

NBSIR 73-205 (R)

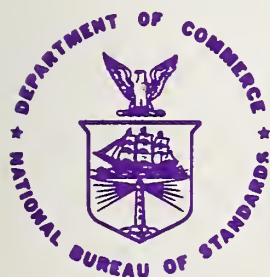
# Projects and Publications of the Applied Mathematics Division

---

Applied Mathematics Division  
Institute for Basic Standards  
National Bureau of Standards  
Washington, D. C. 20234

A Semi-Annual Period

July – December 1972



---

U. S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS



NBSIR 73-205

**PROJECTS AND PUBLICATIONS OF THE  
APPLIED MATHEMATICS DIVISION**

---

Applied Mathematics Division  
Institute for Basic Standards  
National Bureau of Standards  
Washington, D. C. 20234

A Semi-Annual Period

July – December 1972

U. S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary  
NATIONAL BUREAU OF STANDARDS, Richard W. Roberts, Director

## APPLIED MATHEMATICS DIVISION FUNCTIONS

**APPLIED MATHEMATICS DIVISION (205.00):** Conducts research and provides consulting services to the Bureau and other Federal agencies in various fields of mathematics important in science and engineering, including automatic data processing and operations research, with emphasis on statistical, numerical and combinatorial analysis, and on mathematical physics. Develops tools for mathematical work such as mathematical tables, handbooks, manuals, mathematical models and computational methods, and advises on their use. Provides training in disciplines related to these functions.

**MATHEMATICAL ANALYSIS SECTION (205.01):** Conducts research and provides consulting services in core and applied mathematics as they impact on science and engineering, involving research supporting the advancement of computation, particularly in the development of computing algorithms and criteria for their evaluation. Performs mathematical analysis of complex physical systems for the purpose of developing rigorous analytic procedures. Simulates the behavior of physical systems by means of electronic computers, using approximation techniques and semi-analytic methods. Studies the properties and representations of special functions encountered in research and consultation. Develops methods for computing these special functions, and critically evaluates methods commonly used. Collaborates with representatives of government agencies, industry, and universities in an effort of extensive testing, evaluation and documentation of algorithms and computer programs for scientific calculation.

**OPERATIONS RESEARCH SECTION (205.02):** Develops and applies mathematical-computational techniques for and provides consulting services in the analysis, improvement or optimization of complex systems or activity patterns. Conducts investigations in the art of constructing useful mathematical models of complex systems. Studies such systems by applying analytic or simulation methods. Applies these techniques to problems arising in the work of the Bureau or of other Government agencies lacking specialized personnel in this field. Performs research in underlying areas of mathematics, such as linear programming, the theory of linear graphs, and the theory of strategic contests.

**STATISTICAL ENGINEERING SECTION (205.03):** Provides consulting services in the application of mathematical statistics to physical science experiments and engineering tests, particularly in the design of experiments and in the analysis and interpretation of data. Contributes to the development of appropriate statistical techniques on a foundation of research on pertinent topics in probability and mathematical statistics. Conducts studies of computational methods, and prepares reports, manuals, tables, and handbooks to facilitate the application of modern statistical methods.

## Contents

Status of Projects <sup>°</sup> as of December 31, 1972. . . . .	1
1. Mathematical analysis. . . . .	1
2. Operations research. . . . .	8
3. Probability and mathematical statistics. . . . .	11
4. Statistical engineering services. . . . .	13
5. Other activities. . . . .	16
Lectures and technical meetings. . . . .	20
Publication activities. . . . .	22

<sup>°</sup> Only unclassified material is included in this report.

# APPLIED MATHEMATICS DIVISION

July 1, 1972 through December 31, 1972

## TECHNICAL ADVISORY PANEL

Murray S. Klamkin, Ford Scientific Laboratory, Chairman

Carl A. Bennett, Battelle-Northwest  
Richard W. Hamming, Bell Telephone Laboratories  
H. O. Hartley, Texas A. & M. University

M. H. Martin, University of Maryland  
Alan J. Perlis, Yale University  
John Todd, California Institute of Technology

## DIVISION OFFICE

Burton H. Colvin, Ph. D., Chief

Susanne M. Nee, Secretary

Russell A. Kirsch, M. S.  
Anne H. Meininger, B. A.  
Donald J. Orser, B. S.

Irene A. Stegun, M. A.  
Ruth Zucker, B. A.

Morris Newman, Ph. D., Senior Research Mathematician

Doris M. Burrell, Secretary  
Charles R. Johnson, Ph. D.\*\*

Justin C. Schaffert\*  
Mark E. Sheingorn, Ph. D.\*\*

Susan E. Landau\*\*\*

MATHEMATICAL ANALYSIS SECTION, Hans J. Oser, Ph. D., Chief

Jeffrey T. Fong, Ph. D.  
Karl Goldberg, Ph. D.  
Seymour Haber, Ph. D.  
Regina Kopec, Secretary

S. Richard Kraft, Ph. D.  
Joseph Lehner, Ph. D.\*  
Chising A. Liao, M. A.<sup>3</sup>  
Daniel W. Lozier, M. A.  
Frank W. J. Olver, D. Sc.\*

George Reitwiesner, M. S.<sup>2</sup>  
Walter L. Sadowski, Ph. D.  
David J. Sookne, M. A.  
Susan Zuckerman\*\*\*\*

OPERATIONS RESEARCH SECTION, Alan, J. Goldman, Ph. D., Chief

Tyrone B. Ayers, B. S.,<sup>1</sup>  
Jack Edmonds, M. A.<sup>o</sup>  
Judith F. Gilsinn, M. S.  
William G. Hall, B. A.

Dolores E. Harrison, Secretary  
William A. Horn, Ph. D.  
Richard H. F. Jackson, M. A. S.  
Lambert S. Joel, B. A.  
Martin H. Pearl, Ph. D.\*  
Patsy L. Saunders, B. S.  
Timothy L. Shaftel, Ph. D.\*\*\*

STATISTICAL ENGINEERING LABORATORY, Joan R. Rosenblatt, Ph. D., Chief  
Hsien H. Ku, Ph. D., Ass't Chief

Wesley L. Nicholson, Ph. D. IBS Visiting Scientist

Shirley G. Bremer, B. S.\*  
Eleanor S. Brown  
Beverly J. Clipper, Secretary  
Veronica Connor, Secretary  
James J. Filliben, Ph. D.

David Hogben, Ph. D.  
John F. LaBrecque, M. A. T.<sup>oo</sup>  
James A. Lechner, Ph. D.  
Delmas M. Maxwell

Mary G. Natrella, B. A.\*  
Sally T. Peavy, B. S.\*  
Raymond C. Sansing, Ph. D.\*\*  
Peter V. Tryon, Ph. D.<sup>oo</sup>  
Roy H. Wampler, M. A.

\*Part-time

\*\*Postdoctoral Research Associate

\*\*\*Guest Worker

\*\*\*\*Student Trainee

<sup>o</sup> On Leave of Absence

<sup>oo</sup> Located at NBS Boulder Labs.

<sup>1</sup>Detailed from NBS Personnel  
Division (Quality Staffing Program)

<sup>2</sup>Reemployed Annuitant

<sup>3</sup>Presidential Intern



# Status of Projects

## 1. MATHEMATICAL ANALYSIS

### RESEARCH IN MATHEMATICAL ANALYSIS

Task 20540-12-205141/55-57

1550-11

Origin: NBS

Manager: H.J. Oser

Authorized 9/1/54

Full task description: July - September 1954 issue, p. 27

Status: CONTINUED

Kraft completed final editing and revisions on three papers concerned with semi-discrete approximation of linear transport equations; the papers were submitted for publication. The results are of general interest because they establish the convergence and accuracy, and increase the scope and perspicuity of certain numerical algorithms for solution of linear transport equations (which occur pervasively in the area of matter and energy transport). Specifically, this work is being used in Kraft's study of radiant heat transfer in smoke and soot clouds. Knowledge acquired from such studies is useful to the understanding of fire propagation in buildings and efficient energy production in furnaces.

