

Environmental Protection Agency

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(c) You may use any of the following calculations instead of the calculations specified in this subpart G:

(1) Mass-based emission calculations prescribed by the International Organization for Standardization (ISO), according to ISO 8178, except the following:

(i) ISO 8178-1 Section 14.4, NO_x Correction for Humidity and Temperature. See §1065.670 for approved methods for humidity corrections.

(ii) ISO 8178-1 Section 15.1, Particulate Correction Factor for Humidity.

(2) Other calculations that you show are equivalent to within ±0.1% of the brake-specific emission results deter-

mined using the calculations specified in this subpart G.

[70 FR 40516, July 13, 2005, as amended at 73 FR 37324, June 30, 2008; 74 FR 56516, Oct. 30, 2009; 75 FR 23044, Apr. 30, 2010]

§ 1065.602 Statistics.

(a) *Overview.* This section contains equations and example calculations for statistics that are specified in this part. In this section we use the letter “y” to denote a generic measured quantity, the superscript over-bar “-” to denote an arithmetic mean, and the subscript “_ref” to denote the reference quantity being measured.

(b) *Arithmetic mean.* Calculate an arithmetic mean, \bar{y} , as follows:

$$\bar{y} = \frac{\sum_{i=1}^N y_i}{N} \quad \text{Eq. 1065.602-1}$$

Example:
N = 3

$y_1 = 10.60$
 $y_2 = 11.91$
 $y_N = y_3 = 11.09$

$$\bar{y} = \frac{10.60 + 11.91 + 11.09}{3}$$

$\bar{y} = 11.20$

(c) *Standard deviation.* Calculate the standard deviation for a non-biased (e.g., N-1) sample, σ , as follows:

$$\sigma_y = \sqrt{\frac{\sum_{i=1}^N (y_i - \bar{y})^2}{(N-1)}} \quad \text{Eq. 1065.602-2}$$

Example:

N = 3
 $y_1 = 10.60$
 $y_2 = 11.91$
 $y_N = y_3 = 11.09$
 $\bar{y} = 11.20$

$$\sigma_y = \sqrt{\frac{(10.60 - 11.2)^2 + (11.91 - 11.2)^2 + (11.09 - 11.2)^2}{2}}$$

$\sigma_y = 0.6619$

(d) *Root mean square.* Calculate a root mean square, rms_y , as follows:

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$$rms_y = \sqrt{\frac{1}{N} \sum_{i=1}^N y_i^2} \quad \text{Eq. 1065.602-3}$$

Example:

$N = 3$
 $y_1 = 10.60$
 $y_2 = 11.91$
 $y_3 = y_3 = 11.09$

$$rms_y = \sqrt{\frac{10.60^2 + 11.91^2 + 11.09^2}{3}}$$

$rms_y = 11.21$

(e) *Accuracy.* Determine accuracy as described in this paragraph (e). Make

multiple measurements of a standard quantity to create a set of observed values, y_i , and compare each observed value to the known value of the standard quantity. The standard quantity may have a single known value, such as a gas standard, or a set of known values of negligible range, such as a known applied pressure produced by a calibration device during repeated applications. The known value of the standard quantity is represented by y_{refi} . If you use a standard quantity with a single value, y_{refi} would be constant. Calculate an accuracy value as follows:

$$accuracy = \left| \frac{1}{N} \sum_{i=1}^N (y_i - y_{refi}) \right| \quad \text{Eq. 1065.602-4}$$

Example:

$y_{ref} = 1800.0$
 $N = 3$

$y_1 = 1806.4$
 $y_2 = 1803.1$
 $y_3 = 1798.9$

$$accuracy = \left| \frac{1}{3} ((1806.4 - 1800.0) + (1803.1 - 1800.0) + (1798.9 - 1800.0)) \right|$$

$$accuracy = \left| \frac{1}{3} ((6.4) + (3.1) + (-1.1)) \right|$$

$accuracy = 2.8$

(f) *t-test.* Determine if your data passes a *t-test* by using the following equations and tables:

