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COLORIMETRY

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Colorimetry

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Foreword

NBS Circular 478, Colorimetry, was issued 1 March 1950 and has been reprinted several times in response to a continuing demand. Such have been the advances in colorimetry that in 1965, instead of authorizing another reprinting of a work obsolete in many details, I asked Mr. I. Nimeroff to up-date it. I am grateful to him for undertaking this tedious but important task, and I am much pleased with the result.

Deane B. Judd
Assistant Division Chief,
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Colorimetry*

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The definition of color, as a characteristic of light, and the basic principles of its measurement are given. The reduction of spectrophotometric data to three chromaticity coordinates by means of the three-function CIE standard observer system for colorimetry is described. Various methods of direct colorimetry, and visual and photoelectric methods of colorimetry by differences from material standards are treated. The most useful collections of material color standards are described and the most widely used one-dimensional color scales are explained. The limitations of several colorimetric methods: spectrophotometric, photoelectric, and visual, are discussed.

Key Words: Color dictionaries, color vision, colorimeters, photoelectric tristimulus, spectrophotometric colorimetry, visual colorimetry.

1. General Considerations

1.1. Introduction

It is common practice to regard color as a property of objects, and in a limited sense this view is justified. We have color comparators for solutions in which the color is taken as an index of the composition of the solution; and in applying a suitably prepared set of color standards in a color comparator, the color of the unknown behaves as if it were a property of the solution itself, just as the concentrations of the constituents which it indicates. However, this document deals with color for its own sake; and, for this purpose, a broader view is useful. Lights have colors as well as objects. The flame of a Bunsen burner can be changed from bluish purple to orange by the introduction of sodium. And since even objects lose their colors and become invisible unless they reflect, scatter, or transmit radiant energy, or form a part of an illuminated scene, this broader view is that color is a property of light, and of light alone.

As it is possible to measure with a spectrophotometer the spectral energy distribution of any light beam, and as the color of a light correlates closely with its spectral composition, some of the more physically minded people have contended that color is a physical property of radiant energy; but this is not the most useful view. The color change of the Bunsen flame from bluish purple to orange can be shown by a purely physical measurement to be caused by a change in the spectral composition of the emitted energy, but it takes more than physics to decide whether this flame has the same color as the light reflected from the peel of a given citrus fruit. Application of the spectrophotometer to the orange peel will show that the spectral composition of the light reflected from it under daylight illumination is radically different from that emitted by the sodium flame. It has a continuous spectrum relatively strong in the long-wave (550 to 770 nm)[1] portion of the visible spectrum (380 to 770 nm). The visible energy of the sodium flame is nearly all confined to two narrow bands (589.0 and 589.6 nm). Physically, therefore, the two lights are different, but they have closely the same color. The two lights must therefore be identical in some other respect. This identity consists in some aspect of the response made by a normal observer to the sodium flame being the same as the corresponding aspect of the response to the peel of the citrus fruit. The broader view of color must, therefore, include not only the spectral composition of the radiant energy reaching the eye of the observer, but the properties of the observer as well. These properties have been evaluated by finding equivalent stimuli, like the energy of the sodium flame and that reflected from an orange peel, which have different spectral compositions but still manage to stimulate the same color response to the normal observer. Such equivalent stimuli are called *metamers*. In this chapter there will be presented the standard method for

* This monograph is an updating of NBS Circular C-478, Colorimetry, by D. B. Judd (1950). With the permission of Judd, much of the original text has been kept intact where applicable.

[1] The unit nanometers (nm), 10^{-9} meters, was formerly called millimicrons (mμ).

finding by computation whether or not any two lights form equivalent stimuli. The fundamental method of color specification based upon equivalent stimuli plus spectrophotometry will be described in detail together with other methods of obtaining the same numbers. And finally some discussion will be given on the use of the Munsell color system and color dictionaries such as the ISCC-NBS (Inter-Society Color Council-National Bureau of Standards) Method of Designating Color and a Dictionary of Color Names, National Bureau of Standards Circular 553, 1956 [79].[2]

1.2. Definition of Color

The most widely accepted technical definition of color is that given by the Committee on Colorimetry of the Optical Society of America [15]: "Color consists of the characteristics of light other than spatial and temporal inhomogeneities; light being that aspect of radiant energy of which a human observer is aware through the visual sensations which arise from the stimulation of the retina of the eye." It will be noted that this definition relates color and light to radiant energy only in so far as the energy produces a visual effect within an observer. On this account color and light are said to be psychological entities that can be evaluated by means of psychophysical quantities, and in their evaluation it is ordinarily not necessary to pay attention to energy of wavelength shorter than 380 nm, nor longer than 770 nm because the eye is relatievly insensitive to such energy.

1.3. Tridimensionality of Color

If an observer with normal color vision attempts to adjust one element of his visual field whose color is under his control so that it matches a neighboring element, he will ultimately discover that three independent adjustments have to be made. If he is using the red, yellow, and blue paints frequently found in primary grade schools, only by chance will he obtain a match from a mixture of two of them. Even a brown color requires blue in addition to red and yellow. Within the color gamut of the three paints, an exact match for a given paint color is easily possible, but three is the irreducible minimum. Similarly, if he is trying to color-match one spot of light by shining several spotlights of different colors onto the same neighboring spot of a screen, he finds, in general, that either three lights of fixed spectral composition are required, or, if only two lights be added together, not only the amounts of both but also the spectral composition of at least one has to be adjustable. The same rule applies to rotary mixture on a sector disk; four sectors, giving three independent adjustments, are necessary and sufficient.

As the color vision of a normal observer is at least tridimensional, it follows that there must be at least three independent excitations in the optic nerve fibers corresponding to each patch of the visual field. Theories of color vision have been derived mostly from speculation as to the character of these excitations. It also follows that a color specification is expressible by three numbers. For normal observers three numbers are necessary; for partially color-blind observers only two numbers are necessary; and for totally color-blind observers only one is necessary.

In the examples given (paints, spotlights, sector disks), the observer by ajustment of three variables obtains a color match, that is, he has to set up a second stimulus equivalent to the first. Except by accident, however, the ternary or binary mixture does not match the unknown in spectral composition. In the usual case the mixture is equivalent to the unkonwn in color but not in spectral composition, and the unknown and the mixture are said to form a *metameric pair*. There are, however, degrees of difference in spectral composition. If one painted panel be matched by a mixture of red, yellow, and blue paints, the degree of metamerism is likely to be only moderate; but if the paint panel illuminated by daylight be matched by shining on a white card three spotlights each of which contains energy restricted to a narrow wavelength band (such as spectrum red, green, and blue), the degree of metamerism will ordinarily be large.

1.4. Metamerism and the Macular Pigment

Studies of extremely metameric pairs in which mixtures of two parts of the spectrum are set up to color-match other two-part spectrum mixtures have yielded our most valuable knowledge regarding the properties of the average normal eye [1, 44, 84, 99, 158]. An outstanding fact derived from observation of such metamers is that the center of the retina (fovea centralis) has somewhat different properties from that part of the retina immediately surrounding it; that is, a color match set up for the central 2 or 3 deg of the retina becomes an easily detectable mismatch if the eye be turned so as to allow the stimuli forming the metameric pair to affect a portion of the retina, say, 6 deg from the fovea. Furthermore, if the metamers are compared in large patches so as to subtend 6 deg or more at the eye of the observer, this mismatch causes a central spot, known as the Maxwell spot [98], to appear temporarily on either field even though the field is physically uniform. After the spot has faded away, change of fixation to the other field will renew the spot. This dependence of metamerism on the portion of the retina used arises chiefly from the existence of a spot of brownish or yellowish pigment irregularly covering and interpenetrating the central 3 or 4 deg of the normal retina; it is called the *macula lutea* or sometimes the *yellow spot* (see fig. 1). Figure 1 shows a horizontal cross section of the eye. Light enters the tear-film ff, passes through the cornea aa, the aqueous humor B, the pupil bb, the crystalline lens A, the vitreous humor C, and the macula p before reaching the retina i. The macular pigment acts as

[2] Figures in brackets indicate the literature references at the end of this paper.

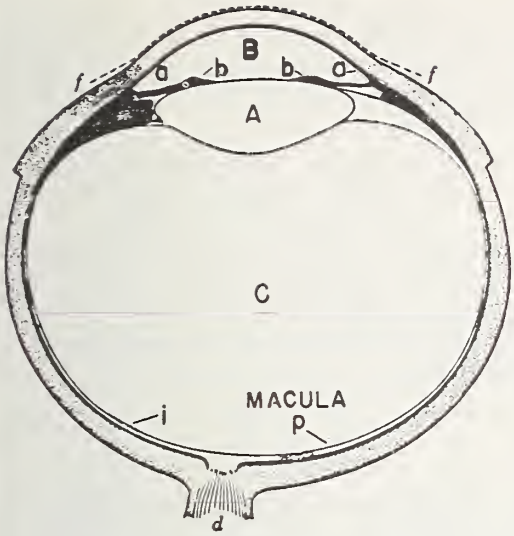


FIGURE 1. *Horizontal cross-section of the normal human eye.*

a selective filter interposed between the vitreous humor C and the retina i. Metamers set up for one normal observer usually fail to hold strictly for anyone else. This failure is ascribable to variations in amount of pigmentation of the eye media (cornea, lens, humors, macula), the macular pigment being one of the chief variables. The properties of the normal eye derived from a small-field study of these extreme metamers therefore refer only to the central 2 deg of the retina, and they refer to an hypothetical average eye. Nobody has been found whose eye differs so little from this average eye that the differences could not be detected. Practically speaking, therefore, nobody has an eye that is colorimetrically normal.

1.5. Grassman's Law

From a knowledge of spectral metamers, it has been possible to summarize concisely the properties of the average normal eye. This summary is made in accord with the principle known as Grassman's law [42] foreshadowed by Newton's laws of color mixture. If a light composed of known amounts of three components (called primaries) is equivalent in color to an unknown light, the three known amounts may be used as a color specification for this light. These amounts are called the *tristimulus values* of the color. Grassman's law states that, when equivalent lights are added to equivalent lights, the sums are equivalent. Thus, if an unknown spot of color were matched by shining on the same spot of a white screen two component spotlights of tristimulus values, X_1, Y_1, Z_1, and X_2, Y_2, Z_2, respectively, by Grassman's law, the tristimulus values, X, Y, Z, of the unknown spot of color would be simply:

$$\begin{aligned} X &= X_1 + X_2 \\ Y &= Y_1 + Y_2 \\ Z &= Z_1 + Z_2 \ . \end{aligned} \qquad (1)$$

Any beam of light, whether it originates from a self-luminous body or comes, by transmission, scattering, or reflection, from a nonself-luminous object, may be considered as made up of a large number of portions of the spectrum. The amounts of these various portions may be determined by spectrophotometry. The spectral values, $\overline{x}(\lambda)$, $\overline{y}(\lambda)$, $\overline{z}(\lambda)$, of each of these portions have been determined for a number of normal observers, and average values are given in table 1 in arbitrary units for a spectrum of unit spectral irradiance.

The principle expressed in Grassman's law has been established by repeated experiment over a wide middle range of retinal illuminances. It breaks down for very high retinal illuminances [159] that begin to approach those sufficient to do the eye permanent harm, and it breaks down if the illumination of the whole retina continues for several minutes to be so slight that vision by the retinal rods (twilight vision) intrudes significantly [84]. Between these two extremes, however, Grassman's law holds independently of the adaptive state of the eye. Thus, if two stimuli of different wavelength distributions of energy be found that are once responded to alike by the eye, they will be seen alike even after exposure of the eye to another stimulus sufficient to change considerably the appearance of the two equivalent stimuli. For example, if a portion of the spectrum near 640 nm (red) be superposed on a portion near 550 nm (yellowish green), it will be found possible to obtain the color of this combination from an intermediate portion of the spectrum, say, 590 nm (orange). If the retina of the eye be highly illuminated by light of wavelength near 640 nm, and its sensitivity to radiant flux of this wavelength region considerably reduced in this way, it is found that, although neither of the equivalent stimuli any longer appears orange, they still give identical colors; for example, they may yield identical yellows or identical greenish yellows. The eye thus cannot be trusted to yield the same color perception from a given stimulus; simultaneous and successive contrast affect it profoundly. But the eye is still a satisfactory null instrument and obeys Grassman's law.

By Grassman's law it is possible to test whether any two beams of light of differing spectral composition form a metameric pair. The condition for metamerism of two beams of light of spectral irradiance, E_1 and E_2, is that simultaneously:

$$\begin{aligned} \Sigma^{\infty}{}_{0}(E_1)_\lambda\overline{x}(\lambda)\Delta\lambda &= \Sigma^{\infty}{}_{0}(E_2)_\lambda\overline{x}(\lambda)\Delta\lambda \\ \Sigma^{\infty}{}_{0}(E_1)_\lambda\overline{y}(\lambda)\Delta\lambda &= \Sigma^{\infty}{}_{0}(E_2)_\lambda\overline{y}(\lambda)\Delta\lambda \\ \Sigma^{\infty}{}_{0}(E_1)_\lambda\overline{z}(\lambda)\Delta\lambda &= \Sigma^{\infty}{}_{0}(E_2)_\lambda\overline{z}(\lambda)\Delta\lambda \end{aligned} \qquad (2)$$

where $\overline{x}(\lambda)$, $\overline{y}(\lambda)$, and $\overline{z}(\lambda)$ characterize the observers' spectral responses. (These equations are written in accord with the CIE notation adopted in 1963. Symbols with subscripted λ, as $E\lambda$, indicate spectral concentration, while symbols with parenthetical λ, as $\overline{x}(\lambda)$, indicate other spectral relationships not critically dependent on choice of wavelength interval.) The wavelength interval,

$\Delta\lambda$, to be used in these summations depends upon the irregularity of the curve of spectral irradiance with wavelength; intervals of 20 nm are sometimes sufficient to yield a significant result; intervals of 10 nm often do; and intervals of 5 nm are usually sufficient except for discontinuous spectral distributions such as those characterising gaseous discharge lamps. De Kerf [27] has reported instances where 1-nm intervals have been required.

The summations of eq (2) form the tristimulus values of the color and are customarily given the symbols X, Y, Z, respectively, so that the condition for a color match would ordinarily be written:

$$\begin{aligned} X_1 &= X_2 \\ Y_1 &= Y_2 \\ Z_1 &= Z_2 \end{aligned} \tag{2a}$$

and would mean that the two colors are identical since their tristimulus values are identical. Thus, to match color one requires X_1 parts of the X primary, Y_1 parts of the Y primary, and Z_1 parts of the Z primary; and because to match color two requires the same amounts of the same primaries as does color one, the two colors are the same.

Any three lights may be used as primaries in a system of tristimulus color specifications, provided only that no one of them is equivalent to a combination of the other two. Tristimulus specifications X, Y, Z, expressed relative to one set of primaries, may be transformed into specifications R, G, B, relative to any other set, by transformation equations of the form:

$$\begin{aligned} R &= K_1X + K_2Y + K_3Z \\ G &= K_4X + K_5Y + K_6Z \\ B &= K_7X + K_8Y + K_9Z \ . \end{aligned} \tag{3}$$

The constants K_1 to K_9 may take on any arbitrary values, positive, negative, or zero, provided they are not such as to make one of the new primaries identical to a combination of the other two; that is, provided that:

$$\begin{vmatrix} K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 \\ K_7 & K_8 & K_9 \end{vmatrix} \neq 0 \ . \tag{3a}$$

As the exceptions that cause the determinant of the system to vanish are trivial, the choice of coordinate system is very wide. The primaries do not even have to correspond to physically realizable lights. Imaginary lights defined by spectral compositions having negative values for some parts of the spectrum are admissible, and indeed are preferred for routine colorimetry because by their use the computation of tristimulus values from spectrophotometric data is somewhat simplified.

2. The 1931 CIE Standard Colorimetric System and 1964 Supplement

2.1. General Principles

In 1931, the Commission Internationale de l'Eclairage (CIE)[3] recommended that all subsequent color data be expressed in terms of the same tristimulus system so that the results would be immediately comparable. The standard observer and coordinate system recommended [49, 65, 135, 143] is defined by the tristimulus values of the spectrum colors given in table 1a and plotted in figure 2. The supplementary observer for large fields, adopted by the CIE in 1964 [59] is given in table 1b. It will be noted that the primaries chosen are such that none of these tristimulus values is less than zero. It is further true that the green primary chosen, whose amounts are designated by Y, is such as to carry all of the luminosity, the other two primaries (red, blue) whose amounts are designated by X and Z, respectively, being unassociated with luminosity. Therefore, the values of $\bar{y}$ for the spectrum correspond to the standard luminosity function, and it is convenient and customary to express the Y value of a luminous area as its *luminance* (photometric brightness) in terms of some recognized unit (such as candles per square meter, millilambert, or foot-lambert). The Y value of an opaque specimen may be conveniently expressed as its *luminous reflectance* (ratio of reflected to incident luminous flux); and the Y value of a transmitting specimen is customarily put in terms of *luminous transmittance* (ratio of transmitted to incident luminous flux).

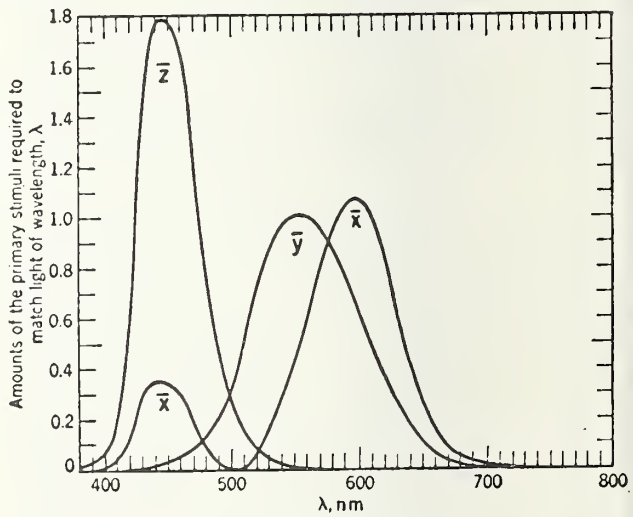


FIGURE 2. *Spectral tristimulus values according to the 1931 CIE standard observer.*

If, as is usual, light combinations are not the chief interest, it is convenient to substitute for the tristimulus values, X, Y, Z, the two ratios, $X/(X+Y+Z)$ and $Y/(X+Y+Z)$, combined with the luminous value, Y. The two ratios are known as *chromaticity coordinates*, x,y, because they serve to

[3] This was formerly called ICI for the initial letters of the English name of this commission.

TABLE 1(a). *The 1931 CIE standard observer (2°)*

Wave-length, nm	Spectral tristimulus values of equal-energy spectrum			Wave-length, nm	Spectral tristimulus values of equal-energy spectrum		
	$\bar{x}_2$	$\bar{y}_2$	$\bar{z}_2$		$\bar{x}_2$	$\bar{y}_2$	$\bar{z}_2$
380	0.0014	0.0000	0.0065	580	0.9163	0.8700	0.0017
385	.0022	.0001	.0105	585	.9786	.8163	.0014
390	.0042	.0001	.0201	590	1.0263	.7570	.0011
395	.0076	.0002	.0362	595	1.0567	.6949	.0010
400	.0143	.0004	.0679	600	1.0622	.6310	.0008
405	.0232	.0006	.1102	605	1.0456	.5668	.0006
410	.0435	.0012	.2074	610	1.0026	.5030	.0003
415	.0776	.0022	.3713	615	0.9384	.4412	.0002
420	.1344	.0040	.6456	620	.8544	.3810	.0002
425	.2148	.0073	1.0391	625	.7514	.3210	.0001
430	.2839	.0116	1.3856	630	.6424	.2650	.0000
435	.3285	.0168	1.6230	635	.5419	.2170	.0000
440	.3483	.0230	1.7471	640	.4479	.1750	.0000
445	.3481	.0298	1.7826	645	.3608	.1382	.0000
450	.3362	.0380	1.7721	650	.2835	.1070	.0000
455	.3187	.0480	1.7441	655	.2187	.0816	.0000
460	.2908	.0600	1.6692	660	.1649	.0610	.0000
465	.2511	.0739	1.5281	665	.1212	.0446	.0000
470	.1954	.0910	1.2876	670	.0874	.0320	.0000
475	.1421	.1126	1.0419	675	.0636	.0232	.0000
480	.0956	.1390	0.8130	680	.0468	.0170	.0000
485	.0580	.1693	.6162	685	.0329	.0119	.0000
490	.0320	.2080	.4652	690	.0227	.0082	.0000
495	.0147	.2586	.3533	695	.0158	.0057	.0000
500	.0049	.3230	.2720	700	.0114	.0041	.0000
505	.0024	.4073	.2123	705	.0081	.0029	.0000
510	.0093	.5030	.1582	710	.0058	.0021	.0000
515	.0291	.6082	.1117	715	.0041	.0015	.0000
520	.0633	.7100	.0782	720	.0029	.0010	.0000
525	.1096	.7932	.0573	725	.0020	.0007	.0000
530	.1655	.8620	.0422	730	.0014	.0005	.0000
535	.2257	.9149	.0298	735	.0010	.0004	.0000
540	.2904	.9540	.0203	740	.0007	.0003	.0000
545	.3597	.9803	.0134	745	.0005	.0002	.0000
550	.4334	.9950	.0087	750	.0003	.0001	.0000
555	.5121	1.0002	.0057	755	.0002	.0001	.0000
560	.5945	0.9950	.0039	760	.0002	.0001	.0000
565	.6784	.9786	.0027	765	.0001	.0000	.0000
570	.7621	.9520	.0021	770	.0001	.0000	.0000
575	.8425	.9154	.0018	775	.0000	.0000	.0000
580	.9163	.8700	.0017	780	.0000	.0000	.0000
				Totals -----	21.3713	21.3714	21.3715

TABLE 1(b). *The 1964 CIE supplementary observer (10°)*

Wave-length, nm	Spectral tristimulus values of equal-energy spectrum			Wave-length, nm	Spectral tristimulus values of equal-energy spectrum		
	$\bar{x}_{10}$	$\bar{y}_{10}$	$\bar{z}_{10}$		$\bar{x}_{10}$	$\bar{y}_{10}$	$\bar{z}_{10}$
380	0.0002	0.0000	0.0007	580	1.0142	0.8689	0.0000
385	.0007	.0001	.0029	585	1.0743	.8256	.0000
390	.0024	.0003	.0105	590	1.1185	.7774	.0000
395	.0072	.0008	.0323	595	1.1343	.7204	.0000
400	.0191	.0020	.0860	600	1.1240	.6583	.0000
405	.0434	.0045	.1971	605	1.0891	.5939	.0000
410	.0847	.0088	.3894	610	1.0305	.5280	.0000
415	.1406	.0145	.6568	615	0.9507	.4618	.0000
420	.2045	.0214	.9725	620	.8563	.3981	.0000
425	.2647	.0295	1.2825	625	.7549	.3396	.0000
430	.3147	.0387	1.5535	630	.6475	.2835	.0000
435	.3577	.0496	1.7985	635	.5351	.2283	.0000
440	.3837	.0621	1.9673	640	.4316	.1798	.0000
445	.3867	.0747	2.0273	645	.3437	.1402	.0000
450	.3707	.0895	1.9948	650	.2683	.1076	.0000
455	.3430	.1063	1.9007	655	.2043	.0812	.0000
460	.3023	.1282	1.7454	660	.1526	.0603	.0000
465	.2541	.1528	1.5549	665	.1122	.0441	.0000
470	.1956	.1852	1.3176	670	.0813	.0318	.0000
475	.1323	.2199	1.0302	675	.0579	.0226	.0000
480	.0805	.2536	0.7721	680	.0409	.0159	.0000
485	.0411	.2977	.5701	685	.0286	.0111	.0000
490	.0162	.3391	.4153	690	.0199	.0077	.0000
495	.0051	.3954	.3024	695	.0138	.0054	.0000
500	.0038	.4608	.2185	700	.0096	.0037	.0000
505	.0154	.5314	.1592	705	.0066	.0026	.0000
510	.0375	.6067	.1120	710	.0046	.0018	.0000
515	.0714	.6857	.0822	715	.0031	.0012	.0000
520	.1177	.7618	.0607	720	.0022	.0008	.0000
525	.1730	.8233	.0431	725	.0015	.0006	.0000
530	.2365	.8752	.0305	730	.0010	.0004	.0000
535	.3042	.9238	.0206	735	.0007	.0003	.0000
540	.3768	.9620	.0137	740	.0005	.0002	.0000
545	.4516	.9822	.0079	745	.0004	.0001	.0000
550	.5298	.9918	.0040	750	.0003	.0001	.0000
555	.6161	.9991	.0011	755	.0002	.0001	.0000
560	.7052	.9973	.0000	760	.0001	.0000	.0000
565	.7938	.9824	.0000	765	.0001	.0000	.0000
570	.8787	.9555	.0000	770	.0001	.0000	.0000
575	.9512	.9152	.0000	775	.0000	.0000	.0000
580	1.0142	.8689	.0000	780	.0000	.0000	.0000
				Totals -----	23.3294	23.3323	23.3343

