of Mathematical Statistics at the New Haven, Connecticut meeting on September 2, 1947.

Magnitude: Class II

Date of Termination: June 30, 1948

Status: INACTIVE. About 60% completed.

Tables and graphs have been prepared showing the operating characteristics of the "exact" and "C.S.M." tests when used as tests of the equality of the parameters of two binomial distributions from which samples of sizes n_1 and n_2, respectively, are drawn independently at random. The ".05" level of significance (in one case, an .0607 level) was adopted, and the tables and graphs were constructed for the cases of $n_1 = n_2 = 3$; $n_1 = 4$, $n_2 = 7$; and $n_1 = n_2 = 7$. A three-dimensional cardboard model of the "power surface" was constructed showing for the case of $n_1 = 4$, $n_2 = 7$, the greater power of the C.S.M. test relative to the exact test when both are used as tests of the equality of the parameters of two binomial distributions - the type of problem for which the C.S.M. test was expressly developed.

Publication: DITTOED copies of a synopsis of Dr. Eisenhart's New Haven address, including an annotated bibliography and the tables and graphs mentioned above can be obtained from the Statistical Engineering Laboratory while the supply lasts. The three-dimensional model showing the power surfaces of the exact and the C.S.M. tests for the case discussed above is available for examination in the Statistical Engineering Laboratory.

Project: 4703-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Power Function of Analysis-of-variance Tests, Requirements for New Tables of.

Origin: Section 11.3, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To formulate requirements for a set of new tables of the integral

and percentage points of Tang's distribution

$$p(u) = \sum_{i=0}^{\infty} \frac{\lambda^i e^{-\lambda}}{i!\, B\left(\frac{f_1+2i}{2}, \frac{f_2}{2}\right)} \left(\frac{u}{1+u}\right)^{\frac{f_1+2(i-1)}{2}} \left(\frac{1}{1+u}\right)^{\frac{f_2+2}{2}}, \quad 0 \leq u \leq \infty,$$

the integral of which furnishes the power function (or, operating characteristics) of analysis-of-variance procedures for making decisions with regard to the presence or absence of fixed (constant) relations of specified form among the means of sub-sets of a statistical population.

Background: During the fall of 1946, Professors Jerzy Neyman (University of California, Berkeley) and Abraham Wald (Columbia University, New York City) discussed with Dr. A. N. Lowan, Chief, The Computation Laboratory (Section 11.2), the possibility of having The Computation Laboratory prepare a set of new tables of the integral and/or percentage points of "the power function of the analysis-of-variance tests". By joint letter dated 31 October 1946, they submitted a specific request to Dr. Lowan. This letter was subsequently referred to the Statistical Engineering Laboratory (Section 11.3) for consideration in the light of the broad, over-all tabulation needs of mathematical statistics.

Magnitude: Class I

Date of Termination: 3rd. Quarter, Fiscal 1948 (Estimated)

Comments: The above estimated date of termination refers to probable date of completion of a final formulation of the requirements of the proposed tables, not to the probable date of completion of the computation of the tables, if authorized as a project of the Computation Laboratory.

Status: INACTIVE. About 95% completed.

Comments of various mathematical and applied statisticians on the tentative final formulation (by C. Eisenhart, J. Laderman, A. N. Lowan and A. Wald, with advice from J. Neyman and J. W. Tukey) not yet collated. A final recommendation, prepared in the light of these comments, will be submitted to the Division in the near future for consideration as a computation project.

Project: 4703-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Formulas for Operating Characteristics and Sample Sizes for Certain Statistical Tests.

Origin: Section 11.3, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To provide a useful collection of formulas for the operating characteristics and the number of observations needed for certain single-sample one-sided tests of statistical hypotheses, with instructions for their application.

Background: Procedures are given in the statistical literature (e.g., in textbooks, journal articles) for determining operating characteristics (discriminating power) and the number of observations needed (cost) for certain single-sample one-sided tests of statistical hypotheses, but they generally require the use of

specialized probability tables. It does not appear to have been generally recognized that many problems of these types can be satisfactorily handled by means of relatively simple approximate formulas requiring for their use only certain readily accessible and easily remembered normal-probability deviates.

Magnitude: Class II

Date of Termination: 3rd. Quarter, Fiscal 1948 (Estimated)

Status: UNDER WAY (CONTINUATION) About 80% completed.

All the necessary formulas have been assembled or developed by Mr. Uttam Chand, guest worker from India, who has incorporated them in a preliminary draft of a paper to be submitted for publication in an appropriate statistical journal and as a manual of the Applied Mathematics Series of the NBS, after preparation of a set of examples illustrating applications and the accuracies of the formulas is completed. Work has begun on determining the accuracies of the formulas.

Publication: The manuscript of the above mentioned draft is available for consultation in the Statistical Engineering Laboratory.

A "Preliminary Note on Accuracy of Certain Formulas for Sample Size and Acceptance Number in the case of Single-Sample Acceptance-Sampling by Attributes" was prepared by J. M. Cameron and C. Eisenhart, and circulated to various interested persons. Dittoed copies may be obtained from the Section Office.

Project: 4803-1 *Priority:* 3 *Date Auth.* 7/15/47

Title: Standard Sampling-Inspection Procedures

Origin: Office of Naval Research, Research and Development Division, War Department.