(1) For an unpaired *t-test*, calculate the *t* statistic and its number of degrees of freedom, *v*, as follows:

$$t = \frac{|\bar{y}_{ref} - \bar{y}|}{\sqrt{\frac{\sigma_{ref}^2}{N_{ref}} + \frac{\sigma_y^2}{N}}} \quad \text{Eq. 1065.602-5}$$

$$v = \frac{\left(\frac{\sigma_{ref}^2}{N_{ref}} + \frac{\sigma_y^2}{N} \right)^2}{\frac{(\sigma_{ref}^2/N_{ref})^2}{N_{ref} - 1} + \frac{(\sigma_y^2/N)^2}{N - 1}} \quad \text{Eq. 1065.602-6}$$

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Example:
 $\bar{y}_{ref} = 1205.3$
 $\bar{y} = 1123.8$
 $\sigma_{ref} = 9.399$
 $\sigma_y = 10.583$
 $N_{ref} = 11$
 $N = 7$

$$t = \frac{|1205.3 - 1123.8|}{\sqrt{\frac{9.399^2}{11} + \frac{10.583^2}{7}}}$$

$t = 16.63$
 $\sigma_{ref} = 9.399$
 $\sigma_y = 10.583$
 $N_{ref} = 11$
 $N = 7$

$$t = \frac{|1205.3 - 1123.8|}{\sqrt{\frac{9.399^2}{11} + \frac{10.583^2}{7}}}$$

$v = 11.76$

(2) For a paired *t*-test, calculate the *t* statistic and its number of degrees of freedom, *v*, as follows, noting that the ϵ_i are the errors (e.g., differences) between each pair of y_{refi} and y_i :

$$t = \frac{|\bar{\epsilon}| \cdot \sqrt{N}}{\sigma_\epsilon} \quad \text{Eq. 1065.602-7}$$

Example:
 $\bar{\epsilon} = -0.12580$
 $N = 16$
 $\sigma_\epsilon = 0.04837$

$$t = \frac{|-0.12580| \cdot \sqrt{16}}{0.04837}$$

$t = 10.403$
 $v = N - 1$
Example:
 $N = 16$

$$F_y = \frac{\sigma_y^2}{\sigma_{ref}^2} \quad \text{Eq. 1065.602-8}$$

Example:

$v = 16 - 1$
 $v = 15$

(3) Use Table 1 of this section to compare *t* to the t_{crit} values tabulated versus the number of degrees of freedom. If *t* is less than t_{crit} , then *t* passes the *t*-test. The Microsoft Excel software package contains a TINV function that returns results equivalent to §1065.602 Table 1 and may be used in place of Table 1.

TABLE 1 OF § 1065.602—CRITICAL *t* VALUES VERSUS NUMBER OF DEGREES OF FREEDOM, *v*¹

<i>v</i>	Confidence	
	90%	95%
1	6.314	12.706
2	2.920	4.303
3	2.353	3.182
4	2.132	2.776
5	2.015	2.571
6	1.943	2.447
7	1.895	2.365
8	1.860	2.306
9	1.833	2.262
10	1.812	2.228
11	1.796	2.201
12	1.782	2.179
13	1.771	2.160
14	1.761	2.145
15	1.753	2.131
16	1.746	2.120
18	1.734	2.101
20	1.725	2.086
22	1.717	2.074
24	1.711	2.064
26	1.706	2.056
28	1.701	2.048
30	1.697	2.042
35	1.690	2.030
40	1.684	2.021
50	1.676	2.009
70	1.667	1.994
100	1.660	1.984
1000+	1.645	1.960

¹Use linear interpolation to establish values not shown here.

(g) *F*-test. Calculate the *F* statistic as follows:

$$\sigma_y = \sqrt{\frac{\sum_{i=1}^N (y_i - \bar{y})^2}{(N-1)}} = 10.583$$

$$\sigma_{\text{ref}} = \sqrt{\frac{\sum_{i=1}^{N_{\text{ref}}} (y_{\text{ref}i} - \bar{y}_{\text{ref}})^2}{(N_{\text{ref}} - 1)}} = 9.399$$

$$F = \frac{10.583^2}{9.399^2}$$

$F = 1.268$

(1) For a 90% confidence F -test, use Table 2 of this section to compare F to the $F_{\text{crit}90}$ values tabulated versus $(N-1)$ and $(N_{\text{ref}}-1)$. If F is less than $F_{\text{crit}90}$, then F passes the F -test at 90% confidence.