TABLE 2(a). *Chromaticity coordinates of the spectrum colors (2°)*

Wave-length, nm	Chromaticity coordinates			Wave-length, nm	Chromaticity coordinates		
	x_2	y_2	z_2		x_2	y_2	z_2
380	0.1741	0.0050	0.8209	580	0.5125	0.4866	0.0009
385	.1740	.0050	.8210	585	.5448	.4544	.0008
390	.1738	.0049	.8213	590	.5752	.4242	.0006
395	.1736	.0049	.8215	595	.6029	.3965	.0006
400	.1733	.0048	.8219	600	.6270	.3725	.0005
405	.1730	.0048	.8222	605	.6482	.3514	.0004
410	.1726	.0048	.8226	610	.6658	.3340	.0002
415	.1721	.0048	.8231	615	.6801	.3197	.0002
420	.1714	.0051	.8235	620	.6915	.3083	.0002
425	.1703	.0058	.8239	625	.7006	.2993	.0001
430	.1689	.0069	.8242	630	.7079	.2920	.0001
435	.1669	.0086	.8245	635	.7140	.2859	.0001
440	.1644	.0109	.8247	640	.7190	.2809	.0001
445	.1611	.0138	.8251	645	.7230	.2770	.0000
450	.1566	.0177	.8257	650	.7260	.2740	.0000
455	.1510	.0227	.8263	655	.7283	.2717	.0000
460	.1440	.0297	.8263	660	.7300	.2700	.0000
465	.1355	.0399	.8246	665	.7311	.2689	.0000
470	.1241	.0578	.8181	670	.7320	.2680	.0000
475	.1096	.0868	.8036	675	.7327	.2673	.0000
480	.0913	.1327	.7760	680	.7334	.2666	.0000
485	.0687	.2007	.7306	685	.7340	.2660	.0000
490	.0454	.2950	.6596	690	.7344	.2656	.0000
495	.0235	.4127	.5638	695	.7346	.2654	.0000
500	.0082	.5384	.4534	700	.7347	.2653	.0000
505	.0039	.6548	.3413	705	.7347	.2653	.0000
510	.0139	.7502	.2359	710	.7347	.2653	.0000
515	.0389	.8120	.1491	715	.7347	.2653	.0000
520	.0743	.8338	.0919	720	.7347	.2653	.0000
525	.1142	.8262	.0596	725	.7347	.2653	.0000
530	.1547	.8059	.0394	730	.7347	.2653	.0000
535	.1929	.7816	.0255	735	.7347	.2653	.0000
540	.2296	.7543	.0161	740	.7347	.2653	.0000
545	.2658	.7243	.0099	745	.7347	.2653	.0000
550	.3016	.6923	.0061	750	.7347	.2653	.0000
555	.3373	.6589	.0038	755	.7347	.2653	.0000
560	.3731	.6245	.0024	760	.7347	.2653	.0000
565	.4087	.5896	.0017	765	.7347	.2653	.0000
570	.4441	.5547	.0012	770	.7347	.2653	.0000
575	.4788	.5202	.0010	775	.7347	.2653	.0000
580	.5125	.4866	.0009	780	.7347	.2653	.0000

TABLE 2(b). *Chromaticity coordinates of the spectrum colors (10°)*

Wave-length, nm	Chromaticity coordinates			Wave-length, nm	Chromaticity coordinates		
	x_{10}	y_{10}	z_{10}		x_{10}	y_{10}	z_{10}
0.380	0.1813	0.0197	0.7990	0.580	0.5386	0.4614	0.0000
.385	.1809	.0195	.7996	.585	.5654	.4346	.0000
.390	.1803	.0194	.8003	.590	.5900	.4100	.0000
.395	.1795	.0190	.8015	.595	.6116	.3884	.0000
.400	.1784	.0187	.8029	.600	.6306	.3694	.0000
.405	.1771	.0184	.8045	.605	.6471	.3529	.0000
.410	.1755	.0181	.8064	.610	.6612	.3388	.0000
.415	.1732	.0178	.8090	.615	.6731	.3269	.0000
.420	.1706	.0179	.8115	.620	.6827	.3173	.0000
.425	.1679	.0187	.8134	.625	.6898	.3102	.0000
.430	.1650	.0203	.8147	.630	.6955	.3045	.0000
.435	.1622	.0225	.8153	.635	.7010	.2990	.0000
.440	.1590	.0257	.8153	.640	.7059	.2941	.0000
.445	.1554	.0300	.8146	.645	.7103	.2898	.0000
.450	.1510	.0364	.8126	.650	.7137	.2863	.0000
.455	.1459	.0452	.8088	.655	.7156	.2844	.0000
.460	.1389	.0589	.8022	.660	.7168	.2832	.0000
.465	.1295	.0779	.7926	.665	.7179	.2821	.0000
.470	.1152	.1090	.7758	.670	.7187	.2813	.0000
.475	.0957	.1591	.7452	.675	.7193	.2807	.0000
.480	.0728	.2292	.6980	.680	.7198	.2802	.0000
.485	.0452	.3275	.6273	.685	.7200	.2800	.0000
.490	.0210	.4401	.5389	.690	.7202	.2798	.0000
.495	.0073	.5625	.4302	.695	.7203	.2797	.0000
.500	.0056	.6745	.3199	.700	.7204	.2796	.0000
.505	.0219	.7526	.2256	.705	.7203	.2797	.0000
.510	.0495	.8023	.1482	.710	.7202	.2798	.0000
.515	.0850	.8170	.0980	.715	.7201	.2799	.0000
.520	.1252	.8102	.0646	.720	.7199	.2801	.0000
.525	.1664	.7922	.0414	.725	.7197	.2803	.0000
.530	.2071	.7663	.0267	.730	.7195	.2806	.0000
.535	.2436	.7399	.0165	.735	.7192	.2808	.0000
.540	.2786	.7113	.0101	.740	.7189	.2811	.0000
.545	.3132	.6813	.0055	.745	.7186	.2814	.0000
.550	.3473	.6501	.0026	.750	.7183	.2817	.0000
.555	.3812	.6182	.0007	.755	.7180	.2820	.0000
.560	.4142	.5858	.0000	.760	.7176	.2824	.0000
.565	.4469	.5531	.0000	.765	.7172	.2828	.0000
.570	.4790	.5210	.0000	.770	.7169	.2831	.0000
.575	.5096	.4904	.0000	.775	.7165	.2835	.0000
.580	.5386	.4614	.0000	.780	.7161	.2839	.0000

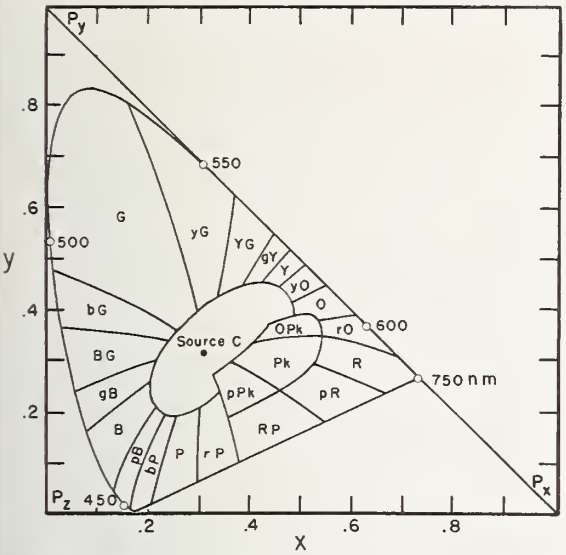


FIGURE 3. *The (x,y)-chromaticity diagram, showing the spectrum locus and the purple boundary.*

Wavelength is indicated in millimicrons. The hue names are those proposed by Kelly (77).

specify the chromatic aspect of the light. The analogous ratio, $Z/(X+Y+Z)$, is also known as a chromaticity coordinate, z, but only two of the three coordinates, x, y, z, give independent information since by definition the sum of all three is unity. Table 2a gives the chromaticity coordinates, x, y, z, of the spectrum colors for 2° and table 2b for 10° fields [59]. Figure 3 shows the points representing the spectrum colors in the (x, y)-chromaticity diagram. This diagram is also known as a *Maxwell triangle* because of Maxwell's first use of such a diagram [98]. Furthermore, it has aptly been called a *mixture diagram* because it indicates in a very simple way the chromaticity of the color resulting from the additive combination of any two lights. The point representing this chromaticity is found on the straight line connecting the points representing two lights. The primary lights are represented by points at the corners of a triangle, and every point within the triangle represents the chromaticity of a mixture of the primary lights whose proportions are indicated by the chromaticity coordinates, x, y, z. The spectrum colors are shown by a smooth curve known as the spectrum locus. A few points on this locus are identified by wavelength in nanometers. It will be noted from figure 3 that the spectrum locus is substantially straight from 540 nm to the long-wave extreme. This means that the standard observer would find binary mixtures of, say, 540 nm with 640 nm, closely equivalent to some intermediate portion of the spectrum. But the spectrum locus from 540 nm to the short-wave extreme is curved outward. This means that for the standard observer a binary mixture of 540 nm with, say, 440 nm would differ importantly in chromaticity from the intermediate parts of the spectrum. By drawing straight lines through any central point (such as $x = y = 1/3$, representing the so-called equal-energy stimulus) and extending them until they cut the spectrum locus, we may find the spectral complementaries relative to a stimulus represented by that point; that is, we may find the two parts of the spectrum that, when combined in proper proportions, will for the standard observer be equivalent to the central stimulus.

The straight line in figure 3 joining the extremes of the spectrum locus represents the chromaticities of the mixtures of the two extremes of the visible spectrum. The area bounded by the closed curve made up of the spectrum locus and this straight line is the locus of all physically realizable chromaticities. Note that the points representing the primaries of the CIE coordinate system, the apices of the triangle ($x = 1, y = z = 0$; $y = 1, x = z = 0$; $z = 1, x = y = 0$), all fall outside this area; that is, the primaries are imaginary. Note also that both the X and Z primaries fall on the line $y = 0$, which is unassociated with luminosity and is known as the *alychne* or *lightless line*. The short-wave extreme of the spectrum locus comes close to this line; this means that, although it has the power to elicit in the standard observer a considerable X and Z response, resulting in a vivid bluish purple color, radiant flux of wavelength 380 to 420 nm is almost unassociated with luminosity. The areas in figure 3 corresponding to common color designations for lights are those proposed by Kelly [77] and will be discussed later.

2.2. Standard Sources

At the time of setting up the standard observer and coordinate system, the International Commission on Illumination [135], Commission International de l'Eclairage (CIE), recommended use of three standard sources for colorimetry; source A, representative of gas-filled incandescent lamps; source B, representative of noon sunlight; and source C, representative of average daylight such as that from a completely overcast sky. Source A is an incandescent lamp operated at a color temperature of 2854 °K, on the international temperature scale ($c_2 = 14{,}380$). Source B is obtained by using this same lamp in combination with a two-cell Davis-Gibson liquid filter designed to give a color temperature of about 5000 °K. Source C is obtained similarly and results in a source of correlated color temperature about 6800 °K. These sources are recommended for general use, or whenever there is no special reason for using some other source. Table 3 gives the relative spectral irradiance of Sources A, B, C, D_{5500}, D_{6500}, and D_{7500}. Sources D_{5500}, D_{6500}, D_{7500} represent several phases of daylight, closely represented by the subscripted correlated color temperatures. Tables 4a and 4b give computation forms for evaluation of the colors of non-self-luminous specimens that transmit, scatter, or reflect incident light for the 2° standard ob-

TABLE 3. *Relative spectral irradiance E of the six standard sources, A, B, C, D_{5500}, D_{6500}, and D_{7500}*

Wave-length nm	E_A	E_B	E_C	E_{5500}	E_{6500}	E_{7500}	Wave-length nm	E_A	E_B	E_C	E_{5500}	E_{6500}	E_{7500}
380	9.79	22.40	33.00	32.60	50.00	66.70	580	114.44	101.00	97.80	97.70	95.80	94.20
385	10.90	26.85	39.92	33.35	52.30	68.35	585	118.08	100.07	95.43	94.55	92.25	90.60
390	12.09	31.30	47.40	38.10	54.60	70.00	590	121.73	99.20	93.20	91.40	88.70	87.00
395	13.36	36.18	55.17	49.50	68.70	85.95	595	125.39	98.44	91.22	92.90	89.35	87.10
400	14.71	41.30	63.30	60.90	82.80	101.90	600	129.04	98.00	89.70	94.40	90.00	87.20
405	16.15	46.62	71.81	64.75	87.15	106.90	605	132.70	98.08	88.83	94.75	89.80	86.65
410	17.68	52.10	80.60	68.60	91.50	111.90	610	136.34	98.50	88.40	95.10	89.60	86.10
415	19.29	57.70	89.53	70.10	92.45	112.35	615	139.99	99.06	88.19	94.65	88.65	84.85
420	21.00	63.20	98.10	71.60	93.40	112.80	620	143.62	99.70	88.10	94.20	87.70	83.60
425	22.79	68.37	105.80	69.75	90.05	107.95	625	147.23	100.36	88.06	92.30	85.50	81.15
430	24.67	73.10	112.40	[illegible]	86.70	[illegible]	630	150.83	100.00	88.00	90.40	83.30	78.70
435	26.64	77.31	117.75	76.75	95.80	112.15	635	154.42	101.56	87.86	91.35	83.50	78.55
440	28.70	80.80	121.50	85.60	104.90	[illegible]	640	157.98	102.20	87.80	91.30	83.70	78.40
445	30.85	83.44	123.45	91.80	110.95	127.25	645	161.51	103.05	87.99	90.60	81.85	76.60
450	33.09	85.40	124.00	98.00	117.00	133.30	650	165.03	103.90	88.30	88.90	80.00	74.80
455	35.41	86.88	123.60	99.25	114.40	132.85	655	168.51	104.59	88.20	89.60	80.10	74.55
460	37.82	88.30	123.10	100.50	117.80	132.40	660	171.96	105.00	87.90	90.30	80.20	74.30
465	40.30	90.08	123.30	100.20	116.35	129.85	665	175.38	105.08	87.22	92.10	81.25	74.85
470	42.87	92.00	123.80	99.90	114.90	127.30	670	178.77	104.90	86.30	93.90	82.30	75.40
475	45.52	93.75	124.09	101.30	115.40	127.05	675	182.12	104.55	85.30	91.95	80.30	73.50
480	48.25	95.20	123.90	102.70	115.90	126.80	680	184.43	103.90	84.00	90.00	78.30	71.60
485	51.04	96.23	122.92	100.40	112.35	122.30	685	188.70	102.84	82.21	84.85	74.00	69.75
490	53.91	96.50	120.70	98.10	108.80	117.80	690	191.93	101.60	80.20	79.70	69.70	63.90
495	56.85	95.71	116.90	99.40	109.10	117.20	695	195.12	100.38	78.24	81.25	70.65	64.50
500	59.86	94.20	112.10	100.70	109.40	116.60	700	198.26	99.10	76.30	82.80	71.60	65.10
505	62.93	92.37	106.98	100.70	108.60	115.15	705	201.36	97.70	74.36	83.80	72.95	66.60
510	66.06	90.70	102.30	100.70	107.80	113.70	710	204.41	96.20	72.40	84.80	74.30	68.10
515	69.25	89.65	98.81	100.35	106.30	111.20	715	207.41	94.60	70.40	77.50	67.95	62.25
520	72.50	89.50	96.90	100.00	104.80	108.70	720	210.36	92.90	68.30	70.20	61.60	56.40
525	75.79	90.43	96.78	102.10	106.25	109.55	725	213.26	91.10	66.30	74.75	65.75	60.30
530	79.13	92.20	98.00	104.20	107.70	110.40	730	216.12	89.40	64.40	79.30	69.90	64.20
535	82.52	94.46	99.94	103.15	106.05	108.35	735	218.92	88.00	62.80	82.15	72.50	66.70
540	85.95	96.90	102.10	102.10	104.40	106.30	740	221.66	86.90	61.50	85.00	75.10	69.20
545	89.41	99.16	103.95	102.55	104.20	105.60	745	224.36	85.90	60.20	78.45	69.35	63.90
550	92.91	101.00	105.20	103.00	104.00	104.90	750	227.00	85.20	59.20	71.90	63.60	58.60
555	96.44	102.20	105.67	101.50	102.00	102.45	755	229.58	84.80	58.50	62.35	55.00	50.60
560	100.00	102.80	105.30	100.00	100.00	100.00	760	232.11	84.70	58.10	52.80	46.40	42.60
565	103.58	102.92	104.11	98.60	98.15	97.80	765	234.59	84.90	58.00	64.35	56.60	52.00
570	107.18	102.60	102.30	97.20	96.30	95.60	770	237.01	85.40	58.20	75.90	66.80	61.40
575	110.80	101.90	100.15	97.45	96.05	94.90	775	239.37	86.10	58.50	73.85	65.10	59.85
580	114.44	101.00	97.80	97.70	95.80	94.20	780	241.67	87.00	59.10	71.80	63.40	58.30

TABLE 4a. *Computation form*

CIE coordinates (2°) Source A (2854 °K.)

Sample_____ Source of Trans. Data_____

Submitted by_____ _____

λ (nm)	$E\alpha\bar{x}_2$	$E\alpha\bar{y}_2$	$E\alpha\bar{z}_2$	τ	$\tau E\alpha\bar{x}_2$	$\tau E\alpha\bar{y}_2$	$\tau E\alpha\bar{z}_2$
380	1		6	0.			
90	5		23	.			
400	19	1	93	.			
10	71	2	340	.			
20	262	8	1256	.			
30	649	27	3167	.			
40	926	61	4647	.			
450	1031	117	5435	.			
60	1019	210	5851	.			
70	776	362	5116	.			
80	428	622	3636	.			
90	160	1039	2324	.			
500	27	1792	1509	.			
10	57	3080	969	.			
20	425	4771	525	.			
30	1214	6322	309	.			
40	2313	7600	162	.			
550	3732	8568	75	.			
60	5510	9222	36	.			
70	7571	9457	21	.			
80	9719	9228	18	.			
90	11579	8540	12	.			
600	12704	7547	10	.			
10	12669	6356	4	.			
20	11373	5071	3	.			
30	8980	3704		.			
40	6558	2562		.			
650	4336	1637		.			
60	2628	972		.			
70	1448	530		.			
80	804	292		.			
90	404	146		.			
700	209	75		.			
10	110	40		.			
20	57	19		.			
30	28	10		.			
40	14	6		.			
750	6	2		.			
60	4	2		.			
70	2			.			
Sums x_w, y_w, z_w	109828 0.4476	100000 0.4075	35547 0.1449	Sums $S = X+Y+Z$	$X =$	$Y =$	$Z =$
x, y, and z ($x = X/S$, $y = Y/S$, $z = Z/S$)							

Planck 2854 °K., $c_2 = 14{,}380$
Planck 2848 °K., $c_2 = 14{,}350$

Computed by_____ Checked by_____

TABLE 4b. *Computation form*

CIE coordinates Source C (Davis-Gibson, 2854 to 6800 °K.)

Sample_____ Source of Trans. Data_____

Submitted by_____ _____

λ (nm)	$E_c\bar{x}_2$	$E_c\bar{y}_2$	$E_c\bar{z}_2$	τ	$\tau E_c\bar{x}_2$	$\tau E_c\bar{y}_2$	$\tau E_c\bar{z}_2$
380	4		20	0.			
90	19		89	.			
400	85	2	404	.			
10	329	9	1570	.			
20	1238	37	5949	.			
30	2997	122	14628	.			
40	3975	262	19938	.			
450	3915	443	20638	.			
60	3362	694	19299	.			
70	2272	1058	14972	.			
80	1112	1618	9461	.			
90	363	2358	5274	.			
500	52	3401	2864	.			
10	89	4833	1520	.			
20	576	6462	712	.			
30	1523	7934	388	.			
40	2785	9149	195	.			
550	4282	9832	86	.			
60	5880	9841	39	.			
70	7322	9147	20	.			
80	8417	7992	16	.			
90	8984	6627	10	.			
600	8949	5316	7	.			
10	8325	4176	2	.			
20	7070	3153	2	.			
30	5309	2190		.			
40	3693	1443		.			
650	2349	886		.			
60	1361	504		.			
70	708	259		.			
80	369	134		.			
90	171	62		.			
700	82	29		.			
10	39	14		.			
20	19	6		.			
30	8	3		.			
40	4	2		.			
750	2	1		.			
60	1	1		.			
70	1			.			
Sums	98041	100000	118103	Sums	$X =$	$Y =$	$Z =$
x_w, y_w, z_w	0.3101	0.3163	0.3736	$S = X+Y+Z$			

x, y, and z
($x = X/S$, $y = Y/S$, $z = Z/S$)

Planck 2854 °K., $c_2 = 14{,}380$
Planck 2848 °K., $c_2 = 14{,}350$

Computed by_____ Checked by_____

TABLE 4c. *10° Distribution coefficients for sources S_A and S_C*

	For S_A			For S_C		
λ (nm)	$E_A\bar{x}_{10}$	$E_A\bar{y}_{10}$	$E_A\bar{z}_{10}$	$E_C\bar{x}_{10}$	$E_C\bar{y}_{10}$	$E_C\bar{z}_{10}$
380			1	1		2
90	3		11	9	1	43
400	25	3	111	103	11	463
10	132	14	605	581	60	2672
20	377	40	1795	1708	179	8122
30	682	83	3368	3011	370	14865
40	968	156	4962	3969	643	20349
450	1078	260	5802	3914	945	21058
60	1005	426	5802	3168	1343	18292
70	737	698	4965	2062	1952	13887
80	341	1076	3274	840	2675	8144
90	76	1607	1968	167	3484	4268
500	20	2424	1150	37	4398	2085
10	218	3523	650	327	5284	976
20	750	4854	387	971	6285	501
30	1644	6086	212	1973	7302	255
40	2847	7267	104	3275	8362	119
550	4326	8099	33	4744	8882	36
60	6198	8766		6322	8941	
70	8277	9002		7653	8322	
80	10201	8740		8444	7235	
90	11967	8317		8874	6168	
600	12748	7466		8583	5027	
10	12349	6327		7756	3974	
20	10809	5026		6422	2986	
30	8583	3758		4851	2124	
40	5992	2496		3226	1344	
650	3892	1561		2014	808	
60	2306	911		1142	451	
70	1277	499		598	233	
80	666	259		293	114	
90	336	130		136	53	
700	167	64		62	24	
10	83	33		28	11	
20	40	15		13	4	
30	19	8		5	3	
40	10	4		3	1	
750	6	2		2	1	
60	2			1		
70	2			1		
	111,159	100,000	32,200	97,298	100,000	116,137

server. Table 4a refers to Source A; table 4b, to Source C. Source B is relatively little used except in Great Britain. Table 4c gives corresponding values for the 10° observer.

2.3. Spectrophotometric Colorimetry

The fundamental nature of the tristimulus specification of color permits it to be used as a common denominator by means of which colorimeters involving color standards of glass, plastic, or solutions, or systems of material color standards, transparent and opaque, may be intercompared. In order to demonstrate how the CIE standard observer and coordinate system may be used for this purpose, four printing inks, red purple, greenish yellow, greenish blue, and blue, have been evaluated, and the steps are reproduced here in detail. Figure 4 shows spectral reflectances of these four printing inks obtained on a recording spectrophotometer. Table 5a gives the spectral reflectances read from the originals of the curves of figure 4. These reflectances apply to the specimens measured except for small wavelength-scale and photometric-scale corrections which have not been applied. Table 5b gives for the greenish yellow specimen the products indicated on the form for computation of luminous reflectance, Y, and chromaticity coordinates, x,y,z, under standard source C; see table 4b. The sums of these products are the tristimulus values, X,Y,Z. The luminous reflectance is found as $Y/100{,}000$; and the chromaticity coordinates, x,y,z, are found by dividing X,Y, and Z, respectively, by the corresponding sum, $X + Y + Z$. Table 5c lists these results for all four printing-ink specimens. Figure 5 is the (x,y)-chromaticity diagram on which have been plotted large dots to represent these chromaticity coordinates, x,y.

Comparison of figure 5 with figure 3 shows that the chromaticity points of the four printing-ink specimens correspond to the hue designations red purple, greenish yellow, and blue. This accords well with the color designations found by visual inspection of the specimens. Furthermore, it will be noted that one of the blues is greener than the other. The position of the chromaticity point for the greener-ink color is in accord with the greener hue of this ink. Note also that the greenish yellow is much lighter than the red purple or either of the blues; this accords with the luminous reflectance determinations (compare 0.74 with 0.221, 0.242, and 0.246 in table 5c).

2.4. Short-Cuts and Automatic Computation

The labor of computing X,Y,Z, or Y,x,y, corresponding to pairs of spectrophotometric curves to see how the colors of the corresponding specimens compare is considerably great. Often the degree of metamerism exhibited by the pair is sufficiently small that the comparison can be made directly from the curves themselves, and much product-control work can be handled in this way. There is still frequent need, in the establishment of color standards and tests for conformity to those standards, to compute the tristimulus values, X,Y,Z, by a short-cut method, the selected ordinate method, to reduce spectrophotometric data.