Project Managers: Dr. Churchill Eisenhart and Mr. Julius Lieblein

Objective: To revise, expand, and extend the present Navy Department manual "Standard Sampling Inspection Procedures" (Part D, Chapter 4, of the Administration Manual of the Material Inspection Service, U.S.N.) so as to be suitable for referencing in all types of Government specifications; particularly, in Federal Specifications.

Background: A task group of the Inspection Advisory Council of the War Department undertook during the fiscal year 1947 to effect a revision of the Navy "Procedures" to adapt them to War Department needs. Additional tables recommended by this task group were computed by Section 11.2 in that fiscal year. It was decided that further mathematical work would be needed on the entire task and that the task should be generalized to include all Federal procurement. Theoretical aspects have been assigned to Section 11.3, while responsibility for practical aspects remains in the Department of National Defense.

Magnitude: Class III

Date of Termination: June 30, 1949

Status: UNDER WAY (CONTINUATION) about 5% completed.

Preliminary meetings have been held with representatives of various Government agencies at which topics pertinent to the proposed revision of the Navy Manual were considered. The first two meetings were held at the Bureau and were concerned with the relative merits of the use of acceptable quality level and average outgoing quality level by an organization such as the Bureau of Ships, as a basis for designation of quality. The other meetings, held at the Navy Department, dealt in particular with a report entitled "Sampling Inspection by Variables", prepared by Professor A. H. Bowker under a Navy contract, and were concerned with matters to be considered in the over-all revision of the Navy Manual. Following these meetings, a tentative plan for a prospectus for the revision project was prepared by J. Lieblein.

In addition, background meetings attended have included several on quality control and a session on the mathematical theory of extreme values.

Project: 47S3-1 *Priority:* 1C *Date Auth.* 7/1/47

Title: Rubber goods; General Specifications

Origin: Section 7.5, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To prepare appropriate acceptance-sampling instructions and plans for inclusion in the revision, now under way, of "Federal Specification ZZ-R-601a; Rubber Goods; General Specifications (Methods of Physical Tests and Chemical Analyses)"; also, to replace the method for rejecting doubtful observations contained in the present edition of the above specification by developing a procedure for "averaging" tensile test measurements that will reduce the effect of a doubtful observation on the reported "average" without introducing significant bias or substantially decreasing the precision of the reported average.

Background: The above Specification is referred to widely as the standard authority on physical tests and chemical analyses of rubber goods, not only in procurement activities of Government agencies, but also in research and development activities. The statistical features of the present edition are obsolete, and it was generally conceded by all concerned that a sound up-to-date approach to the many statistical problems in this Specification was needed.

Magnitude: Class III

Date of Termination: 29 September, 1947

Status: COMPLETED (NOTE: Priority raised from 2B to 1C at beginning of 1st. quarter fiscal 1948.)

Three sets of acceptance-sampling plans corresponding to different

consumption requirements, together with instructions for their use, have been prepared for inclusion in the revision of the Specification. In addition, a procedure has been devised for reducing the effect of a doubtful observation on the reported "average", based on use of the median instead of the arithmetic mean with provision for taking two additional measurements in certain cases. The relevant parts of the text of the Specification were completely rewritten, and a list of the necessary definitions added, by Dr. Eisenhart and Messrs. J. M. Cameron and J. Lieblein of Section 11.3, in collaboration with Messrs. R. F. Tener and J. Mandel of Section 7.5.

Project: 47S3-2 *Priority:* 2B *Date Auth.* 7/1/47

Title: Pre-ignition Rating of Spark Plugs

Origin: Division 3, NBS

Project Manager: Dr. J. H. Curtiss

Objective: To determine statistical tolerance limits for preignition ratings, in 17.6 test engine, of automotive spark plugs of different manufacturers.

Background: The NBS has been requested, by the Electrical Supplies Committee of the Federal Specifications Board, to formulate a classification of automotive spark plugs into groups on the basis of pre-ignition rating in the 17.6 test engine, as a possible substitute for the classification given in para. B-2 of Federal Specification W-P-506.

Magnitude: Class I

Date of Termination: 3rd Quarter, Fiscal Year 1948

Comments: Factors needing study include: (a) possible trends in the successive rating of individual spark plugs; (b) errors inherent in the testing method, e.g. degree of reproducibility of test results in repeated tests on a given machine; and (c) mutual consistency of results of tests conducted by different laboratories. These are to be investigated by means of statistically designed experiments.

Status: INACTIVE. About 20% completed.

A statistical analysis of pre-ignition test data for automotive spark plugs of several nominal types, completed in 3rd, quarter of the fiscal year 1947, showed erratic behavior of the means of a given nominal type, and quite large dispersion of individual readings about these means. These results were not seriously challenged by the industry. It was then pointed out that great difficulties would be encountered in procurement based on any arbitrary classification of spark plugs in view of these chaotic conditions. As a result of a conference with Messrs. Blackburn and Cummings, of Section 3.6, agreement was reached on the appropriateness of an intensive study of the precision of the method of rating.

Project: 4783-3 *Priority:* 2C *Date Auth.* 7/1/47

Title: Clinical Thermometers

Origin: Section 3.1, NBS

Project Managers: Dr. Churchill Eisenhart and Mr. J. M. Cameron

Objective: To substitute appropriate sampling plans for the current practice of 100 per cent inspection and testing.

Background: The Veterans Administration is purchasing large quantities of clinical thermometers on contracts referencing Federal Specification GG-T-311; Thermometers, Clinical, and is using the Bureau as its inspection and testing agency for these thermometers. A backlog of serious proportions has been accumulating in this work at the Bureau. The present project is intended to explore ways of reducing the backlog without loss of protection to the Veterans Administration by instituting adequate and efficient acceptance-sampling procedures.