(2) For a 95% confidence F -test, use Table 3 of this section to compare F to the $F_{\text{crit}95}$ values tabulated versus $(N-1)$ and $(N_{\text{ref}}-1)$. If F is less than $F_{\text{crit}95}$, then F passes the F -test at 95% confidence.

Table 2 of § 1065.602-Critical F values, $F_{\text{crit}90}$ versus $N-1$ and $N_{\text{ref}}-1$ at 90 % confidence

$N-1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	1000+	
$N_{\text{ref}}-1$																				
1	39.86	49.50	53.59	55.83	57.24	58.20	58.90	59.43	59.85	60.19	60.70	61.22	61.74	62.00	62.26	62.52	62.79	63.06	63.32	63.58
2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392	9.408	9.425	9.441	9.450	9.458	9.466	9.475	9.483	9.489	9.491
3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.240	5.230	5.216	5.200	5.184	5.176	5.168	5.160	5.151	5.143	5.134	5.130
4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.936	3.920	3.896	3.870	3.844	3.831	3.817	3.804	3.790	3.775	3.761	3.751
5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.316	3.297	3.268	3.238	3.207	3.191	3.174	3.157	3.140	3.123	3.105	3.100
6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937	2.905	2.871	2.836	2.818	2.800	2.781	2.762	2.742	2.722	2.715
7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703	2.668	2.632	2.595	2.575	2.555	2.534	2.514	2.493	2.471	2.465
8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538	2.502	2.464	2.425	2.404	2.383	2.361	2.339	2.316	2.293	2.288
9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.440	2.416	2.379	2.340	2.298	2.277	2.255	2.232	2.208	2.184	2.159	2.155
10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323	2.284	2.244	2.201	2.178	2.155	2.132	2.107	2.082	2.055	2.052
11	3.225	2.860	2.660	2.536	2.451	2.389	2.342	2.304	2.274	2.248	2.209	2.167	2.123	2.100	2.076	2.052	2.026	2.000	1.972	1.970
12	3.177	2.807	2.606	2.480	2.394	2.331	2.283	2.245	2.214	2.188	2.147	2.105	2.060	2.036	2.011	1.986	1.960	1.932	1.904	1.903
13	3.136	2.763	2.560	2.434	2.347	2.283	2.234	2.195	2.164	2.138	2.097	2.053	2.007	1.983	1.958	1.931	1.904	1.876	1.846	1.846
14	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095	2.054	2.010	1.962	1.938	1.912	1.885	1.857	1.828	1.797	1.797
15	3.073	2.695	2.490	2.361	2.273	2.208	2.158	2.119	2.086	2.059	2.017	1.972	1.924	1.899	1.873	1.845	1.817	1.787	1.755	1.755
16	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028	1.985	1.940	1.891	1.866	1.839	1.811	1.782	1.751	1.718	1.718
17	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001	1.958	1.912	1.862	1.836	1.809	1.781	1.751	1.719	1.686	1.686
18	3.007	2.624	2.416	2.286	2.196	2.130	2.079	2.038	2.005	1.977	1.933	1.887	1.837	1.810	1.783	1.754	1.723	1.691	1.657	1.657
19	2.990	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956	1.912	1.865	1.814	1.787	1.759	1.730	1.699	1.666	1.631	1.631
20	2.975	2.589	2.380	2.249	2.158	2.091	2.040	1.999	1.965	1.937	1.892	1.845	1.794	1.767	1.738	1.708	1.677	1.643	1.607	1.607
21	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.920	1.875	1.827	1.776	1.748	1.719	1.689	1.657	1.623	1.586	1.586
22	2.949	2.561	2.351	2.219	2.128	2.061	2.008	1.967	1.933	1.904	1.859	1.811	1.759	1.731	1.702	1.671	1.639	1.604	1.567	1.567
23	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.890	1.845	1.796	1.744	1.716	1.686	1.655	1.622	1.587	1.549	1.549
24	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877	1.832	1.783	1.730	1.702	1.672	1.641	1.607	1.571	1.533	1.533
25	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866	1.820	1.771	1.718	1.689	1.659	1.627	1.593	1.557	1.518	1.518
26	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855	1.809	1.760	1.706	1.677	1.647	1.615	1.581	1.544	1.504	1.504
27	2.901	2.511	2.299	2.165	2.073	2.005	1.952	1.909	1.874	1.845	1.799	1.749	1.695	1.666	1.636	1.603	1.569	1.531	1.491	1.491
28	2.894	2.503	2.291	2.157	2.064	1.996	1.943	1.900	1.865	1.836	1.790	1.740	1.685	1.656	1.625	1.593	1.558	1.520	1.478	1.478
29	2.887	2.495	2.283	2.149	2.057	1.988	1.935	1.892	1.857	1.827	1.781	1.731	1.676	1.647	1.616	1.583	1.547	1.509	1.467	1.467
30	2.881	2.489	2.276	2.142	2.049	1.980	1.927	1.884	1.849	1.819	1.773	1.722	1.667	1.638	1.606	1.573	1.538	1.499	1.456	1.456
40	2.835	2.440	2.226	2.091	1.997	1.927	1.873	1.829	1.793	1.763	1.715	1.662	1.605	1.574	1.541	1.506	1.467	1.425	1.377	1.377
60	2.791	2.393	2.177	2.041	1.946	1.875	1.819	1.775	1.738	1.707	1.657	1.603	1.543	1.511	1.476	1.437	1.395	1.348	1.291	1.291
120	2.748	2.347	2.130	1.992	1.896	1.824	1.767	1.722	1.684	1.652	1.601	1.545	1.482	1.447	1.406	1.363	1.320	1.265	1.193	1.193
1000+	2.706	2.303	2.084	1.945	1.847	1.774	1.717	1.670	1.632	1.599	1.546	1.487	1.421	1.383	1.342	1.295	1.240	1.169	1.000	1.000