In this method the ordinates of the spectrophotometric curve are read at a series of selected wavelengths different for each source. Instead of multiplying by the tristimulus values of the spectrum of the source, $(E\bar{x})_\lambda$, $(E\bar{y})_\lambda$, $(E\bar{z})_\lambda$, the selected ordinates are spaced proportionately closer in the wavelength regions where the tristimulus values are higher, and the corresponding readings of spectral reflectance are simply added. Tables 6 gives [21, 49] selected ordinates for source A (incandescent lamp light) and source C (average daylight). Table 7 gives the spectral reflectances of the greenish yellow printing-ink specimen read from figure 4b for the selected ordinates for source C together with the sums of these spectral reflectances, both for ten ordinates and for thirty. It will be noted that, after applying the multiplying factors listed in table 6, the tristimulus values, X,Y,Z, for the greenish yellow print-

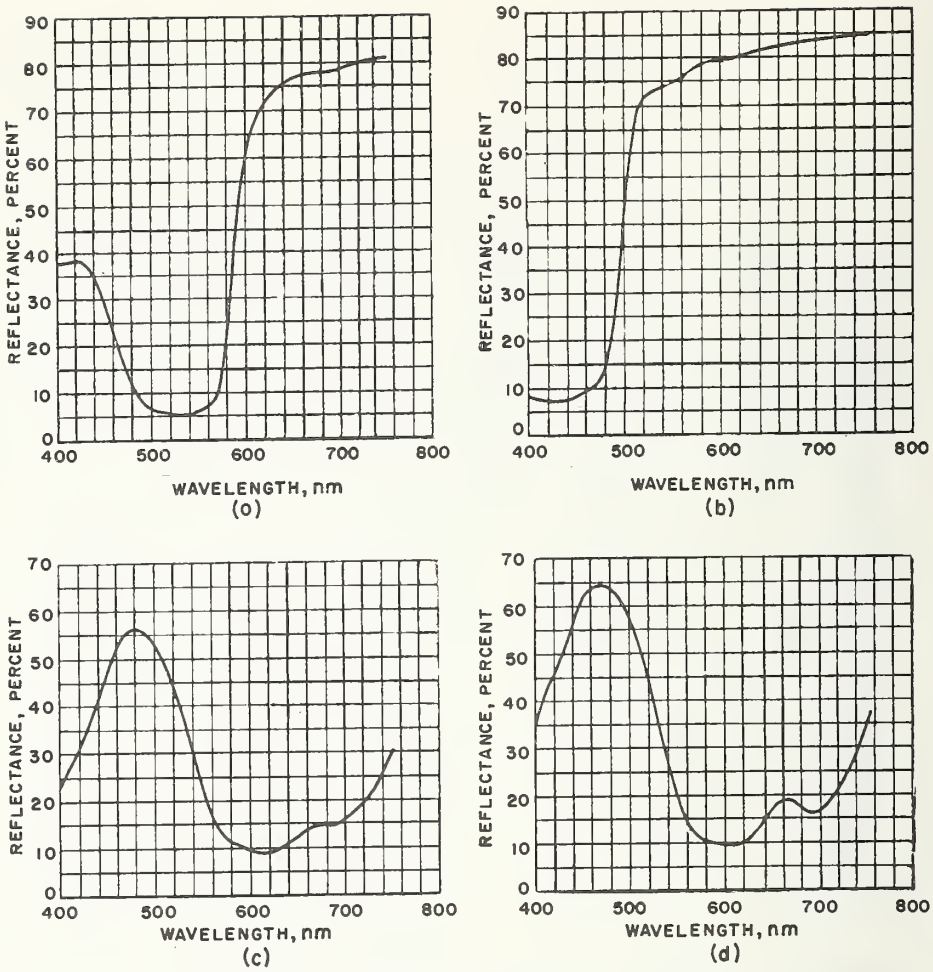


FIGURE 4. *Spectral reflectance of printing-ink specimens: (a) red-purple; (b) greenish yellow; (c) greenish blue; (d) blue.*

ing-ink specimen are found again to a close approximation (compare 0.630, 0.704, and 0.145 from table 7 with 0.631, 0.704, and 0.145, respectively, from table 5c.

Ten selected ordinates sometimes give significant information (see table 7); thirty selected ordinates are often sufficient (as above); and one hundred selected ordinates are sufficient for all but a few very irregular spectral distributions (such as produced by gaseous discharge tubes). These wavelengths are available for many sources in other publications [13, 21, 49]. Nickerson [114] and De Kerf [27] have published studies of the reliability of the selected-ordinate method of computation.

Analog and digital techniques have been developed for use with automatic computing devices to abbreviate the labor of computation required by spectrophotometric colorimetry. In the application of the analog technique [25] data are sent directly from the spectrophotometer to the computer without wavelength-scale and photometric-scale corrections. In one application of the digital technique [12, 95] spectral data are punched directly on cards and fed into the computer, while in another [75] the spectral data are corrected and then punched on cards and fed into the computer.

2.5. Visual Tristimulus Colorimeters

Tristimulus values, X,Y,Z, may be obtained by direct comparison of the unknown light with an optical mixture of three primary lights in a divided photometric field. Since the primaries of the CIE standard colorimetric coordinate system are imaginery, such a tristimulus colorimeter cannot be made to read directly. It must be calibrated by measurements of four known stimuli, and then may yield tristimulus values, X,Y,Z, by a transformation the reverse of that indicated in eq. (3). Since the color matches set up in a tristimulus colorimeter designed to cover any substantial part of all possible colors with a single set of primaries exhibit serious metamerism, the field has to be relatively small, subtrending about 2 deg at the observer's eye. This restriction to a small

TABLE 5a. *Spectral reflectances of four printing-ink specimens*

Wave-length, nm	Spectral reflectance relative to magnesium oxide			
	Red purple	Greenish yellow	Greenish blue	Blue
380	0.375*	0.091*	0.150*	0.230*
390	.375*	.089*	.187*	.293*
400	.376	.085	.228	.354
410	.379	.079	.269	.415
420	.381	.077	.306	.458
430	.373	.076	.353	.505
440	.345	.077	.407	.563
450	.295	.086	.467	.616
460	.235	.095	.520	.639
470	.174	.108	.552	.645
480	.120	.145	.560	.635
490	.083	.250	.548	.608
500	.066	.445	.523	.568
510	.061	.635	.483	.508
520	.057	.708	.432	.438
530	.054	.725	.363	.353
540	.055	.733	.292	.272
550	.062	.743	.220	.198
560	.071	.752	.162	.145
570	.095	.768	.128	.117
580	.220	.782	.113	.106
590	.440	.787	.102	.102
600	.597	.790	.093	.098
610	.676	.793	.088	.097
620	.715	.798	.088	.103
630	.739	.803	.098	.122
640	.756	.809	.110	.147
650	.768	.814	.124	.172
660	.776	.818	.136	.187
670	.780	.822	.145	.186
680	.782	.824	.147	.172
690	.783	.827	.149	.162
700	.788	.829	.160	.169
710	.794	.832	.177	.192
720	.799	.833	.196	.221
730	.805	.835	.218	.256
740	.809	.836	.258	.304
750	.812	.837	.298	.362
760	.815*	.838*	.338*	.422*
770	.817*	.839*	.375*	.484*

*Extrapolated.

angular size of field severely limits the precision of setting compared to what is possible by direct comparison of large specimens in daylight. Furthermore, the metamerism also prevents one normal observer from getting the same reading as another except by accident. If a reasonable approximation to the standard values of X,Y, and Z is to be assured, either the readings of a group of five or ten observers must be averaged, or a color standard yielding a spectral composition similar to that of the unknown specimen must be used. Because of industrial interest in large-field color matching Stiles and Burch [146] and Speranskaya [144] determined the color-matching functions for 10°-field viewing. In this determination either the observers were instructed to ignore the Maxwell spot [73, 98, 103, 104, 156] or it was masked. The color-matching functions thus found are significantly different from the 2°-field functions of the 1931 CIE Standard Observer. The difference is chiefly that expected from the removal of an intervening yellow filter, the macular pigment, from the field of view.

It will be seen that tristimulus colorimeters give only poor information regarding the unknown specimen. Their application to product-control problems is negligible, but because of the ease of calibration and simplicity of the theory they are very useful research tools. Tristimulus colorime-

TABLE 5b. *Computation of tristimulus values, X ,Y, Z, and chromaticity coordinates, x, y, for the greenish-yellow printing-ink specimen under source C*

(The computation form given as table 4b has been used.)

Wave-length, nm	Reflect-ance (ρ)	$\rho E_c\bar{x}$	$\rho E_c\bar{y}$	$\rho E_c\bar{z}$
380	0.091	0		2
390	.089	2		8
400	.085	7	0	34
410	.079	26	1	124
420	.077	95	3	458
430	.076	228	9	1112
440	.077	306	20	1535
450	.086	337	38	1775
460	.095	319	66	1833
470	.108	245	114	1617
480	.145	161	235	1372
490	.250	91	590	1319
500	.445	23	1513	1274
510	.635	57	3069	965
520	.708	408	4575	504
530	.725	1104	5752	281
540	.733	2041	6706	143
550	.743	3182	7305	64
560	.752	4422	7400	29
570	.768	5623	7025	15
580	.782	6582	6250	13
590	.787	7070	5215	8
600	.790	7070	4200	6
610	.793	6602	3312	2
620	.798	5642	2516	2
630	.803	4263	1759	0
640	.809	2988	1167	
650	.814	1912	721	
660	.818	1113	412	
670	.822	582	213	
680	.824	304	110	
690	.827	141	51	
700	.829	68	24	
710	.832	32	12	
720	.833	16	5	
730	.835	7	3	
740	.836	3	2	
750	.837	2	1	
760	.838	1	1	
770	.839	1	0	
Tristimulus values	X,Y,Z	63,076	70,395	14,495
Chromaticity coordinates	x,y,z	0.4263	0.4758	0.0980

TABLE 5c. *Tristimulus values, X, Y, Z, under source C, luminous reflectance relative to magnesium oxide, Y/Y_0, and chromaticity coordinates, x, y, computed from the spectral reflectance of four printing-ink specimens as in table 5b*

Hue designation of specimen	Tristimulus values, arbitrary units			Luminous reflectance, Y/Y_0	Chromaticity coordinates	
	X	Y	Z		x	y
Red purple	39788	22124	30570	0.221	0.430	0.239
Greenish yellow	63076	70395	14495	.704	.426	.476
Greenish blue	19003	24245	54529	.242	.194	.248
Blue	21948	24633	69010	.246	.190	.213

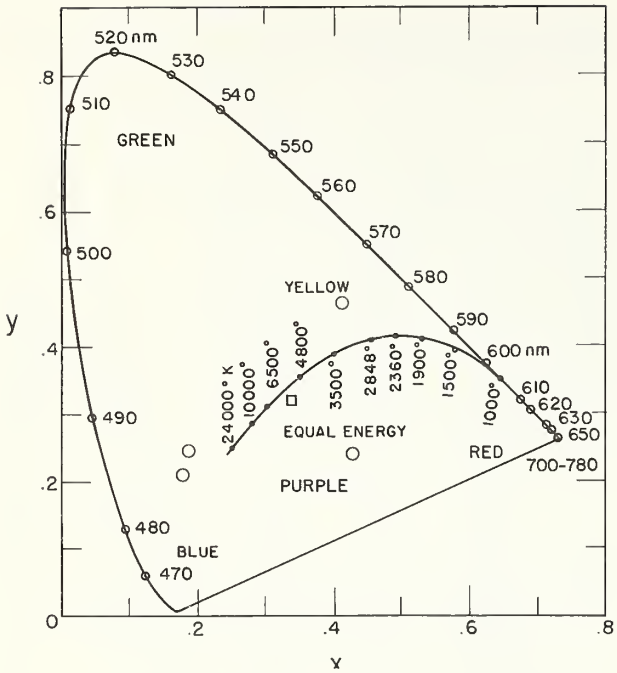


FIGURE 5. *Points representing the colors of four printing-ink specimens whose spectral reflectances are shown in figure 9.4.*

The colors of the ideal closed-cavity radiator are also shown, the temperatures of the radiators being indicated in degrees Kelvin. The smooth curve connecting these points is often called the Planckian locus.

ters have been built and described by Allen [3], Donaldson [28], Guild [43], McAdam [96], Newhall [110], Stiles [145], Verbeek [154], and Wright [157]. The Wright instrument has spectrum primaries; the other four have primaries formed by combining a light source with glass filters. To the Guild and Wright instruments we owe our accurate information regarding the properties of the normal visual system which have been expressed in terms of the standard observer.

The foregoing instruments make up the comparison-field mixture by optical combination of light beams from different sources so that a sum of the separate effects is obtained. A similar optical effect is obtained if the beams are caused to fall upon the same portion of the retina in such rapid succession that a nonflickering spot of color is seen. The effect is that of a time-weighted average of the separate beams. A very simple and widely used tristimulus colorimeter is obtained by taking four disks that have been cut along a radius, interlocking them so as to expose a sector of each, and causing them to rotate on the spindle of a motor so rapidly that neither the separate sectors nor even flicker is perceived. Such an arrangement for combining colors by rotary mixture is called a Maxwell disk. The four disks provide the necessary three degrees of freedom in the adjustment for a match, and if the tristimulus values of the component disks be known (X_1,Y_1,Z_1; X_2,Y_2,Z_2; X_3,Y_3,Z_3; X_4,Y_4,Z_4), the tristimulus values of the mixture can be computed from the fractions of the total area occupied by the respective sectors, f_1,f_2,f_3,f_4:

$$\begin{aligned} X &= f_1X_1 + f_2X_2 + f_3X_3 + f_4X_4 \\ Y &= f_1Y_1 + f_2Y_2 + f_3Y_3 + f_4Y_4 \\ Z &= f_1Z_1 + f_2Z_2 + f_3Z_3 + f_4Z_4. \end{aligned} \tag{4}$$

If the disks are chosen anew for each kind of unknown color to be measured so as to be all fairly similar in color to the unknown, the spectral composition of the mixture color is usually sufficiently nonmetameric that no restriction to the central 2 deg of the retina is required. Furthermore any two normal observers with some experience at making the adjustment can check each other closely. The chief drawback of this simple arrangement for product-control work in color is the time lost in adjustment of the sector disk areas. The motor must be turned off, brought to a stop, the disks loosened and readjusted, the motor turned on and allowed to resume speed several times to obtain a final setting of reasonably good precision.

Nickerson has described a disk colorimeter [105] that avoids the difficulties of the elementary Maxwell disk. Light reflected from the unknown specimen fills one-half of a photometric field, and that from a stationary sector disk fills the other. By having the observer look at the sector disk through a rapidly rotating glass wedge, each sector is presented to view in sufficiently quick succession that no flicker is produced; and at the same time the sectors, since they are stationary, may be continuously adjusted until a color match is obtained. A further advantage is obtained by extending the rotary scanning to the unknown specimen. In this way the average color of a notably nonuniform specimen such as that made up of coarse salt crystals may be obtained. The disk colorimeter has been extensively used by the United States Department of Agriculture for the color-grading of food products and is well adapted to product-control colorimetry of many kinds.

2.6. Dominant Wavelength and Purity

Another way to identify a combination of lights to specify a color, alternate to the tristimulus method, is to determine the luminance (photometric brightness) of one spot of light of fixed spectral composition (such as average daylight) and the luminance of a spot of light of continuously variable spectral composition separately identified (as by wavelength in the spectrum). In this way the requisite three degrees of freedom in adjustment to a color match are supplied. This form of identification leads naturally into a specification in which the luminance [20] of the unknown is given and the chromaticity is specified by two variables in polar coordinates. One of these variables is an angle, the other a radius, and both

TABLE 6. *Selected Ordinates (In Nanometers) for Computing Tristimulus Values, X, Y, Z, for Specimens Under Standard Sources A and C* [20]

Number	Source A			Source C		
	X	*Y*	*Z*	*X*	*Y*	*Z*
1	444.0	487.8	416.4	424.4	465.9	414.1
2*	516.9*	507.7*	424.9*	435.5*	489.4*	422.2*
3	544.0	517.3	429.4	443.9	500.4	426.3
4	554.2	524.1	432.9	452.1	508.7	429.4
5*	561.4*	529.8*	436.0*	461.2*	515.1*	432.0*
6	567.1	534.8	438.7	474.0	520.6	434.3
7	572.0	539.4	441.3	531.2	525.4	436.5
8*	576.3*	543.7*	443.7*	544.3*	529.8*	438.6*
9	580.2	547.8	446.0	552.4	533.9	440.6
10	583.9	551.7	448.3	558.7	537.7	442.5
11*	587.2*	555.4*	450.5*	564.1*	541.4*	444.4*
12	590.5	559.1	452.6	568.9	544.9	446.3
13	593.5	562.7	454.7	573.2	548.4	448.2
14*	596.5*	566.3*	456.8*	577.3*	551.8*	450.1*
15	599.4	569.8	458.8	581.3	555.1	452.1
16	602.3	573.3	460.8	585.0	558.5	454.0
17*	605.2*	576.9*	462.9*	588.7*	561.9*	455.9*
18	608.0	580.5	464.9	592.4	565.3	457.9
19	610.9	584.1	467.0	596.0	568.9	459.9
20*	613.8*	587.9*	469.2*	599.6*	572.5*	462.0*
21	616.9	591.8	471.6	603.3	576.4	464.1
22	620.0	595.9	474.1	607.0	580.5	466.3
23*	623.3*	600.1*	476.8*	610.9*	584.8*	468.7*
24	626.9	604.7	479.9	615.0	589.6	471.4
25	630.8	609.7	483.4	619.4	594.8	474.3
26*	635.3*	615.2*	487.5*	624.2*	600.8*	477.7*
27	640.5	621.5	492.7	629.8	607.7	481.8
28	646.9	629.2	499.3	636.6	616.1	487.2
29*	655.9*	639.7*	508.4*	645.9*	627.3*	495.2*
30	673.5	659.0	526.7	663.0	647.4	511.2
Multiplying factors						
30 ordinates	0.03661	0.03333	0.01185	0.03268	0.03333	0.03938
10 ordinates	.10984	.10000	.03555	.09804	.10000	.11812

*Values for calculation with 10 selected ordinates.

can be computed from the chromaticity coordinates of the fixed spot of light, the variable spot of light, and the unknown, these coordinates serving to locate the respective positions in a chromaticity diagram. If the fixed light is nearly achromatic, the angle often correlates well with the hue of the color perception, and the radius fairly well with its saturation. The most fundamental way to specify the direction on a chromaticity diagram from the point representing the fixed light to the point representing the unknown light is by wavelength of the part of the spectrum required to make the match. If the unknown color can be matched by adding some part of the spectrum to the fixed light, it is said to have a spectral color, and the required wavelength is called *dominant wavelength*. But if a color match is produced for the fixed light by adding some part of the spectrum to the unknown color, the unknown is said to be nonspectral, and the required wavelength is called the *complementary wavelength*. Either dominant wavelength or complementary wavelength may be obtained for the standard observer by drawing on a chromaticity diagram a straight line through the point representing the fixed light and that representing the unknown color, and then by reading the wavelength corresponding to the point at which this line extended intersects the locus of spectrum colors. If the unknown color is plotted between the fixed light and the spectrum, the intersection gives the dominant wavelength; but if the fixed light is represented by a point intermediate to the unknown and the intersection of the straight line with the spectrum locus, the intersection indicates the complementary wavelength.

The degree of approach of the unknown color to the spectrum color is commonly indicated by the ratio of the amount of the spectrum color to the total amount of the two-part combination; this ratio is called purity, and if the amounts are specified in luminance units, the ratio is called *luminance* (formerly colorimetric) *purity*. By far the most common convention, however, is to express the amounts in units of the excitation sum $X + Y + Z$; the resulting ratio is called *excitation purity* and corresponds simply to distance ratios on the chromaticity diagram of a colorimetric coordinate system [49, 64, 133]. Formulas have been derived by Hardy [49] and MacAdam [91] to

TABLE 7. *Spectral Reflectances of Greenish Yellow Printing-Ink Specimen (see fig. 4b) Read for the Selected Ordinates for Source C (see table 6)*

Number	X		Y		Z	
1	0.076		0.102		0.078	
2*	.076	0.076	.240	0.240	.076	0.076
3	.079		.460		.076	
4	.088		.615		.075	
5*	.096	.096	.683	.683	.075	.075
6	.117		.711		.076	
7	.727		.720		.076	
8*	.739	.739	.725	.725	.077	0.77
9	.745		.728		.077	
10	.750		.731		.078	
11*	.758	.758	.735	.735	.080	.080
12	.765		.738		.082	
13	.772		.742		.084	
14*	.779	.779	.745	.745	.086	.086
15	.783		.747		.088	
16	.785		.750		.089	
17*	.787	.787	.755	.755	.091	.091
18	.788		.760		.093	
19	.789		.765		.095	
20*	.790	.790	.772	.772	.097	.097
21	.791		.778		.100	
22	.792		.782		.103	
23*	.793	.793	.785	.785	.106	.106
24	.795		.787		.110	
25	.797		.788		.119	
26*	.800	.800	.790	.790	.133	.133
27	.803		.792		.155	
28	.807		.796		.210	
29*	.812	.812	.802	.802	.342	.342
30	.819		.812		.650	
Totals	19.298	6.430	21.136	7.032	3.677	1.163
Times factor	0.630	0.630	0.704	0.703	0.145	0.137

*Values for calculation with 10 selected ordinates.

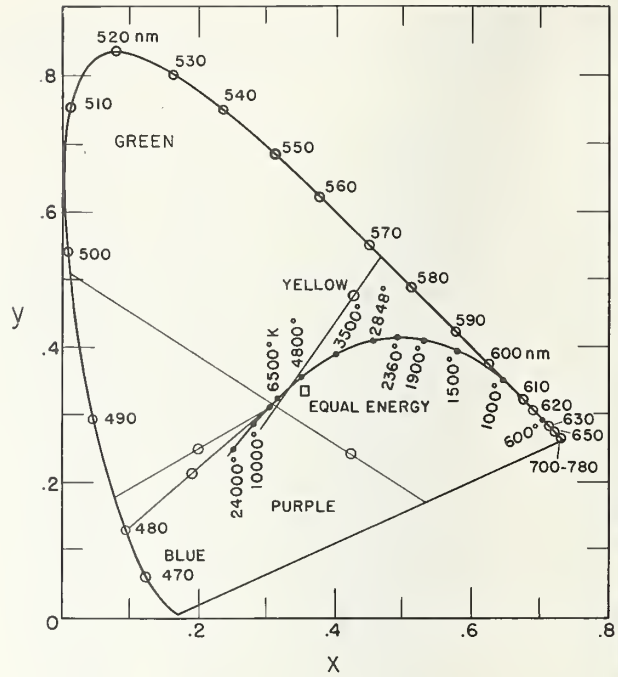


FIGURE 6. *Dominant wavelength and purity evaluated from the (x,y)-chromaticity diagram.*

The four points indicated by circles represent the colors of the four printing-ink specimens shown in figures 9.4 and 9.5.

convert from luminance purity to excitation purity, and the reverse.

Figure 6 indicates how dominant wavelength and excitation purity of the four printing-ink specimens would be found from their chromaticity coordinates x,y, relative to source C taken as the fixed light. Table 8 gives the dominant and complementary (C) wavelengths found as in figure 10 by the intersections of the straight lines with the spectrum locus. Table 8 also gives the excitation purities found by dividing the distance from the fixed point (source C) to the specimen point by the total distance from the fixed point to the boundary (spectrum locus plus straight line connecting its extremes). Large-scale charts for reading dominant wavelength and purity relative to source C are provided in the Hardy *Handbook of Colorimetry* [49].

Apparatus for the direct measurement of dominant wavelength and luminance purity has been designated by Nutting [125] and by Priest [132]. The degree of metamerism ordinarily obtained with such apparatus leaves it open to the same objections as have prevented tristimulus colorimeters with single sets of primaries from being useful for production control. There is a further disadvantage in the direct measurement of luminance purity in that the luminance of the spectrum component has to be determined relative to the luminance of the mixture by separate photometry. Since there is usually a large chromatic difference between these two fields, simple equality-of-brightness settings are not reliable, and an auxiliary flicker photometers, as in Priest's apparatus [132], must be used. This method has been found to exaggerate individual-observer differences; oftentimes observers will differ only slightly in the mixtures of spectrum light and fixed light that they find to be equivalent to an unknown color, but they will disagree importantly in their photomerty of the components.

TABLE 8. *Dominant wavelengths and excitation purities of four printing-ink specimens (see fig. 4)*

Hue designation of specimen	Chromaticity coordinates (from table 5c)		Dominant wavelength, nm	Excitation purity, percent
	x	y		
Red purple	0.430	0.239	498.9C*	53.7
Greenish yellow	.426	.476	573.2	74.0
Greenish blue	.194	.248	483.6	49.6
Blue	.190	.213	479.7	55.1

*C denotes complementary wavelength.

3. Colorimetry by Difference

3.1. General Principles

In the fundamental colorimetry of lights and objects a single standard is used for each class of specimen. Opaque surfaces are referred to the ideal perfect diffuser, or to physically realizable near-perfect diffusers, such as a sufficiently thick layer of magnesium oxide deposited from the smoke of magnesium turnings or ribbon [108] or layers of barium sulfate [141, 6]. Transparent objects, such as gelatin films and crystal or glass plates, are referred to the equivalent thickness of air; transparent solutions, to the equivalent thickness of distilled water or solvent. Self-luminous objects, such as fluorescent lamps, cathode-ray tubes, and incandescent lamps, are measured relative to one of the standard sources, usually source A [19]. The colors of specimens closely resembling the respective standards can be evaluated quite precisely and accurately; those differing radically in spectral composition, only with relative uncertainty. That is, near-white specimens, nearly clear glass plates, and incandescent lamps nearly equivalent to source A present the simplest colorimetric problem; highly selective absorbers and emitters, like the rare-earth glasses and gaseous discharge tubes, present difficult measurement problems. In general, the greater the deviation in spectral composition between the unknown specimen and the standard, the greater the uncertainty of the result obtained by a visual or a photoelectric colorimeter.

Automatic spectrophotometry has greatly extended the application of both visual and photoelectric colorimetry. It has supplied a practical way to calibrate working standards of color. If a fairly large group of specimens is at hand to be measured, say twenty or more, all of similar spectral composition, the most satisfactory way to measure them in the present state of colorimetric science is to evaluate one or two of them carefully by means of the spectrophotometer to serve as working standards, then obtain the color specifications of the remainder by visual or photoelectric determination of the difference between specimen and standard.