Magnitude: Class II

Date of Termination: 3rd. Quarter Fiscal 1948

Status: UNDER WAY (CONTINUATION) About 80% completed.

The contract under which inspection is currently proceeding calls for complete 100 per cent inspection and testing of a delivery with a provision for terminating inspection and testing with rejection of the delivery when the fraction defective among the items tested exceeds a specified limit as given in paragraph F2a of Federal Specification GG-T-311. The probability of rejection of a delivery by this procedure prior to complete inspection and test has been evaluated as a function of the fraction defective of the delivery. A sequential sampling procedure permitting only rejection of a delivery prior to complete inspection and test has been developed, having approximately the same probability-of-rejection curve as the procedure of paragraph F2a of the Specification. In addition, a sequential inspection and testing procedure permitting acceptance or rejection prior to complete inspection and test has been developed, also having approximately the same probability-of-rejection curve. A statistical analysis has been made of the inspection and test records for recent deliveries submitted under V.A. and other contracts, to determine the possibility and practicability of permitting acceptance of a delivery prior to complete inspection and test. At the suggestion of Dr. Eisenhart and Mr. Cameron of Section 11.3, some special tests were run by personnel of Section 3.1 to provide data on (a) the reproducibility of the tests for accuracy and consistency of readings (GG-T-311: Para. F-6g); and (b), on the risks of misclassification. The results of these studies have been organized in a form suitable for presentation at an early conference of personnel of the V.A. and Divisions 3 and 11 of NBS. Write-up of the results has begun.

Project: 47S3-4 *Priority:* 2B *Date Auth.* 7/1/47

Title: Perforated Clay Pipe, Recommended Commercial Standard TS-4368.

Origin: Section 12.5, NBS

Project Managers: Dr. Churchill Eisenhart and Mr. J. M. Cameron

Objective: To prepare an appendix statement evaluating the acceptance-sampling plan specified, for inclusion in the Standard.

Background: The Statistical Engineering Laboratory was requested by Mr. F. W. Reynolds, Section 12.5, to prepare an appendix statement for inclusion in the Standard, setting forth a quantitative evaluation of the operating characteristics of the sampling plan specified, for the information of users of the Standard.

Magnitude: Class I

Date of Termination: 5 August 1947

Status: COMPLETED.

Dr. Eisenhart and Mr. J. M. Cameron of the Laboratory, conferred on several occasions with Messrs. F. W. Reynolds and H. A. Bonnet of Section 12.5, and Mr. F. S. Cresswell, Manager, National Clay Pipe Manufacturers, Inc., with regard to the tentative appendix prepared by the staff of the Laboratory during the last quarter of fiscal 1947, in response to the aforementioned request. In view of certain limitations of the data upon which it was based - the only data available being some results of recent tests of limited scope - it was finally decided to issue the Standard without the proposed "Appendix".

Project: 47S3-5 *Priority:* 2C *Date Auth.* 7/1/47

Title: Physical Constants of Leather

Origin: Section 7.4, NBS

Project Managers: Dr. Churchill Eisenhart and Miss Celia S. Martin

Objective: To advise and assist personnel of Section 7.4 in evaluating the accuracy and precision of methods of measuring the physical constants of leather.

Background: The development and application of statistical methods to industrial process control within the last quarter century has indicated that measurement and testing procedures can fruitfully be regarded as production processes, the "product" being measurements, and that the validity of conclusions based on measurements obtained by a given procedure may be open to question unless the procedure is shown to be in a "state of statistical control". When regarded from this viewpoint, evaluation of the accuracy and precision of a measurement or testing procedure involves (a) showing that it is in a "state of statistical control" (i.e., among other things, that successive measurements are independent, and that the process is free from trends and non-random shifts after adjustment for known or likely sources of bias); and (b) evaluating the parameters of the procedure.

It is expected that projects of this nature will increase the value, for purposes of scientific inference, of measurements taken at the Bureau, and increase the usefulness of test results as a basis for decisions.

Magnitude: Class II
Date of Termination: Latter half of Fiscal 1948
Status: INACTIVE. About 80% completed.

The Statistical Engineering Laboratory is collaborating with Mr. C. E. Weir, Section 7.4, in evaluating the accuracy and precision of measurements of the physical constants of leather by the methods currently used in Section 7.4. Regression analyses have been run, to allow for known trends, and some tests conducted for constancy of variance of the deviations from regression. Computations have been completed. Write-up not yet begun.

Project: 47S3-6 *Priority:* 3 *Date Auth.* 7/1/47

Title: Wool Content of Blankets

Origin: Section 7.5, NBS; Division of Statistical Standards, Bureau of the Budget

Project Manager: Dr. Churchill Eisenhart

Objective: To develop a procedure of sampling and testing a part-wool blanket in order to determine its percent wool by weight with reasonable assurance.

Background: Present specifications for testing textiles provide procedures for determining the percent wool content by weight of a small piece cut from a textile product, but give no instructions regarding the number of pieces to be cut, from a given article, such as a woolen blanket, nor the manner of selecting the pieces for test from the whole article. Thus the interpretation of the phrase "percent wool by weight" as applied to a given article, e.g. a woolen blanket, is left open. This omission is serious since, under the Wool Products Labeling Act of 1939, articles containing wool must bear a label indicating its percent wool content by weight consonant with the normal variations incident to the manufacturing process. It is the aim of this project to determine the variations in percent wool content by weight, from blanket to blanket and from point to point within a blanket, that may reasonably be considered "normal" in the manufacture of part-wool blankets, to furnish a basis for giving an operational meaning to the phrase "percent wool by weight" as applied to an individual blanket.

