Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

October through December 1949

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

AUTHENTICATED
U.S. GOVERNMENT
INFORMATION
GPO

NATIONAL APPLIED MATHEMATICS LABORATORIES

October 1 through December 31, 1949

ADMINISTRATIVE OFFICE

John H. Curtiss, Ph.D., *Chief*
Edward W. Cannon, Ph.D., *Assistant Chief*
Olga Taussky-Todd, Ph.D., *Mathematics Consultant*
Myrtle R. Kellington, M.A., *Technical Aid*
Luis O. Rodriguez, M.A., *Chief Clerk*
John B. Tallerico, B.C.S., *Assistant Chief Clerk*
Jacqueline Y. Barch, *Secretary*
Dora P. Cornwell, *Secretary*
Esther McCraw, *Secretary*
Pauline F. Peterson, *Secretary*

INSTITUTE FOR NUMERICAL ANALYSIS
Los Angeles, California

J. Barkley Rosser, Ph.D. .... Director of Research
Albert S. Cahn, Jr., M.S. .... Assistant to Director

Research Staff

Forman S. Acton, Ph.D. .... Mathematician
George E. Forsythe, Ph.D. .... Mathematician
Magnus R. Hestenes, Ph.D. .... Mathematician
William Karush, Ph.D. .... Mathematician
Cornelius Lanczos, Ph.D. .... Mathematician
Alexander M. Ostrowski, Ph.D. .... Consultant
Otto Szasz, Ph.D. .... Mathematician
Wolfgang R. Wasow, Ph.D. .... Mathematician

Graduate Fellows:

George E. Gourrich, B.S.
Harold Gruen, M.A.
Lloyd K. Jackson, M.A.
Raymond P. Peterson, Jr., M.A.
Marvin L. Stein, M.A.

Machine Development Unit

Harry D. Huskey, Ph.D. .... Mathematician
Biagio F. Ambrosio, B.S. .... Electronic Engineer
Edward L. Lacey, M.S. .... Electronic Engineer
Roselyn Siegel Lipkis, B.A. .... Mathematician
David S. Rutland, M.S. .... Electronic Scientist

Laboratory Electronic Engineers:

Brent H. Alford
Robert V. Barnett
Charles M. Brauer
Arnold Dolmatz
Blanche C. Eidem
Sidney S. Green, B.S.
Harry T. Larson, B.S.
John D. Mack
Michael J. Markakis, B.E.
John L. Newberger
Geraldine Orr
James W. Walsh

Computation Unit

Gertrude Blanch, Ph.D. .... Mathematician
Alfredo Baños, Jr., Ph.D. .... Consultant
David S. Saxon, Ph.D. .... Consultant
Louis L. Bailin, Ph.D. .... Mathematician
Marvin Howard, M.S. .... Mathematician
Everett C. Yowell, Ph.D. .... Mathematician

Computing Staff:

Helen C. Arens, M. A.
Patricia Burton Bremer, B.A.
Leola Cutler, B.A.
Lillian Forthal, B.A.
Fannie M. Gordon, M.B.A.
Elizabeth F. Harding, B.A.
Donald E. Henley
Ruth B. Horgan, B.A.
Gerald W. Kimble, B.A.
Nancy Robbins Mann, M.A.
Shirley L. Marks, M.A.
William O. Paine, Jr.
John A. Postley, B.A.
Everett A. Rea, B.A.
Albert H. Rosenthal
M. Winifred Smith
Rose Tishman, B.A.

Administrative Unit

M. Evelyn Michaud .... Administrative Officer
F. Patricia K. Peed .... Librarian
Ellma Frantz, B.S. .... Library Assistant
Velma R. Huskey, B.A. .... Physical Science Editor

Secretarial Staff:

Elsie E. Aho
Irene M. Everett
Margaret W. Gould
Eleanore J. Harris
Dorothy M. Hibbard
Elsie L. Husman
Dora Kaplan
Tess D. Mazer
Sata Ohashi
Gertrude Z. Reider

COMPUTATION LABORATORY

John Todd, B.S. .... Chief
Franz L. Alt, Ph.D. .... Assistant Chief
Arnold N. Lowan, Ph.D. .... Consultant
Jack Belzer, B.A. .... Mathematician
Donald O. Larson, B.S. .... Mathematician
Joseph H. Levin, Ph.D. .... Mathematician
Herbert E. Salzer, M.A. .... Mathematician
Irene A. Stegun, M.A. .... Mathematician

Computing Staff:

Oneida L. Baylor
Ruth E. Capuano
Natalie Coplan, B.A.
Bernard C. Dove
Mary M. Dunlap, B.S.
Elizabeth F. Godefroy
Wanda P. Gordon, B.A.
Benjamin F. Handy, Jr., M.S.
Robert M. Hayes, M.A.
Bernard Heindish, B.E.E.
Harley A. Hill, B.S.
Viola D. Hovsepian, B.S.
Joseph B. Jordan, B.A.
Henry B. Juenemann, B.A.
Lionel Levinson, B.M.E.
David S. Liepman
Michael S. Montalbano, B.A.
Malcolm W. Oliphant, M.A.
Mary Orr
B. Stanley Prusch
Andress O. Ridgway, B.S.
Max Rosenberg, B.A.
Ruth C. Shepherd, B.S.
Jack Singer
Milton Stein
Bertha H. Walter
Marjorie O. White
Ruth Zucker, B.A.

Florence C. Pettepit .... Secretary
Lillian Sloane .... Secretary

STATISTICAL ENGINEERING LABORATORY

Churchill Eisenhart, Ph.D. .... Chief
W. J. Youden, Ph.D. .... Assistant Chief
Lola S. Deming, M.A. .... Technical Aid
Henry B. Mann, Ph.D. .... Consultant
E. J. Gumbel, Ph.D. .... Consultant
Joseph M. Cameron, M.S. .... Mathematician
Gerald J. Lieberman, M.A. .... Math'l Statistician
Julius Lieblein, M.A. .... Mathematician
Celia S. Martin, B.A. .... Math'l Statistician
Helen S. Petitt .... Computer
Lester H. Gordon .... Computer
Vivian M. Frye, B.A. .... Secretary
Helen M. Herbert .... Secretary

MACHINE DEVELOPMENT LABORATORY

Edward W. Cannon, Ph.D. .... Chief
Samuel Lubkin, Ph.D. .... Electronic Scientist
Merle M. Andrew, Ph.D. .... Mathematician
Ira C. Diehm .... Mathematician
Florence K. Koons, B.A. .... Mathematician
Ethel C. Marden, B.A. .... Mathematician
Edith T. Norris, B.A. .... Mathematician
Ida Rhodes, M.A. .... Mathematician
Otto T. Steiner, B.A. .... Mathematician
Susan A. Moon .... Secretary

U. S. DEPARTMENT OF COMMERCE
Charles Sawyer, Secretary

NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

Projects and Publications
of the
NATIONAL APPLIED MATHEMATICS LABORATORIES

October through December 1949

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from October 1, 1949 through December 31, 1949.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J H Curtiss
Chief

E U Condon

Director
National Bureau of Standards
February 1, 1950

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, and Astronomy.

D. Mathematical Statistics

E. Mathematical Tables

* New projects

Status of Projects

December 31, 1949

I. Institute for Numerical Analysis

(Section 11.1)

1. Research in Numerical Analysis

Key to letter symbols in project numbers:

AE - Algebraic equations
AM - Analytical applied mathematics
CM - Numerical methods in conformal mapping
ODE - Ordinary differential equations
PD - Partial differential equations
PM - Probabilistic methods in numerical analysis

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Project 11.1/1-49-AE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. A search of the literature has been under way. In an effort to insure completeness, a circular was written to workers in the field asking them to suggest little known references. In particular, reference was made to the bibliography "Computational aspects of finite matrix theory" by A. Ostrowski, O. Todd and J. Todd (Ostrowski was formerly with INA, J. Todd is with the Computation Laboratory, and O. Todd is Mathematics Consultant to the NAML). As a result, a larger bibliography is now available, as well as a large compilation of known methods of matrix inversion.

To stimulate interest in the project, a weekly seminar was conducted at which the following persons spoke on the topics listed.

C. Lanczos,	Orthogonalization procedures
J. B. Rosser,	A new iteration scheme
Gertrude Blanch,	Partitioning
H. D. Huskey,	The Choleski method
G. E. Forsythe,	Summary of known methods
R. A. Leibler,	Summary of some numerical results
J. B. Rosser	Exact solutions in the case of integer coefficients

A general iteration scheme was developed which includes as special cases the different methods known as the Gauss-Seidel, Hardy Cross, and Kaczmarz methods,as well as a method which appears new. The basic step of the method is to subtract from a given vector a multiple of another vector in which the coefficient of the second vector is a constant times the inner product of the two vectors. This step is easily programed for a high-speed computer like the INA computer,and could be iterated at high speed. Round-off errors do not accumulate. Estimates of the rate of convergence were obtained and an auxiliary method was found which will greatly speed the convergence in almost all cases of slow convergence. A preliminary report on this method was presented before the American Mathematical Society in New York City on December 27, 1949, by J. B. Rosser.

Publication: A manuscript is in preparation jointly by G.E.Forsythe and J. W. Tukey on the distribution of an n-rowed determinant whose n rows are independent unit vectors.

SOLUTION OF EQUATIONS IN A SINGLE UNKNOWN
Project 11.1/1-49-AE3

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: A. M. Ostrowski

Authorized 4/1/49
Revised 9/15/49
Terminated 12/31/49

Objective: To find new and better ways of dealing with algebraic and transcendental equations in a single unknown.

Background: Such equations arise repeatedly in all branches of applied mathematics.

Status: TERMINATED. The return of the project manager to his regular academic post has resulted in the termination of the project at the Institute for Numerical Analysis.

Publications: (1) "On two problems in abstract algebra connected with Horner's rule," by A. M. Ostrowski; submitted to the American Mathematical Monthly. (2) "Note on Vincent's theorem," by A. M.Ostrowski; to be published in the Annals of Mathematics.

THEORETICAL AND PRACTICAL PROBLEMS IN THE APPLICATION OF A.D.C.M. TO THE DETERMINATION OF ORBITS
Project 11.1/1-49-AM3

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: S. Herrick

Authorized 12/1/48
Terminated 12/31/49

Objective: To find optimum procedures for determining two-body orbits, utilizing a.d.c.m. It is planned to re-examine existing methods and possible new ones in the light of new facilities that will almost certainly alter their present relative value, and to set up calculating machinery specifications (in terms of arithmetic speeds and memory size) to solve this problem, as well as actually program and code the problem for certain anticipated machines.

Background: The first spectacular success in the determination of orbits was the recovery of the lost minor planet Ceres by Gauss nearly a century and a half ago, though Olbers' cometary method and essays by Laplace and Lagrange antedate this time. The methods of Olbers, Guass, and Laplace have been greatly improved in recent years; those of Lagrange and Willard Gibbs also will have to be taken into account in the study, and may prove to be even more applicable than the established ones.

Comments: The study is applicable to preliminary orbits of comets and minor planets, and in part to the rocket problem.

Status: TERMINATED. The project was terminated at the beginning of the quarter when Dr. Herrick returned to his full-time academic duties.

The September 30, 1949 status report described three fields of investigation corresponding to the three publications (1), (2), and (3) below. Item (1) is completed. Item (2) is inactive, awaiting revision of the manuscript on the basis of the project manager's trip to Yale University. Progress was made on the manuscript (3). However, with Dr. Herrick's departure work was terminated by the National Bureau of Standards. The author is expected to continue writing his textbook on rocket navigation in connection with a commercial publishing firm.

Publications: (1) Introduction to "Tables for rectilinear and nearly-rectilinear ('nearly parabolic') orbits," by S. Herrick; to be published in the NBS Applied Mathematics Series. See also project 11.1/2-48-3. (2) "A modification and appraisal of Gibbs' orbit method" by S. Herrick. IN MANUSCRIPT, in possession of the author. (3) Textbook on rocket navigation.

NUMERICAL METHODS IN CONFORMAL MAPPING
Project 11.1/1-49-CM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Managers: Gertrude Blanch, L. K. Jackson
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. All the original project managers have left the Institute, but experimental numerical work supporting their theoretical development was in progress during the quarter. The alternating method recommended by Dr. Ahlfors (see previous quarter's status report) was being given numerical tests by Dr. Blanch under project 11.1/32-49-1. It was being applied to find the conformal map of a rectangle onto a circle. The numerical procedure being used by the computing group was set up by L. K. Jackson.

The mapping of a polygon on the unit circle is given by the formula

$$Z = C\int_{\beta}^{W} \prod_{k=1}^{n} (W - A_k)^{\alpha_k - 1}\, dW ,$$

where the $A_k = e^{i\omega k}$ are the points on the unit circle corresponding to the vertices of the polygon and are unknown. Dr. Ahlfors' paper deals with methods for determining $\omega_{k+1} - \omega_k$, the angle between points on the circle corresponding to successive vertices of the polygon. It can be shown that

$$\omega_{k+1} - \omega_k = 2\pi U_k(Z_o)$$

where $U_k(Z)$ is the function harmonic inside the polygon, equal to one on the side joining Z_k and Z_{k+1}, and equal to zero on the remaining sides. Z_o is the point in the polygon corresponding to the center of the circle. The computing group is testing an alternating process proposed by Dr. Ahlfors for computing $U_k(Z_o)$.