3.2. Chromaticity Spacing, Perceptibility

In the interpretation of the importance of chromaticity differences based upon separation of the points representing the two chromaticities in the (x,y)-diagram a warning is necessary. This diagram is considerably expanded in the green portion relative to the other portions, much as the Mercator projection of the earth's surface is expanded near the poles. Thus, two points separated by a given distance in the green portion of this diagram correspond to chromaticities that are

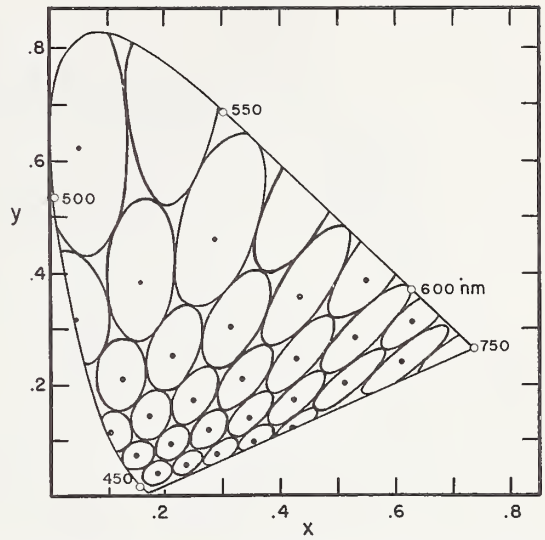


FIGURE 7. *Chromaticity spacing in the* (x,y)*-diagram.* Between any center and any point on the corresponding ellipse there are approximately 100 just noticeable chromaticity steps [67].

much harder to distinguish under ordinary viewing conditions than two chromaticities separated by the same amount in other portions of the diagram. Furthermore, the bluish purple portion of the diagram is correspondingly compressed. The system of ellipses shown on figure 7 serves to indicate approximately the metric properties of the (x,y)-diagram. Under moderately good observing conditions, the distances from the central point of each ellipse to any point on its boundary correspond approximately to one hundred times the chromaticity difference just perceptible with certainty. These ellipses were drawn from a review of the literature in 1936 [66, 67], and subsequent extensive work published by Wright [160, 161] and by MacAdam [92, 94] has corroborated the essential correctness of the indicated chromaticity spacing. Figure 7 not only indicates the extent to which the green portion of the diagram is expanded, and the bluish purple compressed, but also indicates that, in general, the chromatic importance of a distance on the (x,y)-diagram is a function both of the position of the central point and the direction of the deviation from it.

When sets of primaries other than those of the CIE standard observer system are expressed by transformations of the form of eq (3), the chromaticity spacings in the resulting Maxwell triangle may be made to vary widely. There have been several attempts to select primary sets that yield uniform chromaticness scales in which the chromaticity spacing corresponds to perceptibility [14, 55, 66, 90, 144]. The transformation equations for the chromaticity coordinates, r,g, of the uniform-

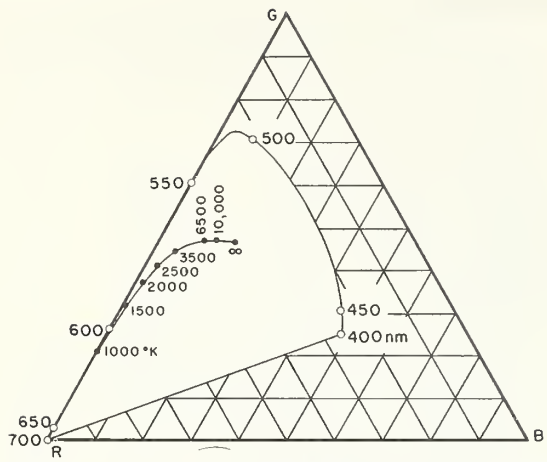


FIGURE 8. *Uniform-chromaticity-scale triangle according to Judd* [66].

The length of the straight line connecting the points representing any two chromaticities on this triangle is approximately proportional to the perceptibility of the chromaticity difference.

chromaticnss–scale (UCS) System [66], from the CIE chromaticity coordinates, x,y, are

$$r = \frac{2.7760x + 2.1543y - 0.1129}{-1.0000x + 6.3553y + 1.5405}$$
$$g = \frac{-2.9446x + 5.0323y + 0.8283}{-1.0000x + 6.3553y + 1.5405} \qquad (5)$$

The triangle resulting from this transformation is shown in figure 8.

In 1960 the CIE adopted a provisional recommendation that the transformation suggested by MacAdam in 1937 [89] be used whenever a spacing perceptually more uniform than that of the (x,y)-diagram is desired. This transformation is:

$$u = \frac{4x}{12y - 2x + 3}$$
$$v = \frac{6y}{12y - 2x + 3} \qquad (6)$$

Although the transformation coefficients are simple, the (u,v)-diagram (fig. 9) has a spacing that closely resembles the UCS diagram.

3.3. Visual Devices

a. Martens Photometer

One of the most useful visual devices for determining relative luminance is the Martens photometer. Figure 10 shows the Martens photometer combined with a diffuse illuminator to form the Priest-Lange reflectometer [134]. This reflectometer is intended for the measurement of luminous reflectance of opaque specimens relative to reflecting standards of similar spectral reflectance. The Priest-Lange instrument is also adaptable to the measurement of luminous transmittance of transparent plates relative to transmitting standards similar in spectral transmittance to the unknown. Finally, the Martens photometer, removed from the mounting, may be used for the determination of the luminance of an unknown self-luminous surface relative to a spectrally similar standard of known luminance. The superior usefulness of the Martens photometer arises from the convenience of the adjustment for equality of brightness between the two halves of the photometer field and from the fact that the dividing line between the half-fields is exceptionally narrow so that it is often invisible when a brightness match has been set.

b. Chromaticity-Difference Colorimeter

The determination of chromaticity coordinates, x,y, by comparison of the unknown specimen with a working standard of similar spectral reflectance can be carried out visually with high precision by means of a colorimeter described by Judd [64]. The adjustment of the chromaticity of the comparison field to match the standard field is by two double wedges, one of greenish and the other of yellowish glass. Since the light from the comparison field must pass through both the yellow and the green wedge, some of the radiant energy being subtracted by each, it is sometimes called a subtractive colorimeter; see figure 11 which gives a schematic diagram. The standard and comparison fields are brought into juxtaposition by means of a Lummer-Brodhun cube having a double-trapezoid pattern subtending $9 \times 13°$ at the observer's eye. The adjustment to near equality of brightness to facilitate detection of chromaticity differences is by movement of the projection lamp that illuminates both standard and comparison surfaces.

A substitution method is usually employed with this colorimeter, a match first being set up between the standard and comparison surfaces by adjustment of the wedges. Then the unknown specimen is substituted for the standard, and the wedges readjusted to restore the match. The differences in wedge settings can be calibrated in terms of differences in the chromaticity coordinates, x,y, of the CIE standard coordinate system, provided the spectral reflectances of the comparison surface are known approximately, from the known spectral transmittances of the wedges. This calibration has been carried out for about 100 widely differing comparison surfaces. It has been found that the calibration is chiefly a function of the chromaticity coordinates, x,y, of the comparison surface; so calibrations for comparison surfaces intermediate in chromaticity to those already calibrated may usually be found satisfactorily by interpolation.

Because of the large patterned field of high luminance and the convenience of the brightness adjustment this instrument takes full advantage of the ability of the observer to detect small chromaticity difference. If specimen and comparison surfaces are similar in spectral composition, the settings for match may be repeated generally within 0.001 in chromaticity coordinates, x,y. The chromaticity-difference colorimeter has the disadvantage, how-

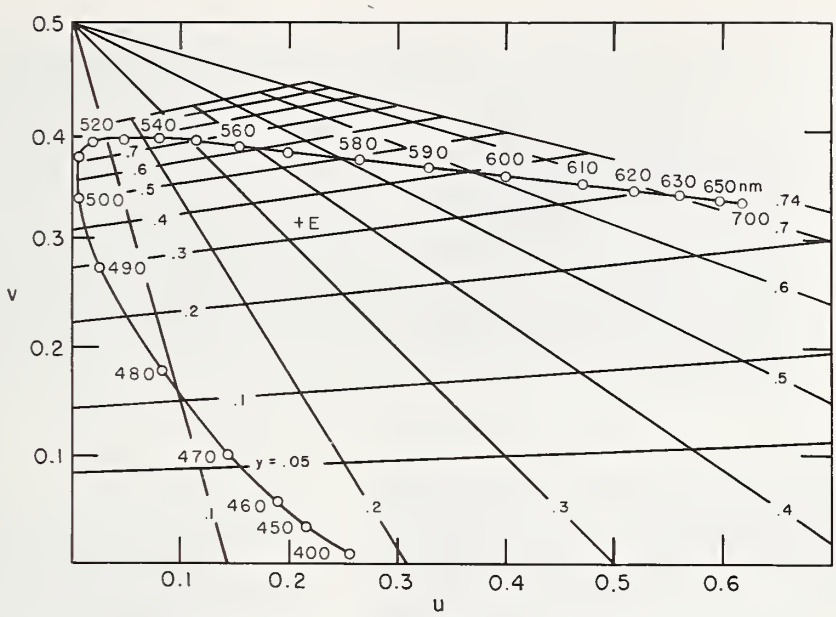


FIGURE 9. *The* (u,v)*-diagram developed by MacAdam* [90].

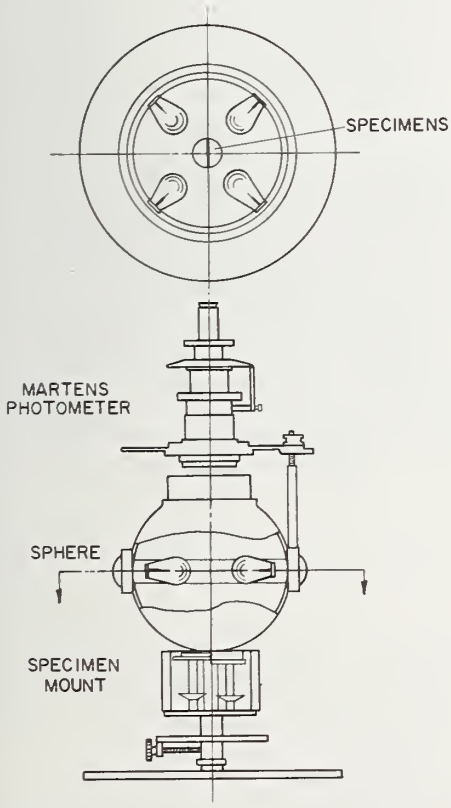


FIGURE 10. *Vertical cross-section of the Priest-Lange* [134] *reflectometer, showing Martens photometer, diffuse illuminator, and specimen holder.*

Horizontal cross section of the illuminator is above.

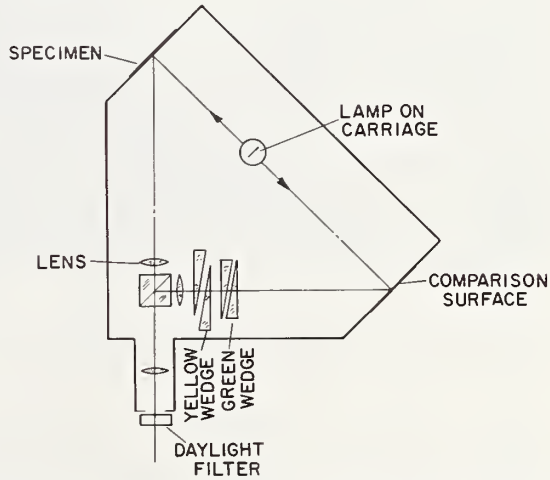


FIGURE 11. *Schematic diagram of chromaticity-difference colorimeter* [68].

ever, of requiring a skilled operator. Furthermore the calibrations, carried out in accord with table 4b, are time-consuming; and the instrument is not applicable to some highly selective samples because nearly homogeneous energy is too little changed in spectral composition by passage through the wedges. This colorimeter has been used in setting up a color standard for ruby mica [70]; for inspection of working standards, transparent and opaque, for conformity to a master standard; and for general colorimetry by difference, for both fluorescent and nonfluorescent specimens [137].

3.4. Photoelectric Devices

If three photocells could be adjusted, as by glass filters, so that their responses were proportional throughout the visible spectrum to some linear combination [as in eq. (3) of the standard CIE distribution curves (see fig. 2)], then they could be used to test whether any two light beams have the same color according to eq. (2) and could be made to yield direct measurements of tristimulus values, X,Y,Z [34, 45]. The X-function filter is the most difficult to design because it has two lobes, one short-wave lobe and one long-wave lobe. Two separate filters to cover a portion of one photocell have been designed for this bilobal function by Barnes [10] and by Nimeroff and Wilson [121]. Figure 12 shows the X-function match achieved in the Nimeroff-Wilson colorimeter. This function has also been fitted by a filter-photocell combination to approximate the long-wave lobe to which a portion of the Z-function is added, either electrically, as in the Color Difference Meter of Hunter [56, 57] and arithmetically, as in the Multipurpose reflectometer of Hunter [55], the Colormaster of Glasser and Troy [38] and the Color Eye of Bentley [11]. In these instruments the transformation equations to obtain X,Y,Z for source C may be represented as:

$$\begin{aligned} X &= 0.80R + 0.18B \\ Y &= 1.00G \\ Z &= 1.18B \end{aligned} \tag{7}$$

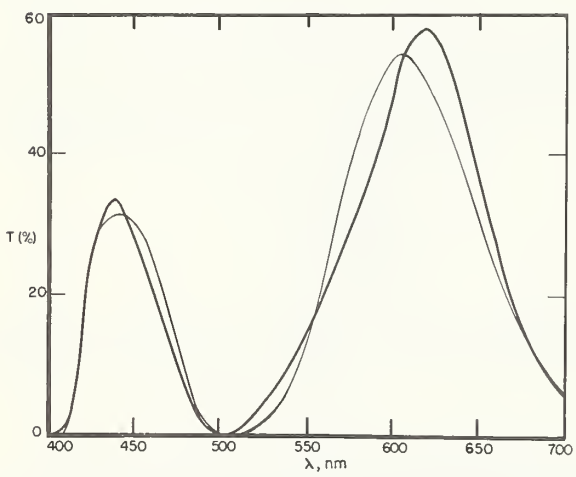


FIGURE 12. *Spectral transmittance of the required (light line) and the approximate filter (heavy line) for CIE function X in the Nimeroff-Wilson colorimeter.*

where R,G, and B are settings with red, green, and blue filters, respectively. In some of these instruments the filter for the long-wave lobe of the X-function may be amber, hence settings with this filter may be designated A. Van den Akker [152] has discussed the incompleteness of success of these colorimeters to duplicate the CIE standard observer system. Recent versions of Color Difference meters and the Color Eye are shown in figure 13.

Figure 14 shows the degree of success achieved by the filters designed by Hunter [55] to duplicate the CIE standard observer and simultaneously to adjust the spectral distribution of a projection lamp to that for CIE source C. Figure 15 shows the discrepancies that the filters of his multipurpose reflectometer introduce. These discrepancies are roughly proportional to the distance from the point representing the magnesium-oxide standard, and are frequently larger than 0.02 in x or y; that is, more than 10 times a reasonable chromaticity tolerance for most colorimetric work. However, for the comparison of near-white surfaecs this degree of duplication is sufficient. Figure 16 refers to the small rectangle near the center of figure 15 and indicates that the discrepancies are less than 0.001 in x or y for comparison of near-white surfaces with magnesium oxide. In general, a similar agreement can be expected in using this photoelectric tristimulus colorimeter for the determination of small chromaticity differences between nonmetameric pairs. And even for measurement of fairly sizable nonmetameric chromaticity differences, such as analyzed spectrophotometrically in the upper portion of figure 17 (BPB 8/2 versus MgO, BG 7/4 versus BG 6/4), and small chromaticity differences with a moderate metameric component, such as shown in the lower left portion of figure 17 (Y_1 versus Y_2), the discrepancy is in the neighborhood of 0.002 in x or y, which is negligible for many purposes. However, for highly metameric pairs, such as shown in the lower right portion of figure 17, the discrepancy may be expected to be in the neighborhood of 0.02, just as it is for large chromaticity differences.

If the limitations of photoelectric tristimulus colorimetry are appreciated, the method is most useful in product-control colorimetry of nonfluorescent specimens by difference from a working standard. The precision of the method is comparable, though perhaps not quite equal, to the best that can be done by eye. No unusual qualifications or extended special training is required by the operator; and, compared to visual colorimetry or to indirect colorimetry by the spectrophotometer, the results are obtained very rapidly.

4. Color Specification by Material Standards

4.1. General Principle

Because of the convenience of material standards of color, they are often used in commerce in preference to specification according to the more fundamental CIE system. Material standards may be carried from place to place, and, if the colors are sufficiently closely spaced in the neighborhood of the unknown color, the nearest match may be found by visual comparison. The color specification con-

FIGURE 13a. *Tristimulus colorimeters by Gardner Laboratory.*

FIGURE 13b. *Tristimulus colorimeters by Hunter Associates Laboratory that give results in nearly uniform color spacing.*

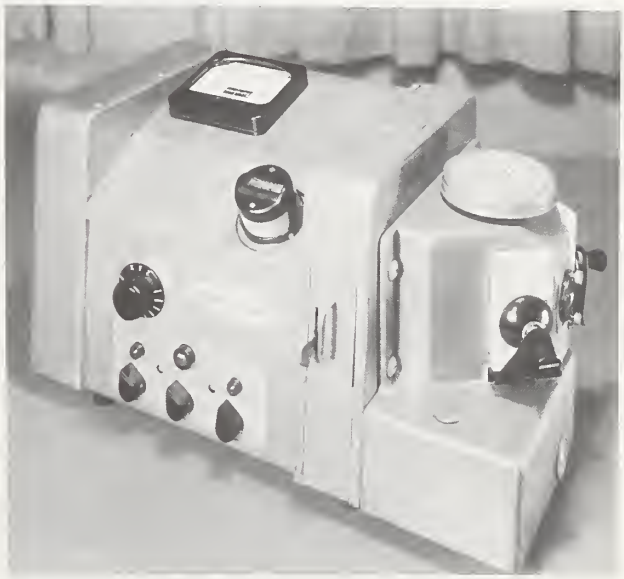


FIGURE 13c. *Tristimulus colorimeters by Instrument Development Laboratories that provide narrow-band interference filters to aid in evaluating the spectral character of a color.*

sists of identifying the particular member of the system yielding a match for the color to be specified. We deal here both with systems of material standards of such scope that a considerable fraction of the colors possible in nonself-luminous objects are represented and with a few special small groups of material standards for particular purposes.

4.2. Transparent Media

Color systems based upon transparent media take advantage of the fact that it is possible with a fixed source to control the color of the transmitted light over a wide range by introducing varying amounts of three absorbing materials. This is done by permitting the light to pass through two or more elements of the absorbing medium instead of through a single element, and is called subtractive combination or mixture because the action of each element is to subtract a certain fraction of each part of the spectrum of the incident light. The color specification consists of the number of unit elements of each of the three absorbing components required to produce the color match.

a. Lovibond Glasses

The Lovibond color system consists of three sets of colored glasses, red, yellow, and blue [86, 87, 88], the principal coloring materials being gold, silver, and cobalt, respectively. The unit of the

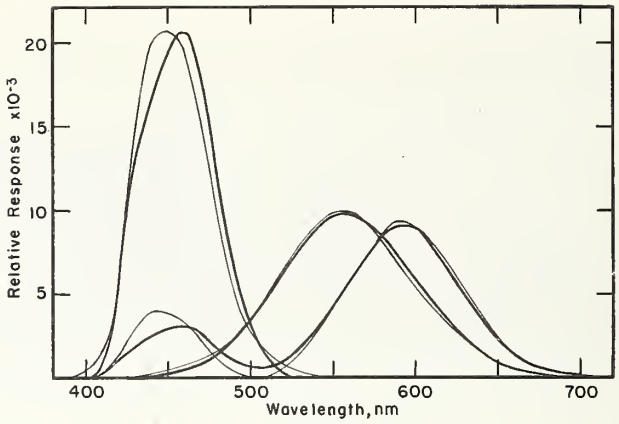


FIGURE 14. *Curves showing the approach of the Hunter tristimulus filters* [55] *combined with incandescent lamp and barrier-layer cell to the CIE spectral tristimulus values for source C.*

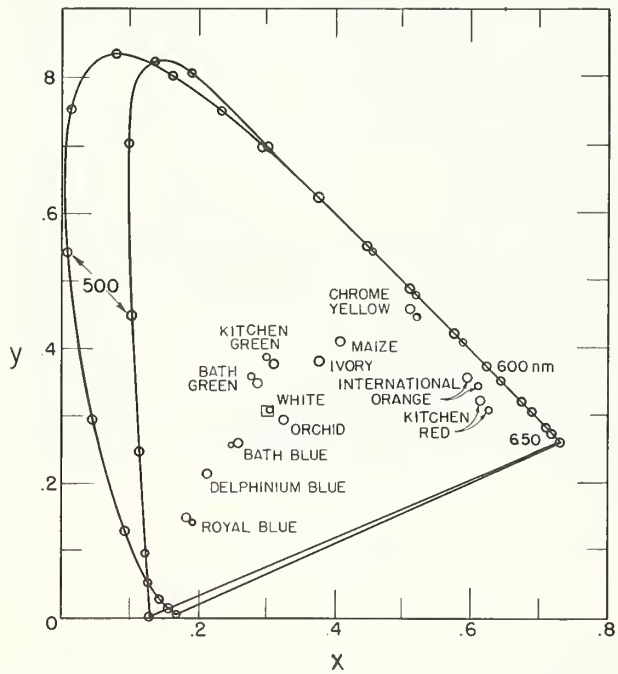


FIGURE 15. *Chromaticity discrepancies expected from the use of the multipurpose reflectometer with Hunter tristimulus filters* [55].

Measurements are all referred to magnesium oxide as the standard white. O, spectrophotometric colorimetry, o, multipurpose reflectometer.

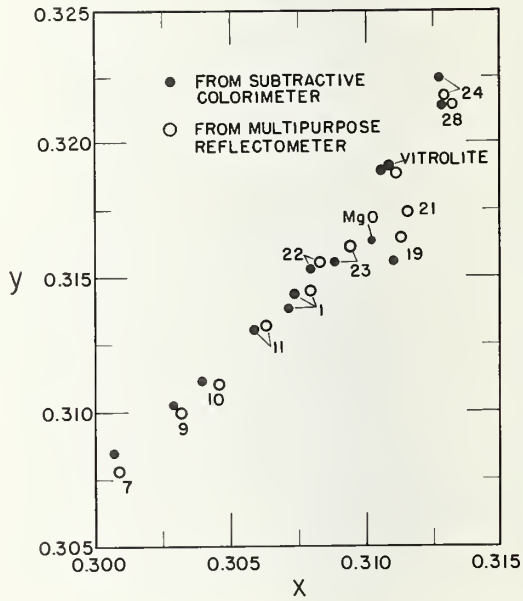


FIGURE 16. *Agreement between multipurpose reflectometer* [55] *and the chromaticity-difference (subtractive) colorimeter for near-white porcelain-enamel specimens measured relative to magnesium oxide.*

scale defined by each set is arbitrary, but the three units are related by being adjusted so that, for observation by daylight, subtractive combination of one unit of each of the red, the yellow, and the blue scales results in a filter perceived as neutral or achromatic. Each scale is exemplified by many glasses, each glass being marked with the number of unit glasses to which it is equivalent. Although the original purpose of the Lovibond color system was to aid in the color control of beer, these glasses are widely used today for other products such as vegetable oils, lubricating oils, and paint vehicles.

A spectrophotometric analysis of the Lovibond color system was made by Gibson and Harris [32], and a scale of the red glasses used in combination with the 35-yellow glass has been constructed by Priest and Gibson [33, 50, 155], having the same unit as the original Lovibond red scale but embodying a closer approach to the principle that the

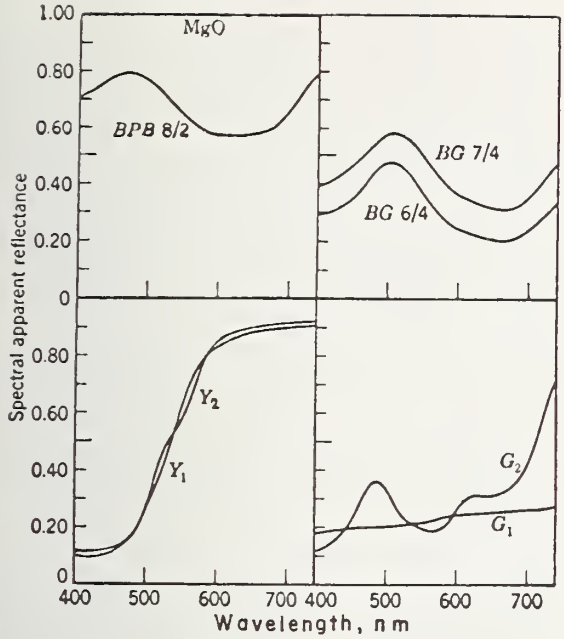


FIGURE 17. *Spectral reflectances of pairs of samples exhibiting various degrees of metamerism.*

Upper two pairs differ considerably in color, but show little or no metamerism. Lower two pairs are near matches; left, moderately metameric; right, strongly.