Magnitude: Class III
Date of Termination: December 1948
Comments: This project stems from a project completed in the fiscal year 1947 dealing with the components of the variance of a wool-content determination based on a small piece taken at random from a part-wool blanket.
Status: UNDER WAY (CONTINUATION) About 5% completed.

A number of conferences have been held, participated in by personnel of various Divisions of NBS, and of the Division of Statistical Standards, Bureau of the Budget. In addition, the matter has been discussed with various persons associated with the American Society for Testing Materials, and a joint ASTM-NBS project tentatively agreed upon. The objectives of this project (e.g. fundamental questions to be answered), and method of approach have been tentatively agreed upon by NBS and ASTM, but there has been virtually no further progress due to

almost complete failure of wool blanket industry to respond to ASTM's effort to bring together at a single conference representatives of the wool blanket industry, ASTM, and NBS.

Project: 47S3-7 *Priority:* 2C *Date Auth.* 7/1/47

Title: Flamability of Textiles

Origin: Section 10.2 (formerly of Section III-6), NBS

Project Managers: Dr. Churchill Eisenhart and Mr. J. M. Cameron

Objective: To evaluate the sampling and testing clauses in "Flamability of Textiles - Recommended Commercial Standard TS 4350" and in "Flameproofing of Textiles" (NBS Circular C 455); to develop alternative and additional procedures as necessary.

Background: Proposed evaluation originally requested by Mr. S. H. Ingberg (of III-6), recently retired, whose work is being carried on by Dr. Marjorie W. Sandholzer of Section 10.2 (formerly of III-6).

Magnitude: Class I

Date of Termination: 3rd. Quarter Fiscal 1948

Status: INACTIVE. About 1% completed.

Awaiting special test results to be furnished by Section 10.2, NBS.

Project: 47S3-8 *Priority:* 2C *Date Auth.* 7/1/47

Title: Effect of Gasoline and Oil Additives on Carbon and Gum Formation

Origin: Division 3

Project Manager: Mr. J. M. Cameron

Objective: To advise and assist personnel of Section 3.5, NBS, with the statistical aspects of the planning and conduct of experiments to determine the effect of gasoline and oil additives on carbon and gum formation of engines.

Background: By utilization of recent advances in the techniques of statistical inference and the principles of experimental design, it is expected that economy and increased efficiency will be effected in this research and testing program. The experiment involves the testing of over 20 additives in combination with a "control" gasoline and oil mixture on 80 similar engines, the problem being to design the most efficient experiment to determine the performance of the various additives.

Magnitude: Class I

Date of Termination: 3rd. Quarter Fiscal 1948

Status: INACTIVE. About 20% completed.

Awaiting synchronization of the engines so that the experiment can be started.

Project: 47S3-9 *Priority:* 3 *Date Auth.* 7/1/47

Title: Teen-age girls' body-measurement study

Origin: Section 12.2, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: Reduction and analysis of certain body-measurement data for "teen-age" girls in order to establish standard "teen-age" size designations for wearing apparel. The analysis proposed will be based chiefly on two bivariate frequency diagrams: hip girth versus stature, and hip girth versus maximum chest girth. A set of representative areas will then be chosen, with attention to practicability and statistical efficiency in regard to "coverage" from which garments, patterns, and forms can be sized to guarantee an accurate fit for a large proportion of the teen-age population.

Background: Various consumer-, distributor-, and producer- groups have for some time indicated displeasure with the current diverse sizing systems for wearing apparel, especially for teen-age girls. It is proposed, therefore, to develop a sensible sizing system for this group by analysis and study of actual data on body measurements - the system to depend on (i.e. be expressed in terms of) a small number of basic body measurements, selected with due regard to practicability and statistical efficiency. For this study a "teen-age girl" is defined as (1) one who is not less than 12 years and not more than 17 years of age, and (2) one who has a "bust development" (i.e. difference between chest girth at arm scye and maximum chest girth) of one centimeter or more.

Some years ago 37 body measurements were taken on approximately 70,000 school girls between the ages of 4 and 17 years by carefully trained anthropometrists, for the Textiles and Clothing Division, Bureau of Home Economics, USDA. The cards on which these data were punched were loaned to NBS; the present study will be based on data from those cards pertaining to teen-age girls as defined above.

Magnitude: Class II(3); Class II(2)

Date of Termination: June 1, 1948 (very approximate since actual work on this project must be arrested from time to time to allow for meetings with interested outside organizations).

Comments: In addition to developing a sizing system that insures accurate fitting of a large proportion of teen-age girls, it is hoped to develop also auxiliary sizing areas for "slims" and "stouts" in order to cover all girls except the very small group admittedly requiring individually made-to-measure clothing.

Status: UNDER WAY (CONTINUATION) About 5% completed.

Preparation of detailed instructions for machine analysis of teen-age body-measurement data completed. Actual machine analysis of the data awaiting installation of IBM equipment.

Project: 48S3-1 *Priority:* 2B *Date Auth.* 12/15/47

Title: Statistical Theory of Diffraction Gratings

Origin: Section 4.0, NBS

Project Manager: Dr. J. H. Curtiss

Objective: To study the distribution of the intensity of spectral lines obtained from a diffraction grating when the spacings between the lines on the grating are subject to random errors.

