NBS TECHNICAL NOTE 910-8

U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

SELF-STUDY MANUAL ON OPTICAL RADIATION MEASUREMENTS

Part 1—Concepts

Chapter 12

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[1] Headquarters and Laboratories at Gaithersburg, MD, unless otherwise noted; mailing address Gaithersburg, MD 20899.

[2] Some divisions within the center are located at Boulder, CO 80303.

[3] Located at Boulder, CO, with some elements at Gaithersburg, MD.

Self-Study Manual on Optical Radiation Measurements: Part 1 — Concepts, Chapter 12

Fred E. Nicodemus, Editor

Chapter 12. Blackbodies, Blackbody Radiation,
and Temperature Scales,
by Joseph C. Richmond and Fred E. Nicodemus

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PREFACE

The background, the objectives, and the content of what has been published and what is planned for the NBS Self-Study Manual on Optical Radiation Measurements have been discussed in detail in the Preface of each of the earlier technical notes in the 910- series with the preceding chapters of this Manual. Accordingly, we will not repeat it all here.

ACKNOWLEDGMENTS. It has not been feasible to segregate the activities on this project in any way that makes it possible to consider that only a certain portion is being carried out under the support of any single agency. All of the following have contributed, directly or indirectly, to all phases of the preparation of the Manual, including this Technical Note 910-8, at some time since 9/30/79, and we want to acknowledge their most welcome support:

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We are also deeply indebted to a great many individuals for invaluable "feedback" concerning the technical content of the Manual, that has encouraged us and has helped us to put these chapters together more effectively. We renew our strong invitation to all readers to submit comments, criticisms, and suggestions. In particular we would welcome illustrative examples and problems from as widely different areas of application as possible.

We sincerely regret that pressure of other work has kept Mrs. Betty Castle from being available again to provide her skillful typing and layout, as she did on all of the earlier issues of the Self-Study Manual. Instead, the Editor is preparing, on a word processor, the camera-ready text as he does the final editing. Mastering the word processor is turning out to be quite an adventure for the first experience on any electronic computer; we hope that satisfactory results can be achieved in the limited time available.

Fred E. Nicodemus, Editor

John B. Shumaker, Project Head

Henry J. Kostkowski, Consultant

January 1985.

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SELF-STUDY MANUAL on OPTICAL RADIATION MEASUREMENTS

Part I. Concepts

This is the eighth in a series of Technical Notes (910-) entitled "Self-Study Manual on Optical Radiation Measurements". It contains Chapter 12 of Part I of this Manual. Additional chapters will continue to be published, similarly, as they are completed. The Manual is a comprehensive tutorial treatment of the measurement of optical radiation that is complete enough for self-instruction. Detailed chapter summaries make it also a convenient authoritative reference source.

In this chapter we review the radiometric treatment, and the significance for radiometric measurements, of blackbodies, blackbody radiation, and temperature scales. Many important and interesting aspects are not treated because our primary interest is in radiometry and radiometric measurements. The emphasis is on ideal blackbodies and laboratory simulators; thermal radiation from other real sources will be treated in a chapter on Thermal Radiation Properties of Materials.

Key Words: blackbody, thermal radiation, temperature, Planck, Wien, Rayleigh-Jeans, Stefan-Boltzmann, ITS, IPTS.

Chapter 12. Blackbodies, Blackbody Radiation, and Temperature Scales

by Joseph C. Richmond and Fred E. Nicodemus

1. In this CHAPTER. Following a brief introduction, which emphasizes the effects of temperature on all radiometric measurements and the importance of a good understanding of temperature scales, and defines the term "blackbody", the important concepts relating to blackbody radiation are presented. The basic Planck law of blackbody radiation is given in terms of most of the important spectral variables. In an appendix, this is extended to still more spectral variables. The other important blackbody radiation laws -- the Stefan-Boltzmann law and the Wien Displacement law -- and the important approximations to the Planck law -- the Wien Distribution law and the Rayleigh-Jeans law -- are presented and discussed in some detail. This is followed by an account of the measurement of temperature and of temperature scales in the SI System (Système Internationale) including thermodynamic temperatures and the International Practical Temperature Scales (IPTS). Additional details about the current IPTS-68 and, for those who need them to cope with earlier literature, about its history and the scales that preceded it, are presented in an appendix along with references to sources for still more complete information.

2. INTRODUCTION.

2.1 BACKGROUND. In Chapter 5 [12.1][1] we introduced the concept of the measurement equation that is central to the entire treatment of radiometry in this Manual. It is the equation that relates, in terms of all of the pertinent parameters, the output signal from a radiometric instrument or device to the incident radiation beam that produces it. There, in Chapter 5 [12.1], we limited the treatment to the radiation parameters of just space (position and direction) and spectrum (wavelength, frequency, wave number, etc.). In Chapter 6 [12.2], it was extended to include the effects of another radiation parameter, polarization. A chapter dealing with the effects of the remaining radiation parameter, time, is

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[1] Figures in brackets indicate literature references listed at the end of this Technical Note.

still in preparation. Furthermore, we mentioned, with a few examples, that the quantities in the measurement equation are also governed by instrumental and environmental parameters, and that, unfortunately, there is no exhaustive list of these, as there is for the radiation parameters.

One ubiquitous and most often significant of these environmental and instrumental parameters is temperature. Both the responsivity and detectivity (noise-level limitation) of almost every radiation detector are functions, and in some cases they are very strong functions, of its temperature. The propagances (transmittances and reflectances) of materials along any optical path are sometimes quite dependent on the temperatures of those materials, and the emissivities of sources are also temperature dependent. Perhaps the most important consideration for radiometric measurements, especially in the middle and longer infrared wavelength regions of the spectrum, is the fact that all matter emits **thermal radiation**[1] as a consequence of its temperature. This includes all parts (optics, housing, chopper blades, etc.) of any measuring instrument or device, and all matter along any optical path, as well as the substances in any radiation source. In fact, in some radiometric measurements, a negative output signal is produced when the thermal radiation emitted by the detector element exceeds that incident on it from the source being measured (so-called "cold-target measurements"). In less extreme cases, it may still be important to recognize that a detector is responding only to the net incident flux in the beam through the instrument throughput, the difference between that from the source and its surroundings and that due to its own emission of thermal radiation toward the source. However, in very many radiometric measurements, particularly in the visible and ultraviolet regions of the spectrum, thermal radiation emitted by the detector is insignificant and may safely be disregarded.

It is clear from this discussion that it is particularly important in radiometry, especially in the infrared region, to be aware of, and to take into account, the effects of thermal radiation. We will devote two chapters to this topic of thermal radiation: (1) In this first chapter, we consider only **blackbody radiation**, including the thermal radiation emitted by an ideal blackbody and also that from real sources employed to approximate as closely as possible to blackbody radiation in actual measurements. We examine, particularly, how that blackbody radiation and its spectral distribution are related to the temperature of the blackbody source and we look also at the way in which this is related to the establishment of temperature scales. This, in turn, leads to a discussion of the closely related considerations of optical pyrometry, where source temperature is measured by evaluating emitted thermal radiation, with the emphasis in this chapter continuing to be on blackbody radiation. (2) In the second chapter, on thermal radiation properties of matter, this treatment is extended to all measurements of thermal radiation in radiometry and optical pyrometry by examining the ways in which thermal radiation is affected by the limitations and properties, such as emissivity, of real sources.

Note that the measurement-equation approach requires continuing emphasis on the basic distribution function, the spectral radiance along each ray, as a function of all of the radiation parameters. As noted below, ideal blackbody radiation is perfectly diffuse and uniform spatially, so that $M_{bb} = \pi \cdot L_{bb}$, making it equally well characterized by spectral exitance M_λ or spectral radiance L_λ. However, it is seldom that exitance is actually measured by collecting and measuring the flux emitted into a full hemisphere. Instead, it is usually inferred from a measurement of the average radiance over a much

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[1] Strictly, "thermal radiation" is the process by which matter emits electromagnetic radiation solely as a result of the thermal motions of the charged particles of which it is composed. The term is also loosely used, as above, to designate the resulting radiant flux.

smaller solid angle. Also, just as soon as consideration is given to real sources, even those designed to simulate blackbodies as closely as possible, we usually find that there is substantial variation of radiance with direction, the approximation being adequate only over a more or less limited solid angle. Accordingly, we continue to emphasize spectral radiance, rather than exitance, as the basic quantity for characterizing a radiation source, including a source of blackbody radiation.

Finally, just as in Chapter 6 [12.2] we made no effort to describe all of the many interesting polarization phenomena that are already very adequately treated in standard textbooks, we will not attempt to include all of the important and fascinating historical development of the theory of blackbody radiation that is already well covered in many standard texts on optics, optical physics, and theoretical physics. Here we start with the Planck law as the basic mathematical model of blackbody radiation, and some of the other "laws" will be presented as useful approximations,[1] even though historically they came first. On the other hand, we will limit this discussion to what we feel are the most essential basic concepts for radiometry. Additional details will be found in such books as Bramson [12.3] and in the Infrared Handbook [12.4]. Our objective is to provide, here, just enough to give the reader the concepts needed to set up the measurement equation and to apply it to the design and execution of a measurement to a desired degree of accuracy.

2.2 DEFINITION of "BLACKBODY".

When used here as a noun: a **blackbody** is a surface (material or geometrical) that absorbs all radiant flux of all wavelengths and polarizations incident upon it from all possible directions (absorptance α equal to one, and reflectance ρ and transmittance τ both equal to zero, regardless of the geometry, spectrum, or polarization of the incident rays).

This definition is in terms of a surface because it is almost always more convenient in radiometric analyses and calculations to consider that absorption takes place at a geometrical surface. For example, even though the physical absorption actually takes place in the matter beyond or behind the geometrical surface, we assign it to the surface across the aperture of a cavity blackbody simulator, or to a smooth surface, that is easily defined geometrically, just above a rough material surface that is not easily specified. This is consistent with our practice of referring ray-radiance distributions to any convenient surface that intersects a beam, as first presented in Chapter 2, especially on pp. 40-42 [12.5].

As an adjective: **blackbody** refers to radiant flux, or to related radiometric quantities or properties, having characteristics like those of a blackbody.

3. BLACKBODY RADIATION.

In order to characterize blackbody radiation, it is useful to jump ahead and consider briefly some topics that strictly belong in the following chapter on the thermal radiation properties of matter. As suggested above, an approximation to a true blackbody is the surface across the aperture of a cavity with completely opaque walls. The approximation improves (1) as the wall absorptance is increased, (2) as the aperture is made smaller with respect to the dimensions of the

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[1] With the general availability and utilization of computers, it is now practically as simple and easy to make the fully accurate calculations, using the complete Planck equation. However, the approximations recognized and included here are still widely present in the literature and can occasionally be useful to facilitate checking the reasonableness of computer results or when simpler analytic expressions can serve a useful purpose.

cavity, and, to the extent that the wall surfaces can be made specularly reflecting[1], (3) as the cavity is configured to increase the number of internal specular reflections of an incident ray before it re-emerges from the aperture. These three conditions all tend to increase the attenuation, by absorption, of radiant energy from an incident ray entering through the aperture. By taking advantage of all three in combination, which may require some sophisticated analysis and good judgment, approximations to one part in a thousand, or even in ten thousand or one hundred thousand, can be achieved for the rays within a beam of limited throughput.

Analysis, based on the second law of thermodynamics, of conditions within an <u>isothermal</u> cavity entirely enclosed within walls that are completely opaque shows that the radiance of every ray of thermal radiation within that isothermal cavity is the same [12.3]. Also, the value of that radiance depends only on the temperature and is completely independent of the material or configuration of the opaque walls. If we make a small opening in the wall, approximating, as closely as desired, a blackbody at the aperture, as discussed in the preceding paragraph, then the small amount of emerging radiation will upset the thermal equilibrium within the cavity to a correspondingly negligible degree. The radiance of the emerging rays from the interior of the cavity will, similarly, approximate that of a blackbody at the temperature of the cavity. Furthermore, by Kirchhoff's law,

$$\alpha(\lambda,\theta,\phi) = \varepsilon(\lambda,\theta,\phi), \tag{12.1}$$

where $\alpha(\lambda,\theta,\phi)$ is the spectral directional absorptance for radiance[2] of wavelength λ incident from direction θ,ϕ and $\varepsilon(\lambda,\theta,\phi)$ is the spectral directional emissivity[3] at wavelength λ in direction θ,ϕ. (Note that this is outward along the ray in direction θ,ϕ, in the opposite sense from that of the incident flux that is absorbed.) This law is valid for any body or surface, and for radiance polarized arbitrarily. Since the absorptance of a blackbody is equal to one for radiance, of any wavelength and any polarization, incident from any direction, its spectral directional emissivity is also equal to one at all wavelengths and in all possible directions, and the emitted radiance is unpolarized. Thus the emitted thermal spectral radiance from such a totally absorbing body is greater than that from any other body for which $\rho \neq 0$ or $\tau \neq 0$ and, hence, is the maximum possible spectral radiance from any body at a given temperature and wavelength. (Note that we are considering only

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[1] For an opaque wall $\tau = 0$ and $\alpha + \rho = 1$, and for real material of high absorptance $\alpha \approx 1$ and the reflectance $\rho \approx 0$ is very small. Nevertheless, if the surface is diffusely reflecting, the directional distribution of that small amount of reflected radiance will include some retroreflection, back into the direction of incidence (right back out of the entrance aperture). However, if the surface is <u>specularly</u> reflecting (mirror-like), and if wall absorptance is high enough and the number of subsequent internal reflections within the cavity can be made large enough, even that small amount of retroreflection can be substantially reduced. Because almost all materials lose specularity upon heating, due to oxidation, recrystallization, etc., most specular cavities are used as traps or absorbers. They are seldom used as blackbody sources except at low temperatures.

[2] Strictly, only flux (power or energy) is absorbed, not radiance. However, since the fraction of a flux element lost by absorption is equal to the corresponding fraction by which its radiance is reduced, we speak loosely of the "absorbed" radiance (see the derivation of directional propagance $\tau^*(\theta,\phi)$ in eq. (2.37) on p. 38 in Chapter 2 [12.5]).

[3] The directional emissivity of a surface element, in a given direction from a point, is defined as the ratio of the thermal radiance emitted in the given direction from that point to the radiance of a blackbody at the same temperature (as that producing the thermal emission): $\varepsilon(\theta,\phi,T) \equiv L(\theta,\phi,T)/L_{bb}(T)$. Similarly, the spectral emissivity is defined as: $\varepsilon(\theta,\phi,\lambda,T) \equiv L_\lambda(\theta,\phi,\lambda,T)/L_{\lambda,bb}(\lambda,T)$. Note that it is unnecessary to specify direction for blackbody radiance and spectral radiance since they are functions of temperature (and wavelength) only.

thermal radiation, resulting from source temperature; luminescence resulting from other sources of energy may, of course, produce "cold light" greatly in excess of the blackbody spectral radiance.)