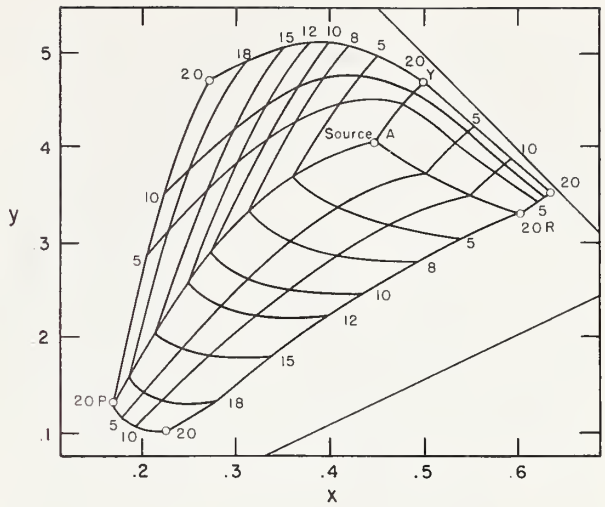


FIGURE 18. *The chromaticities of various two-part combinations of Lovibond red, yellow, and blue glasses for source A as computed by Haupt and Douglas* [51].

Two-part combinations of Lovibond glasses will produce many commercially-important colors.

Lovibond numeral should indicate the number of unit glasses to which the single glass bearing the numeral is equivalent. It is possible to adjust the glasses to slightly lower numerals, if desired, by reducing their thickness slightly [29]. Tintometer, Ltd., makers of the Lovibond glasses, have computed from the published spectrophotometric measurements [32] the chromaticity coordinates, x,y, of all the colors of the ideal Lovibond system [140] produced by illuminating the glasses with CIE standard sources B and C. Computation of these colors for source A has been done at the National Bureau of Standards [51]. Figure 18 shows the Lovibond network for source A. In addition to calibration of red glasses on the Lovibond scale, Tintometer Ltd. will also calibrate them on a scale, not precisely the same, set up in 1961 by agreement with the Color Committee of the American Oil Chemists' Society, based on the Priest-Gibson scale and known as the AOCS scale. Lovibond red glasses defining the AOCS scale are on deposit at Tintometer Ltd and at the National Bureau of Standards. The grading of vegetable oils in this country is carried out by means of this scale.

b. Arny Solutions

The Arny solutions consist of groups of solutions whose concentrations are adjusted to produce the color match. The required concentrations are the specifications of the color. The most used group is a triad consisting of half-normal aqueous solutions of cobalt chloride (red), ferric chloride (yellow), and copper sulfate (blue) in 1 percent hydrochloric acid. This group produces all colors except deep blue and deep red; it is supplemented by a triad of ammoniated aqueous solutions of potassium permanganate and potassium dichromate [8, 9]. Mellon and Martin [101] have reported the spectral transmittances for a number of solutions for colorimetric standards, including the Arny solutions at three or four concentrations, each for the spectral range 440 to 700 nm. By extrapolation of these data it is possible to find approximately the tristimulus values and chromaticity coordinates, x,y, on the standard coordinate system adopted in 1931 for these solutions, just as was done by Mellon [102] for the coordinate system used in America before the international agreement. In this way it is possible to transform color specifications from the standard system into the required concentrations of the Arny solutions, and the reverse. The Arny solutions are used in the 11th edition of the United States Pharmacopoeia as standards for the color of cod-liver oil and in carbonization tests with sulfuric acid for 28 organic compounds.

4.3. Pigmented or Dyed Surfaces

If the material standards are pigmented or dyed surfaces, no automatically convenient notation, such as suggested by additive combination of lights or subtractive combination of absorbing elements, is available. Any systematic aspect of the color specification must be derived from the method of identifying the various members of the set of colored surfaces serving as standards.

a. Color Dictionaries

From a color dictionary are obtained definitions of color names in terms of material standards. The primary aim is therefore to provide an array of

named surface colors adequate for the purpose, and any arrangement or organization of the colors serves only the secondary purpose of assisting the user to find the one which most nearly matches.

The Maerz and Paul Dictionary of Color [97] is the foremost authority on color names. It contains about 7000 different samples of color printed on semiglossy paper, and there are listed about 4000 color names which are keyed to one or another of the color samples. These names are drawn from usage in many fields: paint, textile, ceramic, scientific, technical, and artistic. The samples are also identified by plate, column, and row, and because of their large number and fairly uniform color distribution it is usually possible to find among them a sample approaching what is called a "commercial color match" for any given uniform opaque surface. On this account the Maerz and Paul Dictionary finds a considerable application as a collection of color standards quite separate from its primary function of defining color names. There are noticeable color differences between corresponding samples in different copies of the Dictionary, but the differences have been held to a reasonably small amount by discarding the less satisfactory printed sheets.

The accepted authority for color names in the textile and allied industries is the Color Association of the United States. This association has issued nine editions of a standard color card since 1915, the current edition [149] containing 216 color samples of pure dye silk. Furthermore, the association issues to its members several seasonal cards each year. All colors of these standard and seasonal cards are identified by name and cable number. The standard colors have been measured by spectrophotometric and colorimetric procedures, and luminous reflectance, Y/Y_o, relative to magnesium oxide, and chromaticity coordinates, x,y, for source C have been published [137].

A color dictionary much used for the specification of the colors of flowers, insects, and birds was prepared in 1921 by Ridgway [139]. This outstanding pioneer work contains about 1000 named color samples of paper painted by hand. Each chart shows columns of colors of the same dominant wavelength progressing from each chromatic color at the middle of the column toward white and the top, and toward black at the bottom; and there are five series of such columns, each one encompassing the entire hue circuit, but at different purities. Many of the names were coined at the time of publication to fill in gaps in popular color nomenclature and so have not much descriptive value. Each sample is arbitrarily identified by column, row, and series, however. In addition, there is an alphabetical list of the color names giving this identification.

b. Ostwald System

The notation of the Ostwald system is based on the properties of idealized pigment surfaces having spectral reflectance constant at a certain value between two complementary wavelengths and reflectance constant at a certain other value at other parts of the spectrum [30, 128]. The full colors are those that have the low values of spectral reflectance equal to zero and the high ones equal to 100 percent. The difference between these two reflectances for other idealized pigment surfaces is the full color content, the value of the low reflectance is the white content, and the difference between the high reflectance and 100 percent is the black content. The complete Ostwald notation consists of a number and two letters. The number indicates dominant (or complementary) wavelength on an arbitrary but approximately uniform perceptual scale, and is called Ostwald hue number. The first letter indicates white content, *a* being a white content of 89.13 percent, which is as near to 100 percent as is practicable for usual pigment-vehicle combinations, and other letters in alphabetical sequence indicating decreasing white content on a logarithmic scale. The second letter indicates black content, *a* being a black content of 10.87 percent, which is as near to zero as is practicable, and other letters in alphabetical sequence indicating increasing black content on a logarithmic scale. The logarithmic scales were thought by Ostwald to insure uniform color scales, but this is true only to a rough approximation. Since the percent white content, black content, and full-color content must necessarily add up to 100, no explicit indilation of the latter is required.

The Ostwald ideas have been a considerable aid in thinking about color relationships on the part of those who duplicate colors by mixtures of chromatic pigments with white and black pigments, and they have served as a guide in the selection of combinations of such colors to produce pleasing effects. However, the use of these idealized pigment surfaces as a basis for a system of colorimetry has been hampered by the fact that actual pigment surfaces approximate them rather poorly, and by the fact that not all actual pigment surfaces can be color matched by one of these ideal surfaces. Still, color charts made up more or less in accord with the Ostwald principles have been widely used for color standards and for the selection of harmonizing colors [63, 127, 129, 148]. Of these, the Jacobson *Color Harmony Manual* [63] is pre-eminent not only because of its technical excellence, but also because Foss [30] has given a clear statement of which of the somewhat contradictory Ostwald principles were followed in its construction, and Granville and Jacobson [47] have made a spectrophotometric study of the color chips and have published luminous reflectance, Y/Y_o, relative to magnesium oxide, and chromaticity coordinates, x,y, for every chip. These chips are therefore valuable for use in colorimetry by difference from a working standard (see sec. 3), and the fact that the chip is a member of an orderly arrangement of colors facilitates the selection of a working standard for any particular purpose.

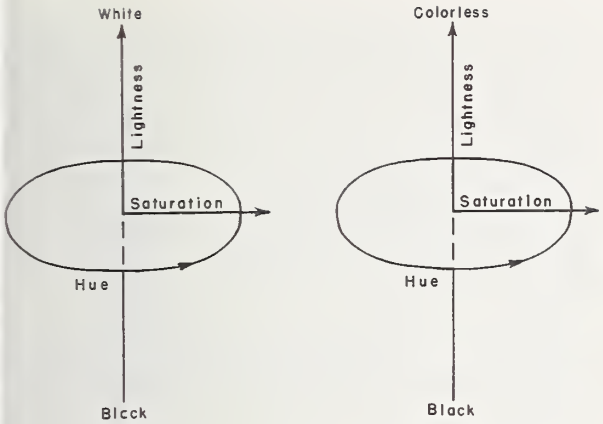


FIGURE 19. *Dimensions of the surface-color solid: (a) opaque surfaces (b) transparent volumes.*

c. Munsell System

The basis of the Munsell system is description of colors perceived to belong to surfaces in terms of hue, lightness, and saturation. Each such tridimensional description can be represented by a point plotted in a space diagram known as the *surface-color solid*, as shown in figure 19. In the surface-color solid the central axis represents the grays extending from black at the bottom to white at the top. *Lightness* of a chromatic (nongray) color determines the gray to which it is equivalent on this scale. Lightness is represented in the color solid by distance above the base plane. *Hue* determines whether a color is perceived as red, yellow, green, blue, purple, or some intermediate; it is represented in the color solid by angle about the central axis. *Saturation* indicates the degree of departure of a surface-color perception from the gray of the same lightness; it is represented by distance from the central gray axis.

The Munsell color system specifies a surface color by giving for usual viewing conditions its position on more or less arbitrary hue, lightness, and saturation scales having perceptually nearly uniform steps [107]. The Munsell term corresponding to lightness is Munsell value, that for saturation is Munsell chroma, and that for hue is Munsell hue. Munsell value is zero for the ideal black surface having luminous reflectance equal to zero, and it is 10 for the ideal white diffusing surface having luminous reflectance equal to 1. Munsell chroma is expressed in arbitrary units intended to be perceptually of the same size regardless of value and hue. The strongest known pigment colors have chromas of about 16; neutral grays have zero chroma as do black and white. Munsell hue is expressed on a scale intended to divide the hue circuit (red, yellow, green, blue, purple, back to red) into 100 perceptually equal steps. According to one convention the 100 Munsell hues are identified simply by a number from 1 to 100, and on this scale hues that differ by 50 are nearly complementary. The

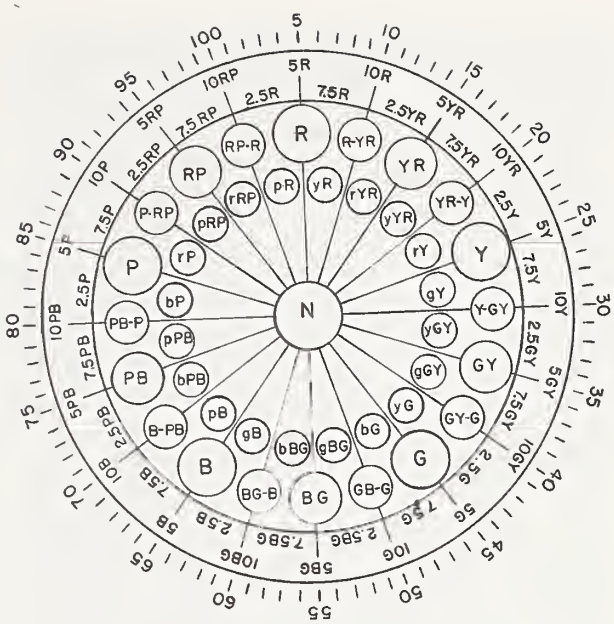


FIGURE 20. *Diagram of hue circle with Munsell hue notation.*

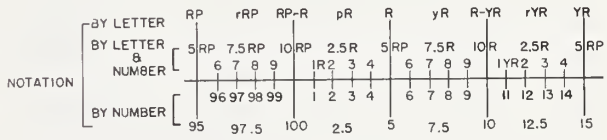


FIGURE 21. *Alternate ways of expressing Munsell hue notation.*

most common convention, however, is to divide these 100 hues into 10 groups of 10 hues each, and identify each group by initials indicating the central member of the group, thus: red R, yellow red YR, yellow Y, green yellow GY, green G, blue green BG, blue B, purple blue PB, purple P, and red purple RP. The hues in each group are identified by the numbers 1 to 10. Thus, the most purplish of the red hues (1 on the scale of 100) is designated as 1R, the most yellowish as 10R, and the central hue as 5R, or often simply as R; see figures 20 and 21. The transition points (10R, 10YR, 10Y, and so on) between the groups of hues are also sometimes designated by means of the initials of the two adjacent hue groups, thus: R-YR = 10R, YR-Y = 10YR, Y-GY = 10Y, and so on; but this convention is little used. The Munsell notation is commonly written: Hue value/chroma, this is, the hue notation, such as 6R, then the value, such as 7, and finally the chroma, such as 4, the latter two being separated by a solidus: 6R 7/4. More precise designations are given in tenths of the arbitrary steps of the scales, thus: 6.2R 7.3/4.4. The grays are indicated by the symbol N for neutral followed by the value notation, thus: N 7/ or N 7.3/; the chroma being zero for neutrals is not specifically noted.

Three representations of the Munsell system

have been published, the original *Atlas* in 1915 [105], now chiefly a collector's item, the *Munsell Book of Color* in 1929 and 1942 [106], and the Munsell Color Standards in High-Gloss Surface in 1957. The book consists of rectangles of mat-finish handpainted paper permanently mounted on charts in a loose-leaf binding. The high-gloss color standards are detachably mounted. The neutrals form a one-dimensional color scale extending from N 1/ to N 9/. Each chromatic sample, of which there are about 1000, takes its place on three color scales: a hue scale, a value scale, and a chroma scale; and the spacing of these scales is intended to be perceptually uniform. The pocket edition, adapted for determining Munsell notation of unknown colors by visual comparison, consists of forty constant-hue charts, so called because all the samples on each chart have the same Munsell hue. These samples are arranged in rows and columns, the rows being chroma scales at constant Munsell value, the columns being value scales at constant Munsell chroma. Comparison of an unknown color with these two families of scales gives by interpolation the Munsell value and Munsell chroma of the unknown. Interpolation between adjacent constant-hue charts gives the Munsell hue. Unknowns not too far outside the range of the Munsell charts may be evaluated with some reliability by extrapolation along the value and chroma scales. Table 9 gives *Munsell Book* notations of the four printing-ink specimens reproduced on figure 4. The greenish yellow specimen was evaluated by interpolation, the other three by extrapolation of varying degrees of uncertainty, the red-purple specimen being furthest outside the range of the Munsell charts and hence the least certainly evaluated by visual estimate.

To facilitate the comparison of the unknown color with those of the permanently mounted paper rectangles on the Munsell charts, either the unknown color must be brought into juxtaposition with the rectangle and held in nearly the same plane, or, if the form of the unknown prevents such juxtaposition, two masks of thin cardboard having a rectangular opening to fit the paper rectangles should be used. One mask should be placed over the unknown color; the other over one or another of the Munsell colors in succession to obtain the interpolated *Munsell Book* notation. Kelly [69, 76] made effective use of a form of mask with three rectangular openings particularly adapted to comparisons involving powdered chemicals and drugs viewed through a cover glass. It is advantageous to have the color of the mask fairly close to that of the unknown, particularly in Munsell value; that is, if the unknown color is dark, the mask should be of a dark color also. Use of a light mask for a dark color prevents the observer from making as precise a visual estimate as he can make with a mask more nearly a color match for the unknown.

Because of the visual uniformity of the scales, the estimates of Munsell notation for unknown colors within the color range of the charts have a reliability corresponding to the use of a much larger collection of unequally spaced color standards. On this account the pocket edition of the *Munsell Book of Color* is widely used as a practical color standard for general purposes.

The standard or library edition shows the same colors as the pocket edition, but it shows them on constant-value charts in a polar-coordinate system, and on constant-chroma charts in a rectangular coordinate system, as well as on contant-hue charts. This edition has full explanatory matter in the text and is adapted particularly for teaching color. It is too bulky for convenient practical use in determining the Munsell notation of an unknown color.

The samples of the 1929 *Munsell Book of Color* have been measured by means of the spectrophotometer twice independently with generally concordant results [39, 78]. In both these studies luminous reflectance, Y/Y_o, relative to magnesium oxide and chromaticity coordinates, x,y, of the roughly 400 samples were computed for source C. Glenn and Killian [39] have also published the dominant wavelength and purity for each of the colors; Kelly, Gibson, and Nickerson [78] have published specifications (Y,x,y) for three additional sources (source A, Macbeth daylight, and limit blue sky. Nickerson and Wilson [118] have extended them to nine sources. Furthermore, they have published [78] a series of (x,y) chromaticity diagrams showing the position of the Munsell colors for each of the Munsell values from 2/ through 8/. From these diagrams it is possible to find the chromaticity coordinates, x,y, correspond-

TABLE 9. *Munsell renotations, luminous reflectances, chromaticity coordinates, Munsell Book Notations, and ISCC-NBS color designations of four printing-ink specimens*

Hue designation of specimen	Luminous reflectance (Y/Y_0)	Chromaticity coordinates		Munsell renotation (hue value/chroma)	Munsell book notation (hue value/chroma)	ISCC-NBS color designation
		x	y			
Red Purple	0.221	0.430	0.239	5.5RP $5.2_5/16.1$	6.0RP 4.8/16	Vivid purplish red
Greenish Yellow	.704	.426	.476	8.3Y $8.6_0/10.6$	7.5Y 9.0/9.5	Brilliant greenish yellow
Greenish Blue	.242	.194	.248	5.6B $5.4_6/8.5$	3.0B 5.4/9	Strong greenish blue
Blue	.246	.190	.213	0.8PB $5.5_0/9.6$	10.0B 5.4/11	Strong blue

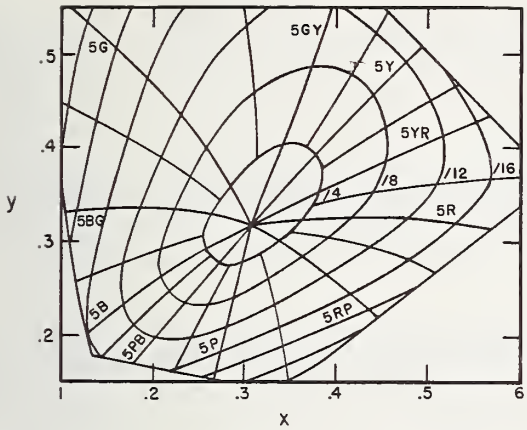


FIGURE 22. *Chromaticities of ideal Munsell colors, value 5/, shown on the (x,y)-diagram.*

This chart serves to define Munsell renotation hue and chroma for colors having $Y/Y_0 = 0.198$. (Prepared by Color Measurement Laboratory, War Food Administration, U.S.D.A.)

ing to any *Munsell Book* notation; and the reverse transformation is also possible. The samples of the 1942 supplement to the *Munsell Book of Color* together with many special Munsell standards have been measured spectrophotometrically by Granville, Nickerson, and Foss [46] and by Nickerson, Tomaszewski, and Boyd [119]. These Munsell standards, together with those of the 1929 *Munsell Book of Color*, number nearly 1500 and comprise the largest systematic set of color standards of known luminous reflectance and chromaticity coordinates ever made. These standards are commercially available separately in disk form and on large sheets, and they make practical the general colorimetry of opaque specimens not only by disk mixture, but also by difference from a standard.

From the charts themselves, luminous reflectance and chromaticity coordinates of an unknown color may also be found quickly and with an accuracy sufficient for many purposes by obtaining first the *Munsell Book* notation of the unknown and then transforming it by reference to (x,y) interpolation charts based on the complete set of standards. Two methods by which to make sure transformations have been described. One, an ASTM method [7a], makes use of tables and charts of corresponding CIE chromaticity coordinates and Munsell notations. The other is an automatic-computer program [75] by which CIE notations are transformed to Munsell notations.

The Munsell color standards may also be used, though less conveniently, in the colorimetry of light-transmitting [69, 76] elements (gelatin films, crystal and glass plates, solutions, and so on); and, conversely, such elements may be given Munsell Book notations from their CIE specifications by means of interpolation charts. Table 12, to be discussed presently in another connection, shows *Munsell Book* notations so derived from the luminous transmittances, T, and chromaticity coordinates, x,y, of the glass standards of the ASTM Union colorimeter. The last four book notations given are relatively uncertain because the colors to be specified are far outside the range of the Munsell standards.

The spacing of the Munsell colors has been examined in detail by a subcommittee of the Colorimetry Committee of the Optical Society of America [111]. This committee work confirmed the many local irregularities in spacing revealed by the spectrophotometric studies and established the need for some more general but minor adjustments to make the colors of the Munsell charts correlate more nearly perfectly under ordinary conditions (adaption to daylight, gray surrounding field, and so on) with the surface-color solid. The subcommittee found it possible from this study to recommend specifications (Y,x,y) on the standard coordinate system defining an ideal Munsell system [112]. It has also given to every Munsell standard a revised notation, called the *Munsell renotation*, indicating exactly in what way and how much each color chip deviates from the ideal. Furthermore, the recommended definition of the ideal Munsell system has been extended beyond the color ranges covered by the present Munsell charts so as to include all colors theoretically producible from nonfluorescent materials [83] under source C. The connection between luminous reflectance, Y, and Munsell renotation value is given in Table 10. Note that N 0/ corresponds to $Y = 0$, N 9.91/ corresponds to magnesium oxide ($Y = 1.00$), and N 5.0/ corresponds to $Y = 0.198$ (quite different from 0.50, the halfway point). Figure 22 shows one of the (x,y)-chromaticity charts (that for Munsell renotation value equal to 5.0) defining the ideal system. From these charts combined with the data given in table 10 it is possible to find the ideal Munsell notation for any color specified in terms of luminous reflectance, Y, and chromaticity coordinates, x,y. Table 9 shows these Munsell renotations for the four printing-ink specimens of figure 4. Note that they agree in a general way, though not perfectly, with the notations found by direct visual comparison with the *Munsell Book of Color*. Some of the discrepancies are ascribable to local irregularities of the color spacing of the Munsell charts, but most of them may be laid to the uncertainty of the visual estimates, all but that for the greenish yellow specimen requiring extrapolation over a considerable color range.