Revisions of Kraft's paper on ordinary differential equations were completed and it is being prepared for WERB. This paper is useful in numerical solution of parabolic and elliptic equations and is related to work being done by Kraft (in collaboration with D. Lewis, 425.01) on boron redistribution in semiconductor processing. It is also of interest because of its connection with a possible physical explanation of the so-called "flashover" phenomenon in building fires.

A paper by Kraft on the integral equations of interreflections was accepted for publication. Steady progress is being made in completing a computer program for the numerical solution of such problems. This work is of interest to workers in the NBS Optical Radiation Section because of its connection with light bulb standardization, radiant flux standardization and spectroscopic instrumentation.

S. Haber continued his empirical investigation of the distribution of the error bounds of certain families of quadrature formulas of the Korobov-Hlawka type. Ways of smoothing the empirical distribution to facilitate comparisons with each other and with known distributions were devised, and calculations carried out.

S. Haber, with O. Shisha of the Naval Research Laboratory, studied the representability of the improper Riemann integral  $\int_0^{\infty} f(x)dx$  as a limit of Riemann sums, together with the related question of the approximability of this integral by finite quadrature formulas. They introduced a new property of functions, similar to boundedness of variation but specific to infinite intervals; and showed that this property is central to the answers to the questions considered.

Walter L. Sadowski completed program development for the moment expansion approach to the solution of the nonlinear Vlasov equation and with D.W. Lozier ran some test cases.

J.T. Fong continues to assist NBS staff members in the mathematical formulation and solution of problems on the mechanical strength and failure mechanism of materials, e.g., toy safety (411.01), extrusion of polymer melts (311.07), transducer calibration (425.03), dynamic force verification in a fatigue testing machine (213.04), etc. Motivated by his earlier experience in design and testing, J.T. Fong continues to make progress on research in the following areas:

(a) Energy Method Based on a Decomposition Postulate: This postulate was proposed by J.T. Fong and J.A. Simmons (312.05) in a paper presented at the International Symposium on Foundations of Plasticity, Polish Academy of Sciences, Warsaw, Poland, Aug. 29 - Sept. 3, 1972. A discussion note on Y. Horie's "On the Thermodynamic States of Plastically Deformed Solids" was also presented there by J.T. Fong at the invitation of Prof. A. Sawczuk, the Symposium Chairman. The first application of the energy decomposition postulate is being made by J.T. Fong on the analysis of Jaske-Mindlin-Perrin's recent experimental data on stainless steel type 304 for cyclic strain history at 538K.

(b) Compressible Material Analysis Based on a Decomposition Theorem: This theorem was proved by J.T. Fong and J.A. Simmons (312.05) in a paper accepted for publication by Z.A.M.P. (Vol. 23, No. 5, 1972, in press). A manuscript entitled "On the Applicability of the Bernstein-Kearsley-Zapas' Theory of Incompressible Media to Materials with Finite Compressibility" is being prepared for review and publication.

(c) Method of Mathematical Formulation Based on "Matrix Calculus": Some elementary formulas in "matrix calculus" was published in a paper by J.T. Fong (NBS J. Res. Vol. 75B, pp. 97-105, 1971). A new manuscript entitled "An Application of Schur's Lemma on Irreducible Sets of Matrices in Continuum Mechanics" is being prepared for review.

F.W.J. Olver has been developing a new asymptotic theory for ordinary differential equations of the second order having two coincident, or nearly coincident, turning points. The object is to find approximations which are uniform with respect to a parameter governing the separation of the turning points, and uniform also in unbounded regions. The results will be of importance in approximating special functions of mathematical physics.

K. Goldberg completed a manuscript entitled: "Distance Coordinates with Respect to a Triangle of Reference". This paper flows from earlier efforts to find efficient position location algorithms in Project WHERE. A number of never-before published identities for triangles, involving barycentric coordinates, are derived.

#### Publications:

- (1) A Decomposition Theorem on the Scalar Potential of a Compressible Bernstein-Kearsley-Zapas Fluid. J.T. Fong and J.A. Simmons. Submitted to a technical journal.
- (2) A Discussion Note on Y. Horie's "On the Thermodynamic States of Plastically Deformed Solids". J.T. Fong. To appear in Proc. International Symp. Foundations of Plasticity, Warsaw, Poland, 1972. (A. Sawczuk, ed.).
- (3) A Non-Equilibrium Thermodynamic Theory of Simple Materials Based on a Single-Integral Entropic Functional. J.T. Fong and J.A. Simmons. Archives of Mechanics (Arch. Mech. Stoc., Warsaw), Vol. 24, 1972, pp. 363-372.
- (4) A Quadrature Formula of Degree Three. L. Flatto and S. Haber. To appear in the Journal of Approximation Theory.
- (5) Experiments on Optimal Coefficients. S. Haber. Applications of Number Theory to Numerical Analysis, Academic Press, N.Y., 1972, pp. 11-37.
- (6) The Existence of Simple Quadrature. S. Haber. Michigan Mathematical Journal, Vol. 19, 1972, pp. 401-410.
- (7) Distance Coordinates with Respect to a Triangle of Reference. K. Goldberg. To appear in NBS J. of Research B., Mathematical Sciences.
- (8) Error Bounds for Stationary Phase Approximations. F.W.J. Olver. To appear in SIAM J. on Mathematical Analysis.



#### Publications:

- (9) An Extension of Riemann's Method of Integration .... S.R. Kraft. To be submitted to Journal of Mathematical Analysis and Application.
- (10) A Priori Inequalities for Semidiscretizations of a Time Dependent Transport Equation. S.R.Kraft. Submitted to a technical journal.
- (11) A Priori Inequalities for a Semidiscrete Stationary Linear Transport Equation. S.R. Kraft. Submitted to a technical journal.
- (12) A Definition of the Fundamental Solution for Systems of O.D.E.'s with Applications. S.R. Kraft. In preparation for submission to NBS review.
- (13) Convergence of Semidiscrete Approximations of Linear Transport Equations. S.R. Kraft. To appear in the Jour. of Mathematical Analysis and Applications.
- (14) Uniqueness and Existence for the Integral Equation of Interreflections. S.R. Kraft. To appear in the SIAM J. on Mathematical Analysis.
- (15) An Existence Theorem for a Linear Transport Equation. S.R. Kraft. In preparation for submission to NBS review.

#### Manuscripts in Preparation:

- (1) J.T. Fong and R.W. Penn, Experimental Definition of Non-Equilibrium Entropy for Aluminum and Stainless Steel.
- (2) J.T. Fong, On the Applicability of the Bernstein-Kearsley-Zapas' Theory of Incompressible Media to Materials with Finite Compressibility.
- (3) J.T. Fong, An Application of Schur's Lemma on Irreducible Sets of Matrices in Continuum Mechanics. Submitted to NBS review for publication in NBS J. Research B.

#### Talks:

- (1) J.T. Fong. A Non-Equilibrium Thermodynamic Theory of Simple Materials Based on a Single-Integral Entropic Functional. International Symposium on Foundations of Plasticity. Polish Academy of Sciences, Warsaw, Poland. August 31, 1972.
- (2) J.T. Fong. A Discussion Note on Y. Horie's "On the Thermodynamic States of Plastically Deformed Solids". International Symposium on Foundations of Plasticity. Polish Academy of Sciences, Warsaw, Poland. August 30, 1972.

PROJECT "WHERE"

Task 20501-2050505/71

6030

Origin: U.S. Army Advanced Materiel Concepts Agency  
Manager: H.J. Oser  
Full Task Description: July-December 1971 issue

Authorized 10/4/71

Status: COMPLETED

Four tasks were carried out during July-Sept.:

1. The algorithm was extended to handle situations where there were not enough ranges to use the basic LSS algorithm, and to utilize smoothing for the altitude coordinate. Various other forms of smoothing and tracking were tried, and compared with simple least squares squared estimates and with a 9-state Kalman filter technique developed at General Dynamics, San Diego, California.
2. Several runs were made on the Boston scenario, with aircraft, to quantify the improvement provided by airborne locators and to evaluate the accuracy of aircraft location for that scenario. This task required conversion and correction of line-of sight data from the Electromagnetic Compatibility Analysis Center in Annapolis, Maryland.
3. A carefully-controlled series of runs was made to develop rules (depending on altitude, geometry, number of locators, etc.) for choosing whether to use the 3D algorithm or a 2D slant-height reduction algorithm utilizing independent height information, for locating aircraft.
4. Miscellaneous support was provided to AMCA. J. Lechner participated in AMCA's briefing of an evaluation panel of independent experts, held at Ft. Monmouth, explaining the NBS study and answering questions; he also provided AMCA with a first draft for the mathematics section of their final report on the feasibility of a micro-navigation position location system. Dr. C. Y. Cho of AMCA was given consultation on his own final report.

