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ABSTRACT
This digest summarizes research findings on best teacher practices in mathematics education. Among the findings are: students can learn both concepts and skills by solving problems; whole-class discussion following individual and group work improves student achievement; and using calculators in the learning of mathematics can result in increased achievement and improved student attitudes. Recommendations based on this summary are discussed in part 2, the companion to this digest. (Contains 16 references.) (MM)

## Improving Student Achievement in Mathematics <br> Part 1: Research Findings

## ERIC Digest

By
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ERIC Clearinghouse for Science, Mathematics, and Environmental Education

# Dígest 

# Improving Student Achievement in Mathematics 

Part 1: Research Findings<br>Douglas A. Grouws \& Kristin J. Cebulla<br>December 2000

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The number of research studies conducted in mathematics education over the past three decades has increased dramatically (Kilpatrick, 1992). Research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics teaching practices. For the classroom implications of the research findings summarized here, please see the companion to this Digest, Improving Student Achievement in Mathematics, Part 2: Recommendations for the Classroom (EDO-SE-00-10)

1. The extent of the students' opportunity to learn mathematics content bears directly and decisively on student mathematics achievement.
Opportunity to learn (OTL) was studied in the First International Mathematics Study (Husén, 1967), where teachers were asked to rate the extent of student exposure to particular mathematical concepts and skills. Strong correlations were found between OTL scores and mean student achievement scores, with high OTL scores associated with high achievement. The link was also found in subsequent international studies, such as the Second International Mathematics Study (McKnight et al., 1987) and the Third International Mathematics and Science study (TIMSS) (Schmidt, McKnight, \& Raizen, 1997).
2. Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.
There is a long history of research, going back to the work of Brownell $(1945,1947)$, on the effects of teaching for meaning and understanding. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations.

## 3. Students can learn both concepts and skills by solving problems.

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later. Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge.
4. Giving students both an opportunity to discover and invent new knowledge and an opportunity to practice what they have learned improves student achievement.
Data from the TIMSS video study show that over $90 \%$ of mathematics class time in the United States 8 th-grade classrooms is spent practicing routine procedures, with the remaining time generally spent applying procedures in new situations. Virtually no time is spent inventing new procedures and analyzing unfamiliar situations. In contrast, students at the same
grade level in typical Japanese classrooms spend approximately $40 \%$ of instructional time practicing routine procedures, $15 \%$ applying procedures in new situations, and $45 \%$ inventing new procedures and analyzing new situations.

Research suggests that students need opportunities for both practice and invention. Findings from a number of studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas.
5. Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.
Student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics.

Structuring instruction around carefully chosen problems, allowing students to interact when solving problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem-solving measures. These gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.
6. Using small groups of students to work on activities, problems and assignments can increase student mathematics achie vement.
Davidson (1985) reviewed studies that compared student achievement in small group settings with traditional wholeclass instruction. In more than $40 \%$ of these studies, students in the classes using small group approaches significantly outscored control students on measures of student performance. In only two of the 79 studies did control-group students perform better than the small group students, and in these studies there were some design irregularities. From a review of 99 studies of cooperative group-learning methods, Slavin (1990) concluded that cooperative methods were effective in improving student achievement. The most effective methods emphasized both group goals and individual accountability.

## 7. Whole-class discussion following individual and group work improves student achievement.

Research suggests that whole class discussion can be effective when it is used for sharing and explaining the variety of solutions by which individual students have solved problems. It allows students to see the many ways of examining a situation and the variety of appropriate and acceptable solutions. Wood (1999) found that whole-class discussion works best when discussion expectations are clearly understood. Students should be expected to evaluate each other's ideas and reasoning in ways that are not critical of the sharer. Students should be
expected to be active listeners who participate in discussion and feel a sense of responsibility for each other's understanding.

## 8. Teaching mathematics with a focus on number sense

 encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important.Number sense relates to having an intuitive feel for number size and combinations, and the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgements. It involves mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the reasonableness of numerical results. Markovits and Sowder (1994) studied 7thgrade classes where special units on number magnitude, mental computation and computational estimation were taught. They determined that after this special instruction, students were more likely to use strategies that reflected sound number sense, and that this was a long-lasting change. In a study of second graders, Cobb (1991) and his colleagues found that students' number sense was improved by a problem-centered curriculum that emphasized student interaction and self-generated solution methods. Almost every student developed a variety of strategies to solve a wide range of problems. Students also demonstrated increased persistence in solving problems.
9. Long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes towards mathematics.
In a review of activity-based learning in mathematics in kindergarten through grade 8, Suydam and Higgins (1977) concluded that using manipulative materials produces greater achievement gains than not using them. In a more recent metaanalysis of sixty studies (kindergarten through postsecondary) that compared the effects of using concrete materials with the effects of more abstract instruction, Sowell (1989) found that the long-term use of concrete materials by teachers knowledgeable in their use improved student achievement and attitudes.