Conceivably, when points on the actual surface of a black (completely absorbing) body are at different temperatures, there must be isotropic radiance of different values emitted from different points. Then the directional distribution of radiance from any field point away from the surface would no longer be isotropic. And, unfortunately, the achievement of uniform temperature is one of the most difficult practical problems and hence likely to be the most seriously limiting factor in the design and fabrication of blackbody simulators. Accordingly, the usefulness of many laboratory blackbody simulators extends only to narrow beams of limited throughput where the rays emerging through the aperture in a cavity all come from a limited area on the back wall of that cavity to minimize the effects of temperature gradients in that wall. Unfortunately, many users are not adequately aware of the extent of this problem.

The most important property of blackbody radiation for our purposes is that, (1) as we will see below in reviewing the Planck law, it has a known spectral radiance as a function of temperature and of the spectral parameter (wavelength, etc.). In addition, it is characterized (2) by isoradiance (Lambertian) distribution throughout the emitted beam (to the extent that the source of the rays is isothermal) and (3) by an absence of polarization.

Blackbody radiation is an important primary standard of total and spectral radiance for radiometry and photometry. All NBS lamp standards of radiance, luminance, irradiance, and illuminance (illumination), and of the corresponding spectral quantities, are based on blackbody radiance and spectral radiance at a known temperature. Blackbody radiation is thus defined as a primary standard for these quantities.

3.1 BLACKBODY RADIATION LAWS

3.1.1 The BASIC PLANCK LAW in TERMS of FREQUENCY ν.

The physical law relating the spectral distribution of the radiant flux emitted by a blackbody radiator to its temperature is the Planck radiation law. The basic equation is

$$L_{e,\nu,bb}(\nu,T) = 2\cdot h\cdot n^2\cdot c_o^{-2}\cdot \nu^3\cdot\{\exp[h\cdot\nu/(k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}}], \tag{12.2}$$

where $L_{e,\nu,bb}(\nu,T)$ $[\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}}]$ is the blackbody spectral radiance at frequency ν [Hz] and at source temperature T [K], along a ray, propagated without attenuation (absorption or scattering out of the beam) in a given direction, at a point in a non-attenuating medium of refractive index n.[1] The basic constants used in eq. (12.2) and their values are:

c_o = velocity of electromagnetic radiation in a vacuum
= $2.99792458(1.2)\cdot 10^8$ $[\mathrm{m\cdot s^{-1}}]$,
h = Planck's constant of action = $6.626176(36)\cdot 10^{-34}$ $[\mathrm{J\cdot s}]$,
k_B = the Boltzmann constant = $1.380662(44)\cdot 10^{-23}$ $[\mathrm{J\cdot K^{-1}}]$.

The other symbols in eq. (12.2) are identified as

ν = (spectral) frequency of the optical radiation [Hz] or $[\mathrm{s^{-1}}]$,

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[1] If there is attenuation, the observed spectral radiance will, of course, be this Planck-law value multiplied by τ^*, the propagance over the path from the source to the point of observation in the medium of index n.

T = absolute or thermodynamic temperature [K].

The values given above for the basic constants are the currently accepted best values published in NBS Special Publication 398, "Fundamental Physical Constants" (August 1974) [12.6]. In each case the number in parentheses following a value is the one-standard-deviation uncertainty in the last digit(s) of the value given. For example,

$$2.99792458(1.2)\cdot 10^{8} \equiv (299\ 792\ 458 \pm 1.2)\ [\mathrm{m\cdot s^{-1}}].$$

3.1.2 The PLANCK LAW in RADIOMETRY in TERMS of WAVELENGTH λ. While eq. (12.2) is the basic equation, because frequency is invariant with index of refraction, the following form of the Planck equation, in terms of wavelength, is more widely used in radiometry:

$$L_{e,\lambda_o,bb}(\lambda_o,T) = 2\cdot h\cdot n^{2}\cdot c_o^{2}\cdot \lambda_o^{-5}\cdot\{\exp[h\cdot c_o/(\lambda_o\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot m^{-1}}], \qquad (12.3a)$$

where $L_{e,\lambda_o,bb}(\lambda_o,T)$ $[\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot m^{-1}}]$ is the blackbody spectral radiance at vacuum wavelength λ_o [m] and temperature T [K], along a ray, propagated without attenuation (absorption or scattering out of the beam) in a given direction, at a point in a non-attenuating medium of refractive index n. In the next section we will deal with the awkwardness that arises from the fact that, here, we are giving areas in $[\mathrm{m^2}]$ and wavelengths in [m] but their product is not a volume $[\mathrm{m^3}]$.

In terms of "local" wavelength $\lambda \equiv \lambda_c/n$ [m], in a medium of refractive index n, this Planck equation becomes

$$L_{e,\lambda,bb}(\lambda,T) = 2\cdot h\cdot n^{-2}\cdot c_o^{2}\cdot \lambda^{-5}\cdot\{\exp[h\cdot c_o/(n\cdot\lambda\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot m^{-1}}]. \qquad (12.4a)$$

3.1.3 SPECTRAL PARAMETERS and UNIT-DIMENSION CONSISTENCY. It is strongly recommended that the simple SI base units [12.7] be used throughout any complex equation or computation, introducing the more convenient multiples only in the final result, as the best way to avoid time-consuming errors. For example, final results might show wavelengths in nanometers [nm] and frequencies in terahertz [THz] while retaining square meters $[\mathrm{m^2}]$ for areas. There is one exception, however, to the foregoing that we urge from the dimensional-analysis standpoint that led to our adoption of the unit-dimensions and unit-dimension-consistency checks in Chapter 1 [12.8]. When length, as wavelength, appears as a spectral parameter, we prefer, most of the time, to use a sub-multiple that is clearly appropriate as a wavelength unit for that spectral unit-dimension, such as, micrometer [μm] (more often in the infrared region of the spectrum) or nanometer [nm]. This helps to keep it distinct, for unit-dimension-consistency checks, from the unit-dimensions of areas given in square meters $[\mathrm{m^2}]$ or square centimeters $[\mathrm{cm^2}]$. Most important, however, it avoids the misleading designation of the unit-dimensions of spectral irradiance E or spectral radiant exitance M in watts per square meter (area) and meter (wavelength) $[\mathrm{W\cdot m^{-2}\cdot m^{-1}}]$ which looks like, but definitely is not, a power density (volume density) in watts per cubic meter $[\mathrm{W\cdot m^{-3}}]$. Instead, show the unit-dimensions of these quantities as, for example, $[\mathrm{W\cdot m^{-2}\cdot nm^{-1}}]$ or $[\mathrm{W\cdot cm^{-2}\cdot \mu m^{-1}}]$. Note, however, that the appearance of multiples or submultiples of the spectral parameters of wavelength and frequency in the same expression requires that the velocity c, that relates them, must be given in consistent units. For example, when wavelength is in nanometers [nm] and frequency in terahertz [THz], c should be in kilometers per second $[\mathrm{km\cdot s^{-1}}]$; thus, $\lambda\cdot\nu = c$ breaks down as

$$\begin{aligned}\lambda[\mathrm{nm}]\cdot\nu[\mathrm{THz}] &= \lambda[10^{-9}\cdot\mathrm{m}]\cdot\nu[10^{12}\cdot\mathrm{s^{-1}}]\\ &= c[10^{3}\cdot\mathrm{m\cdot s^{-1}}] = c[\mathrm{km\cdot s^{-1}}]. \qquad (12.5)\end{aligned}$$

Note the emphasis above on the use of wavelength as a spectral parameter. Spectral parameters are quantities that govern the spectral distribution of a radiometric quantity and, as such, their unit-dimensions must be kept distinct from, and do not cancel or combine with, the unit-dimensions of the radiometric quantity that they govern. The same quantities, however, may appear in the same expressions in other ways where they do cancel and combine with the other unit-dimensions, as when wavelengths govern physical interactions in interference phenomena. It can become a bit difficult to sort all of this out, as illustrated in the next paragraph.

Use of $[\mathrm{nm}]$ for wavelengths complicates the unit-dimension consistency of the Planck equation in the form of eq. (12.3a) or (12.4a). First rewrite eq. (12.3a) with the unit-dimensions of every factor in the appropriate SI base units, as recommended above. For convenience, all dimensionless numerical factors are lumped into a single dimensionless number N.

$$\begin{aligned} L\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot m^{-1}}] &= N \cdot (h\ [\mathrm{J \cdot s}]) \cdot (c_o\ [\mathrm{m \cdot s^{-1}}])^2 \cdot (\lambda_o\ [\mathrm{m}])^{-5} \\ &= N \cdot h \cdot c_o^2 \cdot \lambda_o^{-5}\ [\mathrm{J \cdot s^{-1} \cdot m^{-2} \cdot sr^{-1} \cdot m^{-1}}]. \end{aligned}$$

We may insert $[\mathrm{sr^{-1}}]$ $\{= [\mathrm{m^{-1} \cdot m}]\}$ because this is just a consistency check, not a derivation. Then, since $[\mathrm{W}] \equiv [\mathrm{J \cdot s^{-1}}]$, the unit-dimensions are the same on both sides of the equation. A new value of L, in terms of wavelength in $[\mathrm{nm}]$, is then found as

$$\begin{aligned} L\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}] &= N \cdot h \cdot c_o^2 \cdot \lambda_o^{-5}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot (10^9 \cdot nm)^{-1}}] \\ &= 10^{-9} \cdot N \cdot h \cdot c_o^2 \cdot \lambda_o^{-5}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}] \end{aligned}$$

But λ_o in the factor λ_o^{-5} of this expression is still given in $[\mathrm{m}]$, although it, too, is more likely to have been given initially in $[\mathrm{nm}]$. To take this into account, we should substitute $10^{-9} \cdot \lambda_o$ for λ_o throughout, since

$$\lambda_o\ [\mathrm{nm}] = \lambda_o\ [10^{-9} \cdot \mathrm{m}] = 10^{-9} \cdot \lambda_o\ [\mathrm{m}].$$

Then the last line becomes

$$\begin{aligned} L\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}] &= 10^{-9} \cdot N \cdot h \cdot c_o^2 \cdot (10^{-9} \cdot \lambda_o)^{-5}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}] \\ &= 10^{36} \cdot N \cdot h \cdot c_o^2 \cdot \lambda_o^{-5}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}], \end{aligned}$$

or, more fully,

$$L_{e,\lambda_o,bb}(\lambda_o,T) = 2 \cdot 10^{36} \cdot h \cdot n^2 \cdot c_o^2 \cdot \lambda_o^{-5} \cdot \{\exp[10^9 \cdot h \cdot c_o/(\lambda_o \cdot k_B \cdot T)] - 1\}^{-1}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}], \qquad (12.3b)$$

for $h\ [\mathrm{J \cdot s}]$, $c_o\ [\mathrm{m \cdot s^{-1}}]$, and $\lambda_o\ [\mathrm{nm}]$. Similarly, we can obtain the corresponding expression for eq. (12.4a):

$$L_{e,\lambda,bb}(\lambda,T) = 2 \cdot 10^{36} \cdot h \cdot n^{-2} \cdot c_o^2 \cdot \lambda^{-5} \{\exp[10^9 \cdot h \cdot c_o/(n \cdot \lambda \cdot k_B \cdot T)] - 1\}^{-1}\ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}]. \qquad (12.4b)$$

3.1.4 PLANCK EQUATIONS for DIFFERENT SPECTRAL VARIABLES. In addition to the problems with unit-dimensions, difficulties arise from the fact that the Planck equation is an expression for the spectral derivative of a radiometric quantity (radiance L or exitance M). At this point, some readers may have tried unsuccessfully to check the consistency of eqs. (12.2), (12.3), and (12.4) [reference to eq. (12.3) means either (12.3a) or (12.3b), etc.] by merely substituting

$$\nu = c_o/\lambda_o = c_o/(n \cdot \lambda). \qquad (12.6)$$

This, alone, won't work because, in general,

$$L_{e,\nu,bb} \neq L_{e,\lambda_o,bb} \neq L_{e,\lambda,bb} \quad \text{or} \quad L_\nu \neq L_{\lambda_o} \neq L_\lambda. \qquad (12.7)$$

Instead, we must recognize that we need to equate differentials rather than derivatives, thus:

$$dL = L_\nu \cdot d\nu = L_{\lambda_o} \cdot d\lambda_o = L_\lambda \cdot d\lambda = L_\sigma \cdot d\sigma, \tag{12.8}$$

where, for completeness, we have added an expression in terms of one more commonly used spectral parameter, wave number $\sigma \equiv 1/\lambda_o$ $[\mathrm{cm}^{-1}]$, where λ_o is given in [cm] $\{$and c_o in $[\mathrm{cm}\cdot\mathrm{s}^{-1}]\}$. Accordingly, we also need the corresponding relations between the spectral-parameter differentials

$$d\nu = -c_o \cdot \lambda_o^{-2} \cdot d\lambda_o = -c_o \cdot n^{-1} \cdot \lambda^{-2} \cdot d\lambda = c_o \cdot d\sigma \ [\mathrm{Hz}] \text{ or } [\mathrm{s}^{-1}]. \tag{12.9}$$

Furthermore, since we are primarily interested in magnitudes, and not in whether one quantity increases when another decreases, we will ignore the minus signs in eq. (12.9) hereafter. Incidentally, although the useful forms of these relations are those of eq. (12.9), it may be easier to remember them in the following symmetrical form (ignoring the minus signs):

$$d\nu/\nu = d\lambda_o/\lambda_o = d\lambda/\lambda = d\sigma/\sigma. \tag{12.10}$$

From eqs. (12.6), (12.8), and (12.9), it is relatively straightforward to demonstrate the consistency of eqs. (12.2), (12.3), and (12.4), and to derive a corresponding expression for $L_{e,\sigma,bb}(\sigma,T)$. This topic, including some additional spectral variables, is treated in more detail in Appendix 12-A.