Table 3 of §1065.602—Critical F values, F_{crit} , versus $N-1$ and $N_{\text{ref}}-1$ at 95 % confidence

$N-1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	1000+	
$N_{\text{ref}}-1$																				
1	161.4	199.5	215.7	224.5	230.1	233.9	236.7	238.8	240.5	241.8	243.9	245.9	248.0	249.0	250.1	251.1	252.2	253.2	254.3	
2	18.51	19.00	19.16	19.24	19.29	19.33	19.35	19.37	19.38	19.39	19.41	19.42	19.44	19.45	19.46	19.47	19.47	19.48	19.49	
3	10.12	9.552	9.277	9.117	9.014	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660	8.639	8.617	8.594	8.572	8.549	8.526	
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803	5.774	5.746	5.717	5.688	5.658	5.628	
5	6.608	5.786	5.410	5.192	5.050	4.950	4.876	4.818	4.773	4.735	4.678	4.619	4.558	4.527	4.496	4.464	4.431	4.399	4.365	
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874	3.842	3.808	3.774	3.740	3.705	3.669	
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445	3.411	3.376	3.340	3.304	3.267	3.230	
8	5.318	4.459	4.066	3.838	3.688	3.581	3.501	3.438	3.388	3.347	3.284	3.218	3.150	3.115	3.079	3.043	3.005	2.967	2.928	
9	5.117	4.257	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.937	2.901	2.864	2.826	2.787	2.748	2.707	
10	4.965	4.103	3.708	3.478	3.326	3.217	3.136	3.072	3.020	2.978	2.913	2.845	2.774	2.737	2.700	2.661	2.621	2.580	2.538	
11	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544	2.506	2.466	2.426	2.384	2.341	2.296	
12	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459	2.420	2.380	2.339	2.297	2.252	2.206	
13	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388	2.349	2.308	2.266	2.223	2.178	2.131	
14	4.543	3.682	3.287	3.056	2.901	2.791	2.707	2.641	2.588	2.544	2.475	2.403	2.328	2.288	2.247	2.204	2.160	2.114	2.066	
15	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276	2.235	2.194	2.151	2.106	2.059	2.010	
16	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230	2.190	2.148	2.104	2.058	2.011	1.960	
17	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191	2.150	2.107	2.063	2.017	1.968	1.917	
18	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.156	2.114	2.071	2.026	1.980	1.930	1.878	
19	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124	2.083	2.039	1.994	1.946	1.896	1.843	
20	4.325	3.467	3.073	2.840	2.685	2.573	2.488	2.421	2.366	2.321	2.250	2.176	2.096	2.054	2.010	1.965	1.917	1.866	1.812	
21	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.151	2.071	2.028	1.984	1.938	1.889	1.838	1.783	
22	4.279	3.421	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.128	2.048	2.005	1.961	1.914	1.865	1.813	1.757	