Background: The problem of ruling diffraction gratings has been attacked in the past with a considerable amount of engineering ingenuity, but successful solutions have been rare. To assist in determining a program for the Bureau in diffraction gratings, Division 4 has undertaken to review and develop the theory of such gratings. The present project is a part of the Division 4 study. Proposed by Dr. R. D. Huntoon.

Magnitude: Class II

Date of Termination: 2/28/48

Comments: The mathematical problem is closely related to the problem of "random flights" first proposed by Karl Pearson and later worked on by Lord Rayleigh.

Status: UNDER WAY (NEW)

The mean value and dispersion of the intensity have been computed for one system of random errors, and the theory for another system is being developed.

Project: 48E3-1 *Priority:* 2C *Date Auth.* 9/1/47

Title: Techniques of Statistical Inference

Origin: Educational Committee, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To present an in-hours graduate-level course, with calculus a prerequisite, in modern mathematical statistics and applications, and to prepare a set of official lecture notes for this course.

Magnitude: Class III

Date of Termination: June 1948

Comments: Dr. Eisenhart has devoted a considerable amount of time, both in hours and out of hours, to the preparation of the official notes, a task in which he has been unsparingly assisted by the entire staff of Section 11.3. Also, at the request of the persons enrolled in the course, he has conducted out of hours two one-hour laboratory (or, supervised study) periods, each attended on a voluntary basis by one-half of the class.

Status: UNDER WAY (CONTINUATION). About 20% completed.

The first 20 (of a total of 60) lectures have been delivered, and the students are working on the final-examination problem paper, to be turned in on January 15, 1948.

A total of 15 persons are enrolled in this course: 13 from NBS, 1 from the Naval Medical Research Center, and 1 from the Geophysical Laboratory.

The topics of the lectures delivered to date were as follows:

1. A classification of statistical problems. 2. Fundamental concepts: population,

sample, statistical inference. 3. The hypergeometric distribution. 4. The binomial distribution. 5. The Poisson exponential distribution. 6. The normal distribution and asymptotic normality. 7. Normal-distribution approximations to the binomial, Poisson, and hypergeometric distributions. 8. Consistent and unbiased estimators; theorems on stochastic convergence. 9. Efficient and exhaustive estimators; the method of maximum likelihood. 10. Interval estimation; confidence limits for the parameters of the binomial, Poisson and hypergeometric distributions. 11. Sample sizes needed for estimating parameters with specified assurance of prescribed accuracy. 12. The inverse-sine and square-root transformations. 13. Binomial-probability paper. 14. The nature of a statistical test; admissible decisions, acceptable risks of error, operating characteristics. 15. Lot-by-lot acceptance sampling. 16. Effects of lot-to-lot non-homogeneity on acceptance sampling. 17. Formulas for sample size and operating characteristics. 18. Planning a series of performance tests when the severity of any one test cannot be controlled precisely. 19. The sign test and the partition test. 20. Two-sided versus one-sided tests.

Preparation of the official lecture notes is about 75% completed, but notes for only 5 of the lectures have been dittoed in full to date.

Publications: Dittoed copies are available of the offical notes for lectures 2, 3, 4, 6, and 14. Dittoed copies are available also of certain supplementary material such as computation sheets, brief tables, etc. Such items of particular interest are (1) A set of "Tables Showing the Effect of Lot Size, Lot-fraction Defective, and Sample Size on the Probability of 0, 1, or 2 Defectives" for sample sizes of 5, 10, and 20; lot sizes of 50, 100, 200, 400, and infinity; and lot-fractions defective of .02, .04, .08, and .16. (2) A brief table of the binomial distribution for sample sizes of 5, 10, 20, and 30; and probability P equal to .025(.025).10(.05).90(.025).975. (3) "Table of Confidence Limits for the Parameter of a Binomial Distribution" for sample sizes from 2 to 15 and levels of confidence of .95 and .99. (4) A "Table to Facilitate the Application of the Statistical Sign Test", giving the 1, 5, 10, and 25 percent (two-sided) levels of significance for sample sizes from 1 to 100, large-sample formulas for same, and comments on their use

Section 4. The Machine Development Laboratory

Project: 47D4-1 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Bureau of the Census Computing Machine

Origin: The Bureau of the Census

Project Manager: Dr. E. W. Cannon

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for the preparation of census reports.

Background: As a result of wartime work on electronic computing machines and related developments, it has appeared for some time that a revolution is imminent in methods of compilation and tabulation of statistical data. The construction and successful operation of an electronic digital computing machine, the ENIAC, and other developments in the electronic computing machine field clearly pointed out the possibility of constructing electronic digital equipment to carry out the types of manipulation of data involved in both the regular census compilations and in the newer sampling techniques. It is expected that the proposed electronic equipment will perform at increases in speed over existing equipment of a factor of 10 to 20 in some operations, and up to 100 or 200 or even more for other operations.

Magnitude: Class III

Date of Termination: December, 1949

Comments: This project is related to Project 47D4-2, in that contractors for each project were made aware of the performance specifications for both projects, and were informed that a single model might finally be selected for both projects. Project 47D4-2 involves mathematical work related to the present project as well as to other projects.