3.1.5 COMMON VARIATIONS of PLANCK'S EQUATION. Eqs. (12.3) and (12.4) can be simplified by introducing the **first radiation constant** c_1 (for exitance), or c_{1L} (for radiance), and the **second radiation constant** c_2:

$$\begin{aligned} c_1 &= 2\cdot\pi\cdot h\cdot c_o^2 = 3.741832(20)\cdot 10^{-16}\ [\mathrm{W}\cdot\mathrm{m}^2] \\ &\qquad \text{for all lengths, including wavelengths, in } [\mathrm{m}], \text{ or} \\ &= 2\cdot 10^{36}\cdot\pi\cdot h\cdot c_o^2 = 3.741832(20)\cdot 10^{20}\ [\mathrm{W}\cdot\mathrm{m}^2] \\ &\qquad \text{for areas in } [\mathrm{m}^2] \text{ and wavelengths in } [\mathrm{nm}]; \end{aligned} \tag{12.11}$$

$$\begin{aligned} c_{1L} &= 2\cdot h\cdot c_o^2 = 1.1910621(64)\cdot 10^{-16}\ [\mathrm{W}\cdot\mathrm{m}^2\cdot\mathrm{sr}^{-1}] \\ &\qquad \text{for all lengths, including wavelengths, in } [\mathrm{m}], \text{ or} \\ &= 2\cdot 10^{36}\cdot h\cdot c_o^2 = 1.1910621(64)\cdot 10^{20}\ [\mathrm{W}\cdot\mathrm{m}^2\cdot\mathrm{sr}^{-1}] \\ &\qquad \text{for areas in } [\mathrm{m}^2] \text{ and wavelengths in } [\mathrm{nm}]; \end{aligned} \tag{12.12}$$

$$\begin{aligned} c_2 &= h\cdot c_o\cdot k_B^{-1} = 1.438786(45)\cdot 10^{-2}\ [\mathrm{m}\cdot\mathrm{K}] \\ &\qquad \text{for wavelengths in } [\mathrm{m}], \text{ or} \\ &= 10^9\cdot h\cdot c_o\cdot k_B^{-1} = 1.438786(45)\cdot 10^{7}\ [\mathrm{nm}\cdot\mathrm{K}] \\ &\qquad \text{for wavelengths in } [\mathrm{nm}]. \end{aligned} \tag{12.13}$$

The resulting simplifications of eqs. (12.3b) and (12.4b), respectively, are then

$$L_{e,\lambda_o,bb}(\lambda_o,T) = c_{1L}\cdot n^2\cdot\lambda^{-5}\{\exp[c_2/(\lambda_o\cdot T)] - 1\}^{-1}\ [\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{sr}^{-1}\cdot\mathrm{nm}^{-1}] \text{ and} \tag{12.3c}$$

$$L_{e,\lambda,bb}(\lambda,T) = c_{1L}\cdot n^{-2}\cdot\lambda^{-5}\cdot\{\exp[c_2/(n\cdot\lambda\cdot T)] - 1\}^{-1}\ [\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{sr}^{-1}\cdot\mathrm{nm}^{-1}]. \tag{12.4c}$$

However, in most textbooks and reference books, at least until quite recently, the Planck-law equation is given in terms of spectral radiant exitance as

$$M_{e,\lambda,bb}(\lambda,T) = c_1\cdot\lambda^{-5}\cdot\{\exp[c_2/(\lambda\cdot T)] - 1\}^{-1}\ [\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{nm}^{-1}], \tag{12.14}$$

ignoring the distinction between wavelength in air λ and in vacuum λ_o; in other words setting $n = 1$ in eq. (12.4c) and multiplying by π, since a blackbody is lambertian (isotropically diffuse) and, for such sources, $M_\lambda = \pi \cdot L_\lambda$ (and $c_1 = \pi \cdot c_{1L}$). This neglect of n is treated below.

3.1.6 REFRACTIVE INDEX n in the PLANCK EQUATION.

If a blackbody is radiating into air with a refractive index of about $n = 1.00028$, the difference between λ_o and λ is so small that it can be neglected for all but extremely high-accuracy measurements. Blevin [12.9] has shown that, in radiation pyrometry at an effective wavelength of 665 [nm], failure to correct for the effect of the refractive index of air produces temperature-measurement errors of only -0.07 [K] at 1000 [K], zero at the gold point (1335.58 [K]), and +1.04 [K] at 3000 [K].

As we have just seen in eq. (12.14), whenever the errors resulting from failure to take into account the refractive index n of air do not produce unacceptable values in the final reduced data for a measurement (in air), n can be set at unity. Accordingly, the symbol n is then just omitted from any product in which it is a factor in any equation concerning blackbodies emitting into air. Then, too, $\lambda_o = \lambda$, so the subscript also can be dropped from λ_o.

3.2 The STEFAN-BOLTZMANN LAW.

If the spectral dependence of the index of refraction can be ignored, setting $n(\lambda) = n$, the Planck Law Equation (12.3b) can be integrated over all wavelengths, from zero to infinity, to give the **Stefan-Boltzmann equation**

$$L_{e,bb}(T) = n^2 \cdot \sigma_L \cdot T^4 \; [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}], \tag{12.15}$$

where $L_{e,bb}(T)$ is the spectral-total radiance, at a point in a non-attenuating medium of refractive index n, of any ray from a blackbody at temperature T [K] and

$$\sigma_L = 2 \cdot \pi^4 \cdot k_B^4 / (15 \cdot h^3 \cdot c_o^2) = 1.80492(23) \cdot 10^{-8} \; [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot K^{-4}}] \tag{12.16}$$

is the Stefan-Boltzmann constant for spectral-total radiance. Although this is probably adequate for most applications, the highest accuracy would require that $n(\lambda_o)$ be substituted for n in the integrand with respect to wavelength λ_o taken from the right side of eq. (12.3b).

Like the Planck equation, the Stefan-Boltzmann equation is more often expressed, in texts and references, in terms of exitance M, rather than radiance L. Furthermore, as discussed in the preceding section, the index of refraction is omitted on the assumption that, in air, $n^2 \approx 1$ is sufficiently accurate. Accordingly, we find

$$M_{e,bb}(T) = \sigma_M \cdot T^4 \; [\mathrm{W \cdot m^{-2}}], \tag{12.17}$$

where[1]

$$\sigma_M = \pi \cdot \sigma_L = 5.67032(71) \cdot 10^{-8} \; [\mathrm{W \cdot m^{-2} \cdot K^{-4}}] \tag{12.18}$$

is the Stefan-Boltzmann constant (for exitance).

---

[1] The standard symbol for the Stefan-Boltzmann constant is σ, and the value given here is the value usually listed as a fundamental physical constant [12.6]. However, the standard symbol for wavenumber is also σ. The subscript M is used here to indicate that σ_M is the symbol for the Stefan-Boltzmann constant for radiant exitance M_e and to distinguish it both from σ_L, the Stefan-Boltzmann constant for radiance L_e, and from σ, the symbol for wavenumber.

3.3 The WIEN DISPLACEMENT LAW. The Planck-law equation (12.3c) can be rewritten thus:

$$L_{e,\lambda_o,bb}(\lambda_o \cdot T) = c_{1L} \cdot n^2 \cdot T^5 \cdot f(\lambda_o \cdot T) \; [W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}], \tag{12.19}$$

where the function of the product $\lambda_o \cdot T$ is

$$f(\lambda_o \cdot T) = (\lambda_o \cdot T)^{-5} \cdot \{\exp[c_2/(\lambda_o \cdot T)] - 1\}^{-1} \; [m^{-2} \cdot nm^{-1} \cdot K^{-5}]. \tag{12.20}$$

The spectral distribution of blackbody radiance (or exitance) is, accordingly, a function of the product $\lambda_o \cdot T$. This is known as the **Wien displacement law** [12.10], as distinguished from the Wien radiation law or distribution law, an approximation to the Planck law, that is discussed below. The displacement law is the basis for presenting compact tables and graphs of this spectral distribution, and others derived from it, with the product $\lambda \cdot T$, rather than just λ, as the independent variable. Again, it is more frequently presented in the form for blackbody spectral radiant exitance, usually also ignoring the index n and the distinction between λ_o and λ:

$$M_{e,\lambda,bb}(\lambda \cdot T) = c_1 \cdot T^5 \cdot f(\lambda \cdot T) \; [W \cdot m^{-2} \cdot nm^{-1}], \tag{12.21}$$

where $f(\lambda \cdot T)$ is the same function as that in eq. (12.20), but with λ in place of λ_o.

There are two corollaries of this law which are better known than the law itself [eq. (12.19) or (12.21)]. In fact, the first one [eq. (12.22)] is very often presented alone as the Wien displacement law. These two corollaries are

$$\begin{aligned} n \cdot \lambda_m \cdot T &= b \quad \text{(a constant)} \\ &= 2.8978 \cdot 10^6 \; [nm \cdot K] \end{aligned} \tag{12.22}$$

and

$$\begin{aligned} L_{e,\lambda,bb}(\lambda_m \cdot T) \cdot n^{-2} \cdot T^{-5} &= b' \quad \text{(a constant)} \\ &= 4.0956 \cdot 10^{-15} \; [W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1} \cdot K^{-5}], \end{aligned} \tag{12.23}$$

where λ_m is the wavelength (in a medium of index n) for the maximum value of $L_{e,\lambda,bb}$ [or of $M_{e,\lambda,bb}$]. Again, in air $n \approx 1$ and $\lambda_o \approx \lambda$, so most texts and references ignore n and the distinction between λ_o and λ.

3.4 APPROXIMATIONS to the PLANCK LAW: the WIEN DISTRIBUTION LAW and the RAYLEIGH-JEANS LAW. There are two laws, relating the spectral distribution of blackbody radiation to the blackbody-source temperature, that historically antedate the Planck law. They are of interest, not only for their historical value, but also because they are sometimes useful approximations to the Planck-law equation. They are the **Wien distribution law** or **radiation law**, and the **Rayleigh-Jeans law**. In giving the exact expressions for these laws, we will, nevertheless, use the approximately equals sign $\approx$ to emphasize that they are approximations to the more universal Planck law.

The expression for the Wien distribution law in terms of spectral radiance in a non-attenuating medium of refractive index n is

$$L_{e,\lambda,bb}(\lambda,T) \approx c_{1L} \cdot \{n^2 \cdot \lambda^5 \cdot \exp[c_2/(n \cdot \lambda \cdot T)]\}^{-1} \; [W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}] \tag{12.24}$$

or, in terms of spectral radiant exitance,

$$M_{e,\lambda,bb}(\lambda,T) \approx c_1 \cdot \{n^2 \cdot \lambda^5 \cdot \exp[c_2/(n \cdot \lambda \cdot T)]\}^{-1} \; [W \cdot m^{-2} \cdot nm^{-1}]. \tag{12.25}$$

Note that eqs. (12.24) and (12.25) follow from eqs. (12.4c) and (12.14), respectively, if we set

$$\exp[c_2/(n\cdot\lambda\cdot T)] \approx \{\exp[c_2/n\cdot\lambda\cdot T)] - 1\}, \qquad (12.26)$$

which is true when $n\cdot\lambda\cdot T << c_2$. Accordingly, the Wien distribution law is a good approximation to the exact Planck law when $\lambda\cdot T$ is very small. As always, the index n is often omitted in eqs. (12.24) and (12.25) on the assumption that, <u>in air</u>, $n \approx 1$.

The expression for the Rayleigh-Jeans law in terms of spectral radiance in a non-attenuating medium of refractive index n is

$$L_{e,\lambda,bb}(\lambda,T) \approx (c_{1L}/c_2)\cdot(n\cdot\lambda^4)^{-1}\cdot T \ [W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}] \qquad (12.27)$$

or, in terms of spectral radiant exitance,

$$M_{e,\lambda,bb}(\lambda,T) \approx (c_1/c_2)\cdot(n\cdot\lambda^4)^{-1}\cdot T \ [W\cdot m^{-2}\cdot nm^{-1}] \qquad (12.28)$$

Eqs. (12.27) and (12.28) follow from eqs. (12.4c) and (12.14), respectively, if we use the series expansion of the exponential term:

$$\begin{aligned}\exp[c_2/(n\cdot\lambda\cdot T)] - 1 &= 1 + [c_2/(n\cdot\lambda\cdot T)] + (1/2)\cdot[c_2/(n\cdot\lambda\cdot T)]^2 + \cdot\cdot\cdot\cdot - 1\\ &= [c_2/(n\cdot\lambda\cdot T)], \text{ when } (n\cdot\lambda\cdot T) >> c_2. \qquad (12.29)\end{aligned}$$

Accordingly, the Rayleigh-Jeans law is a good approximation to the exact Planck law only when $n\cdot\lambda\cdot T$ is very large. Again, the index n is usually omitted from eqs. (12.27) and (12.28) on the assumption that, in air, $n \approx 1$. Curves comparing the Planck law to these two approximations to it are shown in Figure 12.1a.

To be more specific and quantitative, the Wien approximation is good to better than 1% for all $(\lambda\cdot T) < 3.1\cdot10^6$ [nm·K] while the Rayleigh-Jeans approximation holds to better than 1% for all $(\lambda\cdot T) > 7.7\cdot10^6$ [µm·K]. For 10% accuracy, these limits are $(\lambda\cdot T) < 6.0\cdot10^6$ [nm·K] and $(\lambda\cdot T) > 7.0\cdot10^4$ [µm·K], respectively. For example, a blackbody source at 6000 [K] (roughly the order of magnitude of the sun) can be approximated by the Wien law within 1% at all wavelengths $\lambda < 500$ [nm] and by the Rayleigh-Jeans law within 1% at all wavelengths $\lambda > 128$ [µm]. For 10% accuracy, the corresponding limits are $\lambda < 1000$ [nm] and $\lambda > 11.7$ [µm], respectively. However between these limiting wavelengths, the stated accuracy requires use of a closer approximation or the full Planck-law formula [12.11]. These limits are graphically depicted in Figure 12.1b.

<u>4. VARIATIONS of BLACKBODY SPECTRAL RADIANCE and SPECTRAL RADIANT EXITANCE with TEMPERATURE.</u> When the Planck equation is differentiated with respect to temperature we get

$$\frac{d\{\ln[L_{\lambda,T,bb}(\lambda)]\}}{d[\ln(T)]} = \frac{dL_{\lambda,T,bb}(\lambda)\cdot T}{L_{\lambda,T,bb}(\lambda)\cdot dT} = \frac{dM_{\lambda,T,bb}(\lambda)\cdot T}{M_{\lambda,T,bb}(\lambda)\cdot dT} = \frac{c_2\cdot\exp[c_2/(\lambda\cdot T)]}{\lambda\cdot T\cdot\{\exp[c_2/(\lambda\cdot T)] - 1\}} \equiv Z. \qquad (12.30)$$

For a small temperature change ΔT [so small that $\lambda\cdot T$ does not change significantly] $L_{\lambda,T,bb}(\lambda)$ or $M_{\lambda,T,bb}(\lambda)$ will change as T^Z, so that the fractional change in $L_{\lambda,T,bb}(\lambda)$ or $M_{\lambda,T,bb}(\lambda)$ can be computed as

$$\frac{\Delta L_{\lambda,T,bb}(\lambda)}{L_{\lambda,T,bb}(\lambda)} = \frac{\Delta M_{\lambda,T,bb}(\lambda)}{M_{\lambda,T,bb}(\lambda)} = \frac{Z\cdot\Delta T}{T}, \qquad (12.31)$$