Munsell renotations, such as these, have a unique usefulness as color specifications. Because of their definition in terms of the standard CIE coordinate system they are capable of nearly the precision of the Y,x,y form of specification, and like that form they may be extended to apply to all object colors, both opaque and transparent objects. For opaque surfaces Y/Y_0 is luminous reflectance relative to magnesium oxide; for transparent objects Y/Y_0 is luminous transmittance relative to an equivalent thickness of air; for solutions Y/Y_0 is luminous

TABLE 10. *CIE (Y) equivalents (in percent relative to MgO) of the recommended Munsell value scale (V) from 0/ to 10/*

V	Yv	V	Yv	V	Yv	V	Yv	V	Yv	V	Yv	V	Yv	V	Yv	V	Yv	V	Yv
10.00	102.56																		
9.99	102.30	9.49	89.77	8.99	78.45	8.49	68.20	7.99	58.92	7.49	50.52	6.99	42.92	6.49	36.07	5.99	29.94	5.49	24.48
8	102.04	8	89.53	8	78.23	8	68.01	8	58.74	8	50.36	8	42.77	8	35.94	8	29.82	8	24.38
7	101.78	7	89.30	7	78.02	7	67.81	7	58.57	7	50.20	7	42.63	7	35.81	7	29.71	7	24.28
6	101.52	6	89.06	6	77.80	6	67.62	6	58.39	6	50.04	6	42.49	6	35.68	6	29.59	6	24.17
5	101.25	5	88.82	5	77.59	5	67.43	5	58.22	5	49.88	5	42.34	5	35.56	5	29.48	5	24.07
9.94	100.99	9.44	88.59	8.94	77.38	8.44	67.23	7.94	58.04	7.44	49.72	6.94	42.20	6.44	35.43	5.94	29.36	5.44	23.97
3	100.73	3	88.35	3	77.16	3	67.04	3	57.87	3	49.56	3	42.06	3	35.30	3	29.25	3	23.87
2	100.47	2	88.12	2	76.95	2	66.85	2	57.69	2	49.41	2	41.92	2	35.17	2	29.13	2	23.77
1	100.21	1	87.88	1	76.74	1	66.66	1	57.52	1	49.25	1	41.77	1	35.04	1	29.02	1	23.67
0	99.95	0	87.65	0	76.53	0	66.46	0	57.35	0	49.09	0	41.63	0	34.92	0	28.90	0	23.57
9.89	99.69	9.39	87.41	8.89	76.32	8.39	66.27	7.89	57.17	7.39	48.93	6.89	41.49	6.39	34.79	5.89	28.79	5.39	23.47
8	99.44	8	87.18	8	76.11	8	66.08	8	57.00	8	48.78	8	41.35	8	34.66	8	28.68	8	23.37
7	99.18	7	86.95	7	75.90	7	65.89	7	56.83	7	48.62	7	41.21	7	34.54	7	28.57	7	23.27
6	98.92	6	86.72	6	75.69	6	65.70	6	56.66	6	48.47	6	41.07	6	34.41	6	28.45	6	23.17
5	98.66	5	86.48	5	75.48	5	65.51	5	56.48	5	48.31	5	40.93	5	34.28	5	28.34	5	23.07
9.84	98.41	9.34	86.25	8.84	75.27	8.34	65.32	7.84	56.31	7.34	48.16	6.84	40.79	6.34	34.16	5.84	28.23	5.34	22.97
3	98.15	3	86.02	3	75.06	3	65.13	3	56.14	3	48.00	3	40.65	3	34.03	3	28.12	3	22.87
2	97.90	2	85.79	2	74.85	2	64.94	2	55.97	2	47.85	2	40.51	2	33.91	2	28.01	2	22.78
1	97.64	1	85.56	1	74.64	1	64.76	1	55.80	1	47.69	1	40.37	1	33.78	1	27.90	1	22.68
0	97.39	0	85.33	0	74.44	0	64.57	0	55.63	0	47.54	0	40.23	0	33.66	0	27.78	0	22.58
9.79	97.14	9.29	85.10	8.79	74.23	8.29	64.38	7.79	55.46	7.29	47.38	6.79	40.09	6.29	33.54	5.79	27.67	5.29	22.48
8	96.88	8	84.88	8	74.02	8	64.19	8	55.29	8	47.23	8	39.95	8	33.41	8	27.56	8	22.38
7	96.63	7	84.65	7	73.82	7	64.01	7	55.12	7	47.08	7	39.82	7	33.29	7	27.45	7	22.29
6	96.38	6	84.42	6	73.61	6	63.82	6	54.95	6	46.92	6	39.68	6	33.16	6	27.34	6	22.19
5	96.13	5	84.19	5	73.40	5	63.63	5	54.78	5	46.77	5	39.54	5	33.04	5	27.23	5	22.09
9.74	95.88	9.24	83.97	8.74	73.20	8.24	63.45	7.74	54.62	7.24	46.62	6.74	39.40	6.24	32.92	5.74	27.12	5.24	22.00
3	95.63	3	83.74	3	72.99	3	63.26	3	54.45	3	46.47	3	39.27	3	32.80	3	27.02	3	21.90
2	95.38	2	83.52	2	72.79	2	63.08	2	54.28	2	46.32	2	39.13	2	32.67	2	26.91	2	21.81
1	95.13	1	83.29	1	72.59	1	62.89	1	54.11	1	46.17	1	39.00	1	32.55	1	26.80	1	21.71
0	94.88	0	83.07	0	72.38	0	62.71	0	53.94	0	46.02	0	38.86	0	32.43	0	26.69	0	21.62
9.69	94.63	9.19	82.84	8.69	72.18	8.19	62.52	7.69	53.78	7.19	45.87	6.69	38.72	6.19	32.31	5.69	26.58	5.19	21.52
8	94.38	8	82.62	8	71.98	8	62.34	8	53.61	8	45.72	8	38.59	8	32.19	8	26.48	8	21.43
7	94.14	7	82.39	7	71.78	7	62.16	7	53.45	7	45.57	7	38.45	7	32.07	7	26.37	7	21.33
6	93.89	6	82.17	6	71.57	6	61.98	6	53.28	6	45.42	6	38.32	6	31.95	6	26.26	6	21.24
5	93.64	5	81.95	5	71.37	5	61.79	5	53.12	5	45.27	5	38.18	5	31.83	5	26.15	5	21.14
9.64	93.40	9.14	81.73	8.64	71.17	8.14	61.61	7.64	52.95	7.14	45.12	6.64	38.05	6.14	31.71	5.64	26.05	5.14	21.05
3	93.15	3	81.50	3	70.97	3	61.43	3	52.79	3	44.97	3	37.92	3	31.59	3	25.94	3	20.96
2	92.91	2	81.28	2	70.77	2	61.25	2	52.62	2	44.82	2	37.78	2	31.47	2	25.84	2	20.86
1	92.66	1	81.06	1	70.57	1	61.07	1	52.46	1	44.67	1	37.65	1	31.35	1	25.73	1	20.77
0	92.42	0	80.84	0	70.37	0	60.88	0	52.30	0	44.52	0	37.52	0	31.23	0	25.62	0	20.68
9.59	92.18	9.09	80.62	8.59	70.17	8.09	60.70	7.59	52.13	7.09	44.38	6.59	37.38	6.09	31.11	5.59	25.52	5.09	20.59
8	91.93	8	80.40	8	69.97	8	60.52	8	51.97	8	44.23	8	37.25	8	30.99	8	25.41	8	20.49
7	91.69	7	80.18	7	69.78	7	60.35	7	51.81	7	44.08	7	37.12	7	30.87	7	25.31	7	20.40
6	91.45	6	79.97	6	69.58	6	60.17	6	51.64	6	43.94	6	36.99	6	30.75	6	25.20	6	20.31
5	91.21	5	79.75	5	69.38	5	59.99	5	51.48	5	43.79	5	36.86	5	30.64	5	25.10	5	20.22
9.54	90.97	9.04	79.53	8.54	69.18	8.04	59.81	7.54	51.32	7.04	43.64	6.54	36.72	6.04	30.52	5.54	25.00	5.04	20.13
3	90.73	3	79.31	3	68.99	3	59.63	3	51.16	3	43.50	3	36.59	3	30.40	3	24.89	3	20.04
2	90.49	2	79.10	2	68.79	2	59.45	2	51.00	2	43.35	2	36.46	2	30.28	2	24.79	2	19.95
1	90.25	1	78.88	1	68.59	1	59.28	1	50.84	1	43.21	1	36.33	1	30.17	1	24.69	1	19.86
0	90.01	0	78.66	0	68.40	0	59.10	0	50.68	0	43.06	0	36.20	0	30.05	0	24.58	0	19.77

TABLE 10. *CIE (Y) equivalents (in percent relative to MgO) of the recommended Munsell value scale (V) from 0/ to 10/ (continued)*

V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V
4.99	19.68	4.49	15.49	3.99	11.935	3.49	8.949	2.99	6.511	2.49	4.580	1.99	3.100	1.49	2.002	0.99	1.196	0.49	0.570
8	19.59	8	15.42	8	11.870	8	8.895	8	6.468	8	4.546	8	3.075	8	1.983	8	1.182	8	.559
7	19.50	7	15.34	7	11.805	7	8.841	7	6.425	7	4.512	7	3.050	7	1.965	7	1.168	7	.547
6	19.41	6	15.26	6	11.740	6	8.787	6	6.382	6	4.479	6	3.025	6	1.947	6	1.154	6	.535
5	19.32	5	15.18	5	11.675	5	8.734	5	6.339	5	4.446	5	3.000	5	1.929	5	1.141	5	.524
4.94	19.23	4.44	15.11	3.94	11.611	3.44	8.681	2.94	6.296	2.44	4.413	1.94	2.975	1.44	1.910	0.94	1.128	0.44	.513
3	19.14	3	15.03	3	11.547	3	8.628	3	6.254	3	4.380	3	2.950	3	1.892	3	1.114	3	.501
2	19.06	2	14.96	2	11.483	2	8.575	2	6.212	2	4.347	2	2.925	2	1.874	2	1.101	2	.489
1	18.97	1	14.88	1	11.419	1	8.523	1	6.170	1	4.314	1	2.901	1	1.856	1	1.087	1	.478
0	18.88	0	14.81	0	11.356	0	8.471	0	6.128	0	4.282	0	2.877	0	1.838	0	1.074	0	.467
4.89	18.79	4.39	14.73	3.89	11.292	3.39	8.419	2.89	6.086	2.39	4.250	1.89	2.853	1.39	1.821	0.89	1.060	0.39	.455
8	18.70	8	14.66	8	11.229	8	8.367	8	6.045	8	4.218	8	2.829	8	1.803	8	1.047	8	.444
7	18.62	7	14.58	7	11.167	7	8.316	7	6.003	7	4.186	7	2.805	7	1.786	7	1.034	7	.432
6	18.53	6	14.51	6	11.104	6	8.264	6	5.962	6	4.154	6	2.781	6	1.769	6	1.021	6	.421
5	18.44	5	14.43	5	11.042	5	8.213	5	5.921	5	4.123	5	2.758	5	1.752	5	1.008	5	.409
4.84	18.36	4.34	14.36	3.84	10.980	3.34	8.162	2.84	5.881	2.34	4.092	1.84	2.735	1.34	1.735	0.84	0.995	0.34	.398
3	18.27	3	14.28	3	10.918	3	8.111	3	5.841	3	4.060	3	2.712	3	1.718	3	.982	3	.386
2	18.19	2	14.21	2	10.856	2	8.060	2	5.800	2	4.029	2	2.688	2	1.701	2	.969	2	.375
1	18.10	1	14.14	1	10.795	1	8.010	1	5.760	1	3.998	1	2.665	1	1.684	1	.956	1	.363
0	18.02	0	14.07	0	10.734	0	7.960	0	5.720	0	3.968	0	2.642	0	1.667	0	.943	0	.352
4.79	17.93	4.29	13.99	3.79	10.673	3.29	7.910	2.79	5.680	2.29	3.938	1.79	2.620	1.29	1.650	0.79	.931	0.29	.341
8	17.85	8	13.92	8	10.612	8	7.860	8	5.641	8	3.907	8	2.598	8	1.634	8	.918	8	.329
7	17.76	7	13.85	7	10.551	7	7.811	7	5.602	7	3.877	7	2.575	7	1.618	7	.906	7	.318
6	17.68	6	13.78	6	10.491	6	7.762	6	5.563	6	3.847	6	2.553	6	1.601	6	.893	6	.306
5	17.60	5	13.70	5	10.431	5	7.713	5	5.524	5	3.817	5	2.531	5	1.585	5	.881	5	.295
4.74	17.51	4.24	13.63	3.74	10.371	4	7.664	2.74	5.485	2.24	3.787	1.74	2.509	1.24	1.569	0.74	.868	0.24	.283
3	17.43	3	13.56	3	10.311	3	7.615	3	5.447	3	3.758	3	2.487	3	1.553	3	.856	3	.272
2	17.34	2	13.49	2	10.252	2	7.567	2	5.408	2	3.729	2	2.465	2	1.537	2	.844	2	.260
1	17.26	1	13.42	1	10.193	1	7.519	1	5.370	1	3.700	1	2.443	1	1.521	1	.832	1	.248
0	17.18	0	13.35	0	10.134	0	7.471	0	5.332	0	3.671	0	2.422	0	1.506	0	.819	0	.237
4.69	17.10	4.19	13.28	3.69	10.075	3.19	7.423	2.69	5.295	2.19	3.642	1.69	2.401	1.19	1.490	0.69	.807	0.19	.225
8	17.02	8	13.21	8	10.017	8	7.375	8	5.257	8	3.613	8	2.380	8	1.475	8	.795	8	.214
7	16.93	7	13.14	7	9.959	7	7.328	7	5.220	7	3.585	7	2.359	7	1.459	7	.783	7	.202
6	16.85	6	13.07	6	9.901	6	7.281	6	5.183	6	3.557	6	2.338	6	1.444	6	.771	6	.191
5	16.77	5	13.00	5	9.843	5	7.234	5	5.146	5	3.529	5	2.317	5	1.429	5	.759	5	.179
4.64	16.69	4.14	12.93	3.64	9.785	3.14	7.187	2.64	5.109	2.14	3.501	1.64	2.296	1.14	1.413	0.64	.747	0.14	.167
3	16.61	3	12.86	3	9.728	3	7.140	3	5.072	3	3.473	3	2.276	3	1.398	3	.735	3	.155
2	16.53	2	12.80	2	9.671	2	7.094	2	5.036	2	3.445	2	2.256	2	1.383	2	.723	2	.143
1	16.45	1	12.73	1	9.614	1	7.048	1	5.000	1	3.418	1	2.236	1	1.368	1	.711	1	.131
0	16.37	0	12.66	0	9.557	0	7.002	0	4.964	0	3.391	0	2.216	0	1.354	0	.699	0	.120
4.59	16.29	4.09	12.59	3.59	9.501	3.09	6.956	2.59	4.928	2.09	3.364	1.59	2.196	1.09	1.339	0.59	.687	0.09	.108
8	16.21	8	12.52	8	9.445	8	6.911	8	4.892	8	3.337	8	2.176	8	1.324	8	.675	8	.096
7	16.13	7	12.46	7	9.389	7	6.866	7	4.857	7	3.310	7	2.176	7	1.310	7	.663	7	.084
6	16.05	6	12.39	6	9.333	6	6.821	6	4.822	6	3.283	6	2.136	6	1.295	6	.651	6	.073
5	15.97	5	12.32	5	9.277	5	6.776	5	4.787	5	3.256	5	2.116	5	1.281	5	.640	5	.061
4.54	15.89	4.04	12.26	3.54	9.222	3.04	6.731	2.54	4.752	2.04	3.230	1.54	2.097	1.04	1.267	0.54	.628	0.04	.049
3	15.81	3	12.19	3	9.167	3	6.687	3	4.717	3	3.204	3	2.078	3	1.253	3	.617	3	.036
2	15.74	2	12.12	2	9.112	2	6.643	2	4.682	2	3.178	2	2.059	2	1.238	2	.605	2	.024
1	15.66	1	12.06	1	9.058	1	6.599	1	4.648	1	3.152	1	2.040	1	1.224	1	.593	1	.012
0	15.57	0	12.00	0	9.003	0	6.555	0	4.614	0	3.126	0	2.021	0	1.210	0	.581	0	.000

transmittance relative to the same thickness of distilled water or solvent. For any of these objects Munsell renotation value, V, may be found from Y/Y_o in accord with table 10. Because of their close correlation with the color solid, Munsell renotations are capable of being quickly understood. Thus the renotation $8.3Y\ 8.6_0/10._6$ for the greenish yellow printing-ink specimen indicates from the letter Y that the specimen is a yellow, from the value 8.6_0 that it is a relatively light color, being close to the top of the value scale, 0 to 10, and from the chroma $10._6$, that it is a strong or nongrayish color, being more than 10 Munsell chroma steps away from the gray of the same Munsell value. Munsell renotation hue and chroma serve more adequately for object colors the purposes formerly served by dominant wavelength and purity. Munsell renotation hue correlates significantly better under ordinary conditions of daylight observation with the hue of the perceived color than does dominant wavelength, and Munsell renotation chroma is by far superior to purity in its correlation to saturation. This correlation with the color-perception solid does not, however, necessarily hold under all observing conditions. Ordinarily this printing-ink specimen will be perceived to have a light, strong greenish yellow color, but it is not so perceived under all conditions. If this specimen be viewed next to a brilliant yellow-green area such as is provided by a fluorescent fabric, it will be perceived to take on a darker color of yellowish orange hue and moderate saturation. Thus the lightness, hue, and saturation of the color perception depend upon the surroundings and upon the adaptive state of the eye; and lightness, hue, and saturation are taken correctly to be psychological terms. But the Munsell renotation refers only to the light that is reflected from the specimen and stays constant regardless of these changes in observing conditions. It is therefore a psychophysical characterization of the specimen according to the light reflected from it, just as are luminous reflectance, Y, and chromaticity coordinates, x,y from which it can be derived.

Another advantage of expressing spectrophotometric results in the form of the Munsell renotation is that the amount and kind of the color difference between two specimens can be found immediately from the two renotations in an easily understandable form. Thus, from the hue difference between the two blue printing-ink specimens ($5.6B$ compared to $0.8PB$), the former is seen to be more greenish (less purplish) by about five Munsell hue steps. From the value difference (5.4_6 compared to 5.5_0) the two are seen to be of the same value to the nearest one-tenth Munsell value step; and from the chroma difference (8.5 compared to 9.6) the greenish blue is seen to be more grayish by one Munsell chroma step. Such differences as these in terms of Munsell hue, value, and chroma may be combined into a single index, I, of color difference [115, 117]:

$$I = \frac{C}{5}\ (2\Delta H) + 6\Delta V + 3\Delta C \qquad (8)$$

where C is Munsell chroma, and ΔH, ΔV, ΔC, are the differences between the two colors in Munsell hue, value, and chroma, respectively. The difference between the colors of the two blue printing-ink specimens would be found by this formula as:

$$I = \frac{9.0}{5}\ (2 \times 5.2) + 6 \times 0.04 + 3 \times 1.1$$

$$= 18.7 + 0.2 + 3.3 = 22.2 \text{ units.}$$

These units are of such size that color differences of less than one unit would ordinarily not be of commercial importance; that is, pairs of colors exhibiting such differences would be considered to be commercial matches. Note that these two blue printing-ink specimens are far from being a commercial match; also note that the hue difference is far more important than the chroma difference, and that the value equivalence is well within commercial toleration.

d. ISCC-NBS System

A method devised at the request of the American Pharmaceutical Association and the United States Pharmacopoeial Convention for designating the colors of drugs and chemicals is coming into use for general purposes. The general plan of the method was worked out by the Inter-Society Color Council, and the details were developed at the National Bureau of Standards; the method is therefore referred to as the ISCC-NBS method of designating colors [69, 79]. This method provides a designation for every color perceived as belonging to an object (either an opaque surface, or a light-transmitting layer), and it has been extended to the colors of self-luminous areas by Kelly [77]; see figure 3. The number of color designations was purposely made small, 267, for the sake of simplicity. Since about ten million surface colors can be distinguished by the normal human observer with optimum observing conditions, the ISCC-NBS method falls far short of supplying a different designation for each distinguishable color, or even for all colors (numbering perhaps half a million) considered to be commercially different.

The plan of the method is to divide the surface-color solid (see fig. 19) arbitrarily into 267 compartments, and assign a designation to each in as good conformity as possible to color nomenclature currently used in art, science, and industry. The compartments embracing the black-white axis are given the following designations: black, dark gray, medium gray, light gray, and white. The compartments adjacent to these are given similar designations formed by adding an adjective indicating the hue, such as yellowish white, dark purplish gray, or greenish black. All other compartments take designations consisting of a hue name (red, orange, yellow, green, blue, purple, pink,

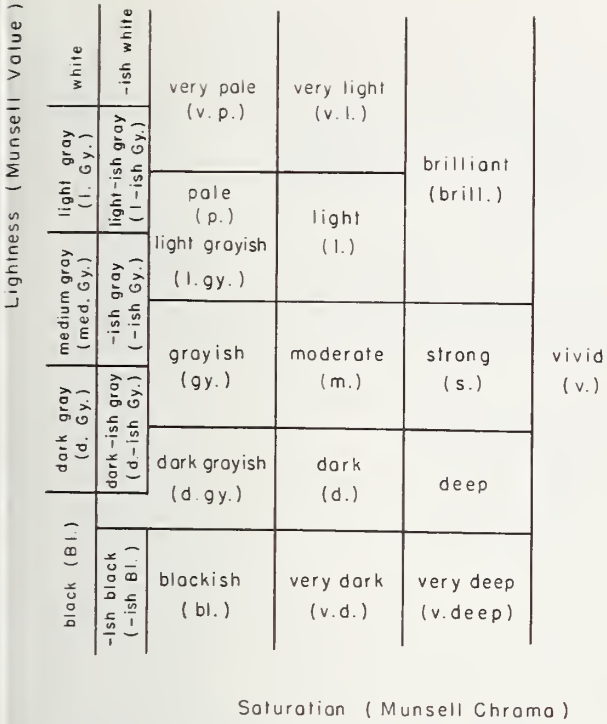


FIGURE 23. *The modifiers used in the color designations of the ISCC-NBS system* [75].

brown, olive) preceded by modifiers (light, dark, grayish, strong) indicating the lightness and saturation of the perceived color. Figure 23 shows the complete list of modifiers used in the ISCC-NBS system; note that *vivid* is a substitute for *very strong; pale*, a substitute for *light grayish; deep*, for *dark* and *strong*, and so on.

The boundaries between the groups of colors known by these designations have been adjusted to accord as closely as possible with common usage and have been expressed in terms of *Munsell Book* notation. Figure 23 also shows these boundaries for colors of purple hue.

The ISCC-NBS designations are not to be considered a substitute for numerical designation of color resulting from application of a suitable colorimetric method, but they do supply a certain precision to verbal color designations that has previously been lacking. Table 11 gives the ISCC-NBS designations of the four printing-ink specimens shown on figure 4 and the ISCC-NBS hue designations have been used throughout the text of this chapter. These color designations are used in the current edition (N.F. VII) of the National Formulary, and they are being inserted into the United States Pharmacopoeia and into a textbook of qualitative analysis [130]. They have been used for describing the colors of building stone [83] and soils [138], and for a considerable variety of research purposes such as the description of mica colors after heat treatment [53]. Kelly [80] has found the Munsell notation for the centroid color

TABLE 11. *Abbreviations for the hue names used in the ISCC-NBS system*

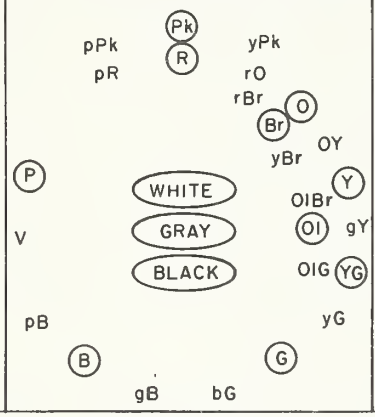
Name	Abbreviation	Name	Abbreviation
Red	R	Purple	P
Reddish orange	rO	Reddish purple	rP
Orange	O	Purplish red	pR
Orange yellow	OY	Purplish pink	pPk
Yellow	Y	Pink	Pk
Greenish yellow	gY	Yellowish pink	yPk
Yellow green	YG	Brownish pink	brPk
Yellowish green	yG	Brownish orange	brO
Green	G	Reddish brown	rBr
Bluish green	bG	Brown	Br
Greenish blue	gB	Yellowish brown	yBr
Blue	B	Olive brown	OlBr
Purplish blue	pB	Olive	Ol
Violet	V	Olive green	OlG

of each compartment assigned an ISCC-NBS color designation and has recommended a system of abbreviations; see table 11.

The ISCC-NBS designations are generally unsuited for use in sales promotion. The method has been approved for color description of drugs and chemicals by the delegates of nine national societies, and it has been recommended for general use by the United States of America Standards Institute [151a].

In 1958 the method was recommended by the Subcommittee on the Expression of Historical Color Usage [60], Inter-Society Color Council, for the statistical expression of color trends. The adaptation of the method for this purpose involved the tabulation of six distinct, but correlated, degrees or levels of accuracy in color description [82]. In the first level the color solid is divided into just 13 parts, ten described by a generic hue name and three neutrals, white, gray and black. In the second level, the color solid is divided into 29 parts, ten of the original 13 parts being further divided and assigned intermediate hue names. In the third level each part of the color solid described by a generic or intermediate hue name and the appropriate modifier descriptive of its lightness and saturation; that is, in the third level of accuracy, the 267 color designations of the ISCC-NBS method are used without abridgement. Level four is illustrated by the Munsell Book of Color [100] with its more than 1,000 uniformly spaced color samples. Level five is illustrated by the interpolated Munsell book notation by which about 100,000 different colors can be reliably specified. Level six, the level in which the greatest accuracy of color identification is possible, is illustrated by two basic methods: the CIE method, and the interpolated Munsell renotations. This degree of accuracy is realizable *only through the measurement of the color with a spectrophotometer*, and is equivalent to the division of the color solid into about 5,000,000 parts. Figure 24 gives further information about the six levels: (A) level of fineness of color identification, (B) number of divisions of color solid, (C) type of color designation, (D) example of color designation, and (E) alternate color-order systems usable in that level.

Through the cooperation of the Inter-Society

A	1	2	3	4	5	6
B	13	29	267	943 to 7056	About 100,000	About 5,000,000
C	Generic hue names and neutrals (circled)	All hue names and neutrals	ISCC-NBS All hue names and neutrals with modifiers (NBS C553)	Color-order system (Collections of color standards sampling the color solid systematically)	Visually interpolated Munsell notation from Munsell Book of Color	CIE (x, y, Y) x = 0.395 y = 0.382 Y = 35.6 or Interpolated Munsell renotation
D	brown	yellowish brown	light yellowish brown (centroid No.76)	10YR 6/4 Munsell 1548	$9\frac{1}{2}$YR $6.4/4\frac{1}{4}$	9.6YR $6.4_5/4.3$
E			TCCA (9th Std.) 216 70128 TTC-595 (Ist Ed) 187 F2310 H.C.C. 800 H407	M&P (Ist Ed) 7056 12H6 Plochere 1248 180 O 5-d Ridgway 1115 XXIX 13" b C.H.M. (3rd Ed) 943 3gc		

Direction for increased fineness of color definition

Direction for statistical expression of color usage (roll up)

A. Level of fineness of color identification
B. Number of divisions of color solid
C. Type of color designation
D. Example of color designation
E. Alternate color-order systems usable in that level

FIGURE 24. *Basis of Universal Color Language.*

Color Council and the Tobey Color Card Co., St. Louis, Mo., the National Bureau of Standards commenced issuing the ISCC-NBS Centroid Color Charts [109] in February 1965. Each set consists of 18 charts displaying samples closely approximating 215 of the 267 centroid colors, and fair approximations of 36 others, making 251 color samples in all. The samples are in the form of one-inch squares of paper coated with glossy-finish paint affixed to a variable gray background so that each color is on a neutral background of approximately its own lightness. The Munsell renotation of each color sample is supplied in the cover pages to the charts. Duplicates of each of the 251 ISCC-NBS centroid colors can be obtained in 9 by 12 in sheets from the Munsell Color Company, Inc. of Baltimore, Md. These centroid colors have been used as standards for a wide variety of purposes. They facilitate color description at level three, and have found their most important application so far in the recording of data for establishing trends of public acceptance of color in various lines of merchandise.