The calculation is being done on a rectangle and the procedure is as follows: The rectangle is covered by a set of overlapping circular sectors for which the Dirichlet problem is solvable. The solution of the problem for the sectors reduces to an integral equation which is solved by iteration with Simpson's Rule. This gives the approximate value of $U_k(Z)$ at a network of points on the circular arcs. Then by solving the Dirichlet problem for the sector containing Z_o we get an approximate value for $U_k(Z_o)$. Since the mapping for a rectangle is given in terms of elliptic integrals, we may calculate $U_k(Z_o)$ directly and thus test the accuracy of our approximation and the rapidity of the convergence of the iterative process.

A method for the mapping of a circle onto an ellipse, proposed by Dr. Ostrowski, is being evaluated by numerical computation in the Washington Computation Laboratory; see project 11.2/33-50-4.

Publications: (1) "Proceedings of a symposium on the construction and application of conformal mapping" edited by E. F. Beckenbach; to be published by the NBS. The volume will include the following papers written in connection with this project: (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A. M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A. Weinstein. (2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W. Seidel; IN MANUSCRIPT in possession of the author (now at the University of Minnesota); publication under consideration. (3) "Numerical methods in conformal mapping," by L. Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The work of Dr. Milne at Oregon State College has been suspended because of illness.

One version of the work of Dr. Ostrowski and Mr. Jackson on Professor Bieberbach's proof is in manuscript form at the Institute for Numerical Analysis.

Dr. Forsythe's method of random round-off proposed in the previous quarter's status report was given a test with IBM equipment on the simple system of differential equations $dx/dt = y$, $dy/dt = -x$. The interval of integration and the integration formula were such that the solution could readily be produced on a tabulator. The use of random digits (instead of the usual rule of '5') for rounding off caused no difficulty in board wiring. The method eliminated the large errors described in Huskey's article (see Jul-Sept 1949 issue). Certain features of the experiment require further study, but it appears that random round-off is a useful numerical procedure in certain problems requiring long cumulations with a limited number of significant figures.

Publications: (1) "A note on the numerical integration of differential equations," by W. E. Milne; to be published in the NBS Journal of Research. (2) "Note on the Runge-Kutta method," by W. E. Milne; to be published in the NBS Journal of Research. (3) "On the remainder of the Runge-Kutta formula in the theory of ordinary differential equations," by L. Bieberbach (edited and revised by A. M. Ostrowski and

L. K. Jackson of the INA); one version IN MANUSCRIPT at the Institute for Numerical Analysis.

NOTE: "Numerical determination of characteristic numbers," by Dr. Milne, previously listed under this project, now appears in the publications for project 11.1/1-50-3.

STUDIES IN NUMERICAL INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PD1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Dr. Milne has temporarily suspended work on his manuscript because of illness.

Dr. Yowell prepared an expository article (publication (2) below) on the testing of one of Dr. Milne's methods on IBM equipment. For an iterative approximate solution of Laplace's equation in a plane Dr. Milne proposed using a rectangular grid and the recursive difference relation

$$(*) \quad \begin{array}{|c|c|c|} \hline 1 & 4 & 1 \\ \hline 4 & 16 & 4 \\ \hline 1 & 4 & 1 \\ \hline \end{array} \cdot f_n(x,y) = 36 f_{n+1}(x,y) \ .$$

Since the difference operator on the left side of (*) is factorable into the product of

$$\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline 1 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{|c|c|c|} \hline 1 & 4 & 1 \\ \hline \end{array} \ ,$$

a deck of cards containing the n-th approximation $f_n(x,y)$ can be used to compute the next approximation $f_{n+1}(x,y)$ with only two passes through the IBM Electronic Calculating Punch Type 604. There is a considerable reduction in card handling over the corresponding work for the Liebmann equation

$$\begin{array}{|c|c|c|} \hline & 1 & \\ \hline 1 & & 1 \\ \hline & 1 & \\ \hline \end{array} \cdot f_n(x,y) = 4 f_{n+1}(x,y) \ .$$

The speed of convergence, wiring diagrams, and the use of boundary values are discussed.

Dr. Yowell presented his exposition at a Computation Forum of the I.B.M. Corporation, Endicott, N. Y., in December.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of the Symposium held at the Harvard Computation Laboratory in September, 1949. (2) "Numerical solution of partial differential equations," by E. C. Yowell, IN MANUSCRIPT in possession of the author.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The greater part of the work on this project has been done by Drs. Wasow and Acton.

As a preparation for the solution of boundary value problems for Laplace's equation in two and more dimensions it was shown that the efficiency of the method can be increased by regarding every intermediate step during a random walk as a starting point for a shorter random walk. The idea of counting each such intermediate point as often as it is visited during the random walk rather than only once was rejected because it was proved that the former method has greater variance than the latter. Numerical tests with one dimensional problems that were run on IBM equipment confirmed this result.

It was shown that a modification of the usual random walk procedure in a quadratic net can be used to find approximate solutions for the first boundary value problem of differential equations of the form $\Delta u + g(x,y)\,u = 0$. In that modification the weight of the wandering point must be multiplied by an appropriate factor at each step and the average weight rather than the relative frequency at the boundary is recorded. It was proved that this procedure converges if, and only if, the difference equation corresponding to $\Delta u + g(x,y)\,u + \lambda u = 0$ has all its eigenvalues positive. Numerical tests are in progress.

Dr. Forsythe assisted in editing "Proceedings of a symposium on the Monte Carlo method, held at the Institute for Numerical Analysis in June 1949;" see publications (1) and (2) below.

Publications: (1)"Various techniques used in connection with random digits," by J.von Neumann (summary written by G. E. Forsythe); to be included in the "Proceedings of a symposium on the Monte Carlo method," to be published by the National Bureau of Standards. (2) "Generation and testing of random digits at the National Bureau of Standards, Los Angeles," by G. E. Forsythe; to be included in the "Proceedings of a symposium on the Monte Carlo method," to be published by the National Bureau of Standards.

DETERMINATION OF EXTREMALS OF FUNCTIONALS
Project 11.1/1-50-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Jul-Sept 1949 issue.

Status: CONTINUED. The method of gradients of functions described in project 11.1/1-50-2 was extended to functionals together with improved criteria for estimating how far to proceed in a given direction before recomputing the gradient.

These results were reported in a talk by Dr. Hestenes before the American Mathematical Society in New York City on December 28, 1949. Already a useful application of these improved criteria has been found, and is reported in connection with project 11.1/1-50-3.

Publications: (1)"Some elementary problems in the calculus of variations," by M. R. Hestenes; submitted to the Mathematics Magazine; also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A. (2) "Quadratic forms in Hilbert space, with applications in the calculus of variations," by M. R. Hestenes; accepted, subject to revision, by the American Journal of Mathematics.

DETERMINATION OF CRITICAL POINTS
Project 11.1/1-50-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Jul-Sept 1949 issue.

Status: CONTINUED. To minimize a function of several variables, one can start at a given point and then try to move in succession to other points where the value of the function is less. The gradient of the function at a given point gives the direction along which the rate of decrease is greatest, and it seems intuitively clear that this is the most efficient direction in which to start out to find a better point. However, the direction of the gradient changes as one proceeds from point to point, so that one should not proceed too far in a given direction. Improved criteria have been found giving an estimate of how far to proceed in a given direction before recomputing the gradient, and numerical tests of these criteria were made. Extensions of these methods are reported in project 11.1/1-50-3.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Project 11.1/1-50-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Managers: G. E. Forsythe, M. Hestenes, W. Karush, C. Lanczos
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The aspects of the project described in the July-Sept 1949 status report under (I), (II),(IV), and (V) were finished with the completion of corresponding Publications (1), (2), (4), (5).

Drs. Feller and Forsythe have been searching the literature in order to compare their work (see Publication (3) below) with that of Morris in England, Semendiaev in Russia, and others. They are also simplifying and condensing a first draft of their matrix reduction.

Drs. Hestenes and Karush applied the Hestenes theory of gradients to finding eigenvalues of symmetric matrices. If x_0 is a given value of the vector x and A is the matrix in question, then the gradient of the quotient

$$\mu = (x^T A x) \div (x^T x)$$

at the value $x = x_0$ has the direction given by

$$\xi_0 = A x_0 - \mu_0 x_0$$

where μ_0 is the value of μ at $x = x_0$. To increase μ we add to x_0 a positive multiple of ξ_0, and to decrease μ we add to x_0 a negative multiple of ξ_0. This gives a new x which we now treat as x_0, and repeat the process. To find the greatest eigenvalue we try to maximize μ at each step, and so always use a positive multiplier; to find the least eigenvalue we try to minimize μ at each step, and so always use a negative multiplier. Two methods of choosing a multiplier are given, each of which insures convergence. One of these has the additional property that as x approaches an extreme eigenvector, ξ approaches the adjacent one. Moreover, for this method an estimate of the rate of convergence is known.

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C.Lanczos;

accepted for publication in the NBS Journal of Research. (2) "Numerical determination of characteristic numbers," by W. E. Milne; to be published in the NBS Journal of Research. (3) A paper embodying results of W. Feller and G. E. Forsythe to be submitted to the Quarterly of Applied Mathematics. (4) "A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation," by M. D. Donsker and M. Kac; to be published in the NBS Journal of Research.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.1/1-50-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. There has been no change in the status of the three publications mentioned in the previous quarter's status report. Final revisions of Publication (2) below neared completion.

In Publications (4) and (5) Dr. Szasz has given the results of projects 11.1/1-49-6 and 11.1/1-49-10, respectively. Full descriptions of these two projects appear in the Oct-Dec 1948 issue of Projects and Publications.

In Publication (6) Dr. Seidel and Dr. Szasz have described the result reported in the July-Sept 1949 issue under project 11.1/1-49-SF1.

Publication (7) by Dr. Beckenbach gives the results of project 11.1/1-49-5. A full description of this project appears in Projects and Publications, Oct-Dec 1948.

Dr. Szasz, in collaboration with Dr. Eugene Lukacs, (see Publication (8) below) has given two necessary conditions for the reciprocal of a polynomial p(t) to be the characteristic function of a probability distribution in one dimension: (i) The polynomial p(t) has no real zero. If b + ia is a zero of p(t), then -b + ia is also. (ii) Assume that p(t) has no multiple zeros. If b + ia is a zero of p(t), then p(t) has a zero $i\alpha$ such that the sign of α is the same as the sign of a and $|\alpha| \leq |a|$.

In another investigation Dr. Szasz is finding conditions for the occurrence of Gibbs phenomenon for the class of convergence factor summability methods in which the generalized sum of the series Σa_n is given by

$$\lim_{t \to 0} \sum_{n=0}^{\infty} a_n \phi(nt).$$

This class inclues Abel summability $[\phi(u) = e^{-u}]$ and Riemann summability $[\phi(u) = (\sin u/u)^k]$. The investigation consists of an investigation of the behavior of

$$\Phi(x,t) = \sum_{n=0}^{\infty} \frac{\sin nx}{n} \phi(nt);$$

as $x \to 0$, $t \to 0$ independently.

Publications: (1) "On the Gibbs phenomenon for Euler means," by O. Szasz; accepted for publication in Acta Litterarum ac. Scient., Szeged., (2) "The Gibbs phenomenon for Hausdorff means," by O. Szasz; in preparation. (3) "On a summation method of O. Perron," by O. Szasz; accepted for publication in Mathematische Zeitschrift. (4) "A generalization of S. Bernstein's polynomials to the infinite interval," by O. Szasz; accepted for publication in the NBS Journal of Research. (5) "On some trigonometric transforms," by O. Szasz; in preparation. (6) "On positive harmonic functions and ultraspherical polynomials," by W. Seidel and O. Szasz; in preparation. (7) "On subharmonic, harmonic, and linear functions of two variables," by E. F. Beckenbach; to be submitted to Revista, Universidad Nacional de Tucuman (Argentina). (8) "Certain Fourier transforms of distributions" by E. Lukacs and O. Szasz; submitted to the Canadian Journal of Mathematics.

INTRODUCTION TO ACCOMPANY TABLES OF THE CHEBYSHEV POLYNOMIALS $C_n(x)$ and $S_n(x)$
Project 11.1/1-50-5

Origin: NBS Authorized 12/1/49
Sponsor: Office of Naval Research, USN Completed 12/31/49
Manager: C. Lanczos

Objective: To prepare a suitable introduction for the forthcoming tables of the Chebyshev polynomials $C_n(x)$ and $S_n(x)$ computed by the NBS Computation Laboratory and to be published in the NBS Applied Mathematics Series.

Background: The Chebyshev polynomials have important analytical properties which make them very useful in numerical mathematics. The project manager in certain published and unpublished works has shown how to use these polynomials in approximating the solution of linear differential equations with rational coefficients, which frequently appear in problems of mathematical physics and engineering.

Comment: See project 11.2/2-50-3.

Status: COMPLETED. The proposed introduction is a 70 page typescript, divided into ten chapters entitled: I. General survey, II. Polynomial expansions of strongest convergence, III. Analytical properties of the Chebyshev polynomials, IV. Algebraic properties of the Chebyshev polynomials, V. Arithmetical properties of the Chebyshev polynomials VI. Functional equations, VII. Increased convergence of a power series if rearranged into the $C_n(x)$ polynomials, VIII. Curve fitting by Chebyshev polynomials, IX. Solution of differential equations with rational coefficients; the τ -method, X. The use of the numerical tables of the Chebyshev polynomials.

The material in chapters II and IX is believed not to have been published heretofore, and consists partly of original research of the project manager with the NBS Mathematical Tables Project in 1944 and subsequent research. In chapter IX the project manager gives a method of approximating the solutions of linear differential equations with rational coefficients. These approximations are by rational functions, and have the advantage over the project manager's earlier approximations that they are valid in one half of the complex plane, instead of just along part of the real axis.