23	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	2.108	2.027	1.984	1.939	1.892	1.842	1.790	1.733	
24	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.237	2.165	2.089	2.008	1.964	1.919	1.872	1.822	1.768	1.711	
25	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.266	2.220	2.148	2.072	1.990	1.946	1.901	1.853	1.803	1.749	1.691	
26	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	2.056	1.974	1.930	1.884	1.836	1.785	1.731	1.672	
27	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	2.041	1.959	1.915	1.869	1.820	1.769	1.714	1.654	
28	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.105	2.028	1.945	1.901	1.854	1.806	1.754	1.698	1.638	
29	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	2.015	1.932	1.887	1.841	1.792	1.740	1.684	1.622	
30	4.160	3.305	2.911	2.678	2.522	2.409	2.322	2.254	2.199	2.153	2.080	2.003	1.920	1.875	1.828	1.779	1.726	1.670	1.608	
40	4.085	3.232	2.839	2.606	2.450	2.336	2.249	2.181	2.126	2.079	2.004	1.925	1.841	1.893	1.844	1.791	1.734	1.674	1.610	
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.099	2.044	1.997	1.919	1.836	1.748	1.799	1.744	1.689	1.631	1.567	1.500	
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.911	1.834	1.751	1.659	1.608	1.554	1.495	1.429	1.352	1.254	
1000+	3.842	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.752	1.666	1.571	1.517	1.459	1.394	1.318	1.221	1.000	

(h) *Slope*. Calculate a least-squares regression slope, a_{1y} , as follows:

$$a_{1y} = \frac{\sum_{i=1}^N (y_i - \bar{y}) \cdot (y_{\text{ref}i} - \bar{y}_{\text{ref}})}{\sum_{i=1}^N (y_{\text{ref}i} - \bar{y}_{\text{ref}})^2} \quad \text{Eq. 1065.602-9}$$

Example:
 N = 6000
 y_i = 2045.8

\bar{y} = 1051.1
 y_{ref 1} = 2045.0
 \bar{y}_{ref} = 1055.3

$$a_{1y} = \frac{(2045.8 - 1050.1) \cdot (2045.0 - 1055.3) + \dots + (y_{6000} - 1050.1) \cdot (y_{\text{ref}6000} - 1055.3)}{(2045.0 - 1055.3)^2 + \dots + (y_{\text{ref}6000} - 1055.3)^2}$$

a_{1y} = 1.0110

(i) *Intercept.* Calculate a least-squares regression intercept, a_{0y}, as follows:

$$a_{0y} = \bar{y} - (a_{1y} \cdot \bar{y}_{\text{ref}}) \quad \text{Eq. 1065.602-10}$$

Example:

\bar{y} = 1050.1
 a_{1y} = 1.0110
 \bar{y}_{ref} = 1055.3
 a_{0y} = 1050.1 - (1.0110 · 1055.3)

a_{0y} = -16.8083

(j) *Standard estimate of error.* Calculate a standard estimate of error, SEE, as follows:

$$SEE_y = \sqrt{\frac{\sum_{i=1}^N [y_i - a_{0y} - (a_{1y} \cdot y_{\text{ref}i})]^2}{N - 2}} \quad \text{Eq. 1065.602-11}$$