R. H. F. Jackson, J. A. Lechner, and D. J. Sookne each presented a paper based on this study, at the Army Design of Experiments Conference, a Mathematical Association of America meeting, and the annual Operations Research Society meeting, respectively. This will appear as an NBS report and be published in the Proceedings of the Army Design of Experiments Conference.

A paper, "A Position-Location System Based on Ranges", was written by R. H. F. Jackson, J. A. Lechner and D. J. Sookne. It will appear as an NBS report and be published in the Proceedings of the 18th Conference on the Design of Experiments in Army Research, Development and Testing. Each of the authors presented a version of the paper.

Publication: Supplement to NBS Final Report on Project WHERE. NBS Report 10940(10/15/72).

Talks:

- (1) J.A. Lechner. A System for Position-Location Based on Ranges. Army Conference on Design of Experiments, Aberdeen, Md. October 26, 1972.
- (2) R.H.F. Jackson. A System for Position-Location Based on Ranges. Annual ORSA meeting, Atlantic City, N. J. November 8-10, 1972.
- (3) D.J. Sookne. A System for Position-Location Based on Ranges. U.S. Naval Academy, Annapolis, Md. November 18, 1972.

# MATHEMATICAL ALGORITHMS

Task 205-12-2050143

Origin: NBS (RTS)  
Manager: H.J. Oser

Authorized 7/1/72

Objective: To gather, collect and disseminate knowledge on mathematical algorithms and their implementations on electronic computers. To develop general methodology to test, validate and certify existing mathematical computer libraries and to apply these methods to a variety of such libraries. And also, to interact with similar groups that are concerned with specific aspects of algorithm proving and validation and to publish and disseminate the results in reports, papers and through conference proceedings.

Background: Initial impetus for this work came from certification of the NBS UNIVAC 1108 FORTRAN mathematics library, an effort that was sponsored by the ICST-CSD (Div. 630).

Status: New

The Bit Comparison Program was extended by D. W. Lozier to handle complex functions and functions of two variables. Certain output formats were also revised and improved.

Work on testing the accuracy of the UNIVAC FORTRAN library was continued by D.W. Lozier, W.L. Sadowski and C.A. Liao. All of the FORTRAN callable mathematical functions have now been tested. Briefly, the results of this testing are the following. The single and double precision routines return very accurate function values in general, but the testing revealed errors in extreme ranges of the argument, incorrect handling of error conditions, or misplaced crossover points that cause local degradation of the accuracy. In general, the accuracy of the complex functions is not as high as of the real functions. The complex logarithm, square root and cube root are the least accurate routines in the library. Testing of the exponentiation routines, which perform  $A**B$  for all combinations of data types, has not yet been completed.

The results of the testing of each library function will be published in a series of NBS Technical Notes. A summary paper presenting these results will be submitted to the AFIPS National Computer Conference in June.

A very complete package of FORTRAN routines for arbitrary precision arithmetic and function evaluation, developed by W.T. Wyatt of Harry Diamond Laboratories, was obtained by D.W. Lozier. However, before general acceptance it requires validation and field testing. Also, its usefulness in general scientific computing would be much enhanced by having a pre-processor which would enable an existing FORTRAN program to be run in "super" precision with a minimum of coding changes. Efforts to secure support from HDL for validation and further development of the package within the Applied Mathematics Division were initiated by D.W. Lozier. Review activities on algorithm papers in professional journals continued throughout the reporting period.

F.W.J. Olver has continued to investigate the construction of algorithms for computing associated Legendre functions for arbitrary real values of the argument and parameters.

G.W. Reitwiesner completed and checked out a FORTRAN computer program that is based on theoretical work by Collins and Heindel. Using modular arithmetic, upper and lower bounds to the real roots of polynomials are found to arbitrary precision. The entire program consists of approximately 1500 FORTRAN statements and is self-contained. Such a program is particularly useful to check out common root-finding programs for accuracy, but it is also indispensable for error analysis and testing ill-conditioned cases.

We have successfully handled such notoriously difficult cases as the Wilkinson polynomial  $\sum_{i=1}^{20} (x-i)$

and polynomial clusters. A paper will appear in the NBS J. Research B that describes the method and the FORTRAN program. This work has now been concluded.

The BESLCI routine certification was redone by D.J. Sookne to correct some deficiencies in the original draft. All the computer runs were redone, and the certification and routine will be submitted to the Communications of the ACM.

D.J. Sookne wrote a program to calculate several series expansions of the Milazzo-Tommasi Diffraction Integral. Sponsor was Dr. Klaus Mielenz, (232.05). The series failed to converge in some cases, and good answers were obtained for only some of the data.

Also by D.J. Sookne, a computer program was written to calculate Legendre functions  $Q_{\nu}^{\mu}(x)$ , for real  $x > 1$ , real  $\nu > -1$ , and integer  $\mu \geq 0$ . Some preliminary accuracy tests were made; the program passed the tests.

D.J. Sookne discovered an error in the  $\sin^{-1}$  routine of an electronic desk calculator. The company and NBS users of such machines were notified.

D.J. Sookne developed a program to calculate coordinates for a numerically controlled milling machine at NBS. The coordinates produced a milled piece of metal to the desired specifications.

#### Publications:

- (1) Performance Test Report on NBS FORTRAN Double Precision Sine and Cosine. D.W. Lozier, L.C. Maximon, and W.L. Sadowski. In preparation for submission to NBS review.
- (2) Performance Tests of the RL 24 UNIVAC 1108 FORTRAN Library of Mathematical Functions. D.W. Lozier, L.C. Maximon and W.L. Sadowski. To appear in the Proceedings of the National Computer Conference in New York City, June 1973.
- (3) A Bit Comparison Program for Algorithm Testing. D.W. Lozier, L.C. Maximon and W.L. Sadowski. Accepted for publication in The Computer Journal.
- (4) A Unified Standards Approach to Algorithm Testing. W.L. Sadowski and D.W. Lozier. Published in Program Test Methods, W. Hetzel, ed., Prentice Hall, 1972.
- (5) Use of Olver's Algorithm to Evaluate Certain Definite Integrals of Plasma Physics Involving Chebyshev Polynomials. W.L. Sadowski and D.W. Lozier. Jour. of Comp. Physics, Vol. 10, No. 3, Dec. 1972, pp. 607-613.
- (6) On Polynomial Real Root Evaluation by Sturm's Theorem and Integer (Modular) Arithmetic. G.W. Reitwiesner. To be submitted to a technical journal.
- (7) Polynomial Real Root Evaluation by Sturm's Theorem Using Modular Arithmetic. G.W. Reitwiesner. NBS Report No. 10883. Oct. 1972.
- (8) Note on Backward Recurrence Algorithms. F.W.J. Olver and D.J. Sookne. Mathematics of Computation, Vol. 26, 1972, pp. 941-947.
- (9) Certification of BESLCI. D.J. Sookne. To be submitted to Communications of ACM.
- (10) Bessel Functions of Real Argument and Integer Order. D.J. Sookne. To be submitted to Communications of ACM.
- (11) Bessel Functions I and J of Complex Argument and Integer Order. D.J. Sookne. To be submitted to Communications of ACM.



Talks:

- (1) D.W. Lozier. Validation of Mathematical Function Routines. Expository Seminar Series of the Applied Mathematics Division. October 12, 1972.
- (2) W.L. Sadowski. Documentation of Mathematical Function Routines. SIAM-SIGNUM Fall Meeting, University of Texas, Austin, Texas, October 18, 1972.
- (3) W.L. Sadowski. A Unified Standards Approach to Algorithm Testing. Invited talk at the Software Alliance Symposium in Granby, Colorado, August 27-29, 1972.