Publication: The manuscript was transmitted to the Computation Laboratory.

TRANSFORMATIONS TO SPEED THE CONVERGENCE OF SERIES
Project 11.1/1-50-6

Origin: NBS Authorized 11/1/49
Sponsor: Office of Naval Research, USN Completed 12/1/49
Manager: J. B. Rosser

Objective: To find methods for more efficient determination of the approximate sum of certain types of infinite series.

Background: The numerical determination of the sum of a series is one of the most frequent computational problems. A great deal of time and expense can be saved when more efficient methods are found.

Status: COMPLETED. Numerical instances are given of the speeding of the convergence of series by the Euler transformation. This is even applied advantageously to certain divergent series, and a rigorous justification is given. An example is given of a series for which use of the Euler transformation is not useful. Instances are given of several less widely known methods. Finally, the method of summation by transformation into a continued fraction is illustrated successfully in the case of certain divergent series. The possibility of applying two different methods in succession to a given series is exploited throughout the paper, in spite of the fact that this often requires summing a divergent series.

Publication: Paper entitled "Transformations to speed the convergence of series," by J. B. Rosser; to be published in NBS Journal of Research.

STUDY OF REAL ZEROS OF REAL DIRICHLET SERIES
Project 11.1/1-50-7

Origin: NBS Authorized 12/1/49
Sponsor: Office of Naval Research, USN
Manager: J. B. Rosser

Objective: To determine for $k \leq 227$ that the corresponding series $L(s,\chi)$ has no positive real zeros.

Background: In the theory of the distribution of primes in arithmetic series, in assigning bounds in the three prime theorem, and in studying the class number of quadratic fields, a knowledge of the location of the real zeros of $L(s,\chi)$ is of value. A long standing conjecture is that there are no positive real zeros for any k. If proved, this result would be of value in each of the fields mentioned above. In the Bull. Am. Math. Soc. 55, 906-913, 1949, the project manager has given a computational procedure for proving the result for individual values of k, and treated each $k \leq 67$.

Status: NEW. The method for dealing with individual values of k, which was referred to in the Background, failed for k=163. Improved methods were developed, but still the case k=163 appeared difficult. Finally a new formula for $L(s,\chi)$ was discovered which made it possible to treat many values of k simultaneously. By means of this formula, the difficult case of k=163 was finally treated adequately. Results have been written up in a paper now being prepared for publication in the NBS Journal of Research.

Publication: "Real roots of real Dirichlet L-Series" by J. B. Rosser; to be submitted to the NBS Journal of Research.

AN ITERATIVE SOLUTION OF FREDHOLM'S INTEGRAL EQUATION
Project 11.1/1-50-8

Origin: NBS Authorized 11/1/49
Sponsor: Office of Naval Research, USN
Manager: C. Lanczos

Objective: To develop a rapidly convergent approximation method for the solution of the general Fredholm type of integral equations.

Background: Fredholm's integral equation is fundamental for the general theory of ordinary differential equations. In spite of the very extensive literature on the analytical behavior of such equations, easily accessible methods of solution are not available. The field of non-symmetric kernels is particularly unexplored.

Comments: The present approach conceives the non-symmetry of the kernel function as the consequence of a skew-angular reference system in which the kernel is analyzed. The double set of biorthogonal functions associated with a non-symmetric kernel are then actually the covariant and contravariant representations of one single set of orthogonal functions. This viewpoint unifies the treatment of symmetric and non-symmetric kernels and the remarkable fact emerges that every non-symmetric kernel can be symmetrized in the proper reference system. In this system the matrix elements of the kernel are either real or purely imaginary numbers; they occupy the diagonal row plus one neighboring row on both sides. The transformation to the principal axes (if such axes exist) and the solution of the given integral equation is now easily accomplished.

Status: NEW. Particular attention was given to that special kernel function $K(x,\xi)$ of the Volterra type, which is equivalent to plain integration between the limits ξ =const. and ξ =x. The associated integral equation defines the Laplace transform of the right side. Since many of the fundamental transcendentals-such as the Gaussian error integral, the exponential integral, the gamma function, the Bessel functions-are defined as Laplace transforms of relatively simple functions, effective rational approximations of these functions are now being investigated, which give rapid convergence in large portions of the complex plane.

Publication: "An iterative solution of Fredholm's integral equation", by C. Lanczos; presented at the American Mathematical Society Meeting, Pasadena, California, November 26th, 1949; final manuscript being prepared for publication.

2. Mathematical Tables

MATHIEU FUNCTIONS II
Project 11.1/2-45-1

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. All the coefficients received from the NBS Computation Laboratory have been checked. The function $Se_r(s,x)$ is about 50% completed.

DECOMPOSITIONS OF ARC TANGENTS
Project 11.1/2-48-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Managers: Gertrude Blanch, John Todd

Authorized 5/25/48
Completed 12/31/49

Objective: To find the integral values of u_i satisfying

$$\arctan (a/b) = \sum_{1}^{k} \arctan u_i ,$$

where $a/b < 1$, and a and b are relatively prime integers for all positive $b \leq 100$.

Background: Mr. Todd is writing a paper explaining the theory of decomposition of arc tangents. This table will be incorporated into the article. The project is being performed as part of the training program of the computation unit in section 11.1.

Status: COMPLETED. Checking of the manuscript will be done under the direct supervision of Mr. Todd who is now associated with the NBS Computation Laboratory in Washington.

Publications: The table will be submitted for publication in the NBS Applied Mathematics Series; the table will include the decompositions for integral arguments $n \leq 2{,}100$. "A problem on arc tangent relations," by John Todd, Am. Math. Mo. 56, 517-528 (Oct. 1949).

SPECIAL TABLE OF BESSEL FUNCTIONS
Project 11.1/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication was in progress.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

ROCKET NAVIGATION TABLES
Project 11.1/2-48-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication was in progress.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLES OF $E_1(z)$, SECOND QUADRANT
Project 11.1/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The auxiliary functions $U_n(z)$ and $T_n(z) = U_n(z)/n$ were completed for $n \leq 26$.

PUNCHED CARD LIBRARY
Project 11.1/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles, 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following tables on punched cards were added to the library:

1. Tables relating to Hankel integrals of order zero. The functions

(a) $J_c(\lambda,x) = \quad J_0(\lambda u) \cos u \, du$

(b) $J_s(\lambda,x) = \quad J_0(\lambda u) \sin u \, du$

(c) $N_c(\lambda,x) = \quad N_0(\lambda u) \cos u \, du$

(d) $N_s(\lambda,x) = \quad N_0(\lambda u) \sin u \, du$

are available for x = 0(.02)2; 6D and x = 0(.1)5; 8D. The auxiliary functions that are needed near the origin

(e) $C_c(\lambda,x) = N_c(\lambda,x) - \frac{2}{\pi} \log \lambda x \, J_c(\lambda,x)$

(f) $C_s(\lambda,x) = N_s(\lambda,x) - \frac{2}{\pi} \log \lambda x \, J_s(\lambda,x)$

are available for x = 0(.02).88; 6D. These tables were published by L. Schwarz in Luftfahrtforschung, 20, No. 12, 341-372 (1943). The tables were differenced and corrected at INA. (They were needed in connection with a problem of the Air Materiel Command.)

2. Table of Bessel-Clifford functions of orders zero and one. This table was reported previously, but a complete description of the particular values available is listed here. The functions

$$C_\nu(x) = x^{-\nu/2} J_\nu(2\sqrt{x}), \quad D_\nu(x) = x^{-\nu/2} Y_\nu(2\sqrt{x}),$$

$$E_\nu(x) = x^{-\nu/2} I_\nu(2\sqrt{x}), \quad G_\nu(x) = x^{-\nu/2} K_\nu(2\sqrt{x}),$$

where $J_\nu(t)$, $Y_\nu(t)$, $I_\nu(t)$ and $K_\nu(t)$, the well-known Bessel functions are available for $\nu = 0,1$: $C_\nu(x)$ and $D_\nu(x)$ for x = 0(.02)1.5(.05)3(.1)13(.2)45(.5)115(1)410; 8D or 9D, and $E_\nu(x)$ and $G_\nu(x)$ for x = 0(.02)1.5(.05)6.2; 8D or 9D. The functions $e^{-2\sqrt{x}} E_\nu(x)$ and $e^{2\sqrt{x}} G_\nu(x)$ are available for x = 6.2(.1)13(.2)36(.5)115(1)160(5)410; 7S or 8S. The table was originally computed at the NBS Computation Laboratory and INA.

3. Tables of spherical Bessel functions. The entries in the two volumns of the NBS by that title, published by Columbia University Press, (New York), are being punched. The punching of volume I was completed; as previously reported; the punching of volume II is 60% complete. The punched cards have not yet been differenced.

3. Development of Automatic Computing Machinery

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.1/22-49-1

Origin: Air Materiel Command, USAF
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The M-R Registers of the arithmetic unit were completed and delivered to INA by the Cole Instrument Company; this represents delivery of half of their contract for registers of the arithmetic unit. The company is at present in the midst of production of the A Registers. The completed M-R Registers were inspected by us for faulty soldering joints, and d-c voltage checking was done. Certain minor modifications were made and these chassis are now ready for functional tests. The racks for holding the arithmetic unit were approximately one-half completed.

A production model of a cathode ray tube memory chassis was completed and checked, and the contract for reproducing forty-five of these units was awarded to the C and B Manufacturing Company of Los Angeles. This company has agreed to deliver the first prototype in approximately two weeks. The cathode ray tube amplifier contract was awarded to the Stolle Engineering and Manufacturing Company of Los Angeles. The first prototype was delivered to us for inspection, and a frequency response curve was taken for this prototype with satisfactory results. Accordingly, this company has been notified to go ahead with the production of the remainder of the cathode ray tube amplifiers.

The control unit was checked and the necessary interconnection wiring was underway. After its completion the control unit will be rechecked to make sure that it is operating properly. This unit will then be available for the testing of the arithmetic and the memory units.

The power supply requirements were determined, and a contract for the d-c power supply units was awarded to the Lee Electric and Manufacturing Company of Los Angeles. The construction of a suitable place to install these power supplies was arranged with the Buildings and Grounds Office of the University of California at Los Angeles. Arrangements were also made to acquire an a-c generator from the University. Negotiations were underway with the University for a motor to use in the motor generator set of the power supply system in order to isolate the computing machine from voltage variations on the supply lines. Schematic drawings of the control wiring for the d-c power supply were in preparation and placement of heater transformers and other components in the power supply cabinet was being planned. The heater transformers themselves were delivered and are ready for use. Preliminary quotations on the ventilating system were requested from several air conditioning firms of Los

Angeles.

Considerable study has beengiven to suitable input-output equipment for the computing machine. A Flexowriter Unit for input-output was ordered and when delivered the necessary circuitry will be constructed to enable direct communication between this unit and the computing machine. The Flexowriter Unit consists of two electric typewriters, one punched paper tape reader, and one paper tape perforator, all of which will be connected with the computing machine. A quotation was obtained from the Raytheon Company for a magnetic tape unit. This unit contains the mechanical features of their tape unit, plus a thyratron flipflop control which determines the direction of motion of the tape. A quotation from Engineering Research Associates was also requested on their photoelectric paper tape reader.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Project 11.1/22-49-2

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Project 11.1/22-50-1

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Improvements were made in the system of semi-automatic instruction for general purpose electronic digital computing machines. These improvements were incorporated in a paper entitled, "Characteristics of the Institute for Numerical Analysis Computer."

Publication: "Characteristics of the Institute for Numerical Analysis Computer" by H. D. Huskey; to be submitted to Mathematical Tables and Other Aids to Computation.

4. Computing Services

SEPARATION OF EXPONENTIALS
Project 11.1/31-48-2

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

HARMONIC ANALYSIS OF A CHEBYSHEV POLYNOMIAL
Project 11.1/31-50-4

Origin: Hughes Aircraft Company
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Authorized 10/1/49
Completed 11/15/49

Objective: To find the unknown constants I_j in the expression

$$T_{47}(z_o \cos u) = \sum_{j=1}^{24} I_j \cos(2j - 1)u$$

for two values of the parameter z_o.

Background: This problem arose in connection with research being performed by the Hughes Aircraft Company under contract with the Air Materiel Command.

Status: COMPLETED.

CALCULATION OF AERODYNAMIC DERIVATIVES BY HASKIND'S METHOD
Project 11.1/31-50-5

Origin: Aircraft Laboratory, Engineering Division, Wright-Patterson Air Base, USAF
Sponsor: Air Materiel Command, USAF
Managers: Gertrude Blanch, E. Yowell

Authorized 8/1/49

Objective: To carry through the computations of aerodynamic derivatives for Mach number M=.7 and eleven values of the parameter μ, by the method outlined in the pamphlet by M. D. Haskind. (Russian; Translation by the Graduate Division of the Applied Mathematics Department of Brown University, A9-T-22, entitled "Oscillations of a Wing in a Subsonic Gas Flow".)

Background: This problem was sent to the Institute for Numerical Analysis via Dr. E. P. Little of the Air Materiel Command. The problem involves an intimate knowledge of Mathieu functions, and in the form submitted, it is suitable for hand calculators, with the aid of some work on IBM equipment.

Comments: Some of the tables relating to Mathieu functions, computed by the National Bureau of Standards and now in process of publication, will save several weeks of computing time for this problem.