Status: UNDER WAY (CONTINUATION). Design specifications have been completed by the Eckert-Mauchly Computer Corporation, successor to the Electronic Control Company (J. W. Mauchly and J. Presper Eckert, Jr., Partners). The design specifications are being evaluated by the Bureau and independently by the National Research Council Committee on High-Speed Calculating Machines, Dr. J. von Neumann, Chairman.

Project: 47D4-2 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Navy Computing Machine

Origin: Mathematics Branch, Office of Naval Research

Project Manager: Dr. E. W. Cannon

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for general mathematical computation.

Background: The project was undertaken to meet the need, recognized by the Mathematics Section of ONR, for faster and more efficient computing machinery than that now existing. Included among the problems at which the machine is aimed are the following: (a) Problems involving the systematic handling of large linear arrays (e.g., determination of the characteristic roots of matrices arising in vibration theory and quantum mechanics; solutions of systems of linear equations such as those which arise in vibration problems, metallurgical problems, weather problems, multivariate statistical analysis); (b) problems involving the solution of linear and non-linear partial differential equations, such as those which arise in the study of supersonic phenomena, turbulent flow, flow of viscous fluids, weather problems, servo-mechanisms, non-linear electrical oscillations and so on.

Magnitude: Class III

Date of Termination: December, 1949

Comments: See Comments for 47D4-1

Status: UNDER WAY (CONTINUATION). Design specifications have been completed by the Raytheon Manufacturing Company and are being evaluated independently by the Bureau and the National Research Council Committee on High-Speed Calculating Machines.

Project: 47D4-3 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Air Comptroller's Computing Machine

Origin: Office of the Air Comptroller, Air Force

Project Manager: Dr. E. W. Cannon

Objective: To develop specifications for, and to construct an automatic-sequenced electronic digital computing machine suitable for use by the U. S. Air Force in program planning and control.

Background: The Air Comptroller's Office requires a high-speed and flexible computing machine to calculate detailed programs consistent with general policy decisions, and to facilitate rapid recomputation of programs to meet budgetary and other limitations. The problems involved are of wide applicability, and a part of the present project consists in formulating them mathematically. It is expected that the primary computation problem to be solved by the machine will consist of finding rapidly the solutions of large systems of simultaneous equations containing up to 1000 unknowns. The computer must be able to store and classify large quantities of data, and to refer rapidly for needed items to huge tables of organization, equipment, supply and other similar data. These tables will contain millions of items. It is required, in addition, that the printing devices associated with the computer will be capable of extremely high-speed printing of the complete details of the Air Force's programs that have been computed.

Magnitude: Class III

Date of Termination: June, 1949

Comments: This project and Projects 47D4-1, 47D4-2, and 47D4-4 are interrelated. Project 47D4-4 serves to coordinate the mathematical direction of the three computing machine projects.

Status: UNDER WAY (CONTINUATION). Work is under way on the mathematical analysis of the principal problems that the Air Comptroller expects to solve on this computing machine. Mrs. John Todd is now studying these problems, and, in addition, Dr. J. von Neumann is assisting Dr. George Dantzig in this mathematical analysis. The applicability of analogue machines is also being studied.

Project: 47D4-4 *Priority:* 2C *Date Auth.* 7/1/47

Title: Programming of Problems for Solution on Automatic Digital Computing Machines

Origin: Bureau of the Census, Navy Department and the Air Force

Project Manager: Mrs. Ida Rhodes

Objective: To program certain general types of mathematical and statistical routines such as sorting, collating, the solution of linear systems, square rooting, etc., which frequently recur in the solutions of larger problems proposed by the Bureau of the Census, the Navy and the U. S. Air Force for solution on automatic digital computing machines. Thereby to detect deficiencies in, and effect improvements in, the design of proposed machines; also to establish a library of routines for the above mentioned types of problems and thus eliminate the necessity for the programmer to repeat the construction of a program whenever he is confronted with certain problems.

Background: The project was primarily undertaken to insure proper coordination and mathematical direction of Projects 47D4-1 and 47D4-2. A secondary justification lies in the fact that when automatic computing machinery becomes generally available, it will be necessary to have collections of programs for the routine mathematical operations, so that problem preparation can be expedited as much as possible.

Magnitude: Class III

Date of Termination: June 30, 1948

Comments: This project serves as the foundation of the mathematical directions of Projects 47D4-1 and 47D4-2. The performance of proposed automatically-sequenced electronic digital computing machines is carefully analyzed. The project is expected to serve as groundwork for the preparation of manuals of operation for the automatic computing machines constructed under the supervision of the Bureau.

Status: UNDER WAY (CONTINUATION). Instruction codes have been prepared for the UNIVAC solution of elementary problems (including division, coding in a floating decimal point, evaluation of the elementary functions, and interpolation), collation and sorting routines. Work has begun on preparation of such instruction codes for integration and the solution of differential equations. Preparation of such instruction codes for the computing machine proposed by the Raytheon Manufacturing Company is under way.

Project: 47D4-5 *Priority:* 2A *Date Auth.* 7/1/47

Title: Magnesium Delay Lines

Origin: Mathematics Section, Office of Naval Research

Project Manager: Dr. E. W. Cannon

Objective: To investigate the possibilities of using the magnesium delay line as the basic high-speed storage element of automatic digital computing machinery.

Background: In the designs of several proposed electronic digital computing machines the basic storage element is the mercury delay line. The mercury delay line stores coded information in the form of acoustic wave packets travelling

through a mercury column. Preliminary investigations have indicated that magnesium might be more satisfactory than mercury for this application. These investigations have revealed that the operating characteristics of magnesium should show advantages over mercury as regards transmission and matching losses. In addition, the simplicity of magnesium delay lines, and the fact that the decreased attenuation characteristics of magnesium might permit a simpler design in the amplifiers associated with each line, make an investigation of magnesium delay lines appear worthwhile.

Magnitude: Class I

Date of Termination: November 11, 1947

Comments: The task is being performed by the Raytheon Manufacturing Company as an extension of the work performed by this Company on design specifications in connection with Project 47D4-2.

Status: TERMINATED. The contractor was unable to improve appreciably the bandwidth characteristics of magnesium delay lines. Tests performed appear to indicate that the performance of magnesium as a high-speed storage element would be inferior to that of mercury.

Project: 47S4-1 *Priority:* 2C *Date Auth.* 7/1/47

Title: The MTAC Section

Origin: Committee on High-Speed Computing of the National Research Council

Project Manager: Miss Edith Norris

Objective: To assemble and edit material for a Section entitled "Automatic Computing Machinery" in the quarterly Mathematical Tables and Other Aids to Computation (MTAC), published by the National Research Council.

Background: This Section is to serve as a news letter, a medium for exchange of opinions, and a vehicle for the publication of shorter technical papers, in the field of automatic high-speed calculating machinery. The need for such a service has been pointed out repeatedly by groups interested in such machinery. The decision to sponsor this task and to assign the basic editorial work to the NAML was made at a joint meeting of the Committees on High-Speed Computing and on Mathematical Tables and Other Aids to Computation in New York in April, 1947.

Magnitude: Class II

Status: UNDER WAY (CONTINUATION). Material for the January, 1948, issue of Mathematical Tables and Other Aids to Computation was compiled and sent to the to the Editors.

Publication: MTAC, January, 1948

3. PUBLICATION ACTIVITIES

3.1 Publications which appeared during the period covered by this report.

3.1.1 Mathematical Tables.

Note: Detailed information concerning the tables published by the National Applied Mathematics Laboratories, including prices and instructions for ordering, is given in Letter Circular LC - 884, which is available from the Bureau upon request.

(1) MT 2, Table of the Exponential Function." Second Edition. United States Government Printing Office.

(2) "Table of the Bessel Functions $J_0(z)$ and $J_1(z)$ for Complex Arguments." Second Edition. Columbia University Press.

(3) "Table of Spherical Bessel Functions, Vol. II." Columbia University Press.

(4) "Table of Circular and Hyperbolic Tangents and Cotangents." Second Edition. Columbia University Press.

(5) "Tables for Facilitating the Use of Chebyshev's Quadrature Formula," by H. E. Salzer. Journal of Mathematics and Physics, Vol. 26 (1947). pp. 191-194.

3.1.2 Manuals, Bibliographies and Indices.

(1) LC - 884, "Mathematical Tables." (A catalogue of the tables prepared to date by the Computation Laboratory. See note at head of 3.1.1.)

(2) "Activities in Applied Mathematics, 1946-1947." (A summary of the work in applied mathematics carried on by a unit of the office of the Director of the Bureau. This unit was the nucleus for the present National Applied Mathematics Laboratories.)

3.1.3 Technical Papers and Reviews

(1) RP 1827, "Acceptance Sampling by Variables, with Special Reference to the case in which quality is measured by the average or dispersion," by J. H. Curtiss. Journal of Research of the National Bureau of Standards, Vol. 39 (1947) pp. 271-290.

(2) "The Checking of Functions Tabulated at Certain Fractional Points," by H. E. Salzer. Mathematical Tables and Other Aids to Computation, Vol. 2 (1947) pp. 318-319.

3.2 Manuscripts in the Process of Publication as of December 31, 1947

3.2.1 Mathematical Tables

(1) "Table of Bessel Functions $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$,". Number 1 of the Applied Mathematics Series of the National Bureau of Standards.

(2) "Table of Coefficients for Obtaining the First Derivative Without Differences." Number 2 of the Applied Mathematics Series of the National Bureau of Standards.

(3) "Tables of the Confluent Hypergeometric Function $F(\frac{1}{2}n, \frac{1}{2}; x)$ and Related Functions." Number 3 of the Applied Mathematics Series of the National Bureau of Standards.

(4) "Table of Coefficients for Interpolating in Functions of Two Variables," to appear in the Journal of Mathematics and Physics.

(5) MT 14, "Table of Probability Functions, Vol. II." Second Edition being printed by the Government Printing Office.

(6) "Bessel Functions of Fractional Orders, Vol. I." Columbia University Press.

(7) "Table of

$$J_0(2\sqrt{u}), \quad Y_0(2\sqrt{u}), \quad \frac{J_1(2\sqrt{u})}{\sqrt{u}}, \quad \text{and} \quad \frac{Y_1(2\sqrt{u})}{\sqrt{u}}$$

To be submitted to the Journal of Research of the National Bureau of Standards.

3.2.2 Manuals, Bibliographies and Indices

(Note: A comprehensive bibliography of the Division has been authorized and a first draft has been prepared. The project has been inactive during the period under review.)

3.2.3 Technical Papers and Reviews

(1) "Theoretical and Numerical Treatment of Diffraction by a Circular Aperature." (Ph.D. Thesis), Groningen, Batavia, Holland); and "On Spheroidal Wave Functions of Order Zero," Journal of Mathematics and Physics, Vol. 24 (1947), pp. 79-92, both by C. J. Bouwkamp. Reviews by G. Blanch for Mathematical Tables and Other Aids to Computation, scheduled for April, 1948 issue.

(2) "A Statistical Analysis of Some Mechanical Properties of Manila Rope," by J. H. Curtiss. To appear in the Journal of Research of the National Bureau of Standards for December, 1947

(3) "Some Trends in Applied Mathematics," by J. H. Curtiss. Being refereed for possible publication in the American Scientist.

(4) "A Federal Program in Applied Mathematics," by J. H. Curtiss. Circulated for comment with a view to publication in Science.

(5) "The Square Root Method for Solving Simultaneous Linear Equations," by J. Laderman. To appear in the January, 1948 issue of Mathematical Tables and Other Aids to Computation.

(6) "A Problem of J. C. P. Miller on Arc Tangent Relations," by John Todd. Submitted to the American Mathematical Monthly.

APPENDIX

Explanation of Project Descriptions

The project descriptions appearing in Sect. 2 of this report are reproductions of the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With the view of making this report more useful, an explanation of certain of the symbols and standard terms used in the Project Forms will now be given.

Project Number. Each project of the Laboratories is identified by a four-location symbol called the Project Number, which appears in the upper left hand corner of the Forms. The first location in the symbol designates the fiscal year in which a Project Form for the project was first prepared; e.g., 45 for 1945, 47 for 1947, etc. (For projects under way as of July 1, 1947, the fiscal year designated is that in which the Form would have been prepared under the present rules.) The letter in the second location denotes the class of project: R stands for research, D for developmental (usually of aids for work in mathematics), S for service, E for educational. The third symbol location denotes the Section of Division 11, to which primary responsibility for the project has been assigned, and the last symbol is a serial number within the section.

Priority. Priority rankings are assigned to each project as a guide for the staff and in recognition of the interests of clients of the Laboratories. The system of rankings is as follows:

Priority 1. This category consists of those projects, the early completion of which is essential to the success of current or impending operations of another division of the Bureau of Standards, or another Government agency, or an important industrial or academic laboratory.

Priority 1A. This priority is assigned only to those projects whose results are immediately needed for purposes of national security.

Priority 1B. This priority is assigned to those projects whose results are, for economic and/or administrative reasons, urgently needed by clients of the National Applied Mathematics Laboratories.

Priority 1C. This priority is assigned to those projects which meet the requirements of the Priority 1 category, but which are not so critically related to the success of current or impending operations of other laboratories.

Priority 2. This category consists of projects of obvious importance, the completion of which will increase the efficiency of and promote economy in the National Applied Mathematics Laboratories, other divisions of the Bureau, other Government agencies, and industrial or academic laboratories.

Priority 2A. This priority is assigned to special and presumably nonrecurrent projects which, if brought to an early and successful conclusion, will almost surely contribute materially to the effectiveness, efficiency and economy of current operations.

Priority 2B. This priority is assigned to special and presumably nonrecurrent projects which will probably contribute to the efficiency and economy of current operations; or which will increase the usefulness of a forthcoming publication of the Laboratories which otherwise has been assigned to Priority 1 or 2, or which is otherwise ready for the printer.

Priority 2C. This priority is assigned to those projects of a routine or recurrent nature, the results of which are integrated with the operations of other laboratories, but which do not satisfy the requirements of Priority 1.

Priority 3. This category consists of projects which have no urgent application to any particular activity of the National Applied Mathematics Laboratories or the clients of the Laboratories, but which are worth while prosecuting, provided that they do not delay problems of higher priority. The base-load projects of the National Applied Mathematics Laboratories, such as the preparation and distribution of major mathematical tables not urgently needed for special work of other laboratories. are assigned to this category.

Date of Authorization. The date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Title. Self-explanatory

Origin. "

Background. "

Magnitude. This is an estimate of the size of a task. At the outset of a project such an estimate will often be only an educated guess, so this entry is subject to change as the work progresses. Four classes are used to designate magnitude: Class I means 5 man-days or less, Class II means more than 5 but not more than 100 man-days, Class III means more than 100 but not more than 1000 man-days, and Class IV means more than 1000 man-days.

Work performed outside the Laboratories on contracts (other than contracts for personal services) is not included in the calculation of magnitude. When more than one Section of the Laboratories is involved in a project, separate entries are made for each Section. The section numbers are then placed in parentheses after the magnitude designations.

Date of Termination. This is the date on which it is estimated that work will terminate. In cases where commitments have been made to outside organizations, the agreed upon completion date is used here. In the case of projects upon which no commitments have been made to outside organizations, this entry is subject to modification as the work progresses, and in certain cases involving low-priority R and D projects, no date of termination is given at all.

Comments. Related projects are mentioned here, together with other relevant information.

Status. Here is given the narrative of the progress to date on the project. In making the entries, certain standard descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to

which the entry applies. These standard terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, but no work of any consequence has been performed as of the date of the report.

"UNDER WAY (NEW)" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, and that work was in progress during that period.

"UNDER WAY (CONTINUATION)" means that the work was initiated more than 3 months preceding the date of the report and was in progress during the 3 months preceding the date of the report.

"INACTIVE" means that the Laboratories made a commitment more than 3 months preceding the date of the report, to work on the project, but no work of any consequence was performed on the project during the last 3 months.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof are to undergo extensive mathematical checks, the designation "COMPLETED" is employed only after these checks have been performed.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "IN MANUSCRIPT" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "COMPLETED" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under Status or Publication to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.