## 2. OPERATIONS RESEARCH

### CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: A.J. Goldman

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Section staff provided miscellaneous consulting and advisory services in 135 recorded instances, 72 involving assistance to NBS staff. The 135 (72) instances totalled to 750 (275) man-hours. Other agencies assisted included the Highway Research Board, NASA, HEW, DOC (International Affairs), UMTA, Forestry Service, and various DOD units (Defense Communications Agency, Army Materiel Systems Analysis Command, Aberdeen Proving Grounds, Eglin AFB).

(2) W.G. Hall continued assistance to the Army Computer Systems Command, serving as official observer during the acceptance testing of a new fire-control computer. (Reported here for convenience; supported under Project 6509425.)

(3) P.B. Saunders completed collaboration in (the continuation of) a study, for the FAA, to study the effectiveness of alternative plans for the assignment of aircraft "beacon" codes. (Reported here for convenience; executed under Project 2050453 set up for overhead-capture purposes as an offshoot of Project 6504453.)

(4) A.J. Goldman continued service as Associate Editor of the Operations Research Society of America's "Transportation Science", and participated in an invitational conference on Forecasting Urban Travel Demand sponsored by the Highway Research Board. He continued as ORSA's representative to the Conference Board of the Mathematical Sciences, and completed a term on the CBMS Executive Council. Goldman also continued as NBS representative to the Mathematical Sciences Division of the National Research Council. At NBS, he continued service as a consultant to the Experimental Technological Incentives Program (ETIP).

#### Publications

- (1) R.D. Elbourne (Div. 650) and P.B. Saunders. Simulation of Air Traffic Control Radar Beacon Code Assignment Plan 3dl. FAA Report FAA-RD-72-103 (9/72).
- (2) A.J. Goldman. The adequacy of management science technology for non-military applications in the Federal government. To appear in Proc., Amer. Soc. Public Admin. Workshop on Management Science in the Federal Government (11/69).
- (3) A.J. Goldman. Product solutions and separation of variables. To appear in Journal of Research NBS 76B (1972).

### COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: A.J. Goldman

Full task description: October-December 1964 issue, p.3; April-June 1962, p.15

Status: CONTINUED

(1) J. Gillsinn completed documenting work with C. Witzgall (Boeing Scientific Research Labs) on comparative evaluation of shortest-path algorithms.

(2) J. Edmonds delivered four lectures on various aspects of combinatorial optimization, and formulated a novel approach to routing in certain communication networks.

(3) W.A. Horn developed simple algorithms to minimize maximum lateness and total delay for single-machine and multi-machine problems concerned with the scheduling of single-operation jobs without penalty for interruption.



## Publications

- (1) J. Edmonds and Peyton Young (CUNY). Matroid designs. Submitted to a technical journal.
- (2) J. Edmonds. Edge-disjoint branchings. Submitted to a technical journal.
- (3) A.J. Goldman. Minimax location of a facility in a network. Transportation Science 6 (1972), 107-118.
- (4) W.A. Horn. Three results for trees, using mathematical induction. Journal of Research NBS, 76B (1972), 39-43.
- (5) W.A. Horn. Single-machine job sequencing with treelike precedence ordering and linear delay penalties. SIAM J. Appl. Math. 23 (1972), 189-202.
- (6) W.A. Horn. Scheduling jobs on a computer. NBS Report 10914 (9/72).
- (7) W.A. Horn. Minimizing average flow time in a multi-machine job shop. To appear in Operations Research.
- (8) W.A. Horn. Optimal design of sorting networks. Submitted to a technical journal.
- (9) W.A. Horn. Some simple scheduling algorithms. Submitted to a technical journal.

## LINEAR AND NON LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: W.G. Hall

Full task description: October-December 1960 issue, p.3

Status: CONTINUED

- (1) Acquisition and assimilation of mathematical programming programs continued. Emphasis was on codes for transportation and capacitated transportation problems. R.H.F. Jackson attended the Advanced Seminar on Mathematical Programming at the University of Wisconsin's Mathematics Research Center.
- (2) T.B. Ayers and R.H.F. Jackson completed computer subroutines for ranging and parametric programming on objective functions and right hand sides of linear programs, to be used with the NBS subroutines (NBS Report 10695, 2/72). Documentation is now undergoing pre-release editing.
- (3) T. Shaftel conducted extensive tests of his continuous multiple modular design algorithm. Shaftel lectured on this material, and surveyed NBS for applications. Implementation of the algorithm into a user-oriented FORTRAN subroutine was carried out by W.G. Hall. Testing is now under way.
- (4) Consultation services continued for users of mathematical programming on the NBS computer installation. This work was mostly by P. Saunders.

## Publications

- (1) A.J. Goldman. Minimax-based selection, adjustment and disaggregation for incompletely specified univariate distributions. Journal of Research NBS 76B (1972), 1-38.
- (2) A.J. Goldman. Scheduling a time shared server to minimize aggregate delay. To appear in Journal of Research NBS 76B (1972).

OPERATIONS RESEARCH SERVICES FOR TECHNICAL ANALYSIS DIVISION

Task 205-12-2050586

Origin: NBS (Div. 431)

Authorized 7/15/71

Sponsor: NBS, various other agencies

Manager: A.J. Goldman

Full task description: July-December 1970 issue, p.8

Status: CONTINUED

(1) Project 4310900: A.J. Goldman participated in a number of project and document reviews for the sponsoring division, and continued editing the proceedings of a conference on cost-effectiveness analysis.

(2) Project 4314518: J. Gilsinn and L.S. Joel continued participation in this study, to obtain estimates of magnitude of the nation's lead-paint poisoning problem, plus means of identifying areas where the problem is likely to be especially severe.

(3) Project 4314552: A.J. Goldman continued consultation on this project, to evaluate the Shirley Highway Express Busway Demonstration Project.

(4) Project 4314487: P.B. Saunders and R.H.F. Jackson continued collaboration in this study, to aid the Army Nurse Corps in personnel-level planning.

(5) Project 4314431: A.J. Goldman, W.G. Hall and W.A. Horn (with Dr. H.J. Oser 205.01) continued participation in this project, to assist DOT's Urban Mass Transportation Administration in several systems-analysis tasks.

Publications

(1) J. Gilsinn. Estimates of the nature and extent of lead paint poisoning in the United States. NBS Technical Note 746 (12/72).

### 3. PROBABILITY AND MATHEMATICAL STATISTICS

#### RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

6030-11

Origin: NBS

Authorized 10/1/62

Manager: Joan Raup Rosenblatt

Full task description: July - December 1962

Status: CONTINUED. Roy H. Wampler has completed a paper on recent developments in linear least squares computations, reporting the comparative performance with respect to accuracy of three algorithms that had previously been found to be generally superior. One of these is the Bureau's ORTHO; the other two were published in ALGOL by Björck and Golub (1967) and Björck (1968). Wampler has developed ANSI FORTRAN versions of the latter two. Using six sets of test problems, he concludes that the two new algorithms are superior to ORTHO, but points out that certain pivoting features of these algorithms are incompatible with requirements to compute quantities needed in statistical analysis of the least squares results. The paper includes an annotated bibliography of 184 items, most of which were published during the past five years, and a table citing publications that include computer programs.

Wesley L. Nicholson, on leave from the Battelle Pacific Northwest Laboratories, has been appointed Institute for Basic Standards Visiting Scientist for the year through July 1973. His principal activities will be implementation and application of "resistive" techniques for exploratory data analysis. Several resistive regression algorithms are being tested, and are being experimentally applied in the analysis of electron diffraction data from crystal-structure investigations.