The translated pamphlet of Haskind was found to have many errors. These are being corrected by L. L. Bailin, M. Howard, and G. Blanch.

Status: NEW. About 95% completed.

SOLUTIONS OF A DIFFERENTIAL EQUATION
Project 11.1/31-50-6

Origin: Hughes Aircraft Company
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Authorized 11/1/49

Objective: (a) To obtain the solution and its first two derivatives of the equation

$$-\frac{d^3y}{dx^3} + \frac{2d^2y}{dx^2} - \frac{dy}{dx} + \frac{ay}{x} = \frac{ae^{isx}}{x}$$

for various values of the parameters, and subject to certain boundary conditions.

(b) To tabulate three independent solutions of the homogeneous equation, and its first two derivatives. (It is known that one solution is a polynomial of degree a.)

(c) To obtain the Wronskian of the three chosen solutions of the homogeneous equation and the six first minors of the Wronskian which involve $y''(x)$, where $y(x)$ is any of the three solutions.

Background: This computation arose in connection with research being performed by the Hughes Aircraft Company under contract to the Air Materiel Command.

Comments: The solutions of the equations involve intimately the "modified" Bessel-Clifford functions of orders 0 and 1.

Status: NEW. About 50% completed.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Project 11.1/31-50-7

Origin: Lockheed Aircraft Corporation — Authorized 12/1/49
Sponsor: Air Materiel Command, USAF
Manager: E. C. Yowell

Objective: (a) To compute coefficients to be used in the design of circular shell-supported frames. Specifically, to tabulate the solutions of the equation

$$(D^6 + 2D^4 + D^2 - d)\, m = 0;$$

where $D = \frac{d}{d\theta}$ for appropriate boundary conditions and for values of the parameter: d = 0, 10, 50, 100, 300, 1000, 2000, 4000, 8000, 25,000.

(b) To make certain auxiliary computations for particular cases where the fuselage skin is supported by a finite number of rings.

Background: This project deals with the single problem of circular shell-supported frames subjected to concentrated loadings. The Lockheed Aircraft Corporation developed the mathematical attack of this problem which has been presented in the form of non-dimensional coefficient curves. These curves, while they are developed for only circular frames, may, by means of approximations, be used for nearly any practical frame which has curvature in the region of applied loading. (Ref. "Analysis of Circular Shell-Supported Frames," Lockheed Aircraft Corp. Report No.42222; "Note on Analyses of Circular Shell-Supported Fuselage Frames," Lockheed Aircraft Corp. Report No. 6821.)

Continuation of the mathematical computations originated by the Lockheed Aircraft Corporation will be accomplished.

Status: NEW. About 25% completed.

SOLUTION OF A NON-LINEAR FIRST ORDER DIFFERENTIAL EQUATION
Project 11.1/31-50-8

Origin: AiResearch Manufacturing Company — Authorized 11/1/49
Sponsor: Air Materiel Command, USAF — Completed 12/5/49
Manager: Gertrude Blanch

Objective: To solve numerically the equation

$$\left\{ \zeta x^{-(1+\cos^2 \alpha_1)} - \mu \right\} \frac{d}{dx} \left(\zeta x^{\sin^2 \alpha_1} \right) = \left(x^{-(1+\cos^2 \alpha_1)} - \mu \right) \frac{d}{dx} x^{\sin^2 \alpha_1}$$

subject to the boundary condition $\zeta = 1.57$ when $x=1$, for the value of the parameters: $\mu = 2.75$, $\alpha = 49^\circ$.

Background: This computation arose in connection with research work being performed by AiResearch Manufacturing Co., under contract with the Air Materiel Command.

Status: COMPLETED. The equation was solved by stepwise integration. Results were transmitted to the AiResearch Manufacturing Company.

COMPUTATIONS RELATING TO A PROBLEM IN ATMOSPHERE DIFFUSION
Project 11.1/31-50-9

Origin: Meteorology Department, University of California at Los Angeles — Authorized 11/15/49
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Objective: Given a set of successive observations of different particles. Let $x_i^{(t)}, y_i^{(t)}$ represent the positions of the i-th particle at time t. The objective is to calculate the angle

$$\theta = \text{arc tangent} \frac{\sum_i (y_i^{(T_i)} - y_i^{(0)})}{\sum_i (x_i^{(T_i)} - y_i^{(0)})}$$

where T_i is the time of the final observation of the i-th particle. Coordinates of the observations must then be calculated in a system rotated through the angle θ.

Background: This computation arose in connection with work being performed by the Meteorology Department of the University of California at Los Angeles under contract to the Watson Laboratory of the Air Materiel Command.

Comments: Some other calculations involving least square solutions were also performed in connection with this problem at the request of the originator.

Status: NEW. About 85% completed.

ANALYSIS OF RAM-JET DATA
Project 11.1/31-50-10

Origin: Marquardt Aircraft Company Authorized 12/20/49
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Objective: To perform certain specified computations with test data furnished by the originator.

Background: These computations are needed for the evaluation of various tests made on ram-jets. The work arises in connection with research performed for the Air Materiel Command.

Comments: New data will be furnished periodically as tests are made.

Status: NEW

FLUTTER ANALYSIS, SWEPT WINGS, II
Project 11.1/31-50-11

Origin: Northrop Aircraft, Inc. Authorized 12/20/49
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch, E. Yowell

Objective: To find the latent roots, whose moduli exceed certain given critical values, for 66 matrices of order 6, composed of complex elements.

Background: This problem arises in connection with a research contract between Northrop Aircraft, Inc. and the Air Materiel Command. The computation is part of the problem of determining critical speeds of aircraft at which flutter phenomena appear.

Status: NEW

SOLUTION OF A SET OF SIMULTANEOUS DIFFERENTIAL EQUATIONS
Project 11.1/31-50-12

Origin: Hughes Aircraft Corporation Authorized 12/20/49
Sponsor: Air Materiel Command, USAF
Manager: Gertrude Blanch

Objective: To obtain the solution of the following set of simultaneous equations:

$$(1) \quad V'' + C_1V' + C_2V = C_3Y' + C_4F(x)W' + C_5G(x)W''$$

$$(2) \quad W''' + A(x)\, W'' + B(x)\, W' = C(x)V$$

$$(3) \quad Z\,Y' = C_6 \sin(\emptyset - Y) - V_m(x) \sin(W - Y)$$

$$(4) \quad Z' = C_6 \cos(\emptyset - Y) - V_m(x) \cos(W - Y)$$

for given boundary conditions. The functions A, B, C, F, G, and V_m involve exponentials; C_i and $\emptyset$ are constants.

Background: The computation arises from a research problem performed by Hughes Aircraft Corporation for the Air Materiel Command.

Status: NEW.

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Project 11.1/32-49-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Among the more extensive computations during this quarter were:

1. Matrix inversion studies for J. B Rosser, C. Lanczos and G. Blanch. (See 11.1/1-49-AE 2.)
2. Studies in characteristic values of matrices for M.R.Hestenes and W. Karush. (See 11.1/1-50-3.)
3. Studies about rounding errors for G. E. Forsythe. (See 11.1/1-49-ODE 2.)
4. Euler transformation studies for J. B. Rosser. (See 11.1/1-50-6.)
5. Computations relating to the zeros of the L-series for J. B. Rosser.(See 11.1/1-50-7.)
6. Calculation in connection with determining parameters in the Schwarz-Christoffel formula. (See 11.1/1-49-CM1.)

REDUCTION OF THEODOLITE DATA
Project 11.1/32-49-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. No flights were evaluated during this quarter.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Project 11.1/32-49-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

DETERMINATION OF ORBITS OF COMETS
MINOR PLANETS, AND SATELLITES
Project 11.1/32-49-6

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. About 35% completed.

TEST OF A VARIATIONAL PROCEDURE FOR DETERMINING THE LOWEST ENERGY VALUE OF A SIMPLE QUANTUM MODEL
Project 11.1/32-50-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: H. Gruen

Authorized 8/1/49
Completed 10/10/49

Objective: To test the feasibility of a variational method for computing quantum states by applying the method to the following problem in nuclear physics.

The operator

$$H = \begin{cases} -\left[\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_0 \right], & |x| \leq a , \\ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} , & |x| \geq a , \end{cases}$$

is the Hamiltonian for a particle of a mass m in a potential well of a depth V_0 and a width 2a. $\hbar$ is Planck's constant divided by 2π.

A trial wave function is chosen as

$$\Phi(x) = \begin{cases} (x^2 - a^2)^9 , & |x| \leq a , \\ 0 , & |x| \geq a . \end{cases}$$

We define

$$F_n(A) = A + \frac{\beta_n(A)}{\beta_{n-1}(A)},$$

where n = 1,3,5,..., and

$$\beta_n(A) = \int_{-a}^{+a} \Phi(x)\ (H - A)^n\ \Phi(x)dx \Big/ \int_{-a}^{+a} \left[\Phi(x)\right]^2 dx.$$

The minimum values of F(A) for A ranging from -500 to +500 for odd values of $n \leq 9$ will be computed and the results compared with the theoretical conjecture (see Background) that an optimum n_0 exists where $\min_A F_{n_0}(A)$ is the best approximation to E_0 obtainable with the given trial wave function.

Background: The variational method together with the conjectured results was suggested by Dr. Edward Gerjuoy of the University of Southern California.

Status: COMPLETED. For the computation carried out(n=1,3,5,7,9), the approximation to E_0 obtained improved for each higher value of n. However no theoretical justification has yet been formulated which indicates that a finite optimum n_0 does or does not exist. The computation, however, led to theoretical investigations which show that with n=3 and with proper restrictions on the nature of the trial wave function $\Phi(x)$ a convergent process can be formulated for this and more general problems.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Project 11.1/32-50-2

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 12/1/49
Sponsor: Bureau of Aeronautics, USN
Manager: Gertrude Blanch

Objective: To solve by stepwise integration a three-dimensional difference equation.

Background: This computation arose in the performance of research work at the Naval Air Missile Test Center at Point Mugu.

Status: NEW. About 15% completed.

SOLUTION OF CERTAIN LINEAR DIFFERENTIAL EQUATIONS
Project 11.1/32-50-3

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 8/1/49
Sponsor: Bureau of Aeronautics, USN Completed 10/31/49
Manager: Gertrude Blanch

Objective: To obtain the solution of an ordinary linear differential equation for thirty-four sets of parameters.

Background: These equations arose during the course of research being performed at the Naval Air Missile Test Center.

Comments: The set of equations will be solved on the U.C.L.A. differential analyzer under the supervision of the Institute for Numerical Analysis. The Institute for Numerical Analysis will also compute completely five of the cases on desk calculators as a check on the graphical solutions of the differential analyzer, and will spot check other values for consistency.

Status: COMPLETED.

COMPUTATIONS ARISING IN THE THEORY OF HYPERSONIC FLIGHT
Project 11.1/32-50-4

Origin: Naval Ordnance Test Station, Pasadena Annex Authorized 11/1/49
Sponsor: Office of Naval Research, USN
Manager: Gertrude Blanch

Objective: (1) To determine by a least squares fit, the coefficients of the approximations

$$XY + A_1 - A_2 Y = 0$$

$$X = b_1 + b_2 Y$$

for certain tabulated data.

(2) To obtain the greatest root of the shock polar equation

$$Y_2 - \gamma_0 M_0^{\,2} (B \cos^2\theta + 2 \sin^2\theta) Y + \gamma_0^2 M_0^{\,4} \sin^2\theta = 0$$

with $\gamma_0 = 1.405$ $B = .891$ $M_0 = 10, 12.5, 15, 17.5, 20, 22.5, 25,$

$B = .833$ $\theta = 10^\circ, 20^\circ, 30^\circ, 45^\circ$

and to obtain the maximum value of θ for which the solutions will be real.

Background: These solutions were needed during the course of research by Dr. E. T. Benedikt of the U.C.L.A. Engineering Department, as a theoretical explanation for the results of experiments conducted by the Naval Ordnance Test Station.

Status: NEW. About 30% completed.

MOMENTS OF ORDER STATISTICS
Project 11.1/32-50-5

Origin: University of Oregon Authorized 10/31/49
Sponsor: Office of Naval Research, USN
Manager: E. C. Yowell

Objective: If we define $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ and $F(x) = \int_{-\infty}^{x} f(t)dt$, to obtain:

(a) $u_1(n,j) = \int_{-\infty}^{+\infty} x\, f(x)(F(x))^{j-1} (1 - F(x))^{n-j}\, dx,$

for n=6(1)20 and $\begin{cases} j = 1(1)\frac{n}{2} \text{ ; n even} \\ j = 1(1)\frac{n-1}{2} \text{ ; n odd ,} \end{cases}$

(b) $u_2(n,j) = \int_{-\infty}^{+\infty} x^2 f(x)(F(x))^{j-1} (1 - F(x))^{n-j}\, dx,$

for n=6(1)20 and $\begin{cases} j = 1(1)\frac{n}{2} \text{ ; n even} \\ j = 1(1)\frac{n-1}{2} \text{ ; n odd ,} \end{cases}$

(c) $u_{1,2}(n,i,j) = \int_{-\infty}^{+\infty}\int_{-\infty}^{y} xy\, f(x)f(y)(F(x))^{i-1}(1-F(y))^{n-j}(F(y)-F(x))^{j-i-1} dx\, dy$

for $n=6(1)20$, $j=2(1)n$, $i=1(1)j-1$.

Values of $u_1, u_2, u_{1,2}$ for all other values $\leq n$ can be easily obtained from these values.