5. One-Dimensional Color Scales

5.1. General Principles

There are many tests analogous to the comparison of a solution of an unknown amount of a constituent with a series of suitably prepared standard solutions to find the concentration of the specimen. In these tests the colors of the unknowns exhibit a one-dimensional change with concentration; and, although this change may be complicated in terms of luminous transmittance and chromaticity coordinates [102], a suitably spaced series of standards over this range of colors will yield the desired concentration either by actual match with one of the standards, or by visual interpolation among them. Such a series of standards is said to constitute a *color scale*. The ideal material from which to make the standards is the constituent of the unknown itself; in this way there is guaranteed not only a perfect color match at some point along the scale, but also a perfectly nonmetameric match so that variation of the illuminant or individual-observer variations are generally unimportant.

However, if the unknown is impermanent, it may become necessary to try to duplicate the desired

colors in a more nearly permanent medium. Glass is a frequent choice because of its generally superior permanence. Some degree of metamerism has then to be tolerated because the standards have coloring constituents not a perfect spectral match for the unknown. It is also rare that a perfect job of color matching for any standard illuminant and observer is done. The observer is then faced with what is often a difficult, and sometimes an impossible, task. He must estimate the position of the unknown color on the scale, and often it will seem to him that the unknown color is not equal to any of the standard colors, nor intermediate between any two of them. The concepts in terms of which the observer perceives these color differences then come into play. If he judges the color difference between the two luminous areas presented to him in terms of hue, brightness, and saturation, as is fairly common, he could estimate the position of the unknown color on the color scale as the point on the scale yielding the same hue, or as that yielding the same brightness or the same saturation; or he could try to estimate the point on the scale yielding the closest color match; or he could disregard brightness differences and try to estimate the point on the scale yielding the closest chromaticity match. The determination becomes an estimate based on what criterion of equivalence is used by the observer, and it depends upon his mental capability in an essentially indescribable way. In spite of these drawbacks, a good color scale is a useful time-saver, as long as it is not used in attempts to provide a one-dimensional solution to what is essentially a multidimensional problem.

Judgments of position on the color scale according to equality of brightness can be expected to correspond to luminous transmittance. Judgments according to equality of hue agree well with the Munsell renotation hue; loci of constant hue for ordinary conditions of observation are indicated on the (x,y) diagram by the curved lines of figure 3 separating the areas corresponding to the various hue names. Judgments according to equality of saturation agree well with Munsell renotation chroma (see fig. 22). If there is only secondary brightness variation along the scale, judgments of nearest chromaticity match may be found approximately by taking the shortest distance on a uniform-chromaticity-scale diagram [14, 23, 55, 66, 141, 142]. Figure 8 shows the uniform-chromaticity-scale triangle according to Judd [66]. If the chromaticity coordinates of a color are (x,y) in the standard CIE system, the color would have chromaticity coordinates (r,g) in this uniform-chromaticity-scale triangle in accord with eq (5). On this diagram the ellipses of figure 7 would be equal tangent circles. If there is primary variation of both luminance and chromaticity, no reliable way of estimating the nearest color match has yet been developed. According to the OSA Colorimetry Committee [24], "The complete experimental clarification of this problem is one of the major programs yet to be undertaken in the field of colorimetric research."

5.2. Color Temperature

Perhaps the most widely used one-dimensional color scale is that of color temperature for classifying light sources. The *color temperature* of a light source is the temperature at which the walls of a furnace must be maintained so that light from a small hole in it will yield the chromaticity of the source to be specified. The color scale thus consists of the series of lights producible by closed-cavity radiation and is specified by temperature on the absolute scale (degrees Kelvin). Working standards of color temperature may consist of an incandescent lamp operating at a fixed voltage combined with a series of amber or blue glasses, like the Lovibond blue glasses; but by far the most common way of producing these chromaticities over moderate ranges of color temperature is by variation of the voltage applied to an incandescent lamp. The locus of these chromaticities (the so-called Planckian locus) is shown on figures 7 and 8. If the chromaticity of the light source is close to, but not exactly equal to, any of the Planckian chromaticities, still it is possible to correlate a color temperature with the source by taking the nearest chromaticity match. Figure 25 shows this correlation [67, 81]. The isotemperature lines, which cut the Planckian locus at varying angles, are all such as to be perpendicular to this curve on figure 26. The x,y and u,v chromaticity coordinates of a number of Planckian radiators (c_2 = 14,380) and standard sources in the 2° and 10° observer systems have been computed by Nimeroff [123]. These are listed in table 12. Since color temperature specifies only the chromaticity of a light, there are many spectral compositions corresponding to the same color temperature. Color temperature of a sonrce is therefore an incomplete and unreliable indication of the rendering of the colors of objects illuminated by it or of the photographic effect of the source. To make color temperature a meaningful and useful basis for comparing two lights it must also be known that they are spectrally similar. Thus, incandescent lamps may be usefully intercompared by means of color temperature, and fluorescent lamps with about the same admixture of mercury spectrum may also be so intercompared, but incandescent lamps may not be intercompared with fluorescent.

5.3. Grading Systems

When the degree of refinement and quality of such products as oils, rosin, and sugars may be characterized on similar one-dimensional color scales which range from dark red through yellow to perfectly colorless, the development of a complex color specification for these products is redundant. These perceived color changes correlate

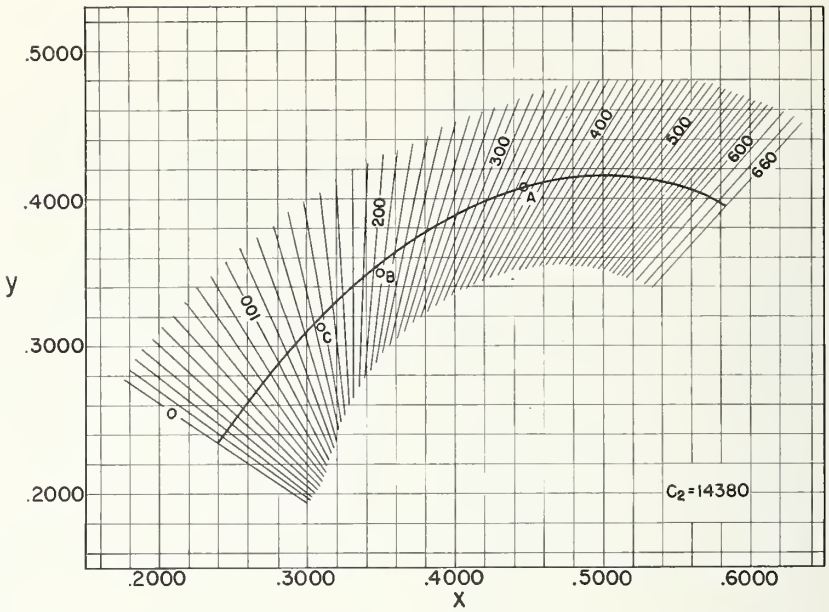


FIGURE 25. *Isotemperature lines for evaluation of correlated color temperatures of non-Planckian colors.*

These lines are perpendicular to the Planckian locus on figure 26.

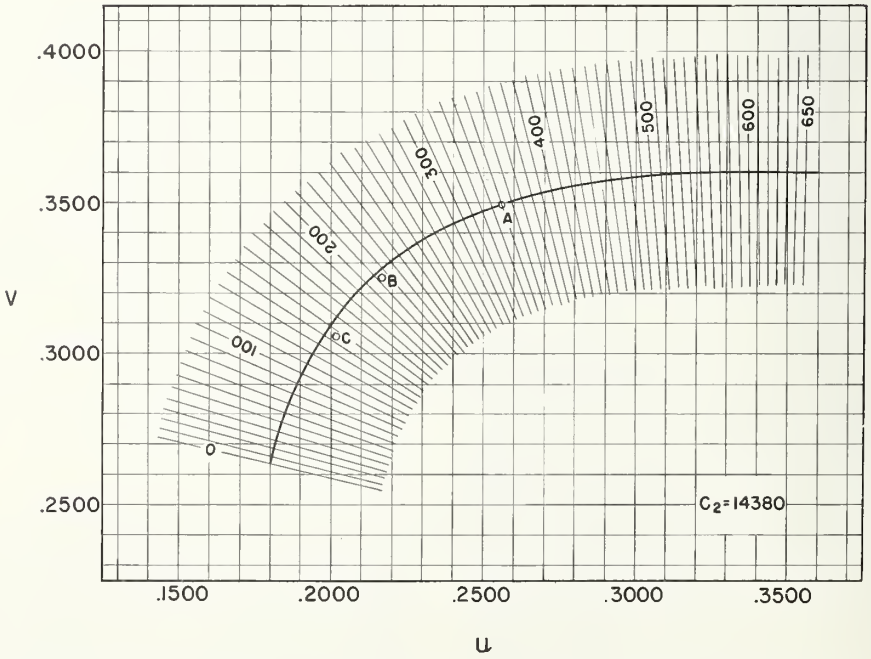


FIGURE 26. *Isotemperature lines of correlated color temperature shown perpendicular on the Planckian locus on the* u,v *chromaticity diagram* [81].

TABLE 12. *Chromaticity coordinates of Planckian radiators (c_2 = 14,380) and standard sources*

Source (°K)	2° Observer system				10° Observer system			
	x_2	y_2	u_2	v_2	x_{10}	y_{10}	u_{10}	v_{10}
1000	0.6526	0.3447	0.4482	0.3547	0.6474	0.3504	0.4383	0.3555
2000	.5266	.4133	.3050	.3590	.5300	.4122	.3078	.3591
3000	.4368	.4041	.2506	.3476	.4403	.4026	.2532	.3476
4000	.3804	.3767	.2251	.3344	.3827	.3759	.2266	.3345
5000	.3450	.3516	.2115	.3231	.3464	.3515	.2119	.3234
6000	.3220	.3317	.2034	.3141	.3227	.3323	.2030	.3146
7000	.3063	.3165	.1982	.3070	.3066	.3176	.1973	.3078
8000	.2952	.3048	.1947	.3014	.2951	.3063	.1934	.3022
9000	.2869	.2956	.1922	.2970	.2866	.2975	.1905	.2979
10000	.2806	.2883	.1904	.2933	.2802	.2905	.1884	.2944
∞	.2399	.2342	.1800	.2636	.2394	.2366	.1786	.2648
A	.4476	.4075	.2560	.3495	.4512	.4059	.2588	.3496
B	.3485	.3517	.2137	.3235	.3498	.3527	.2138	.3241
C	.3101	.3163	.2009	.3073	.3104	.3191	.1994	.3086
E	.3333	.3333	.2105	.3158	.3333	.3333	.2105	.3158

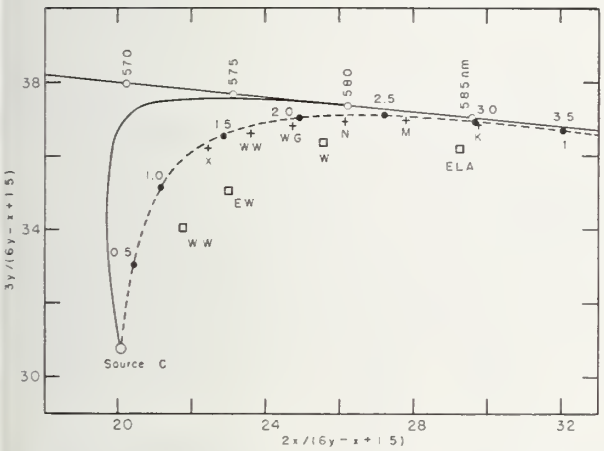


FIGURE 27. *Chromaticities of some standards for grading petroleum products (solid circles), rosin (crosses), and honey (squares).*

The solid curve is generated by an ideal absorption-band sweeping across the spectrum from the short-wave to the long-wave band [40]. The standards have broad absorption bands of different sharpness.

with a shift in the wide absorption band toward the ultraviolet as brown pigments are removed on refining. The differences in scales among these products correspond to differences in sharpness of the absorption band, with oils possessing the sharpest bands, rosins next, and sugars least sharp. Figure 27 compares, on the 1960 CIE-UCS [80] chromaticity diagram, the chromaticity locus generated by varying an ideal absorption band (40) with a few color standards used for petroleum (solid dots), rosin (crosses), and sugar (squares) products.

It is sometimes convenient to state the color of a product in terms of its equivalent on another scale and modified by an adjective that describes the off-locus position. Thus U. S. Rosin Standard X (extra water white), which is equivalent to ASTM petroleum color 1.4 but slightly on the purple side, will be designated 1.4*p*. The letter g will be used to designate departures toward green.

a. Petroleum Products

For more than 30 years the color of lubricating oils and petrolatum has been graded by comparison with the colors of 12 glass standards [7]. The petroleum product in a 33-mm layer and the standard are illuminated by artificial daylight produced by combining an incandescent lamp of color temperature approximately 2750 °K with a filter of Corning Daylite glass specially selected to have spectral transmittances within specified tolerances and further to have for standard source A luminous transmittance, τ_v, and chromaticity coordinates, x,y,z, within the limits:

τ_v	0.107	to	0.160
x	.314	to	.330
y	.337	to	.341
z	.349	to	.329

The specimen holder, the magazine containing the glass color standards, the artificial daylight assembly, and a viewing diaphragm defining the direction of view are mounted together to form a portable instrument known as the Union colorimeter.

Table 13 gives the Lovibond analysis of the glass color standards (6), the luminous transmittance, τ_v, and chromaticity cordinates, x,y, for source C [141], the color names used by the National Petroleum Association, and the nearest chromaticity match on the new ASTM color scale, which was adopted in 1957. This new scale consists of a set of glasses which define a scale that is closer to the range of petroleum colors and with improved step uniformity [48, 72].

Table 14 gives for these glasses the luminous transmittances and chromaticity coordinates for source C in the defining UCS (r,g)-system as well as in the standard CIE (x,y)-system. Also listed are the luminous transmittance tolerances for the glass standards and their equivalents on three other scales.

The grading of naphthas, kerosines, and so on, has for many years been carried out by comparison of the color of rather thick layers (up to 20 in) of the refined oil with the colors of a set of three color standards made of yellowish glass. The Saybolt

TABLE 13. *ASTM Union colorimeter standards, Lovibond analysis, luminous transmittance, τ_v, chromaticity coordinates, x,y, for CIE source C, NPA color names, nearest chromaticity match on the new ASTM color scale, and Munsell Book notations*

Union color Number	Lovibond analysis			Luminous transmittance, τ_v	Chromaticity coordinates		National Petroleum Association names	Nearest chromaticity match on ASTM scale	Munsell Book notations
	Red	Yellow	Blue		x	y			
1.0	0.12	2.4	---------	0.751	0.349	0.382	Lily white	0.55g	10Y 9.6/3.8
1.5	.60	8.0	---------	.654	.400	.446	Cream white	1.05g	8Y 8.8/8.0
2.0	2.5	26.0	---------	.443	.472	.476	Extra pale	1.85p	3Y 7.2/12
2.5	4.6	27.0	---------	.365	.498	.457	Extra lemon pale	2.35p	10YR 6.0/12
3.0	6.9	32.0	---------	.287	.525	.440	Lemon pale	2.85p	7YR 5.7/13.5
3.5	9.4	45.0	---------	.211	.556	.423	Extra orange pale	3.5p	4YR 5.0/14
4.0	14.0	50.0	0.55	.096	.591	.400	Orange pale	4.2p	2YR 3.2/12
4.5	21.0	56.0	.55	.065	.620	.376	Pale	4.9	1YR 2.6/12
5.0	35.0	93.0	---------	.036	.653	.347	Light red	5.8	1YR 2.2/9.5
6.0	60.0	60.0	.55	.017	.676	.323	Dark red	6.5	1YR 1.3/8.5
7.0	60.0	106.0	1.8	.0066	.684	.316	Claret red	6.75	1YR 0.6/4.5
8.0	166.0	64.0	---------	.0020	.714	.286	----------	7.8	1YR 0.2/1.4

TABLE 14. *ASTM color standards for petroleum products. Chromaticity coordinates in the CIE and UCS system with tolerances, luminous transmittances with tolerances, and nearest chromaticity matches on the Union scale, the 35-yellow plus N-red Lovibond scale used for vegetable oils, and the Gardner scale used for paint vehicles*

ASTM color number	Chromaticity coordinates				Luminous transmittance	Nearest chromaticity match on:		
	CIE-system		UCS-system*			Union color scale	Lovibond 35Y + NR scale N	Gardner color scale
	x	y	r	g	τ_v			
0.5	0.3469	0.3739	0.462	0.473	0.86 ± 0.06	0.9 p	----------	5.3p
1.0	.3945	.4359	.489	.475	.77 ± .06	1.45p	----------	8.0p
1.5	.4473	.4781	.521	.454	.67 ± .06	1.8 g	1.5p	10.0
2.0	.4876	.4826	.552	.442	.55 ± .06	2.15g	3.3	11.4g
2.5	.5192	.4711	.582	.416	.44 ± .04	2.65g	5.3g	12.3g
3.0	.5437	.4517	.611	.388	.31 ± .04	3.1 g	7.7g	13.2
3.5	.5648	.4308	.640	.359	.22 ± .04	3.5 g	10.3g	13.9
4.0	.5855	.4102	.671	.328	.152 ± .022	3.85g	13.5g	14.8
4.5	.6053	.3906	.703	.296	.109 ± .016	4.2 g	17.5g	15.6
5.0	.6257	.3742	.736	.264	.081 ± .012	4.55	23 g	16.3
5.5	.6435	.3565	.770	.230	.058 ± .010	4.85	30	17.2
6.0	.6604	.3395	.805	.195	.040 ± .008	5.3	41	18.1
6.5	.6765	.3235	.841	.159	.026 ± .006	6.0	57	----------
7.0	.6913	.3086	.877	.123	.016 ± .004	7.2	82	----------
7.5	.7059	.2941	.915	.085	.0081 ± .0016	7.7	----------	----------
8.0	.7204	.2796	.956	.044	.0025 ± .0006	----------	----------	----------

*Tolerances on chromaticity coordinates, r,g,b ($b = 1-r-g$), are ±0.006.

chromometer is a device for carrying out this comparison. It consists of an artificial daylight lamp meeting the same requirements as given above for the ASTM Union colorimeter, a graduated tube for the oil specimen, a holder for the glass color standards, and a prism eyepiece to bring into juxtaposition the two fields to be compared. There is an open ungraduated tube beneath the holder for the glass standards that serves to duplicate to a degree on the standard side of the instrument the effect of light multiply reflected within the specimen tube. The glass standards consist of two whole disks and one "one-half" disk; these standards must have luminous transmittances, τ_v, and chromaticity coordinates, x,y, for source C, as follows:

	Whole disks	One-half disks
τ_v ----------	0.860 to 0.865	0.888 to 0.891
x ----------	.342 to .350	.327 to .331
y ----------	.367 to .378	.344 to .350
z ----------	.272 to .291	.319 to .330

Oils having colors closely resembling that of distilled water are graded on the Saybolt chromometer by comparison with the half disk; more yellowish oils by one or two whole disks. The depth of oil yielding the closest color match with the glass disk is found by a prescribed procedure [5],

and the color of the oil specimen is designated by a number defined from the disc used and the required depth of oil. Table 15 gives this definition [5] in terms of Saybolt number, Union color, u, and ASTM color, a.

In spite of the fact that the prism eyepiece brings the fields to be compared into juxtaposition and so facilitates the detection of differences in brightness, the change in luminous transmittance with thickness of these refined oils is so slight that it is much less easily detected than the corresponding chromaticity change. The settings of the Saybolt chromometer therefore probably depend essentially on nearest chromaticity match. If the oil sample is turbid, however, not even an approximate match can be obtained, and the method may be inapplicable. In these cases a thickness of the specimen yielding a chromaticity match is much darker than the standard, and no reliable setting of depth of sample yielding nearest match can be found.

Figure 28 shows a portion of the (x,y) chromaticity diagram on which have been plotted the rectangles corresponding to the chromaticity tolerances for Saybolt half disks and whole disks (see above). It will be noted that the tolerances are fairly wide so that variations of ±10 percent of the chromaticity change caused by introduction of the disk into the daylight beam are permissible. The relation between the Union color numbers, u, and

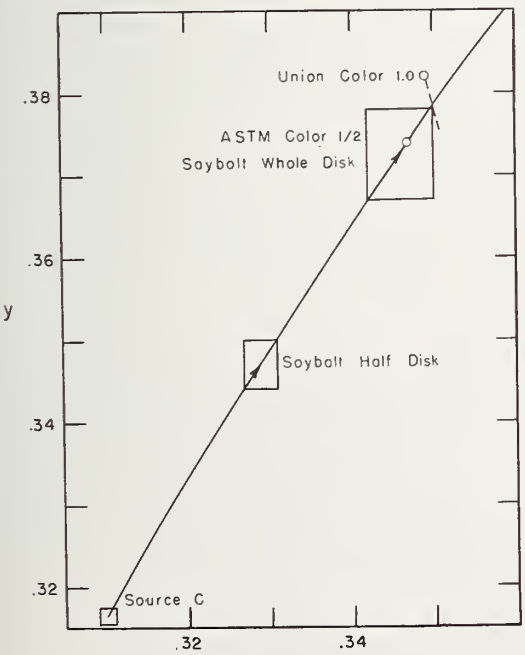


FIGURE 28. *Chromaticities of the Saybolt color standards compared to that for Union No. 1 color, and to those of the yellowish wedge of the chromaticity-difference colorimeter (arrows).*

TABLE 15. *Saybolt numbers and corresponding Union color, u, and ASTM color, a, defined in terms of specimen depth and disk number*

Number of disks n	Depth of oil in	Saybolt number	Union color u	ASTM color a
½	20.00	+30	0.030	0.015
½	18.00	+29	.033	.017
½	16.00	+28	.037	.019
½	14.00	+27	.042	.022
½	12.00	+26	.049	.025
1	20.00	+25	.059	.030
1	18.00	+24	.066	.034
1	16.00	+23	.074	.038
1	14.00	+22	.084	.043
1	12.00	+21	.098	.051
1	10.75	+20	.110	.056
1	9.50	+19	.124	.064
1	8.25	+18	.143	.074
1	7.25	+17	.163	.084
1	6.25	+16	.189	.097
2	10.50	+15	.225	.116
2	9.75	+14	.243	.125
2	9.00	+13	.262	.135
2	8.25	+12	.286	.147
2	7.75	+11	.305	.157
2	7.25	+10	.326	.167
2	6.75	+ 9	.350	.180
2	6.50	+ 8	.364	.187
2	6.25	+ 7	.376	.194
2	6.00	+ 6	.393	.202
2	5.75	+ 5	.411	.211
2	5.50	+ 4	.430	.221
2	5.25	+ 3	.450	.231
2	5.00	+ 2	.474	.243
2	4.75	+ 1	.498	.256
2	4.50	0	.526	.270
2	4.25	− 1	.557	.286
2	4.00	− 2	.592	.304
2	3.75	− 3	.631	.324
2	3.625	− 4	.652	.335
2	3.50	− 5	.677	.347
2	3.375	− 6	.701	.360
2	3.25	− 7	.727	.374
2	3.125	− 8	.758	.388
2	3.00	− 9	.789	.405
2	2.875	−10	.823	.422
2	2.75	−11	.860	.441
2	2.625	−12	.901	.462
2	2.50	−13	.947	.486
2	2.375	−14	.996	.511
2	2.25	−15	1.05	.539
2	2.125	−16	1.11	.571

the Saybolt color expressed in terms of number of whole disks, n, and depth, D, in inches,

$$u = \frac{1.182n}{D},$$

may be derived by reference to figure 28. Two vectors, each corresponding to 300 wedge units define the steps from source C to Saybolt ½ disk to Saybolt whole disk, or 600 wedge units in all. The chromaticity point representing Union Color No. 1 does not fall exactly on the extended line. The intersection of the dotted line and the extended line gives the closest chromaticity match, and shows that the chromaticity of Union Color No. 1 approximates 1.10 Saybolt disk. An oil of Union Color No. 1 will therefore match 1.00 Saybolt disk in a thickness of 33.0/1.10 or 30.0 mm (1.182 in) and will match two whole disks in a thickness of 60.0 mm (2.365 in).