#### Publications:

- (1) Tables of two-associate-class partially balanced designs. Willard H. Clatworthy (State Univ. of New York, Buffalo), with contributions by J. M. Cameron (Office of Measurement Services) and J. A. Speckman. To appear as NBS Applied Math. Series 63.
- (2) W. J. Youden, 1900-1971. Churchill Eisenhart (Senior Research Fellow, Institute for Basic Standards) and Joan R. Rosenblatt. Annals of Mathematical Statistics, Vol. 43 (1972), pp. 1035-1040.
- (3) The percent point function. James J. Filliben. Submitted to a technical journal.
- (4) Coefficients to assist the determination of the moments of non-central t. D. Hogben (with R. S. Pinkham and M. B. Wilk). Table 28 in Biometrika Tables for Statisticians, Vol. 2 (E. S. Pearson and H. O. Hartley, eds.), Cambridge Univ. Press, 1972, page 248. (Reprinted from Biometrika, 1961.)
- (5) Efficient techniques for unbiasing a Bernoulli generator. James A. Lechner. J. Res. NBS-B. Math. Sciences, Vol. 76B (1972), pp. 53-60.
- (6) Design and analysis of experiments. Mary G. Natrella. To appear as a chapter in the third edition of Quality Control Handbook (J. M. Juran, ed.).
- (7) The t-statistic for a double exponential distribution. Raymond C. Sansing. Submitted to a technical journal.
- (8) Covariances of two-sample rank sum statistics. Peter V. Tryon. J. Research NBS, B-Math. Sciences, Vol. 76B (1972), pp. 51-52.
- (9) Some recent developments in linear least squares computations. Roy H. Wampler. To appear in Proc., Computer Science and Statistics: Sixth Annual Symposium on the Interface, Berkeley, California, October 16-17, 1972.

# DEVELOPMENT OF "OMNITAB"

Task 20503-12-2050131

6030-11

Origin and Sponsor: NBS

Authorized 11/1/68

Managers: David Hogben, Sally T. Peavy

Full task description: July - December 1968

Status: CONTINUED. The processing of 80 suggestions and comments from OMNITAB II users was begun.

Sally T. Peavy and David Hogben, in collaboration with Wesley L. Nicholson, are developing a set of portable computer programs for exploratory data analysis. High quality modular subprograms are being developed. Initial work is directed towards a "stem and leaf plot" procedure and a "median polish" procedure for two-way tables. The results will be available for incorporation in OMNITAB and also for general use.

OMNITAB II Version 5.03, correcting a few minor errors and making a few minor improvements, was implemented. Errata sheets 3 and 3A were sent to the purchasers of NBS Tape 1, "OMNITAB II Magnetic Tape and Documentation Parcel."

Sally T. Peavy has completed the development of a comprehensive set of instructions to provide CALCOMP plotting capability.

## Publications:

- (1) OMNITAB II: A computing system for statistical problem solving using Dutch rather than FORTRAN or ALGOL. David Hogben. Statistica Neerlandica, Vol. 26 (1972), pp. 85-99.
- (2) OMNITAB II Segmentation Structure for the SCOPE Operating System. S. T. Peavy and R. N. Varner (Office of Measurement Services). NBS Tech Note 734, June 1972.



#### 4. STATISTICAL ENGINEERING SERVICES

##### COLLABORATION ON STATISTICAL ASPECTS OF

##### NBS RESEARCH AND TESTING

Task 13911-612050950/51-1

9500-11

Origin: NBS

Authorized 7/1/50

Managers: H. H. Ku, J. R. Rosenblatt

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of services performed are the following.

James J. Filliben and John E. McKinney (Polymers Division) developed and implemented a method to determine confidence limits for the abscissa of the intersection of two straight lines, for the case of unequal variances. The problem arose in analysis of thermodynamic data to evaluate the uncertainties in the second order transition temperature and the glass transition temperature. Both transitions are manifested by discontinuities in the variation with temperature of second order properties, and the transition temperature may be determined by the intersection of two straight lines. Satisfactory analysis of McKinney's data required the generalization of existing techniques to the case where the residual variances for the two straight lines cannot be assumed to be equal.

H. H. Ku and Ruth N. Varner (Office of Measurement Services) have completed a note on the use of their computer programs for analysis of contingency tables. The note consists of (1) a brief description of each program to facilitate their selection and use, (2) the control cards, (3) the output and options, and (4) a numerical example which can also serve as a test problem.

James J. Filliben and Harry H. Ku assisted John K. Taylor (Analytical Chemistry Division) in the analysis of daily sulfur dioxide concentration data collected at Hayden, Arizona and Kellogg, Idaho. The results of this analysis will be used by the Dept. of Commerce General Counsel's Office in recommending the degree of emission control that must be required for smelters in certain Western States. One source of uncertainty in the present analysis was the apparent disagreement among records collected by parties having different interests with respect to regulatory requirements.

Mary G. Natrella and Joan R. Rosenblatt are assisting Eric A. Vadelund (Program Manager - Fair Packaging and Labelling, Office of Weights and Measures) to develop a program leading to the issue of documents to supersede NBS Handbook 67, Checking Prepackaged Commodities. The work will be done in collaboration with the U. S. Department of Agriculture and the Food and Drug Administration, with the intention that one set of procedures for checking net quantity will serve the needs of State and local weights and measures officials. NBS Handbook 67 is out of print; Mary Natrella and Joan Rosenblatt have outlined a proposal for replacing it by a system of documents in looseleaf format. This proposal will be discussed further with USDA and FDA and will be the basis for a unified approach to mutually consistent Federal procedures.

H. H. Ku collaborated with H. Bowman (Optical Physics Division) in the determination of volumes of four pieces of single crystal silicon as standards for volume measurements. The experiment consists of three parts:

- 1) Measurements of the length of a quartz etalon from January 1970 thru August 1972 by laser interferometer.
- 2) Measurements of diameters of six near perfect steel balls thru the use of the etalon.
- 3) Comparisons of volumes of steel balls with those of silicon crystals by hydrostatic weighing to obtain sets of ratios:

$$\frac{s_1}{s_2}, \frac{s_1}{x_2 + x_3}, \frac{s_1}{x_4 + x_5}, \frac{s_2}{x_2 + x_3}, \frac{s_2}{x_4 + x_5}, \frac{x_2 + x_3}{x_4 + x_5}$$

where  $S_1$  and  $S_2$  denote volumes of steel balls, and  $X_2, X_3, X_4, X_5$  volumes of four crystals. Sets of ratios were obtained for each of the three pairs of balls.

If the volumes of these crystals can be measured to within an uncertainty of 1 - 2 ppm, they will be used as volume standards for the redetermination of the Avogadro constant.

#### NBS Boulder Laboratories

John F. LaBrecque and Peter V. Tryon assisted H. M. Altschuler (Electromagnetics Division) in review of a draft report on an international intercomparison of microwave low power standards at 10 GHz. The report, prepared at the Electrotechnical Laboratory in Tokyo, was to be discussed at an October meeting of a CCE [Comité Consultatif d'Electricité] Working Group. Begun in 1965, the experiment was planned (in Tokyo) as a complete two-factor experiment (7 labs, 10 standards) but ended up with 5 labs, 9 standards, and 20 missing observations due to loss or breakage of standards. Statistical analysis of these results was followed by work on experiment design for two more intercomparisons for which NBS will be the pilot laboratory. The new experiments will be designed to permit verification of the assumption (that had to be made in the present analyses) that the within-laboratory measurement errors have the same variability at all laboratories.

Peter V. Tryon is participating in the development of an Electromagnetics Division policy on measurement and evaluation of precision connectors and adapters. He pointed out that a connector is inherently a stochastic device having no constant characteristic electrical parameters, and hence must be characterized by a joint statistical distribution of characteristics that might be observed on repeated connections. Accordingly, Tryon proposes that work be initiated to develop practical methods for describing the distribution that characterizes a connector, and notes that practical calibration experiments must be developed for estimating distributional properties rather than physical constants.

#### Publications:

- (1) Confidence limits for the abscissa of intersection of two linear regressions. James J. Filliben and John E. McKinney (Polymers Div.). To appear in NBS J. Res.-B. Math. Sciences.
- (2) A user's guide to the OMNITAB command "STATISTICAL ANALYSIS". H. H. Ku. To appear as an NBS Technical Note.
- (3) Variability of spectral tristimulus values. Isadore Nimeroff (Building Research Division), Joan R. Rosenblatt and Mary C. Dannemiller. Paper 6.2 in Precision Measurement and Calibration: NBS Papers on Colorimetry, NBS Spec. Pub. 300, Vol. 9, pp. 355-363. (Reprinted from J. Research NBS, 1961).
- (4) Material variability as measured by low temperature electrical resistivity. Alan F. Clark (Cryogenics Division) and Peter V. Tryon. To appear in Cryogenics.