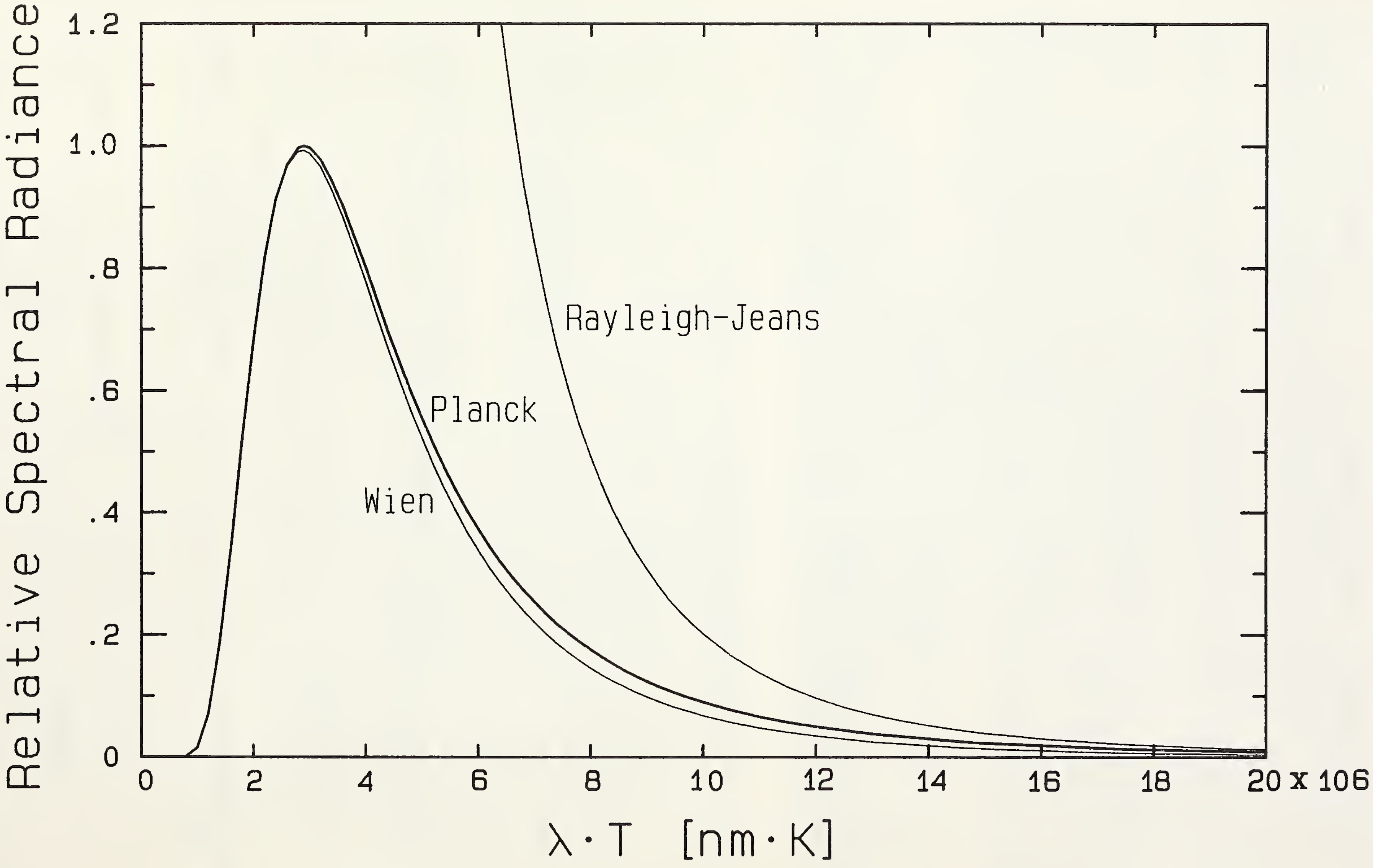

Figure 12.1a. Relative blackbody spectral radiance $\ell_{\lambda,bb}(\lambda\cdot T) = L_{\lambda,bb}(\lambda\cdot T)\cdot T^{-5}\cdot b'^{-1}$ as a function of the product $\lambda\cdot T$ [nm·K]. The heavy curve shows exact values according to the Planck radiation law. The light curves are the approximate values given by the Wien radiation law and the Rayleigh-Jeans law, respectively, showing that the Wien law is a good approximation at the shorter wavelengths at any given temperature while the Rayleigh-Jeans law is similarly a good approximation only at the longer wavelengths.

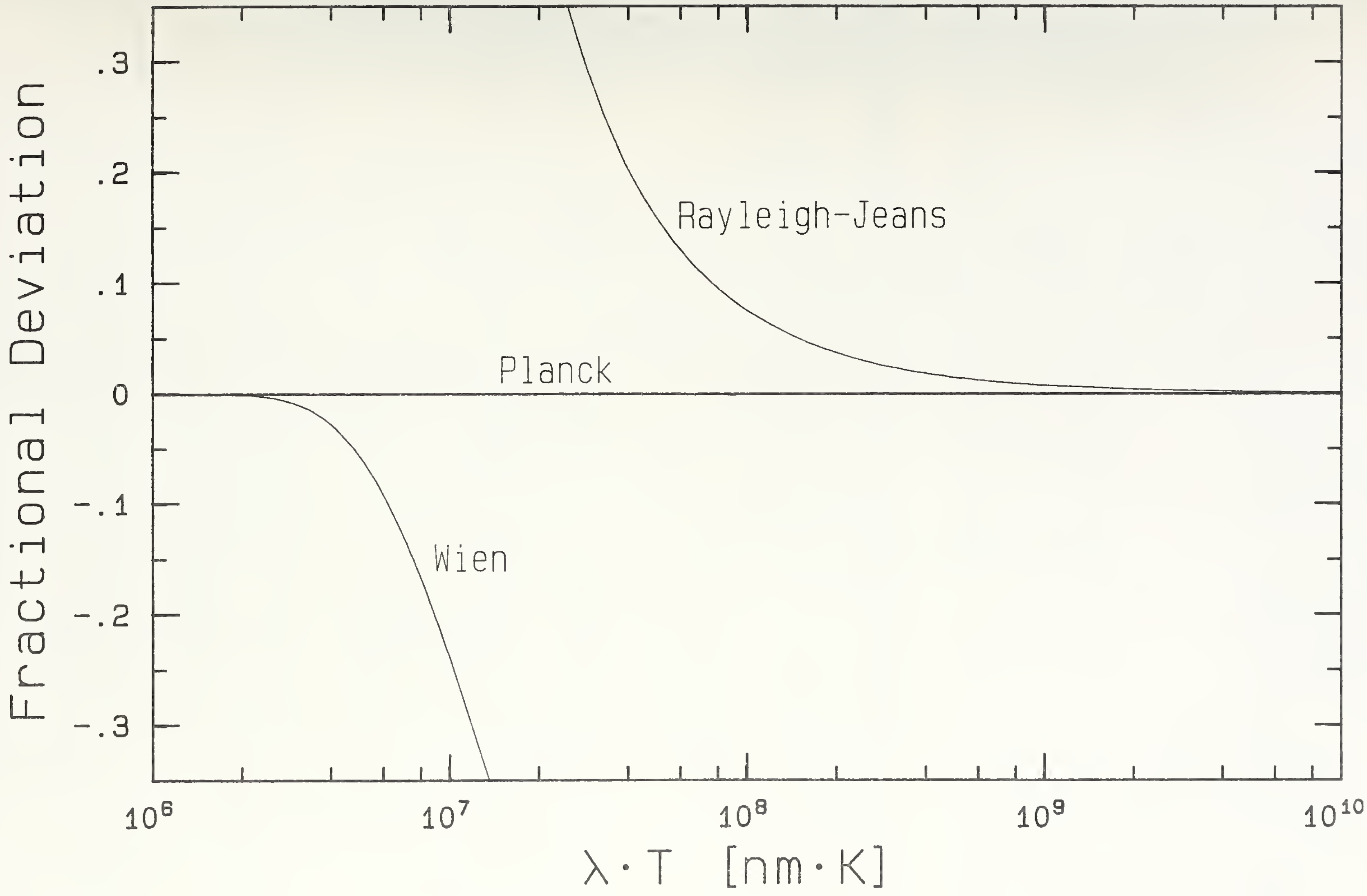

Figure 12.1b. The fractional deviation of the Wien-law and Rayleigh-Jeans-law approximations from the Planck-law exact values of blackbody spectral radiance, spectral exitance, etc., as a function of the product $\lambda \cdot T$ [nm·K].

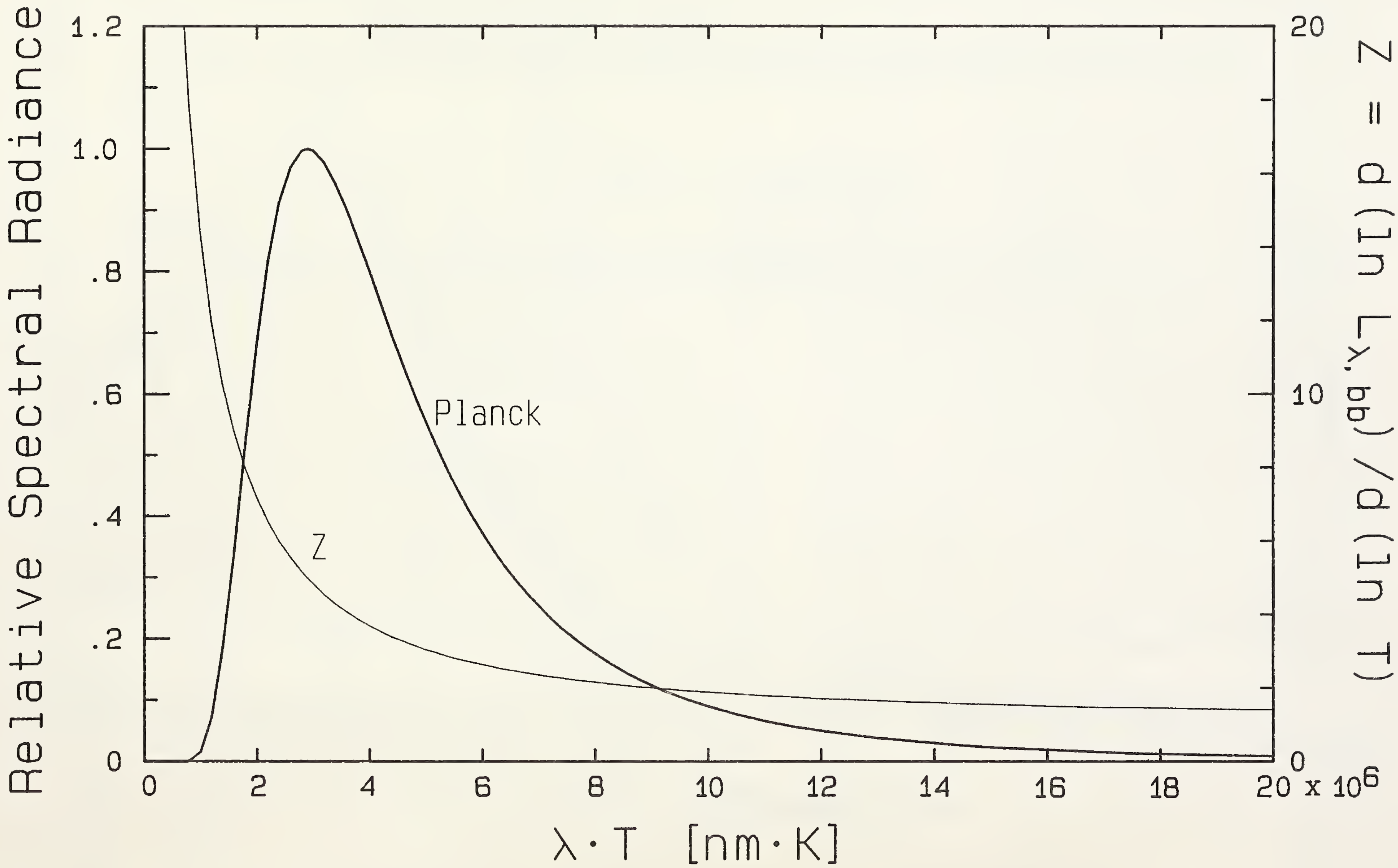

Figure 12.2. The relative spectral distribution of blackbody radiance $\ell_{\lambda,bb}(\lambda\cdot T)$ (heavy line) and the ratio $Z \equiv d[\ell n(L_{\lambda,bb})]/d[\ell n(T)]$ [eq. (12.30)] (light line), both plotted as functions of the product $\lambda\cdot T$ [nm·K].

For $\lambda \cdot T$ less than $3.1 \cdot 10^{-3}$ [m·K], eq. (12.30) can be written as

$$\frac{d\{\ell n[L_{\lambda,T,bb}(\lambda)]\}}{d[\ell n(T)]} = \frac{dL_{\lambda,T,bb}(\lambda) \cdot T}{L_{\lambda,T,bb}(\lambda) \cdot dT} = \frac{dM_{\lambda,T,bb}(\lambda) \cdot T}{M_{\lambda,T,bb}(\lambda) \cdot dT} = \frac{c_2}{\lambda \cdot T} = Z \qquad (12.32)$$

with an error of less than 1%.

The parameter Z in eq. (12.30) approaches infinity as $\lambda \cdot T$ approaches zero and approaches one as $\lambda \cdot T$ approaches infinity.

Figure 12.2 shows a plot of $L_{\lambda,T,bb}(\lambda \cdot T)$ on a normalized scale on which $L_{\lambda,T,bb}(\lambda \cdot T)_{max}$ is set at 1.00, and a plot of Z on a linear scale, both as functions of $\lambda \cdot T$.

Eqs. (12.30), (12.31), and (12.32) are good approximations for most real materials, with any value of emissivity, since spectral emissivity does not change rapidly with temperature if there is no phase change or shift of a critical absorption edge due to the temperature change, as will be discussed in a later chapter on Thermal Radiation Properties of Matter.

5. KOSTKOWSKI'S EQUATION. Another equation, developed by Kostkowski [12.12], can be used to obtain the ratio of the spectral radiances of two arbitrary sources, at any wavelength, when their spectral radiances (so, also, radiance temperatures) are equal at some particular wavelength. The equation is derived from the Wien distribution-law equation [eq. (12.24)], and hence is valid to within 1% at values of $\lambda \cdot T$ less than 3100 [μm·K]. The basic equation is

$$\frac{L_{\lambda,2}(\lambda)}{L_{\lambda,1}(\lambda)} = \frac{\{\varepsilon_1(\lambda_e) \cdot \tau_1(\lambda_e)/[\varepsilon_2(\lambda_e) \cdot \tau_2(\lambda_e)]\}^{(\lambda_e/\lambda)}}{\{\varepsilon_1(\lambda) \cdot \tau_1(\lambda)/[\varepsilon_2(\lambda) \cdot \tau_2(\lambda)]\}} \qquad (12.33)$$

where λ_e is the wavelength at which the spectral radiances of the two sources are equal, and λ is the wavelength at which the ratio of their spectral radiances is given by eq. (12.33). The subscripts 1 refer to the first source and 2 to the second source; ε is emissivity of the source; and τ is the transmittance of the optical path, the only significant factor usually being τ for any window in the path. These calculations are particularly useful in evaluating radiance temperatures in optical pyrometry. For further details see [12.12]. Strictly, this section could be considered more appropriate for the discussion of real materials and their properties in the next chapter. However, it is here because it is so closely related to the basis for optical pyrometry presented in the preceding section.