Example:
 N = 6000
 y_i = 2045.8

a_{0y} = -16.8083
 a_{1y} = 1.0110
 y_{refi} = 2045.0

$$SEE_y = \sqrt{\frac{[2045.8 - (-16.8083) - (1.0110 \cdot 2045.0)]^2 + \dots + [y_{6000} - (-16.8083) - (1.0110 \cdot y_{\text{ref}6000})]^2}{6000 - 2}}$$

SEE_y = 5.348

(k) *Coefficient of determination.* Calculate a coefficient of determination, r², as follows:

$$r_y^2 = 1 - \frac{\sum_{i=1}^N [y_i - a_{0y} - (a_{1y} \cdot y_{refi})]^2}{\sum_{i=1}^N [y_i - \bar{y}]^2} \quad \text{Eq. 1065.602-12}$$

Example:
 N = 6000
 y_i = 2045.8

a_{0y} = -16.8083
 a_{1y} = 1.0110
 y_{refi} = 2045.0
 \bar{y} = 1480.5

$$r_y^2 = 1 - \frac{[2045.8 - (-16.8083) - (1.0110 \times 2045.0)]^2 + \dots [y_{6000} - (-16.8083) - (1.0110 \cdot y_{ref6000})]^2}{[2045.8 - 1480.5]^2 + \dots [y_{6000} - 1480.5]^2}$$

$$r_y^2 = 0.9859$$

(1) *Flow-weighted mean concentration.* In some sections of this part, you may need to calculate a flow-weighted mean concentration to determine the applicability of certain provisions. A flow-weighted mean is the mean of a quantity after it is weighted proportional to a corresponding flow rate. For example, if a gas concentration is measured continuously from the raw exhaust of an engine, its flow-weighted mean concentration is the sum of the products of each recorded concentration times its respective exhaust molar flow rate, divided by the sum of the recorded flow rate values. As another example, the bag concentration from a CVS system is the same as the flow-weighted mean concentration because the CVS system itself flow-weights the bag concentration. You might already expect a certain flow-weighted mean concentration of an emission at its standard based on previous testing with similar engines or testing with similar equipment and instruments. If you need to estimate your expected flow-weighted mean concentration of an emission at its standard, we recommend using the following examples as a guide for how to estimate the flow-weighted mean concentration expected at the standard. Note that these examples are not exact and that they contain assumptions that are not always valid. Use good en-

gineering judgment to determine if you can use similar assumptions.

(1) To estimate the flow-weighted mean raw exhaust NO_x concentration from a turbocharged heavy-duty compression-ignition engine at a NO_x standard of 2.5 g/(kW·hr), you may do the following:

(i) Based on your engine design, approximate a map of maximum torque versus speed and use it with the applicable normalized duty cycle in the standard-setting part to generate a reference duty cycle as described in §1065.610. Calculate the total reference work, *W*_{ref}, as described in §1065.650. Divide the reference work by the duty cycle's time interval, Δ*t*_{dutycycle}, to determine mean reference power, \bar{P}_{ref} .

(ii) Based on your engine design, estimate maximum power, *P*_{max}, the design speed at maximum power, *f*_{max}, the design maximum intake manifold boost pressure, *p*_{inmax}, and temperature, *T*_{inmax}. Also, estimate a mean fraction of power that is lost due to friction and pumping, \bar{P}_{frict} . Use this information along with the engine displacement volume, *V*_{disp}, an approximate volumetric efficiency, η_v, and the number of engine strokes per power stroke (2-stroke or 4-stroke), *N*_{stroke} to estimate the maximum raw exhaust molar flow rate, \dot{n}_{exhmax} .