#### STATISTICAL SERVICES

Task 20503-40-2050132/58-346

5040-11

Origin: Various Agencies

Authorized 3/31/58

Sponsor: NBS

Manager: J. R. Rosenblatt

Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

James J. Filliben has assisted Raymond H. Marcotte, Jr. (Stock Market Statistics Section, SEC) with implementation of Filliben's subroutines for spectral analysis of time series.



Publications:

- (1) Comments on 'Treatment of null responses' by Genevieve L. Meyer and Ronald L. Johnson. James J. Filliben. To appear in Proc. 17th Conf. on Design of Experiments in Army Research, Development and Testing, Oct. 27-29, 1971.
- (2) Statistical methods applicable to counting experiments and evaluation of experimental data. H. H. Ku. To appear in Proc. First International Summer School on Radionuclide Metrology, Herceg Novi, Yugoslavia, August 21-31, 1972.
- (3) Effective statistical tests for detection models. James A. Lechner. To appear in Proc. Military Operations Research Soc. Symposium, Dec. 12-14, 1972, Fort Lee, Virginia.

## 5. OTHER ACTIVITIES

### RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

1540-11

Origin: NBS  
Manager: Morris Newman  
Full task description: July - September 1954 issue, p. 1

Authorized 8/29/54

Status: CONTINUED. M. Newman, in the course of developing and checking his routines for multi-precision and congruential arithmetics, extended his table of the Ramanujan function  $\tau(p)$  modulo  $p$ ,  $p$  prime, to all primes  $p$  less than 20000. In that range the only solutions of  $\tau(p) \equiv 0 \pmod{p}$  are  $p = 2, 3, 5, 7, 2411$ .

Newman also continued investigations in group theory and number theory. In the former field, together with R.C. Lyndon (U. of Michigan), he showed that a product of  $n$  squares in a group  $G$  is in general not expressible as a product of fewer than  $n$  squares in  $G$ . This has been generalized to higher powers by Newman, Lyndon, and T. McDonough (U. of Wales at Aberystwyth). In the latter area, studying Diophantine equations in algebraic number fields, Newman showed that in such a field of degree  $n$  there can be at most  $n$  "consecutive" units, and for each  $n$  this result is best possible.

C.R. Johnson investigated a number of matrix-theory problems relating to eigenvalues and stability theory. The classical theorem of Lyapunov, on "stable" matrices (those with all eigenvalues in the left half-plane), was extended by generalizing the half-plane to an arbitrary angular sector. He derived Gersgorin type bounds for the numerical radius of a matrix (and hence for its eigenvalues), and analyzed sufficient conditions for D-stability. (Matrix  $A$  is "D-stable" if  $DA$  is stable for all positive diagonal  $D$ ; so far this concept has been used mainly in mathematical economics.) Investigating the Hadamard (entrywise) matrix product  $A*B$ , of renewed interest in statistical analysis (cf. G.P.H. Styan, J. Lin. Alg. and Appl. 6 (1973), 217-240), Johnson derived bounds for its eigenvalues and field of values, and gave sufficient conditions for  $A*A^*$  to be positive definite. Johnson also edited and added a bibliography of related work to an oft-requested Bureau report by Helmut Wielandt on eigenvalues of sums and products of matrices; it is to appear in the Journal of Research NBS, Section B.

In work on additive commutators, Johnson showed that for the equation  $A = XY - YX$  in square matrices, with  $\text{Tr}(A) = 0$ , the spectra of  $X$  and  $Y$  can be specified arbitrarily (so long as at least one has no repeated roots). With C.R. DePrima (California Institute of Technology) he began an investigation of the mapping  $A \rightarrow A^{-1}A^*$ , proving that its range consists of those matrices  $T$  with  $T^{-1}$  similar to  $T^*$ .

#### Publications:

- (1) Commutators as products of squares. M. Newman and R.C. Lyndon. To appear in the Proc. Amer. Math. Soc.
- (2) Products of powers in groups. M. Newman, R.C. Lyndon, and T. McDonough. Submitted to a technical journal.
- (3) Modular quotient groups. M. Newman. Submitted to a technical journal.

- (4) Diophantine equations in cyclotomic fields. M. Newman. To appear in J. Reine Angew. Math.
- (5) On the eigenvalues of  $A + B$  and  $AB$ . H. Wielandt. Edited by C.R. Johnson. To appear in the Journal of Research NBS, Section B.
- (6) Gersgorin sets and the field of values. C.R. Johnson. Submitted to a technical journal.
- (7) A note on matrix solutions to  $A = XY - YX$ . C.R. Johnson. Submitted to a technical journal.
- (8) A Lyapunov theorem for angular cones. C.R. Johnson. Submitted to a technical journal.
- (9) Hadamard products of matrices. C.R. Johnson. Submitted to a technical journal.
- (10) The Hadamard product of  $A$  and  $A^*$ . C.R. Johnson. Submitted to a technical journal.
- (11) The range of  $A^{-1}A^*$ . C.R. Johnson and C.R. DePrima. (Work in progress.)

BIOMEDICAL IMAGE AND LANGUAGE PATTERN PROCESSING

Task 20500-2050404

1550-22

Origin: NBS  
Sponsor: National Cancer Institute  
Manager: Russell A. Kirsch  
Full task description: January-June issue 1964, p. 19

Authorized: 1/21/64

Status: TERMINATED. See discussion of project 2050408.

SCANNING MICROSCOPE PROCESSING

Task 20500-2050408

Origin: NBS  
Sponsor: National Cancer Institute  
Manager: Russell A. Kirsch  
Full task description: July-December issue 1968, p. 14

Status: TERMINATED. Final assistance was furnished to the sponsor in the construction of an image processing laboratory at NIH which is now in full scale operation. Documentation was produced for the construction of the image processing electronics.

Publications:

- (1) Programs for the morphological analysis and resynthesis of microscope scanned biological images. Anne H. Meininger. NBS Report 10853. September 1972.
- (2) A technical manual for the SPECTRE II Computer-Controlled Microscope. Philip G. Stein (Division 200.01) and Anne H. Meininger. NBS Report 10962. December 1972.

CHEMICAL BIOLOGICAL INFORMATION PROCESSING

Task 20500-2050410

1550-22

Origin: NBS  
Sponsor: National Institutes of Health, Division of Research Resources  
Biotechnology Resources Branch  
Manager: Russell A. Kirsch  
Full task description: July-December issue 1968, p. 14

Status: Terminated. Advisory work completed.

ARTIFICIAL INTELLIGENCE

Task 20500-2050104

Origin: NBS  
Manager: Russell A. Kirsch

Status: To begin during the next reporting period.

# MATHEMATICAL AIDS

Task 20500-40-2050121/57-216

Origin and Sponsor: NBS

Manager: I. A. Stegun

Full task description: July-December 1964 issue, p.4.

Status: CONTINUED. Consultations continued at the rate of about 10 per month with staff of NBS, other government agencies, industry and universities. Topics covered for the most part were tables and programs for the special functions with emphasis on computing techniques and pitfalls.

Corrections were submitted for incorporation in the 10th printing of AMS 55 - the Handbook of Mathematical Functions. Questioned items are checked and an errata list is maintained covering all printings of the volume.

A paper detailing the automatic methods for computing the exponential integral  $E_n(x)$  has been prepared for publication. The entire range of arguments and/or functional values is covered in the implementing ANSI FORTRAN program which is set up to readily allow for variable precision. Test results are also included to assist in the checking of program modifications.

Further exploratory work continues on automatic integration routines in connection with the computation of special functions, in particular, the Sievert (secant) integral, prominent in radiation-safety calculations and Fresnel integrals, for which current Bureau work in optics requires algorithms in preference to tables and extends into ranges beyond those of past calculations.

Publications: Mathematics Bibliography. Irene A. Stegun and Constance Carter (Library of Congress). Section 1a. American Institute of Physics Handbook, 3rd Edition. McGraw-Hill, Inc., New York, 1972.