Background: The computations were requested specifically by Professor W. J. Dixon. A paper by H. J. Godwin entitled "Some low moments of order statistics," Ann. Math. Stat. 20, 279 (June 1949), carried out the computation for values of n through 10. Professor Dixon is interested in carrying out this computation for values of n through 20. The mean used by Godwin is not the same as that used in our present computation. However, there are simple relations between the functions tabulated by Godwin and those tabulated by the Institute for Numerical Analysis.

Comments: A functional check on the accuracy provided by a formula due to H. L. Jones (Ann. Math. Stat. 19, 273) is being used:

$$\sum_{j=1}^{n} E(x_i x_j) = 1 \ , \quad i = 1,2,\ldots,n$$

where

$$E(x_j) = \frac{n!\ u_1}{(j-1)!\ (n-j)!}$$

$$E(x_j x_j) = \frac{n!\ u_2}{(j-1)!\ (n-j)!}$$

$$E(x_i x_j) = \frac{n!\ u_{1,2}}{(i-1)!\ (j-i-1)!(n-j)!}$$

Status: NEW. About 75% completed.

5. Training

CHARACTERISTIC VALUES OF MATHIEU'S DIFFERENTIAL EQUATION
Project 11.1/4-49-6

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For most recent status report see July-Sept 1949 issue.

COURSES ON AUTOMATIC COMPUTING MACHINERY
Project 11.1/4-50-2

Origin: National Bureau of Standards — Authorized 10/10/49
Sponsor: Various agencies supporting the INA
Manager: H. D. Huskey

Objective: To acquaint the research worker, particularly in the mathematical and physical sciences; with the logical theory and performance characteristics of automatic digital calculators being developed at the present time. Particular emphasis is placed on the machine under construction at the Institute for Numerical Analysis.

Background: A primary aim of this project is to assist in orienting the mathematical research program of the Institute.

Status: NEW. An intensive ten day course was conducted by the project manager from October 10 through October 21. This course was primarily concerned with programing and coding for the Institute for Numerical Analysis Computer.

On October 28 a more general course meeting for one hour a week was inaugurated, and is at present in progress. This course covers electronic features of the computer as well as coding and programing details.

An invitation to attend these courses was issued to interested personnel of various departments of the University of California at Los Angeles and of neighboring scientific and industrial organizations.

II. Computation Laboratory

(Section 11.2)

1. Mathematical Tables

TABLES OF $E_1(z)$, $(z = x + iy)$
Project 11.2/2-43-3

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Project 11.2/2-46-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations of $\log \Gamma(x+iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ were completed. Extension of the table to include values for $x = 0(.1)9$ was underway.

TABLES OF COULOMB WAVE FUNCTIONS
Project 11.2/2-47-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for L = 0 were completed. Extension to L = 1,2,3 was being prepared.

TABLE OF ANTILOGARITHMS
Project 11.2/2-47-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were completed; checking was continued.

TABLES FOR THE OCCASIONAL COMPUTER
Project 11.2/2-47-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1948 issue.

TABLE OF LAGRANGIAN COEFFICIENTS
FOR SEXAGESIMAL INTERPOLATION
Project 11.2/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of 3-, 4-, 5-point interpolation was in progress.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN
HERMITE POLYNOMIALS
Project 11.2/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 is ue.

Status: INACTIVE. For status to date see Jan-Mar 1949 issue.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Project 11.2/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The computations were substantially completed; checking was in progress.

TABLE OF POWERS OF COMPLEX NUMBERS
Project 11.2/2-50-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Page proofs were received and corrected.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 8 in the NBS Applied Mathematics Series.

TABLES TO FACILITATE SEQUENTIAL t-TESTS
Project 11.2/2-50-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Galley proofs were received and corrected; introduction was completed.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 7 in the NBS Applied Mathematics Series.

TABLE OF CHEBYSHEV POLYNOMIALS
Project 11.2/2-50-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Introduction was completed. Galley proofs were being checked.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 9 in the NBS Applied Mathematics Series.

PROBABILITY TABLES FOR EXTREME VALUES
Project 11.2/2-50-4

Origin: NBS, Section 11.3 Authorized 12/31/49
Sponsor: Office of Air Research, AMC, USAF
Manager: H. E. Salzer

Objective: To prepare a set of short tables relating to the statistical theory of extreme values, including:

(1) The inverse of the (cumulative) distribution function of extremes, $y = \log(-\log x)$, $x = .0001(.0001).005(.001).988(.0001).9999$, 5D.

(2) The direct (cumulative) distribution of extremes, $x = \exp(-\exp(-y))$, $y = -3(.1)-2.4(.05)0(.1)4(.2)8(.5)17$, 7D.

(3) Probability density of extremes, $x' = e^{-y} e^{-e^{-y}}$, range of y same as (2), 7D.

(4) Probability density of extremes as a function of the probability, $x' = -x \log x$, $x = .0001(.0001).01(.001).999$, 5D.

(5) Probability points for n'th extremes.

(6) Distribution and density function for the range $\psi(R)$ and $\psi'(R)$, as defined by $\psi'' + \psi' - e^{-R}\psi = 0$; R between -4.6 and 20; 6 or 7D.

Background: The theory of extremes has been developed in recent years, primarily by Dr. E. J. Gumbel. At his suggestion this compilation of pertinent tables has been undertaken. The table listed as (1) above is an extension of the one computed by this Laboratory under project 11.2/33-49-7. Tables (2) to (5) are taken from unpublished manuscripts of A. J. Greenwood, table (6) from an unpublished table computed at Dahlgren Proving Ground, with additions by A. J. Greenwood. All tables are to be checked and edited by this Laboratory. Dr. Gumbel participated in the detailed planning.

Status: NEW. Manuscripts of tables completed elsewhere were secured and were being checked. Computations for the purpose of filling in missing values were underway.

2. Computing Services

GUST ATTACKS ON DELTA WING
Project 11.2/31-50-1

Origin: Aircraft Laboratory, AMC, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations 20% completed.

HEAT CONDUCTION EQUATION
Project 11.2/33-46-1

Origin: Bureau of Ordnance, Department of the Navy
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Project 11.2/33-49-2

Origin: Naval Research Laboratory, USN
Sponsor: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameters a_i, m_i, were performed when requested.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Project 11.2/33-49-5

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Table of heat capacities for molecular hydrogen was completed.

FERMI FUNCTION II
Project 11.2/33-49-10

Origin: NBS, Section 4.4
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Preparation of auxiliary tables was in progress.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Project 11.2/33-49-11

Origin: Lewis Flight Propulsion Laboratory, NACA
Sponsor: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Trial computations for about fifteen cases were completed.

SHOCK WAVE PARAMETERS
Project 11.2/33-49-13

Origin: Bureau of Ordnance, Department of the Navy
Sponsor: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

BASIC IONOSPHERIC DATA
Project 11.2/33-49-14

Origin: NBS, Section 14.1
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

Publications: Current data are published in the monthly CRPL-F reports of the NBS Central Radio Propagation Laboratory; copy prepared from punched cards will replace hand-made copy as soon as procedures and schedules are worked out. The results of statistical computations will be included in scientific papers where appropriate.

RADIO-TELEGRAPH INTERFERENCE
Project 11.2/33-49-17

Origin: NBS, Section 14.4
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

PERCENTAGE POINTS OF THE ARITHMETIC MEAN IN RANDOM SAMPLES FROM THE SECH AND SECH^2 DISTRIBUTIONS
Project 11.2/33-49-18

Origin: NBS, Section 11.3
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations about 80% completed.

COMPUTATION OF NORMAL MODES OF BOX BEAM
Project 11.2/33-49-20

Origin: NBS, Section 6.4
Sponsor: " "
Manager: J. H. Levin

Authorized 6/20/49
Completed 12/31/49

Objective: Computation of three lowest resonant frequencies and associated mode shapes for box beam. For purposes of simplifying the calculation of the mode shapes the box beam is replaced by an idealized structure consisting of discrete mass points connected by weightless springs. The validity of such an approximation is to be tested in Section 6.4 by comparison of observed resonant frequencies and mode shapes for the box beam with calculated frequencies and mode shapes for the substitute structure. The mathematical formulation of this problem leads to the calculation of the three highest characteristic roots and associated modes of a matrix of order 14. If the characteristic roots are λ_i^2, then the desired frequencies are $\omega_i = 1/\lambda_i$.

Background: These studies are of importance in investigating dynamic properties of aircraft, in particular, the dynamic response of airplane wings to transient influences such as landing impacts, gusts, gun blast, and maneuvering.

Status: COMPLETED. Results were transmitted to section 6.4.

TABLES OF CABLE FUNCTIONS
Project 11.2/33-49-21

Origin: David Taylor Model Basin, USN
Sponsor: " " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. On this project, which was reported as completed in the preceding quarter, the originators have requested additional computations to determine the behavior of the functions in the neighborhood of singularities.

WAVE RESISTANCE OF SHIPS
Project 11.2/33-50-1

Origin: David Taylor Model Basin, USN — Authorized 9/1/49
Sponsor: " " " — Completed 12/31/49
Manager: J. H. Levin

Objective: To calculate functions $M_n(\gamma) = \int_0^1 \xi^n x_i \sin\gamma\xi \, d\xi$, $M_n'(\gamma) = \int_0^1 \xi^n \cos\gamma\xi \, d\xi$ for n = 0(1)5(2)11, and for approximately 150 values of γ in the range $1 \leq \gamma \leq 100$; to calculate also specified products of the type $M_i M_k$, $M_i' M_k'$. The above integrals are evaluated by the use of the recursion formulae:

$$M_n(\gamma) = -\frac{\cos\gamma}{\gamma} + \frac{n \sin\gamma}{\gamma^2} - \frac{n(n-1)}{\gamma^2} M_{n-2}$$

$$M_n'(\gamma) = \frac{\sin\gamma}{\gamma} + \frac{n \cos\gamma}{\gamma^2} - \frac{n(n-1)}{\gamma^2} M'_{n-2}$$

Background: For simple forms the wave resistance of a ship is given to a first approximation by Michell's integral:

(1) $$R = \frac{8\rho g}{\pi} \frac{B^2H^2}{L} \int_{\gamma_0}^{\infty} \left\{ I^{+2}(\gamma) + J^{+2}(\gamma) \right\} \frac{(\gamma/\gamma_0)^2}{\sqrt{(\gamma/\gamma_0)^2 - 1}} \, d$$

(2) $$I^+(\gamma) = \int_0^1 \sum n\, a_n \xi^{n-1} \cos\gamma\xi \, d\xi \int_0^1 (1 - \zeta^m)\, e^{-\theta\zeta} \, d\zeta$$

(3) $$I^+(\gamma) = \int_0^1 \sum n\, a_n \xi^{n-1} \sin\gamma\xi \, d\xi \int_0^1 (1 - \zeta^m)\, e^{-\theta\zeta} \, d\zeta$$

where n, m are integers, the quantity preceding the integral sign in (1) is a dimensional factor, and $\theta = \frac{\gamma^2 (\text{constant})}{\gamma_0}$. This approximation can be used as a basis for more complicated cases. An equivalent expression (Havelock's integral) is:

$$R = \frac{8 \rho g}{\pi} \frac{B^2H^2}{L} \int_0^{\pi/2} \left\{ I^{+2} (\gamma_0 \sec \theta) + \mathcal{J}^{+2}(\gamma_0 \sec \theta) \right\} \sec^3\theta \, d\theta$$

where $\gamma_0 \sec \theta$ is substituted for γ. The expressions $M_n(\gamma)$ and $M_n'(\gamma)$ are necessary for the evaluation of either Michell's integral or Havelock's integral.

The indices n of the functions $M_n(\gamma)$, $M_n'(\gamma)$ depend upon exponents of terms in polynomials used for approximating ship forms. With the chosen indices (exponents) the whole range of normal ship forms can be covered.

Besides, the same functions $M_n(\gamma)$, $M_n'(\gamma)$ admit of solving resistance problems of totally submerged bodies near the surface, gliding vessels, and problems of behavior of a ship in a seaway.

Thus these functions are of basic importance in Theoretical Naval Architecture.

Status: COMPLETED. Results were transmitted to the David Taylor Model Basin.

RATING OF WATER CURRENT METERS
Project 11.2/33-50-2

Origin: NBS, Division 6.5
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were performed as requested.

A PROBLEM IN MOLECULAR STRUCTURE
Project 11.2/33-50-3

Origin: Naval Research Laboratory, USN
Sponsor: " " " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were completed for the values of the parameters initially requested and for certain additional values.

CONFORMAL MAPPING OF A CIRCLE ONTO AN ELLIPSE
Project 11.2/33-50-4

Origin: NBS, Section 11.1 — Authorized 10/1/49
Manager: Irene Stegun — Completed 12/31/49

Objective: To evaluate by numerical computation the conformal mapping of the unit circle in the complex plane onto a given ellipse.

Background: The purpose of this project is to gain experience with a method proposed by Dr. A. M. Ostrowski. The essence of the method is as follows: The mapping is given by the function

$$z(u) = \sqrt{A^2 - B^2} \sin \left[\frac{\pi}{2K} \int_0^a \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}} \right]$$

with complex upper limit a, where A and B are the half axes of the ellipse, k the modulus of the corresponding elliptic integral, K the corresponding complete integral and $a = u/\sqrt{k}$. In order to evaluate the integral, the factor $(1 - k^2 t^2)^{-\frac{1}{2}}$ is replaced by a power series; the resulting integral can be expressed in terms of elementary functions of a.

Comments: See project 11.1/1-49-CM1.