6. LIMITATIONS to LABORATORY BLACKBODIES. The only true blackbody would be a fully enclosed cavity with walls that are completely opaque. All of the radiant flux present in the cavity at any instant of time will be completely absorbed by the walls after multiple internal reflections. Then the rates of emission and absorption must be equal at every element of the inner surface of the cavity wall if thermal equilibrium exists. The radiance in such an isothermal cavity, in all possible directions, will conform exactly to the Planck-law equation [eq. (12.3)]. All laboratory blackbody sources emit flux that differs from that emitted by a true blackbody emitter in its amount, geometric distribution, and spectral distribution, because the cavity wall must be breached to view the blackbody radiation in the cavity, or to permit the radiation to emerge, when it is used as a source, and because the walls are never perfectly isothermal. The escaping flux through the aperture is not replaced by internal reflection or re-emission, thus upsetting the thermal equilibrium in the cavity to that extent. This distur-

bance of equilibrium can be kept to a minimum by making the aperture small relative to the internal wall area and, to some extent, also by controlling the configuration and surface of the opaque walls. Directional characteristics are the most difficult to control; practical blackbody simulators are usually good only over limited solid angles that are usually much less than a full hemisphere.

There are two general types of laboratory blackbodies (blackbody simulators), those used as radiation traps to absorb incident radiant flux, and those used as sources (emitters) of blackbody radiation. The design and operation of these two types of blackbodies is quite different, and they will be discussed separately.

6.1 BLACKBODY ABSORBERS. A radiation trap is used to absorb unwanted radiant flux. Black velvet cloth is widely used in optical systems to reduce scattered radiation, even though it is not a very close approximation to a blackbody absorber. For many purposes, this is inadequate and better blackbody absorbers must be used. Examples include a cavity-blackbody absorber in an integrating-sphere reflectometer, used to remove the specularly reflected beam in order to measure the diffuse reflectance of a highly specular sample, such as a mirror. For thermal radiation measurements at the longer infrared wavelengths, this cavity is, strictly, not a "trap" but a blackbody "source"; one that establishes a reference radiation level or background that depends on its temperature.

A second example is a "zero"-reference, or scale-level-reference, source for use with a detector. A typical system employing a "zero" reference has an optical system that alternately directs, first, the radiation beam from the target, and then, that from the "zero"-reference source, onto the detector. This system usually consists of a chopper-mirror that periodically blocks the reference beam with an opaque mirror that reflects the sample beam onto the detector, or vice versa. The "zero" reference usually operates at ambient temperature, particularly for spectrometers (including spectroradiometers and spectrophotometers) with operating ranges that do not exceed about 5 [μm]. The peak of the spectral radiance of a blackbody source at ambient temperature is about 10 [μm], while that of a typical tungsten-filament lamp used as a source in spectrometers is at about 1 [μm]. The ratio of the total radiance of the source to that of the "zero" reference is thus about 10^4, or 10,000 to 1. The fact that the spectral peak of the "zero" reference is at a much longer wavelength than that of the tungsten source increases the ratio of the spectral radiances to about $5 \cdot 10^{20}$ at 1 [μm], $3 \cdot 10^9$ at 2 [μm], $2 \cdot 10^6$ at 3 [μm], $7 \cdot 10^4$ at 4 [μm] and $9 \cdot 10^3$ at 5 [μm], assuming that the tungsten source emits like a 3000-[K] blackbody, which is a rough approximation. At wavelengths below 3 [μm] the ratio of the two radiances is so high that the effect of the "zero" reference cannot be detected with normal systems. At 5 [μm] the ratio of the two signals is within the range where the "zero" reference can give a non-zero reading on most spectrometers. If used as a source at a known ambient temperature, the "zero" source can serve as a reference source of known low radiance, which can be computed from its known temperature by use of the Planck equation. A correction can then be made for the effect of that radiance. As an alternative, the "zero" reference can be cryogenically cooled to a temperature where its radiance cannot be detected by the spectrometers.

6.2 SPECULAR-REFLECTION RADIATION TRAPS. Radiation traps usually achieve their highest absorptance by multiple specular reflections of obliquely incident rays. A non-specular reflecting surface will always return some fraction of an incident ray in the direction of incidence (retroreflection); only a highly specular surface can keep this first-incidence retroreflectance to a minimum. Accordingly, the geometry of the trap must be such that each specularly reflected ray continues to be incident obliquely again on the next surface that it strikes, and so on. Two examples of such radiation traps are: (1) a stack of single-edge razor blades and (2) a curved horn.

Ideally, the interior surface of a cavity used to trap optical radiation should be both highly specular and highly absorbing. However, low absorptance can be compensated for by increasing the number of reflections along the path of an incident ray, entering the cavity opening, before it again emerges through that opening. This obviously depends on both the position and direction of the incident ray as it enters the aperture as well as the configuration, materials, surface conditions, etc. of the cavity behind the aperture. An analysis to determine the effective absorptance of the entire trap as a function of the parameters of an incident beam can clearly become quite involved and we won't attempt it here. However, it may be helpful to look at just a single ray for which the net absorptance is

$$\alpha = 1 - (1-\alpha_W)^{n_r}, \qquad (12.34)$$

where α_W is the wall absorptance and n_r is the number of specular reflections at the cavity wall along the path of the ray before it finally re-emerges from the cavity. Thus, if the wall absorptance is 0.9, a cavity absorptance of 0.99999 for this single ray requires 5 reflections. If the wall absorptance is only 0.8, 8 reflections are required; if it is only 0.7, 17 reflections are required; and if it is only 0.2, 52 reflections are required. Of course, this equation, in which α_W is treated as a constant, is oversimplified for clarity. Actually, the absorptance, and the reflectance $\rho_W = 1 - \alpha_W$ of the opaque wall at any point is a function of, and may vary substantially with, the direction of the incident ray [12.13], the material and surface quality of the wall, the polarization of the incident ray (as affected by any previous reflections within the cavity), etc. Accordingly, it may not be easy to determine the correct value of α_W to use, even as a first approximation in estimating α by eq. (12.34).

The edges of a uniform stack of razor blades form a surface having a series of parallel grooves, each with a highly specular surface and a very small included angle. Such a radiation trap has an absorptance very close to one for a collimated beam of incident radiant flux normal to the plane of this surface, but only within a very limited solid angle. In a plane perpendicular to the grooves, it includes only rays that lie approximately within the angle between the sides of the groove; in a plane parallel to the grooves it is not quite so limited.

A circular-conical cavity will trap a beam filling a larger and more symmetrical cone of directions, but will have to make up for a smaller number of reflections along each ray path by wall coatings of higher absorptance α_W. Furthermore, in any practical fabrication of such a "horn" cavity, there will always be a small but finite surface element at the vertex where specular reflection will be retroreflection. The effect of this is avoided by curving the "tail", as in fig. 12.3, so that this vertex cannot be "seen" or reached directly by an incident ray without several reflections from the curved sides. These traps are usually made of glass, since they can be easily formed by a skilled glass blower. The inner surface is coated with a high-gloss paint, with an absorptance of $\alpha_W = 0.95$ or more.

6.3 BLACKBODY SOURCES. Blackbody sources are more complex than blackbody absorbers. Most of them operate at temperatures of 800 [K] or above. Because the interest is in the spectral radiance of the source, the conditions of a true blackbody (a cavity with isothermal opaque walls) must be approximated as closely as necessary for the application. With careful design the effective emissivity of a laboratory blackbody can approach 0.99999. The deviations of a laboratory blackbody from a true blackbody

Figure 12.3 A blackbody light trap of glass. The inner surface is coated with a high-gloss black paint having an absorptance of at least ninety-five percent ($\alpha \geq 0.95$).

are both geometrical and spectral. An aperture must be made in the opaque wall of the cavity so that some of the radiant flux in the cavity can escape. At the same time, the thermal equilibrium in the cavity must be maintained as closely as possible. Thick walls are sometimes required in order to minimize thermal gradients in the cavity walls, and the aperture is then a small tube through the thick wall. If the cavity walls are isothermal, the radiance escaping through the tube will be lambertian and a very close approximation to blackbody radiance along the axis of the tube and over a small solid angle around the axis, over which there is no vignetting [12.14]. To reduce vignetting and increase the size of the solid angle of lambertian radiance, the tube is usually tapered, with the small end at the inner wall of the cavity. The radiance falls off rapidly with increasing angle from the axis as soon as there is any vignetting.

The emissivity (or spectral emissivity) at the aperture of a cavity blackbody is a complex function of the cavity geometry (which includes its shape and the ratio of the area of the aperture to the total area of the cavity walls including the aperture) and the emissivity (or spectral emissivity) of the cavity wall material (alone, without interreflections between it and other portions of the wall). The details needed to calculate the emissivities of cavity apertures and to design such cavities with desired values of emissivity will be found in The Infrared Handbook [12.15] and references cited there, and in considerable detail in an NBS Monograph now in preparation [12.16].

The thermal gradients in the cavity wall can be minimized in several ways. A thin-walled cavity can be immersed in a low-viscosity, constant-temperature, circulating fluid, or can be immersed in a material undergoing an exothermic or endothermic phase change, such as a condensing gas, a boiling liquid, or a freezing or melting solid. If the cavity is heated by electrical resistance coils, the heating coils should consist of several segments that can be separately controlled, to reduce thermal gradients. Thick walls of a high-conductivity material will also help to reduce thermal gradients. A recent development [12.17] is a pressure-controlled heat-pipe furnace that can be adapted to a laboratory black-body, which holds thermal gradients to less than 0.01 [K] in the range from about 645 to 1300 [K], using cesium from 372 to 671 [°C], or sodium from 626 to 1068 [°C], as the working fluid. In one model the working volume is a cylinder 30 [cm] long and 28 [mm] in diameter in the center of a horizontal tube about 60 [cm] long.

7. TEMPERATURE SCALES.

7.1 EARLY TEMPERATURE SCALES. A brief review of past practices will be helpful in understanding the nomenclature and treatment of temperature in the literature as well as some of the diversity that still persists in current publications.

Temperature may be defined as a measure (1) of the average content of sensible heat per unit mass of a material, or (2) of the average kinetic energy of translation of the particles of which the material is composed.

A temperature scale can be established by assigning values to the temperatures of, and, hence also, to the temperature difference between, two fixed equilibrium temperatures, usually those of phase changes of pure substances. Then other temperature values are defined by interpolation or extrapolation from the fixed points in terms of some physical property of a material that changes with temperature and can be measured easily, such as thermal expansion, electrical resistance, or thermoelectric voltage.

All early temperature scales used the freezing point of water and its boiling point as the fixed points. The **Fahrenheit** scale, with the **degree [°F]** as the unit of temperature difference, was designed so that the temperature of the coldest day in winter would be about 100 [°F] below normal body temperature. The value of 32 [°F] was assigned to the freezing point and 212 [°F] to the boiling point of water. Mentioned only for its historical interest, the Réaumur scale, with the **degree Réaumur [°R]**[1] as the unit of temperature difference, was developed by assigning 0 [°R] to the freezing point and 80 [°R] to the boiling point. The **Celsius** (formerly **Centigrade**) scale, with the **degree Celsius [°C]** as the unit of temperature difference, was developed by assigning values of 0 [°C] to the freezing point and 100 [°C] to the boiling point. Thus 1 [°C] = 0.8 [°R] = 1.8 [°F].

7.2 The THERMODYNAMIC TEMPERATURE SCALE. The concept of **absolute temperature** originated early in the 19th century. The Gay-Lussac law (Charles law[2]), published in 1802 [12.19], states that "at constant pressure, the volume of a fixed mass of gas increases by 1/273 of its volume at 0 [°C] for every degree its temperature is raised". An important alternative form of the law states that the ratio of the volumes of a gas at constant pressure at two temperatures t_1 and t_2, in [°C], is equal to $(t_1 + 273)/(t_2 + 273)$. Hence a new scale of temperature, in which the unit of temperature difference is equal to the [°C] was devised in which 0 [deg] = -273 [°C]. The temperatures on this scale were referred to as absolute temperatures. Later experiments showed that real gases deviate from the Gay-Lussac law at low temperatures, where those real gases liquefy, and most also solidify, before absolute zero is reached. Hence the law applies strictly only to an ideal gas that never liquefies or solidifies, and has zero volume at a temperature of absolute zero. The symbol [°K] and name "degree Kelvin" (subsequently the name of the unit was changed to just the "kelvin") were later adopted for absolute

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[1] The degree Réaumur [°R] should not be confused with the **degree Rankine**, for which the symbol [°R] is also used. The Rankine scale is an absolute-temperature scale, with the degree Rankine equal to the degree Fahrenheit [°F]. On this scale, absolute zero is 0 [°R] = -459.67 [°F]. Hence, any other temperature T [°R] = t [°F] + 459.67. -- taken from [12.18].

[2] In the first published account of this important basic gas law, Gay-Lussac acknowledges the earlier discovery by Charles, who never published it.

temperatures, in honor of Lord Kelvin (William Thomson), who derived an equivalent temperature scale from thermodynamic considerations [12.20]. This scale was based on the **Carnot cycle**, in which an ideal gas undergoes (conceptually) four reversible changes between temperatures T_1 and T_2:

1. A given amount of the ideal gas is compressed in a thermally insulated vessel, with no loss of heat energy (adiabatically), raising its temperature from T_1 to T_2 (pressure has increased; volume has decreased).

2. The vessel, with thermal insulation removed, permits the gas to expand in contact with a heat "source" at a constant temperature T_2 (isothermally), absorbing heat energy Q_2 from the "source" (volume increases; pressure decreases).

3. With thermal insulation again in place, the gas is allowed to expand further, with no loss or gain of heat energy (adiabatically), until it has cooled back to the initial temperature T_1 (volume increases; pressure decreases).

4. With thermal insulation again removed, the gas is compressed with the vessel in contact with a heat "sink" at a constant temperature T_1 (isothermally), down to its initial volume at temperature T_1 at the beginning of step 1, losing heat energy Q_1 to the "sink" (volume decreases; pressure increases).

In this process, the net heat energy absorbed in the cycle, $Q_2 - Q_1$, is transformed into mechanical work by the motion of the gas-containing enclosure against the forces produced by the gas pressure on the walls of that enclosure. Since all parts of the cycle have been postulated to be reversible, the complete cycle can be reversed, making the heat "engine" a "refrigerator" that absorbs heat energy Q_1 from the former "sink" during an isothermal expansion at T_1; increases temperature from T_1 to T_2 by an adiabatic compression; loses heat energy Q_2 to the former "source" during isothermal compression at T_2; and lowers temperature again to T_1 by adiabatic expansion; a cooling cycle that removes Q_1 from the former "sink", to which is added $Q_2 - Q_1$ of mechanical work energy, making a total contribution of Q_2 heat energy to the former "source".

The efficiency η of a Carnot-cycle gas heat engine operating between absolute temperatures T_1 and T_2 can be shown to be

$$\eta = \frac{T_2 - T_1}{T_2}. \tag{12.35}$$

It is evident here [eq. (12.35)] that the engine can have an efficiency of one (100%) only when T_1 is at absolute zero. On this basis absolute temperature, in [°K], was also referred to as **thermodynamic temperature**. This thermodynamic temperature scale was established, independently of the properties of any material substance[1], by defining equal temperature intervals $\Delta T = T_2 - T_1$ as those for which equal amounts of heat energy are transformed into mechanical energy by an ideal heat engine going through a Carnot cycle between temperatures T_2 and $T_1 = T_2 - \Delta T$.