## 10. Using calculators in the learning of mathematics can

 result in increased achievement and improved student attitudes.Studies have consistently shown that thoughtful use of calculators improves student mathematics achievement and attitudes toward mathematics. From a meta-analysis of 79 nongraphing calculator studies, Hembree and Dessart (1986) concluded that use of hand-held calculators improved student learning. Analysis also showed that students using calculators tended to have better attitudes towards mathematics and better self-concepts in mathematics than their counterparts who did not use calculators. They also found that there was no loss in student ability to perform paper-and-pencil computational skills when calculators were used as part of mathematics instruction.

Research on the use of graphing calculators has also shown positive effects on student achievement. Most studies have found positive effects on students' graphing ability, conceptual
understanding of graphs and their ability to relate graphical representations to other representations. Most studies of graphing calculators have found no negative effect on basic skills, factual knowledge, or computational skills.

## References

Brownell, W.A. (1945). When is arithmetic meaningful? Journal of Education Research, 38, 481-98.
Brownell, W.A. (1947). The place of meaning in the teaching of arithmetic. Elementary School Journal, 47, 256-65.
Cobb, P, et al. (1991). Assessment of a problem-centered secondgrade mathematics project. Journal for Research in Mathematics Education, 22, 3-29.
Davidson, N. (1985). Small group cooperative learning in mathematics: A selective view of the research. In R. Slavin (Ed.), Learning to cooperate: Cooperating to learn. (pp.211-30) NY: Plenum.
Hembree, R. \& Dessart, D.J. ((1986). Effects of hand-held calculators in pre-college mathematics education: A meta-analysis. Journal for Research in Mathematics Education, 17, 83-99.
Husén, T. (1967). International study of achievement in mathematics, Vol. 2. NY: Wiley.
Kilpatrick, J. (1992). A history of research in mathematics education. In Grouws, D. A., (Ed.), Handbook of research on mathematics teaching and learning. (pp. 3-38) NY: Macmillan.
Markovit, Z., \& Sowder, J. (1994). Developing number sense: An intervention study in grade 7. Journal for Research in Mathematics Education, 25, 4-29.
McKnight, I.V.S., et al. (1987). The underachieving curriculum. Champaign, IL: Stipes.
Schmidt, W.H., McKnight, C.C., \& Raizen, S.A. (1997). A splintered vision: An investigation of U.S. science and mathematics education. Dordrecht, Netherlands: Kluwer.
Slavin, R.E. (1990). Student team learning in mathematics. In N. Davidson (Ed.), Cooperative learning in math: A handbook for teachers. Boston: Allyn \& Bacon, (pp. 69-102).
Sowder, J. (1992a). Estimation and number sense. In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning. (pp. 371-89) NY: Macmillan.
Sowder, J. (1992b). Making sense of numbers in school mathematics. In R. Leinhardt, R. Putman, \& R. Hattrup (Eds.), Analysis of arithmetic for mathematics education. (pp. 1-51)Hillsdale, NJ: Lawrence Erlbaum.
Sowell, E.J. (1989). Effects of manipulative materials in mathematics instruction. Journal for Research in Mathematics Education, 20, 498-505.
Suydam, M.N. \& Higgins, J. L. (1977). Activity-based learning in elementary school mathematics: Recommendations from research. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
Wood, T. (1999). Creating a context for argument in mathematics class. Journal for Research in Mathematics Education, 30, 171-91.
Other sources of information about best practices:
NCTM Illuminations (http://illuminations.nctm.org/index2.html) National Center for Improving Student Learning and Achievement in Mathematics and Science (http://www.wcer.wisc.edu/ncisla/)
Eisenhower National Clearinghouse for Mathematics and Science Education (http://enc.org)

An expanded version of the ideas presented in this Digest is available online at http://www.ibe.unesco.org/Publications/Practice/prac04e.pdf
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