(iii) Use your estimated values as described in the following example calculation:

$$\bar{x}_{\text{exp}} = \frac{e_{\text{std}} \cdot W_{\text{ref}}}{M \cdot \dot{n}_{\text{exhmax}} \cdot \Delta t_{\text{duty cycle}} \cdot \left(\frac{\bar{P}_{\text{ref}} + (\bar{P}_{\text{frict}} \cdot P_{\text{max}})}{P_{\text{max}}} \right)} \quad \text{Eq. 1065.602-13}$$

$$\dot{n}_{\text{exhmax}} = \frac{P_{\text{max}} \cdot V_{\text{disp}} \cdot f_{\text{nmax}} \cdot \frac{2}{N_{\text{stroke}}} \cdot \eta_V}{R \cdot T_{\text{max}}} \quad \text{Eq. 1065.602-14}$$

Example:

$$e_{\text{NO}_x} = 2.5 \text{ g}/(\text{kW} \cdot \text{hr})$$

$$W_{\text{ref}} = 11.883 \text{ kW} \cdot \text{hr}$$

$$M_{\text{NO}_x} = 46.0055 \text{ g/mol} = 46.0055 \cdot 10^{-6} \text{ g}/\mu\text{mol}$$

$$\Delta t_{\text{duty cycle}} = 20 \text{ min} = 1200 \text{ s}$$

$$\bar{P}_{\text{ref}} = 35.65 \text{ kW}$$

$$\bar{P}_{\text{frict}} = 15\%$$

$$P_{\text{max}} = 125 \text{ kW}$$

$$P_{\text{max}} = 300 \text{ kPa} = 300000 \text{ Pa}$$

$$V_{\text{disp}} = 3.0 \text{ l} = 0.0030 \text{ m}^3$$

$$f_{\text{nmax}} = 2800 \text{ rev/min} = 46.67 \text{ rev/s}$$

$$N_{\text{stroke}} = 4 \text{ l/rev}$$

$$\eta_V = 0.9$$

$$R = 8.314472 \text{ J}/(\text{mol} \cdot \text{K})$$

$$T_{\text{max}} = 348.15 \text{ K}$$

$$\dot{n}_{\text{exhmax}} = \frac{300000 \cdot 0.0030 \cdot 46.67 \cdot \frac{2}{4} \cdot 0.9}{8.314472 \cdot 348.15}$$

$$\dot{n}_{\text{exhmax}} = 6.53 \text{ mol/s}$$

$$\bar{x}_{\text{exp}} = \frac{2.5 \cdot 11.883}{46.0055 \cdot 10^{-6} \cdot 6.53 \cdot 1200 \cdot \left(\frac{35.65 + (0.15 \cdot 125)}{125} \right)}$$

$$\bar{x}_{\text{exp}} = 189.4 \mu\text{mol/mol}$$

(2) To estimate the flow-weighted mean NMHC concentration in a CVS from a naturally aspirated nonroad spark-ignition engine at an NMHC standard of 0.5 g/(kW·hr), you may do the following:

(i) Based on your engine design, approximate a map of maximum torque versus speed and use it with the applicable normalized duty cycle in the

standard-setting part to generate a reference duty cycle as described in §1065.610. Calculate the total reference work, W_{ref} , as described in §1065.650.

(ii) Multiply your CVS total molar flow rate by the time interval of the duty cycle, $\Delta t_{\text{duty cycle}}$. The result is the total diluted exhaust flow of the n_{dexh} .

(iii) Use your estimated values as described in the following example calculation:

$$\bar{x}_{\text{NMHC}} = \frac{e_{\text{std}} \cdot W_{\text{ref}}}{M \cdot \dot{n}_{\text{dexh}} \cdot \Delta t_{\text{duty cycle}}} \quad \text{Eq. 1065.602-15}$$

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Example:

$e_{\text{NMHC}} = 1.5 \text{ g}/(\text{kW}\cdot\text{hr})$
 $W_{\text{ref}} = 5.389 \text{ kW}\cdot\text{hr}$
 $M_{\text{NMHC}} = 13.875389 \text{ g}/\text{mol} = 13.875389 \cdot 10^{-6} \text{ g}/\mu\text{mol}$
 $\dot{n}_{\text{dexh}} = 6.021 \text{ mol}/\text{s}$
 $\Delta t_{\text{duty cycle}} = 30 \text{ min} = 1800 \text{ s}$

$$\bar{x}_{\text{NMHC}} = \frac{1.5 \cdot 5.389}{13.875389 \cdot 10^{-6} \cdot 6.021 \cdot 1800}$$

$\bar{x}_{\text{NMHC}} = 53.8 \mu\text{mol}/\text{mol}$

[70 FR 40516, July 13, 2005, as amended at 73 FR 37324, June 30, 2008; 75 FR 23044, Apr. 30, 2010]

§ 1065.610 Duty cycle generation.