7.2.1 THERMODYNAMIC TEMPERATURE in the CURRENT SI SYSTEM.

As stated above, the thermodynamic temperature of matter is a measure of the concentration of sensible heat in matter, or the average kinetic energy of translation (including vibration) of the particles of which the matter is composed, at that temperature. For this reason temperatures on this scale are often referred to as absolute temperatures. Thermodynamic temperature is used in all computations of radiometric quantities of blackbody radiation as functions of source temperature. The SI quantity symbol for thermodynamic temperature is T, and

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[1] A Carnot cycle of two isothermal, and two alternating adiabatic, reversible steps can be defined, similarly, for any medium that will transform heat energy into mechanical work, including those that produce electrical or magnetic forces as well as gas-pressure forces.

the SI unit of temperature difference is the **kelvin** [K]. On this scale the defining fixed points are (1) the triple point of water (the temperature at which the solid, liquid, and vapor phases of water are in equilibrium) with an assigned value of 273.16 [K], and (2) absolute zero. (This definition is designed to continue the convenient equality between the kelvin [K] and the degree Celsius [°C], since the most careful measurements now place absolute zero at -273.15 [°C] and the freezing point at 0.00 [°C], with the triple point defined at 0.01 [°C].)

7.3 GAS THERMOMETRY. The thermodynamic temperature scale is often realized by gas thermometry, which is based on the ideal gas law

$$P \cdot V = N \cdot R \cdot T \text{ [J] or } [m^2 \cdot kg \cdot s^{-2}] \qquad (12.36)$$

where

P = pressure [Pa] or $[N \cdot m^{-2}]$ or $[m^{-1} \cdot kg \cdot s^{-2}]$

V = volume $[m^3]$

N = quantity of gas in moles [mol]

R = molar gas constant, 8.31441(26) $[J \cdot mol^{-1} \cdot K^{-1}]$

T = temperature [K].

The equation actually used in gas thermometry is

$$\frac{P_1 \cdot V_1}{P_2 \cdot V_2} = \frac{T_1}{T_2} \qquad (12.37)$$

where P_1 and V_1 are pressure and volume at T_1, and P_2 and V_2 are pressure and volume at T_2. This is done in striving for the best accuracy in order to eliminate the constant R, that appears in eq. (12.36), and thus the uncertainty in even the best value for R, given above. Then, by choosing T_1 at the triple-point of water, the measurement becomes a direct comparison to the assigned value of that temperature by which the scale is defined and the only sources of error are those in making the measurement itself.

In a gas-thermometer measurement of temperature, hydrogen or helium is usually used because these gases approximate most nearly in their behavior to the ideal gas. The gas is held at a constant pressure, and the volume is measured at different temperatures; or the gas is held at a constant volume, and the pressure is measured at different temperatures. These measurements are repeated at decreasing pressures, and the ratios are extrapolated to zero pressure, in order to correct for the deviations of the gas used from the ideal gas. Gas thermometry is very difficult and time consuming, and is normally carried out only in a national standards laboratory.

7.4 The INTERNATIONAL PRACTICAL TEMPERATURE SCALE (IPTS). Measurements of thermodynamic temperatures by gas thermometry are not only complex and difficult, as pointed out above, but the uncertainty of even the best of such measurements is larger than the imprecision by which temperatures can be measured by other methods. On the other hand, these other methods can only measure temperature differences with respect to some known temperature; they do not directly measure absolute values of thermodynamic temperature.

This situation has led to the development of the **International Practical Temperature Scale** (IPTS) in which measured thermodynamic temperatures are assigned as exact temperature values to a convenient number of fixed points. Highly consistent results can then be achieved to satisfy the everyday needs for authoritative and consistent values in commerce and science by prescribed methods for realizing these fixed points and for making high-precision interpolation measurements between and beyond them. The IPTS temperatures can also be transformed, when thermodynamic temperatures are needed, by applying published information about the differences between the two scales (IPTS and thermodynamic). Such differences arise as the result of improved measurements of thermodynamic temperatures and from the fact that the interpolation and extrapolation methods of the IPTS do not exactly follow the corresponding "curves" of the thermodynamic scale.

By convention, temperatures are often given in [K] up to 273.15 [K] and in [°C] above 0 [°C] where, of course, 0 [°C] = 273.15 [K] but the IPTS recognizes both practices throughout all temperature ranges. The IPTS is used to ensure uniformity in temperature measurements throughout the world. The present International Practical Temperature Scale (IPTS) is that of 1968 (amended edition of 1975) [12.21]. Temperatures on this scale are approximate thermodynamic temperatures given in kelvins, symbol T_{68} [K], or the corresponding temperatures in degrees Celsius, symbol t_{68} [°C]. By definition the degree Celsius in this scale is identically equal in magnitude to the kelvin, so that

$$T_{68}\ [\mathrm{K}] = (t_{68} + 273.15)\ [^\circ\mathrm{C}]. \tag{12.38}$$

Some representative fixed points of IPTS-68 are given in Table 12.1, and some secondary reference points are listed in Table 12.2.

The standard platinum resistance thermometer is used with the equations given in the text [12.21] to realize the scale from 13.81 [K] to 630.74 [°C], and the standard platinum/platinum-10% rhodium thermocouple thermometer is used with the equations given in the text to realize the scale from 630.74 [°C] to 1064.43 [°C]. Above 1064.43 [°C], the IPTS is defined in terms of the Planck radiation law [eq. (12.4b)], using the gold point at 1064.43 [°C] as the reference temperature and 0.014388 [m·K] for c_2.

It is this last temperature range, above the gold point at 1064.43 [°C], that is of primary interest in radiometry, since it is only here that radiometric measurements are specified as the primary standards. However, those making radiometric measurements in the longer wavelength regions of the infrared spectrum also need to be concerned about thermal radiation from sources at temperatures well below the gold point. This topic will be discussed in more detail in a later chapter on the Thermal Radiation Properties of Matter.

The first International Temperature Scale, the Normal Hydrogen Scale, was adopted by the International Committee on Weights and Measures (CIPM) in 1887 and approved in 1889 by the General Conference on Weights and Measures (CGPM). It was followed by the ITS-27, the ITS-48, which was amended as the IPTS-48 in 1960, and the IPTS-68, currently in effect as amended in 1975. There is now a movement under way to adopt a new IPTS within a few years, perhaps by 1987.

When an IPTS temperature T_{68} has been measured or reported and the corresponding thermodynamic temperature T is needed, it can be obtained by making use of published differences between the two scales $\Delta T = T - T_{68}$ [12.22]. The magnitude of ΔT is less than 0.1 [K] or 100 [m·K] between 0 [°C] and 500 [°C] [12.23]. Above that temperature there is considerable uncertainty and disagreement in ΔT and it is estimated that the thermodynamic temperature at the freezing point of gold may differ from the presently assigned value of 1064.43 [°C] by as much as 0.2 [K] (= 0.2 [°C]). There are a number of projects now under way in different national standards laboratories to make more accurate thermodynamic measurements of the gold point and to establish the estimated accuracy of the results of those measurements with more confidence [12.24].

Note that, since the accuracy of realization of T_{68} is based on the assumption of exactly given fixed-point values (with no uncertainties), the accuracy of $\Delta T = T - T_{68}$ depends primarily on the accuracy or uncertainty of the thermodynamic temperature T, at least up to the gold point at 1064.43 [°C]. Special considerations, including practical notes and sources of error, for establishing the IPTS-68 above 1064.43 [°C] with optical-pyrometry measurements, are presented in section 6 on pp. 125-139 of [12.22]. In addition, for the most accurate measurements, the uncertainty in the universal constant c_2 in the Planck equation might even become significant. In [12.22] (on p. 5), this constant is given as $c_2 = 1.4388 \cdot 10^{-2}$ [m·K]. The most recent value published by NBS [12.6] is $c_2 = 1.438786(45) \cdot 10^{-2}$ [m·K]. These two values, differing by $1.4 \cdot 10^{-7}$ [m·K], are in agreement well within the estimated one-standard-deviation uncertainty of the latter, shown as $4.5 \cdot 10^{-7}$ [m·K]. This uncertainty of only 0.003% will not likely affect an experimental realization of T_{68}.

The more important details of the current amended IPTS-68 have been summarized above. The earlier scales are similarly summarized in Appendix 12-A. More complete authoritative information can be found in reference [12.21].

8. RADIATION THERMOMETRY. The **radiance temperature** of an object is defined as the temperature of a blackbody radiator that has, over a defined spectral pass band, the same radiance as the object. For surfaces having an emissivity less than one, the radiance temperature will be lower than the true temperature. The subject of emissivity and its effect on the radiation emitted by a hot body and, hence, on the relationship between radiance temperature and true temperature, will be discussed in the chapter on the Thermal Radiation Properties of Matter, and will not be treated further here.

8.1 The HUMAN EYE as a RADIATION THERMOMETER. The first **radiation thermometer** was the human eye. The ancient metallurgists, ceramists, glassmakers and others involved with high temperatures learned to estimate temperatures from the brightness and color of a hot object, even though the formalism of a temperature scale did not exist. Even today, a blacksmith judges in this way when a piece of wrought iron heated in a forge is ready for the hammer and glass blowers judge similarly when a piece of glass, heated in a "glory hole" (glass blower's furnace) or by means of a glass blower's torch, is ready for forming.

8.2 The VISUAL OPTICAL PYROMETER. The first optical instrument developed for measuring radiance temperature was the **visual optical pyrometer**. It used the human eye as the detector, but it made it much easier to evaluate the radiance of a hot target by the use of a reference tungsten-filament lamp and a method of converting the observed radiance into a radiance temperature. The optical system superimposes an image of the lamp filament onto that of the target, and the current to the lamp is manually adjusted until the brightness of the filament matches that of the target. (When the match is exact, the image of the filament disappears in the background of the hot-target image, hence it is often called the "disappearing-filament technique".) The wavelength band is restricted to a narrow band centered at about 665 [nm] by the combination of a dark red filter and the spectral response of the normal human eye. The instrument is calibrated by recording the lamp current required to match the known radiance of the filament in a calibrated tungsten strip lamp, or of a blackbody source at a known temperature. The known radiance temperature is plotted as a function of the lamp current producing a calibration curve.

8.3 The PHOTOELECTRIC RADIATION PYROMETER. The **photoelectric radiation pyrometer** is similar to the optical pyrometer, but the human eye is replaced by a suitable radiation detector and the pass band is limited by a narrow-pass-band filter. The detector views the target, and stores the signal in a memory, then views the filament of an internal reference lamp, and automatically adjusts the current to the lamp to produce a signal equal to the target signal. (Both images, target and internal lamp filament, completely fill the instrument throughput, so that the output is directly proportional to the (average) radiance in each beam.) The output may be the current to the lamp, which may be converted to a radiance temperature as is done with a visual optical pyrometer, or the instrument may include a microcomputer which has the calibration stored in its memory and automatically computes the radiance temperature, which may be displayed as a temperature, and/or recorded, e.g., on paper tape. The precision of these devices is of the order of 0.1 to 1 [K].

8.4 The TOTAL-RADIATION PYROMETER. A (spectrally) **total-radiation pyrometer** has a combination of optics (usually focusing mirrors) and a detector that has uniform spectral response over a wide spectral band, and the instrument produces a signal that is a measure of the spectrally total radiance of the target. This instrument is particularly useful for measuring radiance temperatures below the working range of the narrow-pass-band pyrometers. Interpolation between calibration points for these instruments is based on the Stefan-Boltzmann equation [eq. (12.15)].

8.5 The RATIO PYROMETER. The **ratio**, or **"two-color", pyrometer** measures the spectral radiance of the target in two narrow wavelength bands. The ratio of the two radiances is a function of the temperature. Use of this technique reduces errors produced in the other instruments by the emissivity of the target, the propagance over the optical path between the target and instrument, and failure of the target to fill uniformly the complete field of view of the optical systems. If the emissivity of the target, the propagance over the optical path and the fraction of the field of view filled, are the same for both spectral pass bands the inferred temperature is the true temperature of a blackbody source that has the same radiance ratio as the target.

Two-color pyrometry, or, more generally, multi-color pyrometry, is often advocated in the absence of knowledge of the emissivity of the source. However, the accuracy of the technique is extremely sensitive to the values of source emissivity in the instrument pass bands. Careful analysis of the measurement process indicates that, in most cases where the emissivity is not known from independent data, this technique possesses no real advantage over simple single-color pyrometry [12.25]. The principal use of two-color, or multi-color, pyrometry appears to be in situations where average radiance cannot be measured because the source beam fills only a portion of the instrument throughput or where there is strong, spectrally uniform (e.g., caused by smoke or sooty flames) absorption in the optical path.

9. SUMMARY of CHAPTER 12. In this chapter we have started a two-chapter treatment of **temperature**, one of the most important and ubiquitous of the environmental and instrumental parameters that must be considered in the measurement equation. It affects almost every quantity in that equation in one way or another, but we are concerned primarily, in these two chapters, with the way in which it affects radiation sources. Here we focus on **blackbodies** and **blackbody simulators**, and in a later chapter on the

thermal radiation properties of matter, we will look at the ways in which **thermal radiation** is affected by the characteristics of real sources, such as emissivity. Thermal radiation is important as a source of false signals (noise) because all matter above the unattainable absolute zero emits some radiant flux by reason of its temperature. It is also important as a **primary standard** with a known spectral-radiance distribution, given by the **Planck radiation law**.

A blackbody is defined as an opaque surface that absorbs all incident radiant flux of all wavelengths and polarizations incident upon it from all possible directions (reflectance ρ and transmittance τ both equal to zero). By **Kirchhoff's law**,

$$\alpha(\theta,\phi,\lambda) = \varepsilon(\theta,\phi,\lambda), \qquad (12.1)$$

where $\alpha(\theta,\phi,\lambda)$ is the spectral directional absorptance for radiance of wavelength λ incident from direction θ,ϕ and $\varepsilon(\theta,\phi,\lambda)$ is the spectral directional emissivity at wavelength λ in direction θ,ϕ. Then, since $\alpha = 1$ for a blackbody, its emissivity $\varepsilon = 1$ and its thermal radiance at any wavelength and in any direction is the maximum possible for any body at the same temperature. (Emissivity relates only to thermal radiation, resulting from source temperature; other sources of energy may, of course, produce "cold light" greatly in excess of blackbody spectral radiance.)