The corresponding relation for ASTM color number, a, is

$$a = 0.607n/D ,$$

based on the facts that the glass that defines ASTM color number 0.5 is closely equivalent to 1.03 Saybolt disk (as shown on fig. 28) and that thickness on the ASTM scale has been changed to 1.25 in from 33.0 mm [7].

b. Vegetable Oils

In addition to the brown pigments found in petroleum products, edible vegetable oils (corn, cottonseed, olive, peanut, rapeseed, sesame, soybean) usually contain some chlorophyll. Although metamerism exists between the oils and Lovibond glasses [100], the oils are graded, for commercial purposes, by the number (N) of Lovibond red units, added to Lovibond 35-yellow, that are required to match a 5.25-in layer of the oil. Table 16

TABLE 16. *Lovibond 35-yellow plus N-red scale used to color-grade edible vegetable oils. CIE chromaticity coordinates, x,y, and nearest chromaticity match on the ASTM and Gardner color scales*

Lovibond 35Y + NR scale	Chromaticity coordinates		Nearest chromaticity match on:	
	x	y	ASTM color scale	Gardner color scale
0	0.441	0.521	1.3g	9.2g
1	.455	.508	1.4g	9.9g
2	.469	.496	1.6g	10.7g
3	.482	.485	1.9	11.3
4	.494	.474	2.1p	11.7
5	.506	.464	2.4p	12.2p
6	.517	.455	2.6p	12.6p
7	.527	.446	2.8p	12.9p
8	.537	.438	3.0p	13.2p
9	.546	.431	3.3p	13.6p
10	.555	.424	3.5p	13.8p
12	.570	.412	3.8p	14.4p
14	.583	.402	4.0p	14.9p
16	.594	.393	4.3p	15.3p
18	.603	.385	4.5p	15.6p
20	.612	.378	4.7p	15.9p
24	.625	.367	5.1p	16.5p
28	.636	.358	5.4p	17.0p
32	.645	.351	5.6	17.4
36	.652	.345	5.8	17.7
40	.658	.340	6.0	18.0
45	.665	.334	6.1	----------
50	.670	.329	6.3	----------
60	.679	.321	6.6	----------
70	.685	.314	6.8	----------
80	.691	.309	7.0	----------
90	.695	.305	7.1	----------
100	.699	.301	7.3	----------

shows for values of N from 0 to 100 on the $35Y + NR$ scale the CIE source C chromaticity coordinates, x,y, and the nearest chromaticity match on the ASTM scale and on the Gardner scale (the Gardner scale is described in the next section).

Thomson [50] showed that spectral transmittance of oil at wavelengths 460, 550, 620, and 670 nm τ_{460}, τ_{550}, τ_{620}, and τ_{670}) can be expressed in an index that gives excellent correlation with color grading by the Lovibond red units. This index,

$$1.29 \log (1/\tau_{460}) + 69.7 \log (1/\tau_{550}) + 41.2 \log (1/\tau_{620}) - 56.4 \log (1/\tau_{670}),$$

has been found useful for commercial grading of vegetable oils.

c. Rosins and Paint Vehicles

Gum rosin has been graded by color for more than 50 years. Up to 1914 the color standards were made of rosin itself in spite of the relative impermanence of its color, and from 1914 to 1936 standards composed of combinations of Lovibond glasses were used. Brice [15] has described the selection of the present twelve official standards composed of two components of colored glass combined with one component of clear glass, all three cemented together with Canada balsam. The various combinations are given letter designations denoting the grades of rosin delimited by them and have legal status under the Naval Stores Act. The cemented face of the clear glass in each combination is fine ground so as to duplicate the slight turbidity characteristic of molded samples of rosin, which commonly contain traces of fine dirt. The chromaticity spacing was adjusted by means of the uniform-chromaticity-scale triangle of figure 8 so as to progress regularly from small steps for yellow rosins to steps of about four times the initial size for reddish orange rosins. Osborn and Kenyon have studied rosin colors spectrophotometrically [124]. Table 17 gives the names associated with letter designations, luminous transmittance, T, and chromaticity coordinates, x,y, for CIE source C, together with the nearest chromaticity match on the ASTM, $35Y + NR$, and Gardner color scales.

Many special color scales have been set up for the specification of paint vehicles (varnishes, linseed oil, tung oil, etc.). A solution of nickel sulfate and iodine [16] is used to define the darkest color permissible for spar varnish. A color comparator having eighteen glass color standards made by Hellige, Inc., has been used for similar purposes. The Pfund color grader compares a variable thickness of the unknown specimen with a variable thickness of an amber colored glass [131]. The standard is wedge shaped, and the cell for the specimen is likewise wedge shaped. The Parlin (or Cargille) color standards consist of a set of thirty-five solutions. The first ten are Hazen platinum-cobalt solutions [52], developed originally to measure the color of natural water and still used for that purpose under the name of APHA (American Public Health Association) standards [4]. The remainder of the Parlin color standards are caramel solutions. They have been adopted by the ASTM (Designation D365–39) for testing the color of soluble nitrocellulose base solutions. The Pratt and Lambert color standards are varnish mixtures calibrated against the Pfund color grader. The DuPont colorimeter employs six plates as color standards, together with a wedge of the same glass permitting a continuous variation of color between the standards. The Gardner color standards consist of eighteen combinations of the red and yellow Arny solutions. Gardner has determined the Arny and

Table 17. *U.S. standards for rosin. Luminous transmittance, τ_v, chromaticity coordinates, x,y, for CIE source C, and nearest chromaticity match on the ASTM color scale, the 35-yellow plus N-red Lovibond scale, and the Gardner color scale*

Designation		Luminous transmittance	Chromaticity coordinates		Nearest chromaticity match on:		
Letter	Name	τ_v	x	y	ASTM color scale	35Y + NR scale N	Gardner color scale
X	Extra water white	0.609	0.4339	0.4663	1.4p	1.2p	9.7p
WW	Water white	.531	.4579	.4732	1.7p	2.2p	10.8p
WG	Window glass	.466	.4785	.4741	1.9p	3.2p	11.4p
N	Nancy	.396	.5001	.4704	2.2p	4.4	11.9
M	Mary	.322	.5212	.4619	2.6p	6.0g	12.5
K	Kate	.245	.5430	.4483	3.0p	7.8g	13.2p
I	Isaac	.178	.5649	.4310	3.5	10.3g	13.9
H	Harry	.1114	.5879	.4102	4.0	13.8g	14.9
G	George	.0723	.6116	.3874	4.6	18.7g	15.7
F	Frank	.0398	.6364	.3632	5.3	27 g	16.9
E	Edward	.0131	.6640	.3358	6.1	44	18.2
D	Dolly	.0021	.6943	.3057	7.2	88	----------

Table 18. *The Gardner color standards. Chromaticity coordinates, x,y, for CIE source C, nearest chromaticity matches on the ASTM color scale and the 35-yellow plus N-red Lovibond scale, and both nearest lightness matches and nearest chromaticity matches on the Union color scale*

Gardner color number	Chromaticity coordinates		Nearest chromaticity match on:			Nearest lightness match on Union color scale from Gardner (30)
	x	y	35Y + NR scale N	ASTM color scale	Union color scale	
1	0.313	0.324	----------	0.06g	0.12	−1
2	.318	.333	----------	.12g	.22	1+
3	.322	.339	----------	.18g	.33	1–1½
4	.330	.349	----------	.28	.51p	1–1½
5	.342	.371	----------	.47g	.86	1½–2
6	.354	.387	----------	.6	1.05p	1½–2
7	.372	.410	----------	.8	1.25p	2
8	.394	.440	----------	1.0 g	1.45	2–2½
9	.419	.462	0.7p	1.2	1.6	2½
10	.444	.475	1.4p	1.5	1.75	2½–3
11	.467	.482	2.3p	1.75	1.9 g	3–3½
12	.504	.471	4.6	2.3 p	2.45g	3½–4
13	.535	.454	7.1g	2.85	3.0 g	4–4½
14	.567	.427	10.6g	3.55	3.55g	4½+
15	.590	.407	14.2g	4.1	3.9 g	−5
16	.619	.380	21 g	4.8	4.45	6
17	.638	.361	28 g	5.35	4.75	7–8
18	.659	.340	40	6.0	5.25	8+

Lovibond specification for the nearest matches for all the above-mentioned sets of color standards and has also obtained the nearest equivalents in terms of potassium dichromate solutions [31]. From these nearest equivalents it is possible to express color specifications given by any of these means in terms of any other of them, as shown in Table 18 which gives the chromaticity coordinates, x,y, of each of the eighteen Gardner standards for CIE source C, nearest chromaticity match on the Lovibond 35Y + NR scale, the ASTM scale and the Union color scale, together with the nearest lightness match on the Union scale. The Gardner standards define a locus of chromaticities that agrees closely with that of the ASTM standards with only 6 of the 18 standards departing by more than 0.001 as indicated by the letters "p" and "g".

The British Paint Research Station has recommended [151] combinations of Lovibond glasses for color-grading oils and varnishes. Some of the combinations involve colorless or blue glasses to be combined with the oil to match red and yellow glasses, and a device facilitating the setting up of such combinations is also recommended. The Lovibond glasses are mounted in a slide, and the two photometric fields to be compared are brought into juxtaposition by mirrors.

d. Sugar Products

For regulatory purposes, the classification of sugar products (honey, maple syrup, molasses, sugarcane) is conducted visually by comparison with glass chromaticity standards. The source C chromaticity coordinates, x,y, are listed in table 19

Table 19. *U.S. Department of Agriculture chromaticity standards for sugar products. Chromaticity coordinates, x,y, for CIE source C, and nearest chromaticity matches on the ASTM, the Lovibond 35Y + NR, and the Gardner color scales*

Designation and Name	Chromaticity coordinates		Nearest chromaticity match on		
	x	y	ASTM color scale	Lovibond 35Y + NR scale	Gardner color scale
Liquid sugar					
WW(ls), water white	0.3381	0.3529	0.35p	----------	4.2p
W(ls), white	.3772	.3937	.75p	----------	7.0p
EW(h), light amber	.4169	.4245	1.3 p	----------	9.4p
Extracted honey					
WW(h), water white	0.3818	0.3982	0.8 p	----------	7.1p
EW(h), extra white	.4169	.4245	1.3 p	----------	9.4p
W(h), white [1]	.4786	.4531	2.1 p	4.0p	11.7p
ELA(h), extra light amber [2]	.5317	.4450	2.9 p	7.3	13.1p
LA(h), light amber [3]	.6141	.3845	4.7	19.5g	15.85
A(h), amber	.6711	.3279	6.3	51	----------
Maple sirup					
LA(ms), light amber	0.4947	0.4509	2.35p	4.9p	12.1p
MA(ms), medium amber	.5567	.4352	3.35	9.4g	13.7
DA(ms), dark amber	.6041	.3943	4.4	16.8g	15.5
Sugarcane molasses					
No. 1	0.5183	0.4489	2.7 p	6.3	12.7p
No. 2	.6301	.3691	5.15	25 g	16.6
No. 3	.6815	.3179	6.65	64	----------

[1] Same as Sugarcane Sirup No. 1.
[2] Same as Sugarcane Sirup No. 2.
[3] Same as Sugarcane Sirup No. 3.

for the glasses developed by Brice [15] and adopted by the U. S. Department of Agriculture for this purpose. The nearest chromaticity matches on three one-dimensional color scales (ASTM, 35Y+NR, and Gardner) are also given for each standard.

Increasing use is being made of one-dimensional photometric methods [36, 37, 162, 163] for more precise designation of sugar colors. Deitz [26] has developed a one-dimensional specification that is defined as the color difference between the solvent and the sugar solution. The color difference is evaluated in Adams chromatic-value space [2] and is expressed in terms of a so-called NBS unit of sugar color. Measurements of the attenuancy at two wavelengths (420 nm and 560 nm) are used in this evaluation to a good approximation. Brice [16] developed a similar one-dimensional specification that is based on chromaticity differences expressed in MacAdam units [19].

6. Limitations of the Methods

6.1. Spectrophotometric Colorimetry

Spectrophotometric colorimetry, the most fundamental color-measurement technique, suffers from two major sources of difficulty. First, the 1931 standard observer color-matching data were obtained for only some 17 observers [61] under 2° angular subtense and quite low illumination levels. Extrapolation of the use of this system for all observers and conditions may require justification. Secondly, there is usually some lack of precision and accuracy of spectral measurement of exitance, transmittance, and reflectance. Spectrophotometrs are subject to errors of wavelength and photometric scales, stray-energy and slit-width effects, multiple reflections, and errors due to samples that are temperature dependent, wedge-shaped, and translucent. A compilation of these errors and recommendation for their accounting has been published by Gibson [35].

Although the CIE standard observer system for colorimetry has been generally satisfactory since its recommendation in 1931, there have been some difficulties with the system. Most notable is its inability to resolve the chromaticity difference of anatase and rutile titanium dioxide [62, 71] usually viewed with angular subtense much greater than 2°. These difficulties have led the CIE to seek a new system (the 1964 CIE supplementary observer) through the Stiles and Burch [146] and Speranskaya [144] color-matching data of some 75 observers for a 10° photometric field. A complete analysis of these data permits the establishment of estimates of within- and between-observer variability of the system. Estimation of the variability in the color-matching functions, r(red), g(green), and b(blue) requires information about the between-observer deviations, σ_r, σ_g, σ_b, the within-observer deviations, ${}_w\sigma_r$, ${}_w\sigma_g$, ${}_w\sigma_b$, and the covariances, σ_{rg}, σ_{rb}, σ_{bg}. The covariances, σ_{ij}, are the products of the correlation coefficients, ρ_{ij}, and the deviations, σ_i, σ_j; thus $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ where i and j refer to the color-matching functions. Figure 29 shows the between-observer deviations and the correlation coefficients determined by Nimeroff [123]

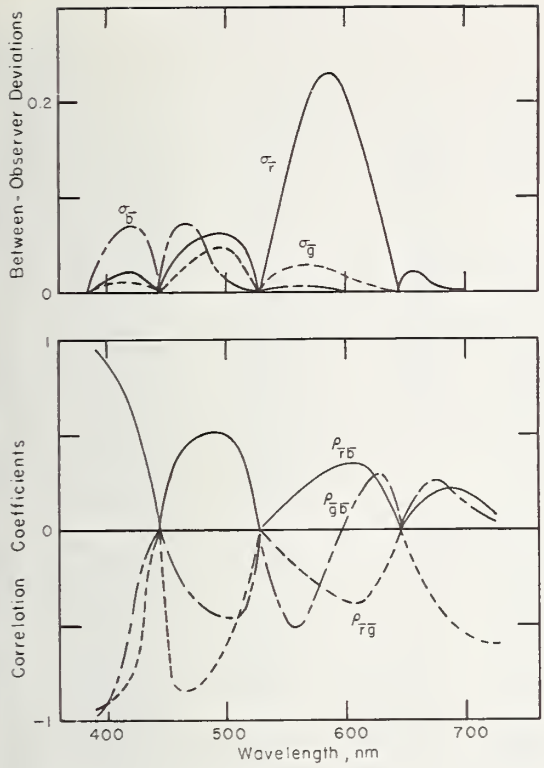


FIGURE 29. *The standard between-observer deviations (upper graph) for 10°-field color-matching functions by Stiles and Burch* [146] *and Speranskaya* [144] *and the correlation-coefficients (lower graph) derived by Nimeroff* [123].

from the Stiles-Burch and Speranskaya data. The average ratio of the between-observer deviations, σ_i, to within-observer deviations, ${}_w\sigma_i$, for the three color-matching functions was estimated as 5.7.

Judd and Kelly [74] have transformed the average color-matching data, $\bar{r}.\bar{g}.\bar{b}.$ to the tristimulus values, $\bar{x}.\bar{y}.\bar{z}$, by the transformation equations:

$$\begin{aligned} \bar{x} &= k_1\bar{r} + k_2\bar{g} + k_3\bar{b} \\ \bar{y} &= k_4\bar{r} + k_5\bar{g} + k_6\bar{b} \\ \bar{z} &= k_7\bar{r} + k_8\bar{g} + k_9\bar{b}. \end{aligned} \tag{9}$$

The variances $V(u)$ and the covariances $C(uv)$ in the transformed system are computed by applying the propagation of error theory, thus:

$$V(u) = \Sigma\left(\frac{\delta u}{\delta i}\right)^2 \sigma^2{}_i + 2\Sigma\left(\frac{\delta u}{\delta i}\right)\left(\frac{\delta u}{\delta j}\right)\sigma_{ij}$$

$$C(u) = \Sigma\left(\frac{\delta u}{\delta i}\right)\left(\frac{\delta v}{\delta i}\right)\sigma^2{}_i + \Sigma\left[\left(\frac{\delta u}{\delta i}\right)\left(\frac{\delta v}{\delta j}\right) + \left(\frac{\delta u}{\delta j}\right)\left(\frac{\delta v}{\delta i}\right)\right]\sigma_{ij}. \tag{10}$$

In table 20 are listed the tristimulus values derived by Judd and Kelly. Also listed are the between-observer variances and covariances in these values derived by Nimeroff [123] on the assumption that the variance and covariance terms involving the constants k_i of the transformation are zero. These data permit estimation of the ranges of color within which matches set by all observers for any one color will fall. Use of the factor 0.175 may be useful in estimating ranges by one observer from the ranges by many observers.

Propagation of error theory has also been applied [120] to solve the problem of finding the limitations of spectral measurements used in evaluating chromaticity coordinates. On the assumption that the standard observer is free from error, uncertainty ellipses for specimens illuminated by CIE source C may be derived from the equation

$$T_\epsilon = \frac{(x\text{-}x_n)^2\sigma^2{}_y + (x\text{-}x_n)(y\text{-}y_n)\sigma_{xy} + (y\text{-}y_n)^2\sigma^2{}_x}{\sigma^2{}_x\,\sigma^2{}_y - \sigma^2{}_{xy}} \tag{11}$$

where $T\varepsilon$ depends on the level of significance, x and y are coordinates of the ellipse, x_n and y_n are the chromaticity coordinates computed from the mean spectral data of a specimen, $\sigma^2{}_x$ and $\sigma^2{}_y$, are the variances, and σ_{xy} is the covariance of the chromaticity coordinates. The results of this investigation are shown in figure 30 compared with the perceptibility ellipses of MacAdam [91]. This error theory has been applied also to the problem of estimating the uncertainty ellipses resulting from imprecision in measuring spectral exitance of fluorescent lamps [122].

Errors introduced by using summation as in eq (2) instead of integration to evaluate tristimulus values $X.Y.Z$, have been treated by Nickerson [114] and by De Kerf [27]. Integration is not possible with the present CIE system because the spectral tristimulus values are specified for small but finite intervals. De Kerf compares colorimetric results from summation with intervals of 1, 5, 10, and 20 nm. The conclusion reached by both investigators is that the size of the interval is determined by the type of problem. The most spectrally selective specimens require the smallest summation intervals.

6.2. Photoelectric Tristimulus Colorimeters

Errors in measurement by photoelectric tristimulus colorimeters result mainly from three failings [58, 153]: (1) to duplicate the geometric conditions of the real situation, (2) to illuminate the specimen with light of applicable spectral character, and (3) to duplicate the required filter-detector spectral sensitivity for the color-matching functions.

a. Angular Conditions

Many photoelectric colorimeters use the standard 45°0° conditions, or the equivalent 0°45° conditions. Errors expected for nonglossy specimens by substitution of 0°-hemispherical condition for the standard 45°0° conditions have been analyzed by Budde and Wyszecki [18].

Table 20. *Spectral tristimulus values and between-observer variances and covariances*

Wave-length nm	$\bar{x}_{10}$	$\bar{y}_{10}$	$\bar{z}_{10}$	$V(\bar{x})$ $\times 10^4$	$V(\bar{y})$ $\times 10^4$	$V(\bar{z})$ $\times 10^4$	$C(\overline{xy})$ $\times 10^4$	$C(\overline{xz})$ $\times 10^4$	$C(\overline{yz})$ $\times 10^4$
400	0.0191097	0.0020044	0.0860109	1.26	0.012	25.58	0.10	0.68	0.47
410	.084736	.008756	.389366	6.61	.11	132.11	.71	29.38	3.21
420	.204492	.021391	.972542	9.36	.26	193.19	.73	42.06	3.50
430	.314679	.038676	1.55348	7.37	.67	154.07	1.06	32.93	5.09
440	.383734	.062077	1.96728	3.85	.45	97.76	.82	18.93	3.55
450	.370702	.089456	1.99480	3.53	.56	81.53	.41	16.14	1.85
460	.302273	.128201	1.75437	10.93	1.00	206.07	.15	45.40	1.36
470	.195618	.185190	1.31756	11.04	2.72	180.40	1.01	42.31	1.32
480	.080507	.253589	.772125	7.16	6.02	60.63	1.08	18.24	.08
490	.016172	.339133	.415254	6.74	10.45	23.38	.63	9.73	.76
500	.003816	.460777	.218502	4.14	12.87	5.93	.006	2.79	2.45
510	.037465	.606741	.112044	3.25	8.29	1.91	.88	1.12	1.00
520	.117749	.761757	.060709	1.82	4.57	.65	1.09	.32	.18
530	.236491	.875211	.030451	1.43	2.53	.27	1.00	.055	.020
540	.376772	.961988	.013676	6.22	5.90	.43	3.01	.13	.28
550	.529826	.991761	.003988	14.31	6.68	1.05	5.28	.53	.70
560	.705224	.997340	.000000	28.28	8.47	1.00	9.60	1.02	.51
570	.878655	.955552		43.72	9.98	.94	14.20	1.48	.11
580	1.01416	.868934		56.88	11.25	.78	18.12	1.84	.38
590	1.11852	.777405		58.85	9.47	.65	19.41	1.94	.81
600	1.12399	.658341		49.31	7.31	.38	16.98	1.48	.72
610	1.03048	.527963		32.43	4.75	.25	11.63	1.02	.52
620	.856297	.398057		15.87	2.40	.12	6.01	.45	.22
630	.647467	.283493		5.75	.92	.059	2.26	.12	.059
640	.431567	.179828		.75	.13	.003	.30	.001	.0007
650	.268329	.107633		.47	.079	.0005	.19	.001	.0003
660	.152568	.060281		.46	.071	.0004	.18	.002	.0009
670	.0812606	.0318004		.13	.019	.0002	.049	.001	.0004
680	.0408508	.0159051		.043	.006	.00002	.016	.0001	.0001
690	.0199413	.0077488		.009	.001	.000004	.003	.00003	.0000
700	.00957688	.00371774		.004	.0005	.000002	.001	9×10^{-6}	$5 \times 10^{-}$
710	.00455263	.00176847		.001	.0002	4×10^{-7}	.0004	1×10^{-6}	$9 \times 10^{-}$
720	.00217496	.00084619		.0002	.00003	4×10^{-8}	7×10^{-5}	3×10^{-8}	$5 \times 10^{-}$

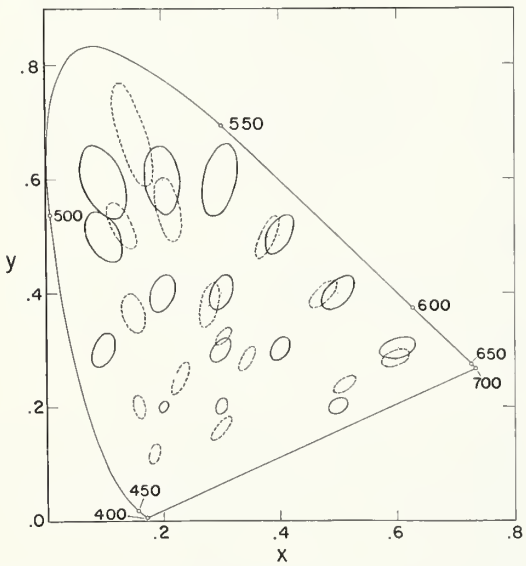


Figure 30. *Uncertainty ellipses compared with MacAdam perceptibility ellipses.*

Correlated uncertainty, solid line ellipses; MacAdam perceptibility, dotted line ellipses. (All ellipses are plotted on a times-ten scale.)

b. Source

Most photoelectric colorimeters are designed for one standard source, usually CIE source C. Such colorimeters do not indicate how closely specimen and standard match for other sources, which depend on the degree of metamerism. Several color imeters now have provision for inserting an auxili ary filter to approximate a source other than the one supplied.

c. Filter-detector combinations

Most photoelectric tristimulus colorimeters use three filter-detector combinations and approximate the short-wave lobe of the $\bar{x}$-function by a suitable fraction of the $\bar{z}$-*function*. Errors encountered in this type of colorimeter have been treated by Van den Akker [152].

There is some degree of failure to duplicate the CIE color-matching functions even in colorimeters with four filters, one for $\bar{y}$, one for $\bar{z}$, and two for the two-lobed $\bar{x}$. This failure restricts utility of these colorimeters to measurement of the color of specimens of spectral characteristics similar to that of the standard.

If the limitations are appreciated, methods using photoelectric tristimulus colorimeters are useful in product-control colorimetry of nonfluorescent specimens. The National Bureau of Standards has developed three sets of color standards, Nonselective standards, KB-Chromatic standards, and S-Chromatic standards, for use with these methods. These standards are now available, either individually or in sets, from the Gardner Laboratory, Inc., Bethesda, Md. 20014, and Hunter Associates Laboratory, Inc., Fairfax, Va. 22030.

6.3. Material Standards of Color

Limitations of visual comparisons result from inability to make precise interpolation between two colors. Use of the Munsell color system requires interpolation of hue, value, and chroma. Experience shows that the best a trained observer does in such interpolations is correct within 0.5 hue step for chromas 4 or more, within 0.2 value step, and 0.4 chroma step.

Visual comparisons are limited also because of the difficulty of choosing permanent materials that have suitable spectral characteristics. The most permanent of these are vitreous enamel and glass, which, however, are so limited in variety of spectral characteristics that metamerism cannot be avoided. This limitation requires control of the spectral character of the light source and disqualifies observers with abnormal vision.

Material standards of glass used in one-dimensional color scales cause difficulty because the luminous-transmittance match with the specimen may not match in chromaticity; see, for example, table 17. Hence, one-dimensional scales as represented by material standards are not always an adequate solution to a problem.

Gross errors in visual judgment may result from marked dissimilarity of surrounds. For example, the orchid standard of the KB-chromatic set issued by the NBS is perceived to have a moderately saturated light reddish-purple color when viewed against a middle gray surround. When it is viewed against a brilliant magenta surround it is seen as greenish gray. Spectrophotometric colorimetry does not account for the influence of surround.

7. References

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(After Helmholtz.) Note that the macular pigment acts as a yellow filter interposed between vitreous humor *C* and retina *i*.

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