Automatic Computing Methods for Special Functions. Part II. The Exponential Integral  $E_n(x)$ . Irene A. Stegun and Ruth Zucker. In process.

## Lectures and Technical Meetings

### Papers and Invited Talks

#### Presented by Members of the Staff at Meetings

#### of Outside Organizations

- COLVIN, Burton H. The Research Administrator. Society of Research Administrators, Sixth Annual Meeting, Seattle, Washington. October 10.
- FILLIBEN, James J. Some Useful Procedures for the Statistical Analysis of Data. Society for Applied Spectroscopy, Dallas, Texas. September 12.
- Some Useful Procedures for the Statistical Analysis of Data. Amer. Statist. Assoc. and Amer. Soc. for Quality Control, Knoxville, Tennessee. October 20.
- Techniques for Tail Length Analysis. Eighteenth Conference on the Design of Experiments in Army Research, Development and Testing, Aberdeen Proving Ground, Maryland. October 26.
- FONG, J. T. A Non-Equilibrium Thermodynamic Theory of Simple Materials Based on a Single-Integral Entropic Functional. International Symposium on Foundations of Plasticity, Polish Academy of Sciences, Warsaw, Poland. August 31.
- A Discussion Note on Y. Horie's "On the Thermodynamic States of Plastically Deformed Solids". International Symposium on Foundations of Plasticity, Polish Academy of Sciences, Warsaw, Poland. August 30.
- GOLDMAN, A. J. Some Aspects of Optimal Facility Location. University of Maryland(Baltimore). November 15.
- JACKSON, R. H. F. A System for Position-Location Based on Ranges. 42nd National Meeting, Operations Research Society of America, Atlantic City, New Jersey. November 10.
- KU, H. H. Statistical Methods Applicable to Counting Experiments and Evaluation of Experimental Data. First International Summer School on Radionuclide Metrology, Herceg Novi, Yugoslavia. August 31.
- Recent Development in Contingency Table Analysis. Amer. Statist. Assoc. and Amer Soc. for Quality Control, Knoxville, Tennessee. October 19.
- LECHNER, James A. A System for Position-Location Based on Ranges. Eighteenth Conference on the Design of Experiments in Army Research, Development and Testing, Aberdeen Proving Ground, Maryland. October 26.
- Effective Statistical Tests for Detection Models. Military Operations Research Society, Fort Lee, Virginia. December 13.
- ROSENBLATT, Joan, R. Discussion of "A Statistical Method of Calibration" by Henry Scheffe. Gordon Research Conference on Statistics in Chemistry and Chemical Engineering, New Hampton, New Hampshire. July 10.
- SADOWSKI, W. L. Unified Standards Approach to Algorithm Testing. Software Alliance Symposium, Granby, Colorado. August 28.
- Documentation of Mathematical Function Routines. SIAM-SIGNUM Fall Meeting, University of Texas, Austin, Texas. October 18.



SHAFTEL, T. L. Computation Experience with Algorithms for the Continuous Modular Design and Related Problems (with G. L. Thompson). 42nd National Meeting, Operations Research Society of America, Atlantic City, New Jersey. November 8.

SOOKNE, David J. A System for Position-Location Based on Ranges. Annual Meeting of the Md.-D. C.-Va. Chapter of the Mathematical Association of America. U. S. Naval Academy, Annapolis, Maryland. November 18.

WAMPLER, Roy H. Some Recent Developments in Linear Least Squares Computations. Computer Science and Statistics: 6th Annual Symposium on the Interface, Berkeley, California. October 17.

#### Applied Mathematics Division Expository Seminar

FLATTO, Leopold (Yeshiva University, New York, New York). A Limit Theorem for Random Coverings of a Circle. September 13.

NEWMAN, Morris Computing with Large Numbers. September 14.

LOZIER, Daniel W. Validation of Mathematical Function Subroutines. October 12.

KUSADA, Tamami (Center for Building Research) Finite Element Method for Steady State Heat Conduction Analyses. November 9.

FATTAL, S. George (Center for Building Research) Finite Element Method for Structural Engineering Applications. December 14.

#### Applied Mathematics Division Lectures

BLEICHER, Michael (University of Wisconsin, Department of Mathematics, Madison, Wisconsin). "Egyptian Fractions". December 11.

#### Operations Research Seminars

EDMONDS, J. R. Four Lectures on Combinatorial Optimization. August.

SHAFTEL, T. L. Cost Reduction through the Design of Standard Subassemblies; Mathematical Formulation. July 13.

A Simplex-like Algorithm for a Class of Geometric Programming Problems, I & II. July 20 & 27.

#### In-Hours Courses, Fall Semester

NATRELLA, Mary G. Experimental Statistics. NBS Graduate School.

TRYON, Peter V. Statistical Methods in Research. University of Colorado.

## Publication Activities

### 1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

#### 1.2 Manuals, Bibliographies

OMNITAB II Segmentation Structure for the SCOPE Operating System, S. T. Peavy and R. N. Varner (Office of Measurement Services). NBS Technical Note 734, June 1972.

Mathematics Bibliography. Irene A. Stegun and Constance Carter (Library of Congress). Section 1a, American Institute of Physics Handbook, 3rd Edition. McGraw-Hill, Inc., New York, 1972.

#### 1.3 Technical Papers

W. J. Youden, 1900-1971. Churchill Eisenhart (Senior Research Fellow, Institute for Basic Standards) and Joan R. Rosenblatt. Annals of Mathematical Statistics, Vol. 43, pp. 1035-1040 (1972).

A Non-Equilibrium Thermodynamic Theory of Simple Materials Based on a Single-Integral Entropic Functional. J. T. Fong and J. A. Simmons. Archives of Mechanics (Arch. Mech. Stoc., Warsaw), Vol. 24, pp. 363-372 (1972).

Minimax-Based Selection, Adjustment and Disaggregation for Incompletely Specified Univariate Distributions. A. J. Goldman. J. of Research NBS 76B, 1-38 (1972).

Minimax Location of a Facility in a Network. A. J. Goldman. Transportation Science 6, 107-118 (1972).

Experiments on Optimal Coefficients. S. Haber. Applications of Number Theory to Numerical Analysis, Academic Press, New York, 1972. (pp. 11-37).

The Existence of Simple Quadrature. S. Haber. Michigan Mathematical Journal, Vol. 19, pp. 401-410 (1972).

Coefficients to Assist the Determination of the Moments of Non-Central  $t$ . D. Hogben (with R. S. Pinkham and M. B. Wilk). Table 28 in Biometrika Tables for Statisticians, Vol. 2 (E. S. Pearson and H. O. Hartley, eds.), Cambridge University Press, 1972, page 248. (Reprinted from Biometrika, 1961.)

OMNITAB II: A Computing System for Statistical Problem Solving Using Dutch Rather than FORTRAN or ALGOL. David Hogben. Statistica Neerlandica, Vol. 26, pp. 85-99 (1972).

Three Results for Trees, Using Mathematical Induction. W. A. Horn. J. of Research NBS 76B, 39-43 (1972).

Single-Machine Job Sequencing with Treelike Precedence Ordering and Linear Delay Penalties. W. A. Horn. SIAM J. Appl. Math. 23, 189-202 (1972).

Efficient Techniques for Unbiasing a Bernoulli Generator. James A. Lechner. J. of Research NBS 76B, 53-60 (1972).

Variability of Spectral Tristimulus Values. Isadore Nimeroff (Building Research Division), Joan R. Rosenblatt and Mary C. Dannemiller. Paper 6.2 in Precision Measurement and Calibration: NBS Papers on Colorimetry, NBS Spec. Pub. 300, Vol. 9, pp. 355-363. (Reprinted from J. of Research NBS, 1961).

Note on Backward Recurrence Algorithms. F. W. J. Olver and D. J. Sookne. Mathematics of Computation, Vol 26, pp. 941-947 (1972).

A Unified Standards Approach to Algorithm Testing. W. L. Sadowski and D. W. Lozier. Program Test Methods. Prentice Hall, 1972.