The spectral radiance as a function of spectral frequency ν [Hz] of a blackbody at thermodynamic or absolute temperature T [K] is given by the **Planck radiation law**

$$L_{e,\nu,bb}(\nu,T) = 2\cdot h\cdot n^2\cdot c_o^{-2}\cdot \nu^3\cdot\{\exp[h\cdot\nu/(k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}}]. \qquad (12.2)$$

This is the spectral radiance from the blackbody source in any given direction at any point in a non-attenuating medium of refractive index n. If there is attenuation, this Planck-law value must be multiplied by the propagance τ^* over the intervening optical path. The other quantities are: the velocity of electromagnetic radiation in vacuum c_o [$\mathrm{m\cdot s^{-1}}$]; **Planck's constant** of action h [J·s]; and the **Boltzmann constant** k_B [$\mathrm{J\cdot K^{-1}}$]. Values of these constants are given.

In terms of vacuum wavelength $\lambda_o \equiv c_o/\nu$ [m] and "local" wavelength $\lambda \equiv \lambda_o/n$ [m] in a medium of index n, the Planck law is given by

$$L_{e,\lambda_o,bb}(\lambda_o,T) = 2\cdot h\cdot n^2\cdot c_o^2\cdot\lambda_o^{-5}\cdot\{\exp[h\cdot c_o/(\lambda_o\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot m^{-1}}], \qquad (12.3a)$$

and

$$L_{e,\lambda,bb}(\lambda,T) = 2\cdot h\cdot n^{-2}\cdot c_o^2\cdot\lambda^{-5}\cdot\{\exp[h\cdot c_o/(n\cdot\lambda\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot m^{-1}}], \qquad (12.4a)$$

respectively. However, the following forms, with areas in [$\mathrm{m^2}$] but wavelengths in [nm] maintain, and make it easier to verify, unit-dimension consistency:

$$L_{e,\lambda_o,bb}(\lambda_o,T) = 2\cdot 10^{36}\cdot h\cdot n^2\cdot c_o^2\cdot\lambda_o^{-5}\cdot\{\exp[10^9\cdot h\cdot c_o/(\lambda_o\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}], \qquad (12.3b)$$

and

$$L_{e,\lambda,bb}(\lambda,T) = 2\cdot 10^{36}\cdot h\cdot n^{-2}\cdot c_o^2\cdot\lambda^{-5}\{\exp[10^9\cdot h\cdot c_o/(n\cdot\lambda\cdot k_B\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}]. \qquad (12.4b)$$

Simpler forms of the Planck equation, in terms of **the radiation constants** c_1 or c_{1L}, and c_2, are often used [these constants are defined and values are given in eqs. (12.11), (12.12), and (12.13)]:

$$L_{e,\lambda_o,bb}(\lambda_o,T) = c_{1L}\cdot n^2\cdot\lambda_o^{-5}\{\exp[c_2/(\lambda_o\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}] \qquad (12.3c)$$

and

$$L_{e,\lambda,bb}(\lambda,T) = c_{1L}\cdot n^{-2}\cdot\lambda^{-5}\cdot\{\exp[c_2/(n\cdot\lambda\cdot T)] - 1\}^{-1}\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}]. \qquad (12.4c)$$

However, in most textbooks and reference books, at least until quite recently, the Planck-law equation is given in terms of spectral radiant exitance as

$$M_{e,\lambda,bb}(\lambda,T) \;=\; c_1\cdot\lambda^{-5}\cdot\{\exp[c_2/(\lambda\cdot T)] - 1\}^{-1} \; [\mathrm{W\cdot m^{-2}\cdot nm^{-1}}], \tag{12.14}$$

ignoring the distinction between wavelength in air λ and in vacuum λ_o; in other words setting $n = 1$ in eq. (12.4c) and multiplying by π, since a blackbody is lambertian (isotropically diffuse) and, for such sources, $M_\lambda = \pi\cdot L_\lambda$ (and $c_1 = \pi\cdot c_{1L}$). The effects and significance of this neglect of the index n are discussed in Section 3.1.6.

Transformation, or reconciliation, between the different forms of the Planck equation expressed in terms of different spectral variables requires equating the different expressions for the differential radiance

$$dL \;=\; L_\nu\cdot d\nu \;=\; L_{\lambda_o}\cdot d\lambda_o \;=\; L_\lambda\cdot d\lambda \;=\; L_\sigma\cdot d\sigma, \tag{12.8}$$

where, for completeness, we have added an expression in terms of one more commonly used spectral parameter, **wave number** $\sigma \equiv 1/\lambda_o$ [cm^{-1}], where λ_o is given in [cm] {and c_o in [cm·s^{-1}]}. Also needed are the relations between the spectral-parameter differentials

$$d\nu \;=\; -c_o\cdot\lambda_o^{-2}\cdot d\lambda_o \;=\; -c_o\cdot n^{-1}\cdot\lambda^{-2}\cdot d\lambda \;=\; c_o\cdot d\sigma \; [\mathrm{Hz}] \text{ or } [\mathrm{s^{-1}}]. \tag{12.9}$$

Furthermore, since we are primarily interested in magnitudes, and not in whether one quantity increases when another decreases, we will ignore the minus signs in eq. (12.9) hereafter. Incidentally, although the useful forms of these relations are those of eq. (12.9), it may be easier to remember them in the following symmetrical form (ignoring the minus signs):

$$d\nu/\nu \;=\; d\lambda_o/\lambda_o \;=\; d\lambda/\lambda \;=\; d\sigma/\sigma. \tag{12.10}$$

Similar relationships are presented, in Appendix 12-A where they are discussed more fully, for three more spectral variables:

vacuum wavenumber $\sigma_o \equiv 1/\lambda_o$ [cm^{-1}] (or [m^{-1}], or [nm^{-1}]); (12A.1)

"local" wavenumber $\sigma \equiv 1/\lambda$ [cm^{-1}] (or [m^{-1}], or [nm^{-1}]), in a medium of refractive index n; (12A.2)

energy per photon $q \equiv h\cdot\nu$ [J·q^{-1}], (where h [J·s·q^{-1}] is Planck's constant of action per photon); more often this is given in units of electron-volts (per quantum) [eV·q^{-1}]. (12A.3)

Ignoring spectral dependence of index [setting $n(\nu) = n$] and integrating the Planck equation, eq. (12.3) with respect to ν from 0 to ∞ gives us the **Stefan-Boltzmann law** for spectral-total radiance as a function of temperature:

$$L_{e,bb}(T) \;=\; n^2\cdot\sigma_L\cdot T^4 \; [\mathrm{W\cdot m^{-2}\cdot sr^{-1}}]. \tag{12.15}$$

And, again, this is often expressed for exitance, and with the refractive index n neglected:

$$M_{e,bb}(T) \;=\; \sigma_M\cdot T^4 \; [\mathrm{W\cdot m^{-2}}], \tag{12.17}$$

The two forms of the **Stephan-Boltzmann constant**, σ_L and σ_M, are defined and values are given in eqs. (12.16) and (12.18), respectively.

From the Planck law [eq. (12.3c)] we also have the **Wien displacement law**

$$L_{e,\lambda_o,bb}(\lambda_o\cdot T) \;=\; c_{1L}\cdot n^2\cdot T^5\cdot f(\lambda_o\cdot T) \; [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}], \tag{12.19}$$

that the spectral distribution of blackbody radiance [or exitance -- see eq. (12.21)] is a function of the product $\lambda_o \cdot T$. A corollary, that is probably more familiar and is often presented as the Wien displacement law, is

$$\begin{aligned} n \cdot \lambda_m \cdot T &= b \quad \text{(a constant)} \\ &= 2.8978 \cdot 10^6 \ [\text{nm} \cdot \text{K}], \end{aligned} \tag{12.22}$$

and another somewhat less familiar corollary is

$$\begin{aligned} L_{e,\lambda,bb}(\lambda_m \cdot T) \cdot n^{-2} \cdot T^{-5} &= b' \quad \text{(a constant)} \\ &= 4.0956 \cdot 10^{-15} \ [\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{nm}^{-1} \cdot \text{K}^{-5}], \end{aligned} \tag{12.23}$$

where λ_m is the wavelength (in a medium of index n) for the maximum value of $L_{e,\lambda,bb}$ [or of $M_{e,\lambda,bb}$]. Again, in air $n \approx 1$ and $\lambda_o \approx \lambda$, so most texts and references ignore n and the distinction between λ_o and λ.

Finally, there are two frequently used approximations to the Planck law that historically antedate it. One is the **Wien distribution law** or **radiation law**

$$L_{e,\lambda,bb}(\lambda,T) \approx c_{1L} \cdot \{n^2 \cdot \lambda^5 \cdot \exp[c_2/(n \cdot \lambda \cdot T)]\}^{-1} \ [\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{nm}^{-1}] \tag{12.24}$$

which is a good approximation to the Planck law when $\lambda \cdot T$ is very small. As always, the index n is often omitted on the assumption that, in air, $n \approx 1$ (approx.). The other is the **Rayleigh-Jeans law**

$$L_{e,\lambda,bb}(\lambda,T) \approx (c_{1L}/c_2) \cdot (n \cdot \lambda^4)^{-1} \cdot T \ [\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{nm}^{-1}] \tag{12.27}$$

which is a good approximation to the exact Planck law only when $n \cdot \lambda \cdot T$ is very large. Again, the index n is usually omitted on the assumption that, in air, $n \approx 1$. The degree of approximation provided by each of these "laws" and their relationship to each other is discussed and is summarized in Figure 12.1.

Radiation pyrometry makes use of the Planck-law relationship between spectral radiance and temperature to measure source temperatures, usually high temperatures, by measurements based on the observed values of radiance of their thermal emissions. Relationships that may be useful in this connection are presented and discussed in sections 4 and 5 and include eqs. (12.30) - (12.33). Important characteristics of blackbody simulators, used as sources of known spectral radiance (when the temperature is accurately known) and as radiation traps to keep stronger beams from interfering with the measurement of weaker beams of radiant flux, are presented and discussed in section 6, including sections 6.1 through 6.3.

The rest of the chapter treats **temperature scales**, including some historical material as background to make the current situation more intelligible and also to help in understanding earlier publications in the literature. The earliest scales were based on more or less arbitrary values assigned to the freezing and boiling points of water, with the **Celsius** (formerly **Centigrade**) **scale** assigning 0 [°C] and 100 [°C], respectively, to these two reference points. Discovery of the **thermodynamic** (or **absolute**) **scale**, based on the **Carnot cycle**, provided a scale that does not require reference to the properties of any particular material. The size of the scale unit has been established by assigning the value of **273.16 [K]** to the **triple point of water**, making it possible to define 1 [°C] ≡ 1 [K] without disturbing earlier measurements in [°C] defined by the assignment of the freezing and boiling points of water at 0 [°C] and 100 [°C], respectively. However, accurate measurements of thermodynamic temperatures are very difficult and time consuming and, even at best, cannot achieve the precision (reproducibility) of most other methods of measuring temperatures relative to consistently reproducible reference points. Accordingly, the **International Practical Temperature Scale** (IPTS) has been developed to facilitate precise comparison between temperature measurements in different scientific and industrial laboratories all over the world. The latest version, the **IPTS-68** (first adopted in 1968 and amended in 1975) is described in some detail, particularly in the ranges pertinent to radiometric measurements, and all, including earlier versions leading up to the IPTS-68, are summarized in Appendix 12-B, with citations of official publications giving complete details for those that need them. The IPTS-68 establishes precisely reproducible fixed points throughout the needed ranges of temperature measurement and also specifies the exact measurement procedures to be used for interpolation and extrapolation between and beyond those fixed points. This consistently reproducible scale is adequate for many requirements. However, for those who need absolute values on the thermodynamic scale, a measured value of IPTS temperature can be converted to the corresponding thermodynamic temperature by means of published tables of the differences between the two scales. Those tables are periodically improved as research provides more accurate measurements on the thermodynamic scale. Finally, there is a brief summary of measurement techniques and types of instrumentation for **radiation thermometry**.

10. ACKNOWLEDGMENTS. In addition to the references cited for specific items, we are indebted to many sources in the literature and among our colleagues for background material and helpful discussions that have been invaluable in broadening the scope of this chapter and in avoiding errors and inadequacies in the preparation of the text. Although it would be impossible to list all of these sources, we have had particularly valuable discussions with John B. Shumaker, Albert T. Hattenburg, Henry J. Kostkowski, James F. Schooley, Martin L. Reilly, Klaus D. Mielenz, and Paul H. E. Meijer. However, responsibility for what has finally been said in the text rests entirely with the authors.

Appendix 12-A. Planck Equations for Additional Spectral Variables.

In addition to the several spectral parameters already treated, the **vacuum wavenumber**[1] σ_o, the **"local" wavenumber** σ, or the **energy per photon** q, may also appear as the spectral variable in the Planck-law equation. They are defined as follows:

$\sigma_o \equiv 1/\lambda_o$ [cm^{-1}] (or [m^{-1}], or [nm^{-1}]), vacuum wavenumber; (12A.1)

$\sigma \equiv 1/\lambda$ [cm^{-1}] (or [m^{-1}], or [nm^{-1}]), "local" wavenumber in a medium of refractive index n; (12A.2)

$q \equiv h\cdot\nu$ [J$\cdot$q^{-1}], energy per photon (where h [J$\cdot$s$\cdot$q^{-1}] is Planck's constant of action per photon); more often this is given in units of electron-volts (per quantum) [eV$\cdot$q^{-1}]. (12A.3)

Note: Don't confuse (1) the spectral-variable quantity q, the quantum-energy that has unit-dimensions [J$\cdot$q^{-1}], with (2) the unit-dimension [q], the quantum (see Table 4-3 in Chapter 4 [12.26])[2]. This is discussed in more detail later on.

These spectral parameters are related to the frequency ν as follows:

$$\nu = c_o\cdot\sigma_o = c_o\cdot\sigma/n = q/h \text{ [Hz]}. \qquad (12A.4)$$

(Here, the length units for c_o and for σ_o or σ must be the same; e.g. if, as is very commonly the case, σ_o and σ are given in reciprocal centimeters [cm^{-1}], c_o must then be in [cm$\cdot$s^{-1}].) Also, as we saw before, we need the corresponding relations between the differentials:

$$d\nu = c_o\cdot d\sigma_o = c_o\cdot n^{-1}\cdot d\sigma = h^{-1}\cdot dq \text{ [Hz]}. \qquad (12A.5)$$

Again, the above relations are the most useful, but it may be easier to remember them in the symmetrical form

$$d\nu/\nu = d\sigma_o/\sigma_o = d\sigma/\sigma = dq/q. \qquad (12A.6)$$

As before, we must start with the equation for the complete differential quantity $dL_{e,bb}$ [and we make use of eq. (12.2)]:

$$\begin{aligned} dL_{e,bb} &= L_{e,\nu,bb}(\nu,T)\cdot d\nu \\ &= 2\cdot h\cdot n^2\cdot c^{-2}\cdot\nu^3\cdot\{\exp[h\cdot\nu/(k_B\cdot T)] - 1\}^{-1}\cdot d\nu \text{ [W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}]. \qquad (12A.7) \end{aligned}$$

Substituting for ν and $d\nu$ the values from eqs. (12A.4) and (12A.5), collecting terms and cancelling all that offset each other, and then dropping the differential, we get:

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[1] Don't confuse the spectral variable quantity σ (unit-dimensions [cm^{-1}]) with the Stefan-Boltzmann constant σ_L or σ_M (unit-dimensions [W$\cdot$m$^{-2}\cdot$K^{-4}]). The context in which it is used will indicate which meaning of σ is intended.

[2] The unit-dimension [q] is not customarily recognized in giving the unit-dimensions of Planck's constant as [J$\cdot$s], although it is always understood that this is the value of action per quantum. Here, where we use this unit-dimension explicitly as [q], it must be added for consistency to the unit-dimensions of all expressions involving h, or any other quantity per quantum.

$$L_{e,\sigma_o,bb}(\sigma_o,T) = 2\cdot h\cdot n^4\cdot c_o^2\cdot\sigma_o^3\cdot\{\exp[h\cdot c_o\cdot\sigma_o/(k_B\cdot T)] - 1\}^{-1} \; [W\cdot m^{-2}\cdot sr^{-1}\cdot(cm^{-1})^{-1}] \tag{12A.8}$$

$$L_{e,\sigma,bb}(\sigma,T) = 2\cdot h\cdot c_o^2\cdot\sigma^3\cdot\{\exp[h\cdot c_o\cdot\sigma/(n\cdot k_B\cdot T)] - 1\}^{-1} \; [W\cdot m^{-2}\cdot sr^{-1}\cdot(cm^{-1})^{-1}] \tag{12A.9}$$

$$L_{e,q,bb}(q,T) = 2\cdot h^{-3}\cdot n^4\cdot c_o^{-2}\cdot q^3\cdot\{\exp[q/(k_B\cdot T)] - 1\}^{-1} \; [W\cdot m^{-2}\cdot sr^{-1}\cdot(J\cdot q^{-1})^{-1}] \tag{12A.10}$$

Note that, since the customary unit-dimension of wavenumber is the reciprocal centimeter $[cm^{-1}]$, the unit-dimension of radiance per unit of this spectral variable is the radiance per unit reciprocal centimeter, or watts per square meter, steradian and reciprocal centimeter $[W\cdot m^{-2}\cdot sr^{-1}\cdot(cm^{-1})^{-1}]$. This could be written in what appears to be a simpler form as $[W\cdot m^{-2}\cdot sr^{-1}\cdot cm]$, but in this form it is not so immediately apparent that this quantity is radiance per wavenumber unit, i.e. per reciprocal centimeter. Similarly, for the spectral variable in units of energy per quantum, or quantum-energy, the spectral radiance is radiance per unit quantum-energy, or watts per square meter, steradian and quantum-energy unit.

Here we have another possibility for confusion. It is essential to distinguish clearly between (1) quantum-energy or energy per quantum q (= $h\cdot\nu$), the quantity used as a spectral parameter, and (2) quantum, the unit-dimension $[q]$, that appears in the unit-dimensions of that quantum-energy q, which are joules per quantum $[J\cdot q^{-1}]$. Accordingly, the unit-dimensions of $L_{e,q,bb}(q,T)$ are $[W\cdot m^{-2}\cdot sr^{-1}\cdot(J\cdot q^{-1})^{-1}]$, as shown in eq. (12A.10). Again, this apparently could be "simplified" {by recognizing that $[W] = [J\cdot s^{-1}]$} to $[q\cdot s^{-1}\cdot m^{-2}\cdot sr^{-1}]$, giving the completely erroneous impression that spectral (energy) radiance in terms of quantum-energy as the spectral parameter

$$L_{e,q} \; [W\cdot m^{-2}\cdot sr^{-1}\cdot(J\cdot q^{-1})^{-1}]$$

is physically identical to photon-flux sterance (radiance)

$$L_p \; [q\cdot s^{-1}\cdot m^{-2}\cdot sr^{-1}]$$

(see Table 4-3 in Chapter 4 [12.26]). This is just another illustration of the statement made earlier (in section 3.1.3) that spectral parameters are quantities that govern the spectral distribution of a radiometric quantity and that their unit-dimensions must be kept distinct from, and do not cancel or combine with, the unit-dimensions of the radiometric quantity that they govern.

We have expressed the Planck law in energy units {flux in $[W]$} in terms of different spectral parameters in eqs. (12.2), (12.3), (12.4), (12A.8), (12A.9), and (12A.10). They can be transformed to the corresponding blackbody photon-flux sterance (radiance), in flux units of quanta per second, e.g. $L_{p,\nu,bb}(\nu,T)$ $[q\cdot s^{-1}\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}]$, by just dividing the energy-unit expression [eq. (12A.7)] by the number of watts per photon-flux unit or $h\cdot\nu$ $[W\cdot(q\cdot s^{-1})^{-1}]$. For example:

$$\begin{aligned} L_{p,\nu,bb} &= (h\cdot\nu)^{-1}\cdot L_{e,\nu,bb} \\ &= 2\cdot n^4\cdot c_o^{-2}\cdot\nu^2\cdot\{\exp[h\cdot\nu/(k_B\cdot T)] - 1\}^{-1} \; [q\cdot s^{-1}\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}] \end{aligned} \tag{12A.11}$$

Similarly, if we start with eq. (12A.10), we have

$$L_{p,q,bb} = (h\cdot\nu)^{-1}\cdot L_{e,q,bb} = q^{-1}\cdot L_{e,q,bb}$$

$$= 2\cdot h^{-3}\cdot n^{4}\cdot c_{o}^{-2}\cdot q^{2}\cdot\{\exp[q/(k_{B}\cdot T)] - 1\}^{-1}\ [q\cdot s^{-1}\cdot m^{-2}\cdot sr^{-1}\cdot(J\cdot q^{-1})^{-1}], \qquad (12A.12)$$

where we have divided by $h\cdot\nu = q\ [W\cdot(q\cdot s^{-1})^{-1}]$ or $[J\cdot q^{-1}]$ because we also want the expression given as a function of the spectral variable $q\ [J\cdot q^{-1}]$.

Eqs. (12A.8) and (12A.9) can also be expressed in terms of c_{1L}, and and c_2 as follows:

$$L_{e,\sigma_o,bb}(\sigma_o,T) = c_{1L}\cdot n^{4}\cdot\sigma_o^{3}\cdot\{\exp[c_2\cdot\sigma_o/T] - 1\}^{-1}\ [W\cdot m^{-2}\cdot sr^{-1}\cdot(cm^{-1})^{-1}] \qquad (12A.8a)$$

and

$$L_{e,\sigma,bb}(\sigma,T) = c_{1L}\cdot\sigma^{3}\{\exp[c_2\cdot\sigma/(n\cdot T)] - 1\}^{-1}\ [W\cdot m^{-2}\cdot sr^{-1}\cdot(cm^{-1})^{-1}] \qquad (12A.9a)$$

Expressing eq. (12A.10) in terms of c_{1L} and c_2 complicates the equation rather than simplifying it.

Appendix 12-B. Summary of Earlier International Temperature Scales.

12B.1 NORMAL HYDROGEN SCALE. The current standard, the IPTS(1968), has been developed over the last century. The first International Temperature Scale based on gas thermometry, the normal hydrogen scale, was adopted by the International Committee on Weights and Measures (CIPM) in 1887, and approved in 1889 by the General Conference of Weights and Measures (CGPM). It covered the range of -25 to 100 [°C], and used the ice point at 0 [°C] and the steam point at 100 [°C] as fixed points. It was transferred to mercury-in-glass thermometers for distribution to other laboratories.

12B.2 ITS-27. The next international temperature scale was adopted by CGPM in 1927, and covered the range from below the boiling point of oxygen, then defined as -182.9 [°C], to above the melting point of gold, then defined at 1063 [°C], and added four intermediate defining fixed points. The platinum resistance thermometer was used to realize the scale from below the boiling point of oxygen to 660 [°C], the platinum/platinum-10% rhodium thermocouple from 660 [°C] to 1063 [°C], and the optical pyrometer above the gold point, with the value of $1.432 \cdot 10^{-2}$ [m·K] for c_2, and using the Wien-distribution-law equation (eq. 12.24) with the n^2 term omitted. Detailed equations were included for the reduction of the resistance-thermometer and thermocouple data to temperatures.

12B.3 ITS-48. The International Temperature Scale of 1948 was adopted by CGPM in 1948. (Note that the designation "International Practical Temperature Scale" did not appear until the amended edition of 1960.) The lower limit of the platinum-resistance-thermometer range was changed to the defined oxygen boiling point, and the division between this range and the thermocouple range became the measured (not defined) antimony freezing point. The silver freezing point was defined as 960.8 [°C], and the gold melting point became the gold freezing point. The Wien-law equation was replaced by the Planck-law equation, with a value of $1.438 \cdot 10^{-2}$ [m·K] for c_2. The equations for use with the resistance thermometer and thermocouple were modified, and the limitation of $\lambda \cdot T$ to optical pyrometry (visible spectrum only) was removed.

The International Practical Temperature Scale of 1948, amended edition of 1960, or IPTS-48, was adopted by CGPM in 1960. The modifications to ITS-48 were: the triple point of water, defined as 0.01 [°C], replaced the freezing point of water; the freezing point of zinc, defined as 419.505 [°C], became a preferred alternate to the sulfur boiling point; and the equations for use with the resistance thermometer and thermocouple were modified.

12B.4 IPTS-68. The International Practical Temperature Scale of 1968 was promulgated by CIPM in 1968. There were extensive changes from IPTS-48. These included changes in the values assigned to the fixed points, to bring them all closer to the thermodynamic scale. The lower limit of the scale was extended to 13.81 [K]. (Although the use of two helium-vapor-pressure scales was recommended for the range 0.5 to 5.2 [K], nothing yet has been adequately standardized to support extension of the IPTS below 13.81 [K].) Six new fixed points were added: the triple point of equilibrium hydrogen, 13.81 [K]; an intermediate liquid-hydrogen point, 17.042 [K]; the boiling point of equilibrium hydrogen, 20.28 [K]; the boiling point of neon, 27.102 [K]; the triple point of oxygen, 54.361 [K]; and the freezing point of tin, 230.9681 [°C]. The boiling point of sulfur was deleted, and the values were changed for the boiling point of oxygen, 90.188 [K]; the freezing point of zinc, 419.58 [°C]; the freezing point of silver, 961.93 [°C]; and the freezing point of gold, 1064.43 [°C]. The equations for reducing resistance-thermometer data became much more complex, and the value for c_2 was changed to $1.4388 \cdot 10^{-2}$ [m·K].

The temperature values in IPTS-68 were not changed in the 1975 amended edition. The changes were primarily in the text, to clarify and simplify use of the scale. The condensation point of oxygen replaced the boiling point of oxygen with no change in the assigned value. The triple point of argon, 83.798 [K], was introduced as a permitted alternative to the condensation point of oxygen.

Table 12.1. Representative Fixed Points of the IPTS-68[a]

Equilibrium state	Assigned value of International Practical Temperature	
	T_{68}(K)	t_{68}(°C)
Equilibrium between the solid, liquid, and vapor phases of equilibrium hydrogen (triple point of equilibrium hydrogen)[b]	13.81	-259.34
Equilibrium between the solid, liquid, and vapor phases of oxygen (triple point of oxygen)	54.361	-218.789
Equilibrium between the solid, liquid, and vapor phases of water (triple point of water)	273.16	0.01
Equilibrium between the liquid and vapor phases of water (boiling point of water)[c]	373.15	100
Equilibrium between the solid and liquid phases of tin (freezing point of tin)[c]	505.1181	231.9681
Equilibrium between the solid and liquid phases of zinc (freezing point of zinc)	692.73	419.58
Equilibrium between the solid and liquid phases of silver (freezing point of silver)	1235.08	961.93
Equilibrium between the solid and liquid phases of gold (freezing point of gold)	1337.58	1064.43

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[a] Representative entries taken from Table 1 of reference [12.21], which gives more points and more details and qualifications or limitations, particularly possible effects of pressure and isotopic abundances.

[b] The term equilibrium hydrogen is defined in section III, 5, of reference [12.21].

[c] The freezing point of tin (t' = 231.9292 [°C], see interpolation eq. 10 and eq. 11a in reference [12.21]) may be used as an alternative to the boiling point of water.

Table 12.2. Representative Secondary Reference Points of the IPTS-68[a]

Equilibrium state	International Practical Temperature	
	T_{68} [K]	t_{68} [°C]
Equilibrium between the solid, liquid, and vapor phases of nitrogen (triple point of nitrogen)	63.146	-210.004
Equilibrium between the liquid and vapor phases of nitrogen (boiling point of nitrogen)	77.344	-195.806
Equilibrium between the solid and vapor phases of carbon dioxide (sublimation point of carbon dioxide)	194.674	-78.476
Equilibrium between the solid and liquid phases of mercury (freezing point of mercury)	234.314	-38.836
Equilibrium between the solid, liquid, and vapor phases of phenoxybenzene (diphenyl ether) (triple point of phenoxybenzene)	300.02	26.87
Equilibrium between the solid, liquid, and vapor phases of benzoic acid (triple point of benzoic acid)	395.52	122.37
Equilibrium between the solid and liquid phases of cadmium (freezing point of cadmium)	594.258	321.108
Equilibrium between the liquid and vapor phases of mercury (boiling point of mercury)	629.81	356.66
Equilibrium between the solid and liquid phases of the copper-aluminum eutectic	821.41	548.26
Equilibrium between the solid and liquid phases of aluminum (freezing point of aluminum)	933.61	660.46
Equilibrium between the solid and liquid phases of nickel (freezing point of nickel)	1728	1455
Equilibrium between the solid and liquid phases of platinum (freezing point of platinum)	2042	1769
Equilibrium between the solid and liquid phases of aluminum oxide, Al_2O_3 (temperature of melting aluminum oxide)	2327	2054
Equilibrium between the solid and liquid phases of tungsten (temperature of melting tungsten)	3695	3422

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[a] Representative entries taken from table 6 of reference [12.21], which gives more points and many more details and qualifications or limitations, including the fact that there are fairly large uncertainties in many of the values tabulated here. Consequently, reference [12.21] should be consulted whenever an attempt is made to use these data in an application calling for any degree of precision or accuracy.

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This is the eighth in a series of Technical Notes (910-) entitled "Self-Study Manual on Optical Radiation Measurements". It contains Chapter 12 of Part I of this Manual. Additional chapters will continue to be published, similarly, as they are completed. The Manual is a comprehensive tutorial treatment of the measurement of optical radiation that is complete enough for self-instruction. Detailed chapter summaries make it also a convenient authoritative reference source.

In this chapter we review the radiometric treatment, and the significance for radiometric measurements, of blackbodies, blackbody radiation, and temperature scales. Many important and interesting aspects are not traeated because our primary interest is in radiometry and radiometric measurements. The emphasis is on ideal blackbodies and laboratory simulators; thermal radiation from other real sources will be treated in a chapter on Thermal Radiation Properties of Materials.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

blackbody; IPTS; ITS; Planck; Rayleigh-Jeans; Stefan-Boltzmann; temperature; thermal radiation; Wien

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