Status: COMPLETED. Results were transmitted to Dr. Ostrowski, who will include them in a forthcoming publication.

VIBRATION OF PLANETARY GEAR TRAIN
Project 11.2/33-50-5

Origin: David Taylor Model Basin, USN Authorized 12/1/49
Sponsor: " " "
Manager: J. H. Levin

Objective: For a set of given matrices to find all real positive characteristic roots lying below a given boundary. The given matrices are of order approximately 15, are symmetric, and have the property that most, but not all, non-zero elements are close to the principal diagonal.

Background: The problem arises in the study of planetary gear trains whose vibrational frequencies are represented by the desired characteristic roots. It is desired to find a rapid mechanical computation procedure which will determine, for any given design of a gear train, those vibrations which fall within a certain dangerous frequency band. In order to make use of the almost diagonal character of the matrix, a generalization of the method of Prohl-Myklestad (J. Appl. Mech. 12, No. 3, Sept. 1945) is used. The latter method, as it stands, applies to matrices in which no non-zero element is more than one step away from the principal diagonal. This is not quite general enough for the present case.

The problem and method of solution were requested specifically by Dr. A. Gleyzal of David Taylor Model Basin.

Status: NEW. A machine procedure was formulated and computations were completed on the data submitted.

IONOSPHERIC WINDS
Project 11.2/33-50-7

Origin: NBS, Section 14.1 Authorized 12/1/49
Sponsor: " "
Manager: B. Heindish

Objective: To analyze certain records of fast-fading ionospheric reflections of radio beams by transcribing them from the original paper tape to punched cards and performing correlation analyses by means of punched card machines.

Background: Measurements of intensity of reflection are recorded at three stations located at the vertices of a right triangle. As an ionospheric disturbance travels over the stations, it causes successive fading of the three records. By cross-correlating the records with various time lags and maximizing the correlation coefficients one obtains the components of the average velocity of travel of such disturbances. The problem was suggested by Mr. C. D. Salsberg of the Ionospheric Research Laboratory.

Status: NEW. Machine procedure was formulated and data were being analyzed as submitted.

CRYSTAL STRUCTURES OF CEMENT COMPOUNDS
Project 11.2/33-50-9

Origin: NBS, Division 9, Portland Cement Research Associate Project — Authorized 12/31/49
Sponsor: Portland Cement Association
Manager: J. H. Levin

Objective: Computation of Fourier series of form

$$\rho(x) = \sum_{h=1}^{20} A_h \cos 2\pi hx + \sum_{h=1}^{20} B_h \sin 2\pi hx$$

for x = 0(1/60)1, and assigned values of the amplitudes A and B.

Background: The table will be used for calculating electron density distributions in crystals from measured intensities of x-ray diffraction maxima. Calculations of electron density in two or three dimensions will be made as an array of numerous one-dimensional syntheses of this type. This work is part of an investigation of the crystal structures of cement compounds which is being carried on by the Portland Cement Association Fellowship.

Comments: The calculations are carried out by punched card methods by the use of a basic deck of cards prepunched with the terms occurring in the series, for h = 0(1)20, x = 0(1/60)15/60 and ± A, ± B = 1(1)5, 10(10)50, 100, 200, 500. The desired terms for a given series are selected manually and summed by machine, with automatic sign changes for the intervals x = 15/60 to 30/60, 30/60 to 45/60, and 45/60 to 1.

Status: NEW. Machine procedure was formulated.

WAVE RESISTANCE OF SHIPS, II
Project 11.2/33-50-10

Origin: David Taylor Model Basin, USN — Authorized 12/31/49
Sponsor: " " " — Completed 12/31/49
Manager: Irene Stegun

Objective: To evaluate by numerical integration the integral

$$g(\theta) = \int_{-\infty}^{+1} \exp\left[-\frac{1}{F^2}\cdot\frac{f}{a}\cdot\frac{1-\lambda}{\cos^2\theta}\right] J^2_{3/2}\left[\frac{\varepsilon}{2F^2}\cdot\frac{1-\lambda}{\cos\theta}\right]\frac{d\lambda}{\lambda},$$

for $f/a = .5$; $\varepsilon = .99$; $F^2 = .09, .1024, .1225, .16$; and $\theta = 0(.2)1.2$. $J_{3/2}$ is the Bessel function of the first kind of order 3/2.

Background: The above integral is the integrand of the "vertical force" integral

$$Z_2 = \int_0^{\pi/2} \frac{g(\theta)}{\cos^3\theta} \, d\theta$$

The problem was requested specifically by Dr. G. Weinblum of the DTMB.

Status: COMPLETED. The results were transmitted to Dr. Weinblum.

WAVE RESISTANCE OF SHIPS, III
Project 11.2/33-50-11

Origin: David Taylor Model Basin, USN Authorized 12/31/49
Sponsor: " " "
Manager: J. H. Levin

Objective: It is proposed to carry out further computations involved in the evaluation of Michell's integral. This phase of the calculation includes the evaluation of the integrals,

$$C \int_{\nu_0}^{\infty} E_n^2 (k\nu^2/\nu_0) \frac{(\nu/\nu_0)^2}{\sqrt{(\nu/\nu_0)^2 - 1}} M_i(\nu) M_j(\nu) \, d\nu,$$

where C is a constant; $E_n(x) = \int_0^1 z^n e^{-xz} dz$; $\nu_0 = 1(.5)15$; $k = 0.1, 0.125, 0.0625$; and $i, j = 0(1)5(2)11$.

Background: The computations were requested specifically by Dr. G. Weinblum of the DTMB.

Comments: This is a continuation of project 11.2/33-50-1.

Status: NEW

STANDARD LORAN TABLES
Project 11.2/34-50-1:Gulf Coast Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " " " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Procedure was formulated and cards were prepared for large scale computations.

EFFECT OF NUCLEAR RADIATIONS ON HUMAN BEINGS
Project 11.2/35-49-1

Origin: Operational Research Office, U.S.Army (Johns Hopkins University)
Sponsor: " " " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were in progress.

EVALUATION OF ANTI-AIRCRAFT FIRE
Project 11.2/35-50-1

Origin: Operations Research Office, U.S.Army (Johns Hopkins University) — Authorized 12/1/49, Completed 12/31/49
Sponsor: " " " "
Manager: D. O. Larson

Objective: To compute and tabulate the function

$$f(n_1, n_2, x) = \frac{n_2 \left[1 - (1 - x/n_2)^{n_1}\right]}{R \cdot x(1-x)^Q + n_2 \; 1 - (1-x)^Q}$$

where Q = largest integer $\leq n_1/n_2$, and $R = n_1 - n_2 \cdot Q$. The function is to be tabulated a) in the case $n_1 = n_2$ for x = 0(.05)1 and n_1 = 1(1)10(5)20, 30, 40, 60, 120, ∞, and b) for $n_2 = 4$, x = 0(.02).1(.05)1 and n_1 = 5(1)13.

Background: This is a function that was encountered in an Operations Research Office project dealing with the overall Evaluation of Anti-Aircraft Fire.

Comments: The formula evaluated was proposed by Mr. M. L. Norden of the Operations Research Office, U. S. Army.

Status: COMPLETED. The results were transmitted to the Operations Research Office.

TABLES FOR COMBAT CREW PLANNING
Project 11.2/36-49-2

Origin: Air Comptroller's Office, USAF
Sponsor: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Preparation for publication continued.

Publication: Tables to be included by the Air Comptroller's Office in the next edition of the Air Force Technical Manual on Combat Crew Planning.

A PROBLEM IN LINEAR PROGRAMING
Project 11.2/36-49-3

Origin: Air Comptroller's Office, USAF
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A satisfactory machine procedure was formulated. Programs were being calculated as requested by the originators.

3. Training

TRAINING OF PERSONNEL IN THE OPERATION OF NEW IBM MACHINES
Project 11.2/4-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. "On the job" training of new staff members continued.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Research in Mathematical Statistics

THE MEAN DEVIATION, STANDARD DEVIATION, AND RANGE AS ESTIMATORS OF SCALE PARAMETERS (MEASURES OF DISPERSION) OF PROBABILITY DISTRIBUTIONS
Project 11.3/1-47-2

Origin: NBS
Full project description appears in Oct-Dec 1948 issue.

Status: CONTINUED. In 1816 Gauss considered the use of the median of the absolute errors of n independent observations from a normal distribution as a convenient estimator of the probable error of the distribution. He noted that, in the case of large samples, 2½ times (more exactly, 2.72 times) as many observations would be needed to achieve a given degree of accuracy with this method as with the root-mean-square (i.e., standard-deviation) method. On the other hand, the two procedures are clearly equivalent for the case of a single observation. The behavior of the median error in very small samples (e.g. $\underline{n} = 3,5,\ldots,15$) was studied during the quarter, using the table of percentage points of the median in random samples of odd sizes from any continuous distribution, developed previously under project 11.3/1-47-1 (see Jan-Mar 1949 issue). The accuracy of the median error is poor compared to that of the root-mean-square-error procedure, but the "inefficiency" of the median error is not quite so conspicuous in small as in large samples.

Publications: (1) Probability center lines for standard deviation and range charts. Churchill Eisenhart. Industrial Quality Control VI, No. 1, 24-26 (July 1949). (2) The relative frequencies with which certain estimators of a normal population tend to underestimate its value. Churchill Eisenhart and Celia S. Martin. To be submitted to the Journal of the American Statistical Association. An abstract appears in Annals of Mathematical Statistics XIX, No. 1, 600 (Dec.1948).

STATISTICAL TESTS OF SIGNIFICANCE FOR 2 x 2 TABLES WHEN THE NUMBER OF OBSERVATIONS IS SMALL
Project 11.3/1-47-3

Origin: NBS
Full project description appears in Oct-Dec 1948 issue.

Status: INACTIVE.

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Project 11.3/1-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Preparation of the manuscript of publication (1) was completed.

Publications: (1) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. To be submitted to the NBS Journal of Research. (2) The fallacy of the best two out of three. For this summary of the results of the study and their practical implications see NBS Tech. News Bull. 33, No. 8, 1(July 1949).

STATISTICAL PROCEDURES FOR INTERPOLATED MEDIANS
Project 11.3/1-49-2

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Project 11.3/1-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.
Manager: Henry B. Mann. (Dr. Mann, who is responsible for technical aspects of this project, undertook the project during a temporary employment by the Bureau from March to June 1949, and is expected to return for brief periods until the work is finished. During his absence Dr. Eisenhart handles inquiries and other matters related to the project.

Status: CONTINUED. H. B. Mann returned to the Bureau for the period December 19-23 to discuss revision of the first draft of his monograph on the elementary theory of stochastic processes in the light of comments received from various experts in the field to whom copies were sent. Plans were made for inclusion of some additional results obtained by Dr. Mann since he left the Bureau in June 1949. Dr. Mann plans to use the monograph in a course on stochastic processes that he will give next spring as a Visiting Professor, at the University of California, Berkeley. It is anticipated that a final draft of the monograph will then be prepared.

2. Manuals of Statistical Methods

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS
Project 11.3/2-47-2

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STANDARD SAMPLING-INSPECTION PROCEDURES
Project 11.3/2-48-1

Origin: Office of Naval Research, and Research and Development Division of the Department of the Army
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For most recent status report see July-Sept 1949 issue.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Project 11.3/2-48-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Additional terms were considered for inclusion in the glossary and work proceeded on the compilation and revision of definitions.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Project 11.3/2-49-1

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Project 11.3/2-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. About 95% completed. The section on interpolation was greatly expanded to include a new method, communicated by Dr. H. O. Hartley, that is especially adapted to the normal probability integral.

3. Statistical Services

WOOL CONTENT OF BLANKETS
Project 11.3/31-47-6

Origin: NBS and Division of Statistical Standards, Bureau of the Budget.
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For most recent status report see July-Sept 1949 issue.

STATISTICAL STUDY OF THE FLOW OF CERTAIN STOCKROOM ITEMS
Project 11.3/31-48-4

Origin: NBS, Section M.4
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STATISTICAL ANALYSIS OF THERMOMETRIC MEASUREMENTS
Project 11.3/31-49-4

Origin: NBS, Section 3.1
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. An experiment involving three types of thermometers (pear, stubby, and oral) under three types of condition (aged, not aged, and aged by a special process) was designed to determine the change in thermometers with time and the efficiency of the special aging process. Five sets of readings are being taken over a period of 6 months, and more than 10,000 readings will be obtained. The first two sets were completed and analysis of the results was started.

Dr. Youden presented a paper on "Comparative tests in a single laboratory" at a joint meeting of the American Society for Testing Materials and the American Statistical Association in New York City on 28 December 1949. This paper relates to earlier tests conducted under this project (see previous status reports).

Publication: "Comparative tests in a single laboratory" by W. J. Youden; to be published in the Bulletin of the American Society for Testing Materials.

WOMEN'S BODY MEASUREMENT STUDY
Project 11.3/31-49-5

Origin: NBS, Section 12.2
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A stature-hip girth bivariate frequency distribution of all the women 18-60 years old was prepared and studied in conjunction with the three (18-29 years, 30-40 years, 41-60 years) bivariate distributions previously considered. It was decided that tolerance limits for stature and hip girth were not to exceed 10 cm. and 5 cm. respectively. Accordingly three stature intervals (shorts 9 cm., regulars 9 cm., and talls 10 cm.) were obtained for each 5 cm. hip interval for each of the three age groups. An increase of 1 cm. in stature was provided for each interval of hip girth. On the basis of stature and hip girth alone this arrangement includes at least 90% of the women in the study. These recommendations have been approved by the requester (a committee of the Mail Order Association of America) and analysis of other variates is being planned.

The task group of the main committee met several times during the quarter in Washington to assist in the step-by-step plans for the project.

PRECISION MEASUREMENTS ON STANDARD TEMPERATURE SOURCES
Project 11.3/31-50-2

Origin: NBS, Section 5.0
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE

PROBABILITY THEORY OF A CERTAIN DOUBLE BRANCHING SYSTEM
Project 11.3/31-50-3

Origin: NBS, Section 4.5
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

STATISTICAL SERVICES TO ARMY FIELD FORCES TEST BOARDS
Project 11.3/32-49-1

Origin: Logistics Division, General Staff, U.S. Army
Sponsor: Research and Development Group, General Staff, U. S. Army
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In October the third visit of the year was made to Fort Bliss. Discussions were held with the Test Board concerning the use of the techniques and tables for approving aiming errors that had been previously furnished by the Statistical Engineering

Laboratory. Further problems were raised by the Test Board and are now being studied.

SOLID PROPELLANT DEVELOPMENT TESTS
Project 11.3/32-49-2

Origin: ORDTU, Research and Development Division, Ordnance Dept., U. S. Army
Sponsor: Research and Development Group, General Staff, U. S. Army
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

PROBABILITY STUDIES OF MISSILE EFFECTIVENESS
Project 11.3/33-49-1

Origin: Bureau of Ordnance, Navy Department, via ElectronicsDivision,NBS
Sponsor: Bureau of Ordnance, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Almost the entire quarter was spent in devising methods of computation, although some work was done in evaluating the data previously obtained. The solution to part of the problem necessitates the use of an electronic analog computer. All the differential equations were derived, and some were actually placed on the machine.

TECHNICAL SYMBOLISM AND TERMINOLOGY
Project 11.3/4-50-1

Origin: NBS Educational Committee
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. 75% completed. Thirty-two persons enrolled for the first term, which began October 4 and ended December 6. Of these, 27 completed the course requirements for the first term.

The topics covered included: Greek alphabet, mathematical symbols, "breaking" of equations when typing, mathematical vocabulary and spelling, significant figures and rounding off figures, handling of subscripts and exponents, use of solidus.

IV. Machine Development Laboratory

(Section 11.4)

1. <u>Design and Construction of Automatic Digital Computing Machines</u>

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Project 11.4/21-47-1

Origin: The Bureau of the Census
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A contract modification incorporating UNIVAC design modifications recommended by the contractor was in process at the end of the quarter. The contractor had completed the technical work preliminary to beginning of production-line UNIVAC construction. The UNIVAC production schedule was disrupted by financing negotiations forced upon the contractor by the action of one of the principal stockholders.

THE NAVY COMPUTING MACHINE
Project 11.4/22-47-2

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The contractor continued work on mercury-line storage and tape-handling units and prototyping of computer components.

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.4/23-49-1

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Performance specifications to be included in bid invitations were prepared and submitted to interested parties for comment.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Project 11.4/24-47-3

Origin: Office of the Air Comptroller, USAF
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

THE NBS INTERIM COMPUTER
Project 11.4/24-49-1

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Construction was proceeding according to schedule. It is expected that the computer will be assembled and placed under test with an eight-word acoustic memory during the next quarter.

ARMY MAP SERVICE COMPUTING MACHINE
Project 11.4/25-49-1

Origin: Army Map Service, U.S.A.
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

DEPARTMENT OF THE ARMY COMPUTER DESIGN
Project 11.4/26-49-1

Origin: Department of the Army
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. No direct work on this machine was performed by the Machine Development Laboratory during this quarter. However, much of the work on the NBS Interim Computer was applicable to this project (see project 11.4/24-49-1).

2. Programing Studies

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Project 11.4/3-47-4

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED.
I. General programing for automatic computing machines. Preliminary investigations of the problem of coding the various mathematical operations involved in the use of matrices were continued. Particular attention was directed to the preliminary phases of the following problems:

1. The use of Lanczos' method for finding the characteristic equation of square matrices.
2. The solution of simultaneous linear inequalities.
3. The use of the Graffe method for finding the roots (both real and complex) of n^{th}-order algebraic equations. These are needed in connection with Lanczos' method for the determination of the eigenvalues and eigenvectors of a matrix.

II. Programing for the NBS Interim Machine. The basic arithmetic operations, the elementary functions (such as $\sqrt{x}$, sin x, and e^x), the conversion of binary numbers to decimal numbers, and the conversion of decimal numbers to binary numbers have been coded for single precision numbers in the fixed binary form and the floating binary form. The basic arithmetic operations and some of the elementary functions have been coded for double precision numbers in fixed binary form. Work continued on the conversion of double precision numbers in fixed binary form.

CODING ON THE E.R.A. COMPUTER
Project 11.4/3-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For most recent status report see July-Sept 1949 issue.

CODING RELATED TO THE UNIVAC SYSTEM
Project 11.4/3-49-2

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Continued investigation of sorting procedures on the UNIVAC was carried out by a working group which included representatives of the Bureau of the Census, the Naval Communications Center, the Office of the Air Comptroller, and the National Bureau of Standards. The comparative analysis of the C-10 and C-7 UNIVAC instruction codes was completed and the acceptance of the code C-10 was recommended. During the month of December two days per week were spent in training the programing staff of the Bureau of the Census.

CODING RELATED TO THE RAYTHEON COMPUTER
Project 11.4/3-49-3

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. Pending submission by the contractor of complete information concerning the computer codes, work on programing of problems for solution on this computer was held in abeyance. For most recent status report see July-Sept 1949 issue.

PERFORMANCE TEST FOR THE BINAC
Project 11.4/3-49-4

Origin: NBS | Authorized 12/1/48
Manager: Ida Rhodes | Completed 12/31/49

Objective: To devise an instruction code for a rigorous test problem on the BINAC.

Background: The BINAC (designed and under construction by the Eckert-Mauchly Computer Corporation) is an automatically-sequenced electronic digital computer embodying many of the important principles of engineering design of the larger and more powerful ENIAC. Satisfactory performance of the BINAC on an extensive test problem has therefore been stipulated by the Bureau as a prerequisite to consideration of the engineering design of the UNIVAC for approval.

Comments: This project is essentially an offshoot of project 11.4/21-47-1.

Status: COMPLETED.

3. Technical Reports on Computing Machinery

THE MTAC SECTION
Project 11.4/4-47-1

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for the Automatic Computing Machinery Section of the April 1950 issue of Mathematical Tables and Other Aids to Computation was assembled and edited. In addition to the bibliography and news items, two technical and two discussion articles were included in this issue. One of the technical papers, "The EDSAC (Electronic Delay Storage Automatic Calculator)", by M. V. Wilkes and W. Renwick, described a serial electronic calculating machine now in operation in Cambridge, England. The machine has 32 ultrasonic storage tanks furnishing 1,024 storage locations in all. It makes use of a single address code, the orders consisting of a functional part defining the operation and a numerical part defining the storage location. Telegraphic punched tape is used for input; the output makes use of a teleprinter system. No special checking facilities are provided for in the machine. The control sequence is divided into two stages: in stage I the order is transferred from the memory to the order tank; in stage II, the order in the order tank is executed. The machine is originally set in operation by push button.

In the second technical article, "Characteristics of the Institute for Numerical Analysis Computer," by Dr. Harry Huskey, the command system of this high-speed electronic digital computer is discussed. Pertinent facts concerning the design of the computer are presented. Details are given of a system of using abbreviated commands which, when appropriate subroutines have been established, greatly simplifies the process of coding a complex problem for solution on such a computer. An example is mentioned showing that any system of 125 or fewer simultaneous linear equations can be coded by using less than thirty abbreviated commands.

The discussion paper, "Note on the condition of matrices," by Olga Taussky-Todd is concerned with the proof of the following theorem: Let A be a real n x n non-singular matrix and A' be its transpose. Then AA' is more "ill-conditioned" than A. It deals specifically with the methods for determining the condition of a matrix proposed by John von Neumann and H. H. Goldstine and also by A. M. Turing. The following two definitions form a basis for proof of the theorem - ill conditioned matrices, in this case, being characterized by very large condition numbers:

(a) P-condition number is $|\lambda_{max}| / |\lambda_{min}|$, where λ_{max} and λ_{min} are the characteristic roots of largest and smallest modulus.

(b) N-condition number is $\frac{1}{n} N(A)N(A^{-1})$, where $N(A) = \left[\sum_{i,k} a_{i,k}^2\right]^{\frac{1}{2}}$

The remainder of the paper proves that the P and N condition numbers of AA' are greater than those of A, i.e., that AA' is more ill-conditioned than A.

The discussion article, "Statistical treatment of values of first 2000 decimal digits of e and of π calculated on the ENIAC," by N. C. Metropolis, G. Reitwiesner, and John von Neumann, discusses some quite serious deviations from uniform distributions of the 10 digits in the case of e which were disclosed in the calculations of Mr. G. Reitwiesner and members of the ENIAC staff at Aberdeen, Maryland. There were no significant deviations in the values of π.

In addition to this material, the galley and page proofs for the October 1949 issue of the journal were corrected and returned to the editor.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Project 11.4/42-49-2

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. This project is pending the results of a suggestion to amalgamate all bibliographies on high-speed automatic computing machinery under the joint auspices of the American Institute of Electrical Engineers and the Institute of Radio Engineers. This joint publication was suggested by J. G. Brainerd, chairman, Committee on Computing Devices, AIEE.

Lectures and Symposia

NBS Applied Mathematics Colloquium Series

GUMBEL, E. J. (Consultant). On the statistical theory and methods of extreme values and their practical applications. A series of four lectures: (1) Survey of practical applications of extreme value theory and methods, (2) Exact distributions of extreme values, (3) The three asymptotic distributions of the extremes, (4) Other extremal order statistics. October 18-21, 1949.

Numerical Analysis Colloquium Series
(Los Angeles, California)

ROSSER, J. B. Transformations to speed the convergence of series. October 3, 1949.

SZASZ, OTTO Gibbs' phenomenon for Hausdorff means. October 10, 1949.

FORSYTHE, G. Analysis of round-off errors. October 17, 1949.

BENEDIKT, E. T. (Engineering Department, UCLA). Random errors in analog computers. October 24, 1949.

BLANCH, GERTRUDE Positioning in matrix calculations. October 31, 1949.

HESTENES, M. R. Gradients of integrals. November 14, 1949.

TUCKER, A. W. (Princeton University and Stanford University). Linear programing. November 23, 1949.

YOWELL, E. C. Inversion of matrices on I.B.M. equipment. November 28, 1949.

CURTISS, J. H. Sampling methods applied to differential equations of the elliptic type. December 12, 1949.

Applied Mathematics Division Technical Meetings

DEMING, LOLA Analysis of some body measurements for the development of a sizing system of apparel for teen-age girls. October 31, 1949.

CURTISS, J. H. Solution of problems involving differential equations by sampling methods; the "Monte Carlo" method. November 7, 1949.

MONTALBANO, M. Scientific computation of optimum programs. November 14, 1949.

LEVIN, J. Characteristic roots and vectors of a matrix. November 21, 1949.

HEINDISH, B. Punched-card analysis of basic ionospheric data. November 28, 1949.

EISENHART, C. The work of the Statistical Engineering Laboratory. December 5, 1949.

SALZER, H. E. Gamma function of complex arguments. December 12, 1949.

CAMERON, J. Sequential sampling plans for JAN standards. December 19, 1949.

Lecture Series on the NBS Interim Computer

LUBKIN, S. A series of lectures on the NBS Interim computer for members of the Bureau's staff, covering the electronic and physical construction, coding problems and different numbers systems. October 27-November 10, 1949.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. (1) Bases for functional values. Presented at the University of Maryland, December 1, 1949. (2) Machine methods for finding the characteristic roots of a matrix. Presented at the Computation Forum of the International Business Machines Corporation, December 5-9, 1949.

EISENHART, C. (1) Certain aspects of accuracy and precision. Presented at a meeting of the Delaware Chapter of the American Society for Quality Control, Wilmington, Delaware, October 6, 1949. (2) The sampling procedure. Presented at the Ordnance School, Aberdeen Proving Ground, Maryland, October 14, 1949.

FORSYTHE, G. E. Round-off errors in numerical integration on automatic machinery. Presented at a meeting of the American Mathematical Society, Pasadena, California, November 26, 1949.

HESTENES, M. R. Gradient integrals. Presented at the American Mathematical Society meeting, New York, N. Y., December 28, 1949.

HUSKEY, H. D. Automatic digital computing machines. Presented at a meeting of the Armed Forces Communication Association, Los Angeles, California, October 13, 1949.

LANCZOS, C. An iterative solution of Fredholm's integral equation. Presented at a meeting of the American Mathematical Society, Pasadena, California, November 26, 1949.

OSTROWSKI, A. M. (1) On Theodorsen's and Garrick's method of conformal mapping. Presented at the Institute of Fluid Dynamics and Applied Mathematics of the University of Maryland, October 6, 1949. (2) Graffe's method in the theory of algebraic equations. (a) Presented at the University of Rochester, October 11, 1949. (b) Presented as the Putnam Memorial Lecture at Harvard University, October 13, 1949. (c) Presented at Princeton University, October 19, 1949.

RHODES, IDA (1) Applications of electronic digital computing machines in factor analysis. Presented at the Symposium on the Role of Electronic High-Speed Computers in Psychological Research, sponsored by the Psychometric Society and held at the meeting of the American Psychological Association, Denver, Colorado, September 6-10, 1949. (2)Idiosyncrasies of automatically-sequenced digital computing machines. Presented at the annual meeting of the American Statistical Association, New York, N. Y., December 30, 1949.

ROSSER, J. B. (1) The axiom of choice. Presented at a meeting of the Logic of Science Group, Rand Corporation, Santa Monica, California, October 14, 1949. (2) Transformations to speed the convergence of series. Presented at the Peripatetic Seminar in Mathematics at the University of California, Los Angeles, November 7, 1949. (3) Development and application of hi-speed computers at the Institute for Numerical Analysis. Presented at a meeting of the Sigma Xi Logic Group at the University of California, Los Angeles, December 6, 1949. (4) A general iteration scheme for solving simultaneous equations. Presented at the American Mathematical Society meeting, New York, N. Y., December 28, 1949.

SZASZ, OTTO (1) On a summation method of O. Perron. Presented by title at the American Mathematical Society meeting, Pasadena, California, November 26, 1949. (2) Generalization of S. Bernstein's polynomials. Presented at the Peripatetic Seminar at the University of Southern California, December 5, 1949.

YOUDEN, W. J. (1) Use of statistics to determine precision of test methods. Presented at the First Pacific Area National meeting of the American Society for Testing Materials, San Francisco, California, October 11, 1949. (2) How statistics improves physical, chemical and engineering measurements. Presented at the Department of Agriculture at the invitation of the Committee on Experimental Design of the Agricultural Research Administration, Washington, D. C., December 14, 1949. (3) Comparative tests in a single laboratory. Presented at a Joint Session of the American Statistical Association and the American Society for Testing Materials, New York, N. Y., December 28, 1949. (4) A statistic for rating diagnostic tests. Presented at a Joint Session of the Biometric Society and the Biometric Section of the American Statistical Association, New York, N. Y., December 30, 1949.

YOWELL, E. C. Numerical solution of partial differential equations. Presented at the Computation Forum of the I.B.M. Corporation, Endicott, N. Y., December 5, 8, 1949.

Publication Activities

1. Publications which appeared during the quarter

1.1 Mathematical Tables

(1) Tables of circular and hyperbolic sines and cosines for radian arguments. NBS Mathematical Table 3. Second edition 1949. Available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., $2.50.

(2) Table of coefficients in numerical integration formulae. NBS Mathematical Table 22. (Originally published in J. Math. Phys. XXII, No. 2, 49-50, June 1943.) Reissued 12/15/49 by the National Bureau of Standards, available from the Superintendent of Documents, Government Printing Office, Washington 25, D.C., 5 cents.

(3) Table of Fourier coefficients. NBS Mathematical Table 23. (Originally published in J. Math. Phys. XXII, No. 3, 136-147, Sept. 1943.) Reissued 11/25/49 by the National Bureau of Standards. Available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 10 cents.

(4) Tables of zeros and weight factors of the first 15 Laguerre polynomials. H. E. Salzer and Ruth Zucker. Bull. Am. Math. Soc. 55, No. 10, 1004-1012 (Oct.1949). Reprints will probably be available.

1.3 Technical Papers

(1) Asymptotic expansions of spheroidal wave functions. M. Abramowitz. J. Math. Phys. 28, No. 3, 195-199, (Oct. 1949). Reprints available.

(2) Exact particle trajectories for nonviscous flow in a plane with a constant coriolis parameter. G. E. Forsythe. J. Meteorology 6, No. 5, 337-346 (Oct. 1949). Reprints available.

(3) Numerical integration for linear sums of exponential functions. R. E. Greenwood. Ann. Math. Stat. 20, No. 4, 608-611 (Dec. 1949). Reprints available.

(4) The remainder in linear methods of approximation. W. E. Milne. NBS J. Res. 43, No. 5, 501-511 (Nov. 1949). Available as RP 2041 from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 10 cents.

(5) On some generalizations of the Cauchy-Frullani integral. A. M. Ostrowski. Proc. Nat. Acad. Sci. 35, No. 10, 612-616 (Oct. 1949). Reprints available.

(6) Formulas for complex Cartesian interpolation of higher degree. H. E. Salzer. J. Math. Phys. 28, No. 3, 200-203 (Oct. 1949). Reprints available.

(7) Polynomials for best approximation over semi-infinite and infinite intervals. H.E. Salzer. Math. Mag. 23, No. 2, 59-69 (Nov-Dec 1949). No reprints available.

(8) A remark concerning the characteristic roots of the finite segments of the Hilbert matrix. O. Taussky-Todd. Oxford Quar. J. 1, No. 20,80-83 (June 1949). Reprints available.

(9) A problem on arc tangent relations. J. Todd. Am. Math. Mo. 56, 517-528 (Oct. 1949). Reprints available.

(10) Operation of voltage-stabilizing elements with current-stabilized supplies. J. J. Gilvarry (Rand Corporation) and D. F. Rutland. The Review of Scientific Instruments, 20, No. 9, 633-637 (Sept. 1949). No reprints available.

1.4 Reviews and Notes

(1) Symposia on conformal mapping and Monte Carlo method. J. H. Curtiss and E. F. Beckenbach. Science 110, 350 (Oct. 7, 1949). Reprints available.

1.5 Miscellaneous Publications

(1) Operational aspects of instrument design. Churchill Eisenhart. Science 110, 343-346 (Oct. 7, 1949). Reprints available.

(2) Some recent trends in applied mathematics. J. H. Curtiss. Amer. Scientist 37, No. 4, 587 (Oct. 1949). Reprints available.

2 Manuscripts in the Process of Publication December 31, 1949.

2.1 Mathematical Tables

(1) Tables of the binomial probability distribution. NBS Applied Mathematics Series 6. Being printed by the Government Printing Office.

(2) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. Being printed by the Government Printing Office.

(3) Tables of powers of complex numbers. NBS Applied Mathematics Series 8. Being printed by the Government Printing Office. Exact values of z^n for $z = x + iy$, $x, y = 0(1)10$, $n = 0(1)25$; exact values of x^n for $x = 0(1)10$, $n = 0(1)25$.

(4) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. Being printed by the Government Printing Office.

(5) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. Being printed by Columbia University Press.

(6) Tables relating to the Mathieu functions. Being printed by the Columbia University Press.

(7) Errata in "Tables relating to Hankel integrals of order zero," by L. Schwarz, published in Luftfahrtforschung, 20, No. 12, 341-372 (1943), and translated by J. Lotsof, Cornell Aeronautics Laboratory, May 1946. A. H. Rosenthal. Submitted to Mathematical Tables and Other Aids to Computation.

2.3 Technical Papers

(1) Tables of integrals of Struve functions. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) A class of mean value functions. E. F. Beckenbach. Accepted by the American Mathematical Monthly.

(3) Metric differential geometry. E. F. Beckenbach. Accepted by the Mathematics Magazine.

(4) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. Submitted to the Bulletin of the American Mathematical Society; also to appear in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(5) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Submitted to the Duke Mathematical Journal.

(6) Table of modified Bernoulli polynomials. G. Blanch and R. Siegel. Accepted for publication in the NBS Journal of Research.

(7) A method for reducing the amount of inspection. J. M. Cameron and W. J. Youden. Submitted to Industrial Quality Control.

(8) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Submitted to the Duke Mathematical Journal.

(9) Forced oscillations in non-linear systems. M.L. Cartwright. Accepted for publication in the NBS Journal of Research.

(10) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(11) Acceptance sampling by variables, with special reference to the case in which quality is measured by average or dispersion. J. H. Curtiss. To appear in a special supplement to the Journal of the American Statistical Association.

(12) A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation. M. D. Donsker and M. Kac. Accepted for publication in the NBS Journal of Research.

(13) Solution of the telegrapher's equation with boundary conditions on only one characteristic. G. E. Forsythe. Accepted for publication in the NBS Journal of Research.

(14) Second order determinants of Legendre polynomials. G. E. Forsythe. Submitted to the Duke Mathematical Journal.

(15) The variance of a certain determinant. G. E. Forsythe. Submitted to the American Mathematical Monthly.

(16) 1949 Ephemeris of Jupiter's ninth satellite. S. Herrick. To be published as a leaflet by the Astronomical Society of the Pacific.

(17) Some elementary problems in the calculus of variations. M. R. Hestenes. Submitted to the Mathematics Magazine; also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A.

(18) Quadratic forms in Hilbert space, with applications in the calculus of variations. M. R. Hestenes. Accepted for publication in the American Journal of Mathematics.

(19) Semi-automatic instruction on the Zephyr. H.D.Huskey. To appear in the "Proceedings of a Symposium on large-scale digital calculating machinery," Harvard Computation Laboratory, September 1949.

(20) Characteristics of the Institute for Numerical Analysis Computer. H.D. Huskey. Submitted to Mathematical Tables and Other Aids to Computation; scheduled to appear in the April 1950 issue.

(21) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Submitted to the Bulletin of the American Mathematical Society.

(22) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. C. Lanczos. Accepted for publication in the NBS Journal of Research.

(23) A note on the numerical integration of differential equations. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(24) Numerical determination of characteristic numbers. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(25) Note on the Runge-Kutta method. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(26) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Submitted to the Bulletin of the American Mathematical Society.

(27) Note on an infinite integral. A. M. Ostrowski. Submitted to the Duke Mathematical Journal.

(28) Note on Vincent's theorem. A. M. Ostrowski. Accepted for publication in Annals of Mathematics.

(29) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to the American Mathematical Monthly.

(30) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(31) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A. M. Ostrowski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(32) Transformations to speed the convergence of series. J. B. Rosser. Accepted for publication in the NBS Journal of Research.

(33) Coefficients for polar complex interpolation. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(34) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Philosophical Magazine.

(35) Formulas for numerical integration of first and second order differential equations in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(36) A bibliography on numerical methods in conformal mapping. W. Seidel. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(37) Inequalities concerning ultraspherical polynomials and Bessel functions. O. Szasz. Submitted to the Bulletin of the American Mathematical Society.

(38) Summation of slowly convergent series with positive terms. O. Szasz. Submitted to the Journal of Mathematics and Physics.

(39) On the Gibbs phenomenon for Euler means. O. Szasz. Submitted by request for publication in the 12th Anniversary volume of the Acta Scientiarum Mathematicarum (Hungary).

(40) On a summation method of O. Perron. O. Szasz. For publication in Mathematische Zeitschrift.

(41) A generalization of S. Bernstein's polynomials to the infinite interval. O. Szasz. Accepted for publication in the NBS Journal of Research.

(42) Certain Fourier transforms of distributions. E.Lukacs (Our Lady of Cincinnati College and Naval Ordnance Test Station, Inyokern) and O. Szasz. Submitted to the Canadian Journal of Mathematics.

(43) Note on the condition of matrices. O. Taussky-Todd. Accepted for publication in April 1950 issue of Mathematical Tables and Other Aids to Computation.

(44) A recurring theorem on determinants. O. Taussky-Todd. Accepted for publication in the American Mathematical Monthly.

(45) The condition of certain matrices. John Todd. Accepted for publication in the Quarterly Journal of Mechanics and Applied Mathematics.

(46) The condition of a certain matrix. John Todd. Accepted for publication in the Proceedings of the Cambridge Philosophical Society

(47) Notes on modern numerical analysis, I. John Todd. Accepted for publication in Mathematical Tables and Other Aids to Computation.

(48) On conformal mapping of variable regions. S.E.Warschawski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping", to be published by the National Bureau of Standards.

(49) Statistics in analytical chemistry. W. J. Youden. Accepted for publication in the Transactions of the New York Academy of Sciences.

(50) A note on the four by four Latin squares. W.J. Youden. Accepted by Biometrics.

(51) Use of statistics to determine accuracy of tentative test methods. W. J. Youden and J. M. Cameron. To be published by the American Society for Testing Materials.

(52) Index for rating diagnostic tests. W. J. Youden. Submitted to Cancer.

2.4 Reviews and Notes

(1) Review of "Proceedings of the Berkeley symposium on mathematical statistics and probability." C. Eisenhart. For publication in Psychometrika.

2.5 Miscellaneous Publications

(1) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

(2) High-speed computing and accounting. H. D. Huskey. and V. R. Huskey. To appear in the Journal of Accounting.

(3) Proceedings of a symposium on the construction and applications of conformal mapping, held at the NBS Institute for Numerical Analysis, Los Angeles, Calif., June 1949. To be published by the National Bureau of Standards.

NOTE: Papers presented at the Symposium on the Monte Carlo Method, held at the NBS Institute for Numerical Analysis on June 29, 30, July 1, 1949, are being assembled for publication.

EXPLANATION OF PROJECT DESCRIPTIONS

The project descriptions appearing in this report are reproduced from the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With a view toward making this report more useful, an explanation of certain terms used in the Project Forms is given here.

Date of Authorization. This is the date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Status. Here is given the narrative of the progress to date on the project. Certain descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to which the entry applies. These terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the quarter covered by the report to work on the project.

"CONTINUED" means that the work was initiated prior to the quarter covered by the report, and was in progress during the quarter.

"INACTIVE" means that the Laboratories made a commitment prior to the quarter covered by the report to work on the project, but no work of any consequence was performed on the project during the quarter.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof is to undergo extensive mathematical checks, the designation "Completed" is employed only after these checks have been performed.

"TERMINATED" means that, although all aspects of the objective were not achieved, it was necessary to terminate the project due to circumstances beyond the control of the Laboratories.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "In Manuscript" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "Completed" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under *Status* or *Publication* to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.