Use of Olver's Algorithm to Evaluate Certain Definite Integrals of Plasma Physics Involving Chebyshev Polynomials. W. L. Sadowski and D. W. Lozier. Jour. of Comp. Physics, Vol. 10, No. 3, pp. 607-613 (1972).

Covariances of Two-Sample Rank Sum Statistics. Peter V. Tryon. J. of Research NBS, pp. 51-52 (1972).

## 1.7 Reports

Scheduling Jobs on A Computer. W. A. Horn. NBS Report 10914. September 1972.

Programs for the Morphological Analysis and Resynthesis of Microscope Scanned Biological Images. Anne H. Meininger. NBS Report 10853. September 1972.

A Technical Manual for the SPECTRE II Computer-Controlled Microscope. Philip G. Stein (Division 200.01) and Anne H. Meininger. NBS Report 10962. December 1972.

Polynomial Real Root Evaluation by Sturm's Theorem Using Modular Arithmetic. G. W. Reitwiesner. NBS Report 10883. October 1972.

Simulation of Air Traffic Control Radar Beacon Code Assignment Plan 3dl. R. D. Elbourne (Division 650) and P. B. Saunders. FAA Report FAA-RD-72-103. September 1972.

Supplement to NBS Final Report on Project WHERE. NBS Report 10940. October 1972.

## 1.8 Technical Notes

Estimates of the Nature and Extent of Lead Paint Poisoning in the United States. J. Gilsinn. NBS Technical Note 746. December 1972.

## 2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

### 2.2 Manuals

A User's Guide to the OMNITAB Command "STATISTICAL ANALYSIS". H. H. Ku. To appear as an NBS Technical Note.

Design and Analysis of Experiments. Mary G. Natrella. To appear as a chapter in the third edition of Quality Control Handbook (J. M. Juran, ed.).

### 2.3 Technical Papers

Material Variability as Measured by Low Temperature Electrical Resistivity. Alan F. Clark (Cryogenics Division) and Peter V. Tryon. To appear in Cryogenics.

Edge-Disjoint Branchings. J. Edmonds. Submitted to a technical journal.

Matroid Designs. J. Edmonds and Peyton Young (CUNY). Submitted to a technical journal.

Comments on 'Treatment of Null Responses' by Genevieve L. Meyer and Ronald L. Johnson. James J. Filliben. To appear in Proc. 17th Conference on Design of Experiments in Army Research, Development and Testing, October 27-29, 1971.

Confidence Limits for the Abscissa of Intersection of Two Linear Regressions. James J. Filliben. To appear in J. of Research NBS, B. Mathematical Sciences.

A Discussion Note on Y. Horie's "On the Thermodynamic States of Plastically Deformed Solids". J. T. Fong. To appear in Proc. International Symposium on Foundations of Plasticity, Warsaw, Poland, 1972. (A. Sawczuk, ed.).

A Decomposition Theorem on the Scalar Potential of a Compressible Bernstein-Kearsley-Zapas Fluid. J. T. Fong and J. A. Simmons. Submitted to a technical journal.

Distance Coordinates with Respect to a Triangle of Reference. K. Goldberg. To appear in J. of Research NBS, B. Mathematical Sciences.

Product Solutions and Separation of Variables. A. J. Goldman. To appear in J. of Research NBS, 76B(1972).

Scheduling a Time Shared Server to Minimize Aggregate Delay. A. J. Goldman. To appear in J. of Research NBS, 76B(1972).

Some Simple Scheduling Algorithms. W. A. Horn. Submitted to a technical journal.

On the Eigenvalues of  $A + B$  and  $AB$ . H. Wielandt. Edited by C. R. Johnson. To appear in the J. of Research NBS, B. Mathematical Sciences.

Gersgorin Sets and the Field of Values. C.R. Johnson. To appear in the J. Math. Anal. and Appl.

A Note on Matrix Solutions to  $A = XY - YX$ . C. R. Johnson. Submitted to the Israel J. Math.

A Lyapunov Theorem for Angular Cones. C.R. Johnson. Submitted to the J. of Research NBS.

Hadamard Products of Matrices. C. R. Johnson. Submitted to the J. Linear and Multilinear Alg.

The Hadamard Product of  $A$  and  $A^*$ . C.R. Johnson. Submitted to a technical journal.

Convergence of Semidiscrete Approximations of Linear Transport Equations. S. R. Kraft. To appear in the Jour. of Mathematical Analysis and Applications.

Uniqueness and Existence for the Integral Equation of Interreflections. S. R. Kraft. To appear in the SIAM J. on Mathematical Analysis.

A Priori Inequalities for Semidiscretizations of a Time Dependent Transport Equation. S.R. Kraft. Submitted to a technical journal.

A Priori Inequalities for a Semidiscrete Stationary Linear Transport Equation. S.R. Kraft. Submitted to a technical journal.

Statistical Methods Applicable to Counting Experiments and Evaluation of Experimental Data. H. H. Ku. To appear in Proc. First International Summer School on Radionuclide Metrology, Herceg Novi, Yugoslavia, August 21-31, 1972.

Effective Statistical Tests for Detection Models. James A. Lechner. To appear in Proc. Military Operations Research Soc. Symposium, Dec. 12-14, 1972, Fort Lee, Virginia.

A Bit Comparison Program for Algorithm Testing. D. W. Lozier, L. C. Maximon and W. L. Sadowski. Accepted for publication in The Computer Journal.

Modular Quotient Groups. M. Newman. Submitted to a technical journal.

Diophantine Equations in Cyclotomic Fields. M. Newman. To appear in J. Reine Angew. Math.

Commutators as Products of Squares. M. Newman and R.C. Lyndon. To appear in the Proc. Amer. Math. Soc.

Products of Powers in Groups. M. Newman, R. C. Lyndon, and T. McDonough. Submitted to a technical journal.

Error Bounds for Stationary Phase Approximations. F. W. J. Olver. To appear in SIAM J. on Mathematical Analysis.

On Polynomial Real Root Evaluation by Sturm's Theorem and Integer (Modular) Arithmetic. G. W. Reitwiesner. To be submitted to a technical journal.

On a Source of Numerical Instability of the Fourier-Hermite Solution of the Nonlinear Vlasov Equation. W.L. Sadowski. Submitted to a technical journal.

The  $t$ -Statistic for a Double Exponential Distribution. Raymond C. Sansing. Submitted to a technical journal.

Some Recent Developments in Linear Least Squares Computations. Roy H. Wampler. To appear in Proc. Computer Science and Statistics: Sixth Annual Symposium on the Interface, Berkeley, California, October 16-17, 1972.



U.S. DEPT. OF COMM. BIBLIOGRAPHIC DATA SHEET		1. PUBLICATION OR REPORT NO. NBSIR 73 205	2. Gov't Accession No.	3. Recipient's Accession No.
4. TITLE AND SUBTITLE Projects and Publications of the Applied Mathematics Division A Semi-Annual Report July Through December 1972			5. Publication Date July -- 1973	
			6. Performing Organization Code	
7. AUTHOR(S) --			8. Performing Organization	
9. PERFORMING ORGANIZATION NAME AND ADDRESS  NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234			10. Project/Task/Work Unit No. --	
			11. Contract/Grant No. --	
12. Sponsoring Organization Name and Address  --			13. Type of Report & Period Covered 6/72-12/72	
			14. Sponsoring Agency Code	
15. SUPPLEMENTARY NOTES  This is a progress report rather than a sponsor report.				
16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)  Descriptions are given of all new division tasks. The status of all tasks is discussed and relevant publications listed. Staff activity in lectures, seminars, courses, etc. is included as well as staff publications.				
17. KEY WORDS (Alphabetical order, separated by semicolons)  Projects;publications;lectures; seminars; courses				
18. AVAILABILITY STATEMENT  <input type="checkbox"/> UNLIMITED.  <input checked="" type="checkbox"/> FOR OFFICIAL DISTRIBUTION. DO NOT RELEASE TO NTIS.		19. SECURITY CLASS (THIS REPORT)  UNCLASSIFIED		21. NO. OF PAGES  228
		20. SECURITY CLASS (THIS PAGE)  UNCLASSIFIED		22. Price