This section describes how to generate duty cycles that are specific to your engine, based on the normalized duty cycles in the standard-setting part. During an emission test, use a duty cycle that is specific to your engine to command engine speed, torque, and power, as applicable, using an engine dynamometer and an engine operator demand. Paragraph (a) of this section describes how to “normalize” your engine’s map to determine the maximum test speed and torque for your engine. The rest of this section describes how to use these values to “denormalize” the duty cycles in the standard-setting parts, which are all published on a normalized basis. Thus,

the term “normalized” in paragraph (a) of this section refers to different values than it does in the rest of the section.

(a) *Maximum test speed, f_{ntest} .* This section generally applies to duty cycles for variable-speed engines. For constant-speed engines subject to duty cycles that specify normalized speed commands, use the no-load governed speed as the measured f_{ntest} . This is the highest engine speed where an engine outputs zero torque. For variable-speed engines, determine the measured f_{ntest} from the power-versus-speed map, generated according to §1065.510, as follows:

(1) Based on the map, determine maximum power, P_{max} , and the speed at which maximum power occurred, f_{nPmax} . If maximum power occurs at multiple speeds, take f_{nPmax} as the lowest of these speeds. Divide every recorded power by P_{max} and divide every recorded speed by f_{nPmax} . The result is a normalized power-versus-speed map. Your measured f_{ntest} is the speed at which the sum of the squares of normalized speed and power is maximum. Note that if multiple maximum values are found, f_{ntest} should be taken as the lowest speed of all points with the same maximum sum of squares.

Determine f_{ntest} as follows:

$$f_{\text{ntest}} = f_{\text{ni}} \text{ at the maximum of } (f_{\text{normi}}^2 + P_{\text{normi}}^2) \quad \text{Eq. 1065.610-1}$$

Where:

f_{ntest} = maximum test speed.
 i = an indexing variable that represents one recorded value of an engine map.
 f_{normi} = an engine speed normalized by dividing it by f_{nPmax} .
 P_{normi} = an engine power normalized by dividing it by P_{max} .

Example:

$(f_{\text{norm1}} = 1.002, P_{\text{norm1}} = 0.978, f_{\text{n1}} = 2359.71)$
 $(f_{\text{norm2}} = 1.004, P_{\text{norm2}} = 0.977, f_{\text{n2}} = 2364.42)$
 $(f_{\text{norm3}} = 1.006, P_{\text{norm3}} = 0.974, f_{\text{n3}} = 2369.13)$
 $(f_{\text{norm1}^2} + P_{\text{norm1}^2}) = (1.002^2 + 0.978^2) = 1.960$
 $(f_{\text{norm2}^2} + P_{\text{norm2}^2}) = (1.004^2 + 0.977^2) = 1.963$
 $(f_{\text{norm3}^2} + P_{\text{norm3}^2}) = (1.006^2 + 0.974^2) = 1.961$
 maximum = 1.963 at $i = 2$
 $f_{\text{ntest}} = 2364.42 \text{ rev}/\text{min}$

(2) For variable-speed engines, transform normalized speeds to reference

speeds according to paragraph (c) of this section by using the measured maximum test speed determined according to paragraph (a)(1) of this section—or use your declared maximum test speed, as allowed in §1065.510.

(3) For constant-speed engines, transform normalized speeds to reference speeds according to paragraph (c) of this section by using the measured no-load governed speed—or use your declared maximum test speed, as allowed in §1065.510.

(b) *Maximum test torque, T_{test} .* For constant-speed engines, determine the measured T_{test} from the power-versus-speed map, generated according to §1065.510, as follows: